# ON THE PREDICTABILITY OF GROWTH AND TRADE

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Timothy J. Kehoe

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# Dedication

To B.

## Abstract

This thesis is composed of three chapters, which are tied together by their focus on discussing factors that affect our ability to predict changes in international trade and economic growth.

The first chapter theoretically and quantitatively evaluates the hypothesis that, due to the existence of large firms (granularity), idiosyncratic shocks to individual firms can lead to significant variation in the growth of countries. I embed granularity, through finiteness in the set of firms, in a general equilibrium environment featuring monopolistic competition, growth, and international trade. Firm productivities grow according to idiosyncratic productivity shocks, which obey a Gibrat's law proportional growth process, and are the only source of growth in the model. I derive an approximate analytic mapping for the standard deviation of GDP growth in this framework, which is non-zero due to granularity. This mapping depends on only a few key parameters, which I estimate for a wide-range of countries using firm-level micro data. My results indicate that idiosyncratic shocks to firms can play a significant role in generating both short-run macroeconomic fluctuations and variation in longer-run growth trends, particularly for countries that engage heavily in international trade. Empirically, I show that the model does well in matching relative differences in GDP volatility across OECD countries.

The second chapter discusses the granular hypothesis and the importance of Pareto tails in generating aggregate uncertainty and instability due violations of the Central Limit Theorem. I argue that the importance of Pareto tails has been significantly overstated and that significant aggregate uncertainty can arise even in the cases where the Central Limit Theorem holds. I revisit the debate on the distribution of firm sizes and show that, when

appropriate statistical methods are used, there is significant variation across countries in whether the distribution of firm sizes follows a Pareto distribution or a lognormal distribution. Despite this variation, I show that these differences are largely irrelevant in determining how much aggregate uncertainty we can expect to arise due to granularity, indicating that the presence of Pareto tails is largely irrelevant and that the pathways through which microeconomic heterogeneity can lead to aggregate uncertainty and instability are more general than previously thought.

The third chapter, joint work with Timothy J. Kehoe and Kim J. Ruhl, develops a methodology for predicting the impact of trade liberalization on exports by industry (3-digit ISIC) based on the pre-liberalization distribution of exports by product (5-digit SITC). We evaluate the ability of our methodology to account for the industry-level variation in export growth by using our model to "predict" the growth in industry trade from the North American Free Trade Agreement (NAFTA). We show that our method performs significantly better than the applied general equilibrium models originally used for the policy evaluation of NAFTA. We find that the most important products in our analysis are not the ones with zero pre-liberalization trade, but those with positive, yet small amounts of pre-liberalization trade.

## Table of Contents

## List of Tables

## List of Figures

Chapter 1

# Good Policy or Good Firms? International Competition and Aggregate Growth in a Granular World

## 1.I. Introduction

Why do countries grow more in some periods than others? Easterly, Kremer, Pritchett, and Summers (1993) show that despite relatively high stability in country characteristics and policy across decades, cross-decade growth is highly unstable. They argue that variation in the growth of countries, therefore, appears to be driven primarily by aggregate shocks, or luck, rather than policy. Cochrane (1994) shows, however, that along with policy shocks, well defined aggregate shocks often have difficulty explaining short-run macroeconomic fluctuations. These results raise the question of what drives variation in the growth of countries. In a recent paper, Gabaix (2011) advances the hypothesis that short-run macroeconomic fluctuations can arise directly from idiosyncratic shocks to large firms. This "granular" hypothesis indicates a break from the view that shocks to individual firms wash out at the aggregate level, due to the large number of firms, and relies on the observation that for many countries a small portion of firms account for a large fraction of overall output. In this chapter, I extend the granular hypothesis, proposing that idiosyncratic shocks to firms can play a significant role in not only generating short run macroeconomic fluctuations, but also variation in longer-run growth trends. Further, I argue that the magnitude of aggregate variation that arises due to idiosyncratic shocks increases as countries engage more heavily in international trade.

Theoretically and quantitatively evaluating this extended granular hypothesis necessitates a break from frameworks in which all firms are infinitesimal relative to the size of an economy, to ones in which individual firms have positive mass and are therefore able to affect macroeconomic aggregates. I embed granularity, through finiteness in the set of firms, in a general equilibrium framework featuring growth, monopolistic competition among firms, and international trade. The only source of growth in the framework is due to idiosyncratic shocks to the productivities of individual firms, which obey a Gibrat's law proportional growth process after Gibrat (1931). Section 1.III specifies the full environment and derives an approximate analytic mapping for the standard deviation of constant price GDP growth, which is non-zero due to granularity. This mapping depends on only a few key parameters, which are straightforward to estimate, and highlight the forces governing the degree to which aggregate variation arises from idiosyncratic shocks to firms in this environment. The primary parameter governing this mapping is how concentrated output is among firms, or how granular the economy is, as measured by the herfindahl index of the economy. The other parameters that govern this mapping are the growth process for firms (in particular the volatility of firm growth), how substitutable output is across firms, and the degree to which a country's largest firms compete internationally versus domestically.

International competition, relative to domestic competition, amplifies the impact individual firms can have on an economy, as when a firm increases its output, its competitors react by lowering their output. If these competitors are domestic firms, this partially negates the aggregate impact of the initial increase. When there is international competition, however, some of the competitors reacting are located outside of the country. When market shares are exchanged internationally, this has a larger aggregate impact than when market shares are reshuffled among firms domestically. The intuition is that for the U.S. economy, the decline of Sears has been largely offset by, and, in fact, has contributed to, the rise of Walmart and Target. For Finland's economy, however, the decline of Nokia has larger implications, even conditioning on Nokia's size relative to Finland's economy, as Nokia's lost sales are not going to other firms in Finland, but rather to firms outside of the country such as Samsung and Apple. International competition, therefore, amplifies the aggregate variation that can arise due to idiosyncratic shocks to firms.

Whereas previous research has focused on the role of granularity in generating short-run fluctuations (quarterly or annual), this chapter extends the granular framework to examine the extent to which idiosyncratic shocks to firms can lead to variation in aggregate growth over longer periods of time, focusing especially on variation in decade-to-decade growth. Due to the Gibrat's law growth process for firm productivities, the effects of idiosyncratic shocks accumulate over time leading to higher variation in the growth of firms and therefore higher variation in GDP growth over longer periods of time. This feature is consistent with the fact that the standard deviation of 10-year firm growth rates is significantly higher than the standard deviation of 1-year firm growth rates, and similarly for 10-year GDP growth rates compared to 1-year GDP growth rates.

In section 1.V, I estimate the parameters of the framework for a wide-range of countries using aggregate data on exports, expenditures, and price indices, as well as firm-level data on sales and gross-margins. I plug these parameter estimates into the analytic mapping derived in section 1.III to quantitatively evaluate the question: "How much variation in GDP growth would we expect in each country if differences in the realizations of idiosyncratic shocks to firms were the only source of aggregate variation?" My quantitative results indicate that idiosyncratic shocks to firms, of the order estimated in firm-level data, can be a significant source of aggregate variation, particularly for countries that engage heavily in international trade. For OECD countries, the model predicts a standard deviation of both 1-year and 10-year GDP growth rates roughly half the magnitude observed in the data. These results suggest that idiosyncratic shocks to firms can potentially play a large role in not only explaining why countries grow more in some years than others, as argued by Gabaix (2011), but also why countries grow more in some decades than others. The granular hypothesis, therefore, provides a potential microfoundation for the aggregate "luck" shocks of Eaton et. al. (1993) in terms of accumulated idiosyncratic productivity shocks to the largest and fastest growing firms in an economy.

In section 1.VI, I show that the predictions of the model are consistent with several stylized facts regarding cross-country differences in GDP growth volatility. In particular, I find that the model does well in matching relative differences in the observed volatility of annual GDP growth rates across OECD countries (R-squared of 0.47), indicating the likely importance of idiosyncratic shocks in generating aggregate fluctuations. Consistent with the predictions of the model, I find that countries which export more as a share of GDP exhibit a higher volatility of annual GDP growth rates, and similarly for countries that are more concentrated in their output as measured by their Herfindahl index. After establishing that the model does well in matching stylized facts regarding annual GDP growth rate volatility, I focus on the model's predictions regarding cross-country variation in growth. I group countries together depending on their predicted variation of 10-year GDP growth rates in the model and show that countries in groups with higher predicted variation in 10-year GDP growth rates do indeed exhibit higher cross-country variation in their growth over 1995–2005, with the model again generating roughly half the variation observed in the data.

### 1.II. Related Literature

This chapter builds on recent research regarding the potential microeconomic origins of aggregate variation. Gabaix (2011) and Carvalho and Gabaix (2013) explore the extent to which idiosyncratic shocks to firms can explain U.S. GDP volatility in granular environments, focusing particularly on the role of linkages across firms and sectors. Carvalho and Graiss (2015) embed granularity in a dynamic Hopenhayn (1992) framework to study how firm dynamics can drive business cycles. Notably, these papers do not require frictions in order to generate aggregate variation from idiosyncratic shocks to firms; instead it arises naturally due to high concentration of output among firms of the magnitude observed in the data. This contrasts with non-granular environments, such as Arellano, Bai, and Kehoe (2012), in which idiosyncratic shocks to firms affect aggregate output only under the presence of financial frictions. A parallel strand of research explored by Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012), Horvath (2000), Dupor (1999), Ataly (2014), Koren and Tenreyro (2007), and Caplin and Leahy (1993) focuses instead on sectoral granularity and the role of sectoral shocks in generating aggregate fluctuations.

This chapter relates to the literature on Gibrat's law based models of firm growth, recent examples of which are papers by Luttmer (2007, 2010) and Arkolakis (2013), by showing how significant aggregate variation arises in these models when there are a finite number of firms. In the models of both Luttmer and Arkolakis, the underlying Gibrat's law based productivity ultimately leads to a right tail that follows a Pareto distribution. Gabaix (2011) notes that when the distribution of firms sizes follows a Pareto distribution, output will remain highly concentrated among firms even when there is a large number of firms overall. Di Giovanni and Levchenko (2012) make use of the insight from Melitz (2003), that lowering barriers to international trade causes small firms to exit and large firms to become larger, to show how international trade increases the concentration of output among firms and therefore increases volatility in granular economies, particularly for smaller countries.

### 1.III. Theoretical Framework

This section lays out the framework of the model. There are two necessary elements required to generate non-zero aggregate variation due to variation in the growth of firms. These elements are finiteness in the set of firms, which Gabaix (2011) refers to as granularity, and idiosyncratic variation in the growth process of firms. To understand how these features translate to aggregate variation in an environment where shocks to firms affect other firms in the economy, I embed these features in an otherwise standard general equilibrium framework featuring international trade and multiple industries, where heterogeneous firms engage in monopolistic competition within each industry ala Krugman (1979) and Dixit and Stiglitz (1977).

### 1.III.A. A Simple Example

Before describing the general equilibrium environment, I present a simple example based on the framework of Gabaix (2011) showing how idiosyncratic shocks to firms can generate aggregate variation due to granularity in an environment where shocks to output are independent across firms. Suppose a firm accounts for 10% of a country's aggregate output, and receives a shock that increases its output by 15%. This shock will therefore increase aggregate output by 1.5% $(= 10\% * 15\%)$. Similarly, the variance in aggregate output growth due to this firm will be $(10\%)^2\sigma_{firm}^2$, where $\sigma_{firm}^2$ is the variance of the firm's shocks. Assuming that shocks are independent across firms, the variance of aggregate output resulting from shocks to individual firm shocks will be $\sigma_{Agg}^2 = \sum_{m=1}^{M}(s_m)^2\sigma_m^2$ where $s_m$ is firm $m$'s share of total output and $\sigma_m^2$ is the variance of firm $m$'s shock. When this variance is identical across firms, the formula simplifies to $\sigma_{Agg}^2 = \sigma_{firms}^2 h$, where $h = \sum_{m=1}^{M}(s_m)^2$ is the Herfindahl index for this set of firms. Therefore in this simple framework, aggregate variation is inherited directly from variation in firm shocks, scaled by a measure of the concentration of output among firms.

### 1.III.B. General Equilibrium Framework

There are $i, j = 1, \ldots, N$ countries, and $k = 1, \ldots, K$ industries, where $K^{TR}$ of the industries are tradable and $K^{NT}$ are non-tradable industries. As an abuse of notation, I also let $K^{TR}$ and $K^{NT}$ denote the set of traded and non-traded industries respectively. There is measure $L_i$ of consumers in country $i$, each with 1 unit of labor, which is supplied inelastically. Consumers have Cobb-Douglas preferences over industries, with period utility at time $t$ given by:

$$U_{i,t} = \sum_{k=1}^{K} \theta_k \log C_{i,k,t},$$

where $C_{i,k,t}$ is consumption of industry $k$'s final output and $\theta_k$ is industry $k's$ expenditure share. Consumers earn income from wages and profits from firms, which are rebated to the consumers in the country the firm is headquartered in. Country $i$'s expenditures on industry $k$ are therefore given by $E_{i,k,t} = \theta_k\left(w_{i,t}L_i + \pi_{i,t}\right)$, where $\pi_{i,t}$ is the total profit of country $i's$ firms and $w_{i,t}$ is the wage of country $i$.

Industries can be either traded or nontraded. In non-traded industries, market clearing ensures that $C_{i,k,t} = Y_{i,k,t}^{i}$, where $Y_{i,k,t}^{i}$ is domestic industry output. In traded industries, industry consumption is a CES bundle of foreign and domestic industry output. Perfectly competitive bundlers solve

$$\min \sum_{j=1}^{N} P_{j,k,t}^{i} Y_{j,k,t}^{i}$$

subject to their technology for creating final consumption output in industry $k$.

$$Y_{i,k,t}^{C} \coloneqq \left(\sum_{j=1}^{N} \left(Y_{j,k,t}^{i}\right)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

Within each industry, there are a finite number of firms each producing a differentiated varieties. Differentiated varieties are transformed into industry output using a constant elasticity of substitution (CES) production function by perfectly competitive set of output bundlers in each industry. Final output producers in country $i$ for industry $k$ minimize costs by selecting output from the $m = 1, \dots, M_{i,k}$ differentiated firms within each industry:

$$\min \sum_{m=1}^{M_{i,k}} y_{i,k,t}^{j,m} y_{i,k,t}^{j,m}$$

subject to the production function

$$Y_{i,k,t}^{j} = \left( \sum_{m=1}^{M_{i,k}} \left( y_{i,k,t}^{j,m} \right)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}$$

Here $\epsilon$ is the elasticity of substitution between products within an industry, $y_{i,k,t}^{j,m}$ is the output of the variety produced by firm $m$ located in country $i$ used in country $j$ at time $t$, and $p_{i,k,t}^{j,m}$ is the price of this output. When the industry is non-traded, $y_{i,k,t}^{j,m} \equiv 0$ for $j \neq i$, as firms are not able to export non-traded varieties to foreign countries.

Firms differ in their productivity and produce their variety using labor, which is supplied inelastically. As in Atkeson and Burstein (2008), there are a finite number of firms in each industry, although here there are a finite number of industries as well. I assume there are no trade costs in traded sectors, therefore output of firm $m$ produced for country $j$ is be given by:

$$y_{i,k,t}^{j,m} = z_{i,k,t}^{m} l_{i,k,t}^{j,m},$$

where $z_{i,k,t}^{m}$ is firm $m$'s productivity, $l_{i,k,t}^{j,m}$ is the amount of labor used to produce output for country $j$. The optimization of industry bundlers yields the quantity demanded of each variety as a function of prices as:

$$y_{j,k,t}^{i,m} = \frac{E_{i,k,t}}{\left( p_{j,k,t}^{i,m} \right)^{\frac{1}{\epsilon}} \left( P_{i,k,t}^{C} \right)^{\epsilon-1}},$$

where $P_{i,k,t}^{C}$ is the overall price index for industry consumption in country $i$

$$P_{i,k,t}^{C} = \left( \sum_{j=1}^{N} \sum_{m=1}^{M_{i,k}} \left(p_{j,k,t}^{i,m}\right)^{\epsilon-1} \right)^{\frac{1}{\epsilon-1}}.$$

A common assumption within this CES framework is that firms either have no effect on (as is the case when there is a continuum of firms), or ignore their effect on, total industry output. When firms face linear output costs this results in firms charging a constant markup over marginal cost. In a granular framework, keeping this assumption implies that, although firms affect the industry price index, they do not take this into account when optimizing. There are two reasons for imposing this assumption. The primary reason is that this chapter is about how having a finite number of firms leads to variation in aggregate variables, and to make the results as comparable as possible to a non-granular framework I hold fixed the pricing behavior of large firms across the granular and non-granular frameworks. The second reason is more practical: this assumption allows me to solve for equilibrium analytically. This assumption can be discarded, and the equilibrium can still be solved numerically. Di Giovanni and Levchenko (2012), however, show that imposing this restriction in unlikely to significantly affect the magnitude of aggregate variation. Still, ignoring the effects of changes in markups is somewhat at odds with the large literature dealing with how markups are affected by international trade, for instance Locker, Goldberg, Khandelwal, and Pavcnik (2012) and Edmond, Midrigan, and Xu (2012). An alternative possibility would be to relax the assumption of constant markups without allowing full Cournot or Bertrand competition across firms. For example, the equilibrium could still be solved analytically if we allow for endogenous markups resulting from either non-linear output costs as in Hottman, Redding, and Weinstein (2014) or misallocation as in Peters (2013). Similarly, it's possible that models that deviate from the standard CES framework, for example through adding non-homotheticities to the CES production functions as in Sato (1977), or to consumer preferences as in Simonovska (2010), may increase the importance of changes in markups.

Under the assumption that firms optimize without taking into account their effect on the industry price index, the firm's price will be a constant markup over its marginal cost:

$$p_{i,k,t}^{m} = \left(\frac{\epsilon}{\epsilon - 1}\right)\frac{w_{i,t}}{z_{i,k,t}^{m}},$$

where the markups depend on the elasticity of substitution across differentiated products within an industry. In this constant markup framework, the profits earned by a firm are proportional to the amount of inputs used by that firm:

$$\pi_{i,k,t}^{m} = \left(\frac{1}{\epsilon - 1}\right)\sum_{j=1}^{N} l_{i,k,t}^{j,m},$$

and firm sales are proportional to the firm's productivity raised to the power $\epsilon - 1$:

$$p_{i,k,t}^{m}\, y_{i,k,t}^{m} = \sum_{j=1}^{N} p_{i,k,t}^{j,m}\, y_{i,k,t}^{j,m} = \left(\frac{z_{i,k,t}^{m}}{w_{i,k,t}}\right)^{\epsilon-1} \frac{\theta_k \left(\frac{\epsilon - 1}{\epsilon}\right)\sum_{j=1}^{N} L_j}{\sum_{j=1}^{N}\sum_{m'=1}^{M_{j,k}}\left(\frac{z_{j,k,t}^{m'}}{w_{j,t}}\right)^{\epsilon-1}}.$$

This last equation indicates that when varieties are highly substitutable, a small increase in productivity leads to a large increase in sales.

### 1.III.C Linking Aggregate Growth to Firm Growth

Starting at an initial time $t_0 = 0$, I compute the constant price GDP growth of country $i$ after $T$ periods as:

$$\Delta_{\mathrm{i,GDP,T}} := \frac{\sum_{k=1}^{K} P_{i,k,0} Y_{i,k,T}}{\sum_{k=1}^{K} P_{i,k,0} Y_{i,k,0}} = \frac{\sum_{k=1}^{K} P_{i,k,0} Y_{i,k,0} \left( \frac{Y_{i,k,T}}{Y_{i,k,0}} \right)}{\sum_{k=1}^{K} P_{i,k,0} Y_{i,k,0}}$$

Where $P_{i,k,0}$ is the initial price index of country $i$'s output, and $Y_{i,k,t} = \sum_{j=1}^{N} Y_{i,k,t}^{j}$ is country $i$'s total output in industry $k$ at time $t$.

### 1.III.C.i Linking Aggregate Growth to Firm Growth: Autarky

If the country is in autarky, then $P_{i,k,0} Y_{i,k,0} = \theta_k E_{i,k,0}$, which means that constant price GDP growth can be expressed as

$$\Delta_{i,GDP,T}^{aut} = \sum_{k=1}^{K} \theta_k \frac{Y_{i,k,T}}{Y_{i,k,0}} = \sum_{k=1}^{K} \theta_k \left( \sum_{m=1}^{M_{i,k}} \frac{\left(z_{i,k,t}^{m}\right)^{\epsilon-1}}{\left(\sum_{m=1}^{M_{i,k}} \left(z_{i,k,0}^{m}\right)^{\epsilon-1}\right)} \right)^{\frac{1}{\epsilon-1}}.$$

Now let $X_{i,k,T}^{m} = z_{i,k,T}^{m} / z_{i,k,0}^{m}$ be firm $m$'s productivity growth after $T$ periods, then in autarky, we can write constant price GDP growth as a weighted power mean of productivity shocks to firms in each industry where the power is $\epsilon - 1$ and each firm's productivity shock is weighted by its share of industry sales for firms located in that country:

$$\Delta_{i,GDP,T}^{aut} = \sum_{k=1}^{K} \left( \sum_{m=1}^{M_{i,k}} \left(X_{i,k,T}^{m}\right)^{\epsilon-1} \left(\omega_{i,k,0}^{m,aut}\right) \right)^{\frac{1}{\epsilon-1}}$$

where $\omega_{i,k,0}^{m,aut}$ is the weight for each firm in autarky

$$\omega_{i,k,0}^{m,aut} := \theta_k \frac{p_{i,k,0}^{m} y_{i,k,0}^{m}}{\sum_{m=1}^{M_{i,k}} p_{i,k,0}^{m} y_{i,k,0}^{m}} = \theta_k \frac{\left(z_{i,k,0}^{m}\right)^{\epsilon-1}}{\sum_{m=1}^{M_{i,k}} \left(z_{i,k,0}^{m}\right)^{\epsilon-1}}$$

and note that $\sum_{k=1}^{K} \sum_{m=1}^{M_{i,k}} \omega_{i,k,0}^{m,aut} = 1$.

### 1.III.C.ii Linking Aggregate Growth to Firm Growth: Trade

When a country engages in trade, expenditures do not necessarily equal production in traded sectors. Instead we have the balanced trade condition:

$$\sum_{k=1}^{K^{TR}} \sum_{\substack{j=1 \\ j \neq i}}^{N} P_{i,k,t} Y_{i,k,t}^{j} = \sum_{k=1}^{K^{TR}} \sum_{\substack{j=1 \\ j \neq i}}^{N} P_{j,k,t} Y_{j,k,t}^{i}$$

while total industry expenditures across countries still equals total industry production:

$$\sum_{j=1}^{N} P_{j,k,t} Y_{j,k,t} = \theta_k \sum_{j=1}^{N} E_{j,t}.$$

In the case with trade, constant price GDP growth becomes (I normalize $w_{i,t} = 1$)

$$\Delta_{\mathrm{i,GDP,T}} = \sum_{k=1}^{K^{NT}} \theta_k \left( \sum_{m=1}^{M_{i,k}} \frac{\left(z_{i,k,t}^{m}\right)^{\epsilon-1}}{\left(\sum_{m=1}^{M_{i,k}} \left(z_{i,k,0}^{m}\right)^{\epsilon-1}\right)} \right)^{\frac{1}{\epsilon-1}}$$

$$+ \left( \sum_{k=1}^{K^{TR}} \theta_k \right) \left( \frac{\sum_{j=1}^{N} E_{j,t}}{\sum_{j=1}^{N} E_{j,0}} \right) \frac{\sum_{k=1}^{K^{TR}} \theta_k \left(s_{i,k,t}^{m}\right) \left( \sum_{m=1}^{M_{i,k}} \frac{\left(z_{i,k,t}^{m}\right)^{\epsilon-1}}{\left(\sum_{m=1}^{M_{i,k}} \left(z_{i,k,0}^{m}\right)^{\epsilon-1}\right)} \right)^{\frac{1}{\epsilon-1}}}{\sum_{k=1}^{K^{TR}} \theta_k \, s_{i,k,0}}$$

where $s_{i,k,t}^{m}$ is firm $m$'s share of sales in industry $k$ for country $i$ in period $t$:

$$s_{i,k,t}^{m} := \frac{p_{i,k,t}^{m} y_{i,k,t}^{m}}{\sum_{j=1}^{N} \sum_{m'=1}^{M_{i,k}} p_{j,k,t}^{m'} y_{j,k,t}^{m'}} = \frac{\left(z_{i,k,t}^{m}\right)^{\epsilon-1}}{\sum_{j=1}^{N} \sum_{m=1}^{M_{i,k}} \left(\frac{z_{j,k,t}^{m}}{w_{j,t}}\right)^{\epsilon-1}}. \qquad (1.1)$$

Again let $X_{i,k,T}^{m} = z_{i,k,T}^{m} / z_{i,k,0}^{m}$ be firm $m$'s productivity growth after $T$ periods, then the first term in the expression for $\Delta_{\mathrm{i,GDP,T}}$, which is for the nontraded industries, will be a weighted power mean as in the case of autarky. The second term, which is for the traded sector, in the expression for $\Delta_{\mathrm{i,GDP,T}}$ is again a weighted power mean, as

$$\sum_{k=1}^{K^{TR}} \frac{\theta_k \left(\sum_{j=1}^{N} E_{j,t}\right)\left(s_{i,k,t}^{m}\right)}{\sum_{k=1}^{K^{TR}} \theta_k \left(\sum_{j=1}^{N} E_{j,0}\right)\left(s_{i,k,0}^{m}\right)} = 1,$$

however, unlike for non-traded industries, the weights are no longer independent of the realizations of productivity shocks. Therefore we can write

$$\Delta_{\mathrm{i,GDP,T}} = \sum_{k=1}^{K} \left( \sum_{m=1}^{M_{i,k}} \left(X_{i,k,T}^{m}\right)^{\epsilon-1} \omega_{i,k,T}^{m} \right)^{\frac{1}{\epsilon-1}}, \qquad (1.2)$$

where

$$\omega_{i,k,T}^{m} = \begin{cases} \theta_k \dfrac{\left(z_{i,k,0}^{m}\right)^{\epsilon-1}}{\sum_{m=1}^{M_{i,k}} \left(z_{i,k,0}^{m}\right)^{\epsilon-1}}, & k \in K^{NT} \\ \left(\sum_{k=1}^{K^{TR}} \theta_k\right) \dfrac{\theta_k \left(\sum_{j=1}^{N} E_{j,t}\right)\left(s_{i,k,T}^{m}\right)}{\sum_{k=1}^{K^{TR}} \theta_k \left(\sum_{j=1}^{N} E_{j,0}\right)\left(s_{i,k,0}^{m}\right)}, & k \in K^{TR} \end{cases} \qquad (1.3)$$

and again $\sum_{k=1}^{K} \sum_{m=1}^{M_{i,k}} \omega_{i,k,T}^{m} = 1$.

This formula is very similar to the autarky case, except now, for traded sectors, the weight for each firm's productivity growth is no longer independent of the productivity growth of firms in the economy. The reason for this, is that international trade allows labor to move from less productive firms in one traded industry to more productive firms in other traded industries. Without trade, constant expenditure shares ensure that a constant fraction of labor is allocated to each sector, regardless of the productivity of the sector. Trade allows countries to specialize in producing the industry varieties that they are more productive in, while importing the industry varieties that they are less productive in. This will influence both the initial Herfindahl of the economy, as countries will specialize in the industries they are most productive in – which is determined by how productive the firms in the industry are.

**1.III.D Productivity Growth Process for Firms**

I assume that that the underlying productivity growth process is common across firms, while the realizations of the productivity growth process are idiosyncratic for each firm. This is a form of Gibrat's Law, which says that the growth rate of each firm is independent of the size of the firm. Productivity growth evolves according to a geometric Brownian motion process:

$$dX_T = \mu X_t dT + \sigma X_T dW_T, \qquad (1.4)$$

$$X_0 = 1,$$

where $\mu$ is the drift (average annual growth rate of logged firm productivity), and $\sigma$ is the volatility (standard deviation of logged firm productivity growth), and $W_T$ is a standard Wiener process. The realizations of the Wiener process are unique across firms. This process implies that productivity growth follows a lognormal distribution with mean and variance after $T$ periods given by:

$$E[X_T] = e^{\mu T}, \qquad (1.5)$$

$$Var[X_T] = e^{2T\mu}\left(e^{T\sigma^2} - 1\right). \qquad (1.6)$$

Empirical research has repeatedly shown that firm growth is well approximated by geometric Brownian motion (GBM), for example see Cabral and Mata (2003), Arkolakis (2013), Singh and Whittington (1975) among many others. Similarly, GBM growth processes are capable of generating many of the stable firm size distributions proposed to fit observed firm sizes. For instance, Stanley et. al. (1995) claims that firm sizes follow a lognormal distribution, which can be generated by a pure GBM process on firm productivity. Axtell (2001) argues that firm sizes are instead best approximated by a Pareto distribution, which can be achieved by adding a lower reflective bound for productivity leads to a Pareto distribution as shown in Gabaix (1999). Other possibilities that can be generated through modifications to an underlying GBM process are a double Pareto distribution as in Arkolakis (2013), or for logged size to follow a mixture of gamma distributions as in Luttmer (2007). I refer the reader to Luttmer (2010) and de Wit (2005) for surveys of the conditions under which GBM produces various distributions. As there is still some controversy and disagreement over what the distribution of firm sizes is in the data, a strength of this framework is that it is not necessary to take a stand on what the initial distribution of firm sizes "should be" or how the initial distribution of firm productivities arises. Instead the framework is compatible with an arbitrary distribution of firm productivities, which can be inferred from firm-level data.

