Phenomenology of Hadrons Containing Heavy Quarks

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Dedication

To my parents, Guizhi Luo and Jiangxiang Li

Abstract

A wide variety of phenomena involving heavy quarks are discussed. The general theoretical tools and methods are introduced in Chapter 2. The heavy quark spin symmetry and QCD multipole expansion are illustrated with simple examples. The peculiar strong dynamics near open heavy flavor thresholds is studied in Chapter 3. A general heavy quark spin decomposition is worked out to understand some unexpected experimental results. Two additional effects of strong dynamics are pointed out: mixing of partial waves and isospin violation. The long-range pion exchange interaction between heavy mesons dominates over other short-distance interaction near the thresholds region, which results in enhanced mixing of partial waves. The isospin violation in S-wave heavy meson pairs is calculated in all orders of isospin breaking parameters and the result is sensitive to a strong interaction parameter which can be used to probe the property of strong interaction. These predicted effects have nontrivial experimental implications, which can be studied in future experiments. Chapter 4 devotes to understand the properties of some exotic resonances discovered recently by the experiments. We propose $Z_b(10610)$, $Z_b(10650)$ and $X(3915)$ as hadronic molecular states, while $Y(4260)$ and $Y(4360)$ as mixed hadro-charmonium states. The ω transition between quarkonium states is also discussed. In Chapter 5, the various decay channels of $h_b(2P)$ are carefully examined. We point out the data may indicate a possible violation of the popular theoretical picture about quarkonium. A particular type of heavy baryon decay is considered in Chapter 6, which may provide a clue about strong diquark correlation and the hidden scale in QCD.

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Chapter 1

Introduction

Quantum chromodynamics (QCD) is the theory of strong interaction. In everyday life, the strong interaction is responsible for binding neutrons and protons together in nuclei. From the discovery of the neutron in 1932 to the birth of QCD in 1974, it took more than 40 years to understand what is the fundamental theory describing the interaction that binds the nuclei together as well as to make sense of a large amount of accelerator data accumulated by that time. QCD, honored by a Nobel Prize in 2004, is now a well-established theory. There are many standard references on the theory of QCD[1]. Although the QCD Lagrangian can be easily written down, its consequences are not well-understood. The strong dynamics in the short distance or high energy can be calculated reliably in the framework of perturbation theory. However, at the long distance or in low energy region the theory is non-perturbative, no reliable analytical approaches derived from the first principles are available. This is the reason why confinement, being a trivial observation from all experiments, is yet to be proven theoretically. We use Λ_{QCD} to denote the energy scale that separates those two regimes. The most successful QCD predictions lie in calculating the observable that is determined by the short distance dynamics such as the inclusive production of hadrons in e^+e^- annihilation at high center-of-mass energies. A vast majority of low energy phenomena, such as the masses of hadrons, transition amplitudes and lifetimes, remain theoretically intractable. It is hoped that the numerical QCD or lattice QCD formulated from the first principles of QCD will predict those observables. However performing numercial QCD calculation on discretized space-time lattice is not as straightforward as doing numerical integral.

Although the progress of lattice QCD has been made throughout the years, it still has its own difficulties, both theoretically(e.g., how to take a continuum limit of zero lattice spacing) and computationally. The results that can be obtained reliably from lattice QCD are limited. I direct the interested readers to some references on numerical QCD[2]. The thesis will focus on the phenomenological and analytic methods to study QCD.

The system of hadrons containing heavy quarks could help us to gain some insights of strong interaction. The presence of a heavy quark, due to its large mass, can simplify the picture, and at least for some cases, enables us to study the intricate strong interaction in a theoretically controllable way. This is because the heavy quark mass (m_Q) is much larger than the typical hadronic energy scale Λ_{QCD}, i.e. $\Lambda_{QCD}/m_Q \ll 1$, so the separation of scales becomes possible. The theoretical methods based on the systematic expansion of the quantity $1/m_Q$ were developed. The heavy quark theory studied intensively in 80s and 90s now becomes common knowledge in our community. The standard textbooks and extensive reviews are available[3]. The heavy quark theory made a series of successful quantitative predictions on the properties of heavy hadrons, such as the heavy hadron masses, their semileptonic weak decay rates and lifetimes.

The simplest nontrivial system in QCD is made of a pair of heavy quark and heavy antiquark, which is called quarkonium. This system plays a fundamental role in understanding QCD. The system made of a pair of charmed quark and antiquark, $c\bar{c}$, is charmonium while the system with bottom quarks, $b\bar{b}$, is bottomonium. It has been more than 40 years since the first charmonium was observed in 1974. The quarkonium system provides an important laboratory to experimentally study the properties of QCD. We can measure the masses of different quarkonium states, their decays and transitions between various states. Many theoretical methods in QCD are directly related to analysis of those experimental results. We now have a good overall picture about how the properties of quarkonium are related to the underlying theory of QCD. For a comprehensive review of quarkonium system, one can read Ref. [4]. One can also find the reviews with the emphasis on the QCD-based theoretical methods[5].

Nowadays, more precise and reliable data become available through the experiments in electron-positron and hadron colliders. The new experiments not only offer further tests to the theory, but also bring us new and unexpected phenomena. For an instance, the charged quarkonium-like resonances show unambiguous evidences for the existence of hadrons containing four valence quarks. The nature of those four-quark states is poorly understood. Another example is the observation of an anomalous breaking of heavy quark spin symmetry. This symmetry is a powerful tool for understanding various processes involving heavy quarks, but some recent experiments contradict severely the predictions from the symmetry. They are not small excesses but with a large statistical significance (5-10σ). The major part of my research is to understand these new experimental results and their implications to the underlying theory of QCD.

The rest of my dissertation is organized as follows. Chapter 2 introduces the QCD-based methods used in understanding experimental results of heavy hadrons. The heavy quark spin symmetry is particularly emphasized and illustrated by different examples involving quarkonium spectroscopy and hadronic transitions. The QCD multipole expansion is also briefly discussed. The heavy quark spin decomposition formulism is discussed in the first section of Chapter 3. The formulism is then applied to explain the enhanced heavy quark spin symmetry breaking happened near the threshold of heavy meson pairs. This method is also useful in the following Chapters as a convenient way to derive the consequence of heavy quark spin symmetry. Two additional subtle effects of strong interaction near threshold are also discussed in Chapter 3, namely enhanced mixing of partial waves and isospin violation. These effects have clear experimental implications and can be observed in future experiments. Chapter 4 studies the nature of the exotic new and old quarkonium-like resonances. They are $Z_b(10610)$ and $Z_b(10650)$ in bottomonium sector,$Y(4260)$, $Y(4360)$, and $X(3915)$ in charmonium. Both hadronic molecules and hadrocharmonium proposals are discussed. The last section in Chapter 4 provides a conventional explanation for a "charmonium-like" resonance. Chapter 5 will use the data of $h_b(1,2P)$ and $\chi_{b1}(1,2P)$ to test a well-accepted theoretical picture of bottomonium annihilation. As a result, the new data indicate a plausible contradiction with this picture. Chapter 6 studies a strangeness changing heavy baryon decay. Due to the recent high precision measurement of b baryon lifetimes, the discussed decay channel may be visible with improved precision in future. This particular channel can be used to test the idea of the strong diquark correlation or the hidden scale of QCD. Chapter 7 concludes the dissertation.

Chapter 2

Heavy Quark Spin Symmetry

The basic aspects of heavy quark theory are discussed in this chapter. There are essentially no new results or perspectives. The purpose is to illustrate the basic idea behind heavy quark theory, especially heavy quark spin symmetry. The discussion is mainly qualitative and phenomenological. More deep and comprehensive reviews of heavy quark theory can be found in Ref. [3].

2.1 Basics of QCD and Heavy Quark Symmetry

Let me briefly review the basics of QCD, the detailed exposition can be found in Ref. [1]. In parallel with the quantum electrodynamics(QED), which is the quantum theory of electromagnetic interaction. QCD is the quantum theory of strong interaction, which is responsible for holding nuclei together. A nucleon is made of quarks. Quark has a quantum number color, which is the "charge" of the strong interaction. Everything with color participates in strong interaction. A quark with a specific color state can be represented by a triplet or a column vector q with three components.

$$q = \begin{pmatrix} q^1 \\ q^2 \\ q^3 \end{pmatrix}, \quad q' = Sq \tag{2.1}$$

One important property of strong dynamics is that it is identical for all three colors. In other words, if one rotates a color state q by a unitary 3-by-3 matrix S ($S^\dagger S = 1$)

to another color state q', then the physical consequences of the strong interaction or observables involving the rotated state should be the same as before. This property is so crucial that we name it as $SU(3)$ gauge symmetry. Any quantity of physical significance must obey this gauge symmetry. Based on the gauge symmetry and Lagrangian field theory formalism, we have to introduce a color vector field $\hat{A}_\mu$ that interacts with any color triplet state and transforms accordingly when a color triplet state get rotated so that the interaction is symmetry under the rotation. It is analogous to the electromagnetic vector field, but $\hat{A}_\mu$ is a 3-by-3 matrix since unlike electromagnetic charges represented by a real number, the color charge is a 3-component vector. The elementary excitation of color vector field is called gluon. The strong dynamics between quarks and gluons are described by the QCD Lagrangian[1]

$$\begin{aligned}\mathcal{L}_{QCD} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu,a} &+ \sum_q \bar{q}i\gamma^\mu(\partial_\mu - igt^aA^a_\mu)q - m_q\bar{q}q \\ &+ \sum_Q \bar{Q}i\gamma^\mu(\partial_\mu - igt^aA^a_\mu)Q - m_Q\bar{Q}Q\end{aligned} \tag{2.2}$$

q and Q are used to represent light and heavy quark color triplets with each component being Dirac 4-component spinor. m_q and m_Q are light and heavy quark masses. The differentiation between heavy quark and light quark will be discussed later. t^a is a 3-by-3 matrix, the $SU(3)$ color generator with the normalization $Tr(t^at^b) = (1/2)\delta^{ab}$ and commutator relation $[t^a, t^b] = if^{abc}t^c$. f^{abc} is the structure constant and repeated index means summation. $a = 1, 2, 3, ..., 8$ is the color index, one example of such a set of matrices are the Gell-Mann matrices. $t^a = \lambda^a/2$. $D_\mu = \partial_\mu - igt^aA^a_\mu$ is the QCD covariant derivative which encodes the interaction between the gluon and quark. g is a numerical number characterizing the strength of the interaction. $\hat{A}_\mu = A^a_\mu \cdot t^a$, since t^a is a matrix, A^a_μ is a number representing a vector field in a particular color state. A gluon, unlike a quark, is represented by an octet in $SU(3)$, and its eight components are A^a_μ. Similar to QED, $G^a_{\mu\nu}$ is the field strength tensor, and it is given by the following expression.

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu \tag{2.3}$$

$G^a_{\mu\nu} = -G^a_{\nu\mu}$ is total antisymmetric Lorentz tensor since f^{abc} is total antisymmetric. We can similarly define chromoelectric field E^a_i and chromomagnetic field B^a_i respectively as

$$E_i^a = -G_{0i}^a, \quad B_i^a = \frac{1}{2}\epsilon_{ijk} G_{jk}^a \tag{2.4}$$

where $i, j, k = 1, 2, 3$ and repeated index means summation, ϵ_{ijk} is total antisymmetric. In addition to Lorentz index i, for each component of chromoelectric or chromomagnetic field, there are eight color components as indicated by the color index a.

One essential property that QCD distinguishes itself from QED is the asymptotic freedom. The QCD coupling constant $\alpha_s = g^2/4\pi$ depends logarithmically on the energy or momentum transfer scale of the physical process. The running of coupling constant at one-loop level is given by the expression.

$$\alpha_s(q) = \frac{\alpha_s(q_0)}{1 + \frac{b\alpha_s(q_0)}{4\pi}\log\frac{q^2}{q_0^2}}, \quad b = 11 - \frac{2}{3}n_q \geq 7 \tag{2.5}$$

$\alpha_s(q)$ is the coupling constant at a specific scale q and n_q is the number of quark flavor which is active at the scale q, $n_q \leq 6$. Eq.(2.5) relates the coupling constants at different scales. One can see for high energies or short distances processes, i.e. for large q, $\alpha_s(q)$ decreases, the strong interaction becomes weak at further short distances. When $q \to \infty$, $\alpha_s(q) \to 0$, hence the quarks become asymptotic free. This is the asymptotic freedom. Eq.(2.5) is derived within the framework of perturbation theory, thus it is only valid for small α_s. At the energy of the Z boson mass, $q = M_Z \approx 90$ GeV, it is measured that $\alpha_s(M_Z) = 0.11 \sim 0.12$. Therefore, at high energies, the perturbative calculation is reliable. However, at low energies where the relevant degrees of freedom are hadrons instead of quarks and gluons, the nonperturbative effects are dominant. The hadronic energy scale Λ_{QCD} that separates those two regions can be estimated from Eq.(2.5). Set the reference energy $q_0 = M_Z$, we solve for the energy scale $q = \Lambda_{QCD}$ where $\alpha_s(\Lambda_{QCD}) \sim 1$.

$$q = q_0 \cdot e^{-\frac{2\pi}{b}\left(\frac{1}{\alpha_s(q_0)} - \frac{1}{\alpha_s(q)}\right)} \tag{2.6}$$

At low energy, $n_q = 3$ then $b = 9$, with $\alpha_s(M_Z) = 0.11 \sim 0.12$, we have estimated

$$\Lambda_{QCD} = 0.3 \sim 0.5\,\text{GeV} \tag{2.7}$$

At a typical size of a hadron, $r \sim 1$ fm, the momentum transfer within a hadron is in the order of Λ_{QCD}, hence the strong coupling constant α_s is large. The strong dynamics of quarks and gluons inside a hadron is very complicated. The theory (QCD) describing hadronic dynamics is in the nonperturbative regime. Quarks strongly interact with each other through emission and absorption of gluons; in addition, gluons can interact with themselves and also create extra quark-antiquark pairs.

However, the presence of a heavy quark in a hadron can simplify the picture. The differentiation between heavy and light quarks is according to the comparison of the mass of quark to the hadronic energy scale Λ_{QCD}, namely $m_q \ll \Lambda_{QCD} \ll m_Q$. The u, d and s quarks are considered to be light quarks. Their masses are small[1] compared to Λ_{QCD}. The c and b quarks[2] are regarded as heavy quarks since their masses are significantly larger than Λ_{QCD}. Indeed, if one uses the mass of the bound state of a heavy quark and anti-quark to estimate the mass of the heavy quark, namely $m_c \approx M_{J/\Psi}/2 \approx 1.5$ GeV and $m_b \approx M_{\Upsilon(1S)}/2 \approx 4.7$ GeV, with “nominally” $\Lambda_{QCD} \sim 0.5$ GeV the separation of scales can be illustrated by the following two ratios.

$$\Lambda_{QCD}/m_c \sim 0.3, \quad \Lambda_{QCD}/m_b \sim 0.1 \tag{2.8}$$

This separation of scales can simplify the physical picture. Consider a hadron containing one heavy quark, the typical momentum transfer between the constituents inside the hadron is of order Λ_{QCD}. In the rest frame of the hadron, the heavy quark has the same magnitude of momentum as the light part, $|\vec{p}_{heavy}| = |\vec{p}_{light}| \sim \Lambda_{QCD}$, thus the velocity of the heavy quark is $v_Q = p_Q/m_Q \sim \Lambda_{QCD}/m_Q$, which is small provided that $m_Q \gg \Lambda_{QCD}$. Then, we can treat the heavy quark as a nonrelativistic object and ignore any fluctuations due to the heavy quark fields, e.g. the effect of producing an additional heavy quark and anti-quark pairs can be safely discarded. Therefore, we can view a hadron containing a heavy quark as a slow moving heavy quark surrounded by a cloud of light quarks and gluons.

Since the momentum exchanged between the heavy quark and its light component is of order Λ_{QCD}, the wavelength associated with the momentum is much larger than the Compton wavelength of the heavy quark, thus the details of the heavy quark are not resolved to the light component. In the limit $\Lambda_{QCD}/m_Q \to 0$, the heavy quark is at rest in a hadron, and the light part interacts with a static color field produced by the heavy

[1] the masses of u and d quarks are a few MeV while s quark mass is about 100 MeV. Although the mass of s quark is significantly larger than the masses of u and d quarks, compared to Λ_{QCD}, they can all be treated as light quarks.

[2] top quark is also heavy but it decays too fast to form a bound state

quark. Although the dynamics of the light component is complicated, it is insensitive to the flavor and spin of the heavy quark. This is the heavy quark symmetry, which states that in the limit $m_Q \to \infty$ the properties of a hadron containing a heavy quark are independent of the spin and flavor of the heavy quark. The heavy quark symmetry is useful in many ways. There are many standard references for this topic, e.g. Ref. [3]. We will focus on the heavy quark spin symmetry (HQSS) in this thesis.

The first line of Eq.(2.2) describes the dynamics of light quarks and gluons. This part is what we call the light component, which is very complicated and beyond theoretical control at hadronic energy scale. On the contrary, the heavy quark part (the second line of Eq.(2.2)) is tractable. We can use nonrelativistic expansion to derive the effective Hamiltonian describing the interaction between heavy quarks and soft gluon fields. The derivation of the Hamiltonian is very similar to the methods used in deriving Pauli Hamiltonian in QED. The details can be found in either Ref. [6] or in section 33 of Ref. [7]. To the first order, the Hamiltonian can be written as

$$H_{int} = m_Q + \frac{(\vec{\sigma}\cdot\vec{\pi})^2}{2m_Q} + H_{light} + O(1/m_Q^2) \tag{2.9}$$

Note that H_{light}, however complicated it might be, is independent of the mass of heavy quark. The Pauli term $\frac{(\vec{\sigma}\cdot\vec{\pi})^2}{2m_Q}$ acts on non-relativistic two component spinor of the heavy quark.

$$\vec{\pi} = \vec{p} - g\vec{A} \tag{2.10}$$

where $A_i = A_i^a \cdot t^a$, as usual, a is the color index, t^a are the color generators. A_i is now a matrix, it does not commute with other components. σ_i are Pauli matrices, which satisfy $\sigma_i\sigma_j = \delta_{ij} + i\epsilon_{ijk}\sigma_k$.

$$\begin{aligned}(\vec{\sigma}\cdot\vec{\pi})^2 = \sigma_i\pi_i\sigma_j\pi_j &= \vec{\pi}^2 + \frac{i}{2}\epsilon_{ijk}\sigma_k[\pi_i,\pi_j] \\ &= \vec{\pi}^2 - g\vec{\sigma}\cdot\vec{B}\end{aligned} \tag{2.11}$$

where $B_i = B_i^a \cdot t^a = \frac{1}{2}\epsilon_{ijk}(\partial_j A_k - \partial_k A_j - ig[A_j, A_k])$, is the chromomagnetic field. Thus, Eq.(2.9) can be rewritten as

$$H_{int} = m_Q + \frac{(\vec{\pi})^2}{2m_Q} - g\frac{\vec{\sigma}\cdot\vec{B}}{2m_Q} + H_{light} + O(1/m_Q^2) \tag{2.12}$$

As we can see from this Hamiltonian, the effect of the heavy quark spin is suppressed by $1/m_Q$. In the limit $m_Q \to \infty$, the role of the heavy quark is producing a static chromoelectric field(gA_0), which only depends on the color charge of the heavy quark. The dynamics of a hadron only depends on the light degrees of freedom, H_{light}, which is independent of the flavor and spin of the heavy quark. In the exact heavy quark symmetry($m_Q \to \infty$), no interaction can change the spin of the heavy quark, while the total angular momentum J of a hadron is conserved, then the spin of heavy quark S_H and the total angular momentum of the light component $J_L = J - S_H$ should be separately conserved. Therefore, we can use the quantum numbers $S_H \otimes J_L$ to classify different states of hadrons containing a heavy quark. In addition, when transitions between different states are considered[8], we have a "selection rule" $\Delta S_H = 0$ and $\Delta J_L = 0$. In the real world, the finite heavy quark mass introduces a correction in the order of Λ_{QCD}/m_Q, which can be taken into account systematically.

2.2 Examples of Heavy Quark Spin Symmetry

Let me illustrate the HQSS with some simple examples. The mass splitting between pseudoscalar and vector mesons is a good example of heavy quark symmetry. In exact HQSS, for the same J_L, there is two-fold degeneracy corresponding to the different spin orientations of the heavy quark, which can be characterized by the total angular momentum $J = J_L \pm \frac{1}{2}$(except for $J_L = 0$). For ground states of charmed and bottom mesons, $J_L = 1/2$, is the spin of the light quark, the mass splitting of pseudoscalar ($J = 0$) and vector ($J = 1$) mesons are

$$M_{D^*} - M_D \simeq 142MeV \quad ; \quad M_{B^*} - M_B \simeq 46MeV \tag{2.13}$$

When compared with their own masses $M_{D^0} \sim 1.9GeV$ and $M_{B^0} \sim 5.3GeV$, the mass splitting indeed is small. What's more, we should expect the correction is of order $1/m_Q$. Thus, we anticipate that differences of squared masses of pseudoscalar(M_0) and vector(M_1) mesons in B and D are the same.

$$M_1^2 - M_0^2 = (M_1 + M_0)(M_1 - M_0) \sim O(m_Q) \cdot O(1/m_Q) \sim \text{constant} \tag{2.14}$$

Indeed, we find

$$M_{D^*}^2 - M_D^2 \simeq 0.55GeV^2 \quad ; \quad M_{B^*}^2 - M_B^2 \simeq 0.49GeV^2 \tag{2.15}$$

When the light part is strange quark, we find

$$M_{D_s^*}^2 - M_{D_s}^2 \simeq 0.58GeV^2 \quad ; \quad M_{B_s^*}^2 - M_{B_s}^2 \simeq 0.53GeV^2 \tag{2.16}$$

The picture of a hadron containing one heavy quark can be readily extended to mesons made of two heavy quarks, like charmonium and bottomonium. We should again expect the dominant interaction to be spin-independent, and the spin-dependent interaction should be suppressed by $1/m_Q$. In fact, if we assume the spin interaction is due to the chromomagnetic moments of two heavy quarks, $\vec{\mu_1} \cdot \vec{\mu_2} \sim \frac{1}{m_Q^2} \vec{s_1} \cdot \vec{s_2}$, the suppression actually is of order $1/m_Q^2$. The suppression of the spin-dependent interaction can be easily seen from the spectrum of charmonium states. The hyperfine splitting are

$$\begin{aligned} E(1^3S_1) - E(1^1S_0) &= M_{J/\psi(1S)} - M_{\eta_c(1S)} \simeq 116MeV \\ E(2^3S_1) - E(2^1S_0) &= M_{\psi(2S)} - M_{\eta_c(2S)} \simeq 49MeV \end{aligned} \tag{2.17}$$

where the standard notation $(n_r+1)^{2S+1}L_J$ is used, n_r is the "radial" quantum excitation number, S is the total spin of the quark pair, L is the orbital angular momentum, J is the total angular momentum. The above energy excitation due to the spin interaction should be compared with the spin-independent one

$$\begin{aligned} E(2^1S_0) - E(1^1S_0) &= M_{\eta_c(2S)} - M_{\eta_c(1S)} \simeq 657MeV \\ E(2^3S_1) - E(1^3S_1) &= M_{\psi(2S)} - M_{J/\psi(1S)} \simeq 590MeV \end{aligned} \tag{2.18}$$

Indeed, we see the spin-dependent interaction is small.

With the help of HQSS, we can qualitatively understand various experimental results without knowing details of the dynamics. For example, let's consider the decays of $\Upsilon(2S)$. The decay $\Upsilon(2S) \to \Upsilon(1S)\eta$ requires to rotate the spin orientation of the $b\bar{b}$ pair while $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$ could proceed without moving the spin. To see this, we notice that $\Upsilon(2S)$ and $\Upsilon(1S)$ are vector mesons while π and η are pseudoscalars. For $\Upsilon(2S) \to \Upsilon(1S)\eta$, the η can only be emitted in P-wave, which mandates a factor of the η meson momentum $\vec{p}$ in the amplitude. Thus, the only invariant amplitude with the appropriate parity in rest frame of $\Upsilon(2S)$ is

$$A(\Upsilon(2S) \to \Upsilon(1S)\eta) = C \cdot (\vec{\epsilon}\,' \times \vec{\epsilon}\,^*) \cdot \vec{p} \tag{2.19}$$

where $\vec{\epsilon}\,'$ and $\vec{\epsilon}$ are the polarization amplitudes of $\Upsilon(2S)$ and $\Upsilon(1S)$. For pure $^3S_1$ states of the quarkonium each polarization $\vec{\epsilon}$ coincides with the total spin of the quark pair. Then the amplitude ((2.19)) requires a spin-dependent interaction that would rotate the total spin of the heavy quark $b\bar{b}$ pair. On the other hand, $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$ can proceed through amplitude

$$A(\Upsilon(2S) \to \Upsilon(1S)\pi\pi) = C' \cdot \vec{\epsilon}\,' \cdot \vec{\epsilon}\,^* \tag{2.20}$$

which does not need to rotate the spin of $b\bar{b}$ pair. Therefore, we should expect the decay $\Upsilon(2S) \to \Upsilon(1S)\eta$ is highly suppressed. It should be suppressed by a factor $\Lambda_{QCD}/m_b \sim 0.1$ in amplitude and a factor 0.01 in rate. Indeed, the ratio from experimental data [9] $\Gamma(\Upsilon(2S) \to \Upsilon(1S)\eta)/\Gamma(\Upsilon(2S) \to \Upsilon(1S)\pi\pi) \sim 10^{-3}$, is very small. Similar suppression is found for $\Upsilon(3S)$, $\Gamma(\Upsilon(3S) \to \Upsilon(1S)\eta)/\Gamma(\Upsilon(3S) \to \Upsilon(1S)\pi\pi) \lesssim 10^{-2}$. In charmonium, due to relative small mass of c quark, the HQSS constraints are weaker, but there is still suppression of the η emission, $\Gamma(\psi(2S) \to J/\psi\eta)/\Gamma(\psi(2S) \to J/\psi\pi\pi) \sim 0.1$. This suppression in fact agrees very well with the expectation since $\Lambda^2_{QCD}/m_c^2 \sim 0.1$.

The above estimates can be made accurate and generalized to other hadronic transitions between quarkonium states by QCD multipole expansion. The applicability of multipole expansion is also due to the large heavy quark mass. The size of the bound state made by a heavy quark and anti-quark pair is small, $r = O(1/m_Q)$. It interacts with soft gluons whose typical wavelength is $\lambda = O(1/\Lambda_{QCD})$. In complete analogy with electromagnetic theory, the multipole expansion is useful when we calculate the atomic transitions under the influence of electromagnetic waves with the wavelength much larger than the atomic size. Because of $\lambda \gg r$, we have similar multipole interaction for QCD. Some leading terms from QCD multipole expansion are the chromoelectric dipole ($E1$), the chromomagnetic dipole ($M1$) and the chromomagnetic quadrupole ($M2$) interaction. Those terms are responsible for nearly all the hadronic transitions between quarkonium states. For a review, one can see Ref. [10], and see also Ref. [11] for QCD multipole expansion. The effective QCD multipole interaction Hamiltonian can be written as [10]

$$H_{E1} = -\frac{1}{2}\xi^a \vec{r} \cdot \vec{E}^a, \quad H_{M1} = -\frac{1}{2m_Q}\xi^a(\vec{\Delta} \cdot \vec{B}^a), \quad \text{and} \quad H_{M2} = -\frac{1}{4m_Q}\xi^a S_j r_i (D_i B_j)^a \tag{2.21}$$

where $\xi^a = t_1^a - t_2^a$ is the difference of the color generators acting on the quark and antiquark (e.g. $t_1^a = \lambda^a/2$ with λ^a being the Gell-Mann matrices), $\vec{r}$ is the vector for relative position of the quark and the antiquark, $\vec{D}$ is the QCD covariant derivative, $\vec{\Delta} = (\vec{\sigma}_Q - \vec{\sigma}_{\bar{Q}})/2$ is the difference of spin operators of the quark-antiquark pair, and $\vec{S}$ is the operator of the total spin of the quark-antiquark pari. One can see H_{M1} and H_{M2} are spin-dependent interactions which is suppressed by $1/m_Q$ as compared to spin-independent interaction H_{E1}. An important difference of multipole interaction in QCD from electromagnetic multipole interaction is that the color operator ξ^a appeared in Eq.(2.21) will change the color quantum number of the state they act on. The physical amplitudes arise in at least the second order in the interaction with the gluon field. The two-pion transition between $^3S_1$ states is generated in the second order in the leading H_{E1} term while the η transition is produced by H_{E1} and H_{M2}. The detailed calculation using chiral algebra and certain low-energy theorems in QCD can be found in Ref. [10]. The structures of the amplitudes are the same as Eq.(2.19) and Eq.(2.20). The theoretical calculation gives the explicit forms of C and C', and the calculated ratio of two amplitudes is in a good quantitative agreement with experimental data.

As we can see from above examples, when we study the phenomena involving heavy quarks the heavy quark spin dependent effects are much smaller than the spin-independent effects. For the leading order estimate, the spin of the heavy quark is a conserved quantity. Any physical process that changes the spin of the heavy quark is suppressed by $1/m_Q$. This is the main lesson from the HQSS. However, a number of experiments show a significant breaking of HQSS which calls for new theoretical ideas and explanations. This is the main subject of the rest thesis.

Chapter 3

Strong Dynamics near Open Heavy Flavor Thresholds

The strong interaction near the threshold region of heavy meson-antimeson pairs is very intriguing. A considerable number of resonances near the thresholds are found by recent experiments. For example, the $Z_b(10610)$ and $Z_b(10650)$ resonances [12] are at respectively the BB^* and B^*B^* thresholds , while the $X(3872)$ [13], $Z_c(3900)$ [14, 15], $Z_c(3885)$ [16], $Z_c(4020)$ [17] and $Z_c(4025)$ [18] are near the DD^* and D^*D^* thresholds. In addition to these unexpected resonances, the HQSS is found to break badly near the threshold region. For an instance, the higher bottomonium state $\Upsilon(4S)$, whose mass is very close to $B\bar{B}$ meson pairs ($M_{\Upsilon(4S)} - M_{B\bar{B}} = 0.02$ GeV), shows enhanced HQSS breaking in its hadronic transitions to $\Upsilon(1S)$ [19], namely,

$$\Gamma(\Upsilon(4S) \to \Upsilon(1S)\eta)/\Gamma(\Upsilon(4S) \to \Upsilon(1S)\pi\pi) \simeq 2.41 \tag{3.1}$$

This ratio is completely unexpected from HQSS. As discussed in Ch. 2, this ratio should be in the order of 0.01. In this Chapter, peculiar features of strong dynamics near threshold will be discussed. Section 3.1 introduces the heavy quark spin decomposition formalism and explains the enhanced HQSS breaking phenomena (such as Eq.(3.1)) near the threshold. The following two sections discuss the subtle effects of strong interaction that have nontrivial experimental consequences. Section 3.2 studies the enhanced mixing of partial waves of heavy meson pairs and Section 3.3 focuses on the isospin violation in production of heavy mesons pairs near threshold. The discussed effects will help us better understand the strong dynamics near the open heavy thresholds.

3.1 Enhanced Heavy Quark Spin Symmetry Breaking due to Mixing with Heavy Meson-antimeson Pairs

Many HQSS predictions of heavy quarkonium assume the quarkonium is in a pure $b\bar{b}$ or $c\bar{c}$ states with definite quantum numbers. We indeed use the similar spectra notations to label those states. However, we need to take into account other degrees of freedom when the mass of quarkonium becomes larger, especially above the mass of heavy meson-antimeson pairs. A straightforward evidence for the presence of other active degrees of freedom is the discovery of charged $Z_b(10610)$ and $Z_b(10650)$ mesons. They are found to decay to bottomonium, which means there is hidden $b\bar{b}$ pair inside. Because of their charge, it must have valence quarks (not quark-antiquark pairs from the quark sea) to break charge neutrality. Due to the active roles of such extra degrees of freedom played in the heavy quarkonium, they possess some unexpected properties, thus we call them quarkonium-like resonances, which is the main subject of next chapter. In this section, we study an important consequence of quarkonium mixing with heavy meson-antimeson pairs. The extra degrees of freedom introduced by the heavy meson pairs will enhance HQSS breaking.

It can be argued on general grounds that the effects of the deviation from HQSS due to heavy meson pairs should be significantly enhanced for the quarkonium-like states in a mass band near the open flavor threshold. Indeed, the typical HQSS breaking scale is the mass splitting μ of the vector and the pseudoscalar mesons. $\mu \simeq 46$ MeV for the B mesons, and $\mu \simeq 140$ MeV for the D mesons. If we denote ΔM as the mass separation between quarkonium-like state and the threshold of the heavy meson pairs, then the HQSS breaking effect due to the admixture with the heavy meson pairs can be estimated by a dimensionless quantity $\mu/\Delta M$. Clearly, this parameter becomes of order one for the quarkonium-like resonances near the threshold region.

In exact HQSS limit, i.e., $m_Q \to \infty$, the spins of the heavy quarks decouple from the dynamics. We can consider any system containing heavy quarks as a direct product of two states: spin state of the heavy quark(s), χ_H and the other state with spinless heavy quark(s), ψ_{SL}. The decomposition can be written in general as

$$|\Psi\rangle = |\chi_H\rangle \otimes |\psi_{SL}\rangle \tag{3.2}$$

χ_H only retains the spin part of the heavy quark(s), any other degrees of freedom, such as momentum of a heavy quark, light quarks and gluons, belongs to ψ_{SL}. The correction to this heavy quark spin decoupled picture is on the order of Λ_{QCD}/m_Q. If the states are labeled by quantum numbers J^{PC}, where J is total angular momentum of the system, P and C are parity and charge conjugate parity respectively, then the conventional quarkonium states below the open heavy flavor thresholds can be decomposed according to Eq.(3.2) as follows.

