THE UNIVERSITY OF MINNESOTA

GRADUATE SCHOOL

Report

of

Committee on Examination

This is to certify that we the undersigned, as a committee of the Graduate School, have given George Warner Swenson final ~~oral~~ examination for the degree of Electrical Engineer ~~Master of~~. We recommend that the degree of Electrical ~~Master of~~ Engineer be conferred upon the candidate.

Minneapolis, Minnesota

May 19 1920

Geo D Shepardson
Chairman

F W Springer

Will F. Holman

THE UNIVERSITY OF MINNESOTA

GRADUATE SCHOOL

Report

of

Committee on Thesis

The undersigned, acting as a Committee of the Graduate School, have read the accompanying thesis submitted by George Warner Swenson for the degree of Electrical Engineer They approve it as a thesis meeting the requirements of the Graduate School of the University of Minnesota, and recommend that it be accepted in partial fulfillment of the requirements for the degree of Electrical Engineer.

Geo L Shepardson
Chairman

F W Springer

Wil. J. Holman

May 19 1930

SPARKING AT RELAY CONTACTS

A THESIS

SUBMITTED IN PARTIAL FULFILLMENT

OF THE REQUIREMENTS FOR THE DEGREE

OF

ELECTRICAL ENGINEER

BY

GEORGE WARNER SWENSON

B.S. UNIVERSITY OF MINNESOTA 1917

JUNE 1920

P R E F A C E

The writing of this thesis on the subject of "Sparking at Relay Contacts" was inspired by work done by the writer in the laboratories of the Western Electric Company in New York City during the summer of 1920.

The various difficulties which arose in connection with machine switching telephony could be traced directly to inductive effects or "kicks" of different relays in the network of circuits.

Several tests of these effects were made at the time with the view of continuing them during the school year (1920-21), but it was found rather impracticable to set up such elaborate circuits, when the principles could just as well be studied with more simple apparatus. For these reasons it was decided to study the inductive effects of a coil resulting in the sparking at the contacts of relays.

Considerable preliminary work was done in the way of getting the Duddell oscillograph into working condition. This was finally accomplished, but it was found that it would be necessary to reconstruct the entire hood to prevent light from the arc light beams from reaching the film. For this reason and because there were only two available elements with which to obtain curves, it was decided to abandon it in favor of the General Electric oscillograph.

I am particularly indebted, for assistance in the laboratory, to Mr.E.E.Clark, Instructor in Mathematics.

June 1920. George Warner Swenson.

C O N T E N T S

I N T R O D U C T I O N

Sparking at contacts of relays, particularly in telegraph and telephone practice, is a matter worthy of serious attention. Such sparking is very common and causes interruptions in service due to the formation of non-conducting substances on the contacts, which necessitates frequent testing and cleaning on the part of maintenance men, and means the expenditure of time and money. Not only is it a question of expense for the operating company, but in case of neglect in inspection, it makes the service undependable where it should be absolutely reliable at all times.

In many cases condensers have been used to absorb the spark energy, and, if properly calculated for the circuit in question, will remove most of the visible sparking. Resistance has also been used, with varying degrees of success, to bridge the "make and break" contact, and also to shunt the inductance in the circuit.

It is the purpose of this work to make experimental studies of various types of circuits, and to use various methods in attempts to reduce the visible sparking, as well as to study the results with the oscillograph. It is further the purpose to find the constants of the circuite in question, and to verify the experimental results by mathematical analysis in the form of differential equations applied to the transient effects caused by changing the circuit conditions.

In the majority of this work interest centers not so much in the sparking at the close of a circuit as at its opening; for, in the latter case, the surge of energy from the inductance is obtained, which raises the e.m.f. across the contact to such a value that it maintains an arc for a comparatively long time after the points of contact have actually separated.

It is possible to completely eliminate the spark in any circuit, by using the proper amounts of capacity and resistance in the right place. In the circuits where speed of operation is important, however, the time constant of the circuit is important and must be considered. That is, the circuit may be arranged so that all sparking is eliminated, but the time constant is too great and the current does not return to normal soon enough. In the case of a telegraph relay circuit, the current must come back to a normal value very quickly, for the releasing value may be the normal current value or slightly above. Therefore, the time constant of a telegraph relay circuit must be small as compared to the time of a signal and still satisfy the conditions for non-sparking. In the case of the time constant of such a circuit being too great, the relay becomes sluggish and difficult of proper adjustment. The same will hold true for local sounder circuits where the current is likely to be comparatively high and the sparking particularly vicious.

As a preliminary study, an ordinary vibrating contact buzzer was taken, to carry through the investigation as outlined above.

First, to operate it with normal current and determine what amounts and positions of capacity and resistance will reduce the visible sparking, and, if possible, will eliminate it entirely. To do this it was necessary to work in a dark room. Second, to use an oscillograph and to study the transient effects at the break of the contact with the various condensers and resistances. Third, to find the constants; namely, the inductance, resistance, capacity, normal working current, and the electromotive force applied. Fourth, to assemble these constants into proper differential equations for solutions, and to determine the condition giving the best results as to time constant and minimum sparking or minimum rise of electromotive force at the break of the circuit.

The second circuit to be studied, will be a local sounder circuit, and the same steps will be followed as above, with more emphasis, perhaps, on the time constant. The third will be a relay circuit where the contact of one relay operates the second relay. The fourth experiment will be with a vibrating or mechanical rectifier used for ringing in small exchanges.

Due to the limitations of the ordinary oscillograph used as a voltmeter, two methods are herein given by which a vacuum tube may be used as a peak voltmeter, or it may be used supplementary to the oscillograph to make the latter practically an electrostatic voltmeter.

INDUCTACE MEASUREMEMT OF VARIABLE AIR-GAP IRON CORE COIL

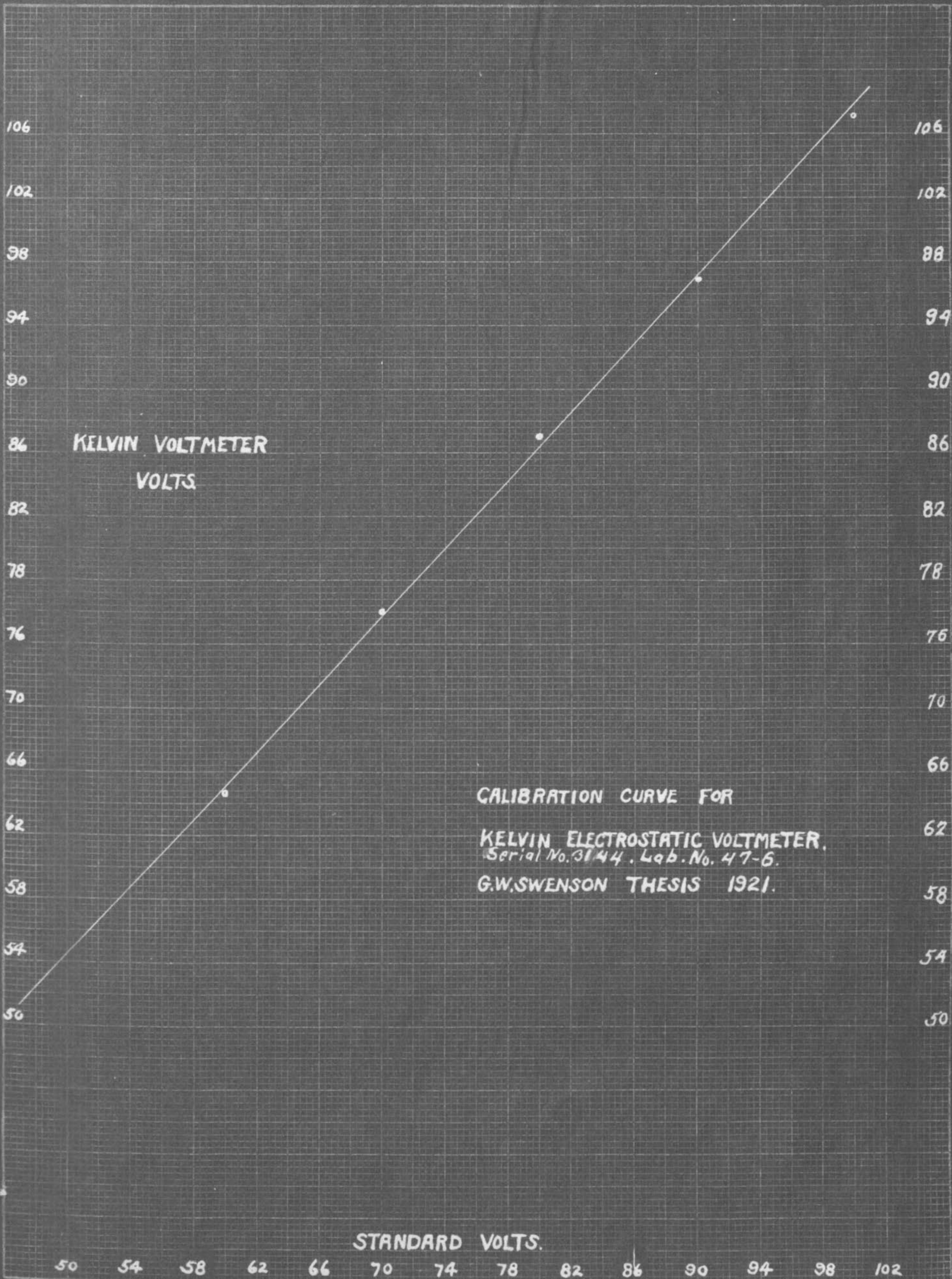
In order to make any calculations as to the probable voltage at the make and break contact, it was necessary to get accurate data on the inductance of any coil to be used. Since the coils studied here are wound on iron cores and the armature varies the inductance at every position, such inductance is a rather hard thing to determine. Furthermore, the current passing through the coil must have the same value as that normally used to operate it. In the case of the Ansonia No. 635 buzzer (laboratory number 4E-3198), it was found that it would operate most satisfactorily at about 150 milli-amperes. This current will vary somewhat with different adjustments, but the adjustment used was about normal, and was kept constant.

Several methods were tried in obtaining the inductance, one being the straight bridge method using standard inductances. It was found that the standards available were not sufficient to determine an unknown inductance of the magnitude here involved. Anderson's modification was then attempted, to allow a comparison of unequal inductances. This method was not at all successful, most likely due to the higher harmonics present in the alternating current used; namely, the Minneapolis General Electric power supply. It would have been possible to screen out these harmonics by means of a filter circuit, but this was not done since the results would not be accurate due to the low flux density in the iron as compared with normal working conditions.

It was then decided to find the required inductances by the voltmeter-ammeter method, and, since the fundamental of the a.c. is very close to a sine wave, it was used for the source of power. Normal alternating current could then be sent through the coil, and the drop across it measured by means of an electrostatic voltmeter, which would take a very small amount of current when shunted across the coil. The large Kelvin voltmeters were used where the voltage reached considerable values; that is, above 20 volts, the lowest reading of the instruments available. The instruments used were very carefully calibrated at the time the readings were taken, so that any variation due to moisture or possible variable magnetic effects from day to day, would not affect the results.

The Kelvin voltmeters would not meet the requirements for low voltage readings, as would be obtained from low inductances, such as the vibrating buzzer and the local sounder. The next resource was the Paul electrostatic deflection voltmeter, which is capable of reading very low values. This instrument, however, was found in rather bad state of repair and it was necessary to do some delicate repair work to straighten the aluminum vanes.

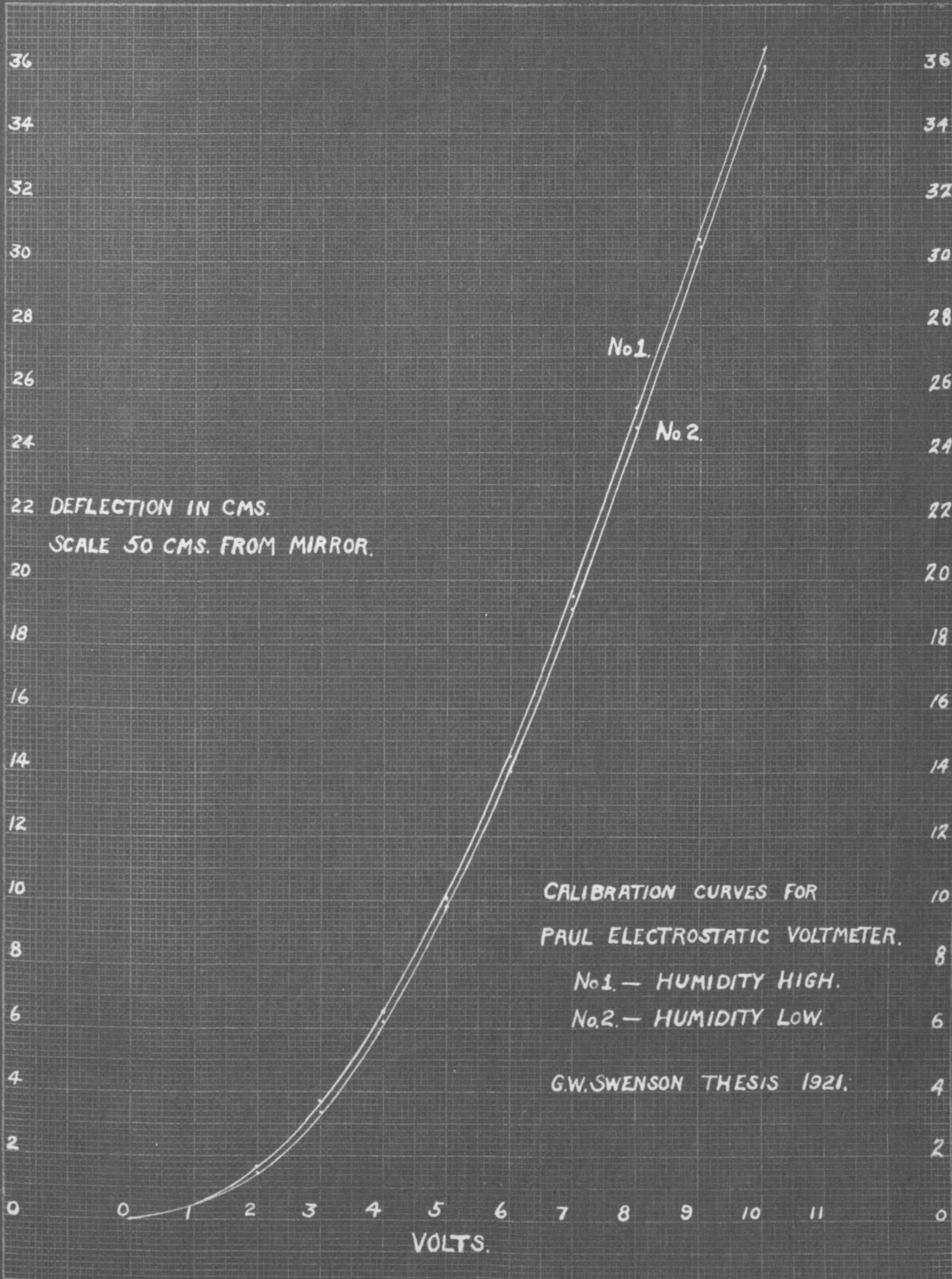
The instructions accompanying the meter specify


KELVIN VOLTMETER
VOLTS.
106 102 98 94 90 86 82 78 76 70 66 62 58 54 50
CALIBRATION CURVE FOR
KELVIN ELECTROSTATIC VOLTMETER.
Serial No. 3144. Lab. No. 47-6.
G.W. SWENSON THESIS 1921.
STANDARD VOLTS.
50 54 58 62 66 70 74 78 82 86 90 94 98 102

a scale at a distance of 98.8 centimeters from the mirror for a reading of 0 to 7 volts, but here a digression was made and the scale set at 50 centimeters from the coil, and, using a circular scale, it gave very distinct vision over the entire scale.

The Paul voltmeter is very sensitive and great care was taken to have it set on a firm foundation, and even then it responds to vibrations of the earth caused by a jar in close proximity to the instrument. The pedestals in the basement laboratory served very well for this purpose. Calibration curves were obtained without a drying agent in the receptacles supplied for that purpose, and the variation for dry and dampdays was found to be constant. The curve for very dry air held very well for all weather after the drying agent was kept in the cups.

The electrostatic voltmeter was carefully calibrated every time it was used and the curve kept constant. Curves will be found on an accompanying print. The range


36
34
32
30
28
26
24
22
20
18
16
14
12
10
8
6
4
2
0
No 1.
No 2.
DEFLECTION IN CMS.
SCALE 50 CMS. FROM MIRROR.
0 1 2 3 4 5 6 7 8 9 10 11
VOLTS.
CALIBRATION CURVES FOR
PAUL ELECTROSTATIC VOLTMETER.
No 1. — HUMIDITY HIGH.
No 2. — HUMIDITY LOW.
G.W. SWENSON THESIS 1921.

was increased to 10 volts and could have been calibrated further if the limits of the scale had been used. The indtrument, then, is good for a maximum of about 14 volts with the above arrangement of scale.

Voltage readings were obtained with normal a.c. through the buzzer coil, and the vibrator clamped in position, with respect to the air gap, as indicated on the following curve showing the variation of the inductance with the air gap.

If the reading, at the point where the vibrator of the buzzer breaks the circuit, is taken, the voltage is indicated by a deflection of 6.4 cm. on the scale, equivalent to 3.85 volts. Carefully measuring the resistance of the coil, it was found to be 5.03 ohms at room temperature of 18^{0} centigrade.

The following formula may then be used to find the inductance:

$$L = \sqrt{(E^2 - R^2 I^2)}/2\pi f I$$

where L = Inductance in henries.

E = Voltage across the coil.

R = Resistance of the coil.

I = Current through the coil.

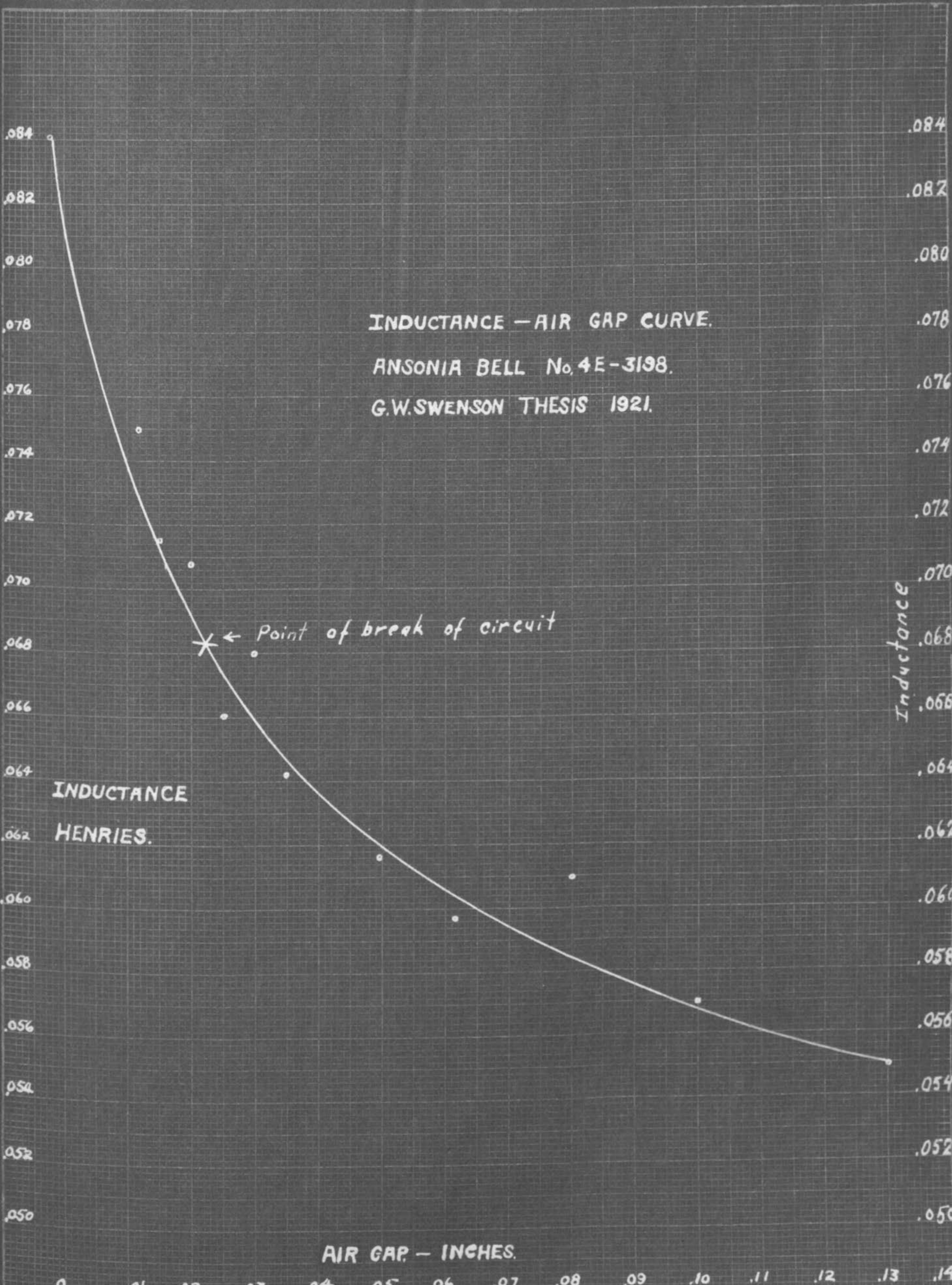
and f = Frequency of the a.c.