### 1.III.E Deriving Variation in GDP Growth

In this section I derive an expression for the standard deviation of constant price GDP growth after $T$ periods, conditional on an arbitrary initial distribution of firm productivities, and the productivity growth process for firms. The distribution of GDP growth is a weighted power mean of lognormal random variables. No simple expression for the density of the sum of lognormal random is known, and, except in the case of autarky where $\epsilon = 2$ leading to linearity in equation (1.2), there is no known closed-form expression for the first two moments of the weighted power mean of lognormal random variables. I instead focus on deriving a simple approximate analytic expression for the mean and standard deviation of GDP growth. There are two benefits to having an analytic expression as opposed to simulating the model. The first is that it makes it clear what role every part of the framework plays in generating and amplifying variation in GDP growth. The second is that is that it makes it easy to quantitatively evaluate the model and apply it across countries, as only a few parameters will need to be estimated for each country.

### 1.III.E.i Deriving Variation in GDP Growth: Autarky

In the case of autarky, the weights of the weighted power mean in equation (1.2) are constant and do not change depending on the shock. An exact closed form expression for the distribution of GDP growth cannot be given, however, an approximation for the mean and variance of GDP growth can be found through a double application of the delta method which makes use of taylor approximations. The approximation yields:

$$E\left[\Delta_{i,GDP,T}^{aut}\right] \approx \xi(T,\sigma,\epsilon)E[X_T], \qquad (1.7)$$

$$Var\left[\Delta_{i,GDP,T}^{aut}\right] \approx \left(\xi(T,\sigma,\epsilon)\right)^2 h_{i,0}^{aut} Var[X_T], \qquad (1.8)$$

where $E[X_T]$ and $Var[X_T]$ are from (1.5–1.6), and $\sigma^2$ is from the GBM process in (1.4),

$$h_{i,0}^{aut} := \sum_{k=1}^{K}\sum_{m=1}^{M_{i,k}} \frac{\left(p_{i,k,t}^m y_{i,k,t}^m\right)^2}{\left(\sum_{k=1}^{K}\sum_{m=1}^{M_{i,k}} p_{i,k,t}^m y_{i,k,t}^m\right)^2}$$

$$= \sum_{k=1}^{K}(\theta_k)^2 \sum_{m=1}^{M_{i,k}} \left(\frac{\left(z_{i,k,0}^m\right)^{\epsilon-1}}{\sum_{m'=1}^{M_{i,k}}\left(z_{i,k,0}^{m'}\right)^{\epsilon-1}}\right)^2, \qquad (1.9)$$

is the sales Herfindahl of the economy in autarky and

$$\xi(T,\sigma,\epsilon) = \left(1 + \frac{(\epsilon-1)(\epsilon-2)}{2}\left(e^{T\sigma^2}-1\right)\right)^{\frac{1}{\epsilon-1}}, \qquad (1.10)$$

is a term that captures the expected additional reallocation that results from non-linearity in the power mean (for $\epsilon \geq 2$). The behavior of this term is illustrated in figure 1.1. The term arises since, as the elasticity of substitution increases, firms that receive high productivity shocks will increase their labor input more, and due to this additional reallocation, constant price GDP grows slightly more than if output were less substitutable across firms. An important result of this analytic mapping is that the sales Herfindahl is a sufficient statistic for how the distribution of initial firm productivities effects the standard deviation of GDP growth. As mentioned previously, this makes it significantly easier to apply the model to make predictions, as we will not require the full distribution of firm sizes, nor even the Herfindahl for each industry.

**1.III.E.i Deriving Variation in GDP Growth: Trade**

In the case of trade, weights for firms in the traded sector are no longer independent from the shocks firms receive, as countries are able to specialize in the industries in which their relatively most productive firms reside. To assist in deriving an approximate mapping for GDP growth in this case I make two simplifying assumptions. First, I assume that there is no uncertainty in aggregate growth for the combined rest of world. This is a limiting case of assuming that the rest of the world is large enough that output is not highly concentrated among firms at a global level. The second assumption is that there is a large number of traded industries with a large number of firms in each industry. The accuracy of the approximate mapping decreases as the number of traded industries decrease, as it under predicts the dampening effects of general equilibrium changes in relative wages. For example, in the case where there is only a single traded industry, trade does not amplify variation in GDP growth at all. With these assumptions imposed, the approximate mapping for GDP growth is derived in a similar way to the autarky with a double application of the delta method. The approximate mapping yields, for $\epsilon \geq 2$:

$$E\left[\Delta_{i,GDP,T}\right] \approx \xi(T,\sigma,\epsilon)E[X_T], \qquad (1.11)$$

$$Var\left[\Delta_{i,GDP,T}\right] \approx \left(\psi\left(\epsilon, S_{i,0}\right)\right)^2 \left(\xi(T,\sigma,\epsilon)\right)^2 Var[X_T]h_{i,0}, \qquad (1.12)$$

where again $E[X_T]$ and $Var[X_T]$ are from (1.5–1.6), $\sigma^2$ is from the GBM process in (1.4),

$$h_{i,0} \coloneqq \sum_{k=1}^{K}\sum_{m=1}^{M_{i,k}} \frac{\left(p_{i,k,t}^m y_{i,k,t}^m\right)^2}{\left(\sum_{k=1}^{K}\sum_{m=1}^{M_{i,k}} p_{i,k,t}^m y_{i,k,t}^m\right)^2}$$

$$= \sum_{k=1}^{K^{NT}} (\theta_k)^2 \sum_{m=1}^{M_{i,k}} \left(\frac{\left(z_{i,k,t}^m\right)^{\sigma_k - 1}}{\sum_{m=1}^{M_{i,k,t}}\left(z_{i,k,t}^m\right)^{\sigma_k - 1}}\right)^2 + \left(\sum_{k=1}^{K^{TR}} \theta_k\right)^2 \frac{\sum_{k=1}^{K^{TR}}(\theta_k)^2\left(s_{i,k,t}^m\right)^2}{\left(\sum_{k=1}^{K^{TR}}(\theta_k)\left(s_{i,k,t}^m\right)\right)^2}, \qquad (1.13)$$

is the sales Herfindahl of the economy, which although defined the same as in autarky, will differ due to specialization in productive traded industries ($h_{i,0} \geq h_{i,0}^{aut}$). We again have $\xi(T,\sigma,\epsilon)$ as defined in equation (1.10),

$$\xi(T,\sigma,\epsilon) = \left(1 + \frac{(\epsilon-1)(\epsilon-2)}{2}\left(e^{T\sigma^2}-1\right)\right)^{\frac{1}{\epsilon-1}},$$

and $\psi\left(\epsilon, S_{i,0}\right)$ captures the additional amplifying effect of international trade outside of its effect on the Herfindahl index of an economy,

$$\psi\left(\epsilon, S_{i,0}\right) = \frac{\left(1+(\epsilon-1)S_{i,0}\right)}{\left(1+S_{i,0}\right)}, \qquad (1.14)$$

where $S_{i,0}$ is the country's initial exports as a share of GDP,

$$S_{i,0} \coloneqq \frac{\sum_{\substack{j=1 \\ j\neq i}}^{N}\left(\sum_{k=1}^{K}\sum_{m=1}^{M_{i,k}} p_{i,k,t}^{j,m} y_{i,k,t}^{j,m}\right)}{\sum_{k=1}^{K}\sum_{m=1}^{M_{i,k}} p_{i,k,t}^{m} y_{i,k,t}^{m}}. \qquad (1.15)$$

Equation (1.12) tells us that international trade has two pathways by which it increases variation in GDP growth. The first pathway is that trade increases the Herfindahl index of the economy through specialization in more productive industries, where the productivity of industries is determined by the productivity of firms within an industry. The second pathway is that it amplifies the impact of shocks to firms in traded industries, which, similarly to the effect on the Herfindahl index, is because trade breaks the restriction that production must be constant within each traded industry. This implies that firms are able to steal the market shares of not only firms within the same industry and country, but also the market shares of firms in other countries; although due to general equilibrium effects and balanced trade, the country must lose market share in other traded industries. Despite the offsetting effect of losing market share in other industries, international trade still amplifies variation in constant price GDP growth, as the firms that gain the most market share, and hence labor input, will be the firms that experience the highest productivity growth, while the firms that lose market share will be firms that experience relatively low productivity growth. This addition amplifying effect is given by $\psi\left(\epsilon, S_{i,0}\right)$, the behavior of which is illustrated in figure 1.2.

**1.IV. Discussion of Theoretical Results**

The mappings derived in section 1.III shed insight on the forces governing how idiosyncratic shocks lead to variation in realized GDP growth due to granularity, or finiteness in the set of firms, in an otherwise standard model of growth and international trade. In these mappings, granularity does not impact the expected growth of an economy, but rather allows realized GDP growth to deviate from expected GDP growth, which does not happen when there is a continuum of firms. The four forces governing the magnitude of variation in GDP growth, as measured by the variance are: i) the growth process for firms, ii) how concentrated output is across firms, iii) how substitutable output is across firms, and iv) how much a country exports as a share of GDP.

Variation in the growth process for firms is essential for allowing granularity to generate variation in growth at the aggregate level. In this chapter, I assume a Gibrat's law type proportional growth process, as this type of growth process matches observed firm growth in the data, is consistent with the observed distribution of firm sizes, and is a standard and flexible way for modeling the growth of firms. That idiosyncratic productivity shocks are both proportional and persistent (here permanent) is the reason why the model predicts higher variation in the growth of firms and therefore higher variation in

GDP growth over longer periods of time as the result of granularity. If instead, shocks were assumed to be completely temporary, the framework not be able to explain why there is higher variation in the growth of firms, nor higher variation in GDP growth over longer periods of time.

How concentrated output is among firms is the same as in the partial equilibrium framework of Gabaix (2011), and is given by the sales Herfindahl of the economy. This is a measure of how granular the economy is, or how big the big firms are. In the case of a continuum of firms, the Herfindahl index of an economy will be equal to zero, implying zero variation in aggregate growth or that realized GDP growth will always equal expected GDP growth. International trade increases the Herfindahl index of the economy by allowing countries to specialize in the industries they are relatively most productive in, which is determined solely by the productivity of firms within each industry. The elasticity of substitution across firms similarly increases the Herfindahl index of an economy, as more productive firms will be larger if output is more substitutable across firms.

Outside of their direct effect on the concentration of output within an economy, both the elasticity of substitution and international trade further increase variation in GDP growth by amplifying the aggregate impact of idiosyncratic shocks to firms. A higher elasticity of substitution implies that firms that receive higher productivity shocks will growth more in terms of their output and labor input. International trade allows countries to allocate more labor to the traded industries which experience the largest productivity growth market and move it away from industries in which firms experiences the lowest productivity growth. This amplifying effect of international trade similarly increases as the elasticity of substitution increases, as the industries with the highest relative productivity growth will acquire more labor.

**1.V. Estimation and Quantitative Results**

In this section, I re-evaluate the question "can the aggregate variation generated by idiosyncratic shocks to firms be large enough to matter in practice?" in the context of the framework developed in this chapter. Relative to previous research in the granularity literature, such as Gabaix (2011) and Carvalho and Grassi (2015), I broaden the scope of this question to focus not only on short-run macroeconomic fluctuations, but also whether idiosyncratic shocks to firms can be important in generating variation in GDP growth over longer periods of time. In particular, I focus on variation in 1 year GDP growth rates and 10 year GDP growth rates.

The goal of this section is to provide an answer for how much variation in GDP growth we might expect in a world where the only source of aggregate variation is due differences in the realizations of idiosyncratic productivity shocks to firms, noting that aggregate variation only arises in this context due to granularity and the existence of large firms. To provide such an answer, I carry out an experiment where I estimate the parameters of the model to construct the standard deviation of forecasted 1-year ahead GDP growth and forecasted 10-year ahead GDP growth for a given base year. I construct the standard deviations of these forecasts separately for each country, holding fixed the parameters governing the growth process for firms and the elasticity of substitution, as the derivation of (1.12) relies on these values being the same in all countries, while allowing the Herfindahl index and exports as a share of GDP to vary across countries.

### 1.V.A Firm-Level Data and Parameter Estimation

Exports as a share of GDP can be taken straight from aggregate data, for which I use the United Nations' (UN) World Development Indicators (WDI) database. To estimate the other parameters of (12), however, requires firm-level data on sales and gross margins. The firm-level data comes from the OSIRIS Industrials database. The database is compiled by Bureau Van Dijk, which is a private specialist provider of information on public and private companies worldwide, with a focus on standardizing data to allow for international comparisons. The dataset contains standardized and reported information on globally listed, as well as major non-listed, companies spanning 190 countries and 20 years and excludes banking and financial firms. The OSIRIS database is a subset of Bureau Van Dijk's ORBIS database, which contains information on 150 million firms worldwide. The coverage of the database is not complete, but offers good coverage for large firms, as documented by Ribeiro, Menghinello, and De Backer (2010), and these are the firms that are most important for the purposes of this chapter.

I select 2005 to be the base year, as it has the largest number of active firms in the OSIRIS database and avoids entangling the parameter estimates with the effects of the 2008-2009 recession. I compute the sales Herfindahl for each country following the definition from (1.13), where the denominator is from aggregate GDP data, again from the UN WDI database, as $\sum_{k=1}^{K}\sum_{m=1}^{M_{i,k}} p_{i,k,t}^{m} y_{i,k,t}^{m} = E_{i,0}$. In my framework, gross output is equal to GDP, however Gabaix (2011) and Hulten (1978) show that sales over GDP, and not sales over gross output or value added over GDP, is the correct measure of firm size for weighting the impact of microeconomic shocks on TFP or GDP when economies feature horizontal linkages across firms. To reduce the effects of a potential outlier in annual revenues for a firm, when computing the Herfindahl index for each country I use the median value of each firm's revenues between 2004–2006. Table 1.1 reports the square root of the estimated Herfindahl index for each country with at least 100 firms with positive revenues in 2005. There is significant cross-country variation in the number of firms with listed data in the OSIRIS database, however, Herfindahl indices computed using only the largest 100 firms for each country are nearly identical to the Herfindahl indices computed using all firms.

The next parameter that must be estimated is the elasticity of substitution across varieties, which I assume to be the same across countries and equal to the estimate for the United States. In my framework, the elasticity of substitution governs markups, and can be estimated using data on gross margins. In particular, we can estimate the elasticity as

$$\epsilon = \frac{1}{GM}, \qquad (1.16)$$

where $GM$ is the gross margin of a firm:

$$GM := \frac{py - wl}{py}. \qquad (1.17)$$

In the data, gross margins vary significantly across firms. Therefore, I estimate the elasticity separately for each firm, and then take a revenue weighted average to arrive at an estimated elasticity of $\tilde{\epsilon} = 3.35$. This estimate is in line with what other papers have found and used, for example by Anderson and van Wincoop (2004), and is stable across different choices of base years.

The final parameters to estimate are the drift and volatility of the productivity growth process for firms, which are assumed to be the same in all countries and again equal to the estimates for the United States. As I do not have firm-level data on quantities and inputs, I am unable to observe productivity directly. Instead, I estimate the productivity process by combining the estimated elasticity of substitution with observed changes in firm-level sales, aggregate expenditures, and consumer price indices. Logged growth in equilibrium sales for firm $m$ as a fraction of industry expenditures is given by:

$$\log\Delta\left(\frac{p_{i,k,T}^{m}y_{i,k,T}^{m}}{\theta_k E_{i,T}}\right)=\log\Delta\left(\left(z_{i,k,T}^{m}\right)^{\epsilon-1}\right)+\log\Delta\left(\sum_{\mathrm{j}=1}^{\mathrm{N}}\sum_{m=1}^{M_{i,k}}\left(z_{i,k,T}^{m}\right)^{\epsilon-1}\right), \qquad (1.18)$$

where

$$\Delta\left(z_{i,k,T}^{m}\right)^{\epsilon-1}:=\frac{\left(z_{i,k,T}^{m}\right)^{\epsilon-1}}{\left(z_{i,k,0}^{m}\right)^{\epsilon-1}}$$

and similarly for the other variables. Growth in the equilibrium consumption price index is given by

$$\log\Delta P_{i,k,T}^{C}=-\left(\frac{1}{\epsilon-1}\right)\log\Delta\left(\sum_{\mathrm{j}=1}^{\mathrm{N}}\sum_{m=1}^{M_{i,k}}\left(z_{i,k,T}^{m}\right)^{\epsilon-1}\right), \qquad (1.19)$$

which substituting into equation (18), yields the equation by which we estimate the productivity process for firms:

$$\log\Delta z_{i,k,T}^{m}=\left(\frac{1}{\epsilon-1}\right)\left(\log\Delta\left(p_{i,k,T}^{m}y_{i,k,T}^{m}\right)-\log\Delta E_{i,T}\right)-\log\Delta P_{i,k,T}^{C}\,. \qquad (1.20)$$

I compute logged annual productivity growth ($T=1$) for each firm using equation (20) between 1995 and 2005. As Davis, Haltiwanger, Jarmin, and Miranda (2007) show, small firms are significantly more volatile than large firms, therefore I restrict my sample to large firms, where I define “large” to be firms with at least one year of over $1 billion USD in total revenue during the sample period. Estimates of the drift, $\tilde{\mu}_{GBM}$, and volatility, $\tilde{\sigma}_{GBM}$, of the geometric Brownian motion productivity process in equation (4) can be computed using the sample mean, $\tilde{\mu}_{\log z}$, and sample standard deviation, $\tilde{\sigma}_{\log z}$, of logged annual productivity growth according to:

$$\tilde{\sigma}_{GBM} = \tilde{\sigma}_{\log z}. \qquad (1.21)$$

$$\tilde{\mu}_{GBM} = \tilde{\mu}_{\log z} + \frac{\tilde{\sigma}_{\log z}^2}{2}. \quad (1.22)$$

To avoid exaggerating the volatility of firm growth due to outliers, I drop the productivity shocks in the highest and lowest decile for each year when computing these estimates. I am then able to plug these estimates into equations (1.5–1.6) to yield estimates for $E[X_T]$ and $Var[X_T]$.

The estimates for $T = 1$ and $T = 10$ are reported in table 1.2, in particular I find a standard deviation for 1 year firm-level productivity growth of 9.00 percent and of 33.99 percent for 10 year productivity growth. The 1 year estimates are similar in magnitude to alternative measures of volatility for large firms found elsewhere in the literature, for example in Gabaix (2011) or Davis, Haltiwanger, Jarmin, and Miranda (2007). If I instead use $T = 10$ in (1.20) to directly compute 10-year productivity growth rates, I find a standard deviation for 10-year firm productivity growth of 33.70 percent, which is nearly identical to the original estimate obtained by substituting $T = 10$ into (1.6) using the drift and volatility from annual firm productivity growth rates. This equivalency highlights the consistency of geometric Brownian motion in matching observed firm growth rates.

### 1.V.B. Quantitative Results and Discussion

To quantitatively evaluate the variation in GDP growth that arises due to granularity and idiosyncratic shocks to firms in my framework, I plug the estimated parameters from the previous section into (1.12) for each country in my sample. I focus on the standard deviation of the of 1-year GDP growth rate forecast to reveal how granularity can generate short-run macroeconomic fluctuations, and the standard deviation of the 10-year GDP growth rate forecast to focus on how granularity can lead to variation in longer-run growth trends.

The way to interpret this exercise, is that I use (1.12) to estimate the standard deviation for the distribution of potential forecasted GDP growth for 2005–2006 (1-year) and 2005–2015 (10-year) for each country, where I assume the only source of aggregate variation is due to granularity and variation in the realizations of idiosyncratic productivity shocks to firms. The results of these exercises are in tables 1.3 and 1.4. Note that in this chapter, I do not exploit the fact that variation in GDP growth can arise due to variation in the growth of foreign countries, as I effectively assume that the world as a whole is non-granular. Therefore, for each country, all relevant information for the state of the rest of the world is contained in the Herfindahl index of a country and exports as a share of GDP.

To put my quantitative results in context, I compare the magnitude of the standard deviations for forecasted GDP growth to the magnitude of observed historic volatility in GDP growth for each country. I use GDP per capita, instead of GDP, to partially adjust for aggregate variation that arises due to population dynamics, which is not an object of focus in my framework. I compute the volatility of 1-year GDP growth rates over the period 1990-2005 and the volatility of 10-year GDP growth rates over the period 1950-

2010 using data on constant price (PPP) GDP per capita from the Penn World Tables. Note that in the data, the base parameters are not fixed over time. In particular, one should expect small changes in the Herfindahl index across periods, meaning this is not a direct comparison to the fixed base year experiment of the model. Nevertheless, this serves as a useful metric for gauging whether the aggregate variation generated by idiosyncratic shocks in the model is large enough to demand attention in practice.

Let $\hat{\sigma}_{i,GDP,T}$ be observed volatility $T$-year GDP growth rates for each country. I compute

$$\gamma_{i,T} = \sqrt{\frac{Var\left[\Delta_{i,GDP,T}\right]}{\hat{\sigma}^2_{i,GDP,T}}}, \qquad (1.23)$$

to represent the relative magnitude of variation in GDP growth generated by idiosyncratic shocks in the model compared to the magnitude of variation observed in the data. Table 1.5 reports the mean and median of these ratios, broken down by whether countries are OECD (Organisation for Economic Co-operation and Development) members or not. For OECD countries, for $T = 1$, I find the average value of $\gamma_{i,1}$ to be 69.1 percent, while for $T = 10$ the average value of $\gamma_{i,10}$ is 44.8 percent. These ratios are lower for non-OECD members, which suggests that idiosyncratic shocks to firms have less potential to explaining the volatility of developing countries compared to developed countries. Table 1.6 shows the full parameterization and results for the United States and Korea, where these countries are chosen as examples due to their significant differences in Herfindahl indices and exports as a share of GDP.

Roughly speaking, my results indicate that, in a world where idiosyncratic shocks to firms are the only source of aggregate variation, on average we would still expect around half the variation in GDP growth rates that we observe in the data for OECD economies, and for both 1-year GDP growth rates and 10-year GDP growth rates. This indicates that idiosyncratic shocks to firms can potentially play a significant role in explaining not only why countries grow more in some years than other, but also why countries can grow more in some decades than others, indicating a much broader role for the aggregate impact of idiosyncratic shocks to firms than previously realized by the granularity literature.

**1.V.C. Discussion on Causality**

My quantitative results establish that idiosyncratic shocks to firms can potentially play a significant role in explaining observed variation in both short-run (1 year) and longer-run (10-year) GDP growth rates. This begs the question: to what extent do idiosyncratic shocks to firms actually explain observed variation in GDP growth rates? Gabaix (2011) and Stella (2014) take opposing viewpoints on this matter in the context of short-run macroeconomic fluctuations for the United States, with Gabaix arguing that idiosyncratic shocks to firms can explain a significant portion of observed 1-year U.S. GDP volatility and Stella arguing that they cannot. This chapter sidesteps this debate, as the contribution of this chapter is not establishing causality, but rather the scope and potential for causality. The question of casualty is best tackled independently for each country and time period, as the answer is likely context specific. For example, it is unlikely that the recent 2008-2009 recession can be significantly explained by idiosyncratic shocks to firms alone. Conversely, it is very plausible that idiosyncratic shocks to Nokia may explain a large portion of Finland's high growth through the early 2000's as well as Finland's subsequent recession, which has coincided with the rapid decline of Nokia.

Rather than attempting to establish causality with regards to whether idiosyncratic shocks to firms explain observed variation in GDP growth across different countries and time periods, the goal of this chapter is to evaluate whether there is theoretical and quantitative merit to this hypothesis in the first place. In my framework, if the magnitude of GDP variation generated by idiosyncratic shocks to firms were insignificant relative to the magnitude of observed GDP variation, this would be evidence that economists are justified in ignoring the potentially causal role of idiosyncratic shocks to firms in explaining aggregate variation. Therefore one could largely bypass the question of causality all together, due to a strong prior that idiosyncratic shocks to firms cannot be responsible for observed aggregate variation, which until recently was the predominant viewpoint in macroeconomics. Instead, my results indicate that idiosyncratic shocks to firms should not be ignored at the aggregate level, and not only can they be an important source of short-run aggregate variation, as argued by Gabaix (2011), but also an important source of aggregate variation in the longer-run as well. This chapter, therefore, provides strong theoretical evidence that ignoring the aggregate impact of idiosyncratic shocks to firms does not appear justified.

### 1.VI. Stylized Facts and Empirical Evidence

In this section, I provide evidence that the predictions of the model are consistent with several stylized facts of the data. In particular, I show that the countries the framework predicts should be more volatile (due to higher standard deviation of forecasted GDP growth) are in fact more volatile. I focus on OECD countries and 1-year GDP growth volatility. I regress the standard deviation of forecasted 1-year GDP growth (model) on observed historic volatility of GDP growth over 1990–2005 (data). The results of this regression are reported in table 1.7. The coefficient is positive and significant, with an R squared of 0.47, and the correlation between the model and the data is 0.69. Table 1.8 shows that when volatility in the data is computed over alternative time periods the correlation between the model and data remains positive. The alternative regression with the intercept forced to zero is illustrated in figure 1.2, the benefit of this regression is that the slope has a natural interpretation. The slope of the no-intercept regression is 1.39, indicating that the standard deviation of GDP growth in the data is, on average, 39% higher than the standard deviation of GDP growth in the model.

Next, I focus on the model's prediction that countries which engage heavily in international trade should, ceteris paribus, exhibit more volatility in their GDP growth. In the model, trade monotonically increases GDP volatility both indirectly, though increasing the Herfindahl index of an economy, as well as directly, by amplifying the aggregate impact of idiosyncratic shocks to firms conditional on the Herfindahl index. The correlations in table 1.8 show that volatility in the data is positively correlated with both exports as a share of GDP in 2005 and the Herfindahl index of a country in 2005. Table 1.9 present the average volatility of annual GDP per capita growth over 1990-2005 when countries are grouped by their share of exports as a percentage of GDP. These results confirm the predictions of the model that, on average, GDP volatility should increase as countries export more a fraction of GDP. Table 1.10 shows that this holds when the sample of countries is restricted to OECD members, which will be the focus moving forward.

To distinguish the mechanisms by which international trade leads to increased aggregate variation from those of di Giovanni and Levchenko (2012), I show that this increase in volatility is not due solely to the effect of international trade on increasing a country's Herfindahl index. Table 1.8 shows that exports as a share of GDP is positively correlated with the Herfindahl index of a country. Table 1.11 groups OECD countries by their Herfindahl index in 2005 and shows that countries with Herfindahl indices experienced higher volatility of annual GDP growth over 1990–2005. To partially disentangle the effects of the Herfindahl index and exports/GDP, I first group countries according to the magnitude of their Herfindahl index, and then within each grouping, I create a subgroup based on how much a country exports as a share of GDP. Table 1.12 presents the results of this exercise and shows that, while countries with higher Herfindahl indices experience higher volatility of GDP growth, countries with higher exports as a share of GDP still experience higher volatility when compared to countries with similar Herfindahl indices.

A key feature of the model is that it predicts higher variation in growth over longer periods of time. Table 1.3 previously established that this prediction is consistent with the data with regards to the relative magnitudes of 10-year GDP growth rate volatility versus 1-year GDP growth rate volatility. Figures 1.4 and 1.5 highlight this prediction of the model by using the parameterized model to create prediction intervals for forecasts of growth for the United States and Korea. The confidence intervals are wider for Korea, indicating the greater uncertainty involved in accurately forecasting growth in an economy that exports more and has more concentrated output among firms. Another way to interpret these forecasts and prediction intervals is that if we have a large number of countries similar to Korea, we should observe more variation in their growth if they are like Korea as opposed to if they are like the United States. To evaluate this prediction, I group OECD countries by the standard deviation of their forecasted 10-year GDP growth rates in the model. I compute GDP growth for each country over the ten year period of 1995–2005 and then take the standard deviation of GDP growth across countries within groups. Table 1.13 compares these results to the average 10-year forecast standard deviation delivered by the model. These results indicate that the set of countries that model predicted should exhibit more variation in 10-year GDP growth did in fact exhibit higher variation in their GDP growth over 1995–2005. Within each grouping, I compute the ratio of the average standard deviation in the model to the standard deviation across countries in the data and these results indicate that we would expect roughly half the variation in 10–year cross-country growth over for OECD countries. These results indicate that, similarly to how granularity and idiosyncratic shocks to firms can potentially explain a significant amount of observed variation across time for a given country as suggested by table 1.5, they can also potentially play a large role in explaining variation in GDP growth across countries for a given time period.

## 1.V. Conclusion

In this chapter, I evaluated the extent to which idiosyncratic shocks to firms can generate variation in GDP growth in a general equilibrium environment featuring monopolistic competition and international trade. Theoretically, I generalized the theoretical results of Gabaix regarding aggregate volatility in granular economies to an environment in which shocks to firms affect the output of other firms in the economy through competitive forces.