Table 3.1: Heavy quark spin decomposition of pure bottomonium

Bottomonium $b\bar{b}$	$\chi_H \otimes \psi_{SL}$	J^{PC}
$\eta_b(1S)$	$0_H^{-+} \otimes 0_{SL}^{++}$	0^{-+}
$\Upsilon(1S)$	$1_H^{--} \otimes 0_{SL}^{++}$	1^{--}
$h_b(1P)$	$0_H^{-+} \otimes 1_{SL}^{--}$	1^{+-}
$\chi_{b0}(1S), \chi_{b1}(1S), \chi_{b2}(1S)$	$1_H^{--} \otimes 1_{SL}^{--}$	$0^{++}, 1^{++}, 2^{++}$
$\Upsilon(1D)$	$1_H^{--} \otimes 2_{SL}^{++}$	1^{--}

For definiteness, only bottomonium without radial excitation are listed in the table. Higher radial excitation does not change this set of quantum numbers. Similar assignments of quantum numbers also work for charmonium. The quantum number J_H^{PC} for $|\chi_H\rangle$ state in quarkonium is determined by the quantum number of a fermion-antifermion system without any spacial motion, while the quantum number J_{SL}^{PC} for the rest degrees of freedom $|\psi_{SL}\rangle$ is determined by the quantum number of a system in which the heavy quarks were treated as scalar or "spinless" particles. This decomposition makes the consequence of the HQSS manifest. In addition to conservation of total quantum numbers, the exact HQSS requires the conservation of the quantum number J_H^{PC} and J_{SL}^{PC} respectively. For example, by the total conservation of parity and angular momentum, the transition $\Upsilon(2S) \to \Upsilon(1S)\eta$ is allowed to proceed through $P-$wave emission of η meson. However, it is suppressed by the HQSS as seen in the previous chapter. It can be also understood in the heavy quark spin decoupled picture here. The exact HQSS leaves the state $|\chi_H\rangle$ unchanged. The quantum number of initial ψ_{SL} is

0^{++}_{SL} while the quantum number of final ψ_{SL} state[1] is 1^{++}_{SL}, thus the process is forbidden by the exact HQSS.

We can also work out the heavy quark spin decomposition of the states involving heavy mesons. In fact, such a decomposition was first used to analyze the heavy quark spin structure of $B^*\bar{B} - B\bar{B}^*$ and $B^*\bar{B}^*$ mesons in explaining the properties of Z_b resonances [20]. The heavy B and B^* mesons are both in the state $\frac{1}{2}^-_H \otimes \frac{1}{2}^+_{SL}$, where the quantum number is J^P, while the anti-mesons $\bar{B}$ and $\bar{B}^*$ are in state $\frac{1}{2}^+_H \otimes \frac{1}{2}^-_{SL}$. They can combine to give a similar spin structure as quarkonium states with definite C parity. The specific decomposition depends on the overall quantum numbers that the heavy meson-antimeson pairs are formed. The complete spin structure including different isospin states of S-wave heavy meson pairs can be found in Refs. [21, 22]. In this section, we are interested in the heavy meson-antimeson pairs that can be directly produced in e^+e^- collision near their thresholds, which requires isospin zero and $J^{PC} = 1^{--}$. The quantum numbers indicate a P-wave relative motion of a meson-antimeson pair. Higher parity-odd orbital angular momenta can be safely neglected since their contribution are small in the near-threshold region. There are four different P-wave states of the heavy mesons with $J^{PC} = 1^{--}$:

$$
\begin{aligned}
B\bar{B}: &\qquad p_i(B^\dagger B) \\
\frac{B^*\bar{B} - \bar{B}^* B}{\sqrt{2}}: &\qquad \frac{i}{2}\epsilon_{ijk} p_j (B^{*\dagger}_k B - B^*_k B^\dagger) \\
(B^*\bar{B}^*)_{S=0}: &\qquad \frac{1}{\sqrt{3}} p_i (B^{*\dagger}_j B^*_j) \\
(B^*\bar{B}^*)_{S=2}: &\qquad \frac{1}{2}\sqrt{\frac{3}{5}} p_k (B^{*\dagger}_i B^*_k + B^{*\dagger}_k B^*_i - \frac{2}{3}\delta_{ik} B^{*\dagger}_j B^*_j)
\end{aligned}
\tag{3.3}
$$

The states$(B^*\bar{B}^*)_{S=0}$ and $(B^*\bar{B}^*)_{S=2}$ are total spin 0 and 2 states of the $B^*\bar{B}^*$ meson pair. The center of mass momentum $\vec{p}$ and the wave functions of the pseudoscalar and vector mesons are used to construct heavy meson pair states with appropriate quantum numbers. The constant factors in the expressions ensure the same normalization for each state. By "the same normalization", we mean the squared absolute value of wave function of each state, after summing over all the polarizations and integrating all directions, is the same. For instance, the normalization of B^* meson state is $\Sigma_{pol} B^{*\dagger}_i B^*_j = \delta_{ij}$.

[1] the final "spinless" state is 0^{++}_{SL} state in $\Upsilon(1S)$ combined with $P-$wave η meson, which gives total 1^{++}_{SL} state

The four states of the meson pairs in Eq.(3.3) are not eigenstates of either the operator of the total spin $\vec{\chi}_H$ of the heavy quark pair, or the operator $\vec{J}_{SL} = \vec{S}_{SL} + \vec{L}$, describing the angular momentum in the limit of spinless b quark. In order to work out the consequence of the HQSS, we need to express above four states in terms of the eigenstates of operators $\vec{\chi}_H$ and $\vec{J}_{SL}$, namely the states $|\chi_H\rangle$ and $|\psi_{SL}\rangle$ in Eq.(3.2). It can be done by using a similar method in Ref. [20]. We can use nonrelativistic two-component spinors to represent the heavy meson states since they move slowly near the threshold region. Concretely, let two-component spinor b ($b^\dagger$) represent the b (anti)quark and two-component spinors q and $q^\dagger$ represent the "rest" degrees of freedom in the mesons, then the meson states can be written as $B \sim b^\dagger q$ and $B_i^* \sim b^\dagger \sigma_i q$, where σ_i are the Pauli matrices. Replacing the heavy meson wave functions in Eq.(3.3) with their spinor representations and performing the Fierz transformation, we obtain the desired spin structures. For an example, with $B\bar{B}$ state,

$$B\bar{B}: \quad p_i(B^\dagger B) \sim p_i(b^\dagger q)(q^\dagger b) = -\frac{1}{2}(b^\dagger \sigma_l b)(q^\dagger \sigma_l q)p_i - \frac{1}{2}(b^\dagger b)(q^\dagger q)p_i \tag{3.4}$$

The Einstein summation rule (the repeated index means summing) is used. One readily identify $b^\dagger \sigma_l b$ is the eigenstate of the total heavy quark spin with $\chi_H = 1$ while $b^\dagger b$ is the state with $\chi_H = 0$. Similarly, $q^\dagger \sigma_l q$ for $S_{SL} = 1$ and $q^\dagger q$ for $S_{SL} = 0$. The total angular momentum J_{SL} of the "rest" degrees of freedom is made of both the total spin S_{SL} of the light quark pair and the orbital angular momentum L which is represented by p_i. Thus, there are four different $|\psi_{SL}\rangle$ states. We can write them explicitly as follows.

$$\begin{aligned}
1_{SL}^{+-}: &\quad p_i(q^\dagger q) \\
0_{SL}^{++}: &\quad \frac{1}{\sqrt{3}} p_i(q^\dagger \sigma_i q) \\
1_{SL}^{++}: &\quad \frac{1}{2}\epsilon_{ijk} p_j (q^\dagger \sigma_k q) \\
2_{SL}^{++}: &\quad \sqrt{\frac{3}{20}}(p_i(q^\dagger \sigma_j q) + p_j(q^\dagger \sigma_i q) - \frac{2}{3}\delta_{ij} p_k (q^\dagger \sigma_k q))
\end{aligned} \tag{3.5}$$

Therefore, any term involving spinors q and momentum p_i can be expressed as a linear combination of above four terms. Combining the four SL states with two spin states of the heavy quark pair, there are total four states of $\chi_H \otimes J_{SL}$ with overall quantum numbers $J^{PC} = 1^{--}$. Labeling the four states as

$$\psi_{10} = 1_H^{--} \otimes 0_{SL}^{++}, \psi_{11} = 1_H^{--} \otimes 1_{SL}^{++}, \psi_{12} = 1_H^{--} \otimes 2_{SL}^{++}, \psi_{01} = 0_H^{-+} \otimes 1_{SL}^{+-} \tag{3.6}$$

Then, the heavy quark spin structure for four heavy meson pair states are

$$\begin{aligned}
B\bar{B}: &\qquad \frac{1}{2\sqrt{3}}\psi_{10} + \frac{1}{2}\psi_{11} + \frac{\sqrt{5}}{2\sqrt{3}}\psi_{12} + \frac{1}{2}\psi_{01} \\
\frac{B^*\bar{B} - \bar{B}^*B}{\sqrt{2}}: &\qquad \frac{1}{\sqrt{3}}\psi_{10} + \frac{1}{2}\psi_{11} - \frac{\sqrt{5}}{2\sqrt{3}}\psi_{12} \\
(B^*\bar{B}^*)_{S=0}: &\qquad -\frac{1}{6}\psi_{10} - \frac{1}{2\sqrt{3}}\psi_{11} - \frac{\sqrt{5}}{6}\psi_{12} + \frac{\sqrt{3}}{2}\psi_{01} \\
(B^*\bar{B}^*)_{S=2}: &\qquad \frac{\sqrt{5}}{3}\psi_{10} - \frac{\sqrt{5}}{2\sqrt{3}}\psi_{11} + \frac{1}{6}\psi_{12}
\end{aligned} \tag{3.7}$$

One can easily check that the matrix of the transformation from the $\chi_H \otimes J_{SL}$ eigenstates to the states of the meson pairs is orthogonal. This is the main result of the current section. It has many phenomenological implications, which have been discussed in detail at Ref. [23]. I only discuss two examples here to show how Eq.(3.7) can be used. The first one is to explain the apparent breaking of HQSS, i.e., Eq.(3.1). The second one is about production of heavy meson pairs in e^+e^- annihilation.

If the quarkonium-like states mix with heavy meson pair states, in addition to the spin structure shown in Table 3.1, the quarkonium-like states will possess new spin structures introduced by heavy meson pairs, as indicated in Eq.(3.7). This mixing can explain the enhanced HQSS breaking effects observed in experiments. One example is $\Upsilon(4S)$.

$\Upsilon(4S)$ is near the threshold of $B\bar{B}$ meson pair ($M_{\Upsilon(4S)} - M_{B\bar{B}} \approx 20$ MeV), and decays dominantly to $B\bar{B}$. It is reasonable to expect that there may be substantially mixing with the heavy meson pair state $B\bar{B}$ inside $\Upsilon(4S)$. The mixing introduces a new spin structure ψ_{11}, which can explain the greatly enhanced rate of $\Upsilon(4S) \to \Upsilon(1S)\eta$. In ψ_{11} component, the polarization amplitude $\vec{\epsilon}\,'$ of $\Upsilon(4S)$ is made of the total spin polarization $\vec{\chi}$ of $b\bar{b}$ pair and $\vec{l}$ of the 1^{++}_{SL} component, i.e., $\epsilon\,'_i = \epsilon_{ijk}\chi'_j l_k$. The polarization amplitude $\vec{\epsilon}$ of $\Upsilon(1S)$ in the final state coincides with the total spin of $b\bar{b}$ pair, $\vec{\epsilon} = \vec{\chi}$. Therefore, the amplitude for the decay $\Upsilon(4S) \to \Upsilon(1S)\eta$ can be written as

$$\begin{aligned}
A(\Upsilon(4S) \to \Upsilon(1S)\eta) &= C(\vec{\epsilon}\,' \times \vec{\epsilon}\,^*) \cdot \vec{p} \\
&= C\epsilon_{ijk}\epsilon\,'_j \chi_k p_i \\
&= C\epsilon_{ijk} \cdot \epsilon_{jlm}\chi'_l l_m \cdot \chi_k p_i \\
&= C(\vec{\chi}\,' \cdot \vec{\chi})(\vec{p} \cdot \vec{l}) - C(\vec{p} \cdot \vec{\chi}\,')(\vec{l} \cdot \vec{\chi})
\end{aligned} \tag{3.8}$$

Thus, as long as $\vec{\chi}\,'$ and $\vec{\chi}$ are not parallel with $\vec{l}$ and $\vec{p}$ respectively, the process does not require a spin-dependent interaction. In other words, setting $\vec{\chi}\,' = \vec{\chi}$ does not result in vanishing amplitude. It can proceed without changing the spin of the heavy quark pair, so there is no HQSS suppression.

Furthermore, we notice that the $B\bar{B}$ component contains a ψ_{01} state which is a spin-singlet $b\bar{b}$ pair. If we assume $\Upsilon(4S)$ has a substantial $B\bar{B}$ admixture, we should also expect an enhancement of decay to heavy quark spin singlet state such as η_b or h_b, which is very strongly suppressed for a pure $b\bar{b}$ state of $\Upsilon(4S)$. The detailed discussion of the $\Upsilon(4S)$ and possible experimental probes of the heavy meson mixing in quarkonium-like states can be found in Ref. [23, 24]. As in the writing of the dissertation, the new observation of the transition $\Upsilon(4S) \to h_b(1P)\eta$ is reported [25]. It turns out that this transition is not at all suppressed, and in fact it is 27 times as large as the two pions transition to $\Upsilon(1S)$ state. This is clearly an evidence for the existence of $B\bar{B}$ component in $\Upsilon(4S)$ state.

We next see what Eq.(3.7) implies for the production of heavy meson pairs in e^+e^- annihilation. The heavy quarks are produced in electromagnetic current, i.e. $\bar{b}\gamma^\mu b$. The pure $b\bar{b}$ is produced in $^3S_1$ state. In non-relativistic near-threshold region, it corresponds to the spin structure $1_H^{--} \otimes 0_{SL}^{++}$. In exact HQSS, the heavy meson pairs can only be produced through the channel ψ_{10}. Then, after taking into account P-wave phase space factor, we obtain a relation between cross sections

$$\begin{aligned}&\frac{\sigma}{v^3}(e^+e^- \to B\bar{B}) : \frac{\sigma}{v^3}(e^+e^- \to B^*\bar{B} + c.c.) : \frac{\sigma}{v^3}[e^+e^- \to (B^*\bar{B}^*)_{S=0}] : \frac{\sigma}{v^3}[e^+e^- \to (B^*\bar{B}^*)_{S=2}]\\ &= (\frac{1}{2\sqrt{3}})^2 : (\frac{1}{\sqrt{3}})^2 : (-\frac{1}{6})^2 : (\frac{\sqrt{5}}{3})^2 = 1 : 4 : \frac{1}{3} : \frac{20}{3} \end{aligned} \tag{3.9}$$

If $S = 0$ and $S = 2$ states of $B^*\bar{B}^*$ are not experimentally resolved, we arrive at the relation

$$\frac{\sigma}{v^3}(e^+e^- \to B\bar{B}) : \frac{\sigma}{v^3}(e^+e^- \to B^*\bar{B} + c.c.) : \frac{\sigma}{v^3}(e^+e^- \to B^*\bar{B}^*) = 1 : 4 : 7 \tag{3.10}$$

This prediction, pointed out long ago [26], is the direct consequence of the exact HQSS. As argued at the beginning of the section, we should expect HQSS to be significantly broken in near-threshold region. Indeed, the known experimental data on production of bottom meson [9] as well as the charmed mesons [26, 27, 28, 29] dramatically contradict the prediction near the corresponding thresholds. For an example, in the observations of the charmonium-like peak $\psi(4040)$, the yield of the vector mesons pairs $D^*\bar{D}^*$, although with the smallest phase space, dominates the production of the charmed mesons. However, when the heavy mesons are produced at center-of-mass energy far away from their thresholds, we should expect HQSS to be restored. The available data on $\Upsilon(5S)$ is a good illustration of this point. The resonance $\Upsilon(5S)$ is well above the thresholds for nonstrange $B^*\bar{B}^*$ meson pairs, $\mu/\Delta M \sim 0.2$, while its mass is close to the thresholds of the strange meson pairs $B_s^*\bar{B}_s^*$, $\mu/\Delta M_s \sim 0.5$. The HQSS should be good for nonstrange mesons and broken significantly for strange mesons. This is indeed what we observed. If we assume that conversion from a pair of strange B_s mesons to a pair of nonstrange B mesons is OZI suppressed. We should expect the relative cross sections obey the Eq.(3.10). The available data [9] give the following result:

$$\sigma(\Upsilon(5S) \to B\bar{B}) : \sigma(\Upsilon(5S) \to B^*\bar{B} + c.c.) : \sigma(\Upsilon(5S) \to B^*\bar{B}^*) \simeq 1 : 2.5 : 7 \tag{3.11}$$

This is not very bad (without a phase space correction). It is much closer to Eq.(3.10) than the similar relation for the production of strange $B_s^{(*)}$ mesons, where the $B_s^*\bar{B}_s^*$ is dominant in the final state. It takes up to 90% [9] of total decay to strange meson pairs.

Although the enhanced HQSS breaking near threshold region is generally expected, the specific mechanism is unclear, for example, why the production of vector meson pairs dominates? Furthermore, the data on charmed mesons [27] show the ratio in Eq.(3.10) displays an intriguing variation depending on the energy above the thresholds. It has been suggested [30, 31] that a possible new resonance is responsible for the observation of the data. Therefore, it is very important to study in detail the strong interaction between the heavy meson pairs near the threshold region, which is the subject of the following two sections.

3.2 Mixing of Partial Waves for Heavy Meson Pairs in e^+e^- Annihilation

It is expected in general that the rescattering between the heavy meson pairs at the energy near the threshold will distort the production ratio predicted by HQSS Eq.(3.9). One extreme case is to consider the scattering of heavy meson pairs at center of mass energy E where $M(B^*\bar{B}) < E < M(B^*\bar{B}^*)$, then the scattering channels are only open for $B^*\bar{B}$ and $B\bar{B}$, the channel $B^*\bar{B}^*$ which is the heavy quark spin partner of the previous two channels is never produced since there is not enough energy to produce it. In general case, the scattering amplitudes between heavy mesons depend on the energy E. When $E \gg M(B^*\bar{B}^*)$, the mass difference between heavy meson pairs can be ignored and scattering amplitudes in different channels obey the relation predicted by HQSS, so the rescattering at higher energy can not provide a mechanism to break the prediction Eq.(3.9). However, for the rescattering near the threshold, the scattering amplitudes generally do not obey HQSS. One specific example of this general argument is the enhanced partial wave mixing of $B^*\bar{B}^*$ mesons pair produced in e^+e^- annihilation which can be reliably calculated by pion-exchange interaction near the threshold. This section is devoted to illustrate the calculation. The main content is based on the paper [32].

The overall quantum numbers $J^{PC} = 1^{--}$ of a pair of $B^*\bar{B}^*$ or $D^*\bar{D}^*$ mesons produced in the e^+e^- annihilation allow for three different combinations of the orbital momentum and the total spin S of the pair: P wave with $S = 0$ as well as a P or F wave with $S = 2$. It is generally expected that the F wave is kinematically suppressed near the threshold, which still leaves unknown the composition of the P-wave production amplitude in terms of the $S = 0$ and $S = 2$ components. This composition, clearly measurable from angular distributions [23], can be quite nontrivial [30] and in fact rapidly varying function of the c.m. energy in the near threshold region. In either case, the actual composition of the amplitude is very likely to be much different from the expectation of HQSS. Namely, the $S = 2$ wave is a factor of 20 in cross section larger than the $S = 0$ wave as indicated by Eq.(3.9).

It is quite clear that the forces between the heavy mesons depending on the spins of light quarks result in a mixing of the partial waves, as discussed [33] for the $S-D$ mixing in the $J^P = 1^+$ channel, and for heavy mesons the effect of these forces is enhanced by the factor of the meson mass M. The enhancement of the effects of the interaction of heavy hadrons through the light degrees of freedom is well known. Indeed, in the limit of large mass M of the heavy quark, the latter interaction, described by a potential V, does not depend on M. Thus, at a given momentum scale p, any effect of the interaction enters through the product MV and thus gets bigger at large M. For an example, in the perturbation theory, the correction to wave function is given by

$$\psi^{(1)} \sim \frac{V}{E_0 - E} = \frac{M \cdot V}{p_0^2 - p^2} \tag{3.12}$$

where E_0, E are the kinetic energies in center of mass frame of heavy meson pairs while $p_0 = |\vec{p}_0|, p = |\vec{p}|$ are corresponding center-of-mass momenta $E = p^2/M$. One can see for the fixed center-of-mass momentum p_0, the effect of interaction V comes with a product MV.

There is likely a part of this interaction at short distances determined by Λ_{QCD} that is currently impossible to analyze in a model independent way. However the interaction also contains a long-distance part due to the pion exchange (see e.g. in Ref. [22]), determined by the strength g of pion interaction with the heavy mesons, known [9] from the decay rate $D^* \to D\pi$. The effects of the pion exchange can be separated from those of the short-distance interaction in the range of c.m. energy above the threshold $E = p^2/M$ where the c.m. momentum p is small compared to Λ_{QCD}, $p^2 \ll \Lambda_{QCD}^2$. Indeed, in the $J^{PC} = 1^{--}$ channel two types of effects are possible: the $P-F$ wave mixing and the mixing of the $S=0$ and $S=2$ P waves. At the short distance where the strength and the range of the potential are determined by Λ_{QCD}, the ratio of the amplitudes that an P-wave state rescatters into a F-wave (the $P-F$ wave mixing) can be estimated as

$$\frac{A_F}{A_P} \propto \frac{Mp^2}{\Lambda_{QCD}^3} \tag{3.13}$$

The estimate generally applies as long as $p^2 \ll \Lambda_{QCD}^2$, however the pion mass μ can be considered small [2] in the scale of Λ_{QCD}. We have two small scales p^2 and μ^2, thus the estimate Eq.(3.13) should be modified for the effect of pion exchange as follows.

$$\frac{A_F}{A_P} \propto g^2 \frac{Mp^2}{\Lambda_{QCD}\mu^2} \quad \text{at } p^2 \ll \mu^2 \tag{3.14}$$

and

$$\frac{A_F}{A_P} \propto g^2 \frac{M}{\Lambda_{QCD}} \quad \text{at } \Lambda_{QCD}^2 \gg p^2 \gg \mu^2 \tag{3.15}$$

Compared to Eq.(3.13), the estimated effect of pion exchange interaction Eq.(3.14) is enhanced by a factor $g^2\Lambda_{QCD}^2/\mu^2$ while Eq.(3.15) is enhanced by $g^2\Lambda_{QCD}^2/p^2$. These

[2] In fact, the pion mass vanishes in the chiral limit, and at small mass of the u and d quarks: $\mu^2 \sim (m_u + m_d)\Lambda_{QCD}$

estimates imply that the pion exchange dominates the wave mixing at the energies corresponding to $p^2 \ll \Lambda_{QCD}^2$ and becomes comparable with the effect of other contributions to the spin-dependent interaction at $p^2 \lesssim \Lambda_{QCD}^2$ where the pion exchange can no longer be separated from those other short distance contributions. The effect of the pion exchange at such momenta is not calculable, due to unknown form factors in both the pion interaction with heavy hadrons (g is not constant anymore) and the short-distance behavior of A_P. For this reason, the specific calculations in this section are limited to the energies above the threshold corresponding to $p^2 \ll \Lambda_{QCD}^2$.

In what follows we calculate the effects arising from the pion exchange in the partial wave mixing for pairs of heavy vector B^* mesons[3] , so that in our numerical estimates we use $M = 5325\,\text{MeV}$. We find that the $P - F$ mixing, although parametrically enhanced at low p, is still of a moderate value and reaches only about 0.1 in the amplitude at the upper end of the applicability of our approach, $p \approx 300\,\text{MeV}$. Thus we confirm the existing expectation that the presence of the F wave in production of heavy vector meson pairs in e^+e^- annihilation is likely insignificant at energies slightly above the threshold.

The mixing of the $S = 0$ and $S = 2$ channels due to the short-distance interaction is generally of order one at any energy near the threshold. However, the energy scale for a variation of this part is set by Λ_{QCD}^2/M, so that no significant change is expected as long as $p^2 \ll \Lambda_{QCD}^2$. In particular, the absorptive part of the $S = 0$ and $S = 2$ mixing amplitude is proportional to $M\,p^3/\Lambda_{QCD}^4$ due to the P wave phase space. On the other hand, the pion exchange contribution experiences a significant variation on a smaller energy scale. Namely we shall argue that the absorptive part of the $S = 0$ and $S = 2$ mixing amplitude behaves as $g^2\,M\,p/\Lambda_{QCD}^2$ at $\mu^2 \ll p^2 \ll \Lambda_{QCD}^2$ and numerically changes from zero at the threshold to a factor of order one at $p \approx 200 - 300\,\text{MeV}$. Thus the expected effect of the pion exchange in the latter mixing is a rapid variation above the threshold in the range of excitation energy up to $15 - 20\,\text{MeV}$.

It can be also noticed that the re-scattering between the channels with two vector mesons and those with one or two pseudoscalar mesons, e.g. $B\bar{B} \to B^*\bar{B}^*$ and $B^*\bar{B} \to B^*\bar{B}^*$, which generally also contributes to the mixing of partial waves of the vector

[3] A similar calculation is applicable also to the $D^*\bar{D}^*$ meson pairs. However the significance of the discussed effects at a fixed c.m. momentum is scaled down by the lighter mass of the charmed mesons.

mesons, should be considered as a short-distance effect, whether it proceeds through the pion exchange or through other forces. Indeed, the momentum transfer in these processes is of the order of $q \sim \sqrt{M\,\Delta}$ with Δ being the mass difference between the vector and pseudoscalar mesons. Since in the heavy quark limit the parametric behavior is $\Delta \sim \Lambda_{QCD}^2/M$, one finds that in the cross-channel re-scattering $q \sim \Lambda_{QCD}$, and at such momentum transfer the pion exchange is indistinguishable from other short-range forces. For this reason in our calculations of the pion exchange we consider only the diagonal re-scattering $B^*\bar{B}^* \to B^*\bar{B}^*$ as shown in Figure 3.1

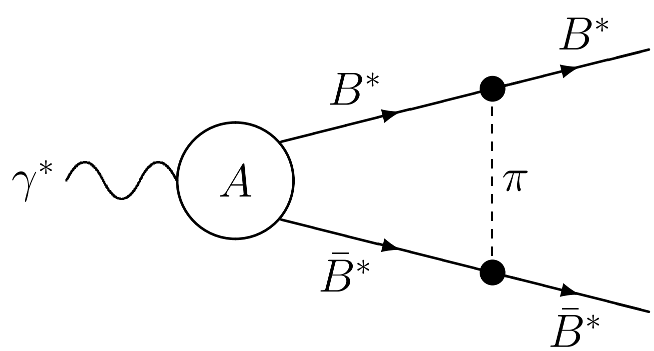

Figure 3.1: Diagonal re-scattering through pion exchange of $B^*\bar{B}^*$ meson pairs produced in e^+e^- annihilation

The diagonal interaction of pions with the isotopic doublet of heavy vector mesons $V = (B^{*+}, B^{*0})$ can be written as

$$H_{int} = i\,\frac{g}{f_\pi}\,\epsilon_{ljk}(V_j^\dagger \tau^a V_k)\,\partial_l \pi^a \ , \tag{3.16}$$

where τ^a are the isotopic Pauli matrices, $f_\pi \approx 132\,\text{MeV}$ is the pion decay constant, and g is a dimensionless coupling constant determined from the $D^* \to D\pi$ decay[4] as

[4] In our normalization, the decay rate is calculated as $\Gamma(D^{*+} \to D^+\pi^0) = g^2 p^3/(6\pi f_\pi^2)$, where $p = 38$ MeV is c.m. momentum of two body decay.

$g^2 \approx 0.18$. π^a represents components of π isotopic triplet.

$$\pi = \begin{pmatrix} \frac{i}{\sqrt{2}}(\pi^+ + i\pi^-) \\ \frac{1}{\sqrt{2}}(\pi^+ - i\pi^-) \\ \pi^0 \end{pmatrix} \tag{3.17}$$

It is assumed in Eq.(3.16) that the nonrelativistic normalization is used for the wave functions of the heavy mesons.

The amplitude for the production of a $B^*\bar{B}^*$ pair in e^+e^- annihilation near the threshold can generally be written in term of three partial wave amplitudes:

$$A(e^+e^- \to B^*\bar{B}^*) = A_0(p^2)\, j_k p_k \cdot \frac{1}{3}\, a_l b_l + j_k \cdot \frac{1}{\sqrt{20}} \left(a_i b_j + a_j b_i - \frac{2}{3}\delta_{ij}\, a_l b_l \right) \times$$
$$\left\{ A_2(p^2)\, \delta_{ki} p_j + \frac{5}{\sqrt{6}}\, A_F(p^2) \left[\frac{1}{p^2}\, p_i p_j p_k - \frac{1}{5}\, (p_k \delta_{ij} + p_j \delta_{ik} + p_i \delta_{kj}) \right] \right\} \ , \tag{3.18}$$

where $\vec{p}(p^2 = \vec{p}^2)$ is the c.m. momentum of one of the mesons, $\vec{a}$ and $\vec{b}$ are the polarization amplitudes for the meson and anti-meson and $\vec{j}$ denotes the polarization amplitude of the virtual photon. The amplitudes A_0 and A_2 are the $S = 0$ and $S = 2$ P-wave amplitudes and A_F is standing for the F-wave one. The relative normalization of the amplitudes in Eq.(3.18) is chosen in such a way that the production cross section is proportional to $p^3\,(|A_0|^2 + |A_2|^2 + |A_F|^2)$. One can also notice that under this normalization the expansion of A_F at small momentum p starts with p^2. The HQSS predicts the following relation between the production amplitudes

$$\text{HQSS:} \quad A_2/A_0 = \sqrt{20}, \quad A_F = 0 \tag{3.19}$$

which will be changed by the re-scattering between heavy mesons.

In what follows we treat the mixing of partial waves induced by the pion as a small effect and we calculate it in the first order of perturbation theory for which we use the nonrelativistic (in heavy mesons) formalism. Proceeding in this way and considering the projection on the F-wave we find the following expression for the amplitude A_F generated after re-scattering through the pion exchange by the P-wave amplitudes A_0 and A_2:

$$A_F(p^2) = \frac{g^2}{f_\pi^2}\,\frac{1}{p^2} \left[\frac{1}{p^2}\, p_i p_j p_k - \frac{1}{5}\, (p_k \delta_{ij} + p_j \delta_{ik} + p_i \delta_{kj}) \right] \times$$
$$\int \frac{d^3 q}{(2\pi)^3} \left[\frac{\sqrt{30}}{2}\, A_0(q^2) + A_2(q^2) \right] \frac{M}{q^2 - p^2 - i\epsilon}\, \frac{q_i\,(q_j - p_j)\,(q_k - p_k)}{(\vec{q} - \vec{p})^2 + \mu^2} \ . \tag{3.20}$$

One can notice that this expression includes the isotopic factor of 3, which corresponds to the pion exchange interaction in the isoscalar state of the $B^*\bar{B}^*$ pairs produced in e^+e^- annihilation. The presence of the F wave projector in Eq.(3.20) implies that only the part of the integral proportional to $p_i p_j p_k$ contributes to the $P-F$ mixing. Namely, if one writes the general expression allowed by the symmetry for the integral

$$\int \frac{d^3q}{(2\pi)^3} \left[\frac{\sqrt{30}}{2} A_0(q^2) + A_2(q^2)\right] \frac{M}{q^2 - p^2 - i\epsilon} \frac{q_i\,(q_j - p_j)\,(q_k - p_k)}{(\vec{q} - \vec{p})^2 + \mu^2} = \\ C_1(p^2)\, p_i\, \delta_{jk} + C_2(p^2)\,(p_j\, \delta_{ik} + p_k\, \delta_{ij}) + C_3(p^2)\, p_i p_j p_k\ , \tag{3.21}$$

only the structure proportional to C_3 contributes to the expression (3.20) for the amplitude A_F:

$$A_F(p^2) = \frac{2}{5} \frac{g^2}{f_\pi^2}\, p^2\, C_3(p^2)\ . \tag{3.22}$$

Clearly, if the amplitudes A_0 and A_2 are smooth functions as q^2 varies on the scale of p^2 or μ^2 the integral for C_3 converges and is determined by the range of q such that $q^2 \sim p^2$, or $q^2 \sim \mu^2$ if $p^2 < \mu^2$. In order to estimate the numerical significance of the $P - F$ mixing we approximate the amplitudes A_0 and A_2 by constants, in which case the integral is calculated analytically and the result reads as

$$A_F(p^2) = r_{FP}(p) \left[\frac{\sqrt{30}}{2} A_0 + A_2\right] \tag{3.23}$$

with the mixing function $r_{FP}(p)$ given by

$$r_{FP}(p) = \frac{g^2}{20\pi} \frac{M\,\mu}{f_\pi^2} \left\{ \frac{5 + 18\,t + 8\,t^2}{16 t^{5/2}} \arctan(2\sqrt{t}) - \frac{15 + 34t}{24t^2} \right. \\ \left. +i \left[\frac{15 + 24\,t - 4\,t^2}{24\,t^{3/2}} - \frac{5 + 18\,t + 8\,t^2}{32\,t^{5/2}} \log(1 + 4\,t)\right]\right\}\ , \tag{3.24}$$

where $t = p^2/\mu^2$.

The plot for the function $r_{FP}(p)$ is shown in Figure 3.2. One can see that the discussed $P-F$ wave mixing is quite small. Indeed, at the upper end of the applicability range of our calculation, at $p \approx 300\,\text{MeV}$, the mixing function is still less than 0.05, corresponding to only of order 0.1 mixing with the $S = 0$ P-wave amplitude, and smaller for the $S = 2$ P-wave amplitude.

