Essays in Macroeconomics of the Labor Market

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Dedication

To my parents Yongjin and Ok Yee, my wife Nahyun, and my son Jueun

Abstract

This dissertation consists of three chapters. All chapters are related to business cycle issues in the labor market with search frictions. In Chapter 1, I examine the effect of medical re-evaluations for disability insurance (DI) recipients on the 1981 recession and its fast recovery. In the US, the recovery in the employment rate of men from the 1981 recession was faster than any other recovery since 1965. During the 1981 recession and at the beginning of its recovery, the number of disability insurance applicants and recipients dropped while the numbers increased in all other recessions. This decrease is attributed to the fact that the most stringent medical re-evaluations for DI recipients occurred between 1981 and 1983. Medical re-evaluation is a policy that periodically terminates benefits of ineligible DI recipients. This paper examines the role of medical re-evaluation in the 1981 recession and its fast recovery. To this end, I build a general equilibrium business-cycle search and matching model with health, DI and unemployment insurance (UI) eligibility. Medical re-evaluations affect the number of people who search for jobs (direct effect) and job-finding probabilities for all unemployed people (general equilibrium effect). The overall effect of the policy depends on the willingness of firms to hire workers. The main experiment shows that the change in stringency of medical re-evaluations during the 1981 recession made the recession deeper and the recovery faster.

In Chapter 2, my coauthor, John Seliski, and I develop a model with both frictional labor markets and financial frictions to explore how the dynamics of real and financial variables are affected by financial shocks. Financial shocks affect the borrowing capacity of firms in the economy. In particular, we evaluate how important the inclusion of financial shocks is in accounting for labor market fluctuations by using a standard RBC matching model as a benchmark. We find that the inclusion of financial frictions and financial shocks improves a standard matching model's ability to account for the observed dynamics of labor market variables. Financial frictions are able to generate more volatile hours per worker, labor shares, and employment relative to our benchmark matching model, bringing simulated moments closer to observed fluctuations.

In Chapter 3, I study an alternative mechanism of wage negotiations in an environment where a firm hires more than one worker and the firm faces diminishing marginal product of labor (MPL). When Nash bargaining with a marginal worker breaks down, a firm negotiates wages with existing workers collectively and produces with them. Due to diminishing MPL, the breakdown of the negotiation with the marginal worker negatively affects the bargaining position of the firm with existing workers (one fewer workers) since MPL is higher with one fewer workers. How much the firm internalizes this negative effect depends on stochastic bargaining powers of existing workers which can be identified through labor share data. The stochastic bargaining power of existing workers provides an additional margin to increase the volatility of labor market variables. In contrast to the prediction of Ríos-Rull and Santaeulalia-Llopis (2010), in which the effect of productivity shocks is dampened when labor share overshoots due to huge wealth effects from the overshooting property, this paper presents a model in which the labor share overshoots and the volatility of employment closely matches that of US data.

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Chapter 1

The Effect of Medical Re-evaluations for Disability Insurance Recipients on Aggregate Employment Dynamics

1.1 Introduction

In the US, the recovery in the employment rate of men from the 1981 recession was faster than any other recovery since 1965. During the 1981 recession and at the beginning of its recovery, the number of disability insurance (DI) applicants and recipients dropped while the numbers increased in all other recessions. This decrease is attributed to the fact that the most stringent medical re-evaluations for DI recipients occurred between 1981 and 1983. Medical re-evaluation is a policy that periodically terminates the benefits of ineligible DI recipients. This paper examines the role of medical re-evaluation[1] for DI recipients in the 1981 recession and its fast recovery.

Based on the abovementioned facts, I build a general equilibrium business-cycle

[1] It is called the Continuing Disability Reviews (CDRs) in reality.

search and matching model with health (in terms of work limitation), DI, and unemployment insurance (UI) eligibility to quantify the effect of medical re-evaluations on the 1981 recession and its recovery. In the model, after receiving health shocks, risk-neutral employed people can quit their jobs in order to apply for DI. Unemployed people first decide whether to apply for DI, and then, choose whether to search for jobs while collecting UI benefits if they are eligible for UI. DI applicants[2] must wait for five months until the acceptance decision is made. During this period, they can also search for jobs if they want, and collect UI benefits. They can be accepted for DI with some probability and, if accepted, start to collect DI benefits and do not search for jobs. DI recipients receive medical re-evaluations every period with some probability and their benefits can be terminated with some probability. Each firm hires only one worker. Workers who have different productivity compete in the same labor market and firms do not know who will be matched with them when they post vacancies. Therefore, a firm decides whether to post a vacancy based on the expected value of posting a vacancy, which depends on the distribution of unemployed people. Worker flows into and out of the DI program affect the distribution of unemployed people. In turn, it affects a firm's decision to post a vacancy and job finding probabilities. Lastly, after meeting workers, firms immediately learn the worker's productivity and health status, and wages are determined by Nash bargaining based on this information.

The model is calibrated to match key features of the US economy, including a distribution of health status among employed people, unemployed people, and DI recipients. I use the Panel Study of Income Dynamics (PSID), Current Population Survey (CPS), and public Social Security Administration (SSA) data in the calibration. A key feature of computation is that the model has aggregate productivity shocks and heterogeneous workers are randomly matched in the same labor market. Therefore, the measure of unemployed people is one of the aggregate state variables. Krusell-Smith (1998) approximation is used to solve for the model outside of the steady state. In this paper, the measure of unemployed people is replaced with the total number of employed people.

More stringent medical re-evaluation induces more people to look for jobs (direct effect) because more DI recipients are terminated and start to look for jobs while less people apply for DI. If job-finding probabilities are fixed, then the direct effect will

[2] People who have applied for DI

increase the employment rate. However, the increase in the number of people who look for jobs results in changes in job-finding probabilities for all unemployed people (general equilibrium effect). Whether job-finding probabilities decrease or increase depends on how much firms want to hire workers and on the state of the economy. A firm's decision to post a vacancy depends on the following two effects. If more people look for jobs, then it is easier for firms to find workers. This increases a firm's incentive to post a vacancy. On the other hand, DI recipients are more likely to have lower productivity. Therefore, the inflow of terminated DI recipients into the unemployment pool increases the probability of firms meeting less productive workers. In sum, when more people look for jobs due to more stringent medical re-evaluations, firms face a trade-off between a higher probability of finding workers and a lower probability of meeting more productive workers. So, it is ambiguous whether a firm will post a vacancy. Consequently, the general equilibrium effect which is the change in job finding probabilities is also ambiguous. The overall effect of the change in stringency of the policy on employment rate will be determined by the direct effect and the general equilibrium effects.

To determine the effect of the policy change during the 1981 recession, I perform an experiment which is similar to what happened during the 1981 recession. This is an unexpected one-time increase in the frequency[3] of medical re-evaluations during a recession. The experiment shows that more frequent medical re-evaluations during the 1981 recession made the recession deeper and the recovery faster. During the recession, since more DI recipients are terminated, more people look for jobs. However, firms do not want to hire workers during a recession. More importantly, since DI recipients are more likely to have lower productivity, the inflow of terminated DI recipients into the unemployment pool lowers the expected value of hiring workers. Therefore, a firm's incentive to post a vacancy further decreases. In sum, during the recession, the effect of the drop in job-finding probabilities dominates the effect of the increase in the number of people who look for jobs. Therefore, the recession becomes deeper compared to the recession where no change in the frequency of the policy occurs. However, as the economy recovers, job-finding probabilities start to increase because more jobs are posted. Consequently, the effect of the increase in the number of people who look for

[3] It is measured as a proportion of DI recipients who received medical re-evaluations in the given year. Therefore, it can be interpreted as a probability of receiving medical re-evaluations.

jobs outweighs the effect of job-finding probabilities, which accelerates the recovery.

This paper makes several contributions in terms of documentation of facts, model, and quantitative analysis. I document three facts from the U.S. data. First, I show that the recovery in the employment of men from the 1981 recession was faster than any other recovery since 1965. In literature, recoveries before 1990 are considered faster than those after 1990. However, when I look at the employment rate for men, only the recovery from the 1981 recession was faster than any other recovery since 1965. In this sense, the 1981 recession is unique. Second, during the 1981 recession and at the beginning of its recovery, the number of DI applicants and recipients dropped while the numbers increased in all other recoveries. This result is attributed to the fact the most stringent medical re-evaluations occurred between 1981 and 1983. The period 1981-1983 is unique in the sense that the stringency (in terms of both frequency and tightness[4]) of medical re-evaluations was significantly higher than that in other periods. The main change in stringency of the policy during this period was the dramatic increase in frequency, whereas tightness remained high throughout 1978 to 1983. Third, I provide evidence on the importance of medical re-evaluations. When the policy became more stringent, prospective DI applicants were more willing to look for jobs rather than to apply for DI. Drastic variations in frequency and tightness of medical re-evaluations allow us to learn the relation between the decisions for DI applications and the stringency of the policy. Many papers have studied how other DI policies[5] affect the behavior of people, yet the importance of the medical re-evaluation policy has been overlooked.

In terms of model, this paper makes three contributions. First, to the best of my knowledge, this is the first business-cycle model with DI. Second, in my model, all unemployed people, including those who do not have any work limitation can be affected by DI policies through changes in job-finding probabilities (general equilibrium effect). Although many have used empirical methods or structural life-cycle models to study the behavior of prospective DI applicants or of rejected DI applicants, no one has studied the effect of DI policies on people who do not have any work limitation. If the

[4] It is measured as a probability that DI benefits were ceased conditional on the medical re-evaluation

[5] I can group them into three; 1) a policy that makes DI applicants hard to enter the DI program, 2) a policy that affects the amount of DI benefits, and 3) a policy that makes DI recipients hard to maintain their eligibility. Most papers focus on the first two policies.

size of worker flow in and out of the DI program is not negligible, these movements can affect the job-finding probabilities for all unemployed people who look for jobs in the same labor market. This mechanism works in my model through general equilibrium effects. In addition, because DI applications are sensitive to changes in job-finding probabilities, general equilibrium feedback effects are quantitatively relevant. Third, every structural model in literature assumes that DI applicants cannot search for jobs, even though there is no clear evidence for this assumption. The assumption about the behavior of DI applicants is quantitatively relevant because the incentives and timing of DI applications are affected by whether or not they can search for jobs while applying for DI. Therefore, in this paper, DI applicants have an option to search for jobs and collect UI benefits if they are eligible.

The quantitative contribution of this paper is twofold. First, I examine the role of medical re-evaluations on the 1981 recession and its fast recovery. No one has studied the link between the change in DI policies and the fast recovery from the 1981 recession. This paper examines the effect of the increase in frequency of medical re-evaluations during the 1981 recession on the recession and its recovery through 1) a direct effect, namely, an increase in the number of people who look for jobs, and 2) a general equilibrium effect, namely, a change in job-finding probabilities for all unemployed people. The experiment shows that more frequent medical re-evaluations during a recession lead to a deeper recession and a faster recovery.

Second, I use the model to examine the effect of the extended length of time people collect UI benefits in the presence of DI. Without DI, the extended UI benefits decrease employment rate. This is because workers look for jobs less intensively[6] and firms have less incentives to hire workers due to higher wages resulting from higher outside options of workers[7] . However, in the presence of DI, the extended UI benefits could increase employment rates. This is because there is one more important channel in the presence of DI. The extended UI benefits induces more people to look for jobs by delaying their DI applications until UI benefits are expired. According to the experiment, the extension of UI benefits from 26 weeks to 52 weeks leads to a deeper recession and slower

[6] See Nakajima (2012)

[7] See Hagedorn et al. (2015)

recovery. This is because the number of people who look for jobs by delaying DI applications do not increase much while job-finding probabilities remain lower throughout the recession and recovery. However, if the duration is further extended from 52 weeks to 99 weeks, unemployed people who have work limitation are more willing to look for jobs by delaying their DI applications, and this effect dominates the effect of the drop in job-finding probabilities during the recovery. Therefore, the extended UI benefits lead to a faster recovery. This result implies that in the presence of DI, the extended UI benefits during recessions can expedite recoveries if the timing of extension is well designed considering the state of the economy.

This paper is related to several strands of literature on DI. In terms of policy, Moore (2014) examines the employment effect of terminated DI recipients after the 1996 removal of drug and alcohol addictions as qualifying conditions, and finds that the employment effect from the policy change was large. The policy reform in Moore (2014) is similar to that in my paper in the sense that it terminated a subset of DI recipients from the DI program. However, Moore (2014) uses empirical methods to study the employment effect only for terminated DI recipients. In my paper, DI policies affect all the unemployed, including people who have no work limitation, through a change in job-finding probabilities. In terms of models with DI, several papers have a structural life-cycle model with DI and search friction in the labor market for steady state analysis. Benitez-Silva et al. (2011) study the effect of a policy that induces DI recipients to return to work. Low and Pistaferri (2012) estimate the disability risks that individuals face and the parameters governing the DI program. Kitao (2014) and Kim (2014) examine the role of Medicare in the DI program on the life-cycle labor supply. In aforementioned papers, job-finding probabilities are fixed because they do not model firms. In contrast, this paper builds a general equilibrium business-cycle search and matching model with DI, where DI application decisions are affected by changes in job-finding probabilities over the business cycle. This paper is also related to literature on the relationship between UI policies and DI applications. Mueller et al. (2015) identify the effect of UI exhaustion on DI application and find no evidence that expiration of UI benefits causes DI applications. Lindner and Nichols (2012) examine whether or not participating in temporary assistance programs, including UI, influences DI applications, and find evidence that increased access to UI benefits reduces DI applications. Rutledge (2012) empirically investigates the effect of UI extensions on DI applications, and whether UI eligibility, extension, and exhaustion affect the timing of DI applications. Rutledge (2012) finds that jobless individuals are significantly less likely to apply for DI during UI extensions, and significantly more likely to apply when UI is exhausted. My paper examines the role of extended UI in the presence of DI during a recession with a general equilibrium model where DI applications are affected by UI policies as well as changes in job-finding probabilities.

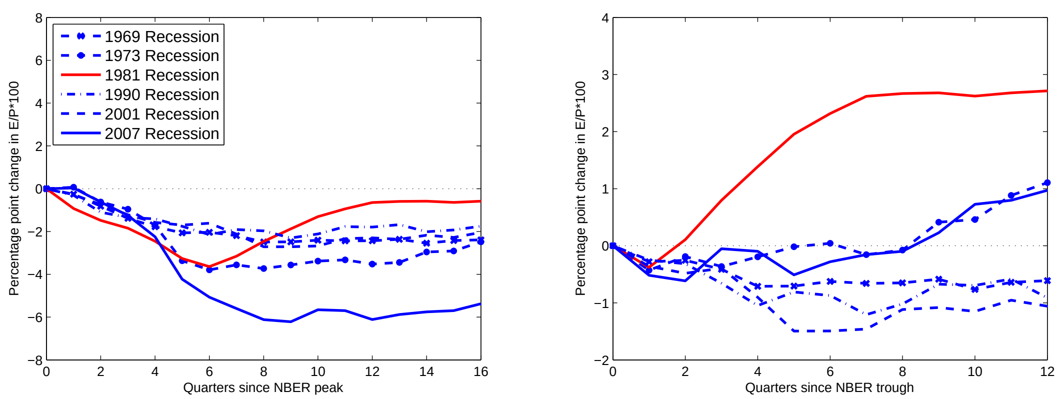
This paper proceeds as follows. Section 2 documents facts about the 1981 recession, its fast recovery, and medical re-evaluations. Section 3 describes the model and Section 4 presents the calibration. Section 5 shows the results of quantitative analysis. Lastly, Section 6 concludes.

1.2 Facts

In this section, I document several facts about the 1981 recession, its fast recovery, and medical re-evaluations.

1.2.1 Fast recovery in the employment rates from the 1981 recession

Figure 1.1: Recessions and recoveries in the employment rate of men since 1965



(a) Recessions and recoveries

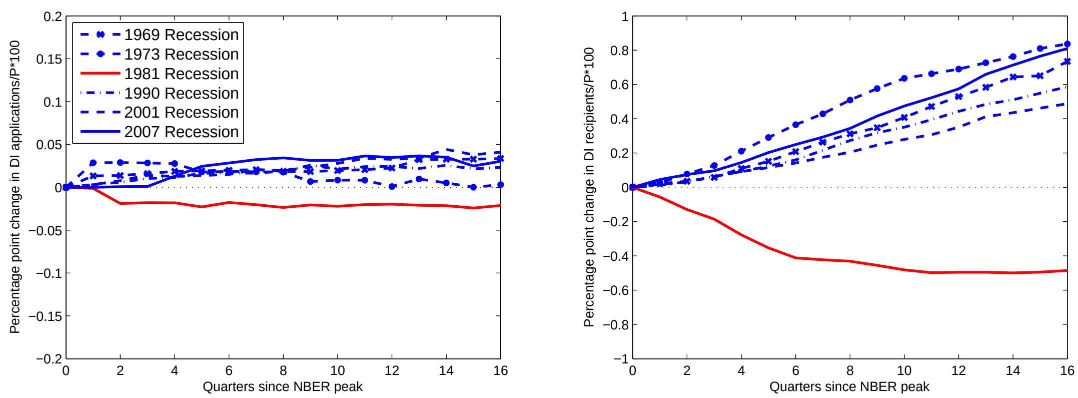
(b) Recoveries

Note: Figure 1.1 shows percentage point changes in the employment rate of men (age 25-64). All series are computed from the monthly CPS and they are not filtered.

In literature, recoveries before 1990 are considered faster than those after 1990. However, when I look at the employment rate for men, only the recovery from the 1981 recession was faster than any other recovery since 1965. In this sense, the recovery from the 1981 is unique. Figure 1.1(a) shows percent point changes in the employment rate[8] of men[9] since NBER peak[10] and Figure 1.1(b) shows percent point changes in the employment rate of men since NBER trough. From Figure 1.1(b) we can clearly see that the recovery from the 1981 recession was significantly faster than others.

1.2.2 Most stringent medical re-evaluation policy during 1981-1983

Figure 1.2: DI applicants and DI recipients



(a) DI applicants/Population

(b) DI recipients/Population

Note: Figure 1.2 shows percentage point changes in DI applicants per population and DI recipients per population for men. All series are computed from the public Social Security Administration (SSA) data. Since the SSA do not publish DI application data by sex, DI applications for men are calculated with a total number of monthly DI applications and a share of men in DI awards each month.

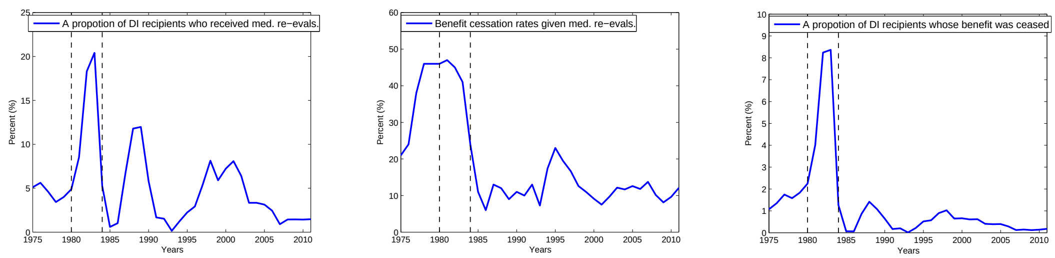
During the 1981 recession and at the beginning of its recovery, the number of DI applicants and recipients dropped while the numbers increased in all other recoveries as we can see in Figure 1.2. The period 1981-1983 is unique in the sense that stringency of

[8] The definition of the employment rate in this paper is the employment-population ratio.

[9] The recovery from the 1981 recession is still fastest when I look at data including women as in Figure A.1 in Appendix A. The reason why I only look at data for men is that woman's labor force participation steadily increased between 1970s and 1980s and I want to control this factor in the analysis.

[10] I exclude the 1980 recession because it was shortly followed by the 1981 recession.

Figure 1.3: Three measures for stringency of medical re-evaluations



(a) Frequency (b) Tightness (c) Frequency and tightness

Note: Figure 1.3 shows three measures of stringency of medical re-evaluations. They are calculated with the data from Government Accountability Office (GAO) reports (1997) and Annual Report of Continuing Disability Review (2011).

medical re-evaluations was significantly higher than that in other periods. I define three different measures for stringency of medical re-evaluations. Figure 1.3(a) shows the frequency of medical re-evaluations, which is measured as a proportion of DI recipients who received medical re-evaluations in the given year. The annual frequency increased from 4.9% in 1980 to 20.4% in 1983 mainly due to the Social Security Disability Amendments of 1980. Before the amendments, medical re-evaluations were conducted only for selected DI recipients whose medical condition was expected to be improved. However, after the amendments, the Congress required SSA to conduct medical re-evaluations on all DI recipients at least once every three years except for DI recipients expected to be permanently disabled. Therefore, during 1981-1983 approximately 1.2 million medical re-evaluations were conducted and benefits of 0.5 million recipients were ceased[11] . This stringent medical re-evaluations led to public outcry which resulted in a nation-wide moratorium on medical re-evaluations during 1983-1984 and the Social Security Disability Benefits Reform of 1984. In 1985, medical re-evaluations resumed on a gradual basis, employing the new medical improvement review standard mandated by the Congress in the 1984 Amendments. However, the frequency of medical re-evaluation varied based on budget availability afterward. Figure 1.3(b) shows the tightness of medical re-evaluations, which is measured as a probability that DI benefits were ceased conditional on the medical re-evaluation. The tightness was highest throughout 1978 to

[11] Those numbers include both men and women.

1983. After the reform in 1984, tightness significantly dropped from 41% in 1983 to 11% in 1985 because the newly introduced medical review standard made SSA harder terminate the benefits of DI recipients. Finally, Figure 1.3(c) shows both the frequency and tightness of medical re-evaluations, which is measured as a a proportion of DI recipients whose benefit was ceased by the medical re-evaluation. We can see a big spike during 1981-1983, which shows the most stringent medical re-evaluations occurred during this period. The main change in stringency of the policy during this period was the dramatic increase in frequency, whereas tightness remained high throughout 1978 to 1983.

1.2.3 Importance of medical re-evaluations

Table 1.1: Correlations between stringency of medical re-evaluations and behavior of the unemployed

1994-2006 (excluding recessions)	U w/ work limitation	U w/o work limitation
Corr(stringency, DI applications/pop)	-0.51	
Corr(stringency, Pr[U→E])	0.47	-0.02

Note: Stringency of medical re-evaluations denotes the third measure of stringency in Figure 1.3(c). DI applications are computed from the public SSA data. Transition probabilities from the unemployed to the employed by health status (self-reported work limitation) are calculated from the March CPS (men, 25-64). I choose the period 1994-2006 because there were major changes in the CPS in 1994. I exclude the recession periods because DI applications and the transition probabilities are sensitive to the recessions. However, when we include the recession periods, the signs and magnitudes are similar to the numbers in Table 1.1. I am working on the same table with the PSID so that I can use samples before 1994.

Many papers have studied how other DI policies[12] affect the behavior of people, yet the importance of the medical re-evaluation policy has been overlooked. I document that when the policy became more stringent, prospective DI applicants were more willing to look for jobs rather than to apply for DI. Drastic variations in frequency and tightness of medical re-evaluations allow us to learn the relation between the decisions for DI application and the stringency of the policy. Table 1.1 shows correlations between stringency of medical re-evaluations and behavior of prospective DI applicants and of the

[12] I can group them into three; 1) a policy that makes DI applicants hard to enter the DI program, 2) a policy that affects the amount of DI benefits, and 3) a policy that makes DI recipients hard to maintain their eligibility. Most papers focus on the first two policies.

unemployed. The correlation between the stringency of the policy and DI application per population is -0.51. The correlation between the policy and the transition probabilities from the unemployed to the employed for the unemployed who have work limitation is 0.47 whereas the correlation for the unemployed who have no work limitation is almost zero. This implies when the medical re-evaluations became more stringent, prospective DI applicants were more willing to look for jobs rather than to apply for DI.

1.3 Model

1.3.1 Environment

The model period is assumed to be a month. The economy consists of a continuum of risk-neutral workers and firms. The total measure of workers is normalized to one. Workers have an ex-ante heterogeneous individual productivity $x \in [\underline{x}, \overline{x}]$ which does not change over time. At the beginning of each period, workers receive a health shock (in terms of work limitation) γ. After receiving a health shock, employed people can quit in order to apply for DI. Unemployed people decide whether to apply for disability insurance (DI), and then choose whether to search for jobs while collecting unemployment insurance (UI) benefits b^U if they are eligible. DI applicants[13] must wait for 5 months until the acceptance decision is made. During this period, they can also search for jobs if they want, and collect UI benefits[14] . They can be accepted with probability $\pi_a(\gamma)$ which depends on the level of work limitation and if they are accepted they start to collect DI benefits b^D and do not search for jobs. At the beginning of each period, DI recipients can voluntarily leave the program to find jobs and if they choose to stay, at the end of the period they receive medical re-evaluations with probability π_r[15] and their DI benefits can be terminated with probability $\pi_t(\gamma)$. Each firm hires only one worker and firms do not know the worker's individual productivity and health status until they

[13] People who have applied for DI

[14] In reality, if DI applicants collect UI benefits, it might lower the probability of being accepted to the DI program. In the current version of the model, I assume that collecting UI benefits during the 5-month waiting period does not affect the probability of being accepted to the DI program.

[15] For more appropriate analysis, I should make the probability of receiving medical re-evaluations depend on the health status when DI recipients were accepted to the DI program as in reality. I can model it, but it is not easy to pin down these parameters due to lack of data. For this reason, I assume that this probability is the same for every DI recipient, but this assumption can be relaxed if I find proper targets later.

meet. Therefore, search is random in the sense that all types of workers compete in the same labor market and wages are determined by Nash Bargaining. I assume that wages does not depend on a worker's DI application status a, and months after DI application m for simplicity. The number of new matches is determined by the matching function $M = M(U, V)$. I can define the market tightness $\theta \equiv \frac{V}{U}$, job-finding probability for workers $p(\theta) \equiv \frac{M(U,V)}{U}$, and job-filling probability for firms $q(\theta) \equiv \frac{M(U,V)}{V}$. Firms can enter the market by posting a vacancy at the cost of κ.

1.3.2 Timing of the model

1. Aggregate labor productivity shocks and health shocks are realized
2. A worker's decision is made:
   - The employed decide to quit
   - The unemployed decide whether to apply for DI, then choose whether to search for jobs
   - DI recipients decide whether to leave the DI program to find jobs
3. Production takes place and vacancies are posted / search and matching occurs
4. DI acceptance decision is made and DI recipients are terminated through medical re-evaluations
5. The employed are exogenously separated

1.3.3 Worker's problem

The individual states of a worker are represented by (l, γ, a, m, e). $l \in \{E, U, D\}$ represents labor force status which includes the employed (E), the unemployed (U), and DI recipients (D) who are not in the labor force. $\gamma \in \{\gamma_n, \gamma_m, \gamma_s\}$ denotes the level of work limitation which lowers the individual productivity of the worker by γ. γ_n, γ_m, and γ_s denote no work limitation, moderate work limitation, and severe work limitation, respectively. Since DI applicants must wait for 5 months until the acceptance decision is made, I should keep track of DI application status $a \in \{0, 1\}$, and months after DI application $m \in \{1, 2, 3, 4, 5\}$. Lastly, $e \in \{0, 1\}$ indicates whether a worker is eligible for UI benefits or not.

The aggregate states of the economy are represented by (z, ψ) where z is an aggregate labor productivity and ψ is a measure of workers. Workers and firms should keep track of the measure of workers ψ because heterogeneous workers search for jobs in the same market and they are randomly matched to firms, which makes the job-finding probability $p(\theta(z,\psi))$ and job-filling probability $q(\theta(z,\psi))$ depend on the measure of workers ψ as well as on the aggregate labor productivity z.

Since workers have a different individual productivity x, every value function is indexed by x.

Employed workers: Employed workers can quit in order to apply for DI at the beginning of each period.

$$W^x(E,\gamma,0,0,e;z,\psi) \quad = \quad \max\Big[\underbrace{W_c^x(E,\gamma,0,0,e;z,\psi)}_{work},\underbrace{W^x(U,\gamma,0,0,e;z,\psi)}_{quit}\Big]$$

If they choose to work, they can be exogenously separated with probability χ at the end of the period and become unemployed. The stochastic process of the level of work limitation γ is governed by a transition probability matrix Π^γ. The employed without UI eligibility stochastically become eligible and the stochastic process is governed by a transition probability matrix Π_E^e. Wages depend on the individual productivity x, the level of work limitation γ, and UI eligibility e, which will be described later more in detail in the calibration section. I assume disutility from labor force participation $c_p(\gamma)$ for the employed and the unemployed who search for jobs.

$$\begin{aligned}
W_c^x(E,\gamma,0,0,e;z,\psi) \quad &= \quad w^x(\gamma,e,\theta(z,\psi)) - c_p(\gamma) \\
(\textit{not separated}) \quad &+ \quad \beta E_{z,\gamma,e}\left[(1-\chi)W^x\left(E,\gamma',0,0,e';z',\psi'\right)\right. \\
(\textit{separated}) \quad &+ \quad \left.\chi W^x\left(U,\gamma',0,0,e';z',\psi'\right)\right] \\
s.t. \quad & \\
& \quad log\, z' = \rho\, log\, z + \varepsilon',\ \psi' = T(z,\psi),\ \gamma' = \Pi^\gamma(\gamma),\ e' = \Pi_E^e(e)
\end{aligned}$$

Unemployed workers: Unemployed workers decide whether to apply for DI at an application cost of c_a at the beginning of the period.

$$W^x(U,\gamma,0,0,e;z,\psi) \quad = \quad \max\Big[\underbrace{W_c^x(U,\gamma,0,0,e;z,\psi)}_{not\,apply},\underbrace{-c_a + W_a^x(U,\gamma,1,1,e;z,\psi)}_{apply\ for\ DI}\Big]$$

Once they made the decision for DI application, they choose whether to search for jobs.