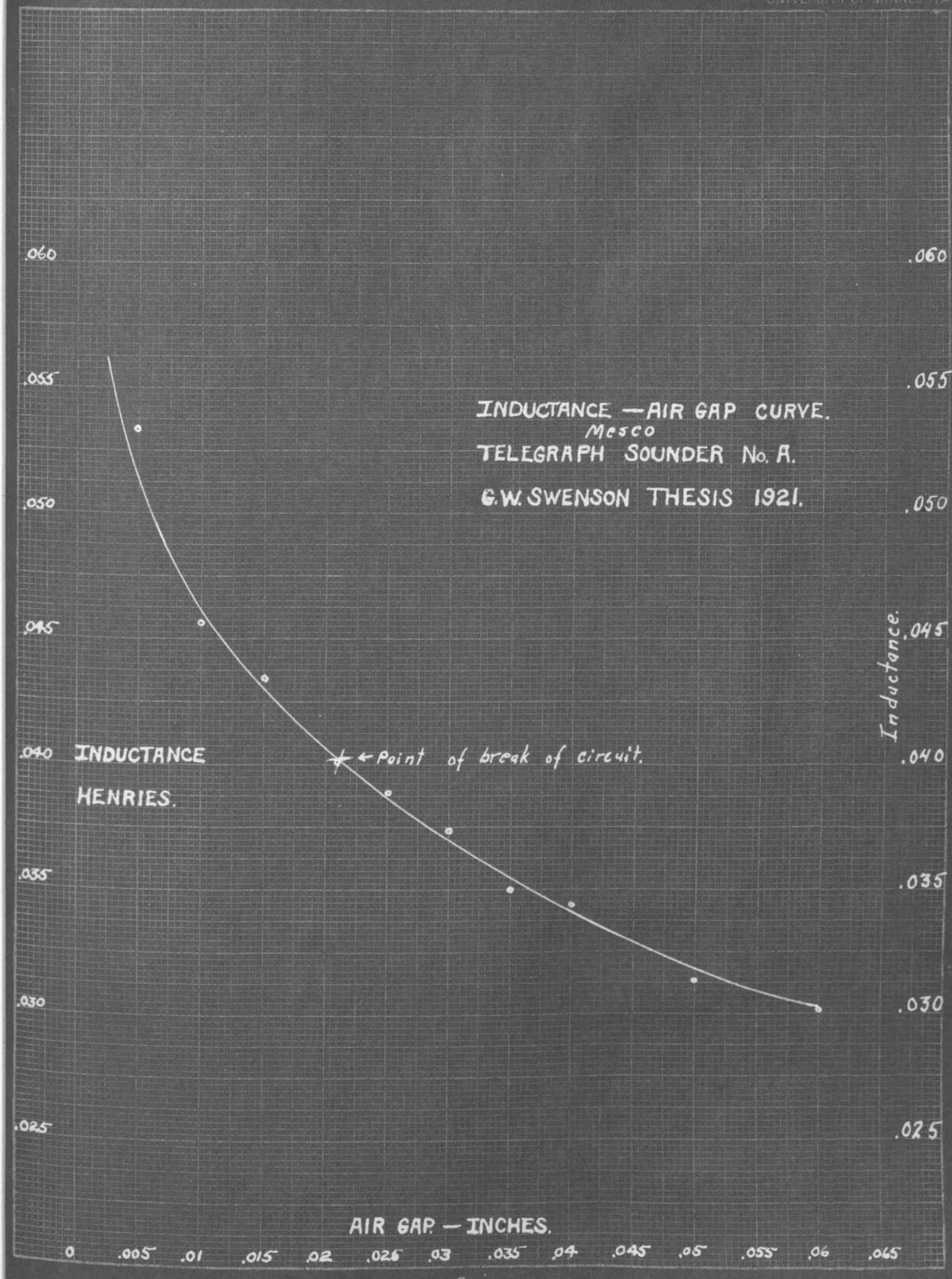
The frequency is very constant at 60 cycles. Solving the equation for the various values of E, the inductance is found for that condition. In this case, at the point of breaking the circuit, the inductance is 0.06835 henry or 68.35 millihenries and is the value to be used in the transient equation.

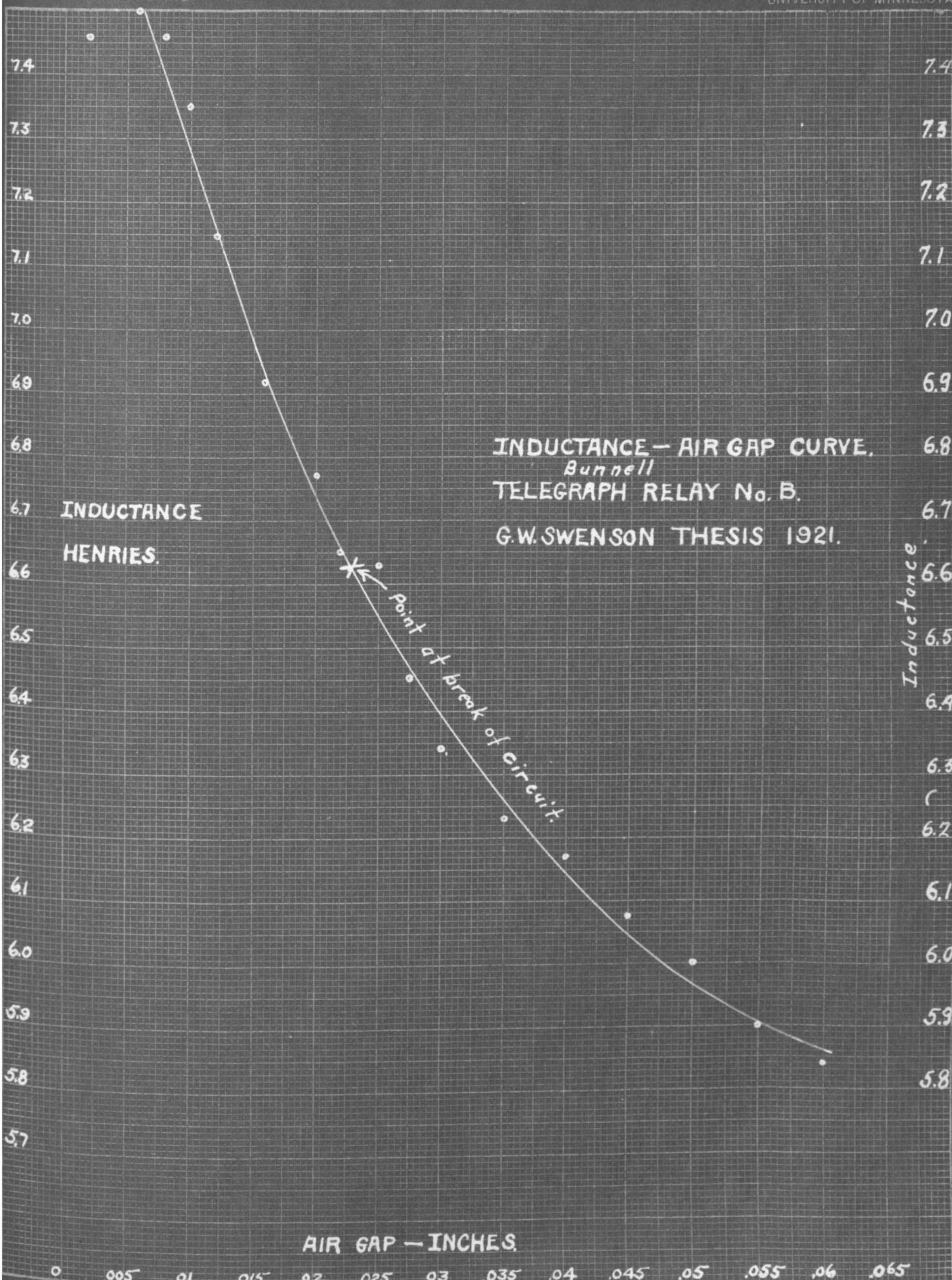
Readings were taken for several air-gap settings of the buzzer, to show the variation of the inductance with lengthening and shortening the air-gap. A curve is plotted, on a following page, for these values.

Similar data were taken for a Mesco sounder (property of the U.S. Signal Corps), for purposes of this work, called sounder No."A". Another set of data was obtained for the Bunnell 250 ohm relay, also property of the Signal Corps and, for this work, called relay No."B".

These curves will be utilized in the transient calculation and the value chosen will depend upon the setting of the length of air-gap and the time of breaking the circuit.


INDUCTANCE — AIR GAP CURVE.
ANSONIA BELL No. 4E-3198.
G.W. SWENSON THESIS 1921.
← Point of break of circuit
INDUCTANCE
HENRIES.
Inductance
AIR GAP — INCHES.
.084
.082
.080
.078
.076
.074
.072
.070
.068
.066
.064
.062
.060
.058
.056
.054
.052
.050
0
.01
.02
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
.14


INDUCTANCE — AIR GAP CURVE.
Mesco
TELEGRAPH SOUNDER No. A.
G.W. SWENSON THESIS 1921.
INDUCTANCE
HENRIES.
Inductance.
← Point of break of circuit.
AIR GAP — INCHES.
.060
.055
.050
.045
.040
.035
.030
.025
0
.005
.01
.015
.02
.025
.03
.035
.04
.045
.05
.055
.06
.065


INDUCTANCE – AIR GAP CURVE.
Bunnell
TELEGRAPH RELAY No. B.
G.W. SWENSON THESIS 1921.
INDUCTANCE
HENRIES.
Inductance
Point at break of circuit.
AIR GAP – INCHES.
7.4
7.3
7.2
7.1
7.0
6.9
6.8
6.7
6.6
6.5
6.4
6.3
6.2
6.1
6.0
5.9
5.8
5.7
0
.005
.01
.015
.02
.025
.03
.035
.04
.045
.05
.055
.06
.065

B U Z Z E R

M A X I M U M   V O L T A G E   V A R I A T I O N

W I T H   N O   C O N D E N S E R

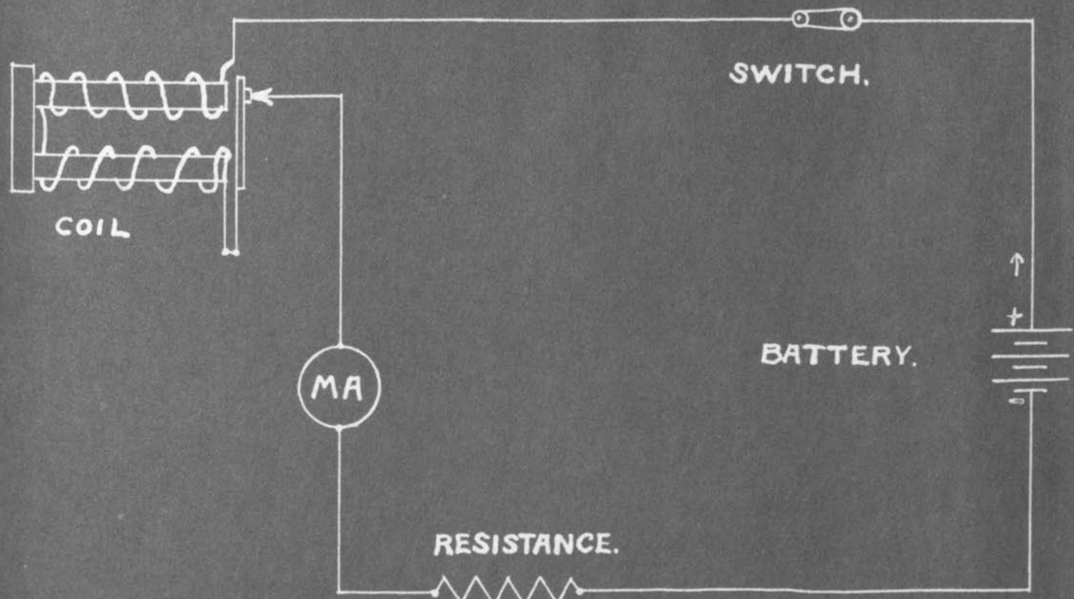
When there is no change made in the buzzer circuit; i.e., when the circuit is left in its original state of manufacture, there is vicious sparking at the vibrating contact.

The circuit, as shown in Fig.1, under such conditions, is one of resistance and inductance in series.

The total resistance, when the circuit is closed is 30 ohms, and when opened, is a rather indeterminate thing, due to the spark at the contact, which ionizes the air, and consequently gives a varying resistance that will have finite values in place of being infinite, as might be inferred at the first glance. It may, however, be considered that the circuit is not actually broken until the spark has been extinguished and the resistance of the gap is made infinite.

Using the transient equation:

ANSONIA BUZZER No. 635.



COIL R – 5.03 ω.
" L – .06835 H.
RESISTANCE – 24.97 ω.
BATTERY – 4.5 VOLTS.
CURRENT – .15 AMPERES.

FIG. ONE.

GEO. W. SWENSON THESIS 1921.

$$i = I_p + (I_o - I_p)\epsilon^{-\frac{Rt}{L}}$$

for the opening of the circuit shown in Fig.1, it may be developed to obtain the maximum voltage induced across the coil, which may be considered as the voltage across the gap plus the battery voltage.

In the above equation, where I_p is the permanent value of current and where R_{gap} is infinite, the first term I_p is so infinitely small that it may be neglected.

$$\text{or} \quad i = (I_o - I_p)\epsilon^{-\frac{Rt}{L}}$$

$$\text{and } di/dt = -R/L(I_o - I_p)\epsilon^{-\frac{Rt}{L}}$$

$$E = Ldi/dt = -R(I_o - I_p)\epsilon^{-\frac{Rt}{L}}$$

But di/dt = maximum when $t = 0$

and since I_p is extremely small and the exponential term is unity:

$$E = -R\,I_o$$

If R is infinite, E will also be infinitely large.

Even though the resistance should not actually become infinite, it necertheless, from the nature of air as an insulator, reaches a very high value after the spark has been extinguished.

If it be assumed that the resistance reaches a definite and conservative value of 200 ohms, the electromotive force across the gap would still be of the nature of

$$E = -10^2 \times 2 \times I_0 \qquad \text{where } I_0 = .15$$

$$E = -2 \times 10^2 \times .15$$

$$E = -30 \text{ volts,}$$

which is still too high and will cause bad sparking.

Another difficulty is encountered in the use of the oscillograph in the circuit. In order to obtain the voltage variation across the "make and break" contact, it is necessary to bridge the element across it, which introduces a permanent finite resistance in parllel with the resistance of the arc, and, eventually, the infinite resistance of the air gap.

While, theoretically, the voltage curve would rise to an infinite value, the resistance of the oscillograph would, in any case, prevent this from being shown on the oscillogram.

The resistance of the oscillograph element and the series resistance in the case of oscillogram No.1 was 520 ohms. Applying this resistance in the formula for inductance and resistance in series, and assuming the gap resistance to be infinite

$$I_p = 4.5/520 = 0.00865 \text{ ampere}$$

which may be neglected.

$$I_o = 4.5/30 = 0.15 \text{ ampere}$$

$$E = -520 \times 0.15 = 77 \text{ volts.}$$

This is a prohibitive voltage and causes sparking at the contact.

Referring to oscillogram No.1 which was taken with the Duddell oscillograph, and, consequently, is not a very good print, it is seen that the e.m.f. across the gap does rise to a very high value and by close examination of the film, it was found that the fine rising line actually ran off the film, indicating a high electromotive force at the break.


#1

The connections used in taking oscillogram No.1 were made as shown in Fig.2.

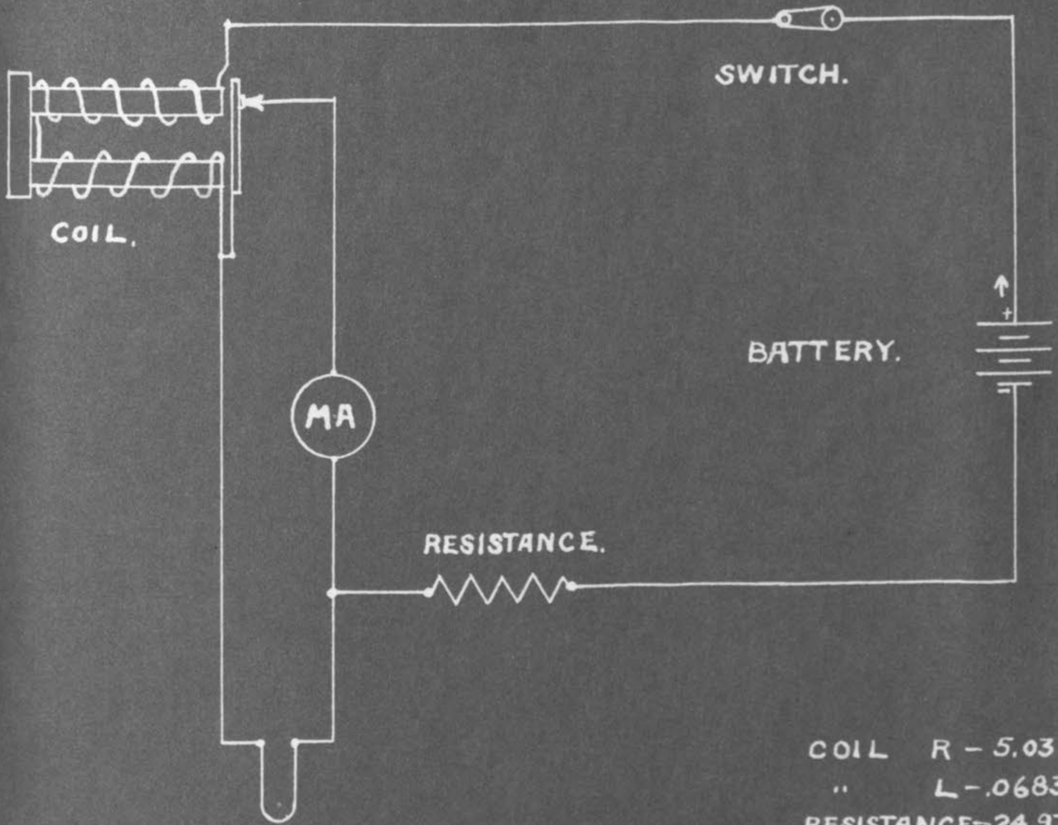
The electromotive force was kept constant so that the current did not vary.

