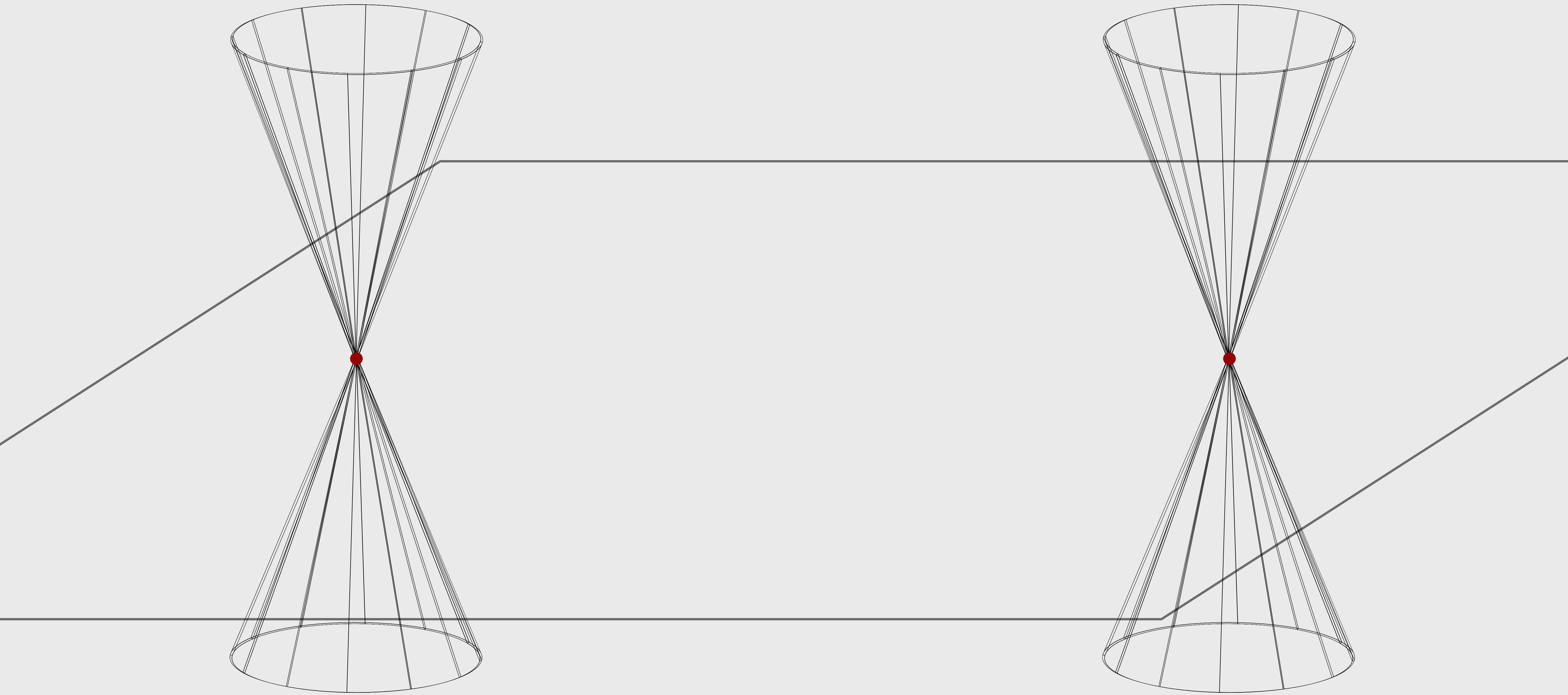
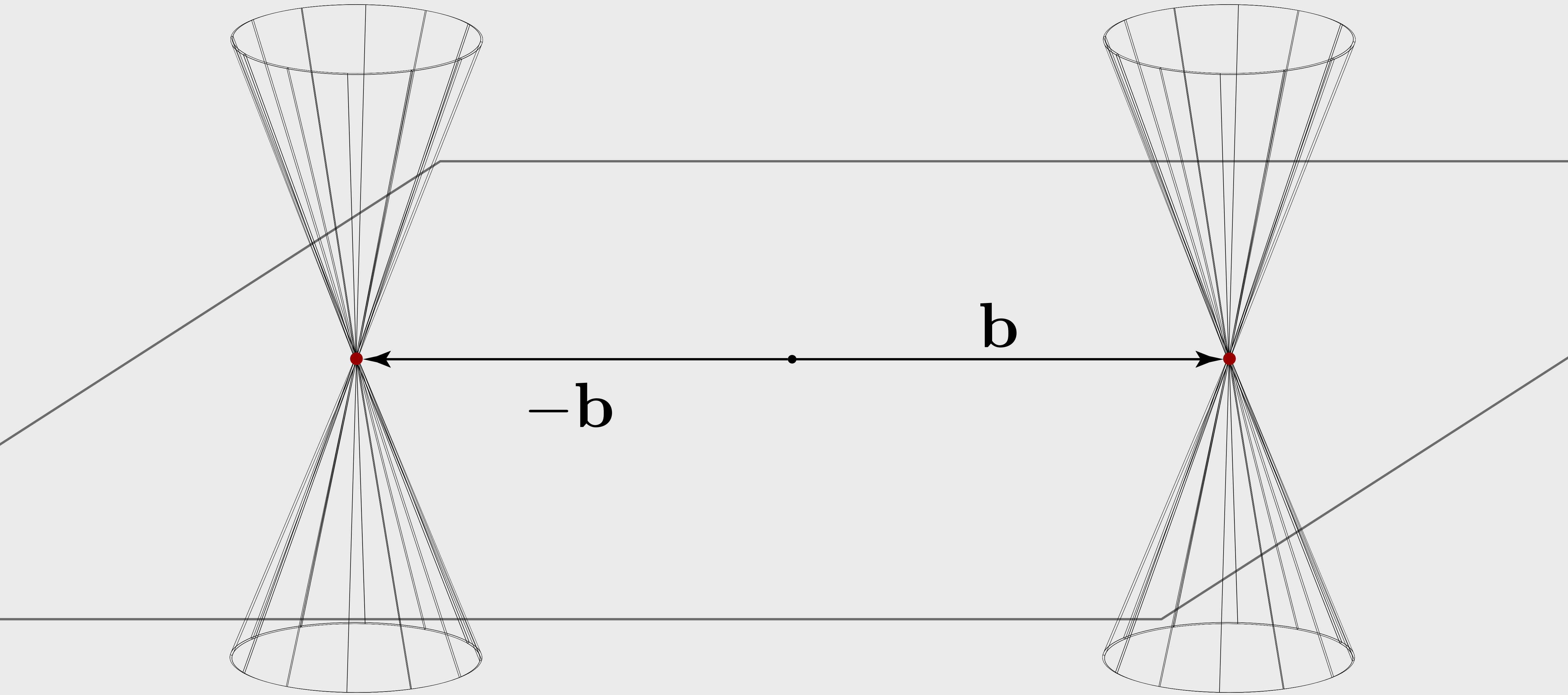