My theoretical results reveal novel pathways through which idiosyncratic shocks to firms can directly generate and amplify aggregate volatility. In particular, the degree to which aggregate fluctuations arise from idiosyncratic shocks to firms depends not only on the concentration of output among firms, as shown by Gabaix and di Giovanni and Levchenko (2012), but also on how substitutable output is among firms and the degree to which firms compete internationally versus domestically. My framework thus offers an explanation for why countries that engage heavily in international trade more are more volatile, and suggests a potential avenue for explaining why some sectors exhibit more volatility than others as noted by di Giovanni and Levchenko (2009) and Grassi and Imbs (2014).

Quantitatively, I find that, given observed firm sizes and dynamics, even if idiosyncratic shocks to firms were the only potential source of variation in GDP growth, we would still expect roughly half the magnitude of variation in GDP growth observed in the data across a wide range of countries. Notably, this holds not only for variation in annual GDP growth rates, but also for variation in decade-to-decade GDP growth rates. This indicates that idiosyncratic shocks can be an important source of not only short-run macroeconomic fluctuations, as noted by Gabaix (2011) and Carvalho and Grassi (2015), but also variation in longer-run growth trends. This suggests that when economists are interested in why countries grow more in some periods than others, or why countries may react differently to similar policy interventions, a first order thing that needs to be considered is differences in the growth of a country's largest and fastest growing firms. To what extent, and how, the growth of these firms is shaped by political, social, economic, and environmental factors not addressed in this chapter remains an exciting area for future research.

**Table 1.1**
**Herfindahl Indices of Countries in OSIRIS Database**

| Country | Code | Firm Count | Herf. Index | Largest Firm Share | Country | Code | Firm Count | Herf. Index | Largest Firm Share |
|---|---|---|---|---|---|---|---|---|---|
| Argentina | ARG | 253 | 0.063 | 0.03 | Jordan | JOR | 171 | 0.225 | 0.17 |
| Australia | AUS | 1665 | 0.104 | 0.04 | Japan | JPN | 7042 | 0.115 | 0.04 |
| Belgium | BEL | 182 | 0.132 | 0.06 | Korea | KOR | 2316 | 0.207 | 0.09 |
| Brazil | BRA | 837 | 0.141 | 0.09 | Kuwait | KWT | 155 | 0.046 | 0.03 |
| Canada | CAN | 4334 | 0.103 | 0.02 | Sri Lanka | LKA | 163 | 0.032 | 0.01 |
| Switzerland | CHE | 260 | 0.301 | 0.18 | Mongolia | MNG | 157 | 0.035 | 0.03 |
| Chile | CHL | 711 | 0.239 | 0.09 | Mexico | MEX | 279 | 0.164 | 0.11 |
| China | CHN | 1920 | 0.080 | 0.04 | Malaysia | MYS | 1073 | 0.268 | 0.25 |
| Colombia | COL | 206 | 0.087 | 0.06 | Nigeria | NGA | 114 | 0.021 | 0.01 |
| Cyprus | CYP | 113 | 0.109 | 0.06 | Netherlands | NLD | 245 | 0.226 | 0.09 |
| Germany | DEU | 921 | 0.143 | 0.07 | Norway | NOR | 225 | 0.304 | 0.19 |
| Denmark | DNK | 167 | 0.151 | 0.13 | N. Zealand | NZL | 156 | 0.099 | 0.07 |
| Egypt | EGY | 736 | 0.058 | 0.04 | Oman | OMN | 113 | 0.037 | 0.02 |
| Spain | ESP | 198 | 0.103 | 0.05 | Peru | PER | 239 | 0.062 | 0.03 |
| Finland | FIN | 143 | 0.339 | 0.20 | Philippines | PHL | 213 | 0.078 | 0.04 |
| France | FRA | 953 | 0.153 | 0.08 | Pakistan | PAK | 329 | 0.057 | 0.04 |
| U.K. | GBR | 2456 | 0.275 | 0.16 | Russia | RUS | 371 | 0.147 | 0.07 |
| Greece | GRC | 288 | 0.067 | 0.03 | Sweden | SWE | 486 | 0.189 | 0.08 |
| Hong Kong | HKG | 229 | 0.327 | 0.17 | Singapore | SGP | 675 | 0.406 | 0.34 |
| Indonesia | IDN | 303 | 0.030 | 0.02 | Thailand | THA | 530 | 0.154 | 0.13 |
| Ireland | IRL | 113 | 0.216 | 0.08 | Turkey | TUR | 236 | 0.062 | 0.04 |
| Israel | ISR | 756 | 0.112 | 0.04 | U.S.A. | USA | 7742 | 0.057 | 0.03 |
| India | IND | 3284 | 0.055 | 0.04 | Vietnam | VNM | 341 | 0.022 | 0.02 |
| Iran | IRN | 179 | 0.028 | 0.02 | S. Africa | ZAF | 363 | 0.141 | 0.08 |
| Italy | ITA | 297 | 0.095 | 0.05 | | | | | |

The column *Herf Index* reports the square root of the estimated Herfindahl index for each country in the OSIRIS database with over 100 firms with positive revenues in 2005. The column *Largest Firm Share* reports the 2005 revenues of the largest firm in each country divided by the 2005 GDP of that country. *Firm Count* lists the number of firms with positive 2005 revenues for each country. The average Herfindahl index among the countries in table 1.1 is 0.136, while the revenues of the largest firm in each country are on average 7.9 percent of GDP.

In total, the OSIRIS database contains 51,287 firms headquartered in 136 countries with positive revenues in 2005. The total revenues of these firms sum up to 95.2 percent of 2005 World GDP and yield a global Herfindahl index of 0.028.

**Table 1.2**

**Estimated Productivity Growth Process for Firms**

| **Estimated Object** | **Variable** | **Estimated Value** |
|---|---|---|
| Mean Logged Annual Productivity Growth | $\tilde{\mu}_{\log z}$ | 0.0105 |
| St.Dev. of Logged Annual Productivity Growth | $\tilde{\sigma}_{\log z}$ | 0.0885 |
| Drift of GBM Process | $\tilde{\mu}_{GBM}$ | 0.0145 |
| Volatility of GBM Process | $\tilde{\sigma}_{GBM}$ | 0.0885 |
| Expected 1-year Firm Growth | $E[X_1] - 1$ | 1.46% |
| St.Dev. of Forecasted 1-Year Firm Growth | $\sqrt{Var[X_1]}$ | 9.00% |
| Expected 10-year Firm Growth | $E[X_{10}] - 1$ | 15.55% |
| St.Dev. of Forecasted 10-Year Firm Growth | $\sqrt{Var[X_{10}]}$ | 32.99% |

Table 1.2 reports the estimated parameters governing the productivity growth process for firms. Logged annual productivity growth for each firm between 1995–2005 is computed using equation (1.20) using an estimated elasticity of substitution of 3.35. When computing the sample mean and sample standard deviation, the sample is restricted to U.S. firms with at least 1 year of greater than $1 Billion USD in sales over the sample period. These estimates are used along with equations (1.21) and (1.22) to estimate the drift and volatility of the geometric Brownian motion process in equation (1.4). The drift and volatility is then used to estimate the expectation and standard deviation of productivity growth for each firm after both 1-year and 10-years using equations (1.5) and (1.6).

If I instead used logged 10-year productivity growth rates directly to estimate the standard deviation of 10-year firm growth I arrive at an estimate of 33.70 percent, which is nearly identical to the estimated 10-year standard deviation above of 32.99 percent, which uses annual growth rates.

## Table 1.3

### Standard Deviation of Forecasted 1-Year GDP Growth

### Versus Historic 1-Year GDP Growth Volatility (1990-2005)

| Country | Herf. Index | Exports /GDP | Pred St.Dev. | Data Vol. | Country | Herf. Index | Exports /GDP | Pred St.Dev. | Data Vol. |
|---|---|---|---|---|---|---|---|---|---|
| Argentina | 0.063 | 20.95 | 0.71 | 10.97 | Jordan | 0.225 | 52.95 | 2.99 | 9.86 |
| Australia | 0.103 | 18.22 | 1.13 | 1.65 | Japan | 0.115 | 14.56 | 1.22 | 2.30 |
| Belgium | 0.132 | 73.37 | 1.87 | 3.64 | Korea | 0.207 | 37.42 | 2.56 | 5.02 |
| Brazil | 0.141 | 15.31 | 1.51 | 7.18 | Kuwait | 0.046 | 62.15 | 0.63 | 53.65 |
| Canada | 0.103 | 36.63 | 1.26 | 2.54 | Sri Lanka | 0.032 | 32.6 | 0.39 | 3.79 |
| Switzerland | 0.301 | 54.07 | 4.01 | 1.83 | Mexico | 0.164 | 26.79 | 1.90 | 4.50 |
| Chile | 0.239 | 39.56 | 2.99 | 5.58 | Mongolia | 0.035 | 59.67 | 0.47 | 13.52 |
| China | 0.080 | 36.77 | 0.99 | 3.65 | Malaysia | 0.268 | 113.49 | 3.98 | 7.24 |
| Colombia | 0.087 | 17.09 | 0.95 | 3.39 | Nigeria | 0.021 | 34.98 | 0.26 | 21.26 |
| Cyprus | 0.109 | 48.04 | 1.42 | 6.52 | Netherlands | 0.226 | 66.35 | 3.14 | 3.42 |
| Germany | 0.143 | 38.17 | 1.78 | 1.58 | Norway | 0.304 | 43.75 | 3.88 | 3.96 |
| Denmark | 0.151 | 47.24 | 1.96 | 2.51 | N. Zealand | 0.099 | 29.43 | 1.18 | 2.40 |
| Egypt | 0.058 | 29.51 | 0.69 | 7.55 | Oman | 0.037 | 55.28 | 0.49 | 7.47 |
| Spain | 0.103 | 24.91 | 1.19 | 2.37 | Pakistan | 0.057 | 15.16 | 0.61 | 3.26 |
| Finland | 0.339 | 40.67 | 4.27 | 5.02 | Peru | 0.062 | 26.32 | 0.72 | 4.95 |
| France | 0.153 | 26.48 | 1.78 | 2.76 | Philippines | 0.078 | 47.1 | 1.01 | 6.15 |
| U.K. | 0.275 | 25.98 | 3.18 | 2.62 | Russia | 0.147 | 34.45 | 1.79 | 12.14 |
| Greece | 0.067 | 21.10 | 0.75 | 2.6 | Singapore | 0.406 | 224.17 | 6.03 | 12.66 |
| Hong Kong | 0.327 | 194.37 | 4.85 | 7.48 | Sweden | 0.189 | 45.84 | 2.44 | 3.56 |
| Indonesia | 0.030 | 32.44 | 0.37 | 7.36 | Thailand | 0.154 | 72.64 | 2.19 | 7.91 |
| India | 0.055 | 19.30 | 0.61 | 2.63 | Turkey | 0.062 | 22.69 | 0.70 | 7.16 |
| Ireland | 0.216 | 78.45 | 3.11 | 3.62 | U.S.A. | 0.057 | 10.09 | 0.58 | 1.43 |
| Iran | 0.028 | 31.52 | 0.33 | 5.69 | Vietnam | 0.022 | 62.11 | 0.30 | 3.66 |
| Israel | 0.112 | 40.48 | 1.41 | 3.66 | S. Africa | 0.141 | 27.94 | 1.65 | 2.67 |
| Italy | 0.095 | 24.97 | 1.10 | 2.31 | | | | | |

Column *Pred St.Dev.* reports the standard deviation (percent) of forecasted 1-year GDP growth in my framework where idiosyncratic shocks to firms are the only source of variation/ uncertainty in GDP growth. These values are computed using equation (1.12) using $T = 1$, where the parameters governing the growth process for firms are listed in table 1.2 and the estimated elasticity of substitution is 3.35. *Herf. Index* lists the Herfindahl indices from table 1.1 for each country. *Exports/GDP* is the share of exports as a percentage of GDP in 2005, and comes from the UN's WDI database. *Data Vol.* is the observed volatility (st.dev.) of annual GDP growth rates between 1990–2005 (percent). These are computed using annual constant price (PPP) GDP per Capita data from the Penn World Tables (PWT 8.0).

## Table 1.4

### Standard Deviation of Forecasted 10-Year GDP Growth

### Versus Historic 10-Year GDP Growth Volatility (1950-2010)

| Country | Herf. | Exports /GDP | Pred St.Dev. | Data Vol. | Country | Herf. | Exports /GDP | Pred St.Dev. | Data Vol. |
|---|---|---|---|---|---|---|---|---|---|
| Argentina | 0.063 | 20.95 | 2.71 | 38.59 | Jordan | 0.225 | 52.95 | 11.48 | 36.51 |
| Australia | 0.103 | 18.22 | 4.34 | 12.22 | Japan | 0.115 | 14.56 | 4.67 | 69.36 |
| Belgium | 0.132 | 73.37 | 7.19 | 15.27 | Korea | 0.207 | 37.42 | 9.82 | 40.24 |
| Brazil | 0.141 | 15.31 | 5.79 | 22.33 | Kuwait | 0.046 | 62.15 | 2.41 | 98.93 |
| Canada | 0.103 | 36.63 | 4.85 | 14.30 | Sri Lanka | 0.032 | 32.60 | 1.48 | 24.88 |
| Switzerland | 0.301 | 54.07 | 15.40 | 13.53 | Mexico | 0.164 | 26.79 | 7.31 | 20.65 |
| Chile | 0.239 | 39.56 | 11.49 | 17.78 | Mongolia | 0.035 | 59.67 | 1.80 | 63.92 |
| China | 0.08 | 36.77 | 3.80 | 44.10 | Malaysia | 0.268 | 113.49 | 15.28 | 19.30 |
| Colombia | 0.087 | 17.09 | 3.63 | 22.95 | Nigeria | 0.021 | 34.98 | 0.99 | 101.61 |
| Cyprus | 0.109 | 48.04 | 5.44 | 31.89 | Netherlands | 0.226 | 66.35 | 12.07 | 15.68 |
| Germany | 0.143 | 38.17 | 6.83 | 34.69 | Norway | 0.304 | 43.75 | 14.91 | 15.43 |
| Denmark | 0.151 | 47.24 | 7.53 | 15.00 | N. Zealand | 0.099 | 29.43 | 4.51 | 11.75 |
| Egypt | 0.058 | 29.51 | 2.65 | 43.19 | Oman | 0.037 | 55.28 | 1.89 | 42.80 |
| Spain | 0.103 | 24.91 | 4.56 | 27.37 | Pakistan | 0.057 | 15.16 | 2.33 | 19.38 |
| Finland | 0.339 | 40.67 | 16.39 | 17.59 | Peru | 0.062 | 26.32 | 2.76 | 30.53 |
| France | 0.153 | 26.48 | 6.82 | 20.64 | Philippines | 0.078 | 47.10 | 3.86 | 17.61 |
| U.K. | 0.275 | 25.98 | 12.23 | 11.95 | Russia | 0.147 | 34.45 | 6.87 | -- |
| Greece | 0.067 | 21.10 | 2.87 | 38.64 | Singapore | 0.406 | 224.17 | 23.15 | 41.62 |
| Hong Kong | 0.327 | 194.37 | 18.62 | 30.74 | Sweden | 0.189 | 45.84 | 9.35 | 14.16 |
| Indonesia | 0.03 | 32.44 | 1.41 | 22.77 | Thailand | 0.154 | 72.64 | 8.40 | 35.85 |
| India | 0.055 | 19.30 | 2.32 | 35.62 | Turkey | 0.062 | 22.69 | 2.68 | 13.53 |
| Ireland | 0.216 | 78.45 | 11.93 | 25.49 | U.S.A. | 0.057 | 10.09 | 2.23 | 10.06 |
| Iran | 0.028 | 31.52 | 1.28 | 81.72 | Vietnam | 0.022 | 62.11 | 1.15 | 32.89 |
| Israel | 0.112 | 40.48 | 5.41 | 53.20 | S. Africa | 0.141 | 27.94 | 6.33 | 17.70 |
| Italy | 0.095 | 24.97 | 4.21 | 29.91 | | | | | |

Column *Pred St.Dev.* reports the standard deviation (percent) of forecasted 10-year GDP growth in my framework where idiosyncratic shocks to firms are the only source of variation/ uncertainty in GDP growth. These values are computed using equation (1.12) using $T = 10$, where the parameters governing the growth process for firms are listed in table 1.2 and the estimated elasticity of substitution is 3.35. *Herf.* lists the Herfindahl indices from table 1.1 for each country. *Exports/GDP* is the share of exports as a percentage of GDP in 2005, and comes from the UN's WDI database. *Data Vol.* is the observed volatility (percent) of 10-year GDP growth rates between 1950–2010 (GDP growth between 1950–1960, 1960–1970,…). These are computed using annual constant price (PPP) GDP per Capita data from the Penn World Tables (PWT 8.0). Observed volatility is not computed for Russia, as it has only two decades of data availability, while every other country listed has at least four decades of data availability.

**Table 1.5**

**Summary for Ratios of Model/Historic GDP Volatility**

**1-Year vs 10-Year and OECD vs non-OECD**

| Sample | Object | Variable | Mean Value | Median Value |
|---|---|---|---|---|
| All<br>(45 Countries) | 1-Year Ratio<br>10-Year Ratio | $\gamma_{i,1}$<br>$\gamma_{i,10}$ | 47.4%<br>33.6% | 42.3%<br>23.4% |
| OECD<br>(24 Countries) | 1-Year Ratio<br>10-Year Ratio | $\gamma_{i,1}$<br>$\gamma_{i,10}$ | 69.1%<br>44.8% | 53.3%<br>35.5% |
| Non-OECD<br>(21 Countries) | 1-Year Ratio<br>10-Year Ratio | $\gamma_{i,1}$<br>$\gamma_{i,10}$ | 22.6%<br>20.9% | 18.6%<br>12.0% |

Table 1.5 reports the mean and median values of the ratio of the standard deviation of forecasted GDP growth to observed historic volatility of GDP growth for both 1-Year GDP growth rates and 10-year GDP growth rates. These ratios are computed for each country according to (1.23). The standard deviation of forecasted GDP growth is computed using equation (12) with $T = 1$ for 1-year GDP growth rates and $T = 10$ for 10-year GDP growth rates. Observed historic 1-year GDP volatility is for 1990-2005, while 10-year GDP volatility is for 1950-2010, these values are reported in tables 1.3 and 1.4, respectively. When computing the mean and median of the ratios, I exclude the countries in the top decile of observed GDP volatility as outliers (all non-OECD). These outlier countries have an average volatility over 500 percent greater than the volatility of non-outlier countries for 1-year GDP growth rate volatility and over 250 percent greater for 10-year GDP growth rate volatility.

A full list of OECD member countries is available at: http://www.oecd.org/about/membersandpartners/list-oecd-member-countries.htm

**Table 1.6**

**Parameter Estimates and Quantitative Results for the United States and Korea**

| Parameters/Results | Variable | United States | Korea |
|---|---|---|---|
| Drift of GBM Process | $\tilde{\mu}_{GBM}$ | 0.015 | 0.015 |
| Volatility of GBM Process | $\tilde{\sigma}_{GBM}$ | 0.089 | 0.089 |
| Elasticity of Substitution | $\epsilon$ | 3.35 | 3.35 |
| Herfindahl index | $\sqrt{h_{i,0}}$ | 0.057 | 0.207 |
| Exports/GDP | $S_{i,0}$ | 0.101 | 0.374 |
| St.Dev. of 1-Year Forecast | $\sqrt{Var[\Delta_{i,GDP,1}]}$ | 0.58% | 2.56% |
| 1-Year Volatility (1990–2005) | -- | 1.43% | 5.02% |
| Ratio of Forecasted/Historic | $\gamma_{i,1}$ | 0.405 | 0.510 |
| St.Dev. of 10-Year Forecast | $\sqrt{Var[\Delta_{i,GDP,10}]}$ | 2.23% | 9.38% |
| 10-Year Volatility (1950–2010) | -- | 10.06% | 40.24% |
| Ratio of Forecasted/Historic | $\gamma_{i,10}$ | 0.212 | 0.233 |

Table 1.6 reports the estimated parameters and standard deviation of forecasted 1-year and 10-year GDP growth using equation (1.12) for the cases of Korea and the United States. The Herfindahl index and Exports/GDP are from table 1.3, for a base year of 2005. The drift and volatility of the GBM process for firms are from table 1.2, and the elasticity of substitution is estimated using (1.16). Observed historic volatility for both 1-year GDP growth rates and 10-year GDP growth rates are from tables 1.3 and 1.4, respectively, and computed using data on GDP per capita (PPP) from the Penn World Tables. The ratios are computed according to (1.23) and give the standard deviation of forecasted GDP growth in the model (which is non-zero due to granularity) divided by observed historic GDP volatility.

**Table 1.7**

**Predicted Versus Observed Annual GDP Volatility for**

**OECD Countries, (1990–2005)**

| Coefficient | Estimate | Standard Error |
|---|---|---|
| Model | 0.771 | 0.183 |
| Constant | 1.572 | 0.004 |
| Observations | | 22 |
| R-squared | | 0.47 |

Table 1.7 lists the estimated coefficients and standard errors of the regression: $data = \beta_0 + \beta_1 * model$. *Data* refers to observed annual GDP volatility (percent) between 1990–2005, while *model* refers to the standard deviation of forecasted 1-year GDP growth in the model, where both are from table 1.3.

Turkey and Switzerland are excluded from this regressions and following OECD-only tables as outliers. Turkey experienced low GDP growth between 1990–2000 followed by explosive GDP growth from 2000–2005, and this tale of two halves leads Turkey to have the highest GDP volatility of any OECD country over 1990–2005 by a significant amount. Switzerland is excluded to as its unusually low GDP volatility appears to be the result of being in a sustained great depression, see Kehoe and Ruhl (2005).

**Table 1.8**

**Correlation with Observed Annual GDP Volatility**

**for OECD Countries, Various Time Periods**

| Time Period | Correlation with Model | Correlation with Exports | Correlation with Herf. |
|---|---|---|---|
| 1990–2005 | 0.686 | 0.427 | 0.672 |
| 1980–2010 | 0.593 | 0.356 | 0.577 |
| 1950–2010 | 0.245 | 0.013 | 0.257 |
| Correlation between Trade and Herf. | | | 0.418 |
| Observations | | | 22 |

*Correlation with Model* lists the Pearson's correlation coefficients between the standard deviation of forecasted 1-year GDP growth (2005 base year) using equation (1.12), which are listed in table 1.3, and observed annual GDP growth volatility (per capita, constant price PPP) over alternative time periods for OECD countries. Switzerland and Turkey are excluded as outliers for this and subsequent tables as discussed in the footnote for table 1.7. *Correlation with Exports* gives the correlation coefficient between observed annual GDP growth volatility and Exports/GDP. *Correlation with Herf.* gives the correlation coefficient between observed annual GDP growth volatility and the Herfindahl index of a country. Both the Herfindahl index and Exports as a share of GDP for each country are listed in table 1.3. As predicted by the model, countries that engage heavily in trade tend to have larger Herfindahl indices.

**Table 1.9**

**Volatility of Annual GDP Growth (1990-2005):**

**Countries Grouped by Share of Exports as % of GDP**

| Set of Countries (by Exports/GDP) | Exports Cutoff | Observations | Mean Volatility | Median Volatility |
|---|---|---|---|---|
| Top 25% | > 47% | 12 | 5.43% | 5.09% |
| Middle 50% | between | 22 | 4.52% | 3.87% |
| Bottom 25% | < 25% | 11 | 4.09% | 2.63% |
| Top 50% | > 32% | 23 | 5.13% | 3.96% |
| Bottom 50% | < 32% | 22 | 4.16% | 3.01% |

See footnote under table 1.12 for description of tables 1.9–1.12.

**Table 1.10**

**Volatility of Annual GDP Growth (1990-2005):**

**OECD Countries Grouped by Share of Exports as % of GDP**

| Set of Countries (by Exports/GDP) | Exports Cutoff | Observations | Mean Volatility | Median Volatility |
|---|---|---|---|---|
| Top 25% | > 41% | 6 | 3.45% | 3.59% |
| Middle 50% | between | 11 | 3.45%* | 2.76% |
| Bottom 25% | < 25% | 5 | 2.07% | 2.30% |
| Top 50% | > 36% | 11 | 3.78% | 3.64% |
| Bottom 50% | < 36% | 11 | 2.50% | 2.40% |

*Mean volatility of the middle 50% is skewed upward by high observed volatilities for Israel and Chile. Both countries are near the cutoff with exports/GDP of approximately 40% in 2005.

**Table 1.11**

**Volatility of Annual GDP Growth (1990–2005):**

**OECD Countries Grouped by Herfindahl Indices**

| Set of Countries (by Herf. Index) | Herf. Cutoff | Observations | Mean Volatility | Median Volatility |
|---|---|---|---|---|
| Top 25% | > 0.21 | 6 | 4.04% | 3.79% |
| Middle 50% | between | 11 | 3.05% | 2.76% |
| Bottom 25% | < 0.10 | 5 | 2.26% | 2.40% |
| Top 50% | > 0.14 | 11 | 3.87% | 3.62% |
| Bottom 50% | < 0.14 | 11 | 2.41% | 2.37% |

**Table 1.12**

**Volatility of Annual GDP Growth (1990–2005):**

**OECD Countries Grouped by Herfindahl and Share of Exports as % of GDP**

| Set of Countries (by Herf. Index) | Herf. Cutoff | Set of Countries (by Exports/GDP) | Exports Cutoff | Obs. | Mean Volatility | Median Volatility |
|---|---|---|---|---|---|---|
| Top 50% | > .014 | Top 50% | > 36% | 8 | 4.09% | 3.79% |
| | | Bottom 50% | < 36% | 3 | 3.30% | 2.75% |
| Bottom 50% | < .014 | Top 50% | > 36% | 3 | 2.96% | 3.63% |
| | | Bottom 50% | < 36% | 8 | 2.20% | 2.33% |

Tables 1.9–1.12 present cross-tabulations reporting the means and medians across countries of observed volatility in annual GDP growth from 1990–2005 taken from table 1.3. I separate countries by quartiles based on both their share of exports as a percentage of GDP in 2005 and, for OECD economies, their Herfindahl index in 2005. I then compute the mean and median under different groupings of these quartiles. The results show that countries that export more and countries with higher Herfindahl indices tend to exhibit higher volatility.

**Table 1.13**

**Variation in Cross-Country Growth (1995–2005):**

**OECD Countries Grouped by St.Dev. of Forecasted 10-Year GDP Growth**

| Set of Countries (by St.Dev. of Forecasted Growth) | Obs. | Average St.Dev. of Forecasted 10-year GDP Growth | St.Dev. of Growth Across Countries | Ratio |
|---|---|---|---|---|
| Top 25% | 6 | 13.2% | 28.3% | 0.47 |
| Middle 50% | 11 | 6.8% | 14.3% | 0.48 |
| Bottom 25% | 5 | 3.6% | 10.1% | 0.36 |
| Top 50% | 11 | 10.9% | 22.5% | 0.48 |
| Bottom 50% | 11 | 4.7% | 12.3% | 0.38 |

Table 1.13 groups countries into quartiles by the standard deviation of their forecasted 10-year GDP growth rates from table 1.4 and arranges these quartiles into alternative groupings. *Average St.Dev. of Forecasted 10-Year GDP Growth* reports the average value of these standard deviations within each grouping.

For each country, I then compute observed constant price (PPP) GDP per capita growth over 1995-2005 from the Penn World Tables. *St.Dev. of Growth Across Countries* reports the standard deviation of these observed growth rates across countries within each grouping.

*Ratio* reports the ratio of the average standard deviation of forecasted 10-year GDP growth to the standard deviation of observed GDP growth over 1995–2005 across countries.

**Figure 1.1**

**Behavior of Amplifying Curvature Term: $\xi(T, \sigma, \epsilon)$**

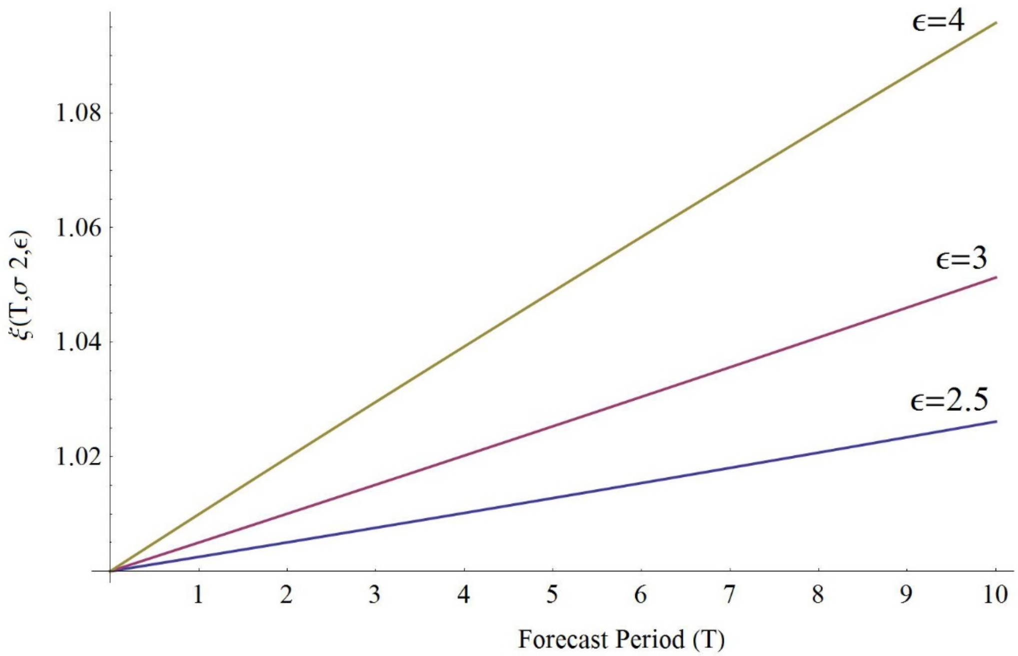


Figure 1.1 illustrates the behavior of $\xi(T, \sigma, \epsilon)$ from equation (1.10) for different values of $\epsilon$ and $T$. The volatility of the geometric Brownian motion process for firms is fixed at $\sigma = 0.10$. This figure indicates that the term is close to 1 and increases near linearly with $T$ over short time periods, where the slope depends on the elasticity of substitution. If $\epsilon = 2$ then $\xi(T, \sigma, \epsilon) = 1$.

