REPORT

in

COMMITTEE ON THESIS

The undersigned, acting as a Committee of the Graduate School, have read the accompanying thesis submitted by Ralph Sylvester Underwood for the degree of Master of Arts. They approve it as a thesis meeting the requirements of the Graduate School of the University of Minnesota, and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Arts.

Minneapolis, Minnesota

~~May~~ June 1 1917

F. P. Leavenworth
Chairman

Henry A. Erikson

Wm. O. Beal

REPORT

of

COMMITTEE ON EXAMINATION

This is to certify that we the undersigned, as a Committee of the Graduate School, have given Ralph Sylvester Underwood final oral examination for the degree of Master of Arts. We recommend that the degree of Master of Arts be conferred upon the candidate.

Minneapolis, Minnesota

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An Investigation of the Proper Motion of Faint Stars Near Sirius.

A thesis submitted to the

Faculty of the Graduate School of the

University of Minnesota

by

Ralph S. Underwood

In partial fulfillment

of the requirements for the degree of

Master of Arts.

June 1917

Introduction.

A comparison with a stereo-comparator of two photographs of the region about Sirius, taken in 1897 and 1915 respectively at the Observatory of the University of Minnesota, showed an annual proper motion of about two-tenths of a second in the case of one relatively faint star. Since appreciable proper motion in a faint star is more unusual and, for reasons which will be given later, more indicative of unique attributes of the star itself than the same proper motion in a brighter star, it was deemed worth while to determine accurately the star's motion and position. The measurements necessary to accomplish this determination would make it possible at the same time to search carefully the field about Sirius for more proper motion stars, including any having the motion of Sirius itself and too distant and faint to be liable to detection by direct observation. Incidentally, any other bodies of interest in the region examined, such as variable stars or faint nebulae, would be revealed by such an investigation; so that the results should comprise a detailed and comparatively authoritative summary of the celestial objects in the selected area, at least down to the twelfth magnitude. Since enough early photographs were available at the Observatory to make an investigation of this kind practicable, at the suggestion of Professor Leavenworth I made it the subject of my thesis.

Discussion.

The reason that proper motion in a faint star has special significance is that it indicates either a high absolute velocity or a low luminosity,--perhaps in the majority of cases the latter. If the star is as distant as the average star of the same magnitude, its velocity must exceed the average; while if it is relatively as near as most stars having equal apparent motion it must radiate less light than they, either on account of small mass or of intrinsic condition such as a half-cooled state due to stellar old age. Of course the result may be, and probably in most cases is, due to a combination of these circumstances rather than to any one of them; but it nevertheless establishes in each case the fact that the star has in some measure one or more of these unique features. It is obvious that faint proper motion stars make fairly promising objects of parallactic investigation -- being equal, perhaps, in a general way, to stationary bright stars in this respect--, and that they are, other things being equal, unquestionably the most likely of all the stars to yield high radial velocities whenever by present or improved methods sufficient light can be obtained from them to make these quantities determinable. This will be the more apparent when we consider that the highest radial velocities would naturally be found, by the law of chance, in the class of stars numerically the greatest,--in other words, the faint stars; and that of this class those having the greater apparent motion would in general have the greater radial and actual motion. Owing to their faintness, however, such stars are not likely to be discovered except by systematic micrometical measurements of many photographs, involving much labor for ms meagre results; and for this reason few of the recorded proper motion stars are fainter than the ninth magnitude. Since a given proper motion acquires importance with decrease in magnitude of the star, or conversely, the fainter the star, the smaller the proper motion needed to give it prominence, there are probably many unrecorded stars among the millions revealed by the camera which are entitled by virtue of proper motion to a catalog position. They should be of prime importance in investigations of the larger cosmic problems, such as the matter of star streams and the motion of the solar system; and they might also furnish in some cases, when the practical observational difficulties due to their faintness have been overcome, valuable spectroscopic data in regard to "old" suns.

