12263
32

THE TEMPERATURE COEFFICIENTS OF THE MOVING COIL GALVANOMETER.

A THESIS

Submitted to the Faculty of the

Graduate School

of the

University.

May 20, 1909.

by

OLAF HOVDA.

In Partial Fulfillment of the Requirements

for the degree of Master of Arts.

[Reprinted from the PHYSICAL REVIEW, Vol. XXVIII., No. 4, April, 1909.]

THE TEMPERATURE COEFFICIENTS OF THE MOVING COIL GALVANOMETER.

BY ANTHONY ZELENY AND O. HOVDA.

THE increasing use of the moving coil galvanometer for deflection work of precision renders a knowledge of its temperature coefficients desirable. The coefficients, however, differ greatly in different instruments so that no values can be determined which are applicable in general with any high degree of accuracy, but their nature and order of magnitude can be learned from the values that have been obtained for one type of the instrument.

As no similar determinations appear to have been published, the method employed in obtaining the values of the coefficients is described and the deductions are given at some length. This is done to show also some peculiarities of the instrument, which may not be generally known.

All observations were made on two galvanometers[1] having chilled cast-iron magnets.

DEC 2 - 1909

§ 1. THE TEMPERATURE COEFFICIENT FOR CURRENT MEASUREMENTS.

The diagram of the apparatus employed for determining the temperature coefficient for current measurements is shown in Fig. 1. The galvanometer G was inclosed in a double walled jacket J, having a small window W for the taking of observations. The constancy of the temperature of the water circulating between the walls of the jacket was maintained automatically. The air surrounding the galvanometer was kept in motion by two fans operated by a small motor, and the temperature of the galvanometer was determined by means of a mercury thermometer. To insure that variations in the results were not due to manipulation in obtaining a constant current, two independent circuit arrangements were

[1] Leeds and Northrup Co., "P Type."

78085

employed as shown in the diagrams A and B of Fig. 1. These were connected to the galvanometer, in succession, by means of the commutator K_1. The current flowing through the galvanometer was maintained constant during each set of readings for all the observations taken at the various temperatures. When the circuit B was employed, the resistance R_5 was 500,000 ohms when the upper suspension was of phosphor bronze and 300,000 ohms when it was of steel. This high resistance rendered negligible the change in

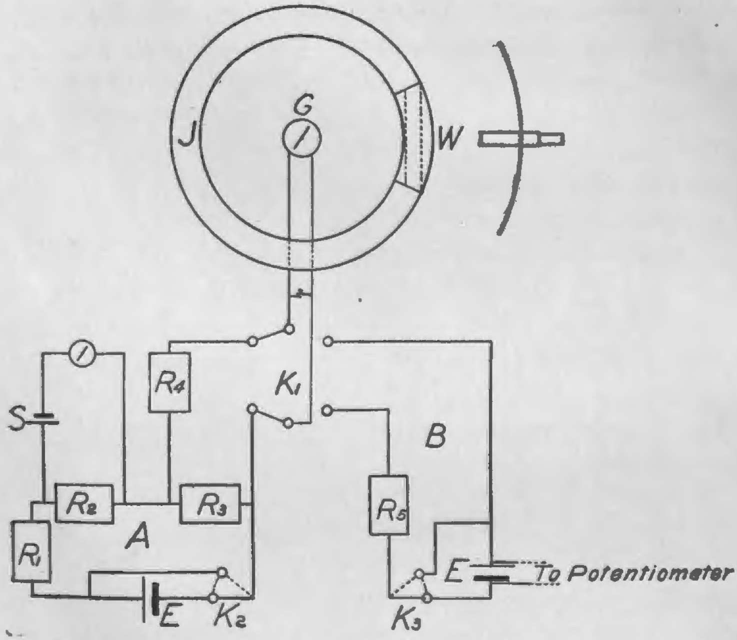

Fig. 1.

resistance of the galvanometer. When the circuit A was employed, the constancy of the main current was maintained by means of the resistance R_1 and the secondary circuit containing the cadmium cell S, while the resistance of the galvanometer branch was maintained constant by altering the resistance in the box R_4 to correspond to the known change in the resistance of the galvanometer.

The same galvanometer coil and lower phosphor-bronze spiral were employed in all determinations, while the upper suspension was either of phosphor bronze or of steel.

In the preliminary determinations it was found that between 0° and 60° C. the results obtained for the temperature coefficient were, to the limit of accuracy attained, independent of the temperatures employed.