**Figure 1.2**

**Behavior of Amplifying Trade Term: $\psi(\epsilon, S)$**

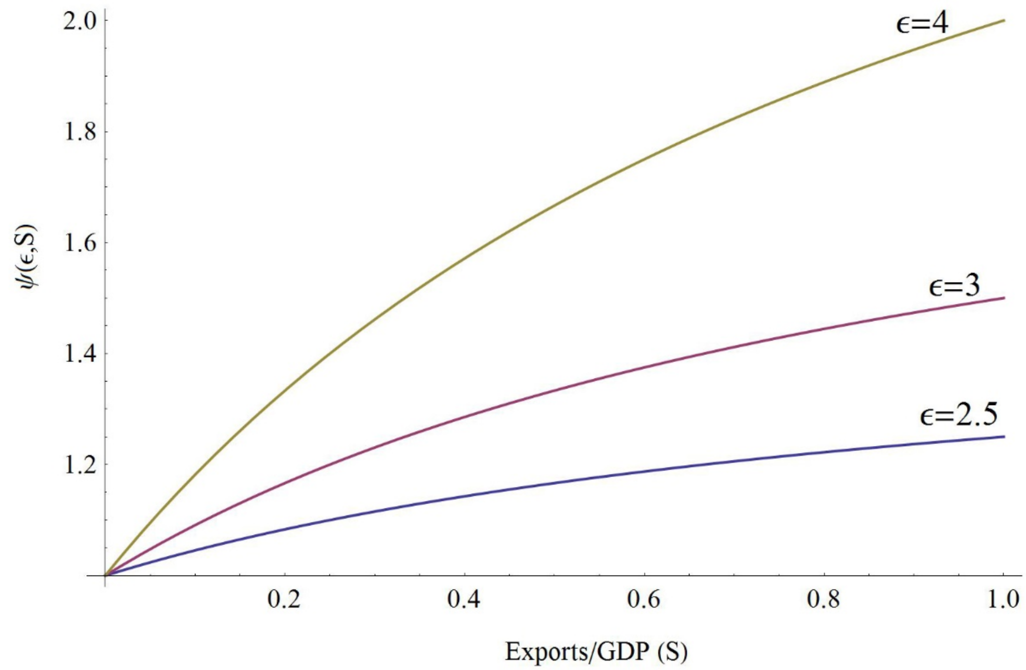


Figure 1.2 illustrates the behavior of $\psi(\epsilon, S)$ from equation (1.14) for different values of $\epsilon$ and $S$. International trade amplifies the variation in GDP growth by both increasing the Herfindahl index, which is not captured in this graph, and by amplifying the aggregate impact of idiosyncratic shocks to firms conditional on the Herfindahl index, which is given by this term  This figure indicates that the term is monotonically increasing with a decreasing rate of increase as $S$ approaches 1. The term increases as $\epsilon$ increases, indicating trade has a larger amplifying effect when output is more substitutable across firms. If $S = 0$ then there is no trade, indicating that $\psi(\epsilon, S) = 1$. The accuracy of this term depends on there being multiple traded sectors, as discussed in section 1.III.E.i. If there is only a single traded sector then there is no amplifying effect of trade.

**Figure 1.3**

**Predicted Versus Observed Annual GDP Volatility for**

**OECD Countries, (1990–2005)**

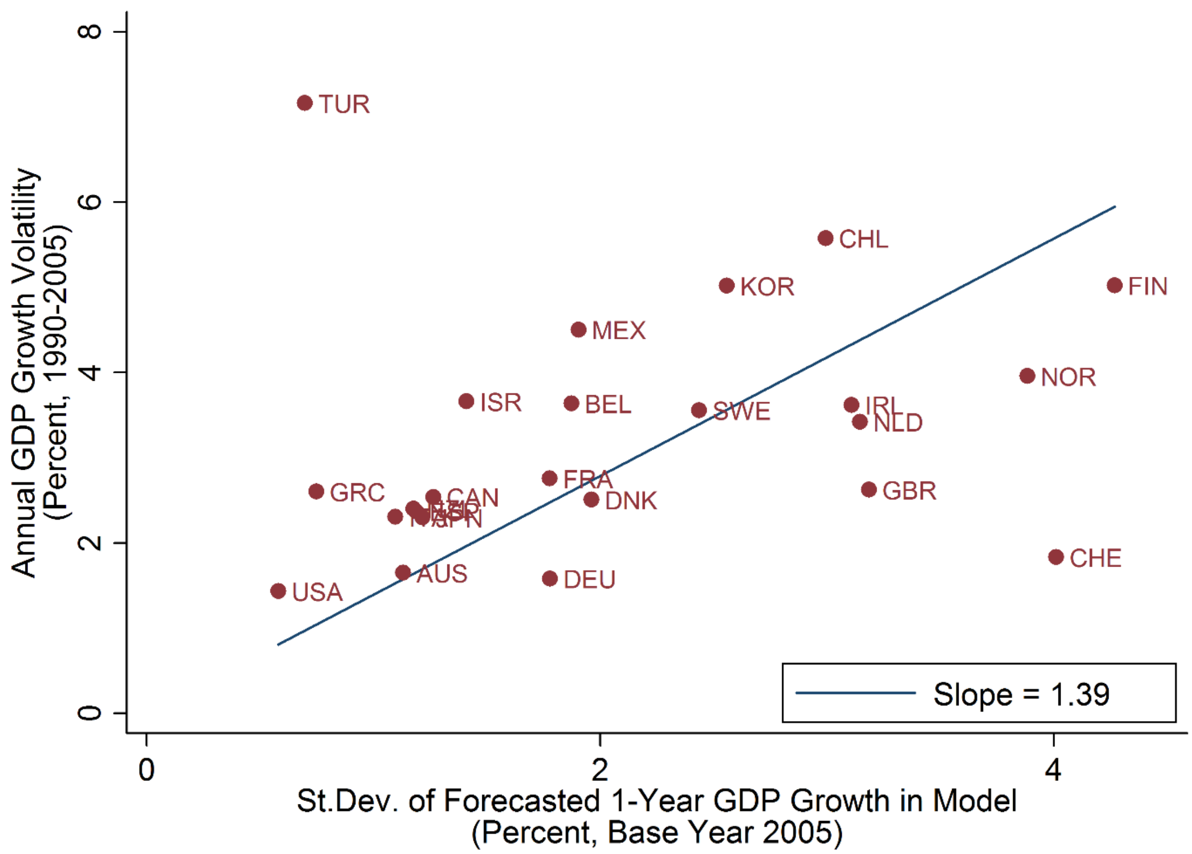


Figure 1.3 plots observed annual GDP growth volatility (percent) between 1990–2005 versus the standard deviation of forecasted 1-year GDP growth in the model, where both are from table 1.3.

The fit line is from the following regression: $data = \beta_1 * model$. There is no constant in this regression. The interpretation of the slope is that the standard deviation of GDP growth in the data is, on average, 39% higher than the standard deviation of GDP growth in the model.

As detailed in table 1.7, Turkey and Switzerland are excluded from this regression as outliers. Turkey experienced low GDP growth between 1990–2000 followed by explosive GDP growth from 2000–2005, and this tale of two halves leads Turkey to have the highest GDP volatility of any OECD country over 1990–2005 by a significant amount. Switzerland is excluded as its unusually low GDP volatility appears to be the result of being in a sustained great depression, see Kehoe and Ruhl (2005).

**Figure 1.4**

**Forecasted GDP Growth for the United States**

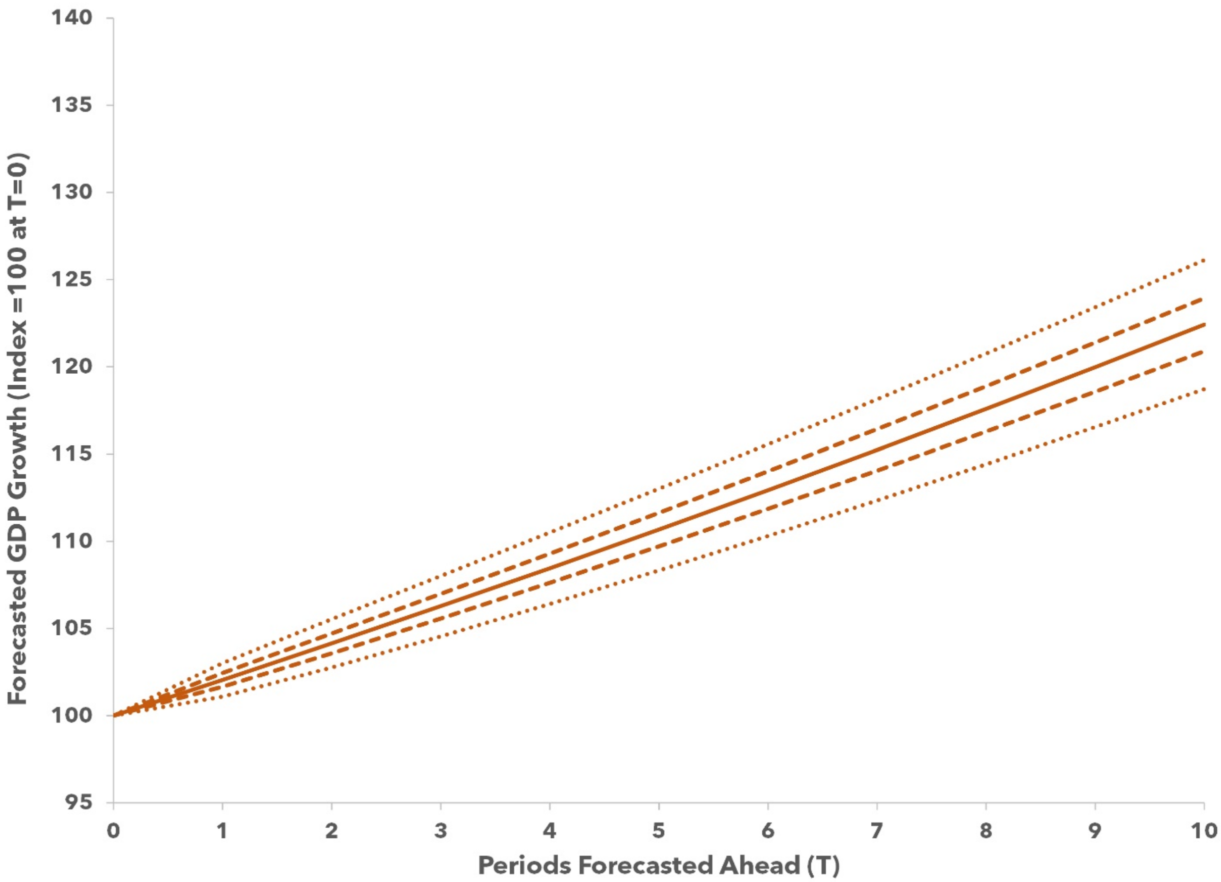


Figure 1.4 presents forecasted growth using the model parameterized to the United States. The parameters used are taken from table 1.6 with a base year of 2005. Expected GDP growth is given by the solid line, and the dotted lines represent the prediction confidence intervals for the forecast. One interpretation of these intervals is as follows. Suppose we have a large number of countries exactly like the United States at $T = 0$ and sort them by realized GDP growth after 10 years. Due only to granularity and differences in the realizations of idiosyncratic shocks to individual firms, the country in the 95th percentile will have grown by 26% (2.3% annualized), the median country by 22% (2.0% annualized), and the country in the 5th percentile by 18% (1.7% annualized).

**Figure 1.5**

**Forecasted GDP Growth for Korea**

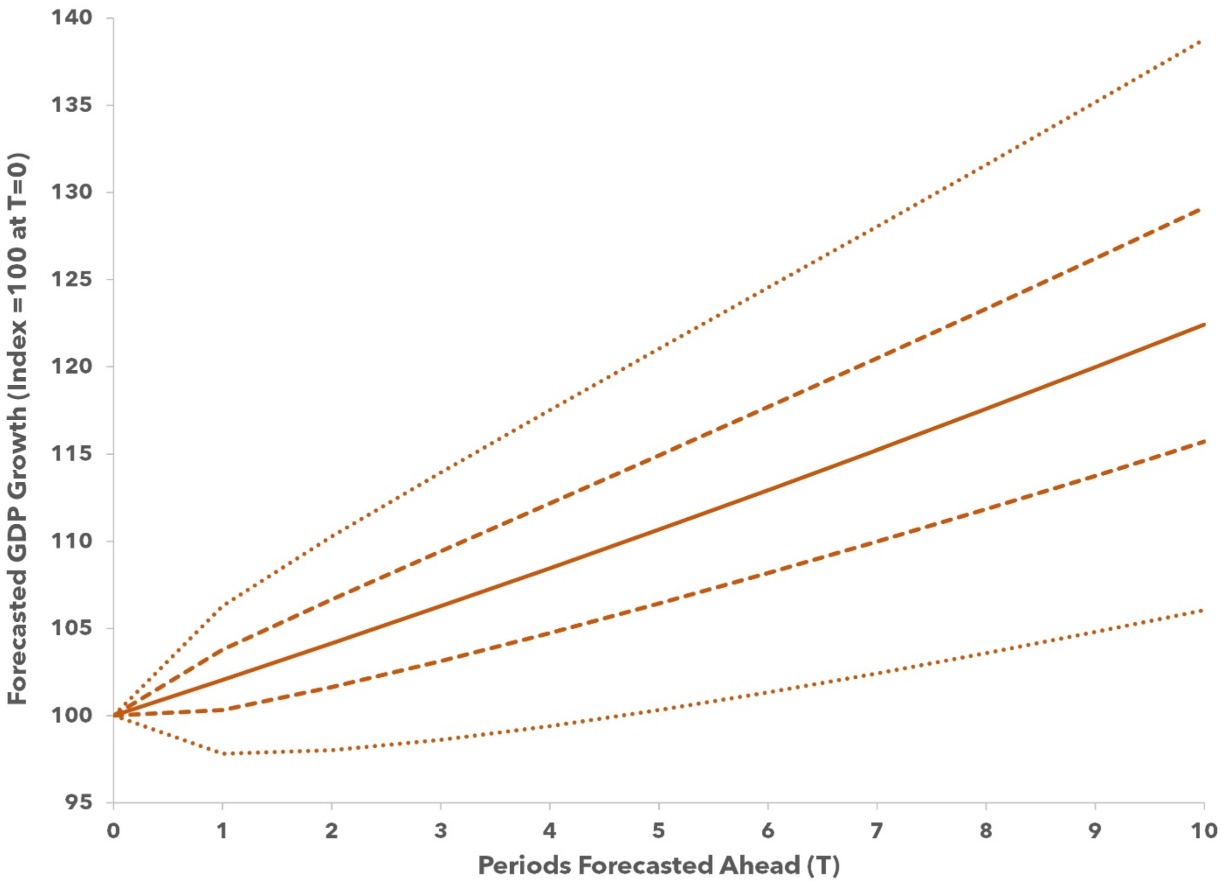


Figure 1.5 presents forecasted growth using the model parameterized to Korea. The parameters used are taken from table 1.6 with a base year of 2005. Expected GDP growth is given by the solid line, and the dotted lines represent the prediction confidence intervals for the forecast. One interpretation of these intervals is as follows. Suppose we have a large number of countries exactly like Korea at $T = 0$ and sort them by realized GDP growth after 10 years. Due only to granularity and differences in the realizations of idiosyncratic shocks to individual firms, the country in the 95th percentile will have grown by 38% (3.3% annualized), the median country by 22% (2.0% annualized), and the country in the 5th percentile by 5% (0.5% annualized).

**Figure 1.6**

**Forecasted GDP Growth for Non-Granular Economy**

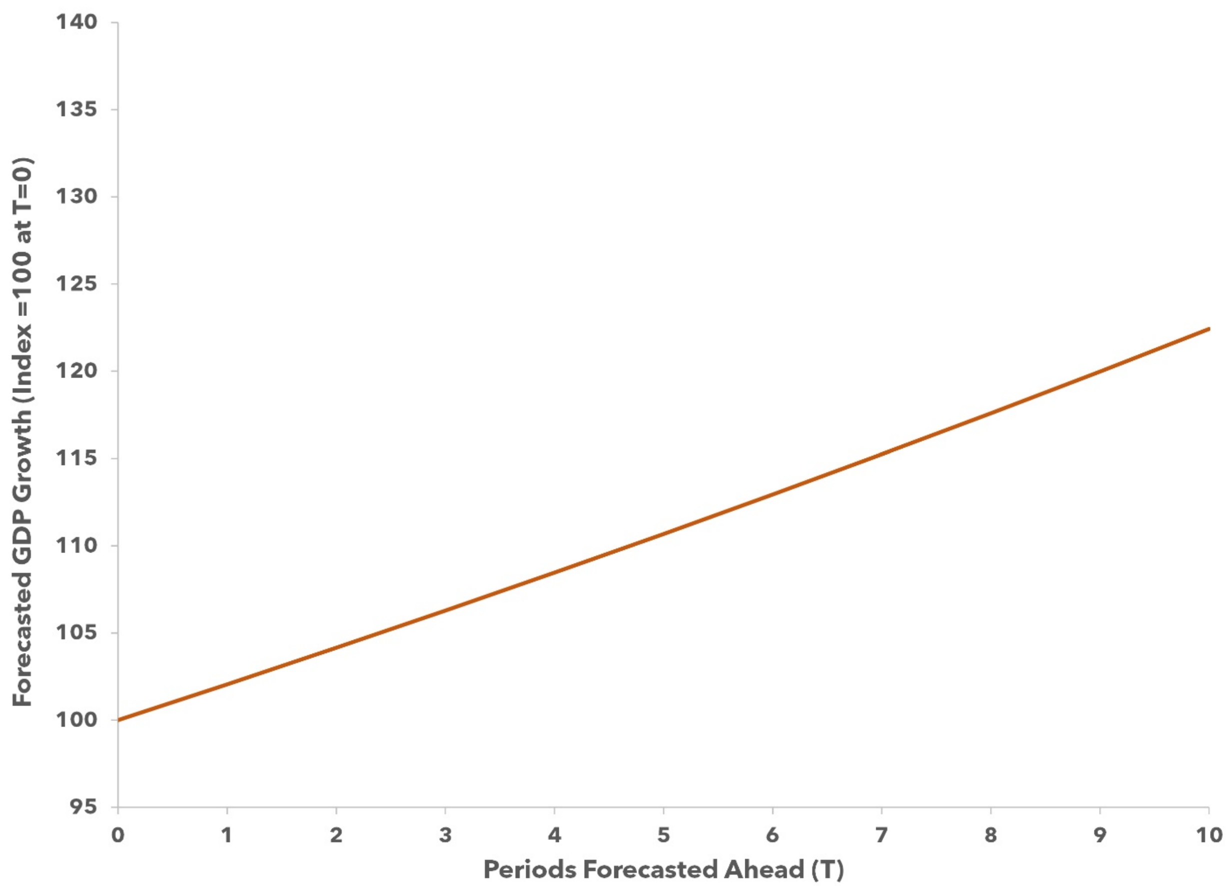


Figure 1.6 presents forecasted growth in the model for a non-granular economy ($h_{i,0} = 0$) with the parameters for the growth process for firms and elasticity of substitution taken from table 1.6. In the absence of granularity, the model generates zero variation in GDP growth, which grows deterministically at 2.0 percent per year.

Chapter 2

# On The Irrelevance of Pareto Tails

## I. Introduction

Over the past 15 years there have been significant advances in understanding the implications of firm dynamics and firm heterogeneity as it applies to international trade[1] and macroeconomic fluctuations[2]. Prominently featured in these advances, has been the exploitation of functional form assumptions regarding the underlying sources of firm heterogeneity. As these frameworks typically deliver predictions regarding the relationship between the underlying firm heterogeneity and observable firm characteristics, such as firm size as measured in gross revenues or the number of employees, these functional form assumptions effectively manifest as assumptions regarding the distributions of these firm-level characteristics[3], which can then be empirically examined to judge the applicability of the functional form assumptions.

In this chapter, I focus on the assumption that the right tail of firm sizes follows a Pareto distribution, and the importance of this assumption for the granular hypothesis (the hypothesis that idiosyncratic shocks to firms can generate significant aggregate

[1] The seminal works in this literature being the monopolistically competitive framework of Melitz (2003) and the Ricardian framework of Eaton and Kortum (2002).
[2] See for example the 'granularity' literature spawned by Gabaix (2011).
[3] For simplicity, I therefore refer to functional form assumptions on then underlying sources of firm-level heterogeneity as assumptions regarding the firm-level characteristics themselves.

fluctuations as shocks to large firms can significantly impact the overall economy) discussed in Chapter 1. This chapter proceeds in three sections. In the first section, I review the reasoning put forward by Gabaix (2011) for why Pareto tails are commonly thought to be essential to the granular hypothesis due to violating the finite variance condition necessary for the Central Limit Theorem (CLT) to hold. I present an argument for why Pareto tails, and, more generally, distributions with infinite second moments, are not necessary for the granular hypothesis to hold in practice by contrasting their convergence with that of the lognormal distribution, which is highly skew, yet obeys the CLT and has finite variance.

In the second section of this chapter, I revisit the empirical debate surrounding whether the right tail of the distribution of firm sizes appears to follow a Pareto or lognormal distribution. I review the standard methods for distinguishing between the two distributions, such as log-log plots of the cumulative distribution function. Through Monte Carlo simulations, I compare the effectiveness of these methods with the uniformly most powerful test (UMPT) of Del Castillo and Puig (1999) and show that the UMPT performs superiorly to the standard methods most commonly used.

In the third section of this chapter, I apply the UMPT to a dataset containing firm-level microdata on gross revenues for large firms in 43 countries and show that there is significant cross-country variation in whether the right tail of the distribution of firm sizes appears to follow a Pareto distribution or a lognormal distribution. Despite this variation, I show that the distribution of firm sizes is not predictive for how much volatility a country faces. I show that there is a large role for granularity even in countries with a lognormal right tail for the distribution of firm sizes, providing evidence that the importance of whether the right tail follows a Pareto distribution or not is overstated.

### 2.I. Pareto Tails and the Granular Hypothesis

In this section, I review the reasoning behind claims that the existence of a Pareto tail in firm sizes is necessary for the granular hypothesis to hold.[4] I then show that by accounting for scaling constants in limiting behavior and by focusing exclusively on the right tail separately from the body of the firm size distribution, that these arguments show that granularity is more general than previously thought as it does not, in fact, require the existence of a Pareto right tail.

### 2.I.A. The Argument for the Necessity of Pareto Tails

The general idea for the necessity of Pareto tails for granularity, is that for shocks to individual firms to matter in the aggregate, it is necessary for the Central Limit Theorem to be violated to prevent the impact of individual shocks from quickly averaging out as the number of firms in the economy increases. As will be discussed in section 2.III, the most commonly proposed distributions to explain the tail of the distribution of firm sizes are the lognormal distribution and the Pareto distribution. While a distribution featuring a lognormal tail will always satisfy the CLT, this is not necessarily the case doesn't hold due to infinite variance depending on the parameter governing the slope of the Pareto tail. For this reason, it is thought to be essential that the distribution of firm sizes exhibit a Pareto tail rather than lognormal tail if the granular hypothesis is to have any merit.

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[4] See the papers by Gabaix (2011), Carvalho and Grassi (2015), and Di Giovanni and Levchenko (2012).

To illustrate the above argument, consider again the simple example drawn from Gabaix (2011) and discussed in section 1.III.A. Assume that there are $i = 1, \dots, N$ firms in the economy each with output $Y_{i,t}$ so that the total output of the economy is $Y_t = \sum_{i=1}^{N} Y_{i,t}$. Now assume that each firm receives an independent and identically distributed (iid) random shock to sales $\xi_{i,t}$ so that $Y_{i,t+1} = \xi_{i,t} Y_{i,t}$ and the total output of the economy becomes $Y_{i,t+1} = \sum_{i=1}^{N} \xi_{i,t} Y_{i,t}$. Due to the linearity of variance, this means that $Var\left[Y_{i,t+1}\right] = \sum_{i=1}^{N} \left(Y_{i,t}\right)^2 Var\left[\xi_{i,t}\right]$, and if $\xi_{i,t}$ is iid with $Var\left[\xi_{i,t}\right] = Var[\xi_t]$, then the standard deviation of the percentage growth of total output is

$$Var\left[\frac{Y_{t+1} - Y_t}{Y_t}\right] = Var[\xi_t] h_t, \qquad (2.1)$$

where

$$h_t = \sum_{i=1}^{N} \left(\frac{Y_{i,t}}{\sum_{i=1}^{N} Y_{i,t}}\right)^2 \qquad (2.2)$$

is the Herfindahl index of the economy[5]. That $\xi_{i,t}$ is iid across firms is known as Gibrat's law and seems to be a good approximation for large firms as shown by Cabral and Mata (2003) and Singh and Whittington (1975) and discussed above in section 1.III.D. Taking the square root of equations (2.1) shows that the standard deviation for the growth rate of aggregate output scales with the standard deviation of the iid shocks and the square root Herfindahl index of the economy, where the latter scales with the number of firms, $N$. For example let $h_{t,N}$ be the Herfindahl index for an economy with $N$ equal sized firms, then

[5] The Herfindahl index is sometimes referred to as the Hirschman-Herfindahl index after Hirschman (1964).

$\sqrt{h_{t,N}} = \frac{1}{\sqrt{N}}$. If there is only a single firm in the economy, then $h_{t,1} = 1$, and if there are 1000 firms $\sqrt{h_{t,1000}} \approx 0.03$ and if there are six million firms, which is the approximate number of employer firms in the United States in 2008 according to the U.S. Census Bureau[6], then $\sqrt{h_{t,6000000}} \approx 0.0004$ indicating that there would be no room for idiosyncratic shocks to have a measurable aggregate impact if all firms were the same size. Of course, firms are not all the same size, however, the Central Limit Theorem states that $i = 1, \ldots, N$ firms are drawn iid from a distribution with finite mean and variance each with size $Y_{i,t}$, then

$$\sqrt{h_{t,N}} \to \frac{1}{\sqrt{N}} \frac{\sqrt{E\left[Y_{i,t}^2\right]}}{E\left[Y_{i,t}\right]}, \qquad (2.3)$$

for large $N$.[7] **Proof of Equation 2.3:** Starting with the definition of the Herfindahl index we have

$$h_{t,N} = \frac{\sum_{i=1}^{N} Y_{i,t}^2}{\left(\sum_{i=1}^{N} Y_{i,t}\right)^2} = \frac{N\left(\frac{1}{N}\sum_{i=1}^{N} Y_{i,t}^2\right)}{\left(N\frac{1}{N}\sum_{i=1}^{N} Y_{i,t}\right)^2} = \frac{\overline{Y_{i,t}^2}}{N\left(\overline{Y_{i,t}}\right)^2} \to \frac{1}{N}\frac{E\left[Y_{i,t}^2\right]}{\left(E\left[Y_{i,t}\right]\right)^2},$$

where → follows from the Law of Large Numbers and $E\left[Y_{i,t}^2\right]$ exists and is finite. Taking a square roots delivers equation 2.3. ■

[6] http://www.census.gov/econ/smallbus.html

[7] Equation (2.3) can be derived without the Central Limit Theorem, relying only on the Law of Large Numbers and that $E\left[Y_{i,t}^2\right]$ exists and is finite, however, the CLT can be used to create confidence intervals around $h_{t,N}$.

As only $\frac{1}{\sqrt{N}}$ changes with the number of firms in the economy, this means that for a lognormal tail, the rate of decay for the aggregate impact of idiosyncratic shocks is still approximately $\frac{1}{\sqrt{N}}$ for large $N$.