Oscillogram No.1 also indicates that the transient effect dies out completely before the next vibration begins. The rise in the value of electromotive force across the gap when the circuit is made, is due to the resistance of an imperfect contact. The fact that the transient effect dies out completely may be determined by use of the time constant, which, for a series inductance and resistance circuit, is L/R in seconds, and is the time it takes foe the envelope to reach 1/e of its maximum value. The time constant of the circuit, then, is

$$0.06835/30 = 0.002278 \text{ seconds;}$$

and,since the rate of vibration is about 60 per second or .0166 seconds per vibration, and the time of"make" is about equal to the time of "break" or .0083 secon, it may safely be said that the transient effect has practically disappeared at each vibration.

ANSONIA BUZZER No. 635.



COIL R – 5.03 ω.
" L – .06835 H.
RESISTANCE – 24.97 ω.
BATTERY – 4.5 VOLTS.
CURRENT – .15 AMPERES

FIG. TWO.

GEO. W. SWENSON THESIS 1921.

BUZZER

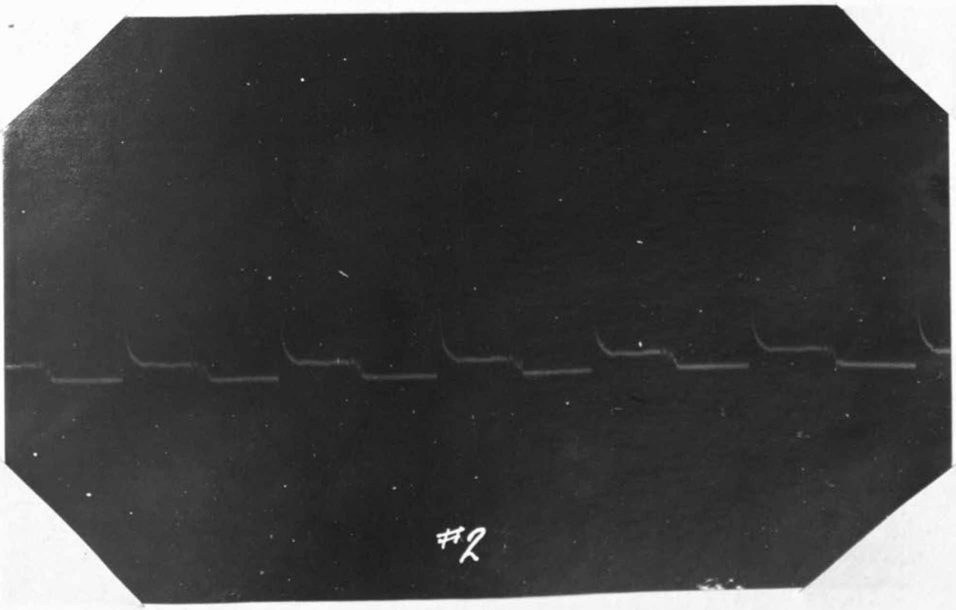
MAXIMUM VOLTAGE VARIATION WITH CONDENSER ACROSS GAP

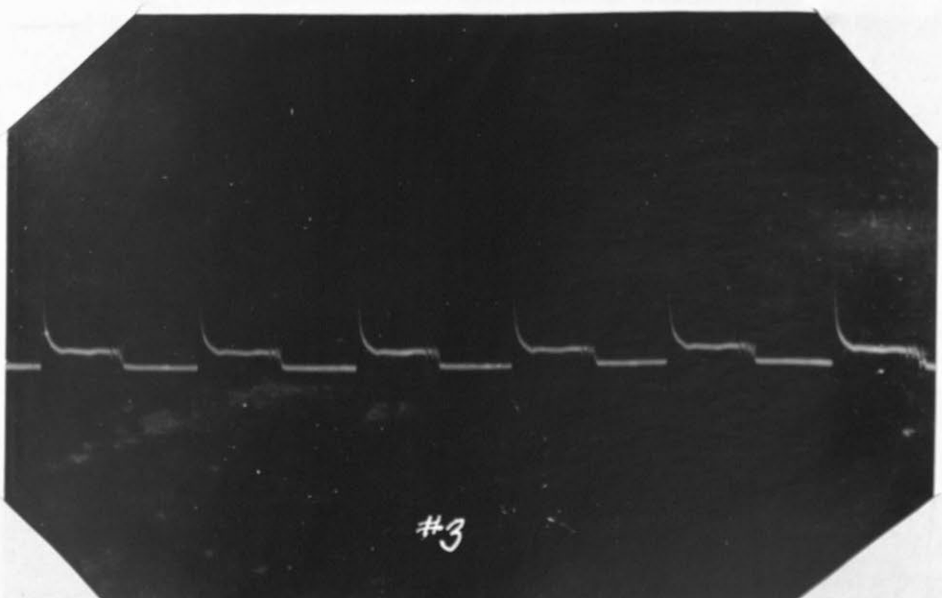
In an attempt to reduce the sparking and still not to impair the efficiency or the time constant or the circuit too much, various amounts of capacity were inserted across the air-gap, and the oscillograms were taken.

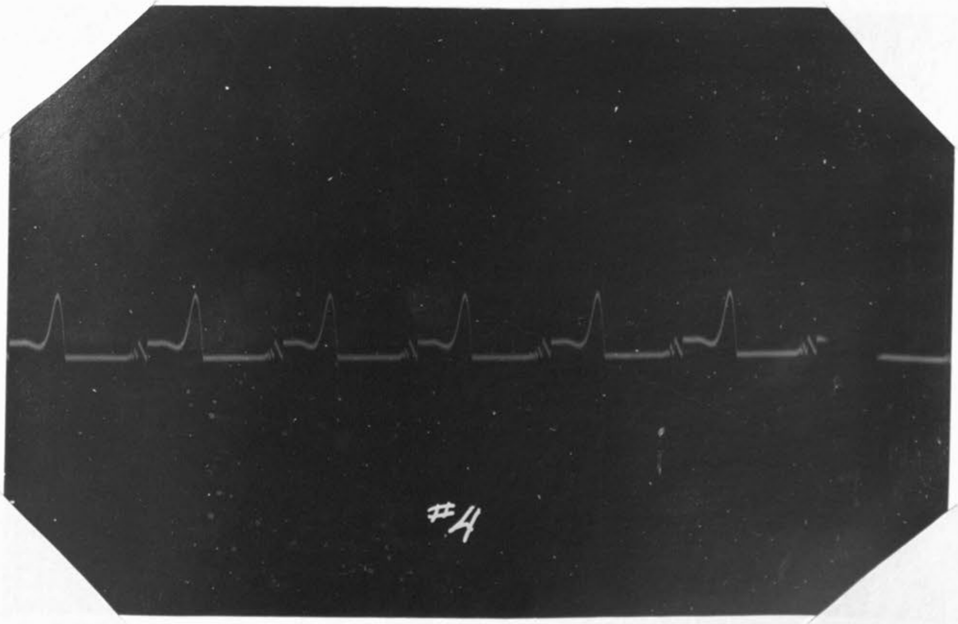
Oscillograms, Nos. 2, 3, 4, and 6 are for condensers across the gap. These oscillograms were taken before any mathematical work was done to determine how much capacity would be required to cut the e.m.f. across the gap to an allowable limit.

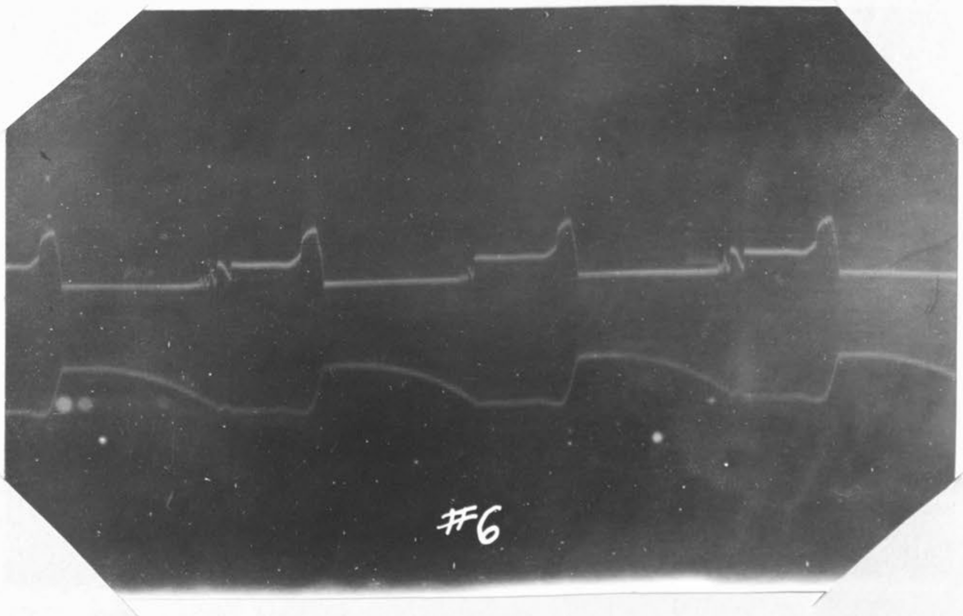
The following condenser values were used with the buzzer:

Oscillogram	No.2.....	0.38	microfarad
"	No.3.....	0.20	"
"	No.4.....	20.28	"
"	No.6.....	15.38	"


#2


#3

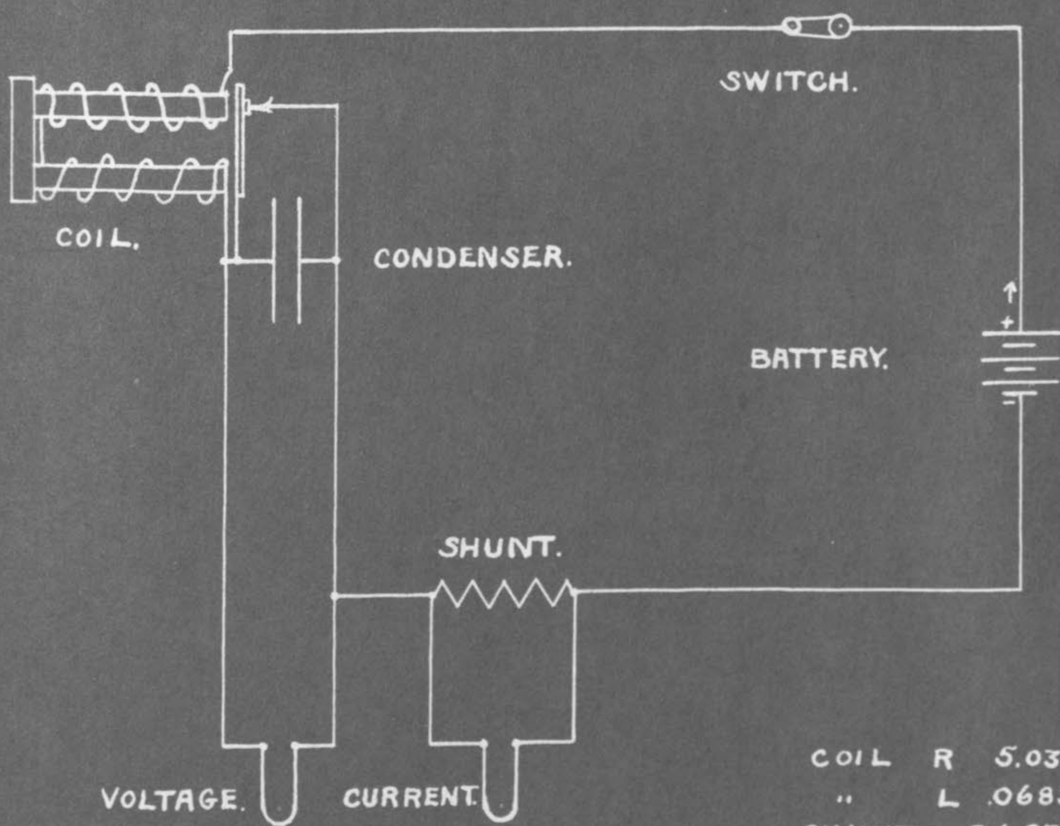

#4


#6

By the indications of the oscillograms, the small capacities cut doun the voltage to some extent but the larger capacities reduce the peak more noticeably.

The circuit for the buzzer and the connections to the oscillograph were such as are shown in Fig.3. This circuit indicates that the maximum voltage across the gap would be the same as the maximum voltage across the condenser "C". In the above mentioned circuit, neglecting the oscillograph elements, the voltage equation may be written for a circuit containing resistance, inductance and capacity in series, as shown on the following page.

ANSONIA BUZZER NO. 635.



COIL R 5.03 ω.
" L .06835 H.
SHUNT 24.97 ω.
BATTERY 4.5 VOLTS.

FIG. THREE.

GEO. W. SWENSON THESIS 1921.

$$E = L\frac{di}{dt} + Ri + \frac{q}{C} \qquad ①$$

differentiating $0 = L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} \qquad ②$

Equation ② is of the form

$$\frac{d^ny}{dx^n} + P_1\frac{d^{n-1}y}{dx^{n-1}} + P_2\frac{d^{n-2}y}{dx^{n-2}} - \cdots P_ny = X$$

In this case $X = 0$

This equation is satisfied by $y = y_1$

$\therefore$ $y = C_1y_1$, $y = C_2y_2$, $y = C_ny_n$ are solutions

as well as $y = C_1y_1 + C_2y_2 + \text{-----} + C_ny_n$.

Let $y = \epsilon^{mx}$ be a solution.

Example: Let $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - 54y = 0$ be an equation in which the values of m are desired, It is of the form

$$m^2 + 3m - 54 = 0$$

$$\therefore m = 6 \text{ or } -9.$$

$\therefore$ the general solution is

$$y = C_1\epsilon^{6x} + C_2\epsilon^{-9x}$$

where C_1 and C_2 are determined from initial conditions.

G.W.S. '21

This solution has the general form

$$i = B_1 \epsilon^{m_1 t} + B_2 \epsilon^{m_2 t} \qquad (3)$$

Apply this to the problem at hand which is of the form

$$Lm^2 + Rm + \frac{1}{C} = 0 \qquad (a)$$

Since from equation $am^2 + bm + c = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

in equation (a) $\quad m = \dfrac{-R \pm \sqrt{R^2 - \frac{4L}{C}}}{2L}$

Let $-m_1 = a_1$ and $-m_2 = a_2$

then $a_1 = \dfrac{R - \sqrt{R^2 - \frac{4L}{C}}}{2L}$ and $a_2 = \dfrac{R + \sqrt{R^2 - \frac{4L}{C}}}{2L}$

$$\text{Let } \sqrt{R^2 - \frac{4L}{C}} = S.$$

then (3) becomes $\quad i = B_1 \epsilon^{-\frac{R-S}{2L}t} + B_2 \epsilon^{-\frac{R+S}{2L}t} \qquad (4)$

When $t = 0$ $\quad i = I_0$ = value of current in the inductance.

Also $e_c = E_{c0}$ = value of e.m.f. across the condenser.

$$e_c = -\frac{q}{C} = -E + Ri + L\frac{di}{dt}$$

G.W.S. '21

From (4) $\frac{di}{dt} = -\frac{R-S}{2L} B_1 \epsilon^{-\frac{R-S}{2L}t} - \frac{R+S}{2L} B_2 \epsilon^{-\frac{R+S}{2L}t}$ (5)

$$\therefore e_c = -E + R\left(B_1 \epsilon^{-\frac{R-S}{2L}t} + B_2 \epsilon^{-\frac{R+S}{2L}t}\right) - L\left(\frac{R-S}{2L} B_1 \epsilon^{-\frac{R-S}{2L}t} + \frac{R+S}{2L} B_2 \epsilon^{-\frac{R+S}{2L}t}\right)$$

Combining terms

$$e_c = -E + B_1 \epsilon^{-\frac{R-S}{2L}t} \frac{R+S}{2} + B_2 \epsilon^{-\frac{R+S}{2L}t} \frac{R-S}{2} \qquad (6)$$

When $t = 0$ $\quad e_c = E_{co}$

$$\therefore e_c = E_{co} = -E + B_1 \frac{R+S}{2} + B_2 \frac{R-S}{2}$$

$$E_{co} + E = B_1 \frac{R+S}{2} + B_2 \frac{R-S}{2} \qquad (7)$$

When $t = 0$ $\quad i = I_0$

Then from (4) $\quad I_0 = B_1 + B_2$

or $B_1 = I_0 - B_2$

From (6) $\quad E_{co} + E = I_0 \frac{R-S}{2} - B_2 \frac{R+S}{2} + B_2 \frac{R-S}{2}$

or $E_{co} + E - I_0 \frac{R-S}{2} = B_2\left(\frac{R-S}{2} - \frac{R+S}{2}\right) = -S B_2$

$$\therefore B_2 = -\frac{E_{co} + E - I_0 \frac{R+S}{2}}{S} \qquad (8)$$

$$B_1 = \frac{E_{co} + E - I_0 \frac{R-S}{2}}{S} \qquad (9)$$

Substituting these values for B_1 and B_2 in

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equations (4) and (6)

$$i = \frac{E_{c0} + E - I_0\left(\frac{R-S}{2}\right)}{S}\,\epsilon^{-\frac{R-S}{2L}t} + \frac{-E_{c0} - E + I_0\frac{R+S}{2}}{S}\,\epsilon^{-\frac{R+S}{2L}t} \qquad (10)$$

$$e_c = -E + \frac{R+S}{2}\left(\frac{E_{c0} + E - I_0\frac{R-S}{2}}{S}\right)\epsilon^{-\frac{R-S}{2L}t} + \frac{R-S}{2}\left(\frac{-E_{c0} - E + I_0\frac{R+S}{2}}{S}\right)\epsilon^{-\frac{R+S}{2L}t} \qquad (11)$$

which are the complete and general equations for the dead beat case where $S = \sqrt{R^2 - \frac{4L}{C}}$

The above equations will hold only when R^2 is greater than $\frac{4L}{C}$, i.e. they are the equations for the dead beat case, When R^2 is less than $\frac{4L}{C}$ the circuit is oscillatory and the radical S becomes imaginary. In the case of the problem at hand, the radical S will not become real until a value of capacity of over 300 microfarads is used, and since it is not proposed to use much over 20 microfarads, it will be necessary to treat the equations on the basis of an oscillatory circuit.