The effect of thermo-electric currents was eliminated by taking the null readings while the keys K_2 or K_3 were turned so as to cut out the battery and leave the galvanometer circuit closed.

The observations and results obtained, when using a 3 mil phosphor-bronze suspension 13 centimeters in length, are given in Tables I. and II. The observations were taken on a circular scale at a distance of 50 centimeters.

TABLE I.

Galvanometer No. 6788. Phosphor-Bronze Suspension.

Temperatures.		Circuit.	Deflections at		Difference.	Temperature Coefficient for Deflections.
t'	t''		t'	t''		
5°.1	54°.7	*A*	21.748	21.909	+0.161	+0.00015
5.2	54.7	*B*	18.539	18.634	0.095	.00010
54.7	5.2	*A*	21.894	21.748	0.146	.00014
54.7	5.1	*B*	18.631	18.532	0.099	.00010
4.9	54.4	*A*	18.454	18.590	0.136	.00015
5.0	54.4	*B*	18.420	18.545	0.125	.00014
54.4	5.0	*A*	18.593	18.472	0.121	.00013
55.4	5.0	*B*	18.560	18.430	0.130	.00014
						+0.00013

TABLE II.

Galvanometer No. 6737. Phosphor-Bronze Suspension.

Temperatures.		Circuit.	Deflections at		Difference.	Temperature Coefficient for Deflections.
t'	t''		t'	t''		
56°.5	5°.0	*A*	17.850	17.680	+0.170	+0.00019
56.4	5.0	*B*	17.959	17.743	0.216	.00024
34.7	14.8	*A*	17.791	17.717	0.074	.00021
34.7	14.8	*B*	17.842	17.749	0.093	.00026
18.8	44.2	*A*	17.738	17.853	0.115	.00026
18.8	44.2	*B*	17.745	17.855	0.110	.00025
44.3	20.6	*A*	17.861	17.775	0.086	.00020
44.2	20.5	*B*	17.867	17.781	0.086	.00020
						+0.00023

The average of the above two values is assumed to be the approximate temperature coefficient for this type of galvanometer when its upper suspension is a 3 mil phosphor bronze strip. The expression for the relation of the deflections for equal currents at different temperatures then may be written

$$d_t' = d_{20}' \left[1 + 0.00018\,(t - 20)\right]. \qquad (1)$$

In Table III. are given the observations and results obtained for galvanometer No. 6788 when the upper suspension was a steel strip. The figure of merit at 20° was now 5.48×10^{-7} in place of 2.26×10^{-7} for the phosphor-bronze strip.

TABLE III.

Galvanometer No. 6788. Steel Suspension.

Temperatures.		Circuit.	Deflections at		Difference.	Temperature Coefficient for Deflections.
t'	t''		t'	t''		
2.°7	53.°7	*A*	12.702	12.706	+0.004	+0.00001
2.5	53.9	*B*	12.695	12.692	−0.003	−0.00001
53.7	3.5	*A*	12.672	12.671	+0.001	.00000
53.9	3.6	*B*	12.679	12.677	+0.002	.00000
						0.00000

This shows that the temperature coefficient for this galvanometer with a steel suspension is practically negligible. See also calculated value of the coefficient for galvanometer No. 6737, in Table VIII, which makes the average value for the two galvanometers + 0.0000

SOURCES OF ERROR.

The large variation in the individual results obtained when using the phosphor-bronze suspension are due to several causes :

1. *Zero Shift.* — On changing the temperature, the null reading changed by an amount varying from two to ten millimeters. This introduced an error of unknown magnitude due to the change of position of the coil within the field and to the eccentricity of the scale. Precautions were taken to minimize this error as far as possible.

2. The assumption, based on the preliminary experiments, that the coefficient is the same at all temperatures is not strictly correct.

This is inferred from the variation of the temperature coefficient of the magnetic field, shown in Table VI.

3. *Magnetic Hysteresis*[1] *Due to Magnetic Impurities in the Coil.* — Errors due to this were eliminated as far as possible by short-circuiting the galvanometer on the return of the coil to the null reading so as to render it aperiodic; also, while deflecting, the coil was damped by means of rapid tapping of the short-circuiting key so that it did not move beyond the point of the permanent deflection. See also § 5.

4. Molecular changes[2] within the suspension fiber may have changed the torsional moment with time.

5. The lag in the field strength of the magnet was eliminated by keeping the galvanometer at a constant temperature for about three hours before the observations were taken.