In contrast, if there is a Pareto tail, meaning that $P(Y_{i,t} > y) \sim y^{-a}$ as $y \to \infty$, and $a < 2$, then the distribution that the $Y_{i,t}$ are drawn from will have infinite variance, as well as infinite mean if $0 < \alpha \leq 1$, and therefore equation (2.3) will not hold. Instead Gabaix (2011) shows that in this case

$$\sqrt{h} \to \begin{cases} \dfrac{\nu_a}{\ln N}, & for\ a = 1 \\ \dfrac{\nu_a}{N^{1-\frac{1}{a}}}, & for\ 1 < a < 2 \end{cases}, \qquad (2.4)$$

where $\nu_a$ is a random variable that depends on $a$, but does not change with $N$. If $a \geq 2$ then equation (2.3) continues to hold. Contrasting equation (2.4) with equation (2.3) makes it clear why the Pareto tails are thought to be essential to the granular hypothesis. As figure 2.1 shows, the Herfindahl index converges to zero much faster when the underlying distribution for firm sizes has a lognormal tail instead of a Pareto tail with $\alpha = 1$, which is a special case of the Pareto tail referred to as a Zipf distribution. In fact, as discussed by Axtell (2001) and Luttmer (2007), the distribution of firm sizes in the United States is often though to follow a Pareto tail with $a \approx 1$, although estimates range from between 1 and 2 (in section 2.III, I estimate $\alpha \approx 1.46$). Therefore with 6 million firms in the United States, equation (2.4) shows that, ignoring the constants, a Zipf tail permits a herfindahl index roughly 360 times larger $\left(=\left(\frac{1}{\ln 6000000}\right)/\left(\frac{1}{\sqrt{6000000}}\right)\right)$ than a lognormal distribution.

### 2.I.B. Granularity without Pareto Tails: Important Elements

The previous section relies on limiting arguments for the necessity of Pareto tails, in particular that $\frac{1}{\log N}$ converges to zero much slower than $\frac{1}{\sqrt{N}}$. Here I show that after accounting for two factors, constants in the limiting behavior and a distinction between the body and tail of the distribution, there is not a significant difference in the rate of convergence towards zero for a Herfindahl index generated from a Pareto tail versus a lognormal right tail.

### 2.I.B.i. The Importance of Scaling Constants

Consistent with standard practice in asymptotic analysis, previous research has focused only on the dominant terms governing the rate of decay for equations (2.3–2.4), while ignoring the scaling constants in each equation. The reason for this, is that as $N \to \infty$, the impact of the constants in determining relative rates of convergence approaches zero. Here I argue, however, that for practical values of $N$, such as $N = 6000000$, we are sufficiently far away from the limiting behavior of the Herfindahl indices, that it is remains essential to take into account the scaling constants when comparing rates of convergence for Pareto and lognormal tails.

In both equations (2.3) and (2.4), the constants scaling the rate of decay for the Herfindahl index depends on the parameters of the respective distributions. I estimate these parameters for the United States[8] by fitting each distribution to firm level data on gross revenues, and plot the fit in figure 2.2. For the lognormal distribution I estimate $\mu = 11.98$ and $\sigma = 2.83$, which yields an estimated constant of $\sqrt{E[Y_i^2]}/E[Y_i] \approx 54.84$. For the Pareto distribution, I compute the associated constant for both my estimated tail parameter, and for the special case where $\alpha = 1$. When $\alpha = 1$, the scaling constant is $\nu_{\alpha=1} = 1.28$, and for my estimated coefficient from section 2.II of $\alpha = 1.46$, the constant is $\nu_{\alpha} = 0.57$. As we can see the estimated constant for the lognormal tail is significantly larger than the estimated constants for the Pareto tails. Figure 2.3 replots the estimated Herfindahl indices using equations (2.3–2.4) with these constants taken into account.[9] From this plot, we see that the difference in convergence rates for the Herfindahl index is considerably smaller than when constants are ignored. Indeed, for $N = 6000000$,after taking into account the constants, the difference in estimated Herfindahl indices between a Pareto tail with $\alpha = 1$ and a lognormal tail shrinks from a factor of 360 to a factor 6.5.

### 2.I.B.ii. Tails versus Bodies

Although Axtell (2011) claims that both the body and tail of the distribution of firm sizes is well approximated by a Pareto distribution, as reported by Head, Mayer, and Thoenig (2014), it has become increasingly accepted that the lognormal distribution provides a better approximation for the body of the distribution of firm sizes while disagreement has primarily focused on the right tail. In addition to this, the granularity of an economy is

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[8] The data is described in section 2.III.

[9] That the estimated Herfindahl index is greater than one for small values of $N$ is discussed in section 2.I.B.ii.

governed almost entirely by the firms in the right tail, as a firms impact on the the Herfindahl index declines at a quadratic rate. For those reasons, it makes sense to consider the body of the distribution of firm sizes fixed and focus only on the tail when comparing the granularity of an economy with a Pareto right tail versus a lognormal right tail. To that extent, it is possible to rewrite equation (2.2) in terms of the Herfindahl index of the tail, scaled by the portion of total output accounted for by the body of the distribiton. In particular, we can write:

$$h_N = \left(\frac{\sum_{i=N-\tau}^{N} x_i}{\sum_{i=1}^{N} x_i}\right)^2 h_\tau, \qquad (2.5)$$

where $h_N$ is the Herfindahl index of the entire economy, $h_\tau$ is the Hefindahl index of the largest $\tau$ firms in the tail of the economy, and $\frac{\sum_{i=N-\tau}^{N} x_i}{\sum_{i=1}^{N} x_i}$ is the portion of output accounted for by firms outside of the tail. If we consider the top 1000 firms in the economy, $\tau = 1000$, then $\frac{1}{\log \tau}$ is only 4.5 times larger than $\frac{1}{\sqrt{\tau}}$, and after accounting for constants, equations (2.3) and (2.4) would actually predict that an economy with a lognormal right tail would be more granular than if the economy had a Pareto right tail with $\alpha = 1$.

A legitimate concern, however, is that we cannot apply equations (2.3–2.4) if we are focusing on the tail alone, as they describe limiting behavior which may not apply, as $\tau = 1000$ is too small of a sample. Figure 2.3 confirms the validity of this concern, as it shows that at $\tau = 1000$ we would have $h_\tau > 1$, which is not possible. To address this concern, I construct samples from a Pareto distribution and a lognormal distribution so that the empirical Cumulative Distribution Function (CDF) of each sample perfectly matches the true underlying CDF of the distribution. I then directly compute the Herfindahl indices generated by each sample, for sample sizes ranging from $N = 10^1$ to $N = 10^8$. Figure 2.4 plots these Herfindahls, and shows us that, while the limiting behavior displayed in figure 2.3 is not a good approximation until the sample size is at least $N = 10{,}000$, with a tail size of $1000$, we would still expect an economy with a lognormal right tail to be more granular than an economy with an economy with a Pareto right tail, assuming the portion of output accounted for by the body of the distribution is held constant.

**2.II. Distinguishing between Pareto and Lognormal Tails**

In this section, I revisit the empirical debate over the whether the distribution of firm sizes, as measured by gross revenues, is better described as having a lognormal right tail, as argued by Stanley (1995) and Cabral and Mata (2003), or a Pareto fat tail as argued by Axtell (2001), Luttmer (2007). In this paper, I focus exclusively on the right tail as those are the firms that determine how granular an economy is, as mentioned in section 2.I.B.ii, and due to the fact that there is significantly better data availability for firms in the right tail.

It's worth acknowledging that this argument over the distribution of firm sizes mirrors the debate of lognormal versus Pareto with regards to the distribution of city sizes by Eeckhout (2004,2009), Levy (2009), and Malevergne, Pisarenko, and Sornette (2009). Similarly to how I focus on product development as the source for the emergence of a fat-tailed distribution, in the context of city sizes, Gabiax (1999) highlights the existence of reflective lower bound as another potential mechanism for generating a Pareto distribution from a Gibrat's law growth process. For a history of the development of these types of models and the Pareto vs lognormal debate in other disciplines, I refer the reader to Mitzenmacher (2004).

### 2.II.A. The Uniformly Most Powerful Unbiased Test and Alternative Methods

Much of the difficulty in reaching a consensus over the distribution of firm sizes is due to surprising similarities between the Pareto and lognormal distributions that makes it difficult to discern between the two distributions. Although there is widespread acknowledgement that traditional goodness of fit tests and visual comparisons suffer from a lack of power (Eeckhout 2009), these tests are still widely used in practice. In fact, perhaps the most common method for telling the difference between the lognormal and Pareto distributions is the visual method employing log-log plots as in Axtell (2001). The basis of this method rests on noting that the logarithm of the complementary cumulative distribution function (CCDF) for a Pareto distribution is

$$\log\big(1 - CDF(x)\big) = a \log k - a \log x,$$

which implies a linear relationship between the CCDF and firm size, if firm size follows a Pareto distribution. Therefore telling whether firm sizes follows a lognormal or Pareto distribution should be as easy as looking for the presences or absence of curvature in the log-log plot as Figure 2.5 shows. However, as Figure 2.6 shows, lognormal distributions can appear linear over finite ranges, especially when we truncate the distribution as we do when focusing on the right tail. Some authors such as Eeckhout (2004) have suggested that this means we should restrict ourselves to only looking at the complete distribution, and if the body displays curvature to conclude a lognormal right tail. However, distributions that display a lognormal body with a Pareto right tail such as in Reed (2002) and the importance of fat tails limits the degree of faith we should have in identifying the tail according to the body.

To understand why it can be so hard to tell the two distributions apart in practice, Sornette and Cont (1997) note that we can re-write the lognormal distribution pdf as

$$f(x) = \mathcal{C}x^{-1+\gamma(x)}, \ \ for\ x > 0,$$

where

$$\mathcal{C} = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-\mu^2}{2\sigma^2}} \text{ and } \gamma(x) = \frac{\log x}{2\sigma^2} - \frac{\mu}{\sigma^2}.$$

This is similar in structure to the pdf of the Pareto distribution, where

$$f(x) = ak^a x^{-1-a}, \ \ x > k,$$

which indicates that the Pareto distribution can be thought of as special case of the lognormal distribution in which $\gamma(x)$ is constant instead of increasing with $x$, which leads to the hump we see in log-log plots for the lognormal distributions when we look at a wide enough range. When $\sigma^2$ is relatively large, however, as it is for the distribution of firm sizes or city sizes, $\log x / \sigma^2$ may appear approximately constant over finite ranges of $x$, giving the illusion of linearity and a Pareto distribution.

Just as it is difficult to visually tell the distributions apart due to the similarity in structure, it is also difficult for most standard statistical tests to distinguish between them without large sample sizes. In response to this there has been a movement towards methods that are capable of empirically distinguishing between the Pareto and the lognormal distribution such as maximum entropy methods, as in Bee (2011), and the uniformly most powerful unbiased test (UMPT) from Castillo and Puig (1999), which makes use of the above similarity in structure and is the method employed in this paper.

Del Castillo and Puig (1999) show the relevant test statistics for the uniformly most powerful unbiased test of whether $\psi = 0$ is the sample coefficient of variation. This is due to the theoretical coefficient of variation always being equal to 1 for an exponential distribution and less than or equal to one for a truncated normal distribution. For example in the special case where $\mu = 0$ the truncated normal distribution has a theoretical coefficient of variation equal to $\sqrt{2/\pi}$. For sample sizes with $N = 100$ or greater, we can determine the critical point of the test noting that the sample coefficient of variation obeys the Central Limit Theorem and is approximately normally distributed. Alternatively, for small sample sizes the critical point can be computed with high accuracy using saddlepoint methods laid out in their paper.

Table 2.1 shows the results of a Monte Carlo experiment comparing the accuracy of the UMPT with three standard alternatives, a test of curvature, a Residual Sum of Squares (RSS) Test, and a Kolmogorov–Smirnov (K-S) test in terms of accurately classifying simulated Pareto and lognormal distributions. The curvature test tests for curvature by examining the statistical significance of a quadratic term in the log-log plot. If the quadratic term is not statistically significant, the test classifies the tail as being Pareto. The Residual Sum of Squares (RSS) test operates by fitting both a Pareto and lognormal distribution to the data, and picking the distribution that delivers the lowest RSS as having the best fit. The Kolmogorov-Smirnov test is a goodness of fit tests, which classifies the tail by seeing if it is possible to reject the null hypothesis that the tail in the data is generated by a Pareto (lognormal) tail, while failing to reject the null hypothesis for the alternative tail. If the test rejects the null hypothesis, or fails to reject the null hypothesis, for both a Pareto and the lognormal tail, then it fails to classify the distribution as either.

Table 2.1 reports the results of these Monte Carlo experiments. It shows that with a sample size of 100, the UMPT is accurate 94.9% of the time compared to accuracy ranging between 29.7% and 74.7% for the other three tests. When we have a sample size of 1000, the accuracy of the C-P method increases to 97.5%, and while the RSS and K-S test perform well with the larger sample sizes, and the curvature test continues to perform poorly, with an accuracy rate of only 52.5% due to almost always classifying the sample as having a lognormal tail regardless of the true underlying tail distribution. In light of these Monte Carlo results, there appears to be no reason to choose any of the three standard tests over the UMPT, and further, any research employing these standard methods with relatively small sample sizes, or a curvature test with a sample of any size, should be viewed as highly suspect, as should reliance on visual methods.

### 2.III. Empirical Results

In this section, I apply the UMPT test to an internationally comparable dataset featuring firm-level microdata on revenues for large firms worldwide. The data comes from the Million Dollar Directory compiled by Dunn & Bradstreet[10], which is a specialist provider of credit and marketing data, and contains information on 34 million public and private firms worldwide. The database has good coverage of large firms globally, as well as

[10] Previous research has used data from Dunn and Bradstreet to study topics such as vertical integration, for example by Alfaro, Conconi, Fadinger, and Newman (2015) and Acemoglu, Johnson, and Mitton (2005), as well as firm composition and cross country income differences, e.g. by Alfaro, Charlton, and Kanczuk (2008).

complete coverage in the United States and Canada. As my focus is on the right tail, I select only firms that are non-subsidiary headquarter locations with greater than $10 million USD in gross revenues[11] and restrict my analysis to countries with at least 100 firms remaining in the sample. After these restrictions, I am left with a sample of 162,000 firms across 43 countries.

Table 2.2 reports the results of the UMPT test applied to each of the 43 countries in the sample. I find significant cross-country variation in terms of whether the distribution of firm sizes appears to have a Pareto tail or a lognormal tail. In particular, I find that 24 of the 43 countries appear to have a Pareto right tail in their distribution of firm sizes[12], while the other 19 countries display a lognormal right tail. I believe this is the first paper to find such variation, a fact which I attribute to using an internationally comparable database as well as a statistical test with enough power to successfully distinguish a Pareto tail from a lognormal tail. For each country, I then compute the Herfindahl index using equation (2.2) and report it in table 2.2, where I use reported firm revenues in the numerator and each country's GDP as the denominator following Gabaix (2011). It is important to note that, to the extent that my sample is missing small firms, these firms to not have a significant impact on the Herfindahl index.

Table 2.3 compares average Herfindahl indices for countries with a Pareto right tail to those for countries with a lognormal right tail. Surprisingly, these results show countries

[11] My results are robust to alternative tail specifications, including allowing the tail cutoff to fluctuate by country. I focus on non-subsidiary headquarter locations to avoid double-counting, as revenues are reported on a corporate-wide basis.
[12] Consistent with previous research by Axtell (2011) and Luttmer (2007), I find the distribution of firm sizes in the United States to display a Pareto right tail.

with a Pareto right tail actually exhibit slightly lower Herfindahl indices on average, which provides strong evidence against the claim that Pareto tails are necessary for a country to exhibit granularity. One concern might be, however, that these results are being driven by an outside factor, and perhaps after accounting for this factor countries with Pareto tails would exhibit higher Herfindahl indices. To partially alleviate this worry, I regress the country's Herfindahl indices on an indicator variable that equals one if the country is classified as having a Pareto right tail, and include the country's GDP and number of firms included in the sample as control variables. Table 2.4 reports the results of these regressions, and in all cases I find the regression coefficient on the indicator for a Pareto right tail to be negative and insignificant, whereas we would expect it to be positive and significant if the existence of a Pareto tail was essential to the granular hypothesis.

### 2.IV. Conclusion

In this chapter, I argued that the standard justification given for the necessity of Pareto tails for the granular hypothesis falls apart after focusing on the tail separately from the rest of the distribution and after taking into account constants in expressions governing limiting behavior. Empirically, I showed that, when appropriate statistical methods are used, there is significant cross-country variation in whether the distribution of firm sizes has a Pareto right tail or a lognormal right tail, however whether the tail is Pareto or lognormal is not predictive of how granular an economy will be. Rather than being a necessity, my results show that the existence of a Pareto tail is irrelevant in practice, and therefore the concept of granularity applies more generally than previous research suggested.

## Table 2.1

### Distinguishing between Pareto and Lognormal Tails: Monte Carlo Results

| *(correct %)* | **UMPU Test** | | **Curvature Test** | | **RSS Test** | | **K-S Test** | |
|---|---|---|---|---|---|---|---|---|
| **Sample N** | 100 | 1000 | 100 | 1000 | 100 | 1000 | 100* | 1000 |
| **Pareto** | 95.7 | 95.0 | 15.1 | 5.0 | 97.7 | 100 | 32.8 | 95.1 |
| **lognormal** | 94.2 | 100.0 | 95.8 | 100 | 51.8 | 86.5 | 26.6 | 95.2 |
| **Total** | 94.9 | 97.5 | 55.4 | 52.5 | 74.7 | 93.2 | 29.7 | 95.2 |

Samples are drawn either from a Pareto distribution with both tail and cutoff equal to 1 or a lognormal distribution with mean 0, variance 1, and a truncation point of 1. The numbers in each cell represent the percent of the time the test correctly classified each simulation.

Testing procedure is as follows:

Step 1: Draw a random samples from the Pareto and lognormal distributions

Step 2: Run the UMPU Test, curvature test, and RSS test on each sample and classify whether the distribution chose the correct distribution.

-The curvature test involves testing for curvature in the log-log plot by adding a quadratic term and testing its significance. If the quadratic coefficient is significantly different from zero, classify the distribution as lognormal, otherwise as Pareto. This test offers the closest comparison to the informal visual test of the four tests.

-The Residual Sum of Squares (RSS) test involves fitting a sample to both a Pareto distribution and a lognormal distribution and picking the one with the lowest Residual Sum of Squares. In this simulation I fit using nonlinear least squares, however MLE proves similar results.

Step 3: For the K-S test, which differs from the other tests in that it is a goodness-of-fit test rather than a comparative test, I run the test against each distribution. If sample is Pareto, test the null-hypotheses of it being Pareto and lognormal. If the test rejects the lognormal null-hypothesis, but fails to reject the Pareto null-hypothesis classify the test as correct.

*Says Pareto when Pareto = 95.9%. Says Pareto when lognormal =70.0%. Says lognormal when lognormal 95.6%. Says lognormal when Pareto 63.7%.

**Table 2.2**

**Tail Classifications and Herfindahl Index Estimates**

| Country | Firms | Tail | Herf | Country | Firms | Tail | Herf |
|---|---|---|---|---|---|---|---|
| Australia | 2439 | Pareto | 0.157 | Japan | 41356 | Pareto | 0.101 |
| Austria | 1284 | Pareto | 0.151 | Korea | 1290 | Lognormal | 0.149 |
| Belgium | 1395 | Lognormal | 0.076 | Mexico | 924 | Pareto | 0.075 |
| Brazil | 1956 | Lognormal | 0.009 | Netherlands | 1178 | Lognormal | 0.159 |
| Bulgaria | 189 | Pareto | 0.021 | Norway | 750 | Pareto | 0.187 |
| Canada | 4950 | Pareto | 0.165 | Poland | 1380 | Pareto | 0.040 |
| China | 3716 | Lognormal | 0.015 | Portugal | 580 | Pareto | 0.018 |
| Colombia | 1579 | Lognormal | 0.029 | Romania | 179 | Pareto | 0.036 |
| Croatia | 236 | Pareto | 0.032 | Russia | 3624 | Lognormal | 0.058 |
| CzechRep. | 535 | Pareto | 0.037 | Serbia | 221 | Pareto | 0.178 |
| Denmark | 839 | Lognormal | 0.178 | Singapore | 514 | Lognormal | 0.145 |
| England | 6219 | Lognormal | 0.227 | Slovakia | 162 | Pareto | 0.066 |
| Finland | 1013 | Pareto | 0.101 | Slovenia | 226 | Pareto | 0.155 |
| France | 7182 | Pareto | 0.033 | South Africa | 192 | Lognormal | 0.164 |
| Germany | 10516 | Lognormal | 0.121 | Spain | 4263 | Pareto | 0.016 |
| Greece | 845 | Pareto | 0.031 | Sweden | 2128 | Pareto | 0.127 |
| Hong Kong | 639 | Lognormal | 0.266 | Switzerland | 968 | Pareto | 0.296 |
| Hungary | 252 | Pareto | 0.023 | Taiwan | 617 | Lognormal | 0.154 |
| India | 3820 | Lognormal | 0.063 | Turkey | 771 | Lognormal | 0.101 |
| Ireland | 445 | Lognormal | 0.233 | Ukraine | 550 | Lognormal | 0.054 |
| Israel | 459 | Lognormal | 0.074 | USA | 40135 | Pareto | 0.064 |
| Italy | 10324 | Pareto | 0.035 | | | | |

**Table 2.3**

**Average Herfindahl Index by Tail Classification**

| Tail Classification | Number of Countries | Average Herfindahl |
|---|---|---|
| **Pareto** | 24 | 0.089 |
| **Lognormal** | 19 | 0.119 |
| **Total** | 43 | 0.102 |

**Table 2.4**

**Regression Results: Herfindahl Index by Tail Classification**

| Regression Coefficient | Estimate | Standard Error |
|---|---|---|
| Pareto Indicator | −0.035 | 0.024 |
| Firm Count | 1.63e-6 | 1.10e-6 |
| GDP | −7.819 | 3.696 |
| Constant | 0.120 | 0.016 |
| Observations | | 43 |
| R-squared | | 0.04 |

**Figure 2.1**

**Limiting Herfindahl Decay Rates: 1/LogN vs 1/SqrtN (Indexed)**

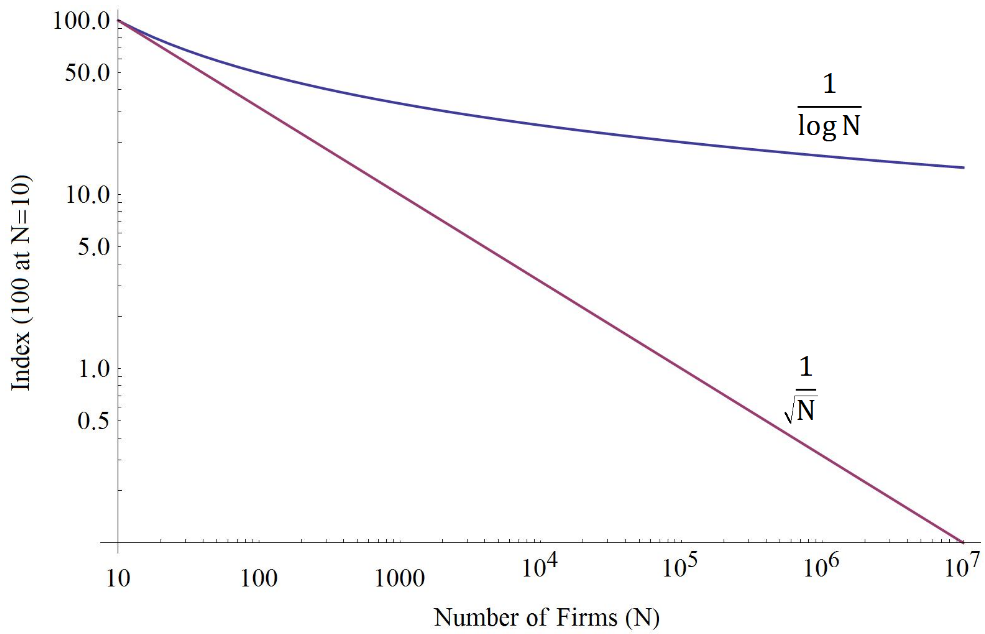



**Figure 2.2**

**Pareto vs Lognormal Tail Fit for United States**

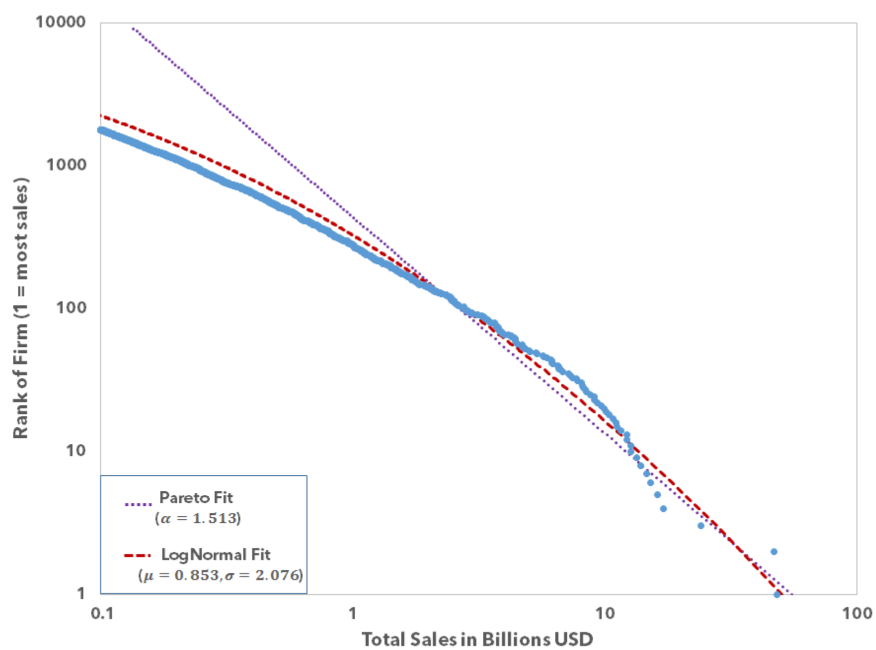


**Figure 2.3**

**Limiting Herfindahl Decay Rates: Constants Included**

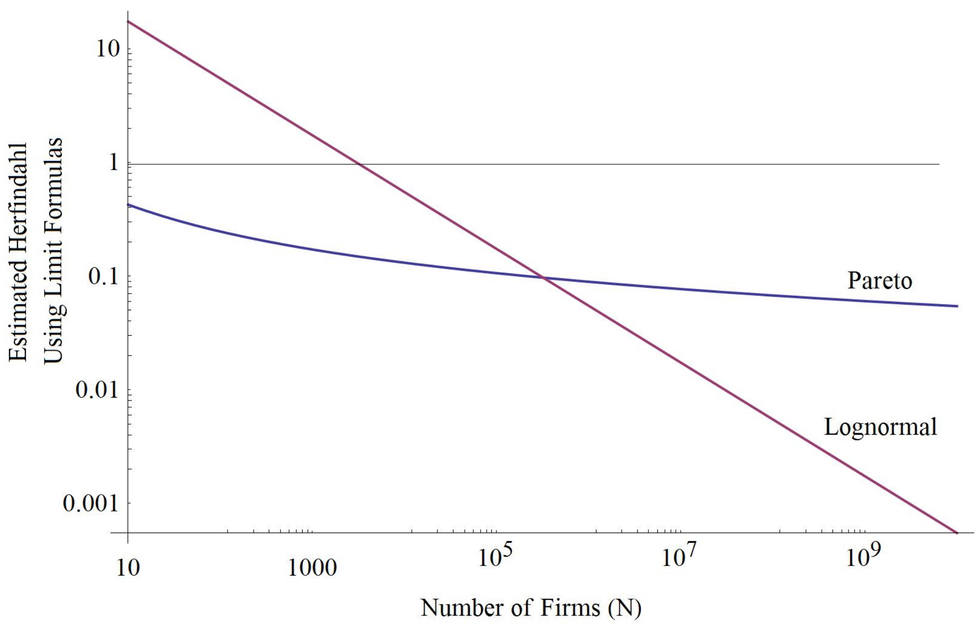



**Figure 2.4**

**Herfindahl Decay Rates: Computed Directly**

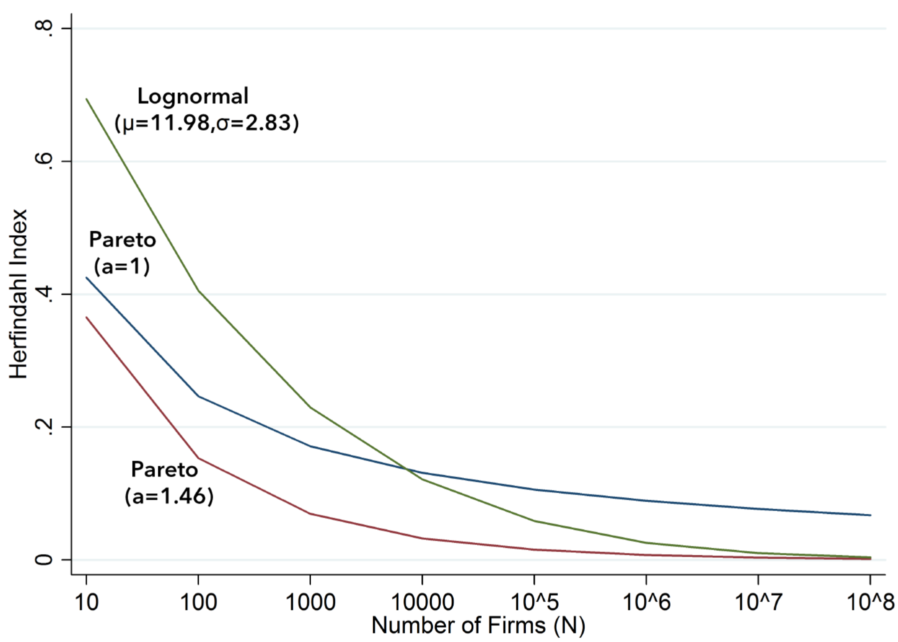


**Figure 2.5**

**Ideal Log-Log Plots for the Pareto and Lognormal Distributions**

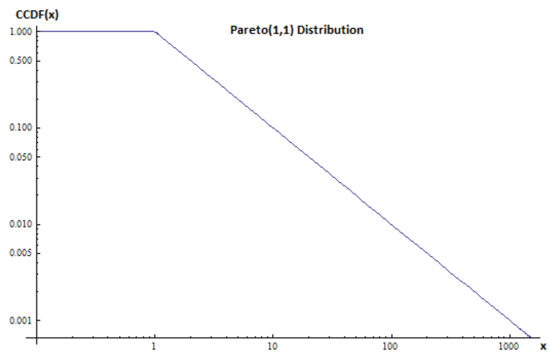
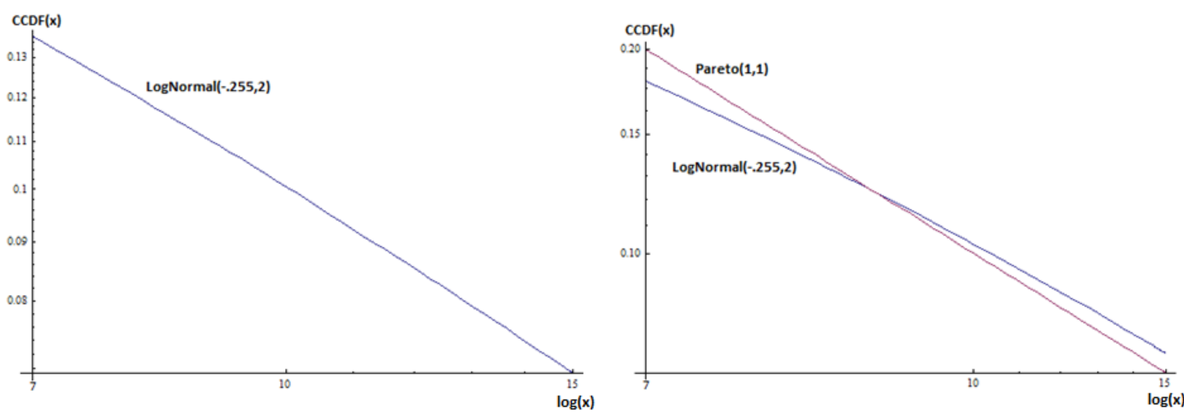



**Figure 2.6**

**Log-Log Plots for the Pareto and Lognormal Distributions Over a Limited Range**



Chapter 3

# Using the New Products Margin to Predict the Industry-Level Impact of Trade Reform[13]

## 3.I. Introducion

When policy makers debate trade liberalization, the worry is not over the aggregate increase in trade, but over the unequal impact of freer trade across industries: Which industries will expand and which will contract? When policy makers have turned to economic models for answers to these questions — most notably during the lead up to the North American Free Trade Agreement — they have been given industry-level forecasts (USITC, 1992) that were largely inaccurate (Kehoe, 2005). Can we improve our ability to forecast the industry-level impacts of trade policy?