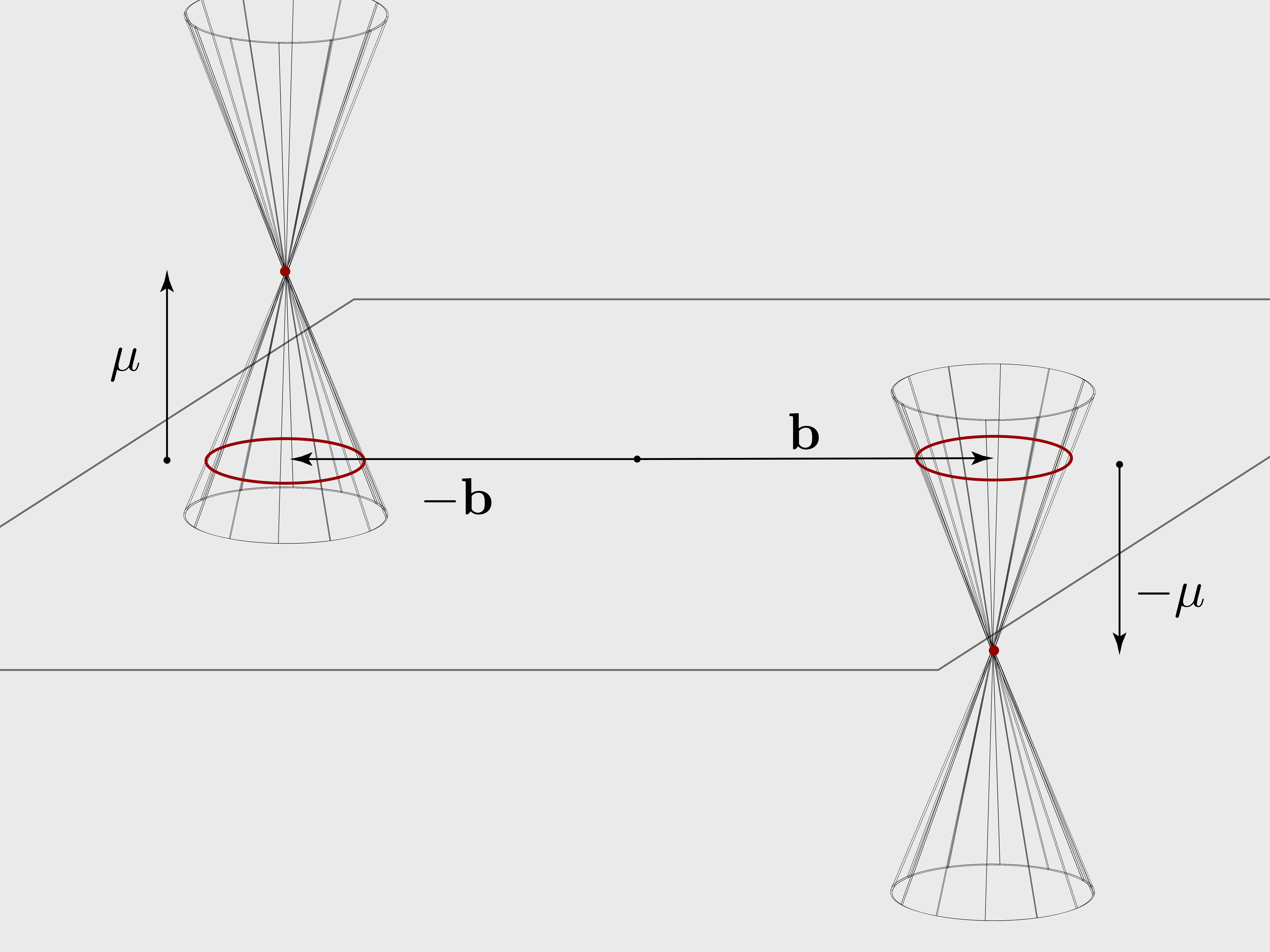
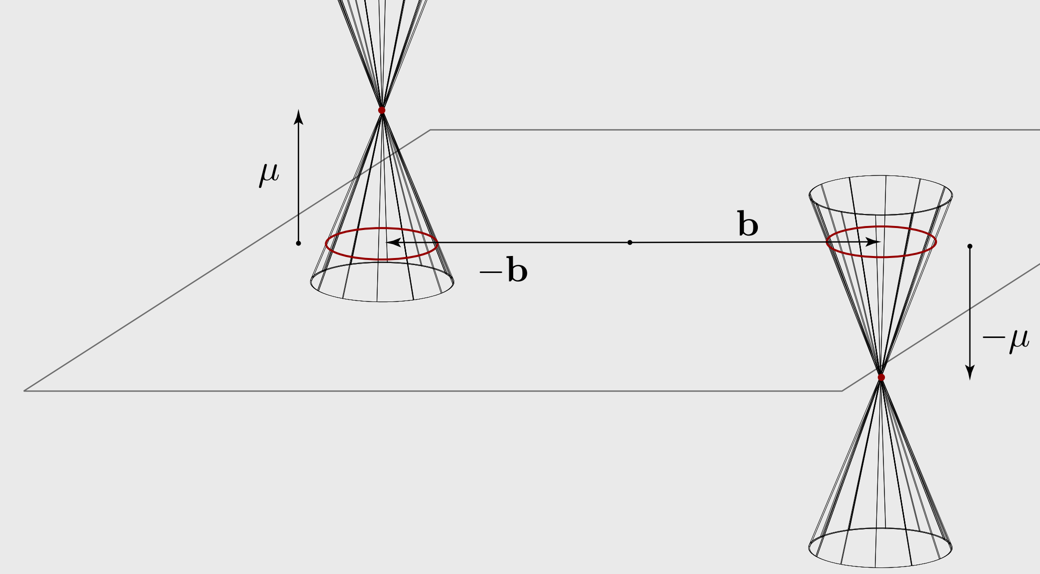