§ 2. Temperature Coefficient for Potential Measurements.

The temperature coefficient for potential measurements is determined from the deflection coefficient for current measurements and the resistance coefficient of the galvanometer circuit. The latter must be known for each particular case, for the galvanometer resistance is only a fraction of the total.

To enable the calculation of the resistance coefficient of galvanometers to be made, the coefficients of the suspension fibers, spirals, and the coil, were determined separately.

The following resistance equations were obtained:

For phosphor-bronze spirals or suspensions,

$$R_t = R_{20}\,[1 + 0.00063\,(t - 20)]. \quad (2)$$

For steel suspensions, $$R_t = R_{20}\,[1 + 0.00356\,(t - 20)]. \quad (3)$$

For the galvanometer coil, $$R_t = R_{20}\,[1 + 0.00394\,(t - 20)]. \quad (4)$$

In the galvanometer employed, the resistence of the upper phosphor-bronze suspension was 3.34 ohms at 20° C., and of the spiral, 17.25 ohms; that of the steel suspension was 5.71 ohms, and of the galvanometer coil, 96 ohms.

[1] A. Zeleny, Phys. Rev., Vol. 23, p. 400, 1906.

[2] K. E. Guthe, Bulletin of the Bureau of Standards, Vol. 2, p. 53, 1906.

Representing the resistance coefficient of the whole circuit by B, the deflections for equal differences of potential for the above galvanometer are derived from the following equations:

With phosphor-bronze suspension,

$$d_t = d_{20}[1 + (0.00018 - B)(t - 20)]. \quad (5)$$

With steel suspension, $d_t = d_{20}[1 + (0.00005 - B)(t - 20)]$. (6)

§ 3. TEMPERATURE COEFFICIENT FOR BALLISTIC MEASUREMENTS.

The temperature coefficient for ballistic throws was obtained by discharging through the galvanometer, at different temperatures, equal quantities of electricity from a mica condenser charged by means of cadmium cells. The observations and results are given in Table IV.

TABLE IV.

Galvanometer.	Suspension.	Temperatures.		Throws at		Coefficient for Ballistic Throws.
		t'	t''	t'	t''	
No. 6788	Phosphor-bronze.	54°.0	4°.0	18.491	18.708	−0.00023
		54.2	5.3	18.082	18.254	−0.00019
						−0.00021
No. 6788	Steel.	2.6	53.8	12.310	12.179	−0.00021
		53.8	4.9	12.179	12.322	−0.00024
						−0.00022
No. 6737	Phosphor-bronze.					−0.00012[1]

The value of the coefficient in the cases given is practically independent of the kind of suspension, but varies with the galvanometer employed. See also Table IX. The approximate temperature coefficients for ballistic measurements for this type of galvanometer having either a phosphor-bronze or a steel suspension, is assumed to be the average of the above values, and the relation of the ballistic throws for equal quantities at different temperatures is expressed by

$$d_t = d_{20}[1 - 0.00017(t - 20)]. \quad (7)$$

§ 4. CALCULATION OF THE TEMPERATURE COEFFICIENTS FROM THE COEFFICIENTS OF THE VARIOUS PARTS OF THE GALVANOMETER.

The temperature coefficients of a moving coil galvanometer are due to the variation of several quantities with temperature. The

[1] The average of six independent sets of observations.

temperature coefficients of these quantities may be determined separately, and, from the relation of the temperature coefficients of the galvanometer to these coefficients, the former may be calculated from the latter. A comparison of the values obtained directly by experiment with those thus calculated gives an index to their reliability and shows the magnitude of the effect produced by the various causes which combined to produce errors in the observed results.

It was necessary to determine the temperature coefficients for the strength of the magnetic field and for the period of vibration of the coil system. The values of the expansion coefficients of the copper in the coil and of the iron in the magnet were taken from tables.

(*a*) *The Temperature Coefficient for the Strength of the Magnetic Field.* — To determine the variation in the strength of the magnetic field with the temperature, the coil was removed from the galvanometer, and the magnet and iron core were set forward from the wooden supporting frame a distance of about one centimeter. Two rectangular coils, each consisting of 45 turns of silk-covered manganin wire, were made to slide one on each side of the iron core, and were of such size and so placed that when withdrawn each loop cut all the lines of force once. These coils were connected in series with a ballistic galvanometer through the secondary of a standard mutual inductance coil,[1] and the number of lines in the field, at different temperatures, was determined by comparison with this standard whose coefficient of mutual induction was 0.017653 henry. The current in the primary of the standard coil was measured by

TABLE V.

