Surfaces of 3d symmetry protected phases

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with F. Burnell, X. Chen, A. Vishwanath

Plan:

1) Symmetry Protected Topological phases (SPTs)

- Surfaces of SPTs

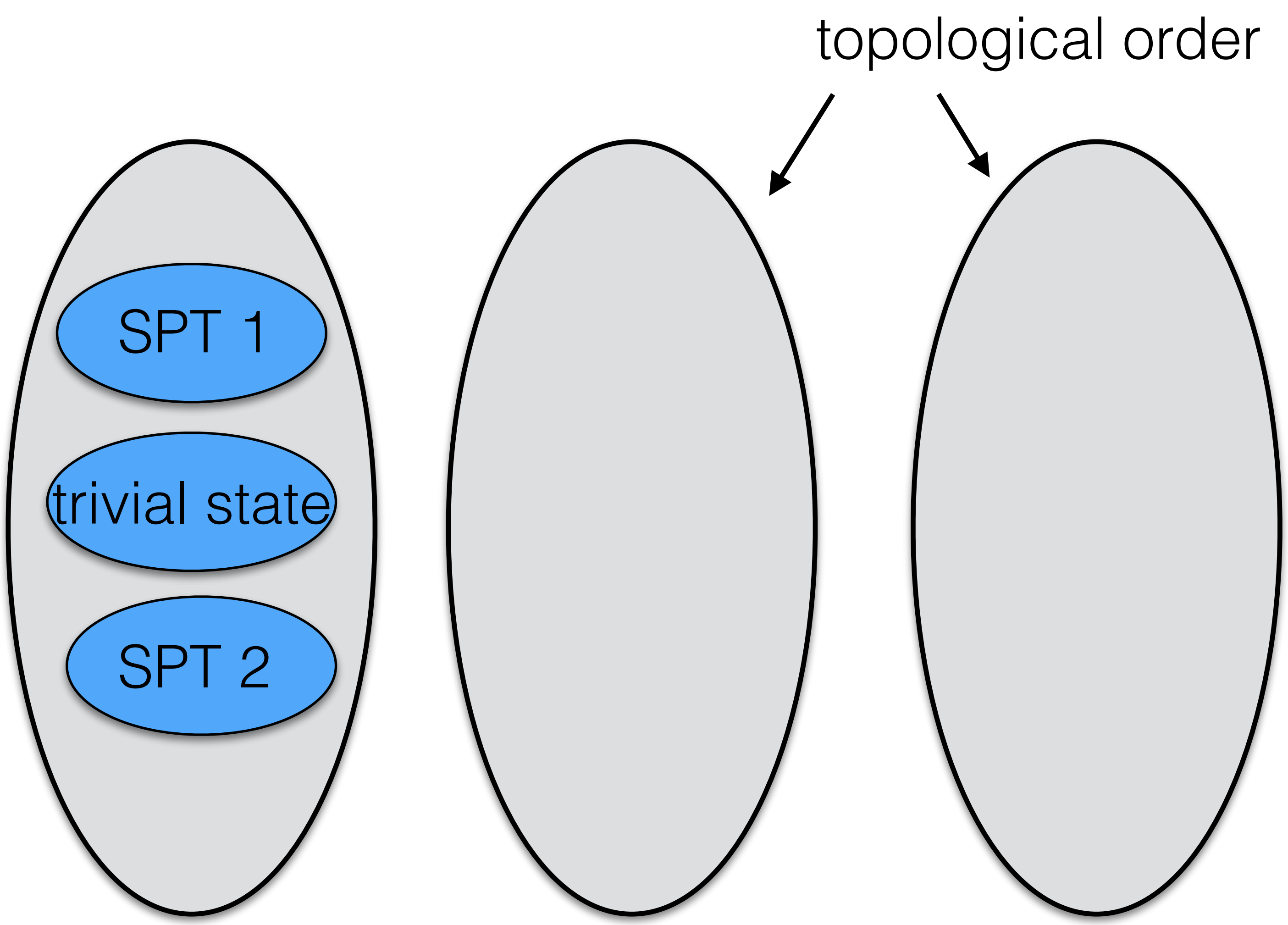
2) Fully gapped interacting surfaces

(c.f. Vishwanath, Senthil)

Symmetry protected topological phases

Setting: gapped Hamiltonians invariant under symmetry group G (e.g. time reversal, spin rotation)

Formal definition: A Hamiltonian H defines a non-trivial SPT if it can be continously connected to a 'trivial' symmetric Hamiltonian without closing the gap, but only at the expense of breaking the symmetry