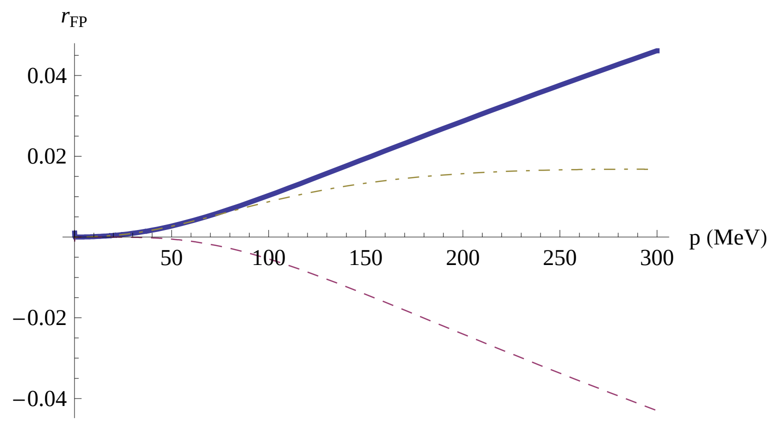

Figure 3.2: The function r_{FP} describing the mixing of the P and F waves: the absolute value (solid), and the real (dotdashed) and imaginary (dashed) parts.

The available data [34, 35] do not indicate a presence of a resonance at the $B^*\bar{B}^*$ threshold in e^+e^- annihilation. However the data are not yet conclusive and a threshold resonance may show up in future studies. In this case our approximation of smooth (constant) amplitudes A_0 and A_2 would generally be not applicable literally. For this reason we have verified that using in Eq.(3.20) these amplitudes with a Breit-Wigner shape instead of constants does not qualitatively change the conclusion that the $P-F$ mixing is small and remains at the level of 0.1 or less for a broad range of the resonance parameters.

We shall argue that the effect of the pion exchange is significantly larger numerically for the mixing of the two P-wave amplitudes A_0 and A_2. Proceeding to a calculation of this effect we notice that the mixing $A_0 \to A_2$ is the same as $A_2 \to A_0$ by reversibility, so that it is sufficient to consider the mixing only 'in one direction', e.g. the $S = 2$ amplitude A_2 generated by the $S = 0$ production amplitude A_0. Using Eq.(3.16) for the heavy meson - pion interaction and our definition in Eq.(3.18) of the production amplitudes, we find that for a pair initially produced by the amplitude A_0 an $S = 2$

state is generated by rescattering with the amplitude

$$A_{\pi 2} = j_k \, \frac{1}{\sqrt{20}} \, \left(a_i b_j + a_j b_i - \frac{2}{3} \delta_{ij} \, a_l b_l \right) \times$$
$$\frac{\sqrt{5}\, g^2}{f_\pi^2} \int \frac{d^3 q}{(2\pi)^3} \, A_0(q^2) \, \frac{M}{q^2 - p^2 - i\epsilon} \, \frac{q_k \, (q_i - p_i) \, (q_j - p_j)}{(\vec{q} - \vec{p})^2 + \mu^2} \; . \tag{3.25}$$

If one writes the general expression for the three-index integral in terms of partial waves,

$$\int \frac{d^3 q}{(2\pi)^3} \, A_0(q^2) \, \frac{M}{q^2 - p^2 - i\epsilon} \, \frac{q_k \, (q_i - p_i) \, (q_j - p_j)}{(\vec{q} - \vec{p})^2 + \mu^2} = D_1(p^2) \, p_k \, \delta_{ij}$$
$$+ D_2(p^2) \, (p_j \, \delta_{ik} + p_i \, \delta_{jk}) + D_3(p^2) \, \left[\frac{1}{p^2} \, p_i p_j p_k - \frac{1}{5} \, (p_k \delta_{ij} + p_j \delta_{ik} + p_i \delta_{kj}) \right] , \tag{3.26}$$

it can be readily seen that the $S = 2$ projector in Eq.(3.25) selects only the part proportional to the invariant function D_2. As a result the expression for the generated $S = 2$ amplitude δA_2 can be written as

$$\delta A_2(p^2) = \frac{g^2}{\sqrt{5}\, f_\pi^2} \int \frac{d^3 q}{(2\pi)^3} \, A_0(q^2) \, \frac{M}{q^2 - p^2 - i\epsilon} \, \frac{1}{p^2} \, \frac{3\, (\vec{p} \cdot \vec{q} - p^2) \, (q^2 - \vec{p} \cdot \vec{q}) - (\vec{p} \cdot \vec{q}) \, (\vec{q} - \vec{p})^2}{(\vec{q} - \vec{p})^2 + \mu^2} \; . \tag{3.27}$$

Unlike the integral for the $P - F$ mixing in Eq.(3.20) this expression does not converge for a constant $A_0(q^2)$ and thus is not determined by the intermediate momentum $q^2 \sim p^2 \ll \Lambda_{QCD}^2$ if the amplitude $A_0(q^2)$ varies at the scale of Λ_{QCD}^2. Thus the pion exchange at small momentum transfer does not dominate the discussed mixing of the P waves, and one should take into account other interactions at short distances. However the significance of the mixing generated by the pion exchange at longer distances can still be evaluated from Eq.(3.27) by considering the absorptive part of the mixing determined by $q^2 = p^2$. The calculation of the absorptive part of the mixing amplitude is done by replacing in Eq.(3.27) the propagator $(q^2 - p^2 - i\epsilon)^{-1}$ with $\pi \, \delta(q^2 - p^2)$, and one finds

$$\delta A_2(p^2)\big|_{\rm abs} = r_{20}(p) \, A_0(p^2) \tag{3.28}$$

with the absorptive mixing function r_{20} given by

$$r_{20}(p) = \frac{g^2}{16 \sqrt{5}\, \pi} \, \frac{M \, p}{f_\pi^2} \left[-6 + \frac{\mu^2}{p^2} + \left(\frac{\mu^2}{p^2} - \frac{\mu^4}{4 \, p^4} \right) \, \log \left(1 + \frac{4 \, p^2}{\mu^2} \right) \right] \; . \tag{3.29}$$

The plot of the function r_{20} is shown in Figure 3.3. One can see that this function changes between zero at the threshold to rather large values of about 0.75 at the upper end of the applicability of our calculation. This significant and rapid variation of the mixing in fact justifies a consideration of the absorptive part alone, since the dispersive part and any other effects arising from short distances are expected to exhibit a variation on the momentum scale of order Λ_{QCD}, which scale is parametrically larger than the range of the plot in Figure 3.3. Thus any possible cancellation in the absorptive part due to the short-distance processes cannot take place in the entire range of momenta, at which our approach is applicable.

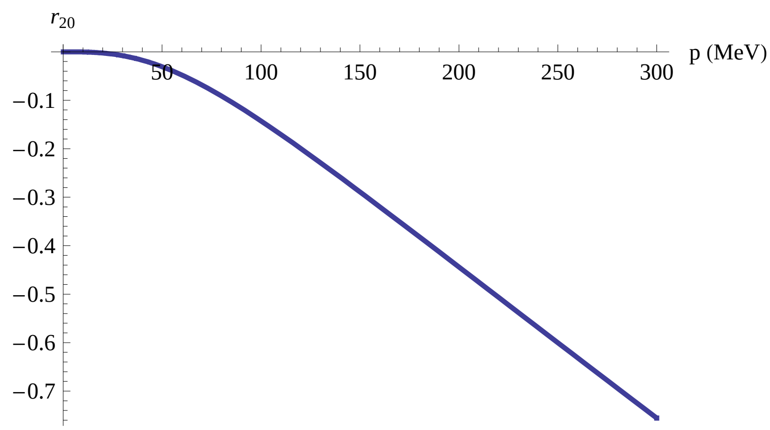

Figure 3.3: The function r_{20} describing the absorptive part of the mixing between the $S = 0$ and $S = 2$ P-waves.

In this section, we have considered the effect of the pion exchange on the mixing between three partial waves of $B^*\bar{B}^*$ mesons produced in e^+e^- annihilation at energy near the threshold. The pion exchange is calculable and dominates the mixing of P and F waves as long as the c.m. momentum p of the mesons is small as compared to Λ_{QCD}, which restricts the range of the excitation energy of the meson pair to at most $E \approx 15 \div 20\,\text{MeV}$. We find that the $P - F$ mixing is rather small and should not exceed approximately 0.1 in the amplitude. The mixing effect is however significantly larger numerically for the mixing of the two P-wave states, corresponding to the total spin of the meson pair $S = 0$ and $S = 2$. Only for the absorptive part of this mixing the dominance of the pion exchange can be ensured, while the full effect generally depends on the unknown interaction at short distances determined by Λ_{QCD}. We find that in the latter case the absorptive part of the mixing rapidly changes with energy from zero at the threshold to about 0.75 at the upper end of the range where our approach is applicable. We thus conclude that the partial wave composition of the produced pairs of vector B^* mesons should exhibit a nontrivial behavior near the threshold in e^+e^- annihilation, which can be studied experimentally.

3.3 Isospin Violation in the Yield of Heavy Meson Pairs

Isospin violation in the production of heavy meson pairs is another interesting effect of strong interaction near the threshold. The isospin violation can be probed by the observable quantity called charged-to-neutral ratio $R^{c/n}$, which can be defined generally as follows.

$$R^{c/n} = \frac{\text{the yield of charged meson pairs}}{\text{the yield of neutral meson pairs}} \tag{3.30}$$

The heavy meson pairs can be produced in both S and P wave. We shall see $R^{c/n}$ is sensitive to strong interaction parameters which provides a way to study the strong dynamics near threshold region. The P-wave heavy meson pairs can be produced directly in e^+e^- annihilation and $R^{c/n}$ can be written as the ratio

$$R^{c/n} = \frac{\sigma\left(e^+e^- \to X^+X^-\right)}{\sigma\left(e^+e^- \to X^0\bar{X}^0\right)} \tag{3.31}$$

where X stands for pseudoscalar or vector heavy mesons, i.e., $D^{(*)}$ or $B^{(*)}$. The calculation of Eq.(3.31) and how it compared to the experimental data was discussed in detail at Ref. [36].

In this section, we address the charged-to-neutral ratio $R^{c/n}$ of the yield of heavy meson pairs in the processes of the type $e^+e^- \to \pi^0\, D^*\bar{D}^{(*)}$ and $e^+e^- \to \gamma\, D^{(*)}\bar{D}^{(*)}$ and the similar processes with the $B^{(*)}$ mesons very near and exactly at the threshold for the corresponding pair of charged mesons. Namely, calculate the following observable

$$R^{c/n} = \frac{\sigma\left(e^+e^- \to \pi/\gamma\ X^+X^-\right)}{\sigma\left(e^+e^- \to \pi/\gamma\ X^0\bar{X}^0\right)} \tag{3.32}$$

Due to additional emission of π or γ, unlike in Eq.(3.31), the heavy meson pairs are produced in S-wave, which is a more interesting channel to study, since the formation of the resonant structure (either bound state or virtual state) of heavy meson pairs is more likely in S-wave. The discussion in this section is based on the paper [39].

The deviation of this ratio from the value determined by the isotopic properties of the source of the pairs due to the isospin-violating mass differences within the isotopic doublets of heavy mesons and due to the Coulomb interaction between the charged mesons is most significant near the threshold [5] .Furthermore, it has been argued previously [38, 36, 37] that the specific expressions for the isospin-violating effects in the charged-to-neutral yield ratio are sensitive to the strong-interaction scattering phases and can thus serve as a probe of the force between the heavy mesons. The formulas for the dependence of $R^{c/n}$ on the scattering phases were found for a P-wave [36] and for an S-wave [37] production in the first order in the isospin-violating mass and electric charge differences. Given that the effect of these differences grows as the energy decreases towards the threshold, the available expressions are not applicable very near or at the threshold where this effect is the largest and, hopefully, is more readily measurable. For this reason we derive here the expressions for $R^{c/n}$ that are valid to all orders in the isotopic mass difference and in the Coulomb interaction between the charged mesons. Our treatment is complementary to the previous studies in that it is applicable at low energy above the threshold in the center of mass of the heavy meson pair, where the scattering between the mesons can be described in the S-wave within the small interaction radius approximation [40] in terms of the scattering lengths a_0 and a_1 in the channels with definite isospin. We find in particular that due to the Coulomb attraction the yield of pairs of charged mesons does not go to zero at exactly the threshold, but rather starts with a finite step, at which the ratio $R^{c/n}$ for charmed mesons is generally comparable to one with the specific value being determined by the isotopic mass difference and the appropriate strong scattering length.

In what follows we start with describing the approach to the problem and derive the expression for $R^{c/n}$ in the case where the meson pair is produced by an isotopically pure source, such as e.g. $I = 1$ source in $e^+e^- \to \pi^0\, D^* \bar{D}^{(*)}$. There is also a case of an isotopically mixed source such as in the process $e^+e^- \to \gamma\, D^{(*)} \bar{D}^{(*)}$ where the photon can be emitted by the current of light quarks. The formula for isotopically pure source can be readily generalized to that for mixed source. The explicit formula is not given here but can be found in Ref. [39].

[5] Clearly the isotopic mass difference is not essential for the B and B^* mesons where it is known to be very small, but the Coulomb interaction may give rise to a significant effect in $R^{c/n}$ [37].

In the small interaction radius approximation (see e.g. in the textbook [40]) it is assumed that the strong interaction is limited to distances r between the mesons shorter than an effective radius r_0. At $r < r_0$ the potential for the strong interaction is assumed to be much larger than the isospin violating terms and also much larger than the variation of the center of mass energy E from the threshold in the range where the discussed approximation is considered. The former assumption implies that the interaction at $r < r_0$ depends only on the isospin, and that in this 'inner' region the system with a fixed orbital momentum is described by the radial wave functions with definite isotopic spin: $\phi_0(r)$ for $I = 0$ and $\phi_1(r)$ for $I = 1$. In the absence of any sources these functions should be regular at $r = 0$. We use the notation $\phi(r) = r\,R(r)$ with $R(r)$ being the radial part of the solution of the three-dimensional Schrödinger equation in a given partial wave, so that in the S-wave the regularity implies $\phi_{0,1}(0) = 0$. These solutions to the 'inner' problem are to be matched to the 'outer' wave functions at $r = r_0$, so that the matching conditions are determined by their logarithmic derivatives at r_0:

$$\phi_0'(r)/\phi_0(r)|_{r=r_0} = -\kappa_0, \qquad \phi_1'(r)/\phi_1(r)|_{r=r_0} = -\kappa_1. \tag{3.33}$$

In the energy range that is much smaller than the potential of the strong interaction in either of the isotopic channels the dependence of the wave functions ϕ_0 and ϕ_1 on the specific value of the energy can be neglected, so that the constants $\kappa_{0,1}$ can be considered as independent of the energy. At $r > r_0$ the strong force can be entirely neglected, and the wave functions for the meson pairs are described either by a free particle Schrödinger equation for neutral particles, or by the motion in the Coulomb potential for the charged mesons. The solutions to these 'outer' equations are to be used for matching at $r = r_0$ the logarithmic derivatives in Eq.(3.33). In the limit where the mass differences and the Coulomb interaction are neglected, the isotopic symmetry is exact for the outer problem as well, and the solutions for the outer problem are simply the plane waves $\exp(\pm ipr)$ with p being the momentum of each of the mesons in the center of mass frame. Furthermore, at low energy above the threshold the momentum is also small, so that $pr_0 \ll 1$ and (for the 'outer' functions) the matching conditions can be considered as shifted from $r = r_0$ to $r = 0$. In this limit one thus readily finds the well known expressions for the scattering amplitudes in the channels with definite isospin [40]

$$f_0 = -\frac{1}{\kappa_0 + i\,p}, \qquad f_1 = -\frac{1}{\kappa_1 + i\,p} , \tag{3.34}$$

so that the scattering length [defined as $-f(p \to 0)$] in each channel is the inverse of the corresponding κ:

$$a_{0,1} = \frac{1}{\kappa_{0,1}} . \tag{3.35}$$

The expressions (3.34) correspond to the scattering phases δ_0 and δ_1 in the isotopic channels given as

$$\cot \delta_{0,1} = -\frac{\kappa_{0,1}}{p} . \tag{3.36}$$

This relation shows the deficiency of using the scattering phases for discussion of isospin violating effects beyond the first order. Indeed, once e.g. the isotopic mass difference between the mesons is taken into account, the momentum p for the pair of neutral mesons, p_n, is different from that for the pair of charged ones, p_c. As a result the notion of the scattering phases for definite isospin becomes ambiguous, and this ambiguity is determined by the ratio of the mass difference to the excitation energy above the threshold. On the contrary, the isotopic parameters $\kappa_{0,1}$ (or, equivalently, the scattering lengths) are stable as long as the isospin violating terms are small in comparison with the energy of the strong interaction in the 'inner' region and can be neglected.

Proceeding to our derivation of the formulas for $R^{c/n}$ we start with neglecting the Coulomb effect and considering only the isospin violation by the isotopic mass difference in a process, where the heavy meson pair is produced by an isotopically pure source. For definiteness we consider a source producing the heavy meson pairs in the $I = 1$ state, as is the case, we believe to a good accuracy, for the processes e.g. $e^+e^- \to \pi^0 D\bar{D}^*$ and $e^+e^- \to \pi^0 D^*\bar{D}^*$. Unlike in the scattering problem, the relevant for the production process wave functions contain only outgoing waves. Denoting $\phi_c(r)$ ($\phi_n(r)$) the 'outer' wave function for the pair of charged (neutral) mesons, one can formulate the problem as that of finding the solution that up to an overall normalization factor (common for ϕ_n and ϕ_c) at $r > r_0$ reads as

$$\phi_c(r) = b_1 \exp(ip_c r), \qquad \phi_n(r) = \exp(ip_n r) , \tag{3.37}$$

where the notation b_1 implies that this coefficient arises in a situation where the source is a pure isovector. The ratio of the outgoing fluxes in the waves in Eq.(3.37) determines the ratio $R^{c/n}$:

$$R^{c/n} = \frac{p_c}{p_n} |b_1|^2 ~, \tag{3.38}$$

so that the problem reduces to finding the coefficient b_1.

As different from the scattering problem, in the production process there is an isovector source coupled to the $I = 1$ function ϕ_1. We consider here the case where the source is located within the region of the strong interaction, i.e. at $r < r_0$. This appears to be a reasonable assumption for the practical experimental conditions. Indeed, the emission of the pion is a strong interaction process with the corresponding distance scale, and the only 'smearing' of the source could arise for soft pions from the pion Compton wave length. In the actual measurements at the e^+e^- energy $\sqrt{s} \approx 4.26\,\text{GeV}$ and above the pion momentum for the production of the charmed meson pairs at the threshold is at least as large as 0.2 GeV, so that the corresponding characteristic distance is also comparable to the scale of the strong interaction. Thus the isovector 'inner' wave function ϕ_1 is not a solution of the 'inner' problem without a source, and the second of the boundary conditions in Eq.(3.33) should not be used. However the isoscalar channel is not affected at $r < r_0$ by the source, and the isoscalar function $\phi_0(r)$ is not changed and satisfies the first of the relations in Eq.(3.33). Thus the isoscalar combination of the wave functions (3.37), $\phi_0 = \phi_c + \phi_n$ should still satisfy this boundary condition at $r = r_0$. Shifting, as before, the matching point to $r = 0$, one readily finds the expression for the coefficient b_1:

$$b_1 = -\frac{\kappa_0 + i p_n}{\kappa_0 + i p_c} ~, \tag{3.39}$$

so that

$$R^{c/n} = \frac{p_c}{p_n} \frac{\kappa_0^2 + p_n^2}{\kappa_0^2 + p_c^2} ~. \tag{3.40}$$

Including the effect of the Coulomb attraction between the charged mesons amounts to replacing the outgoing plane wave $\exp(i p_c r)$ in the 'c' channel with the exact solution in the Coulomb potential $V_C = -\alpha/r$ which asymptotically at $r \to \infty$ is an (appropriately normalized) outgoing wave. This solution is well known (and can be found e.g. in the textbook [40]) and is given by

$$g(r) = -2i\, p_c r \, \exp(i p_c r) \, \left[\frac{1 - \exp(-2\pi\lambda)}{2\pi\lambda} \right]^{1/2} \Gamma(1 - i\,\lambda)\, U(1 - i\,\lambda, 2, -2i\, p_c r) ~, \tag{3.41}$$

where $U(a,b,z)$ is the standard confluent hypergeometric function of the second kind, and λ stands for the Coulomb parameter, $\lambda = m\,\alpha/p_c$, with m being the reduced mass for the pair of charged mesons, e.g. $m \approx 1.05\,\text{GeV}$ for the $D^{*+}D^{*-}$ pair and $m \approx 0.97\,\text{GeV}$ for D^+D^{*-}. The solution in Eq.(3.41) at large r describes an outgoing wave with the flux normalized to p_c, while its expansion at small r reads as

$$g(r) = i\,\xi\,p_c r + \frac{1}{\xi}\,\{1 - 2p_c r\lambda\,[\log(2p_c r) + 2\gamma_E - 1 + \text{Re}\,\psi(i\,\lambda)]\} + O(r^2\log r)\ , \quad (3.42)$$

where $\gamma_E \approx 0.5772$ is the Euler constant, $\psi(z) = \Gamma'(z)/\Gamma(z)$ is the logarithmic derivative of the Gamma function, and ξ is introduced for notational simplicity as

$$\xi = \left[\frac{2\pi\lambda}{1-\exp(-2\pi\lambda)}\right]^{1/2}\ , \quad (3.43)$$

so that ξ^2 is the well known Sommerfeld factor for the Coulomb attraction.

As it can be seen from Eq.(3.42) the derivative of the real part of the wave function $g(r)$ has a logarithmic singularity at $r \to 0$. Therefore in the matching conditions the distance r_0 in this logarithmic term should be kept finite, while in the rest of the terms it can still be replaced by zero. The coefficient b_1 is then readily found from the matching conditions as

$$b_1 = -\xi\,\frac{\kappa_0 + ip_n}{\kappa_0 - 2m\alpha\,[\log(2p_c r_0) + 2\gamma_E + \text{Re}\,\psi(i\,\lambda)] + ip_c\xi^2}\ . \quad (3.44)$$

The final expression for the ratio $R^{c/n}$ with a purely isovector source is thus given by

$$R^{c/n} = \frac{p_c}{p_n}\,\frac{2\pi m\alpha/p_c}{1-\exp(-2\pi m\alpha/p_c)} \times \quad (3.45)$$
$$\frac{\kappa_0^2 + p_n^2}{\{\kappa_0 - 2m\alpha\,[\log(2p_c r_0) + 2\gamma_E + \text{Re}\,\psi(i\,m\alpha/p_c)]\}^2 + p_c^2\,(2\pi m\alpha/p_c)^2/[1-\exp(-2\pi m\alpha/p_c)]^2}\ .$$

The discussed treatment can be readily adapted for the case of an isoscalar source of the heavy meson pairs. Indeed, in this case it is the isovector wave function ϕ_1 which satisfies at $r < r_0$ the Schrödinger equation without source, so that the isovector combination $\phi_c - \phi_n$ of the wave functions in the two channels, describing at $r > r_0$ the outgoing waves,

$$\phi_c(r) = b_0\,g(r)\ , \qquad \phi_n(r) = \exp(ip_n r)\ , \quad (3.46)$$

has to satisfy at $r = r_0$ the second boundary condition in Eq.(3.33). Using the explicit expression (3.42) for the Coulomb modified wave function, one then readily finds

$$b_0 = \xi \, \frac{\kappa_1 + i p_n}{\kappa_1 - 2m\alpha \, [\log(2p_c r_0) + 2\gamma_E + \mathrm{Re}\, \psi(i\,\lambda)] + i p_c \xi^2} \tag{3.47}$$

and, accordingly, the ratio $R^{c/n}$ is given by the same expression as in Eq.(3.45) with κ_0 replaced by κ_1.

The expressions (3.44) and (3.47) are valid to all orders in the isotopic mass difference, i.e. the difference between p_c and p_n at a given energy, and in the Coulomb interaction between the charged mesons. It can be mentioned that in the limit of a perfect isotopic symmetry ($p_c = p_n$ and $\alpha \to 0$) one finds $b_1 = -1$, $b_0 = 1$ and $R^{c/n} = 1$ in both cases. If the isotopic mass difference is nonzero, but in the limit, where there is no strong interaction (formally corresponding to a zero scattering length, i.e. $\kappa \to \infty$) the usual phase space ratio $R^{c/n} = p_c/p_n$ is recovered, provided that the Coulomb interaction is neglected. If the Coulomb attraction is accounted for in this limit, one recovers the well known Sommerfeld factor for the Coulomb enhancement of the production rate. At finite κ_0 the first order term of expansion of the expression (3.45) in the isotopic mass difference and in α matches the previously known formula [37], if the relation (3.36) for the phase shifts $\delta_{0,1}$ in terms of $\kappa_{0,1}$ is also used. Furthermore, in the discussed here approach it is quite natural that the isospin-violating effects in the ratio $R^{c/n}$ for production by an isotopically pure source, i.e. with either $I = 0$ or $I = 1$, are influenced by the strong interaction in the orthogonal isospin channel, i.e. $I = 1$ or $I = 0$, respectively. This property, found in the first-order treatment [36, 37], persists in all orders in the discussed isospin breaking terms, and can be used for studying the strong interaction between the heavy mesons in the isotopic states that may not be readily accessible [37].

It is instructive to illustrate the significance of the discussed effects in $R^{c/n}$ with numerical estimates. We present here such estimates for the case of the charmed meson pairs produced in the processes $e^+e^- \to \pi^0 + D^*\bar{D}^*$ and $e^+e^- \to \pi^0 + (D\bar{D}^* + c.c.)$. In these reactions the heavy meson pair is produced in the $I = 1$ state, so that the charged-to-neutral yield ratio is determined by the strong interaction parameters κ_0 and r_0 according to the relation (3.45). In fact the dependence in Eq.(3.45) on the effective radius r_0 is very weak as long as r_0 is much smaller than the 'Bohr radius' for the system of $D^{(*)}$ mesons: $r_0 \ll 1/m\alpha \approx 27\,\mathrm{fm}$, which is certainly the case. For this reason we fix r_0 at $1\,\mathrm{fm}$, and make estimates for a 'reasonable' range of values for the parameter κ_0, which is presently totally unknown, and which is generally different for $D^*\bar{D}^*$ and $D\bar{D}^*$ systems. The resulting behavior of the ratio $R^{c/n}$ near the threshold for $D^*\bar{D}^*$ is shown in Figure 3.4.

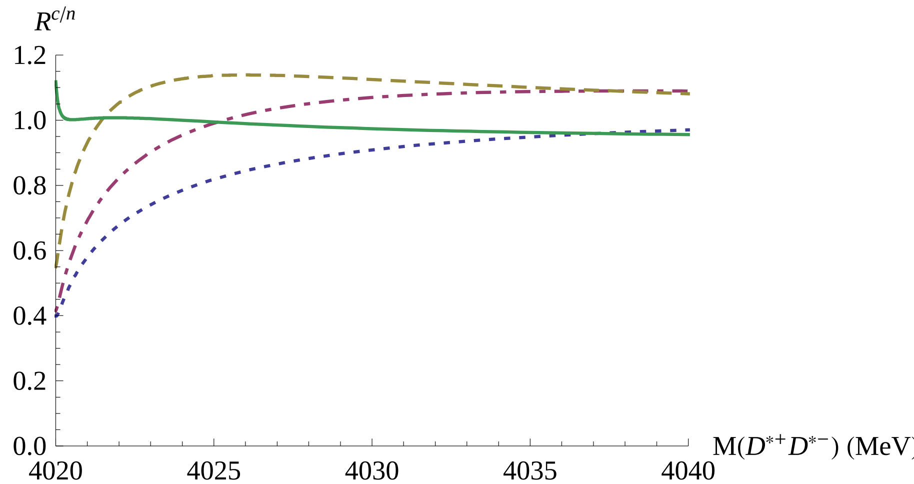

Figure 3.4: The charged-to-neutral yield ratio $R^{c/n}$ for the $D^*\bar{D}^*$ pairs produced in $e^+e^- \to \pi^0 D^*\bar{D}^*$ near the threshold. The plots are calculated using Eq.(3.45) with $r_0 = 1\,\mathrm{fm}$ and a set of different values of κ_0: -100 MeV (solid), 100 MeV (dashed), 200 MeV (dot-dashed). The dotted curve is for the limit of no strong interaction between the mesons, formally corresponding to $\kappa_0 \to \infty$.

The most characteristic feature, as readily seen from the plots, is that the ratio $R^{c/n}$ does not go to zero at exactly the threshold in the invariant mass of the meson pair [6] , but rather starts with a finite step. The particular value of $R^{c/n}$ at this point depends on κ_0 as illustrated in Figure 3.5, and also weakly depends on the radius r_0, which is fixed at $r_0 = 1\,\mathrm{fm}$ in Figure 3.5. One can readily see that the behavior of the starting threshold values of $R^{c/n}$ for the two types of the charmed meson pairs is very close. Thus any experimental measured significant difference of these values would reveal a

[6] Clearly, this essentially due to the fact that the product $p_c\xi^2$ at $p_c \to 0$ approaches a finite value $2\pi m\alpha \approx 45\,\mathrm{MeV}$, which is not small in comparison with the momentum of the neutral pair p_n at the same invariant mass (e.g. $p_n \approx 115\,\mathrm{MeV}$ for $D^{*0}\bar{D}^{*0}$ at the threshold of $D^{*+}D^{*-}$).

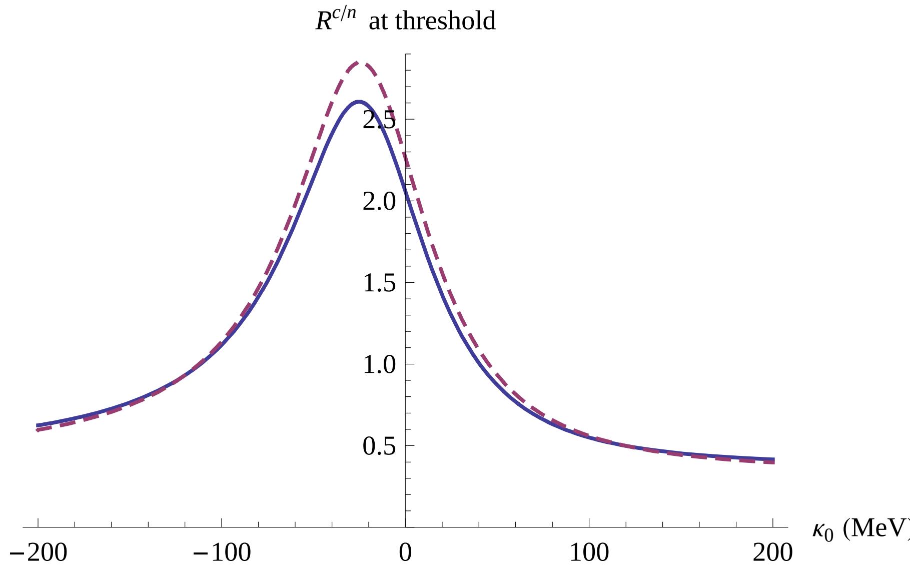

Figure 3.5: The dependence on κ_0 of the value of $R^{c/n}$ at exactly the threshold for $D^{*+}D^{*-}$ (solid) and for D^+D^{*-} (D^-D^{*+}) (dashed).

dissimilarity in the strong interaction scattering between the isoscalar channels $D\bar{D}^*$ and $D^*\bar{D}^*$ with the quantum numbers $J^{PC} = 1^{+-}$.

To summarize the current section, we have considered the interplay between the isospin violating mass differences, the Coulomb interaction and the strong scattering in the threshold behavior of the charged-to-neutral yield ratio $R^{c/n}$ for the S-wave production of overall neutral pairs of heavy $D^{(*)}$ or $B^{(*)}$ mesons. The expressions (3.44), (3.47) and (3.45) for this behavior take into account all orders in the mass differences and in the Coulomb interaction and are derived in the limit where the strong scattering of heavy mesons can be described by the scattering lengths in the isoscalar and isovector channels, which description is appropriate for the near-threshold behavior in the S wave. The considered here production processes can be observed experimentally in the reactions such as $e^+e^- \to \pi^0\, D^*\bar{D}^{(*)}$ and $e^+e^- \to \gamma\, D^{(*)}\bar{D}^{(*)}$, a study of which appears to be well within the capabilities of the current BESIII experiment. In particular we find that the onset of the yield of pairs of charged mesons starts with a finite step at the threshold. The height of the step is sensitive to a strong scattering length, and its measurement can be used as a probe of strong interaction between the heavy mesons. Indeed, if one finds the magnitude of κ_0 is small, e.g. $|\kappa_0| \ll 200$ MeV, it means that it has a large scattering length. This is a clear signature of the existence of resonant structure at $I = 0$ channel, even though it is not directly probed in the experiment[7] . Furthermore, the nature of the resonance is unambiguously determined by the sign of κ_0 [40]. Explicitly, with small $|\kappa_0|$, if $\kappa_0 > 0$, then it is a shallow bound state, otherwise it is a virtual state. During the writing of dissertation, a recent measurement in the process $e^+e^- \to \pi^0 D^* \bar{D}$ was published by BESIII[41]. The data clearly indicate the ratio $R^{c/n}$ starts at a finite step. The careful analysis of the data can be done in future.

3.4 Summary

In this chapter, we have studied the interesting properties of strong interaction near the threshold of heavy mesons pairs. The mixing with the heavy meson pairs will introduce enhanced HQSS breaking. The method used in section 3.1 to derive heavy quark spin structure is also very useful in other context. We will see it used in next chapter to elucidate the consequence of HQSS. It is general expected that the pion exchange interaction will dominate other complicated short-distance interaction near the threshold region. This long-distance pion exchange is calculable, which results in an enhanced partial wave mixing as discussed in section 3.2. In a recent paper [42], its effect to line shape of near threshold resonance is discussed. It is now clear that this pion exchange should be taken into account carefully when we are dealing with strong interaction near the threshold region. The isospin violation discussed in section 3.3 provides a model-independent way to analyze the strong dynamics between the heavy mesons near the threshold. All the complicated strong interaction at small distance is encoded in the parameters such as $\kappa_{0,1}$, which can be extracted from experiments. This approach is very helpful to discern the nature of a number of resonances discovered near the threshold region of heavy mesons. It is complementary to the model interpretations of those exotic resonances. In next chapter, we will study those resonances directly and focus on explaining the nature or internal dynamics of the quarkonium-like states.