Current in Primary of Standard Coil.	Throw from Standard Coil.	Throw Due to Cutting Field of Magnet.	Temperature of Magnet. Degrees.	Number of Lines of Force in the Field.
0.16669	18.784	19.642	2.9	3419.2
0.16702	18.837	19.575	14.8	3404.8
0.16655	18.782	19.524	20.9	3396.2
0.16652	18.768	19.350	38.8	3367.7
0.16590	18.672	19.130	60.6	3334.2

[1] A. Zeleny, PHYS. REV., Vol. 23, p. 411, 1906; E. B. Rosa, PHYS. REV., Vol. 24, p. 241, 1907.

means of a potentiometer and standard resistance. The observations and results for the magnet of galvanometer No. 6788 are given in Table V.

These results are plotted in Fig. 2.

The average temperature coefficients for various ranges, as determined from the curve, are given in Table VI. The temperature coefficients for the field strength were obtained from the temperature coefficients for the total number of lines and the expansion of the pole faces of the magnet.

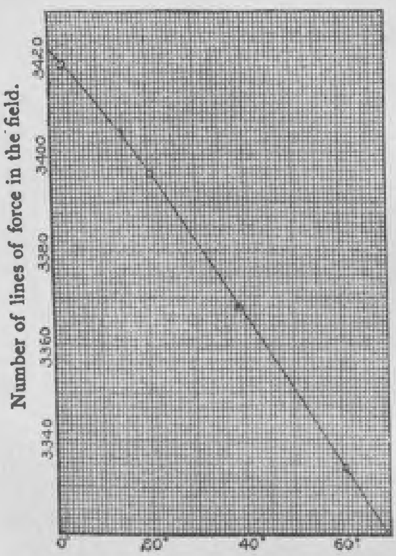

Fig. 2.

The temperature coefficient for the field strength of galvanometer No. 6737 between 15° and 30° C. was found to be − 0.00035, the total number of lines at 20° C. being 3,172. The area of each pole face was 8 sq. cm.

(*b*) *Temperature Coefficient for the Time of Vibration of the Coil.* — The temperature coefficient for the time of vibration was obtained with the coil both outside and within the field of the magnet. The results are given in Table VII.

TABLE VI.

Range of Temperature.	Temperature Coefficient for the Number of Lines.	Temperature Coefficient for the Field Strength.
0° to 20° C.	−0.00037	−0.00039
20° to 40°	−0.00046	−0.00048
40° to 60°	−0.00046	−0.00048
0° to 60°	−0.00043	−0.00045
15° to 30°	−0.00044	−0.00046

The difference in the values of the coefficient, within and without the field, is due to the presence of magnetic impurities within the coil.

TABLE VII.

Galvanometer.	Suspension.	Coil Outside of Magnetic Field.	Coil Within the Magnetic Field.
No. 6788	Phos. bronze.	+0.00027	+0.00032
"	Steel.	+0.00021	+0.00023
No. 6737	Phos. bronze.		+0.00031

(*c*) *Calculation of the Coefficient for Current Measurements.*—From the equation for the time of vibration of the coil

$$t = 2\pi\sqrt{\frac{I}{T+M}},$$

$$\frac{(T+M)_t}{(T+M)_{20}} = \frac{t_{20}^2 I_t}{t_t^2 I_{20}} \equiv \frac{t_{20}^2 D_t^2}{t_t^2 D_{20}^2}, \tag{8}$$

where T, M, t, I, D, are respectively the torsional moment, magnetic moment due to impurities, period of vibration, moment of inertia, and the width of the coil. The subscripts represent temperatures.

Assuming the field to be practically uniform, the relation between the current and the deflection is expressed by

$$\tfrac{1}{10} FLnDi \cos\theta \equiv (T+M')\,\theta = (T+M')\,kd', \tag{9}$$

where F, i, d', M', represent respectively, the strength of the magnetic field, the current, the deflection, and the moment due to the magnetic impurities, which may differ somewhat from M. The effective length L of the coil does not vary with the expansion of the copper, for both ends of the coil extend beyond the field of the magnet, but does vary with the linear expansion of the iron of the magnet.