b
−b



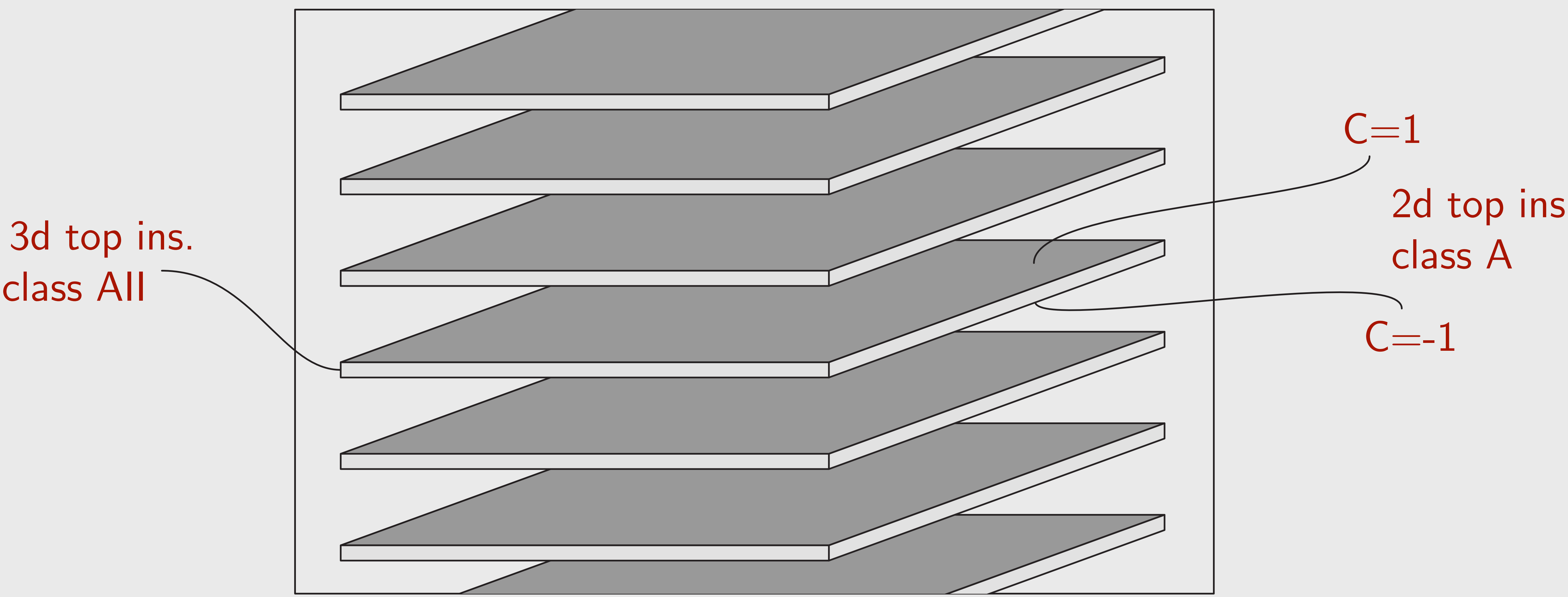
μ
$\mathbf{b}$
$-\mathbf{b}$
$-\mu$



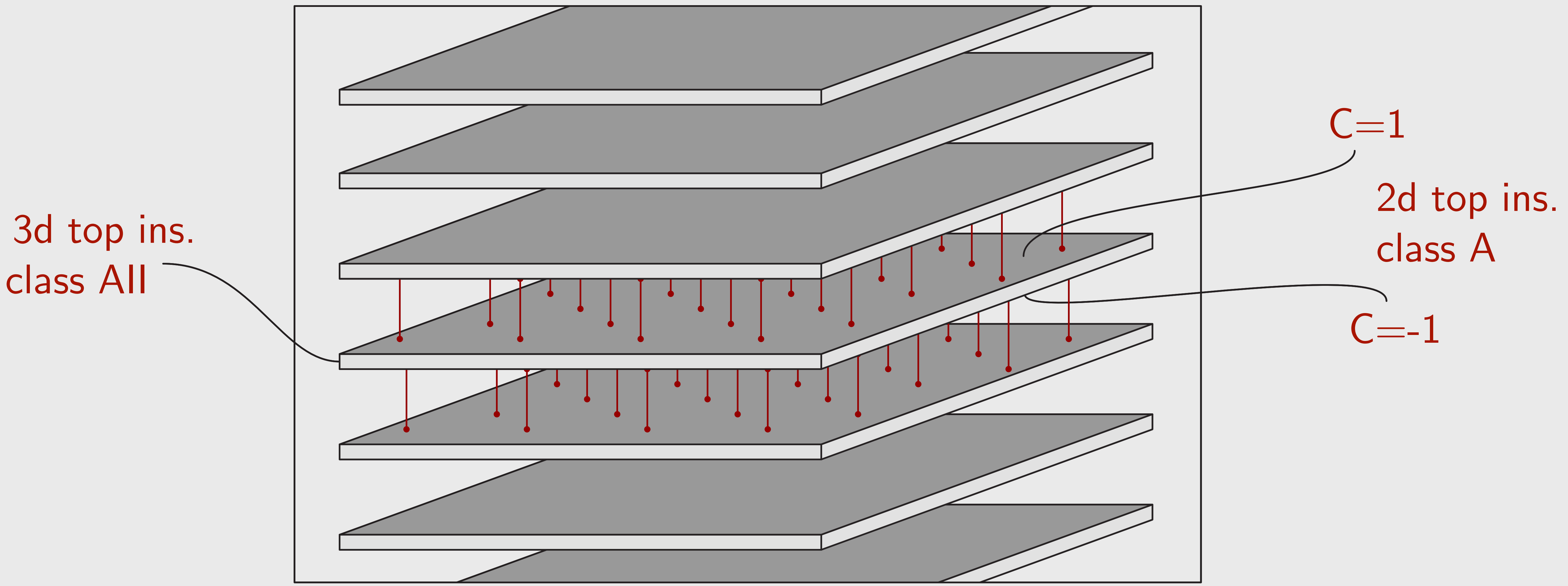
nodes protected against gapping

can be moved in energy/momentum but
not individually destroyed; in general: topological Fermi surfaces

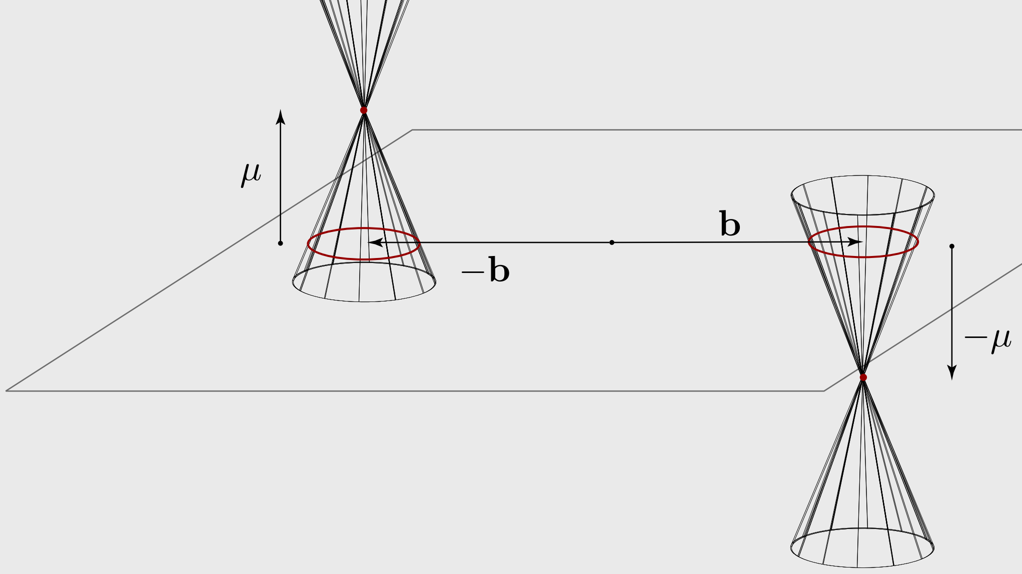
physical realization: stacked 2d topological insulators



Burkov and Balents, 11



Burkov and Balents, 11



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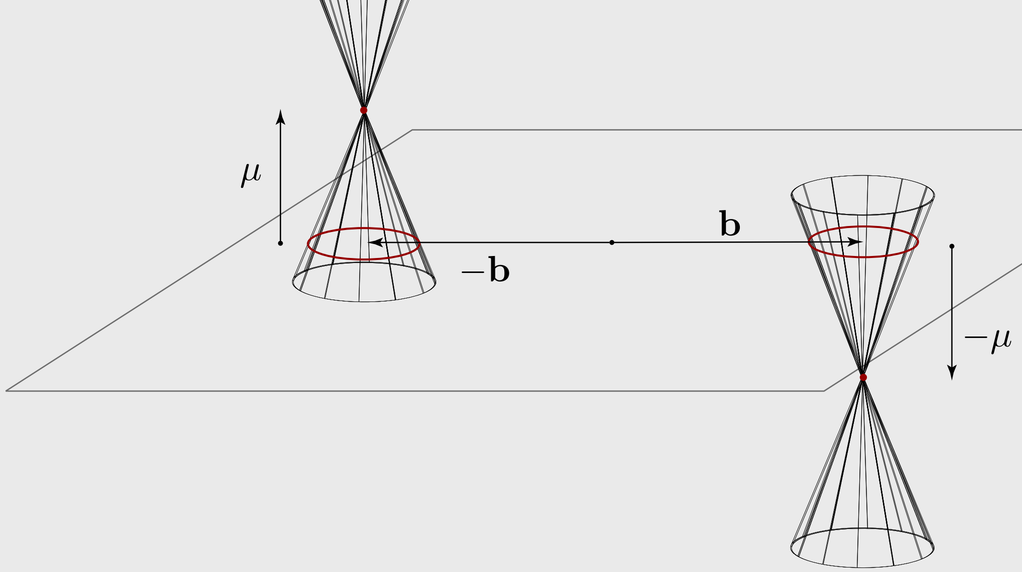