When $R^2 < \frac{4L}{C}$ $\qquad S = \sqrt{(-1)\left(\frac{4L}{C} - R^2\right)}$

Let $\sqrt{\frac{4L}{C} - R^2} = Q$ and $\sqrt{-1} = j$

$$\therefore S = jQ$$

Substitute this value of S in equation (4)

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$$i = B_1 \epsilon^{-\frac{R-jQ}{2L}t} + B_2 \epsilon^{-\frac{R+jQ}{2L}t}$$

From equation 6

$$e_c = -E + B_1 \frac{R+jQ}{2} \epsilon^{-\frac{R-jQ}{2L}t} + B_2 \frac{R-jQ}{2} \epsilon^{-\frac{R+jQ}{2L}t}$$

Combining terms

$$i = \epsilon^{-\frac{R}{2L}t}\left(B_1 \epsilon^{\frac{jQ}{2L}t} + B_2 \epsilon^{-\frac{jQ}{2L}t}\right) \quad (12)$$

$$e_c = -E + \epsilon^{-\frac{R}{2L}t}\left(\frac{R+jQ}{2} B_1 \epsilon^{\frac{jQ}{2L}t} + \frac{R-jQ}{2} B_2 \epsilon^{-\frac{jQ}{2L}t}\right) \quad (13)$$

From Trigonometry

$$\epsilon^{\pm jQ} = \cos Q \pm j \sin Q$$

Substituting the values of $\epsilon^{\pm jQ}$ in equations (12) and (13)

$$i = \epsilon^{-\frac{R}{2L}t}\left[(B_1 + B_2)\cos\frac{Q}{2L}t + j(B_1 - B_2)\sin\frac{Q}{2L}t\right]$$

$$e_c = -E + \epsilon^{-\frac{R}{2L}t}\left[\frac{R(B_1+B_2) + jQ(B_1-B_2)}{2}\cos\frac{Q}{2L}t + \frac{jR(B_1-B_2) - Q(B_1+B_2)}{2}\sin\frac{Q}{2L}t\right]$$

Let $B_1 + B_2 = A_1$

$j(B_1 - B_2) = A_2$

$$\therefore \quad i = \epsilon^{-\frac{R}{2L}t}\left[A_1 \cos\frac{Q}{2L}t + A_2 \sin\frac{Q}{2L}t\right] \quad (14)$$

$$e_c = -E + \epsilon^{-\frac{R}{2L}t}\left[\frac{RA_1 + QA_2}{2}\cos\frac{Q}{2L}t + \frac{RA_2 - QA_1}{2}\sin\frac{Q}{2L}t\right] \quad (15)$$

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When $t = 0 \qquad I_0 = A_1$ and $e_c = E_0$

$$\therefore E_0 = -E + \frac{RA_1 + QA_2}{2}$$

Then $$A_2 = \frac{2(E_0 + E) - RI_0}{Q}$$

Where $Q = \sqrt{\frac{4L}{C} - R^2}$

Substitute the values of A_1 and A_2 in (14) and (15)

$$i = \epsilon^{-\frac{R}{2L}t}\left[I_0 \cos\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t + \frac{2(E_0+E) - RI_0}{\sqrt{\frac{4L}{C} - R^2}} \sin\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t\right] \qquad (16)$$

$$e_c = -E + \epsilon^{-\frac{R}{2L}t}\left\{(E_0+E)\cos\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t + \frac{R(E_0+E) - \frac{2L}{C}I_0}{\sqrt{\frac{4L}{C} - R^2}} \sin\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}\,t\right\} \qquad (17)$$

This is the most general equation for the e.m.f. across the condenser when the circuit is oscillatory.

Since it is impractical indeed to think of using a condenser of the size of 300 microfarads for a buzzer, and since it is not necessary to reduce the sparking absolutely, R will be very small compared with $2\sqrt{\frac{L}{C}}$, the first term of the current equation and parts of the

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e.m.f. equation will drop out, being insignificant, and there remains

$$i = \sqrt{\frac{C}{L}}\, E\, \epsilon^{-\frac{R}{2L}t} \sin \frac{1}{\sqrt{LC}}\, t \qquad (18)$$

$$e_c = E\left\{1 - \epsilon^{-\frac{R}{2L}t}\left(\cos \frac{1}{\sqrt{LC}}\, t + \frac{R}{2}\sqrt{\frac{C}{L}} \sin \frac{1}{\sqrt{LC}}\, t\right.\right. \qquad (19)$$

The only value of e_c, in which there need be any interest taken in this case, is its maximum value, which, from the nature of damped vibrations, will occur some time during the first quarter of the first cycle of the oscillations. It is possible, now, to differentiate the expression for e_c and to equate it to zero, for at that point, the slope will be zero, and indicate a maximum value.

$$\frac{d}{dt} e_c = \frac{d}{dt}\left\{E - E\epsilon^{-\frac{R}{2L}t}\left(\cos \frac{1}{\sqrt{LC}}\, t + \frac{R}{2}\sqrt{\frac{C}{L}} \sin \frac{1}{\sqrt{LC}}\, t\right)\right\}$$

$$= 0 - E\left[-\epsilon^{-\frac{R}{2L}t} \frac{1}{\sqrt{LC}} \sin \frac{t}{\sqrt{LC}} - \frac{R}{2L}\epsilon^{-\frac{R}{2L}t} \cos \frac{t}{\sqrt{LC}} + \right.$$

$$\left.\left(\epsilon^{-\frac{R}{2L}t}\right)\frac{R}{2}\sqrt{\frac{C}{L}}\, \frac{1}{\sqrt{LC}} \cos \frac{t}{\sqrt{LC}} - \frac{R}{2L}\epsilon^{-\frac{R}{2L}t} \frac{R}{2}\sqrt{\frac{C}{L}} \sin \frac{t}{\sqrt{LC}}\right] = 0$$

Collecting terms

$$E\epsilon^{-\frac{R}{2L}t}\left\{\left(\frac{1}{\sqrt{LC}} + \frac{R^2}{4L}\sqrt{\frac{C}{L}}\right) \sin \frac{t}{\sqrt{LC}} + \left(\frac{R}{2L} - \frac{R}{2L}\right) \cos \frac{t}{\sqrt{LC}}\right\} = 0$$

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Since $E\epsilon^{-\frac{R}{2L}t}$ cannot equal zero,

$$\left(\frac{1}{\sqrt{LC}} + \frac{R^2}{4L}\sqrt{\frac{C}{L}}\right)\sin\frac{t}{\sqrt{LC}} = \left(\frac{R}{2L} - \frac{R}{2L}\right)\cos\frac{t}{\sqrt{LC}}$$

$$\text{or} \quad \tan\frac{t}{\sqrt{LC}} = \frac{\frac{R}{2L} - \frac{R}{2L}}{\frac{1}{\sqrt{LC}} + \frac{R^2}{4L}\sqrt{\frac{C}{L}}} = 0$$

$$\therefore \frac{t}{\sqrt{LC}} = \tan^{-1} 0$$

$$\text{or} \quad t = \sqrt{LC}\,\tan^{-1} 0$$

But $\tan^{-1} 0 = 0, \pi, 2\pi, 3\pi \cdots n\pi$ radians.

Since the value of the tangent in the first half cycle is the only one of interest here, it may be said that $t = \pi\sqrt{LC}$. Then for varying values of C, the exact time of this maximum peak can be found in seconds.

If the value, $t = \pi\sqrt{LC}$, is now substituted in equation (19), the maximum value of voltage may be obtained

$$e_{c_{Max.}} = E - E\epsilon^{-\frac{R}{2L}\pi\sqrt{LC}}\left(\cos\frac{1}{\sqrt{LC}}\cdot\pi\sqrt{LC} + \frac{R}{2}\sqrt{\frac{C}{L}}\sin\pi\right)$$

$$\cos\pi = -1 \qquad \sin\pi = 0$$

$$\therefore e_{c_{Max.}} = E + E\epsilon^{-\frac{\pi R}{2}\sqrt{\frac{C}{L}}}$$ (See page 95)

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This equation, when the constants of the circuit are applied, will give the maximum value of voltage for the case that has been cited.

The following are the constants of the circuit:

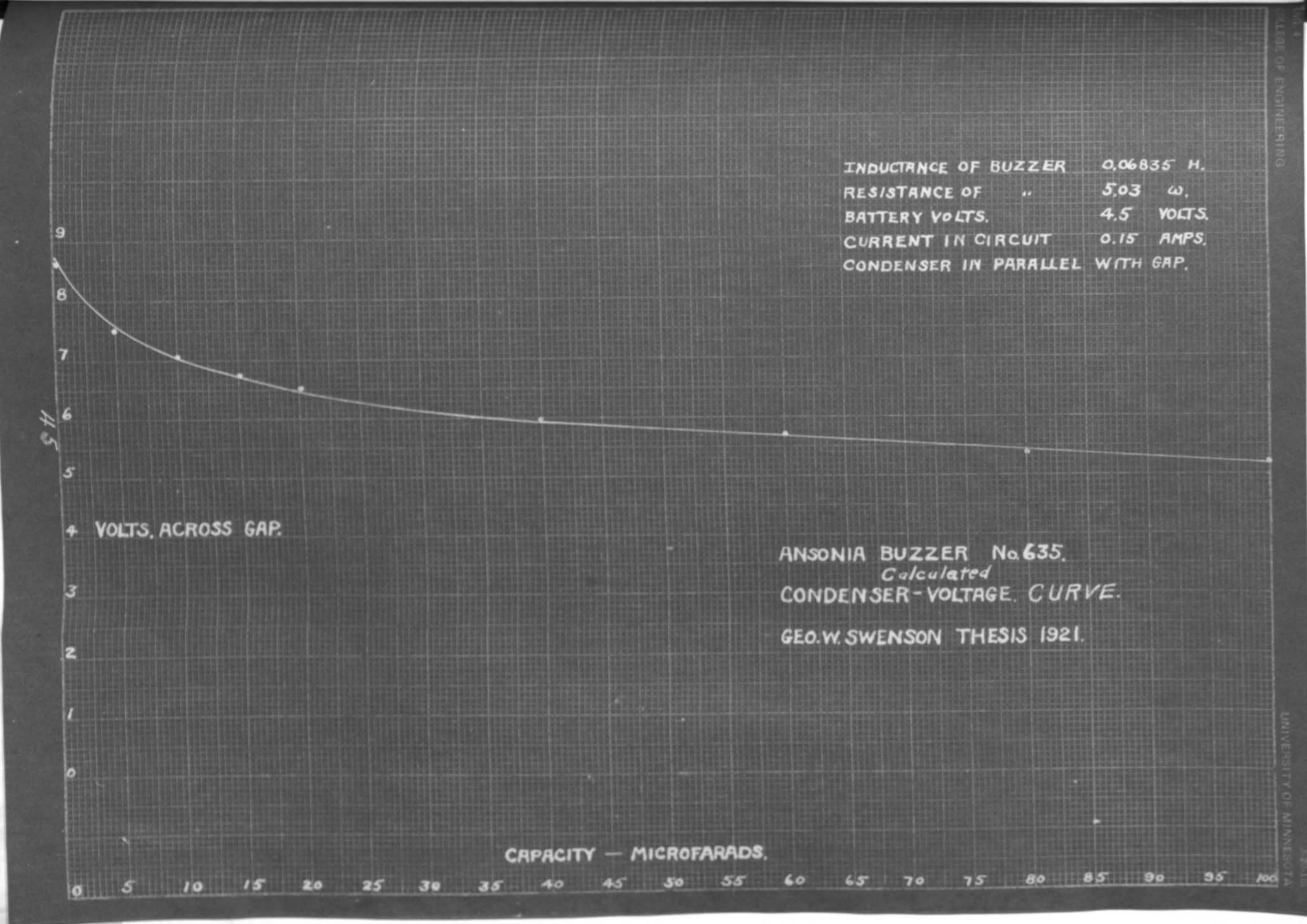
$E = 4.5$ volts

$R = r + R = 30$ ohms

$L = 0.06835$ henry

For varying values of capacity, the following values of voltage across the condenser are found from the above formula:

Capacity (m.f.)	e_c (volts)
100	5.25
80	5.40
60	5.70
40	5.95
20	6.52
10	6.75
5	7.50
1.0	8.25
0.1	8.64


COLLEGE OF ENGINEERING
UNIVERSITY OF MINNESOTA
INDUCTANCE OF BUZZER 0,06835 H.
RESISTANCE OF " 5,03 ω.
BATTERY VOLTS. 4.5 VOLTS.
CURRENT IN CIRCUIT 0.15 AMPS.
CONDENSER IN PARALLEL WITH GAP.
VOLTS. ACROSS GAP.
ANSONIA BUZZER No. 635.
Calculated
CONDENSER-VOLTAGE. CURVE.
GEO. W. SWENSON THESIS 1921.
CAPACITY — MICROFARADS.
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100
0 1 2 3 4 5 6 7 8 9
45

These values are plotted on a following page, and show that the voltage cannot reach twice the battery voltage with capacity in the circuit. This statement can hold true only for those cases where the resistance of the circuit is very small compared with the factor $2\sqrt{L/C}$.

The visible sparking is not overcome, however, until at least 5 microfarads have been placed across the gap. The curve indicates that there is not much to be gained by increasing the capacity to more than 10 m.f. since all visible sparking has disappeared before this point is reached.

BUZZER

MAXIMUM VOLTAGE VARIATION
CONDENSER ACROSS INDUCTANCE

In the study of the buzzer with capacity in parallel with the inductance, the same Ansonia buzzer No. 635, was used as in the preceding experiments.

When capacity is shunted around the inductance, as shown in Fig.4, two parallel paths are offered to the surge of current when the circuit is broken, both of which may be oscillatory. The condenser circuit will be highly oscillatory for reasonable values of capacity, and the battery circuit will probably be slightly oscillatory, as, in fact, it is shown to be by the oscillogram.

Oscillograms were taken, with the various capacity values across the coil and it was found that the visible sparking could not be eliminated except with very high values of capacity as compared with those used across the spark gap for the same purpose.

The mathematical analysis becomes rather involv-

ANSONIA BUZZER NO 635. — CIRCUIT WITH CAPACITY IN PARALLEL WITH THE COIL.

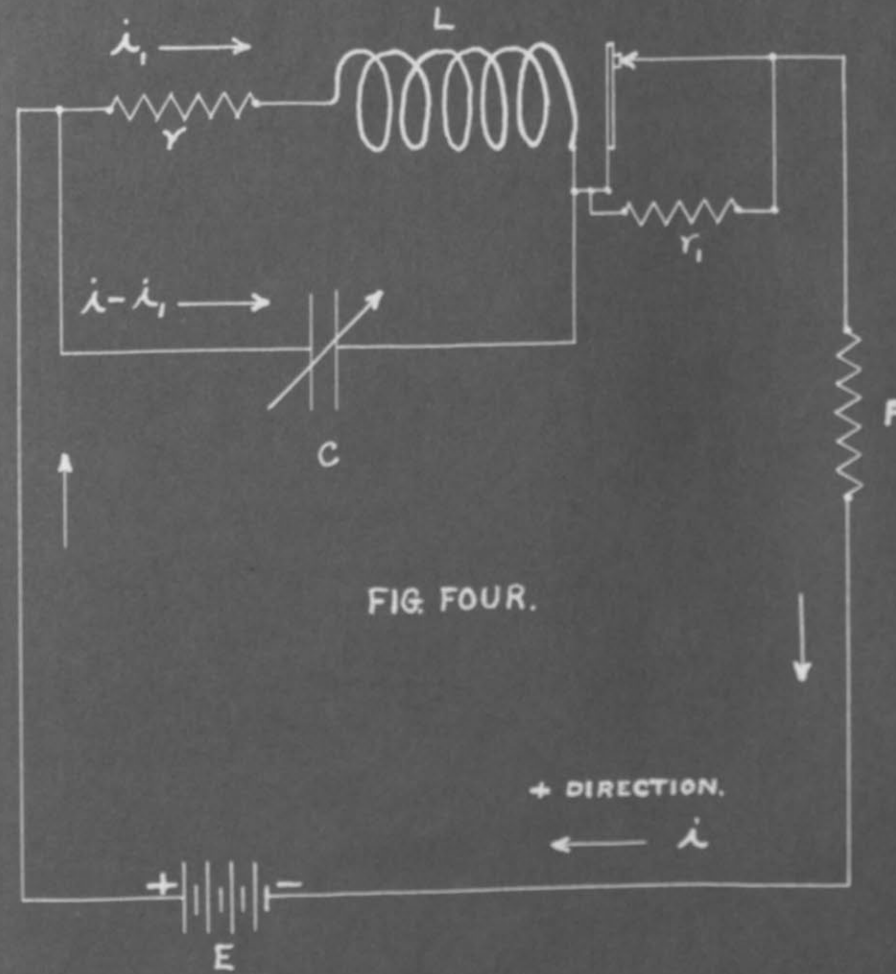


FIG. FOUR.

L.	0.06835 H.
r.	5.03 ω.
r_1.	VARIABLE.
C.	"
R.	24.97 ω.
E.	4.5 VOLTS.

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ed, with branched circuits, but a solution is offered here for the circuit, which may be used in further study of the nature of the arc as to its resistance.

In Fig.4, let r_1 be the changing resistance of the contact, r the resistance of the coil, R the resistance of the outer circuit, L the inductance of the coil, C the capacity of the condenser, i the current through the outer circuit, i_1 the current through the inductance, $i - i_1$ the current through the condenser, E the battery voltage, and q the charge on the condenser. For convenience, let the operator $\frac{d}{dt}$ be called p,

For the main or outer circuit and through the inductance, the electromotive force equation is

$$E - Ri - r_1 i - r i_1 - Lpi = 0 \qquad (1)$$

For the outer circuit and through the condenser the equation of the electromotive forces is

$$E - Ri - r_1 i - \frac{q}{C} = 0$$

$$\text{or} \quad E - Ri - r_1 i - \frac{1}{p}\frac{i - i_1}{C} = 0 \qquad (2)$$

Subtract (2) from (1)

$$r i_1 + Lpi_1 = \frac{1}{p}\frac{i - i_1}{C}$$

$$\text{or} \quad i_1(p^2LC + prC + 1) = i$$

$$\therefore \quad i_1 = \frac{i}{p^2LC + prC + 1} \qquad (3)$$

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Substitute ③ in ①

$$Ri + ri_1 + \frac{ri + Lpi}{p^2LC + prC + 1} = E \qquad ④$$

or $p^2RLCi + p^2r_1LCi + pRrCi + prr_1Ci + pLi +$

$$Ri + r_1 i + ri = E(p^2LC + prC + 1)$$

One is justified in saying that $\frac{d^2}{dt^2}LC +$ $\frac{d}{dt}rC = 0$, since L, C and R are constants

$$\therefore \left\{p^2(RLC + r_1LC) + p(RrC + r_1rC + L) + (R + r_1 + r)\right\} i = E \qquad ⑤$$

Then the equation is of the form

$$(A\frac{d^2i}{dt^2} + B\frac{di}{dt} + Ci = K$$

Divide through by A

$$\left(\frac{d^2}{dt^2} + a\frac{d}{dt} + b\right) i = C_1 \quad \text{where}$$

$$a = \frac{B}{A}, \quad b = \frac{C}{A}, \quad C_1 = \frac{K}{A}$$

Let D be the operator $\frac{d}{dt}$

$$\therefore D^2 i + Dai + bi = C_1$$

$$i(D^2 + aD + b) = C_1$$

To factor, put $D^2 + aD + b = 0$

$$\therefore D = \frac{a \pm \sqrt{a^2 - 4b}}{2}$$

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or $D^2+aD+b = \left(\frac{a-\sqrt{a^2-4b}}{2}\right)\left(\frac{a+\sqrt{a^2-4b}}{2}\right)$

$$\therefore\ i = \frac{C_1}{\left(D+\frac{a+\sqrt{a^2-4b}}{2}\right)\left(D+\frac{a-\sqrt{a^2-4b}}{2}\right)}$$

As partial fractions

$$i = C_1\left\{\frac{\frac{1}{\sqrt{a^2-4b}}}{D+\frac{a-\sqrt{a^2-4b}}{2}} - \frac{\frac{1}{\sqrt{a^2-4b}}}{D+\frac{a+\sqrt{a^2-4b}}{2}}\right\}$$

Let $N_1 = \frac{C_1}{\sqrt{a^2-4b}}$ $\qquad N_2 = -\frac{C_1}{\sqrt{a^2-4b}}$

$-m_1 = \frac{a-\sqrt{a^2-4b}}{2}$ $\qquad -m_2 = \frac{a+\sqrt{a^2-4b}}{2}$

Then $i = \frac{N_1}{D-m_1} + \frac{N_2}{D-m_2}$

$$\therefore\ i = N_1\epsilon^{m_1 t}\int\epsilon^{-m_1 t}\,dt + N_2\epsilon^{m_2 t}\int\epsilon^{-m_2 t}\,dt$$

$$i = N_1\cancel{\epsilon^{m_1 t}}\left(\frac{-\cancel{\epsilon^{-m_1 t}}}{m_1}\right) + N_2\cancel{\epsilon^{m_2 t}}\left(\frac{-\cancel{\epsilon^{-m_2 t}}}{m_2}\right)$$

$$i = -\frac{N_1}{m_1} - \frac{N_2}{m_2}$$

Substitute the above values of N_1, N_2, m_1, m_2 in the last equation

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is obtained, which resolves itself into the same solution that was obtained for the series circuit; i.e., the general solution is

$$i = B_1 \epsilon^{-m_1 t} + B_2 \epsilon^{-m_2 t}$$

$$m_1 = \frac{B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$\therefore m_1 = \frac{B + \sqrt{B^2 - 4AC}}{2A} \quad \text{and} \quad m_2 = \frac{B - \sqrt{B^2 - 4AC}}{2A}$$

$$\therefore \quad i = B_1 \epsilon^{-\frac{B+\sqrt{B^2-4AC}}{2A}t} + B_2 \epsilon^{-\frac{B-\sqrt{B^2-4AC}}{2A}t} \qquad (6)$$

At this point it would be well to inspect equation (5) and see if it is possible to eliminate some of the terms. It is known that the resistance r_1 will be very high, and consequently, one is justified in thowing out the terms not containing r_1, since they will be, comparatively, very small. Then the equation takes the following form:

$$\left[p^2(r_1 LC) + p(r r_1 C) + r\right] i = 0 \qquad (7)$$

for the complimentary solution.

and $i = \frac{E}{r_1}$ is the particular solution.