From equations (8) and (9), assuming $(T+M) \equiv (T+M')$,

$$\frac{d_t'}{d_{20}'} \equiv \frac{D_t F_t L_t (T+M)_{20}}{D_{20} F_{20} L_{20} (T+M)_t} = \frac{F_t t_t^2 L_t D_{20}}{F_{20} t_{20}^2 L_{20} D_t}, \tag{10}$$

from which

$$d_k' \equiv F_k + 2t_k + L_k - D_k, \tag{11}$$

where d'_k, F_k, t_k, L_k, D_k are the temperature coefficients respectively for the deflections, field strength, time of vibration, linear expansion of cast iron and that of copper.

The calculations and comparisons are made in Table VIII.

Table VIII.

Galvanometer.	Suspension.	F_k	$2t_k$	$L_k - D_k$	Temp. Coef. for Deflections.	
					Calculated.	Observed.
No. 6788	Phos. bronze.	−0.00045	+0.00064	−0.00001	+0.00018	+0.00013
No. 6737	" "	−0.00035	+0.00062	−0.00001	+0.00026	+0.00023
					+0.00022	+0.00018
No. 6788	Steel.	−0.00045	+0.00046	−0.00001	0.00000	0.00000
No. 6737	"	−0.00035	(+0.00046)	−0.00001	+0.00010	
					+0.00005	

(*d*) *Calculation of the Coefficient for Ballistic Throws.* — From the equation for the constant for ballistic measurements, $K = K't\sqrt{\rho}/2\pi$, the relation between the deflections for equal quantities of different temperatures is, for open circuit work where $\sqrt{\rho}$ is small, expressed by

$$\frac{d_t}{d_{20}} = \frac{K_{20}}{K_t} = \frac{K'_{20}t_{20}}{K'_t t_t} = \frac{d'_t t_{20}}{d'_{20} t_t}, \tag{12}$$

where d, d', and t, represent ballistic throws, deflections for continuous currents, and periods of vibration, at the temperatures t° and 20° C. From this

$$d_k \equiv d'_k - t_k, \tag{13}$$

where d_k, d'_k, and t_k, represent the temperature coefficients respectively for the ballistic throws, deflections for continuous currents, and the periods of vibration. The calculations and comparisons are made in Table IX.

The agreement of the calculated with the observed values, in Tables VIII. and IX., is as close as can be expected when the sources of error given in § 1 and § 5 are considered; in addition to these sources of error it was necessary, in § 4 (*c*), to assume the magnetic moment due to the impurities in the coil to be the same whether the coil is vibrating or is deflected and at rest.

TABLE IX.

Galvanometer.	Suspension.	d_k'	$-t_h$	Temp. Coef. for Ballistic Throws.	
				Calculated.	Observed.
No. 6788	Phos. bronze.	+0.00013	−0.00032	−0.00019	−0.00021
No. 6737	" "	+0.00021	−0.00031	−0.00010	−0.00012
				−0.00015	−0.00017
No. 6788	Steel.	0.00000	−0.00023	−0.00023	−0.00022
No. 6737	"	+0.00010	−0.00023	−0.00013	
				−0.00018	

§5. Magnetic Impurities within the Coil.

1. The galvanometer coil, on open circuit, was suspended between the poles of an uncharged electromagnet, and when the latter was charged, the coil turned through a considerable angle toward parallelism with the lines of force, showing the presence of magnetic impurities.

2. A galvanometer coil, on open circuit, turns on its axis when the magnetic field of the galvanometer is short-circuited. This also shows the presence of a directive moment due to magnetic impurities in the coil. When the field is weakened, the directive moment of the impurities is lessened and the coil is allowed to return nearer to the position it would occupy if there were no impurities. In twelve galvanometers in which the fields were short-circuited, the coils turned through angles represented by deflections varying from a few tenths to several millimeters on the scale. In one case only was there no appreciable deflection.

3. The restoring moment in a vibrating galvanometer coil is produced in part by these magnetic impurities within the coil. The value of the restoring moment due to the impurities as compared with the torsional moment of the suspension was determined from the periods of vibration of the system when in the magnetic field and when outside of it. The ratio of M/T is then obtained from

$$t = 2\pi \sqrt{\frac{I}{T+M}},$$

where T and M represent respectively the torsional and magnetic moments per unit angle of displacement. The magnetic moment produced by the earth's magnetic field acting on the magnetic impurities was considered negligible. The observations and results are given in Table X.

TABLE X.