In the work about to be described five independent photographs of the Sirius region were employed. They were taken respectively on February 24 and 26, 1897, February 15, 1915, February 26, 1916, and March 23, 1917. The observational work on each plate consisted of finding by means of a measuring machine the coordinates in millimeters of twenty stars, taken along axes parallel to declination and right ascension circles, called the x-axis and y-axis respectively. The measured stars included the one suspected of proper motion, Sirius, and nine others found in the A.G. Washington catalog which were near enough to be available for the determination of absolute position. The first four of the plates mentioned were employed for this purpose. Then, as the plan took on a more ambitious scope, I repeated the process, substituting for the 1916 plate the one taken in 1917 by myself. in order to get the full benefit of the elapsed time since 1897, and measuring more than one hundred faint stars in the immediate neighborhood of Sirius. The field thus examined intensively was rectangular in shape and lay between right ascension circles 16 hr. 39 m. 30 s. and 16 hr. 42 m. 0 s., and declination circles -16° 25' and -16° 45'.

The plates were then all reduced to a single plate by use of the formulae (using X and Y to indicate the x and y coordinates of a single star)

$$a + Xb + Yc + M_x = 0, \quad \text{and}$$

$$a + Xb + Yc + M_y = 0,$$

in which, when M_x is the difference between the x coordinates of a given star on the two plates compared, a is the difference in x of the respective plate centers, b is a constant factor of X due to any difference in scale which the two plates may have, and c is a constant expressing the angular difference of orientation. These constants automatically take care of variations in the plates due to precession, nutation, and differential refraction. The formula for transformation in declination is, as shown, precisely similar in form to that for right ascension, but the b and c, while keeping their original position, interchange functions, the former now standing for orientation error and the latter for scale error. This arrangement makes it possible to shorten the labor of reduction by writing the two formulae together thus

$$a + Xb + Yc + M = 0 \quad \ldots + M' = 0$$

since the coefficients of the unknown quantities a, b, and c are in each case the same. In order to facilitate the computation, the twenty-eight comparison stars used in the reduction were so chosen as to make the sums of their plus and minus coordinates in both right ascension and declination approximately equal to zero, and to make the number of positive and negative signs exactly equal in both x and y. The equations thus formed were easily reducible to three equations in the three unknowns by adding them first with all signs unchanged, and then with the signs of the proper equations changed to give the same signs successively to all the X's and all the Y's. By this means the constant a was eliminated from the last two equations, and the coordinates of the other two unknowns were unequally balanced, giving to their determination an accuracy approximating that of the least square solution.

After the four plates first measured had been reduced to a common one in this way, and the average coordinates of the stars common to both sets of measurements obtained, the same formulae were used to reduce these average coordinates of the first set to those of the second. Since not enough stars common to both groups were available to make the abridged solution described above sufficiently accurate, the standard least square solution was employed. In this way the average coordinates of all the stars measured were obtained, referred to a single plate. The constants of reduction from this plate to the sky were then found as follows: The X and Y coordinates of the measured stars having known positions were reduced to time and arc respectively, by multiplying the former by $(59.8/15 \sec.d_0)^s$ and the latter by $(59.8)''$, d_0 being the estimated declination of the center of the plate, and $(59.8)''$ the approximate value of one millimeter of scale. These values were reduced to the sky by means of the usual tables for this purpose, and compared with the values obtained by subtracting the estimated right ascension and declination of the center of the plate from the catalog positions of the stars. The differences constitute the M's in equations of the type given above, and the least square solution of these equations gives the constant errors of scale, orientation, and estimated position of the plate center. These constants, applied to any pair of measured coordinates expressed in time and arc, give, when the tables of correction from plate to sky are subsequently applied, the celestial coordinates of the star in question. The requisite data for the reduction of each of the stars measured is given below.

Those stars which gave evidence of proper motion were reduced to their probable position in 1900 by means of the formula

$$Z_0 + ut - Z = 0,$$

in which t is the time interval in years between 1900 and the date of the photograph, u is the annual proper motion, Z is the measured coordinate in x or y, and Z_0 is the corresponding coordinate for 1900. The five dates on which the measured photographs were taken made possible five observation equations, weighted according to the number and quality of observations from which the average coordinates for each date were derived. These equations were solved by the method of least squares, and the unknown quantities u and Z_0, together with their probable errors, were thus obtained for each proper motion star. The value of u thus obtained for Sirius, which was included among these stars as a check upon the method, agreed as well as might be expected with the extremely accurate value known.

The positive results of the investigation comprised the determination of the position and proper motion of three stars besides Sirius, including one faint catalog star (2243 A.G.Washington). The most important negative result was the failure to find a star having the motion of Sirius itself. This negative result was, however, vitiated to some extent by the "halation" effect, or darkening of the plate about Sirius, which obscured many of the faint stars near the brighter object; but this effect applies only to such an immediate neighborhood of the much-observed star that any objects within it would probably have already been measured directly with the micrometer eye-piece.