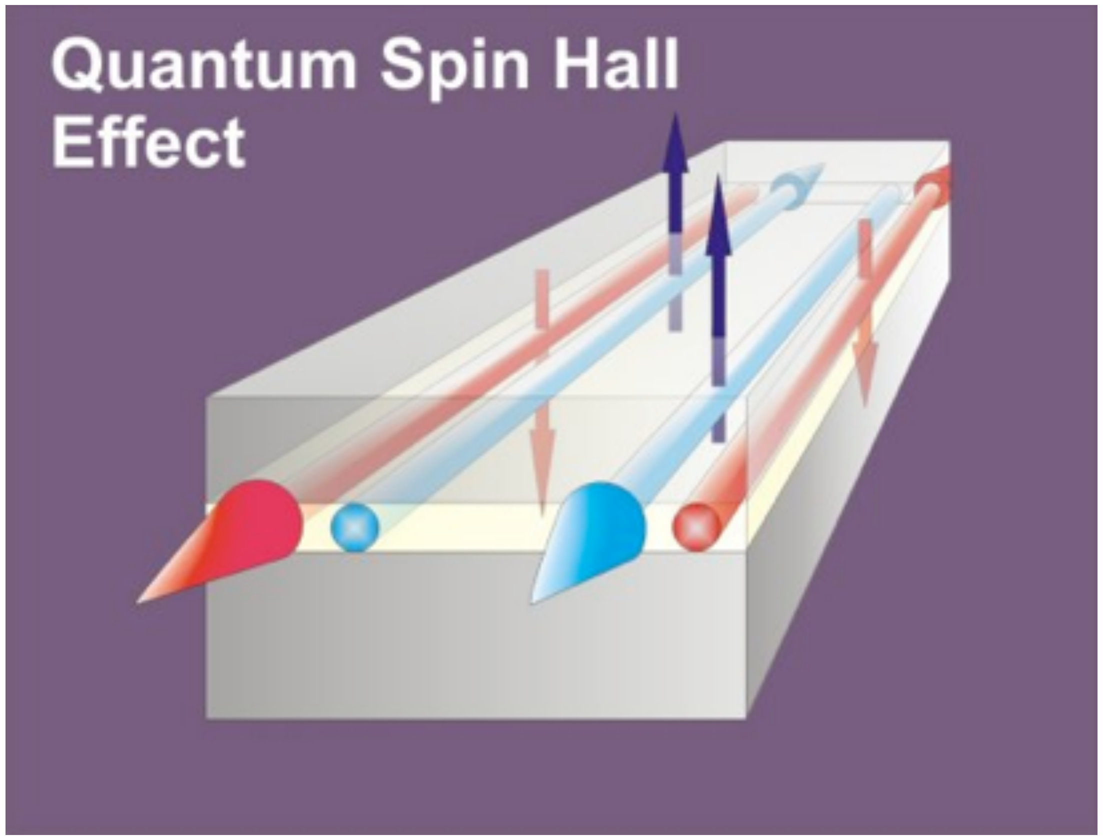
Example: Haldane phase of spin 1 antiferromagnet