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[7] recall that the parameter κ_0 can be extracted from the experimental data that is produced by $I = 1$ source

Chapter 4

Exotic Heavy Quarkonium-like Resonances

The heavy quarkonium is the bound state of a heavy quark and a heavy anti-quark. The gross features of quarkonium energy levels, namely its spectroscopy, can be well understood within the framework of the non-relativistic potential model, e.g., the Cornell model [43]. However, recent experiments have discovered a series of resonances that couple to the known quarkonium states but their properties are unexpected from the conventional picture of quarkonium. These resonances are named as heavy exotic mesons or quarkonium-like states. The comprehensive review of those exotic heavy quarkonium-like states is not attempted here, instead some simple examples are used to show their unexpected properties. One can find a comprehensive review in Ref. [4], or more recently and shorter review in Ref. [44].

The exotic resonance states are usually observed above the open heavy flavor threshold whose properties can not be explained by the conventional picture that the non-relativistic heavy quark and anti-quark pair $Q\bar{Q}$ interact with each other through some potential. Extra degrees of freedom must play an active role. For example, in the decay of $X(3872)$, the strong isospin violation is observed [9]

$$\frac{\Gamma(X(3872) \to J/\psi\omega \to J/\psi\pi^+\pi^-\pi^0)}{\Gamma(X(3872) \to J/\psi\pi^+\pi^-)} = 0.8 \pm 0.3 \tag{4.1}$$

Clearly, the above two transitions have different G parity since $G(\omega) = -1$ while $G(\pi^+\pi^-) = +1$. Thus, the $X(3872)$ is not in a definite isospin state, which can not be explained in a conventional quarkonium picture whose quantum number is always $I^G = 0^+$. Another more straightforward example mentioned at the beginning of the previous chapter is the discovery of charged quarkonium-like states, e.g.,

$$Z_b(10610)^{\pm} \to \Upsilon(1S)\pi^{\pm}, \quad Z_c(3900)^{\pm} \to J/\psi\pi^{\pm} \tag{4.2}$$

Those states decaying to quarkonium contain a hidden $Q\bar{Q}$ inside, but they are charged, thus the additional valence quarks are needed and they can not be a pure $Q\bar{Q}$ state. A third surprising property from those states is enhanced HQSS breaking. From Table 3.1, each quarkonium state is in a definite heavy quark spin state. The transition between different heavy quark spin state is suppressed. However, the apparent breaking of HQSS is observed, e.g.

$$\frac{\Gamma(Z_b(10610)^{\pm} \to \Upsilon(2S)\pi^{\pm})}{\Gamma(Z_b(10610)^{\pm} \to h_b(1P)\pi^{\pm})} \sim 1.6 \tag{4.3}$$

For the last example of the unusual property from the exotic state, let's consider the relative strength of two types of decays. For the quarkonium state above the heavy meson pairs, it can decay into a pair of meson and anti-meson as well as decay into a lower quarkonium state. With a higher excited $Q\bar{Q}$ state, the conventional wisdom tells us it prefer to decay into a pair of meson and anti-meson. This is indeed the case for the well-established quarkonium states, e.g. $\psi(3770)$, but not the case for certain exotic resonance, e.g. $Y(4260)$. The difference is enormous as shown in data [9, 27].

$$\frac{\Gamma(Y(4260) \to D\bar{D})}{\Gamma(Y(4260) \to J/\psi\pi^+\pi^-)} < 4.0, \quad \frac{\Gamma(\psi(3770) \to D\bar{D})}{\Gamma(\psi(3770) \to J/\psi\pi^+\pi^-)} \approx 400 \tag{4.4}$$

I hope now the word "exotic" is fully justified. This chapter is devoted to explain some aspects of those exotic resonances. The nature of $Z_b(10610)$ and $Z_b(10650)$ is discussed in section 4.1. The calculation based on the hadronic molecule picture agrees very well with experimental data as illustrated in section 4.1.1. In section 4.1.2, besides their decay to lower bottomonium state with emission of pion, other decay channels are pointed out. It can be studied with the available data of the transition $\Upsilon(5S) \to \pi\pi\pi\chi_b$. The mystery of $Y(4260)$ and $Y(4360)$ is studied in section 4.2. In addition to the puzzling preferred decay to lower charmonium state, i.e., Eq.(4.4), the enhanced HQSS breaking similar to Eq.(4.3) is also observed in new experiments for both two states. Two interpretations are compared and discussed. $X(3915)$ recently identified as $\chi_{c0}(2P)$

state by particle data group (PDG) based on the spin-parity assignment $J^P = 0^+$ is actually difficult to interpret as a conventional charmonium state. We argue in section 4.3 that $X(3915)$ may be a bound state of a $D_s\bar{D}_s$ meson pair, which explains the peculiarity of the experimental data about $X(3915)$. The last section 4.4 discusses a conventional explanation of an unusual peak near center of mass energy 4230 MeV in the process $e^+e^- \to \chi_{c0}\omega$. We argue that the "unexpected" property of the experimental data can be explained by a conventional quarkonium state $\psi(4160)$.

4.1 $Z_b(10610)$ and $Z_b(10650)$

The isovector 'twin' resonances $Z_b = Z_b(10610)$ and $Z_b' = Z_b(10650)$, found by the Belle Collaboration [12] near the respective thresholds $B^*\bar{B}$ and $B^*\bar{B}^*$, are naturally interpreted [20, 22, 48] as molecular states made of the heavy meson-antimeson pairs. Each of the two new resonances is observed through the decay to either $\Upsilon(nS)\,\pi$ with $n = 1, 2$, or 3, or to $h_b(kP)\,\pi$ with $k = 1$ or 2. Moreover, the decays to the states of ortho- and para- bottomonium are found to have comparable strength with no suppression of either of them by HQSS. This behavior is natural within the interpretation of the Z_b resonances as being molecular S-wave states of the heavy mesons: $Z_b \sim B^*\bar{B} - \bar{B}^*B$ and $Z_b' \sim B^*\bar{B}^*$, since the total spin of the $b\bar{b}$ quark pair within a meson system is not fixed [20]. We can apply the similar heavy quark spin decomposition to this situation and obtain the spin structure as follows.

$$Z_b \sim B^*\bar{B} - \bar{B}^*B \sim 1_H \otimes 0_{SL} + 0_H \otimes 1_{SL} \tag{4.5}$$

$$Z_b' \sim B^*\bar{B}^* \sim 1_H \otimes 0_{SL} - 0_H \otimes 1_{SL} \tag{4.6}$$

Therefore, Z_b or Z_b' decays to different heavy quark spin states with comparable probability. In this section, we will study the properties of Z_b resonances in the molecular picture. Its decay to lower bottomonium states are calculated and its contribution to the process in $\Upsilon(5S) \to \pi\pi\pi\chi_b$ is pointed out. The discussion is based on Ref. [45, 46].

4.1.1 Decay to Bottomonium plus Pion

Although at present the type of the threshold singularity in the heavy meson - antimeson channel (bound, virtual, or resonant state) corresponding to the Z_b peaks is not known, it appears clear that the Z_b peaks result from a strong dynamics of very slowly moving mesons near the threshold. The general picture implied by molecular assumption is that the heavy b and $\bar{b}$ quarks at the Z_b resonances are moving at distances longer than the characteristic size of bottomonium, which is also in a qualitative agreement with the available data [47] on the yield of different radial excitations of bottomonium in the decays of these resonances to $\Upsilon(nS)\,\pi$ and $h_b(kP)\,\pi$. Namely, the yield does not diminish with the number of excitation in spite of kinematical suppression for production of heavier states. This implies, at a qualitative level, that the overlap of the bottomonium states with a widely separated heavy quark pair in the initial state increases with the excitation number due to larger spatial size of the excited states.

In this subsection, we calculate the relative rates of decay of the Z_b resonances to various radial excitations of bottomonium with emission of light mesons, and thus to quantify the theoretical estimates of the relative strength of the observed transitions $Z_b^{(\prime)} \to \Upsilon(nS)\,\pi$ and $Z_b^{(\prime)} \to h_b(kP)\,\pi$. We assume that the bottomonium $b\bar{b}$ system in the final state is pure color singlet and is sufficiently compact, so that its interaction with soft gluon field can be considered within the multipole expansion in QCD [11, 10] with the leading term being the chromo-electric dipole. The transition of the heavy $b\bar{b}$ pair from the initial 'molecular' state to bottomonium is due to this interaction at short distances whose scale is set by the bottomonium size, while the (soft) gluon field induces the transition of the light quark-antiquark-gluon components of the initial 'molecule' to the light hadron(s) in the final state. In this picture the specific form of the heavy $b\bar{b}$ 'overlap' amplitude is set by the wave function of the initial state, the chromo-electric dipole interaction, and the wave function of the bottomonium state. Therefore, given a model for the latter wave function for various radial excitations, one can evaluate the relative strength of the transitions to those excited states of bottomonium. In our estimates in this section we use the simple model with the Cornell potential [43]. As for the initial state wave function of the $b\bar{b}$ pair we use the short-distance part of that for a slowly moving pair. Moreover, the chromo-electric interaction links the color singlet finite state to a color-octet initial $b\bar{b}$ pair. Clearly, such state is present in a molecular heavy meson-antimeson system. Indeed, in the colorless B or B^* mesons the color of the b quark is correlated with the color of the light antiquark. Then in a well separated meson-antimeson system the color of $\bar{b}$ in the meson is fully uncorrelated with that of b

in the antimeson, so that a color-octet $b\bar{b}$ pair is present with the statistical weight 8/9 and the statistical weight of a colorless $b\bar{b}$ state is 1/9. At short distances there is a weak (Coulomb-like) repulsion between b and $\bar{b}$ in the color octet state. Although suppressed by the color factor $1/(N_c^2 - 1) = 1/8$ the effect of this repulsion is noticeable at a small momentum of the heavy quarks, and we take it into account. Furthermore, it is important for the discussed approach that, even though the orbital angular momentum of the heavy mesons in a molecular system can be fixed (S wave in the Z_b resonances), the orbital angular momentum of the $b\bar{b}$ pair in such system is generally not fixed [20, 21] due to the motion inside the mesons. Thus the chromo-electric dipole transitions to the S-wave $\Upsilon(nS)$ of bottomonium proceed from the P-wave state of the initial $b\bar{b}$ pair, while those transitions to the P-wave $h_b(kP)$ levels are dominantly from the initial $b\bar{b}$ S-wave pair, since the wave function for a D-wave is suppressed at short distances.

In the calculation in this section we use the Hamiltonian for the chromo-electric dipole interaction in the form

$$H_{E1} = -\frac{1}{2}\,\xi^a\,\vec{r}\cdot\vec{E}^a(0)\ , \tag{4.7}$$

where $\xi^a = t_1^a - t_2^a$ is the difference of the color generators acting on the quark and antiquark (e.g. $t_1^a = \lambda^a/2$ with λ^a being the Gell-Mann matrices), $\vec{r}$ is the vector for relative position of the heavy quark and the antiquark. Finally, $\vec{E}$ is the chromo-electric gluon field strength. We therefore write the amplitudes of the discussed decays in the form

$$\begin{aligned} \langle \Upsilon(nS)\,\pi | H_{E1} | Z_b\rangle &= C_S\, A_{nS}\, E_\pi\,(\vec{Z}\cdot\vec{\Upsilon}) \\ \langle h_b(kP)\,\pi | H_{E1} | Z_b\rangle &= C_P\, A_{kP}\, \left(\vec{p}_\pi\cdot[\vec{Z}\times\vec{h}]\right)\ , \end{aligned} \tag{4.8}$$

where E_π ($\vec{p}_\pi$) is the pion energy (momentum), $\vec{Z}$, $\vec{\Upsilon}$, and $\vec{h}$ are the polarization amplitudes of the initial and final resonances, and C_S, C_P are constants that do not depend on the excitation number of the final state, while this dependence is contained in the amplitudes A_{nS} and A_{kP} describing the overlap integrals with the dipole interaction (4.7):

$$A_{nS} = \int R_{nS}(r)\, r\, R_P^{(8)}(r)\, r^2\, dr\ , \tag{4.9}$$

$$A_{kP} = \int R_{kP}(r)\, r\, R_S^{(8)}(r)\, r^2\, dr\ . \tag{4.10}$$

In the latter expressions R_{nS} (R_{kP}) are the radial wave functions of the bottomonium S (P) wave states and $R_S^{(8)}$ ($R_P^{(8)}$) are the radial wave functions of the color-octet $b\bar{b}$ pair at small momentum above the threshold in the corresponding partial wave.

It can be noted that the constants C_S and C_P encode the information about the amplitudes for the $b\bar{b}$ pair to be in the corresponding color and orbital state as well as the amplitude for the conversion by the gluon operator $\vec{E}$ of the initial light quark-antiquark-gluon 'environment' into the final pion. Clearly, these constants are beyond present theoretical control, and for this reason it is not possible within the present approach to establish a quantitative relation between transitions to S- and P-wave states of bottomonium. The only guidance on the behavior of the light-hadron part of the amplitudes in Eq.(4.8) is provided by the soft-pion properties, which mandate the factor E_π in the transitions to $\Upsilon(nS)$ and the factor $\vec{p}_\pi$ in the transitions to $h_b(kP)$ [20]. Once these factors are accounted for as in Eq.(4.8), all the dependence on the excitation number of the specific final bottomonium state is contained in the overlap integrals (4.9) and (4.10).

In order to evaluate the latter overlap integrals we use the potential model of heavy quarkonium with the Cornell potential [43]

$$V = -\frac{\kappa}{r} + \frac{r}{a^2} \tag{4.11}$$

with $\kappa = 0.52$ and $a = 2.34\,\mathrm{GeV}^{-1}$, and calculate numerically the eigenfunctions R_{nS} and R_{kP} (we also set $m_b = 5\,\mathrm{GeV}$). We further consider the relative momentum q of the b and $\bar{b}$ quarks in the initial state as small. In the limit, where the Coulomb-like repulsion in the octet state is neglected, in the limit of small q the radial function in the S-wave state can be considered as constant $R_S^{(8)}(r) \approx \mathrm{const}$, while that in the P can be set as proportional to r: $R_P^{(8)}(r) \approx \mathrm{const}\, r$. (Clearly, the overall normalization of these functions is not important for calculation of the ratios of the amplitudes A_{nS} with different n and the ratios of A_{kP} with $k = 1$ and 2.) In what follows we consider the modification of the overlap integrals in Eqs. (4.9) and (4.10) by the short-dsitance effect of the Coulomb-like repulsion between b and $\bar{b}$ in the color octet state. This repulsion is described by the potential

$$V_8(r) = \frac{\kappa_8}{r} \ , \tag{4.12}$$

and we use in our estimates the value of the coefficient κ_8 related to that in the color singlet (Eq.(4.11)) as in a one gluon exchange: $\kappa_8 = \kappa/8 = 0.065$.

One point related to the Coulomb-like repulsion that can be mentioned is that if the potential (4.12) was applicable at all distances then the continuum wave functions would vanish in the limit $q \to 0$ at any finite r due to impenetrability (from long distances) of the Coulomb barrier at zero energy. However neither the expression (4.12) is applicable at long distances, nor the momentum q is set to be literally zero. At longer distances the motion in the molecular state is described by that of heavy mesons, rather than individual heavy quarks, and also the typical values of q in the considered problem are small but finite and are set by the inverse size of the molecule. The overlap integrals are determined by the behavior of the wave function of the $b\bar{b}$ quark pair at short distances, i.e. at the characteristic size of bottomonium. At these distances the r dependence of the small-momentum wave function can still be calculated in the potential (4.12), while the normalization of the wave function is determined by the long-range modification of the interaction (4.12). Since the normalization of the functions cancels in the discussed here ratios of the amplitudes, one can use in a calculation of the integrals in Eqs. (4.9) and (4.10) either the small momentum limit of the continuum wave functions in terms of their short-distance expansion:

$$R_S^{(8)} = 1 + \frac{\kappa_8 \, m_b}{2} \, r + O(r^2) \; , \qquad R_P^{(8)} = r \, \left[1 + \frac{\kappa_8 \, m_b}{4} \, r + O(r^2) \right] \; , \tag{4.13}$$

or introduce a small but finite q, and use the exact Coulomb functions (see e.g. in the textbook [?])

$$R_S^{(8)} = \text{const} \, e^{iqr} \, {}_1F_1 \left(1 + i \frac{m_b \, \kappa_8}{2q}; 2; -2i \, q \, r \right) \; , \quad R_P^{(8)} = \text{const} \, r \, e^{iqr} \, {}_1F_1 \left(2 + i \frac{m_b \, \kappa_8}{2q}; 4; -2i \, q \, r \right) \tag{4.14}$$

with ${}_1F_1(a;b;z)$ being the Kummer confluent hypergeometric function. We apply both approaches and find that they result in similar estimates of the ratios of the considered overlap amplitudes. In particular we find that these ratios only weakly depend on q at $q < 200\,\text{MeV}$.

The numerical results of our calculation and the experimental data are presented in the Tables 4.1 and 4.2. One can readily see that our estimates are within the range allowed by the current data. It is clear however, that there is much room for improvement of the data as well as for refinement of the theoretical approach. In particular, on the theoretical side, the specific numbers are fully dependent on the model wave functions

Table 4.1: Ratios of decay rates for $Z_b(10610)$

Ratio	$\kappa_8 = 0$	Eqs. (4.13)	Eqs. (4.14), $q = 0 \div 0.2\,\mathrm{GeV}$	Experiment [47]
$\frac{\Gamma[Z_b \to \Upsilon(1S)\pi]}{\Gamma[Z_b \to \Upsilon(2S)\pi]}$	0.11	0.09	$0.10 \div 0.11$	0.073 ± 0.029
$\frac{\Gamma[Z_b \to \Upsilon(3S)\pi]}{\Gamma[Z_b \to \Upsilon(2S)\pi]}$	0.62	0.74	$0.70 \div 0.60$	0.49 ± 0.19
$\frac{\Gamma[Z_b \to h_b(2P)\pi]}{\Gamma[Z_b \to h_b(1P)\pi]}$	0.58	0.78	$0.72 \div 0.63$	1.54 ± 0.95

Table 4.2: Ratios of decay rates for $Z_b(10650)$

Ratio	$\kappa_8 = 0$	Eqs. (4.13)	Eqs. (4.14), $q = 0 \div 0.2\,\mathrm{GeV}$	Experiment [47]
$\frac{\Gamma[Z_b' \to \Upsilon(1S)\pi]}{\Gamma[Z_b' \to \Upsilon(2S)\pi]}$	0.10	0.08	$0.09 \div 0.10$	0.10 ± 0.04
$\frac{\Gamma[Z_b' \to \Upsilon(3S)\pi]}{\Gamma[Z_b' \to \Upsilon(2S)\pi]}$	0.86	1.02	$0.97 \div 0.83$	0.68 ± 0.24
$\frac{\Gamma[Z_b' \to h_b(2P)\pi]}{\Gamma[Z_b' \to h_b(1P)\pi]}$	0.73	0.99	$0.91 \div 0.80$	1.99 ± 1.11

for the bottomonium states and on a general picture of the motion in a near threshold 'molecule'. Once more precise data might become available this may contribute to a better understanding of the structure of both bottomonium and of the molecular states of heavy mesons. Also, in our estimates we used a soft-pion approximation, and ignored any effects of a possible form factor depending on the momentum of the pion, which effects can be especially significant in the transitions to the final state $\Upsilon(1S)\pi$. Such effects may arise from the unknown at present amplitude of the conversion of the light component of the meson-antimeson pair to pion (in the factors C_S and C_P) as well as from the recoil factors in the dipole matrix elements A_{nS} and A_{kP}. The recoil factor is in fact determined by the process of conversion, namely by the fraction of the pion momentum transferred to individual heavy quark or antiquark as opposed to the recoil against the pair $b\bar{b}$ as a whole. (This is different from e.g. a photon emission, where the entire photon momentum is transferred to an individual quark or antiquark.) We are not aware at present of a proper way of including and estimating these momentum-dependent factors, and for this reason we chose to neglect these altogether. The fact that our numerical result for the relative yield of $\Upsilon(1S)\pi$ is in a reasonable agreement with the data appears to indicate that the form factor effect should not be dramatic. One can also readily notice that our estimates for the yield of $h_b(2P)$ relative to that of $h_b(1P)$ are about twice smaller than the central values of the experimental data and the agreement is only due to the currently large experimental uncertainty. If future more precise data would change this to a meaningful disagreement, this could possibly indicate an enhanced contribution of dipole transitions to the P-wave bottomonium from a continuum D-wave of the color-octet $b\bar{b}$ pair.

4.1.2 Contribution to $\Upsilon(5S) \to \pi\pi\pi\chi_b$

The recent results [49] of analysis of the Belle data on the decays of the $\Upsilon(5S)$ bottomonium resonance to $\pi^+\pi^-\pi^0\chi_{bJ}$ with $J = 1, 2$ present an observation of the hadronic transitions with emission of the ω resonance: $\Upsilon(5S) \to \omega\chi_{bJ}$. However the data also reveal a significant non-ω background in the invariant mass distribution of the three pions, which distribution is enhanced at the higher end of the spectrum around 0.9 GeV. We suggest here that the non-ω part of the process may be in fact due to the contribution of the bottomonium-like isovector Z_b and Z_b' resonances through the cascade transitions $\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi\rho\chi_{bJ}$, naturally resulting in the enhancement in the three-pion invariant mass spectrum around 0.9 GeV, although the presence of the Z_b resonances can be better studied by, say the energy distribution of a single pion.

As seen in the previous subsection, the $Z_b^{(\prime)}$ resonances can be well understood in as a S-wave molecule state of the heavy meson pair. In such molecular states the spins of the heavy quark and antiquark are not correlated with each other, but rather with the spins of the light (anti)quark in the heavy meson. As a result, the resonances Z_b and Z_b' are (orthogonal) mixed states with respect to the total spin of the $b\bar{b}$ quark pair:

$$Z_b \sim 1_H^- \otimes 0_{SL}^- + 0_H^- \otimes 1_{SL}^- \,, \qquad Z_b' \sim 1_H^- \otimes 0_{SL}^- - 0_H^- \otimes 1_{SL}^- \,, \tag{4.15}$$

The quantum numbers and the heavy quark spin structure of the $Z_b^{(\prime)}$ states suggest that besides the observed single pion transitions to the lower bottomonium states there should exist [20] similar transitions with emission of two pions: $Z_b^{(\prime)} \to \rho\eta_b(1S) \to \pi\pi\eta_b(1S)$ and $Z_b^{(\prime)} \to \rho\chi_{bJ}(1P) \to \pi\pi\chi_{bJ}(1P)$. In most of the latter transitions to the $\chi_{bJ}(1P)$ levels the energy is slightly below the nominal mass of the ρ meson [except for the transition $Z_b' \to \rho\chi_{b0}(1P)$], but the resulting kinematical suppression is not very strong due to large width of the ρ resonance. Clearly, the latter transitions would contribute to the observed signal for the decay $\Upsilon(5S) \to \pi\pi\pi\chi_{bJ}(1P)$ as a non-ω background.

In the limit of HQSS all six transitions from both Z_b and Z_b' resonances to the three χ_{bJ} levels proceed due to the part of the spin wave function (4.15) containing the 1_H^- component, and their amplitudes $A(Z_b^{(\prime)} \to \rho \chi_{bJ})$ can be described by one coupling g_ρ:

$$A(Z_b^{(\prime)} \to \rho \chi_{bJ}) = g_\rho \, \left(Z_i^a + Z_i^{a\prime} \right) \, q_j \, \rho_k^a \, \left[\frac{1}{\sqrt{3}} \epsilon_{ijk} \, \chi^{(0)} + \frac{1}{\sqrt{2}} \, \left(\delta_{ij} \, \chi_k^{(1)} - \delta_{ik} \, \chi_j^{(1)} \right) + \epsilon_{jkl} \, \chi_{il}^{(2)} \right] \, , \tag{4.16}$$

where a is the isotopic triplet index, $\vec{q}$ is the momentum of the ρ meson, $\vec{Z}$, $\vec{Z}'$ and $\vec{\rho}$ are the polarization amplitudes of respectively the Z_b, Z_b' and ρ resonances, and $\chi^{(0)}$, $\chi_i^{(1)}$ and $\chi_{ij}^{(2)}$ stand for the amplitudes of the final χ_{bJ} states with respectively $J = 0, 1$ and 2. The latter amplitudes are assumed to be normalized to the number of polarization states: $\chi^{(0)} \chi^{(0)*} = 1$, $\chi_i^{(1)} \chi_i^{(1)*} = 3$, $\chi_{ij}^{(2)} \chi_{ij}^{(2)*} = 5$, and the spin-2 amplitude $\chi_{ij}^{(2)}$ is symmetric and traceless.

It can be noted that the equal coupling of the Z_b and Z_b' resonances in Eq.(4.16) is also a consequence of HQSS, since these resonances become degenerate in the limit where the interaction due to the heavy quark spin is turned off. This assumption is known to be in a reasonable agreement (within the current experimental uncertainty) with the available data [12] on the relative strength of the Z_b and Z_b' peaks in the channels $\pi \Upsilon(nS)$ and $\pi h_b(kP)$.

We also emphasize here that an application of HQSS to the discussed transitions from the $Z_b^{(\prime)}$ states has a quite different status than in the case of the decays from $\Upsilon(5S)$, where, e.g. in the decays $\Upsilon(5S) \to \omega \chi_{bJ}$ the measured [49] yield of χ_{b1} is approximately three times larger than of χ_{b2}, while a straightforward application of HQSS and treating $\Upsilon(5S)$ as a pure $b\bar{b}$ quarkonium would imply that relative yield should be 3:5 (i.e. proportional to the number of spin states for χ_{bJ}), modulo minor kinematical corrections. However there are reasons to conclude [23] that the resonance $\Upsilon(5S)$ is a more complicated object, likely due to a mixing with states of heavy meson pairs, whose thresholds are close to its mass. In particular the reported [12] presence of the $f_2(1270)$ tensor resonance in the dipion channel in the decay $\Upsilon(5S) \to \pi\pi\Upsilon(1S)$ is likely an indicator of the presence of in the $\Upsilon(5S)$ of a state with light degrees of freedom, denoted in Ref. [23] as ψ_{12}, where the heavy $b\bar{b}$ pair has total spin one, while the light degrees of freedom are in a $J^{PC} = 2^{++}$ state. For the decay of such state to $\omega \chi_{bJ}$, the HQSS would predict the ratio of the rates for $\chi_{b0} : \chi_{b1} : \chi_{b2} = 15 : 20 : 1$. Clearly, the experimentally observed [49] ratio of $\chi_{b1} : \chi_{b2}$ can well be a result of a combined effect of the pure $b\bar{b}$ and ψ_{12} components of the $\Upsilon(5S)$. The situation is different for the $Z_b^{(\prime)}$ resonances. Their quantum numbers limit the possible internal spin structures to only those included in Eq.(4.15) and it is only the part containing 1_H^- that, according to HQSS, gives rise to transitions to the spin-triplet quarkonium levels, including the decays to $\rho\chi_{bJ}$. It can also be mentioned that this spin structure of the $Z_b^{(\prime)}$ resonances requires the statistical weight enhancement 5:3 for the yield of the χ_{b2} state relative to that of the χ_{b1} which appears to agree with the data indicating a somewhat larger non-ω background in the decays to $J = 2$ state as compared to $J = 1$, i.e. in a strong variance with the ratio for the resonant ω yield.

The contribution of the $Z_b^{(\prime)}$ resonances to the decays of the $\Upsilon(5S)$ depend on the coupling f in the amplitude of the decay $\Upsilon(5S) \to \pi Z_b$, which amplitude has the form

$$A[\Upsilon(5S) \to \pi Z_b^{(\prime)}] = f \left[\vec{\Upsilon} \cdot \left(\vec{Z}_b^a + \vec{Z}_b^{a\prime} \right) \right] \pi^a \, E_\pi \ , \tag{4.17}$$

where the factor of the pion energy, E_π, is mandated by the chiral properties of soft pions. In order to avoid the uncertainty related to the absolute value of this coupling f, we consider the ratio of the rate for the cascade decay $\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi\rho\chi_{bJ}$ to the known one for the process $\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi\pi h_b(1P)$, in which ratio the coupling f cancels. Instead, this ratio depends on the relation between the constant g_ρ in Eq.(4.16) and a similar constant g_π describing the transitions $Z_b^{(\prime)} \to \pi h_b(1P)$ whose amplitude can be written as

$$A[Z_b^{(\prime)} \to \pi h_b(1P)] = g_\pi \, \epsilon_{ijk} \, \left(Z_i^a - Z_i^{a\prime} \right) \, p_j \, h_k \pi^a \ , \tag{4.18}$$

where $\vec{h}$ is the polarization amplitude of the $h_b(1P)$ bottomonium and $\vec{p}$ stands in this case for the pion momentum. [It can be noticed that the relative sign between the amplitudes for the Z_b and Z_b' resonances in the transitions involving spin-singlet $b\bar{b}$ pair in Eq.(4.18) is opposite to that for the transitions between spin-triplet states as in the amplitudes (4.16) and (4.17).]

Moreover, it is the ratio of the constants g_ρ/g_π that can be of a particular interest. Indeed, in the HQSS limit the spin-singlet bottomonium $h_b(1P)$ and the spin-triplet states $\chi_{bJ}(1P)$ are described by the same spatial wave function. Therefore this ratio of the constants is determined by the relation between the wave functions for the triplet and the singlet in the spin of the $b\bar{b}$ pair parts of the $Z_b^{(\prime)}$ resonances in Eq.(4.15) and the relation between the amplitudes for conversion of the 0^-_{SL} and 1^-_{SL} light quark states into respectively ρ or a pion in the discussed hadronic transitions. The numerical estimates, to be discussed further, suggest a tantalizing possibility of the constants g_ρ and g_π being (approximately) equal. At present we can offer no a priori motivation for such an equality to hold, however should it be established experimentally, it may indicate some kind of symmetry in the dynamics of the molecular states.

For the purpose of our numerical estimates we treat the ρ meson as a simple Breit-Wigner resonance with fixed width $\Gamma_\rho \approx 150\,\mathrm{MeV}$, thus neglecting the variation of its width parameter at the invariant mass of the $\pi\pi$ pair, q^2, being not equal to the nominal value of m_ρ^2. As is well known, such variation is process dependent, and, if required, can be studied and taken into account when (and if) more detailed data on the process $\Upsilon(5S) \to \pi^+\pi^-\pi^0\chi_{bJ}$ become available. We also note that the three terms arising in the amplitude from the isotopic permutation of the pion emerging from the first transition in the cascade, $\Upsilon(5S) \to \pi Z_b^{(\prime)}$, with one of the pions emerging from the ρ resonance do not interfere in the total rate. This is due to that the pion in the first transition is emitted in the S-wave in the rest frame of the bottomonium [cf. Eq.(4.17)], while each of the pions from the decay of the ρ resonance is in the P wave in this frame, when the ρ emission is described by the amplitude in Eq.(4.16). After these preliminary remarks we write the expression for the ratio of the rates for the processes $\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi^+\pi^-\pi^0\chi_{bJ}$ and $\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi^+\pi^- h_b$ as follows

$$\frac{\Gamma[\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi^+\pi^-\pi^0\chi_{bJ}]}{\Gamma[\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi^+\pi^- h_b]} = \frac{2J+1}{2}\,\frac{|g_\rho|^2}{|g_\pi|^2}\,\frac{I_\rho}{I_\pi}\;, \tag{4.19}$$

where the phase space integrals I_ρ and I_π are given by

$$I_\rho = \int \frac{dE_\pi\, dq^2}{\pi} \left| \frac{1}{E_1 - E_\pi + i\Gamma_1/2} + \frac{1}{E_2 - E_\pi + i\Gamma_2/2} \right|^2 E_\pi^2\, |\vec{k}_\pi|\, |\vec{q}|^3\, \frac{m_\rho\, \Gamma_\rho}{(q^2 - m_\rho^2)^2 + m_\rho^2\, \Gamma_\rho^2} \tag{4.20}$$

and

$$I_\pi = \int dE_\pi \left| \frac{1}{E_1 - E_\pi - i\Gamma_1/2} - \frac{1}{E_2 - E_\pi + i\Gamma_2/2} \right|^2 E_\pi^2\, |\vec{k}_\pi|\, |\vec{p}|^3 \;. \tag{4.21}$$

Here the following notations are used: Γ_1 and Γ_2 are the widths of the Z_b and Z_b' resonances, E_1 and E_2 are the corresponding resonance energies for the pion emitted in the $\Upsilon(5S) \to \pi Z_b^{(\prime)}$ transition, $E_1 \approx 258\,\mathrm{MeV}$, $E_2 \approx 213\,\mathrm{MeV}$, E_π is the energy of this pion, and $\vec{k}_\pi$ is its momentum. Furthermore, q^2 is the squared invariant mass of the two pions emerging from the ρ decay, and $\vec{q}$ is the spatial part of q, $|\vec{q}|^2 = (\Delta - E_\pi)^2 - q^2$, with Δ being the total energy release in the transition from the initial $\Upsilon(5S)$ to the final $1P$ state of bottomonium. Finally $\vec{p}$ in Eq.(4.21) is the momentum of the 'second' pion, i.e. of the one emitted in the transition $Z_b^{(\prime)} \to \pi h_b$. The kinematical boundaries in the integral (4.20) are $4m_\pi^2 < q^2 < (\Delta - E_\pi)^2$ for the (first) integration over q^2 and $m_\pi < E_\pi < (\Delta - 2m_\pi)$ for E_π, while in the integral I_π the integration limits are set by $m_\pi < E_\pi < (\Delta - m_\pi)$ (although in practice both integrals are dominated by the values of E_π in the vicinity of the resonances at E_1 and E_2). It can be also pointed out that the overall numerical factor in Eq.(4.19) corresponds to the normalization to the rate of the dipion transition to $h_b(1P)$ with only charged pions, for which rate the data are available [12, 9] [1] . Moreover, unlike the dipion transitions from $\Upsilon(5S)$ to $\Upsilon(nS)$ levels, which contain a nonresonant background not associated with the Z_b and Z_b' resonances, the transitions to the h_b states are exclusively given by the resonance contribution, which is theoretically justified by the notion that the $Z_b^{(\prime)}$ resonances provide the only significant mechanism for the apparent HQSS breaking and which behavior is in a very good agreement with the data [12]. Thus the relation (4.19) applies to only the resonant process $\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi\pi\pi\chi_{bJ}$ rather than to any additional nonresonant background that may be present in $\Upsilon(5S) \to \pi\pi\pi\chi_{bJ}$.

