The Impact of the Flipped Classroom Model of Instruction on Fifth Grade Mathematics Students

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Dedication

It is with great honor that I dedicate this dissertation to my husband, Kim Wiley, who made this whole journey possible. Thank you.

Abstract

The National Council of Teachers of Mathematics (NCTM) calls for "high quality mathematics instruction for *all* students". This declaration, along with the present achievement gap, has driven many innovations in mathematics teaching and curriculum in the past 15 years, including the Flipped Classroom model. Currently, this model is being implemented at all levels of schooling and academic areas, yet there is very little research as to its effectiveness. This model, as enacted in this study, has the students watch a video lecture at home and then come to class and do the traditional homework during the class period. This study attempts to expand this body of research by looking at the Flipped Classroom model as it is implemented in fifth grade mathematics classrooms.

This study uses a convergent concurrent mixed methods design to develop an understanding of the impact that this model of instruction has on elementary students. The participants were 112 fifth grade students from four classrooms in a Midwestern suburban school district. Qualitative data was collected through class room observations, and student and teacher interviews over the course of two units of instruction on decimals and fractions. Quantitative data was collected from two unit posttests and an attitude survey at the end of the study. The NCTM Mathematics Practices were used as a framework to analyze the teaching practices and research on students' conceptual understanding of decimals and fractions formed the basis for understanding student thinking during the interviews.

The qualitative data suggests that this model, as enacted in this study, strongly encourages the use of rules and procedures, not always accurately, to the detriment of developing conceptual understanding. The quantitative data shows that most students did appear to meet the state standards in the area of decimals while many more did not in the area of fractions. Of equal concern is that low achieving students had less access to the videos at home and more frequently found them frustrating or confusing. Continued research on teaching practices and equity issues within the Flipped Classroom model would help further address these issues.

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Chapter 1 – Introduction

The Achievement Gap in mathematics is at the forefront of most educators' minds and is driving the movement to change what is happening in classrooms around the United States in terms of standards and daily practices. In 2000, the National Council of Teachers of Mathematics (NCTM) updated their existing standards document with a new publication entitled, *Principles and Standards for School Mathematics.* It is this document that serves as one of the foundational pieces for the development of the Common Core State Standards (2010) as well as for several individual states who still comply with their own standards for the teaching and learning of mathematics. In its vision, the NCTM calls for "a classroom, a school, or a district where all students have access to high-quality, engaging mathematics instruction" (NCTM, 2000, p. 3). It is this vision that has driven many innovations in teaching including the development of the "Flipped Classroom" model of instruction.

The Flipped Classroom model takes the classroom lecture and puts it on video for students to watch at home and then uses the class time for working on problems that were originally part of the homework assignment or other tasks that the teacher can assist students with during the class period (Bishop & Verlager, 2013). Advancements in technology make this possible and with greater availability and access to technology in schools, the idea of "flipping" the instructional model traditionally used in many classrooms has become an increasingly popular trend.

While there are an increasing number of teachers nationwide trying this method in their classrooms, there is a limited amount of research documenting the impact of this model on students. The research that is available has been conducted at the secondary and post-secondary level and primarily in science classes (Bishop & Verlager, 2013). In addition, the findings are inconsistent in that some studies show positive results in achievement while an approximately equal amount show no change or negative results (i.e. Strayer, 2012; Toto &Nguyen, 2009; Day & Foley, 2006). Interestingly, more and more elementary classroom teachers are now incorporating the Flipped Classroom model for their mathematics instruction with no published research at this instructional level to support this model of instruction.

There is a need for research on these new classroom models of instruction. At this point in time there is no peer reviewed research on the Flipped Classroom model at the elementary level nor is there research that looks at effective mathematical teaching practices occurring within the Flipped Classroom model for instruction. Children at the elementary level are cognitively and socially very different than older teenagers and young adults (AMLE, 2010). The National Council of Teachers of Mathematics (NCTM) calls for high quality mathematics instruction for all students (NCTM, 2000). At this point in time it is not agreed upon or truly known whether using a flipped model of instruction is developmentally or pedagogically appropriate for elementary mathematics students or meets the NCTM vision for high quality mathematics instruction.

This study addresses the influence of a model of Flipped Classroom instruction on student attitudes toward and achievement in mathematics with a specific emphasis on procedural versus conceptual understanding. Furthermore, this study looks at how instruction is delivered in the classroom and its alignment or misalignment with the NCTM teaching practices for instruction to support both conceptual and procedural understanding. A convergent concurrent mixed methods design (Creswell & Plano Clark, 2011) was used in order to collect both quantitative survey data on attitudes and achievement and qualitative interview and classroom observation data on these same constructs simultaneously. The qualitative data will be compared with the findings of the empirical quantitative data. The study takes place in a Midwestern suburban school district with approximately 112 5th grade mathematics students in four different classrooms over a three month period of time. Teachers in these four classrooms used a flipped classroom model in their mathematics classes.

Conceptual Framework

This conceptual framework section begins by defining the “Flipped Classroom” and offering a broad overview of its history and development. From there, the NCTM Principles and the Mathematics Teaching Practices (NCTM, 2014) that apply directly to the Flipped Classroom model in this study are discussed. “These principles describe particular features of high-quality mathematics education” (NCTM, 2000, p. 11). The research that supports the NCTM Principles and the Effective Teaching Practices is described in detail in Chapter 2. This section concludes with an overview of existing gaps in the evaluation of Flipped Classroom models in mathematics education and how this study addresses these gaps through the research questions.

What is Flipped Learning*?*

If one were to Google the definition of "Flipped Classroom" or "Flipped Learning", numerous entries would be listed providing a wide array of descriptions and definitions. The Flipped Learning Network (FLN) is a non-profit organization for educators that serves as a professional learning community to support teachers and researchers who are working with Flipped Learning or creating a Flipped Classroom. The organization has a large membership (20,000 +) and appears to serve as one of the significant leaders in the Flipped Learning and Classroom movement. They define Flipped Learning as:

> "a pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter" (FLN, 2014).

While there are others who discuss various alternative definitions (Bishop & Verleger, 2013) this study uses the FLN definition as a basis for understanding the instructional model. Other definitions and other studies interchange the terms "Flipped Classroom" and "Flipped Learning" or simply refer to the idea that the traditional in class activities and out of class activities are reordered or "flipped" (Lage, Platt, Treglia, 2000). In addition, this study will include the specific use of instructional video viewed outside of the mathematics classroom as the mode of direct instruction identified in the above stated definition.

The theoretical basis for the flipped classroom model of instruction is set in the importance of student centered versus teacher centered learning environments. "Student centered learning approaches derive from constructivist views of education, in which the construction of knowledge is shared and learning is achieved through students' engagement with activities in which they are invested" (Kain, 2002, p. 104). A major component of student-centered learning are active learning strategies in which the students are constructing knowledge by working together cooperatively (Michael, 2006). Peer instruction (Crouch & Mazur, 2001) along with Priming or Pre-training, also known as advanced organizers (Ausubel, 1960) play an important theoretical role in the purpose of the video lectures students view as homework within the Flipped Classroom model of instruction. The video lectures are designed to prepare and instruct the students on their classwork for the next day and take the place of the lecture that would have occurred in class. Eliminating the lecture portion of the class period theoretically opens up the in-class time for student centered approaches to instruction that would involve problem solving and cooperative learning experiences.

The National Council of Teachers of Mathematics Principles

"Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction" (NCTM, 2000, p.3). It is this vision that is driving innovation in instructional design and curriculum. The six NCTM Principles, Equity, Curriculum, Teaching, Learning, Assessment and Technology, are the foundation to high-quality mathematics programs regardless of the vehicle being used to deliver instruction. "The Principles should be useful as perspectives on which educators can base decisions that affect school mathematics" (NCTM, 2000, p.7). In a recent published document entitled "Principles to Actions: Ensuring Mathematical Success for All" (NCTM, 2014). NCTM creates a detailed description that explicitly establishes what the NCTM Principles look like in classrooms as well as outlines eight "Mathematics Teaching Practices [which] provide a framework for strengthening the teaching and learning of mathematics" (NCTM, 2014, p. 9).

These eight practices are noted below:

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematics discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking.

(NCTM, 2014, p. 10)

These eight practices are based on the past four decades of research in mathematics education (NCTM, 2014). Teachers working to improve their instruction and the learning that takes place in their classrooms can use these eight practices to develop an effective model of instruction regardless of the vehicle through which the instruction is delivered. How these effective practices are or are not enacted in Flipped Classrooms has yet to be studied.

This study will look specifically at five of the practices:

a. *Implement tasks that promote reasoning and problem solving*

b. *Use and connect mathematical representations*
c. *Facilitate meaningful mathematical discourse*
d. *Build procedural fluency from conceptual understanding*
e. *Elicit and use evidence of student thinking*

These five practices were chosen because they can be observed from an outside perspective. Specifically, this study will look at the types of activities that are occurring in the classrooms and what types of instructional or pedagogical actions the teacher is making during these activities. High quality mathematics instruction, based on these practices, supports the development of conceptual understanding which will be a focus during both the classroom observations and the student interviews.

Gaps in the Research

At this point in time, while the scope of content matter being flipped is vast, "Flipped Learning" is primarily being researched at the post-secondary and secondary level. Based on FLN membership, the group of teachers attempting to flip their classrooms begins all the way down at the kindergarten level (FLN, 2014). According to the FLN blogs, at the elementary level, teachers are flipping math courses and science courses and reporting both positive and negative impacts on their students. There is a significant gap in the literature in this area that needs to be addressed in order to support what teachers are reporting. Currently, it does not appear that there are any peer-reviewed research studies published at the elementary classroom level and specifically in mathematics. In addition, similar to the existing peer-reviewed studies at the post-secondary level, the classroom teacher is serving the dual role as the teacher and researcher. The need for research to be done from a participant observer perspective is needed in order to establish the context of the setting, understand what is routine that may go unnoticed by the teacher, and uncover things that no one has noticed before (Patton, 2002). The participant observer "can move beyond the selective perceptions" (Patton, 2002, p.264) of the teachers and develop a rich, accurate picture of what is actually happening in the classroom.

An additional area that has not been addressed in the research on Flipped Learning is how the use of the Flipped Classroom model can support or impede the NCTM Principles. While classroom teachers appear to be concerned about student achievement and motivation in flipped classrooms there does not seem to be a connection made to the research supported Principles of the NCTM for high quality mathematics for all students. The research that has been done reports course grades or compares course grades to traditional course grades but does not analyze specific teaching practices that occur in these courses.

Due to the gaps in the literature this study will address the following research questions.

- To what extent does the observed model of Flipped Classroom instruction align with the NCTM Principles to Action in five of the eight Mathematics Teaching Practices for high quality mathematics instruction in four 5th grade classrooms?

    a. Implement tasks that promote reasoning and problem solving
    b. Use and connect mathematical representations
    c. Facilitate meaningful mathematical discourse
    d. Build procedural fluency from conceptual understanding
    e. Elicit and use evidence of student thinking

- How is student achievement on the decimal and fraction units affected by the model of Flipped Classroom instruction in this study?
    a. Do the students meet the MN State Standards for decimal and fraction concepts as measured by the district created post unit tests?
    b. To what extent do student understandings reflect conceptual knowledge of decimals and fractions based on research on student thinking in the areas of decimals and fractions?

- To what extent is there a relationship between student achievement and student attitudes towards mathematics in the Flipped Classroom model of instruction?

Overview of the Dissertation

Chapter two offers an overview of the limited peer reviewed research that is currently available in regards to the Flipped Classroom or Flipped Learning as well as the underpinning theoretical basis for the development of this model of instruction. Additionally, what is known about how students' create a conceptual understanding of fractions and decimals will be reviewed. The NCTM Principles and the eight Mathematics Teaching Practices, as developed by the NCTM, are discussed as they are the foundation of the conceptual framework for this study. Chapter three will elaborate on the methodology of this study and provide support for the need for multiple methods of data collection in this study. Finally chapters four and five will analyze and interpret the findings from this study in an attempt to put forward detailed understandings and new knowledge about the impact of the Flipped Classroom model of instruction on 5th grade elementary students in regards to their mathematical understandings as well as the instructional practices that are occurring in these classrooms using the Flipped Classroom model.

Chapter 2 – Literature Review

The research questions asked in this study require a review of the literature that reflects the current research on Flipped Classroom models of instruction, students' procedural and conceptual understanding of fractions and decimals and the development of the National Council of Teachers of Mathematics Mathematics Teaching Practices (NCTM, 2014).

- To what extent does the observed model of Flipped Classroom instruction align with the NCTM Principles to Action in five of the eight Mathematics Teaching Practices for high quality mathematics instruction in four 5th grade classrooms?
- How is student achievement on the decimal and fraction units affected by the model of Flipped Classroom instruction in this study? (Procedural and conceptual)
- To what extent is there a relationship between student achievement and student attitudes towards mathematics in the Flipped Classroom model of instruction?

This literature review will support what is known in these areas of study in order to create a specific context for the findings in this study. Further, an understanding of the framework for the NCTM Mathematics Teaching Practices will assist in connecting what is known about high-quality mathematics teaching pedagogy to the use of the Flipped Classroom model of instruction and the pedagogy and student understanding outcomes that occurred in the classrooms in this study.

Research on the Flipped Classroom

A review of the research on the Flipped Classroom or Flipped Learning shows two distinct types of research. After completing a thorough search of the literature a very small number of studies have been published in peer reviewed journals or as conference proceedings (Bishop & Verlager, 2013; Hamden, McKnight, McKnight & Arfstrom, 2013). While this may seem concerning it is important to keep in mind that the type of Flipped Learning models studied in this project have only been possible in the past seven to ten years due to advancements and availability of technology. This does not provide a large window of time for scholarly research to be completed and published. The second and largest body of research has been done by teachers and professors as self-studies and has been reported at conferences, in practioner journals and self-published books and blogs.

Peer-reviewed published studies conducted at the post-secondary level dominate the body of research on the Flipped Classroom. The content areas in which the studies were conducted vary from computer courses that teach Excel or information system spreadsheets (Frydenberg, 2013; Davis, Dean, & Ball, 2013) to computer science classes for engineering students (Foertsch, Moses, Strikwerda &Litzkow, 2002; Day & Foley, 2006) and introductory statistics courses (Strayer, 2012). The specific details of how the Flipped instruction model is enacted varied; however each study consistently used some sort of video component as the "homework" portion of the course which delivered the lecture that would have been traditionally done in the classroom.

Two areas of focus were found in the findings of these studies; student attitudes toward the course or materials, and student achievement. Both areas found mixed results. Strayer (2012) conducted a study comparing an inverted (Flipped) introductory statistics course to a traditional introductory statistics course at the post-secondary level. He found that students felt a disconnect between what was viewed online outside of class and the activities in class. He attributed this to the course being a beginning level course in which the students may not have developed a strong interest in the subject matter. As the teacher of both courses, he also noted that the students in the inverted class became more open to cooperative learning activities and innovative teaching strategies. On the other hand, an increase in motivation and an overall feeling that the lecture portion of the course was useful was found to be a theme in several other studies (Foertsch et. al, 2002; Frydenberg, 2013). Significant student achievement gains were documented by two studies (Day and Foley, 2006; Pierce & Fox, 2012) one in a foundational pharmaceutics course and the other in a human-computer interaction course. Interestingly, the other studies encouraged future research into student achievement but they did not comment on achievement levels in the classes that were studied or it was stated that the achievement levels hadn't changed from previous courses.

The research designs in all of the peer-reviewed studies were very similar. The researcher and their team members were the instructors of the course and developed the videos and syllabus for the course. If the study was a comparison type of case study, another professor in the department frequently taught the same course in the traditional manner while the researcher taught the "flipped" course. Generally, achievement data was taken from course final exams and the attitude data was collected from course evaluations. The course design either used the class time to do the practice exercises that would have been traditionally assigned as homework or it was used to extend lab activity time in the science courses.

In a single study by Papadopoulos and Roman (2010), in an undergraduate electrical engineering course, a pretest and posttest were administered which did show a statistically significant difference in achievement by those students in the flipped classroom compared to the traditional course. The pretest data showed no significant difference between the scores (18.3% and 17.1% correct respectively) in the two courses however the posttest showed that the students in the flipped course outperformed the students in the traditional course (31.2% and 24.1% correct respectively).

While the peer-reviewed studies are currently at the post-secondary level, teacher self-studies have been primarily done at the secondary level and specifically in science courses. A Google search will again lead to literally hundreds of teachers' stories about their experiences flipping their classrooms. The most highly cited by these teachers, and frequently modeled work comes from Jonathon Bergman and Aaron Sams who are chemistry teachers in Colorado and now active bloggers and members of FLN. Their book, "Flip Your Classroom: Talk to Every Student in Every Class Everyday" (2012) highlights their experiences and what they feel has led to higher levels of student achievement and motivation (Hamden, e.al. 2013).

In addition to those who have seen positive benefits to flipping their classroom there are many cautionary tales told as well. Some teachers have not seen a change in achievement level and noted student frustration with the model (Toto & Ngyen, 2009, Hamden et al. 2013, Strayer, 2012). The teachers noted that the students reported preferring live-in-person lectures over video lectures and that the videos were too long. The majority of the concerns expressed about the Flipped Classroom model can be found on blog posts as those are not generally people who continue with the model or choose to speak at conferences. These concerns include, emphasizing the importance of lectures and homework instead of active learning, unequal access to technology at home to view the videos and possible mediocre quality of videos that the students are viewing (Hamden, et. al., 2013).

These studies cited in this literature review all base the use of the Flipped Classroom model of instruction on the teacher/researcher's desire to create a student-centered environment and to make the most of one-on-one interactions between teachers and students. The idea is to shift the delivery of information from the traditional lecture given by the teacher to learning through individual conversation or interactions between the teacher and the students or between groups of students. "In effect, student-centered learning environments emphasize constructing personal meaning by relating new knowledge to existing conceptions and understandings; technology promotes access to resources and tools that facilitate construction" (Hannafin & Land, pp.170, 1997). Specifically, in the case of the Flipped Classroom model, the video that the students watch the night before class provides the new knowledge or conceptions which can then be unpacked during the class time interactions with the teacher and other students. What actually happens during the class time can vary greatly from students working on the traditional pencil-paper tasks (i.e. Demetry, 2010; Mason, Shuman, & Cook, 2013; Fulton, 2012) or assignments to cooperative learning experiences or problem solving activities requiring critical thinking and teamwork (i.e. Day & Foley, 2006; Lage, et al. 2000). The conclusion made by most of these studies is that the teacher/researcher felt that by using a more student-centered approach, they "gained time" in class to actually talk to students individually or in small groups and therefore had a better understanding of where their students were at academically. This research tends to focus on the positive perspectives of the teacher or the social/emotional reactions of the students. It is important - however absent in the current research - to consider how students are learning and more specifically whether they are gaining rich conceptual understandings or procedural or rote understandings of concepts.

Defining "Developing Conceptual Understanding"

The National Council of Teachers of Mathematics (NCTM) Learning Principle states "Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge" (NCTM, 2000, pp.20). The idea of "meaningful mathematics" is generally connected to the work of Brownell (1935) who wrote extensively on the importance of teaching for understanding or meaning. While a balance of meaning and skill are needed to be successful in mathematics (Brownell, 1956), what it means to truly "understand" needs to be defined. Skemp (1976) defines two types of understanding, relational understanding and instrumental understanding. Relational understanding involves knowing the "why" behind what one is doing whereas instrumental understanding involves knowing the "rules". It is relational understanding that the NCTM Learning Principle (2000) emphasizes so that procedural or instrumental understanding is done with accuracy and purpose.

Understanding relationships in mathematics comes from creating and internalizing mental models and making connections between these mental representations (Hiebert & Carpenter, 1992; Hiebert & Grouws, 2007). "Understanding occurs as representations get connected into increasingly structured and cohesive networks" (Hiebert & Carpenter, 1992, pp.69). These mental models or representations are created over time and through experiences. Jerome Bruner, a cognitive psychologist, and Zoltan Dienes, a mathematician, each developed a three step learning theory, or cycle, in regards to developing conceptual understandings. The first step of each theory rests in the understanding that initial learning begins with a concrete experience that can appear to be unorganized and play like. Bruner refers to this first mode of development as the "enactive" mode (Bruner, 1966). A student must have an opportunity to work with materials in a hands-on situation which in many cases is similar to Dienes first level of his Dynamic Principle which he refers to as the "play stage" (Dienes, 1960). This play stage requires manipulatives that offer a hands-on opportunity to enter into a learning situation that is relatively unstructured. The Constructivity Principle supports these stages in that "construction should always precede analysis" (Post, 1992, pp.10).