nodes protected against gapping

can be moved in energy/momentum but
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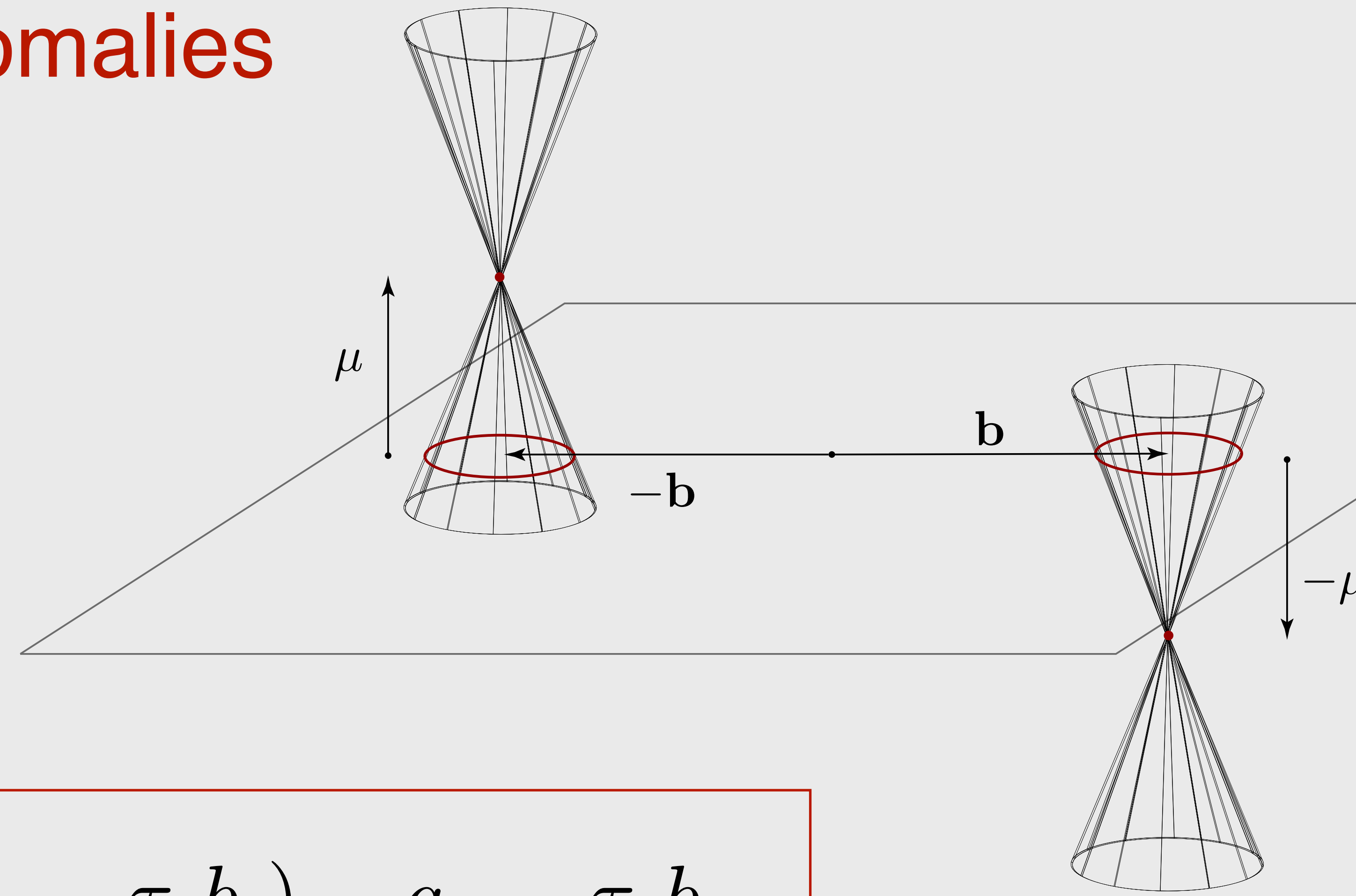
physical realization: stacked 2d topological insulators

surface states (‘Fermi arcs’)

non-conservation of charge at individual nodes (axial anomaly)

generates unconventional transverse response: chiral magnetic effect, CME and anomalous Hall effect, AHE

transverse response from anomalies



$$\boxed{H = \tau_3 \otimes \sigma^i (k_i - a_i - \tau_3 b_i) - a_0 - \tau_3 b_0}$$

τ_3 acts in nodal space

$a = (a_0, a_i)$ is external vector potential

$b = (b_0, b_i)$ is (3+1)-dimensional constant 'internal' vector potential. Coupled to axial current. Can be 'gauged out' by anomalous gauge transformation

transverse response from anomalies cont'd

effective gauge field action (Burkov & Zyusin)

$$S[a,b] = \frac{1}{32\pi}\int d^3x dt\, \theta\, \epsilon^{\mu\nu\rho\sigma} f_{\mu\nu} f_{\rho\sigma} =$$

$$= -\frac{1}{8\pi}\int d^3x dt\, \epsilon^{\mu\nu\rho\sigma}\partial_\mu \left(\theta\, a_\nu \partial_\rho a_\sigma\right)$$

$$\theta = \theta(x,t) = 2b_i x^i - 2b_0 t$$

transverse response from anomalies cont'd

effective gauge field action (Burkov & Zyusin)

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$$\theta = \theta(x,t) = 2b_i x^i - 2b_0 t$$

variation $\delta_b S[a,b]$ yields axial current non-conservation

$$\partial_\mu j^{\mu,a} = \frac{1}{16\pi^2}\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

transverse response from anomalies cont'd

effective gauge field action (Burkov & Zyusin)

$$S[a,b] = \frac{1}{32\pi}\int d^3x dt\, \theta\, \epsilon^{\mu\nu\rho\sigma} f_{\mu\nu} f_{\rho\sigma} =$$

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$$\theta = \theta(x,t) = 2b_i x^i - 2b_0 t$$

variation $\delta_a S[a,b]$ yields current response

$$\underline{j} = \frac{1}{4\pi^2}\,\mu \underline{B}$$ CME (?)

$$\underline{j} = \frac{1}{4\pi^2}\,\underline{b}\times\underline{E}$$ AHE

Effective theory of the disordered Weyl metal

Minneapolis , May 1st, 2015

Alexander Altland, Dmitry Bagrets (:(not here…)