Using the value $10.865\,\mathrm{GeV}$ for the central energy at which the $\Upsilon(5S)$ data are collected by Belle, and also the current experimental central values of the widths for the $Z_b^{(\prime)}$ resonances, $\Gamma_1 = 18.6\,\mathrm{MeV}$ and $\Gamma_2 = 11.5\,\mathrm{MeV}$, we estimate numerically: $I_\pi \approx 2.94\,\mathrm{GeV}^5$, $I_\rho(\chi_{b0}) \approx 0.231\,\mathrm{GeV}^5$, $I_\rho(\chi_{b1}) \approx 0.152\,\mathrm{GeV}^5$ and $I_\rho(\chi_{b2}) \approx 0.120\,\mathrm{GeV}^5$. Therefore the relative yield of $\chi_{b0} : \chi_{b1} : \chi_{b2}$ bottomonium states in the discussed cascade process is estimated as

$$1 : 2.0 : 2.6 \; . \tag{4.22}$$

[1] The simple model for the ρ peak with q^2 independent width, which we use in our estimates, can be readily modified by replacing in Eq.(4.20) the constant Γ_ρ with $\Gamma_\rho(q^2)$.

In terms of the ratio of the rate to that of $\Upsilon(5S) \to \pi^+\pi^- h_b(1P)$ we find

$$\frac{\Gamma[\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi^+\pi^-\pi^0\chi_{b1}]}{\Gamma[\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi^+\pi^- h_b]} \approx 0.78\,\frac{|g_\rho|^2}{|g_\pi|^2}\,,$$
$$\frac{\Gamma[\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi^+\pi^-\pi^0\chi_{b2}]}{\Gamma[\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi^+\pi^- h_b]} \approx 0.10\,\frac{|g_\rho|^2}{|g_\pi|^2}\,. \tag{4.23}$$

The branching fraction for the decay $\Upsilon(5S) \to \pi^+\pi^- h_b$ is currently measured [9] as $\left(3.5^{+1.0}_{-1.3}\right) \times 10^{-3}$ which, using the estimates (4.23), results in the predictions

$$\mathcal{B}[\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi^+\pi^-\pi^0\,\chi_{b1}] \approx \left(2.7^{+0.8}_{-1.0}\right) \times 10^{-4}\frac{|g_\rho|^2}{|g_\pi|^2}\,,$$
$$\mathcal{B}[\Upsilon(5S) \to \pi Z_b^{(\prime)} \to \pi^+\pi^-\pi^0\,\chi_{b2}] \approx \left(3.5^{+1.0}_{-1.3}\right) \times 10^{-4}\frac{|g_\rho|^2}{|g_\pi|^2}\,. \tag{4.24}$$

Experimentally the non-ω background in the Belle data [49] on the decays $\Upsilon(5S) \to \pi^+\pi^-\pi^0\,\chi_{bJ}$ corresponds to the branching fraction of about 4×10^{-4} for χ_{b1} and 7×10^{-4} for χ_{b2} with an error in each value apparently amounting to between 2 and 3 units times 10^{-4}.

The data [49], even though with a large uncertainty, suggest the possibility that the contribution of the Z_b and Z_b' resonances can be close to what one would estimate from Eq.(4.23) with $|g_\rho| \approx |g_\pi|$. Such a relation could imply a spin independence not only for the heavy quarks, but also (an approximate one) for the light ones, in the processes of conversion of molecular states into a light meson and heavy quarkonium. In view of this intriguing possibility it appears very interesting to study these processes in more detail. In particular, the contribution of the $Z_b^{(\prime)}$ states to the decays $\Upsilon(5S) \to \pi^+\pi^-\pi^0\chi_b(1P)$ can be evaluated from the data by the distribution of the smallest of the energies of the three pions in the decay, whose distribution should contain the resonance peaks at E_1 and E_2.

4.2 $Y(4260)$ and $Y(4360)$

The charmonium-like resonances $Y(4260)$ and $Y(4360)$ in e^+e^- annihilation present a considerable challenge for interpretation of their internal structure due to their unusual decay properties. Namely these resonances were mostly observed through their pionic transitions to either J/ψ or ψ' charmonium states: $Y(4260) \to J/\psi\pi\pi$ [50, 51, 52, 53] and $Y(4360) \to \psi'\pi\pi$ [54, 55]. The most surprising feature of these resonances is that, unlike for other known states above the open charm threshold, e.g. $\psi(3770)$, or $\psi(4040)$, the decays to final states containing pairs of charmed mesons are not dominant. Several models for the structure of $Y(4260)$ have been discussed in the literature: a $c\bar{c}g$ hybrid [57], a $cs\bar{c}\bar{s}$ tetraquark [58], hadrocharmonium [10, 56], and, more lately as an S-wave molecular system containing an excited $D_1(2420)$ meson and a D meson [59]. Most recently the new results from the BESIII experiment have added to the intrigue of the properties of the $Y(4260)$ and $Y(4360)$ resonances and may in fact hold a clue to understanding the structure of these states. Namely, in addition to the observation of isovector peaks $Z_c^{\pm}(3900)$ [60] and $Z_c^{\pm}(4025)$ [61] in the decays $Y(4260) \to Z_c(3900)\pi$ and $Y(4260) \to Z_c(4025)\pi$ the BESIII collaboration reported [62] an observation of production of the final state $h_c\pi^+\pi^-$ at both $\sqrt{s} = 4.26\,\text{GeV}$ $[\sigma(e^+e^- \to h_c\pi^+\pi^-) = 41.0 \pm 2.8 \pm 4.7\,\text{pb}]$ and $\sqrt{s} = 4.36\,\text{GeV}$ $[\sigma(e^+e^- \to h_c\pi^+\pi^-) = 52.3 \pm 3.7 \pm 9.2\,\text{pb}]$, with a yield comparable to that of e.g $J/\psi\pi^+\pi^-$ at the peak of $Y(4260)$: $\sigma(e^+e^- \to J/\psi\pi^+\pi^-) = 62.9 \pm 1.9 \pm 3.7\,\text{pb}$. The latter behavior clearly implies a significant breaking of the HQSS. It may appear at first that this behavior is reminiscent of the known production of the $h_b(1P)$ and $h_b(2P)$ [63] bottomonium spin singlet states in the two-pion transitions from $\Upsilon(10890)$. In the bottomonium case this apparent breaking of the heavy quark symmetry is entirely associated with the $Z_b(10610)$ and $Z_b(10650)$ isovector resonances [12] and the observed properties of these transitions are in agreement with the molecular picture for the Z_b resonances. The data however indicate that for the charmonium-like resonances the dominant contribution to the transitions to $h_c\pi\pi$ is continually spread over the phase space, rather than being associated with an intermediate Z_c resonance. Therefore one has to explain these transitions either by a heavy quark symmetry breaking within the $Y(4260)$ and $Y(4360)$ resonances, or in the mechanism for their decay.

In this section, two different interpretations of $Y(4260)$ and $Y(4360)$ states will be discussed. The first one is hadronic molecule picture, which regards $Y(4260)$ as S-wave molecule state of $D_1(2420)\bar{D}$ meson pair, while the second one is hadrocharmonium picture. In the end, we will argue the latter picture may be more credible. The main content is based on Ref. [64, 65].

4.2.1 Molecular Interpretation Weakened by HQSS

In the orbitally excited heavy $Q\bar{q}$ mesons ($D_1(2420)$ and $D_2(2460)$ with $Q = c$ and $B_1(5721)$ and $B_2(5747)$ with $Q = b$) the light (anti)quark is in the state with the quantum numbers $J^P_{SL} = (3/2)^+$. The HQSS thus requires that in their decay into the ground state heavy mesons and a pion, e.g. $D_1(2420) \to D^*\pi$, $D_2(2460) \to D\pi$, $D_2(2460) \to D^*\pi$, the pion is emitted in the D-wave. For this reason these excited heavy mesons are relatively narrow, having the width of about 25 MeV, unlike the other pair of the orbitally excited mesons, where the light antiquark is in the $J^P_{SL} = (1/2)^+$ state, e.g. $D_0(2400)$ and $D_1(2430)$, whose widths are in the ballpark of 300 MeV due to similar decays with the pion emission in the S-wave [9]. The $J^P = (3/2)^+$ mesons can be combined with the ground-state (anti)mesons in the S-wave to form pairs with the quantum numbers $J^{PC} = 1^{--}$ matching those necessary for direct production in the e^+e^- annihilation. This property has lead to the suggestion [66, 67, 68, 69] that the observed in the e^+e^- resonance $Y(4260)$ is a near-threshold S-wave bound 'molecular' state of ($D_1(2420)\bar{D}$ - c.c.) charmed meson pair. If correct, this picture would generally imply, due to the HQSS, an existence of whole slew of similar threshold charmonium states with either the D meson replaced by D^* or the $D_1(2420)$ replaced by $D_2(2460)$. Some of these charmonium-like states with $J^{PC} = 1^{--}$ should be directly observable as resonances in the e^+e^- annihilation at the total energy (4.3 - 4.4) GeV, namely molecular states of $D_2(2460)\bar{D}^*$ and $D_1(2420)\bar{D}^*$. Moreover a similar 'suite' of bottomonium-like molecular resonances would be expected in e^+e^- annihilation near 11.0 GeV.

The purpose of this subsection is to show that in fact the production of the discussed $J^{PC} = 1^{--}$ S-wave pairs with one orbitally excited $(3/2)^+$ meson is forbidden by the HQSS. This conclusion casts doubt on the interpretation of the resonance $Y(4260)$ as a D_1D molecular state, since it effectively removes the important argument that being an S-wave state it carries no significant threshold suppression for its yield in the e^+e^- annihilation. Although, indeed there is no kinematical suppression, the production amplitude is suppressed by the inverse of the heavy quark mass, which is the parameter for breaking the HQSS. For the charmed quark one would generally expect the latter suppression to amount to a factor of order 0.1, or stronger, in the rate. In view of our result, an alternative interpretation of the structure of $Y(4260)$ resonance, e.g. as hadro-charmonium [10, 56], may be more credible.

The claimed selection rule for production of the heavy meson pairs is strictly valid in the limit of HQSS. Indeed, in this limit the spin of both the heavy quark and the antiquark is strictly conserved. In other words, in the conversion of the heavy quark-antiquark pair $\bar{Q}\,Q$ produced by the electromagnetic current into the final state of heavy mesons the spin of the heavy quarks is not dynamical, and one can consider instead spinless heavy quarks. Then the considered orbitally excited mesons have the quantum numbers $J^P_{SL} = (3/2)^+$, and the ground-state heavy mesons have $J^P_{SL} = (1/2)^-$. The electromagnetic current $(\bar{Q}\gamma_\mu Q)$ for slow heavy quarks has the nonrelatistic form $\bar{Q}\sigma_i Q$ and couples only to the spin of the heavy quarks. Once the spin of the heavy quarks is removed the equivalent operator generating the heavy quark pair is equivalent to a unit operator (with the point-like spatial structure $\delta^3(\vec{r})$) corresponding to the quantum numbers $J^{PC} = 0^{++}$. Clearly, is is impossible to make the total spin 0 state out of an S-wave pair of mesons with spin 3/2 and 1/2. Thus no S-wave state can be produced, and this conclusion is valid for any combination of the polarizations of the mesons. In the real situation, where the heavy quarks have spin, these polarization states combine with the spin of the $\bar{Q}Q$ pair to form the discussed here three types of the meson-antimeson pairs with $J^{PC} = 1^{--}$. Thus no such pair can be produced in the S wave.

An alternative proof of our conclusion, where the spin of the heavy quarks is not removed and is explicitly traced, goes as follows. A state of the heavy meson pair can be decomposed [20, 23] in terms of the total spin of the $\bar{Q}\,Q$ pair, χ_H, and the total angular momentum J_{SL} of the rest ('light') degrees of freedom, where the latter includes both the total spin of the light quark pair $\bar{q}\,q$ and any orbital momentum. The states with $J^{PC} = 1^{--}$ are generally a combination of four eigenstates of the operators $\vec{\chi}_H$ and $\vec{J}_{SL}$:

$$\psi_{10} = 1^{--}_H \otimes 0^{++}_L\,, \quad \psi_{11} = 1^{--}_H \otimes 1^{++}_L\,, \quad \psi_{12} = 1^{--}_H \otimes 2^{++}_L\,, \quad \text{and} \quad \psi_{01} = 0^{-+}_H \otimes 1^{+-}_L\,. \tag{4.25}$$

Clearly, in the $(3/2)^+$ mesons the light degrees of freedom carry angular moment 3/2, while in the ground state mesons they carry the angular momentum 1/2. Thus combining two such mesons in the S-wave would never produce a state with $J_{SL} = 0$, and the component $1^{--}_H \otimes 0^{++}_{SL}$ should be absent in the decomposition of the meson pair states in terms of $\chi_H \otimes J_{SL}$ eigenstates. This can be also readily verified by performing the transformation explicitly:

$$\begin{aligned}
(D_1\bar{D} - \bar{D}_1 D) \quad &: \quad \frac{1}{2\sqrt{2}}\,\psi_{11} + \frac{\sqrt{5}}{2\sqrt{2}}\,\psi_{12} + \frac{1}{2}\,\psi_{01} \\
(D_1\bar{D}^* - \bar{D}_1 D^*) \quad &: \quad \frac{3}{4}\,\psi_{11} - \frac{\sqrt{5}}{4}\,\psi_{12} + \frac{1}{2\sqrt{2}}\,\psi_{01} \\
(D_2\bar{D}^* - \bar{D}_2 D^*) \quad &: \quad \frac{\sqrt{5}}{4}\,\psi_{11} + \frac{1}{4}\,\psi_{12} - \frac{\sqrt{5}}{2\sqrt{2}}\,\psi_{01}
\end{aligned} \tag{4.26}$$

(It can be noticed that the meson-pair wave functions and the functions ψ_{ij} are assumed to be normalized to one, so that the transformation matrix is orthogonal.) The missing $\chi_H \otimes J_{SL}$ eigenstate ψ_{10} is exactly the one (and the only one) produced by the electromagnetic current $(\bar{Q}\sigma_i Q)$. Thus the production of each of the states in Eq.(4.26) is forbidden in the limit of HQSS where χ_H is conserved.

One might argue that the HQSS breaking can be enhanced in the threshold region [23], where the mass splitting between the mesons related by this symmetry is comparable to the distance to the thresholds. We can note however, that this is very unlikely to be the case for our selection rule. Indeed at the threshold for each of the considered meson-antimeson channel the major effect is the mass splitting μ between the D^* and D mesons, $\mu \approx 140\,\text{MeV}$, which is approximately the same as the energy gap between the thresholds. This could generally lead to a significant violation of HQSS due to rescattering between the channels. However in the discussed case *neither* of the three channels can be produced, so that the rescattering between them cannot give rise to a non-zero production amplitude. The distance to the other thresholds is parametrically of order Λ_{QCD}, so that the effects of the HQSS breaking can be estimated as being of order μ/Λ_{QCD}.

Another effect of HQSS breaking could be a mixing between the orbitally excited $(3/2)^+$ and $(1/2)^+$ spin 1 mesons, e.g. a mixed structure of the $D_1(2420)$ and $D_1(2430)$. However, phenomenologically this mixing is apparently very small, as one can deduce from the small total width of $D_1(2420)$ as compared to that of $D_1(2430)$. Had the mixing been substantial, it would have introduced a large S-wave component in the amplitude of the decay $D_1(2420) \to D^*\pi$ resulting in a considerable enhancement of the rate of this decay.

One might also argue that, in the case of the charmonium region, the mass of $Y(4260)$ is sufficiently high above the charm threshold, and the energy-related breaking of the HQSS can enhance the yield of $D_1\bar{D}$ pairs in the e^+e^- annihilation. It should be noted, however, that the excitation of the $(3/2)^+$ mesons over the ground state ones is parametrically given by Λ_{QCD}. Thus, the parameter for the HQSS breaking at the threshold of $D_1\bar{D}$ is still $\Lambda_{QCD} m_c \sim 0.3$ in the amplitude, corresponding to suppression ~ 0.1 in the rate, unless there is some currently unknown mechanism for enhancing such breaking in this particular case. A simple way to illustrate that the energy 4.26 GeV is not large enough to produce significant energy induced spin effects for the charmed quark, one can consider the D-wave production in e^+e^- annihilation of the charmed quarks in the limit of free quarks (so that the spin breaking is a pure energy-related effect). Using the textbook expressions for the plane-wave Dirac spinors for free fermions, one readily finds the D/S ratio of the production rates at the total c.m. energy $\sqrt{s} = E$:

$$\left|\frac{A_D}{A_S}\right|^2 = \frac{(E-2m)^2}{2(E+m)^2} \tag{4.27}$$

Using also, for a numerical estimate, $m_c = M_{J/\psi}/2$, so that $E = 4.26\text{GeV}$ corresponds to $E/m_c = 2.75$, one estimates the ratio of the rates as $|D/S|^2 = 0.02$, which indicates that the kinematical effect is still quite small at such energy.

We thus conclude that the yield in the e^+e^- annihilation of the discussed $(3/2)^+ + (1/2)^-$ S-wave pairs of heavy mesons, forbidden in the limit of HQSS, should be significantly suppressed when the effects of the spin symmetry breaking are also taken into account.

4.2.2 Mixed Hadrocharmonium

Having argued against the molecular interpretation, we still must turn to the other mechanism to explain the observed enhanced breaking of HQSS. In fact the splitting of the discussed resonances by about 100 MeV can be compared with the characteristic scale of the HQSS breaking in the charm sector for which a representative value is the mass splitting between D^* and D mesons of about 140 MeV. In this subsection we suggest and explore the possibility that the resonances $Y(4260)$ and $Y(4360)$ form a pair of mixed states containing both a spin-triplet and a spin-singlet $c\bar{c}$ pair. Clearly, such mixing is possible only in the presence of other degrees of freedom, i.e. of the light quarks and/or gluons, and is generally possible in either of the discussed models of hybrid or four quark systems. We would like to discuss the mixing within the hadrocharmonium model, where a relatively compact colorless $c\bar{c}$ pair is embedded in a mesonic excitation of light quarks, the binding between the two parts is provided by van der Waals-like force[2] .

In the hadrocharmonium model one can naturally expect a mixing between an embedded $^3S_1$ charmonium state and a $^1P_1$ state. Indeed, in terms of the multipole expansion in QCD, the leading interaction depending on the spin of the heavy quarks is the chromomagnetic dipole (M1), described by the Hamiltonian

$$H_{M1} = -\frac{1}{4m_c}\,\xi^a\,(\vec{\Delta}\cdot\vec{B}^a)\ , \tag{4.28}$$

where $\vec{B}^a$ is the chromomagnetic field, $\xi^a = t^a_c - t^a_{\bar{c}}$ is the difference of the color generators acting on the quark and antiquark, and $\vec{\Delta} = \vec{\sigma}_c - \vec{\sigma}_{\bar{c}}$ is a similar difference for the spin operators. The leading effect in transitions between colorless states of a nonrelativistic $c\bar{c}$ pair induced by this term arises through its interference with the chromoelectric dipole (E1) interaction

$$H_{E1} = -\frac{1}{2}\,\xi^a\,(\vec{r}\cdot\vec{E}^a)\ , \tag{4.29}$$

where $\vec{E}^a$ is the chromoelectric field and $\vec{r}$ is the vector of the relative position between the quark and the antiquark. Clearly, the combined action of the terms (4.28) and (4.29) changes the orbital angular momentum by one unit and the total spin of the pair by one unit, $\Delta L = 1$ and $\Delta S = 1$, and thus links a $^3S_1$ state of charmonium to the $^1P_1$.

In the present discussion we denote Ψ_3 the wave function of a hadrocharmonium state with the quantum numbers $J^{PC} = 1^{--}$ containing a $^3S_1$ $c\bar{c}$ pair, and denote Ψ_1 that for the $J^{PC} = 1^{--}$ state with an embedded $^1P_1$ $c\bar{c}$ pair[3] . We suggest that the observed $Y(4260)$ and $Y(4360)$ resonances arise as a result of mixing between these two states due to the spin dependent interaction:

$$Y(4260) = \cos\theta\,\Psi_3 - \sin\theta\,\Psi_1\ , \qquad Y(4360) = \sin\theta\,\Psi_3 + \cos\theta\,\Psi_1 \tag{4.30}$$

with θ being the mixing angle.

[2] although the two parts are colorless, they can still attract to each other due to residual strong interaction just like two neutral molecules can still form a bound state by a residual electromagnetic force

[3] Clearly the required overall quantum numbers $J^{PC} = 1^{--}$ with a $c\bar{c}$ pair in the $^1P_1$ state can arise only in the hadrocharmonium system due to the contribution of the light degrees of freedom.

Assuming that the mixing is the dominant source of the heavy quark spin symmetry breaking, the model of the mixed states described by Eq.(4.30) implies a distinctive pattern of production in the e^+e^- annihilation of the final states $J/\psi\pi\pi$, $\psi'\pi\pi$ and $h_c\pi\pi$ in the energy region of the $Y(4260)$ and $Y(4360)$ resonances including the interference effects. Indeed, due to the heavy quark spin symmetry only the Ψ_3 state is produced by the electromagnetic current, and the decays to the final states with either the J/ψ or ψ' charmonium are due to the same Ψ_3. Thus the amplitudes for production of these final states can be written as

$$A[e^+e^- \to J/\psi(\psi')\pi\pi] \propto \left(\cos^2\theta \, BW_1 + \sin^2\theta \, BW_2\right) \, A[\Psi_3 \to J/\psi(\psi')\pi\pi] \; , \qquad (4.31)$$

where BW_1 and BW_2 stand for the Breit-Wigner resonance factors for respectively $Y(4260)$ and $Y(4360)$, $BW(E) = (E - M + i\,\Gamma/2)^{-1}$ with E being the c.m. energy. On the other hand, the production of the final state $h_c\pi\pi$ is exclusively due to the mixing with Ψ_1, so that the production amplitude reads as

$$A[e^+e^- \to h_c\pi\pi] \propto \cos\theta \, \sin\theta \; (BW_1 - BW_2) \; A[\Psi_1 \to h_c\pi\pi] \; . \qquad (4.32)$$

The proportionality coefficients in Eqs. (4.31) and (4.32) depend on unknown couplings, so that these formulas can be used to describe the behavior of the yield in each channel in the resonance region, but not, say, the relative yield for different channels.

It can be noted that the relative sign between the two Breit-Wigner factors in Eqs. (4.31) and (4.32) is uniquely determined by the inherent in the discussed hadrocharmonium model assumption that the structures $Y(4260)$ and $Y(4360)$ arise from the mixing of two states with definite total spin of the $c\bar{c}$ pair, i.e. Ψ_3 with $S_{c\bar{c}} = 1$ and Ψ_1 with $S_{c\bar{c}} = 0$. This implies that in Ψ_3 and Ψ_1 the heavy quark and antiquark are correlated with each other, rather than each having a strong correlation with the light constituents, which would be the case in a molecular, tetraquark, or hybrid picture. In the latter models individual states would be mixed in the total spin of the $c\bar{c}$ pair, so that the interference pattern between $Y(4260)$ and $Y(4360)$ in the discussed final channels would generally be different.

The behavior of the amplitudes $A[\Psi_3 \to \psi'\pi\pi]$ and $A[\Psi_1 \to h_c\pi\pi]$ is in all likelihood somewhat different from that of $A[\Psi_3 \to J/\psi\pi\pi]$. Namely, the energy released in the pion pair in the former two processes is sufficiently low, and one can rely on the chiral low energy regime, where the amplitudes for these decays are bilinear in the energy or momentum of the two pions [70, 71], and this behavior is in agreement with the reported [54, 55] pion spectra in the transitions $Y(4360) \to \psi' \pi\pi$. Therefore, neglecting the pion mass, one can approximate the rate for these decays is being proportional to the seventh power of the energy release,

$$\Gamma[\Psi_3 \to \psi' \pi\pi] \propto [E - M(\psi')]^7 \; , \Gamma[\Psi_1 \to h_c \pi\pi] \propto [E - M(h_c)]^7 \; . \tag{4.33}$$

Clearly, the strong dependence on energy enhances the rates for the higher peak $Y(4360)$, and the effect is most significant for the emission of ψ' where the available energy is the smallest. On the other hand, the energy release in the transitions to $J/\psi\pi\pi$ exceeds 1.1 GeV, so that the low energy chiral limit is not applicable. Rather one would expect that these latter transitions are dominated by the $f_0(980)$ resonance in the dipion channel, which expectation is supported by the available data [50, 51, 52]. Thus the decay can be approximated as a two-body process: $\Psi_3 \to J/\psi f_0$, so that there is very little phase space kinematical dependence over the energy range of the $Y(4260)$ and $Y(4360)$ resonances.

The approximation in Eq.(4.33) for the phase space integration, as well as the treatment of the transitions to J/ψ as two-body decay can and should be refined by using the actual experimental pion spectra, once more detailed data become available. For the purpose of the present discussion we use the described simplifications and illustrate in Figure 4.1 the energy behavior of the yield in each decay channel in the suggested model. Clearly, the shape of the curve for the $h_c\pi\pi$ channel does not depend on the mixing angle, and is sensitive only to the widths of the two resonances and the mass splitting between them. In the plots of Figure 4.1 , the masses of the resonances are fixed at 4.26 GeV and 4.36 GeV, and we find that choosing $\Gamma[Y(4260)] = 80\,\text{MeV}$ and $\Gamma[Y(4360)] = 100\,\text{MeV}$ produces a ratio of the production rates at 4.26 GeV and 4.36 GeV for the channel $h_c\pi\pi$, that is in a reasonable agreement with the recently reported data [62]. These chosen values of the resonance widths do not exactly coinside with the central values in the Tables [9] ($108 \pm 12\,\text{MeV}$ and $74 \pm 18\,\text{MeV}$), but are compatible with the data, given their present uncertainty. The relative yield at the two peaks in each of the channels $J/\psi\pi\pi$ and $\psi'\pi\pi$ is sensitive to the mixing angle θ, and we find that the value $\theta = 40^\circ$ used in the plots appears to not contradict the current data.

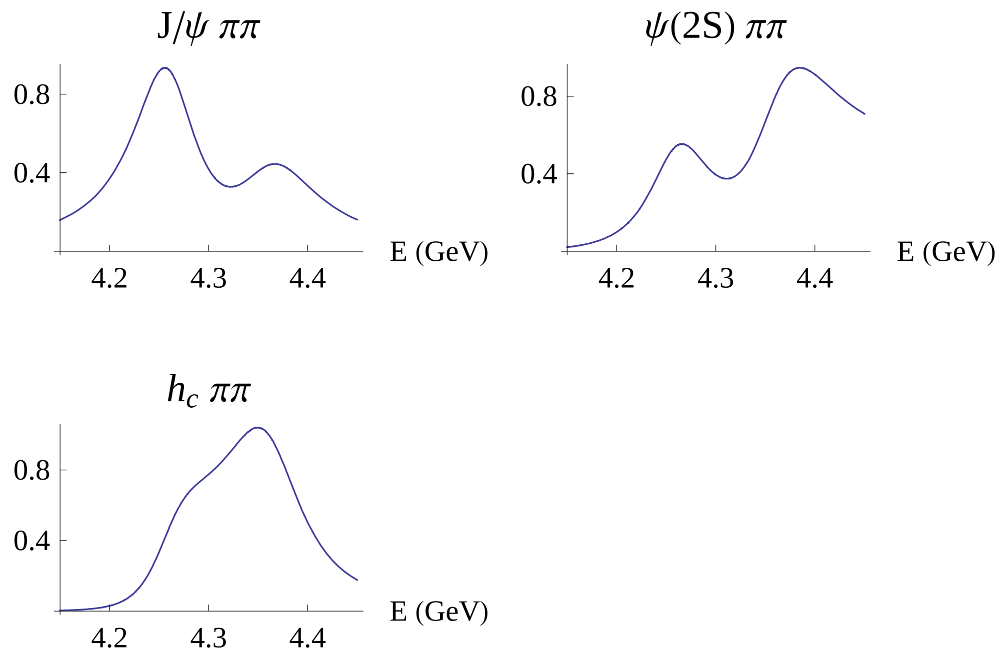

Figure 4.1: The energy dependence of the yield of the final states $J/\psi\pi\pi$, $\psi'\pi\pi$ and $h_c\pi\pi$ (arbitrary units) in the region of the $Y(4260)$ and $Y(4360)$ in the discussed model with mixing of two states.

Although we make no attempt here to analyze the relative production rates between different channels, one may notice that the final state $\psi'\pi\pi$ is strongly kinematically suppressed in comparison with $J/\psi\pi\pi$. Therefore, in order to explain the production of $\psi'\pi\pi$ at the $Y(4360)$ peak with a rate comparable to that of $J/\psi\pi\pi$ at $Y(4260)$, the coupling of the state Ψ_3 to $\psi'\pi\pi$ should be significantly stronger than to $J/\psi\pi\pi$. In the hadrocharmonium picture, this implies that the state Ψ_3 contains mostly ψ', rather than J/ψ, which is quite natural, given the larger spatial size of the excited charmonium. In other words, the so far observed 'affinity' of $Y(4260)$ to J/ψ is a superficial kinematical effect and the underlying hadronic structure in fact contains mostly ψ'.

One can also notice the discussed here mixing model predicts a definite pattern of the interference between the resonances. Namely, the phase between the Breit-Wigner factors is 0° for the heavy quark spin conserving channels $J/\psi\pi\pi$ and $\psi'\pi\pi$ [Eq.(4.31)], and is 180° for the spin violating final state $h_c\pi\pi$ [Eq.(4.32)]. In particular, this phase relation results in the absence of a visible dip in the production rate of the latter final state at energies between the resonances, as can be seen in Figure 4.1.

The large value of the mixing angle, $\theta \approx 40^\circ$, justifies considering the mixing to be the dominant source of the heavy quark spin symmetry breaking and neglecting other possible (smaller) effects of violation of this symmetry, e.g. in the decay amplitudes. Simultaneously the large mixing implies that the unmixed states Ψ_3 and Ψ_1 are very close in mass. Indeed one can readily solve the two state mixing, and find that at $\theta = 40^\circ$ the diagonal masses of Ψ_3 and Ψ_1 should be approximately 4.30 GeV and 4.32 GeV, while the heavy quark spin symmetry breaking mixing amplitude is $\mu \approx 50\,\mathrm{MeV}$. The latter amplitude is of a normal scale expected for the spin symmetry violating effects in the charm sector. On the other hand, the proximity of the unmixed states in mass to within about 20 MeV may appear acidental, but to the best of our knowledge cannot be ruled out. In this context the state Ψ_3 can be viewed e.g. as ψ' embedded in a light-quark mesonic excitation with quantum numbers $J^{PC} = 0^{++}$, while Ψ_1 is an h_c bound in an excited 0^{-+} light-quark mesonic state. This possible picture of the unmixed hadrocharmonium states, might require a clarification, regarding the quantum numbers of the light degrees of freedom in the discussed two-pion transitions. For the state $\Psi_3 \sim (1^{--})_{c\bar{c}} \otimes (0^{++})_{q\bar{q}}$ the picture of the transition is quite straightforward: both pions in the decay to $J/\psi\pi\pi$, or $\psi'\pi\pi$ can be emitted in the S-wave by the 0^{++} component, so that no transfer of angular momentum to the $c\bar{c}$ pair is necessary. The picture is however necessarily different for the decay $\Psi_1 \to h_c\pi\pi$. Indeed, for soft pions the amplitude of the latter decay has the form

$$A(\Psi_1 \to h_c\pi\pi) \propto \epsilon_{ijk}\, h_{ci}\Psi_{1j}(E_2 p_{1k} + E_1 p_{2k}) \; , \tag{4.34}$$

where $\vec{\Psi}_1$ and $\vec{h}_c$ are the polarization amplitudes of the initial Ψ_1 and the final h_c, and $\vec{p}_1$, $\vec{p}_2$ (E_1, E_2) are the momenta (energies) of the two pions. One can thus see that the angular momentum of the $c\bar{c}$ pair has to be rotated. This however does not imply a violation of the heavy quark spin symmetry, since it is not the spins of the heavy quarks but rather their (P-wave) angular momentum that is rotated. The amplitude (4.34) can thus be represented as arising from the action on the initial state of the operator $\mathcal{O} = \ell_i(E_2 p_{1i} + E_1 p_{2i})$, involving the operator $\vec{\ell}$ of the angular momentum of the heavy quark pair. Clearly, the operator $\mathcal{O}$ has the quantum numbers 0^{-+}, so that the simplest hadrocharmonium configuration, linked by this operator to the final state $h_c\pi\pi$, is $\Psi_1 \sim (1^{+-})_{c\bar{c}} \otimes (0^{-+})_{q\bar{q}}$ [4] .

It can be noted that the possible latter structure of the state Ψ_1 also suggests that there can be a substantial yield in the not yet observed channel $e^+e^- \to h_c\eta$ in the same energy range of the $Y(4260)$ and $Y(4360)$ resonances. Due to the present uncertainty in understanding the conversion of the light degrees of freedom in hadrocharmonium into light mesons it is difficult to offer a specific prediction for the cross section. It is quite possible however that the yield of the $h_c\eta$ final state can be comparable to that of $h_c\pi\pi$.

Furthermore, as previously discussed, combining the light-matter excitations with the states of charmonium generally gives rise to a number of new charmonium-like resonances. In particular, in the discussed here picture, besides the states $\Psi_3 \sim (1^{--})_{c\bar{c}} \otimes (0^{++})_{q\bar{q}}$ and $\Psi_1 \sim (1^{+-})_{c\bar{c}} \otimes (0^{-+})_{q\bar{q}}$ one might expect existence of hadrocharmonium states with the structure $(1^{--})_{c\bar{c}} \otimes (0^{-+})_{q\bar{q}}$ (with a mass approximately 3.9 GeV), and $(1^{+-})_{c\bar{c}} \otimes (0^{++})_{q\bar{q}}$ (at approximately 4.7 GeV). It is clear however that these isoscalar resonances should have quantum numbers $J^P = 1^{+-}$ and would not be directly produced in e^+e^- annihilation, or in single pion transitions from the states produced in e^+e^- annihilation.