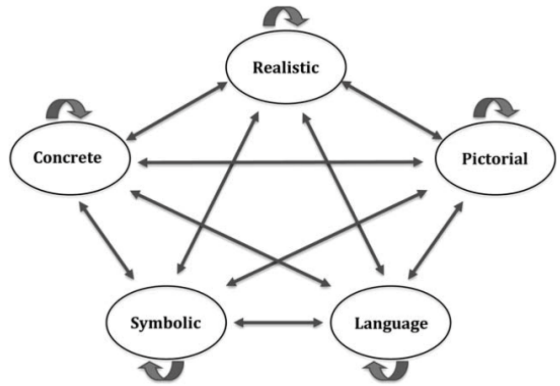
The second stage or level of each learning progression works under the assumption that the unstructured activity is heading toward a more structured view of the concept to be learned. The word structure in this setting refers to the amount of teacher involvement as well as the shape or strength of the concept being developed in the activity. Dienes (1960) refers to this next level of the Dynamic Principle as "structured activities". Manipulatives still play a very important role in the activity however there is a more prescribed purpose in their use. Burner (1966) refers to the second level as the "ikonic" mode. The student is able to organize their thoughts or conceptions from the hands-on mode to a visual or pictorial representation. This mode still requires students to manipulate or create pictures or visuals and communicate their meaning through either written or spoken language.

The final level is the end goal or what Dienes (1960) refers to as the emergence of a mathematical concept. Bruner (1966) addresses this last stage by calling it the "symbolic" mode that in essence is the written or spoken form of an abstract concept that could be communicated to another person. While not identical but very similar, each of these theorists has developed a model of learning that requires a social element and then the physical manipulation of objects which eventually leads to the creation of a structure. This is internalized and finally can be communicated in an abstract or symbolic manner. If students are allowed to operate in a mathematical environment that includes the opportunity to progress through each of the three stages, theoretically they would be able to develop important mathematical structures within their thinking and understanding. Understanding mathematical structure is central to learning and understanding. "Grasping the structure of a subject is understanding it in a way that permits many other things to be related to it meaningfully" (Bruner, 1960, p. 7). When a student understands the structure they are more likely able to transfer their learning to new situations and less likely to forget what they have learned (Bruner, 1960). Dienes also places an emphasis on the structure of mathematics and states, "Mathematics will be regarded as a structure of relationships, the formal symbolism being merely a way of communicating parts of the structure from one person to another" (1960, p. 31).

Dienes' Perceptual Variability Principle and the Mathematical Variability Principle both suggest that conceptual understanding is developed through multiple experiences with different representations (1960). The Perceptual Variability Principle calls for the use of multiple embodiments to maximize learning. In other words the activities should differ in appearance with different materials but possess the same inner goal for structural development. Similarly, the Mathematical Variability Principle suggests that other activities should be incorporated that vary irrelevant variables in a situation but maintain a consistency in the relevant variables. An example given of this principle is that of learning the structure of a parallelogram. The teacher may vary the size or color of the parallelograms but not the concept of opposite sides being parallel (Post, 1992).

Expanding on Bruner's and Dienes work, Lesh, Post and Behr (1987) developed a translation model to illustrate multiple modes of representation. (Figure 1)

.Figure 1. *Lesh Translation Model*



This model illustrates not only what types of learning modes a student needs to experience to develop conceptual understandings but also the translations that a student should be able to demonstrate if they have a conceptual understanding of a topic. These theories of conceptual understanding create a framework to further understand the research on children's thinking around decimals and fractions. This framework guides this research study by identifying effective teaching practices that support student conceptual understanding versus procedural understanding as well as establishing what conceptual understanding looks like through students actions and conversations.

Research on Teaching for Conceptual Understanding of Decimals

Beginning in fourth grade, the Minnesota mathematics standards require students to be able to read and write decimal numbers according to their place value, compare and order decimal numbers and round decimal numbers. By fifth grade students are expected to also add and subtract decimal numbers. These concepts can all be done procedurally but how students come to truly understand the relative size or quantity of decimal numbers develops gradually over time (Hiebert, Wearne & Tabor, 1991). Decimals can be interpreted in two ways; as an extension of the base-10 place value system or as a part-whole model for rational numbers. Because of this, students need many experiences with both interpretations to develop conceptual understandings.

Wearne and Hiebert (1989) studied the development of students' conceptual understanding of decimals by using base-10 blocks to develop the decimal place value interpretation with two fourth grade classrooms. After working with the blocks for multiple lessons students were asked to represent what they built with written symbols. Through student interviews the researchers could establish that the students were in fact using quantitative reasoning to get correct answers versus just following a rule or procedure. By helping the students construct meaning for decimal numbers through the use of the blocks, they were then able to develop procedures for accurately operating with symbolic representations.

Keeping in mind the importance of multiple representations and translations from cognitive learning theories, the Rational Number Project created lessons involving hundreds grids that students used to represent the part-whole interpretation of decimal numbers (Cramer, Behr, Post & Lesh, 1997). In a series of teaching studies one significant finding centered on student language in naming decimals correctly (tenths, hundredths, thousandths). When students were able to connect the name of a decimal to the colored representation on a hundreds grid they were able to grasp the relative size of the decimal and use the image to support their ideas about order and equivalence (Cramer, Monson, Wyberg, Leavitt & Whitney, 2009). Additionally, students were able to use these mental images that they established with their initial work with the hundreds grids to successfully add and subtract decimals in symbolic form. It was noted that after the first teaching study students needed more time to accurately add and subtract decimals symbolically so this was taken into account in later studies (Cramer et al, 2009). Hiebert, Wearne and Tabor (1991) also found similar results in a study with a class of 25 fourth grade students. Through the use of physical representations students were able to gradually create mental images which they used to support their symbolic work with decimals.

In contrast to the research done to establish what supports the development of conceptual understanding of decimals several studies have analyzed the types of misconceptions students have in relation to the relative size of decimal numbers (Resnick, Nesher, Leonard, Magone, Omanson & Peled, 1989 and Steinle & Stacey, 1998 and 2001). By understanding the buggy rules that students apply it is possible to analyze what teaching practices appear to hinder true conceptual understanding of decimals. In Resnick et al.'s (1989) study 113 students in fourth through sixth grade from three different countries were interviewed and asked questions that required them to compare two decimal numbers. Four different student created "rules" were established in connection to student misconceptions; the whole number rule, the zero rule, the fraction rule and apparent experts. In the case of the first two rules, students applied whole number thinking and the notion that a zero is a place holder to decimal numbers. Students who used their knowledge of fractions frequently got answers correct if they demonstrated that they saw decimals as an extension of the base-10 place value system.

For example they would say something like “0.4 is the same as four over 10”. However, if they knew that fractions could be expressed as decimals but did not understand the place value aspect they would do things like making ¾ into 0.3 or 0.34. Those who appeared to be “experts” were students who could get answers correct but would state a rule or procedure with no connection as to why it worked. Steinle and Stacey (1998) replicated this study and broke down each of these rules into specific student actions to give a more detailed view of these misconceptions. By analyzing the various types of misconceptions that students use in comparing fractions it becomes clear that the use of multiple representations of decimals is critical to developing conceptual understandings of decimals.

A recent study by Cramer et al. (in press) using the Rational Number Project decimal curriculum with fourth graders, combines the research on using multiple representations of decimals and decimal misconceptions to create five indicators of conceptual understanding. These indicators are (1) using precise mathematical language when working with decimals, (2) accurately using models to represent decimals, (3) describing how to compose and decompose decimals based on mental images of the models or place value while ordering decimals, (4) using an understanding of the relative size of decimals to guide their estimation with operations with decimals, and (5) using a model and their ability to compose and decompose decimals to interpret addition and subtraction operations and build meaning for work with symbols. Through student interviews, it was determined that students who could be categorized as meeting all five indicators did have strong conceptual understanding while students that only met some or none of the indicators had partial to no conceptual understanding of decimals. Those students who used their mental images and could make connections between those images and symbolic representations were able to accurately operate with decimals. This study will use these indicators to determine decimal conceptual understanding.

Research on Teaching for Conceptual Understanding of Fractions

Children begin exploring fractions at a much earlier age than they do decimals. The Minnesota state math standards at third grade requires that students be able to read, write and order fractions with common denominators using the part-whole construct of fractions as well as identify points on a number line which uses the fractions as a measure construct. By fifth grade students are expected to add and subtract fractions with unlike denominators. Clearly, the success that students will have at this level will be determined to some degree by their prior experiences with fractions and the depth of their conceptual understanding of the part-whole construct and fractions as a measure construct.

Rational number sense, as defined by Lamon (2007), is:

> "[when students] have an intuitive feel for the relatives sizes of rational numbers and the ability to estimate, to think qualitatively and multiplicatively, to solve proportions and to solve problems, to move flexibly between interpretations and representations, to make sense, and to make sound decisions and reasonable judgment" (pp.636).

It is this definition that is the basis for the importance of developing a conceptual understanding of fractions. An initial study done by Behr, Lesh, Post and Silver (1983) used the Rational Number Project curriculum with three groups of fourth and fifth graders and focused on the part-whole construct of fractions. They found that while it is generally understood that students should use manipulatives to build a conceptual understanding of fractions, the ideas that children developed were different depending on what type of manipulative they used. Therefore it is critical that students have many opportunities with different types of manipulatives as they are building understandings and moving from concrete work with fractions to symbolic work with fractions. English and Halford (1995) also observed the difficulties children have as they are developing understandings for the various meanings of fractions and specifically concluded that students need experiences with both continuous area type models (circles or fractions strips) as well as discrete models (chips) in order to form a “cohesive mental model” (pp.143).

Partitioning a whole unit is foundational to understanding fractions. Pothier and Sawada (1983) found that students developed the ability to partition accurately over time and moved from breaking a whole unit into 2 and then continued halvings to working with odd numbers of partitions. Behr and Post (1992) bring out the importance for teachers to recognize the difference between continuous and discrete manipulative models when developing the notion of equal partitions with students. It is one task to fold a paper strip in half and then fourths but a completely different task to find a half of a dozen pencils or a fourth of that same group. This directly links to the importance of using multiple models for instruction.

Comparing and ordering fractions according to their relative size is another key component to developing rational number sense (Behr, Wachsmuth, Post & Lesh, 1984). As a part of another Rational Number Project teaching experiment it was found that students who have a strong conceptual understanding of fractions have four methods for comparing or ordering fractions that do not rely on a procedural skill such as finding common denominators. These four strategies were identified as same numerator, same denominator, transitive and residual. Students using the same numerator strategy talked about the "size of the piece", actually referring to the denominator because they knew they had the same number of pieces. Students who use the same denominator strategy also refer to the "size of the piece" but now it is in relation to the number of pieces that they have which are the same size. The transitive strategy was used when students compared two fractions to a "½" benchmark. That is, comparing whether or not the fraction in question is a little more than a half or a little less than a half. Finally, the residual strategy was used when the fractions were being compared to a whole unit. Students would talk about the size of the piece missing which would get them to a whole. Using this information, or mental image, the student was able to determine that the smaller the missing piece the larger the fraction. In a large study involving 1600 fourth and fifth graders comparing the Rational Number Project (RNP) curriculum to a commercial curriculum, Cramer, Post and delMas (2002) found that students who had developed a conceptual understanding of fractions through their experience with the RNP curriculum "had statistically higher mean scores on the posttest and retention test" (pp. 111) in the areas of concepts, order, transfer and estimation compared to students who were in classes using a commercial curriculum. The students, through extensive exposure to a variety of models, were able to develop mental images and construct meaning from their experiences to employ the earlier noted strategies to successfully compare and order fractions.

Eventually students need to be able to add and subtract fractions in the symbolic form. Accurately being able to estimate sums and differences of fractions is essential to reasoning about the procedure to add and subtract fractions as well as questioning the validity of the answer. Students who jump to converting fractions to those with common denominators, in their head, are following a procedure and likely have not developed conceptual strategies for estimating the sum or difference of two fractions. Cramer and Wyberg (2007) found that students who had developed strategies to order fractions or could estimate which fraction was larger, could not necessarily estimate the sum or difference of two fractions. The students were in a school district using a standards-based NSF funded curricula and took both a written test and participated in one-on-one interviews with the researchers. They found that depending on the strategies the student used, determined whether they were successful with the estimating tasks. Students could use whole number thinking strategies and possibly get the answer correct on a written test for ordering fractions (i.e. ¾ is bigger than 2/3 because the numbers are bigger) but then get answers to estimating sums and differences wrong on the same test. It would appear that the student did have a conceptual understanding of the size of the fractions but were not able to use that knowledge in an estimating task. This type of whole number thinking as a strategy is common (Mack, 1990; Ball, 1990) and can lead to very mixed-up understandings of the relative size of fractions. The reality is, that without knowing what the student is thinking we don't know if they truly have a conceptual understanding of fractions. This study emphasizes the importance of eliciting student thinking to get at their understandings and misconceptions. From there, teachers can develop future activities to support further conceptual development. Students need mental images created from their experiences with various fraction models to help them accurately estimate not only the relative size of fractions but also the sum or difference of fractions.

Cramer and Whitney (2010) suggest that because of the complexity of rational numbers, building fraction number sense early is essential. Students need a variety of models and contexts to develop a clear understanding of the relative size of fractions and fraction equivalence. Estimation skills will support conceptual understanding and should be developed prior to formal symbolic work with fractions in operations. Based on these suggestions and the other research previously cited, this study will use the following indicators for fraction conceptual understanding:

1. Understand the relative size of fractions and be able to compare them by describing mental images based on models. (Rational Number Sense)
2. Use the understanding of the relative size of fractions to accurately estimate when doing operations with fractions. (Student is able to explain with mental images or use multiple representations to explain how they estimated)
3. Use symbolic representation to compute with fractions and be able to explain the process with mental images or models.

The National Council of Teachers of Mathematics Principles and Mathematics Teaching Practices

"Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction" (NCTM, 2000 p. 3). It is this vision that is driving innovation in instructional design and curriculum. The six NCTM Principles (Equity, Curriculum, Teaching, Learning, Assessment and Technology) are the foundation to high-quality mathematics program regardless of the vehicle being used to deliver instruction: "The Principles should be useful as perspectives on which educators can base decisions that affect school mathematics" (NCTM, 2000, p. 7).

The Equity Principle states, "Excellence in mathematics education requires equity – high expectations and strong support for all students" (NCTM, 2000, p. 12). All students should have access to a highly qualified teacher, a challenging mathematics curriculum and the support that they need to be successful (NCTM, 2000). How this principle is enacted will depend on the needs of the students and will require "accommodating differences to help everyone learn mathematics" (NCTM, 2000, p. 13)

"Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (NCTM, 2000, p. 16). The Teaching Principle addresses the knowledge of the teacher, both content and pedagogy, as well as understanding how students learn and the environment that makes learning possible (NCTM, 2000). While it is clear that there are many forms of effective teaching, "effective teachers recognize that the decisions they make shape students' mathematical dispositions and can create rich settings for learning" (NCTM, 2000, p. 18).

The Learning Principle is student focused and emphasizes that, “Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge” (NCTM, 2000, p. 20). It is critical that students learn mathematics with understanding (NCTM, 2000). This understanding is established through challenging tasks and interactive discourses that revolve around problem solving and interaction with classmates and teachers (NCTM, 2000).

The Technology Principle identifies that, “Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students’ learning” (NCTM, 2000, p. 24). Technology can support inquiry in all areas of mathematics and support students in their reasoning and problem solving (NCTM, 2000). “Technology can help teachers connect the development of skills and procedures to the more general development of mathematical understanding” (NCTM, 2000, p. 26).

The remaining two Principles, Assessment and Curriculum, are critical to a successful mathematics program but are not necessarily major components of the Flipped Classroom model of instruction. This can be observed by analyzing the wide range of content areas that are currently employing this model and the wide variety of assessment techniques used within each content area. The classrooms in this study are using the same curriculum and assessment as all other fifth grade classrooms in the school district.

To bring these principles to life, the NCTM recently published a document entitled, “Principles to Actions: Ensuring Mathematical Success for All” (2014). It is the Eight Mathematics Teaching Practices in this document that creates the framework “for strengthening the teaching and learning of mathematics” (2014) and thus the framework for this study. The research-informed effective practices outlined in the document are not only illustrated through the teacher's actions but also through the student's actions. The Flipped Classroom model for instruction grounds itself in the importance of student-centered learning therefore it is important to pay attention to both the student and the teacher actions. "These Eight Mathematics Teaching Practices represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics" (NCTM, 2014, pp. 9).

Mathematics Teaching Practices
Establish mathematics goals to focus learning. Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.
Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.
Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.
Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.
Pose purposeful questions. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.
Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.
Support productive struggle in learning mathematics. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.
Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

(NCTM, 2014, pp.10)

The specific teacher actions and student actions for each practice can be found in Appendix A. How these actions are taken up by teachers and in turn their impact on students is what meant by high quality instruction for all students (NCTM, 2000).

Conclusion

The research base for the Flipped Classroom model of instruction is small and focused only at the secondary and post-secondary level (Hamden, et. al, 2013). It is important to understand what is taking place at these levels of education because that is what is being replicated at the elementary level (Ingram et. al., 2014). Elementary teachers are looking at how the Flipped Classroom model is enacted at the secondary and post-secondary level and recreating that in their own classrooms without purposefully considering the unique needs of elementary age students. Teachers and school district administrators are making instructional decisions based on limited research and in many cases with no regard for what is known about teaching and learning, specifically in mathematics. The importance of student-centered learning is stressed by many Flipped Classroom studies but to what degree that actually happens is not consistent or necessarily content and context driven. How research based effective teaching practices are or are not used in the Flipped Classroom model and what types of student learning are occurring have not been specifically researched, particularly in mathematics.

The importance of conceptual understanding in mathematics is a central tenant of the National Council of Teachers of Mathematics Principles and Standards (2000). What conceptual understanding is and how it can be developed, specifically in mathematics, becomes important to understand as we look at models of instruction. The importance of all students receiving high quality instruction is central to creating innovative instructional models that support students in developing conceptual understandings.

This study addresses the need for research on a Flipped Classroom model of instruction at the elementary level. By using a participant observer perspective on what is happening in the Flipped Classrooms, this study will be able to view the impact of this model of instruction through the lens of effective mathematics teaching practices, specifically in the areas of decimals and fractions. An understanding of what these practices are and the theories that they are based on will aid in the interpretation of what is happening and how this Flipped model of instruction is impacting students.

Chapter 3 – Research Methodology

This study was designed to examine what teaching practices were existing in elementary classrooms using a Flipped Classroom instructional model and how these practices were affecting the attitude towards and achievement in mathematics of the students in these classrooms. This study took place during two fifth grade curriculum units of instruction, decimals and fractions, which occurred over eight weeks of time in four classrooms. The classroom teachers taught all of the lessons and administered all the assessments that included the curriculum posttests and an attitude survey. The posttests covered the mathematics content for each unit and were developed by the curriculum authors. The attitude survey measured students' feelings toward mathematics, working in groups and the use of technology. This study is unique to the current body of research on the Flipped Classroom in that all the other published work put the researcher in the role of the teacher while in this study the researcher is an outside observer. Based on the existing body of research on the Flipped Classroom and teaching for conceptual understanding of decimals and fractions the following research questions were addressed.

1. To what extent does the observed model of Flipped Classroom instruction align with the NCTM Principles to Action in five of the eight Mathematics Teaching Practices for high quality mathematics instruction in four 5th grade classrooms?

    a. Implement tasks that promote reasoning and problem solving
    b. Use and connect mathematical representations
    c. Facilitate meaningful mathematical discourse
    d. Build procedural fluency from conceptual understanding
    e. Elicit and use evidence of student thinking

2. How is student achievement on the decimal and fraction units affected by the model of Flipped Classroom instruction used in this study?