disorder qualitative

field theory construction

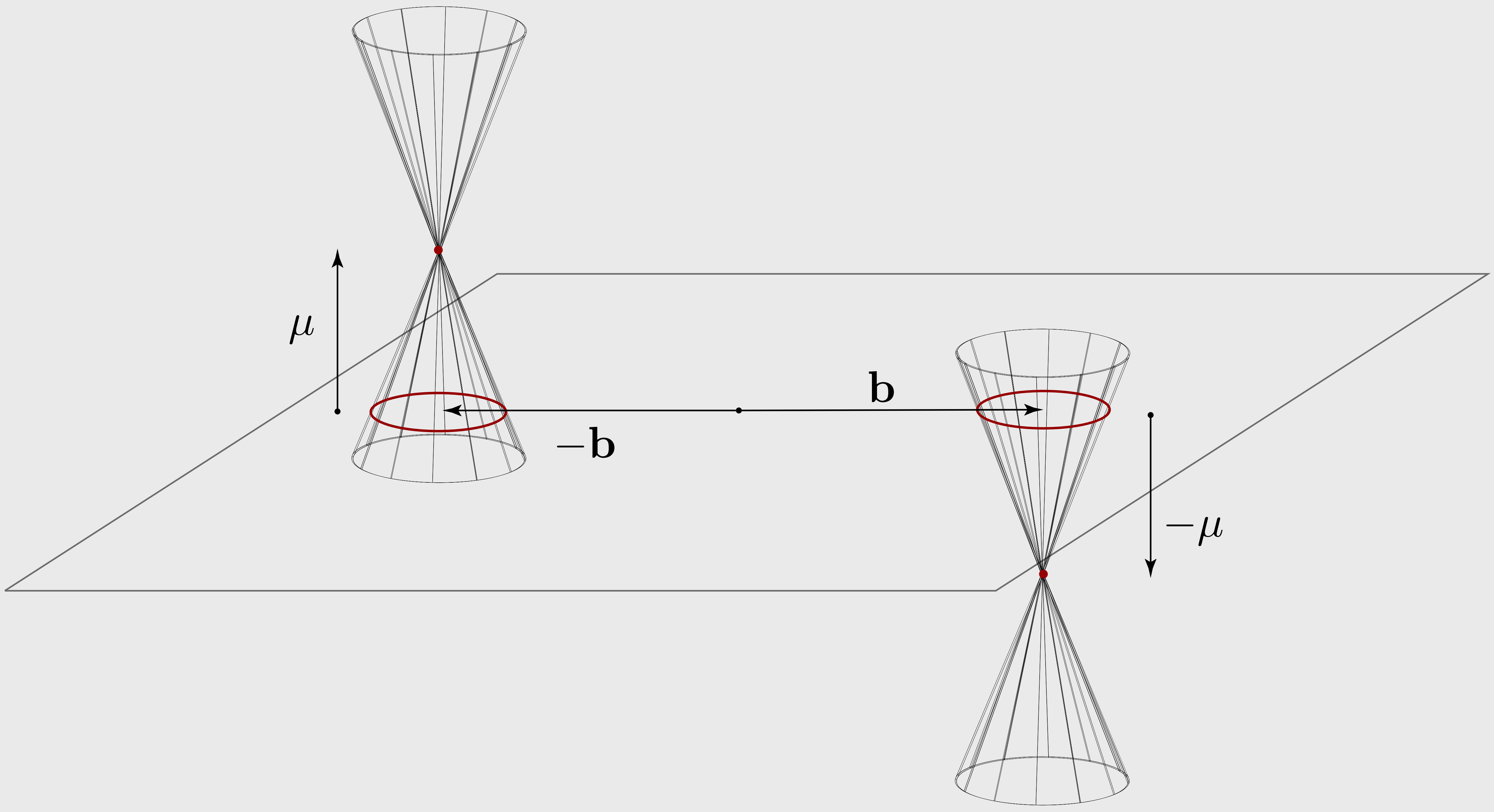
discussion

arXiv:1501.06537, PRL to be published

disorder qualitative

diffusive transport

Q: do anomalies survive internode scattering? (Burkov et al. 14, Son & Spivak 13, $\mu \gg \tau^{-1}$)



diffusive transport

Q: do anomalies survive internode scattering? (Burkov et al. 14, Son & Spivak 13, $\mu \gg \tau^{-1}$)

charge density

charge/axial charge coupling ~B

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial z^2} + \Gamma\frac{\partial n_a}{\partial z}$$

axial charge density

$$\frac{\partial n_a}{\partial t} = D\frac{\partial^2 n_a}{\partial z^2} + \Gamma\frac{\partial n}{\partial z} - \frac{n_a}{\tau_a}.$$

internode coupling

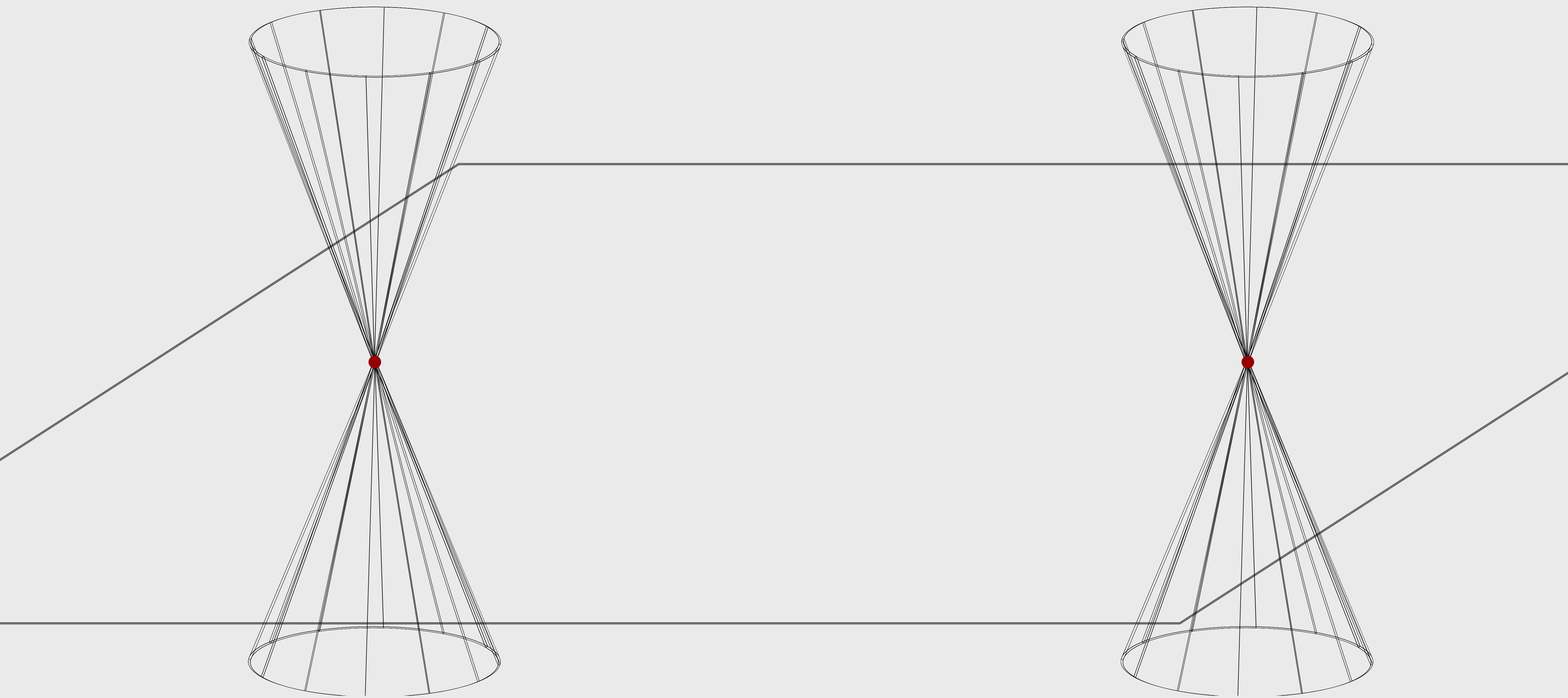
substitution of first into second eq.

$$\sigma = \sigma_0 + \frac{B^2\tau_a}{4\pi^2\nu}$$

DoS

disorder at the Weyl nodes

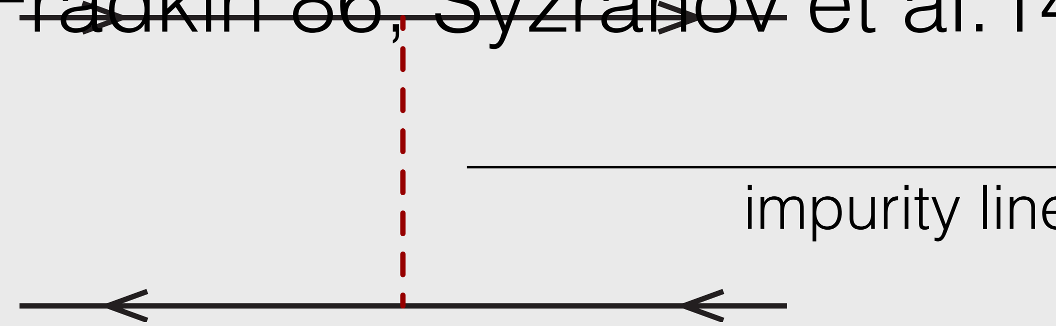
Q: how does disorder affect the nodes? (Fradkin 86, Syzranov et al.14)



disorder at the Weyl nodes

Q: how does disorder affect the nodes? (Fradkin 86, Syzranov et al.14)

$$\langle V(\mathbf{x})V(\mathbf{x}')\rangle = \delta(x-x')\gamma_0$$

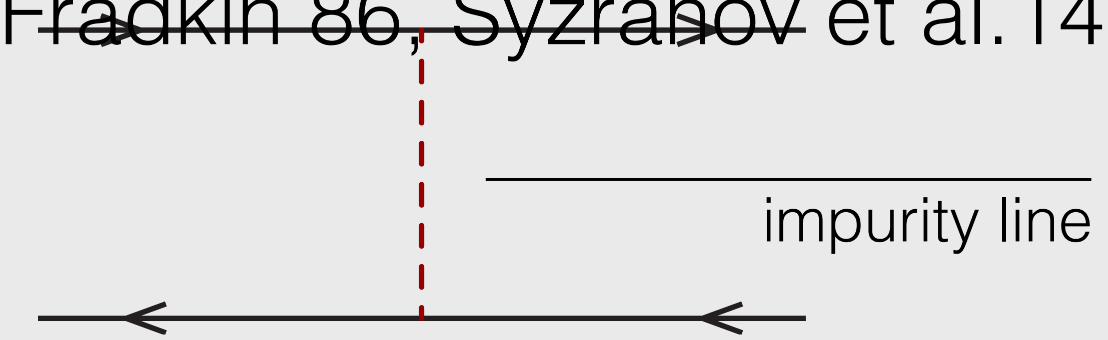


dimensional analysis: disorder irrelevant in d=3

$$\frac{d\gamma}{dl} = -\gamma$$

disorder at the Weyl nodes

Q: how does disorder affect the nodes? (Fradkin 86, Syzranov et al.14)