Very shortly after our hadro-charmonium models was published in the arXiv, the BESIII experiment made available [72] the data on the energy dependence of the cross section for the process $e^+e^- \to h_c\pi^+\pi^-$ in the range of E from 4.19 GeV to 4.42 GeV. We thus subsequently attempt at fitting the data within the suggested two resonance model, with the interference between the resonances described by Eq.(4.32). In performing the fit we allowed the masses and the widths in the two Breit-Wigner factors to float, as well as the overall normalization factor, thus resulting in the total of five fit parameters. Furthermore we included only the statistical experimental errors in our calculation of χ^2, and not included the reported systematical errors. We believe that this is the proper procedure, since the systematical errors in the data [72] arise from the uncertainty in

[4] One can also notice that the quantum numbers of the emitted dipion in its center of mass frame are 0^{++} and 2^{++}, and these combine with the angular momentum in the rest frame of the heavy quarkonium to ensure the conservation of the overall angular momentum and the parity.

the normalization and are in fact proportional to the central values of the data at each energy point, thus being strongly correlated. Since the overall normalization is one of our fit parameters, the experimental uncertainty in the normalization is absorbed into the definition of this overall factor It should be also noted that using the low energy approximation for the pion emission amplitude introduces an uncertainty at higher energies. A better procedure would require the data on the actual pion spectra at each energy. Lacking such data, we estimate the effect of the possible inaccuracy of our treatment of the higher energies by comparing the results of the fit to all the available data and to the same data with the highest energy point excluded. We find that the extracted parameters of the lower mass resonance and the overall quality of the fit are quite stable under this variation of the procedure, while the most affected is the width Γ_2 of the heavier resonance. Also with the limited experimental information available to us at present, we do not attempt to evaluate the errors in the extracted parameters and quote here only the 'central' values corresponding to the minimum of χ^2.

The result of our fit, using all ten data points from 4.19 GeV to 4.42 GeV, for the masses and the widths of the resonances is $M_1 = 4213\,\text{MeV}$, $\Gamma_1 = 69\,\text{MeV}$, $M_2 = 4379\,\text{MeV}$, $\Gamma_2 = 160\,\text{MeV}$ with $\chi^2/N = 6.0/5$, where N is the number of degrees of freedom. The fit with the data point at $E = 4.42\,\text{GeV}$ excluded yields $M_1 = 4214\,\text{MeV}$, $\Gamma_1 = 61\,\text{MeV}$, $M_2 = 4351\,\text{MeV}$, $\Gamma_2 = 117\,\text{MeV}$ with $\chi^2/N = 3.6/4$. The input data and the fit curves are shown in Figure 4.2.

One can readily notice that our fit results in a lower, than the table value, mass of the lower resonance $Y(4260)$, $M_1 \approx 4215\,\text{MeV}$. This low value is compatible within the errors with the one reported in Ref. [52], but does not appear to agree with Refs. [50, 51, 53]. It should be noted however that all the previous determinations were done using the final state $J/\psi\pi\pi$. The production of this final state, as well as of $\psi'\pi\pi$ requires no violation of the heavy quark spin symmetry and may receive an unsuppressed contribution from the non resonant continuum. The interference between the $Y(4260)$ resonance and the continuum amplitude may generally result in a shift of the apparent position of the resonance. It is not clear however whether this shift can be large enough to explain the difference between the results of our fit to the data [72] and the previous determinations. In either case the significance of this discrepancy can possibly be understood with a more detailed set of data.

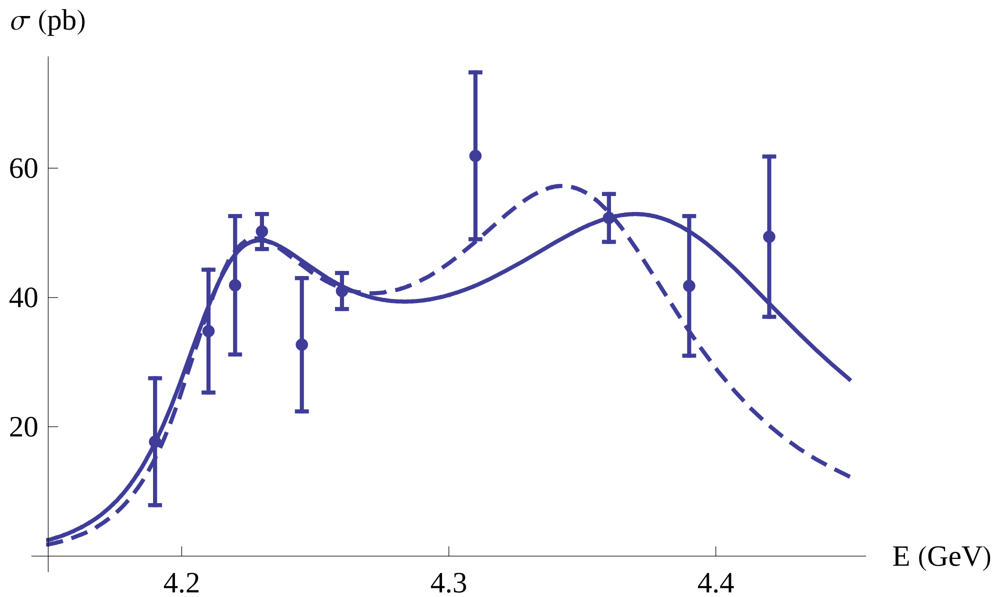

Figure 4.2: The energy dependence of the cross section $\sigma(e^+e^- \to h_c\pi^+\pi^-)$. The data points from Ref. [72] are shown with the statistical errors only. The curves show the behavior in the two-resonance model with the parameters determined from the fit with all the data points (solid) and with the highest energy data point excluded (dashed).

4.3 $X(3915)$

The charmoniumlike state $X(3915)$ presents a considerable challenge for understanding its internal structure. This resonance with the mass [9] $M = 3918.4 \pm 1.9\,\text{MeV}$ and width $\Gamma = 20 \pm 5\,\text{MeV}$ has been observed only in the decay mode $X(3915) \to \omega J/\psi$ in B decays, $B \to K\,X(3915) \to K\,\omega J/\psi$ [73, 74], with the combined branching fraction measured as

$$\mathcal{B}\left[B^+ \to K^+\,X(3915)\right]\,\mathcal{B}\left[X(3915) \to \omega J/\psi\right] = \left(3.0^{+0.9}_{-0.7}\right) \times 10^{-5}\ , \tag{4.35}$$

and in two photon production [75, 76] with the yield described by [9]

$$\Gamma\left[X(3915) \to \gamma\gamma\right]\,\mathcal{B}\left[X(3915) \to \omega J/\psi\right] = 54 \pm 9\,\text{eV}\ . \tag{4.36}$$

Furthermore, a BaBar angular analysis [76] of the two photon production favors the spin-parity assignment $J^P = 0^+$ for $X(3915)$. For this reason an assignment [77] of this resonance as a radially excited charmonium ${}^3P_0$ state $\chi_{c0}(2P)$ was hastily adopted by BaBar [76] and later by PDG [9]. However it has been convincingly argued [78, 79] that the observed properties of the resonance $X(3915)$ are highly unlikely for a $\chi_{c0}(2P)$ state, suggesting [78] that the actual $\chi_{c0}(2P)$ lies at the mass of about 3840 MeV and has a broad width, $\Gamma \sim 200\,\mathrm{MeV}$ dominated by the decay to $D\bar{D}$ pairs, and thus leaving the resonance $X(3915)$ to be interpreted as an exotic state.

In this section we discuss the possibility that this resonance is a bound state of a $D_s\bar{D}_s$ meson pair, whose threshold is the nearest in mass to $X(3915)$. We argue that such interpretation might explain, although currently with a considerable uncertainty, the unusual observed properties of this resonance and that it can be tested experimentally, e.g. by studying the decays $B \to K\, D_s\bar{D}_s$ near the threshold of the strange charmed meson-antimeson pair. The discussion is based on the Ref. [80]. We start with briefly review peculiarities in the measured properties of $X(3915)$ pointed out in Refs. [78, 79] that, in particular, preclude the $\chi_{c0}(2P)$ assignment and then discuss how those properties can be implemented in the $D_s\bar{D}_s$ molecular model.

A major peculiarity of the resonance $X(3915)$ is the large combined branching fraction in Eq.(4.35) considered together with the general understanding that for a pure charmonium the rates of transitions with emission of light hadrons are typically quite small with the largest rate [that of $\psi(2S) \to \pi\pi\, J/\psi$] being of the order of 0.1 MeV. Indeed, if one conservatively caps the rate of the decay $X(3915) \to \omega J/\psi$ at 1 MeV, corresponding to $\mathcal{B}\,[X(3915) \to \omega J/\psi] < 5\%$, the branching fraction for the B decay, $\mathcal{B}\,[B^+ \to K^+\, X(3915)]$ would be estimated as being larger than approximately 6×10^{-4}, which would make the $X(3915)$ one of the most abundantly produced charmonium states in the decays of B meson. In particular, this estimate would exceed the measured yield of the $\chi_{c0}(1P)$ state, $\mathcal{B}\,[B^+ \to K^+\, \chi_{c0}(1P)] = (1.50^{+0.15}_{-0.14}) \times 10^{-4}$, which is hardly acceptable in any reasonable theoretical scheme.

Another unusual property of $X(3915)$ is the apparent absence of the dominance of its decay into $D\bar{D}$ meson pairs. Namely, the current measurements show no signal for such decay, and using the available Belle data [81] Olsen [79] estimates $\mathcal{B}\left[X(3915) \to D^0\bar{D}^0\right] < 1.2\,\mathcal{B}\,[X(3915) \to \omega J/\psi]$, which, given the isotopic symmetry, translates to

$$\mathcal{B}\left[X(3915) \to D\bar{D}\right] < 2.4\,\mathcal{B}\,[X(3915) \to \omega J/\psi]\ . \tag{4.37}$$

This behavior is in a remarkable contrast with expectation for a 0^+ charmonium and with the one observed for the spin-2 resonance identified [9] as $\chi_{c2}(2P)$ with mass 3927.2 ± 2.6 MeV, whose width $\Gamma = 24 \pm 6$ MeV is practically saturated by its decay into $D-$wave $D\bar{D}$ meson pairs. [5]

Clearly, the conclusion regarding an unacceptably high rate of the decay $B \to K\,X(3915)$ derived from the experimental number in Eq.(4.35) is invalidated if the branching fraction $\mathcal{B}\,[X(3915) \to \omega J/\psi]$ is significantly larger than 5%, say about 30%, while the upper bound in Eq.(4.37) could be satisfied if one could identify a mechanism for suppression of the decay $X(3915) \to D\bar{D}$. We will argue here that both these requirements might be fulfilled if the resonance $X(3915)$ is an $S-$wave bound molecular state of the $D_s\bar{D}_s$ meson pair. The binding energy Δ in this case is

$$\Delta = 2M(D_s) - M_X = 18.2 \pm 1.9\,\mathrm{MeV}\ , \tag{4.38}$$

so that the bound state is moderately shallow.

The argument involving the perceived relative suppression of the decay $X(3915) \to \omega J/\psi$ is based on invoking the Okubo-Zweig-Iizuka (OZI) rule. For a $D_s\bar{D}_s$ state both this decay and the decay to $D\bar{D}$ would be OZI suppressed and may thus have a comparable rate. As for the absolute rate of $X(3915) \to \omega J/\psi$, it can be recalled that the original application of the OZI rule was for an explanation of the suppression of the decay $\phi \to \rho\pi$. While the latter decay is indeed OZI suppressed relative to the decay into $K\bar{K}$, in absolute terms its width is approximately 0.65 MeV, even though this is a $P-$wave decay with a moderate c.m. momentum of 179 MeV. The decay $X(3915) \to \omega J/\psi$ is an $S-$wave process where, in the discussed picture, the $s\bar{s}$ pair converts to ω with no apparent kinematical suppression since the momentum in this decay is about 220 MeV. It thus does not seem unreasonable to suggest that even though the latter decay is OZI suppressed, similarly to $\phi \to \rho\pi$, the $S-$wave amplitude is enhanced in comparison with the $P-$wave in the ϕ meson decay, so that an order of magnitude larger decay rate, $\Gamma\,[X(3915) \to \omega J/\psi] \sim 6$ MeV, required for making the branching fraction of approximately 30%, might in fact be not untypical.

The decay into the pairs of non-strange D mesons, $X(3915) \to D\bar{D}$, is similarly OZI suppressed, so that it should come as no surprise that its rate is comparable to

[5] The very small mass splitting of only about 9 MeV between the χ_{c2} and $X(3915)$ is used [78] as an additional argument against the interpretation of $X(3915)$ as $\chi_{c0}(2P)$ charmonium state.

that of the decay to $\omega J/\psi$. Furthermore the OZI rule also keeps $X(3915)$ as a $D_s\bar{D}_s$ molecule from a significant mixing with ${}^3P_0$ states of charmonium, which mixing would also certainly give rise to enhanced decays into $D\bar{D}$.

A possible caveat in the discussed scheme for $X(3915)$ and its decays is related to the decay $X(3915) \to \eta\eta_c$, which in fact is the only possible process for a $D_s\bar{D}_s$ molecule that is not OZI suppressed. A recent search for this process by Belle [82] resulted in the upper bound (at 90% CL)

$$\mathcal{B}\left[B^+ \to K^+\, X(3915)\right]\, \mathcal{B}\left[X(3915) \to \eta\eta_c\right] < 3.3 \times 10^{-5} \; . \tag{4.39}$$

By comparing this bound with the number in Eq.(4.35) one can readily see that the decays $X(3915) \to \omega J/\psi$ and $X(3915) \to \eta\eta_c$ can have comparable rates. It should be pointed out that there is at least one possible factor that somewhat mitigates the OZI suppression of the former process in comparison with the latter. This factor is related to the heavy quark spin structure of an $S-$wave of two pseudoscalar mesons $D_s\bar{D}_s$. In the mesons the spin of the heavy quark (antiquark) is fully correlated with the spin of the light antiquark (quark). As a result the meson-antimeson system is a mixed state with regards to the total spin of the $c\bar{c}$ pair. The heavy quark spin decomposition for the $S-$wave pair of pseudoscalar mesons [21] reads as

$$\left(D_s\bar{D}_s\right)_{0^+} = \frac{1}{2}\, 0_H \otimes 0_L - \frac{\sqrt{3}}{2}\, 1_H \otimes 1_L \; , \tag{4.40}$$

so that in the discussed $D_s\bar{D}_s$ molecule the probability for the $c\bar{c}$ pair to be in a spin triplet state is 3/4, while the probability for a spin singlet is 1/4. Since the heavy quark spin is approximately conserved, the transition to the spin triplet J/ψ level is enhanced by a factor of three over that to the spin singlet η_c. Furthermore, the OZI allowed transition of the $s\bar{s}$ pair to the η meson, $\eta = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$ in the limit of flavor $SU(3)$ symmetry, carries another factor of 2/3 in the probability. As a result the ratio of the transition rates can be factorized as

$$\frac{\Gamma(D_s\bar{D}_s \to \eta\eta_c)}{\Gamma(D_s\bar{D}_s \to \omega J/\psi)} = \frac{2}{9}\, \frac{p_\eta}{p_\omega}\, \frac{1}{R}\, \left|\frac{F(p_\eta)}{F(p_\omega)}\right|^2 \; , \tag{4.41}$$

where $R < 1$ is the OZI suppression factor, $p_\omega \approx 220\,\mathrm{MeV}$ ($p_\eta \approx 665\,\mathrm{MeV}$) is the momentum of ω (η) in the decay, and $F(p)$ is the form factor accounting for the absorbed recoil momentum by the charmonium ground state. One can see that, although a larger momentum in the η transition gives a larger phase space, the reduction of the amplitude by the form factor $F(p)$ can be more significant, since p_η is comparable with the typical momentum in the ground state charmonium. Clearly, lacking a better description of the discussed transitions, it appears impossible to reliably estimate the ratio of the rates in Eq.(4.41), and the question as to whether the η transition can have a rate not larger than the decay to $\omega J/\psi$ remains open, and one has to turn to other ways of testing the suggested hidden strangeness molecular structure of $X(3915)$.

One characteristic signature of a bound $D_s\bar{D}_s$ $S-$wave state arises if one treats it as shallow bound state dominating the near threshold $D_s\bar{D}_s$ scattering amplitude. At the binding energy as in Eq.(4.38) the characteristic under-threshold momentum is $\kappa = \sqrt{M_D\Delta} \approx 190\,\text{MeV}$, so that the amplitude can be described by the scattering length

$$a = \frac{1}{\kappa} \approx 1\,\text{fm}\ , \tag{4.42}$$

which also determines the characteristic distances at which the mesons move in the bound state. It should be mentioned that, most likely, the 1 fermi length scale is near the limit at which the effective radius approximation can be applied to the amplitude, or, in other words, the bound state at $18\,\text{MeV}$ is at the limit of being considered as 'shallow'. Also such distances are comparable with the size of the mesons, so that considering them as point objects is only marginally applicable. However in lieu of a more detailed approach we use here the approximation of a 'large' scattering length for description of the near threshold dynamics of a $D_s\bar{D}_s$ pair. In this approximation the $D_s\bar{D}_s$ scattering amplitude at a momentum k above the threshold can be written as

$$f = -\frac{1}{\kappa + i\gamma + ik} \tag{4.43}$$

where the imaginary shift $i\gamma$ for the position of the pole in k takes into account the inelasticity due to the width Γ of the resonance $X(3915)$, $\kappa + i\gamma = \sqrt{M_D(\Delta + i\Gamma/2)}$ (numerically $\gamma \approx 50\,\text{MeV}$).

In the processes, where the $D_s\bar{D}_s$ pair is produced by a source localized at distances shorter than a, the production near the threshold can be considered as dominated by a resonance process, and the usage of the resonance amplitude in the form (4.48) is sometimes referred to as the Fattè parametrization [83]. Applying this parametrization to the decays $B \to K D_s\bar{D}_s$, one readily finds that the distribution in the invariant mass

$M_{D\bar{D}}$ of the $D_s\bar{D}_s$ pairs near the threshold is given by

$$\frac{d}{dM_{D\bar{D}}}\Gamma(B \to K D_s \bar{D}_s) = C\,\frac{k}{\kappa^2 + (\gamma + k)^2} \tag{4.44}$$

where C is a constant and $k = \sqrt{M_D(M_{D\bar{D}} - 2M_D)}$ is the relative momentum of the mesons in their center of mass frame. In the expression (4.44) we neglect the effect of the Coulomb interaction between the mesons, which effect itself depends on the meson-antimeson strong interaction [39] and becomes significant only in the immediate vicinity of the threshold. Clearly, the resonance corresponding to a shallow bound state enhances the yield of the meson pairs near the threshold as illustrated in Figure 4.3 and such enhancement can be tested in the existing or future data on the B decays.

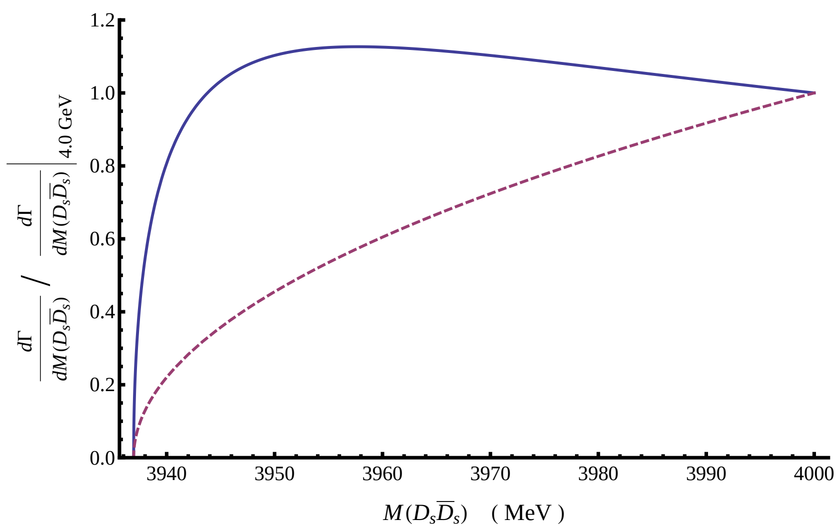

Figure 4.3: The spectrum of invariant mass for the $D_s\bar{D}_s$ system near the threshold with the enhancement due to a bound state (solid) as described by Eq.(4.44), compared with the $S-$wave phase space (dashed). Both curves are normalized to the same value at $M(D_s\bar{D}_s) = 4.0\,\text{GeV}$.

According to the discussed picture of the $X(3915)$ resonance, the bulk of its total width is thus the sum of the decay rates to $\omega J/\psi$, $\eta\eta_c$ and $D\bar{D}$ all contributing a comparable amount. One more type of decay, possibly contributing a slightly smaller but still noticeable amount to the total width is the decay to light hadrons due to the annihilation of the charmed quark and antiquark. In order to estimate (very approximately) the rate of such decays we start with a discussion of $X(3915)$ decay into two photons whose rate enters Eq.(6.7). For an $S-$wave nonrelativistic $D_s\bar{D}_s$ meson pair in a state described by a wave function $\psi(r)$ the width for annihilation into $\gamma\gamma$ is given by

$$\Gamma(D_s\bar{D}_s \to \gamma\gamma) = \frac{2\pi\,\alpha^2}{M_D^2}\,|\Phi|^2\,|\psi(0)|^2 \ , \tag{4.45}$$

where Φ is the form factor for the vertex $D_s\bar{D}_s\gamma\gamma$, normalized so that $\Phi = 1$ for point particles of unit charge. Using for an (very approximate) estimate $|\psi(0)|^2 \sim \kappa^3/\pi$ in the bound state $X(3915)$, one gets

$$\Gamma[X(3915) \to \gamma\gamma] \sim 0.2\,|\Phi|^2\,\mathrm{keV} \tag{4.46}$$

which is in the same ballpark as the experimental number (6.7), if $|\Phi|$ is not significantly less than one and if, as we discuss here, $\mathcal{B}\,[X(3915) \to \omega J/\psi] \sim 0.3$.

The annihilation of the $c\bar{c}$ in from the molecular state can proceed either through two gluons for the heavy quark pair in a spin singlet state, or through a one-gluon mediated process $c\bar{c} \to q\bar{q}$ for the $c\bar{c}$ pair in a color-octet spin-triplet state. For the known charmonium states: η_c, χ_{c0}, χ_{c2}, that decay both to two photons and to light hadrons through two gluons, the ratio of the $\gamma\gamma$ decay rate to that of the hadronic decay is around 2×10^{-4}. Allowing for the additional contribution of the annihilation through one gluon from the molecular state, it appears reasonable to approximately estimate the similar ratio for $X(3915)$ as

$$\frac{\Gamma[X(3915) \to \gamma\gamma]}{\Gamma[X(3915) \to \text{light hadrons}]} \sim 10^{-4} \ , \tag{4.47}$$

so that in absolute terms the annihilation width of $X(3915)$ is likely to be about (1.5 - 2) MeV and contribute about 10%, or slightly less to its total width. Naturally, one should expect, due to the presence of the $s\bar{s}$ quark pair in $X(3915)$, an enhanced K/π ratio in such annihilation decays. However it is not clear whether this enhancement can be detected over the background, since these decays contribute only a rather small fraction of the total width of the resonance.

To summarize this section, we argue that the unusual observed properties of the $X(3915)$ resonance, in particular its apparently large decay rate to $\omega J/\psi$ and a suppressed decay into $D\bar{D}$ pairs may be understood if this resonance is a molecular bound state of a $D_s\bar{D}_s$ meson pair. Unavoidably, our estimates of the properties of such molecular object are subject to large uncertainties because of the current lack of a reliable description of the strong multiquark dynamics. However the possibility that $X(3915)$ is an under threshold pole in the $D_s\bar{D}_s$ scattering amplitude located close to the physical region can be tested experimentally by searching for the enhanced yield of these meson-antimeson pairs near the threshold, e.g. in the decays $B \to K D_s\bar{D}_s$.

4.4 $\chi_{c0}\,\omega$ Production in e^+e^- Annihilation through $\psi(4160)$

The recently reported [84] BESIII data on production of the final states $\chi_{cJ}\,\omega$ in the e^+e^- annihilation at $\sqrt{s}$ from 4.21 to 4.42 GeV indicate a peak in the cross section for $\chi_{c0}\,\omega$ at about 4.23 GeV and apparently no corresponding peaks above the thresholds for $\chi_{cJ}\,\omega$ with $J = 1$ and $J = 2$. The best fit to the $\chi_{c0}\,\omega$ data with a resonance curve yields the parameters of the resonance[84]: $M = (4230 \pm 8)$ MeV and $\Gamma = (38 \pm 12)$ MeV, which parameters do not correspond to any of the previously known charmonium-like states. It is not unusual recently that new quarkonium-like resonances are revealed in various channels with a hidden heavy flavor, and the newly observed peak could indicate an existence of another such state. In this section, however, we explore a somewhat more routine interpretation of the observed peak as being due to a well known charmonium $J^{PC} = 1^{--}$ resonance, namely the $\psi(4160)$. The detailed discussion can be found in Ref. [85].

Despite the notation $\psi(4160)$ the actual mass of this state is in fact [9] $M_\psi = (4191 \pm 5)$ MeV (and $\Gamma_\psi = (70 \pm 10)$ MeV. The shift in the mass from the initial data to the current higher value is mostly due to the re-analysis [86] of the interference with the $\psi(4040)$ peak. We argue that a close proximity of the actual mass of $\psi(4160)$ to the observed enhancement of the $e^+e^- \to \chi_{c0}\,\omega$ cross section makes our interpretation well compatible with the data. The suppression of the production of χ_{c1} and χ_{c2} in similar processes above their thresholds as well as of χ_{c0} in that range of higher energy is then merely due to larger distance from the resonance peak.

If confirmed by the future data, the discussed interpretation would imply that the process $e^+e^- \to \chi_{c0}\,\omega$ is not as much of direct relevance to the searches for new charmonium-like states, possibly of a complex structure, but rather falls into another very interesting class of hadronic transitions between quarkonium states, more specifically, of the transitions with the emission of the ω meson. Such transitions between the $J^{PC} = 1^{--}$ and the χ_J states were observed in both charmonium: $\chi_{c0}(2P) \to J/\psi\,\omega$ [87], and bottomonium: $\chi_{b1,b2}(2P) \to \Upsilon(1S)\,\omega$ [88], and most recently [89] at the $\Upsilon(5S)$ resonance: $\Upsilon(5S) \to \chi_{b1}\,\omega$ (and also an indication of $\Upsilon(5S) \to \chi_{b2}\,\omega$). These processes are not suppressed by any (approximate) symmetries in QCD and are allowed to proceed in the S wave [6] . In the limit of exact heavy quark spin symmetry the transitions for the χ_J states with different J are related by the generic expression [90] for the S-wave amplitude:

$$A(\psi\chi_J\omega) = g_\omega \left(\psi_i\omega_i\chi_0 + \sqrt{\frac{3}{2}}\,\epsilon_{ijk}\psi_i\omega_j\chi_k + \sqrt{3}\,\psi_i\omega_j\chi_{ij} \right) , \tag{4.48}$$

where the nonrelativistic limit for heavy quarkonium is assumed with the nonrelativistic normalization for the heavy states, $\vec{\psi}$ and $\vec{\omega}$ stand for the polarization amplitudes for the 1^{--} state (ψ) and the ω meson, and χ_0, $\vec{\chi}$ and χ_{ij} are the amplitudes for the χ_J states. The dimensionless constant depends on the specific 1^{--} state and on the considered multiplet of the χ_J states. In our normalization the rate of e.g. the decay $\psi(4160) \to \chi_{c0}\,\omega$ is given by

$$\Gamma[\psi(4160) \to \chi_{c0}\,\omega] = g_\omega^2 \frac{p_\omega}{2\pi} \tag{4.49}$$

with p_ω being the momentum of the emitted ω.

If one neglects the widths of the ω and χ_{c0} resonances and approximates the $\psi(4160)$ resonance by a simple Breit-Wigner shape, the cross section for the process $e^+e^- \to \psi(4160) \to \chi_{c0}\,\omega$ is given by

$$\sigma(e^+e^- \to \chi_{c0}\,\omega) = g_\omega^2 \,\frac{3}{2M^2}\, \frac{\Gamma_{ee}\,p_\omega(E)}{(E-M)^2 + \Gamma^2/4} \tag{4.50}$$

where $E = \sqrt{s}$ is the center of mass energy, M and Γ are the mass and the total width of the $\psi(4160)$ resonance, and Γ_{ee} is its partial width of decay to e^+e^- currently measured with a considerable uncertainty [9], $\Gamma_{ee} = (0.48 \pm 0.22)$ keV. Assuming the central value for the latter width, the expression in Eq.(4.50) reproduces the experimentally measured

[6] Within the multipole expansion in QCD [11] these processes arise in the third order in the leading $E1$ interaction [90]. This illustrates absence of a suppression, even though the multipole expansion is hardly applicable at a quantitative level to the processes with highly excited quarkonium states.

cross section of approximately 55 pb at $E = 4230$ MeV, at which point the largest statistics is available and the experimental errors are the smallest, if $g_\omega^2 \approx 4 \times 10^{-2}$. Naturally, the uncertainty in this estimate is large, about 50%, mainly due to poor knowledge of the Γ_{ee}.

For a more detailed fit to the data of the shape of the energy dependence of the cross section described by the discussed resonance mechanism, especially at the lower end of the relevant energy range, we have included the effects of the finite widths of the ω resonance (8.5 MeV) and of the χ_{c0} charmonium state (10.5 MeV). Since we neglect any possible non-resonant background and any variation of the width of $\psi(4160)$ at energies well above the resonance, the only parameter in the fit is the overall normalization, i.e. the coupling g_ω^2. A comparison of our fit with the data is shown in Figure 4.4. The quality of the fit is $\chi^2/N_{d.o.f.} = 9.9/8$, which although is likely worse than the fit to a new resonance in Ref. [84], but is still compatible with the data within one standard deviation. It can be noted that most likely the simple Breit-Wigner approximation has to be modified at the higher end of the measured energy range. Thus if we fit only the data at 4.31 GeV and below (i.e. not including the three highest energy points) the figure of merit for our fit improves to $\chi^2/N_{d.o.f.} = 4.0/5$.

Also shown in Fig. 1 are the expected from the formulas in Eqs. (4.48) and (4.50) curves for the cross section in the channels $\chi_{c1}\,\omega$ and $\chi_{c2}\,\omega$. It should be noted however that a straightforward application of these formulas may suffer from relatively large modifications. Such modifications can arise both from a deviation at higher energies of the resonance curve from the Breit-Wigner approximation, and from possible other non-resonant contributions to the production mechanism, e.g. mediated by the heavy meson-antimeson pairs [23, 91]. In particular, the latter contributions can potentially violate the heavy quark symmetry prediction for the ratio of the yield σ/p_ω in the channels $\chi_{cJ}\,\omega$ with different J: $\chi_{c0} : \chi_{c1} : \chi_{c2} = 1 : 3 : 5$.

We have also examined a possible contribution of the higher charmonium-like resonance $\psi(4415)$. The data [84] indicate no enhancement at the energy of this state. Neither our fit showed any improvement with inclusion of this additional resonance. We find that its contribution to the production of $\chi_{c0}\,\omega$ should be less than 10% of that of $\psi(4160)$ in the amplitude. This probably indicates that the resonances $\psi(4160)$ and $\psi(4415)$ have significantly different internal dynamics.

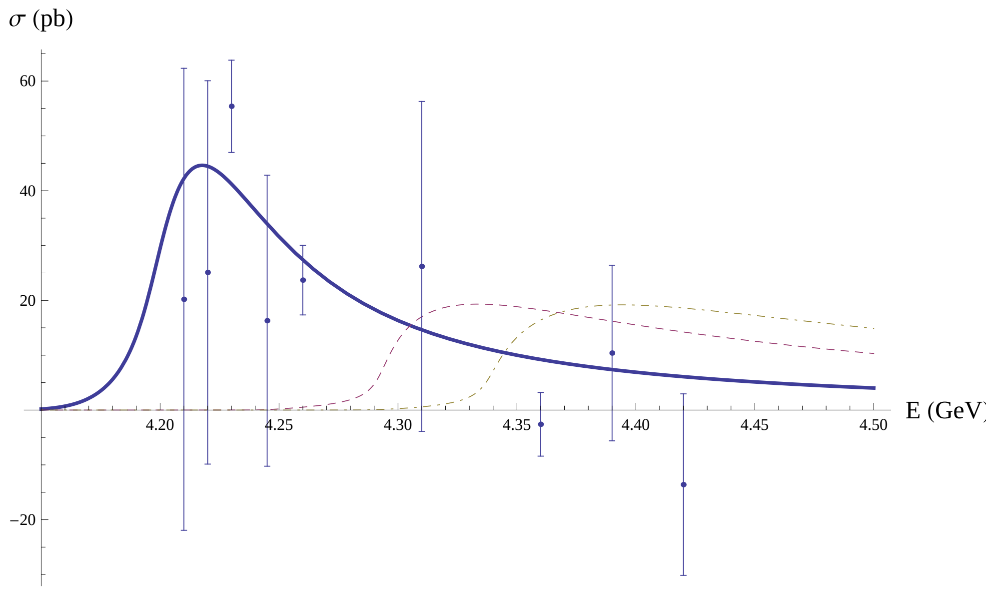

Figure 4.4: The fit of the $\psi(4160)$ resonance contribution to the data [84] for $\sigma(e^+e^- \to \chi_{c0}\,\omega)$ (solid). Also shown are the curves for the cross section in the channels $\chi_{c1}\,\omega$ (dashed) and $\chi_{c2}\,\omega$ (dot-dash).