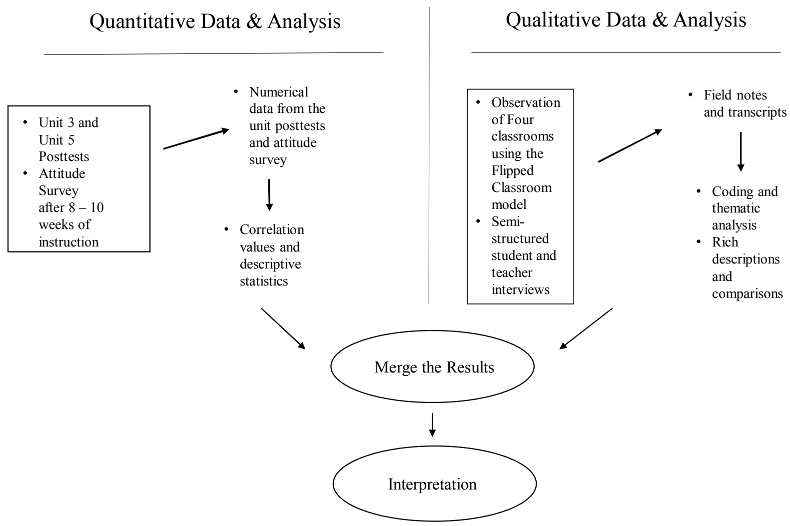
    a. To what extent do the students meet the MN State Standards for decimal and fraction concepts as measured by the curriculum post unit tests?
    b. To what extent do student understandings reflect conceptual knowledge of decimals and fractions based on research on student thinking in the areas of decimals and fractions?

3. To what extent is there a relationship between student achievement and student attitudes towards mathematics in the Flipped Classroom model of instruction?

Research Design

Figure 2 is an overview of how this study used a convergent concurrent mixed method design (Creswell & Plano Clark, 2011) to understand the impact of the Flipped Classroom model of instruction on students and to describe the teaching practices in these flipped classrooms. Specific procedures are identified with the product that was produced from both the quantitative and qualitative data. Each type of data was analyzed separately and then merged for both complementarity and completeness purposes (Happ, Dabbs, Tate, Hrlick & Erlin, 2006).

Figure 2. *Overview of Research Design*



In order to document the actions of the teachers and students, classroom observations recorded as field notes were completed during 32 class periods. Post-tests were given to all students after the completion of each unit and an attitude survey was completed by each student at the end of the second unit. Additionally, student interviews were conducted with 20 students at the end of each unit (40 students total) that addressed both their experiences in the Flipped Classroom and their conceptual understandings of the content from each unit. Teacher interviews were also conducted at the conclusion of the study to gain the teacher's perspective on the flipped Classroom model. This pragmatic approach of combining both qualitative and quantitative data to answer the research questions allows for rich descriptions to be developed of what is taking place in the Flipped classrooms in this study. This approach "attempts to provide evidence that meets the epistemological standard of what John Dewey called warranted assertability" (Johnson & Christensen, 2012, pp. 432). The data generated from the themes found in the classroom observations and student interviews is put in concert with quantifiable data such as activity frequencies, unit test scores and survey results to establish an accurate picture of what is happening in these classrooms using the Flipped Classroom model of instruction and the impact that this model is having on student attitudes as well as their mathematical understandings and achievement.

Participants

This study took place in a suburban school district outside of a large Midwestern metropolitan area. This district began its fourth year of Flipped Classroom mathematics instruction at the fourth and fifth grade levels. The district had participated in a broad study of the use of the Flipped Classroom model of instruction the previous year with the CAREI Center at the University of Minnesota (Ingram, Wiley, Miller & Wyberg, 2014) and agreed to this study for the purposes of creating a deeper understanding of how the Flipped Classroom model is impacting students in the district. Four classrooms of fifth grade students, selected by the district Technology Facilitator participated in this study. The teachers of the classrooms involved in this study were recruited based on the Facilitator's knowledge of their interest in the Flipped Classroom and their previous experiences teaching within this model. This purposeful sampling has provided "information-rich cases for study in depth" (Patton, 2002, p. 230). The schools' demographics are relatively different from each other although they are in the same district. Because the Flipped Classroom model is used throughout the district this may allow for a broader understanding of the impact this model of instruction has on students.

The demographics of the students in the study from the two schools as well as the district are shown in Table 1.

Table 1. *Demographics of School Enrollment*

Subgroup	School District N = 8,800	Southside Elementary N = 88	Central Elementary N = 29
Amer. Indian/Alaskan	.7%	0.8%	0.5%
Asian/Pacific Islander	5.2%	9.5%	7.2%
Hispanic	3.4%	9.5%	1%
Black, not of Hispanic origin	4.4%	13.8%	1.5%
White, not of Hispanic origin	86.2%	66.3%	89.7%
ELL	2.0%	13%	1.7%
Special Ed.	13.9%	13.6%	14.3%
Free/Reduced Lunch	16.5%	30.3%	5.2%

Southside Elementary

Three classrooms of 5th grade students and their teachers from Southside Elementary participated in this study. There were 88 students from this school in the study. Of the two schools in this study, Southside is more diverse than the other school and more diverse than the district as a whole. All students participated in the math classes with the exception of three students with significant learning disabilities who received their mathematics instruction outside of the main classroom. All three classrooms were a heterogeneous mix of abilities for all subjects. Students received their math instruction from their homeroom teacher during the math period. Within each classroom students were split into two groups based on their pretest scores prior to each math unit. Students who passed the pretest at a 90% level worked independently in an alternative curriculum. All other students used the district adopted curriculum, Math Expressions. The videos that students watched were available through the district Moodle site and were made by both Southside teachers and teachers from across the district.

Central Elementary

One classroom of 29 fifth grade students along with their teacher, participated in the study from Central Elementary. The 5th grade at this school uses a block schedule so all the 5th graders received their math instruction from the same teacher each day and then rotated to other teachers for the other subjects. Due to time constraints of the researcher, only one section was observed for this study. This school and class population was much more homogeneous than the other school and the district with the exception of the number of Special Education students serviced. All students were taught by the 5th grade math teacher except two that received their math instruction outside the mainstream classroom. Similar to Southside Elementary, the videos that the students watched were available on the district Moodle site and were made by the Central Elementary math teacher and other teachers across the district. Summary information on the teachers' backgrounds is provided in Table 2.

Table 2. *Teachers and their Teaching Experience*

Teacher	School	Years Teaching	Years "Flipping"
Kim Anderson	Southside Elementary	14 years	4 years*
Tracy Williams	Southside Elementary	8 years	2 years
Liz Hansen	Southside Elementary	18 years	4 years*
Julie Meyer	Central Elementary	17 years	3 years

(*) Teacher was one of the original members of the team of teachers using the Flipped Classroom model in the school district.

Flipped Classroom Model

As with all instructional models, there can be a variety of ways that the Flipped Classroom model is enacted. The formal definition of a Flipped Classroom used in this study states that a Flipped Classroom or Flipped Learning is a,

> "a pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter" (FLN, 2014).

The classrooms in this study use what could be considered a "traditional" model for flipping their math classes. That is, the students watch a video at home for homework which would have been the in-class lecture and then during the math class in school they work out of the homework workbook provided by the curriculum. In this sense, the students are literally doing their traditional homework in school. With that said, there is an added component in that based on the unit pretest, students who score 90% or higher work out of different text book independently and watch the videos created by the textbook publisher. The start of each class period may begin with warm-up problems or a review lecture but not every day necessarily. Additionally, the teacher may pull a small group of students who need more support based on quiz scores or the teacher's knowledge about the students. Again, this does not happen every day. In alignment with the definition of the Flipped Classroom model, the teacher moves around the room while the students are working to offer individual support as needed.

The videos that the students watch are created by various teachers around the school district and stored on a Moodle site (See Appendix G). They vary in length but typically are approximately 10 minutes long and correlate to the lessons in the district curriculum. On the video, the students hear the teacher's voice as they watch him or her work through problems on the screen, similar to Khan Academy videos. There may be a drawing of a fraction bar to support the conceptual understanding of the relative size of a fraction followed up with the step by step procedure of how to make common denominators between two fractions in order to add or subtract them. Several example problems are generally provided that model the procedure repeatedly. At the end of the video is a short quiz on the content that was just on the video. Typical questions on the quiz would be, "Turn these mixed numbers into improper fractions:
3 2/5, 2 1/4, 2 2/3, and 4 1/6". The score the student receives is available to the teacher immediately through a link on the Moodle site. Generally, students are expected to take notes in a notebook during the video and show their work for the quiz questions.

Curriculum

The school district adopted the Math Expressions (ME) curriculum published by Houghton Mifflin Harcourt (2009) in 2009 and they are now using the 2011 edition. This curriculum was developed by The Children's Math Worlds Research Project and was directed and authored by Dr. Karen C. Fuson (Fuson, 2011, pp. xix). The curriculum was developed from previous research that was funded in part by several grants from the National Science Foundation.

The philosophy of this curriculum states,

> "Math Expressions incorporates the best practices of traditional and reform mathematics curricula. The program strikes a balance between promoting children's natural solution methods and introducing effective procedures" (Teachers Edition, 2011, pp. xix).

The curriculum is based on 10 years of research conducted by Dr. Fuson. The curriculum's intent is to develop conceptual understandings and problem solving skills through strategies using various representations to develop children's natural solution methods and then progress quickly to efficient strategies for solving problems (Fuson, 2011, pp. xix). Students are expected to work with partners or small groups to develop communication skills and increase student understandings. The lessons are sequenced based on research on effective learning progressions (Fuson, 2011, pp. xix).

This is the main curriculum that the teachers in the classrooms in this study used on a daily basis. The students worked out of a Student Activity Workbook (Vol. 1) and a Homework and Remembering Workbook (Vol. 1) during this study. Unit 3: Addition and Subtraction of Whole Numbers and Decimals as well as Unit 5: Addition and Subtraction of Fractions were the two units of study during this research project. Each unit included 22 and 21 lessons respectively along with quizzes dispersed throughout the unit and a final assessment at the end of the unit. The questions on the final assessment have been aligned to the Minnesota State standards (Appendix D).

Unit 3: Addition and Subtraction of Whole Numbers and Decimals, uses bar models, hundreds grids and money as models for conceptual understanding of decimals in the first three lessons in the Student Activity book. The next 19 lessons are application and practice in adding, subtracting and rounding whole numbers and decimals. The models shown in the first three lessons are not present in the rest of the lessons. The only model shown in the Homework and Remembering book is in the first lesson of chapter 3 and it is a chart showing money equivalences to decimals (i.e. ones = $1.00, tenths = dimes, hundredths = pennies). Students in this study primarily worked out of the Homework and Remembering book in all the classrooms for this unit.

Unit 5: Adding and Subtracting Fractions, uses fraction bars and number lines as models for developing conceptual understandings of fractions in the Student Activity book. The Homework and Remembering book also included fraction circles as a model in three lessons in addition to the fraction bars and number lines which each occurred in three lessons. Sometimes these models were used in combination so a total of six out of the 21 lessons used one or more of these three models. Again, the students in this study primarily worked out of the Homework and Remembering book. For both Unit 3 and Unit 5, the Teacher's Edition suggests the use of a manipulative called a

"MathBoard" which the students can put various fraction pieces, number lines or decimal cards on and manipulate to compare, order and add or subtract fractions and decimals. This component of the curriculum was not observed in the classrooms during this study. A summary of each unit and the corresponding Minnesota State Standards is in Table 3.

Table 3. Minnesota 5th Grade State Standards for Fractions and Decimals

Minnesota 5th Grade Number and Operation Academic Standards & Benchmarks	Unit 3 Lesson Correlation*	Unit 5 Lesson Correlation*
5.1.2.1 Read and Write Decimals using place value to describe decimals in terms of groups from millionths to millions.	3.1 – 3.3, 3.5, 3.15	5.18
5.1.2.3 Order fractions and decimals, including mixed numbers and improper fractions, and locate on a number line.	3.2, 3.3	5.2, 5.11, 5.13, 5.14, 5.18, 5.19
5.1.2.4 Recognize and generate equivalent decimals, fractions, mixed numbers, and improper fractions		5.11 – 5.14, 5.16, 5.18, 5.19, 5.21
5.1.2.5 Round numbers to the nearest 0.1, 0.01, 0.001	3.15	
5.1.3.1 Add and subtract decimals and fractions, using efficient and generalizable procedures, including standard algorithms.	3.4, 3.7 – 3.9	5.1, 5.3 – 5.10, 5.14 – 5.17, 5.19 – 5.21
5.1.3.2 Model addition and subtraction of fractions and decimals using a variety of representations.	3.4, 3.7, 3.9	5.1, 5.3 – 5.7, 5.9, 5.14, 5.17, 5.19
5.1.3.3 Estimate sums and differences of decimals and fractions to assess the reasonableness of results		5.20
5.1.3.4 Solve real-world and mathematical problems requiring addition and subtraction of decimals, fractions and mixed numbers, including those involving measurement, geometry and data.	3.4, 3.8, 3.9, 3.21	5.1, 5.3 – 5.5, 5.9 – 5.11, 5.14, 5.19, 5.21

*Math Expressions Teachers Edition (2011).

Students who passed the unit pretests with a 90% or higher score were given an alternative textbook to work from while the rest of the class was using the Math Expressions. These students are included in this study because they received the same instruction from the teacher and took the same posttests and attitude survey as the other students in their class even through their independent work was from a different textbook. The textbook was Mathematics Course 1 published by Holt McDougal (2010) and is designed to be used at the sixth grade level. This curriculum was developed to emphasize conceptual understanding, focus on critical thinking and reasoning and integrate mathematical modeling (www.hmhco.com). Each unit includes nine or ten lessons and the textbook looks like a traditional middle school math book in which the students worked on 35 to 50 exercises a day in their notebook. Interspersed throughout the chapters were "Hand On Labs" which incorporated hundreds grids, number lines, bar models and area models to develop a conceptual understanding but these activities were not observed as occurring during this study. Students, identified by their teachers, in the "Red Book Group" worked on Chapter 3: Decimals and Chapter 5: Fraction Operations in the text book and also did the quizzes and final unit tests from Math Expressions with the rest of their class. If students completed all the work in both chapters they would have gone beyond the Minnesota fifth grade standards and worked on material covered in the sixth grade standards for decimals and fractions.

Data Collection and Analysis Procedures

The data collection and analysis methods for this study will be explained as it applies to each of the research questions.

RQ #1: *To what extent does the observed model of Flipped Classroom instruction align with the NCTM Principles to Action in five of the eight Mathematics Teaching Practices for high quality mathematics instruction in four 5th grade classrooms?*

a. *Implement tasks that promote reasoning and problem solving*
b. *Use and connect mathematical representations*
c. *Facilitate meaningful mathematical discourse*
d. *Build procedural fluency from conceptual understanding*
e. *Elicit and use evidence of student thinking*

Qualitative Data Collection and Analysis

Classroom observations and field notes were documented one to two times a week in each classroom for a total of 32 lesson observations. The field notes consisted of descriptions of classroom activities and the amount of time dedicated to each activity (See Table 5 in Chapter 4). Student and teacher interactions were also recorded along with specific conversations when possible. In addition, student on-task behavior was counted in ten minute increments and recorded as a percentage of the class on-task behavior throughout each class period. On-task was defined as what is observable; looking at the speaker, following the directions of the teacher, following the directions of the activity etc. As the researcher, I served as a participant observer while the regular classroom teachers maintained their traditional role in delivering the instruction. This allowed me to "employ multiple and overlapping data collection strategies: being fully engaged in experiencing the setting while at the same time observing and talking with other participants about whatever is going on" (Patton, 2002, p. 265-266). This role allowed me to focus on the impact that the Flipped Classroom models of instruction have on students' attitudes and achievement during instruction and as well as the classroom activities. Details of the learning activities that occurred in the classrooms were also recorded so that they may or may not be linked to the students' attitudes and achievement in the class as well as to the NCTM Mathematics Teaching Practices (2014).

Action comparison tables (see Appendix A) based on the "Teacher and Student Actions" in Principles to Actions: Ensuring Mathematical Success for All (NCTM, 2014) for each of the five practices to be observed were used to develop codes and themes from the classroom observations. The first cycle of coding used Descriptive Coding to create an inventory of the contents of the field notes (Saldana, 2013). The second cycle of coding narrowed the inventory into categories and patterns of actions. This process supports the development of rich narrative descriptions of what is happening in each classroom as well as create supporting details for the achievement and attitude survey data. These comparison descriptions include student activities during the class periods, teacher activities during the class period and descriptions of student engagement. The time during each class period for each activity was recorded so that the order and duration of the activities can be included in the description. Themes and quotations from the semi-structured interviews were brought into the tables as they converge or diverge with the field note observations. The use of the action comparison tables to organize and analyze the themes that emerge in the classrooms demonstrate the degree of alignment with the NCTM Mathematics Teaching Practices (NCTM, 2014) and will be discussed in Chapter 4 of the study.

RQ #2: *How is student achievement on the decimal and fraction units affected by the Flipped Classroom model of instruction in this study?*

a. *Do the students meet the MN State Standards for decimal and fraction concepts as measured by the post unit tests?*
b. *To what extent do student understandings reflect conceptual knowledge of decimals and fractions based on research on student thinking in the areas of decimals and fractions?*

RQ #3: *To what extent is there a relationship between student achievement and student attitudes towards mathematics in the Flipped Classroom model of instruction?*

Quantitative Data Collection Analysis

Descriptive statistics were created from the posttest achievement data and attitude survey data (see Appendix B and C). The descriptive statistics include frequencies and percentages as well as measures of central tendency. The attitude survey was administered at the end of this study. The Student Attitude Survey (SAS) (Brookstein, Hegedus, Dalton, Moniz & Tapper, 2011) was designed by researchers at the Kaput Center for Research and Innovation in STEM Education at the University of Massachusetts, Dartmouth to "explore students deeply held beliefs about mathematics and learning mathematics" (Brookstein et al. 2011, p.1). (See Appendix B) The attitude survey is made up of 27 questions of which 23 the authors, through item principle components analysis and factor loading, have grouped into four components (Brookstein et al. 2011, pp. 6-7). . The four components identified by the authors of the survey are Deep affect: Positivity towards learning mathematics and school, Working collaboratively and related effect, Working privately and Use of technology. Questions #2 and #7 were rewritten to remove the words “In middle school” due to the fact that it was fifth graders taking the survey. This tool was created using the Principled Assessment Design which helped to support both concurrent and predictive validity of the instrument (Brookstein et al. 2011). In addition, it was compared to the Fennema-Sherman Mathematics Attitude Scale (1976) which has been extensively researched to determine its validity and reliability.

In this study, each of the four component score totals was correlated separately to the post test achievement scores. Bivariate correlations using a Spearman rho correlated coefficient show the strength and the direction of the relationship between each attitude component and the Unit 5 Decimal post test score. The significance values may highlight areas of convergence or divergence between achievement and attitudes. The students were assigned a number which was recorded with their posttest and surveys in order to preserve confidentiality but allow for comparison analysis. The correlation coefficients were calculated using SPSS software. In order to achieve the desired statistical power of 0.8 with a moderate effect size of Cohen[3] of 0.3 and probability level of 0.05 using a two-tailed test, a total sample of population of 90 and group size of 45 is required (Statistical Calculator 3.0). Because this is a correlation analysis the correlation coefficient will be squared (the coefficient of determination). A moderate effect size would be 0.09 or 9% of the variability is accounted for with this method. This study has a total sample size of

112 and a group size of approximately 50 therefore the desired statistical power level will be reached.

Alignment of the Math Expressions unit tests to the Minnesota State Mathematics Standards (Appendix D) was used to determine whether or not students are meeting the Minnesota State Standards for decimal and fraction understanding. The questions on the tests were inclusive of the grade level state standards which represent the content knowledge that the students should have gained. All students in the study took the same unit test after the completion of each unit.