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Divide ⑦ by r_1

Then $A = LC$, $B = rC$, and $C = 1$

Substitute these values in ⑥

$$\therefore\ i = B_1 \epsilon^{-\frac{rC + \sqrt{r^2C^2 - 4LC}}{2LC}t} + B_2 \epsilon^{-\frac{rC - \sqrt{r^2C^2 - 4LC}}{2LC}t}$$

$$\text{or}\quad i = B_1 \epsilon^{-\left(\frac{r + \sqrt{r^2 - \frac{4L}{C}}}{2L}\right)t} + B_2 \epsilon^{-\left(\frac{r - \sqrt{r^2 - \frac{4L}{C}}}{2L}\right)t}$$

The complete solution is the complimentary solution plus the particular solution.

$$\therefore\ i = B_1 \epsilon^{-\left(\frac{r + \sqrt{r^2 - \frac{4L}{C}}}{2L}\right)t} + B_2 \epsilon^{-\left(\frac{r - \sqrt{r^2 - \frac{4L}{C}}}{2L}\right)t} + \frac{E}{r_1} \qquad ⑧$$

This is the current in the outer circuit.

When $t = 0$, $i_1 = I_0$, current in the inductance.

Also when $t = 0$, $i_1 = i = I_0$

and $e_c = E_{co}$, the voltage across the cond.

Since $e_c = \frac{q}{C} = L\frac{di_1}{dt} + ri_1$

$$\therefore\ E_{co} = L\frac{di}{dt} + ri \qquad ⑨$$

From ⑧

$$\frac{di}{dt} = -\frac{r+s}{2L}B_1 \epsilon^{-\frac{r+s}{2L}t} - \frac{r-s}{2L}B_2 \epsilon^{-\frac{r-s}{2L}t}$$

where $s = \sqrt{r^2 - \frac{4L}{C}}$

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$\therefore$ (9) becomes

$$E_{co} = L\left(-\frac{r+s}{2L}\right)B_1\epsilon^{-\frac{r+s}{2L}t} - L\left(\frac{r-s}{2L}\right)B_2\epsilon^{-\frac{r-s}{2L}t} +$$

$$rB_1\epsilon^{-\frac{r+s}{2L}t} + rB_2\epsilon^{-\frac{r+s}{2L}t} + \frac{rE}{r_1}$$

When $t = 0$, $\epsilon^{-\frac{r+s}{2L}t} = \epsilon^{+\frac{r-s}{2L}t} = 1$

$$\therefore E_{co} - \frac{rE}{r_1} = \left(r - \frac{r+s}{2}\right)B_1 + \left(r - \frac{r-s}{2}\right)B_2$$

$$\text{or } E_{co} - \frac{rE}{r_1} = \frac{r-s}{2}B_1 + \frac{r+s}{2}B_2 \qquad (10)$$

From equation (8), when $t = 0$,

$$I_0 = B_1 + B_2 + \frac{E}{r_1}$$

$$B_1 = I_0 - B_2 - \frac{E}{r_1}$$

Substitute this value of B_1 in (10)

$$\therefore B_2 = \frac{E_{co} - \frac{E}{2r_1}(r+s) - \frac{I_0}{2}(r-s)}{s}$$

$$B_1 = \frac{-E_{co} + \frac{E}{2r_1}(r-s) + \frac{I_0}{2}(r+s)}{s}$$

Then the complete equation for i, with the constants determined, is

$$i = \frac{-E_{co} + \frac{E}{2r_1}(r-s) + \frac{I_0}{2}(r+s)}{s}\epsilon^{-\frac{r+s}{2L}t} +$$

$$\frac{E_{co} - \frac{E}{2r_1}(r+s) - \frac{I_0}{2}(r-s)}{s}\epsilon^{-\left(\frac{r-s}{2L}\right)t} + \frac{E}{r_1} \qquad (11)$$

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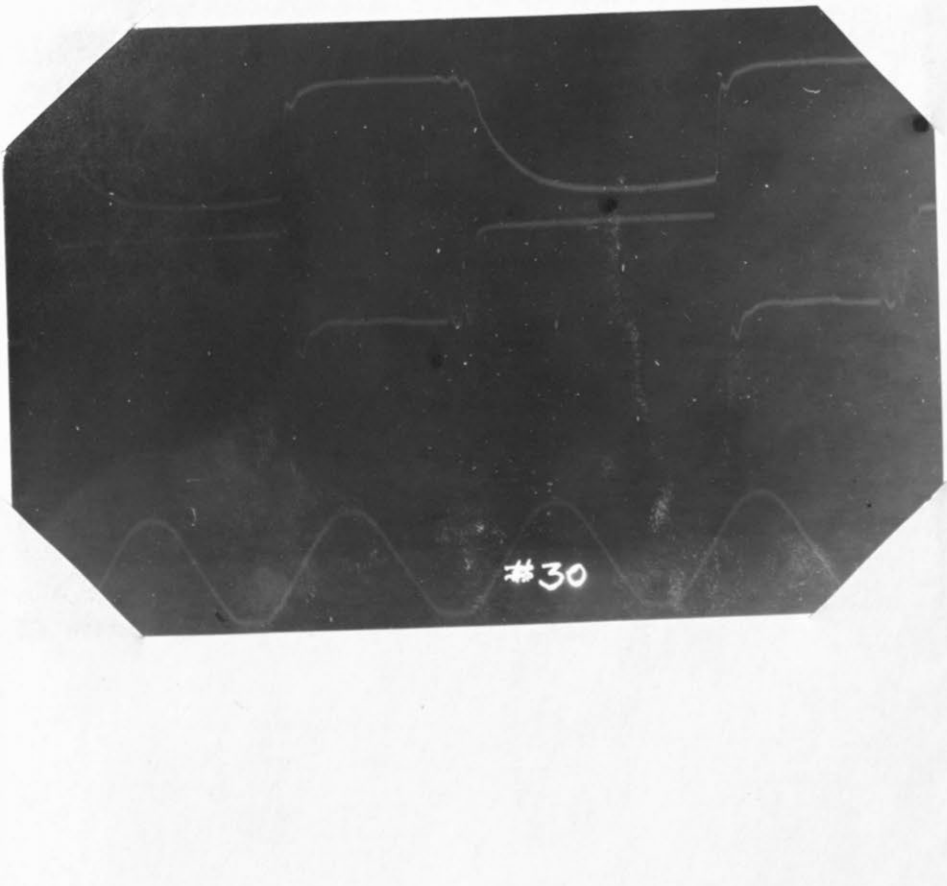
oscillograph across the spark gap, and it was found that with 7.95 microfarads, the maximum peak of the voltage was still about twice the normal value of the e.m.f. This is shown on oscillogram No.30. The effect of 1.17 microfarads in this position is shown by oscillogram No. 30.

In either of these cases it is evident that the capacity across the inductance is not as satisfactory as that across the gap. In other words it takes more capacity across the inductance than it does across the gap to accomplish the sameresults as to sparking.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#31

S O U N D E R   C I R C U I T

Resistance Across Inductance

A Mesco 5-ohm sounder, property of the U. S. Signal Corps, and, for the purpose of this work called sounder No. A, was next set up under normal working conditions. At the adjustment used, i. e., an air gap of .021 inch at the "making" position, the inductance will be found, by reference to the curve in a previous chapter on the measurement of inductance, to be 40.45 millihenries, and the resistance to be3.75 ohms at room temperature of 18 degrees Centigrade.

The normal operating current value at the adjustment given above, was 0.275 ampere when the circuit was connected without spark reducing devices. When resistance is added in parrallel with the coil, sufficient increase in current must be provided to keep the current through the coil at 0.275 ampere under steady conditions. In this way the sounder can be made to

MESCO SOUNDER NO. A.

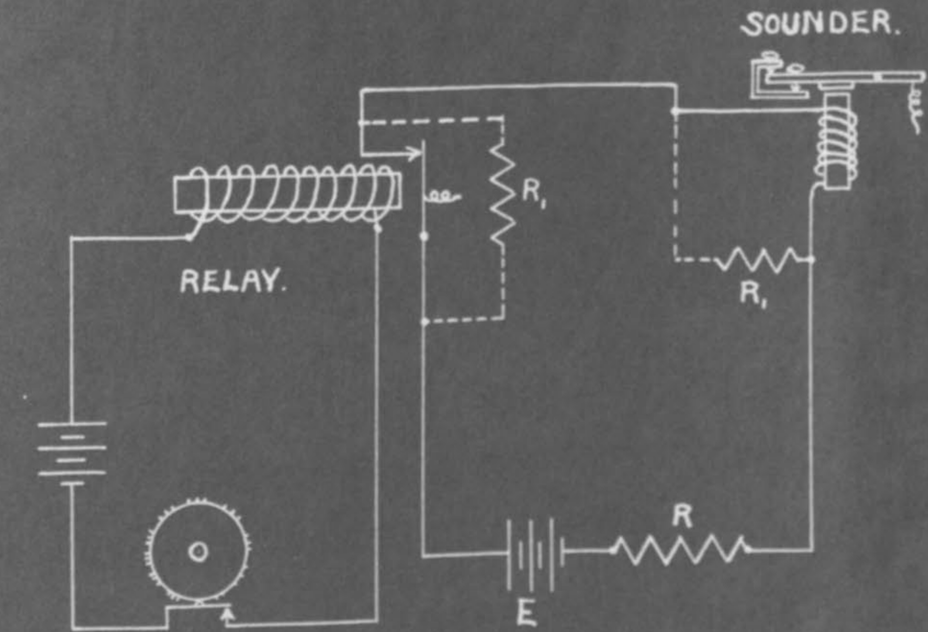

FIG. FIVE.

SOUNDER	RES.	3.75 ω.
..	IND.	40.45 M.H.
R		20 ω.
R_1		VARIABLE.
E.		6.5 VOLTS.

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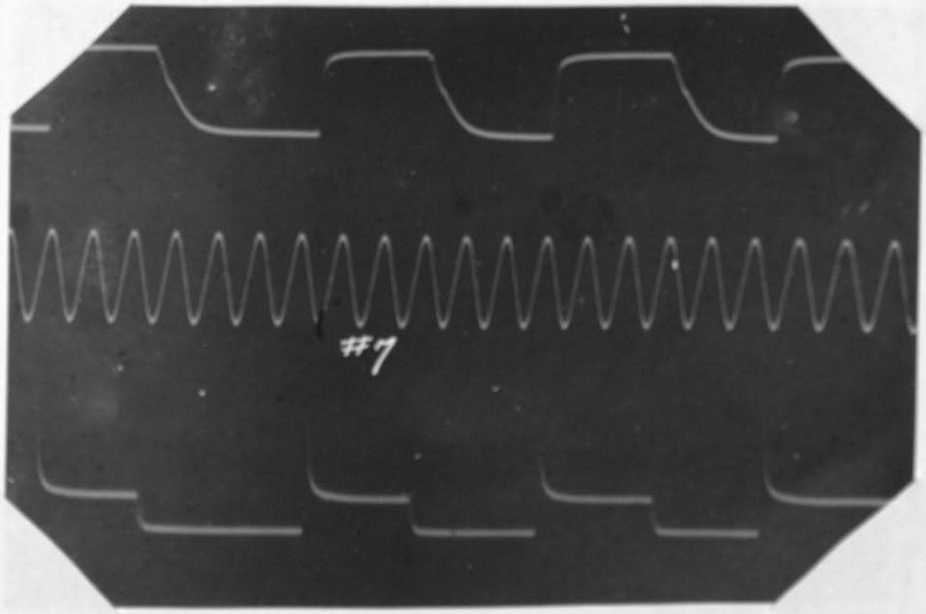
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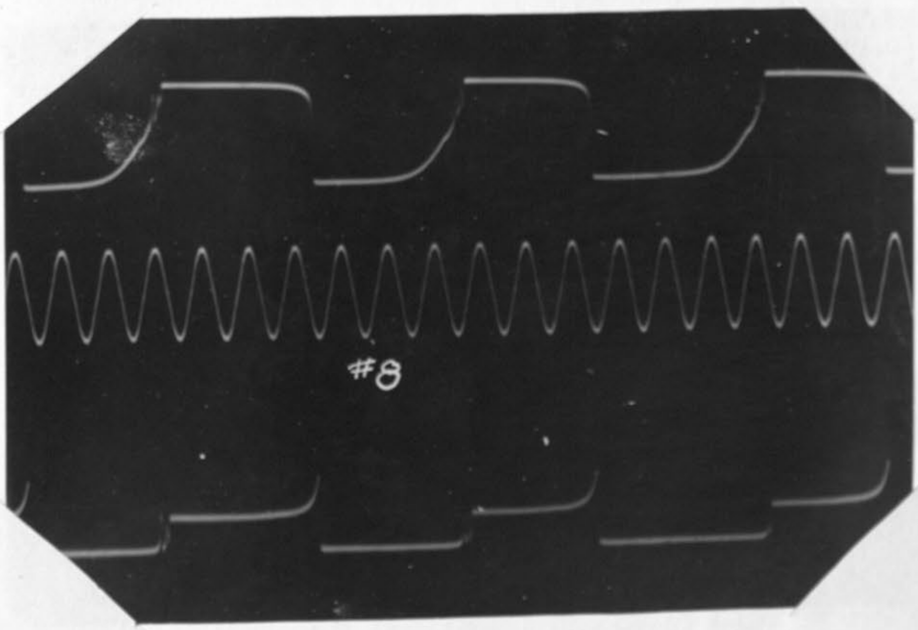
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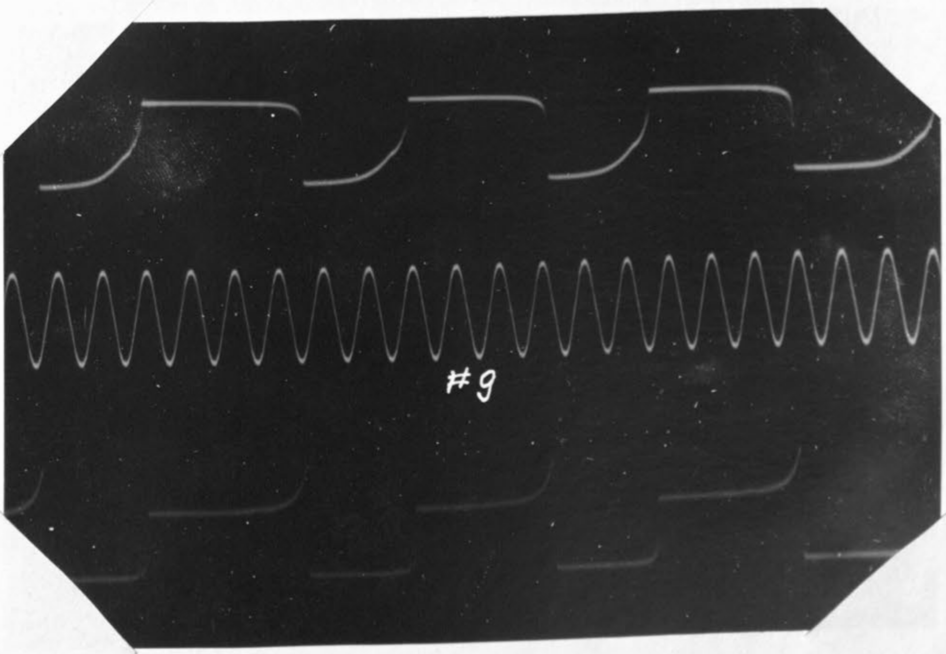
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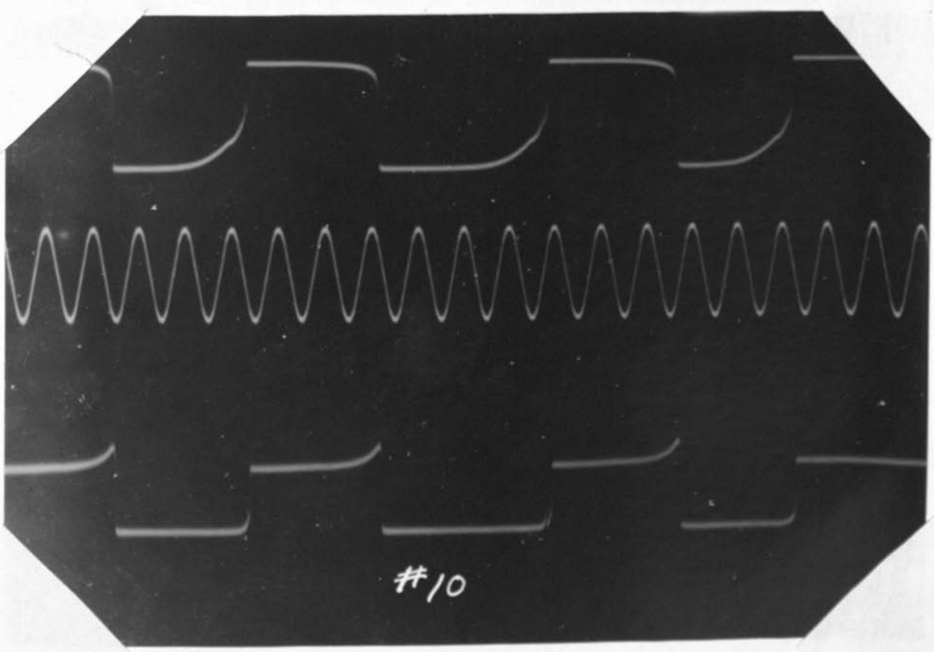

#10

ed by such an arrangement. The time constant of the circuit in this case is expressed by $\frac{L}{R}$.

The effect of the oscillograph will be neglected for the present, and will be studied in a later chapter.

The time constant in the case of no resistance, and shown by oscillogram No. 71, is $\frac{.04045}{23.75} = .0017$ sec.

Twenty five words per minute in American Morse, corresponds to 371 signals per minute or 10.3 signals per second, based upon the law of averages for the length of words. This then, increases to 20.6 "makes" or "breaks" per second, and the time of each is .0485 second, assuming the "marking" and "spacing" intervals to be the same. The signal sent for the oscillograms shown was the letter S, and by comparing the speed of the signals with the 60 cycle wave it is found that the speed used would give "marking" and "spacing" intervals of about .0525 seconds, which is fairly close to the American Morse rate.

In the case which has been cited as about proper for reduction of the spark; namely, 10 ohms in parallel with the coil, the time constant would be changed, due to there being two parallel paths for the current. The total resistance, then, is 3.75 1/(1/10 1/20) = 10.41 ohms. The time constant = .04045/10.41 = .00388 second. When compared with .0525 second, the time constant for this condition seems sufficiently small to insure the disappearance of the transient effect, so that it will not disturb the next signal.