1) Unemployed workers who have not applied for DI

$$W_c^x(U,\gamma,0,0,e;z,\psi) \quad = \quad \max\Big[\underbrace{W_{c,ns}^x(U,\gamma,0,0,e;z,\psi)}_{not\,search},\underbrace{W_{c,s}^x(U,\gamma,0,0,e;z,\psi)}_{search}\Big]$$

If they do not search, then they just wait for one month.

$$W_{c,ns}^x(U,\gamma,0,0,e;z,\psi) \quad = \quad \beta E_{z,\gamma,e}\left[W^x\left(U,\gamma',0,0,e';z',\psi'\right)\right]$$

If they search for jobs, they collect UI benefits b^U if they are eligible even while applying for DI. If they find a job at the end of the period, they become employed. Otherwise, they remain unemployed. The unemployed with UI eligibility stochastically loose their eligibility and the stochastic process is governed by a transition probability matrix Π_U^e. $\mathbb{I}_{(e=1)}$ is an indicator function which has 1 if they are eligible for UI benefits.

$$\begin{aligned}
W_{c,s}^x\left(U,\gamma,0,0,e;z,\psi\right) &= b^U\left(x,\gamma\right)\mathbb{I}_{(e=1)} - c_p\left(\gamma\right) \\
(find\ a\ job) \quad &+ \beta E_{z,\gamma,e}\left[p\left(\theta\left(z,\psi\right)\right)W^x\left(E,\gamma^{'},0,0,e^{'};z^{'},\psi^{'}\right)\right. \\
(not\ find\ a\ job) \quad &+ \left.\left(1-p\left(\theta\left(z,\psi\right)\right)\right)W^x\left(U,\gamma^{'},0,0,e^{'};z^{'},\psi^{'}\right)\right] \\
&s.t. \\
&\quad log\, z^{'} = \rho\, log\, z + \varepsilon^{'},\ \psi^{'} = T\left(z,\psi\right),\ \gamma^{'} = \Pi^{\gamma}(\gamma),\ e^{'} = \Pi_U^e(e)
\end{aligned}$$

2) Unemployed workers who have applied for DI

$$W_a^x\left(U,\gamma,1,m\in\{1,2,3,4,5\},e;z,\psi\right) = \max\Big[\underbrace{W_{a,ns}^x\left(U,\gamma,1,m,e;z,\psi\right)}_{not\,search},\underbrace{W_{a,s}^x\left(U,\gamma,1,m,e;z,\psi\right)}_{search}\Big]$$

Before the 5th month of DI application, if they do not search, they just wait for one month as DI applicants.

$$W_{a,ns}^x\left(U,\gamma,1,m\in\{1,2,3,4\},e;z,\psi\right) = \beta E_{z,\gamma,e}\left[W_a^x\left(U,\gamma^{'},1,m+1,e^{'};z^{'},\psi^{'}\right)\right]$$

At the 5th month of DI application, if they are accepted, they become DI recipients. Otherwise, they remain unemployed.

$$\begin{aligned}
W_{a,ns}^x\left(U,\gamma,1,m=5,e;z,\psi\right) &= \\
(accepted) \quad & \beta E_{z,\gamma}\left[\pi_a\left(\gamma\right)W^x\left(D,\gamma^{'},0,0,e^{'};z^{'},\psi^{'}\right)\right] \\
(not\,accepted) \quad &+ \left(1-\pi_a\left(\gamma\right)\right)W^x\left(U,\gamma^{'},0,0,e^{'};z^{'},\psi^{'}\right)\Big]
\end{aligned}$$

Before the 5th month of DI application, if they search and find a job at the end of the period, they become employed or keep waiting for the decision. Otherwise, they remain DI applicants.

$$\begin{aligned}
W_{a,s}^x\left(U,\gamma,1,m\in\{1,2,3,4\},e;z,\psi\right) &= b^U\left(x,\gamma\right)\mathbb{I}_{(e=1)} - c_p\left(\gamma\right) \\
(find\ a\ job) \quad &+ \beta E\left[p\left(\theta\left(z,\psi\right)\right)\max\left[W^x\left(E,\gamma^{'},0,0,e^{'};z^{'},\psi^{'}\right),\right.\right. \\
&\qquad \left. W_a^x\left(U,\gamma^{'},1,m+1,e^{'};z^{'},\psi^{'}\right)\right] \\
(not\ find\ a\ job) \quad &+ \left.\left(1-p\left(\theta\left(z,\psi\right)\right)\right)W_a^x\left(U,\gamma^{'},1,m+1,e^{'};z^{'},\psi^{'}\right)\right] \\
&s.t. \\
&\quad log\, z^{'} = \rho\, log\, z + \varepsilon^{'},\ \psi^{'} = T\left(z,\psi\right),\ \gamma^{'} = \Pi^{\gamma}(\gamma), \\
&\quad e^{'} = \Pi_U^e(e)
\end{aligned}$$

At the 5th month of DI application, if they are accepted and find a job at the same time, they choose whether to become employed or DI recipients. If they are accepted but they do not get a job, they become DI recipients. If they are not accepted but find a job, they become employed. Lastly, if they are not accepted and do not find a job, they remain unemployed.

$$
\begin{aligned}
W^x_{a,s}(U,\gamma,1,5,e;z,\psi) &= b^U(x,\gamma)\,\mathbb{I}_{(e=1)} - c_p(\gamma) \\
(\textit{accepted \& find a job}) \quad &+ \beta E\Big[\pi_a(\gamma)\,p(\theta(z,\psi))\max\Big[W^x\left(E,\gamma',0,0,e';z',\psi'\right), \\
&, \qquad W^x\left(D,\gamma',0,0,e';z',\psi'\right)\Big] \\
(\textit{accepted \& not find a job}) \quad &+ \pi_a(\gamma)(1-p(\theta(z,\psi)))\,W^x\left(D,\gamma',0,0,e';z',\psi'\right) \\
(\textit{not accepted \& find a job}) \quad &+ (1-\pi_a(\gamma))\,p(\theta(z,\psi))\,W^x\left(E,\gamma',0,0,e';z',\psi'\right) \\
(\textit{not accepted \& not find a job}) \quad &+ (1-\pi_a(\gamma))(1-p(\theta(z,\psi)))\,W^x\left(U,\gamma',0,0,e';z',\psi'\right)\Big] \\
&s.t. \\
&\quad log\, z' = \rho\, log\, z + \varepsilon',\ \psi' = T(z,\psi),\ \gamma' = \Pi^{\gamma}(\gamma),\ e' = \Pi^e_U(e)
\end{aligned}
$$

DI recipients: DI recipients decide whether to stay in the DI program or voluntarily leave it to find jobs at the beginning of the period.

$$
W^x(D,\gamma,0,0,e;z,\psi) = \max\Big[\underbrace{W^x_c(D,\gamma,0,0,0;z,\psi)}_{stay\ in\ DI}, \underbrace{W^x(U,\gamma,0,0,e;z,\psi)}_{leave\ DI}\Big]
$$

If they choose to stay, they do not search for jobs while collecting DI benefits b^D. At the end of the period, they receive medical re-evaluations with probability π_r , and conditional on the medical re-evaluation, their benefit can be terminated with probability $\pi_t(\gamma)$. Once they start to collect DI benefits, they loose their UI eligibility with probability 1.

$$
\begin{aligned}
W^x_c(D,\gamma,0,0,0;z,\psi) &= b^D(x) \\
(\textit{terminated}) \quad &+ \beta E_{z,\gamma}\Big[\pi_r\pi_t(\gamma)\,W^x\left(U,\gamma',0,0,0;z',\psi'\right) \\
(\textit{not terminated}) \quad &+ (1-\pi_r\pi_t(\gamma))\,W^x\left(D,\gamma',0,0,0;z',\psi'\right)\Big] \\
&s.t. \\
&\quad log\, z' = \rho\, log\, z + \varepsilon',\ \psi' = T(z,\psi),\ \gamma' = \Pi^{\gamma}(\gamma)
\end{aligned}
$$

If they choose to leave the program, they become unemployed.

1.3.4 Firm's problem

The individual states of a firm are represented by (γ, e) and the aggregate states are represented by (z,ψ). Each firm hires only one worker. Firms do not know the worker's individual productivity and the level of work limitation until they meet.

Firms matched with (x,γ,e)-type workers: At the end of the period, a worker can be exogenously separated or endogenously separated by quitting. Firms take the worker's decision for quitting s' as given. If the worker is separated, the firm becomes unmatched.

$$\begin{aligned} J^{x}(\gamma,e;z,\psi) &= zx(1-\gamma)-w^{x}(\gamma,e,\theta(z,\psi)) \\ (\textit{not separated}) &+ \beta E\Big[(1-\chi)\left(1-s^{'}\right)J^{x}\left(\gamma^{'},e^{'};z^{'},\psi^{'}\right) \\ (\textit{separated}) &+ \left(1-(1-\chi)\left(1-s^{'}\right)\right)V\left(z^{'},\psi^{'}\right)\Big] \end{aligned}$$

s.t.

$$log\, z^{'}=\rho\, log\, z+\varepsilon^{'},\ \psi^{'}=T(z,\psi),\ \gamma^{'}=\Pi^{\gamma}(\gamma),\ e^{'}=\Pi_{E}^{e}(e)$$

$$s^{'}=g_{q}^{x}\left(E,\gamma^{'},0,0,e^{'};z^{'},\psi^{'}\right)=\begin{cases}1 & if\ W_{c}^{x}\left(E,\gamma^{'},0,0,e^{'};z^{'},\psi^{'}\right)<W^{x}\left(U,\gamma^{'},0,0,e^{'};z^{'},\psi^{'}\right)\\ 0 & if\ otherwise\end{cases}$$

Unmatched firms: Since firms do not know the worker's individual productivity x and the level of work limitation γ, they have to take into account a type distribution of the unemployed who search for jobs when they decide to enter the market.

$$\begin{aligned} V(z,\psi) &= -\kappa \\ (\textit{matched}) &+ \beta\Big[q(\theta(z,\psi))\int E_{z,\gamma,e}\left[\left(1-s^{'}\right)J^{x}\left(\gamma^{'},e^{'};z^{'},\psi^{'}\right)\right]\times \\ & \quad \frac{\psi_{s}(x,U,\gamma,a,m,e)}{\int\psi_{s}(x,U,\gamma,a,m,e)\,d(x,U,\gamma,a,m,e)}d(x,U,\gamma,a,m,e) \\ (\textit{not matched}) &+ (1-q(\theta(z,\psi)))E_{z}\left[V\left(z^{'},\psi^{'}\right)\right]\Big] \\ s.t. & \\ & log\, z^{'}=\rho\, log\, z+\varepsilon^{'},\ \psi^{'}=T(z,\psi),\ \gamma^{'}=\Pi^{\gamma}(\gamma),\ e^{'}=\Pi_{U}^{e}(e) \\ & g_{s}^{x}(U,\gamma,a,m,e;z,\psi)\ \text{ is a decision rule for searching for jobs} \\ & \psi_{s}(x,U,\gamma,a,m,e)=\mathbb{I}_{\left(g_{s}^{x}(U,\gamma,a,m,e;z,\psi)=1\right)}\psi(x,U,\gamma,a,m,e) \end{aligned}$$

Free entry condition: With the free entry condition $V(z,\psi)=0$, we have

$$\begin{aligned} \kappa &= \beta q(\theta(z,\psi))\int E_{z,\gamma,e}\left[\left(1-s^{'}\right)J^{x}\left(\gamma^{'},e^{'};z^{'},\psi^{'}\right)\right]\times \\ & \frac{\psi_{s}(x,U,\gamma,a,m,e)}{\int\psi_{s}(x,U,\gamma,a,m,e)\,d(x,U,\gamma,a,m,e)}d(x,U,\gamma,a,m,e) \end{aligned}$$

$$\text{where } \psi_{s}(x,U,\gamma,a,m,e)=\mathbb{I}_{\left(g_{s}^{x}(U,\gamma,a,m,e;z,\psi)=1\right)}\psi(x,U,\gamma,a,m,e)$$

Note that the market tightness $\theta(z,\psi)$ depends on the measure of workers ψ as well as on the aggregate labor productivity z.

1.3.5 Nash bargained wages

Wages are determined by Nash bargaining problem.

$$w^x(\gamma, e, \theta(z,\psi)) \quad = \quad arg\max_w \left(W_c^x(E,\gamma,0,0,e;z,\psi) - W^x(U,\gamma,0,0,e;z,\psi)\right)^{\mu} \left(J^x(\gamma,e;z,\psi)\right)^{1-\mu}$$

Or equivalently,

$$w^x(\gamma, e, \theta(z,\psi)) \quad s.t. \quad (1-\mu)\left(W_c^x(E,\gamma,0,0,e;z,\psi) - W^x(U,\gamma,0,0,e;z,\psi)\right) = \mu\left(J^x(\gamma,e;z,\psi)\right)$$

I assume that wages do not depend on a worker's DI application status a, and months after DI application m for simplicity. μ denotes the bargaining power of workers.

1.3.6 Equilibrium

Definition (Recursive Competitive Equilibrium): A recursive competitive equilibrium is a set of value functions for workers $W^x(l,\gamma,0,0,e;z,\psi)$,$W_c^x(l,\gamma,0,0,e;z,\psi)$, $W_{c,ns}^x(U,\gamma,0,0,e;z,\psi)$,$W_{c,s}^x(U,\gamma,0,0,e;z,\psi)$,$W_{a,ns}^x(U,\gamma,a,m,e;z,\psi)$, $W_{a,s}^x(U,\gamma,a,m,e;z,\psi)$,value functions for firms, $J^x(\gamma,e;z,\psi)$,$V(z,\psi)$, decision rules for quitting $g_q^x(E,\gamma,0,0,e;z,\psi)$,applying for DI $g_a^x(U,\gamma,0,0,e;z,\psi)$, searching for jobs $g_s^x(U,\gamma,a,m,e;z,\psi)$, leaving the DI program $g_l^x(D,\gamma,0,0,e;z,\psi)$, the market tightness $\theta(z,\psi)$, wages $w^x(\gamma,e,\theta(z,\psi))$, and a law of motion for the measure $\psi' = T(z,\psi)$ such that:

1. Given the market tightness and wages, decision rules for workers solve the worker's problems.
2. The market tightness is consistent with the free entry condition.
3. Wages are the solutions to the Nash bargaining problem.
4. The law of motion for the measure is consistent with optimal decision rules and stochastic processes of z, γ, and e.

1.4 Calibration

I assume the following matching function, a variant of Haan et al. (2000)

$$M(u,v) = \phi_1 \frac{uv}{\left(u^{\phi_2} + v^{\phi_2}\right)^{1/\phi_2}}$$

I put an additional scale parameter ϕ_1 as in Wiczer (2014) because without the scale parameter it is difficult to match both the level of the job-finding probability and the elasticity of job-finding probabilities with respect to the market tightness in data. The transition probability matrix of UI eligibility for the employed Π_E^e and the unemployed Π_U^e are assumed to be

$$\Pi_E^e = \begin{bmatrix} 1-\pi_E^{0,1} & \pi_E^{0,1} \\ 0 & 1 \end{bmatrix}, \ \Pi_U^e = \begin{bmatrix} 1 & 0 \\ \pi_U^{1,0} & 1-\pi_U^{1,0} \end{bmatrix}$$

The employed without UI eligibility stochastically become eligible with probability $\pi_E^{0,1}$ and the unemployed with UI eligibility stochastically lose their eligibility with probability $\pi_U^{1,0}$.

1.4.1 Predetermined parameters

Table 1.2: Predetermined parameters

Parameter	Description	Value	Remark
β	Discount factor	0.9967	Annual interest rate = 4%
μ	Bargaining power of workers	0.50	
ϕ_2	Elasticity param. in the matching function	1.6	Schaal (2012)
$\pi_U^{1,0}$	Prob. of losing UI eligibility for U	0.1538	Duration of UI: 26 weeks[16]
$\pi_a(\gamma_n)$	Prob. of acceptance for no WL	0.00	
ρ	Persistence of aggregate labor prod.	0.97	Hagedorn & Manovskii (2011)
σ_ϵ	Standard dev. of aggregate labor prod.	0.006	Hagedorn & Manovskii (2011)

I choose a monthly discount factor β of 0.9967 which implies that the annualized interest rate is 4%. The bargaining power of workers is set at 0.50. The elasticity parameter in the matching function is set at 1.6 as in Schaal (2012). The probability of losing UI eligibility for the unemployed is set at 0.1583 which implies that the average length of time people collect UI benefits is 26 weeks. I assume that workers who do not have work limitation cannot be accepted to the DI program. I use the same shock process for the aggregate labor productivity as that in Hagedorn and Manovskii (2011). The amount of unemployment benefits $b^U(x,\gamma)$ is determined by the following equation.

$$b^U(x,\gamma) = 0.40 \times w^x\left(\gamma, 1, \overline{\theta}\right)$$

The replacement rate of UI benefits is set at 0.40 as in Shimer (2005) and the steady state wage $w\left(x,\gamma,1,\overline{\theta}\right)$ is used when I calculate the benefits. DI benefits $b^D(x)$ is calculated by the same formula as the Social Security benefits. First, I need to compute the Average Indexed Monthly Earnings (AIME) which is the average of past highest earnings up to 35 years. Since the model does not have a life-cycle structure, I assume that the AIME can be approximated by the steady state wage for people who do not have work limitation and have UI eligibility, which is the highest wage among x-type workers in the model

$$\text{Average Indexed Monthly Earnings (AIME)} \equiv w^x\left(\gamma_n, 1, \overline{\theta}\right)$$

Based on the AIME, I compute the Primary Insurance Amount (PIA) by the following formula.

$$\text{PIA} = \begin{cases} 0.9 \times \text{AIME} & if\ \text{AIME} \leq \$316.25 \\ \$284.63 + 0.32 \times (\text{AIME} - \$316.25) & if\ \$316.25 < \text{AIME} \leq \$1905.50 \\ \$793.19 + 0.15 \times (\text{AIME} - \$1905.50) & if\ \$1905.05 < \text{AIME} \end{cases}$$

I use the bend points in 1986 because every nominal wage in the calibration is discounted by the Consumer Price Index (CPI) given the base year 1986. Finally, the PIA is capped by the maximum amount of benefits which depends on the PIA. I also use the bend points in 1986.

$$\text{Max of benefits} = \begin{cases} 1.5 \times \text{PIA} & if\ \text{PIA} \leq \$403.75 \\ \$605.63 + 2.72 \times (\text{PIA} - \$403.75) & if\ \$403.75 < \text{PIA} \leq \$583.25 \\ \$1093.87 + 1.34 \times (\text{PIA} - \$583.25) & if\ \$583.25 < \text{PIA} \leq \$760.50 \\ \$1331.38 + 1.75 \times (\text{PIA} - \$760.50) & if\ \$760.50 < \text{PIA} \end{cases}$$

Therefore, the DI benefits $b^D(x)$ in this paper can be expressed by

$$b^D(x) = \min\left[\text{PIA, Max of benefits}\right]$$

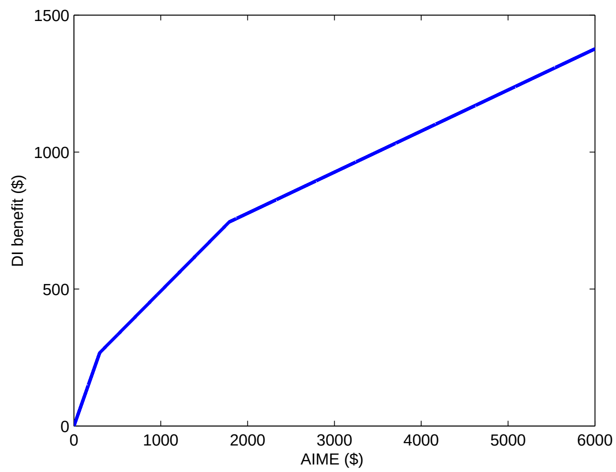
Figure 1.4 summarizes the relation between the AIME and DI benefits in the model.

1.4.2 Parameters estimated outside of the model

I define a discrete variable for health status (the level of work limitation) following Low and Pistaferri (2012). In the PSID, people are asked three different questions about their work limitation:

1. Do you have any physical or nervous condition that limits the type of work or the amount of work you can do? (possible answers: yes / no)
   : If the answer is yes, then interviewer asks the second question.

Figure 1.4: Average Indexed Monthly Earnings (AIME) and DI benefits in the model



2. Does this condition keep you from doing some types of work? (possible answers: yes / no / can do nothing)
   : If the answer is yes or no, then interviewer asks the third question.

3. For what work you can do, how much does it limit the amount of work you can do? (possible answers: a lot / somewhat / just a little / not at all)

Table 1.3 shows how I define the level of work limitation based on the three questions above.

Table 1.3: Definition of the level of work limitation

	Ans. to the 1st Q.	Ans. to the 2nd Q.	Ans. to the 3rd Q.
No work limitation	No	-	-
	Yes	Yes / no	Not at all
Moderate work limitation	Yes	Yes / no	Somewhat / just a little
Severe work limitation	Yes	Can do nothing	-
	Yes	Yes / no	A lot

The monthly transition matrix for health status is estimated from the PSID by using the discrete variable for work limitation in Table 1.3. Given the assumption that a shock process for work limitation is stable throughout a year, an annual transition matrix Π_A^γ can be directly estimated from the PSID for 1986-1992[17] . Once the annual transition

[17] Data for DI recipients along with other variables are only available for this period.

matrix is estimated, it can be converted to the monthly transition matrix Π^{γ} such that $(\Pi^{\gamma})^{12} = \Pi_A^{\gamma}$. Table 1.4 represents the estimated monthly transition matrix Π^{γ}.

Table 1.4: Monthly transition matrix for health status

	No work lim.	Moderate work lim.	Severe work lim.
No work limitation	0.995	0.004	0.001
Moderate work limitation	0.048	0.935	0.017
Severe work limitation	0.015	0.031	0.954

1.4.3 Parameters calibrated in the model

17 remaining parameters are calibrated to match key features of the US economy for 1986-1992 by using 17 targets. 11 parameters among 17 parameters are related to health and the DI program: the penalty rate of productivity with moderate work limitation γ_m and severe work limitation γ_s, probability of acceptance for moderate work limitation $\pi_a(\gamma_m)$ and severe work limitation $\pi_a(\gamma_s)$, probability of termination given medical re-evaluation for no work limitation $\pi_t(\gamma_n)$, moderate work limitation $\pi_t(\gamma_m)$, and severe work limitation $\pi_t(\gamma_s)$, disutility from labor force participation for no work limitation $c_p(\gamma_n)$, moderate work limitation $c_p(\gamma_m)$, severe work limitation $c_p(\gamma_s)$, and DI application cost c_a. 6 remaining parameters are related to the labor market: exogenous separation rate χ, a minimum value of individual productivity $\underline{x}$, a maximum value of individual productivity $\overline{x}$, the scale parameter in the matching function ϕ_1, cost of posting vacancies κ, and probability of getting UI eligibility for the employed $\pi_E^{0,1}$.

Targets

Table 1.5: Distribution of labor force status (2 targets)

Employed/Population	Unemployed/Population	DI recipients/Population	Sum
0.917	0.049	0.034*	1

Note: table 5 shows a distribution of labor force status for 1986-1992. The numbers of the employed, the unemployed, and population are calculated from the monthly CPS (men, 25-64). The number of DI recipients is calculated from the public SSA data. *: it is not used because the distribution of labor force status adds up to one.

Table 1.6: Distribution of health status by labor force status (6 targets)

	No work lim.	Moderate work lim.	Severe work lim.	Sum
Employed	0.935	0.055	0.010^{*}	1
Unemployed	0.775	0.185	0.040^{*}	1
DI recipients	0.148	0.189	0.663^{*}	1

Note: table 6 shows distribution of health status by labor force status for 1986-1992. The numbers are calculated from the PSID (men, 25-64). *: they are not used because the distribution of health status given a specific labor force status adds up to one.

First, I use a distribution of labor force status (Table 1.5) and a distribution of health status among the employed, the unemployed, and DI recipients (Table 1.6). Table 1.7 shows the rest of targets.

Table 1.7: Other targets (9 targets)

Target	Value	Remark
New DI applicants/P×100	0.109	SSA, 1986:Q1-1992:Q4
Average prob. of termination given med. re-evals.	0.106	SSA, 1986:Q1-1992:Q4
Rate of wage drops with moderate work lim.	0.251	PSID, 1986-1992
Rate of wage drops with severe work lim.	0.450	PSID, 1986-1992
5 percentile of monthly wages	641	PSID, 1986-1992
95 percentile of monthly wages	5866	PSID, 1986-1992
Average job-finding probability	0.430	Shimer's data, 1986:Q1-1992:Q4
Elasticity of job-finding prob. w.r.t. the tightness	0.300	Shimer (2007)
Share of the unemployed who receive UI benefits	0.362	Nakajima (2012)

Note: the average job-finding probability is computed with the data constructed by Robert Shimer as part of Shimer (2012)

The number of new DI applications per population is used to pin down the DI application cost. The penalty rates of productivity with work limitations can be determined by wage differences between people who have no work limitation and people who have work limitation. In order to pin down the minimum and maximum values of individual productivity, I use the 5 percentile of monthly wages and 95 percentile of monthly wages from the annual wages in the PSID. The average job-finding probability and the elasticity of job-finding probabilities with respect to the market tightness are used to pin down the scale parameter in the matching function and the cost of posting vacancies.

Calibrated parameters

Table 1.8 summarizes 17 calibrated parameters and Table 1.9 represents moments from data and the model. The model successfully matches most of the targets, but the distributions of health status by labor force status are relatively difficult to match.

Table 1.8: 17 parameters calibrated in the model

Parameter	Description	Calibrated value
γ_m	Penalty rate of productivity with moderate work lim.	0.282
γ_s	Penalty rate of productivity with severe work lim.	0.645
$\pi_a(\gamma_m)$	Prob. of acceptance for moderate work lim	0.150
$\pi_a(\gamma_s)$	Prob. of acceptance for severe work lim.	0.810
$\pi_t(\gamma_n)$	Prob. of termination given med. re-evals. for no work lim.	0.420
$\pi_t(\gamma_m)$	Prob. of termination given med. re-evals. for moderate work lim.	0.180
$\pi_t(\gamma_s)$	Prob. of termination given med. re-evals. for severe work lim.	0.010
$c_p(\gamma_n)$	Disutility from labor force participation for no work lim.	1032
$c_p(\gamma_m)$	Disutility from labor force participation for moderate work lim.	1210
$c_p(\gamma_s)$	Disutility from labor force participation for severe work lim.	1425
c_a	DI application cost	1950
χ	Exogenous separation rate	0.017
$\underline{x}$	Minimum value of individual productivities	2070
$\overline{x}$	Maximum value of individual productivities	6130
ϕ_1	Scale parameter in the matching function	0.538
κ	Cost of posting vacancies	2113
$\pi_E^{0,1}$	Prob. of getting UI eligibility for the employed	0.006

1.5 Quantitative results

1.5.1 Steady state equilibrium

In this section, I compare main statistics in the steady states under different DI and UI policies.

Table 1.9: Calibration results: data vs. model

Target	Data	Model
Employed/Population	0.917	0.918
Unemployed/Population	0.049	0.049
New DI applicants/P×100	0.109	0.118
Proportion of no work lim. among the employed	0.935	0.929
Proportion of moderate work lim. among the employed	0.055	0.064
Proportion of no work lim. among the unemployed	0.775	0.737
Proportion of moderate work lim. among the unemployed	0.185	0.134
Proportion of no work lim. among DI recipients	0.148	0.150
Proportion of moderate work lim. among DI recipients	0.189	0.210
Average prob. of termination given med. re-evals.	0.106	0.107
Rate of wage drops with moderate work lim.	0.251	0.249
Rate of wage drops with severe work lim.	0.450	0.449
5 percentile of monthly wages	641	708
95 percentile of monthly wages	5866	5870
Average job-finding probability	0.430	0.430
Elasticity of job-finding prob. w.r.t. the tightness	0.300	0.302
Share of the unemployed who receive UI benefits	0.362	0.361

Comparison of statistics: more stringent DI policies

Table 1.10 compares main statistics in the steady states under 5 different DI policies.

1. Baseline (1986-1992)
2. Tighter medical re-evaluations
3. Tighter and more frequent medical re-evaluations
4. Lower probability of acceptance
5. Less amount of DI benefits

In general, more stringent DI policies result in less DI recipients and higher employment rates. However, the results for the number of DI applications are not trivial. The lower probability of acceptance and less amount of DI benefits induce less people to apply for DI. In contrast, under more stringent medical re-evaluations, ironically more people apply for DI. When the medical re-evaluations become more stringent, workers have less incentives to apply for DI because of higher termination risks from the DI program. However, higher terminations from the DI program increase the number of unemployed people and the number of prospective DI applicants. The overall effect will depend on the magnitude of these conflicting effects. Given the calibration, when medical re-evaluations become more stringent, the latter effect dominates the former effect in the steady state. Consequently, slightly more people apply for DI whereas the number of DI recipients drops due to higher outflows from the DI program.

Table 1.10: Statistics in the steady states: more stringent DI policies

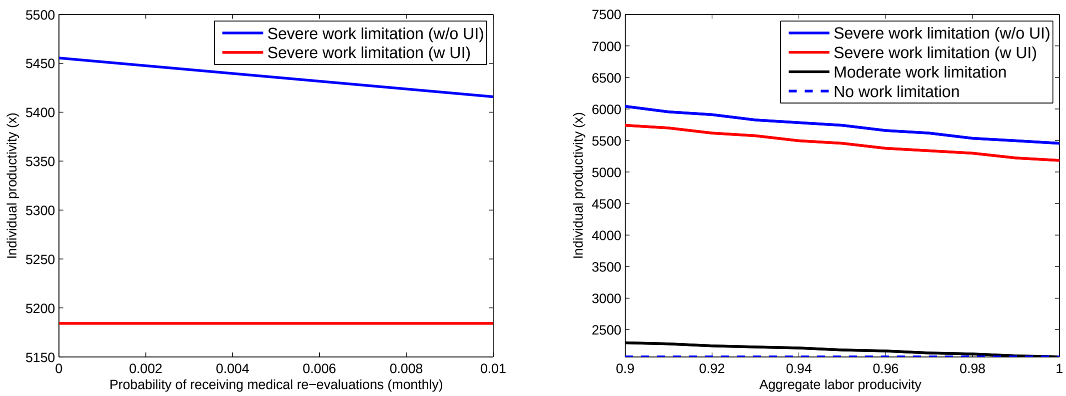
Variables	(1)	(2)	(3)	(4)	(5)
Aver. prob. of acceptance	73.3%	73.3%	73.3%	58.7%[18]	73.3%
Frequency of med. re-evals.	0.48%[19]	0.48%	1.61%	0.48%	0.48%
Aver. tightness of med. re-evals.	10.00%	44.00%	44.00%	10.00%	10.00%
Amount of DI benefits	-	-	-	-	20% ↓
Employed/Population	0.9184	0.9206	0.9239	0.9203	0.9262
Unemployed/Population	0.0491	0.0496	0.0495	0.0492	0.0481
DI recipient/Population	0.0324	0.0298	0.0266	0.0305	0.0258
New DI applicant/Population×100	0.1181	0.1188	0.1200	0.1153	0.1172
Job-finding probability	0.4296	0.4294	0.4292	0.4295	0.4291
Vacancies×100	6.6916	6.7050	6.7152	6.6802	6.7086
Average monthly wages ($ in 1986)	3498	3494	3488	3495	3485

Cutoff productivity for DI applications

Figure 1.5 shows cutoff productivity for DI application in the steady state when the tightness is 44%[20] . In the steady state, only unemployed people who have severe work limitation apply for DI. Whether or not the unemployed are eligible for UI benefits is important for DI applications. People who are not eligible for UI benefits are more likely to apply for DI. Figure 1.5(b) shows how cutoff productivity changes as the aggregate labor productivity changes. As productivity drops, more people apply for DI. In particular, people who have moderate work limitation start to apply for DI.

[20] I will use this economy in the main experiment in this paper

Figure 1.5: Cutoff productivity for DI applications



(a) Frequency of medical re-evaluations

(b) Aggregate labor productivity

Comparison of statistics: more generous UI policies

Table 1.11: Statistics in the steady states: more generous UI policies

Variables	(1)	(2)	(2)	(3)	(3)
Prob. of losing UI eligibility	26 (weeks)[21]	26	26	52[22]	99[23]
Replacement rate	40%	50%	60%	40%	40%
Employed/Population	0.9184	0.9289	0.9128	0.9172	0.9248
Unemployed/Population	0.0491	0.0452	0.0611	0.0503	0.0478
DI recipient/Population	0.0324	0.0260	0.0261	0.0325	0.0238
DI applicant/Population×100	0.1181	0.1071	0.1067	0.1188	0.0949
Monthly Job-finding rate	0.4296	0.4279	0.3994	0.4232	0.4120
Vacancies×100	6.6916	6.5901	7.4640	6.4843	6.0869
Average monthly wages ($ in 1986)	3498	3482	3479	3506	3494

Table 1.11 compares main statistics in the steady states under 5 different UI policies.