Qualitative Data Collection and Analysis

Data from the semi-structured student interviews (see Appendix F) on conceptual understanding of decimals and fractions were coded by indicators (Appendix E) of conceptual understanding developed through research from the Rational Number Project at the University of Minnesota. Based on this research, each response to each item in the interview has been coded as “Conceptual” or “Procedural”. Total response types were calculated for each question. In addition, data from the semi-structured student interviews regarding attitudes towards mathematics were coded in alignment with the four components from the SAS survey in order to provide a deeper understanding of what may affect student attitudes towards mathematics or the Flipped Classroom model.

Mixed Methods Data Collection and Analysis

In order to compare the results of the quantitative and qualitative data, cross tabulation displays were created. These displays compare the qualitative themes generated by the classroom observations and semi-structured interviews to the quantitative results from the student achievement and SAS survey data. These displays support the analysis by highlighting convergent and discrepant findings as well as serving as tools during the interpretation phase of the study to establish complementarity and completeness. These displays address each research question and their subcomponents as a method to establish validity for the merging of the data to answer the questions in this study (Creswell & Plano Clark, 2011).

The students chosen for the semi-structured interviews represented a group that the classroom teachers identified as "low achieving" and a group that the classroom teachers identified as "high achieving" based on the unit test scores. These stratified groups will "illustrate characteristics of particular subgroups in order to facilitate comparison" (Patton, 2002, p. 244). Four to six students in each subgroup from each classroom participated in the semi-structured interviews. Twenty student interviews (10 high achieving and 10 low achieving students) occurred after each unit for a total of 40 interviews. Questions for the student semi-structured interviews were derived from the SAS survey questions as well as from the curriculum content and prior research findings on the Flipped Classroom model of instruction. Eight questions were linked to the unit posttests in order to determine the depth of conceptual understanding of the mathematics that the students just studied. Questions for the teacher interviews were developed from the field note observations during class sessions and after the posttest assessments have been scored. The interviews with both the students and the teachers were audio recorded and transcribed. The student interviews attempted to capture a more detailed account of what the students are thinking and feeling that may be missed or unclear in the SAS data and the achievement data and specific to the Flipped Classroom model. The teacher interview responses were linked to the observation data and student interview and SAS data to determine the alignment of teacher perceptions to what is actually taking place in their classroom in terms of the instruction and the impact the Flipped Classroom model has on their students.

Summary - Validity and Reliability

During this study, 32 class period observations were completed, 40 student interviews and 4 teacher interviews occurred and results from 112 post tests and attitude surveys were compiled. In order to establish trustworthiness, this quantity of data, and importantly the variety of types of data, were collected in order to develop rich and detailed descriptions of what is happening in these classrooms using a Flipped Classroom model for instruction and what impact this model may have on the students in these classrooms. Using Methods Triangulation as well as Data Triangulation in this study, "areas of convergence increase confidence in the findings. Areas of divergence open windows to better understanding of the multifaceted, complex nature of a phenomenon" (Patton, 2002, pp.559). By collecting both qualitative and quantitative data to address the same questions the validity of the data collection process and the analysis process is strengthened. Both types of data joined in multiple displays attempt to minimize the threats to the validity and creditability of the data analysis (Creswell & Plano Clark, 2011). Chapter 4 details the findings from this data and chapter 5 analyzes and discusses the implications of the results of these findings in relation to the research questions of this study.

Chapter 4 – Results

This study uses a concurrent convergent mixed-method design to study the teaching practices used in a Flipped Classroom model of instruction and the impact this model of instruction has on students' attitudes and achievement in mathematics both procedurally and conceptually. Qualitative data on the teaching practices used in this study as well as student data on procedural and conceptual understandings of the content taught were collected. Quantitative data on student attitudes towards mathematics and their achievement in mathematics based on posttest assessments were also collected from 112 students in the study. The results from both the qualitative and the quantitative data as well as their convergence or divergence are presented in this chapter as they provide evidence to answer each of the research questions in this study.

Qualitative Data Results and Analysis – Research Question #1

Field notes were taken during 32 classroom observations of four fifth grade math classrooms in order to develop an understanding of the routines and teaching practices that are enacted in each classroom using the Flipped Classroom model of instruction in this study. The field notes were coded and themes were developed to make a comparison between the NCTM Mathematics Teaching Practices (2014) and what is actually happening in these classrooms. The results of this data provide evidence to answer the first research question in this study:

1. *To what extent do the observed models of Flipped Classroom instruction align with the NCTM Principles to Action in five of the eight Mathematics*

Teaching Practices for high quality mathematics instruction in four 5th grade classrooms?

a. *Implement tasks that promote reasoning and problem solving*
b. *Use and connect mathematical representations*
c. *Facilitate meaningful mathematical discourse*
d. *Build procedural fluency from conceptual understanding*
e. *Elicit and use evidence of student thinking*

To establish an overview of the daily routines that occur consistently in these classrooms using a Flipped Classroom model of instruction, the field notes were initially coded by "activity type". From this coding, Table 4 outlines the two different types of daily instructional models that occurred during the math periods in all four classrooms.

Table 4. *Two Types of Observed Instructional Models*

Instructional Model A	Instructional Model B
• Students begin the class period with a warm-up or review problems. • Teacher gives a 5 – 10 minute lecture based on the video from the previous night. • Students work on workbook pages individually or with a partner (informal). • Teacher circulates around the room assisting individual students.	• Students work on workbook pages individually or with a partner (informal). • Teacher pulls a small group of students together for a short mini-lesson based on need. • Teacher circulates around the room assisting individual students.

Within both instructional models there could be slight variations if the teacher chose to give a quiz or planned an alternative activity but this only happened on an occasional basis during the lessons that were observed. Consistently, however, it appeared that the classroom teachers primarily relied on the video from the previous night to deliver the main content instruction. When a short lecture or mini-lesson occurred it typically consisted of practice problems that the teacher referenced to the video.

To further understand the details of what was happening within each classroom, the total instructional time over the course of the study was calculated and within that total time, the amount of time dedicated to various specific activities was calculated as a percentage of the total class time. Figure 3 graphically represents this data for each classroom. Below Figure 3 is a detailed description of each activity in Table 5.

Figure 3. *Class Period Activities by Percent of Total Class Time*

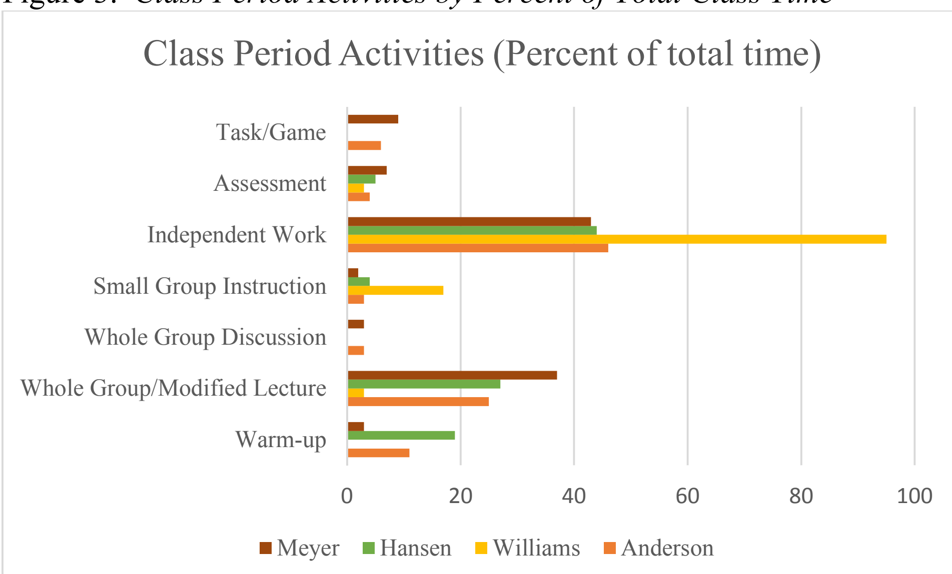


Table 5. *Class Period Activity Descriptions*

Activity	Activity Description
Task/Game	A whole class activity such as a problem solving task, playing a game from the curriculum or playing a game or doing skills practice on a computer.
Assessment	A "quick quiz" from the curriculum.
Independent Work	The time that students are working out of their workbook or textbook.
Small Group Instruction	The teacher purposefully calls 3-8 students together to review specific content.

Whole Group Discussion	A whole class session during which time students are sharing their strategies, offering new strategies, asking questions of each other and the teacher – conversation like.
Whole Group/Modified Lecture	Teacher demonstrates a procedure and sometimes asks procedural questions in an IRE (initiate, respond, evaluate) type dialogue. i.e. “What is 3 x 4?” during the procedure to make common denominators.
Warm-up	Either a commercially made worksheet with one problem from each math strand or several practice problems on the Smart Board connected to the video from the previous night.

From Figure 2, it appears that three of the four teachers; Meyer, Hansen and Anderson, use a relatively similar set of classroom routines. Ms. Williams had a significantly larger amount of time dedicated to independent work and small group instruction and was not observed doing any sort of warm-up activity and a minimal amount of modified lecture. She more frequently than her colleagues, pulled small groups of students to the side for specialized instruction although this did not happen during every observation. In all the classrooms, the small group instruction occurred while the other students in the room were working independently on their workbook or textbook assignments. Because of this, some instructional minutes were counted twice which explains why the total percentage of instructional minutes slightly exceeds 100% for each teacher. Overall, in all the classrooms, there was very little time used for whole group discussion or alternate tasks compared to the amount of time students worked in their workbook or textbook during independent work time.

Within each type of activity that occurred in the classrooms during the observations, specific teacher actions and student actions were noted and coded in relation to the NCTM Mathematics Teaching Practices (2014) in five specific areas;

a. *Implement tasks that promote reasoning and problem solving*
b. *Use and connect mathematical representations*
c. *Facilitate meaningful mathematical discourse*
d. *Build procedural fluency from conceptual understanding*
e. *Elicit and use evidence of student thinking*

How these practices were actually enacted in these classrooms do not necessarily match with the NCTM descriptions of the teacher actions that would demonstrate these practices. Through the coding of the field notes themes emerged which were then linked as closely as possible to each of the NCTM Mathematics Practices to analyze the alignment or misalignment of what was actually observed in relation to the NCTM Mathematics Practices. The following tables illustrate the NCTM (2014) descriptions of teacher actions for each practice in relation to the actual observed teacher and student actions during this study. The NCTM suggested actions in these tables are limited to what could be observed in the classroom. See Appendix A for a complete listing of NCTM suggested actions for each teaching practice. Following each table is a narrative description of the observed actions that are associated with each teaching practice.

NCTM Mathematics Practice A: Implement tasks that promote reasoning and problem solving

Table 6. *Comparison of Observations on NCTM Mathematics Practice A*

Implement tasks that promote reasoning and problem solving	
NCTM Suggested Actions	Observed Actions
• Teachers motivate students' learning through opportunities for exploring and solving problems	• Teachers demonstrated how to solve problems similar to those viewed on the video using a standard procedure or algorithm.

that build and extend their current mathematical understanding. • Teachers select tasks that provide multiple entry points through the use of varied tools and representations • Teachers pose tasks that require a high level of cognitive demand • Teachers encourage students to use varied approaches and strategies	• Teachers assigned pages in the Homework and Remembering workbook that corresponded with the video from the previous night. • Teachers suggested using a chart on the wall or an alternate strategy if a student was unclear how to solve a problem • During the modified lecture time, a teacher sometimes solicited an alternate strategy or explanation.

During each observation the tasks that the students worked on were always from the Math Expressions curriculum or were teacher created problems similar to the problems from the lesson video. If the problems were teacher created or from the hardcover curriculum book, the students would copy them into their notebook to solve. At times the teacher would put the page from the student Activity workbook or the Homework and Remembering workbook on the Smart Board and the whole class would work through the first few problems on the page together. Below is an example of this type of instruction from the field notes:

(From Lesson 5.12 – Equivalent fractions)

> T: Who can tell me what an equivalent fraction is?
> S1: umm, I'm guessing but two fractions with the same denominator?
> T: (calls on another student)
> S2: Two fractions worth the same amount
> T: (writes ½ and 3/6 on the board) These two fractions are equal – they show the same amount. Now we need to find the multiplier – the factor that we are going to multiple both the numerator and the denominator by to get the equivalent fraction. (Teacher writes a small x3 next to the numerator and denominator of ½)
> T: (Writes 5/6 = 10/12 on the board) What do you multiply each number in 5/6 by to get 10/12?
> S: (Chorally) 2
> T: So if you have 15/18 = 5/6 (writes this on the board) what is the divisor?
> S3: 3

*Teacher continues with two more examples this time having the students do this in their notebooks and then check with their neighbors about the multipliers. After a few minutes the teacher calls on a couple of students to give the answers – she writes the answers in on the board.

This type of modified lecture would generally take ten minutes at which point the students would then work on their own or with a partner of their choice on the assigned Homework and Remembering pages.

The NCTM suggests that the tasks teachers pose for their students build or extend on their current mathematical understandings which might appear to be happening in these classrooms if the students are understanding what was on the video. However, the tasks posed in the observed classroom may not be engaging to all students or have multiple entry points. Generally only one strategy or procedure was observed except in the case of subtracting mixed numbers that required borrowing. In this instance, students were told they could make the fractions into improper fractions first and then find the common denominator which eliminated the borrowing issue. Multiple representations or physical representations were typically only observed if they were on the workbook page that was projected on the Smart Board.

NCTM Mathematics Practice B: Use and connect mathematical representations

Table 7. *Comparison of Observations on NCTM Mathematics Practice B*

Use and connect mathematical representations	
NCTM Suggested Actions	Observed Actions
• Teacher selects tasks that allow students to decide which representations to use • Teachers allocate substantial instruction time for students to use, discuss and make connections between representations	• Teacher models using a pictorial representation when it is on the workbook page. • Teacher points towards a poster on the wall that has equivalent fraction bars on it.

• Teachers ask students to make drawings to justify or explain their reasoning • Teachers focus students' attention on the structure of the mathematical idea regardless of the representation	• Teacher suggests thinking about a pizza or a candy bar to help solve a problem. • Student draws a picture to explain an answer to a question.

The observed use of mathematical representations in this study was restricted to the pictorial representations (fraction circles, fraction bars or number lines) in the curriculum workbooks or to suggested mental images by the teacher (pizza or candy bars). Students were not observed using hands-on manipulatives such as fraction circle pieces or paper strips nor were they observed making connections between any type of representation. In a few cases, during the semi-structured interviews which will be discussed in detail later in this chapter, students drew pictures to explain why they answered a question a certain way but this was not a consistent action by all students. In several instances, documented in the field notes, students were unsure how to draw a pictorial representation and were told by their teacher to "skip that part" as long as they knew the answer to the question. The NCTM suggested actions imply that students should have many experiences with many different types of representations and should make connections (or translations, Lesh et al. 1987) between them. These types of actions were not observed in any of the classrooms during this study. The representations used in these classrooms were in pictorial form provided in the curriculum workbook and were typically used with the first couple questions on the page.

NCTM Mathematics Practice C: Facilitate meaningful mathematical discourse

Table 8. *Comparison of Observations on NCTM Mathematics Practice C.*

Facilitate meaningful mathematical discourse	
NCTM Suggested Actions	Observed Actions
• Teachers engage students in purposeful sharing of ideas, reasoning and approaches using varied representations. • Teachers select and sequence student approaches for whole class analysis and discussion. • Teachers facilitate discourse among students by positioning them as authors of ideas who can explain and defend their approaches. • Teachers ensure progress toward mathematical goals by making explicit connections to student approaches and reasoning	• Teacher solicits, from students, several procedures to solve a problem. • Teachers use the "Turn and talk" protocol to have students share an answer to a question with their neighbor. • Students play the role of teacher demonstrating a procedure for the class

Observations of classroom discourse took on several forms in the classrooms in the study. Most frequently students were asked to provide the steps in a procedure or an alternate way to solve a problem and then would walk the class through their steps in a "show and tell" manner. The second form of discourse occurred when the students were told to "turn and talk" to their neighbor about what they thought the answer should be or how they solved a problem. In one instance there was an interesting dialogue between several students out loud during the whole group instruction time:

(The class is looking at a bar graph showing various insect sizes)

T: Explain to your neighbor how to find the difference in heights between the insects.

S. (Talk with each other – some pointing at the graph)

T: About how many times bigger is the June Bug compared to the Fire Fly?

S1: About 1 and a half

T: Can you explain why you think that?

S1: ummm….I might be wrong
S2: It should be 1 – I found it by subtracting
S3: I think it is 2 times bigger but I don't know why
S4: Well, if you add 1.3 and 1.3 that is 2.6 which is close to 2.5 so it would be 2 times bigger.

While mathematically the student explanations are not completely correct nor did the teacher follow up to have students explain their thinking further, these students were comfortable sharing their thinking. This was the only observed instance that students facilitated their own conversation.

The NCTM suggested actions encourage the teacher to select, sequence and make connections between student representations and responses. These types of teacher actions were not observed during this study. Students were observed explaining the procedural steps that they followed to obtain a solution and occasionally took on the role of the teacher in front of the class in this process. The "show and tell" method of sharing appears to be the most frequent type of dialogue in these classrooms.

NCTM Mathematics Practice D: Build procedural fluency from conceptual understanding

Table 9. *Comparison of Observations on NCTM Mathematics Practice D.*

Build procedural fluency from conceptual understanding	
NCTM Suggested Actions	Observed Actions
• Teachers provide students with opportunities to use their own reasoning strategies and methods • Teachers ask students to discuss and explain why the procedures that they are using work • Teachers connect student generated strategies to more efficient procedures	• Students are called on to share the procedure they used to solve the problem • Teachers referenced a poster with multiple fraction bars on it during instruction on equivalent fractions • Teachers referenced "rules" multiple times • Students spent a majority of the class time working in the workbook on practice problems.

• Teachers use visual models to support students' understanding of general methods • Teachers provide students with opportunities for practice of procedures	

The actions of the teachers and students in this study appear to be slightly more aligned to the suggested actions for this NCTM Mathematics Practice than for the other practices. While there were not any observations of students explaining "why" they chose to use the procedure they chose, there were many instances of students following a procedure and reciting the rules for a procedure (i.e. "whatever you do to the top you do to the bottom" or "line up the decimal points"). Teachers repeatedly demonstrated the standard procedure or traditional algorithm and asked students to demonstrate or talk through their own use of these procedures. The students had a significant opportunity for practice due to the amount of time dedicated to independent work and the large number of problems on the workbook and textbook pages.

An important NCTM action that did appear to be missing was the link from a conceptual idea to the standard or efficient procedure. During student interviews, students consistently answered the question, "Why do you think that?" with responses like, "That is the rule" or "Because the video (or teacher) told me to". During the teacher interviews, all four teachers shared that they found the videos important because the student was able to watch the procedure (or steps) multiple times, the Flipped Classroom offers more practice time and most students "can do math if you just tell them the procedure". The idea of moving from a conceptual understanding to a procedural understanding was not an action that the teachers talked about or was observed in the classrooms.

NCTM Mathematics Practice E: Elicit and use evidence of student thinking

Table 10. *Comparison of Observations on NCTM Effective Practice E.*

Elicit and use evidence of student thinking	
NCTM Suggested Actions	Observed Actions
• Teachers elicit and gather evidence of student understanding • Teachers make in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold and extend • Teachers reflect on evidence of student learning to inform planning • Students reveal their understanding through written work and classroom discourse	• Teachers moved around the room during independent work time assisting students who raised their hand • Teachers pulled small groups of students for focused instruction based on quiz scores or common questions • Teachers called on students during whole group time to share procedures or answers • Teachers used up thumbs-up, thumbs down for understanding as a formative assessment after explaining a procedure

Interestingly, during the teacher interviews all of the teachers made a statement regarding how much better they felt they knew their students using the Flipped Classroom model. When pressed about the meaning of that, responses centered on the idea that because they had time to talk to individual students everyday they felt that they had a better grasp of where the students were academically. While the observed actions only partially align with the NCTM suggested actions it would seem that eliciting student thinking is occurring on a regular basis in these classrooms since the teachers feel they have a better understanding of their students' needs. As was noted in connection with the practices discussed earlier, it was observed that students regularly shared the procedure to answer a question and teachers asked questions to get the students to talk about the steps in a procedure.