Results.

In designating the stars I shall keep the numbers employed throughout the computations, except when another designation is clearly more desirable. The numbers greater than 2200 are from the A.A. Washington Catalog. The decimal place of the probable error is determined by the last place of the proper motion value.

Proper Motion Stars.

Star	R.A.	u	Dec.	u	Magn.
I	$6^h 40^m 29^s.92$	$+0^s.0122 \pm 8$	$-16° 25' 3''.1$	$+0''.024 \pm 12$	10.5†
II	6 40 15.24	+0.0004 ± 4	-16 39 36.4	+0.065 ± 8	9.5†
2243	6 40 30.74*	+0.0002 ± 3	-16 41 36.0*	-0.046 ± 7	8.8
S.	6 40 44.60*	-0.0383 ± 7*	-16 34 43.0*	-1.190 ± 7*	-2.0

* My own values.
† Estimated by myself from micrometical measurements of the image-diameters.

Residuals For Computed Positions of Catalog Stars.

	2258	2238	2234	2232	2253	2264	2249	2250	2243	2245(S)
R.A.	$0^s.00$	-.07	+.02	+.05	-.04	+.05	-.05	+.06	+.02	-.01
Dec.	$+0''.7$	+0.3	-0.3	-0.1	+0.4	-0.8	+0.1	+0.3	+0.3	+0.6

Coordinates of Measured Stars.

In this list the columns labeled 1897, 1916, etc., contain the deviations of the corresponding plate values from the average values on the left. The sum of these deviations is not zero in most cases because in the second set of measurements the plates called 1897 and 1897' were weighted 2/3 and 4/3 respectively. The deviations are given in order to throw light upon any proper motions which might later appear, and also to give a general idea of the accuracy of the work. When the deviations are large the cause may usually appears in the data either in the form of low weight for the observation or of one large discrepant value, like the first value in 87 Y. The column labeled "W" gives the weight of the coordinate values estimated on a scale of ten, 1 indicating an extremely poor image for measuring and 10 a perfect image. The column labeled "Magn." contains a rough estimate of the magnitudes. These may have rather large systematic errors in the higher numbers; but they are probably fairly consistent.

Constants for Reduction to Sky.

R.A. a -0.00235 b +0.00245 c -0.00696 Center $6^h 40^m 47^s.28$

Dec. a +1.0954 b +0.1808 c +0.1065 Center $-16^\circ 34' 52''.9$

No. Stars Common to Both Groups

No.	Av. X (mm.)	Deviations (1st group) (mm./10,000)				Deviations (2gr.) (mm./10,000)				Magn.
		1897	1897'	1915	1916	1897	1897'	1915	1917	
2238	-8.7034	- 26	-58	-19	+98	- 13	+15	+23	-11	8.4
45	-7.0819	- 7	+23	+63	-68	- 42	+97	+16	-40	9.5
1	+0.1334	- 25	+98	-12	+ 6	- 3	+13	-68	- 8	10.0
2	+0.9186	- 10	+10	+ 1	+14	+ 18	- 5	0	-32	10.5
3	-4.0095	- 24	-18	+ 9	-11	- 17	+10	+23	+36	10.0
8	-4.3660	- 5	-36	+ 7	+30	+ 19	- 4	-26	+ 4	10.0
9	+8.5341	- 14	- 7	- 2	-23	+ 26	+ 2	+25	-17	10.0

No.	Av. Y									W
2238	-2.6958	-28	- 7	+13	+47	-24	+13	+ 4	-10	8
45	+9.5129	+15	-90	+80	-83	+10	-68	+12	+38	7
1	-1.0634	-26	-44	+97	+ 8	-47	+10	+70	-49	9
2	-1.0711	-25	0	+26	- 4	- 4	+ 2	+15	- 6	9
3	-8.4672	-16	+23	-15	-38	- 6	+41	- 7	+31	9
8	+6.3191	+41	0	-11	-23	+34	+53	-52	-39	9
9	+7.1150	- 2	- 7	+26	-24	-19	+25	+ 6	+10	9

Remaining Stars.

First Group.