It may also be instructive to compare the relative strength of the coupling in the ω transitions in the charmonium and bottomonium sectors. The observed [89] rate of the transition $\Upsilon(5S) \to \chi_{b1}\,\omega$ corresponds to $g_\omega^2(5S \to 1P) \approx 3 \times 10^{-4}$, i.e. an order of magnitude smaller in the amplitude than the discussed coupling in charmonium. It is not clear however to what extent this transition is representative for bottomonium, since for the transitions from $\Upsilon(5S)$ the data indicate a strong violation of the heavy quark symmetry prediction $(5:3)$ for the relative yield of χ_{b2} and χ_{b1}. Whatever the mechanism responsible for the enhanced spin symmetry breaking is (e.g. a contribution of meson-antimeson states [23]), it would also affect the strength of the coupling for the ω emission. Perhaps, a more sensible comparison could be done using the data on the decays $\chi_{b1,b2}(2P) \to \Upsilon(1S)\,\omega$ for which the data on their relative rate do not contradict the HQSS. However the absolute rate of the ω transitions from the $\chi_{bJ}(2P)$ bottomonium to $\Upsilon(1S)$ is unknown since the total widths of the $\chi_{bJ}(2P)$ states is not measured. Using the theoretical estimates [92, 93] of the radiative and the total widths of the bottomonium $2P$ states one may deduce e.g. $\Gamma[\chi_{b1}(2P) \to \Upsilon(1S)\,\omega] \approx 1.5\,\mathrm{keV}$, and thus estimate $g_\omega^2(2P \to 1S) \approx 2 \times 10^{-5}$. Clearly, these estimates point to a much weaker coupling for the ω transitions in bottomonium as compared to charmonium. Qualitatively, a weaker coupling for the $b\bar{b}$ system would be expected on general grounds.

In summary, we find that a minimalistic description of the recent BESIII data on $e^+e^- \to \chi_{cJ}\,\omega$ in terms of the effect of the known resonance $\psi(4160)$ is quite compatible with the measurements of the cross section. This description implies that the coupling of the $\psi(4160)$ to the channels $\chi_{cJ}\,\omega$ is of the same order as in another known charmonium process $\chi_{c0}(2P) \to J/\psi\,\omega$. In both these cases the interaction of the ω meson is significantly stronger than in similar processes in bottomonium — a behavior that is generally expected, but not yet described quantitatively.

Chapter 5

A Possible Violation of Parton Picture in Decays of $h_b(2P)$

As the experimental results accumulated for quarkonium, the detailed analysis of the data become available. It allows us to test our theoretical understanding more carefully. This chapter will discuss an anomaly revealed in decays of $h_b(2P)$. The spin-singlet P wave $^1P_1$ states of bottomonium $h_b(1P)$ and $h_b(2P)$ provide ample opportunities to study the QCD dynamics of heavy quarkonium. These resonances were observed by Belle [94] in two-pion transitions from $\Upsilon(5S)$ and also an evidence of the transition $Upsilon(3S) \to h_b(1P)\,\pi^0$ was presented by BaBar [95]. The theoretical expectations for the masses and decay properties of these resonances were previously considered in the literature, and the most detailed compllation and discussion can be found in Ref. [93]. The theoretical treatment of the $^1P_1$ states is facilitated by their relation, within the nonrelativistic description of bottomonium, to the spin-triplet $\chi_{bJ}(1P)$ and $\chi_{bJ}(2P)$ states. The expected masses of the h_b resonances are determined by the 'center of gravity' of the corresponding triplet states (and are in a remarkable agreement with the measured values), while the decay properties of the $h_b(1P)$ and $h_b(2P)$ particles are most naturally related to those of the corresponding $^3P_1$ bottomonium states $\chi_{b1}(1P)$ and $\chi_{b1}(2P)$. Indeed, these latter $J^{PC} = 1^{++}$ resonances are the closest in mass to their spin-singlet counterparts, so that the kinematical differences in some decays are minimal, and also they share the property of having relatively small annihilation rates, since in the inclusive 'parton' picture of the annihilation both types of states annihilate in the order α_s^3: $\chi_{b1} \to q\bar{q}g$ and $h_b \to 3g$, with g standing for gluon and q for a light quark. The straightforward theoretical picture with a similarity between the decay properties of the $^1P_1$ and $^3P_1$ has been recently put to test by the Belle data [96] on the dominant radiative transitions from the h_b resonances. The reported branching fraction for such decay of the $h_b(1P)$ resonance, $\mathcal{B}[h_b(1P) \to \eta_b(1S)\,\gamma] = (49.2 \pm 5.7^{+5.6}_{-3.3})\%$ arguably compares reasonably well (accounting also for a difference in the photon energy) with the known similar fraction for the $\chi_{b1}(1P)$ [9]: $\mathcal{B}[\chi_{b1}(1P) \to \Upsilon(1S)\,\gamma] = (33.9 \pm 2.2)\%$. However, the central values of the data [96] for the transitions form $h_b(2P)$: $\mathcal{B}[h_b(2P) \to \eta_b(1S)\,\gamma] = (22.3 \pm 3.8^{+3.1}_{-3.3})\%$ and $\mathcal{B}[h_b(2P) \to \eta_b(2S)\,\gamma] = (47.5 \pm 10.5^{+6.8}_{-7.7})\%$ are significantly higher than for the spin-triplet 'analog' $\chi_{b1}(2P)$ [9]: $\mathcal{B}[\chi_{b1}(2P) \to \Upsilon(1S)\,\gamma] = (9.2 \pm 0.8)\%$ and $\mathcal{B}[\chi_{b1}(2P) \to \Upsilon(2S)\,\gamma] = (19.9 \pm 1.9)\%$. Indeed, the reported central values of the radiative decay rates for $h_b(2P)$ indicate that the annihilation decay rate $\Gamma_{ann}[h_b(2P)]$ of the $h_b(2P)$ may be significantly suppressed in comparison with the rate expected from the similarity relation $\Gamma_{ann}[h_b(2P)]/\Gamma_{ann}[h_b(1P)] = \Gamma_{ann}[\chi_{b1}(2P)]/\Gamma_{ann}[\chi_{b1}(1P)]$.

In this section, we quantify the possible contradiction with the similarity and argue that it gets even stronger, if the hadronic transitions from $h_b(2P)$ to lower bottomonium states are taken into account. Namely, the $h_b(2P)$ resonance has the decay mode $h_b(2P) \to \eta\,\Upsilon(1S)$, which is kinematically forbidden for the $h_b(1P)$ and has no analog for the $\chi_{b1}(2P)$ state. We argue that the branching fraction for this decay, although uncertain, can be significant (up to $O(10\%)$), which would further reduce the fractional probability remaining for the annihilation decays of $h_b(2P)$. In what follows, we proceed in two steps. The first step is to use available data and HQSS to estimate the following "the ratio of the ratios"

$$r = \frac{\{\Gamma_{ann}[h_b(2P)] + \Gamma[h_b(2P) \to \Upsilon(1S)\,\eta]\}\,/\Gamma_{ann}[h_b(1P)]}{\Gamma_{ann}[\chi_{b1}(2P)]/\Gamma_{ann}[\chi_{b1}(1P)]}\ . \tag{5.1}$$

If $\Gamma[h_b(2P) \to \Upsilon(1S)\,\eta]$ can be ignored, then $r = 1$ from the parton picture. If the η transition is estimated to be large, then one should expect $r > 1$. So the second step is to estimate the decay rate of $h_b(2P) \to \Upsilon(1S)\,\eta$. The discussion here is based on the Ref. [97].

5.1 The Ratio of the Annihilation Ratios

The absolute rates of the transitions between the P- and S-wave states of the spin-singlet bottomonium are related to those for the spin-triplet states by HQSS within the nonrelativistic description of the $b\bar{b}$ system. The rates of the radiative electric-dipole transitions are related as

$$\Gamma[h_b(kP) \to \eta_b(nS)\,\gamma] = \frac{\omega_{kn1}^3}{\omega_{kn3}^3}\,\Gamma[\chi_{b1}(kP) \to \Upsilon(nS)\,\gamma]\ , \tag{5.2}$$

where ω_{kn1} (ω_{kn3}) is the photon energy in the transition between the spin-singlet (spin-triplet) states. [1] Using Eq.(5.2) one readily estimates

$$\begin{aligned}
\Gamma[h_b(1P) \to \eta_b(1S)\,\gamma] &\approx 1.5\,\Gamma[\chi_{b1}(1P) \to \Upsilon(1S)\,\gamma];\\
\Gamma[h_b(2P) \to \eta_b(1S)\,\gamma] &\approx 1.25\,\Gamma[\chi_{b1}(2P) \to \Upsilon(1S)\,\gamma];\\
\Gamma[h_b(2P) \to \eta_b(2S)\,\gamma] &\approx 1.44\,\Gamma[\chi_{b1}(2P) \to \Upsilon(2S)\,\gamma]\ .
\end{aligned} \tag{5.3}$$

Unlike the $1P$ states, the heavier $2P$ ones also undergo hadronic transitions to lower levels of bottomonium. Two types of such of such transitions, with emission of either ω resonance or two pions, are common for $\chi_{b1}(2P)$ and $h_b(2P)$ and their absolute rates can be related:

$$\Gamma[h_b(2P) \to \eta_b(1S)\,\omega] = \frac{p_{\omega 1}}{p_{\omega 3}}\,\Gamma[\chi_{b1}(2P) \to \Upsilon(1S)\,\omega] \approx 2.6\,\Gamma[\chi_{b1}(2P) \to \Upsilon(1S)\,\omega]\ , \tag{5.4}$$

where $p_{\omega 1}$ ($p_{\omega 3}$) is the ω momentum in the transition between the spin-single (spin-triplet) states, and

$$\Gamma[h_b(2P) \to h_b(1P)\,\pi\pi] \approx \Gamma[\chi_{b1}(2P) \to \chi_{b1}(1P)\,\pi\pi]\ . \tag{5.5}$$

Indeed, both types of transitions are induced by the chromo-electric dipole interaction of the heavy quark pair with soft gluon field. The former transition arises in the third order in this interaction [90], while the two-pion emission arises in the second order [11, 10].

[1] In a strict sense, the account for the difference in the factor ω^3 is beyond the accuracy of the lowest order in the breaking of the heavy quark spin symmetry. However we follow the tradition of including this factor, since this factor is dictated by the QED gauge invariance, and since the effect of the spin-dependence is somewhat enhanced in this factor. Our conclusions would not change qualitatively, if this factor is omitted.

In either process the heavy quark spin decouples, and the relation (5.4) takes into account the difference in the phase space of the S-wave processes, which difference is quite essential, since the decay $\chi_{b1}(2P) \to \Upsilon(1S)\,\omega$ is close to the threshold. In Eq.(5.5) any kinematical difference can be neglected since the energy released in the two related decays is essentially the same within the (small) experimental uncertainty.

Using the relations (5.3), (5.4) and (5.5) one can readily find the estimates for the branching fractions for the yet unobserved hadronic transitions in terms of the experimentally measured quantities:

$$\begin{aligned}
&\mathcal{B}[h_b(2P) \to \eta_b(1S)\,\omega] = \\
&\frac{\Gamma[h_b(2P) \to \eta_b(1S)\,\omega]}{\Gamma[\chi_{b1}(2P) \to \Upsilon(1S)\,\omega]}\,\frac{\Gamma[\chi_{b1}(2P) \to \Upsilon(2S)\,\gamma]}{\Gamma[h_b(2P) \to \eta_b(2S)\,\gamma]}\,\frac{\mathcal{B}[\chi_{b1}(2P) \to \Upsilon(1S)\,\omega]}{\mathcal{B}[\chi_{b1}(2P) \to \Upsilon(2S)\,\gamma]} \times \\
&\mathcal{B}[h_b(2P) \to \eta_b(2S)\,\gamma] \approx (7 \pm 2)\%\ ,
\end{aligned} \tag{5.6}$$

where the uncertainty is in fact dominated by the experimental errors in $\mathcal{B}[\chi_{b1}(2P) \to \Upsilon(1S)\,\omega]$, and

$$\begin{aligned}
&\mathcal{B}[h_b(2P) \to h_b(1P)\,\pi\pi] = \\
&\frac{\Gamma[h_b(2P) \to h_b(1P)\,\pi\pi]}{\Gamma[\chi_{b1}(2P) \to \chi_{b1}(1P)\,\pi\pi]}\,\frac{\Gamma[\chi_{b1}(2P) \to \Upsilon(2S)\,\gamma]}{\Gamma[h_b(2P) \to \eta_b(2S)\,\gamma]}\,\frac{\mathcal{B}[\chi_{b1}(2P) \to \chi_{b1}(1P)\,\pi\pi]}{\mathcal{B}[\chi_{b1}(2P) \to \Upsilon(2S)\,\gamma]} \times \\
&\mathcal{B}[h_b(2P) \to \eta_b(2S)\,\gamma] \approx (1.5 \pm 0.3)\%\ ,
\end{aligned} \tag{5.7}$$

With these estimates one can evaluate the balance of the total widths of the discussed bottomonium states and test whether the remaining fraction of the decays of the $h_b(2P)$ resonance can be made compatible with the expected similarity between the annihilation decay rates of the P-wave states. In doing so one can notice that for the lower P-wave levels the annihilation decay and the discussed radiative transitions $\chi_{b1}(1P) \to \Upsilon(1S)\,\gamma$ and $h_b(1P) \to \eta_b(1S)\,\gamma$ exhaust the total probability of decay, modulo extremely minor decay modes like e.g. $\chi_{b1}(1P) \to \chi_{b0}\,\gamma$, or $h_b(1P) \to \Upsilon(1S)\,\pi^0$, which can be safely neglected. For the $\chi_{b1}(2P)$ state the discussed hadronic transitions to lower bottomonium with emission of ω or two pions also contribute to the total decay rate. However, their total contribution is at the level of about two percent and can be readily taken into account (or neglected altogether). The counting is apparently somewhat different for the $h_b(2P)$ state. Indeed, as estimated in Eqs. (5.6) and (5.7) the ω and two-pion transitions can contribute together about 8.5% of the total decay rate, which is not negligible compared to the fraction remaining after accounting for the radiative decays. Furthermore we will argue that a potentially significant additional contribution can arise from the transition $h_b(2P) \to \Upsilon(1S)\,\eta$, further reducing the estimated annihilation rate for the $h_b(2P)$ resonance.

We thus use here the following numbers for the annihilation branching fraction $\mathcal{B}_{ann}$ in the evaluation of the balance of decay rates of the discussed P-wave states:

$$
\begin{aligned}
&\mathcal{B}_{ann}[\chi_{b1}(1P)] = 1 - \mathcal{B}[\chi_{b1}(1P) \to \Upsilon(1S)\,\gamma] = (66.1\ \pm 2.2)\%\ ;\\
&\mathcal{B}_{ann}[\chi_{b1}(2P)] = 1 - \mathcal{B}[\chi_{b1}(2P) \to \Upsilon(1S)\,\gamma] - \mathcal{B}[\chi_{b1}(2P) \to \Upsilon(2S)\,\gamma]\\
&-\mathcal{B}[\chi_{b1}(2P) \to \Upsilon(1S)\,\omega] - \mathcal{B}[\chi_{b1}(2P) \to \chi_{b1}(1P)\,\pi\pi] = (68.4 \pm 2.1)\%\ ;\\
&\mathcal{B}_{ann}[h_b(1P)] = 1 - \mathcal{B}[h_b(1P) \to \eta_b(1S)\,\gamma] = (50.8 \pm 8)\%\ ;\\
&\mathcal{B}_{ann}[h_b(2P)] + \mathcal{B}[h_b(2P) \to \Upsilon(1S)\,\eta] = 1 - \mathcal{B}[h_b(2P) \to \eta_b(1S)\,\gamma] - \mathcal{B}[h_b(2P) \to \eta_b(2S)\,\gamma]\\
&-\mathcal{B}[h_b(2P) \to \eta_b(1S)\,\omega] - \mathcal{B}[h_b(2P) \to h_b(1P)\,\pi\pi] \approx (22 \pm 15)\%\ . \qquad (5.8)
\end{aligned}
$$

Using these numbers and the relations (5.3) between the radiative decay rates one can estimate the "ratio of the ratios" of the absolute decay rates in Eq.(5.1). The value of r corresponding to the current data can be estimated by rewriting it as

$$
\begin{aligned}
r = &\frac{\{\mathcal{B}_{ann}[h_b(2P)] + \mathcal{B}[h_b(2P) \to \Upsilon(1S)\,\eta]\}\,/\mathcal{B}[h_b(2P) \to \eta_b(2S)\,\gamma]}{\mathcal{B}_{ann}[h_b(1P)]/\mathcal{B}[h_b(1P) \to \eta_b(1S)\,\gamma]} \times\\
&\frac{\mathcal{B}_{ann}[\chi_{b1}(1P)]/\mathcal{B}[\chi_{b1}(1P) \to \Upsilon(1S)\,\gamma]}{\mathcal{B}_{ann}[\chi_{b1}(2P)]/\mathcal{B}[\chi_{b1}(2P) \to \Upsilon(2S)\,\gamma]} \times\\
&\frac{\Gamma[h_b(2P) \to \eta_b(2S)\,\gamma]}{\Gamma[\chi_{b1}(2P) \to \Upsilon(2S)\,\gamma]}\,\frac{\Gamma[\chi_{b1}(1P) \to \Upsilon(1S)\,\gamma]}{\Gamma[h_b(1P) \to \eta_b(1S)\,\gamma]} \approx 0.25 \pm 0.25\ . \qquad (5.9)
\end{aligned}
$$

The indicated error includes only the uncertainty in the experimental data. As discussed, the possible corrections to the theoretical input, based on the relations (5.2), (5.4) and (5.5), are likely smaller than the current experimental errors. If the similarity of the annihilation decay of the P-wave states holds, the quantity r should be equal to 1 if the rate of the decay $h_b(2P) \to \Upsilon(1S)\,\eta$ is negligible, and in general should be greater than 1. Indeed, in the picture of bottomonium annihilation at short distances the similarity relation $\Gamma_{ann}[h_b(2P)]/\Gamma_{ann}[h_b(1P)] = \Gamma_{ann}[\chi_{b1}(2P)]/\Gamma_{ann}[\chi_{b1}(1P)]$ should hold up to corrections due to nonfactorization of the spin and coordinate degrees of freedom, which are at least as small as v^2/c^2 in the nonrelativistic expansion and should not exceed a few percent in bottomonium (recall $v_Q \sim \Lambda_{QCD}/m_Q \sim 0.1$ for b quark). The similarity relation is certainly exact in the standard calculation of the annihilation rates, see e.g., Ref. [93].

Clearly the estimate of r in Eq.(5.9) shows that the current data on the radiative decays of $h_b(1P)$ and (especially of) $h_b(2P)$ are on the verge of a dramatic contradiction with the similarity of annihilation processes of the P-wave states of bottomonium. The disagreement may become even worse if the contribution of the decay $h_b(2P) \to \Upsilon(1S)\,\eta$ to the total width of $h_b(2P)$ is not small.

5.2 The Estimate of Decay $h_b(2P) \to \Upsilon(1S)\,\eta$

Within the multipole expansion in QCD the transition $h_b(2P) \to \Upsilon(1S)\,\eta$ arises as a combined effect of the chromoelectric dipole ($E1$) and the chromomagnetic dipole ($M1$) interaction described by the following terms in the effective Hamiltonian

$$H_{E1} = -\frac{1}{2}\xi^a\,\vec{r}\cdot\vec{E}^a \ , \quad H_{M1} = -\frac{1}{2\,m_b}\,\xi^a\,(\vec{\Delta}\cdot\vec{B}^a) \ , \tag{5.10}$$

where the notations are the same as before. The assumed here normalization convention is that the QCD coupling g is absorbed into the definition of the gluon field strength.

The presence of the heavy quark mass in the denominator in H_{M1} reflects the fact that the spin-dependent chromomagnetic interaction is suppressed by the HQSS. In the considered process of emission of the η meson, this suppression however is somewhat compensated [98, 99] by the enhancement due to the axial anomaly relation [100, 101]:

$$\epsilon^{\mu\nu\lambda\sigma}\,\langle\eta|G_{\mu\nu}G_{\lambda\sigma}|0\rangle = 16\pi^2\sqrt{\frac{2}{3}}\,f_\eta\,m_\eta^2 \ , \tag{5.11}$$

where f_η is the η "decay constant", equal to the pion decay constant $f_\pi \approx 130\,MeV$ in the limit of exact flavor SU(3) symmetry, and f_η is likely to be larger due to effects of the SU(3) violation.

The calculation of the transition rate is fully analogous to that for the $\Upsilon(3S) \to h_b(1P)\,\pi^0$ decay in Ref. [99] (also in the review [10]), and the resulting expression can be written as

$$\Gamma[h_b(2P) \to \Upsilon(1S)\,\eta] = \left(\frac{\pi^2}{27}\,f_\eta\,m_\eta^2\right)^2\,|I(2P \to 1S)|^2\,\frac{p_\pi}{3\pi} \ , \tag{5.12}$$

where p_η is the momentum of the η meson and $I(2P \to 1S)$ is the heavy quarkonium matrix element:

$$I(2P \to 1S) = \langle 1S | \mathcal{G}_S \, r + r \, \mathcal{G}_P | 2P \rangle \tag{5.13}$$

containing the partial-wave Green function of the heavy quark pair $\mathcal{G}_S$ and $\mathcal{G}_P$ in the color octet state.

Currently the matrix element (5.13) cannot be evaluated with any reliability, and one has to resort to indirect arguments. In particular the rate of the discussed transition can be compared to that of a similar decay $\Upsilon(3S) \to h_b(1P)\,\pi^0$. The latter decay involves isospin violation, which in terms of the chiral anomaly is expressed through the difference of the masses of the u and d quarks. The relation between the rates of these two processes takes the form

$$\frac{\Gamma[h_b(2P) \to \Upsilon(1S)\,\eta]}{\Gamma[\Upsilon(3S) \to h_b(1P)\,\pi^0]} = \frac{1}{3} \left(\frac{m_u + m_d}{m_u - m_d} \, \frac{f_\eta \, m_\eta^2}{f_\pi \, m_\pi^2} \right)^2 \frac{p_\eta}{p_\pi} \left| \frac{I(2P \to 1S)}{I(3S \to 1P)} \right|^2 \approx$$
$$1.3 \times 10^3 \times \left| \frac{I(2P \to 1S)}{I(3S \to 1P)} \right|^2 \; . \tag{5.14}$$

If the BaBar evidence [95] for the decay $\Upsilon(3S) \to h_b(1P)\,\pi^0$ is taken at face value, their signal corresponds to the absolute rate of this transition in the ballpark of 20 eV. Thus if the matrix elements in Eq.(5.14) for the $2P \to 1S$ and $3S \to 1P$ transitions were the same, the absolute rate of the decay $h_b(2P) \to \Upsilon(1S)\,\eta$ would be about 25 keV and would thus exceed the estimate [93] (~ 15 keV) for the rate of the radiative transition $h_b(2P) \to \eta_b(2S)\,\gamma$. One can possibly argue, however, that the spatial size of the initial and the final bottomonium states in the transition $2P \to 1S$ is smaller than in $3S \to 1P$, so that the amplitude $I(2P \to 1S)$ should be somewhat suppressed as compared to $I(3S \to 1P)$ (although this argument does not take into account the possible effect of an extra oscillation in the $3S$ wave function). Allowing a factor of $\sim 1/2 - 1/3$ for this suppression one can very approximately estimate the rate $\Gamma[h_b(2P) \to \Upsilon(1S)\,\eta]$ to be about one quarter of $\Gamma[h_b(2P) \to \eta_b(2S)\,\gamma]$ within a factor of two or so, corresponding to $\mathcal{B}[h_b(2P) \to \Upsilon(1S)\,\eta] \sim O(10\%)$.

It is quite clear that the presented arguments involve a great uncertainty, and for this reason it would be very interesting if the transition $h_b(2P) \to \Upsilon(1S)\,\eta$ could be found in the existing Belle data at the $\Upsilon(5S)$ energy, or an upper limit on the branching fraction for this process could be established. The current data result in the estimate in Eq.(5.9) which is in a really poor agreement with the 'parton' picture of annihilation of the P-wave bottomonium, even if the contribution of this decay is negligible. An observation of the transition $h_b(2P) \to \Upsilon(1S)\,\eta$ at a noticeable level would make the situation with the (non)similarity of the annihilation of the spin-singlet and spin-triplet $J = 1$ bottomonium states quite dramatic and present an interesting riddle for theoretical interpretation.

Chapter 6

Decays $\Xi_b \to \Lambda_b \pi$ as a Probe to Diquark Correlations in Hyperons

The lifetime of heavy baryons has been measured with a high accuracy, more subtle effect can be studied. The lifetime of weakly decaying heavy hadrons is dominantly determined by the decay of the heavy quark. However the strange heavy hyperons can also decay without the destruction of the heavy quark, but rather due to the weak decay of the strangeness [103, 104], e.g. $\Xi_b^0 \to \Lambda_b \pi^0$ and $\Xi_b^- \to \Lambda_b \pi^-$. The rate of these decays is expected to be in the same ballpark as that of the ordinary strange hyperons, i.e. of the order of $0.01\,\mathrm{ps}^{-1}$. Furthermore, the heavy baryons are likely to be more spatially compact systems than the light ones, so that a certain enhancement of the weak pion transitions for the heavy hyperons would be quite natural. It is expected that these decays contribute only at a permille level to the lifetimes of the charmed hyperons, while their contribution to the total decay rate of the strange b baryons can be essential at a level of one or few percent and their effect on the lifetimes of Ξ_b^0 and Ξ_b^- may become visible once the experimental data become sufficiently precise. The purpose of this chapter is to point out that in addition to a possible relevance to studies of the Heavy Quark Expansion (HQE), the weak strangeness changing pion transitions for the b hyperons are interesting on their own and provide a testing ground for the properties of $J^P = 0^+$ light diquarks which, according to some theoretical suggestions (e.g. in Ref. [105]), can contain a large QCD scale that would enhance the rate of such transitions. The detailed discussion can be found in Ref. [102].

In this chapter, the estimate of the decay rate $\Xi_b \to \Lambda_b \pi$ can be decomposed into two steps. The first step involves in expressing the decay amplitude in terms of one unknown matrix element by applying HQE, which is model independent. Then, different models are used to estimate this matrix element, thus obtain the final decay rate. Therefore, the direct measurement can fix this matrix element so the possible model assumption or parameter can be tested or determined.

6.1 The Decay $\Xi_b \to \Lambda_b \pi$ and Heavy Quark Expansion

The lifetimes of the weakly decaying b baryons, Λ_b, Ξ_b^0 and Ξ_b^-, are traditionally a subject of a significant interest in connection with the (HQE) for the inclusive decay rates of heavy hadrons. Recent improvements in the accuracy and reliability of the experimental data on these lifetimes [106] have to a great extent resolved the tensions of the early LEP measurements of $\tau(\Lambda_b)$ with the predictions from HQE. Namely, the latest data with the highest reported precision by the LHCb experiment [107, 108] result in the average value [106] $\tau(\Lambda_b) = 1.468 \pm 0.009 \pm 0.008$ ps, and the recent results, albeit of a lower accuracy, from other experiments [109, 110, 111, 112] are consistent with this value. These measurements place the ratio of the lifetimes $\tau(\Lambda_b)/\tau(B_d^0)$ well inside the range 0.95 - 1.0, in agreement with the original HQE prediction [113] as well as with the more recent theoretical estimates [114]. Now, as the Λ_b lifetime issue between the HQE and the experiment is apparently resolved, the theory can be put to further test against the currently improving data [109, 115, 116] on the lifetimes of the rest of the weakly decaying b baryons, which may also provide a quantitative insight in the internal structure of baryons. In this respect of a special interest are the differences of the inclusive decay rates within the flavor SU(3) (anti)triplet of the b baryons Λ_b, Ξ_b^0 and Ξ_b^- as well as their comparison with similar differences in the antitriplet of the charmed hyperons Λ_c, Ξ_c^0 and Ξ_c^+.

The HQE predicts a very small difference between the inclusive rates of the beauty decay in the Λ_b and Ξ_b^0 hyperons, similar to the prediction of a less than one percent difference between the lifetimes of the B_d^0 and B_s^0 mesons, due to the approximate (but in fact quite accurate) U spin symmetry of the leading term in HQE that depends on the flavors of the spectator light quarks [1] . The current data [115] do not contradict this prediction: $\tau(\Xi_b^0)/\tau(\Lambda_b) = 1.006 \pm 0.018 \pm 0.010$. However it is also not excluded, within the errors, that the Ξ_b^0 decay rate receives a few percent contribution from the weak transition $\Xi_b^0 \to \Lambda_b \, \pi^0$.

It is also expected [113] from HQE that the lifetime of Ξ_b^- is noticeably longer than that of Λ_b. A quantitative prediction depends on hadronic matrix elements of the four-quark operators arising in the expansion for the inclusive weak decay rate which matrix elements were studied purely theoretically [117, 118] and also phenomenologically [119] from the measured lifetime differences for the charmed baryons. The latter determination of the relevant baryonic matrix elements results in the estimated difference between the inclusive beauty decay rate for Λ_b and Ξ_b^- equal to $0.11 \pm 0.03\,\mathrm{ps}^{-1}$. This value is substantially larger than the two available experimental numbers for the difference of total decay rates for these b baryons: the reported CDF result [109], $\tau(\Xi_b^-) = 1.32 \pm 0.14 \pm 0.02\,\mathrm{ps}$ gives the opposite sign for the difference compared to HQE prediction, while the latest LHCb number $\tau(\Xi_b^-) = 1.55^{+0.10}_{0.09} \pm 0.03\,\mathrm{ps}$ corresponds to $\tau^{-1}(\Lambda_b) - \tau^{-1}(\Xi_b^-) = 0.03 \pm 0.04\,\mathrm{ps}^{-1}$, and is formally compatible with the expected difference of the inclusive decay rates only due to the large experimental errors and the theoretical uncertainty.

Given that the experimental situation with the lifetimes of the strange b baryons is still in flux, one can consider several options. One is that an application of the leading terms in HQE to the charmed hyperons is not quantitatively reliable due to insufficiently heavy mass of the charmed quark, so that the estimate [119] of the relevant baryonic matrix elements from the lifetime differences of the charmed hyperons results in an exaggerated prediction for the difference of the beauty decay rates between Λ_b and Ξ_b^-. Although this is not excluded, there are arguments [114] against such reasoning. Another possibility, based on the history of measurements of $\tau(\Lambda_b)$, is that future more accurate experimental data will eventually settle at a longer lifetime for the Ξ_b^- in agreement with the HQE estimate in Ref. [119]. It is however also possible that the actual value of $\tau(\Xi_b^-)$ is somewhere half way between the current data and the theoretical expectation,

[1] A discussion of this property can be found e.g. in Ref. [114].

and the strangeness changing weak pion transitions are enhanced in comparison with the ordinary hyperons, so that there is a noticeable addition (at the level of few percent) to the total decay rate of the Ξ_b hyperons. It can be noticed that in the decays $\Xi_b \to \Lambda_b \, \pi$ the $\Delta I = 1/2$ rule should hold to a high accuracy [104], so that

$$\Gamma(\Xi_b^- \to \Lambda_b \, \pi^-) = 2 \, \Gamma(\Xi_b^0 \to \Lambda_b \, \pi^0) \;, \tag{6.1}$$

and an additional decay rate for Ξ_b^0 being still compatible with the current data may translate to a few percent shortening of the lifetime of Ξ_b^-. Certainly, a direct observation of the transitions $\Xi_b \to \Lambda_b \, \pi$ would definitely solve the issue, however it is not clear whether such observation is feasible in the LHC environment.

In the strangeness changing weak transitions between b baryons the heavy b quark is a spectator, and the process is determined entirely by dynamics of the light diquark which has quantum numbers $J^P = 0^+$. It can be mentioned, that a mixing with the b baryons from the flavor SU(3) sextet in which the light diquark is in the $J^P = 1^+$ state is suppressed by both the flavor SU(3) symmetry and by the heavy quark spin symmetry, so that the suppression factor in the mixing amplitude is $O(m_s/m_b)$, and considering the diquark in a pure 0^+ state makes a very good approximation. In this limit the discussed decays of strangeness are thus $0^+ \to 0^+$ transitions with emission of a pion, for which only the parity violating S wave amplitude is allowed [104]. This amplitude can be approximated by its value at vanishing pion momentum, where it is given by the current algebra reduction formula

$$\langle \Lambda_b \, \pi_i(p=0) \, | H_W | \, \Xi_b \rangle = \frac{\sqrt{2}}{f_\pi} \, \langle \Lambda_b \, | \left[Q_i^5, \, H_W \right] | \, \Xi_b \rangle \;, \tag{6.2}$$

where H_W is the weak interaction Hamiltonian, π_i is the pion triplet in the Cartesian notation, and Q_i^5 is the corresponding isotopic triplet of axial charges. The constant $f_\pi \approx 130 \, MeV$, normalized by the charged pion decay, is used here, which results in the the coefficient $\sqrt{2}$ in Eq.(6.2). Clearly, the expression in the r.h.s. in Eq.(6.2) is nonvanishing only for the part of the Hamiltonian corresponding to $\Delta I = 1/2$ which gives the relation (6.1).

The strangeness changing weak Hamiltonian is well studied. At a normalization point μ below the charmed quark mass, $\mu \ll m_c$ this Hamiltonian has the general form [120]

$$H_W = \sqrt{2}\, G_F\, \cos\theta_c\, \sin\theta_c \sum_{i=1}^{7} C_i(\mu)\, O_i(\mu)\ , \tag{6.3}$$

where θ_c is the Cabibbo angle, and the explicit form of the operators O_i can be found e.g. in Ref. [120]. In the present discussion the main part is being played by the operator

$$O_1 = (\overline{u}_L\, \gamma_\mu\, s_L)\, (\overline{d}_L\, \gamma_\mu\, u_L) - (\overline{d}_L\, \gamma_\mu\, s_L)\, (\overline{u}_L\, \gamma_\mu\, u_L)\ . \tag{6.4}$$

Indeed, this is the only $\Delta I = 1/2$ operator having a large coefficient in the expansion (6.3): at a low μ one finds [120] $C_1 \approx 2.5$. Other $\Delta I = 1/2$ operators in the sum in Eq.(6.3) have small coefficients (less than 0.1) and their matrix elements are not especially enhanced. For this reason, at the present level of accuracy, it is sufficient to retain in the expansion for the Hamiltonian only the term with the operator O_1.