In the last 20 years, several important advancements in trade theory have revolved around the idea that trade liberalization not only brings about trade in products already being traded, but trade in new kinds of products as well — what we call the extensive trade margin. Less work, however, has been done in incorporating these insights into models

[13] Joint work with Timothy J. Kehoe (University of Minnesota, Federal Reserve Bank of Minneapolis, and National Bureau of Economic Research) and Kim J. Ruhl (Stern School of Business, New York University). This chapter has been published in the Journal of International Economics (July 2015), Volume 96, Issue 2, under the same title, and appears in this dissertation with permission from Elsevier.

that can be used to predict the impacts of trade liberalization for use in policy analysis[14]. In this chapter, we show that a very simple predictive model that incorporates the extensive trade margin performs quite well — beating several workhorse models — in accounting for the industry-level response of trade following the North American Free Trade Agreement (NAFTA).

Our methodology is based on the finding in Kehoe and Ruhl (2013) that products that were not traded or were traded very little before liberalization, what we call the *least traded products*, grow faster than the relatively heavily traded products following trade liberalization. Our model posits that industries — a collection of products — with relatively more of these least traded products will grow faster than industries with relatively fewer least traded products. Our model can be written as a linear function with two parameters, and we show how to find these two parameters using cross-sectional variation in trade data.

We evaluate our model by "forecasting" the industry-level effects of NAFTA using only data that would have been available in 1989 — several years before the implementation of NAFTA. We compare our forecasts with the actual growth in trade that occurred from 1989 to 2009 and find that the model does quite well: the weighted correlation between our forecasts and the data averages 0.39 across all six NAFTA country pairs. This result is even more striking when we compare our forecasts with those from general equilibrium models actually used to forecast the effects of NAFTA, whose

[14] A notable exception is Yaylaci and Shikher (2014), which uses a model based on Eaton and Kortum (2002) to make predictions about the Korea-U.S. free trade agreement. We discuss their forecasts, and compare them to our own forecasts of the Korea U.S.-free trade agreement, in the appendix.

weighted correlation with the data averages 0.00. We repeat this exercise using alternative measures of accuracy, and the conclusion remains the same. Our simple statistical model performs substantially better.

The failure of the general equilibrium models so often used in policy analysis is driven by the underlying structure that typically does not allow for an extensive margin. Models built from Armington (1969) assumptions about national product differentiation imply that countries already import all the available products from other countries. This leaves no role for products with little or no trade to have an impact on the model outcomes. Instead, the response to trade liberalization in these models is set by home bias parameters and elasticities of substitution between products.

In the workhorse heterogeneous agent trade models — variants of Eaton and Kortum (2002) and Melitz (2003) — the extensive trade margin is made up of goods that go from zero trade to positive trade following liberalization. In our model, a product with zero trade in the base year will not show up in the share of least traded products in an industry, so products with no trade in the base year do not factor into our predictions. What matters for our model are the products that are traded in small amounts before liberalization. This means that these workhorse trade models are not the theoretical analogue of our statistical model. Instead, a model like that in Arkolakis (2010), in which small firms react more to falling trade costs, is appropriate. Similar ideas can be found in Eaton et al. (2014) or Ruhl and Willis (2013), which focus on the growth of small firms growing larger in export markets.

In the following section, we describe our methodology and show how to estimate the parameters of the model using only pre-liberalization data. In section 3, we use the model to "forecast" the effects of NAFTA on industry-level exports, and we evaluate our forecasts using the observed changes in trade flows. In section 4, we compare our forecasts to those from general equilibrium models that were actually used to forecast NAFTA in the 1990s, and we show that our simple model outperforms the

### 3.II. A Predictive Model Based on the Extensive Margin

In this section, we develop a methodology based on the insight from Kehoe and Ruhl (2013): much of the growth in trade following a trade liberalization occurs within the set of products that were previously not traded or were traded very little. We refer to growth in trade from products that were not previously traded or were traded very little as growth on the *extensive margin* or the *new products margin*. We refer to growth in trade from products that were previously traded in relatively large amounts as growth on the *intensive margin*. Our methodology, based on that of Kehoe and Ruhl (2013), allows the cutoff for what products we consider to be least traded to vary across country pairs in order to take into account the relative importance of each product for a country's trade.

We define a *product* to be a 5-digit SITC Rev. 2 code. We sort all of the products from lowest to highest by their average value of trade over the first three years in our sample. (We average over three years to minimize the measure's dependence on any particular year.) Starting with the products with the least trade in the first three years, we then sum the value of trade in the base year until we accumulate a set of products that accounts for 10 percent of total trade in the base year. If a product is in that set, we classify it as a *least traded product*. In the appendix, we show that our results are robust to using 5 percent or 20 percent of total trade as the cutoff instead of 10 percent. Within the set of least traded products are products from different industries, where an industry is a collection of products. Adapting a concordance developed by Muendler (2009), we map each of the 1,836 5-digit SITC products into one of 37 3-digit ISIC industries. In what follows, we use the industry classification system from Brown, Deardorff, and Stern (1995) to keep our results comparable to theirs. This classification system is a more aggregated version of the 3-digit ISIC.

Once we have mapped the products to industries, we can compute the share of trade in each industry that is accounted for by least traded products within the industry. How prevalent are these least traded products across industries? Consider the Canada-U.S. trade relationship before NAFTA, which we study in section 3 as way to evaluate our methodology. In table 3.4 we report the fraction of trade in an industry accounted for by the least traded products in 1989, the year before NAFTA was implemented. There are substantial differences across industries. For example, least traded products made up 77 percent of total textile exports from Canada to the United States in 1989, but only 1 percent of exports in the wood products industry.

How is the share of least traded products in an industry related to the growth in trade in that industry following liberalization? Kehoe and Ruhl (2013) show that growth in least traded products can be explosive after liberalization, so it follows that industries with more least traded products would be expected to grow faster after liberalization than industries with fewer least traded products. Our prediction is that industries with higher shares of least traded products will experience more growth than industries with lower shares of least traded products.

We formulate our model of trade growth by industry as a simple linear function of the share of exports accounted for by least traded products in that industry. Specifically, we predict that the growth between periods $T_0$ and $T_1$ in industry $j$ will be

$$z_{ij}^k = (1 - s_{ij}^k)\alpha_i^k + s_{ij}^k(\alpha_i^k + \beta_i^k), \tag{3.1}$$

where $z_{ij}^k$ is the growth in exports, $x_{ij,t}^k$, from country $i$ to country $k$ in industry $j$ deflated by the growth in GDP, $y_{it}$, of the exporting country,

$$z_{ij}^k = 100\left(\frac{x_{ijT_1}^k / y_{iT_1}}{x_{ijT_0}^k / y_{iT_0}} - 1\right), \tag{3.2}$$

$s_{ij}^k$ is the share of exports accounted for by least traded products in that industry, and $\alpha_i^k$ and $\beta_i^k$ are constants. Here $\alpha_i^k$ is the average growth rate of non–least traded products, and $\beta_i^k$ is the additional growth generated by least traded products.

Notice that as long as $\beta_i^k > 0$, all values of $\alpha_i^k$ and $\beta_i^k$ give the same predictions for the relative ordering of growth across industries. This means that any series of predictions by industry of the form (3.1) generates the same correlation with a series of observations by industry if $\beta_i^k$ is positive. Therefore, correlations offer a way of evaluating the general merit of our simple statistical model in a way that does not depend on its particular parameterization. If the correlation was low, that would indicate there is little hope for success regardless of the parameterization[15]. As we will show, however, our predictions

[15] In addition to using correlation coefficients to evaluate the model, we consider other metrics in sections 3 and 4.

perform much better in terms of correlation with observed changes than the general equilibrium models originally used to predict the effects of NAFTA. In the following sections, we lay out our methodology for parameterizing (3.1) and show that, as indicated by these correlations, our methodology does indeed deliver substantially improved industry-level predictions in the case of NAFTA.

### 3.II.A. Parameterization

We need to choose values for $\alpha_i^k$ and $\beta_i^k$ in order to use (3.1) to predict industry-level trade growth. Given our interest in industry-level outcomes, we develop a method to parameterize the model so that its aggregate predictions are consistent with the aggregate data. In particular, we require that $\alpha_i^k$ and $\beta_i^k$ generate aggregate predictions that match the two features of the data described below.

First, the predicted total trade growth of our model must match the total trade growth predicted by the gravity equation. This condition is

$$\alpha_i^k + 0.1\beta_i^k = \hat{z}_i^k, \tag{3.3}$$

where the left-hand side of (3.3) is our model's prediction of total trade growth (the 0.1 follows from the 10 percent threshold in the least traded products definition), and the right-hand side is the predicted trade growth from the gravity equation, which we specify below. The motivation for this restriction is straightforward: we would like our industry-level predictions to be consistent with an aggregate trade prediction, and the gravity equation is a simple and effective way to generate an aggregate trade prediction.

We require an elasticity of aggregate trade growth with respect to tariffs. One approach would be to simply use the elasticities reported in the literature, a survey of which can be found in Head and Mayer (2013). This approach takes advantage of the econometric sophistication of modern gravity equation research, but does not allow us to control the samples from which the estimates arise. In particular, we demand that only pre-liberalization data are used in estimating our model. For our purposes, we specify a simpler gravity equation, but one that allows for the primary determinants found in the gravity literature,

$$\log \frac{x_i^k}{y_i} = \lambda_\tau \log\left(1+\tau_i^k\right) + \lambda_2 \log y_i + \lambda_3 \log y_k + \lambda_4 \log d_i^k + \lambda_{10} + \varepsilon_i^k , \tag{3.4}$$

where $x_i^k$ is exports from country $i$ to country $k$, $\tau_i^k$ is the average tariff rate, $y_i$ is GDP of the exporting country, $y_k$ is GDP of the importing country, and $d_i^k$ is the distance between countries.[16] We also provide estimates where we include variables indicating whether the countries share a border, common language, or colonial relationship. With an estimate of $\lambda_\tau$, we can predict the increase in trade corresponding to a decrease in tariffs from $\tau_i^k$ to $\tau_i^{k\prime}$ as

$$\hat{z}_i^k = \exp\left(-\lambda_\tau \left[\log(1+\tau_i^k) - \log(1+\tau_i^{k\prime})\right] - 1\right) \times 100 . \tag{3.5}$$

[16] Our gravity model is admittedly simple. We work with this specification because the data needed for our NAFTA exercise in the next section are severely limited by the tariff data. This restricts the number of observations, making it difficult to include, for example, country-level fixed effects to control for multilateral resistance. Despite these limitations, our model produces an estimated trade elasticity in the range of estimates given by Simonovska and Waugh (2014), who use a more state-of-the-art approach.

Our gravity equation only takes into account tariff reductions when predicting changes in trade. Trade liberalizations, however, often include not just reductions in bilateral tariffs, but reductions in non-tariff trade barriers as well. Measuring these nontariff barriers is a difficult task that is outside the scope of this study. See Baier and Bergstrand (2007) and Head and Mayer (2013) for discussions about measuring the impact of free trade agreements using binary variables.

Our second restriction requires that the model's predicted aggregate growth in least traded products, as a share of total trade growth, be consistent with the cross-country evidence. This restriction can be written as

$$\frac{\alpha_i^k + \beta_i^k}{\alpha_i^k + 0.1\beta_i^k} = \gamma, \tag{3.6}$$

where $\gamma$ is the cross-country average of the ratio of the growth in trade of least traded products to overall growth in trade. Following Kehoe and Ruhl (2013), $\gamma$ is estimated from

$$\Delta_t x_i^k = \gamma \Delta_t \tilde{x}_i^k + \varepsilon_i^k, \tag{3.7}$$

where $x_i^k$ is total exports from $i$ to $k$, $\tilde{x}_i^k$ is exports of least traded products, and $\Delta_t$ is the 10-year growth rate operator. This equation is estimated using data on as many bilateral pairs as possible, but using a time period (to compute growth rates) that ends before our period of interest, $T_0$.

In (3.3) and (3.6), the left-hand side quantities are predictions of our model, and the right-hand side quantities are independent data moments. Given values of $\gamma$ and $\hat{z}_i^k$, we solve the system of equation defined by (3.3) and (3.6) for $\alpha_i^k$ and $\beta_i^k$. We are then ready to use (3.1) to make predictions about industry-specific growth rates.

### 3.III. Evaluating the Methodology: NAFTA

In this section, we evaluate the predictive power of our methodology by using it to "predict" the impact of NAFTA. We select 1989 as our base year and we use 2009 as our endpoint, since that is the year of full implementation of NAFTA. Our results are robust to selecting 2007 as our endpoint in order to avoid entangling the effects of NAFTA with the effects of the 2008–2009 recession and the fall in trade that accompanied it.

To parameterize the NAFTA model, we follow the procedure laid out in section 2.1, and we restrict our estimation to use only data that would have been available in 1989. We estimate the gravity equation in (3.4) using 1989 data (results reported in table 3.1) and find $\lambda_\tau = -2.76$. This value is somewhat lower than the range of trade elasticities reported in the gravity literature, as surveyed in Head and Mayer (2013). This discrepancy appears to be largely due to our use of earlier data. If we reestimate the gravity equation using 2005 data, we estimate a trade elasticity very much in line with the carefully constructed estimates of Simonovska and Waugh (2014). We use (3.7) to estimate $\gamma$ using 1978–1987 data and find that $\gamma = 3.65$, which is nearly identical to the estimate in Kehoe and Ruhl (2013), which uses 1995–2005 data. Table 3.2 reports the average initial tariffs, the predicted aggregate changes in trade based on our gravity model, and the estimated $\alpha_i^k$ and $\beta_i^k$ that we use with equation (3.1) to make our NAFTA predictions for the United States, Canada, and Mexico. Tariffs for Mexican imports are unavailable for 1989, so we use Mexican tariffs in 1991 as the initial tariff levels.

The predictions for the NAFTA country pairs are reported in tables 3.4–3.6. For each country pair we report, by industry: the observed growth rate from the data, 1989–2009; the least traded products' share of trade in 1989; and the predictions from our least traded products model (LTP). In what follows, we evaluate our predictions against both the observed export growth rates and the predictions from general equilibrium models used to forecast NAFTA export growth.

### 3.III.A. Evaluating the Parameterization Methodology

We have estimated $\left(\alpha_i^k, \beta_i^k\right)$ for the NAFTA countries following the procedure in section 2.1, which uses only data that were available in 1989. Using the growth in trade that actually occurred following NAFTA, we can find the best possible coefficients given our statistical model's form and compare them with our estimates. These optimal coefficients are the solution to

$$\tilde{\alpha}_i^k, \tilde{\beta}_i^k = \arg\min \sum\nolimits_{j=1}^{38} \omega_{ij}^k \left(\alpha_i^k + \beta_i^k s_{ij}^k - z_{ij}^{k,data}\right)^2, \tag{3.8}$$

where $\omega_{ij}^k$ is industry $j$'s share of total exports from country $i$ to $k$ in 1989 and $s_{ij}^k$ is the share of least traded products in each industry. The observed industry-level growth rate is $z_{ij}^{k,data}$. The optimal coefficients are reported in table 3.2. Take, for example, trade between Canada and the United States: setting $\alpha_{usa}^{can} = -34.54$ and $\beta_{usa}^{can} = 175.84$ would have been the best possible linear prediction based on the least traded products data for U.S. exports to Canada, indicating that between 1989 and 2009, exports in the least traded set grew by 141.3 ($=-34.54+175.84$) percent more than U.S. GDP, whereas other exports grew 34.54 percent less than U.S. GDP.

In table 3.2, we compare the optimal coefficients with our estimated coefficients that we used to make our NAFTA predictions. Overall, we find a correlation of 0.65 for $\tilde{\alpha}_i^k$ and $\alpha_i^k$ and a correlation of 0.63 for $\tilde{\beta}_i^k$ and $\beta_i^k$. These differences indicate that, while there remains room for improvement, our methodology for estimating $\alpha_i^k$ and $\beta_i^k$ performs well.

**3.III.B. Controlling for Tariff Size**

A potential concern is that larger growth rates for least traded products may be driven by larger decreases in tariffs. As evident in table 3.3, the least traded products are indeed subject to larger initial tariffs, and larger declines in tariffs, when compared with non–least traded products.[17] Also clear from the table is that our assumption that a free trade agreement means the complete elimination of tariffs is a good approximation of NAFTA. To investigate whether accounting for tariffs eliminates the finding that least traded products grow faster following trade liberalization, we estimate a modified version of (3.8) that incorporates tariff data,

$$\tilde{\alpha}_i^k, \tilde{\beta}_i^k = \arg\min \sum_j \omega_{ij}^k \left(\alpha_i^k + \beta_i^k s_{ij}^k + \zeta_i^k \tau_{ij}^k - z_{ij}^{k\,data}\right)^2, \tag{3.9}$$

[17] We use 1991 tariff data for Mexican imports because tariff data are not available in 1989. We exclude the code 02242 (Dry Milk) for Canadian exports to Mexico as an outlier, because it experiences a tariff rate increase from 1.05 to 1.37, which upwardly skews the average tariff rate for non–least traded products.

where $\tau_{ij}^{k}$ is the average tariff rate in 1989.

The resulting estimates for $\tilde{\alpha}_i^k$ and $\tilde{\beta}_i^k$ are reported in table 3.3 and show that accounting for tariffs does not have a significant impact on the degree to which least traded products grow more than non–least traded products. The only exception is for Mexican exports to the United States, where our estimate of $\tilde{\beta}_i^k$ falls significantly when estimated using industry-level data. The correlation between the coefficients estimated in (3.8) and the coefficients that control for initial tariff rates is −0.53 for $\tilde{\alpha}_i^k$ and 0.62 for $\tilde{\beta}_i^k$. This indicates that while tariffs may explain much of the average growth in trade across all products, differences in tariffs do not appear to be responsible for the higher growth rates of least traded products.

There is room for future research to investigate other ways in which the new products margin may interact with other traditional determinants of trade flows. For example, it may be that least traded products exhibit more growth when an exporter exhibits a revealed comparative advantage in those products.

### 3.IV. Predictions of NAFTA Models

In the previous section, we compared our results with the best possible linear forecasting model. In this section, we compare our results with those from general equilibrium models that were used to forecast the effects of NAFTA. To develop a baseline for judging our predictions for NAFTA, we follow Fox (1999) and Kehoe (2005) and evaluate the performance of one of the most prominent of the models built to analyze NAFTA, the Brown-Deardorff-Stern (BDS) model (Brown 1992, 1994; Brown, Deardorff, and Stern, 1992, 1995; Brown and Stern, 1989). In this section, we compare the predictions made by the BDS model and our forecasting model with the observed growth in trade following NAFTA. In the appendix, we perform similar comparisons for two alternative models of NAFTA.

### 3.IV.A. The BDS Model and the Data

The BDS model made predictions at the industry level, where each of their 23 industries is defined as an aggregate of ISIC 3-digit industries: this is the classification we adopted in section 3, in order to make our model comparable with the BDS model. We compute the percentage growth in exports for each industry deflated by GDP growth in accordance with (3.2). We report the industry-level export growth rates from the data and the predictions of the BDS model in tables 3.4–3.6.

We use two criteria to compare the predictions of the BDS model with the data: the weighted correlation coefficient between the model predictions and the data, where the weights are the 1989 trade volumes, and the estimated coefficients $a_i^k$ and $b_i^k$ from the regression

$$\min_{a_i^k, b_i^k} \sum_{j=1}^{23} \omega_{ij}^k \left( a_i^k + b_i^k z_{ij}^{k,model} - z_{ij}^{k,data} \right)^2, \tag{3.10}$$

where, again, $j$ indexes the industry. The deviation of the coefficient $b_i^k$ from 1 indicates how poorly the model does in predicting the signs and the absolute magnitudes of the changes in the data. In particular, $b_i^k$ tells us whether the differences across industries are underpredicted $(b_i^k > 1)$, overpredicted ($0 < b_i^k < 1$), or predicted in the wrong direction ($b_i^k < 0$).

We report the estimated coefficients from (3.10) for the NAFTA pairs at the bottom of tables 3.4–3.6. Take, for example, exports from Canada to the United States, which we report in table 3.4. The BDS model does a poor job of predicting Canadian exports to the United States: the weighted correlation between the model's predictions and the data is negative (−0.28), and the coefficients from (3.10) that come closest to the data involve multiplying all of the predicted growth rates by −3.33 and adding 21.82.

### 3.IV.B. The BDS Model and Our Predictions

Using the methodology laid out in section 2.1, we form our predictions for the NAFTA countries and report our results in tables 3.4–3.6 alongside the results from the BDS model. We find that our predictions based on the initial fraction of least traded products considerably outperform the BDS model for each country pair. Returning to the example of Canadian exports to the United States (see table 3.4), we see that our predictions perform better, with a weighted correlation of 0.39, and the linear function of the prediction that comes closest to the data involving multiplying all of the predicted growth rates by 3.53 and subtracting 50.21. That our predictions significantly outperform the predictions in BDS for Canadian exports to the United States can be seen in figures 1 and 2, which plot the regression lines resulting from (3.10), where the size of each data point corresponds to that industry's weight.

Table 3.7 summarizes our findings and reports the corresponding statistics for all six of the bilateral North American country pairs. Notice that the BDS model had almost no predictive power for the impact of NAFTA by industry. When we estimate (3.10) pooling the data for all six pairs, the coefficient $b$ put on the predictions of the BDS model is 0.17, and when we allow $b$ to differ by country pair, the weighted average is −0.94 compared with 2.72 and 3.53 for our predictions.

Table 3.8 compares the overall accuracy of our predictions with the accuracy of the BDS predictions. In this table we compare the average absolute percentage difference between the predictions and the actual growth over 1989–2009:

$$\chi_i^k = \frac{1}{23}\sum_{j=1}^{23}\left\|\frac{z_{ij}^k - \hat{z}_{ij}^k}{\hat{z}_{ij}^k}\right\|. \tag{3.11}$$

Our predictions significantly outperform the BDS predictions — by an order of magnitude in most cases. We also compare the accuracy of "predictions" based on the optimal coefficients from (3.8). Although these are not true predictions since they make use of ex-post data, they establish an upper bound of how useful predictions of the form (3.1) can be. We see that the best fit estimates perform very well, especially for bilateral trade involving Mexico. This suggests that there are large potential gains in accuracy from an improved methodology for estimating $\alpha_i^k$ and $\beta_i^k$.

It is worth stressing that this failure of the BDS model is not specific to this particular model. We focus on the BDS model because it is a widely used and well-documented model built to analyze the impact of NAFTA, and it has predictions for all directions of bilateral trade between Mexico, Canada, and the United States. Kehoe (2005) argues that two other models that were very prominent in policy discussions of NAFTA, the Cox-Harris model of Canada (Cox, 1994, 1995; Cox and Harris 1985, 1992a, 1992b) and the Sobarzo model of Mexico (Sobarzo, 1992a, 1992b, 1994, 1995), also perform poorly in this sort of exercise. In the appendix, we show that we achieve similar results with the Sobarzo model and the Cox-Harris model as well. It is also important to note that the sorts of models used to analyze NAFTA are still being employed to analyze trade policies around the world, so understanding the limitations of this class of models remains important. See, for example, Brown, Kiyota, and Stern (2005), Ciuriak and Chen (2007), DeRosa and Gilbert (2004), Francois, Rivera, and Rojas-Romagosa (2008), Lips and Rieder (2005), U.S. International Trade Commission (2004), as well as Kiyota and Stern (2007). In the appendix, we show how our model's predictions compare to Kiyota and Stern (2007), which forecasts the effects of the Korea-U.S. free trade agreement — a liberalization that is currently underway. Our predictions are substantially different than theirs, but we cannot, however, perform an ex-post analysis of the models until sometime in the future.

Why do these models have difficulty explaining industry trade patterns after liberalization? As Kehoe (2005) explains, the models used to predict the impact of NAFTA could not pick up increases in exports on the extensive margin, or new products margin, because of the assumptions made in these models. In particular, the sorts of Armington aggregators and Dixit-Stiglitz utility functions used in these models, along with no fixed costs of exporting, allowed only increases on the intensive margin.

### 3.IV.C. Discussion of Results

To get some idea of what drives our results, we examine an industry where the simple least traded products exercise does better than the BDS model: Canadian exports of chemicals to the United States, which grew 99.6 percent while the BDS model predicted −3.1 percent. The disaggregated data for this industry show that the chemicals industry is made up of 318 5-digit SITC categories. Of the 318 categories, 296 are least traded Canadian exports. Compared with Canadian GDP, the least traded chemicals increased by 187 percent, whereas the other non–least traded chemicals increased by only 47 percent. The growth in least traded products is far from uniform: for example, exports of code 51571 (Sulphonamides) increased by 3,424 percent more than Canadian GDP, 58241 (Polyamides in primary forms) increased by 4 percent, and 58241 (Chlorine) decreased by 28 percent compared with Canadian GDP.

Products that report zero trade in 1989 are classified as least traded products, and if they report positive trade in 2009, that trade is counted toward the growth rate for least traded products. Notice, however, that the number of zero-traded products has no influence on our shares $s_i$ of least traded products in each industry in 1989. In the appendix we show that our results are unchanged even when we completely ignore growth from products that report zero trade in 1989. This means that the essential products in terms of generating any predictive power from our exercise are not products reporting zero trade, but products that are positively traded, although with very small amounts of trade. Arkolakis (2010) shows that the importance of products with small, nonzero trade to overall trade growth can be explained by marketing costs and the number of consumers that purchase a product. This additional margin for growth is diminishing for products with large amounts of trade and causes products with small yet positive trade to experience higher levels of growth.

### 3.V. Conclusions

This chapter provides a methodology for predicting changes in bilateral trade at the industry level following a trade liberalization. We evaluate our methodology in the context of NAFTA and show that our methodology — which exclusively focuses on least traded products — would have yielded better predictions than the general equilibrium models employed at the time. Our results suggest that researchers should include the new products margin in any analysis of the impact of trade reform. We hope this finding will spur the development of models that are consistent with the expansion of trade on the new products margin so that we can improve our ability to predict the effects of trade reforms and so that we can perform counterfactual analyses of alternative reforms.