No.	Av. X	Deviations (mm./10,000)				Magn.	W.
		1897	1897'	1915	1916		
2258	+12.5355	0	+ 49	- 91	+ 42	8.5	7
2234	-21.5118	+128	-149	+ 84	-63	9.2	6

2232	-22.8205	+ 63	-300	+198	+ 40	7.8	6
2253	+ 9.2835	- 18	+ 68	- 44	- 12	9.2	8
2264	+19.1720	- 58	+165	- 78	- 35	9.2	7
2249	+ 5.0192	+ 18	+109	- 80	- 46	9.1	7
2250	+ 5.7435	+ 2	+165	-126	- 42	9.2	7
III	-11.1563	- 18	- 63	+ 19	+ 63	10.0	8
IIII	-11.1074	- 16	- 63	+ 61	+ 17	10.0	8
	Av.Y.						
2258	+12.2964	+101	- 24	- 43	- 35		
2234	-21.5465	- 6	- 97	+ 25	+ 77		
2282	+38.0171	- 90	- 48	+ 85	+ 49		
2253	+45.0099	- 82	+ 45	- 14	+ 51		
2264	+35.8198	- 14	+ 84	- 45	- 26		
2249	-12.2261	- 2	+ 13	- 11	0		
2250	-14.1853	+ 6	+ 95	- 89	- 12		
III	- 1.9591	+ 3	- 10	+ 7	+ 2		
IIII	+ 3.6876	+ 29	- 55	+ 76	- 50		

Second Group.

No.	Av.X	Deviations 1897[1]	1897[2]	1915	1917	Av.Y	Deviations 1897[1]	1897[2]	1915	1917	Magn.	W
2	+ 0.0187	+ 23	- 45			- 4.6746	- 3	+ 6			12.0	1
3	-0.4348	- 33	+ 65			- 6.1434	- 33	+ 64			13.0	1
4	-0.4762	+ 22	- 47	+ 2		-11.5700	+ 12	- 22	- 2		13.0	6
5	-0.5074	+ 52	-103			+ 4.1983	- 7	- 16			13.0	1
6	-0.1332	- 25	+ 50			+ 4.4137	+ 44	- 90			13.0	2
7	+0.0663	- 16	- 52		+ 75	+ 3.2917	- 37	+ 11		+ 41	11.5	5
8	--0.2162	+ 11	- 28	+ 12	- 6	+10.2127	- 3	+ 54	+ 35	- 66	11.5	6
9	-1.6641	+ 15	- 12	- 92	+ 81	+17.3321	- 98	+ 18	- 38	+136	11.5	4
10	-1.2199	- 33	+270	-118	- 17	+15.6754	+ 33	+ 16	- 9	- 45	12.0	1
11	-1.[illegible]135	- 15	+ 14	+ 4	+ 9	+12.3834	+ 9	+ 40	+ 18	- 59	10.5	7
12	-0.9711	+ 3	- 34	+117	- 98	+ 8.4832	+ 78	- 18	- 77	- 14	11.0	3
13	-1.2832	+ 13	+ 10		- 25	+ 4.6035	- 46	+ 27		+ 42	13.0	4
14	-2.1710:	+ 12	0	- 8	- 6	-15.3695	+ 10	- 96	+ 70	- 20	10.0	7
15	-1.9974	- 7	+ 14			+ 4.4846	+ 6	- 10			12.5	3
16	-2.0705	- 2	+ 9	+ 15	- 18	+11.7924	- 6	- 10	+ 9	+ 4	9.5	9
17	-3.3955	+ 37	- 67	+ 5	- 12	+21.6852	- 14	+ 75	- 9	- 20	10.0	7
18	-3.2744	+ 72	+ 47	- 55	- 72	+19.3850	+ 18	+197	-101	- 52	11.5	3
19	-3.3693	+ 25	- 48			+ 0.4789	+ 41	- 84			12.5	2
20	-3.0458	- 20	- 19	+ 25	+ 16	-5.1402	- 27	+ 50	- 25	+ 28	9.5	8
21	-3.6225	+ 12	- 16	- 11	+ 9	- 9.4349	+ 18	- 1	+ 66	- 88	10.5	6
22	-3.6373	+ 17	+ 23	- 8	- 29	-12.3555	- 4	- 38	+ 29	+ 1	11.0	6
25	-4.2459	- 35	+ 68			- 2.1110	- 12	+ 22			13.0	1
27	-4.1481	- 10	- 38	+ 46	- 8	+12.1156	+ 45	- 25	- 14	- 28	9.5	8