1. Baseline (1986-1992)
2. More amount of DI benefits (replacement rate: 50%)
3. More amount of DI benefits (replacement rate: 60%)
4. Longer length of time people collect UI benefits (52 weeks)
5. Longer length of time people collect UI benefits (99 weeks)

More generous UI policies give us non-trivial results for the employment rate. For example, the employment rate of the economy where the replacement rate is 50%, is higher than that of the baseline economy (40%). However, the employment rate of the economy where the replacement rate is 60%, is lower than that of the baseline economy. More generous UI policies affect the employment rate in two different ways. First, more people search for jobs without applying for DI, which will increase the employment rate. Second, it becomes more difficult for the unemployed to find jobs because more people look for jobs but firms have less incentives to hire workers due to higher wages resulting from higher outside options for workers. This will decrease the employment rate. The overall effects depend on the relative magnitude of these two effects. If the replacement rate is sufficiently high, then the latter effect outweighs the former effect. As a result, we have lower employment rates. In terms of the duration of UI, the employment rate of the economy where the maximum length of time people collect UI benefits is 52 weeks, is lower than that of the baseline economy (26 weeks). However, the employment rate of the economy where the maximum length of time is 99 weeks, is higher than that of the baseline economy. If the length of time is sufficiently longer, such as 99 weeks, the former effect dominates the latter effect and the employment rate can be higher than that of the baseline economy. In sum, more generous UI policies can lead to higher employment rates in the presence of DI as opposed to literature in which more generous UI policies generally decrease the employment rates due to less incentives for workers to search for jobs or less incentive for firms to post vacancies.

1.5.2 Equilibrium with aggregate labor productivity shocks

Krusell-Smith (1998) approximation

$$\begin{aligned}\kappa &= \beta q\left(\theta\left(z,\psi\right)\right)\int E_{z,\gamma,e}\left[\left(1-s^{'}\right)J^{x}\left(\gamma^{'},e^{'};z^{'},\psi^{'}\right)\right]\times \\ &\quad \frac{\psi_s\left(x,U,\gamma,a,m,e\right)}{\int\psi_s\left(x,U,\gamma,a,m,e\right)d(x,U,\gamma,a,m,e)}d(x,U,\gamma,a,m,e)\end{aligned}$$

$$\text{where } \psi_s\left(x,U,\gamma,a,m,e\right)=\mathbb{I}_{\left(g_s^x\left(U,\gamma,a,m,e;z,\psi\right)=1\right)}\psi\left(x,U,\gamma,a,m,e\right)$$

Given that all unemployed people look for jobs in the same labor market, the market tightness $\theta\left(z,\psi\right)$ depends on the measure of workers ψ as well as on the aggregate labor productivity z as we can see in the free entry condition above. In this case, it is not possible to solve for the equilibrium with aggregate shocks outside of the steady state because the measure is an infinite dimensional state variable. Krusell and Smith (1998) approximate the equilibrium with aggregate shocks by replacing a measure with finite moments of the economy under the assumption of bounded rationality. Following their method, the measure ψ is replaced with the aggregate employment E in this paper. Therefore, the aggregate state variables in this economy are $\{z, E\}$ in stead of $\{z, \psi\}$. In order to predict the market tightness θ and the future value for the aggregate employment $E^{'}$, I assume simple log-linear prediction functions for the market tightness $\theta(z, E)$ and the aggregate employment $E^{'}$:

$$log(\theta) = b_{\theta,0} + b_{\theta,1} log(E) + b_{\theta,2} log(z)$$
$$log(E') = b_{E,0} + b_{E,1} log(E) + b_{E,2} log(z)$$

The details about the computation is described in the Appendix A. The following is the converged prediction functions and their accuracies for the baseline model:

$$log(\theta) = 0.5340 + 0.1033\, log(E) + 0.8329\, log(z), \;\; R^2 = 0.9524$$
$$log(E') = -0.0001 + 0.9993\, log(E) + 0.0015 log(z), \;\; R^2 = 0.9987$$

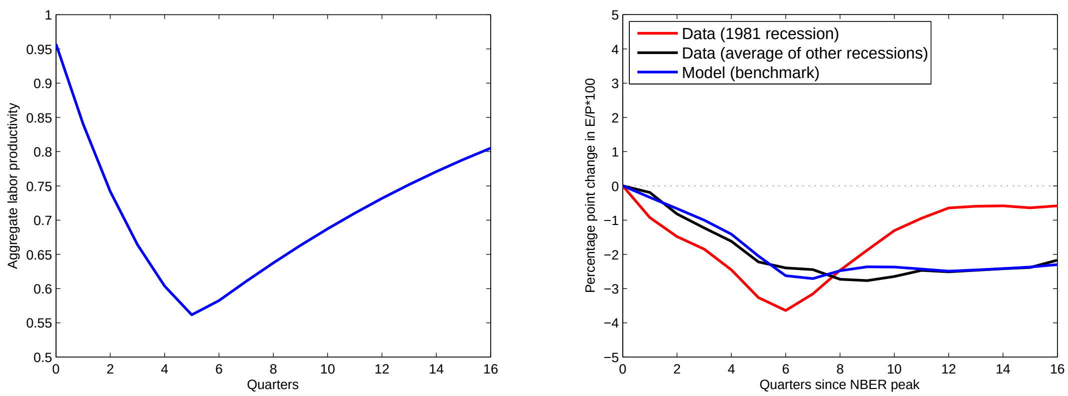
Change in frequency of medical re-evaluation during a recession

To determine the effect of the policy change during the 1981 recession, I perform a simple experiment[24] : an unexpected one-time increase in the frequency of medical re-evaluations. Figure 1.6 shows a series of aggregate labor productivities (Figure 1.6(a)) to generate a benchmark recession (Figure 1.6(b)) which is the average of other recessions since 1965 except the 1981 recession. Given that aggregate labor productivity shocks cannot generate sufficient variations in unemployment and employment in search and matching models as noted in Shimer (2005), I need a big drop of aggregate labor productivites to generate an appropriate magnitude of the recession in data. Given 44% of tightness of medical re-evaluations[25] , I only change the frequency of medical re-evaluations from 0.48% to 1.61% (monthly frequency) at the end of the 3rd quarter

[24] This experiment is not exactly the same as what happened during the 1981 recession because there might be several changes in the frequency during 1981-1983. I am working on a better experiment.
[25] This is the average for the period 1981-1983. There was no significant change in the tightness before and during the 1981 recession.

after the onset of the recession. Figure 1.7 shows the inputs for the experiment. I assume that this change is unexpected and it occurs after the decisions of the workers are made in that period.

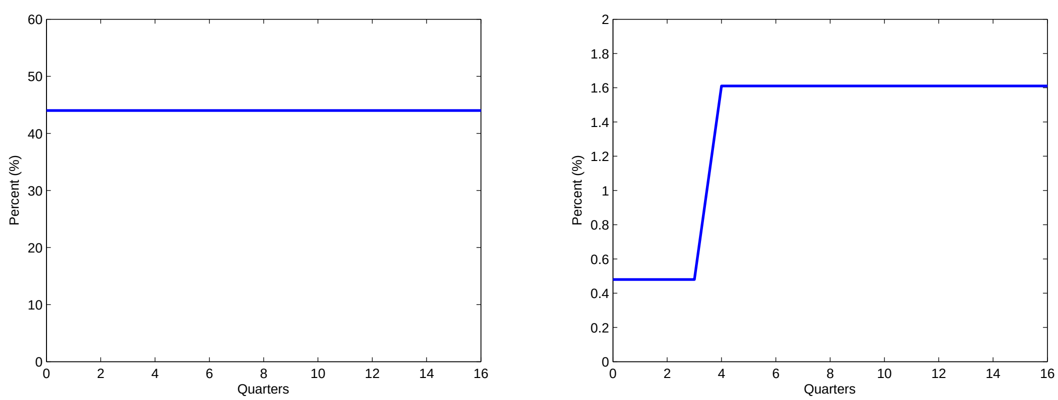
Figure 1.6: Aggregate labor productivity and paths of employment rates in the baseline recession



(a) Aggregate labor productivity (b) Benchmark recession

Note: All series from the model are converted to quarterly series by averaging three consequent monthly series.

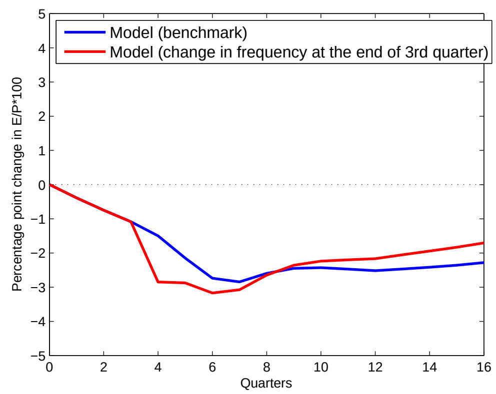
Figure 1.7: Inputs for the experiment: change in the frequency of med. re-evals during a recession



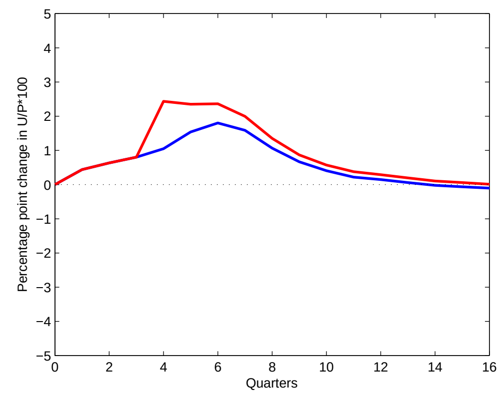
(a) Tightness of med. re-evals. (b) Frequency of med. re-evals.

Figure 1.8 shows the main result for the experiment. As we can see Figure 1.8(a), the model generates a deeper recession and faster recovery afterward. When I compare the beginning and the trough of the recession, the employment rate is lower by 0.3 percentage points when the policy becomes more stringent. When I compare the start of the recovery and after 2 years from that, the employment rate is higher by 0.9 percentage points when the policy becomes more stringent.

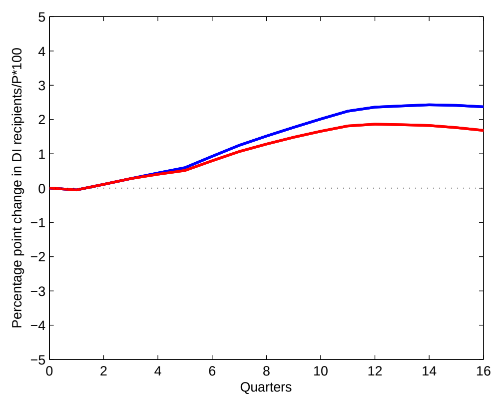
Figure 1.8: Results: change in frequency of medical re-evaluations



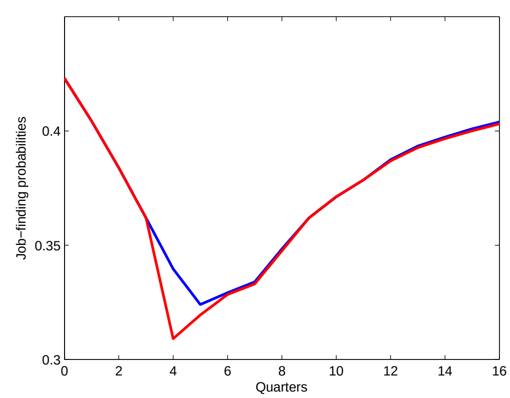
(a) Employed/Population



(b) Unemployed/Population



(c) DI recipients/Population

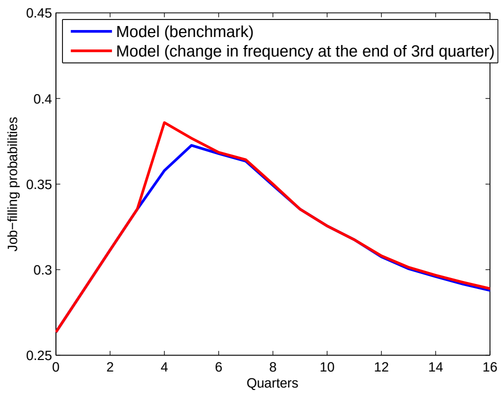


(d) Job-finding probabilities

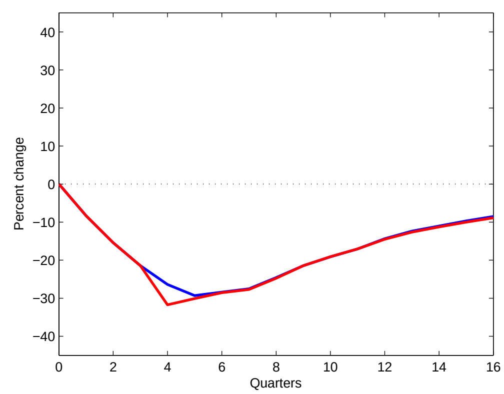
Note: All series from the model are converted to quarterly series by averaging three consequent monthly series.

Note that the recovery in the model is not as fast as the one in the data. There might be other reasons for the faster recovery than medical re-evaluations, such as expansionary monetary policies and tax cuts during the Reagan administration. Since the aggregate productivity measured as total factor productivity (TFP) or aggregate labor productivity is not that different across different recoveries as in Figure A.4, the difference in productivity during recoveries does not seem to be the main reason.

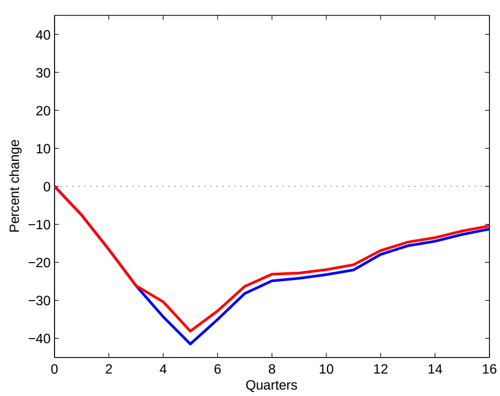
Figure 1.9: Results: change in frequency of medical re-evaluations (continued)



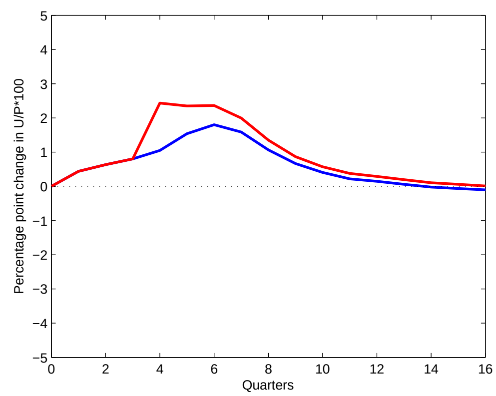
(a) Job-filling probabilities



(b) Expected value of hiring



(c) Vacancies



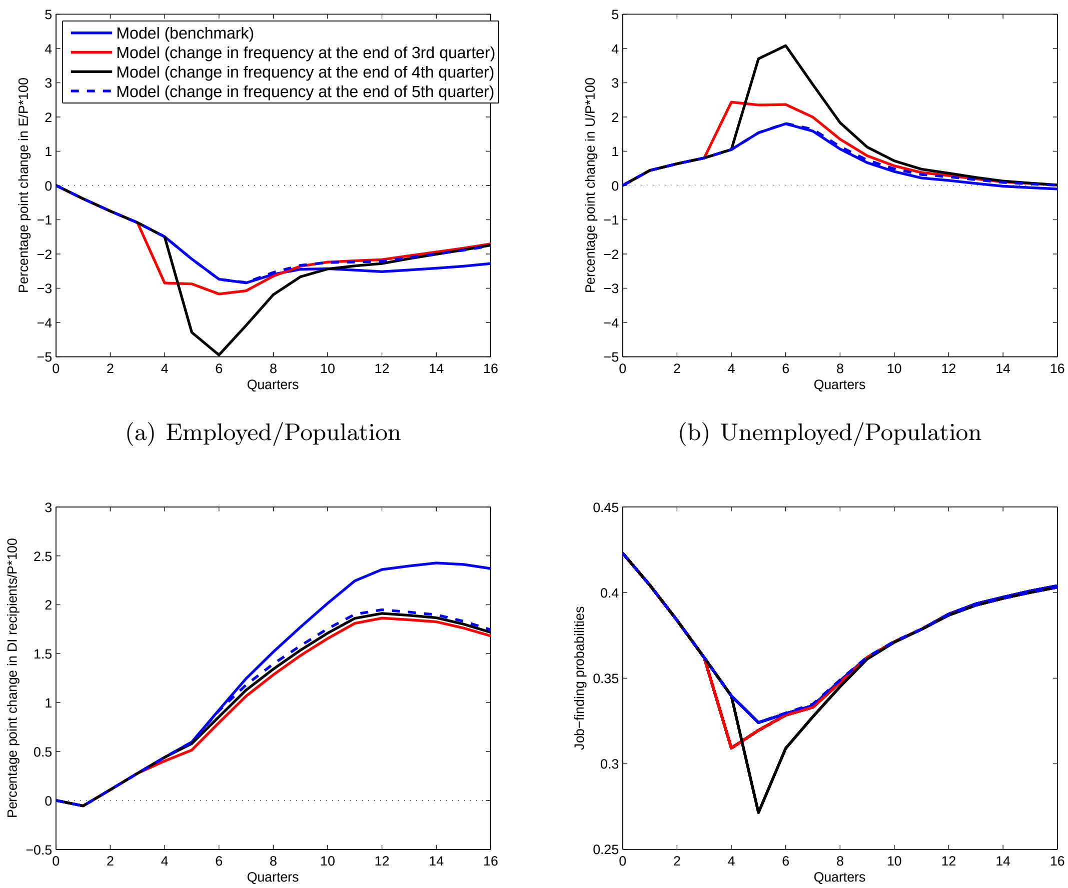
(d) Unemployed/Population

Note: All series from the model are converted to quarterly series by averaging three consequent monthly series.

Aside from the other reasons like expansionary monetary policies and tax cuts during the Reagan administration, the model has a problem regarding a magnitude of the increase in DI recipients during the recession and its recovery as we can see in Figure 1.8(c). The number of DI recipients increases much more sharply compared to the one in data. This problem may be related to the Shimer puzzle. As we discussed before as in Figure 1.6(a), aggregate labor productivity shocks cannot generate sufficient variations in unemployment and employment in search and matching models. Therefore, we need a big drop of aggregate labor productivity to generate an appropriate magnitude of the recession in data. The problem is that more unemployed people apply for DI when the productivity drops as in Figure 1.5(b), and the big drop of aggregate labor productivites makes more unemployed people apply for DI in the model than in data. This problem can be addressed by using alternative calibrations proposed by Hagedorn and Manovskii (2008), where a bargaining power of workers (0.052) is significantly lower than that of the benchmark calibration (0.50).

The main mechanism for the deeper recession and faster recovery in the model is as follows. Medical re-evaluations affect the number of people who search for jobs (direct effect) and job-finding probabilities for all unemployed people (general equilibrium effect). When the frequency unexpectedly increases at the end of the 3rd quarter, the number of unemployed people significantly increases as in Figure 1.8(b). The increase in the number of the unemployed results in a sharp decrease in job-finding probabilities as we can see in Figure 1.8(d). As more people look for jobs, the probability of firms meeting workers increases as in Figure 1.9(a). However, the expected value of hiring a worker decreases as in Figure 1.9(b) because the inflow of terminated DI recipients into the unemployment pool decreases the probability of meeting good (more productive) workers. Since the former effect slightly dominates the latter effect, firms post more vacancies compared to the economy where there is no change in the frequency of the policy. Although firms post more vacancies, the increase in the number of people who look for jobs outweighs the increase in the number of vacancies during the recession. Consequently, the job-finding probabilities drops as we can see in Figure 1.8(d). In sum, the effect of the drop in job-finding probabilities dominates the effect of the increase in the number of people who look for jobs. Therefore, the recession becomes deeper compared to the recession where no change in the frequency of the policy occurs. However, as the economy recovers, job-finding probabilities start to increase because more jobs are posted as we can see in Figure 1.8(d). Consequently, the effect of the increase in the number of people who look for jobs outweighs the effect of job-finding probabilities, which leads to a faster recovery.

Figure 1.10: Results: change in frequency of medical re-evaluations (different timing)



(a) Employed/Population

(b) Unemployed/Population

(c) DI recipients/Population

(d) Job-finding probabilities

Note: All series from the model are converted to quarterly series by averaging three consequent monthly series.

The timing of changing in frequency is also relevant. Figure 1.10 shows the results for changes in frequency of medical re-evaluations at three different quarters. When the policy changes at the end of the 4th quarter which is close to the trough of the recession, the drop in job-finding probabilities is the biggest as in Figure 1.8(d). This is because firms are not likely to hire workers during this period. On the other hand, when the policy changes at the end of the 5th quarter where the aggregate labor productivity starts to recover, the drop in job-finding probabilities is negligible and the employment rate does not drop at the time of the policy change.

Extension of duration of UI during a recession

In literature where disability insurance is not explicitly considered, the extension of the length of time people collect UI benefits decreases employment rates. This is because workers look for jobs less intensively[26] and firms have less incentives to hire workers due to higher wages resulting from higher outside options of workers[27] . However, in the presence of DI, the extended UI benefits could increase employment rate. This is because there is one more important channel in the presence of DI. The extended UI benefits induces more people to look for jobs by delaying their DI applications until UI benefits are expired.

In this section, I perform an experiment in which the length of time people collect UI benefits is extended during a recession. In this experiment, we compare three different recessions:

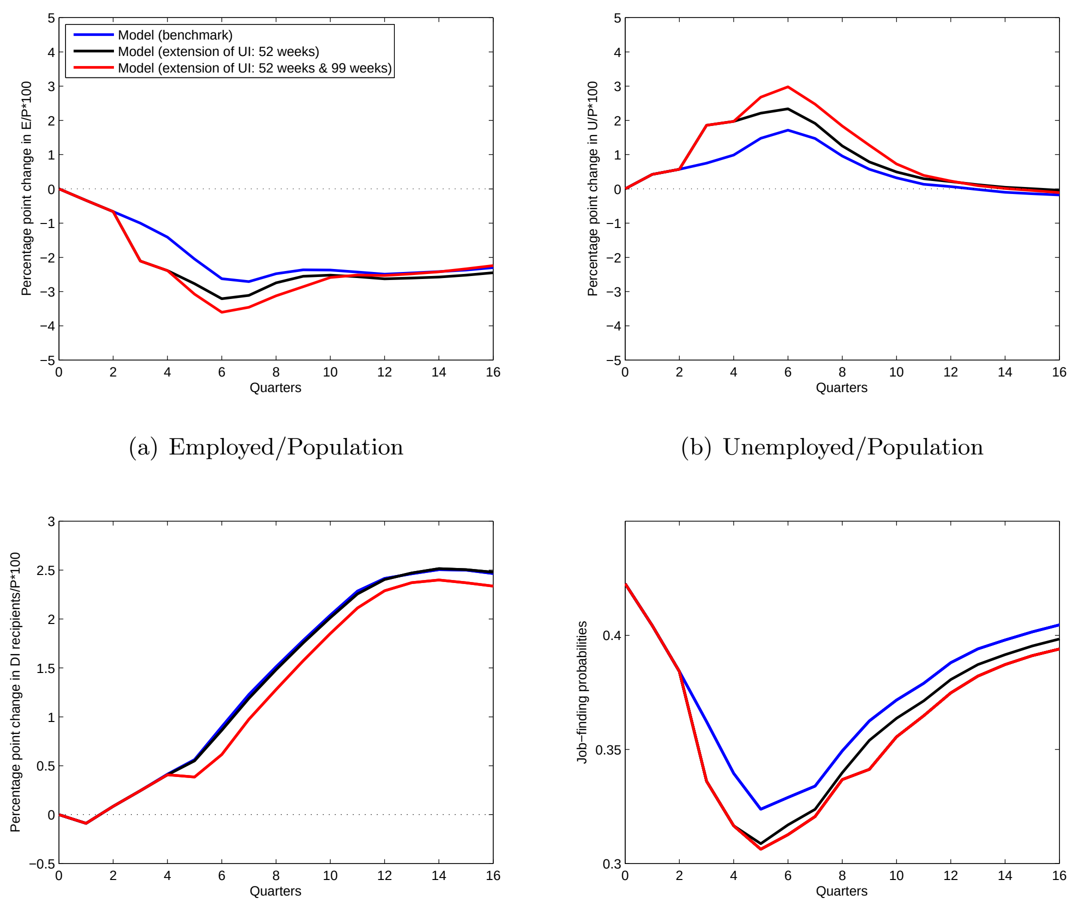
1. Benchmark: no extension of the duration of UI benefits (26 weeks)
2. One time extension of the duration of UI benefits (26 weeks → 52 weeks at the 3rd quarter)
3. Further extension of the duration of UI benefits (26 weeks → 52 weeks at the 3rd quarter → 99 weeks at the 6th quarter)

Figure 1.11 shows the results of the experiment. The extension of the duration of UI benefits from 26 weeks to 52 weeks during a recession leads to a deeper recession and slower recovery because it becomes more difficult for the unemployed to find jobs throughout the recession and its recovery whereas the number of people who look for jobs by delaying their DI applications does not increase much. However, if the duration is further extended from 52 weeks to 99 weeks, the unemployed who have work limitation

[26] See Nakajima (2012)
[27] See Hagedorn et al. (2015)

are more willing to look for jobs by delaying DI applications, and this effect dominates the effect of the drop in job-finding probabilities during the recovery. Consequently, the extended UI benefits lead to a faster recovery. This result implies that the extended UI benefits during recessions can expedite recoveries in the presence of DI if the timing of extension is well designed considering the state of the economy.

Figure 1.11: Results: extension of duration of UI



(a) Employed/Population

(b) Unemployed/Population

(c) DI recipients/Population

(d) Job-finding probabilities

Note: All series from the model are converted to quarterly series by averaging three consequent monthly series.

1.6 Conclusion

In the US, the recovery in the employment rate of men from the 1981 recession was faster than any other recovery since 1965. During the 1981 recession and at the beginning of its recovery, the number of disability insurance (DI) applicants and recipients dropped while the numbers increased in all other recessions due to the most stringent medical re-evaluations between 1981 and 1983. Medical re-evaluation is a policy that periodically terminates benefits of ineligible DI recipients. This paper examines the role of medical re-evaluation for DI recipients in the 1981 recession and its fast recovery.

This paper makes several contributions. I document the fast recovery in the employment rate of men from the 1981 recession and the most stringent medical re-evaluations during 1981-1983. In terms of model, to the best of my knowledge, I build the first general equilibrium business-cycle model with DI. We can use the model to analyze how DI policies affect unemployed people, including those who have no work limitation, through changes in job-finding probabilities. Quantitatively, this paper examines the effect of medical re-evaluations on the 1981 recession and its fast recovery and the role of an extended duration of unemployment insurance (UI) benefits in the presence of DI during recessions.

I build a general equilibrium business-cycle search and matching model with health, DI, and UI eligibility. The model is calibrated to match key features of the US economy for 1986-1992 by using the PSID, CPS, and public SSA data. Given that the model has aggregate labor productivity shocks and heterogeneous workers are matched randomly in the labor market, Krusell-Smith (1998) approximation is used to solve for the model outside of the steady state.

To determine the effect of the policy change during the 1981 recession, I perform a simple experiment: an unexpected one-time increase in the frequency of medical re-evaluations during a recession. The experiment shows that more frequent medical re-evaluations during the 1981 recession made the recession deeper and the recovery faster. Lastly, aside from the main experiment, I use the model to examine the role of the extended length of time people collect UI benefits in the presence of DI during a recession. The experiment shows that in the presence of DI, the extended UI benefits during recessions can expedite recoveries if the timing of extension is well designed considering the state of the economy.

Chapter 2

Labor Market Fluctuations and the Role of Financial Shocks

2.1 Introduction

The financial turmoil that began with the subprime mortgage crisis in 2007 brought about not only one of the largest decreases in real GDP in the US since the Great Depression, but also a substantial increase in the rate of unemployment. The unemployment rate jumped from 4.7% in 2007:Q4 to 9.9% in 2009:Q4 while real GDP decreased at an astonishing -1.7% annualized rate over the same time period. High unemployment has persisted and continues to be a challenge today, even after real GDP has recovered to pre-recession levels. It seems natural to assess the role credit markets have played in the sharp decrease in employment and its sluggish recovery to pre-recession levels.

The financial crisis and resulting Great Recession have fostered renewed interest in the incorporation of financial frictions in macroeconomic models. Many recent studies have emphasized the importance of employing such frictions to account for macroeconomic fluctuations in key variables over the business cycle. In particular, so called 'financial shocks' have been deemed significant contributing factors for the observed dynamics of real and financial variables over the business cycle. Financial shocks directly affect the financial sector of the economy as opposed to standard productivity shocks that are merely propagated through the financial sector. However, applicable studies have been silent about how unemployment and job postings interact with the deterioration of credit market conditions. In order to address this shortcoming, we evaluate just how important financial shocks are in accounting for movements in key labor market variables by using a standard real business cycle (RBC) matching model which incorporates financial frictions via an enforcement constraint. We assess the importance of incorporating financial shocks into our model by comparing our results to those of a standard matching model without financial frictions. We take our benchmark matching model without financial frictions to be the model developed by Andolfatto (1996) (simply Andolfatto hereafter). We refer to this as the standard matching model throughout.

While analyzing the role of the financial sector over the business cycle is not a new topic, most previous studies utilized the credit channels formalized by Bernanke and Gertler (1989), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999) and treated the financial sector as an accelerator of productivity shocks. This standard credit channel differs from those developed more recently by Perri and Quadrini (2011) and Jermann and Quadrini (2012) (JQ hereafter), which incorporate financial shocks that directly affect the financial sector's ability to lend. That is, the financial sector not only propagates productivity shocks originating from other sectors of the economy, but it also acts as a source of the business cycle itself via financial shocks. The latter studies have emphasized the impact of financial shocks in their explanations for labor market fluctuations but offer no means for analyzing the extensive margin of employment in their framework.

Some authors have already highlighted the need for addressing the role of financial frictions on unemployment. Petrosky-Nadeau (2014) uses asymmetric information and costly state verification between financial intermediaries and borrowers which increases both the magnitude and persistence of unemployment fluctuations relative to a standard neoclassical growth model. Chugh (2013) uses a similar credit channel but builds a model with capital accumulation. Monacelli, Trigari, and Quadrini (2012) use a model with linear utility and no capital accumulation and show that borrowing more from financial intermediaries shifts bargaining weight from the worker to the firm which can explain why firms cut hiring after a negative financial shock even in the absence of a liquidity shortage. Our study departs from previous approaches and employs the credit channel used in JQ in order to compare the gains of adding financial frictions over a

Table 2.1: Business cycle statistics, 1984:Q1-2012:Q1

Variable (x)	$\sigma_x\%$	$\rho\,(x, Output)$	$\rho\,(x_t, x_{t-1})$
Output	1.12	–	0.87
Total Hours	1.26	0.85	0.89
Employment	0.88	0.82	0.93
Hours per Worker	0.45	0.77	0.61
Wages	0.91	-0.18	0.77
Labor Productivity	0.66	0.07	0.59
Labor Share	0.73	-0.28	0.78
Vacancies	11.26	0.86	0.91
Equity Payouts/GDP	1.39	0.69	0.91
Debt Repurchases/GDP	2.23	-0.84	0.93

standard matching model as developed by Merz (1995) and Andolfatto (1996). Our model framework is somewhat related to that of Garin (2012), but his study neither utilizes the intensive margin nor compare the results to a standard RBC matching model. This distinction is important since the response along the intensive margin to financial frictions and shocks in our model is quite different from that along the extensive margin.