The major observed difference between the NCTM suggested actions for eliciting student thinking and the observed actions was the act of making instructional decisions based on student understanding. Every day the next lesson video was listed as homework on the board and the class room lesson the following day matched the content from the video the night before. This was consistent in all four classrooms regardless of the topic or student questions. If students finished their workbook early they were told they could work ahead in the next lesson even though it did not appear that an adult had looked through their work to assess their level of understanding or struggle. The one exception occurred in Ms. Meyer's room at the beginning of unit 5 on fractions. She commented to the students that she was concerned that some students weren't getting their work done so she switched up the classroom routine. Prior to starting the usual independent work she gave a short quiz that the students would bring to her and get corrected immediately. Based on how they did on the quiz determined what type of independent work they did. This could be the regular workbook pages or it could be a remedial or challenge sheet. It was observed that more students seemed to stay on task however it did not change the pace at which the lessons were delivered.

Overall, in all the class rooms, the lessons were done in the order and at the pace dictated by the Math Expressions curriculum and its pacing guide. Students who had gotten 90% or better on the unit pretest were able to work in the advanced textbook but they followed each lesson in order as well with no observed direct instruction. While teachers were observed spending a significant amount of time assisting individual students and in some cases working with small groups, the understandings or lack of understanding elicited by students did not appear to affect instructional decision making in terms of pedagogy or pacing.

Summary

In general, the dominate observed teaching practices in the Flipped Classrooms in this study centered on teaching and learning procedures and individually talking with students. This would be consistent with one of the purposes of the Flipped Classroom in that a teacher is able to “talk to every student every day” (Bergmann & Sams, 2012) because they are not spending time giving a traditional lecture in class. The procedure oriented lecture has been moved to the video which the students watch at home or outside of the math class time. The need for increased practice time of these procedures was mentioned by all of the teachers in this study which was one benefit they felt the Flipped Classroom model provided to students and was clearly observed during all the lessons. There was not an emphasis placed on using multiple representations or facilitating mathematical discourse during the lessons observed. While the importance of developing conceptual understanding is clearly established throughout the NCTM Mathematics practices by using multiple representations, making connections between these representations and procedures and bringing these conceptual understandings out through discussion and problem solving tasks, these actions were not regularly observed in the class rooms in this study.

Quantitative Data Results and Analysis – Research Questions #2

2. How is student achievement on the decimal and fraction units affected by the model of Flipped Classroom instruction?

a. To what extent do the students meet the MN State Standards for decimal and fraction concepts as measured by the curriculum post unit tests?
b. To what extent do student understandings reflect conceptual knowledge of decimals and fractions based on research on student thinking in the areas of decimals and fractions?

Unit posttests were administered by the classroom teachers for both Unit 3: Adding and Subtracting Decimals and Unit 5: Fractions, to all the students in their math class rooms. The questions on the unit posttests were developed by the Math Expressions curriculum and were correlated to the Minnesota State Standards for fifth grade in each of these content areas (see Appendix D) in order to assure that the students were being assessed on the content that was required to be taught by the state of Minnesota. Semi-structured student interviews provided an additional layer of qualitative data on students' conceptual and procedural understandings which will be addressed later in this chapter.

The posttest tests were given to 112 fifth grade students in the classrooms in this study. The Unit 3 decimal written test was made up of 38 questions based on the lessons in the curriculum and the Unit 5 fractions written test had 20 questions. All the teachers in the study used the same test after teaching all the lessons in the unit. The tests were corrected by the classroom teachers and the students' scores were recorded in the form of percent correct. The descriptive statistics for each posttest is in Table 11.

Table 11. *Mean Performance Scores on the Unit 3 and the Unit 5 Posttests*

Posttests	N	Mean (%)	S.D.	Min.	Max.
Unit 3 Posttest - Decimals	112	91.45	6.91	71.00	100.00
Unit 5 Posttest - Fractions	112	81.31	15.86	37.50	100.00

The mean score for the Unit 3 posttest and the Unit 5 posttest show a difference of approximately 10%. What is important to note is the difference in the range of the scores for each test. By looking at the minimum and maximum scores we see that the range for the Unit 3 test is 29 while the range for the Unit 5 test is 62.5. In order to look more closely at how the posttest scores for each unit break out into percentage groups, a frequency table (Table 12) showing the number of students who received a score in each percentage range for each posttest is shown below.

Table 12. *Frequency Table of Student Scores for the Unit 3 and Unit 5 Posttests*

Score	Unit 3 Posttest N = 112	Unit 5 Posttest N = 112
90 - 100%	75	48
80 – 89.9%	30	23
70 – 79.9%	7	19
60 – 69.9%		9
50 – 59.9%		7
40 – 49.9%		2
30 – 39.9%		4

Based on the frequency table it would appear that 105 students, or 94 % of the total number of students, received a score of 80% or higher on the Unit 3 posttest. It would seem likely that these students have demonstrated an understanding of the decimal content from the unit and may have therefore met the Minnesota state standards in the area of decimals.

In looking at the Unit 5 posttest scores, 71 students or 63.4% of the total number of students, received a score of 80% or higher. This appears to be a much lower number of students who likely have met the Minnesota state standards in the area of fractions. The cut-off of 80% was determined to be a practical score in that students receiving a score lower than this clearly have some understanding of the content but also have misconceptions which lead to errors on the test and therefore have likely not completely met the standards for a fifth grader in the content area at this time. By analyzing the results from the student semi-structured interviews involving students' conceptual and procedural understandings of fractions and decimals, a clearer explanation of what may be happening can be developed.

Mixed Methods Data Results and Analysis – Research Question #2

Twenty students were interviewed singularly after each unit posttest for a total of 40 different students in the study. The students were selected by their classroom teacher and represented both students who were considered high achieving and those considered low achieving as determined by their test scores and their teacher. Each student was asked the same set of questions (See Appendix F) and their responses were recorded both on paper and with a digital recorder. The questions were developed based on the research cited in chapter 2 of this study on what it means to have a conceptual understanding of decimals and fractions as well as being similar in type to what was on the Math

Expressions posttests. The student responses were evaluated as being either correct or incorrect and then whether the student demonstrated conceptual understanding of the answer or procedural understanding of the answer from their explanations. Data from each set of interviews will be analyzed separately with examples of typical conceptual and procedural responses included for various question types.

Unit 3 Decimal Interview

The decimal interview consisted of seven questions that required the students to name given decimals, determine their relative size in comparison to other decimals, estimate with decimals and compute with decimals. These are the same skills with decimals that the students were asked to demonstrate on their unit test and are required by the Minnesota state standards at the fifth grade level. Many of the questions were modeled after interview questions in a recent study (Cramer et.al in press) which appeared to show differences in the types of understandings that a student may have about decimals. In order for a student to show conceptual understanding they must make reference to using a mental image or draw a pictorial representation of their answer or give a detailed explanation that goes beyond a rule. A procedural answer typically will be given by explaining a rule or a procedure, not necessarily correctly, without a connection to a representation. Table 13 gives example responses from students, during the interviews, of conceptual and procedural answers for some types of the questions in the decimal interview.

Table 13. *Examples of Conceptual and Procedural Student Interview Responses*

Which decimal is larger 0.103 or 0.13

Correct – Conceptual

- 0.13 because a hundredth is larger than a thousandth and you would have only ten hundredths with the other one and three little thousandths

Correct – Procedural

- 0.13 because if you add a 0 to the end then 130 is larger than 103

Incorrect – Conceptual

- 0.103 because you would color in the whole hundreds grid

Incorrect – Procedural

- 0.103 because it has more numbers in it

Estimate the answer to 0.37 + 0.4

Correct – Conceptual

- Thirty seven hundredths is the same as three tenths and seven hundredths. Seven hundredths is close to another tenth so that would be four tenths and another four tenths would be eight tenths

Correct – Procedural

- The seven is more than 5 so I round up so that would make the 3 into a 4 and four and four is 8 so 8 tenths

Incorrect – Conceptual

- (student drew a 10 x10 grid and colored 41 of the squares)

Incorrect – Procedural

- 41 because I added 37 and 4

Picture 0.57. If you took away 0.009 would your answer be more than a half or less than a half?

Correct – Conceptual

- More than a half because a thousandth is a little tiny piece so that means you wouldn't be taking much away from the fifty seven hundredths which is already more than a half.

Correct – Procedural

- More than a half because I added a 0 to the 0.57 which made it 570 and then just took 9 away from that.

Incorrect – Conceptual

- Less than a half because if I picture 57 squares and cross out 9 of them that would be less than 50 which is a half.

Incorrect – Procedural

- (incorrect arithmetic) More than half because if you line up the decimals and subtract you get 0.579 which is more than half

The responses as correct or incorrect and by type, conceptual or procedural, for each group of students and for each of the decimal interview question are in the tables (Table 14 and 15) below. Below each table is a narrative analysis of the results.

Table 14. *Decimal Interview Responses by Type from Low Achieving Students*

Low Achieving Students (N = 10)				
Questions	Correct	Incorrect	Conceptual	Procedural
07 or 0.4 Which is larger?	9	1	1	9
0.103 or 0.13 Which is larger?	5	5	1	9
Put these decimals in order from Least to greatest: 0.245, 0.025, 0.249, 0.3	5	5	0	10
Estimate 0.37 + 0.4	1	9	0	10
Picture 0.57. If you took 0.009 away Would the amount left be more than a ½ or less than ½?	2	8	0	10
Solve 0.375 + 2.5	9	1	0	10
Solve 4.85 – 0.437	8	2	0	10

This table illustrates the general strategy that the low achieving students used of following a procedure or rule to answer the questions over using a conceptually based thought to answer the questions. In general, the procedures used by this group of students were based on whole number thinking such as, “7 is more than 4” or “the one with more numbers in it is bigger”. The students were able to get correct answers in many situations using this line of thinking. The two questions that required estimation, which had very few correct responses, demonstrated that these student likely do not have a strong conceptual understanding of the relative size of decimal numbers and their use of whole number thinking therefore did not get them to a correct answer. The students were able to compute with decimals correctly because they all stated that they “needed to add zeros to make the numbers the same size” and then they just “lined up the decimals” and added or subtracted “like normal”. In this sense they were able to get many answers correct with whole number solution strategies and questionable conceptual understandings of decimals.

Table 15. *Decimal Interview Responses by Type from High Achieving Students*

High Achieving Students (N = 10)				
Questions	Correct	Incorrect	Conceptual	Procedural
0.7 or 0.4 Which is larger?	10	0	1	9
0.103 or 0.13 Which is larger?	10	0	0	9
Put these decimals in order from Least to greatest: 0.245, 0.025, 0.249, 0.3	10		0	10
Estimate 0.37 + 0.4	8	2	0	10
Picture 0.57. If you took 0.009 away Would the amount left be more than a ½ or less than ½?	7	3	4	6
Solve 0.375 + 2.5	10	0	0	10
Solve 4.85 – 0.437	10	0	0	10

Overall, the high achieving students were able to answer more questions correctly which would be expected. Consistent with the low achieving students, this group relied on a rule or procedure to answer most of the questions and many of their rules were based on whole number thinking as well. Again, the students that answered the two estimating questions correctly tended to state the rule that “if the last number is more than 5 round up” (0.37 rounded to 0.4) and then they added 0.4 and 0.4 in their head for an estimated answer of 0.8. They also talked about visualizing adding a zero to 0.57 to be able to use the algorithm to subtract 0.009 from 0.570 to get their estimate. In this case though, four students were able to talk about the fact that the nine-thousandths was a very tiny amount so they conceptually knew that taking that away from 57 hundredths wouldn’t change the number very much and therefore were able to not only give a correct answer but explain it with conceptual understanding. This entire group of students answered the computation questions the same way as the other group stating, “line up the decimals” and “add zeros” then they just added or subtracted.

Based on the interviews with these 20 students, it appears that they are able to use rules or procedures, frequently based on whole number thinking, to answer questions involving decimals correctly. It would also appear that there is a lack of conceptual understanding of the relative size of decimals based on the number of incorrect answers to the estimating decimals questions. In this case using a rule that involved whole number thinking did not result in a correct answer. These two factors could explain the higher level of achievement, compared to the Unit 5 test on fractions, on the written Unit

3 test on decimals. Even though there is a Minnesota standard regarding decimal estimation:

> 5.1.3.3 Estimate sums and differences of decimals and fractions to assess the reasonableness of results

The Math Expressions curriculum does not include any lessons on estimation nor were there any questions on the Unit 3 posttest involving estimating with decimals. All of the questions on the test could be solved using a rule or procedure and there were not any visual representations of decimal numbers which would have required a conceptually based answer

Unit 5 Fraction Interview

The Unit 5 fraction student interview was made up of nine questions based on previous Rational Number Project studies (Cramer, Post, delMas, 2002; Cramer & Wyberg 2007). The questions were designed to allow students to demonstrate their understanding of the relative size of fractions, compare and order fractions and compute with fractions. Each of these concepts are included in the Minnesota State Standards for fifth grade and were included in the Math Expressions Unit 5 lessons. Twenty students from across the four classrooms participated in the interviews; ten designated as high achieving and ten designated as low achieving based on their test score and teacher knowledge. The student responses were initially evaluated as “correct” or “incorrect” and then coded as a “conceptual” or “procedural” response. Examples of each type of student response, given by students during the interviews, for some of the types of questions are in Table 16.

Table 16. *Examples of Conceptual and Procedural Student Interview Responses*

Put these fractions in order from least to greatest; 1/5, 1/3, ¼

Correct – Conceptual

- If you cut a pizza into 5 pieces each piece is going to be smaller than if you cut it into 4. If you cut it into only 3 pieces you would get the biggest pieces

Incorrect – Conceptual

- 1/5 is the biggest because you would need 4 more pieces to get to a whole and you would need less pieces to get to a whole for ¼ and 1/3

Correct – Procedural

- 1/5, ¼, 1/3 because the bigger the denominator the smaller the fraction

Incorrect – Procedural

- 1/3, ¼, 1/5 because 3 is the smallest and 5 is the biggest

Which is larger 4/5 or 11/12?

Correct – Conceptual

- 11/12 because you need one more piece to get to a whole in each but 1/12 is a lot smaller piece than 1/5 so 11/12 is closer to a whole

Incorrect – Conceptual

- They are the equal because they both need one more to get to a whole

Correct – Procedural

- (student found a common denominator) 55/60 is more than 48/60 so 11/12

Incorrect – Procedural

- (wrong reasoning) 11/12 because the numbers are bigger

Estimate 7/8 + 12/13

Correct – Conceptual

- 2 – each fraction is almost one whole so 1 plus 1 is 2

Incorrect – Conceptual

- 1 – they are each about a whole

Correct – Procedural

- (found common denominators and added first) so now if I round it, it will be 2

Incorrect – Procedural

- It will be 19 over something but I can’t find a common denominator

The responses from the low achieving students are shown below in Table 17. The total number of correct and incorrect responses are shown followed by the total number of conceptual or procedural response were given.

Table 17. *Fraction Interview Responses by Type from Low Achieving Students*

Low Achieving Students (N = 10)				
Questions	Correct	Incorrect	Conceptual	Procedural
Put these fractions in order; 1/5, 1/3, 1/4	6	4	6	4
Which fraction is larger 4/5 or 11/12?	0	10	4	6
Which fraction is smaller 1/20 or 1/17?	5	5	7	3
Are these fractions equal or is one less, 5/12 or ¾?	7	3	3	7
Are these fractions equal or is one less, 6/4 or 6/5?	6	4	2	8
2/5 + ¾ = 5/9 Do you agree?	9	1	0	10
Estimate: 7/8 + 12/13	1	9	1	9
Solve: 2 1/5 + 1 ¾ =	0	10	0	10
Solve: 4 1/8 – 2 2/4 =	0	10	0	10

The low achieving students did demonstrate some conceptual understanding of fractions when they were able to explain their answer using a pizza analogy or a fraction bar analogy. If they did not seem to have these types of mental images they gave responses based on rules or procedures that may or may not have worked. For example, when the fractions were unit fractions (a 1 in the numerator), the rule “the larger the denominator the smaller the piece” worked. However, they tended to not apply this rule or apply it incorrectly when the numerators were larger than one and different from each other. This situation typically resulted in an incorrect answer.

For all of the questions that appeared to be computation questions, including the estimation question, the students that knew the rule about finding common denominators attempted to apply this procedure first. Because the denominators of 8 and 13 in the estimation question are difficult to work with using this procedure, it was typically abandoned or the wrong common denominator was used and an incorrect estimate response was given. This would lead to the conclusion that the students do not have a strong conceptual understanding of the relative size of a fraction or know the purpose for finding common denominators. Interestingly, while the notion that they needed common denominators to add or subtract fractions correctly was articulated by all the students most were not able to follow through and do this procedure correctly. This would also indicate a lack of conceptual understanding of equivalent fractions.

These same interview questions were asked to ten high achieving students. The total correct and incorrect responses as well as the total number of conceptual and procedural response from these students are in Table 18.

Table 18. *Fraction Interview Responses by Type from High Achieving Students*

High Achieving Students (N = 10)				
Questions	Correct	Incorrect	Conceptual	Procedural
Put these fractions in order; 1/5, 1/3, 1/4	10	0	8	2
Which fraction is larger 4/5 or 11/12?	8	2	4	6
Which fraction is smaller 1/20 or 1/17?	10	0	7	3
Are these fractions equal or is one less, 5/12 or ¾?	9	1	1	9

Are these fractions equal or is one less, 6/4 or 6/5?	10	0	4	6
2/5 + ¾ = 5/9 Do you agree?	10	0	2	8
Estimate: 7/8 + 11/12	7	3	7	3
Solve: 2 1/5 + 1 ¾ =	10	0	0	10
Solve: 4 1/8 – 2 2/4 =	10	0	1	9

As expected, these students were again able to respond correctly to most of the questions. There appears to be more conceptual understandings in this group of students based on the number of responses that included either a drawing of a circle or fraction bar with their explanations or a description of what these fractions looked like as mental images. This aided more students in answering the estimation question correctly as well. Interestingly, most of the students reverted to a procedure to compare 5/12 and ¾. They spent time finding a common denominator to answer the question. The one student who did answer it conceptually stated that "5/12 is a little bit less than ½ and ¾ is a little bit more than half so 5/12 is smaller". This is a good example of transitive thinking and demonstrates strong conceptual understanding of the relative size of fractions.

The rule regarding common denominators was stated by all the students in this group and they could all complete this skill accurately which lead to the correct answer to the questions. When asked "Why do we need to find the common denominators?" Most of the students said things like, "well, that's just the rule" or "I don't really know". Two students were able to explain that "you can't add different size pieces together and call it the same thing so we need to make them the same size pieces". This type of an explanation demonstrates a conceptual understanding of the need for common denominators and equivalent fractions while the other responses were clearly done to follow the rules.

In this set of twenty interviews, students attempted to use a conceptual explanation when comparing and ordering fractions and understanding the relative size of fractions. The low achieving students did not seem to do this as accurately or as often as the high achieving students however. When the students chose to use a rule or procedure, the high achieving students were able to do this correctly more often than the low achieving students. This large discrepancy of the use of accurate conceptual reasoning along with the correct application of rules and procedures could explain the much larger range of test scores on the Unit 5 written test. The Unit 5 written test included questions on comparing and ordering fractions, comparing the relative size of fractions and adding and subtracting fractions. There was one lesson in the unit on estimating with fractions and it is a Minnesota state standard but there were not any test questions that asked students to do this. While all of the students knew rules and procedures for working with fractions their ability to do so correctly obviously affected the outcome of their test score. The low achieving students did not demonstrate strong conceptual understandings nor were they able to follow the rules and procedures correctly. The high achieving students seem to have developed stronger conceptual understandings with mental images of fractions however still relied on rules and procedures regardless of knowing why they work.

Quantitative Data Results and Analysis – Research Questions #3

3. To what extent is there a relationship between student achievement and student attitudes towards mathematics in the Flipped Classroom model of instruction?