[1] Weighted 4/3
[2] Weighted 2/3

No.	Av. X	Deviations				Av. Y	Deviations				Magn.	W
30	-5.3880	-31	-81	+136	-73	- 5.7739	+22	+36	-85	+35	12.5	2
31	-6.2082	+ 8	-24	+26	-19	-12.4 668	-25	+8	+ 2	+28	9.5	8
32	-5.9030	-33	+66	- 6	+ 7	- 9.6511	+ 8	-40	-18	+34	10.0	7
33	-5.9509	- 8	+42	+22	- 5	-9.5628	- 4		+ 2	+ 4	13.0	3
34	-5.9643	+ 5	-10			- 9.1049	-10	+38			13.0	1
36	-6.3303	+43	-84			- 0.3593	+ 3	- 8			13.0	1
37	-6.2171	+46	+29	-148	+66	+ 0.8357	-48	+33	+18	+16	12.0	2
38	-6.2357	-30	-63	+67	+19	+ 6.5555	- 4	+54	+ 2	-34	11.5	4
39	-6.5836	- 4		+37	-27	+ 6.3157	- 5		+ 1	+ 9	10.5	8
42	-7.0308	+10	+56	-58	+ 6	- 0.1557	0	+89	-44	-13	13.0	1
43	-6.9065	+36	-72			+ 0.4167	-10	+22			13.0	1
44	-7.6358	- 3	- 4	+18	-18	+ 3.7383	+63	+61	- 7	-119	11.0	7
46	-8.3409	+15	+30	+37	-77	+13.6983	+117	-49	+45	-170	12.0	4
47	-8.4961	-48	-86	+26	+94	+10.5546	-42	+22		+41	12.0	1
48	-8.5340	-75	-113	+193	-20	+10.3849	-56	+22	+28	+24	12.0	3
49	-8.0248	+ 9	- 4	+17	-24	+ 9.3633	-23	+19	+22	- 4	11.0	6
50	-7.9850	+ 9	+35	- 5	-29	+ 6.5496	+ 6	+23	-10	-13	10.5	7
52	-7.9440	0	- 1			- 5.9094	-10	+81			11.5	5
53	+0.6221	-28	+50	- 9	+12	-11.5717	+ 6	- 1	-35	+27	11.0	6
54	+0.5208	- 6	+22	+56	-44	-10.4895	-10		+14	+12	10.0	7
56	+0.6620	-19	-28		+43	- 6.6216	-82	-37		+135	10.5	2
57	+0.4564	+20	-16		-15	- 1.9424	-46	- 6		+65	13.0	1
58	+0.3479	+12	-37		+ 9	- 1.9603	+42	+26		-74	13.0	4
59	+0.6033	-32	+135		-46	+0.0673	-30	-12		+50	12.0	3
60	+0.3380	-11		+52	-30	+30.4737	- 6		+56	-45	11.0	7