We start by documenting the cyclical properties of key variables for the US economy over the period 1984:Q1-2012:Q1 in Table 2.1. We chose this period for our analysis since JQ have argued that 1984 corresponds to a break in the volatility in many business cycle variables and that this time period also saw the stabilization of structural change in US financial markets compared to previous periods. All variables are deflated by population, logged (except debt repurchases and equity payouts), and HP-filtered. Debt repurchases and equity payouts statistics are computed after detrending with a band-pass filter that preserves cycles of 1.5-8 years (Lawrence J. Christiano and Terry J. Fitzgerald (2003)). Wages are defined as real labor compensation per labor-hour. A detailed description of the data used in Table 2.1 and throughout our study can be found in the Appendix B.

A few elements in Table 2.1 deserve some discussion. First, employment is much more volatile than hours worked per worker. While total hours fluctuate more than output itself, most of this is adjusts along the extensive margin. The relative contribution of variance in hours per worker to total hours worked is 32%. Thus, the intensive margin is one that should be incorporated into any model seeking to understand fluctuations in total hours worked in the US economy. Employment and total hours tend to lag output by one quarter while hours worked and vacancies are coincident variables which suggests firms are able to adjust the intensive margin and post vacancies quicker than they can adjust the stock of employees. We will incorporate this fact into our model. Second, real wages are almost as volatile as output, but are surprisingly countercyclical over our sample period. Third, the labor share is countercyclical, implying that during periods of expansion, labor is allocated relatively less of the gains. Finally, note that equity payouts are strongly procyclical while debt repurchases are strongly countercyclical. As JQ pointed out, there seems to be substitutability between equity payouts and debt repurchases over the business cycle. It is our goal to see what gains can be made in accounting for the fluctuations in the variables reported in Table 2.1 once financial shocks are incorporated into a standard matching model.

The paper is structured as follows. Section II proposes a model with labor market frictions, financial frictions, and financial shocks. Section III discusses the calibration of the models. Section IV studies the quantitative properties of our benchmark model and our proposed model. Section IV studies the importance of financial shocks by comparing our model's results to those of a standard matching model. Section VI concludes

2.2 Model

Our model framework follows closely the models developed by JQ and Andolfatto. Since the Andolfatto model has a matching framework but no financial frictions, we take this to be our benchmark model to compare our results to. We will refer to the benchmark model as the Andolfatto model, the standard matching model, or simply Andolfatto. Note that the equations characterizing the solution to our model with financial frictions can quickly be mapped into the Andolfatto model by shutting down both the financial shock processes and the Lagrange multipliers on the enforcement constraint. For this reason, we do not lay out the Andolfatto model explicitly but choose to develop our model with financial frictions first.

2.2.1 Matching

Time is discrete and goes on forever. The timing of our model is as follows: (i) shocks are realized, (ii) wages and hours are bargained over, (iii) firms take our intra-period loans, (iv) production takes place and vacancies are posted, and then (v) separations and matches occur.

Labor markets are frictional and the law of motion of total employment, N, depends on the number of matches that occur at the end of each period. We take one model period to be one quarter. We assume that the number of matches is dictated by a constant returns-to-scale matching technology which depends on the total number of unemployed, $U \equiv 1 - N$, and on the total number of vacancies, V, posted by firms: $M(V, 1-N)$. Defining $V/(1-N) \equiv \theta$ as labor market tightness, we then define the job-finding rate as $\Psi(\theta) = M(\theta, 1)$ and the job-filling rate as $\Phi(\theta) = M(1, 1/\theta)$. Assuming that jobs are destroyed at the exogenous rate $\chi \in (0, 1)$, it follows that employment evolves according to:

$$N' = (1-\chi)N + \Psi(\theta)(1-N)$$

2.2.2 Households

There is a continuum of identical and infinitely lived households each of measure one. Each household is endowed with a unit of time to split between working hours and leisure hours and each household derives utility from consumption and leisure. Households discount the future by the factor $\beta \in (0, 1)$. We model a representative household similar to Merz (1995) and Andolfatto (1996), which allows for perfect unemployment insure across households. This, along with the assumption that there are no search costs, implies that every unemployed household will always be searching for a job. Households trade uncontingent bonds, a_H, and shares in firms, s. Unemployed households receive the unemployment benefit $b \geq 0$ from the government and each household pays the lump-sum tax T. We can then write the program of the representative household as:

$$V(S, s_H) = \max_{c, s', a'_H} \{u(c) + n\nu(1-h) + (1-n)\nu(1) + \beta E[V(S', s'_H)]\}$$

s.t.

$$c+\frac{a_H'}{1+r\left(S\right)}+p\left(S\right)s'=w\left(S,s_H\right)nh\left(S,s_H\right)+(1-n)b+a_H+\left[p\left(S\right)+d\left(S\right)\right]s-T\left(S\right)$$

$$n'=(1-\chi)n+\Psi\left(S\right)\left(1-n\right)$$

$$S'=G(S),\ c\geq 0,\ \text{No-Ponzi condition}$$

The aggregate state of the economy is given by $S=\{z,\xi;K,B,N,D_{-}\}$, where z is total factor productivity and ξ is the financial shock which both evolve stochastically. K is the aggregate capital stock, B is total bond holdings of the household sector, N is total employment, and D_{-} is the amount of dividends paid out last period. $s_H=\{s,a_H,n\}$ is the individual state, and G is the law of motion for aggregate state variables. d is the dividend paid to shareholders, and p is the share price of the representative firm.

Wages and hours are the result of a Nash-bargaining problem between workers and the firm at the beginning of each period, so from the household's perspective $w\left(S,s_H\right)nh\left(S,s_H\right)$ is given before any consumption or savings decisions take place. Since we have assumed separable utility between consumption and leisure, the intra-household consumption level doesn't depend on employment status as noted in Merz (1995) and Andolfatto (1996). Note that this has the implication that unemployed households are better off than those that are employed since they receive the same consumption level as those that are employed but enjoy all the leisure. This implication is discussed in detail in Cheron and Langot (2004). The first order conditions (dropping the dependence on states) from the household's problem give:

$$\begin{aligned}1 &= E\left[m'\left(1+r\right)\right]\\ 1 &= E\left[m'\left(\frac{p'+d'}{p}\right)\right]\end{aligned}$$

where $m'=\beta u_c\left(c'\right)/u_c\left(c\right)$ is the stochastic discount factor. These equations taken together simply give us the no-arbitrage condition between shares and bonds. All derivations of first order conditions for all agents can be found in the Appendix B.

2.2.3 Firms

We model the firm and derive an enforcement constraint similar to JQ. There exists a representative firm with gross revenue $F(z,k,nh)$, where z is the stochastic level of aggregate productivity. Capital evolves according to the standard law of motion $k' = (1-\delta)\,k + i$, where i is investment and $\delta \in [0,1]$ is the rate of depreciation. Firms discount the future via the stochastic discount factor m' and pay the fixed cost $c_v > 0$ to post a vacancy. The firm also pays the equity payout cost $\varphi(d, d_{-})$ to pay dividends to shareholders. We impose this dividend adjust cost to capture the observation that firms tend to smooth dividends as well as to formalize the financial friction. Firms use equity and debt with debt preferred to equity due to the subsidy $\tau \in (0,1)$. Therefore, the *effective* gross interest rate that the representative firm faces every period is given by $R = 1 + r\,(1-\tau)$.

After negotiating wages and hours, firms take out the intra-period loan l_t to finance working capital. Before receiving any revenue from production, the firm pays the wage bill wnh, chooses investment, chooses the equity payout d and the associated adjustment cost, the number of vacancies v to post, and new intertemporal debt a'_F. Since all payments are done before the realization of revenues, the firm must take out the intra-period loan:

$$l = wnh + i + c_v v + \varphi(d, d_{-}) + a_F - \frac{a'_F}{R}$$

The firm's budget constraint every period is

$$i + a_F + \varphi(d, d_{-}) = F(z, k, nh) - wnh - c_v v + \frac{a'_F}{R}$$

It follows that the intra-period loan is simply total expected revenue, $l = F(z, k, nh)$.

The firm has the option to default after total revenues are realized but before the working capital loan l is paid back. At this moment in time, the firm holds liquidity l and total liabilities $l + a'_F/\,(1+r)$. Since firms can easily abscond with the liquidity l, the lender can only recover the firm's physical capital stock k' with probability ξ, which is stochastic. With probability $(1-\xi)$, the lender's recovery value is zero. One can be interpret this probability as the probability of finding a buyer of the firm's capital stock.

In the case of default, the lender and the firm can negotiate a payment after the liquidation value of the capital stock is realized. We assume that the firm has all the bargaining power in this negotiation process and the lender will only get the threat value.

If the liquidation value is zero, the lender will not shutdown the firm because it is better off waiting for the intertemporal loan a'_F to come due. The firm keeps the liquidity l in this case. Therefore, the total ex-post value of default in the case when the liquidation value is zero is:

$$l + E\left[m'J'\right]$$

where m' is the stochastic discount factor and J' is the value of the firm tomorrow. That is, $E\left[m'J'\right]$ is the expected present value of the firm if the firm continues to operate.

If the liquidation value is k', the firm will negotiate the payment P to prevent the lender liquidating the firm. The net surplus to the firm of avoiding liquidation is:

$$l + E\left[m'J'\right] - P$$

The lender's net surplus of reaching an agreement is:

$$P + \frac{a'_F}{1+r} - k'$$

Assuming the firm holds all the bargaining power, the firm must pay $P = k' - a'_F/\left(1+r\right)$ to avoid liquidation. It follows that the total net surplus of reaching an agreement is:

$$l + E\left[m'J'\right] + \frac{a'_F}{1+r} - k'$$

Since the liquidation value is not known until after the default takes place, when the intra-period loan is contracted, the expected total net surplus to the firm (since they have all the bargaining power) is

$$\begin{aligned} & \xi\left(l + E\left[m'J'\right] + \frac{a'_F}{1+r} - k'\right) + (1-\xi)\left(l + E\left[m'J'\right]\right) \\ = \quad & \xi\left(\frac{a'_F}{1+r} - k'\right) + l + E\left[m'J'\right] \end{aligned}$$

Incentive compatibility requires that the expected surplus of defaulting not exceed the value of not defaulting. This requires that

$$\begin{aligned} E\left[m'J'\right] \quad & \geq \quad \xi\left(\frac{a'_F}{1+r} - k'\right) + l + E\left[m'J'\right] \\ \xi\left(k' - \frac{a'_F}{1+r}\right) \quad & \geq \quad l = F(z,k,nh) \end{aligned}$$

The firm's ability to borrow is limited by the enforcement constraint derived above. Higher debt in the form of either inter-temporal or intra-temporal loans is associated with a tighter enforcement constraint while a higher capital stock loosens the enforcement constraint. Since employment (due to the lack of endogenous separations), productivity, the probability ξ, and the capital stock are given, the firm only has control over k', a_F', and the intensive margin h. We refer to innovations in ξ as 'financial shocks' since it directly affects the firm's capacity to borrow from lenders. Negative innovations can be viewed as a deterioration in credit market conditions.

We can then write the program of the representative firm as:

$$J(S, s_F) = \max_{d,k',a_F',v,n'} \{d + E[m' J(S', s_F')]\}$$

s.t.

$$k' + a_F + \varphi(d, d_-) = F(z, k, nh\,(S, s_F)) + (1-\delta)k - w\,(S, s_F)\,nh\,(S, s_F) - c_v v + \frac{a_F'}{R\,(S)}$$

$$\xi\left(k' - \frac{a_F'}{1 + r\,(S)}\right) \geq F(z, k, nh\,(S, s_F))$$

$$n' = (1-\chi)n + \Phi\,(S)\,v$$

$$S' = G(S),\; k', v \geq 0$$

where $s_F = \{k, a_F, n, d_-\}$ is the individual state, $R = 1 + r(1-\tau)$, and the firm's equity payout cost is $\varphi(d, d_-)$. Once again note that wages and hours are bargained at the beginning of the period and are treated as given in the program described above.

The first order conditions (dropping state dependencies) to the firm's problem gives:

$$\begin{aligned}
1 &= \lambda\varphi_d + E[m'\lambda'\varphi_{d_-}'] \\
\lambda c_v &= \Phi E\left[m' J_n'\right] \\
\lambda - \gamma\xi &= E\left[m'\left[\left(\lambda' - \gamma'\right)F_k' + (1-\delta)\,\lambda'\right]\right] \\
\lambda\,(1+r) - \gamma R &= R\,(1+r)\,E\left[m'\lambda'\right]
\end{aligned}$$

where λ and $\gamma \geq 0$ are the Lagrange multipliers on the budget constraint and enforcement constraint, respectively. To see how these equations relate to the Andolfatto model, simply consider the equations above and set $R = 1 + r, \lambda = 1$, and $\gamma = 0$.

2.2.4 Nash bargaining

Wages and hours are bargained over at the beginning of each period via a Nash bargaining problem between the representative household and the representative firm. Employing the notation from above, the value of an additional worker to the representative household is (in terms of consumption units):

$$\frac{V_n}{u_c} = \frac{\nu\,(1-h) - \nu\,(1)}{u_c} + wh - b + (1 - \chi - \Psi)\,\beta\left[\frac{V_n'}{u_c}\right]$$

The value to the representative firm of an additional worker is:

$$J_n = (\lambda - \gamma) F_{nh} h - \lambda w h + (1 - \chi) E \left[m' J'_n \right]$$

where λ and γ are, again, the Lagrange multipliers on the firm's budget constraint and enforcement constraint, respectively. Following Andolfatto (1996), it is assumed that the each worker is so small such that $F_{nh} \equiv \partial F / \partial (nh)$ is taken as given by both the household and the firm during the bargaining process. Given the worker's bargaining weight $\mu \in (0, 1)$, the wage and hours are the result of the Nash bargaining problem:

$$(w, h) = \arg\max_{w,h} \left(\frac{V_n}{u_c} \right)^{\mu} (J_n)^{1-\mu}$$

Taking the derivatives with respect to wages and hours gives us the sharing rule of the production surplus and the static condition determining the number of hours.

$$\begin{aligned} \mu J_n &= \lambda (1 - \mu) \left(\frac{V_n}{u_c} \right) \\ \frac{\nu_{(1-h)} (1 - h)}{u_c} &= \left(1 - \frac{\gamma}{\lambda} \right) F_{nh} \end{aligned}$$

Using the sharing rule, $\mu J_n = \lambda (1 - \mu) (V_n / u_c)$, along with the definition of V_n / u_c and J_n, gives the wage bill per worker:

$$\begin{aligned} wh &= \mu \left[\left(1 - \frac{\gamma}{\lambda} \right) F_n + (1 - \chi) E \left[m' \frac{J'_n}{\lambda} \right] + \frac{V}{1 - N} \Phi E \left[m' \frac{J'_n}{\lambda'} \right] \right] \\ &\quad + (1 - \mu) \left[\frac{\nu (1) - \nu (1 - h)}{u_c} + b - (1 - \chi) E \left[\beta \frac{V'_n}{u_c} \right] \right] \end{aligned}$$

This is simply a weighted average of (i) the effective marginal productivity of a worker plus the expected future value of maintaining the match plus the average discounted savings to the firm of not having to post a vacancy next period and (ii) the endogenous outside option of the worker which is simply the forfeited leisure in terms of consumption units as well as the unemployment benefit b minus the future value of maintaining the match. The marginal productivity of each worker F_n is driven down by the effective tightness of the enforcement constraint γ / λ. This is the key equation driving our results.

According to Hagedorn and Manovskii (2008) (HM hereafter), in order to increase the volatility of vacancies and employment, we need to increase the volatility of the firm's surplus per worker. In order to achieve this, they calibrate a low bargaining weight and a high value of the outside option for workers. The low value of the bargaining weight of workers makes the wage bill per worker less volatile in response to the marginal productivity of each worker F_n. The workers' higher outside option makes the firm's surplus small. These two properties taken together makes the firm's surplus per worker more sensitive to the marginal productivity of each worker F_n, which means firms have a greater incentive to post vacancies. Financial frictions have a similar effect by generating an additional wedge between the wage bill per worker and the marginal productivity of each worker F_n. When financial frictions are present, capital is more 'valuable' to the firm than an additional worker since capital has the added benefit of loosening the enforcement constraint in this model.

In our setup, positive financial shocks and negative productivity shocks will increase the outside option of workers endogenously. For these shocks, firms will choose to increase hours per worker since both shocks will relax the enforcement constraint and hours can be increased instantly unlike the stock of employees or capital. Since workers will work more on average, the outside option of not working increases. As a result, the firm's surplus per worker becomes more sensitive to the marginal productivity of each additional worker F_n, which gives the firm more of an incentive to change vacancy postings in response to shocks.

To see the effect of the enforcement constraint on the wage bill more clearly, consider the case in which the equity payout is simply $\varphi(d, d_-) = d$. In this case, there are no costs associated with adjusting the dividend and $\varphi_d = 1/\lambda = 1$. It follows that we can write the wage bill in (2.12) as

$$\begin{aligned} wh &= \mu\left[(1-\gamma)F_n + \left(\frac{V}{1-N}\right)c_v\right] \\ &\quad + (1-\mu)\left[\frac{\nu(1)-\nu(1-h)}{u_c} + b\right] \end{aligned}$$

Since $\gamma \geq 0$, the tighter the enforcement constraint, the lower the effective marginal productivity of each worker to the firm becomes. That is to say, in situations in which the shadow price of the enforcement constraint increases, the bargaining weight shifts away from workers to the firm due to the fact that the firm would like to decrease the number of employees in order to loosen the enforcement constraint. However, since there are no endogenous separations, the firm is inhibited from decreasing either the capital stock or the stock of workers and must do so along the intensive margin. The shadow price of our enforcement constraint will increase during positive shocks to total factor productivity and in situation in which the credit market conditions deteriorate.

If there were no credit market frictions in our environment or during situations in which our enforcement constraint becomes nonbinding ($\gamma = 0$), our wage bill would collapse to the standard matching model sharing rule:

$$\begin{aligned} wh &= \mu \left[F_n + \left(\frac{V}{1-N} \right) c_v \right] \\ &\quad + (1-\mu) \left[\frac{\nu(1) - \nu(1-h)}{u_c} + b \right] \end{aligned}$$

This last equation will correspond to the wage bill in the Andolfatto benchmark model. The derivation of the equations above is detailed in the Appendix B.

2.2.5 Government

The government in this model simply raises revenue in order to subsidize firm's borrowing and to pay out the unemployment benefits b to the mass of unemployed households. This is simply:

$$T(S) = \left(\frac{1}{R(S)} - \frac{1}{1+r(S)} \right) a'_F(S, s_F) + (1-N) b$$

where S once again denotes the aggregate state.

2.2.6 Equilibrium

A *recursive competitive equilibrium* is defined as a set of functions for (i) the household's policies $c(S, s_H), s'(S, s_H)$, and $a'_H(S, s_H)$; (ii) the household's value function $V(S, s_H)$; (iii) the firm's policies $d(S, s_F), k'(S, s_F), a'_F(S, s_F)$, and $v(S, s_F)$; (iv) the firm's value function $J(S, s_F)$; (v) aggregate prices $r(S), R(S), p(S)$, and $m'(S, S')$; (vi) taxes $T(S)$; (vii) the law of motion for aggregate states $S' = G(S)$. Such that: (i) the household's policies are optimal and $V(S, s_H)$ satisfies the Bellman's equation (2.1); (ii) the firm's policies are optimal and $J(S, s_F)$ satisfies the Bellman's equation (2.4); (iii) $m' = \beta u_c(c') / u_c(c)$; (iv) the government's budget is balanced; (v) wages and hours $(w(S, s_H, s_F), h(S, s_H, s_F))$ is the solution to the bilateral Nash bargaining problem given by equation (2.9); (vi) markets clear, $s' = 1, a'_F = a'_H$; (vii) the law of motion $G(S)$ is consistent with individual decisions and the stochastic processes for z and ξ.

2.3 Calibration of the model

We must now specify some functional forms in order to evaluate our model's quantitative results. We define the matching technology, the aggregate production technology and the equity payout cost to be:

$$\begin{aligned} M(V, 1-N) &= \omega V^{\psi}(1-N)^{1-\psi} \\ F(z, K, Nh) &= zK^{\alpha}(Nh)^{1-\alpha} \\ \varphi(d, d_{-}) &= d + \kappa(d - d_{-})^2 \end{aligned}$$

where $\psi \in (0,1)\,, \alpha \in (0,1)$ and $\kappa \geq 0$. The representative household's preferences take the form:

$$u(c) = \log(c)$$

$$\nu(\ell) = \begin{cases} \phi\frac{\ell^{1-\eta}}{1-\eta} & \text{if } \ell \in [0,1) \\ \phi_u & \text{if } \ell = 1 \end{cases}$$

and the stochastic processes follow an autoregressive system:

$$\begin{aligned} \begin{pmatrix} z' \\ \xi' \end{pmatrix} &= \mathbf{A}\begin{pmatrix} z \\ \xi \end{pmatrix} + \begin{pmatrix} \varepsilon_z \\ \varepsilon_\xi \end{pmatrix} \\ \begin{pmatrix} \varepsilon_z \\ \varepsilon_\xi \end{pmatrix} &\sim N(0, \Sigma) \end{aligned}$$

where ε_z and ε_ξ are normally distributed innovations with variance-covariance matrix Σ. We now left to determine twenty-one parameters in the model.

Our parameters can be categorized into three groups based on the way we chose to calibrate them. The first set of parameters are predetermined outside model. The second group is a set of parameters for the shock processes which are estimated from the constructed Solow residual and financial shock series. The last group of parameters consists of parameters determined endogenously in the model. We calibrate these parameters using simulated method of moments with a number of targets to be matched. To jointly choose this group of parameters, we minimize the distance between seven moments in the data and the in the model.

2.3.1 Predetermined parameters

We set the unemployment benefit $b = 0$, so this plays no role in our analysis. We basically follow Andolfatto (1996) for the discount factor $\beta = 0.99$, the depreciation rate $\delta = 0.025$, the separation rate $\chi = 0.15$ and the matching elasticity $\psi = 0.60$. Since we focus on an economy where the wage in the labor market is determined in a non-competitive fashion, we cannot use labor share data to pin down α. Rather, we choose a value for $\alpha = 0.64$, which is common across the macroeconomic literature and it is also the same as Andolfatto (1996). We choose the tax benefit of debt in a similar to JQ, $\tau = 0.35$. Finally, we set the bargaining weight of workers $\mu = 0.35$, which is a middle of HM (2008) and Shimer (2005). To summarize:

Table 2.2: Predetermined parameters

Parameter	*Description*	*Value*	*Remarks*
β	Discount factor	0.99	annual rate of return 4%
δ	Depreciation rate	0.025	Andolfatto (1996)
χ	Job-separation rate	0.15	Andolfatto (1996)
ψ	Matching elasticity	0.60	Andolfatto (1996)
α	CD parameter for capital	0.36	Andolfatto (1996)
τ	Tax benefit (subsidy)	0.35	JQ (2012)
μ	Bargaining weight	0.35	middle of HM and Shimer

All these parameters, except τ, will also be used in the Andolfatto model.

2.3.2 Parameters for the shock processes

We construct our z series using the definition of our aggregate production function. In order to construct a series of the measured Solow residual, we must first specify a series for Y_t, K_t, N_t, and h_t. We use Current Population Survey data on the level of employment (N_t) and the average weekly hours worked (h_t). Y_t is simply real GDP taken from the Bureau of Economic Analysis. We construct our capital stock using Flow of Funds data for the nonfinancial business sector and deflate the level of investment each period by the business GDP price index taken from the Bureau of Economic Analysis. Depreciation is taken to be the consumption of fixed capital of nonfinancial business Since we only have flows of net capital expenditures and not a level, we pick K_0 in 1952 such that the capital-output ratio displays no trend. Since we begin the recursion in 1952 and our analysis begins in 1984:Q1, it is not relevant for our results based on the time period for our analysis. Log-linearizing our aggregate production function gives:

$$\hat{z}_t = \hat{y}_t - \alpha \hat{k}_t - (1-\alpha)\,\hat{N}_t - (1-\alpha)\,\hat{h}_t$$

where hats denote log-deviations from a linear trend for each variable estimated over the period 1984:Q1-2012:Q1. We normalize $\bar{z} = 1$.

For the construction of our financial shocks, we make the assumption that the enforcement constraint is *always* binding. Of course, the validity of this assumption is critical for the construction of our financial shock series. We verify ex-post: after constructing the series for the shocks and feeding them into the model to verify that the Lagrange multiplier is always strictly greater than zero. This assumption is strong and open for debate. However, we feel that viewing the nonfinancial business sector in the aggregate as always being constrained is not an outrageous assumption to make. Log-linearizing the enforcement constraint (equation (2.4)), gives us:

$$\hat{\xi}_t = \frac{\bar{\xi}\bar{b}^e}{\bar{y}}\hat{b}^e_{t+1} - \frac{\bar{\xi}\bar{k}}{\bar{y}}\hat{k}_{t+1} + \hat{y}_t$$

where we construct $\hat{b}^e_{t+1}$ using Flow of Funds data for net borrowing in credit market instruments in the nonfinancial business sector deflated by the business GDP price index. $\hat{y}_t$ in this case is not total GDP but real business GDP. Details of the data can be found in the Appendix B. The capital stock is as defined previously. We fix $\bar{b}^e/\bar{y} = 3.37$ to match the liabilities-output ratio over our sample period. This, in turn, gives us $\bar{\xi}\bar{k}/\bar{y} = 1.4362$ and $\bar{\xi}\bar{b}^e/\bar{y} = 0.4361$. We then use the constructed series for $\hat{z}_t$ and $\hat{\xi}_t$ and estimate a vector-autoregressive process over the time period 1984:Q1-2012:Q1. This gives us the matrix of coefficients and the variance-covariance matrix:

$$\begin{aligned}
\mathbf{A}\begin{pmatrix} z \\ \xi \end{pmatrix} &= \begin{pmatrix} 0.9910 & -0.0351 \\ 0.2403 & 0.8978 \end{pmatrix}\begin{pmatrix} z \\ \xi \end{pmatrix} \\
\Sigma &= \begin{pmatrix} 0.0050^2 & 0.000027 \\ 0.000027 & 0.0079^2 \end{pmatrix}
\end{aligned}$$

For our Andolfatto benchmark model without financial frictions, we simply have a AR(1) process for the productivity given by:

$$\begin{aligned}
\rho_z &= 0.9426 \\
Var\,(\varepsilon_z) &= 0.0051^2
\end{aligned}$$

2.3.3 Parameters determined using targets

For our remaining seven parameters, we use the simulated method of moments to minimize the distance between seven moments from the data and from the model. Our seven targets are:

1. Frisch elasticity of hours for those employed: 0.5
2. Steady-state employment to population ratio: 62%
3. Steady-state hours per worker: 0.39 (weekly potential hours are assumed to be 100)
4. Steady-state job-filling rate: 90%
5. Vacancy expenditures-output ratio: 2.18%
6. Debt to GDP ratio: 3.37
7. Standard deviation of the equity payout-GDP ratio: 1.39

According to Silva and Toledo (2009), the average cost of time spent hiring one worker is approximately 3.6%-4.3% of total labor costs. We target the median, 3.9%, of these estimates. In terms of our model, this implies $\frac{c_v v}{\Phi w n h} = 0.039$, which in turn gives $\frac{c_v v}{y} = 0.218$ given our targets for the job-filling rate $\Phi = 0.9$ and the labor share $= 0.62$, which is the average labor share over our sample period. κ is chosen to have a standard deviation of the equity payout-GDP ratio generated by the model equal to that of data.

For the calibration of the Andolfatto model, we omit the last two targets listed above from the calibration since κ and $\bar{\xi}$ are not present in that model environment. These targets give the following set of parameters for both our model (KS model, which stands for Kim and Seliski) and the Andolfatto model:

2.4 Results and discussion

We solve both the KS and Andolfatto models using 2nd order approximation around the steady-state. The derivation of the nonlinear equations characterizing both models' equilibriums can be found in the Appendix B. We first show the resulting impulse

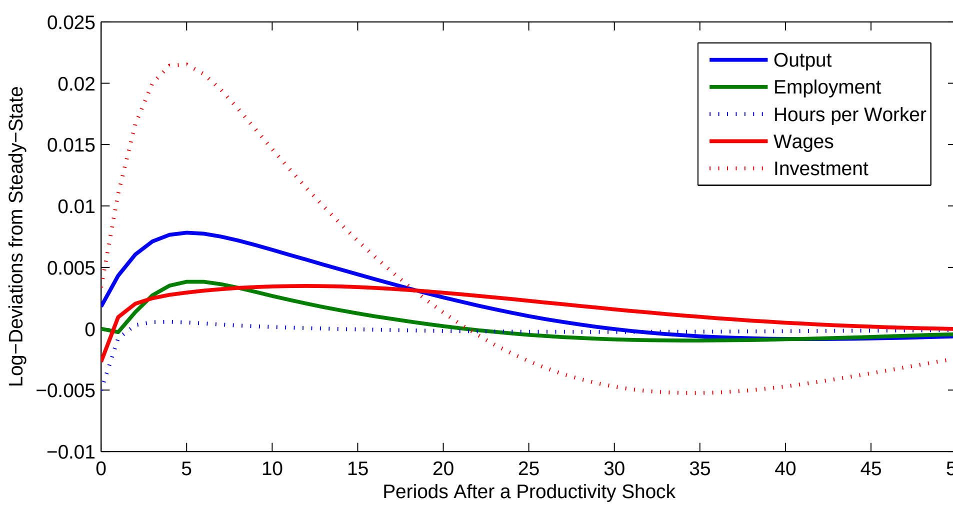
Table 2.3: Calibrated parameters

Parameter	*Description*	*KS*	*Andolfatto*
η	Curvature parameter for leisure	3.1166	3.1166
ϕ	Scale parameter for leisure	0.7814	0.7797
ϕ_u	Leisure for unemployed	0.2525	0.2554
c_v	Cost of posting a vacancy	0.1960	0.1875
ω	Matching efficiency	0.5349	0.5349
$\bar{\xi}$	Mean of credit process	0.1294	–
κ	Equity payout cost	0.1460	–

response functions for the KS model in order to develop some intuition underlying our results.