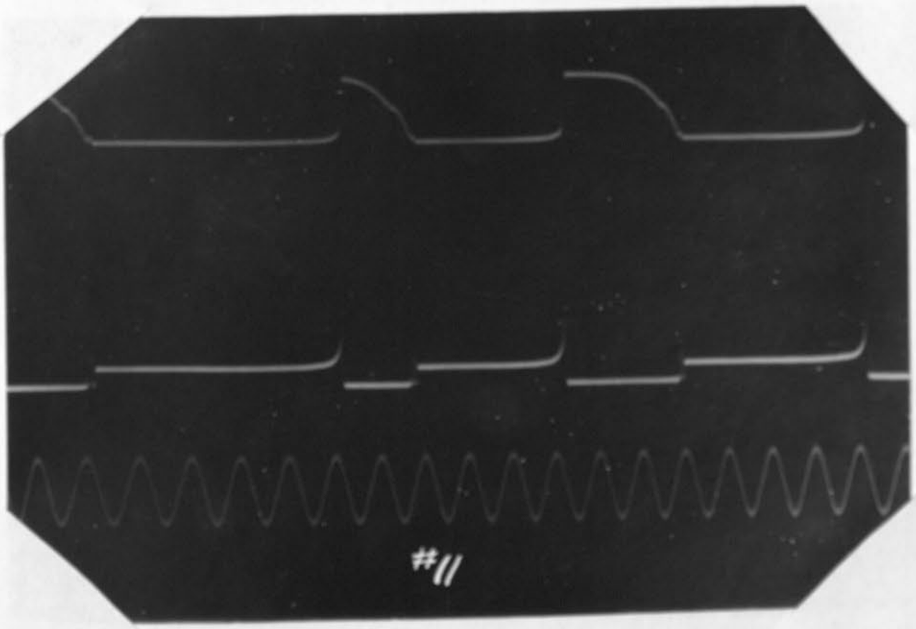
Equations could be set up for this case as well as for the one where the capacity was shunted across the inductance, but they would also involve more knowledge of the varying spark resistance than seems available at the present time.

Shunting the inductance with resistance, in the case of the sounder, seems to be a very satisfactory method of eliminating the sparking at the relay contact.

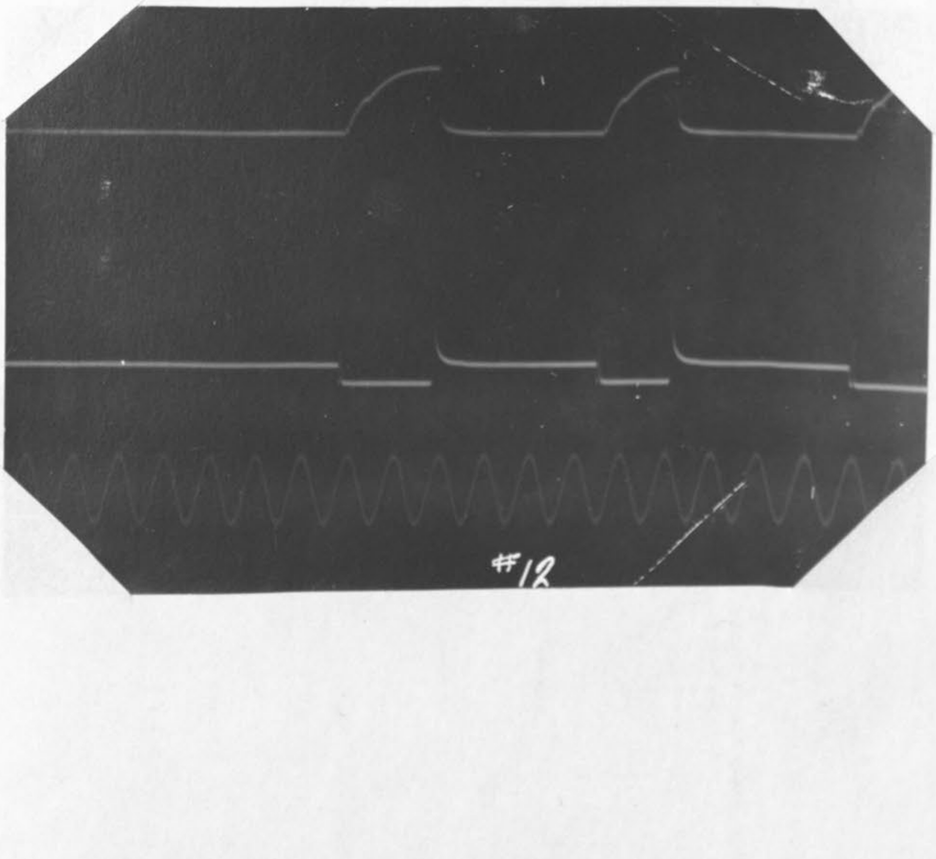
Resistance Across the Gap

Oscillograms, Nos. 11, 12, 13, and 14 were taken with the circuit as shown in Fig.5, and with the resistance across the spark gap. The resistances used were, respectively, 1137, 405, 80, and 38.5 ohms.

From the outset, the objection to this arrangement is that currnet is drawn from the battery at all times. The oscillograms indicate that no material reduction of sparking results and also that the voltage is not greatly reduced until such a low value is arrived at that the steady flow of current at non-operating intervals becomes very objectionable. Although this experiment revealed no satisfactory results, they are inserted here as a matter of record.



#11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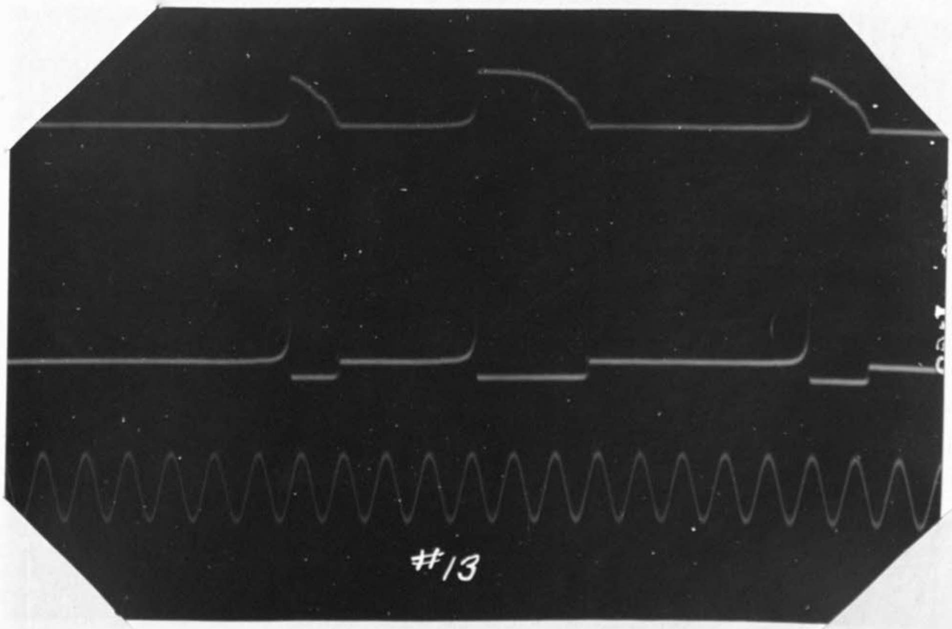
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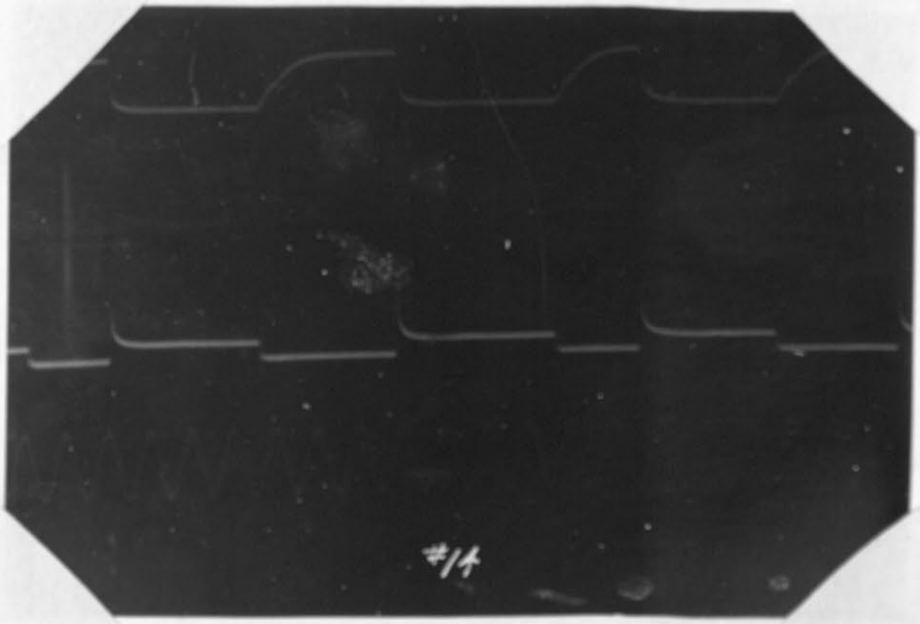

#14

Mesco Sounder

Capacity Across the Gap

With the condenser across the spark gap of the local sounder circuit, the conditions will be the same as those of the buzzer circuit, providing the circuit is oscillatory. The circuit for this condition is shown in Fig.6, where the dotted condenser around the relay contact is the one in use.

The circuit is oscillatory when R^2 is less than $4L/C$ and, since $R^2 = 400$ where $4L/C = 1618$ for $C = 100$ microfarads, itis safe to say that the circuit is oscillatory. If smaller capacities are used the difference will be still greater, but it is impractical to think of using even 100 m.f. Then the same equation which was developed with the buzzer for maximum voltage will hold here. The equation $e_c = E + E\epsilon^{-\frac{R\pi}{2}\sqrt{\frac{C}{L}}}$ expresses the maximum voltage peak across the condenser for any value of capacity.

The following table of values is plotted on a

MESCO SOUNDER NO. A.

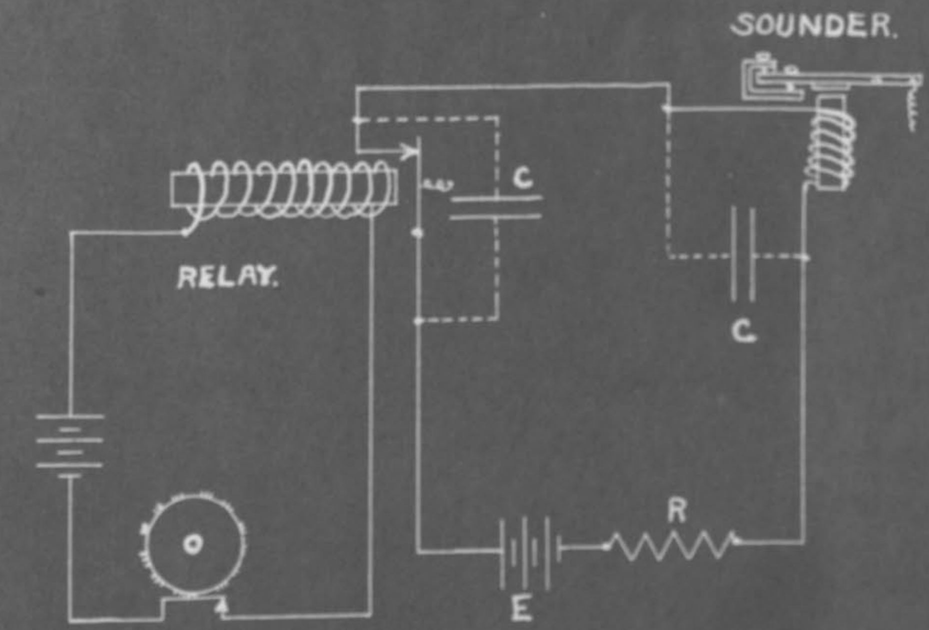


FIG. SIX.

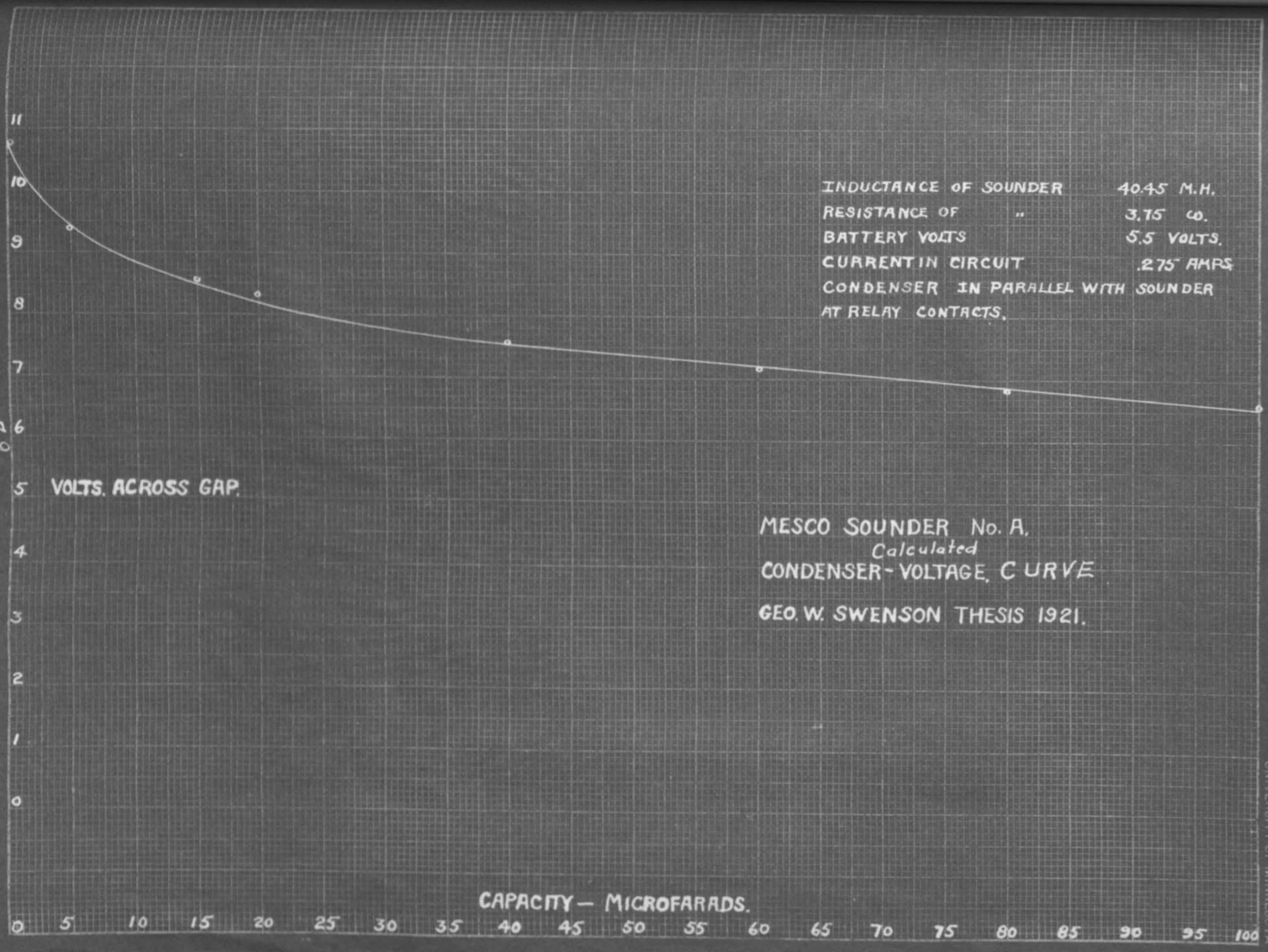
SOUNDER	RES.	3.75 ω.
"	IND.	40.45 M.H.
R		16.25 ω.
C		VARIABLE.
E		5.5 VOLTS.

GEO. W. SWENSON THESIS 1921.

curve to show the maximum voltage values for this circuit.

Cspacity (m.f.)	e_c (volts)
100	6.65
80	6.35
60	7.15
40	7.56
20	8.32
10	8.85
5	9.40
1.0	10.20
0.1	10.75

The equation, as well as the curve, would indicate that the voltage across the gap is not reduced to a minimum untill an infinite capacity was introduced. The visible sparking, however, is eliminated with a capacity of about 1 or 2 microfarads and the curve shows that nothing appreciable is to be gained by going beyond 5 microfarads.



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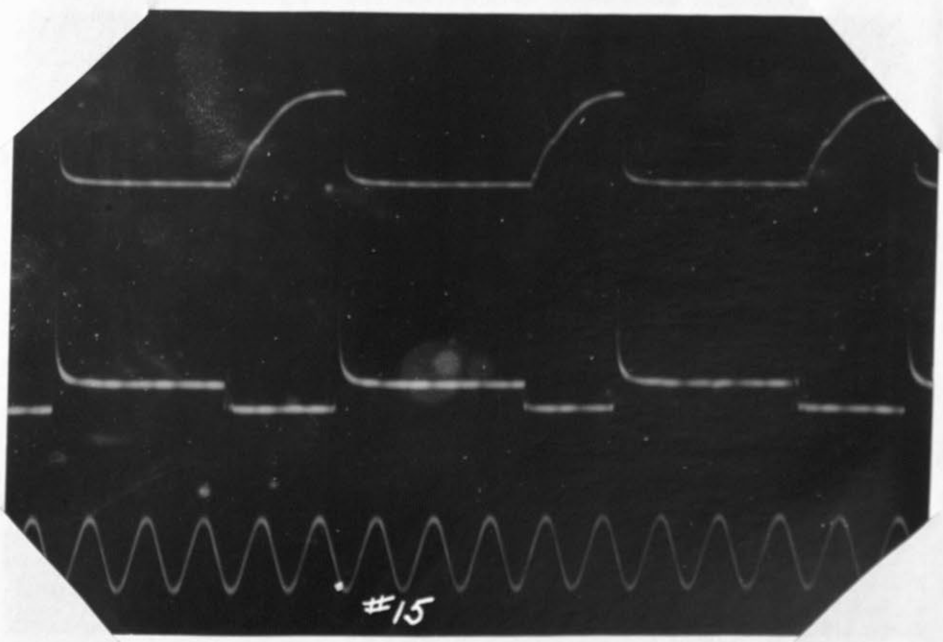
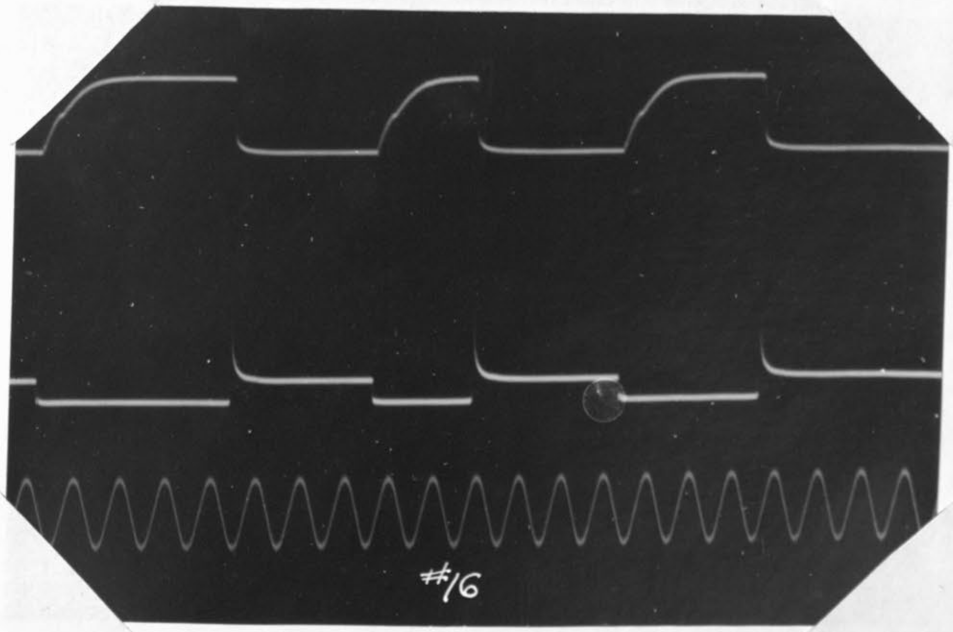
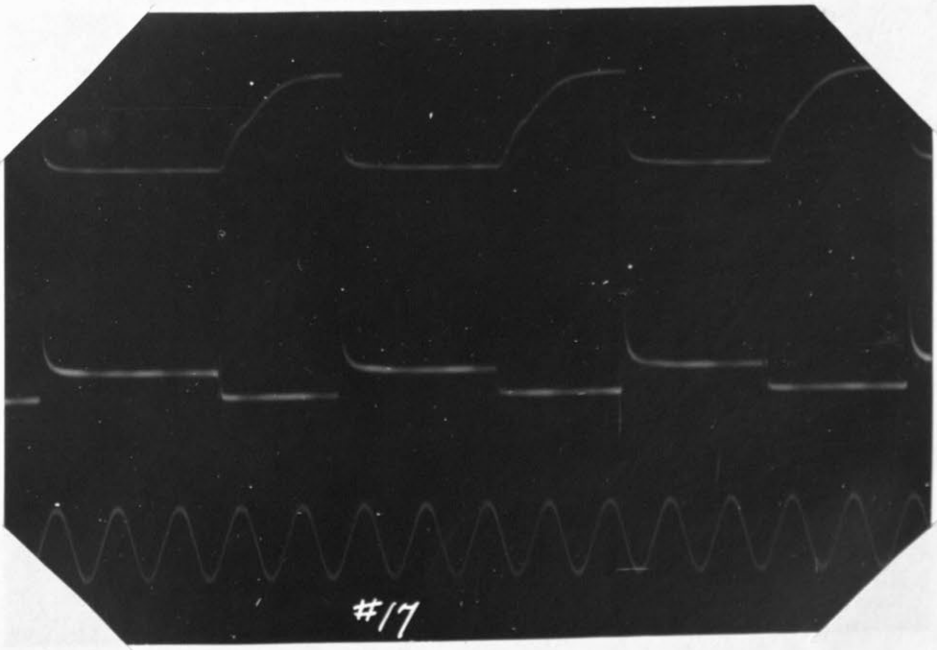
INDUCTANCE OF SOUNDER 40.45 M.H.
RESISTANCE OF " 3.75 ω.
BATTERY VOLTS 5.5 VOLTS.
CURRENT IN CIRCUIT .275 AMPS.
CONDENSER IN PARALLEL WITH SOUNDER AT RELAY CONTACTS.
MESCO SOUNDER No. A.
Calculated
CONDENSER-VOLTAGE CURVE
GEO. W. SWENSON THESIS 1921.
VOLTS. ACROSS GAP.
CAPACITY — MICROFARADS.
0 1 2 3 4 5 6 7 8 9 10 11
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100