No.	Av. X	Deviations				Av. Y	Deviations				Magn.	W
61	+1.3216	+ 9	-45	+24	- 7	+13.3733	+ 3	+47	+ 7	-45	11.0	7
62	+1.6013	-63	+10		+75	+ 3.4423	-39	-14		+32	12.5	4
63	+2.0672	- 5	+ 8			+ 2.3562	-12	+26			12.5	5
64	+2.0372	- 9	-27	+64	-45	-12.3800	+42	-39	+ 2	-31	12.0	4
65	+3.1037	-50	+25	+63	-13	+13.3775	- 3	-48	+36	+14	11.5	7
66	+3.4161	- 5	+14		- 4	+14.6123	-68	-29	-31	+129	12.0	1
67	+3.4031	-62	+52	-74	+100	+12.2332	+73	+60	-138	- 5	11.0	6
69	+3.4650	-26	-12	+26	+12	+ 8.1776	-21	+ 9	+21	+16	10.0	6
71	+3.3451	+46	-18		-50	+ 2.7702	-71	-59		+135	12.0	1
72	+3.3305	-33	-12	+60		+ 1.6604	0	+19	-11		13.0	2
73	+3.2714	+ 2	- 3			+ 0.8444	-43	+37			12.0	1
74	+3.4766	-87	- 4	+102	-11	- 3.6617	+31	+47	-122	+52	12.0	6
75	+5.0253	+56	+20	-70	-16	-12.2864	-38	+58	+31	-10	8.5	9
76	+4.4923	+27	+23	+113	-165	+22.8424	+24	+40	+43	-100	13.0	1
77	+4.7888	+3	-32	+58	-28	+ 8.5526	+25	-86	- 1	+25	12.5	5
78	+5.0193	+26	+17	-14	+22	+14.7729	+ 6	0	-61	+41	11.0	7
79	+4.2100	-18	+46	-87	+55	+14.5527	+37	+38	-49	-24	10.0	8
80	+4.9540	+10	+39	-126	+83	+10.6043	+60	+80	-199	+56	11.5	4
81	+5.3727	+29	-29			+12.5518	+ 6	-14			12.5	1
83	+5.3780	+10	-10	+ 1	-11	+ 9.8258	+15	-54	+21	- 5	9.5	8
84	+6.0222	-49	+18	+33	+21	+ 7.0040	+29	-76	- 5	+17	11.5	4
85	+5.8560	+ 7	- 2	+12	-17	+ 2.6993	- 9	+ 3	+12	- 1	10.0	7
86	+5.8070	-29	+ 9	+36	- 3	- 1.3316	-16	-16	+26	+ 6	12.0	4
87	+5.9175	+40	-24	- 1	-26	- 2.7434	+113	-52	-84	-60	11.5	4

No.	Av. X	Deviations				Av. Y	Deviations				Magn.	W
89	+7.0594	-18	+ 2		+24	- 2.0975	-49	-128	+124	+26	13.0	3
90	+6.7760	+17	-46	+ 2	+ 6	- 0.2924	+18	+14	-35	+ 2	11.0	7
91	+6.9308	+19	+12	-30	- 4	+ 3.8719	+55	+22	-38	-53	11.5	6
92	+6.5097	+1	+60	-29	- 9	+11.5975	+ 1	+17	+ 3	-14	11.0	6
93	+6.7731	-17	+57	+21	-29	+14.8769	+24	-16	- 5	-17	9.5	7
94	+6.9398	-15	+ 2	- 2	+21	+18.3745	+26	+67	-13	-66	11.5	7
95	+6.4782	+ 6	+12	-20	+ 5	+20.8023	0	+ 3	+ 1	- 2	10.0	8
96	+7.8595	-16	+47	+17	-27	+12.2056	+19	+13	-32	- 1	11.0	6
97	+7.5666	-14	-14	- 9	+36	- 8.0101	-13	-16	+24	+ 5	9.5	7
98	+7.3162	+40	-31	-39	+ 5	+ 0.0005	-16	-52	- 4	+61	10.0	8
99	+9.0304	+ 7	-13	-38	+36	- 3.0406	-44	+24	+22	+20	11.5	5
100	+8.2073	-29	+10	-50	+80	- 5.5006	+11	- 2	-13	+ 1	11.0	7
101	+8.7244	+ 4	-42	+32	-11	+13.7361	-14	-22	-44	+78	11.0	5
102	+8.6690	-15	+28	-20	+21	+15.5249	-24	-40	+27	+30	10.5	6
103	+8.9680	+10	+20	-46	+21	+13.8302	+15	-62	+24	- 3	11.0	6
104	+10.1430	+27	+27	-43	-12	+ 6.9149	+ 8	-52	+11	+15	9.5	8
105	+9.8775	+ 1	+ 2	- 1	- 1	+ 1.4018	+23	-39	- 4	- 1	10.5	7
106	+9.8282	+17	-21	-21	+10	- 1.0177	+10	-22	+50	-49	10.0	8

Summary

Micrometical measurements of about one hundred and twenty faint stars in the neighborhood of Sirius resulted in the discovery of three stars having certain proper motion, one of which moves nearly two-tenths of a second of arc a year in right ascension. No star was found with a proper motion equal to or appreaching that of Sirius, and no variables were detected. All stars down to about the twelfth magnitude were measured.

In conclusion, I wish to acknowledge my indebtedness to Professor F.P. Leavenworth not only for general oversight of the work, but also for many practical suggestions born of experience which materially lessened the labor of computation. I also received some helpful suggestions from Assistant Astronomer William O. Beal.