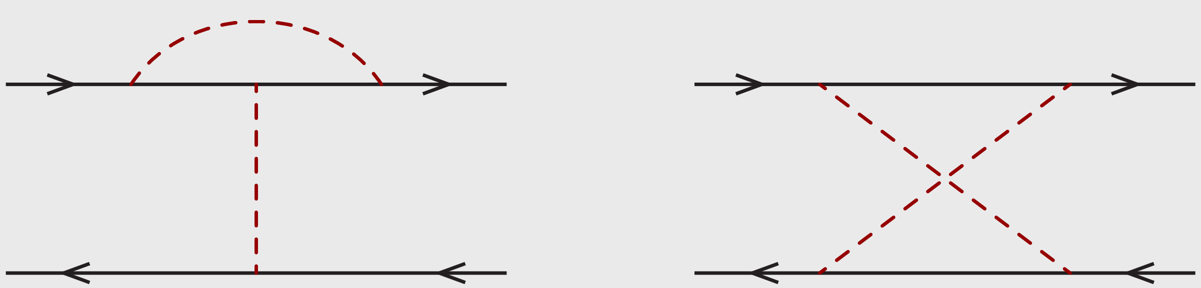


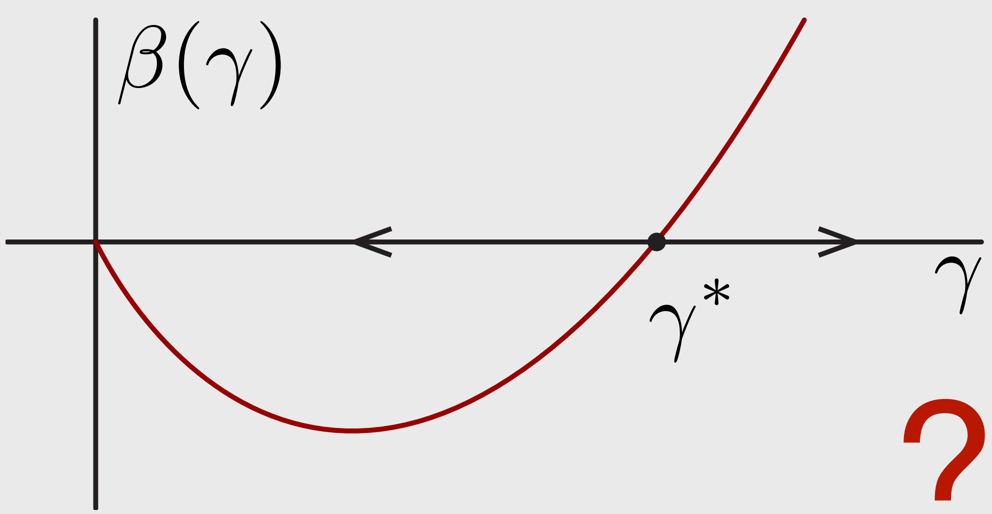
$$\langle V(\mathbf{x})V(\mathbf{x}')\rangle = \delta(x-x')\gamma_0$$

dimensional analysis: disorder irrelevant in d=3

$$\frac{d\gamma}{dl} = -\gamma$$

… but only at the clean fixed point …





$$\frac{d\gamma}{dl} = -\gamma + \frac{\gamma^2}{\gamma^*} \qquad \gamma^* \sim \Lambda^{-1}$$

?

this talk

describe physics at large distance scales in the supercritically disordered system

suspect: 3d Anderson metal

with added topological signatures

field theory

field theory

(I) replica functional and disorder averaging

(II) stationary phase, symmetry breaking, Goldstone modes

(III) anomaly and gauge structure

(IV) gradient expansion

field theory

(I) replica functional and disorder averaging

(II) stationary phase, symmetry breaking,

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replica functional & disorder averaging

replica index
$a = 1, \dots, R$

$$\left\langle G^{+} G^{-} \dots \right\rangle \longleftrightarrow Z = \int D(\bar{\psi}, \psi) \left\langle e^{i\bar{\psi}^{a}(\hat{\mu}+i\delta\tau_3-\hat{H})\psi^{a}} \right\rangle$$

discriminates
between retarded and advanced

replica rotation symmetry: action invariant under

$$\boxed{\psi \to T\psi, \qquad T \in \mathrm{U}(2R)}$$

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field theory

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stationary phase analysis

disorder averaged functional

$$Z = \int D(\bar{\psi},\psi) DT \, e^{i\bar{\psi}(\hat{\mu}+i\kappa T\tau_3 T^{-1}-\hat{H})\psi}$$

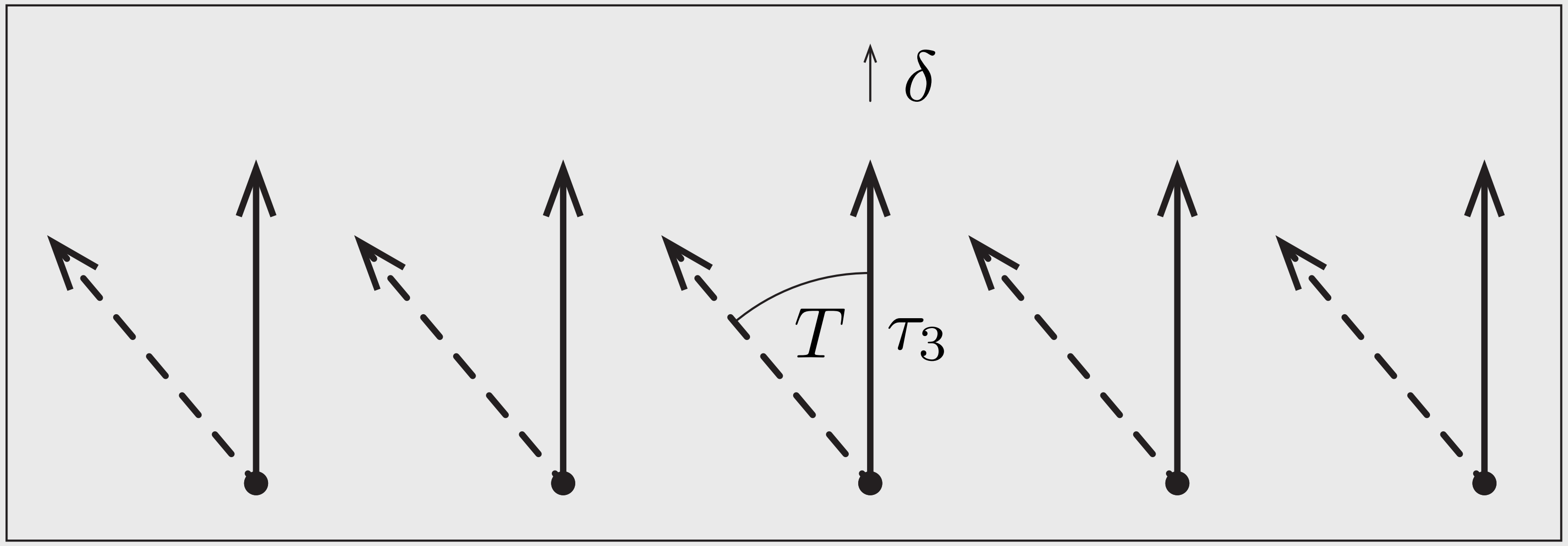
$\kappa = (2/\pi)v\Lambda(1-\gamma^*/\gamma)$

Fradkin 86

replica rotation symmetry spontaneously broken *if* system is supercritically disordered

Goldstone modes

$$Z = \int D(\bar{\psi}, \psi) DT \, e^{i\bar{\psi}(\hat{\mu} + i\kappa T \tau_3 T^{-1} - \hat{H})\psi}$$



action invariant under

$T \longrightarrow T_0 T, \qquad [T_0, \hat{H}] = 0$ change of ‘magnetization axis’