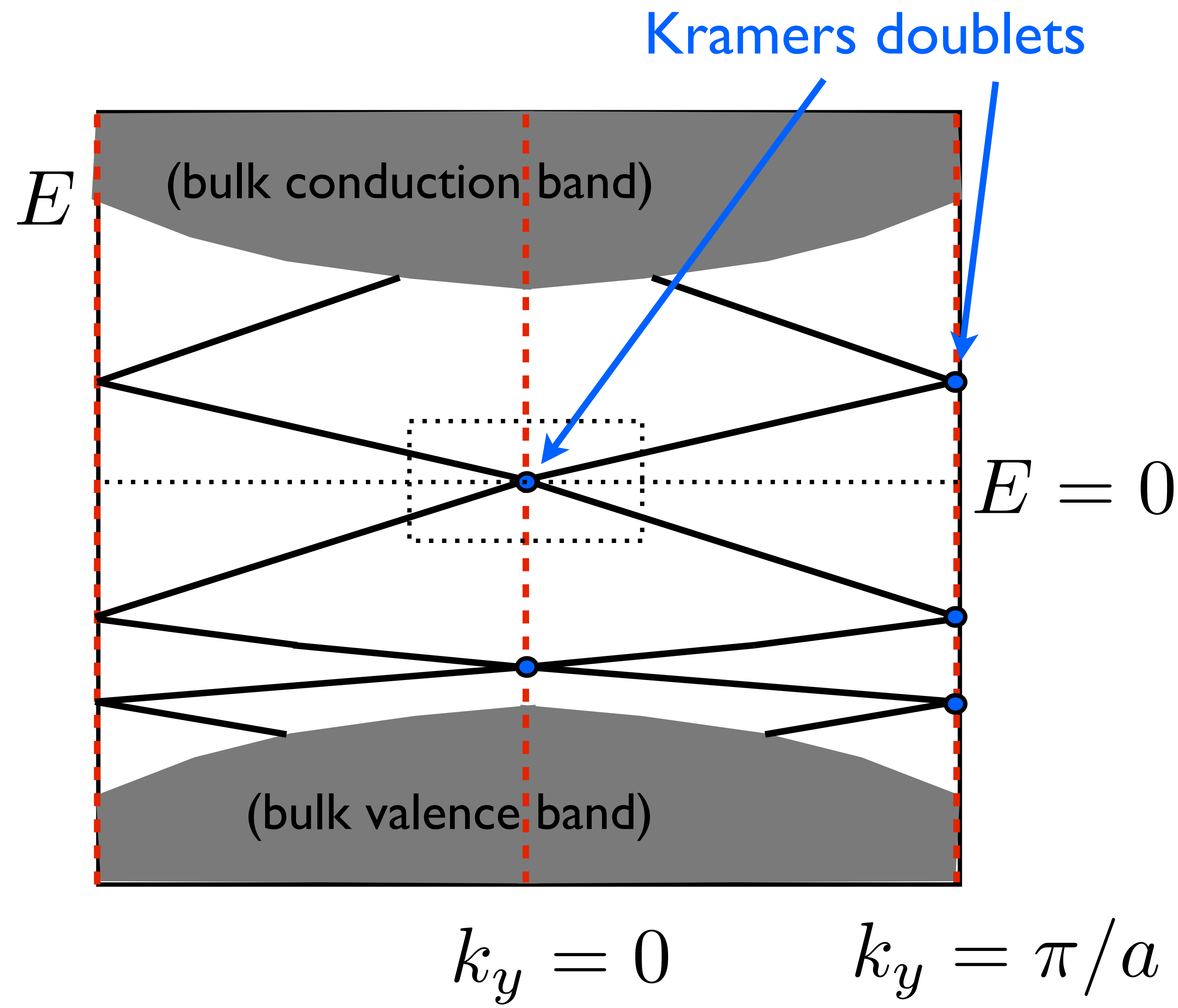
 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)$

 $= |+\rangle\langle\uparrow\uparrow| + |0\rangle\frac{\langle\uparrow\downarrow| + \langle\downarrow\uparrow|}{\sqrt{2}} + |-\rangle\langle\downarrow\downarrow|$

1d SPTs are characterized by protected degeneracies at their endpoints

Example: 2d quantum spin Hall effect



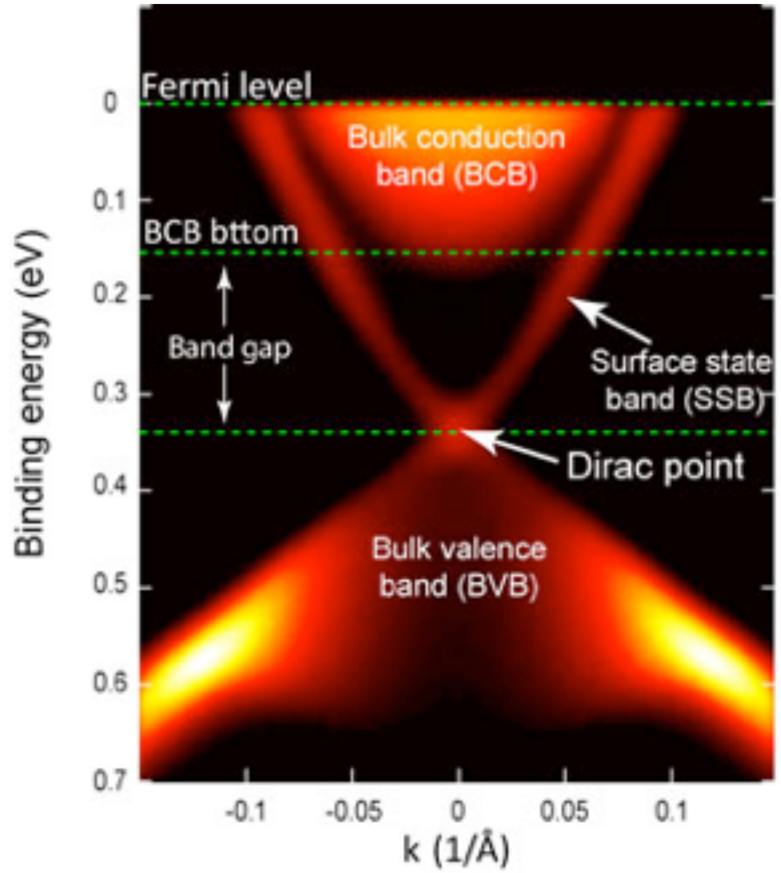


2 options for 2d SPTs:

1) gapless edge states

2) spontaneous symmetry breaking

Example: 3d topological insulator



3 options for 3d SPTs:

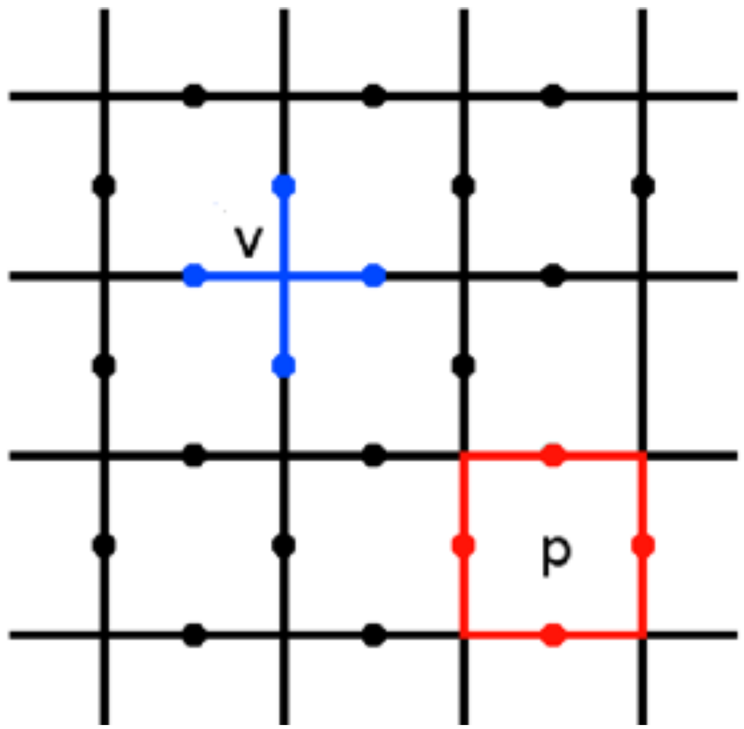
1) gapless surface states

2) spontaneous symmetry breaking

3) gapped, symmetric surface with topological order

String nets: 2D warmup

- $\mathbb{Z}_2$ gauge theory (toric code):

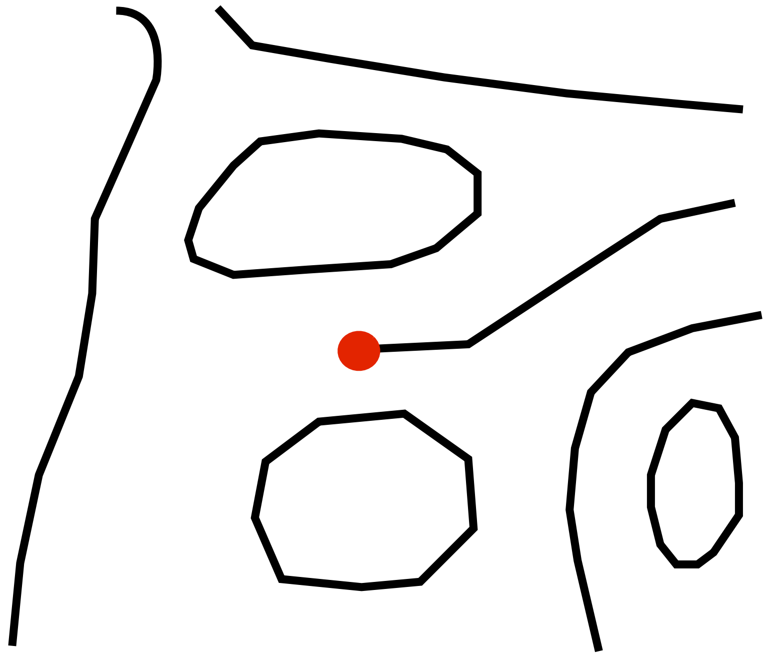


$$A_v = \prod_{i \in v} \sigma_i^x, \;\; B_p = \prod_{i \in p} \sigma_i^z.$$

$$H_{TC} = -J \sum_v A_v - J \sum_p B_p, \;\; J > 0.$$

- loop model:

$$|\Psi\rangle = \sum_{\text{loop configs L}} |L\rangle$$



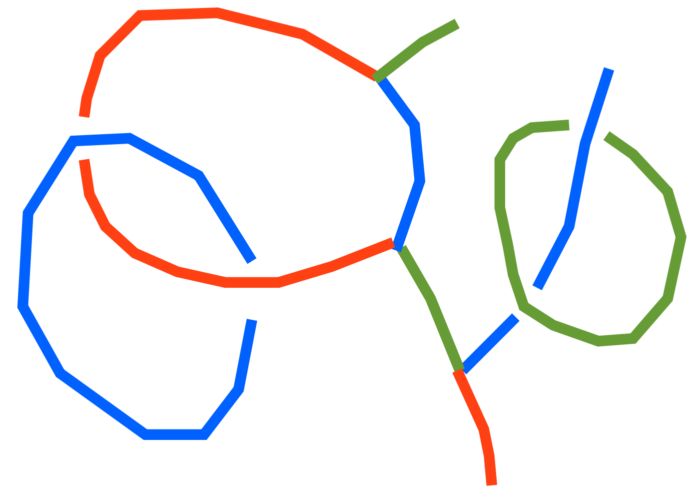
String net model for *bosonic* 3D time reversal SPT

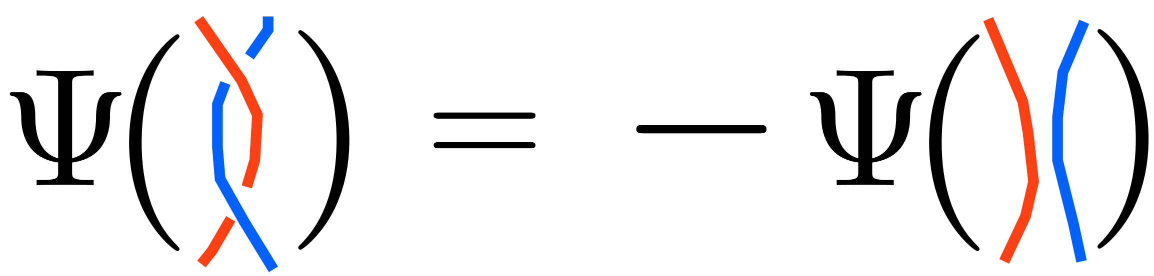
bosonic T-SPT:

Senthil & Vishwanath, Wang & Senthil, Kitaev

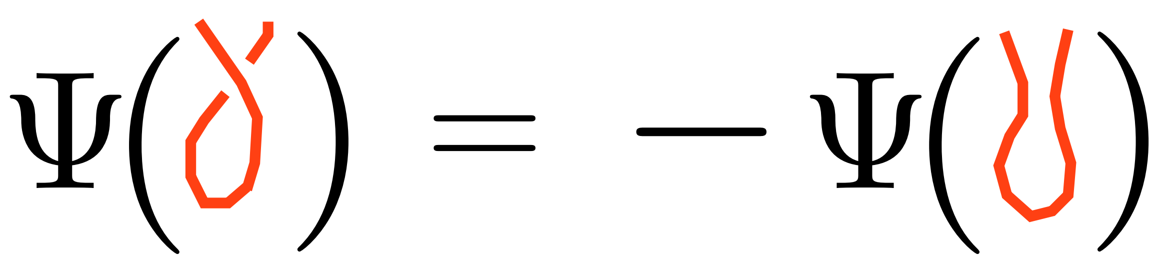
Burnell, Chen, Fidkowski, & Vishwanath

(based on Walker-Wang construction, see also von Keyserlingk, Burnell, Simon)

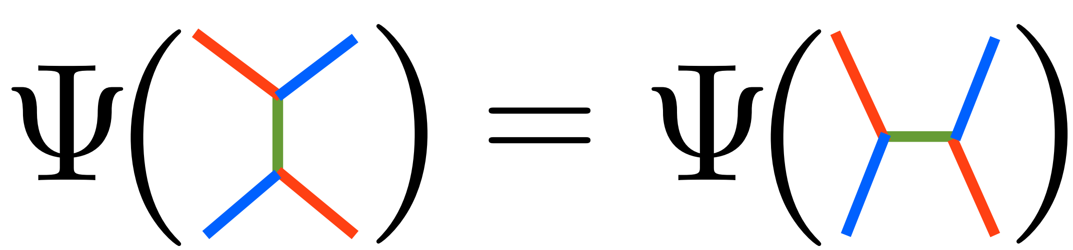




(and for red/green & blue/green braids)