A bivariate correlation was used between the Unit 5 Fraction written posttest score and each of the four components on the Student Attitude Survey (Brookstein et. al., 2011) to determine if there is a relationship between student achievement and the student responses in the four components on the attitude survey. The SAS was given one or two days after the Unit 5 posttest. The Unit 5 Fraction posttest scores did not appear to meet the assumptions for normality in that the histogram (Figure 4) showed a negative skew which aligned with the skew statistic of

-1.069. The box plot revealed several outliers as well. The Q-Q Plot (Figure 5.) also shows the scores are not in a reasonably straight line. To further determine that the normality assumption has been violated the Kolmogorov-Smirnov test was run on SPSS. The significance value of $p < 0.01$ further suggests a violation of the assumption of normality.

Figure 4. *Histogram from Unit 5 Posttest*

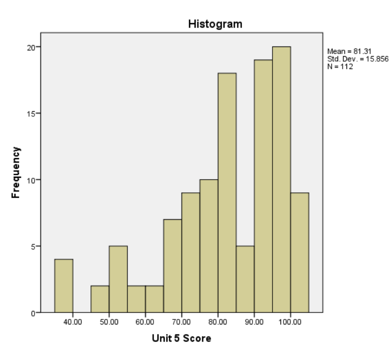
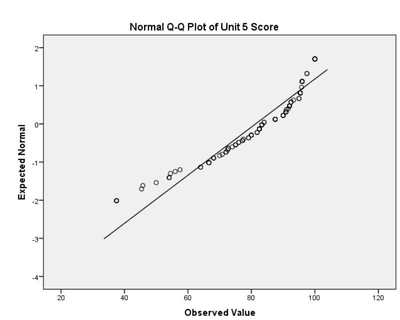


Figure 5. *Q-Q Plot from Unit 5 Posttest*



These same tests for the assumptions of normality were applied to the four component areas of the Student Attitude Survey responses. Based on the visual observation of the histograms and the Q-Q plots for each component it would appear that this data is reasonably normal. Because both sets of data do not meet the assumptions for normality the Spearman Correlation Coefficient was found. The Spearman Correlation does not require both sets of data to be normally distributed.

Component #1 – Deep Affect*: Positivity towards learning mathematics and school*

This area asked questions that "gave a sense of relatively stable student beliefs and attitudes towards math and school that we predict will not change over the course of the short intervention" (Brookstien et. al., 2011, pp. 6). A short intervention in both the design of the survey and this study includes experiences less than 10 weeks in length. Responses to seven of the 23 questions on the survey were combined to create this component score.

A Spearman rho correlation coefficient was calculated for the relationship between the students' Unit 5 Fraction posttest score and component #1 of the SAS. A moderate correlation was found (rho (110) = 0.367, $p < 0.01$) indicating a significant relationship between the posttest score and component #1. The coefficient of determination (rho^2) for Cohen[3] is then 0.135 which also implies a moderate relationship between the two variables accounting for 13.5% of the variability in the relationship. (See

Table 19.). Students who scored higher on the posttest tend to have more positive feelings towards mathematics and school.

Component #2 – Working collaboratively and related effect

This component is made up of nine questions out of the 23 total questions on the SAS. This component "illustrated student perceptions and motivations within the classroom" (Brookstien et. al., 2011, pp. 6) paying specific attention to how the student feels about working in groups.

A Spearman rho correlation coefficient was calculated for the relationship between the students' Unit 5 Fraction posttest score and component #2 of the SAS. An extremely weak correlation that was not significant was found ($r\ (110) = -.148$, $p > 0.05$). The coefficient of determination (rho^2) for Cohen[3] is then 0.022 which also implies a weak relationship between the two variables accounting for only 2.2% of the variability in the relationship. (See Table 19.). The student Unit 5 Fraction posttest scores are not related to how they feel about working in groups.

Component #3 – Working privately

Component #3, Working privately looks at how students feel about working alone in class. Three of the 23 questions on the survey were combined for this component score. Compared to the other components this seems like a relatively small number of questions but the Principle Component Analysis done by the authors of the survey found this to be a valid group of questions to form this component.

A Spearman rho correlation coefficient was calculated for the relationship between the students' Unit 5 Fraction posttest score and component #3 of the SAS. An extremely weak correlation that was not significant was found ($r(110) = 0.163$, $p > 0.05$). The coefficient of determination (rho^2) for Cohen[3] is then 0.027 which also implies a weak relationship between the two variables accounting for only 2.7% of the variability in the relationship. (See Table 19.). The student Unit 5 Fraction posttest scores are not related to how they feel about working alone during class time.

Component #4 – Use of technology

This component illustrates students' attitudes towards technology when doing math. Four of the 23 questions on the survey asked about how students felt when they used technology to do math or if they felt it helped with their math.

A Spearman rho correlation coefficient was calculated for the relationship between the students' Unit 5 Fraction posttest score and component #4 of the SAS. An extremely weak correlation that was not significant was found ($r(110) = -0.068$, $p > 0.05$). The coefficient of determination (rho^2) for Cohen[3] is then 0.005 which also implies a weak relationship between the two variables accounting for less than 1% of the variability in the relationship. (See Table 19.). The student Unit 5 Fraction posttest scores are not related to how they feel about using technology to do math.

Table 19. *Summary of Bivariate Correlations between the Unit 5 Posttest and the Four Attitude Components*

Variable	Comp. #1 Deep Affect	Comp. #2 Working Coll.	Comp. #3 Working Alone	Comp. #4 Technology
Unit 5 Cor. Coef. (rho)	0.367**	-0.148	0.163	-0.068
rho^2	0.135	0.022	0.027	0.005
Sig. (2-tailed)	<0.01	0.120	0.087	0.474
N	112	112	112	112

**Correlation is significant at the 0.01 level (2-tailed)

It would seem obvious that the better you do in math the more you would like math which is supported by the moderate correlation between the two variables. Interestingly, the fact that there is not a significant correlation between the other variables and how a student does in math implies that there are other factors involved in a student's success in math and their feelings towards math. This aligns with responses from the student interviews in that both low achieving and high achieving students indicated that they like to work with friends with a few of the high achieving students stating that they liked to be able to work ahead. This may or may not be working alone or with a partner because it was observed and noted in the field notes that students in both the Math Expressions workbook and the Holt textbook choose to do both; work alone sometimes and with a partner other times. It was equally noted in the student interviews that students in both the high achieving and the low achieving groups had mixed feelings about the computer videos with a few more low achieving students expressing frustration or dislike compared to the high achieving students. The technology component did show a "close to" significant relationship ($p = 0.068$) but was not strong enough to determine a level of correlation. Overall, it would appear that the Flipped Classroom model and its routines do not have a major impact on students' attitudes in the areas of working collaboratively, working independently and using technology in correlation to their achievement in mathematics.

Qualitative and Mixed Methods Data Results and Analysis – Research Question #3

The first portion of the semi-structured student interviews, which all 40 students interviewed participated in, involved five questions that asked for student input on how they feel about math class, how they feel about the Flipped Classroom model and student videos and their access to technology. These questions prompted responses that align with the four components of the attitude survey. As stated earlier the students were identified by their teacher as either being low achieving or high achieving in math. There were 20 students from each category interviewed.

Generally, with just a few low achieving student exceptions, the students that were interviewed stated that they liked math. In particular they liked having the opportunity to work with their friends and they liked doing their Homework and Remembering workbook or pencil paper work in class instead of at home. A few of the low achieving students commented that they thought the tests were hard or in general stated that they felt things were confusing. There was a significant correlation between attitude towards math and achievement on the Unit 5 Fraction test so these types of comments from the low achieving students interviewed is likely representative of many of the low achieving students that took the Unit 5 test and the attitude survey. Likewise, there was not a significant correlation between achievement levels and students' attitudes towards working in groups, working alone or using technology which again, is supported by the responses from both groups during the student interviews.

The major differences in feedback between the low achieving and the high achieving students occurred when they were asked questions about the videos and their access to a computer and the internet. While there was a mix of positive and negative feedback from both groups of students, the specific details of their comments seemed to align to their achievement level. Details of this feedback is in Table 20.

Table 20. *Video and Technology Access Student Interview Responses*

	Low Achieving Students	High Achieving Students
Positive Feedback	• Videos are helpful • Liked video homework better than workbook homework	• Liked video homework • Videos tell you how to do it
Negative Feedback	• Videos are too long • Videos are confusing and go too fast • Prefers lesson in class so you can ask questions • Misses having a teacher • Didn't like missing class to watch the video	• Videos are boring
Re-watching Videos	• 8 out of 20 have re-watched a video	• 11 out of 20 have re-watched a video
Computer Access	• Most have only one device in their home to watch the videos • 6 out of 20 students reported that they do not have internet access at home • About half reported a slow connection	• Most have multiple devices to watch the videos • Most report that they have a good internet connection

While there were some students in both groups that liked the videos or found them helpful, the number of specific negative concerns stated by the low achieving students is very different than that of the high achieving students. These students wanted to be able to ask questions when they didn't understanding things immediately, they talked about being confused by the multiple steps they thought they had to follow to solve a problem, and they didn't like missing class if they had to watch the video in school. Interestingly, this same group of students was less likely to re-watch a video and when asked why the responses generally centered on frustration or that it was just too confusing. It could be surmised that the students who had to watch the videos in school did not re-watch them because that would keep them away from their classroom even longer.

The issue of accessibility also appears to be different between the high achieving and low achieving groups of students. The high achieving students generally had multiple devices such as an IPad, a laptop and desk top computers to watch the videos at home. They also all claimed to usually have no internet connection issues and the speed of the connection did not bother them. On the other hand, while some of the low achieving students had no issue with access to the internet, it was more common to hear a student say that they only had one computer in the house, if it all, and that it usually had to be shared between older siblings or a parent. These students also complained of slower internet connections or troubles getting the videos to play. The students who had to watch the videos in school either did not have a computer at home or it was inconsistent because they lived in two different households, one with a computer and one without, or they didn't get a "turn" because other family members were using the computer. The significant correlation between achievement and attitude toward math is again supported through these student interviews when looking at student feedback on the videos and technology access. Students who struggle both following the mathematical procedures on the video and with computer access issues would likely not do as well on the posttest.

Conclusion

The analysis of the qualitative and the quantitative data in this study have created a detailed picture of the types of teaching practices occurring in the classrooms in this study using the Flipped Classroom model as well as putting forth a deeper understanding of the impacts this model of instruction has on the fifth grade students. The quantitative data from the posttests and the attitude survey provided an overhead view of the performance and feelings of the students in this study while the qualitative data from the classroom observations and interviews fills in the details from a more intimate perspective.

The teachers in this study appeared to rely on the video as the main vehicle for instruction which they supplemented with procedural practice support in the classroom. The video told the procedure and worked through several example problems then the teacher provided more practice opportunities with the procedure in class. The teachers followed the curriculum pages explicitly therefore the use of multiple representations was dependent upon what was provided within the pages of the book or workbook. Teachers spent the majority of the class time assisting individual students that had questions while they worked independently or pulled small groups of students based on quiz scores or common needs. The students generally worked in informal pairs or alone, during the class period, in their Homework and Remembering workbook or the Holt textbook.

These types of teaching practices appear to be in limited alignment with the NCTM Mathematics Practices in the areas of developing procedural fluency and eliciting student thinking. Building conceptual understanding through the use of multiple representations, classroom discourse and engaging tasks was not typically observed during this study. Linking conceptual understanding to procedural fluency was therefore also not observed. There appeared to be no alignment to the NCTM Mathematics Practices in these areas. The teachers all felt that by being able to have the time to talk with students individually thus eliciting student thinking instead of spending time lecturing, gave them a stronger grasp of the students' thinking and they felt that they could better meet the individual needs of the students in the classroom.

Most students did appear to meet the Minnesota State Standards for decimals based on the Unit 3 posttest however the results of the Unit 5 posttest on fraction showed a much greater discrepancy in student understandings. From the student interview data, most students used procedures based on whole number thinking to answer the decimal questions which typically got them the correct answer but did not necessarily demonstrate conceptual understandings of decimals. Because whole number thinking becomes a very buggy strategy when working with fractions, many more students were not able to answer the fraction questions correctly on the posttest or in the student interview. Students who could follow the procedure for making common denominators were generally successful computing with fractions but were not always able to estimate or compare fractions with unlike denominators. Some students did talk about mental images or drew pictures to explain their thinking but this was not consistent among all the students nor was it consistent between questions within the individual interviews.

Overall there was a moderate statistical significance in the correlation between achievement on the Unit 5 Fraction posttest at the end of the study and student attitudes about math and their math class. Students who performed better on the posttest tended to have more positive feelings towards math. The student interviews suggested that most of the students like math and their math class, especially working with friends and doing their traditional workbook homework in class. This would tend to support the results of the attitude survey which showed no significant correlation between achievement and feelings toward working in groups vs. working alone or using technology. Stronger differences became apparent when looking at the feedback about the videos and student access to technology. There were some students in both groups who liked the videos however the lower achieving students shared more concerns about being frustrated and confused while higher achieving students liked the fact the video told them what to do and they could work ahead. Lower achieving students also faced more challenges accessing the videos due to less home equipment and slower internet connections. These specific impacts were not apparent from the attitude survey or posttest achievement data alone and play an extremely important role in understanding the overall impact the Flipped Classroom model in this study has on students' conceptual understandings and attitudes towards and achievement in math. Further conclusions and implications based on the results of this study will be discussed in Chapter 5.

Chapter 5 – Summary, Conclusions and Implications

The National Council of Teachers of Mathematics states, “All students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding. There is no conflict between equity and excellence” (NCTM, 2000, pp. 5). This statement in conjunction with the prevailing achievement gap in mathematics has given rise to innovations and research on teaching and learning mathematics in an effort to truly provide “high quality mathematics instruction for all students” (NCTM, 2000). This study sought to exam one of these innovations, the Flipped Classroom model of instruction as it is enacted in fifth grade elementary classrooms. This chapter provides a summary of this study and the significant findings from the data and analysis presented in Chapter 4. It will conclude with implications for action and future research in this area of mathematics education.

Summary

The purpose of this study was to exam how the Flipped Classroom model of instruction impacts fifth grade students in both their attitudes towards and achievement in mathematics with a particular focus on conceptual understanding versus procedural understanding. This study also examined teacher practices within the Flipped Classroom model enacted in the classrooms in this study and their alignment or misalignment to the NCTM Mathematics Teaching Practices (NCTM, 2014).

Research in the area of the Flipped Classroom model has been done at the secondary and post-secondary level and typically in the areas of science and mathematics with the instructor serving the dual role as the researcher. Achievement on final exams and measurements of attitude based on course reviews have served as the major pieces of evaluation data in most of these studies. Based on the current body of research, there is a need for research on the Flipped Classroom model at the elementary level specifically in mathematics with attention to specific teaching practices and the learning outcomes. Due to the gaps in the research this study addressed the following questions:

1. To what extent does the observed model of Flipped Classroom instruction align with the NCTM Principles to Action in five of the eight Mathematics Teaching Practices for high quality mathematics instruction in four 5th grade classrooms?

   a. Implement tasks that promote reasoning and problem solving
   b. Use and connect mathematical representations
   c. Facilitate meaningful mathematical discourse
   d. Build procedural fluency from conceptual understanding
   e. Elicit and use evidence of student thinking

2. How is student achievement on the decimal and fraction units affected by the model of Flipped Classroom instruction in this study?

   a. Do the students meet the MN State Standards for decimal and fraction concepts as measured by the district created post unit tests?
   b. To what extent do student understandings reflect conceptual knowledge of decimals and fractions based on research on student thinking in the areas of decimals and fractions?

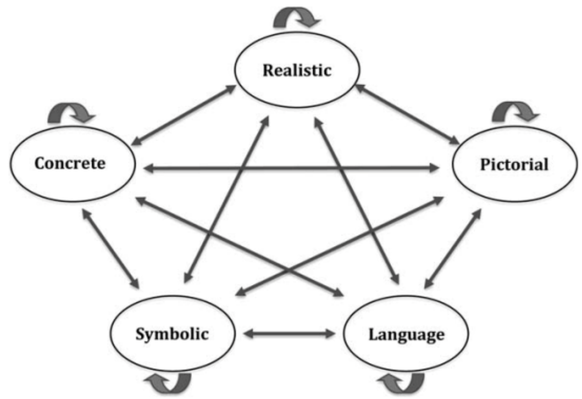
3. To what extent is there a relationship between student achievement and student attitudes towards mathematics in the Flipped Classroom model of instruction?

The NCTM Principles (2000) and the Mathematics Teaching Practices (2014), the Lesh Translation Model (1987) and its links to research on students' conceptual understandings of decimals and fractions and the Flipped Learning Network's definition of Flipped Learning, create the conceptual framework for this study.

The NCTM Principles (2000) of Equity, Curriculum, Teaching, Learning, Assessment, and Technology "describe particular features of high-quality mathematics education" (NCTM, 2000, pp. 11). The Eight Mathematics Teaching Practices (NCTM, 2014) "represent a core set of high-leverage practices and essential teaching skills necessary to promote deep learning of mathematics" (pp.9). Teacher and student actions have been outlined by NCTM to guide the development of these high-leverage practices and support the development of conceptual understanding of mathematics that students need to acquire.

Conceptual understanding, or Relational Understanding (Skemp, 1976), involves understanding the "why" behind a mathematical concept. Heibert and Carpenter (1992) further suggest that in order to truly understand a concept a student needs to create and internalize mental models and make connections between these models and other representations. The Lesh Translation Model (see Figure 6.) (Lesh, Post and Behr, 1987) establishes the types of representations and translations that students must experience in order to support the development of conceptual understanding. For example, when learning about the relative size of fractions students could use fraction circles or fold paper strips to see them concretely. Then draw pictures and describe them to their classmates and finally record various equivalent sized fractions symbolically.

Figure 6. Lesh Translation Model



Research explaining what it means to conceptually understand decimals and fractions include the use of mental models and translations between models. (i.e. Hiebert, Wearne & Tabor, 1991; Cramer, Behr, Post & Lesh, 1997; Cramer et.al. 2009; Cramer et. al, in press; Behr et. al. 1984; Cramer, Post and delMas, 2002). Researchers are concerned that in order to understand the relative size, compare and order and compute accurately with fractions and decimals, students need to have many experiences with a variety of representations and make connections between these representations in order to develop a deep understanding of fractions and decimals.

The definition of "Flipped Learning" or the "Flipped Classroom" used in this study was developed by members of the Flipped Learning Network (2014). It states that Flipped Learning is a:

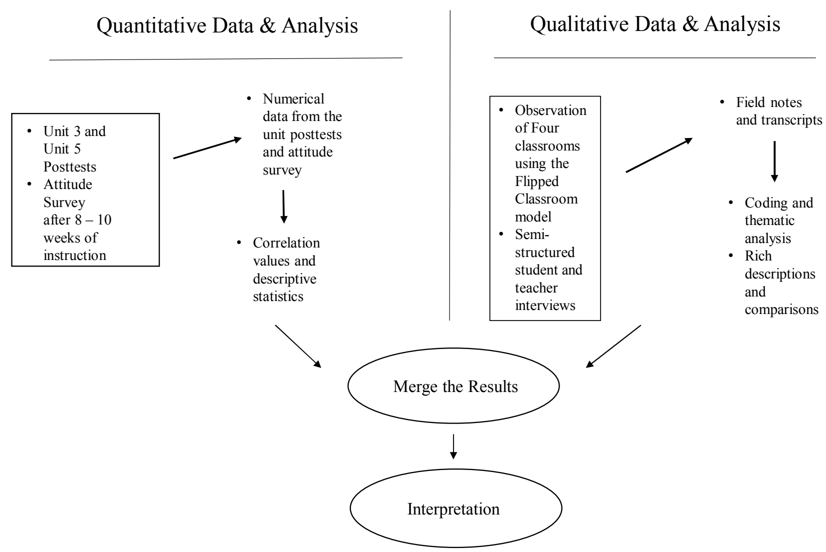
> "a pedagogical approach in which direct instruction moves from the group learning space to the individual learning space, and the resulting group space is transformed into a dynamic, interactive learning environment where the educator

> guides students as they apply concepts and engage creatively in the subject matter” (FLN, 2014).