$T \longrightarrow T k(x), \qquad [k(x), \tau_3] = 0$ rotation around ‘magnetization axis’

field theory

(I) replica functional and disorder averaging

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field theory

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anomaly and gauge structure

$$Z = \int D(\bar{\psi}, \psi) DT \, e^{i\bar{\psi}(\mu + i\kappa T \tau_3 T^{-1} - \hat{H})\psi}$$

$$= \int D(\bar{\psi}, \psi) DT \, e^{i(\bar{\psi}T)\left(\mu + i\kappa\tau_3 - \hat{H} - T^{-1}[\hat{H}, T]\right)(T^{-1}\psi)}$$

anomaly and gauge structure

$$Z = \int D(\bar{\psi},\psi)DT\, e^{i\bar{\psi}(\mu+i\kappa T\tau_3 T^{-1}-\hat{H})\psi}$$

$$= \int D(\bar{\psi},\psi)DT\, e^{i(\bar{\psi}T)\left(\mu+i\kappa\tau_3-\hat{H}-T^{-1}[\hat{H},T]\right)(T^{-1}\psi)}$$

-> anomaly

anomaly and gauge structure

$$Z = \int D(\bar\psi,\psi) DT\, e^{i\bar\psi(\mu + i\kappa T\tau_3 T^{-1} - \hat H)\psi}$$

$$= \int D(\bar\psi,\psi) DT\, e^{i(\bar\psi T)\left(\mu + i\kappa\tau_3 - \hat H - T^{-1}[\hat H, T]\right)(T^{-1}\psi)}$$

-> anomaly

generator of
Goldstonemode fluctuations

Goldstone mode generators $T^{-1}[\hat H, T] \longrightarrow T^{-1}\partial_i T \equiv A_i$

Transformation $T \to Tk$ implies (cf. Volovik & Yakovenko 89)

$$\boxed{A_i \to k^{-1}(A_i + k\partial_i k^{-1})k}$$

non-abelian gauge theory with gauge group $\mathrm{U}(R) \times \mathrm{U}(R)$

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field theory

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gauge field expansion

note: regulator action essential (cf. Redlich 84)

$$S_{\rm reg}[T] = -S[T]\big|_{\kappa\searrow 0} \longrightarrow \begin{cases} -S[T] & k\to\infty \quad \text{UV} \\ 0 & k\to 0 \quad \text{IR.} \end{cases}$$

PHYSICAL REVIEW D VOLUME 29, NUMBER 10 15 MAY 1984

Parity violation and gauge noninvariance of the effective gauge field action in three dimensions

A. N. Redlich

Center for Theoretical Physics, Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

(Received 17 January 1984)

The effective gauge field action due to fermions coupled to SU(N) gauge fields in three dimensions is found to change by $\pm\pi|n|$ under a homotopically nontrivial gauge transformation with winding number n. This gauge noninvariance can be eliminated by adding a parity-violating topological term to the action, or by regulating the theory in a way which produces this term automatically in the effective action. The Euler-Heisenberg effective action is calculated in the SU(2) theory and in QED.

gauge field expansion

note: regulator action essential (cf. Redlich 84)

$$S_{\rm reg}[T] = -S[T]\Big|_{\kappa\searrow 0} \longrightarrow \begin{cases} -S[T] & k\to\infty \quad \text{UV} \\ 0 & k\to 0 \quad \text{IR.} \end{cases}$$

lowest order expansion

$$\boxed{\begin{aligned} S_{\rm d}[A] &= \frac{\sigma_{xx}}{8}\sum_i \int d^3x\, {\rm tr}([A_i,\tau_3]^2), \\ S_{\rm top}[A] &= -\frac{\sigma_{xy}}{2}\epsilon^{3ij}\int d^3x\, {\rm tr}(\tau_3\partial_i A_j), \end{aligned}}$$

discussion I

layered quantum Hall action

$$S_{\mathrm{d}}[A] = \frac{\sigma_{xx}}{8} \sum_i \int d^3x \, \mathrm{tr}([A_i, \tau_3]^2),$$

$$S_{\mathrm{top}}[A] = -\frac{\sigma_{xy}}{2} \epsilon^{3ij} \int d^3x \, \mathrm{tr}(\tau_3 \partial_i A_j),$$

layered quantum Hall action

$$S_{\mathrm{d}}[A] = \frac{\sigma_{xx}}{8}\sum_i \int d^3x \, \mathrm{tr}([A_i, \tau_3]^2),$$

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$$\downarrow$$

$$\boxed{\begin{aligned} S_{\mathrm{d}}[Q] &= \frac{\sigma_{xx}}{8} \int d^3x \, \mathrm{tr}(\partial Q^2) \\ S_{\mathrm{top}}[Q] &= -\frac{\sigma_{xy}}{8}\epsilon^{3ij} \int d^3x \, \mathrm{tr}(Q\partial_i Q \partial_j Q) \end{aligned}}$$

layered quantum Hall action

$$S_{\mathrm{d}}[Q] = \frac{\sigma_{xx}}{8} \int d^3x \, \mathrm{tr}(\partial Q^2)$$

$$S_{\mathrm{top}}[Q] = -\frac{\sigma_{xy}}{8} \epsilon^{3ij} \int d^3x \, \mathrm{tr}(Q \partial_i Q \partial_j Q)$$

coupling constants

$$\sigma_{xx} = \frac{\mu^2 + 3\kappa^2}{6\pi\kappa v} \longrightarrow \begin{cases} \frac{\kappa}{2\pi v} & \mu \to 0 \\ \frac{1}{3} v^2 \gamma & \mu \gg \kappa \end{cases}$$

longitudinal conductivity

$$\sigma_{xy} = \frac{1}{\mathrm{Vol}} \partial_B N \longrightarrow \frac{1}{L_z} \sum_n C_n \xrightarrow{L_z b \gg 1} \frac{b}{2\pi}$$

Hall conductivity

disorder averaged Chern
number of n-th layer

layered quantum Hall action

$$S_{\mathrm{d}}[Q] = \frac{\sigma_{xx}}{8} \int d^3x \, \mathrm{tr}(\partial Q^2)$$

$$S_{\mathrm{top}}[Q] = -\frac{\sigma_{xy}}{8} \epsilon^{3ij} \int d^3x \, \mathrm{tr}(Q \partial_i Q \partial_j Q)$$

action describes layered quantum Hall systems

Three-Dimensional Disordered Conductors in a Strong Magnetic Field: Surface States and Quantum Hall Plateaus

J. T. Chalker

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A. Dohmen

Institut für Theoretische Physik, Universität zu Köln, Zülpicher Strasse 77, 50937 Köln, Germany

(Received 6 July 1995)

We study localization in layered, three-dimensional conductors in strong magnetic fields. We demonstrate the existence of three phases—insulator, metal, and quantized Hall conductor—in the two-dimensional parameter space obtained by varying the Fermi energy and the interlayer coupling strength. Transport in the quantized Hall conductor occurs via extended surface states. These surface states constitute a subsystem at a novel critical point, which we describe using a new, directed network model.

layered quantum Hall action

$$\boxed{\begin{aligned} S_{\mathrm{d}}[Q] &= \frac{\sigma_{xx}}{8}\int d^3x\,\mathrm{tr}(\partial Q^2) \\ S_{\mathrm{top}}[Q] &= -\frac{\sigma_{xy}}{8}\epsilon^{3ij}\int d^3x\,\mathrm{tr}(Q\partial_i Q\partial_j Q) \end{aligned}}$$

Localization and Metal-Insulator Transition in Multilayer Quantum Hall Structures

Ziqiang Wang

Department of Physics, Boston College, Chestnut Hill, Massachusetts 02167

(Received 10 June 1997)