**Table 3.1**

**Gravity Equation Estimates**

| **Variable** | **1989** | **1989** | **2005** | **2005** |
|---|---|---|---|---|
| Initial Tariffs | −2.755 | −2.085 | −3.511 | −3.371 |
| | (0.454) | (0.617) | (0.411) | (0.440) |
| Exporter GDP | 1.191 | 1.112 | 1.252 | 1.215 |
| | (0.026) | (0.032) | (0.011) | (0.014) |
| Importer GDP | 0.743 | 0.690 | 0.986 | 0.994 |
| | (0.040) | (0.048) | (0.013) | (0.015) |
| Distance | −1.135 | −0.987 | −1.520 | −1.312 |
| | (0.098) | (0.105) | (0.034) | (0.036) |
| Exporter GDP per Capita | | 0.260 | | 0.093 |
| | | (0.049) | | (0.020) |
| Importer GDP per Capita | | 0.100 | | −0.032 |
| | | (0.078) | | (0.023) |
| Border | | 1.108 | | 1.208 |
| | | (0.382) | | (0.168) |
| Common Language | | 0.820 | | 0.988 |
| | | (0.162) | | (0.065) |
| Colonial Relationship | | −0.517 | | 0.638 |
| | | (0.270) | | (0.143) |
| Constant | 6.173 | 3.164 | 4.413 | 2.177 |
| | (1.101) | (1.271) | (0.334) | (0.383) |
| Observations | 1,280 | 1,280 | 9,833 | 9,833 |
| R-squared | 0.641 | 0.660 | 0.640 | 0.651 |

Notes: Standard errors are reported in parentheses. Bilateral aggregate trade data from UN Comtrade. *Initial tariff* is the effectively applied average tariff rates obtained from the TRAINS (Trade Analysis and Information System) database accessed through WITS (World Integrated Trade Solution). All other gravity variables are from the CEPII Gravity Dataset from Head, Mayer, and Ries (2010) and Head and Mayer (2013). *Common language* is equal to one if a common language is spoken by at least 9 percent of the population. *Colonial relationship* is equal to one if a colonial relationship has ever existed. Sample includes all countries with data for all variables in the given year.

## Table 3.2

### Estimated and Optimal Coefficients, NAFTA

| Exporter | Importer | Initial Tariff | Predicted Growth | Estimated | | Optimal | |
|---|---|---|---|---|---|---|---|
| | | | | $\alpha$ | $\beta$ | $\tilde{\alpha}$ | $\tilde{\beta}$ |
| CAN | MEX | 13.85 | 42.96 | 30.31 | 126.54 | 254.23 | 4468.37 |
| CAN | USA | 4.26 | 12.18 | 8.59 | 35.88 | –20.42 | 185.24 |
| MEX | CAN | 7.27 | 21.33 | 15.05 | 62.83 | 115.16 | 286.39 |
| MEX | USA | 5.62 | 16.26 | 11.47 | 47.89 | 51.52 | 77.54 |
| USA | CAN | 7.81 | 23.02 | 16.24 | 67.81 | –34.54 | 175.84 |
| USA | MEX | 13.70 | 42.44 | 29.94 | 125.01 | 62.31 | 265.44 |
| **Correlation with Estimated Coefficients** | | | | | | 0.65 | 0.63 |

Growth in trade is measured as the change in trade deflated by the GDP growth rate. The estimated coefficients are the solution to (3) and (6). The optimal coefficients are the solution to (8).

**Table 3.3**

**Tariff Rates by Product Type, NAFTA**

| | | Average Tariff Rates (Percent) | | | | Optimal Coefficients | |
|---|---|---|---|---|---|---|---|
| | | Least Traded Products | | Non-Least Traded Products | | Conditional On Initial Tariffs | |
| **Exporter** | **Importer** | **1989** | **2009** | **1989** | **2009** | $\tilde{\alpha}$ | $\tilde{\beta}$ |
| CAN | MEX | 13.22 | 0.00 | 6.29 | 0.00 | −306.08 | 3890.50 |
| CAN | USA | 2.62 | 0.05 | 0.78 | 0.02 | 23.57 | 273.88 |
| MEX | CAN | 6.20 | 0.09 | 4.17 | 0.00 | 101.94 | 354.89 |
| MEX | USA | 4.73 | 0.01 | 4.11 | 0.00 | 36.71 | 3.86 |
| USA | CAN | 6.30 | 0.65 | 5.57 | 0.18 | −32.78 | 111.59 |
| USA | MEX | 12.99 | 0.21 | 11.80 | 0.16 | 61.61 | 241.03 |
| **Correlation with Estimated Coefficients** | | | | | | −0.53 | 0.62 |

The optimal coefficients conditional on initial tariff rates are the solution to (9). The correlation with estimated coefficients uses the estimated coefficients given in table 2. These optimal coefficients are computed using 3-digit ISIC codes as industries instead of the industry classification from Brown, Deardorff, and Stern (1995) that is used elsewhere in the paper. We do this because WITS provides tariff data directly at the 3-digit ISIC level.

## Table 3.4

## Changes in Canada-U.S. Industry-level Trade

| Industry | Canada to United States: Data | BDS | Least Traded Share | LTP | United States to Canada: Data | BDS | Least Traded Share | LTP |
|---|---|---|---|---|---|---|---|---|
| Agriculture | 12.5 | 3.4 | 0.26 | 17.8 | −6.4 | 5.1 | 0.19 | 28.8 |
| Mining and Quarrying | 237.6 | 0.4 | 0.05 | 10.4 | 51.3 | 1.0 | 0.16 | 26.8 |
| Food | 101.2 | 8.9 | 0.24 | 17.0 | 124.1 | 12.7 | 0.25 | 33.3 |
| Textiles | 42.4 | 15.3 | 0.77 | 36.0 | −35.9 | 44.0 | 0.52 | 51.8 |
| Clothing | 50.2 | 45.3 | 0.59 | 29.6 | −3.0 | 56.7 | 1.00 | 84.1 |
| Leather Products | −67.7 | 11.3 | 1.00 | 44.5 | −64.0 | 7.9 | 0.61 | 57.5 |
| Footwear | −49.9 | 28.3 | 1.00 | 44.5 | −67.2 | 45.7 | 0.34 | 39.6 |
| Wood Products | −54.5 | 0.1 | 0.01 | 9.0 | −30.6 | 6.7 | 0.07 | 20.7 |
| Furniture and Fixtures | −46.6 | 12.5 | 0.00 | 8.6 | 22.5 | 35.6 | 0.00 | 16.2 |
| Paper Products | −65.9 | −1.8 | 0.04 | 10.1 | 13.7 | 18.9 | 0.15 | 26.1 |
| Printing and Publishing | 0.7 | −1.6 | 0.12 | 12.9 | −19.6 | 3.9 | 0.05 | 19.6 |
| Rubber Products | 45.8 | 9.5 | 0.10 | 12.0 | 30.2 | 19.1 | 0.05 | 19.9 |
| Chemicals | 99.6 | −3.1 | 0.38 | 22.1 | 50.2 | 21.8 | 0.24 | 32.8 |
| Petroleum Products | −79.8 | 0.5 | 0.07 | 11.2 | −43.1 | 0.8 | 0.13 | 25.3 |
| Glass Products | −45.7 | 30.4 | 0.40 | 22.9 | −20.0 | 4.4 | 0.23 | 31.6 |
| Nonmetal Minerals | −0.4 | 1.2 | 0.38 | 22.4 | −1.9 | 11.9 | 0.59 | 56.1 |
| Iron And Steel | −12.7 | 12.9 | 0.36 | 21.5 | 53.5 | 11.6 | 0.28 | 35.2 |
| Nonferrous Metals | −20.9 | 18.5 | 0.07 | 11.0 | −20.8 | −6.7 | 0.11 | 23.6 |
| Metal Products | 17.7 | 15.2 | 0.20 | 15.7 | −5.3 | 18.2 | 0.16 | 27.0 |
| Nonelectrical Machinery | −8.4 | 3.3 | 0.21 | 16.0 | −38.9 | 9.9 | 0.08 | 21.5 |
| Electrical Machinery | −16.4 | 14.5 | 0.15 | 13.9 | −42.6 | 14.9 | 0.05 | 19.7 |
| Transportation Equip. | −44.3 | 10.7 | 0.01 | 8.8 | −37.8 | −4.6 | 0.01 | 16.6 |
| Misc. Manufactures | 56.1 | −2.1 | 0.45 | 24.8 | −19.2 | 11.5 | 0.15 | 26.6 |
| **Weighted Corr. with Data** | | −0.28 | | 0.30 | | 0.39 | | 0.54 |
| **Regression Coefficient $a$** | | 21.82 | | −64.78 | | –26.62 | | −76.65 |
| **Regression Coefficient $b$** | | −3.33 | | 5.16 | | 1.34 | | 2.59 |
| **BDS-LTP Weighted Correlation** | | | | −0.11 | | | | 0.70 |

The column *Data* reports the growth rates of industry exports deflated by the exporter's GDP growth rate, 1989−2009. The column *BDS* reports the predictions from Brown, Deardorff, and Stern (1995). The column *Least traded share* reports the share of industry exports accounted for by least traded products in 1989. The column *LTP* reports the predictions from the least traded products model in (1). The regression coefficients $a$ and $b$ are the solution to (10).

## Table 3.5

### Changes in Canada-Mexico Industry-level Trade

| Industry | Canada to Mexico: Data | BDS | Least Traded Share | LTP | Mexico to Canada: Data | BDS | Least traded share | LTP |
|---|---|---|---|---|---|---|---|---|
| Agriculture | 410.8 | 3.1 | 0.04 | 34.9 | 105.5 | −4.1 | 0.11 | 22.0 |
| Mining and Quarrying | 6.9 | −0.3 | 0.03 | 33.8 | 77.8 | 27.3 | 0.03 | 16.8 |
| Food | 181.2 | 2.2 | 0.02 | 33.0 | 175.3 | 10.8 | 0.22 | 28.9 |
| Textiles | 656.4 | −0.9 | 0.49 | 92.2 | −39.2 | 21.6 | 0.29 | 33.1 |
| Clothing | 3553.9 | 1.3 | 1.00 | 156.8 | 703.5 | 19.2 | 1.00 | 77.9 |
| Leather Products | 165.1 | 1.4 | 1.00 | 156.8 | 71.5 | 36.2 | 1.00 | 77.9 |
| Footwear | 23.6 | 3.7 | 1.00 | 156.8 | −41.2 | 38.6 | 0.15 | 24.5 |
| Wood Products | 16636.0 | 4.7 | 0.97 | 153.4 | 419.1 | 15.0 | 1.00 | 77.9 |
| Furniture and Fixtures | 12913.0 | 2.7 | 1.00 | 156.8 | 1402.1 | 36.2 | 0.01 | 15.9 |
| Paper Products | 214.7 | −4.3 | 0.04 | 34.9 | 46.1 | 32.9 | 0.14 | 24.0 |
| Printing And Publishing | 1887.8 | −2.0 | 1.00 | 156.8 | 2412.5 | 15.0 | 1.00 | 77.9 |
| Rubber Products | 3185.0 | −1.0 | 0.23 | 59.6 | 1416.2 | −6.7 | 1.00 | 77.9 |
| Chemicals | 1249.4 | −7.8 | 0.25 | 61.5 | 272.7 | 36.0 | 0.91 | 72.5 |
| Petroleum Products | 489.2 | −8.5 | 1.00 | 156.8 | - | - | - | - |
| Glass Products | 519.7 | −2.2 | 1.00 | 156.8 | −13.0 | 13.3 | 0.10 | 21.2 |
| Nonmetal Minerals | 1497.6 | −1.8 | 1.00 | 156.8 | 143.8 | 5.7 | 0.45 | 43.2 |
| Iron and Steel | 190.2 | −15.0 | 0.02 | 33.1 | 52.3 | 19.4 | 1.00 | 77.9 |
| Nonferrous Metals | 442.0 | −64.7 | 0.07 | 39.4 | −50.9 | 138.1 | 0.07 | 19.7 |
| Metal Products | 2843.9 | −10.0 | 0.73 | 122.6 | 276.9 | 41.9 | 0.45 | 43.3 |
| Nonelectrical Machinery | 1360.5 | −8.9 | 0.19 | 55.0 | 124.0 | 17.3 | 0.05 | 18.1 |
| Electrical Machinery | 2293.0 | −26.2 | 0.23 | 59.7 | 263.7 | 137.3 | 0.08 | 20.1 |
| Transportation Equip. | 6352.2 | −4.4 | 0.27 | 64.2 | 119.3 | 3.3 | 0.00 | 15.3 |
| Misc. Manufactures | 409.9 | −12.1 | 0.18 | 53.0 | 523.4 | 61.1 | 0.55 | 49.4 |
| **Weighted Corr. With Data** | | −0.10 | | 0.55 | | 0.06 | | 0.33 |
| **Regression Coefficient $a$** | | 645.29 | | −815.91 | | 135.79 | | 46.57 |
| **Regression Coefficient $b$** | | −7.94 | | 35.31 | | 0.16 | | 4.56 |
| **BDS-LTP Weighted Correlation** | | | | −0.12 | | | | 0.02 |

The column *Data* reports the growth rates of industry exports deflated by the exporter's GDP growth rate, 1989−2009. The column *BDS* reports the predictions from Brown, Deardorff, and Stern (1995). The column *Least traded share* reports the share of industry exports accounted for by least traded products in 1989. The column *LTP* reports the predictions from the least traded products model in (1). The regression coefficients $a$ and $b$ are the solution to (10).

## Table 3.6

## Changes in Mexico-U.S. Industry-level Trade

| Industry | Mexico to United States: Data | BDS | Least Traded Share | LTP | United States to Mexico: Data | BDS | Least Traded Share | LTP |
|---|---|---|---|---|---|---|---|---|
| Agriculture | −20.1 | 2.5 | 0.07 | 14.8 | 46.6 | 7.9 | 0.10 | 41.9 |
| Mining and Quarrying | 27.0 | 26.9 | 0.01 | 12.0 | 86.2 | 0.5 | 0.18 | 52.0 |
| Food | 119.5 | 7.5 | 0.27 | 24.6 | 129.5 | 13.0 | 0.17 | 51.4 |
| Textiles | 89.6 | 11.8 | 0.72 | 45.9 | 125.7 | 18.6 | 0.43 | 84.0 |
| Clothing | 449.4 | 18.6 | 0.42 | 31.4 | 63.9 | 50.3 | 0.24 | 60.0 |
| Leather Products | −66.8 | 11.7 | 0.53 | 36.9 | 58.4 | 15.5 | 0.67 | 113.8 |
| Footwear | −62.1 | 4.6 | 0.03 | 12.9 | −58.5 | 35.4 | 0.10 | 42.3 |
| Wood Products | −74.8 | −2.7 | 0.12 | 17.0 | −21.6 | 7.0 | 0.09 | 41.2 |
| Furniture and Fixtures | 64.9 | 7.6 | 0.00 | 11.5 | 6.6 | 18.6 | 0.00 | 29.9 |
| Paper Products | −61.0 | 13.9 | 0.23 | 22.7 | 29.4 | −3.9 | 0.07 | 38.2 |
| Printing and Publishing | 212.3 | 3.9 | 1.00 | 59.4 | 194.9 | −1.1 | 0.13 | 46.0 |
| Rubber Products | 147.1 | −5.3 | 0.43 | 32.2 | 165.9 | 12.8 | 0.06 | 37.0 |
| Chemicals | 27.9 | 17.0 | 0.59 | 39.6 | 208.2 | −8.4 | 0.23 | 58.2 |
| Petroleum Products | −98.0 | 34.1 | 0.12 | 17.0 | −71.6 | −7.4 | 0.06 | 37.5 |
| Glass Products | 12.1 | 32.3 | 0.16 | 19.3 | 53.8 | 42.3 | 0.39 | 78.1 |
| Nonmetal Minerals | −19.5 | 3.7 | 0.26 | 24.1 | 57.8 | 0.8 | 0.57 | 100.7 |
| Iron And Steel | 18.5 | 30.8 | 0.28 | 24.7 | 84.0 | −2.8 | 0.24 | 60.4 |
| Nonferrous Metals | 53.8 | 156.5 | 0.12 | 17.3 | 104.6 | −55.1 | 0.12 | 44.7 |
| Metal Products | 80.4 | 26.8 | 0.30 | 25.8 | 84.7 | 5.4 | 0.14 | 47.1 |
| Nonelectrical Machinery | 171.3 | 18.5 | 0.14 | 18.3 | 102.8 | −2.9 | 0.09 | 41.3 |
| Electrical Machinery | 46.5 | 178.0 | 0.02 | 12.4 | 59.5 | −10.9 | 0.01 | 31.0 |
| Transportation Equip. | 127.0 | 6.2 | 0.02 | 12.6 | 79.3 | 9.9 | 0.02 | 32.5 |
| Misc. Manufactures | 92.8 | 43.2 | 0.24 | 23.0 | 96.6 | −9.4 | 0.13 | 46.1 |
| **Weighted Corr. with Data** | | −0.13 | | 0.17 | | −0.06 | | 0.47 |
| **Regression Coefficient $a$** | | 66.64 | | 32.95 | | 88.47 | | -1.26 |
| **Regression Coefficient $b$** | | −0.11 | | 1.62 | | −0.24 | | 2.12 |
| **BDS-LTP Weighted Correlation** | | | | −0.32 | | | | 0.21 |

The column *Data* reports the growth rates of industry exports deflated by the exporter's GDP growth rate, 1989−2009. The column *BDS* reports the predictions from Brown, Deardorff, and Stern (1995). The column *Least traded share* reports the share of industry exports accounted for by least traded products in 1989. The column *LTP* reports the predictions from the least traded products model in (1). The regression coefficients $a$ and $b$ are the solution to (10).

**Table 3.7**

**Measures of Model Fit, NAFTA**

| | | BDS Model Predictions | | | LTP Model Predictions | | |
|---|---|---|---|---|---|---|---|
| **Exporter** | **Importer** | **Correlation with Data** | ***a*** | ***b*** | **Correlation with Data** | ***a*** | ***b*** |
| CAN | MEX | −0.10 | 645.29 | −7.94 | 0.55 | −815.91 | 35.31 |
| CAN | USA | −0.28 | 21.82 | −3.33 | 0.30 | −64.78 | 5.16 |
| MEX | CAN | 0.06 | 135.79 | 0.16 | 0.33 | 46.57 | 4.56 |
| MEX | USA | −0.13 | 66.64 | −0.11 | 0.17 | 32.95 | 1.62 |
| USA | CAN | 0.39 | −26.62 | 1.34 | 0.54 | −76.65 | 2.59 |
| USA | MEX | −0.06 | 88.47 | −0.24 | 0.47 | −1.26 | 2.12 |
| **Weighted Average** | | −0.00 | 19.83 | −0.94 | 0.39 | −50.21 | 3.53 |
| **Pooled Regression** | | 0.06 | 10.53 | 0.17 | 0.24 | −41.41 | 2.72 |

The regression coefficients $a$ and $b$ are the solution to (10). The weighted average uses the 1989 bilateral total trade values as weights for each country-pair. The pooled regression is the results of the regression with the data for all six country-pairs pooled together.

**Table 3.8**

**Mean Absolute Percentage Difference between Predictions and Data, NAFTA**

| | | Predictions | | LTP Model with Optimal Coefficients |
|---|---|---|---|---|
| **Exporter** | **Importer** | **BDS** | **LTP** | |
| CAN | MEX | 14929.4 | 1251.4 | 46.9 |
| CAN | USA | 8417.9 | 604.2 | 382.5 |
| MEX | CAN | 1517.5 | 709.3 | 52.4 |
| MEX | USA | 567.4 | 370.5 | 74.5 |
| USA | CAN | 617.7 | 273.2 | 156.3 |
| USA | MEX | 1734.5 | 128.0 | 33.7 |
| **Weighted Average** | | 3942.2 | 395.8 | 223.6 |

The mean absolute percentage change is defined in (11). The weighted average uses the 1989 bilateral total trade values as weights for each country-pair.

**Figure 3.1. Predicted versus Actual Growth Rates, BDS Model**

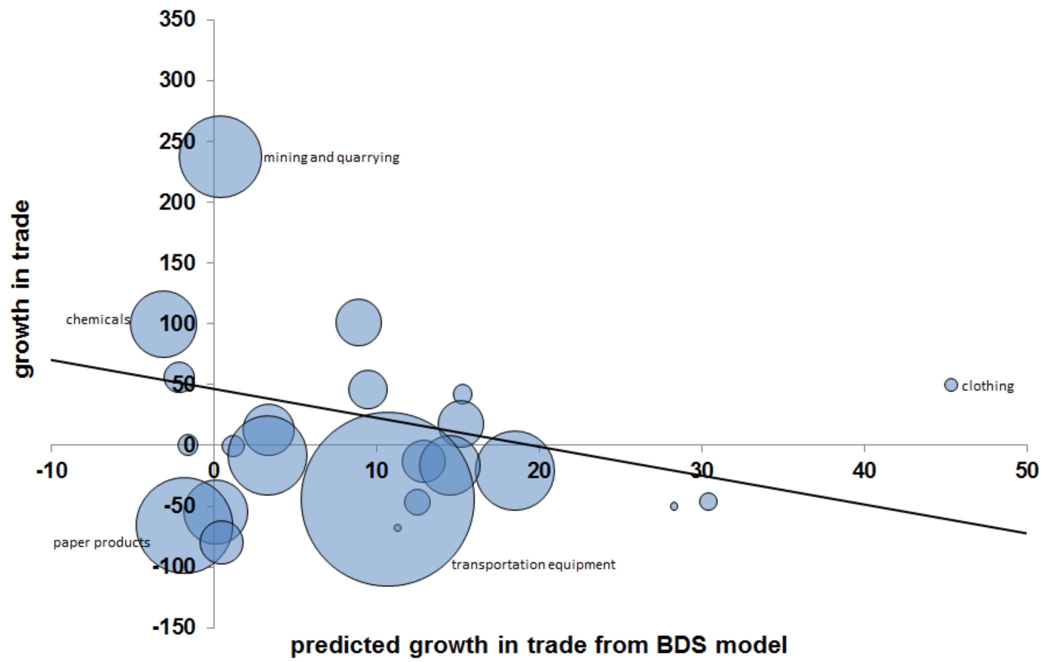
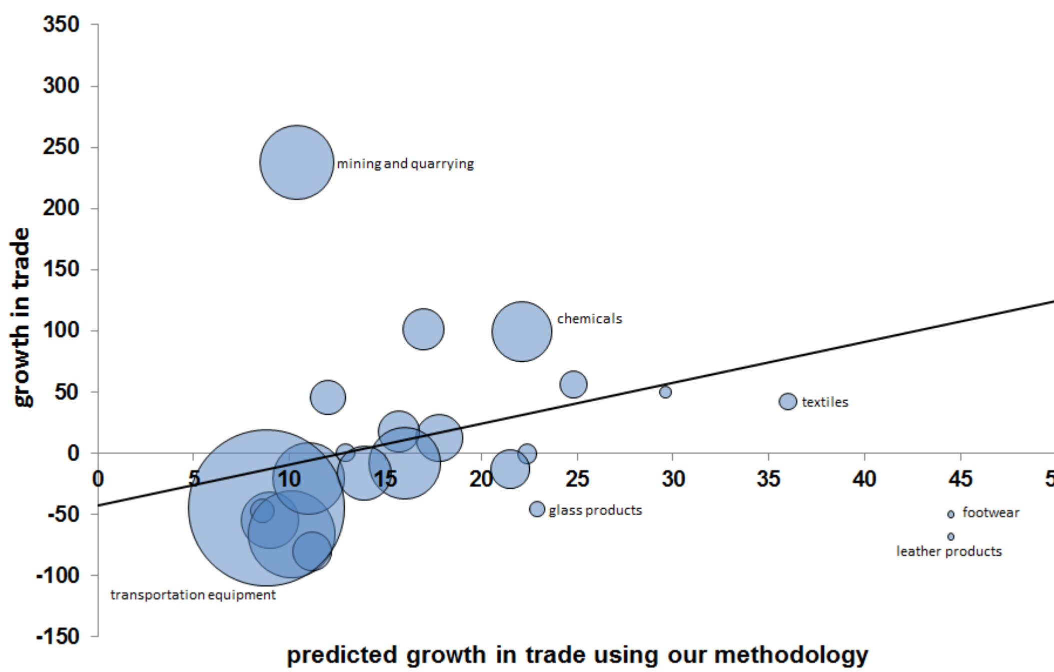



**Figure 3.2. Predicted versus Actual Growth Rates, LTP Model**



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## Appendix

## Robustness Checks and KORUS Predictions for Chapter 3

### Appendix A. Robustness Checks

In this section we offer several robustness checks for our results with respect to NAFTA.

#### A.1 Other models of NAFTA

As shown in Kehoe (2005), the poor predictions of the BDS model are not unique, and other applied general equilibrium models predicting the effects of NAFTA performed in a similarly poor fashion. To show that our results extend beyond just the BDS model of NAFTA, we examine the Sobarzo model of Mexico (Sobarzo, 1992a, 1992b, 1994, 1995) and the Cox-Harris model of Canada (Cox, 1994, 1995; Cox and Harris 1985, 1992a, 1992b).

The Cox-Harris model predicted the changes in exports and imports between Canada and the World for 14 different industries. Since a concordance from the ISIC classification to the Cox-Harris industries is not provided in the original paper, we adapt the one provided in Kehoe (2005). We use imports and exports from Canada to the World, both as reported by Canada, from Comtrade as our base data and follow the same methodology as we used for evaluating the BDS model. Since the Cox-Harris predictions are for total imports and exports for Canada, we follow the same procedure as we did for the Kiyota and Stern (2007) predictions for Korea, using the World as a trading partner. In our results shown in table A.1, we see that the results are similar to what we found when evaluating the BDS model. We report our predictions instead of the share of least traded products, since the two are no longer perfectly correlated. The Cox-Harris model had very little predictive power for both imports and exports, whereas using the share of least traded products in each industry performed significantly better at matching the relative changes in industry trade, achieving a weighted correlation of the data of 0.18 for exports and 0.33 for imports compared with the weighted correlations of 0.06 and 0.04, respectively, for the Cox-Harris model.

The Sobarzo model predicted the changes in imports and exports between Mexico and North America for 21 different industries, where North America is considered to be Canada and the United States. Since Sobarzo does not provide a concordance between ISIC and its industries, we adapt the concordance given in Kehoe (2005). We use the same base 5-digit SITC data as we did for the BDS exercise, constructing the share of least traded products for imports and exports between Mexico and the United States and Mexico and Canada separately. We then predict the increase in trade by industry separately for Mexico-U.S. and Mexico-Canada before combining the predictions to generate overall predictions for growth in imports and exports between Mexico and North America. After that we again follow the same methodology as we did for the BDS exercise, and we find that the Sobarzo model does poorly in predicting both imports and exports between North America and Mexico. As summarized in table A.2, the weighted correlation between the Sobarzo model's predictions and the data is negative (−0.12) for imports from North America to Mexico, whereas the correlation between the share of least traded products in an industry and the industry's growth is much higher (0.47). For exports to North America from Mexico, the correlation between the predictions and the data is much better (0.43) and in fact does better than using the share of least traded products (0.05). The poor performance of the least traded exercise seems to stem from considering the predictions for Mexico with

Canada and the United States jointly rather than Mexico-Canada and Mexico-U.S. separately. In particular, in table 3.18 we see that when we consider them separately, the weighted correlation between our predictions and the data is 0.33 for Mexican exports to Canada and 0.17 for Mexican exports to the United States. When the predictions are computed separately and then aggregated together, however, the weighted correlation drops to 0.05 (see table A.2). How the aggregation of individual countries into regions affects the predictions from our least traded exercise is something that merits further study.

### A.2 Alternative Thresholds

One might be concerned that our results depend on our choice to define the set of least traded products using a cutoff of 10 percent of total trade, which may appear somewhat arbitrary. Table A.3 reports our optimal coefficients and correlation with the data while using a cutoff of 5 percent or 20 percent of total trade when defining least traded products. Under all cutoffs, the correlation between observed changes in the data and the share of least traded products in each industry remains high relative to competing models. The optimal coefficients change slightly to adjust for the fact that least traded products account for a different share of total trade, however, the correlation between the optimal coefficients under the various cutoffs is very high, ranging from 0.91 to approximately 1.00. This indicates that our results are largely robust to plausible alternative cutoffs and that the exact cutoff does not play a large role as long as it is not too small and not too large. These findings parallel those of Kehoe and Ruhl (2013), which show that the new products margin operates largely the same even with alternative cutoffs for defining the set of least traded products. Therefore, we choose a cutoff of 10 percent because we think this cutoff performs well in capturing the new products margin and is the simplest and most straightforward to understand and remember.

### A.3 Ignoring Non-Traded Products

As mentioned in the main chapter, the number of zero traded products has no influence on our shares   of least traded products in each industry in 1989. This means that the our extensive margin is not the typical extensive margin of products that are previously not traded at all, but rather products with very small amounts of trade. To highlight this fact, we recompute the optimal coefficients and correlations between observed growth in the data and the share of least traded products and completely ignore growth in products that initially reported zero trade. Table A.4 reports the results of this exercise and shows that our results are largely unchanged. This lends further support to our claim that the essential products in terms of generating any predictive power from our exercise are not products reporting zero trade, but products that are positively traded, although with very small amounts of trade.