The students watch a video lecture at home for homework and come to class the next day to work on problems or activities related to the video. This can be enacted in a variety of ways with the most “traditional” model being that the students do the original pencil-paper homework in class. The teacher is then present to assist with these practice problems. This traditional model of the Flipped Classroom is the model observed in the classrooms in this study. The students watched a video each night made by district teachers and based on a lesson in the curriculum and then came to class the next day and work on the corresponding lesson pages in their Homework and Remembering workbook.

This study used a convergent concurrent mixed-methods design (Creswell & Plano Clark, 2011) to collect both qualitative and quantitative data from four fifth grade classrooms in two schools (112 students) in a Midwest suburban school district. Qualitative data was collected from classroom observations, and student and teacher semi-structured interviews. Quantitative data was collected from two unit posttests on decimals and fractions as well as a Student Attitude Survey (Brookstien et. al., 2011). The data was analyzed separately and then merged to develop a more complete picture of the impact the Flipped Classroom model has on teaching and learning in the classrooms in this study. (See Figure 7)

Figure 7. *Overview of Research Design*



The classroom observations were conducted during two units of study, Unit 3: Decimals and Unit 5: Fractions, over the span of approximately eight weeks. The classroom teachers used the district adopted Math Expressions curriculum for the majority of the students and an alternative Holt 6th grade textbook for those who passed the unit pretest with a score of 90% or better. Field notes were taken during these observations and then coded first by activity and then by teaching practices and student actions observed. Themes developed that illustrate the routines and practices that were typical in these classrooms. Most lessons began with warm-up problems and then a mini-lecture which typically lasted 5 – 10 minutes, based on the video the night before. The rest of the class period was devoted to independent work time in the student workbooks. The video typically walked the students through the steps in a procedure and then modeled several practice problems (See Appendix G). The mini-lecture did the same thing using sample problems based on the problems and procedures in the video. Students could work with a partner or by themselves and the teacher circulated around the room assisting individuals as needed. At times small groups were pulled to work on a specific skill based on quiz scores or common student questions. It appeared, and was also shared in the teacher interviews, that the teachers depended on the video as the main vehicle to deliver the instruction.

Also from the field notes, teacher and student actions were coded based on the NCTM Mathematics Teaching Practices. These practices were then aligned, where possible, to the suggested NCTM actions. Overall, there was a weak alignment between the observed actions in the classrooms and the suggested NCTM actions; most distinctly in the practices *Uses and Connect Mathematical Representations* and *Facilitate Meaningful Mathematical Discourse.* The mathematical representations that were used were all in pictorial form and appeared only when they were present in the curriculum materials. Typically this would be in the first few lessons of each unit. There was a fraction bar poster in each classroom that was occasionally referenced by the teacher but was not actually used as a tool by any student. Further, there were no observations of connections being made between any of the pictorial representations. In part this could be because of the types of conversation that was observed in these classrooms. The majority of the dialogue heard involved the steps in procedures with short student responses. Students were asked to contribute the correct answer to the next step in the process or share how they answered a question, but this generally involved the steps used, not the reasoning behind the steps. The "turn to your neighbor" protocol was frequently observed but again, what was shared was a single answer or procedure to solve the problem. Students did appear comfortable sharing their thinking both with their partner and with the whole class and in several instances were observed modeling a procedure in front of the class in the role of the teacher.

The practice of *Implementing Tasks that Promote Reasoning and Problem Solving* could be slightly aligned to the work that the students did in their workbook or notebook from the textbook. The problems in these materials were based on the procedures taught in the video and reviewed in class. The NCTM suggested actions call for engaging problems with multiple entry points. The types of problems in the curriculum materials were procedural type questions used to practice what the students observed on the video and in class. While there could be multiple entry points, a variety of strategies were not observed being discussed so likely were not used by the students. In addition, it is difficult to assess the types of reasoning and problem solving that the students used because this was not typically discussed in relation to their independent work. The type of tasks that the NCTM calls for and the subsequent teacher and student actions were not typically observed in these classrooms.

Building procedural fluency was observed in all classrooms however *Building Procedural Fluency from Conceptual Understanding* was generally not observed. The emphasis was clearly on learning rules or procedures and then practicing these procedures. A great deal of class time was devoted to student practice independently which stemmed from the instruction in the video and the mini-lecture at the beginning of each class period. Because of the limited use of multiple representations and connections through meaningful discourse in the classrooms, it is difficult to ascertain what level of conceptual understanding the students were using to do the work, compared to memorized rules and procedures.

Two main actions were linked to the practice *Elicit and Use Evidence of Student Thinking*. In all the classrooms, teachers spent a great deal of time talking with students individually and in some cases pulled small groups of students who needed similar support. Through the teacher interviews, all the teachers felt very strongly that with the Flipped Classroom model, they now had the time to better understand where their students were at mathematically because they were not devoting as much time to the traditional in-class lecture. However, beyond talking with individual students, instructional decisions on the pacing or order of lessons appeared to be dictated by the curriculum. Every day the video for the next lesson was posted as homework and the in-class work the next day was the lesson workbook pages that went with it. The exception to this was the students in the Holt textbook worked at their own pace so they could move ahead if they completed the work. Occasionally the teacher would announce that if a student had finished their assigned Math Expressions workbook pages they could go on to the next lesson as well or do some other worksheets that may or may not be more challenging. Eliciting student thinking centered on student questions or needs based on their ability to complete the questions in their workbooks or on quizzes accurately. Using that elicited thinking was only observed in how the teacher pulled groups together or allowed them to work in the alternate textbook and work ahead.

Each unit culminated with a posttest designed by the Math Expressions curriculum. The tests, as well as the lessons in each unit, were aligned to the Minnesota State Standards for fifth grade in the areas of decimals and fractions (See Appendix D and Table 3). All students in the classrooms in this study took the same posttest. This study used the score of 80% or greater as the cutoff to likely meet the Minnesota State standards. This was a practical decision in that anything less than 80% clearly showed some understanding of the topic however misconceptions or errors were taking place which could limit the students ability to meet the standards in that area at this time. The Unit 3: Decimal test had a mean score of 91.45% with 94% of the 112 students receiving a score of 80% or higher. The Unit 5: Fraction test had a mean score of 81.31% with 63.4% of the students receiving a score of 80% or higher. Possible explanations for the difference in student achievement could be explained in the analysis of procedural versus conceptual understandings found during the student interviews after each unit.

Interviews were conducted with 20 different students after each unit for a total of 40 different students. The students were selected by their teacher and were identified as either high achieving or low achieving based on their test scores and the teacher's knowledge of the student. During the decimal interviews the use of whole number thinking was observed across the group of students. This type of procedural thinking, 0.7 is greater than 0.4 because seven is more than four or add zeros and line up the decimals, typically allowed the students to get the answers correct while not necessarily understanding what they were doing. Only one student referred to a mental image of a grid and bar to explain how they got their answer. Both high achieving and low achieving students struggled with the two estimating questions because their whole number thinking became a buggy strategy. The relative size of the decimal, in the case of 0.009, was not generally thought of as being very small so would have little change to the original number of 0.57. Most students who did get this question correct explained that they imagined doing the problem in their head by adding a zero behind the 0.57 and then lining up the decimals to subtract. The unit posttest did not have any estimating questions on it therefore the use of whole number thinking and following rules likely allowed many students to get answers correct regardless of whether they had a conceptual understanding of the relative size of the decimal number.

The interviews after the fraction unit test showed more use of mental images or pictorial representations to explain some answers however they were not used consistently or to support estimation with fractions. Further, all the students could recite the rule about making common denominators prior to adding or subtracting fractions however very few knew why they should do that and only 10 of the 20 students could do it accurately. Most students could explain how the denominator relates to the size of a piece of pizza or a candy bar when looking at unit fractions or fractions with a common numerator. Some students described this while others drew a simple picture. However, this same type of thinking tended to not be used when asked to compare fractions with unlike numerators. For example, “which is greater 4/5 or 11/12?” Many students said they were equal because they were both one piece away from a whole or the correct answer was 11/12 because the numbers were bigger. Regardless of the type of question, the students typically tried to find the common denominators before comparing, estimating or computing with fractions. This frequently resulted in the wrong answer or a correct answer based on a procedure versus any demonstration of the conceptual understanding of the relative size or equivalence of a fraction. The inconsistent demonstration of conceptual understanding and consistent, but frequently inaccurate, use of procedures could correspond to the wider range of test scores on the fraction unit test as well as the smaller number of students receiving a score of 80% or greater compared to the decimal posttest. Based on the student interviews it would appear that a limited number of students have developed a conceptual understanding of fractions as described by the research.

The Unit 5: Fraction posttest scores were correlated to the four component scores on the attitude survey to look for a relationship between achievement and the students' feelings toward math and school, working in groups, working alone and using technology. A Spearman rho bivariate correlation coefficient was found due to the fact that the Unit 5 posttest scores were not normally distributed; the scores in each of the four component areas did meet the assumptions for normality. The Spearman rho correlation does not require both sets of data to be normally distributed. A moderate correlation was found (rho (110) = 0.367, $p < 0.01$) indicating a significant relationship between the posttest score and component #1; how students feel towards math and school. Students who scored higher on the posttest tend to have more positive feelings towards mathematics and school. There was not a significant correlation between the posttest scores and any of the other three component scores on the attitude survey. This is supported by feedback during the student interviews. Most students, in both the high achieving and low achieving groups, did report liking to work in groups and it was noted during the classroom observations that students regularly chose to work with a partner or by themselves. Negative opinions regarding the videos were shared by the low achieving students but they may not have thought of this as "technology" when answering the survey questions.

All 40 students (20 high achieving and 20 low achieving) interviewed were asked the same five questions about their feelings toward math and specific aspects of the Flipped Classroom model. As stated earlier many students like math to some degree, liked working with friends and having a video for homework instead of pencil-paper homework. The differences emerged when asked about the videos and their home computer and internet access. The high achieving students generally liked how the videos told you what to do but the low achieving students frequently reported the videos to be confusing. Many of these students also reported frustration with not being able to ask their teacher a question during the video and typically did not re-watch a video as often as the high achieving students. This difference in re-watching the videos could be linked to the fact that some of the low achieving students had to watch the videos at school because they did not have computer access at home. A few shared that they didn't like to miss class to watch the video therefore re-watching the video could make this a worse situation. In general, the high achieving students reported the use of multiple home devices to watch the videos and good internet connections. The low achieving students typically had one device at home with mixed comments on their internet connections. The interview data suggests that there are discrepancies in access to computers and the internet as well as experiences with the videos between low achieving and high achieving students.

Overall, the qualitative data suggests that the Flipped Classroom model enacted in this study supports the teaching of rules and procedures and has a weak alignment to the NCTM Mathematics Teaching Practices. During the student interviews, the use of rules or procedures dominated the processes used by the students, although not always accurately. When merging these qualitative findings with the quantitative posttest data it would suggest that students were able to demonstrate their ability to meet the Minnesota State Standards more frequently in the area of decimals when taking a test based on the use of procedures. Conversely, when the students were less able to utilize the procedures and had limited conceptual understandings they did not perform as well, in the case of the posttest on fractions in which a fewer number of students are likely to meet the state standards at this time. In general students reported liking math which was significantly correlated to their achievement on the Fraction posttest. Further, the data from the student interviews suggests that lower achieving students tended to be more frustrated by the videos, did not re-watch the videos as often and had more access issues to computers and the internet compared to their high achieving classmates. The implications for these findings will be discussed in the following sections.

Conclusion

The definition of the Flipped Classroom used in this study begins with the language, "a pedagogical approach" (FLN, 2014). This implies that what the teacher does within the model is critical to success or failure of the model and that of the students. Further, the definition describes a classroom that is a "dynamic, interactive learning environment where the educator guides students as they apply concepts and engage creatively in the subject matter" (FLN, 2014). These ideas would appear to align with the NCTM expectations for high quality mathematics instruction for all students. The teacher is responsible to intentionally and purposefully choose engaging tasks with multiple entry points, offer many experiences with multiple representations and make connections between the representations, and then make instructional decisions based on elicited student thinking. The purpose in these actions, based on research, would support the deep learning of mathematics both conceptually and procedurally.

Based on the data collected in this study, the traditional style of "teach by telling" is still maintained in the Flipped Classroom even though the teachers feel, as stated in their interviews, that they no longer teach this way. The researched based practices of the NCTM were generally not employed nor would the FLN description, stated at the beginning of this section, appear to describe the classrooms observed. Teacher beliefs about teaching and learning mathematics can be productive or unproductive (NCTM, 2014) and greatly influence what happens in the classroom. The importance of doing this study from an outside observer perspective allowed these findings to appear. The teachers in this study are highly dedicated to their students and the students' success yet their beliefs about the importance of learning procedures and practicing them inhibited a change in their pedagogy when they switched from a "traditional" math class to the Flipped Classroom. The belief that the Flipped Classroom model provided better support for each individual student compared to the traditional model is held by these teachers and therefore may have limited their desire to truly change their pedagogy and teach for conceptual understanding instead of focusing on procedures. Teaching for conceptual understanding, as described in Chapter 2, requires an extensive use of a variety of manipulatives, translations between manipulatives and models, and discussions that share student thinking and reasoning. These types of practices, based on research, were not observed.

Of equal concern is the issues surrounding the differences between high achieving students and low achieving students in regards to their reactions to the videos and their access to computers at home along with the internet connection. From an adult perspective, including secondary and post-secondary students in other studies, the opportunity to be able to watch a video over and over again is very appealing when working with challenging material. Elementary age students in this study do not appear to share this same feeling. Due to computer access issues this may be especially true for those students in need of the most support mathematically. The use of video at home may be supporting the disparity in achievement of some students in this study instead of being a useful tool for learning, as perceived by adults. As the NCTM Equity Principle states, "Access to technology must not become yet another dimension of educational inequity" (NCTM, 2000, pp.14).

These overarching conclusions are based on the data collected in this study and supported by the research behind the conceptual framework of this study and research on conceptual understanding. However this study does have several limitations. First, the literature on the Flipped Classroom suggests that there are many ways that the model can be enacted. This study only observed one such model, therefore other versions of the Flipped Classroom may offer different outcomes or results. Secondly, every student in each classroom was not interviewed so there may be perspectives from the "average" student not represented in these findings. Finally, the duration of the study was limited to approximately eight weeks of instruction and not every lesson in every classroom during those eight weeks was observed. It would be possible that over a longer period of time, different observations could lead to additional supportive or conflicting findings.

Implications for Action and Future Research

The idea of "Flipping" the classroom has become very popular across all levels of education and many content areas. This study demonstrates that teachers who choose to implement this model in their classrooms need to be very intentional with their pedagogy within this model just as they would within the traditional classroom model. The idea or structure of just "Flipping" the classroom does not appear to support students any more, and possibly less, than the traditional model. The intentional use of effective practices in whatever field of study is being taught is one critical element to the success of the students.

Future research on Flipped Classroom models should look how the intentional or purposeful use of effective teaching practices is supported or thwarted within the structure of "flipping". For instance, to what extent does the use of procedural videos at home support or impede the development of conceptual understandings in the classroom?

In addition, what supports are need for teachers implementing a Flipped Classroom model that encourage the use of high-leverage tasks with students?

Research on the Flipped Classroom needs to continue to be conducted using a larger variety of methods with students of all ages. More research conducted by those not also serving as the teacher will enlighten a deeper perspective of what is actually happening in these classrooms. Instruments beyond final exams and course evaluations will provide greater details to enrich the descriptions and possibly evaluate more accurately the effectiveness of these models. Finally, there is a need for studies involving students of all ages. The needs and understandings, thus the impact of the model, on an elementary age student may prove to be very different than that of a high school or college age student as this study has suggested. While the Flipped Classroom model has become very popular the body of research is still developing. If this is a viable pedagogical structure or model, more knowledge is needed to support teachers who want to implement this model.

Concluding Remarks

Implementing the NCTM Mathematics Practices (2014), changing pedagogy and beliefs and creating a new learning structure or environment are very complex tasks that teachers are undertaking. The intent is to provide students with excellent instruction so that all students have the opportunity to succeed. This study, using the research behind the NCTM Mathematics Practices, the Lesh Translation model and research on conceptual understanding to evaluate the Flipped Classroom model, offers insight into a very popular, yet minimally researched, attempt to do this. This study highlights the importance of teaching practices that affect student learning beyond just changing the structure of the class period as well as the impact that changing the structure has on specific groups of students. The Flipped Classroom model will continue to be implemented across the United States therefore it is critically important to continue to support the development of research based effective teaching practices as well as encourage an acute awareness of newly created issues of equity based on the use of technology. Research based effective practices that support high quality mathematics instruction for *all* students as well as equitable learning environments are necessary regardless of the teaching model if we are going to close the achievement gap.

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Appendix A

Teacher and Student Actions for Mathematics Teaching Practices

(NCTM, 2014)

Implement tasks that promote reasoning and problem solving **Teacher and student actions**	
What are *teachers* doing?	**What are *students* doing?**
Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. Selecting tasks that provide multiple en- try points through the use of varied tools and representations. Posing tasks on a regular basis that require a high level of cognitive demand. Supporting students in exploring tasks without taking over student thinking. Encouraging students to use varied approaches and strategies to make sense of and solve tasks.	Persevering in exploring and reasoning through tasks. Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas. Using tools and representations as need- ed to support their thinking and problem solving. Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.

(NCTM, 2014, pp. 24)

Use and connect mathematical representations Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Selecting tasks that allow students to decide which representations to use in making sense of the problems. Allocating substantial instructional time for students to use, discuss, and make connections among representations. Introducing forms of representations that can be useful to students. Asking students to make math drawings or use other visual supports to explain and justify their reasoning. Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	Using multiple forms of representations to make sense of and understand mathematics. Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other 118epresent-tations. Making choices about which forms of representations to use as tools for solving problems. Sketching diagrams to make sense of problem situations. Contextualizing mathematical ideas by connecting them to real-world situations. Considering the advantages or suitability of using various representations when solving problems.

(NCTM, 2014, pp. 29)

Facilitate meaningful mathematical discourse Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Presenting and explaining ideas, reason- ing, and representations to one another in pair, small-group, and whole-class discourse. Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments. Seeking to understand the approach- es used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. Identifying how different approaches to solving a task are the same and how they are different.

(NCTM, 2014, pp. 35)

Build procedural fluency from conceptual understanding Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Providing students with opportunities to use their own reasoning strategies and methods for solving problems. Asking students to discuss and explain why the procedures that they are using work to solve particular problems. Connecting student-generated strategies and methods to more efficient procedures as appropriate.	Making sure that they understand and can explain the mathematical basis for the procedures that they are using. Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems. Determining whether specific approaches generalize to a broad class of problems.
What are *teachers* doing?	**What are *students* doing?**
Using visual models to support students' understanding of general methods. Providing students with opportunities for distributed practice of procedures.	Striving to use procedures appropriately and efficiently.

(NCTM, 2014, pp. 47-48)

Elicit and use evidence of student thinking Teacher and student actions	
What are *teachers* doing?	**What are *students* doing?**
Identifying what counts as evidence of stu- dent progress toward mathematics learning goals. Eliciting and gathering evidence of student understanding at strategic points during instruction. Interpreting student thinking to assess mathematical understanding, reasoning, and methods. Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend. Reflecting on evidence of student learning to inform the planning of next instructional steps.	Revealing their mathematical under- standing, reasoning, and methods in written work and classroom discourse. Reflecting on mistakes and misconcep- tions to improve their mathematical understanding. Asking questions, responding to, and giving suggestions to support the learning of their classmates. Assessing and monitoring their own progress toward mathematics learning goals and identifying areas in which they need to improve.