Using the reduction formula (6.2), one readily finds the expression for the amplitude of the transition $\Xi_b^- \to \Lambda_b\, \pi^-$ in the form

$$\langle \Lambda_b\, \pi^-(p=0)\, | H_W |\, \Xi_b^- \rangle = \frac{\sqrt{2}}{f_\pi}\, G_F\, \cos\theta_c\, \sin\theta_c\, C_1\, X\ , \tag{6.5}$$

with X being the hadronic matrix element

$$X = \langle \Lambda_b\, | (\overline{u}_L\, \gamma_\mu\, s_L)\, (\overline{d}_L\, \gamma_\mu\, d_L) - (\overline{d}_L\, \gamma_\mu\, s_L)\, (\overline{u}_L\, \gamma_\mu\, d_L) |\, \Xi_b^- \rangle\ . \tag{6.6}$$

Using the nonrelativistic normalization condition for the states of heavy baryons, one can write the rate of the pion transition in terms of the amplitude X as

$$\Gamma(\Xi_b^- \to \Lambda_b\, \pi^-) = \cos^2\theta \sin^2\theta\, C_1^2\, \frac{G_F^2\, |X|^2\, p_\pi}{\pi\, f_\pi^2} \approx 1.3 \times 10^{-2}\, \text{ps}^{-1} \left(\frac{C_1}{2.5} \right)^2 \left| \frac{X}{0.01\, \text{GeV}^3} \right|^2 , \tag{6.7}$$

where $p_\pi \approx 100\, \text{MeV}$ is the pion momentum. Correspondingly, the expression for the branching fraction in terms of X reads as

$$\mathcal{B}(\Xi_b^- \to \Lambda_b\, \pi^-) \approx 2\% \left(\frac{C_1}{2.5} \right)^2 \left| \frac{X}{0.01\, \text{GeV}^3} \right|^2\ . \tag{6.8}$$

Up to now, apart from the unknown matrix element X, the estimate Eq.(6.7) or Eq.(6.8) is very accurate considering the accuracy of current algebra and HQE. The largest uncertainty comes from X which can not be estimated model independently. We next consider two different models for evaluating X.

6.2 Estimate of Matrix Element X

6.2.1 The Diquark Picture

It has been suggested in the literature [121, 105] that the scalar diquarks may have special properties resulting in an enhancement of short-distance correlations inside them similar to the matrix element X. One theoretical argument [105] for such behavior is that in QCD with two colors instead of three, and in the chiral limit the (colorless) diquark would be a Goldstone boson and exist on equal footing with a pion. Thus their respective decay constants should be the same. Extending this behavior to the actual QCD with three colors one can introduce[121] the 'diquark decay constant' g_D for a color-antitriplet diquark D_i (e.g. made of u and d quarks),

$$\langle 0 \,|\epsilon_{ijk}(\overline{u^c}^j \gamma_5 d^k)|\, D_l\rangle = \sqrt{\frac{2}{3}}\, g_D\, \delta_{il} \tag{6.9}$$

with i, j, k, l being the triplet color indices, and compare it with an equivalent pion coupling to the pseudoscalar density

$$\langle 0\,|(\overline{u}\gamma_5 d)|\,\pi\rangle = g_\pi, \quad g_\pi = \frac{f_\pi\, m_\pi^2}{m_u + m_d} \approx 0.2\,\mathrm{GeV}^2 \;. \tag{6.10}$$

(The factor $\sqrt{2/3}$ in Eq.(6.9) accounts for the different number of colors contributing in a pion and in a diquark of a fixed color.) If such extension from two colors to the actual three-color QCD is of relevance, one expects [121] the approximate relation $g_D \approx g_\pi$.

Clearly, such diquark picture has intrinsically embedded the color antisymmetry:

$$\langle \Lambda_b\,|(\overline{u}_L\, \gamma_\mu\, s_L)\,(\overline{d}_L\, \gamma_\mu\, d_L)|\, \Xi_b^-\rangle = -\langle \Lambda_b\,|(\overline{d}_L\, \gamma_\mu\, s_L)\,(\overline{u}_L\, \gamma_\mu\, d_L)|\, \Xi_b^-\rangle \;. \tag{6.11}$$

Using then the Fierz identity $(\overline{u}_L\, \gamma_\mu\, s_L)\,(\overline{d}_L\, \gamma_\mu\, d_L) = 2\,(\overline{d}_{Li} u^c_{Rj})\,(\overline{s^c}^j_R d^i_L)$ one finds in the vacuum insertion dominance approximation

$$X = \frac{g_D^2}{6\, m_D} \;, \tag{6.12}$$

where m_D stands for the 'diquark mass'. Naturally, it is not quite clear what value should one use for m_D. However, the study [121] of the constant g_D by the QCD sum rule method has produced a strong correlation between m_D and g_D, such that the ratio

g_D^2/m_D, entering Eq.(6.12) depends only weakly on the assumed value of m_D. Using Eq.(6.12) and the results presented in Ref. [121], we estimate

$$X \approx 0.01\,\mathrm{GeV}^3 \tag{6.13}$$

This estimate understandably suffers from a considerable uncertainty, partly from the usage of speculative properties of 0^+ diquarks, and partly, within this usage, from the usual uncertainties of the QCD calculation. In particular, the calculation of Ref. [121] is carried out in the standard way and takes into account the first perturbative terms in the correlator of diquark densities and the terms with the quark and gluon vacuum condensates. However it has been pointed out later [105] that such correlator should also receive a significant contribution from direct instantons, which would enhance the constant g_D, so that the actual value of the matrix element X can be larger than estimated in Eq.(6.13).

6.2.2 The Bag Model

In order to probe how reasonable the value (6.13) is, one can take an alternative approach and compare X with similar four-quark matrix elements describing the light quark density at the heavy quark, rather than the density of light quarks at coinciding point:

$$x = -\langle \Lambda_b \,|(\overline{b}\,\gamma_\mu\, b)(\overline{u}\,\gamma_\mu\, s)|\, \Xi_b^- \rangle, \qquad y = -\langle \Lambda_b \,|(\overline{b}_i\,\gamma_\mu\, b_k)(\overline{u}_k\,\gamma_\mu\, s_i)|\, \Xi_b^- \rangle \tag{6.14}$$

The matrix elements x and y are related [104] by the heavy quark symmetry and the flavor SU(3) to the same quantities describing the lifetime differences of the charmed hyperons [119]. In a simplistic nonrelativistic picture these can be related [113] to the decay constant f_B for the B mesons: $y = -x = f_B^2 m_b/12 \approx 0.016\,\mathrm{GeV}^3$. However, for these quantities the color antisymmetry relation $y = -x$ cannot be correct since it is not preserved by renormalization at $\mu \ll m_b$. Namely, the amplitude y depends on the normalization scale μ in this range, while the amplitude x does not. The amplitude x can be determined [119] from the lifetime differences between Λ_c, Ξ_c^0, and Ξ_c^+, and the updated value is $x = -0.042 \pm 0.005\,\mathrm{GeV}^3$, while the updated value of y is small ($y < 0.01\,\mathrm{GeV}^3$) across the range of μ below m_c. Unlike for the heavy-light correlators, the color antisymmetry for the light diquark densities in Eq.(6.11) is preserved by renormalization. For this reason it is not quite clear how to compare the heavy-light and light-light correlations. In lieu of a better procedure, we replace the two terms by their average, and find for the color antisymmetric combination $y - x \approx 0.03\,\text{GeV}^3$ both from the relation in the simplistic quark model and using the values extracted from the lifetimes of charmed hyperons.

One may expect that the density of light quarks on the heavy quark as in x and y in a baryon is somewhat larger than the density of two light quarks on top of each other as in X. Indeed, the heavy quark is a static 'center of force' at the 'center' of the baryon, while the light quarks are each spread around inside the baryon. In order to estimate the numerical reduction factor for collisions between the light quarks as compared to the collisions of each with the center, we use the simple original bag model [122], and consider the heavy quark as static, i.e. in the limit of infinite mass. Using the wave functions for massless light quarks in the bag, we find

$$X \approx 0.62\,(y - x) \approx 0.02\,\text{GeV}^3 \ , \tag{6.15}$$

with the numerical factor arising from the ratio of the integrals over the quark wave functions: $\int_0^{r_1} \left[j_0^2(r) + j_1^2(r)\right]^2 r^2\,\mathrm{d}r / \int_0^{r_1} \left[j_0^2(r) + j_1^2(r)\right] r^2 \mathrm{d}\,r \approx 0.62$, where $j_0(r)$ and $j_1(r)$ are the standard spherical Bessel functions, and $r_1 \approx 2.043$ is the smallest positive solution to $j_0(r_1) = j_1(r_1)$.

One can see, from the estimates presented here, that there is a considerable uncertainty in the value of the baryonic matrix element X determining the rate of the weak decays $\Xi_b \to \Lambda_b\,\pi$. The range of these estimates $X \sim 0.01 - 0.02\,\text{GeV}^3$ corresponds to the branching fraction for such decay (2 - 8)% for Ξ_b^- and (1 - 4)% for Ξ_b^0. At such level the contribution of these beauty-conserving decays can produce an effect on the lifetime differences between weakly decaying b baryons that would be visible on top of the effects of HQE in the b decay processes. Furthermore, since in these b baryons the light quark is to a high accuracy a pure 0^+ state, a measurement of the rates of these pion transitions and thus of the value of the matrix element X, may have very interesting implications for understanding the dynamics of 0^+ diquarks. In particular, it would test the existing in the literature ideas about effects of short distance scales in such systems.

Chapter 7

Conclusion

The advance of the experiments in last five years has uncovered a large number of exotic quarkoniumlike resonances that challenge the conventional theoretical interpretation. The first exotic bottomoniumlike state $Z_b^\pm(10610)$ and $Z_b^\pm(10650)$, unambiguously containing four valence quarks , were found by the Belle Collaboration at 2011. Two years later at 2013, the BESIII reported a similar charmoniumlike resonance $Z_c^\pm(3900)$ and it was soon confirmed by the Belle. In the same year, two structures $Z_c^\pm(4025)$ and $Z_c^\pm(4020)$ were found by the BESIII in different decay channels. The question whether they are the same state remains open. The BESIII also found another charmoniumlike state $Z_c^\pm(3885)$. The electric neutral partners of those resonances were also found in 2014 and 2015. Besides discovering the new resonances, new decay channels of some old states such as $Y(4260)$ and $Y(4360)$ were also measured. These new experimental results have expanded our knowledge about heavy hadron spectroscopy and provided an ideal laboratory to study the intriguing properties of strong interaction. With these new experimental input, we have investigated the strong dynamics near threshold region of heavy mesons, try to resolve the nature of some observed exotic resonances, and reexamine the updated data on heavy quarkonium and heavy baryon to test our theoretical understanding.

We have discussed in Chapter 2 the general theoretical ideas and QCD-based methods of understanding the structure of heavy quarkonium spectroscopy as well as their hadronic transitions. The heavy quark spin symmetry and QCD multipole expansion are proven to be powerful theoretical tools to understand various phenomena of heavy quarkonium. They give accurate quantitative description on hadronic transitions between heavy quarkonium states, especially for low radial excitation states. However, for heavy quarkonium near the thresholds of the heavy mesons, the enhanced HQSS breaking is expected.

In Chapter 3, peculiar effects of strong dynamics near threshold region are discussed. Section 3.1 introduces the heavy quark spin decomposition which allows one to separate the heavy quark spin from the rest degree of freedom. This formulism has been successfully applied to analyze the consequences of the HQSS in different settings. The heavy quark spin structure of the heavy meson pairs in $J^{PC} = 1^{--}$ channel is worked out in details and used to explain the enhanced HQSS due to quarkoniumlike state mixing with the heavy meson. It is well known that the effects of the interaction of heavy hadrons through the light degrees of freedom is enhanced. We point out in Section 3.2 the effect of enhanced mixing of partial waves of heavy meson pairs is actually calculable near the threshold region. The P-F partial wave mixing although parametrically enhanced is still small and can be safely neglected, but the mixing of spin $S = 0$ and $S = 2$ partial waves is significant. The absorptive part of mixing amplitude changes between zero at the threshold to 0.75 at 20 MeV above the threshold. One consequence of this mixing is to break the spin composition of the heavy meson pairs predicted by HQSS, which provides an illustration how HQSS is broken due to rescattering of heavy meson pairs near the threshold. Section 3.3 studies another subtle effect near the threshold, namely isospin violation. The observable charge-to-neutral ratio $R^{c/n}$ is calculated in all orders of isospin breaking parameters. The interplay between Coulomb interaction and strong interaction is strongest near the threshold region which is sensitive to the strong interaction parameters. The calculated expression can be used to extract important information about strong dynamics of scattering between two heavy mesons.

Some of the exotic heavy quarkonium-like resonances are considered in Chpater 4. The properties of $Z_b^{\pm}(10610)$ and $Z_b^{\pm}(10650)$ can be well understood in the framework of hadronic molecular states. We have analyzed the heavy quark spin structure and their decay patterns to bottomonium plus pion. The prediction from the molecular assumption agrees the experiment very well. Thus, we conclude that $Z_b^{\pm}(10610)$ and $Z_b^{\pm}(10650)$ are S-wave $B^*\bar{B} - \bar{B}^*B$ and $B^*\bar{B}^*$ molecules respectively. Section 4.2 addresses the old puzzle of $Y(4260)$ and $Y(4360)$. With new experimental input that they both decay, with comparable rates, to spin-singlet and spin-triplet $c\bar{c}$ pairs, we propose they are actually mixed hadrocharmonium. The model predicts a distinctive feature that is compatible with the current data. We also point out the alternative molecular interpretation of $Y(4260)$ is disfavored by HQSS. We believe our hadrocharmonium model is more credible. $X(3915)$ although assigned as $\chi_{c0}(2P)$ should be considered as an exotic state. We carefully analyze its decay properties and argue that the relative large decay rate to J/ψ and small rate to $D\bar{D}$ are strongly at variance with the conventional charmonium, hence its identity needs to be studied more carefully. We suggest that the $X(3915)$ is in fact the bound state of $D_s\bar{D}_s$ with binding energy about 18 MeV. This interpretation could well explain the current experimental data of $X(3915)$. The decay rates in other decay channel are also estimated whose value can be well accommodated with its total decay rate 20 MeV. In section 4.4, we provide a conventional interpretation of a possible charmonium-like resonance observed in the final state $\chi_{cJ}\omega$ in the e^+e^- annihilation at center of mass energy from 4.21 to 4.42 GeV. The observed the peak is the consequence of a nearby well-known charmonium $\psi(4160)$. The magnitude of the coupling constant that needs to produce the observed peak is within the expectation from other similar charmonium transitions.

The new and improved measurement on some old heavy hadron states provides an opportunity to carefully test our understanding of QCD dynamics. The parton picture of bottomonium is examined in Chapter 5 while a possible enhanced diquark correlation is suggested in Chapter 6.

The possible violation of parton picture of bottomonium annihilation is illustrated by using the decay data of $h_b(1,2P)$ and $\chi_{b1}(1,2P)$. We consider hadronic transitions from the $h_b(2P)$ bottomonium resonance to lower states of bottomonium with emission of either ω meson, or two pions, or η meson. With the help of heavy quark spin symmetry and QCD multipole expansion, we are able to estimate the fraction of the $h_b(2P)$ total decay rate remaining for the annihilation rate, which is on a verge of contradiction with the expectation from the parton picture. The transition $h_b(2P) \to \Upsilon(1S)\,\eta$ is also estimated and including this transition will deviate further away from the parton picture.

The decays $\Xi_b \to \Lambda_b \pi$ are strangeness changing weak transitions involving only the light diquark in the baryon. Thus these decays can test the properties of such diquarks, namely enhanced correlations in $J^P = 0^+$ light diquarks. We revisit the estimates of the rates of these decays and point out that with the enhanced correlation their branching fraction can reach a few percent and may become visible in the precision measurements of differences of the lifetimes of b baryons.

References

[1] I. J. R. Aitchison and A. J. G. Hey, "Gauge Theories in Particle Physics." Graduate Student Series in Physics (IOP Publishing Ltd, Bristol, 1989); A. Smilga, "Lectures on Quantum Chromodynamics." (World Scientific, 2001); W. Greiner, S. Schramm and E. Stein, "Quantum Chromodynamics." 3rd Ed (Springer, 2007); B. L. Ioffe, V. S. Fadin and L. N. Lipatov, "Quantum Chromodynamics: Perturbative and Nonperturbative Aspects." (Cambridge University Press, 2010)

[2] see a review on lattice QCD in K.A. Olive et al.(Particle Data Group), Chin. Phys. C38, 090001 (2014); T. Degrand and C. DeTar, "Lattice Methods for Quantum Chromodynamics." (World Scientific, 2006); C. Gattringer and C. B. Lang, "Quantum Chromodynamics on the Lattice: An Introductory Presentation." (Springer, 2009); L. Lellouch et al. "Modern Pespectives in Lattice QCD: quantum field theory and high performance computing." Lecture notes of the Les Houches Summer School, Vol. 93 eds. (Oxford University Press, 2011)

[3] A. V. Manohar and M. B. Wise, "Heavy Quark Physics." (Cambridge University Press, 2000); M. Neubert, Physics Reports, 245, 5-6, 259 (1994); M. B. Voloshin, Surveys in High Energy Physics, 8, 1-4 (1995); M. Shifman, "Lectures on Heavy Quarks in Quantum Chromodynamics." [arXiv:9510377 [hep-ph]]; I. Bigi, M. Shifman and N. Uraltsev, Ann. Rev. Nucl. Part. Sci. 47:591-661 (1997); N. Uraltsev, "Topics in the Heavy Quark Expansion." [arXiv:0010328 [hep-ph]]

[4] N. Brambilla et al.(Quarkonium Working Group), "Heavy quarkonium physics." CERN Yellow Report, CERN-2005-005,487, (2005); [arXiv:0412158v2 [hep-ph]];

"Heavy quarkonium: progress, puzzles, and opportunities." Eur. Phys. J. C71:1534, (2011) [arXiv:1010.5827 [hep-ph]]

[5] V. A. Novikov, L. B. Okun, M. A. Shifman, A. I. Vainshtein, M. B. Voloshin and V. I. Zakharov, Physics Reports, 41, No.1, 1-133 (1978); M. B. Voloshin, Prog. Part. Nucl. Phys. 61:455-511 (2008)

[6] J. D. Bjorken and S. D. Drell, "Relativistic Quantum Mechanics." (Mc Graw-Hill, 1964)

[7] V. B. Berestetskii, E. M. Lifshitz and L. P. Pitaevskii, "Quantum Electrodynamics." 2nd Ed (Pergamon Press,Oxford, 1982)

[8] N. Isgur and M. B. Wise, Phys. Rev. Lett. **66**, 1130 (1991)

[9] K. A. Olive *et al.* (Particle Data Group Collaboration), Chin. Phys. C **38**, 090001 (2014).

[10] M. B. Voloshin, Prog. Part. Nucl. Phys. **61**, 455 (2008) [arXiv:0711.4556 [hep-ph]].

[11] K. Gottfried, Phys. Rev. Lett. 40, 598 (1978). M. B. Voloshin, Nucl. Phys. B154, 365 (1979).

[12] A. Bondar *et al.* [Belle Collaboration], Phys. Rev. Lett. **108**, 122001 (2012) [arXiv:1110.2251 [hep-ex]].

[13] S. -K. Choi *et al.* [Belle Collaboration], Phys. Rev. Lett. **91**, 262001 (2003); D. Acosta *et al.* [CDF II Collaboration], Phys. Rev. Lett. **93**, 072001 (2004); V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. Lett. **93**, 162002 (2004); T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **103**, 152001 (2009); R. Aaij *et al.* [LHCb Collaboration], Eur. Phys. J. **C72**, 1972 (2012);

[14] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **110**, 252001 (2013) [arXiv:1303.5949 [hep-ex]].

[15] Z. Q. Liu *et al.* [Belle Collaboration], Phys. Rev. Lett. **110**, 252002 (2013) [arXiv:1304.0121 [hep-ex]].

[16] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **112**, 022001 (2014) arXiv:1310.1163 [hep-ex].

[17] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **111**, 242001 (2013) arXiv:1309.1896 [hep-ex].

[18] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **112**, 132001 (2014) arXiv:1308.2760 [hep-ex].

[19] B. Auber *et al.* [BABAR Collaboration], Phys. Rev. D **78**, 112002 (2008)

[20] A. E. Bondar, A. Garmash, A. I. Milstein, R. Mizuk and M. B. Voloshin, Phys. Rev. D **84**, 054010 (2011) [arXiv:1105.4473[hep-ph]]

[21] M. B. Voloshin, Phys. Rev. D **84**, 031502 (2011) [arXiv:1105.5829[hep-ph]]

[22] T. Mehen and J. W. Powell, Phys. Rev. D **84**, 114013 (2011) [arXiv:1109.3479 [hep-ph]].

[23] M. B. Voloshin, Phys. Rev. D **85**, 034024 (2012) [arXiv:1201.1222 [hep-ph]].

[24] M. B. Voloshin, Mod. Phys. Lett. A **26**, 773 (2011)

[25] U. Tamponi *et al.* [Belle Collaboration], Phys. Rev. Lett. **115**, 142001 (2015)

[26] A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. Lett. **38**, 317 (1977).

[27] D. Cronin-Hennessy *et al.* [CLEO Collaboration], Phys. Rev. D **80**, 072001 (2009)

[28] G. Pakhlova *et al.* [Belle Collaboration], Phys. Rev. D **77**, 011103 (2008); Phys. Rev. D **83**, 011101 (2011)

[29] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. D **79**, 092001 (2009); P. del Amo Sanchez *et al.* [BABAR Collaboration], Phys. Rev. D **82**, 052004 (2010)

[30] S. Dubynskiy and M. B. Voloshin, Mod. Phys. Lett. A **21**, 2779 (2006) [hep-ph/0608179].

[31] S. Dubynskiy and M. B. Voloshin, Phys. Rev. D **78**, 116014 (2008)

[32] Xin Li, M. B. Voloshin, Phys. Rev. D **87**, 094033, (2013)

[33] M. B. Voloshin, Phys. Rev. D **87**, 074011 (2013) arXiv:1301.5068 [hep-ph].

[34] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **102**, 012001 (2009) [arXiv:0809.4120 [hep-ex]].

[35] A. E. Bondar [on behalf of the Belle Collaboration], Talk at the 36th Int. Conf. on High Energy Physics (ICHEP2012), Melbourn, July 2012. URL: http://belle.kek.jp/belle/talks/ICHEP12/A.Bondar.pdf

[36] S. Dubynskiy, A. Le Yaouanc, L. Oliver, J. -C. Raynal and M. B. Voloshin, Phys. Rev. D **75**, 113001 (2007) [arXiv:0704.0293 [hep-ph]].

[37] M. B. Voloshin, Phys. Rev. D **86**, 034013 (2012) [arXiv:1204.1945 [hep-ph]].

[38] M. B. Voloshin, Mod. Phys. Lett. A **18**, 1783 (2003) [hep-ph/0301076].

[39] Xin Li and M. B. Voloshin, Phys. Rev. D **89**, 014013 (2014)

[40] L. D. Landau and E. M. Lifshits, *Quantum Mechanics (Non-relativistic Theory)*, Third Edition, Pergamon, Oxford, 1977.

[41] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **115**, 222002 (2015)

[42] M. B. Voloshin, Phys. Rev. D **92**, 114003 (2015)

[43] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. Lett. **36**, 500 (1976); K. D. Lane and E. Eichten, Phys. Rev. Lett. **37**, 477 (1976) [Erratum-ibid. **37**, 1105 (1976)]. E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D **17**, 3090 (1978) [Erratum-ibid. D **21**, 313 (1980)]; E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. M. Yan, Phys. Rev. D **21**, 203 (1980).

[44] S. L. Olsen, Front. Phys. **10**, 101401 (2015)

[45] Xin Li and M. B. Voloshin, Phys. Rev. D **86**, 077502 (2012)

[46] Xin Li and M. B. Voloshin, Phys. Rev. D **90**, 014036 (2014)

[47] A. Bondar [for Belle Collaboration]. Talk at the ICHEP 2012 Conference, July 7, Melbourne, Australia. [arXiv:1209.6450 [hep-ex]]

[48] M. Cleven, F. -K. Guo, C. Hanhart, U. -G. Meissner, Eur. Phys. J. **A47**, 120 (2011)

[49] X. H. He et al. [Belle Collaboration], Phys. Rev. Lett. **113**, 142001 (2014)

[50] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **95**, 142001 (2005) [hep-ex/0506081].

[51] Q. He *et al.* [CLEO Collaboration], Phys. Rev. D **74**, 091104 (2006) [hep-ex/0611021].

[52] C. Z. Yuan *et al.* [Belle Collaboration], Phys. Rev. Lett. **99**, 182004 (2007) [arXiv:0707.2541 [hep-ex]].

[53] J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **86**, 051102 (2012) [arXiv:1204.2158 [hep-ex]].

[54] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **98**, 212001 (2007) [hep-ex/0610057].

[55] X. L. Wang *et al.* [Belle Collaboration], Phys. Rev. Lett. **99**, 142002 (2007) [arXiv:0707.3699 [hep-ex]].

[56] S. Dubynskiy and M. B. Voloshin, Phys. Lett. B **666**, 344 (2008) [arXiv:0803.2224 [hep-ph]].

[57] S. L. Zhu, Phys. Lett. B **625**, 212 (2005); F. E. Close and R. P. Page, Phys. Lett. B **628**, 215 (2005); E. Kou and O. Pene, Phys. Lett. B **631**, 164 (2005).

[58] L. Maiani, V. Riquer, F. Piccinini and A. D. Polosa, Phys. Rev. D **72**, 031502 (2005) [hep-ph/0507062].

[59] G. -J. Ding, Phys. Rev. D **79**, 014001 (2009) [arXiv:0809.4818 [hep-ph]]; M. -T. Li, W. -L. Wang, Y. -B. Dong and Z. -Y. Zhang, arXiv:1303.4140 [nucl-th]; Q. Wang, C. Hanhart and Q. Zhao, arXiv:1303.6355 [hep-ph]; F. -K. Guo, C. Hanhart, U. -G. Meissner, Q. Wang and Q. Zhao, arXiv:1306.3096 [hep-ph]; X. -H. Liu and G. Li, arXiv:1306.1384 [hep-ph].

[60] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **110**, 252001 (2013) [arXiv:1303.5949 [hep-ex]].

[61] M. Ablikim *et al.* [BESIII Collaboration], arXiv:1308.2760 [hep-ex].

[62] Changzheng Yuan, Talk at the XXVI International Symposium on Lepton and Photon interactions at High Energies, San Francisco, June 2013.

[63] I. Adachi *et.al.* [Belle Collaboration], arXiv:1209.6450 [hep-ex].

[64] Xin Li and M. B. Voloshin, Phys. Rev. D **88**, 034012 (2013)

[65] Xin Li and M. B. Voloshin, Mod. Phys. Lett. A **29**, 1450060 (2014)

[66] G. -J. Ding, Phys. Rev. D **79**, 014001 (2009) [arXiv:0809.4818 [hep-ph]].

[67] M. -T. Li, W. -L. Wang, Y. -B. Dong and Z. -Y. Zhang, arXiv:1303.4140 [nucl-th].

[68] Q. Wang, C. Hanhart and Q. Zhao, arXiv:1303.6355 [hep-ph].

[69] F. -K. Guo, C. Hanhart, U. -G. Meissner, Q. Wang and Q. Zhao, arXiv:1306.3096 [hep-ph].

[70] L. S. Brown and R. N. Cahn, Phys. Rev. Lett. **35**, 1 (1975).

[71] M. B. Voloshin, JETP Lett. **21**, 347 (1975) [Pisma Zh. Eksp. Teor. Fiz. **21**, 733 (1975)].

[72] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **111**, 242001 (2013) arXiv:1309.1896 [hep-ex].

[73] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **94**, 182002 (2005) [hep-ex/0408126].

[74] B. Aubert *et al.* [BaBar Collaboration], Phys. Rev. Lett. **101**, 082001 (2008) [arXiv:0711.2047 [hep-ex]] and P. del Amo Sanchez *et al.* [BaBar Collaboration], Phys. Rev. D **82**, 011101 (2010) [arXiv:1005.5190 [hep-ex]].

[75] S. Uehara *et al.* [Belle Collaboration], Phys. Rev. Lett. **104**, 092001 (2010) [arXiv:0912.4451 [hep-ex]].

[76] J. P. Lees *et al.* [BaBar Collaboration], Phys. Rev. D **86**, 072002 (2012) [arXiv:1207.2651 [hep-ex]].

[77] X. Liu, Z. G. Luo and Z. F. Sun, Phys. Rev. Lett. **104**, 122001 (2010) [arXiv:0911.3694 [hep-ph]].

[78] F. K. Guo and U. G. Meissner, Phys. Rev. D **86**, 091501 (2012) [arXiv:1208.1134 [hep-ph]].

[79] S. L. Olsen, Phys. Rev. D **91**, no. 5, 057501 (2015).

[80] Xin Li and M. B. Voloshin, Phys. Rev. D **91**, 114014 (2015).

[81] J. Brodzicka *et al.* [Belle Collaboration], Phys. Rev. Lett. **100**, 092001 (2008) [arXiv:0707.3491 [hep-ex]].

[82] A. Vinokurova *et al.* [Belle Collaboration], arXiv:1501.06351 [hep-ex].

[83] S. M. Flatte, Phys. Lett. B **63**, 224 (1976).

[84] M. Ablikim *et al.* [BESIII Collaboration], Phys. Rev. Lett. **114**, 092003 (2015) arXiv:1410.6538 [hep-ex].

[85] Xin Li and M. B. Voloshin, Phys. Rev. D **91**, 034004 (2015)

[86] M. Ablikim *et al.* [BES Collaboration], eConf C **070805**, 02 (2007) [Phys. Lett. B **660**, 315 (2008)] [arXiv:0705.4500 [hep-ex]].

[87] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **94**, 182002 (2005) [hep-ex/0408126].

[88] H. Severini *et al.* [CLEO Collaboration], Phys. Rev. Lett. **92**, 222002 (2004) [hep-ex/0307034].

[89] X. H. He *et al.* [Belle Collaboration], Phys. Rev. Lett. **113**, 142001 (2014) [arXiv:1408.0504 [hep-ex]].

[90] M. B. Voloshin, Mod. Phys. Lett. A **18**, 1067 (2003) [hep-ph/0304165].

[91] L. Ma, X. H. Liu, X. Liu and S. L. Zhu, Phys. Rev. D. **91**, 034032 (2015) arXiv:1406.6879 [hep-ph].

[92] W. Kwong and J. L. Rosner, Phys. Rev. D **38**, 279 (1988).

[93] S. Godfrey and J. L. Rosner, Phys. Rev. D **66**, 014012 (2002) [hep-ph/0205255].

[94] I. Adachi *et al.* [Belle Collaboration], Phys. Rev. Lett. **108**, 032001 (2012) [arXiv:1103.3419 [hep-ex]].

[95] J. P. Lees *et al.* [BABAR Collaboration], Phys. Rev. D **84**, 091101 (2011) [arXiv:1102.4565 [hep-ex]].

[96] R. Mizuk *et al.* [Belle Collaboration], Phys. Rev. Lett. **109**, 232002 (2012) arXiv:1205.6351 [hep-ex].

[97] Xin Li and M. B. Voloshin, Phys. Rev. D **86**, 094013 (2012)

[98] M. B. Voloshin and V. I. Zakharov, Phys. Rev. Lett. **45**, 688 (1980).

[99] M. B. Voloshin, Sov. J. Nucl. Phys. **43**, 1011 (1986) [Yad. Fiz. **43**, 1571 (1986)].

[100] D. J. Gross, S. B. Treiman and F. Wilczek, Phys. Rev. D **19**, 2188 (1979).

[101] V. A. Novikov, M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B **165**, 55 (1980).

[102] Xin Li and M. B. Voloshin, Phys. Rev. D **90**, 033016 (2014).

[103] S. Sinha and M. P. Khanna, Mod. Phys. Lett. A **14**, 651 (1999).

[104] M. B. Voloshin, Phys. Lett. B **476**, 297 (2000) [hep-ph/0001057].

[105] M. Shifman and A. Vainshtein, Phys. Rev. D **71**, 074010 (2005) [hep-ph/0501200].

[106] S. Stone, arXiv:1406.6497 [hep-ex].

[107] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **111**, 102003 (2013) [arXiv:1307.2476 [hep-ex]].

[108] R. Aaij *et al.* [LHCb Collaboration], Phys. Lett. B **734**, 122 (2014) [arXiv:1402.6242 [hep-ex]].

[109] T. A. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. D **89**, 072014 (2014) [arXiv:1403.8126 [hep-ex]].

[110] V. M. Abazov *et al.* [D0 Collaboration], Phys. Rev. D **85**, 112003 (2012) [arXiv:1204.2340 [hep-ex]].

[111] S. Chatrchyan *et al.* [CMS Collaboration], JHEP **07**, 163 (2013) [arXiv:1304.7495 [hep-ex]].

[112] G. Aad *et al.* [ATLAS Collaboration], Phys. Rev. D **87**, no. 3, 032002 (2013) [arXiv:1207.2284 [hep-ex]].

[113] M. A. Shifman and M. B. Voloshin, Sov. Phys. JETP **64**, 698 (1986) [Zh. Eksp. Teor. Fiz. **91**, 1180 (1986)].

[114] A. Lenz, arXiv:1405.3601 [hep-ph].

[115] R. Aaij *et al.* [LHCb Collaboration], arXiv:1405.7223 [hep-ex].

[116] R. Aaij *et al.* [LHCb Collaboration], arXiv:1405.1543 [hep-ex].

[117] P. Colangelo and F. De Fazio, Phys. Lett. B **387**, 371 (1996) [hep-ph/9604425].

[118] M. Di Pierro *et al.* [UKQCD Collaboration], Phys. Lett. B **468**, 143 (1999) [hep-lat/9906031].

[119] M. B. Voloshin, Phys. Rev. D **61**, 074026 (2000) [hep-ph/9908455].

[120] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Sov. Phys. JETP **45**, 670 (1977) [Zh. Eksp. Teor. Fiz. **72**, 1275 (1977)].

[121] H. G. Dosch, M. Jamin and B. Stech, Z. Phys. C **42**, 167 (1989).

[122] A. Chodos, R. L. Jaffe, K. Johnson and C. B. Thorn, Phys. Rev. D **10**, 2599 (1974).