2.4.1 Innovations to productivity

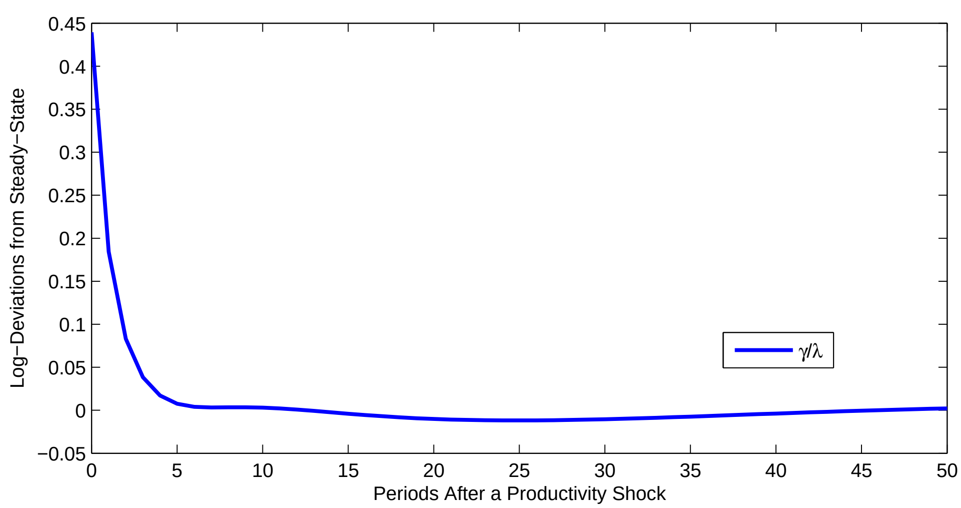
Figure 2.1: IRFs to a one standard deviation shock to TFP



When a productivity shock hits the KS model economy, the enforcement constraint instantly tightens. Since the stock of employees and capital are fixed, firms can only loosen the constraint via hours per worker and investing in a higher k_{t+1}. Hours in the model respond immediately because they can substitute for bodies that cannot be increased due to the nature of the hiring process. Once employees are separated exogenously, hours recovers back to its steady-state level.

In response to a positive productivity shock, the firm allocates resources away from labor input by decreasing both wages and hours and allocating the savings to investment. This is consistent with the countercyclical nature of the labor share reported earlier. The shift in bargaining power is due to the shock increasing the ratio of the Lagrange multipliers, effectively lowering the marginal product of each worker to the firm. The reason for the firm allocating more resources to capital is clear. After a tightening of the enforcement constraint, capital is deemed more ‘valuable’ to the firm because investment in capital tomorrow loosens the constraint. That is, labor and capital are imperfect substitutes not only due to their role in the production process, but also due to the added benefit of the higher capital stock loosening the enforcement constraint. The firm wishes to build up capital initially to loosen the constraint for future periods in order to take advantage of the persistence in the positive productivity shock. After employment begins to move (since it cannot move immediately), both wages and hours recover after the firm has effectively loosened the enforcement constraint by accumulating a higher capital stock.

Figure 2.2: IRFs to a one standard deviation shock to TFP



To visually see what is going on with the effective marginal product per worker, recall from the wage bargaining solution that $(1 - \gamma/\lambda) F_n$ is the effective benefit to the firm of employing an additional worker. The interpretation of γ/λ is the shadow price of the enforcement constraint discounted by the firm's marginal cost of financing operations via equity. We plot the deviations of the effective shadow price below.

The kinks are due to the frictional nature of employment (employment cannot adjust when the shock is initially realized). While the shadow price associated with the constraint is quite high initially, it quickly drops off as the firm accumulates capital in order to loosen the constraint. Once the constraint has been loosened, due to the higher k_{t+1}, the firm begins to accumulate employees once again by posting vacancies.

Figure 2.3: IRFs to a one standard deviation shock to TFP

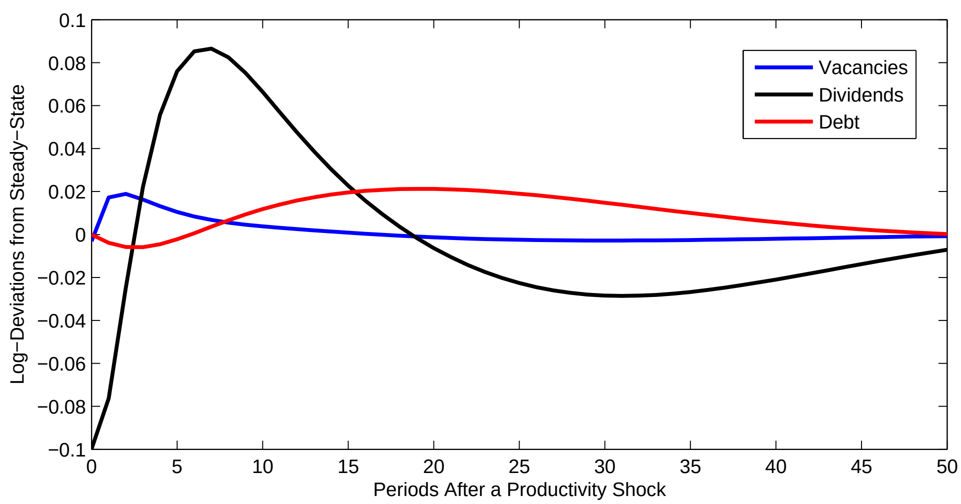


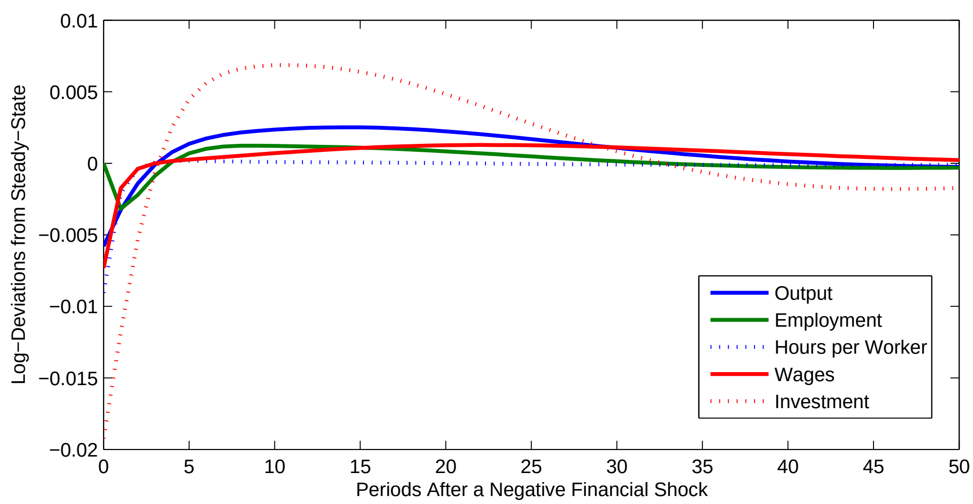
Figure 2.3 shows how the firms finance their operations and how much of their resources are devoted to hiring purposes after a productivity shock. Once again, the kink is the result of the lagged nature of employment. Initially, the firm finance their capital accumulation not only by reducing labor inputs and labor costs, but also via reductions in equity payouts. The firms use internal finances briefly to accumulate capital resources. It is noteworthy that equity payouts reach their peak over a year after the TFP innovation. This can be viewed as the firm paying out the highest dividends once it has adjusted both employment and capital to a situation in which

the enforcement constraint's shadow price reaches its minimum deviation. Dividend payouts reach its peak around the same time that γ/λ reaches its minimum deviation. That is, the opportunity cost associated with diverting resources to dividend payments is at its lowest level.

2.4.2 Innovations to credit conditions

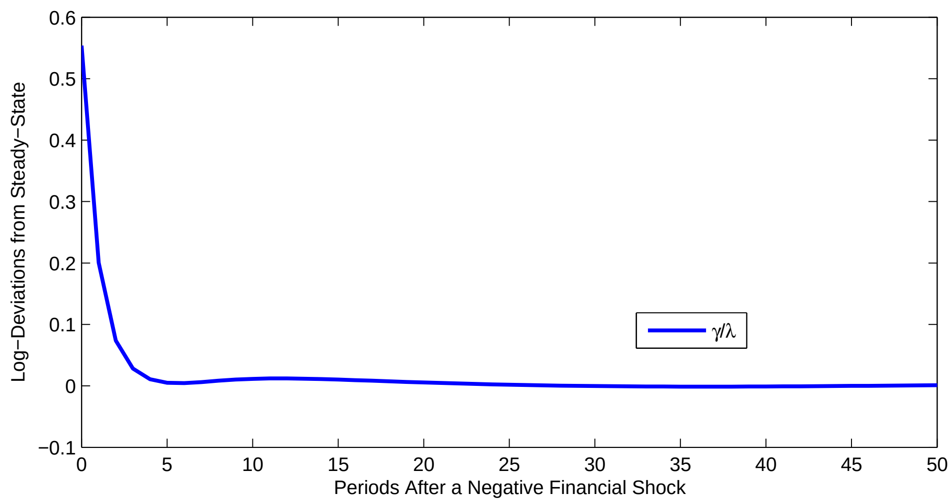
We now consider the situations in which our model economy is hit by a *negative* financial shock.

Figure 2.4: IRFs to a negative one standard deviation financial shock



Similar to the productivity case, investment is hit hardest by an innovation to the financial process. As the firm faces a tighter enforcement constraint due to the negative financial shock, it immediately cuts hours, wages and investment. Since the firm cannot immediately adjust employment, employment does not drop until the period after the shock. One of the key differences between the financial shock and the productivity shock, is the speed at which the economy recovers to its steady-state levels. This is in contrast to many findings that periods of financial distress lead to prolonged recessions. As in the positive productivity case, a negative financial shock shifts bargaining power away from the worker. Again, this is due to the tightness of the borrowing constraint driving down the effective marginal product of an additional worker to the firm.

Figure 2.5: IRFs to a negative one standard deviation financial shock

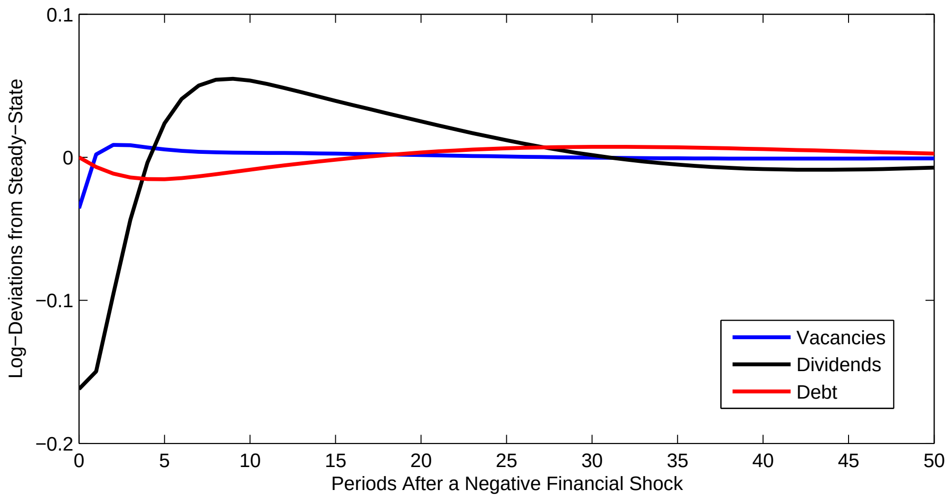


The effective shadow price of the enforcement constraint displays a very similar pattern to the positive productivity case but drops off faster to return near to its steady-state ratio. Workers quickly recover their bargaining position as the ratio of the Lagrange multipliers returns near its steady-state level.

The most pronounced difference between the productivity and financial shock cases is the movement of financial variables, which one would expect. The firm decreases its debt position and continues to decrease it for some time after the financial shock. The firm also reduces its equity payouts but eventually increases them after some time. This is consistent with the observation that both equity payouts and debt positions are reduced during periods of financial turmoil as reported in JQ. To highlight the contributions of each shock to key variables, we report the variance decomposition of each shock.

Financial shocks have a substantial impact on the volatility of both hours per worker and the labor share. The effects of financial shocks on the volatility of output and employment are relatively low. Despite equity and debt being financial variables, the impact of financial shocks on these is relatively similar to productivity shocks. While productivity shocks are still the main source of fluctuations along the extensive margin

Figure 2.6: IRFs to a negative one standard deviation financial shock



and seem to be the key driver in overall business cycle fluctuations, the impact of financial shocks is far from negligible on hours worked per worker. Financial shocks account for 36% of the volatility in total hours worked, mostly due to the impact of financial shocks on hours worked per worker. This, along with the fact that vacancies, hours, and the labor share are quite sensitive to financial shocks, provides evidence that incorporating financial shocks into a matching model results in a measurable improvement in the overall understanding of labor market fluctuations.

2.4.3 Comparing results

We now compare our results to the Andolfatto model (model without financial frictions) to see what gains and what shortcomings the incorporation of financial frictions provides. Both the KS and Andolfatto models are simulated for 350 periods 500 times. Eighty-eight periods of data are burned in order to strip out the importance of initial values.Variables are then logged and HP-filtered (except debt repurchases and equity payouts).

Table 2.4: Variance decomposition (percent)

Variable	$\hat{z}$	$\hat{\xi}$
Output	86.20	13.80
Total Hours	63.99	36.01
Employment	83.36	16.64
Hours per Worker	8.44	91.56
Wages	76.38	23.62
Labor Productivity	85.73	14.27
Labor Share	19.15	80.85
Vacancies	61.09	38.91
Equity Payouts/GDP	45.74	54.26
Debt Repurchases/GDP	48.69	51.31

Table 2.5: Business cycle moments

	$\sigma_x\%$			$\rho\,(x, Output)$			$\rho\,(x_t, x_{t-1})$		
Variable (x)	Data	*KS*	*Andolfatto*	Data	*KS*	*Andolfatto*	Data	*KS*	*Andolfatto*
Output	1.12	1.15	0.94	–	–	–	0.97	0.78	0.81
Total Hours	1.26	0.94	0.50	0.85	0.86	0.93	0.89	0.66	0.90
Employment	0.88	0.60	0.45	0.82	0.77	0.81	0.93	0.83	0.86
Hours per Worker	0.45	0.64	0.14	0.77	0.54	0.72	0.61	0.15	0.48
Wages	0.91	0.62	0.41	-0.18	0.78	0.96	0.77	0.33	0.67
Labor Productivity	0.66	0.58	0.51	0.07	0.58	0.94	0.59	0.59	0.58
Labor Share	0.73	0.79	0.13	-0.28	0.18	-0.67	0.78	0.27	0.44
Vacancies	11.26	3.73	2.60	0.86	0.71	0.72	0.91	0.31	0.45
Equity Payouts/GDP	1.39	1.39	–	0.69	0.70	–	0.91	0.89	–
Debt Repurchases/GDP	2.23	2.07	–	-0.84	-0.78	–	0.93	0.88	–

The addition of financial shocks into the matching model has a marked impact on key labor market variables. While the Andolfatto model generates high employment volatility, it is still orders of magnitude less than the data. The KS model improves the model's performance along this dimension. We are able to match the volatility of total hours and labor productivity quite well. However, our model performs poorly in replicating movements in wages and capturing the countercyclical nature of the labor share. Additionally, while the data has the intensive margin accounting for 32% of the variation in total hours worked, the KS model delivers 53%, overstating the importance of hours worked per worker while the Andolfatto model delivers only 14%.

Despite these shortcomings, our results comport to a greater extent with actual data than the Andolfatto model, indicating that the addition of financial frictions and financial shocks have a positive impact on matching moments from the data. This, taken together with the variance decomposition implies that financial shocks are an important dimension to incorporate into standard matching models. Our credit channel shows up through the multipliers associated with the enforcement constraint which drives down the marginal benefit of employees to firms. Financial frictions generate an additional wedge between the wage bill per worker and the marginal productivity of each worker F_n. Capital is more 'valuable' to the firm than an additional worker in this environment since capital has the additional benefit of loosening the enforcement constraint. This makes both investment and hours per worker sensitive to shocks originating in the financial sector or from TFP. Despite improving some labor market variables' volatilities via financial shocks, we are still quite far off from replicating the volatility displayed in the data, especially for vacancies.

2.5 Conclusion

Does the incorporation of financial shocks into a standard matching model better our understanding of fluctuations in hours, employment, and wages? Our analysis suggests that there are gains to be made by accounting for such shocks in a standard matching model. We proposed a model that uses Andolfatto as a our benchmark matching model and incorporate financial frictions and shocks into the environment similar to JQ. Within our model, we show that the credit channel has marked impacts on key labor variables via the shifting of bargaining power from workers to firms through the effective shadow price on the enforcement constraint.

Comparing our results to the Andolfatto model, calibrated to hit the same targets, demonstrates that our model can better replicate business cycle moments. Moreover, a variance decomposition of the shocks suggests that financial shocks play an important role in the fluctuation of both hours per worker and the labor share. While our results still support the notion that business cycle fluctuations are still largely due to productivity shocks, it also suggests that future research that employs a matching model framework should seriously consider the incorporation of financial shocks as well as the intensive margin to account for movements in key labor market variables. Without the incorporation of financial shocks, movements in employment, hours per worker, and the labor share are relatively muted over the business cycle.

Chapter 3

Wage Negotiations in Multi-worker Firms and Stochastic Bargaining Powers of Existing Workers

3.1 Introduction

In literature, two different bargaining protocols are used in the search and matching model where a firm hires more than one worker and the firm faces diminishing marginal product of labor (MPL). One is the Stole and Zwiebel (1996) type bargaining protocol as in Hawkins (2011), Elsby and Michaels (2013), Acemoglu and Hawkins (2014), and Fujita and Nakajima (2014). In these papers, a breakdown of a negotiation with a marginal worker negatively affects the bargaining position of the firm with other workers (one fewer workers) since MPL is higher with one fewer workers. The other is a standard bargaining protocol as in Merz (1995), Andolfatto (1996), and Cheron and Langot (2004). In these papers, a breakdown of a negotiation does not affect the bargaining with other workers because they implicitly assume that MPL does not change when the firm bargains with other workers. I interpret these two bargaining protocols as two extreme cases: in terms of relative bargaining powers between other workers and a firm.

I will call other workers existing workers. If existing workers have all the bargaining powers[1] , then the firm has to fully internalize the negative effects from the breakdown of the negotiation with a marginal worker. However, if the firm has all the bargaining powers[2] , the firm does not internalize any negative effects from the breakdown by ignoring that MPL is higher with one fewer workers. Given the two extreme cases, I am looking at cases between the two extremes by introducing stochastic bargaining powers of existing workers.

In this paper, when Nash bargaining with a marginal worker breaks down, a firm negotiates wages with existing workers. The bargaining powers of existing workers are stochastic. Due to diminishing MPL, the breakdown of the negotiation with the marginal worker negatively affects the bargaining position of the firm with existing workers (one fewer workers) since MPL is higher with one fewer workers. How much the firm internalizes this negative effect depends on the stochastic bargaining powers of existing workers which can be identified through labor share data. During expansions, it is relatively difficult for the firm to hire workers, so existing workers might have higher bargaining powers. If the firm fails to hire a marginal worker due to a breakdown of negotiations, the firm has to pay higher wages to existing workers. Since the failure to hire marginal workers is more costly during expansions, the firm has more incentives to hire marginal workers by offering higher wages to forgo the higher cost associated with the breakdown. During recessions, the opposite happens. Through this mechanism, the stochastic bargaining powers of existing workers provide an additional margin to increase the volatility of labor market variables. The calibrated model generates more volatile total hours, employment, hours per worker while labor share overshoots in response to productivity shocks as documented in Ríos-Rull and Santaeulalia-Llopis (2010). In particular, the volatility of employment in the model is similar to the actual US data. In contrast to the prediction of Ríos-Rull and Santaeulalia-Llopis (2010), in which the effect of productivity shocks is dampened when labor share overshoots due to huge wealth effects from the overshooting property of labor share, this paper presents a model in which the labor share overshoots in response to productivity shocks and the volatility of employment closely matches that of US data.

[1] This is my interpretation of the Stole and Zwiebel type bargaining protocol

[2] This is my interpretation of the standard bargaining protocol

Table 3.1: Business cycle moments in data and models over 1960:Q1-2012:Q1

	$\sigma_x\% \left(\frac{\sigma_x}{\sigma_{Output}}\right)$			$\rho\,(x, Output)$			$\rho\,(x_t, x_{t-1})$		
Variable (x)	Data	Andolfatto	KL	Data	Andolfatto	KL	Data	Andolfatto	KL
Output	1.54 (1.00)	1.31 (1.00)	1.31 (1.00)	-	-	-	0.86	0.82	0.82
Total Hours	1.38 (0.90)	0.70 (0.53)	0.70 (0.53)	0.85	0.92	0.92	0.88	0.91	0.91
Employment	1.00 (0.65)	0.68 (0.52)	0.69 (0.53)	0.81	0.78	0.78	0.91	0.89	0.88
Hours per Worker	0.49 (0.32)	0.19 (0.15)	0.19 (0.15)	0.74	0.55	0.54	0.56	0.58	0.58
Wages	0.91 (0.59)	0.62 (0.47)	0.64 (0.49)	0.34	0.94	0.94	0.69	0.65	0.64
Labor Productivity	0.82 (0.53)	0.72 (0.55)	0.72 (0.55)	0.45	0.92	0.92	0.57	0.62	0.62
Labor Share	0.74 (0.48)	0.12 (0.09)	0.09 (0.07)	-0.08	-0.72	-0.70	0.78	0.51	0.50
Vacancies	13.23 (8.59)	3.65 (2.79)	3.69 (2.82)	0.90	0.80	0.80	0.91	0.54	0.54

1) All data are in logs and filtered using the HP filter with a smoothing parameter of 1600.

2) In the Andolfatto model, I use the standard bargaining protocol

3) In the KL model, I use the Stole and Zwiebel type bargaining protocol

This paper is related to several studies which can be classified into three groups. First, the baseline model is based on Andolfatto (1996). His model embeds search and matching framework into an otherwise standard RBC model, and has both extensive margins and intensive margins. By incorporating search and matching framework in labor markets, the model improves the standard RBC model along several dimensions. However, the volatility of labor market variables is still far lower than that of actual data. The Andolfatto model also has highly pro-cyclical real wages and labor productivity, which have weakly pro-cyclical counterparts in actual data. Several papers have addressed these problems. Nakajima (2012) analyzes several volatility problems by explicitly distinguishing between leisure and unemployment benefits for the outside options of households. This distinction is consistent with the calibration proposed by Hagedorn and Manovskii (2008). However, the main focus of Nakajima (2012) is unemployment and vacancies than employment and hours per worker, which are my main interest. Cheron and Langot (2004) address the second failure of Andolfatto (1996) by using non-separable preference between consumptions and leisure such that the outside options of households can move counter-cyclically. This proposal results in less pro-cyclical real wages and labor productivity. However, this paper is not interested in the volatility of labor market variables in general.

The second branch of papers related to my paper is literature on the Stole and Zwiebel type bargaining and its applications to business cycle dynamics[3] . Krause and Lubik (2007) (KL, henceforth) incorporate the Stole and Zwiebel type bargaining protocol into a simple RBC search and matching model to evaluate the quantitative effects of the bargaining protocol on business cycle dynamics. They show that the aggregate effects of the bargaining protocol are negligible. Table 3.1 summarizes business cycle moments for the modified KL model[4] . The performance of KL model is almost the same as the Andolfatto model, and both models perform poorly in replicating moments

[3] Hawkins (2011), Elsby and Michaels (2013), Acemoglu and Hawkins (2014), and Fujita and Nakajima (2014) also study the labor marker fluctuations with the Stole and Zwiebel type bargaining, but the main focus of these papers is unemployment and vacancies than employment and hours per worker.

[4] The original model in KL does not have capital and intensive margins. Therefore, I add the Stole and Zwiebel type bargaining to the Andolfatto model rather than to the original model in KL in order to assure fair comparison of the two models.

along several dimensions. In contrast to KL, this paper introduces the stochastic bargaining with existing workers when the match with a marginal worker fails, and the bargaining powers of existing workers vary stochastically. The time-varying incentives to hire workers for firms, resulting from the stochastic bargaining, provide a new margin to increase the volatility of labor market variables. Later, it turns out that Andolfatto and KL are two extreme cases where bargaining powers of existing workers are fixed at 0, and 1, respectively, in the baseline model.

Lastly, this paper is also related to papers studying labor share. Recently, Ríos-Rull and Santaeulalia-Llopis (2010) document several properties of labor share dynamics based on US data. In particular, they propose redistributive shocks that can be identified by using labor share data in the US, and point out the importance of the dynamic property of labor share (overshooting). They showed that labor share overshoots in response to productivity shocks, and the dynamic overshooting response of labor share drastically dampens the role of productivity shocks on labor markets due to huge wealth effects. My model also generates the overshooting property of labor share, but total hours, employment, and hours per worker are still more volatile than the benchmark Andolfatto model. Different from Ríos-Rull and Santaeulalia-Llopis (2010), the search and matching framework weakens wealth effects from the overshooting of labor share and more incentives for firms to hire workers offset the huge reduction of total hours.

The main contribution of this paper is as follows. First, I incorporate the stochastic bargaining with existing workers into the Andolfatto model, which has both extensive and intensive margins in the labor market[5] . To the best of my knowledge, I first study the effect of the stochastic bargaining powers of existing workers on wage negotiations in multi-worker firms. The bargaining powers of existing workers can be time-varying because when labor markets are tighter, mostly in booms, existing workers are more valuable as the firm will have difficulty finding new workers. However, when the labor market is less tight, mostly in recessions, existing workers become less attractive to firms, which could easily find new workers. This reason makes the bargaining powers of existing workers possibly pro-cyclical with some lags. Another possible explanation

[5] I include intensive margins for two reasons. First, labor share is important for identifying bargaining shocks, and for the labor share in the model to be consistent with actual data, I need to include intensive margins. Second, bargaining shocks directly affect intensive margins because the bargaining powers of existing workers affect the relative usefulness of intensive and extensive margins for the firm.

could be related to the entry and exit of firms. In booms, firms compete with each other because of higher entry rates of new firms and lower exit rates of existing firms. These situations reduce the monopolistic or bargaining powers of firms over existing workers. However, during recessions, the opposite happens. Given this explanation, the bargaining powers of existing workers move pro-cyclically with some lags based on the pro-cyclical entry and the counter-cyclical exit rates. The inclusion of the stochastic bargaining with existing workers improves the capacity of the standard RBC search and matching model, especially in the volatility of total hours, employment, hours per worker, and labor share.

Second, I identify bargaining shocks by using labor share data. I provide the link between the bargaining powers of workers and the movement of labor share in the US. In addition, my model generates an overshooting property of labor share, but the effect of productivity shocks on labor market variables are still significant in contrast to the prediction of Ríos-Rull and Santaeulalia-Llopis (2010). In their model, the effect of productivity shocks is dampened when labor share overshoots because of huge wealth effects from the overshooting property. In contrast to their model, the baseline model has a search and matching framework, and the nature of this framework weakens wealth effects resulting from the overshooting of labor share. On top of these differences, more incentives for firms to hire workers offset the huge reduction of total hours in booms.

The remainder of the paper is structured as follows. Section 2 introduces the baseline model with the stochastic bargaining powers of existing workers. Section 3 discusses the calibration of the baseline model. Section 4 shows quantitative analysis of the model. Section 5 discusses the robustness of the baseline model. Finally, Section 6 concludes and proposes the further research.

3.2 Model

I develop a model based on a standard RBC search and matching model, the Andolfatto (1996) model. The main difference between this paper and the Andolfatto model is the outside option of a firm in the bargaining with a marginal worker. I explicitly consider the outside option of a firm when the firm bargains with a marginal worker. The outside option of the firm is bargaining with existing workers and producing goods with them. The issue with bargaining with existing workers is the wages the firm pays. In this paper, these wages depend on the bargaining powers of existing workers. If the bargaining powers of existing workers are high, then existing workers will receive higher wages, but if the bargaining powers of existing workers are low, then they will receive lower wages. Note that these wages are not realized if the match with the marginal worker is successful while they still affect the equilibrium wages. In this paper, matches are always successful because the surplus of a new match is always positive. Therefore, wages bargained with existing workers would not be realized in equilibrium. Furthermore, I assume the bargaining powers of existing workers stochastically evolves. Except for the stochastic bargaining with existing workers, the baseline model is similar to Andolfatto (1996), and Cheron and Langot (2004).

3.2.1 Matching

I assume that the period in the model is a quarter. The timing of my model is as follows: (1) shocks are realized, (2) wages and hours per worker are bargained over with marginal workers, (3) if matches are not successful, the firm bargains wages with existing workers (4) workers are matched with the firm (5) production takes place and the firm posts vacancies, and (6) separations occur.

Since labor markets are frictional, the unemployed search for jobs and firms post vacancies to hire workers. The number of matches is determined by constant returns to scale matching function $M = M(V, 1-N)$, which depends on the total number of vacancies, V, and the total number of the unemployed, $U \equiv 1-N$. For later use, I define $\theta = V/(1-N)$ as market tightness in labor markets. Also, I define the job-finding rate $\Psi(\theta) \equiv M/(1-N) = M(\theta, 1)$ and the job-filling rate $\Phi(\theta) \equiv M/V = M(1, 1/\theta)$. Finally, I assume that workers are separated at the exogenous and constant rate $\chi \in (0,1)$. Therefore, we have the following law of motion of total employment.

$$N^{'} = (1-\chi)N + M(V, 1-N)$$

3.2.2 Household

There is a continuum of identical and infinitely lived households of measure one. The measure of members in each household is also normalized to 1. The aggregate states in this economy are given by $S = \{z, \gamma; K, N\}$, where z is the aggregate productivity and γ is the bargaining power of existing workers, which varies stochastically. K is the aggregate capital stock, and N is total employment. The individual state variables of the household are $s_H = \{a, n\}$, where a is the amount of assets they hold and n is the measure of the employed in household. I can write the household problem as follows:

$$\Omega\left(S, s_H\right) = \max_{c,a^{'}} u\left(c\right) + nu^{l}\left(1 - h\left(S, s_H\right)\right) + \left(1 - n\right)u^{l}\left(1\right) + \beta E\left[\Omega\left(S^{'}, s_H^{'}\right)\right]$$

$$\begin{aligned} &s.t. \\ c + a^{'} + T\left(S\right) &= w\left(S, s_H\right)h\left(S, s_H\right)n + \left(1 - n\right)b + \left(1 + r(S)\right)a + \Pi\left(S\right) \\ n^{'} &= \left(1 - \chi\right)n + p\left(S\right)\left(1 - n\right) \\ S' &= G\left(S\right) \end{aligned}$$

where $u\left(c\right)$ is utility from consumption, a is the assets household holds, $u^{l}\left(\cdot\right)$ is utility from leisure, $T(S)$ is the lump-sum tax, $\Pi\left(S\right) = F\left(z, k, nh\right) - w\left(S, s_F\right)h\left(S, s_F\right)n - \left(r\left(S\right) + \delta\right)k - \kappa v$ is the dividend which will be defined in the firm's problem. $p\left(S\right) = M/\left(1 - N\right)$ is the job-finding rate and $G\left(S\right)$ is the law of motion of aggregate state variables. Household takes wages $w\left(S,\, s_H\right)$ and hours per worker $h\left(S, s_H\right)$ as given. They are jointly determined via Nash bargaining.

The household consumes (c), accumulate assets (a) which they rent to a firm, and supplies labor. The n fraction of members in each household is matched with the firm and employed. And the $1 - n$ fraction of members is unemployed, searches for jobs, and they collect unemployment benefits (b) from the government. I assume that there is no search cost, and so every member who is not employed searches for the job.[6] I also assume that there is a perfect insurance for unemployment within the household as noted in Andolfatto (1996).[7] As a result, every member receives the same consumption level. Note that this implies unemployed members are better off than those who are employed since they receive the same consumption level but the unemployed enjoy a full amount of leisure.[8]

---

[6] In this sense, u in my model is the non-employed. I do not distinguish between the unemployed and the non-employed like Andolfatto (1996). Since the measure of the unemployment rate in model and data are inconsistent, I do not report any statistics regarding unemployment in this paper.

[7] Separable utility functions over consumption and leisure satisfy this assumption.