We study the phase structure and Hall conductance quantization in weakly coupled multilayer electron systems in the integer quantum Hall regime. We derive an effective field theory and perform a two-loop renormalization group calculation. It is shown that (i) finite interlayer tunnelings (however small) give rise to successive metallic and insulating phases and metal-insulator transitions in the unitary universality class; (ii) the Hall conductivity is not renormalized in the metallic phases in the 3D regime; and (iii) the Hall conductances are quantized in the insulating phases. In the bulk quantum Hall phases, the effective field theory describes the transport on the surface. [S0031-9007(97)04575-4]

layered quantum Hall action

$$S_{\mathrm{d}}[Q] = \frac{\sigma_{xx}}{8} \int d^3x \, \mathrm{tr}(\partial Q^2)$$

$$S_{\mathrm{top}}[Q] = -\frac{\sigma_{xy}}{8} \epsilon^{3ij} \int d^3x \, \mathrm{tr}(Q \partial_i Q \partial_j Q)$$

$$\frac{dg_{xx}}{d \ln L} = g_{xx} - \frac{1}{3\pi^4 g_{xx}}, \qquad \frac{dg_{xy}}{d \ln L} = g_{xy},$$

$$g_{\mu\nu} = \sigma_{\mu\nu} \Lambda^{-1}$$

Ohmic scaling (Wang, 97)

discussion II

CS action

higher order expansion in A/derivatives

$$S_{\mathrm{CS}}[A] = -\frac{i\epsilon^{ijk}}{8\pi} \sum_{s=\pm} s \int d^3x \, \mathrm{tr}\left(A_i P^s \partial_j A_k P^s + \frac{2}{3} A_i P^s A_j P^s A_k P^s \right)$$

projector onto
retarded/advanced sector

a strange animal: not a Hopf term, but also no genuine CS-term

building faith in CS action

gauge (non-)invariance: under $A_i \to k^{-1}(A_i + k\partial_i k^{-1})k$ CS action changes as

$$S_{\mathrm{CS}}[A] \longrightarrow S_{\mathrm{CS}}[A] + i\pi(\#_{k-\mathrm{windings}})$$

compensated by regulator action

building faith in CS action

gauge (non-)invariance: under $A_i \to k^{-1}(A_i + k\partial_i k^{-1})k$ CS action changes as

$$S_{\mathrm{CS}}[A] \longrightarrow S_{\mathrm{CS}}[A] + i\pi(\#_{k-\mathrm{windings}})$$

compensated by regulator action

quadratic expansion reproduces axial diffusion propagator

$$A_i = \partial_i W - \frac{1}{2}[W, \partial_i W] + \ldots$$

$$W = \begin{pmatrix} & B \\ -B^\dagger & 0 \end{pmatrix}$$

quadratic expansion con't

quadratic action

from CS action

Pauli in
nodal space

DoS

$$S^{(2)}[B, B^\dagger] = \frac{\nu}{2\pi} \int d^3x \, \mathrm{tr} \left(B \left(D\partial^2 + i\omega + \frac{1}{\tau_s}(1 + \tau_1^{\mathrm{n}}) - i\frac{B}{\nu}\tau_3^{\mathrm{n}} \right) B^\dagger \right)$$

diffusion
constant

internode scattering
time

produces axial diffusion mode

$$\frac{\partial n}{\partial t} = D\frac{\partial^2 n}{\partial z^2} + \Gamma \frac{\partial n_a}{\partial z}$$

$$\frac{\partial n_a}{\partial t} = D\frac{\partial^2 n_a}{\partial z^2} + \Gamma \frac{\partial n}{\partial z} - \frac{n_a}{\tau_a}.$$

Bulk CME

substitute $A = a, a = \{a_i\}$ into action, where

$$a_3 = a(x)\tau_3, \qquad a_i = \frac{1}{2}\epsilon^{3ij} B x_j,\ i = 1,2$$

source field

static magnetic field B in 3-direction

$$\lim_{R\to 0} \frac{i}{4\pi R}\delta_{a(x)} Z = \langle j_3(x)\delta(\epsilon - H)\rangle = \frac{1}{4\pi^2} B \longrightarrow \frac{B\mu}{2\pi^2}$$

two nodes & $\int d\epsilon$

summary

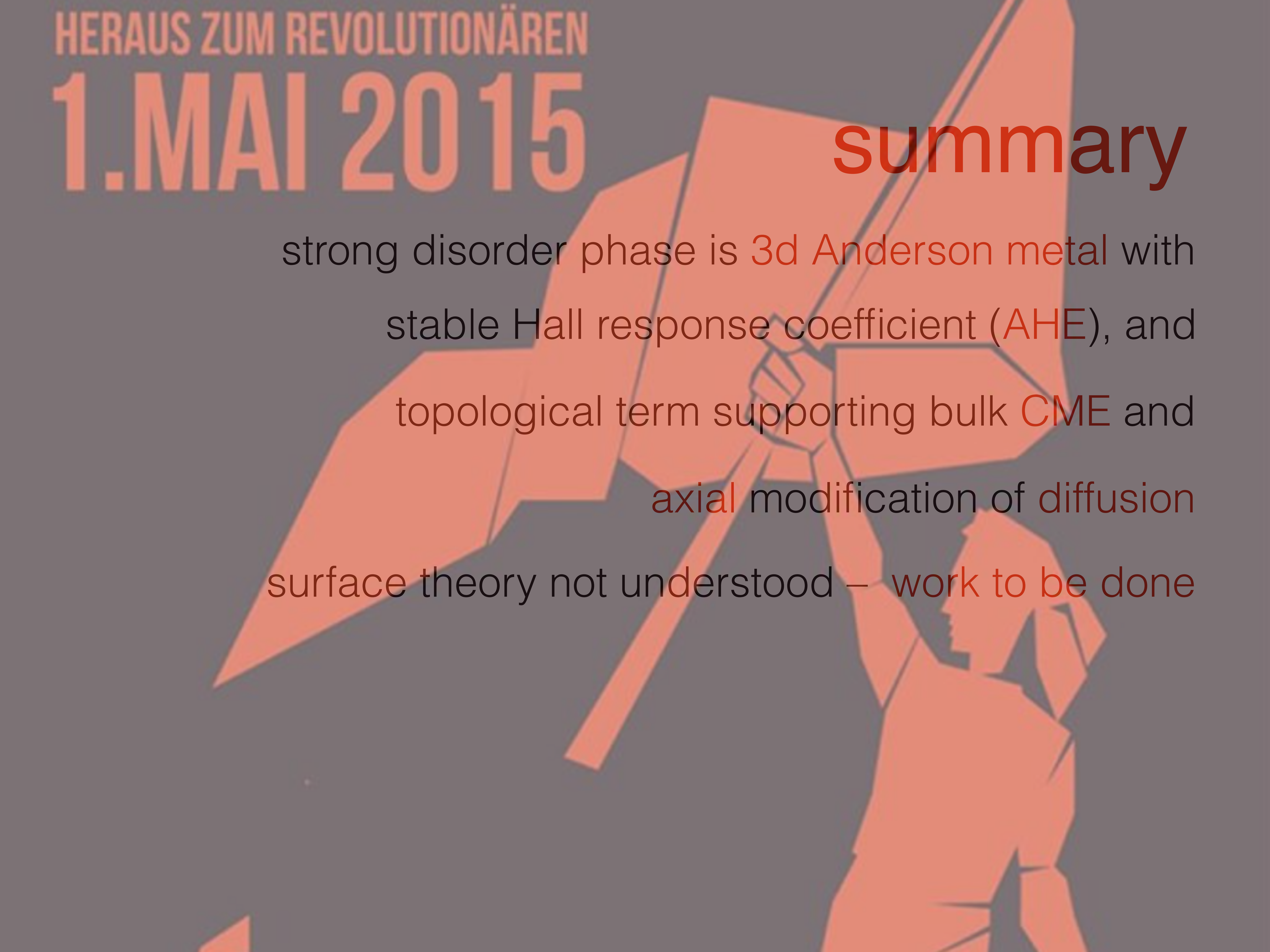
strong disorder phase is 3d Anderson metal with

stable Hall response coefficient (AHE), and

topological term supporting bulk CME and

axial modification of diffusion

surface theory not understood – work to be done



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