79

Oscillograms, Nos. 15, 16, and 17 were taken with 0.1, 0.2, and 0.7 microfarads, respectively, across the gap, and altho the first two gave considerable sparking it is quite evident that the peak value of the voltage is being reduced with the increasing capacities.


#15


#16

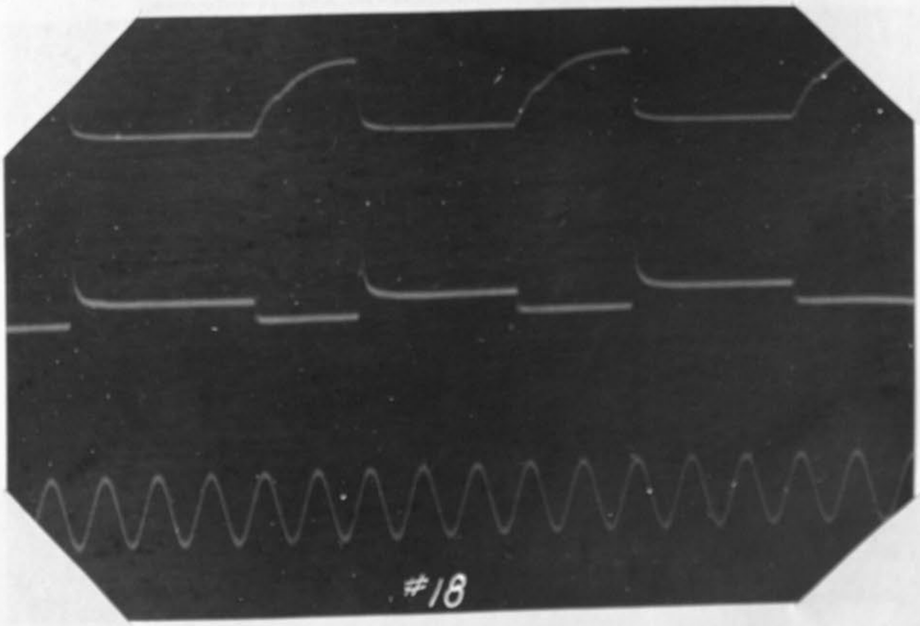

#17

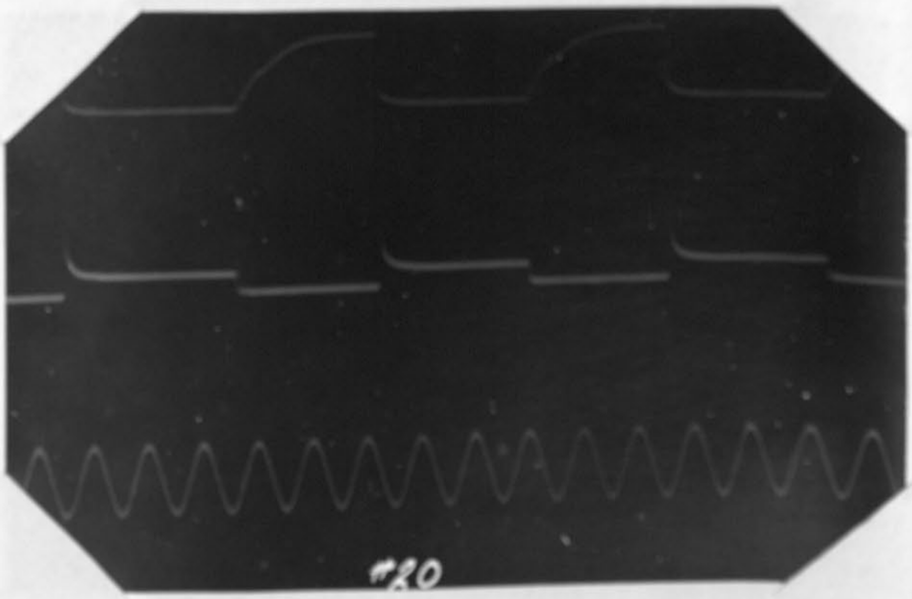
Mesco Sounder

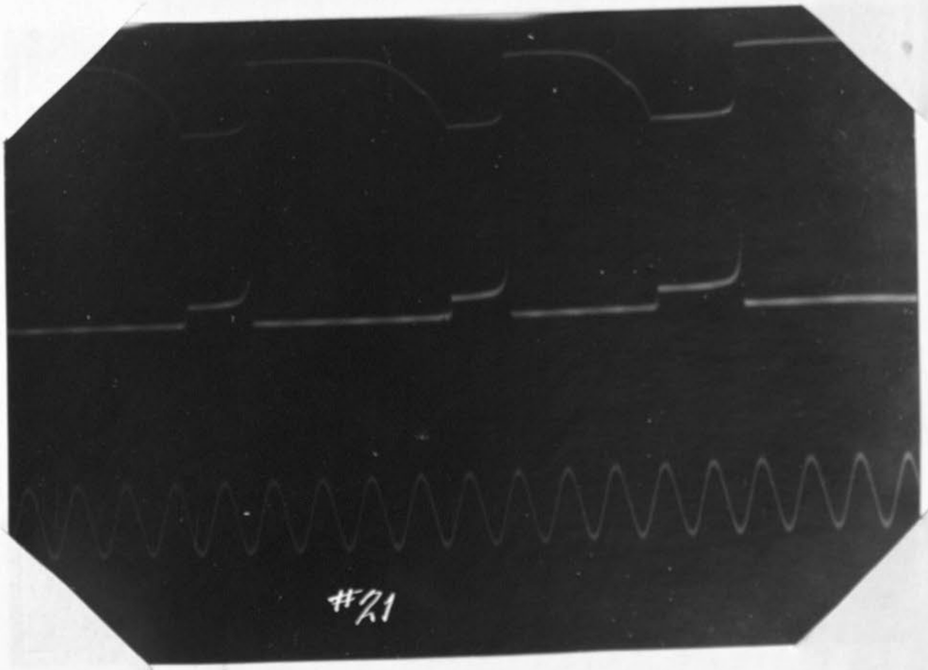
Capacity Across the Inductance

With capacity across the inductance, as indicated in Fig.6, by the dotted lines, it is evident that the values of capacity are somewhat higher to reduce the sparking to the same amount as was accomplished by the condenser across the gap. Oscillograms, Nos. 18, 20, and 21 were taken with 0, 0.2, and 0.5 microfarads, respectively, and altho they show no appreciable decrease in voltage with increasing capacities, there was a slight decrease as will be more clearly pointed out later.

The conclusion from these experiments would be that resistance across the inductance is the most satisfactory method of reducing the sparking at the relay contact.


#18


#20


#21

THE VACUUM TUBE-OSCILLOGRAPH VOLTMETER

It will be recalled that considerable difficulty has been encountered with the oscillograph, due to the fact that there were no means of obtaining the exact peak voltages across the arcing contact. The oscillograph elements have comparatively low resistances, and, if placed across an arcing contact, will not give the results desired. If high resistance is placed in series with the element, the amplitude of the curve becomes too small to be seen, even with the most sensitive element at hand.

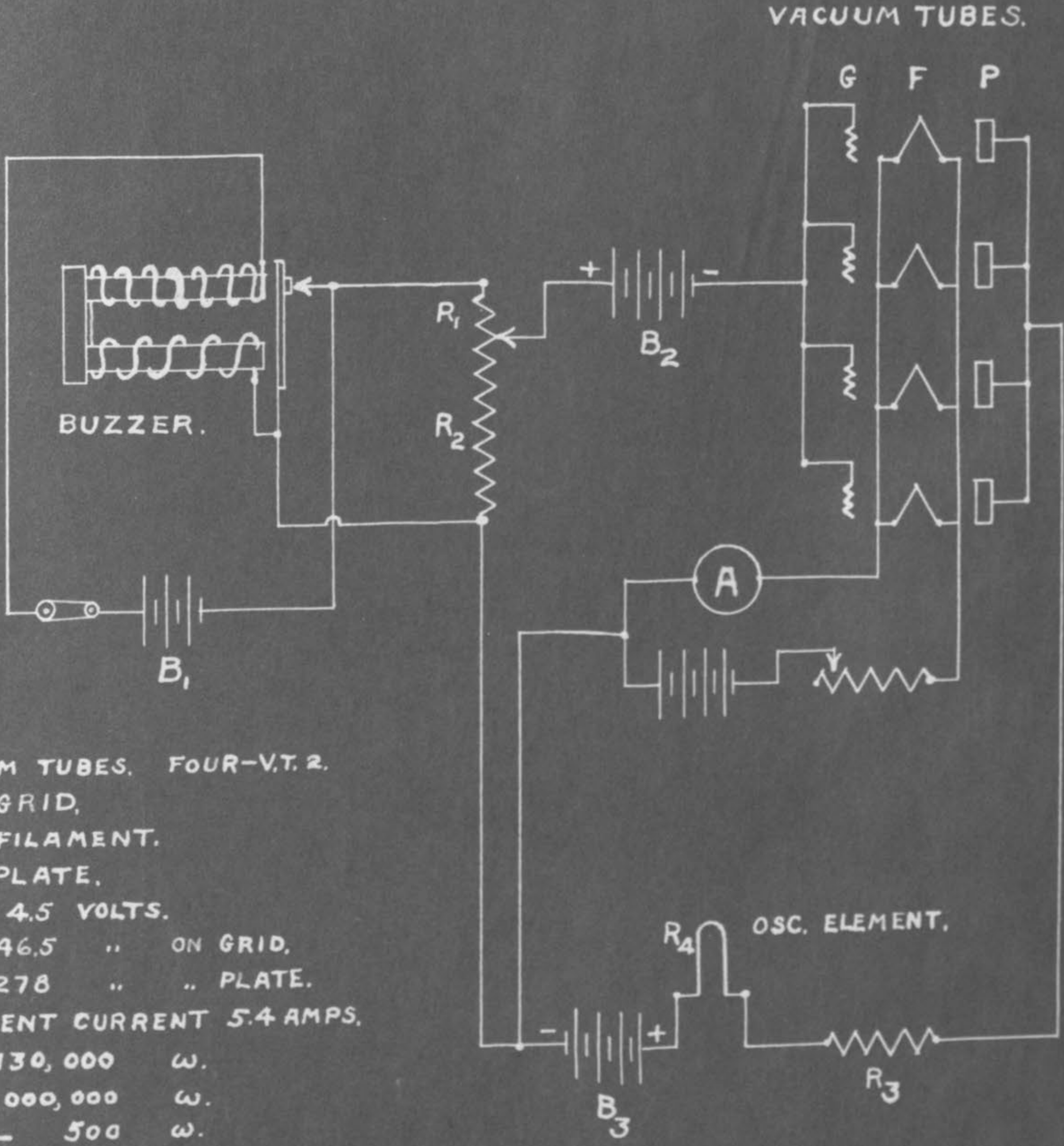
To overcome this difficulty, a vacuum tube circuit was set up to be used in connection with a sensitive oscillograph element. From the characteristics of a three element vacuum tube, it is known that very small changes of current in the grid circuit will produce comparatively large variations in the plate current; i.e., it may be used as an amplifier. It is also known that the grid circuit may have very high resistance, almost infinite, in fact, if it were not for the necessity of having a leakage path, where the excess negative electrons which have a tendancy to"pile up" on the grid, may leak off between the impulses. This leak, however, may be of the nature of a megohm or two, depending largely upon the use to which the tube is put. The leak, then, will determine the resistance which will be placed across the spark gap when it is connected for a test.

The circuit used in the experiments is shown in Fig.7. A high resistance pencil mark was used for the grid leak, and was in this case(as closely as could be measured) about two megohms. A resistance of two megohms will have very little effect upon the resistance of the arc when placed in parallel with it.

It was doubtful whether one tube would have sufficient capacity for the experiments, and it was, therefore, decided to use four Western Electric Type VT-2 tubes in multiple, to insure all the capacity that might be required.

VACUUM TUBE OSCILLATOR VOLTMETER.

HIGH RESISTANCE VOLTMETER.



VACUUM TUBES. FOUR-V.T. 2.

G - GRID.

F - FILAMENT.

P - PLATE.

B_1 - 4.5 VOLTS.

B_2 - 46.5 " ON GRID.

B_3 - 278 " " PLATE.

FILAMENT CURRENT 5.4 AMPS.

R_1 - 130,000 ω.

R_2 - 2,000,000 ω.

R_3 —— 500 ω.

R_4 —— 20 ω.

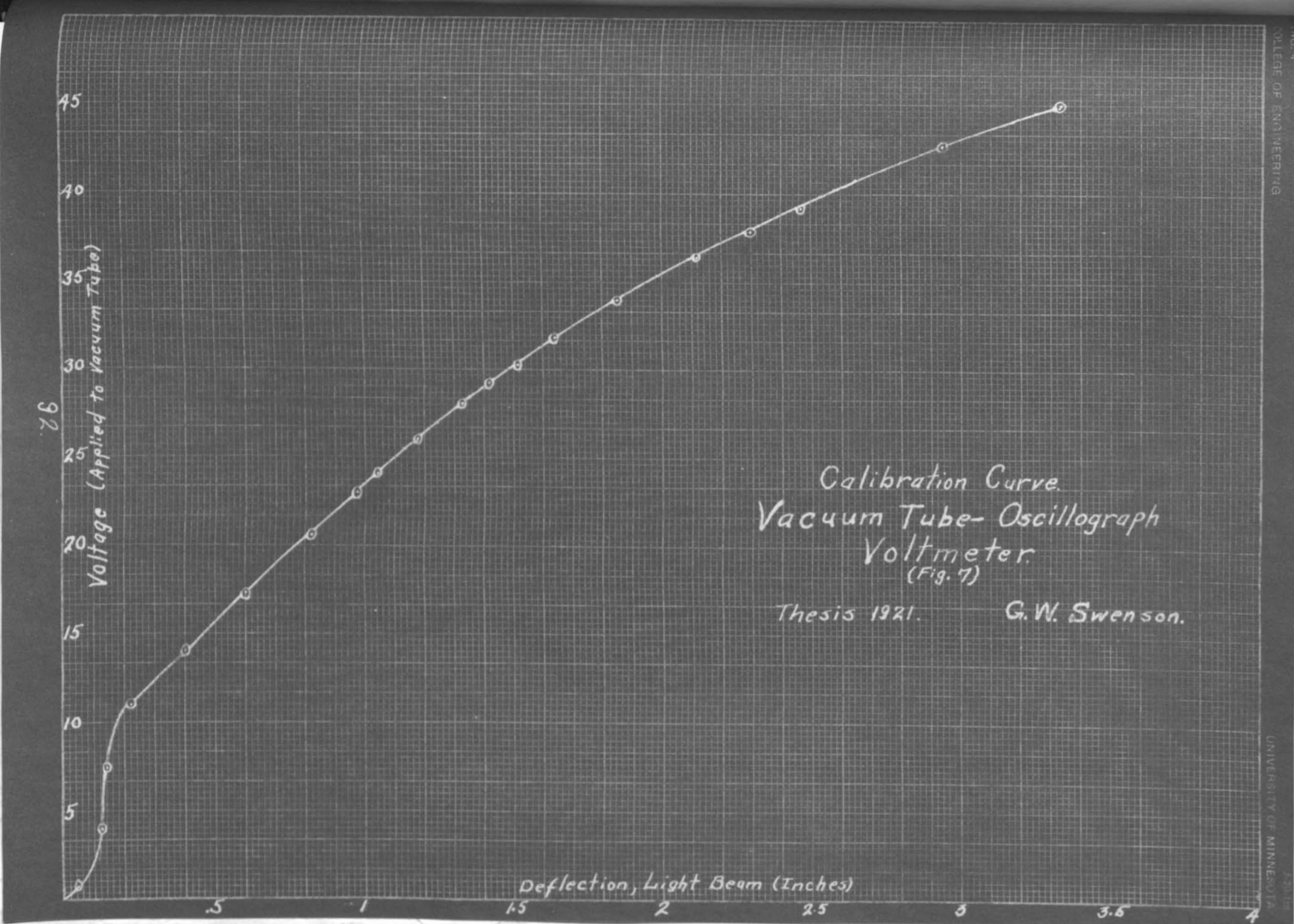
FIG. SEVEN.

GEO. W. SWENSON THESIS 1921.

It was found desirable to use rather high plate voltage, and, consequently, a relatively high negative grid voltage, to bring the characteristic curve to zero. The best results were obtained with a plate resistance of about 520 ohms, including the oscillograph element, which is in series with the plate circuit. The normal filament current of 1.35 amperes per tube, or 5.4 amperes for the four tubes, was used throughout the test.

It was necessary to calibrate the tubes, together with the oscillograph, very carefully, and this was done by measuring the deflection of the light beam on the tracing table of the oscillograph with varying voltages applied to the grid. The data are omitted here, but a curve follows which gives the variation of the light ray deflections with varying grid voltages. This curve is not exactly linear, because of the low plate resistance. This is not a serious matter as long as the curve is smooth, and it will prove satisfactory for this purpose.