[8] I can relax this assumption. As noted in Cheron and Langot (2004), Nakajima (2012), if I use non-separable utility functions over consumption and leisure, the employed receive higher levels of consumption than the unemployed. Consequently, the employed are better off in equilibrium. If I use non-separable utility functions, the performance of the model would be better, especially for labor productivity and real wages. However, I do not use these utility functions because I prefer to setup the baseline model in a more parsimonious way.

The first order conditions of household's problem give[9]

$$E\left[\beta\frac{u_c^{'}}{u_c}\left(1+r^{'}\right)\right] = 1$$

This is a standard Euler equation for the household.

3.2.3 Firm

There exists a representative firm. The firm produces goods using a constant returns to scale production technology $F(z,k,nh)$, where z is the aggregate productivity . Given the aggregate state S, and the individual state variable of the firm $s_F = \{n\}$, I can write firm's recursive problem as follows:

$$\begin{aligned} J(S,s_F) &= \max_{v,k,n'} \Pi(S) + E\left[m'\left(S,S'\right)J\left(S',s_F'\right)\right] \\ &= \max_{v,k,n'} F(z,k,nh) - w(S,s_F)\,h(S,s_F)\,n - (r(S)+\delta)\,k - \kappa v + E\left[m'\left(S,S'\right)J\left(S',s_F'\right)\right] \\ s.t. & \\ n' &= (1-\chi)\,n + q(S)\,v \\ S' &= G(S) \end{aligned}$$

where $m^{'}\left(S,S^{'}\right) = \beta u_c\left(c\left(S^{'}\right)\right)/u_c\left(c\left(S\right)\right)$ is the stochastic discount factor, κ is the cost of posting vacancies, and $q(S) = M/V$ is the job-filling rate. Again, $G(S)$ is the law of motion of aggregate state variables. The firm hires workers and rent capital from the households, and posts vacancies to hire more workers in the next period. Firms also take wages $w(S,s_F)$ and hours per worker $h(S,s_F)$ as given. They are jointly determined via Nash bargaining. From the first order conditions, we have two equilibrium conditions.

$$\begin{aligned} r &= F_k - \delta \\ \kappa &= qE\left[m^{'}J_n^{'}\right] \end{aligned}$$

The first condition is an equation for the equilibrium rental rate. The second equation is a job creation condition, which implies the firm posts vacancies up to the point where the marginal cost of posting vacancies equals to the value of an additional worker discounted by the probability that the firm meets a marginal worker.

3.2.4 The bargaining with a marginal worker

As stated before, wages, w, and hours per workers, h, are jointly determined via Nash bargaining between a worker and a firm each period. Formally, Nash bargaining problem can be written as follows:

[9] I will drop state variables for simple notations.

$$
\begin{aligned}
(w,h) &= arg\max_{w,h} \left(\Omega^m\right)^{\mu} \left(J^m\right)^{1-\mu} \\
&= arg\max_{w,h} \left(\frac{V^E - V^U}{u_c}\right)^{\mu} \left(\lim_{\Delta\to 0} \frac{J\left[n+\Delta\right] - J^B\left[n\right]}{\Delta}\right)^{1-\mu}
\end{aligned}
$$

The first component, Ω^m, denotes the marginal value of employment for the worker[10] and the second component, J^m, represents the marginal value of an additional employee to the firm. μ is the bargaining power of a marginal worker. V^E is the value of employment for the worker and V^U is the value of unemployment for the worker, which is the outside option of the worker. $J\left[n+\Delta\right]$ is the value of the firm when the match with the $(n+\Delta)$-th worker is successful and $J^B\left[n\right]$ is the value of the firm when the negotiation breaks down, which is the outside option of the firm. The only difference between the bargaining problem in this paper and the standard Nash bargaining is the outside option of the firm, $J^B\left[n\right]$, which is defined within the marginal value of an additional employee to the firm.

The marginal value of employment for the worker

I can define the marginal value of employment for the worker as follows.

$$
\begin{aligned}
\Omega^m = \frac{V^E - V^U}{u_c} &\equiv \left[wh + \frac{u(c)}{u_c} + \frac{u^l\left(1-h\right)}{u_c} + \left(1-\chi\right)\beta E\left[\frac{V^{E'}}{u_c}\right] + \chi\beta E\left[\frac{V^{U'}}{u_c}\right]\right] \\
&- \left[b + \frac{u(c)}{u_c} + \frac{u^l\left(1\right)}{u_c} + p\beta E\left[\frac{V^{E'}}{u_c}\right] + \left(1-p\right)\beta E\left[\frac{V^{U'}}{u_c}\right]\right] \\
&= wh - b + \frac{u^l\left(1-h\right) - u^l\left(1\right)}{u_c} + \left(1-\chi-p\right)\beta E\left[\frac{V^{E'} - V^{U'}}{u_c}\right]
\end{aligned}
$$

Note that the bracket in the first line is the value of working which includes the wage bill, utility from consumption, utility from leisure, and the continuation value of employment for the worker. The bracket in the second line is the outside option of the worker which consists of unemployment benefits, utility from consumption, utility from leisure, and the continuation value of unemployment for the worker. From the value function of the household, we also have

$$
\frac{\Omega_n}{u_c} = wh - b + \frac{u^l\left(1-h\right) - u^l\left(1\right)}{u_c} + \left(1-\chi-p\right)E\left[\beta\frac{\Omega_n'}{u_c}\right]
$$

[10] Note that this value is discounted by the marginal utility of consumption so that the unit of this term can be converted to consumption goods

From the above equations, we have

$$\Omega^m = \frac{\Omega_n}{u_c}$$

Therefore, the marginal value of employment for the worker that I defined before is the same as the partial derivative of the value function of the household with respect to the number of the employed in household, n.

The marginal value of an additional employee to the firm

The marginal value of an additional employee to the firm is not trivial because the outside option of the firm can be defined in different ways. The outside option of the firm in the bargaining with a marginal worker is bargaining wages with existing workers and producing goods with them. The key component of the outside option for the firm is the wages the firm pays to existing workers. Let w^e be the wages negotiated between the firm and existing workers when the match with a marginal worker breaks down[11]. I can define the value of an additional employee to the firm, J^m, as follows:

$$\begin{aligned} J^m &\equiv \lim_{\Delta\to 0} \frac{J[n+\Delta] - J^B[n]}{\Delta} \\ &= \lim_{\Delta\to 0} \frac{1}{\Delta}\left[\left(F(z,k,(n+\Delta)h) - w[n+\Delta](n+\Delta)h - (r+\delta)k - \kappa v + \beta E\left[\frac{u'_c}{u_c} J\left[(n+\Delta)'\right]\right]\right)\right. \\ &\left. -\left(F(z,k,nh) - w^e[n]nh - (r+\delta)k - \kappa v + \beta E\left[\frac{u'_c}{u_c} J^B[n']\right]\right)\right] \\ &= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0} \frac{[w[n+\Delta](n+\Delta)nh - w^e[n]nh]}{\Delta} + (1-\chi)\beta E\left[\frac{u'_c}{u_c} J^{m\prime}\right] \end{aligned}$$

$J[n+\Delta]$ denotes the value of the firm when the match with a marginal worker is successful[12]. I assume that if the match is successful today, then it is also successful afterward. $J^B[n]$ denotes the value of the firm when the negotiation with the marginal worker breaks down, which is the outside option of the firm. In this case, the firm do not hire new workers and continues to produce goods with existing workers by continuing wage negotiations with them afterward. $w[n+\Delta]$ is Nash bargained wages with

[11] As I mentioned before, these wages, w^e, would not be realized in equilibrium. These wages show up in the outside option of the firm, but the match with a marginal worker is always successful in this paper because the match surplus is always positive. Consequently, the wages for existing workers are not realized in equilibrium while they still affect equilibrium wages and other variables.

[12] I drop aggregate state variables for simple notations here

the $(n+\Delta)$-th worker and $w^e[n]$ is wages for existing workers when the match breaks down. The second line is the value of the firm when the firm hires $\triangle$ more workers, which includes the level of output less wage bills with workers including newly hired ones and costs of posting vacancies, and the continuation value of the firm. The third line is the outside option of the firm, which consists of the level of output less wage bills with existing workers and costs of posting vacancies, and the continuation value of the firm. The derivation of the last equation can be found in the Appendix C.

If the firm has all the bargaining powers, then the firm does not internalize any negative effects from the breakdown of the negotiation with the marginal worker by ignoring that MPL is higher with one fewer workers. In this case, the firm pays existing workers the same wages as the firm would have paid the marginal worker . Then, we have $w^e[n] = w[n+\Delta]$.

Proposition 1

Suppose $w^e[n] = w[n+\Delta]$. Then, the marginal value of an additional employee to the firm reduces to

$$J^m = \frac{\partial F(z,k,nh)}{\partial n} - w[n]h + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m\prime}\right]$$

Proof. See Appendix C. □

This is the standard marginal value of an additional employee to the firm in literature where wages are determined via the standard bargaining protocol as in Merz (1995), Andofatto (1996), and Cheron and Langot (2004). Also, note that this equation can be directly derived by differentiating the firm's value function J with respect to n, under the assumption that wages are not a function of n.

On the other hand, if existing workers have all the bargaining powers, then the firm should fully internalize the negative effects from the breakdown. In this case, the firm continues Nash bargaining with one fewer workers, and we have $w^e = w[n]$, where $w[n]$ is Nash bargained wages with n-th worker.

Proposition 2

Suppose $w^e[n] = w[n]$. Then, the marginal value of an additional employee to the firm reduces to

$$J^m = \frac{\partial F(z,k,nh)}{\partial n} - w[n]h - \frac{\partial w[n]}{\partial n} nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c} J^{m'}\right]$$

Proof. See Appendix C. □

This is the marginal value of an additional employee to the firm when wages are determined via the Stole and Zwiebel bargaining protocol as in Hawkins (2011), Elsby and Michaels (2013), Acemoglu and Hawkins (2014), and Fujita and Nakajima (2014). Note that this equation can be directly derived by differentiating the firm's value function J with respect to n, under the assumption that wages are an explicit function of n. The partial derivative term $\frac{\partial w}{\partial n}$ will be turned out to be negative later.

In this paper, I assume that wages, $w^e[n]$, are determined based on the bargaining powers of existing workers, γ. More specifically, I assume $w^e[n] \equiv \gamma w[n] + (1-\gamma) w[n+\Delta]$. For example, if existing workers have higher bargaining powers, they receive wages more close to $w[n]$, and if they have lower bargaining powers, they receive wages more close to $w[n+\Delta]$.

Proposition 3

Suppose $w^e[n] = \gamma w[n] + (1-\gamma) w[n+\Delta]$. Then, the marginal value of an additional employee to the firm reduces to

$$J^m = \frac{\partial F(z,k,nh)}{\partial n} - w[n]h - \gamma\frac{\partial w[n]}{\partial n} nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c} J^{m'}\right]$$

Proof. See Appendix C. □

By construction, if $\gamma = 0$, then *Proposition 3* reduces to *Proposition 1* (standard bargaining protocol), and if $\gamma = 1$, then *Proposition 3* reduces to *Proposition 2* (Stole and Zweibel bargaining protocol). Note that the marginal value of an additional employee to the firm depends on the stochastic bargaining power of existing workers through the term, $\gamma\frac{\partial w[n]}{\partial n}nh$. This is the main contribution of this paper. The inclusion of the stochastic bargaining bargaining with existing workers provides an additional margin to increase the volatility of labor market variables basically through the term, $\gamma\frac{\partial w[n]}{\partial n}nh$ within the the marginal value of an additional employee to the firm.

Stochastic bargaining powers of existing workers, γ

The bargaining power of existing workers, $\gamma \in [0, 1]$, can be time-varying because when labor markets are tighter, mostly in booms, existing workers are more valuable as the firm will have difficulty finding new workers. However, when the labor market is less tight, mostly in recessions, existing workers become less attractive to firms, which could easily find new workers. This reason makes the bargaining power of existing workers possibly pro-cyclical with some lags. Another possible explanation could be related to the entry and exit of firms over business cycles, which are abstracted from in this paper. In booms, several firms compete with a specific firm because of higher entry rates of new firms and lower exit rates of existing firms. These situations reduce the monopolistic or bargaining powers of the firm over existing workers. However, during recessions, the opposite happens. Given this explanation, the bargaining power of existing workers moves pro-cyclically with some lags based on the pro-cyclical entry and the counter-cyclical exit rates.

Since the baseline model does not have any endogenous mechanism to generate time-varying bargaining power of existing workers, I will assume that γ varies stochastically and call innovations to γ bargaining shocks. I will show, in the calibration section, that bargaining shocks can be identified by using labor share data from US once we have the solution to the first order differential equation from the wage bill equation. I set a fixed bargaining power for marginal workers, μ, while I allow the bargaining powers of existing workers, γ, to vary over time. In the robustness section, I show the time-varying bargaining power of a marginal worker is quantitatively not an important factor given the constructed shock series of bargaining power of a marginal worker, μ_t, by using labor share data. I will discuss it more in the robustness section.

Solutions to the bargaining with a marginal worker

Now, we turn to the bargaining problem which is the same as standard Nash bargaining given the marginal value of employment for the worker and the marginal value of an additional employee to the firm.

$$\Omega^m = wh - b + \frac{u^l(1-h) - u^l(1)}{u_c} + (1-\chi-p)E\left[\beta\frac{u'_c}{u_c}\Omega^{m'}\right]$$

$$J^m = \frac{\partial F(z,k,nh)}{\partial n} - wh - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]$$

Given the bargaining power of the marginal worker, $\mu \in [0,1]$, and the bargaining powers of existing workers , $\gamma \in [0,1]$, wages and hours per worker are determined via the following standard bargaining problem.

$$(w,h) = arg\max_{w,h}(\Omega^m)^\mu (J^m)^{1-\mu}$$

I will write w instead of $w[n]$ for simple notations hereafter. From the first order conditions with respect to w and h, we have the following two equations.

$$wh = \mu\left(\frac{\partial F(z,k,nh)}{\partial n} - \gamma\frac{\partial w}{\partial n}nh + \frac{V}{1-N}\kappa\right) + (1-\mu)\left(\frac{u^l(1) - u^l(1-h)}{u_c} + b\right)$$

$$\frac{u^l_{(1-h)}(1-h)}{u_c} = F_{nh} + \gamma\frac{\partial w}{\partial n}n$$

where $F_{nh} = \frac{\partial F(z,k,nh)}{\partial(nh)}$[13]. The first equation is the wage bill equation and the second equation is an intra-temporal condition for hours per worker. Note that we have additional terms, $\gamma\frac{\partial w}{\partial n}nh$ and $\gamma\frac{\partial w}{\partial n}n$ in the both equations compared to the standard bargaining case. The term $\frac{\partial w}{\partial n}$ can be calculated by solving the first order differential equation, which will be defined from the wage bill equation shortly. The first equation is similar to the wage bill equation as in Cheron and Langot (2004) except for the second term in the right hand side, $\gamma\frac{\partial w}{\partial n}nh$. I can rewrite the wage bill equation as the first order differential equation with respect to wages w. Assuming a Cobb-Douglas production function, $F(z,k,nh) = e^z k^\alpha (nh)^{1-\alpha}$, the solution to the first order differential equation is given as

$$w = \mu\left(\frac{(1-\alpha)}{1-\mu\gamma\alpha}e^z k^\alpha h^{-\alpha} n^{-\alpha} + \frac{V}{1-N}\kappa h^{-1}\right) + (1-\mu)\left(\frac{u^l(1) - u^l(1-h)}{u_c} + b\right)h^{-1}$$

From the above equation, we have

$$\frac{\partial w}{\partial n} = -\frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha}e^z k^\alpha h^{-\alpha} n^{-\alpha-1} < 0$$

$$\gamma\frac{\partial w}{\partial n} = -\frac{\mu\gamma\alpha(1-\alpha)}{1-\mu\gamma\alpha}e^z k^\alpha h^{-\alpha} n^{-\alpha-1} < 0$$

Using the derivative term, we can rewrite two important conditions as follows:

$$\frac{u^l_{(1-h)}(1-h)}{u_c} = \frac{(1-\alpha)}{1-\mu\gamma\alpha}e^z k^\alpha (nh)^{-\alpha}$$

[13] Following Andolfatto (1996), it is assumed that each worker is so small such that $F_{nh} = \frac{\partial F(z,k,nh)}{\partial(nh)}$ is taken as given by both the worker and the firm during the bargaining.

$$wh = \mu\left(\frac{(1-\alpha)}{1-\mu\gamma\alpha}e^{z}k^{\alpha}h^{1-\alpha}n^{-\alpha} + \frac{V}{1-N}\kappa\right) + (1-\mu)\left(\frac{u^{l}(1) - u^{l}(1-h)}{u_c} + b\right)$$

Stochastic bargaining power of existing workers, γ, shows up in the equations for both intensive and extensive margins. This implies that bargaining shocks possibly increase the volatility of both margins. If $\gamma = 0$, we have similar conditions as in literature which uses standard bargaining protocol.

$$\begin{aligned}\frac{u^{l}_{(1-h)}(1-h)}{u_c} &= (1-\alpha)\, e^{z}k^{\alpha}\,(nh)^{-\alpha} \\ wh = \mu\left((1-\alpha)\, e^{z}k^{\alpha}h^{1-\alpha}n^{-\alpha} + \frac{V}{1-N}\kappa\right) &+ (1-\mu)\left(\frac{u^{l}(1) - u^{l}(1-h)}{u_c} + b\right)\end{aligned}$$

If $\gamma = 1$, the the conditions become similar to ones in KL[14] .

$$\begin{aligned}\frac{u^{l}_{(1-h)}(1-h)}{u_c} &= \frac{(1-\alpha)}{1-\mu\alpha}e^{z}k^{\alpha}\,(nh)^{-\alpha} \\ wh = \mu\left(\frac{(1-\alpha)}{1-\mu\alpha}e^{z}k^{\alpha}h^{1-\alpha}n^{-\alpha} + \frac{V}{1-N}\kappa\right) &+ (1-\mu)\left(\frac{u^{l}(1) - u^{l}(1-h)}{u_c} + b\right)\end{aligned}$$

3.2.5 Government

The government simply raises revenue in order to pay out unemployment benefits b to unemployed members within the household. Therefore, the government budget constraint is

$$T(S) = (1-n)\, b$$

3.2.6 Equilibrium

A *recursive competitive equilibrium* is a set of functions; the household's value function $\Omega(S, s_H)$, the household's policy functions $c(S, s_H), a'(S, s_H)$, the firm's value function $J(S, s_F)$, the firm's policy functions $v(S, s_f), k(S, s_f)$, aggregate prices $r(S), m(S, S')$, taxes $T(S)$, dividends $\Pi(S)$, and the law of motion for aggregate state variables $G(S)$ such that

(1) Household's policy functions solve the household's problem

(2) Firm's policy functions solve the firm's problem

(3) $m(S, S') = \beta u(c(S'))/u_c(c(S))$

(4) Wages and hours per worker $(w(S), h(S))$ are the solution to the bargaining problem

(5) All markets clear

(6) The government budget constraint is balanced

(7) The law of motion $G(S)$ is consistent with individual decisions

[14] Since KL does not have intensive margins, they do not have equation for intensive margins.

3.3 Calibration

First, I define the matching function and the aggregate production function to be

$$\begin{aligned} F(z,k,nh) &= e^{z}k\,(nh)^{1-\alpha} \\ M &= \omega V^{\psi}\,(1-N)^{1-\psi} \end{aligned}$$

where $\alpha \in (0,1)\,, \psi \in (0,1)$. I specify the household's utility function as follows

$$\begin{aligned} u\,(c) &= log\,(c) \\ u^{l}(1-h) &= \phi_e \frac{(1-h)^{1-\eta}}{1-\eta} \end{aligned}$$

Including the parameters in the functions defined above, I have 19 parameters to be calibrated. Parameters can be categorized into three groups based on the way to calibrate them. The first set of parameters are predetermined parameters outside the model. The second set of parameters are parameters for shock processes, which will be estimated from constructed shock processes from US data. The last group of parameters is parameters to be determined in the model by using the steady state conditions and relevant targets.

3.3.1 Predetermined parameters

I basically follow Andolfatto (1996) for the discount factor $\beta = 0.99$, the separation rate $\chi = 0.15$, the depreciation rate $\delta = 0.025$, the Cobb Douglas parameter for capital $\alpha = 0.36$, and the coefficient for vacancies in the matching function $\psi = 0.60$. Note that since the labor market is not competitive in this paper, I cannot use labor share data to calibrate α. Table 3.2 summarizes predetermined parameters.

Table 3.2: Predetermined parameters

Parameters	Description	Value	Source
β	Discount factor	0.99	Annual rate: 4%
χ	Separation rate	0.15	Andolfatto (1996)
δ	Depreciation rate	0.025	Andolfatto (1996)
α	CD parameter for capital	0.36	Andolfatto (1996)
ψ	Coef. for vacancies in matching function	0.60	Andolfatto (1996)

3.3.2 Parameters for the shock processes

Productivity shocks can be constructed as a series of the measure Solow residual. From the aggregate production function, we have

$$\hat{z}_t = \hat{y}_t - \alpha\hat{k}_t - (1-\alpha)\,\hat{n}_t - (1-\alpha)\,\hat{h}_t$$

where hats denote log-deviations from a linear trend for each variable over the period 1960:Q1-2012:Q1. I normalize $\bar{z} = 1$.

For bargaining shocks, we can use the solution to the first order differential equation we solved before

$$\begin{aligned}
\frac{\partial w}{\partial n} &= -\frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha} z k^{\alpha} h^{-\alpha} n^{-\alpha-1} \\
\frac{\partial w}{\partial n}\frac{n}{w} &= -\frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha}\frac{y}{nhw} = -\frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha}\frac{1}{labor\,share} \\
labor\,share &= \frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha}\frac{1}{(-\epsilon_{w,n})}
\end{aligned}$$

where $\epsilon_{w,n} \equiv \frac{\partial w}{\partial n}\frac{n}{w}$.[15] I assume the elasticity $\epsilon_{w,n}$ does not move much around the steady-state value $\overline{\epsilon_{w,n}} \equiv \overline{\frac{\partial w}{\partial n}\frac{n}{w}} = -\frac{\mu\alpha(1-\alpha)}{1-\mu\bar{\gamma}\alpha}\frac{\bar{y}}{\bar{n}\bar{h}\bar{w}}$, and I will show this assumption is innocuous in the quantitative analysis section. From equation (36), I can construct series of γ_t given series of the labor share data from US.

$$\begin{aligned}
(labor\,share)_t &= \frac{\mu\alpha(1-\alpha)}{1-\mu\gamma_t\alpha}\frac{1}{(-\overline{\epsilon_{w,n}})} \\
\gamma_t &= \frac{1}{\mu\alpha} - \frac{(1-\alpha)}{(labor\,share)_t\,(-\overline{\epsilon_{w,n}})}
\end{aligned}$$

The series of labor share are constructed from US data. The detail can be found in the appendix C. Given signs of parameters and $\overline{\epsilon_{w,n}} < 0$, there exists the positive relationship between bargaining shocks γ_t and labor share. This implies that the higher labor share is related to the higher bargaining powers of existing workers.

$$\frac{\partial\,(labor\,share)}{\partial\gamma} = \frac{(\mu\alpha)^2(1-\alpha)}{(1-\mu\gamma\alpha)^2(-\overline{\epsilon_{w,n}})} > 0$$

Based on several information criteria such as FPE, AIC, HQIC, and SBIC, I specify VAR(1) system for detrended shock series $\hat{z}, \hat{\nu}$ to estimate shock processes.

$$\begin{aligned}
\begin{pmatrix}\hat{z}'\\ \hat{\nu}'\end{pmatrix} &= \begin{pmatrix}\rho_{zz} & \rho_{\gamma z}\\ \rho_{z\gamma} & \rho_{\gamma\gamma}\end{pmatrix}\begin{pmatrix}\hat{z}\\ \hat{\gamma}\end{pmatrix} + \begin{pmatrix}\varepsilon_z'\\ \varepsilon_\gamma'\end{pmatrix} \\
\begin{pmatrix}\varepsilon_z\\ \varepsilon_\gamma\end{pmatrix} &\sim N\left(0, \begin{pmatrix}\sigma_{zz}^2 & \sigma_{z\gamma}\\ \sigma_{z\gamma} & \sigma_{\gamma\gamma}^2\end{pmatrix}\right)
\end{aligned}$$

[15] Note that $\epsilon_{w,n} < 0$.

Table 3.3: Shock processes

Parameters	Value	Remarks
ρ_{zz}	0.9616	P-value = 0.000
$\rho_{\gamma z}$	-0.0053	P-value = 0.169
$\rho_{z\gamma}$	0.3937	P-value = 0.000
$\rho_{\gamma\gamma}$	0.9336	P-value = 0.000
σ_z	0.0070	-
σ_γ	0.0399	-
$\sigma_{z\gamma}$	-0.0001	$\rho\left(\varepsilon_z, \varepsilon_\gamma\right)$ =-0.3643

Table 3.3 summarizes parameters estimated using VAR(1) system above. All coefficient parameters except for $\rho_{\gamma z}$ are significant. Note that we have $\rho_{z\gamma} = 0.3937$, which means today's productivity shocks increase tomorrow's bargaining powers of existing workers. This is key mechanism that the inclusion of the stochastic bargaining makes total hours, employment and hours per workers more volatile in addition to productivity shocks.

3.3.3 Parameters determined using targets

Table 3.4: Targets

Target	Value	Source
Frisch elasticity of hours for those employed	0.50	Andolfatto (1996)
Steady-state employment to population ratio	0.60	Data (1960:Q1-2012:Q1)
Steady-state hours per worker[16]	0.39	Data (1960:Q1-2012:Q1)
Steady-state job-filling rate	0.90	Andolfatto (1996)
Vacancy expenditure to output ratio[17]	0.0218	Silva & Toledo (2009)
Replacement ratio	0.40	Shimer (2005)
Bargaining power of a marginal worker, $\mu = \bar{\gamma}$[18]	μ	-

I choose the remaining 7 parameters using equilibrium conditions in the steady state and targets from the literature and data from US over 1960:Q1-2012:Q1. The targets I used are summarized in Table 3.3. First, I set Frisch elasticity of hours for employed to 0.50, the steady state job-filling rate to 0.90 as in Andolfatto (1996), which is common across the literature. According to Silva and Toledo (2009), the average cost of time spent hiring one worker is approximately 3.6%-4.3% of total labor costs. I take the

Table 3.5: Parameters determined using targets

Parameters	Description	Baseline
η	Curvature parameter for leisure	3.0940
ϕ_e	Scale parameter for leisure	0.9136
κ	Cost of posting vacancies	0.1905
ω	Matching efficiency	0.5156
b	Unemployment Benefits	0.4080
μ	Bargaining power of a marginal worker	0.5697
$\overline{\gamma}$	Bargaining power of existing workers	0.5697

target the mid point of those range, 3.9%, which gives vacancy expenditure to output ratio $\frac{\kappa v}{y} = 0.0218$.[19] . I use 40 percent as the value of unemployment benefits following Shimer (2005). In Shimer, this value implicitly includes the value of leisure, but in this paper I explicitly consider the leisure in the utility function, so unemployment benefits, b, are purely unemployment benefits as in Nakajima (2012). Targets and parameters determined using these targets are listed in Table 3.4 and Table 3.5 respectively. I set the mean value of the bargaining power of existing workers, $\overline{\gamma}$, to be 0.5605 which is the same as the bargaining power of a marginal worker, μ, calibrated in the model.[20]

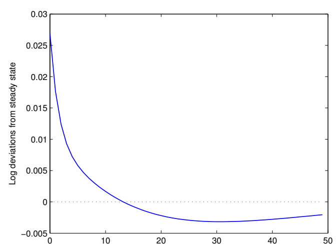
Since the parameter $\overline{\gamma}$ is a free parameter and there is no clear way to pin down this parameter, I calibrate it such that $\overline{\gamma} = \mu$ in the steady state. As I will discuss in the robustness section later, lower values of $\overline{\gamma}$ generates more volatile labor market variables. However, I set $\overline{\gamma} = \mu = 0.5697$ which gives almost the least volatility among $\overline{\gamma} \in (0, 1)$ in the baseline model. In this regard, I think the choice of $\overline{\gamma} = 0.5697$ is innocuous and parsimonious. Also, note that calibrated value for the bargaining power of a marginal worker, μ, is 0.5697, which guarantees quantitative results of the baseline model are not a direct result from a low value of μ as noted in Hagedorn and Manovskii (2008).

[19] This value is calculated based on job-filling rate $\Phi = 0.90$ and labor share = 0.62.

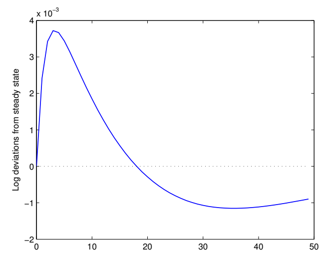
[20] The mean of the bargaining power of existing workers, $\overline{\nu}$, and the bargaining power of a new worker, μ, are jointly determined in the steady state.

3.4 Quantitative analysis

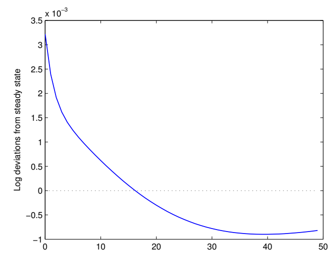
Figure 3.1: IRFs to the positive one standard deviation bargaining shock



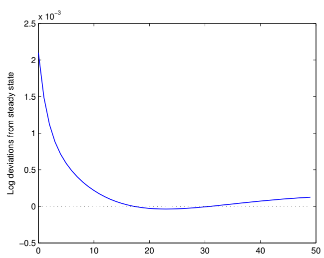
(a) Vacancies



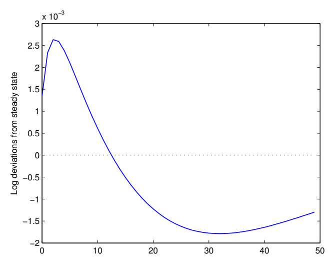
(b) Employment



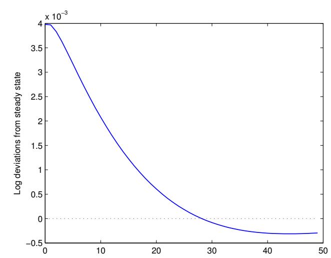
(c) Wages



(d) Hours per worker



(e) [Output



(f) Labor share

3.4.1 Impulse response functions for positive bargaining shocks

Figure 3.1 shows the impulse response of key labor market variables to the positive one standard deviation bargaining shock. When positive bargaining shocks hit the economy, the bargaining power of existing workers instantly increases. Since bargaining powers of existing workers are higher then before, a firm has more incentives to hire marginal workers by offering higher wages to forgo the higher cost associated with the failure to hire marginal workers. Therefore, the firm instantly posts more vacancies, and employment increases one period later due to the nature of search frictions. Since the firm pays higher wages, hours per worker increase and higher total hours yield higher outputs in the equilibrium. Higher employment, hours per worker, and wages results in an increase in labor share by offsetting an increase in outputs.