### A.4 Prices and Quantities

Although our exercises look at changes in the value of trade, our results are primarily driven by changes in quantities rather than changes in prices. To show this, we examine all products for which we have quantity data and decompose the changes in real value into changes in price and changes in quantity, where real value is taken to be the reported level of trade converted to the exporting country's national currency and then deflated by the exporting country's producer price index. We then compute a weighted average of this decomposition, using the initial trade value as each product's weight. To reduce the effect of outliers, we do not include products in the top and bottom fifth percentile of products in terms of the percentage of growth accounted for by changes in quantity. These results are shown in table A.5, where we see that on average, 86.9 percent of all changes in value are due to changes in quantities. When more than 100 percent of the change is due to changes in quantities, this indicates that prices decreased while the total value of trade increased or vice versa.

**Appendix B. Predictions for U.S.-Korea FTA**

In this section we use our model to make predictions about a trade liberalization that is currently under way, the U.S.-Korea free trade agreement (KORUS). The United States and Korea signed a free trade agreement, KORUS, in 2007, which was enacted in 2012. To make our predictions for the effects of KORUS, we follow the methodology laid out in section 2. We take the base year to be 2005. To parameterize the model, we need an elasticity of total trade to tariffs and the cross-country coefficient relating aggregate trade growth to newly traded products growth. We reestimate the gravity equation in (3.4), this time using data from 2005. Columns 3 and 4 of table B.1 report the estimates. The value for $\gamma$ comes from table 3.2 in Kehoe and Ruhl (2013). They estimate that $\gamma$ is 3.59 using data on 1,913 bilateral country pairs covering 1995–2005.

For simplicity, we assume that KORUS reduces tariffs to zero. Table B.1 reports our results from equations (3.3) and (3.6). We predict that total growth in bilateral trade will be 43.57 percent for U.S. exports to Korea and 14.14 percent for Korean exports to the United States. The difference in predicted trade growth is the result of initial tariffs that are significantly higher on U.S. exports to Korea than they are on Korean exports to the

United States. For our industry-level predictions using (3.1), we set $\alpha_{kor}^{usa} = 9.06$ and $\beta_{kor}^{usa} = 50.78$ for Korean exports to the United States and $\alpha_{usa}^{kor} = 27.92$ and $\beta_{usa}^{kor} = 156.46$ for Korean imports from the United States. In table B.2 we report the fraction of least traded products by industry for Korean exports to the United States and U.S. exports to Korea. Again, we see significant variation in this share, ranging from 0.01 to 1.00.

### B.1 Our KORUS Predictions

Table B.2 reports our predictions for each industry (a list of each 3-digit ISIC industry code and its description is given in table B.3). Our predictions vary widely across industries: for Korean exports to the United States, our predictions range from an increase of 10.1 percent in exports of furniture and fixture (ISIC code 332) to an increase of 50.8 percent in exports of industries that are completely made of up least traded products, such as pottery and glass products (ISIC codes 361 and 362). For U.S. exports to Korea, our predictions are substantially higher, ranging from an increase of 31.0 percent for paper products (ISIC code 341) to 156.5 percent for industries such as apparel and footwear (ISIC codes 322 and 324).

### B.2 Predictions from Other Models

We first compare our predictions with the predictions from two alternative models. First, we compare predictions with those from Kiyota and Stern (2007), which are based on the methodology and assumptions of the Brown-Deardorf-Stern model discussed in section 4. Then we compare our results with those from Yaylaci and Shikher (2014), which are based on the Ricardian framework of Eaton and Kortum (2002).

Kiyota and Stern (2007) predict the changes in total imports and exports for 14 industries, as well as two service industries that we ignore, for Korea and the United States following liberalization. To make our results comparable to theirs, we aggregate the ISIC industries into their industries and compute the share of least traded products in each of those industries. Kiyota and Stern do not provide an exact concordance between the ISIC codes and their industries, so we develop one. Kiyota and Stern focus on trade flows between Korea and the World and the United States and the World, whereas our methodology predicts bilateral trade flows. To make our predictions comparable to theirs, we assume that exports from the United States to the World, excluding Korea, grow by the factor $\alpha$ and similarly for exports from Korea to the World, excluding the United States. This assumption allows us to keep our predictions of the form (3.1) using U.S.-Korea data. We use these data on trade flows to identify the set of least traded products. Mechanically, we multiply $\alpha$ by the fraction of trade accounted for by the United States for Korea, and by Korea for the United States. For example, for predicting Korean exports to the World, we set $\alpha$ to 2.27 $(=(15.55)(0.146))$ since Korean exports to the United States account for 14.6 percent of Korean exports to the World in 2005. Tables B.4 and B.5 compare our results with those of Kiyota and Stern. As we see, there are significant differences in the predictions between the two methods, especially for U.S. exports: our predictions have correlations with theirs that range from –0.23 for U.S. exports to 0.82 Korean exports.

Yaylaci and Shikher (2014) predict the changes in bilateral trade for 15 manufacturing industries between the United States and Korea following liberalization. Yaylaci and Shikher lack predictions for the agricultural industry, so we exclude it from our predictions after classifying products as least traded. Table B.6 compares our predictions with the predictions of Yaylaci and Shikher. Again, there are significant differences between them, with a correlation of 0.43 between our predictions and those of Yaylaci and Shikher for Korean exports to the United States and a correlation of 0.19 for U.S. exports to Korea. The largest difference is for Korean exports to the United States: we predict the paper industry will experience the most growth, whereas Yaylaci and Shikher predict it will experience the least growth of all industries. There are similarities as well: Yaylaci and Shikher predict, for example, that for Korean exports to the United States, food will exhibit the most growth, whereas we predict it will exhibit the second highest growth. Similarly, we predict textiles will exhibit the most growth in U.S. exports to Korea, and Yaylaci and Shiker predict it will exhibit the second highest growth.

## Table A.1

## Changes in Canadian Trade Relative to Canadian GDP (Percent):

## Observed Changes versus Cox-Harris Predictions and LTP Based Predictions

| | Exports to World | | | Imports from World | | |
|---|---|---|---|---|---|---|
| **Industry** | **Data** | **Cox-Harris** | **LTP** | **Data** | **Cox-Harris** | **LTP** |
| Agriculture | 39.1 | −4.1 | 5.8 | −7.6 | 7.2 | 19.0 |
| Chem. and Misc. Man. | 70.9 | 28.1 | 14.5 | 29.7 | 10.4 | 21.0 |
| Fishing | −30.9 | −5.4 | 5.9 | 8.3 | 9.5 | 17.2 |
| Food, Bev., and Tobacco | 95.5 | 18.6 | 10.9 | 52.0 | 3.8 | 16.4 |
| Forestry | −24.8 | −11.5 | 11.7 | −14.8 | 7.1 | 24.0 |
| Machinery and Appl. | 11.7 | 57.1 | 12.2 | −23.9 | 13.3 | 14.1 |
| Mining | 117.0 | −7.0 | 5.9 | 65.4 | 4.0 | 9.0 |
| Nonmetallic Minerals | 20.9 | 31.8 | 22.3 | −15.8 | 7.3 | 27.4 |
| Refineries | −67.8 | −2.7 | 10.0 | −77.1 | 1.5 | 10.7 |
| Rubber and Plastics | 107.3 | 24.5 | 15.4 | 27.1 | 13.8 | 14.5 |
| Steel and Metal Products | 6.6 | 19.5 | 10.3 | 8.5 | 10.0 | 18.3 |
| Textiles and Leather | 18.4 | 108.8 | 26.1 | −20.1 | 18.2 | 14.1 |
| Transportation Equip. | −37.5 | 3.5 | 8.5 | −34.6 | 3.0 | 13.4 |
| Wood and Paper | −58.5 | 7.3 | 6.6 | −8.1 | 7.2 | 17.9 |
| **Weighted Corr. with Data** | | 0.06 | 0.18 | | 0.04 | 0.33 |
| **Regression Coefficient $a$** | | 2.00 | −25.40 | | −10.57 | −55.51 |
| **Regression Coefficient $b$** | | 0.16 | 3.24 | | 0.24 | 3.07 |
| **CH-LTP Weighted Correlation** | | | 0.81 | | | 0.22 |

The column *Data* reports the growth rates of industry exports deflated by the exporter's GDP growth rate, 1989−2009. *Cox-Harris* reports the predictions from Cox (1995), which are based on the methodology of Cox and Harris (1985). The column *LTP* reports the predictions from the least traded products model in (1). The regression coefficients $a$ and $b$ are the solution to (10). Fraction least traded is for U.S.-Korea trade, not total trade with the world.

**Table A.2**

**Changes in Mexican Trade Relative to Mexican GDP (Percent):**

**Observed Changes versus Sobarzo Predictions and LTP Based Predictions**

| | **Exports to North America** | | | **Imports from North America** | | |
|---|---|---|---|---|---|---|
| **Industry** | **Data** | **Sobarzo** | **LTP** | **Data** | **Sobarzo** | **LTP** |
| Agriculture | −15.3 | −11.1 | 15.1 | 61.0 | 3.4 | 41.6 |
| Beverages | 161.8 | 5.2 | 12.1 | 189.0 | −1.8 | 70.0 |
| Chemicals | 34.1 | −4.4 | 40.4 | 218.5 | −2.7 | 58.2 |
| Electrical Machinery | 54.7 | 1.0 | 12.7 | 66.3 | 9.6 | 31.1 |
| Food | 100.8 | −6.9 | 31.4 | 128.8 | −5.0 | 49.2 |
| Iron and Steel | 19.6 | −4.9 | 26.4 | 92.0 | 17.7 | 58.3 |
| Leather | −64.6 | 12.4 | 37.5 | 60.0 | −0.4 | 114.1 |
| Metal Products | 86.2 | −4.4 | 26.3 | 94.8 | 9.5 | 47.4 |
| Mining | 27.7 | −17.0 | 12.1 | 79.4 | 13.2 | 50.4 |
| Nonelectrical Machinery | 166.5 | −7.4 | 18.2 | 115.8 | 20.7 | 41.4 |
| Nonferrous Metals | 36.8 | −9.8 | 17.7 | 113.9 | 9.8 | 44.6 |
| Nonmetallic Min. Prod. | −16.0 | −6.2 | 24.5 | 64.3 | 10.9 | 100.9 |
| Other Manufactures | 88.4 | −4.5 | 22.9 | 96.7 | 4.2 | 49.4 |
| Paper | −35.9 | −7.9 | 25.8 | 49.7 | −4.7 | 38.8 |
| Petroleum | −98.0 | −19.5 | 17.0 | −71.2 | −6.8 | 37.6 |
| Rubber | 158.9 | 12.8 | 32.6 | 178.2 | −0.1 | 37.1 |
| Textiles | 69.5 | 1.9 | 43.9 | 131.3 | −1.2 | 84.0 |
| Tobacco | −61.3 | 2.8 | 59.4 | 575.5 | −11.6 | 155.0 |
| Transportation Equip. | 126.1 | −5.0 | 12.9 | 97.7 | 11.2 | 32.6 |
| Wearing Apparel | 197.2 | 30.0 | 23.3 | 29.2 | 4.5 | 54.7 |
| Wood | 30.8 | −8.5 | 13.6 | 2.9 | 11.7 | 36.5 |
| **Weighted Corr with Data** | | 0.43 | 0.05 | | −0.12 | 0.47 |
| **Regression Coefficient** $a$ | | 81.13 | 56.62 | | 104.22 | 6.41 |
| **Regression Coefficient** $b$ | | 3.06 | 0.43 | | −0.77 | 2.17 |
| **Sobarzo-LTP Weighted Correlation** | | | 0.20 | | | −0.32 |

The column *Data* reports the growth rates of industry exports deflated by the exporter's GDP growth rate, 1989−2009. *Sobarzo* reports the predictions from Sobarzo (1995). The column *LTP* reports the predictions from the least traded products model in (1). The regression coefficients $a$ and $b$ are the solution to (10). Fraction least traded is for U.S.-Korea trade, not total trade with the world.

**Table A.3**

**Correlations and Optimal Coefficients with Alternative Cutoffs**

| | | 5% LTP Cutoff | | | 20% LTP Cutoff | | |
|---|---|---|---|---|---|---|---|
| **Exporter** | **Importer** | **Correlation with Data** | $\tilde{\alpha}$ | $\tilde{\beta}$ | **Correlation with Data** | $\tilde{\alpha}$ | $\tilde{\beta}$ |
| CAN | MEX | 0.36 | 474.86 | 4524.16 | 0.48 | 200.78 | 2501.44 |
| CAN | USA | 0.30 | −18.83 | 338.66 | 0.33 | −26.22 | 121.57 |
| MEX | CAN | 0.32 | 122.13 | 433.39 | 0.21 | 117.31 | 132.43 |
| MEX | USA | 0.08 | 55.96 | 66.28 | 0.10 | 53.37 | 29.50 |
| USA | CAN | 0.52 | −30.64 | 273.64 | 0.61 | −41.69 | 123.66 |
| USA | MEX | 0.45 | 62.46 | 527.87 | 0.56 | 51.58 | 186.33 |
| **Weighted Average** | | 0.37 | −2.74 | 311.31 | 0.42 | −11.65 | 122.31 |
| **Pooled Regression** | | 0.23 | −2.21 | 300.51 | 0.26 | −11.03 | 119.23 |
| **Correlation with 10% Estimates** | | | 0.91 | 1.00 | | 0.99 | 1.00 |

The optimal coefficients are the solution to (8). These coefficients are calculated using alternative cutoffs for defining the share of least traded products and compared correlation to the coefficients in the paper with the original 10 percent cutoff definition by computing the correlation between them. The weighted average uses the 1989 bilateral total trade values as weights for each country-pair. The pooled regression is the results of the regression with the data for all six country-pairs pooled together.

**Table A.4**

**Correlations and Optimal Coefficients with Zeros Ignored**

| | | Original | | | Ignoring Zeros | | |
|---|---|---|---|---|---|---|---|
| **Exporter** | **Importer** | **Correlation with Data** | $\tilde{\alpha}$ | $\tilde{\beta}$ | **Correlation with Data** | $\tilde{\alpha}$ | $\tilde{\beta}$ |
| Can | Mex | 0.36 | 254.23 | 4468.37 | 0.61 | 165.14 | 3719.32 |
| Can | Usa | 0.30 | –20.42 | 185.24 | 0.28 | −20.09 | 175.52 |
| Mex | Can | 0.32 | 115.16 | 286.39 | 0.25 | 107.86 | 210.37 |
| Mex | Usa | 0.08 | 51.52 | 77.54 | 0.05 | 52.64 | 20.16 |
| Usa | Can | 0.52 | –34.54 | 175.84 | 0.54 | −34.63 | 176.08 |
| Usa | Mex | 0.45 | 62.31 | 265.44 | 0.41 | 63.31 | 227.14 |
| **Weighted Average** | | 0.37 | −5.74 | 185.67 | 0.36 | −5.59 | 168.58 |
| **Pooled Regression** | | 0.24 | −5.30 | 181.18 | 0.24 | −4.95 | 162.14 |
| **Correlation with Original Optimal Coefficients** | | | | | | 0.97 | 1.00 |

The optimal coefficients are the solution to (8). The column *Original* corresponds to the coefficients reported in the main paper in table 2. The column *Ignoring Zeros* reports the optimal coefficients when growth in products that are originally not traded at all is ignored. The weighted average uses the 1989 bilateral total trade values as weights for each country-pair. The pooled regression is the results of the regression with the data for all six country-pairs pooled together.

## Table A.5

### Changes in North American Trade Deflated by Exporter's PPI:

### Growth Decomposed into Changes in Quantities and Changes in Prices (Percent)

| | | | average share of total growth | |
|---|---|---|---|---|
| **Exporter** | **Importer** | **Period** | ***P*** | ***Q*** |
| Canada | Mexico | 89–09 | −9.1 | 109.1 |
| Canada | United States | 89–09 | 32.3 | 67.7 |
| Mexico | Canada | 89–09 | 24.4 | 75.6 |
| Mexico | United States | 89–09 | 8.9 | 91.1 |
| United States | Canada | 89–09 | −3.2 | 103.2 |
| United States | Mexico | 89–09 | −1.3 | 101.3 |
| **Weighted Average** | | | 13.1 | 86.9 |
| **Pooled** | | | 16.2 | 83.8 |

This table decomposes growth in exports into changes in prices, column *P*, and changes in quantities, column *Q*. These are the weighted averages across products for which both quantity and price data are available, where the weight for each product is its trade value in 1989 and varies by country-pair.

## Table B.1

### Estimated Coefficients, U.S.-Korea Trade

| Exporter | Importer | Initial Tariff | Predicted Growth | Estimated $\alpha$ | Estimated $\beta$ |
|---|---|---|---|---|---|
| United States | Korea | 10.85 | 43.57 | 31.03 | 125.43 |
| Korea | United States | 3.84 | 14.14 | 10.07 | 40.72 |

Growth in trade is measured as the change in trade deflated by the GDP growth rate. The estimated coefficients are the solution to (3) and (6). The optimal coefficients are the solution to (8).

## Table B.2

### Predicted Growth in Korea-U.S. Trade Relative to Exporter's GDP (Percent)

| Korea to United States | | | | | | United States to Korea | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **ISIC code** | **LTP** | **Least traded share** | **ISIC code** | **LTP** | **Least traded share** | **ISIC code** | **LTP** | **Least traded share** | **ISIC code** | **LTP** | **Least traded share** |
| **111** | 50.8 | 1.00 | **342** | 29.1 | 0.47 | **111** | 37.9 | 0.05 | **342** | 50.9 | 0.16 |
| **113** | 50.8 | 1.00 | **351** | 24.0 | 0.34 | **113** | 35.9 | 0.04 | **351** | 53.4 | 0.18 |
| **121** | 50.8 | 1.00 | **352** | 28.9 | 0.46 | **121** | 35.1 | 0.03 | **352** | 47.1 | 0.13 |
| **122** | 50.8 | 1.00 | **353** | 13.4 | 0.08 | **122** | 50.1 | 0.15 | **353** | 35.3 | 0.03 |
| **130** | 50.8 | 1.00 | **354** | 50.8 | 1.00 | **130** | 53.3 | 0.18 | **354** | 156.5 | 1.00 |
| **210** | - | - | **355** | 14.2 | 0.10 | **210** | 31.0 | 0.00 | **355** | 102.4 | 0.57 |
| **220** | 50.8 | 1.00 | **356** | 12.3 | 0.05 | **220** | 33.3 | 0.02 | **356** | 34.7 | 0.03 |
| **230** | 50.8 | 1.00 | **361** | 50.8 | 1.00 | **230** | 38.5 | 0.06 | **361** | 40.0 | 0.07 |
| **290** | 50.8 | 1.00 | **362** | 50.8 | 1.00 | **290** | 76.0 | 0.36 | **362** | 89.5 | 0.47 |
| **311*** | 33.5 | 0.58 | **369** | 15.1 | 0.12 | **311*** | 53.8 | 0.18 | **369** | 106.6 | 0.60 |
| **313** | 50.8 | 1.00 | **371** | 14.4 | 0.11 | **313** | 84.8 | 0.43 | **371** | 106.2 | 0.60 |
| **314** | 10.1 | 0.00 | **372** | 26.9 | 0.41 | **314** | 156.5 | 1.00 | **372** | 43.7 | 0.10 |
| **321** | 28.7 | 0.46 | **381** | 26.7 | 0.41 | **321** | 115.1 | 0.67 | **381** | 51.0 | 0.16 |
| **322** | 23.0 | 0.32 | **382** | 13.2 | 0.08 | **322** | 156.5 | 1.00 | **382** | 42.8 | 0.09 |
| **323** | 50.8 | 1.00 | **383** | 10.9 | 0.02 | **323** | 55.3 | 0.19 | **383** | 35.6 | 0.04 |
| **324** | 50.8 | 1.00 | **384** | 10.4 | 0.01 | **324** | 156.5 | 1.00 | **384** | 34.8 | 0.03 |
| **331** | 50.8 | 1.00 | **385** | 30.1 | 0.49 | **331** | 57.4 | 0.21 | **385** | 37.7 | 0.05 |
| **332** | 14.3 | 0.10 | **390** | 24.0 | 0.34 | **332** | 71.5 | 0.32 | **390** | 63.6 | 0.26 |
| **341** | 50.8 | 1.00 | | | | **341** | 40.0 | 0.07 | | | |

*311 is the single Major Group 311–312. The column *Least traded share* reports the share of industry exports accounted for by least traded products in 2005. The column *LTP* reports the predictions from the least traded products model in (1). The regression coefficients $a$ and $b$ are the solution to (10).

**Table B.3**

**ISIC Industry Codes and Descriptions**

| ISIC Code | Industry Name |
|---:|---|
| 111 | Agriculture and livestock production |
| 113 | Hunting, trapping and game propagation |
| 121 | Forestry |
| 122 | Logging |
| 130 | Fishing |
| 210 | Coal mining |
| 220 | Crude petroleum and natural gas production |
| 230 | Metal ore mining |
| 290 | Other mining |
| 311–312* | Food manufacturing |
| 313 | Beverage industries |
| 314 | Tobacco manufactures |
| 321 | Manufacture of textiles |
| 322 | Manufacture of wearing apparel, except footwear |
| 323 | Manufacture of leather and products of leather, leather substitutes and fur |
| 324 | Manufacture of footwear |
| 331 | Manufacture of wood and wood and cork products, except furniture |
| 332 | Manufacture of furniture and fixtures, except primarily of metal |
| 341 | Manufacture of paper and paper products |
| 342 | Printing, publishing and allied industries |
| 351 | Manufacture of industrial chemicals |
| 352 | Manufacture of other chemical products |
| 353 | Petroleum refineries |
| 354 | Manufacture of miscellaneous products of petroleum and coal |
| 355 | Manufacture of rubber products |
| 356 | Manufacture of plastic products not elsewhere classified |
| 361 | Manufacture of pottery, china and earthenware |
| 362 | Manufacture of glass and glass products |
| 369 | Manufacture of other non-metallic mineral products |
| 371 | Iron and steel basic industries |
| 372 | Non-ferrous metal basic industries |
| 381 | Manufacture of fabricated metal products |
| 382 | Manufacture of machinery except electrical |
| 383 | Manufacture of electrical machinery apparatus, appliances and supplies |
| 384 | Manufacture of transport equipment |
| 385 | Manufacture of professional and scientific equipment |
| 390 | Other manufacturing industries |

*311–312 is considered by the United Nations to be one Major Group (3-digit code) within the Division (2-digit code) 31, Manufacture of Food, Beverages and Tobacco, within the Major Division (1-digit code) 3, Manufacturing. It has eleven Groups (4-digit codes).

## Table B.4

### Predicted Growth in Korean trade, Kiyota-Stern Model

| | Korean Exports to World | | | Korean Imports from World | | |
|---|---|---|---|---|---|---|
| **Industry** | **Kiyota-Stern** | **LTP** | **Least traded share** | **Kiyota-Stern** | **LTP** | **Least traded share** |
| Agriculture | −0.6 | 5.8 | 1.00 | 10.6 | 11.4 | 0.05 |
| Chemicals | 1.0 | 1.9 | 0.25 | 3.5 | 9.5 | 0.15 |
| Food, Bev., and Tobacco | 6.9 | 4.3 | 0.50 | 7.6 | 9.5 | 0.19 |
| Leather and Footwear | 7.7 | 2.5 | 1.00 | 0.6 | 3.0 | 0.34 |
| Machinery and Equip. | −0.2 | 1.8 | 0.04 | 1.8 | 6.6 | 0.06 |
| Metal Products | 0.4 | 2.1 | 0.24 | 1.7 | 3.7 | 0.18 |
| Mining | −1.8 | 5.3 | 1.00 | 1.0 | 0.3 | 0.08 |
| Misc. Manufactures | 5.3 | 1.9 | 0.27 | 4.2 | 8.1 | 0.08 |
| Natural Resources | 0.6 | 2.5 | 1.00 | 1.3 | 5.1 | 0.16 |
| Nonmetallic Min. Prod. | 0.2 | 4.2 | 0.47 | 3.4 | 8.4 | 0.46 |
| Textiles | 8.6 | 2.6 | 0.44 | 3.6 | 5.4 | 0.67 |
| Transportation Equip. | 2.7 | 2.1 | 0.01 | 2.1 | 10.3 | 0.03 |
| Wearing Apparel | 27.7 | 10.5 | 0.33 | −6.0 | 2.6 | 1.00 |
| Wood Products | 0.2 | 3.5 | 0.39 | 2.0 | 8.1 | 0.11 |
| **KS-LTP weighted correlation** | | | **0.82** | | | **0.63** |

The column *Kiyota-Stern* reports the predictions from Kiyota and Stern (2007). The column *Least traded share* reports the share of industry exports accounted for by least traded products in 2005. The column *LTP* reports the predictions from the least traded products model in (1). The regression coefficients $a$ and $b$ are the solution to (10). Fraction least traded is for U.S.-Korea trade, not total trade with the world.

## Table B.5

### Predicted Growth in United States, Kiyota-Stern Model

| | U.S. Exports to World | | | U.S. Imports from World | | |
|---|---|---|---|---|---|---|
| **Industry** | **Kiyota-Stern** | **LTP** | **Least traded share** | **Kiyota-Stern** | **LTP** | **Least traded share** |
| Agriculture | 4.4 | 1.4 | 0.07 | 0.2 | 0.1 | 1.00 |
| Chemicals | 0.4 | 1.6 | 0.15 | 0.0 | 0.4 | 0.26 |
| Food, Bev., and Tobacco | 2.0 | 1.8 | 0.19 | 0.1 | 0.2 | 0.56 |
| Leather and Footwear | 0.4 | 2.2 | 0.23 | −0.1 | 0.1 | 1.00 |
| Machinery and Equip. | 0.3 | 1.5 | 0.05 | 0.0 | 0.6 | 0.04 |
| Metal Products | 0.3 | 1.6 | 0.24 | 0.0 | 0.4 | 0.22 |
| Mining | 0.1 | 0.6 | 0.15 | 0.0 | 0.0 | 1.00 |
| Misc. Manufactures | 0.5 | 1.4 | 0.08 | 0.0 | 0.3 | 0.40 |
| Natural Resources | 0.4 | 4.1 | 0.15 | 0.0 | 0.3 | 1.00 |
| Nonmetallic Min. Prod. | 0.6 | 3.0 | 0.71 | 0.0 | 0.3 | 0.27 |
| Textiles | −0.1 | 1.4 | 0.64 | −0.4 | 1.0 | 0.46 |
| Transportation Equip. | 0.0 | 0.7 | 0.02 | −0.1 | 0.5 | 0.01 |
| Wearing Apparel | −0.1 | 1.7 | 0.70 | −0.5 | 0.4 | 0.32 |
| Wood Products | 0.1 | 1.2 | 0.23 | 0.0 | 0.2 | 0.58 |
| **KS-LTP weighted correlation** | | | **0.22** | | | **−0.23** |

The column *Kiyota-Stern* reports the predictions from Kiyota and Stern (2007). The column *Least traded share* reports the share of industry exports accounted for by least traded products in 2005. The column *LTP* reports the predictions from the least traded products model in (1). The regression coefficients $a$ and $b$ are the solution to (10). Fraction least traded is for U.S.-Korea trade, not total trade with the world.

**Table B.6**

**Predicted Growth in Korean Trade, Yaylaci-Shikher Model**

| | Korea to United States | | | United States to Korea | | |
|---|---|---|---|---|---|---|
| **Industry** | **Yaylaci-Shikher** | **LTP** | **Least traded share** | **Yaylaci-Shikher** | **LTP** | **Least traded share** |
| Chemicals | 28.2 | 24.7 | 0.36 | 30.3 | 51.4 | 0.16 |
| Electrical machinery | 15.5 | 10.9 | 0.02 | 41.0 | 35.6 | 0.04 |
| Food | 70.1 | 33.0 | 0.56 | 422.3 | 55.0 | 0.19 |
| Other machinery | 8.9 | 13.2 | 0.08 | 31.9 | 42.8 | 0.09 |
| Medical | 9.9 | 30.1 | 0.49 | 45.0 | 37.7 | 0.05 |
| Metals | 9.3 | 15.5 | 0.13 | 17.0 | 56.5 | 0.20 |
| Nonmetals | 20.5 | 21.2 | 0.27 | 38.7 | 89.1 | 0.46 |
| Other | 11.8 | 24.0 | 0.34 | 28.5 | 63.6 | 0.26 |
| Paper | 1.4 | 37.6 | 0.68 | 5.5 | 42.1 | 0.09 |
| Petroleum | 2.2 | 15.1 | 0.12 | 7.2 | 35.3 | 0.03 |
| Metal products | 14.2 | 26.7 | 0.41 | 33.8 | 51.0 | 0.16 |
| Rubber | 19.8 | 13.7 | 0.09 | 48.0 | 52.7 | 0.17 |
| Textile | 56.3 | 26.0 | 0.39 | 63.5 | 112.8 | 0.65 |
| Transportation Equip. | 23.3 | 10.4 | 0.01 | 33.9 | 34.8 | 0.03 |
| Wood | 7.9 | 18.4 | 0.21 | 21.1 | 62.0 | 0.25 |
| **KS-LTP weighted correlation** | | | **0.43** | | | **0.19** |

The column *Yaylaci-Shikher* reports the predictions from Yaylaci and Shikher (2014). The column *Least traded share* reports the share of industry exports accounted for by least traded products in 2005. The column *LTP* reports the predictions from the least traded products model in (1). The regression coefficients $a$ and $b$ are the solution to (10).