Field.	Period of Vibration.	M/T
Earth's field.	8.534″	(0.00)
Galvanometer magnet.	8.289	0.07
Strong electromagnet.	6.03	1.00

The results show how large a part the magnetic impurities play in the directive action upon the coil and how much they diminish the sensitiveness of the galvanometer.

4. The deflection, in the particular galvanometer employed, changed about one tenth of a millimeter during the first minute after the coil came to rest immediately following a previous deflection in the same direction. On account of the large influence of the magnetic impurities, this may be ascribed to the lag in the change of direction and strength of the magnetism in the magnetic impurities rather than to a change in the torsional moment or the fatigue of the suspension.

The above considerations help to explain the irregularities in the results obtained for the temperature coefficients. The hysteresis of the magnetic impurities and the effect of temperature upon them must produce disturbing effects.

§ 6. SUMMARY AND DISCUSSION.

The moving coil galvanometer has three temperature coefficients, one each for current, potential, and ballistic measurements. On account of magnetic hysteresis and other disturbing causes, it was found impossible to determine the values of these coefficients for an individual instrument with as high a degree of precision as it is possible to read deflections or throws. The values of the coefficients for different galvanometers of the same construction and materials, are found to differ considerably, so that only approximate values can be given for any type of the instrument.

The averages of the sets of values obtained for the two galvanometers are taken as the approximate temperature coefficients for all moving coil galvanometers having chilled cast-iron magnets. These values are summarized in Table XI., where B, as before, represents the temperature coefficient for the resistance of a particular circuit.

TABLE XI.

Kind of Measurement.	Suspension.	Temperature Coefficient.
Current.	Phos. bronze (3 mil).	+0.00018
"	Steel.	+0.00005
Potential.	Phos. bronze (3 mil).	+0.00018−B
"	Steel.	+0.00005−B
Ballistic.	Phos. bronze (3 mil).	−0.00017
"	Steel.	−0.00017

These values are very nearly correct for galvanometers whose magnets have a field intensity of 425 units and a temperature coefficient of − 0.00040, and in which $M/T = 0.07$ approximately.

The influence of a variation in the value of M/T upon the temperature coefficient of the galvanometer can be estimated from the difference in the temperature coefficients for the time of vibration within and without the field, given in Table VII. The value of the temperature coefficient for the time of vibration with the 3 mil phosphor-bronze fiber is 0.00005 larger within than outside the field. If there were no magnetic impurities in the coil, or, more correctly, if the amount of the magnetic impurities present was just sufficient to neutralize the effect due to the diamagnetic properties of the copper, the temperature coefficient for the time of vibration in the field of the magnet would be the same as that determined without the field. In such a case, as is seen from equation (11) and in Table VIII., the temperature coefficient for current measurements would be obtained by subtracting 0.00010 from the value given in Table XI., and the temperature coefficient for ballistic throws by adding − 0.00005 to the corresponding value.

With a 1.5 mil phosphor-bronze suspension, M/H was found to have a value of 0.9 and the temperature coefficient for the time of vibration a value of + 0.00039 in the field of the galvanometer magnet. The latter value gives for the temperature coefficient of the galvanometer for current measurements a value of + 0.00032 and for ballistic throws a value of − 0.00007.

It is inferred from the values of the temperature coefficients for the total number of lines obtained by B. O. Peirce[1] for several chilled cast-iron magnets, that the temperature coefficient for the field strength of any particular magnet of this type is not liable to exceed much the limits of the two here used.

The temperature coefficients of a galvanometer with a magnet other than chilled cast iron can be calculated from the known temperature coefficient of its field strength by the aid of equations (11) and (13) and the values given in Table XI. If F_k' be the value of the temperature coefficient of the field strength of any galvanometer magnet and K represents any one of the temperature coefficients of the galvanometer with a chilled cast-iron magnet, the value of the corresponding temperature coefficient for a galvanometer with the former magnet is represented by

$$K' = K + (F_k' + 0.00040), \qquad (14)$$

where the proper algebraic signs for K and F_k' must be used. The temperature coefficients of the field strengths for magnets made of various materials can be computed from the temperature coefficients for the total number of lines, determined for such magnets by B. O. Peirce in the paper cited above.

PHYSICAL LABORATORY,
UNIVERSITY OF MINNESOTA,
Nov. 27, 1908.

[1] Proceedings of Am. Acad. of Arts and Sciences, Vol. 38, p. 551, 1903; Vol. 40, p. 701, 1905.