3.4.2 Business cycle moments

Table 3.6 summarizes quantitative results of the baseline model. I compare the baseline model to the Andolfatto model to see what gains and what shortcomings the inclusion of stochastic bargaining and bargaining shocks gives. Again, all data are in log and HP filtered. First of all, the baseline model generates a high (relative) volatility of employment, 0.67, which almost close to the actual U.S. data, 0.65. This is a remarkable success and the main contribution in this paper. Since employment is very volatile, total hours is much volatile than the Andolfatto model. Hours per worker and vacancies are slightly more volatile than Andolfatto, but the differences are small. The moments for labor share are almost similar to the actual US data. This result might be a direct result of the identification strategy for bargaining shocks from labor share data. However, the moments for labor share in the model, along with the overshooting property I will discuss shortly, justify the assumption for the identification of bargaining shocks; $\epsilon_{w,n} \equiv \frac{\partial w}{\partial n}\frac{n}{w}$ does not move much around the steady state.

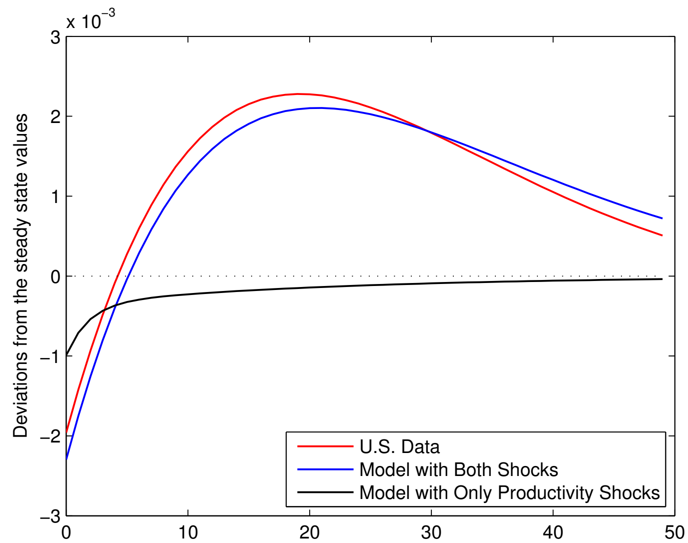
The main mechanism generates more volatile labor market variables is that the impact of productivity shocks is amplified by changes in bargaining powers of existing workers in addition to the impact of each shock. Recall that the estimated parameter for ρ_{zv} is 0.3937, which means that as the productivity shocks today positively affect the bargaining powers tomorrow, and as the bargaining power of existing workers increases, the firm will have more incentives to hire marginal workers by offering higher wages to forgo the higher cost associated with the failure to hire marginal workers. This dynamic interaction between productivity shocks and bargaining shocks amplifies the volatility of labor market variables, especially employment.

I now consider shortcomings of the baseline model relative to Andolfatto. The baseline model generates the higher volatility of labor productivity, weak pro-cyclicality of total hours, employment and hours per worker. Also, labor productivity is more persistent and vacancies are less persistent than the Andolfatto model and actual US data. Despite of these shortcomings, the baseline model performs better than Andolfatto model in general.

Table 3.6: Business cycle moments in data and models over 1960:Q1-2012:Q1

	$\sigma_x\% \left(\frac{\sigma_x}{\sigma_{Output}}\right)$			$\rho\,(x, Output)$			$\rho\,(x_t, x_{t-1})$		
Variable (x)	Data	Baseline	Andolfatto	Data	Baseline	Andolfatto	Data	Baseline	Andolfatto
Output	1.54 (1.00)	1.36 (1.00)	1.31 (1.00)	-	-	-	0.86	0.86	0.82
Total Hours	1.38 (0.90)	0.99 (0.73)	0.70 (0.53)	0.85	0.81	0.92	0.88	0.93	0.91
Employment	1.00 (0.65)	0.91 (0.67)	0.68 (0.52)	0.81	0.77	0.78	0.91	0.92	0.89
Hours per Worker	0.49 (0.32)	0.25 (0.18)	0.19 (0.15)	0.74	0.40	0.55	0.56	0.56	0.58
Wages	0.91 (0.59)	0.68 (0.50)	0.62 (0.47)	0.34	0.91	0.94	0.69	0.69	0.65
Labor Productivity	0.82 (0.53)	0.80 (0.59)	0.72 (0.55)	0.45	0.70	0.92	0.57	0.66	0.62
Labor Share	0.74 (0.48)	0.64 (0.47)	0.12 (0.09)	-0.08	0.09	-0.72	0.78	0.73	0.51
Vacancies	13.23 (8.59)	4.36 (3.21)	3.65 (2.79)	0.90	0.76	0.80	0.91	0.59	0.54

Figure 3.2: Impulse response function of labor share to productivity innovation



3.4.3 Implication on labor share

Ríos-Rull and Santaeulalia-Llopis (2010) first document the overshooting property of labor share. They showed that labor share overshoots in response to productivity shocks, and the dynamic overshooting response of labor share drastically dampens the role of productivity shocks on labor markets due to huge wealth effects. Figure 3.3 shows the overshooting of labor share in the baseline model. If I abstract from bargaining shocks, the model no longer generates the overshooting of labor share. The reason the model with bargaining shocks features the overshooting of labor share might be a direct result of the identification strategy of bargaining shocks. Again, the fact that labor share overshoots in the baseline model, along with other moments for labor share are almost the same as those in actual data, justifies the assumption I pose to identify bargaining shocks; $\epsilon_{w,n} \equiv \frac{\partial w}{\partial n}\frac{n}{w}$ does not move much around the steady state.

More importantly, the baseline model generates the overshooting property of labor share, but the effect of productivity shocks is still significant on labor markets in contrast to the prediction of Ríos-Rull and Santaeulalia-Llopis (2010) in which the effect of productivity shocks is dampened when labor share overshoots because of huge wealth effects from the overshooting property. In contrast to their model, the baseline model has a search and matching framework, and the nature of this framework weakens wealth effects resulting from the overshooting of labor share. On top of these differences, more incentives for firms to hire workers offset the huge reduction of total hours in booms. In response to positive productivity shocks output instantly increases, but employment does not increase because of search frictions, which cause an instant drop in labor share. As the productivity shocks today positively affect the bargaining shocks tomorrow, $\rho_{zv} = 0.3937$, and as the bargaining power of existing workers increases, the firm will have more incentives to hire marginal workers. Consequently, employment, wages, and hours per worker will increase by offsetting an increase in outputs. This increase explains the overshooting of labor share in response to positive productivity shocks.

3.4.4 The role of productivity shocks and bargaining shocks

Now I consider how productivity shocks and bargaining shocks differently affect the model predictions. When the economy has only productivity shocks, z, the model predictions are almost the same as the Andolfatto model. Comparing to the baseline model which has both shocks, the volatility of employment, labor share, and vacancies is dampened, but correlations between labor market variables and outputs get close to the actual data. Auto-correlations are almost the same as the baseline case.

When the economy has only bargaining shocks, the volatility of outputs significantly drops, which means bargaining shocks cannot be the main driving source of output fluctuations. On the other hand, the volatility of total hours, employment, hours per worker remarkably increases, which is far beyond the volatility in the baseline model. Also, total hours, employment, hours per worker, and labor shares are strongly procyclical. However, auto-correlations are almost the same as the baseline case.

Table 3.8 shows the variance decomposition. Bargaining shocks have a substantial impact on the volatility of total hours, employment, hours per worker, vacancies, and labor share. While bargaining shocks play a remarkable role in the labor markets, productivity shocks seem to be still the main driving force of business cycles given productivity shocks account for about 90% of output fluctuations. This result is also consistent with the finding in moments in Table 3.7.

Table 3.7: Business cycle moments in models with different shocks

	$\sigma_x\%\left(\frac{\sigma_x}{\sigma_{Output}}\right)$			$\rho\,(x, Output)$			$\rho\,(x_t, x_{t-1})$		
Variable (x)	Both	Only z	Only γ	Both	Only z	Only γ	Both	Only z	Only γ
Output	1.36 (1.00)	1.31 (1.00)	0.43 (1.00)	-	-	-	0.86	0.82	0.91
Total Hours	0.99 (0.73)	0.70 (0.53)	0.68 (1.58)	0.81	0.92	0.99	0.93	0.91	0.91
Employment	0.91 (0.67)	0.69 (0.53)	0.60 (1.40)	0.77	0.78	0.96	0.92	0.88	0.88
Hours per Worker	0.25 (0.18)	0.19 (0.15)	0.21 (0.49)	0.40	0.54	0.45	0.56	0.58	0.57
Wages	0.68 (0.50)	0.63 (0.48)	0.43 (1.00)	0.91	0.94	0.66	0.69	0.65	0.60
Labor Productivity	0.80 (0.59)	0.72 (0.55)	0.25 (0.58)	0.70	0.92	-0.96	0.66	0.62	0.91
Labor Share	0.64 (0.47)	0.10 (0.08)	0.63 (1.47)	0.09	-0.71	0.83	0.73	0.50	0.76
Vacancies	4.36 (3.21)	3.67 (2.80)	3.23 (7.51)	0.76	0.80	0.49	0.59	0.54	0.54

Table 3.8: Variance decomposition (in percent)

Variable	z	ν
Output	89.69	10.31
Total Hours	48.15	51.85
Employment	58.35	41.65
Hours per Worker	9.86	90.14
Wages	68.22	31.78
Labor Productivity	90.58	9.42
Labor Share	27.44	72.56
Vacancies	52.42	47.58

3.5 Robustness

3.5.1 Stochastic bargaining power of a marginal worker, μ_t

Now I assume the bargaining power of a marginal worker varies stochastically while the bargaining power of existing workers is fixed at $\gamma = \overline{\gamma} = \overline{\mu}$. Again, I identify series of μ_t by using the solution to the first order differential equation, and series of the labor share data from U.S.

$$\begin{aligned} (labor\,share)_t &= \frac{\mu_t \alpha (1-\alpha)}{\mu_t \bar{\gamma}\alpha} \frac{1}{\left(-\overline{\epsilon_{w,n}}\right)} \\ \mu_t &= \frac{1}{\bar{\gamma}\alpha + \frac{\alpha(1-\alpha)}{(labor\,share)_t \left(-\overline{\epsilon_{w,n}}\right)}} \end{aligned}$$

where $\epsilon_{w,n} \equiv \frac{\partial w}{\partial n}\frac{n}{w}$. Again, I assume the elasticity $\epsilon_{w,n}$ does not move much around the steady-state value $\overline{\epsilon_{w,n}} \equiv \overline{\frac{\partial w}{\partial n}\frac{n}{w}} = -\frac{\mu\alpha(1-\alpha)}{1-\mu\bar{\gamma}\alpha}\frac{\bar{y}}{\bar{n}\bar{h}\bar{w}}$. Table 3.9 shows the comparison of business cycle moments. Stochastic bargaining power of a marginal worker cannot quantitatively improve the Andolfatto model, even moments for labor share which is used for identifying shock series μ_t.[21]

3.5.2 Different calibrations for $\overline{\gamma}$

I now simulate the baseline model with different values for $\overline{\gamma} = 0.3$ (an example of low values[22]), 0.5697 (a middle value and the calibrated value for the baseline model such that $\mu = \overline{\gamma}$), and 0.9 (an example of high values[23]). Table 3.10 shows business cycle moments for each case. If I set $\overline{\gamma} = 0.3$, then volatility of employment and hours per workers significantly increases than the baseline calibration case, $\overline{\gamma} = 0.5697$. However, if I set $\overline{\gamma} = 0.9$, then moments are almost the same as those of the baseline calibration case, $\overline{\gamma} = 0.5697$. Mechanically, low values of $\overline{\gamma}$ increase the volatility of total hours, employment, hours per worker. $\overline{\gamma}$ is a free parameter in this paper and there is no clear way to pin down this parameter. However, the choice of $\overline{\gamma} = 0.5697$ in the baseline model seems innocuous and parsimonious in the sense that I think setting the same values for the mean of bargaining powers of existing workers and bargaining powers of new workers, $\overline{\gamma} = \mu$, is a reasonable given there is no information on $\overline{\gamma}$, and $\overline{\gamma} = 0.5697$ yields the least volatility of labor market variables among $\overline{\gamma} \in (0, 1)$.

[21] This results do not change with different values of $\bar{\gamma}$

[22] In this case, the calibrated value of μ is 0.5512

[23] In this case, the calibrated value of μ is 0.5956

Table 3.9: Business cycle moments in the model: shocks on μ

	$\sigma_x\%\left(\frac{\sigma_x}{\sigma_{Output}}\right)$		$\rho\,(x, Output)$		$\rho\,(x_t, x_{t-1})$	
Variable (x)	Shock on μ	Andol.	Shock on μ	Andolfatto	Shock on μ	Andol.
Output	1.40 (1.00)	1.31 (1.00)	-	-	0.85	0.82
Total Hours	0.79 (0.57)	0.70 (0.53)	0.93	0.92	0.92	0.91
Employment	0.76 (0.54)	0.68 (0.52)	0.82	0.78	0.89	0.89
Hours per Worker	0.19 (0.14)	0.19 (0.15)	0.57	0.55	0.59	0.58
Wages	0.62 (0.44)	0.62 (0.47)	0.94	0.94	0.68	0.65
Labor Productivity	0.73 (0.52)	0.72 (0.55)	0.91	0.92	0.63	0.62
Labor Share	0.15 (0.11)	0.12 (0.09)	-0.56	-0.72	0.56	0.51
Vacancies	4.00 (2.86)	3.65 (2.79)	0.76	0.80	0.55	0.54

Table 3.10: Business cycle moments in the model with different values for $\overline{\gamma}$

	$\sigma_x\%\left(\frac{\sigma_x}{\sigma_{Output}}\right)$			$\rho\,(x, Output)$			$\rho\,(x_t, x_{t-1})$		
Variable (x)	$\overline{\gamma} = 0.3$	0.5697	0.9	0.3	0.5697	0.9	0.3	0.5697	0.9
Output	1.39 (1.00)	1.36 (1.00)	1.36 (1.00)	-	-	-	0.86	0.86	0.86
Total Hours	1.13 (0.81)	0.99 (0.73)	0.98 (072)	0.80	0.81	0.81	0.93	0.93	0.93
Employment	1.02 (0.73)	0.91 (0.67)	0.90 (0.66)	0.77	0.77	0.77	0.91	0.92	0.92
Hours per Worker	0.32 (0.23)	0.25 (0.18)	0.24 (0.18)	0.39	0.40	0.41	0.55	0.56	0.56
Wages	0.75 (0.54)	0.68 (0.50)	0.67 (0.49)	0.87	0.91	0.91	0.67	0.69	0.70
Labor Productivity	0.83 (0.60)	0.80 (0.59)	0.80 (0.59)	0.58	0.70	0.70	0.67	0.66	0.66
Labor Share	0.83 (0.60)	0.64 (0.47)	0.63 (0.46)	0.21	0.09	0.08	0.73	0.73	0.73
Vacancies	5.03 (3.62)	4.36 (3.21)	4.30 (3.16)	0.70	0.76	0.77	0.56	0.59	0.59

3.6 Conclusion

This paper studies an alternative mechanism of wage negotiations in multi-worker firms that face diminishing MPL. When Nash bargaining with a marginal worker breaks down, a firm negotiates wages with existing workers collectively and produces with them. The bargaining powers of existing workers are stochastic. Due to diminishing MPL, the breakdown of the negotiation with the marginal worker negatively affects the bargaining position of the firm with existing workers (one fewer workers) since MPL is higher with one fewer workers. How much the firm internalizes this negative effect depends on the stochastic bargaining powers of existing workers which can be identified through labor share data. During expansions, it is relatively difficult for the firm to hire workers, so existing workers might have higher bargaining powers. If the firm fails to hire a marginal worker due to a breakdown of negotiations, the firm has to pay higher wages to existing workers in order to produce goods with them. Since the failure to hire marginal workers is more costly during expansions, the firm has more incentives to hire marginal workers by offering higher wages to forgo the higher cost associated with the breakdown. During recessions, the opposite happens. Through this mechanism, the stochastic bargaining powers of existing workers provide an additional margin to increase the volatility of labor market variables. The calibrated model generates more volatile total hours, employment, hours per worker while labor share overshoots in response to productivity shocks as documented in Ríos-Rull and Santaeulalia-Llopis (2010). In particular, the volatility of employment in the model is similar to the actual US data. In contrast to the prediction of Ríos-Rull and Santaeulalia-Llopis (2010), in which the effect of productivity shocks is dampened when labor share overshoots due to huge wealth effects from the overshooting property of labor share, this paper presents a model in which the labor share overshoots in response to productivity shocks and the volatility of employment closely matches that of US data.

In this paper, I assume the bargaining power of existing workers to be exogenous. The quantitative results show that the time-varying bargaining power of existing workers is an important margin to understand the fluctuations of total hours, employment, hours per workers, and labor share, and the overshooting property. However, this paper abstracts from the endogenous mechanism for the time-varying bargaining power of existing workers. Therefore, coming up with an endogenous mechanism for bargaining shocks would be worthwhile for future research. One possible theory could be related to the entry and exit of firms over business cycles. In booms, several firms compete with a specific firm because of higher entry rates of new firms and lower exit rates of existing firms, which reduce the monopolistic or bargaining power of firms over existing workers. However, during recessions, the opposite happens. By incorporating the entry and exit decision of firms, I might be able to explain the endogenous movements of the bargaining power of existing workers. Moreover, this paper does not focus on unemployment because the baseline model treats the unemployed and the non-employed who are out of the labor force similarly, and the measure of unemployment is inconsistent with the data. In this regard, I could extend the baseline model by distinguishing between the unemployed and the non-employed to obtain a proper measure of unemployment.

References

[1] D. Acemoglu and W. Hawkins. Search with multi-worker firms. *Theoretical Economics*, 9(3):583–628, 2014.

[2] D. Andolfatto. Business cycles and labor-market search. *American Economic Review*, 86(1):112–132, 1996.

[3] R. Barnichon. Building a composite Help-Wanted Index. *Economics Letters*, 109(3):175–178, 2010.

[4] H. Benitez-Silva, M. Buchinsky, and J. Rust. Using a life cycle model to predict induced entry effects of a $1 for $2 benefit offset in the SSDI program. *Working paper*, 2011.

[5] B. Bernanke and M. Gertler. Agency costs, net worth, and business fluctuations. *American Economic Review*, 79(1):14–31, 1989.

[6] B. Bernanke, M. Gertler, and S. Gilchrist. The financial accelerator in a quantitative business cycle framework. *the Handbook and Macroeconomics*, 1999.

[7] M. Bils, Y. Chang, and S. Kim. Worker heterogeneity and endogenous separations in a matching model of unemployment fluctuations. *American Economic Journal: Macroeconomics*, 3(1):128–154, 2011.

[8] P. Cahuc, F. Marque, and E. Wasmer. A theory of wages and labor demand with intra-firm bargaining and matching frictions. *International Economic Review*, 49(3):943–972, 2008.

[9] A. Cheron and F. Langot. Labor market search and real business cycles: Reconciling Nash bargaining with the real wage dynamics. *Review of Economic Dynamics*, 7(2):476–493, 2004.

[10] S. K. Choi and J. V. Ríos-Rull. Understanding the dynamics of labor share: The role of noncompetitive factor prices. *Annales d'Economie et Statistique*, 95/96, 2009.

[11] L. J. Christiano and T. J. Fitzgerald. The band pass filter. *International Economic Review*, 44(2):435–465, 2003.

[12] S. K. Chugh. Costly external finance and labor market dynamics. *Journal of Economic Dynamics and Control*, 37:2882–2912, 2013.

[13] M. Elsby and R. Michaels. Marginal jobs, heterogenous firms, and unemployment flows. *American Economic Journal: Macroeconomics*, 5(1):1–48, 2013.

[14] S. Fujita and M. Nakajima. Worker flows and job flows: A quantitative investigation. *Working paper*, 2014.

[15] J. Gali, F. Smets, and R. Wouters. Unemployment in an estimated New Keynesian model. *NBER Working Paper*, No. 17201, 2011.

[16] J. Garin. Borrowing constraints, collateral fluctuations, and the labor market. *Working paper*, 2012.

[17] Government Accountability Office. SSA is making progress toward eliminating continuing disability review backlogs. *American Economic Journal: Macroeconomics*, GAO/T-HEHS-97-222, 2007.

[18] D. Haan, J. Wouter, G. Ramey, and J. Watson. Job destruction and propagation of shocks. *American Economic Review*, 90:482–498, 2000.

[19] M. Hagedorn and I. Manovskii. The cyclical behavior of equilibrium unemployment and vacancies revisited. *American Economic Review*, 98(4):1692–1706, 2000.

[20] M. Hagedorn and I. Manovskii. Productivity and the labor market: Co-movement over business cycle. *International Economic Review*, 52(3):603–619, 2011.

[21] M. Hagedorn, F. Karahan, I. Manovskii, and K. Mitman. Unemployment benefits and unemployment in the Great Recession: The role of macro effects. *Working paper*, 2015.

[22] W. Hawkins. Do large-firm bargaining models amplify and propagate aggregate productivity shocks? *Working paper*, 2011.

[23] N. Jaimovich and H. Siu. The trend is the cycle: Job polarization and jobless recoveries. *Working paper*, 2015.

[24] U. Jermann and V. Quadrini. Macroeconomic effects of financial shocks. *American Economic Review*, 102(1):238–271, 2012.

[25] S. Kim. The labor supply and welfare effects of early access to Medicare through Social Security disability insurance. *Working paper*, 2014.

[26] S. Kitao. A life cycle model of unemployment and disability insurance. *Journal of Monetary Economics*, 68:1–18, 2014.

[27] N. Kiyotaki and J. H. Moore. Credit cycles. *Journal or Political Economy*, 105(2):211–248, 1997.

[28] M. Krause and T. Lubik. Does intra-firm bargaining matter for business cycle dynamics? *Deutsche Bundesbank Discussion Paper*, No. 17, 2007.

[29] P. Krusell and A. Smith. Income and wealth heterogeneity in the macroeconomy. *Journal of Political Economy*, 106(5):867–896, 1998.

[30] S. Lindner and A. Nichols. The impact of temporary assistance programs on disability rolls and re-employment. *Working paper*, 2012.

[31] H. Low and L. Pistaferri. Disability insurance and the dynamics of the incentive-insurance tradeoff. *Working paper*, 2012.

[32] T. MaCurdy. An empirical model of labour supply in a life cycle setting. *Journal or Political Economy*, 89(61):1059–1085, 1981.

[33] M. Merz. Search in the labor market and real business cycle. *Journal or Monetary Economics*, 36:269–300, 1981.

[34] T. Monacelli, V. Quadrini, and A. Trigari. Financial markets and unemployment. *NBER Working Paper*, No. 17389, 2012.

[35] T. Moore. The employment effect of terminating disability benefits. *Working paper*, 2014.

[36] A. Mueller, J. Rothstein, and T. Wachter. Unemployment insurance and disability insurance in the Great Recession. *Working paper*, 2015.

[37] M. Nakajima. Business cycles in the equilibrium model of labor market search and self-insurance. *International Economic Review*, 53(2):399–432, 2012.

[38] M. Nakajima. A quantitative analysis of unemployment benefit extensions. *Journal of Monetary Economics*, 59(7):686–702, 2012.

[39] F. Perri and V. Quadrini. International recessions. *NBER Working Paper*, No. 17201, 2012.

[40] N. Petrosky-Nadeau and E Wasmer. The cyclical volatility of labor markets under frictional financial markets. *American Economic Journal: Macroeconomics*, 5(1):193–221, 2013.

[41] N. Petrosky-Nadeau. Credit, vacancies and unemployment fluctuations. *Review of Economic Dynamics*, 17(2):191–205, 2014.

[42] J. V. Ríos-Rull and R. Santaeulalia-Llopis. Redistributive shocks and productivity shocks. *Journal of Monetary Economics*, 57:931–948, 2010.

[43] M. Rutledge. The impact of unemployment insurance extensions of disability insurance application and allowance rates. *Working paper*, 2012.

[44] R. Shimer. The cyclical behavior of equilibrium unemployment and vacancies. *American Economic Review*, 95(1):25–49, 2005.

[45] R. Shimer. Mismatch. *American Economic Review*, 97(4):1074–1101, 2007.

[46] R. Shimer. Reassessing the ins and outs of unemployment. *Review of Economic Dynamics*, 15:127–148, 2012.

[47] J. Silva and M. Toledo. Labor turnover costs and the cyclical behavior of vacancies and unemployment. *Macroeconomic Dynamics*, 13:76–96, 2009.

[48] Social Security Administration. Annual report of continuing disability reviews: Fiscal year 2011. 2013.

[49] L. Stole and J. Zwiebel. Intra-firm bargaining under non-binding contracts. *Review of economic Studies*, 63(3):375–410, 1996.

[50] D. Wiczer. Long-term unemployment: Attached and mismatched? *Working paper*, 2014.

Appendix A

Appendix to Chapter 1

A.1 Computation

A.1.1 Steady state equilibrium

In the steady state, the aggregate labor productivity is a constant. Therefore, the measure of workers ψ does not change over time. The computational algorithm for the steady state equilibrium is as follows:

1. Guess the market tightness θ
2. Given θ, Nash bargained wages and value functions for workers and firms can be solved
3. Given converged value functions, stationary measures of economy can be computed
4. Compute θ^{new} satisfies the free entry condition along with value functions for firms and measure of the economy
$$\kappa = \beta q\left(\theta\left(z,\psi\right)\right)\int E_{z,\gamma,e}\left[\left(1-s^{'}\right)J^{x}\left(\gamma^{'},e^{'};z^{'},\psi^{'}\right)\right]\mathbb{I}_{\left(g_{s}^{x}(U,\gamma,a,m,e;z,\psi)=1\right)}d\psi\left(x,U,\gamma,a,m,e\right)$$
5. If θ and θ^{new} are close enough, then I found the steady state. Otherwise repeat from step 2 to step 4 with a new guess for θ
$$\theta = \lambda_{\theta}\theta + \left(1-\lambda_{\theta}\right)\theta^{new}$$

A.1.2 Equilibrium with aggregate shocks: approximated equilibrium

Following Krusell and Smith (1998), the measure ψ is replaced with the aggregate employment E in this paper. Therefore, the aggregate state variables in this economy are $\{z, E\}$ in stead of $\{z, \psi\}$. In order to predict the market tightness θ and the future value for the aggregate employment E, I assume simple log-linear prediction functions for the market tightness $\theta(z, E)$ and the aggregate employment E[1] :

$$log(\theta) = b_{\theta,0} + b_{\theta,1} log(E) + b_{\theta,2} log(z)$$
$$log(E') = b_{E,0} + b_{E,1} log(E) + b_{E,2} log(z)$$

Given the aggregate state variables $\{z, E\}$ and the prediction rules, I can solve the approximated equilibrium as follows:

1. Guess a set of coefficients in the prediction functions

$$b = b^{old} \equiv \left(b^{old}_{\theta,0}, b^{old}_{\theta,1}, b^{old}_{\theta,2}, b^{old}_{E,0}, b^{old}_{E,1}, b^{old}_{E,2}\right)$$

2. Given prediction rules, Nash bargained wages and value functions for workers and firms can be solved. I linearly interpolate the value functions with respect to $E^{'}$

3. Given converged value functions, I run a simulation of 10500 periods with an artificial series of $\{z_t\}_{t=1}^{10500}$ in order to generate a set of series $\{\theta_t, E_t\}_{t=1}^{10500}$. I can compute θ_t and E_t by using converged value functions in step 2, the prediction rules, and the free entry condition.