(NCTM, 2014, pp. 56)

Appendix B

Adapted Student Attitude Survey (Brookstein et. al., 2011)

0	1	2	3	4
Strongly Disagree	Disagree	Neutral/Undecided	Agree	Strongly Agree

1. I think mathematics is important in life.	0	1	2	3	4
2. In school, my math teachers listened carefully to what I had to	0	1	2	3	4
3. I learn more about mathematics working on my own.	0	1	2	3	4
4. I do not like to speak in public.	0	1	2	3	4
5. I prefer working along rather than in groups when doing	0	1	2	3	4
6. I get anxious in school.	0	1	2	3	4
7. In school, I learn more from talking to my friends than from listening to my teacher.	0	1	2	3	4
8. Technology can make mathematics easier to understand.	0	1	2	3	4
9. Cell phones are an important technology in my life.	0	1	2	3	4
10. I like my own space outside school the majority of the time.	0	1	2	3	4
11. I enjoy being part of large groups outside school.	0	1	2	3	4
12. I do not participate in many group activities outside school.	0	1	2	3	4
13. I do not like school.	0	1	2	3	4
14. I like math.	0	1	2	3	4
15. I feel confident in my abilities to solve mathematics problems.	0	1	2	3	4
16. In the past, I have not enjoyed math class.	0	1	2	3	4
17. I receive good grades on math tests and quizzes.	0	1	2	3	4
18. When I see a math problem, I am nervous.	0	1	2	3	4
19. I am not eager to participate in discussions that involve	0	1	2	3	4
20. I enjoy working in groups better than alone in math class.	0	1	2	3	4
21. I like to go to the board or share my answers with peers in math	0	1	2	3	4
22. I enjoy hearing the thoughts and ideas of my peers in math class.	0	1	2	3	4
23. Mathematics interests me.	0	1	2	3	4
24. I sometimes feel nervous talking out-loud in front of my	0	1	2	3	4
25. I enjoy using a computer when learning mathematics.	0	1	2	3	4
26. When using technology for learning mathematics, I feel like I am in my own private world.	0	1	2	3	4
27. I am not comfortable using technology in math class.	0	1	2	3	4

Appendix C

Sample Questions from the Math Expressions Post Unit Tests (Fuson, 2011)

Decimal Unit Test

Write each number.

1. thirty-seven thousand, five hundred sixty 37,560

2. three million, six hundred two thousand, eight hundred twenty-four 3,602,824

3. seven tenths 0.7

4. five hundred twenty-eight thousandths 0.528

Compare. Write > (greater than) or < (less than).

5. 789,261 < 798,612

6. 3,491,652 > 3,419,652

7. 0.741 > 0.714

8. 0.08 < 0.6

Add or subtract each pair of numbers. Then show how to estimate to check your answer. Check students' explanations.

9. 652,721 + 201,054 = 853,775

10. 1.392 + 0.85 = 2.242

11. 794,627 − 322,069 = 472,558

12. 6.418 − 1.37 = 5.048

Solve.

Show your work.

19. Eliza is making a snack. She plans to mix 1.2 pounds of almonds and 0.75 pounds of cashews. How many pounds of nuts is that altogether?

1.95 pounds

20. Extended Response The distance between the library and the park is 1,563 feet. The distance between the library and the bank is 528 feet. The distance between the library and the fruit stand is 296 feet less than the distance between the library and the bank. Explain how to find how much greater the distance between the library and the park is than the distance between the library and the fruit stand.

First find the distance between the library and the fruit stand, 528 − 296 = 232 feet. Then subtract the distance from 1,563. 1,563 − 232 = 1,331. The distance between the

Fraction Unit Test

Add or subtract. Simplify your answers.

1. **a.** $\frac{2}{5} + \frac{1}{5} =$ $\frac{3}{5}$

b. $\frac{5}{6} - \frac{1}{3} =$ $\frac{1}{2}$

2. **a.**

$$\begin{array}{r} 5\frac{3}{8} \\ -\ 4\frac{5}{8} \\ \hline \frac{3}{4} \end{array}$$

b.

$$\begin{array}{r} 2\frac{3}{4} \\ +\ 3\frac{1}{8} \\ \hline 5\frac{7}{8} \end{array}$$

3. Find *s*.

$\frac{1}{s} + \frac{1}{s} + \frac{1}{s} + \frac{1}{s} = \frac{5}{5}$ $s =$ 4

4. Circle the greater fraction. Then write > or < between the fractions. Explain your thinking.

$\frac{3}{8}$ < $\frac{3}{7}$

Sevenths are larger than eighths since it takes fewer of them to make a whole. Each fraction has a numerator of three, so three sevenths is greater.

5. Write the mixed number as an improper fraction. Show your work.

$3\frac{1}{3} =$ $\frac{10}{3}$ $3\frac{1}{3} = 1 + 1 + 1 + \frac{1}{3} = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} + \frac{1}{3} = \frac{10}{3}$

6. Write these fractions in order from **least to greatest.**

$\frac{11}{12}$, $1\frac{2}{12}$, $\frac{2}{3}$, $\frac{8}{6}$, $\frac{3}{3}$

$\frac{2}{3}$, $\frac{11}{12}$, $\frac{3}{3}$, $1\frac{2}{12}$, $\frac{8}{6}$

7. Write these numbers in order from **greatest to least.**

$\frac{10}{5}$, 2.02, $\frac{1,000}{1,000}$, $2\frac{1}{10}$, $\frac{4}{5}$

$2\frac{1}{10}$, 2.02, $\frac{10}{5}$, $\frac{1,000}{1,000}$, $\frac{4}{5}$

8. Circle the fraction that is equivalent to $\frac{3}{5}$. Show your work.

$\frac{33}{50}$ $\frac{12}{20}$ $\frac{15}{35}$ $\frac{3}{5} \times \frac{4}{4} = \frac{12}{20}$

10. **Extended Response** Kim played in the park for $\frac{2}{3}$ hour. Later, Simone played for $\frac{1}{5}$ hour more than Kim. How many hours did they play altogether?

First calculate how long Simone played by adding both fractions together: $\frac{2}{3} + \frac{1}{5} = \frac{13}{15}$. Then add the amount of time that Kim played to the amount of time that Simone played: $\frac{13}{15} + \frac{2}{3} = 1\frac{8}{15}$. together

Appendix D

Alignment of Assessments to the MN State Standards

Decimal Unit Test (MN Standard followed by applicable test items)

MN Standard 5.1.1.3 Estimate solutions to arithmetic problems in order to assess the reasonableness of the results.

No test items matched this standard

MN Standard 5.1.1.4 Solve real-world and mathematical problems requiring addition, subtraction, multiplication and division of multi-digit whole numbers. Use various strategies, including inverse relationships between operations, the use of technology, and the context of the problem to assess the reasonableness of results.

20. The distance between the library and the park is 1,563 feet. The distance between the library and the bank is 528 feet. The distance between the library and the fruit stand is 296 feet less than the distance between the library and the bank. Explain how to find how much greater the distance between the library and the park is than the distance between the library and the fruit stand.

MN Standard 5.1.2.1 Read and write decimals using place value to describe decimals in terms of groups form millionths to millions.

1. Thirty-seven thousand, five hundred sixty __________
2. Three million, six hundred two thousand, eight hundred twenty four _____________
3. Seven tenths _________
4. Five hundred twenty-eight thousandths __________

MN Standard 5.1.2.3 Order fractions and decimals, including mixed numbers and improper fractions, and locate on a number line.

7. 0.741 _______ 0.714 8. 0.08 _______ 0.6 (insert < or > symbol)

MN Standard 5.1.3.1 Add and subtract decimals and fractions, using efficient and generalizable procedures, including standard algorithms.

10. 1.392 + 0.85 = 12. 6.418 – 1.37 =

MN Standard 5.1.3.4 Solve real-world and mathematical problems requiring addition and subtraction of decimals, fractions and mixed numbers, including those involving measurement, geometry and data.

19. Eliza is making a snack. She plans to mix 1.2 pounds of almonds and 0.75 pounds of cashews. How many pounds of nuts is that altogether?

Fraction Unit Test (MN Standard followed by applicable test item)

MN Standard 5.1.2.3 Order fractions and decimals, including mixed numbers and improper fractions, and locate on a number line.

4. Circle the greater fraction. Then write < or > between the fractions. Explain your thinking. 3/8 __________ 3/7

6. Write these fractions in order from least to greatest. 11/12, 1 2/12, 2/3, 8/6, 3/3

7. Write these numbers in order from greatest to least. 10/5, 2.02, 1000/1000, 2 1/10, 4/5

MN Standard 5.1.2.4 Recognize and generate equivalent decimals, fractions and mixed numbers and improper fractions in various contexts.

5. Write the mixed number as an improper fraction, show your work. 3 1/3 = _______

8. Circle the fraction that is equivalent to 3/5. Show your work. 33/50, 12/20, 15/35

MN Standard 5.1.3.1 Add and subtract decimals and fractions, using efficient and generalizable procedures, including standard algorithms.

1.a. 2/5 + 1/5 =

1.b. 5/6 – 1/3 =

2.a. 5 3/8 – 4 5/8 =

2.b. 2 ¾ + 3 1/8 =

3. 1/s + 1/s +1/s + 1/s + 1/s = 5/5 s = _________

MN Standard 5.1.3.4 Solve real-world and mathematical problems requiring addition and subtraction of decimals, fractions and mixed numbers, including those involving measurement, geometry and data.

9. Kim played in the park for 2/3 hour. Later, Simone played for 1/5 hour more than Kim. How many hours did they play altogether?

Appendix E

Indicators of Conceptual Understanding

Indicators of Decimal Understanding:

(Hiebert & Wearne, 1986; Steinle, Vicki, & Stacey, 2001; Cramer et al. in press)

1. Using precise mathematical language when working with decimals.
2. Accurately using models to represent decimals.
3. Describing how to compose and decompose decimals based on mental images of the models or place value while ordering decimals.
4. Using an understanding of the relative size of decimals to guide their estimation with operations with decimals.
5. Using a model and their ability to compose and decompose decimals to interpret addition and subtraction operations and build meaning for work with symbols.

Indicators of Fraction Understanding:

(Cramer, Post, delMas, 2002; Cramer, Wyberg, 2007; Cramer & Whitney, 2010; Lamon, 2007)

1. Understand the relative size of fractions and be able to compare them by describing mental images based on models. (Rational Number Sense)
2. Use the understanding of the relative size of fractions to accurately estimate when doing operations with fractions. (Student is able to explain with mental images or use multiple representations to explain how they estimated)
3. Use symbolic representation to compute with fractions and be able to explain the process with mental images or models.

Appendix F

Semi-Structured Student Interviews

Attitude and background questions

1. Tell me about your math class (What about it do you like?, What about it do you not like?)
2. Tell me about the videos that you watch for homework (How do they help you? What do you like or not like about them? Do you watch them alone or with an adult? How often do you watch them?)
3. Tell me about a time that you re-watched a video (why? Did this help you?)
4. Do you have access at home to a computer (high speed internet? If not at home how do you watch the videos?)
5. How does this class differ from previous years? (Is what the teacher doing different? Do you talk to your friends in class more or less? What do you talk about?)

Conceptual Understanding Questions

After the decimal unit:

1. Name these decimals: 0.7 and 0.40 Which decimal is larger? Explain your thinking.
2. Name these decimals: 0.103 and 0.13 Which decimal is larger? Explain your thinking
3. Put these decimals in order from smallest to largest: 0.245, 0.025, 0.249, 0.3 Explain your thinking.

4. Estimate the answer: 0.37 + 0.4 = Explain your thinking.
5. Picture 0.57 If you took away 0.009 would the amount left be more than ½ or less than ½? Explain your thinking without finding the exact answer.
6. Show how you would solve 0.375 + 2.5 Explain your thinking
7. Show how you would solve 4 .85 - 0.437 Explain your steps

After the fraction unit test:

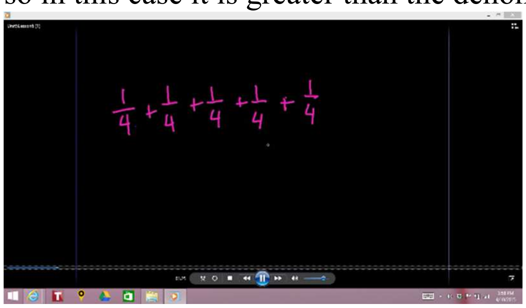
1. Put these fractions in order from the least to the greatest: (Explain your thinking)

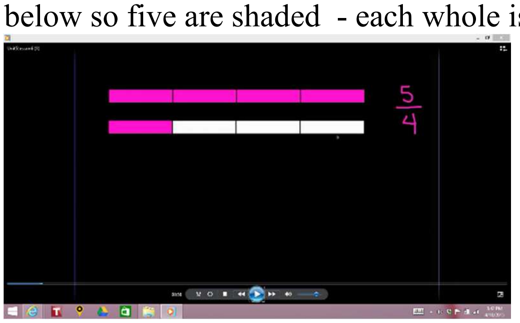
   1/5, 1/3, 1/4

2. Which fraction is larger? 4/5 or 11/12 (Explain how you know)
3. Which fraction is smaller? 1/20 or 1/17 (Explain how you know)
4. Are these fractions equal or is one less? 5/12 or ¾ (Explain how you know)
5. Are these fractions equal or is one less? 6/4 or 6/5 (Explain how you know)
6. 2/5 + 3/4= 5/9 Do you agree? (Explain your response)
7. Estimate:7/8 + 12/13 = (Explain your thinking)
8. Solve: 2 1/5 + 1 ¾ =
9. Solve: 4 1/8 – 2 2/4 =

Appendix G
Sample Video Transcript

Video Unit 5 Lesson 6 (15:58 minute long) **words in italics indicate teacher or video actions*
Objectives: *(opening screen)*
1. Represent improper fractions and mixed numbers numerically and with drawings
2. Explore ways to convert between mixed numbers and improper fractions.

Teacher talking:
(0:00 – 0:55) Hello this is Mrs. XXX from XXXX, this is Unit 5 Lesson 6 *(reads objectives)*
Let's take a look at this chain of unit fractions that I've got here. One fourth plus one fourth plus one fourth plus one fourth plus one fourth. What do you get when you add them together? *(short pause)* Well we get five fourths (*writes this in*). Five fourths is an improper fraction which means the numerator is greater than or equal to the denominator so in this case it is greater than the denominator.


(0:56 – 1:19) Here is another example with a drawing *(two fraction bars are shown divided into fourths)* Here is one, two, three, four and then five with the fifth one down below so five are shaded - each whole is broken up into four units.


(1:20 – 2:12)Here we go back with our original chain of unit fractions *(circles four ¼ s)* so here are four which is one whole and then there is one more so we have one plus one fourth equals one and one fourth. We call one and one fourth a mixed number. It's called a mixed number because it is a mixture of a whole number and a fractional amount so a whole number and a fraction together is called a mixed number.

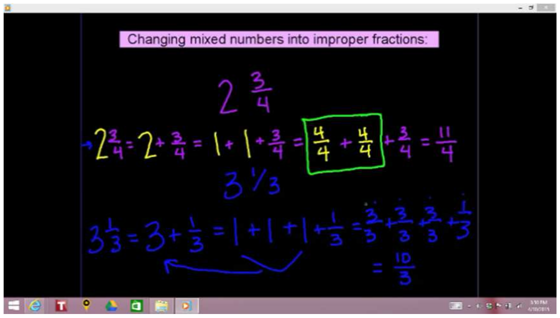
(2:13 – 3:20) *Teacher repeats with another problem using the unit fraction of 1/3 with no picture, only a chain of five 1/3s.*

(3:21 – 4:31) Here I have a picture of four circles. Express this picture one as an improper fraction and two as a mixed number. Pause the video for a second and answer those two questions *(picture shows four circles each divided in half)* Here is what you should have gotten. Remember that an improper fraction is when the numerator is greater than or equal to the denominator so each of my whole circles is divided into two so that is my denominator (writes this down) and I have one, two, three, four, five, six, seven halves *(writes this over the two)* . Now the mixed number is, I've got one whole, two wholes, three wholes, *(writes a big 3)* plus one half (*writes a ½ next to the 3*). My mixed number is three and a half, my improper fraction is seven halves to match the circles above.

(4:32 – 5:34) *Screen says Your Turn – tells the students to make up their own fraction chain of unit fractions and then write it is a mixed number and an improper fraction – teacher reads this out loud and says to pause the video and do this. Reminds them that a unit fraction is any fraction with a one on top. Flashes back to the ¼ chain as an example.*

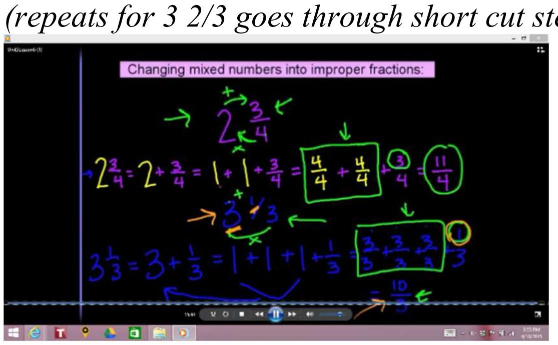
(5:35 – 6:19) There could have been a number of ways that you could have done this. I'm going to choose the fraction one fifth. Now if my denominator is five I am going to need to have five of these in order to have a whole number so here I have a chain of unit fractions (6 all together) and I am going to circle five of them that equals one whole plus one fifth so my mixed number is one and one fifth. My improper fraction is (counts the chain out loud) six fifths. Notice that my denominator never changes.

(6:20 – 7:47) All right now we are going to change a mixed number into an improper fraction. We have here two and three fourths and we are going to change it into an improper fraction *(see top half of the screen shot)* This great big long chain of numbers here in fractions looks a little intimidating but I am going to go through it here and I color coded it for a reason. My whole numbers are always going to be yellow and then as it changes into a fraction the fraction is representing a whole number. Two and three fourths is equal to two plus three fourths, which is equal to one plus one, I get the one plus one from the two, the three fourths stays the same, now the one is the same as four fourths and I need to have a fraction that equals one. So four fourths plus four fourths, which came from my ones, I just converted it to a fraction so that I can get my answer and then I am still adding my three fourths. Now I have three fractions that I can add together, four fourths plus four fourths plus three fourths which equals eleven fourths. My improper fraction is eleven fourths.



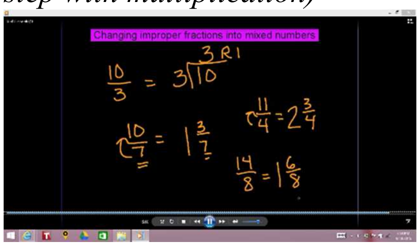
(7:48 – 10:39) *Writes another example and goes through the same procedure for 3 1/3. Uses the same long chain of fractions (see bottom half of screen shot)* See if you can see any patterns so we can figure out a short cut so we don't have to write out this lone line of fractions. Maybe you came up with this four fourths two times – four times two is eight then I just added that last fraction to get eleven. Down here I got three thirds three times, three times three is nine then I add my last one third gives me ten thirds *(while going through this she is drawing a multiplication sign and addition sign around the mixed number)*

(10:40 – 11:51) Let's look at two and a half. Let's do this without the long line of fractions. My short cut is if I take my bottom number times my whole number, add the top number. Remember my bottom number never changes so I have two times two is four and add one which is five over the same denominator is two so I have five halves. So I have turned this mixed number into an improper fraction. Let's practice one more *(repeats for 3 2/3 goes through short cut steps)*



(11:52 – 12:42) *Your Turn screen comes up – teacher reads it out loud and tells them to pause the screen and do the work – see screen shot for problems)* Alright three and two fifths, I have five times three is fifteen plus two is seventeen fifths. Remember bottom number never changes. Two and one fourth, four times two is eight plus one is nine, bottom number is four so it is nine fourths. Number three, three times two is six plus two is eight, eight thirds. Number four, four and one sixth, six times four is twenty four and one is twenty five sixths.

(12:43 – 15:18) All right now we are going to go the other way and change improper fractions into mixed numbers. Let's look back at this screen *(see previous screen shot)* and we started with our mixed number and got to improper fraction. Now what if we went backwards and we want to start with this improper fraction if we work backwards to the mixed number. We can see that ten thirds we had three three's in there plus one more. A fraction bar really means division so ten thirds really means ten divided by three *(shows traditional algorithm)* so three goes into ten three times with a remainder of one. Look back here – here is my three with a remainder of one. When I work backwards going from an improper fraction to a mixed number I am dividing. *(continues with example of 10/7 same procedure, again with 11/4, and again with 14/8)* I can always check it because eight times one is eight plus six is fourteen eighths (*repeats backward step with multiplication)*



(15:19 -15:58) *Screen with objectives comes back up.* Okay and that is the end of the lesson *(teacher rereads the objective out loud – flashes back to previous screens as she reads)*