Peak voltages on all of the circuits which have been discussed in this paper were checked by the vacuum


ENG. 4
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Calibration Curve.
Vacuum Tube- Oscillograph
Voltmeter.
(Fig. 7)
Thesis 1921.
G. W. Swenson.
Voltage (Applied to Vacuum Tube)
45
40
35
30
25
20
15
10
5
Deflection, Light Beam (Inches)
.5
1
1.5
2
2.5
3
3.5
4
92.

tube-oscillograph method in an attempt to prove or disprove any theories or conclusions already arrived at. Oscillographic records might have been taken of all these conditions, but due to lack of time, accurate measurements of the deflections in each case were made directly from the tracing table and recorded.

The first circuit to be investigated was the buzzer circuit without capacity (Fig.2) with the exception that the circuit of Fig.7 was substituted for the oscillograph element there shown, to measure the voltage across the gap. The deflection registered, under conditions of normal operation as described before, was 1.35 inches, which, from the calibration curve, corresponds to 27.5 volts.

The above peak voltage across the gap is by no means infinite when the maximum voltage is obtained. The assumption made on page 24, that the resistance of the gap was about 200 ohms, was a very close one, since it gives a maximum peak voltage of thirty volts.

The next circuit to be checked was that made by placing the various values of capacity across the gap, as shown in Fig.3, and the following results were obtained:

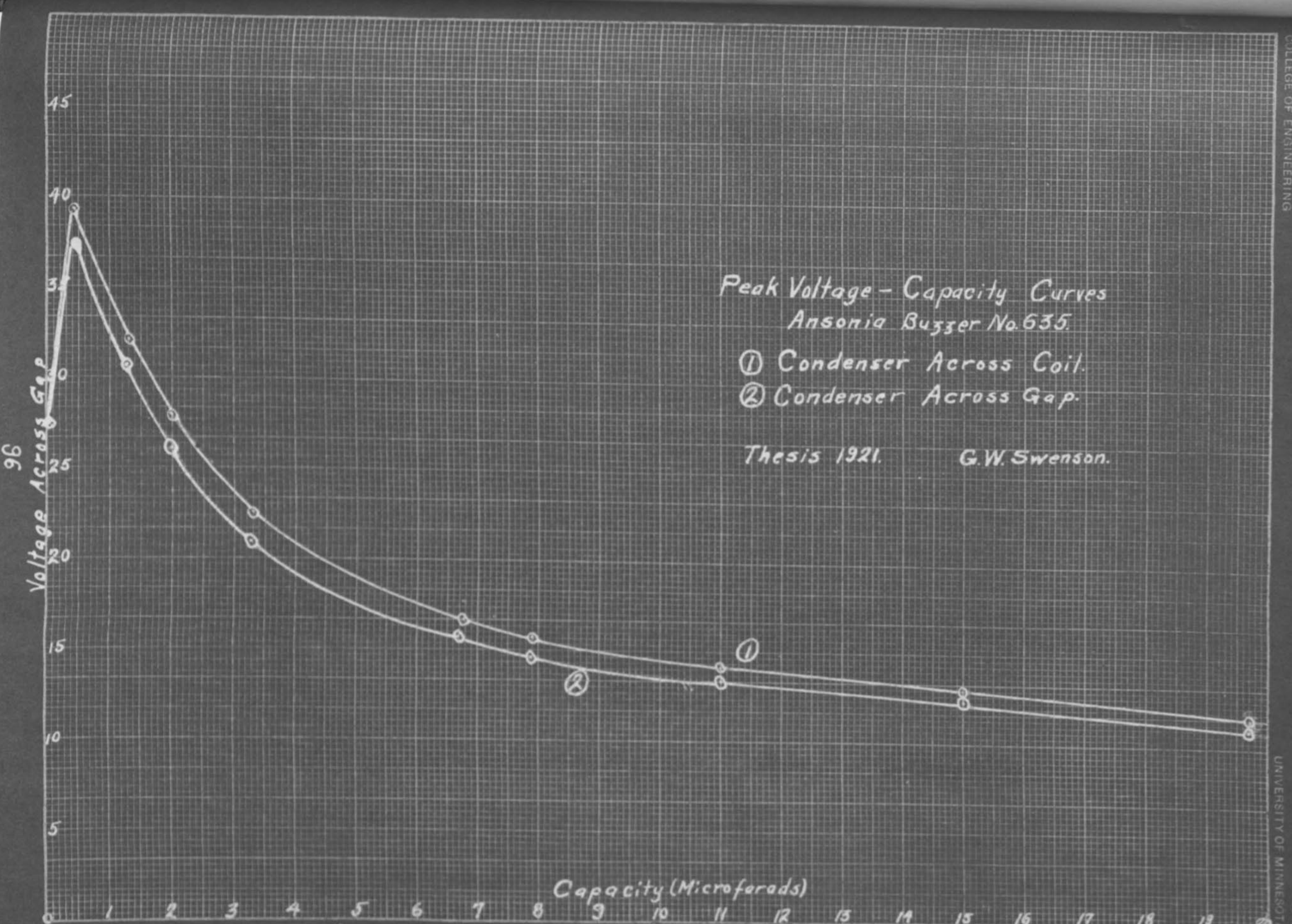
Capacity (m.f.)	Peak voltage
0.0	27.5
0.47	37.4
1.23	30.7
1.91	26.0
3.21	20.0
6.70	15.8
7.88	14.3
11.09	13.5
15.05	12.4
19.70	11.0

The last circuit with the buzzer is the one where capacity is shunted across the coil, the arrangement being indicated in Fig.4. The readings are given below:

Capacity (m.f.)	Peak voltage
0.0	27.5
0.47	39.2
1.23	32.0
1.91	27.7
3.21	22.5
6.70	16.0
7.88	15.5
11.09	14.0
15.05	12.8
19.70	11.5

The above tabulations are plotted in curves on page 96, both on the same page for comparison.

Somewhat contrary to the mathematical treatment, the voltage rises higher than is shown by the formula (See page 43). The reason for this is that no account was taken of the resistance of the arc in parallel with the condenser, which has the effect of raising the voltage above what it would be with the condenser alone. The general shapes of the curves are the same, with the excep-



Peak Voltage - Capacity Curves
Ansonia Buzzer No. 635.
① Condenser Across Coil.
② Condenser Across Gap.
Thesis 1921.
G.W. Swenson.
Voltage Across Gap
Capacity (Microfarads)

tion that the voltage rise is higher upon adding capacity of any very small amount, because the arc has been partially extinguished and the resistance raised. Then the current does not seem to follow, inversely, the change of resistance, but tends to maintain a higher comparative value, thus increasing the voltage with added resistance. This does not hold true for larger values of capacity, since the current has a better path through the condenser, and, consequently, the current dies doun more rapidly as the resistance is increased.

It will be recalled* that a statement was made to the effect that more capacity was required for reducing the voltage across the gap with condensers around the inductance, than with condensers across the gap. By referring to these last curves, one finds this statement still holds true, but that the difference is very slight, and it is a question whether it can be said that one method may be preferred to the other.

*See page 84.

Mesco Sounder Circuit

The four different conditions used with the Mesco sounder; namely, capacity across the "make and break" contact of the relay, capacity across the sounder inductance, resistance across the inductance, and resistance across the gap were next set up for test. The arrangements used were those shown in Figs. 5 and 6 of the last chapter. Normal operating conditions were maintained throughout the test.

The data for these experiments are given below:

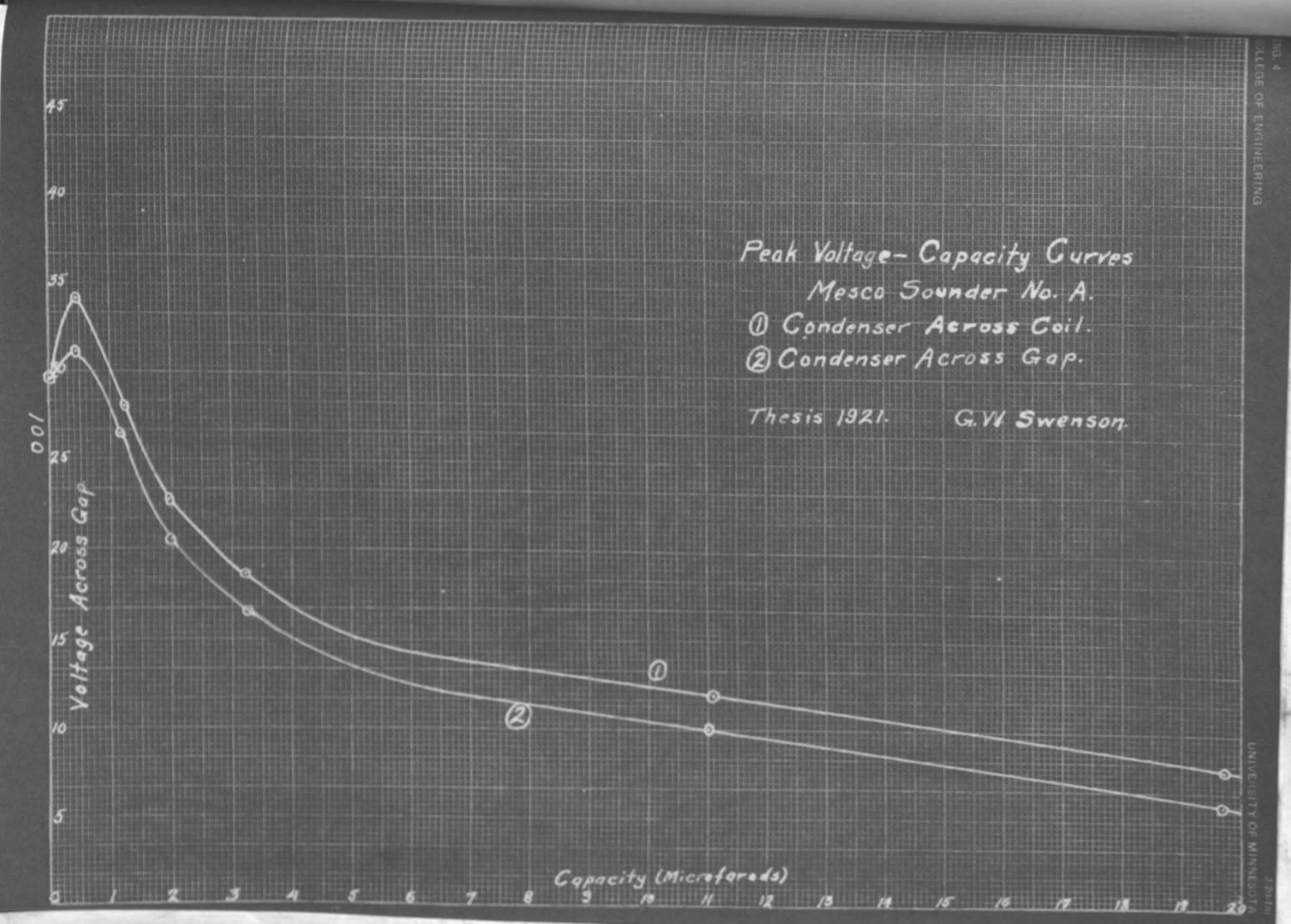
Condenser Across the Gap

Capacity (m.f.)	Peak voltage
0.0	29.7
0.47	31.2
1.23	26.5
1.91	20.0
3.21	16.5
11.09	10.0
19.70	6.0

Condenser Across the Coil

Capacity (m.f.)	Peak voltage
0.0	29.7
0.47	34.0
1.23	28.0
1.91	22.7
3.21	17.0
11.09	12.0
19.70	8.0

These values are plotted on the following page, and, like the buzzer, give the maximum peak voltages with a small condenser, in place of with no condenser as might be expected. They also show that, practically, there is not much to be gained by going beyond four or five m.f. capacities for this case, as far as sparking and high voltage are concerned. Again, the curves show clearly that the capacity across the coil must be slightly higher than that across the gap to effect the same reduction in sparking.



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Peak Voltage- Capacity Curves
Mesco Sounder No. A.
① Condenser Across Coil.
② Condenser Across Gap.
Thesis 1921.
G.W. Swenson.
Voltage Across Gap
Capacity (Microfarads)
100

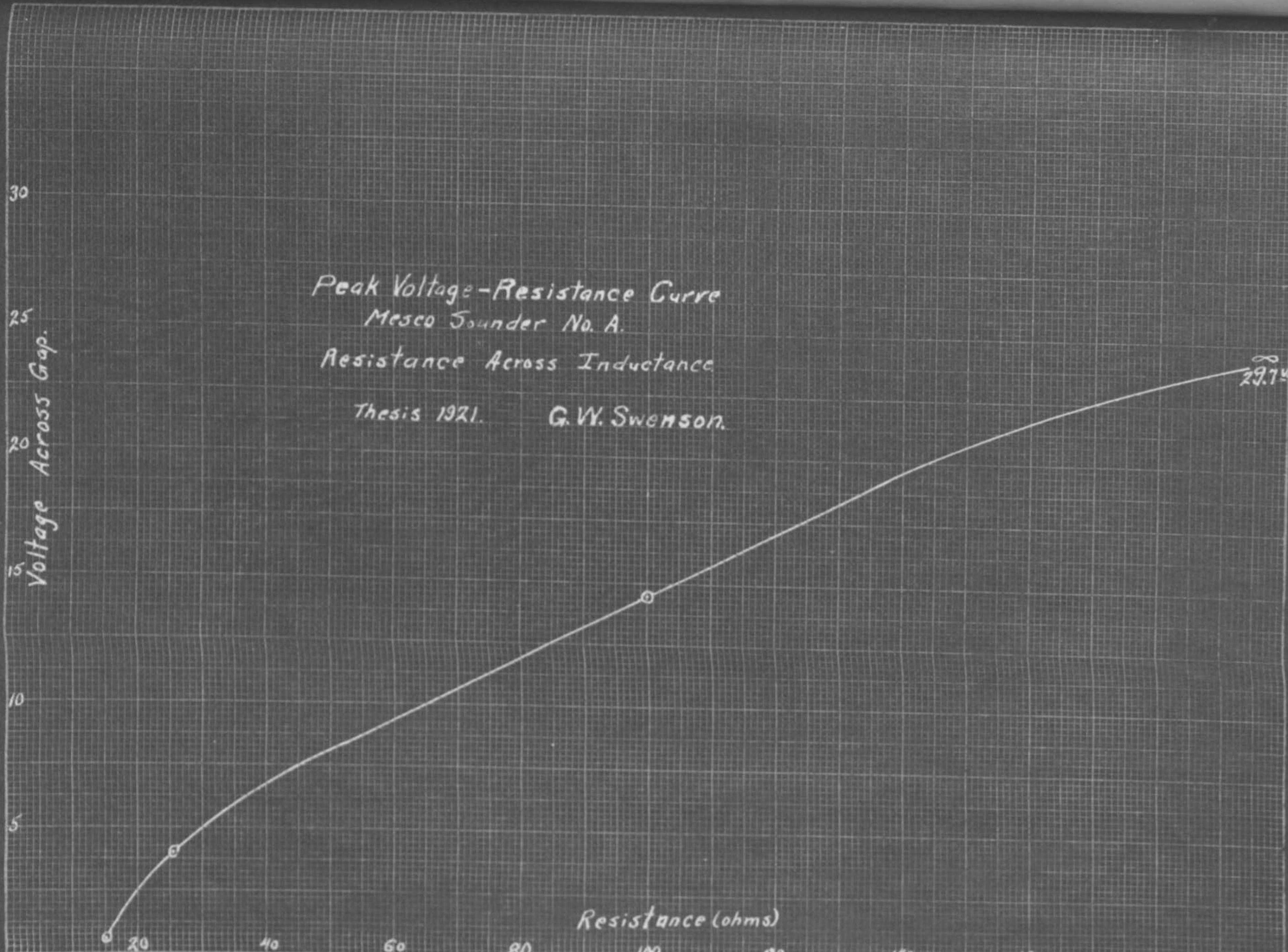
Resistance Across the Inductance

Resistance (ohms)	Peak voltage
	29.7
100	14.4
25	4.0
15	0.5
0	0.0

Resistance Across the Gap

Resistance (ohms)	Peak voltage
	29.7
1137.0	28.0
405.1	25.5
79.4	20.7
38.6	11.6
0.0	0.0

The curves for these data are plotted on the following pages. The cases where resistance is placed across the gap are, of course, barred out for all practical purposes, altho they will reduce the spark for small values of re-



Peak Voltage-Resistance Curve
Mesco Sounder No. A.
Resistance Across Inductance
Thesis 1921.
G.W. Swenson.
Voltage Across Gap.
Resistance (ohms)
30
25
20
15
10
5
20
40
60
80
100
120
140
160
180
200
∞
29.7v

sistance. The permanent current, flowing at all times is too great. The resistance across the inductance, however, proves very satisfactory, and a great voltage reduction is effected by 20 or 30 ohms across the inductance, which does not disturb the action of the sounder appreciably.

These last experiments have been very interesting, and the results are gratifying indeed. A great deal of work could yet be done in this direction, and, in fact further study is contemplated.

S U M M A R Y

Altho the study of sparking contacts has not, by any means, been exhausted, and considerable porfitable study could still be done on the different phases of it, several points of interest have been brought out.

1. In the study of measuring inductance, a method was devised to measure, accurately, iron core inductances, especially small ones, with normal working saturation of the iron.

2. Curves have been plotted to show the variation of inductance of various types of apparatus, by variation of the length of the air-gap.

3. The limitations of the oscillograph as a voltage measuring device have been pointed out.

4. The mathematical treatment of the voltage across the arc has been proved to hold with a fairly high degree of accuracy.

5. Time constants of several circuits have been calculated to prove that the transient effects decrease

to a very low value before the next impulse is due.

6. Transient equations have been completely developed for the case of inductance, resistance and capacity in series, and an equation for the maximum peak voltage has been evolved.

7. Values for several different cases, have been given for the amount of capacity or resistance necessary to reduce sparking, and to reduce the voltage across the spark gap.

8. Transient equations for the resistance in series with a parallel circuit of inductance and capacity have been developed, which apply in the case of steady resistance. With variable resistance further study is necessary to complete them.

9. It has been shown that non-inductive resistance in parallel with the inductance is a very desirable method of reducing sparking and peak voltages.

10. Curves have been plotted to prove that slightly higher values of capacity are necessary in the

use of the condenser across the inductance, compared with those across the spark gap to reduce the sparking and voltage to the same value.

11. A vacuum tube circuit is shown which is very useful, in connection with the oscillograph, to measure voltage by drawing very little current indeed.

12. Among other points of interest, brought out in this study, is the evidence of comparatively high invisible conductivity, after the spark breaks, indicating that there is still considerable ionization.

13. It has been shown that the voltage across the arc rises, when very small values of capacity are placed across the arc or the inductance of buzzer and sounder circuits, above that without condensers.

S U G G E S T I O N S

Several points have been discovered, in the work for this paper, which are deserving of further study. A few suggestions are given here as a guide to future study on this problem.

1. The principles herein outlined for small inductances might be applied to larger inductances, such as telephone and telegraph relays.

2. A further study of the vacuum tube-oscillograph voltmeter here used should be made to perfect and develope it so that it may be used upon a moment's notice for any voltage measurement where the consumption of current by the voltmeter must be a minimum. Oscillographic records should be made to show the constancy or the inconstancy of the peaks at each succeeding impulse.

3. A further study of the exact nature and resistance of the spark and of the gap after the arc is extinguished, would, most likely, lead to very interesting results. In this connection a further investigation

should be made as to the reason for the peak voltage rising with small capacities in parallel with the gap or the inductance.

4. A further study could be carried on with reference to the experimental and theoretical peak values of voltage as shown in this paper. The general slopes of the curves are the same, but the values do not check.

5. A great deal of work could still be done in setting up exact equations for the voltage across the gap with capacity in parallel with it, all in series with the inductive circuit. Allowance will have to be made for the fact that the spark gap resistance is variable.