4. Once I have the set of series $\{\theta_t, E_t\}_{t=1}^{10500}$, I can update the coefficients in the prediction functions $b^{new} = \left(b^{new}_{\theta,0}, b^{new}_{\theta,1}, b^{new}_{\theta,2}, b^{new}_{E,0}, b^{new}_{E,1}, b^{new}_{E,2}\right)$ by running OLS regressions with $\{\theta_t, E_t\}_{t=501}^{10500}$. Note that I drop the first 500 observations for the regression

5. If b and b^{new} are close enough, then I found converged prediction functions. Otherwise repeat from step 2 to step 4 with a new guess for b

$$b = \lambda_b b^{old} + (1 - \lambda_b) b^{new}$$

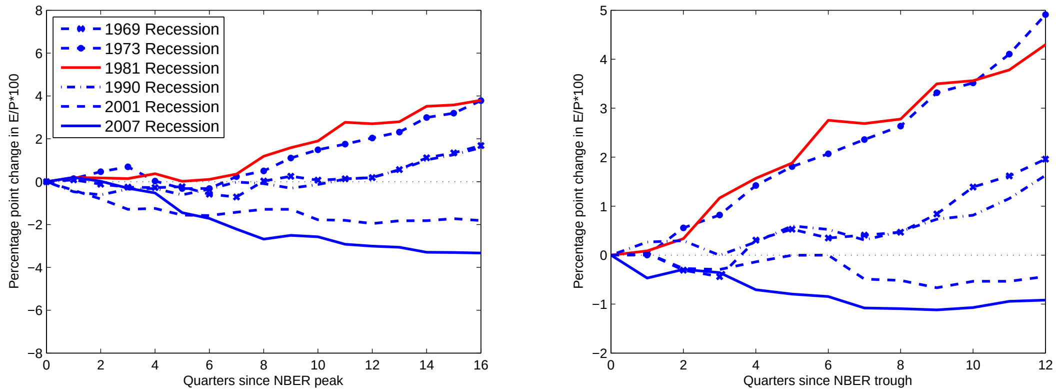
6. I use R^2 for the measure for accuracies of the prediction functions. The following are the converged prediction functions and their accuracies for the baseline model:

$$log(\theta) = 0.5340 + 0.1033\, log(E) + 0.8329\, log(z), \ R^2 = 0.9524$$
$$log(E') = -0.0001 + 0.9993\, log(E) + 0.0015 log(z), \ R^2 = 0.9987$$

[1] This is a simpler version of prediction functions in Bils et al. (2011)

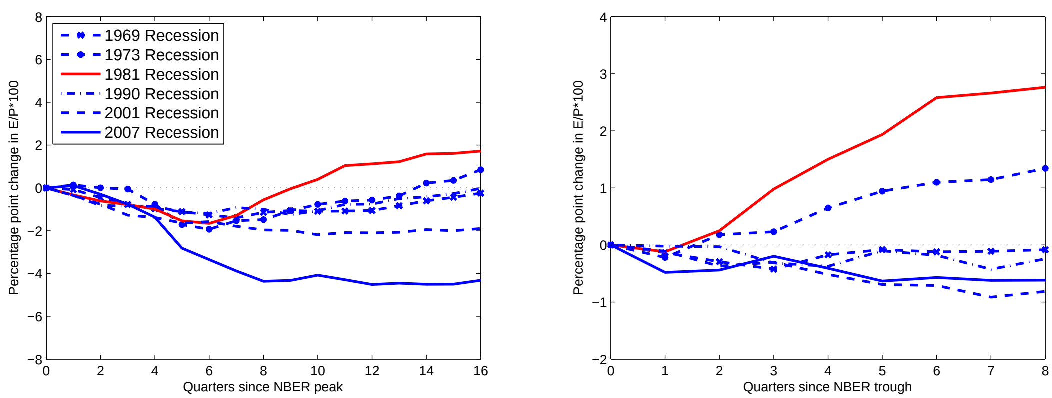
A.2 Graphs

Figure A.1: Recessions and recoveries in the employment rate of women since 1965



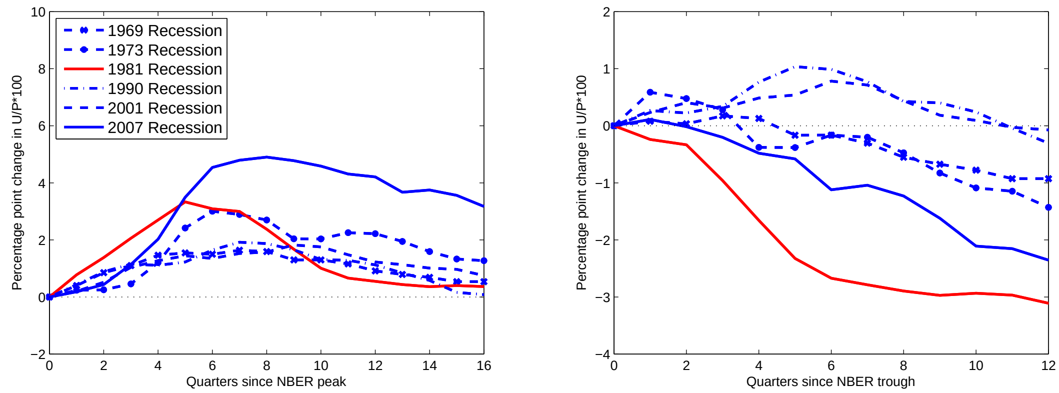
(a) Recessions and recoveries (b) Recoveries

Figure A.2: Recessions and recoveries in the employment rate of men and women since 1965



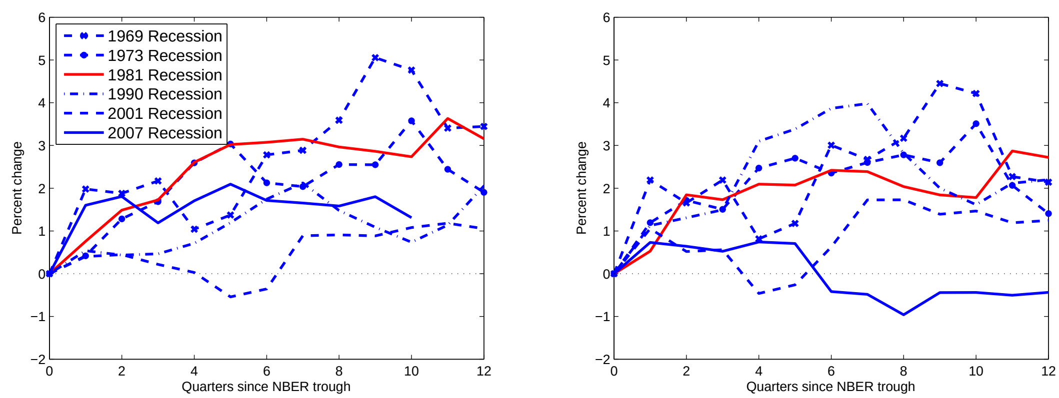
(a) Recessions and recoveries (b) Recoveries

Figure A.3: Recessions and recoveries in unemployment-population ratio of men since 1965



(a) Recessions and recoveries (b) Recoveries

Figure A.4: Aggregate productivity during recoveries (HP-filtered)



(a) Total factor productivity (TFP) (b) Average labor productivity

Appendix B

Appendix to Chapter 2

B.1 Derivation of the equilibrium conditions

Households solve following dynamic programming problem.

$$V(S, s_H) = \max_{c, s', a'_H} \{u(c) + n\nu(1-h) + (1-n)\nu(1) + \beta E[V(S', s'_H)]\}$$

$$\text{s.t.}$$

$$c + \frac{a'_H}{1+r(S)} + p(S)\, s' = w(S, s_H)\, nh(S, s_H) + (1-n)b + a_H + [p(S) + d(S)]\, s - T(S)$$

$$n' = (1-\chi)n + \Psi(S)(1-n)$$

$$S' = G(S),\ c \geq 0,\ \text{No-Ponzi condition}$$

Let λ_H and π_H be the Lagrange multipliers on budget constraint and law of motion for employment respectively. Then, we have the following first order conditions:

$$\begin{array}{cc} [c] & u'(c) - \lambda_H = 0 \\ [s'] & \beta E[V'_s] - \lambda_H p = 0 \\ [a'_H] & \beta E[V'_{a'_H}] - \lambda_H \frac{1}{1+r} = 0 \end{array}$$

Also, from the envelope conditions we have

$$\begin{aligned} V_{a_H} &= \lambda_H \\ V_s &= \lambda_H (p + d) \end{aligned}$$

By combining the first order conditions and envelope conditions, we get the following the no-arbitrage condition between shares and bonds.

$$
\begin{aligned}
1 &= E\left[m'\left(1+r\right)\right] \\
1 &= E\left[m'\left(\frac{p'+d'}{p}\right)\right]
\end{aligned}
$$

where $m' = \beta u_c\left(c'\right)/u_c\left(c\right)$ is the stochastic discount factor.

Now, the representative firm solves following problem.

$$
J(S, s_F) = \max_{d,k',a'_F,v,n'} \{d + E[m'J(S', s'_F)]\}
$$

s.t.

$$
k' + a_F + \varphi(d, d_-) = F(z, k, nh\left(S, s_F\right)) + (1-\delta)k - w\left(S, s_F\right)nh\left(S, s_F\right) - c_v v + \frac{a'_F}{R\left(S\right)}
$$

$$
\xi\left(k' - \frac{a'_F}{1+r\left(S\right)}\right) \geq F(z, k, nh\left(S, s_F\right))
$$

$$
n' = (1-\chi)n + \Phi\left(S\right)v
$$

$$
S' = G(S),\ k', v \geq 0
$$

Let λ, γ, and π be the Lagrange multipliers on the budget constraint, enforcement constraint, and law of motion for employment respectively. Then, we have the following first order conditions.

$$
\begin{array}{cc}
[d] & 1 + E[m'J'_{d_-}] - \lambda\varphi_d = 0 \\
[k'] & E[m'J'_k] - \lambda + \gamma\xi = 0 \\
[a'_F] & E[m'J'_{a_F}] + \lambda\frac{1}{1+r(1-\tau)} - \gamma\xi\frac{1}{1+r} = 0 \\
[v] & -\lambda c_v + \pi\Phi = 0 \\
[n'] & E[m'J'_n] = \pi
\end{array}
$$

Also, from the envelope conditions we have

$$
\begin{aligned}
J_k &= \left(\lambda-\gamma\right)F_k + \left(1-\delta\right)\lambda \\
J_{a_F} &= -\lambda \\
J_n &\equiv (\lambda-\gamma)zF_{nh}h - \lambda wh + (1-\chi)E[m'J'_n] \\
J_{d_-} &= -\lambda\varphi_{d_-}
\end{aligned}
$$

By combining the first order conditions and envelope conditions, we simply get the following first order conditions for the firm.

$$\begin{aligned}
1-E[m'\lambda'\varphi'_{d_}] &= \lambda\varphi_d \\
\lambda c_v &= \Phi E\left[m' J'_n\right] \\
\lambda-\gamma\xi &= E\left[m'\left[(\lambda'-\gamma')\,F'_k+(1-\delta)\,\lambda'\right]\right] \\
\lambda\,(1+r)-\gamma\xi R &= R\,(1+r)\,E\left[m'\lambda'\right]
\end{aligned}$$

where $R = 1 + r(1-\tau)$ is the *effective* gross interest rate.

B.2 Derivation of Nash bargaining solutions

Given the worker's bargaining weight $\mu \in (0,1)$, the wage and hours are solutions to the Nash bargaining problem:

$$(w,h) = \arg\max_{w,h} \left(\frac{V_n}{u_c}\right)^{\mu} (J_n)^{1-\mu}$$

The first order conditions for this problem are

$$\begin{array}{cc}
[w] & \mu J_n = (1-\mu)\lambda(\frac{V_n}{u_c}) \\
 & \mu J_n\left(-\frac{v_{(1-h)}(1-h)}{u_c}+w\right) = -(1-\mu)\frac{V_n}{u_c}((\lambda-\gamma)F_{nh}-\lambda w) \\
[h] & (1-\mu)\left(\frac{V_n}{u_c}\right)\left(-\frac{v_{(1-h)}(1-h)}{u_c}+w\right) = -(1-\mu)\left(\frac{V_n}{u_c}\right)((\lambda-\gamma)F_{nh}-\lambda w) \\
 & \left(-\frac{v_{(1-h)}(1-h)}{u_c}+w\right) = -(1-\frac{\gamma}{\lambda})F_{nh}+w \\
 & \frac{v_{(1-h)}(1-h)}{u_c} = \left(1-\frac{\gamma}{\lambda}\right)F_{nh}
\end{array}$$

The equilibrium wage bill can be derived from the sharing rule and the definition of $\frac{V_n}{u_c}$ and J_n.

$$\mu J_n = (1-\mu)\lambda\left(\frac{V_n}{u_c}\right)$$

$$\begin{aligned}
\mu((\lambda-\gamma)F_{nh}h-\lambda wh+(1-\chi)E[m'J'_n]) &= \\
(1-\mu)\lambda\left(\frac{v(1-h)-v(1)}{u_c}+(wh-b)+(1-\chi-\Psi)E\left[\beta\frac{V'_n}{u_c}\right]\right)&
\end{aligned}$$

$$\begin{aligned}
wh &= \mu\left(\left(1-\frac{\gamma}{\lambda}\right)F_{nh}h\right)+(1-\mu)\left(\frac{v(1)-v(1-h)}{u_c}+b\right)+ \\
&\quad \mu(1-\chi)E\left[m'\frac{J'_n}{\lambda}\right]-(1-\mu)(1-\chi-\Psi)E\left[\beta\frac{V'_n}{u_c}\right] \\
&= \mu\left(\left(1-\frac{\gamma}{\lambda}\right)F_{nh}h\right)+(1-\mu)\left(\frac{v(1)-v(1-h)}{u_c}+b\right)+ \\
&\quad \mu(1-\chi)E\left[m'\frac{J'_n}{\lambda}\right]+\mu\Psi E\left[m'\frac{J'_n}{\lambda'}\right]-(1-\mu)(1-\chi)E\left[\beta\frac{V'_n}{u_c}\right]
\end{aligned}$$

Using the sharing rule $\mu J_n = \lambda(1-\mu)(V_n/u_c)$ and $F_{nh}h = F_n$, along with the definition of V_n/u_c and J_n, gives the wage bill per worker:

$$\begin{aligned} wh &= \mu\left[\left(1-\frac{\gamma}{\lambda}\right)F_n + (1-\chi)E\left[m'\frac{J_n'}{\lambda}\right] + \frac{V}{1-N}\Phi E\left[m'\frac{J_n'}{\lambda'}\right]\right] \\ &\quad + (1-\mu)\left[\frac{\nu(1)-\nu(1-h)}{u_c} + b - (1-\chi)E\left[\beta\frac{V_n'}{u_c}\right]\right] \end{aligned}$$

B.3 Data appendix

Data for Employment, Average Weekly Hours Worked and the Labor Force are taken from the Bureau of Labor Statistics. Total GDP and business GDP are taken from the National Income and Product Accounts (NIPA) published by the Bureau of Economics Analysis. Real wages are defined as labor compensation to plus labor's share of proprietors income deflated by the GDP deflator and divided by total hours (employment multiplied by average weekly hours). Labor productivity is defined as total GDP divided by total hours. Vacancies are constructed using the Conference Board's Help-Wanted Index and the composite Help-Wanted Index by Barnichon (2010).

Equity Payouts and Debt Repurchases are taken from the Flow of Funds published by the Federal Reserve Board. Equity Payouts are defined as Net dividends of nonfinancial business minus Net increase in corporate equities of nonfinancial business minus Proprietors' net investment of nonfinancial business. Debt Repurchases are the negative of Net increase in credit markets instruments of nonfinancial business. Both Equity payouts and Debt repurchases are divided by business GDP from NIPA. Total GDP is used to compute the correlations reported in Table 1.

The capital stock is constructed similar to JQ. Using the law of motion of capital

$$k_{t+1} = k_t + Investment - Depreciation$$

we define *Depreciation* as Consumption of fixed capital in nonfinancial corporate business plus Consumption of fixed capital in nonfinancial noncorporate business taken from the Flow of Funds. *Investment* is measured as Capital expenditures in non financial business, also from the Flow of Funds. Both variables are deflated by the price index for business GDP from NIPA. The initial capital stock is chosen so that the capital-output ratio in the business sector does not display any trend over the period 1952:Q1-2012:Q1.

The stock of debt is constructed (again, similar to JQ) using the law of motion

$$b_{t+1}^e = b_t^e + Net\,New\,Borrowing$$

where *Net New Borrowing* is defined as the Net increase in credit markets instruments of non-financial business taken from the Flow of Funds. $b^e_{t+1} = b_{t+1}/(1 + r_t)$ since this is the model equivalent of the end-of-period debt reported in the data. We take the initial value of the stock of debt to be the nonfinancial business sector's stock of debt in 1952:Q1 from the balance sheet data reported in the Flow of Funds. We deflate the constructed series by the price index for business GDP from NIPA.

Appendix C

Appendix to Chapter 3

C.1 Marginal value of an additional employee to the firm

$$
\begin{aligned}
J^m &= \lim_{\Delta\to 0}\frac{1}{\Delta}\left[\left(F(z,k,(n+\Delta)h) - w[n+\Delta](n+\Delta)h - (r+\delta)k - \kappa v + \beta E\left[\frac{u_c^{'}}{u_c}J\left[(n+\Delta)^{'}\right]\right]\right)\right. \\
&\quad \left. -\left(F(z,k,nh) - w^e[n]nh - (r+\delta)k - \kappa v + \beta E\left[\frac{u_c^{'}}{u_c}J^B\left[n^{'}\right]\right]\right)\right] \\
&= \lim_{\Delta\to 0}\frac{F(z,k,(n+\Delta)h) - F(z,k,nh)}{\Delta} - \lim_{\Delta\to 0}\frac{[w[n+\Delta](n+\Delta)nh - w^e[n]nh]}{\Delta} \\
&\quad + E\left[\beta\frac{u_c^{'}}{u_c}\lim_{\Delta\to 0}\frac{J[(1-\chi)(n+\Delta)] - J^B[(1-\chi)n]}{\Delta}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0}\frac{w[n+\Delta](n+\Delta)nh - w^e[n]nh}{\Delta} \\
&\quad + (1-\chi)E\left[\beta\frac{u_c^{'}}{u_c}\lim_{\Delta\to 0}\frac{J[(1-\chi)n + (1-\chi)\Delta] - J^B[(1-\chi)n]}{(1-\chi)\Delta}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0}\frac{w[n+\Delta](n+\Delta)nh - w^e[n]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right]
\end{aligned}
$$

C.1.1 Proof of proposition 1

Under $w^e[n] = w[n+\triangle]$, the equation can be rewritten as

$$\begin{aligned}
J^m &= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0}\frac{w[n+\Delta](n+\Delta)h - w[n+\Delta]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c'}{u_c}J^{m'}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0}\frac{w[n+\Delta]nh + w[n+\Delta]\Delta h - w[n+\triangle]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c'}{u_c}J^{m'}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0} w[n+\Delta]h + (1-\chi)\beta E\left[\frac{u_c'}{u_c}J^{m'}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - w[n]h + (1-\chi)\beta E\left[\frac{u_c'}{u_c}J^{m'}\right]
\end{aligned}$$

C.1.2 Proof of proposition 2

Under $w^e[n] = w[n]$, the equation can be rewritten as

$$\begin{aligned}
J^m &= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0}\frac{w[n+\Delta](n+\Delta)h - w[n]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c'}{u_c}J^{m'}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0}\frac{w[n+\Delta]nh + w[n+\Delta]\Delta h - w[n]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c'}{u_c}J^{m'}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0}\frac{w[n+\Delta] - w[n]}{\Delta}nh - \lim_{\Delta\to 0} w[n+\Delta]h + (1-\chi)\beta E\left[\frac{u_c'}{u_c}J^{m'}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - \frac{\partial w[n]}{\partial n}nh - w[n]h + (1-\chi)\beta E\left[\frac{u_c'}{u_c}J^{m'}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - w[n]h - \frac{\partial w[n]}{\partial n}nh + (1-\chi)\beta E\left[\frac{u_c'}{u_c}J^{m'}\right]
\end{aligned}$$

C.1.3 Proof of proposition 3

Under $w^e[n] = \gamma w[n] + (1-\gamma) w[n+\Delta]$, the equation can be rewritten as

$$\begin{aligned}
J^m &= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0} \frac{w[n+\Delta](n+\Delta)h - (\gamma w[n] + (1-\gamma) w[n+\Delta])nh}{\Delta} + \\
&\quad (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0} \frac{w[n+\Delta]nh + w[n+\Delta]\Delta h - \gamma w[n]nh - (1-\gamma) w[n+\Delta]nh}{\Delta} + \\
&\quad (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0} \frac{w[n+\Delta]\Delta h - \gamma w[n]nh + \gamma w[n+\Delta]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0} w[n+\Delta]h - \lim_{\Delta\to 0} \gamma \frac{w[n+\Delta] - ww[n]}{\Delta} nh + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right] \\
&= \frac{\partial F(z,k,nh)}{\partial n} - w[n]h - \gamma \frac{\partial w[n]}{\partial n} nh + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c} J^{m'}\right]
\end{aligned}$$

C.2 Derivation of the equilibrium conditions

Household solves the following dynamic programming problem.

$$\Omega(S, s_H) = \max_{c,a^{'}} u(c) + n u^l(1 - h(S, s_H)) + (1-n) u^l(1) + \beta E\left[\Omega(S', s_H')\right]$$

$$\begin{aligned}
&s.t. \\
c + a' + T(S) &= w(S, s_H) h(S, s_H) n + (1-n) b + (1 + r(S)) a + \Pi(S) \\
n' &= (1-\chi) n + p(S)(1-n) \\
S' &= G(S)
\end{aligned}$$

Let λ_H and μ_H be the Lagrange multiplier on budget constraint, and law of motion for employment respectively. Then we have the following first order conditions.

$$\begin{aligned}
u_c &= \lambda_H \\
E\left[\beta \Omega_a'\right] &= \lambda_H
\end{aligned}$$

From the envelope condition with respect to a, we get

$$\Omega_a = (1+r)\lambda_H$$

Taking a derivative with respect to n', we have

$$\mu_H = E\left[\beta \Omega_n'\right]$$

By combining equations above, we get the standard Euler equation .

$$E\left[\beta\frac{u_c'}{u_c}\left(1+r'\right)\right] = 1$$

Now, firms solve the following problem.

$$\begin{aligned}
J(S,s_F) &= \max_{v,k} \Pi(S) + E\left[m'\left(S,S'\right)J\left(S',s_F'\right)\right] \\
&= \max_{v,k} F(z,k,nh) - w(S,s_F)\,h(S,s_F)\,n - (r(S)+\delta)\,k - \kappa v + E\left[m'\left(S,S'\right)J\left(S',s_F'\right)\right] \\
s.t. & \\
n' &= (1-\chi)\,n + q(S)\,v \\
S' &= G(S)
\end{aligned}$$

where $m'(S,S') = \beta u_c(c(S'))/u_c(c(S))$ is the stochastic discount factor and $q(S) = M/V$ is the job-filling rate.

Let μ_F be the Lagrange multipliers on law of motion of employment. Then, we have the following first order conditions for firms.

$$\begin{aligned}
\kappa &= \mu_F q(S) \\
r+\delta &= F_k
\end{aligned}$$

From the definition of the marginal value of an additional employee to the firm, the following condition should hold.

$$E\left[m'J^{m'}\right] = \mu_F$$

By combining equations above, we have an equation for the rental rate and a job creation condition.

$$\begin{aligned}
r &= F_k - \delta \\
\kappa &= q\beta E\left[m'J^{m'}\right]
\end{aligned}$$

C.3 Solutions to the bargaining problem with a marginal worker

Now, we turn to the bargaining problem which is the same as standard Nash bargaining given the marginal value of employment for the worker and the marginal value of an additional employee to the firm.

$$\begin{aligned}
\Omega^m &= wh - b + \frac{u^l(1-h) - u^l(1)}{u_c} + (1-\chi-p)\,\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right] \\
J^m &= \frac{\partial F(z,k,nh)}{\partial n} - wh - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\,\beta E\left[\frac{u_c'}{u_c}J^{m'}\right]
\end{aligned}$$

Given the bargaining power of the marginal worker, $\mu \in [0,1]$, and the bargaining powers of existing workers , $\gamma \in [0,1]$, wages and hours per worker are determined via the following standard bargaining problem .

$$
\begin{aligned}
(w,h) &= arg\max_{w,h} (\Omega^m)^\mu (J^m)^{1-\mu} \\
&= arg\max_{w,h} \left(wh - b + \frac{u^l(1-h) - u^l(1)}{u_c} + (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right]\right)^\mu \\
&\times \left(\frac{\partial F(z,k,nh)}{\partial n} - wh - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right)^{1-\mu}
\end{aligned}
$$

The first order condition with respect to w gives the following sharing rule.

$$\mu J^m = (1-\mu)\Omega^m$$

By plugging the definitions of Ω^m and J^m, we have

$$
\begin{aligned}
&\mu\left(\frac{\partial F(z,k,nh)}{\partial n} - wh - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right) \\
&= (1-\mu)\left(wh - b + \frac{u^l(1-h) - u^l(1)}{u_c} + (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right]\right)
\end{aligned}
$$

It can be rewritten as

$$
\begin{aligned}
wh &= \mu\left(\frac{\partial F(z,k,nh)}{\partial n} - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right) + \\
&\quad (1-\mu)\left(\frac{u^l(1) - u^l(1-h)}{u_c} + b - (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right]\right) \\
&= \mu\left(\frac{\partial F(z,k,nh)}{\partial n} - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right] - (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right) + \\
&\quad (1-\mu)\left(\frac{u^l(1) - u^l(1-h)}{u_c} + b\right) \\
&= \mu\left(\frac{\partial F(z,k,nh)}{\partial n} - \gamma\frac{\partial w}{\partial n}nh + p\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right) + (1-\mu)\left(\frac{u^l(1) - u^l(1-h)}{u_c} + b\right) \\
&= \mu\left(\frac{\partial F(z,k,nh)}{\partial n} - \gamma\frac{\partial w}{\partial n}nh + p\frac{\kappa}{q}\right) + (1-\mu)\left(\frac{u^l(1) - u^l(1-h)}{u_c} + b\right) \\
&= \mu\left(\frac{\partial F(z,k,nh)}{\partial n} - \gamma\frac{\partial w}{\partial n}nh + \frac{V}{1-N}\kappa\right) + (1-\mu)\left(\frac{u^l(1) - u^l(1-h)}{u_c} + b\right)
\end{aligned}
$$

The sharing rule, $\mu J^m = (1-\mu)\Omega^m$ is used in the second line and the optimal condition for vacancies, $\kappa = q\beta E\left[m' J^{m'}\right] = q\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]$ is used in the last line.

The first order condition with respect to h gives the following intra-temporal condition for hour per worker .

$$\mu J^m \left(w - \frac{u^l_{(1-h)}(1-h)}{u_c}\right) = (1-\mu)\Omega^m \left(-\frac{\partial F(z,k,nh)}{\partial n \partial h} + w + \gamma\frac{\partial w}{\partial n}n\right)$$

Since $\mu J^m = (1-\mu)\,\Omega^m$ holds from the first order condition with respect to w,

$$\begin{aligned} w - \frac{u^l_{(1-h)}(1-h)}{u_c} &= -\frac{\partial F(z,k,nh)}{\partial n \partial h} + w + \gamma \frac{\partial w}{\partial n} n \\ \frac{u^l_{(1-h)}(1-h)}{u_c} &= \frac{\partial F(z,k,nh)}{\partial n \partial h} + \gamma \frac{\partial w}{\partial n} n \\ &= \frac{\partial F(z,k,nh)}{\partial (nh)} + \gamma \frac{\partial w}{\partial n} n \end{aligned}$$

Following Andolfatto (1996), it is assumed that each worker is so small such that $\frac{\partial F(z,k,nh)}{\partial(nh)}$ is taken as given by both the worker and the firm during the bargaining.

C.4 Solutions to the first order differential equation

Given the production function and utility function, the sharing rule, the intra-temporal condition, and the wage bill can be written as

$$\begin{aligned} \mu J^m &= (1-\mu)\,\Omega^m \\ \phi_e (1-h)^{-\eta} c &= (1-\alpha)\, zk^\alpha (nh)^{-\alpha} \\ wh &= \mu \left((1-\alpha)\, zk^\alpha h^{1-\alpha} n^{-\alpha} - \gamma \frac{\partial w}{\partial n} nh + \frac{V}{1-N}\kappa \right) + (1-\mu) \left(\frac{u^l(1) - u^l(1-h)}{u_c} + b \right) \end{aligned}$$

We can rewrite the wage bill as the first order differential equation as follows.

$$\begin{aligned} \mu\gamma nh \frac{\partial w}{\partial n} + hw &= \mu \left((1-\alpha)\, zk^\alpha h^{1-\alpha} n^{-\alpha} + \frac{V}{1-N}\kappa \right) + (1-\mu) \left(\frac{u^l(1) - u^l(1-h)}{u_c} + b \right) \\ \frac{\partial w}{\partial n} + \frac{1}{\mu\gamma n} w &= \frac{1}{\mu\gamma nh} \left(\mu \left((1-\alpha)\, zk^\alpha h^{1-\alpha} n^{-\alpha} + \frac{V}{1-N}\kappa \right) + (1-\mu) \left(\frac{u^l(1) - u^l(1-h)}{u_c} + b \right) \right) \end{aligned}$$

So, the integrating factor is

$$e^{\int \left(\frac{1}{\mu\gamma n}\right) dn} = e^{\frac{1}{\mu\gamma} ln(n)} = n^{\frac{1}{\mu\gamma}}$$

By multiplying both sides of the equation by $n^{\frac{1}{\mu\gamma}}$ and integrating both sides with respect to n, we have

$$\begin{aligned} w &= n^{-\frac{1}{\mu\gamma}} \int n^{\frac{1}{\mu\gamma}} \frac{1}{\mu\gamma nh} \times \\ & \left[\mu \left((1-\alpha)\, zk^\alpha h^{1-\alpha} n^{-\alpha} + \frac{V}{1-N}\kappa \right) + (1-\mu) \left(\frac{u^l(1) - u^l(1-h)}{u_c} + b \right) \right] dn + Dn^{-\frac{1}{\mu\gamma}} \\ &= n^{-\frac{1}{\mu\gamma}} \int n^{\frac{1}{\mu\gamma}} \frac{1}{\mu\gamma nh} \times \\ & \left[\mu \left((1-\alpha)\, zk^\alpha h^{-\alpha} n^{-\alpha} \right) + \mu \left(\frac{V}{1-N}\kappa + \frac{(1-\mu)}{\mu} \left(\frac{u^l(1) - u^l(1-h)}{u_c} + b \right) \right) \right] dn + Dn^{-\frac{1}{\mu\gamma}} \\ &= n^{-\frac{1}{\mu\gamma}} \left[\frac{\mu(1-\alpha)}{1-\mu\gamma\alpha} zk^\alpha h^{-\alpha} n^{-\alpha + \frac{1}{\mu\gamma}} + n^{\frac{1}{\mu\gamma}} \frac{\mu}{h} \left(\frac{V}{1-N}\kappa + \frac{(1-\mu)}{\mu} \left(\frac{u^l(1) - u^l(1-h)}{u_c} + b \right) \right) \right] + \\ & Dn^{-\frac{1}{\mu\gamma}} \\ &= \frac{\mu(1-\alpha)}{1-\mu\gamma\alpha} zk^\alpha h^{-\alpha} n^{-\alpha} + \frac{\mu}{h} \left(\frac{V}{1-N}\kappa + \frac{(1-\mu)}{\mu} \left(\frac{v(1) - v(1-h)}{u_c} + b \right) \right) + Dn^{-\frac{1}{\mu\gamma}} \end{aligned}$$

where D is a constant of the integration of the homogeneous equation. By assuming the total wage bill wnh has to remain finite as employment becomes small as in Hawkins (2011) or alternatively by assuming $\lim_{n \to 0} wnh = 0$ as in Cahuc, Maroque, and Wasmer (2008), we have $D = 0$. From the equation above, we also have

$$\frac{\partial w}{\partial n} = -\frac{\mu\alpha\,(1-\alpha)}{1-\mu\gamma\alpha} zk^{\alpha}h^{-\alpha}n^{-\alpha-1} < 0$$

C.5 Equilibrium conditions

The equilibrium of the model is characterized by the following conditions under functional forms specified in the calibration section.

$$\begin{aligned}
E\left[\beta\frac{C}{C'}\left(1+r'\right)\right] &= 1 \\
r &= F_k - \delta = \alpha\frac{Y}{K} - \delta \\
N^{'} &= (1-\chi)N + \omega V^{\psi}\left(1-N\right)^{1-\psi} \\
q &= \omega V^{\psi-1}\left(1-N\right)^{1-\psi} \\
Y &= C + I + \kappa V \\
I &= K^{'} - \left(1-\delta\right)K \\
Y &= F\left(z,K,Nh\right) = e^{z}K^{\alpha}\left(Nh\right)^{1-\alpha} \\
\frac{\kappa}{q} &= E\left[\beta\frac{C}{C'}\left[\left(1-\alpha\right)\frac{Y^{'}}{N'} - w^{'}h^{'} + \frac{\mu\alpha\left(1-\alpha\right)}{1-\mu\gamma\alpha}\frac{Y^{'}}{N'} + \left(1-\chi\right)\frac{\kappa}{\Phi'}\right]\right] \\
\phi_e\left(1-h\right)^{-\eta}C &= \left(1-\alpha\right)\frac{Y}{Nh} \\
wh &= \mu\left(\left(1-\alpha\right)F_n + \frac{V}{1-N}\kappa\right) + \left(1-\mu\right)\left(\left(\phi_e\frac{1}{1-\eta} - \phi_e\frac{\left(1-h\right)^{1-\eta}}{1-\eta}\right)C + b\right) + \\
&\quad \frac{\mu\alpha\left(1-\alpha\right)}{1-\mu\gamma\alpha}F_n
\end{aligned}$$

C.6 Data appendix

C.6.1 Raw data

1. Employment, Average Weekly Hours Worked, Population: Bureau of Labor Statistics (BLS).

2. Real GDP, GDP, Compensation of Employees, Proprietors Income, GDP deflator: National Income and Product Accounts (NIPA) published by the Bureau of Economics Analysis. (BEA)

3. Vacancies: Conference Board's Help Wanted Index and the Composite Help Wanted Index by Barnichon (2010)

4. Consumption of Fixed Capital, Capital Expenditure in non-financial non-corporate business: Flow of Funds

C.6.2 Constructed data

1. Employment $= \frac{\text{Employment}}{\text{Population}}$

2. Hours per Worker $= \frac{\text{Average Weekly Hours Worked}}{20\times 5}$

3. Total Hours = Employment $\times$ Hours per Worker

4. Labor Share $= \frac{\text{Compensation of Employees}}{\text{GDP-Proprietors Income}}$

5. Real Wage $= \frac{\text{Labor Share}\times\text{Real GDP}}{\text{Tolal Hours}}$

6. Labor Productivity $= \frac{\text{Real GDP}}{\text{Tolal Hours}}$

7. Vacancies = Conference Board's Help Wanted Index and the Composite Help Wanted Index by Barnichon (2010)

8. Investment = Capital Expenditure deflated by GDP deflator

9. Depreciation = Consumption of Fixed Capital deflated by GDP deflator

10. Capital Stock is constructed by the perpetual inventory method using follow law of motion

$$k_{t+1} = k_t + \text{Investment} - \text{Depreciation}$$

    Initial capital stock is chosen so that the capital-output ratio does not display any trend over the period 1960Q1-2012Q1.