(and for green & blue)

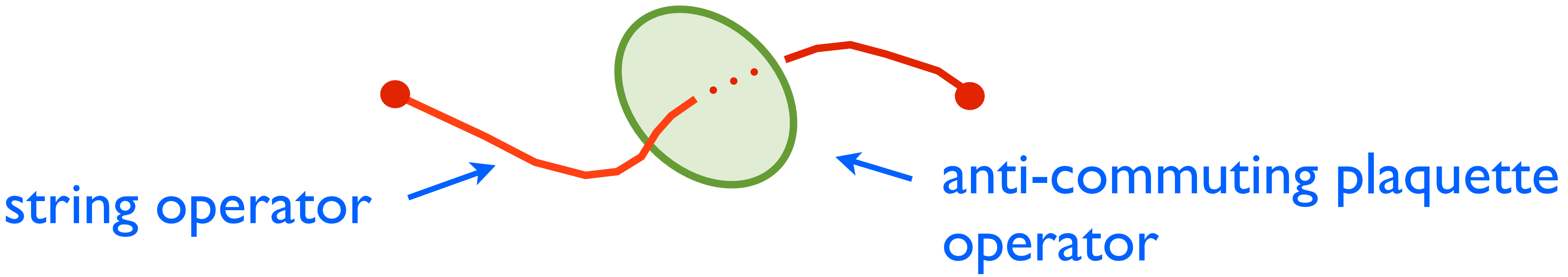


(and for all other admissible color combinations)

Properties of bosonic topo SC:

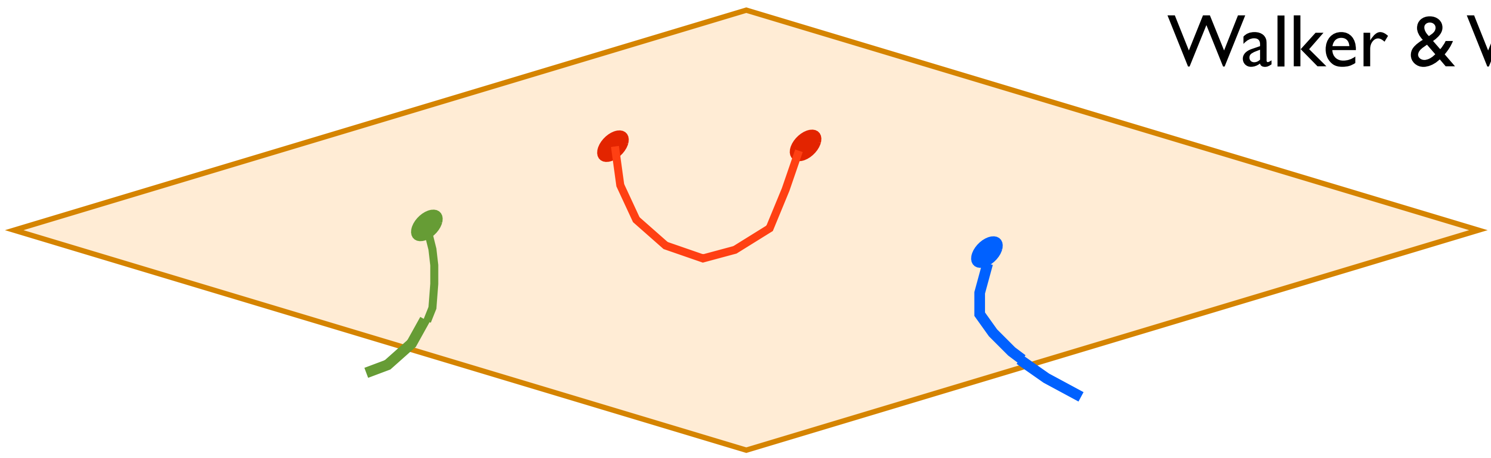
1) time reversal symmetric

2) no bulk deconfined excitations:



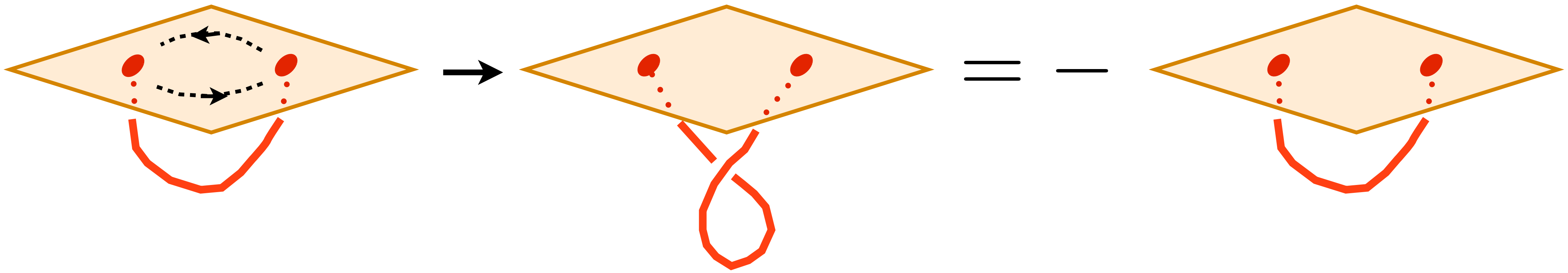
3) surface topological order:

Walker & Wang, Burnell & Simon

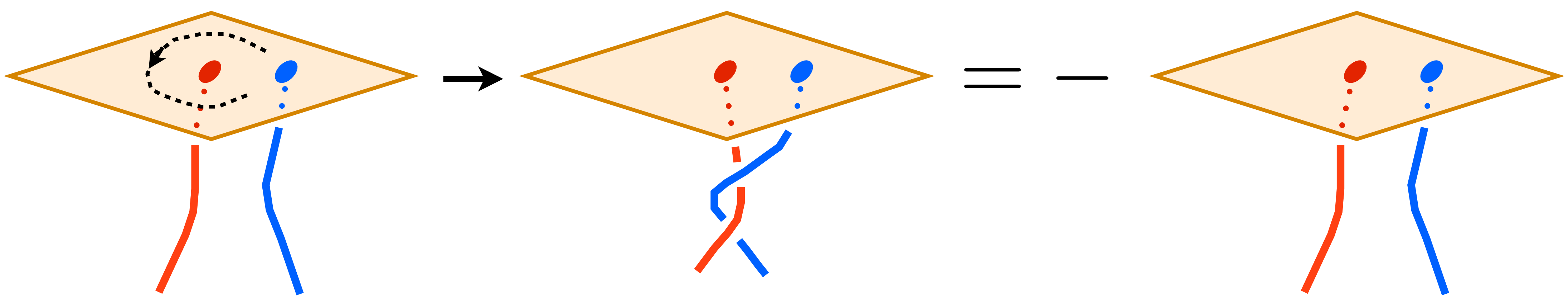


Surface topological order:

- quasiparticles are fermions:



- and mutual semions:



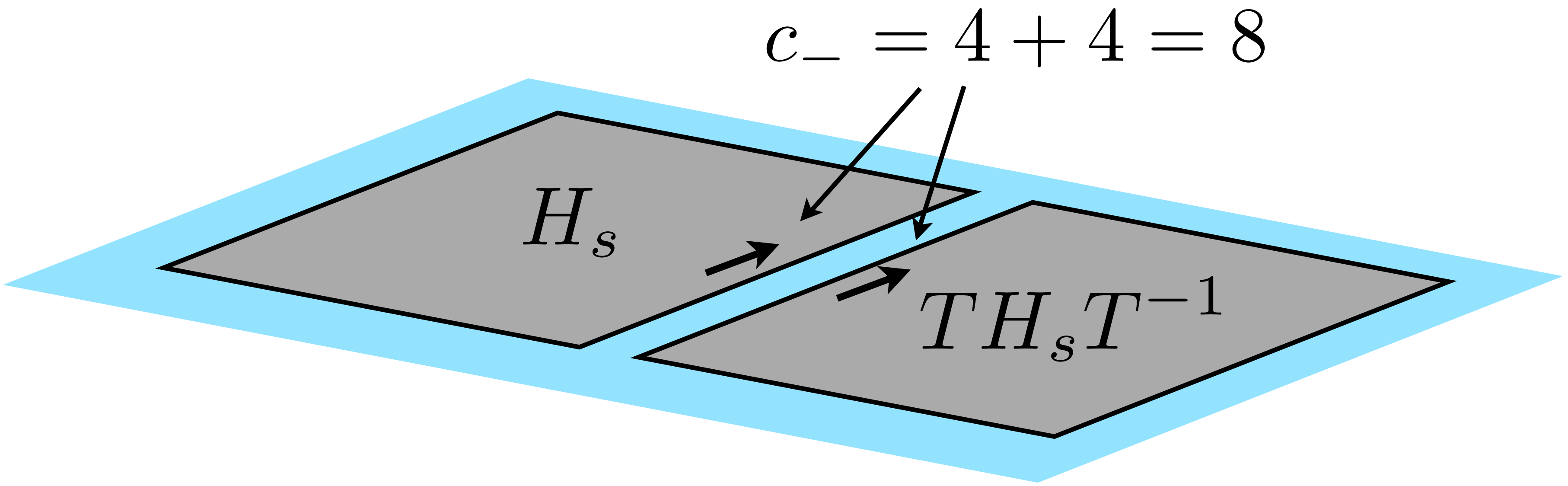
A well known topological order:

$U(1)$ Chern-Simons theory with $K = \begin{pmatrix} 2 & -1 & -1 & -1 \\ -1 & 2 & 0 & 0 \\ -1 & 0 & 2 & 0 \\ -1 & 0 & 0 & 2 \end{pmatrix}$

- chiral

Anomalous nature of T-symmetric surface state:

- remove topological order by coupling in time reversed copies of chiral theory:



- "1/2 of E_8 state"

TI surface = 1/2 of $\nu = 1$ IQHE $(c_- = 1)$

Topo SC surface = 1/2 of p+ip SC $(c_- = 1/2)$

General idea: bootstrap surface state to 3d SPT

2D anyon theory (braided fusion category)	$\longrightarrow$	3D model based on Walker-Wang construction

Guess surface theory for 3d topo SC? Constraints:

1) fermionic theory: include electron as *bulk* excitation

2) electron must be Kramers doublet $(T^2 = -1)$

3) "1/2 of p+ip": $c_- = 1/4$

Examples of fermionic 2d topological orders:

- $\nu = 1/3$ Laughlin state: (6 quasiparticles)

  not T-symmetric, wrong $c_-(=1)$

- $\nu = 5/2$ Moore-Read state: (12 quasiparticles)

  not T-symmetric, wrong $c_-(=0 \bmod 1/2)$

"Simplest" fermionic 2d topological order:

- $SO(3)_3$ Chern-Simons: (4 quasiparticles)
  - T-symmetric
  - electron *must* be Kramers doublet
  - central charge $c_- = 9/4 = 1/4 \bmod 1/2$

$SO(3)_3$ Chern-Simons Theory:

- integral spin sub-theory of $SU(2)_6$:

$$\boxed{0} \quad \cancel{\frac{1}{2}} \quad \boxed{1} \quad \cancel{\frac{3}{2}} \quad \boxed{2} \quad \cancel{\frac{5}{2}} \quad \boxed{3}$$

trivial q.p. I — $\theta_I = 1$

semion s — $\theta_s = i$

$\xleftrightarrow{T}$

semion $\tilde{s}$ — $\theta_{\tilde{s}} = -i$

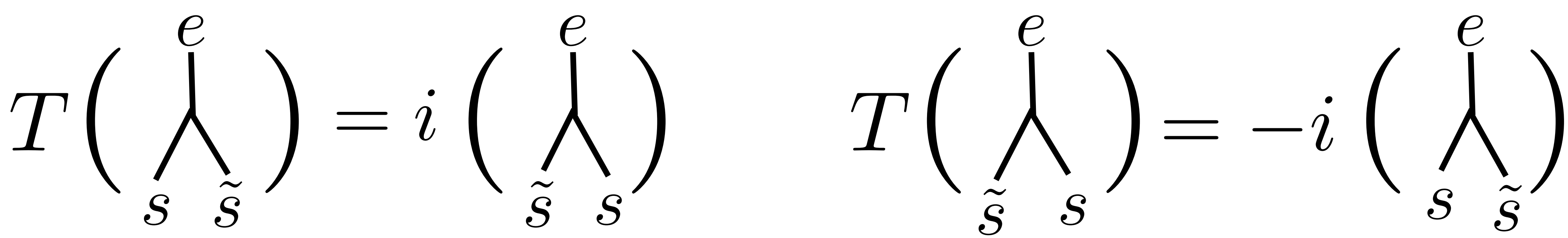
electron e — $\theta_e = -1$

- non-abelian:

$$s \times s = I + s + \tilde{s}$$
$$s \times e = \tilde{s}$$

$$d_s = d_{\tilde{s}} = 1 + \sqrt{2}$$

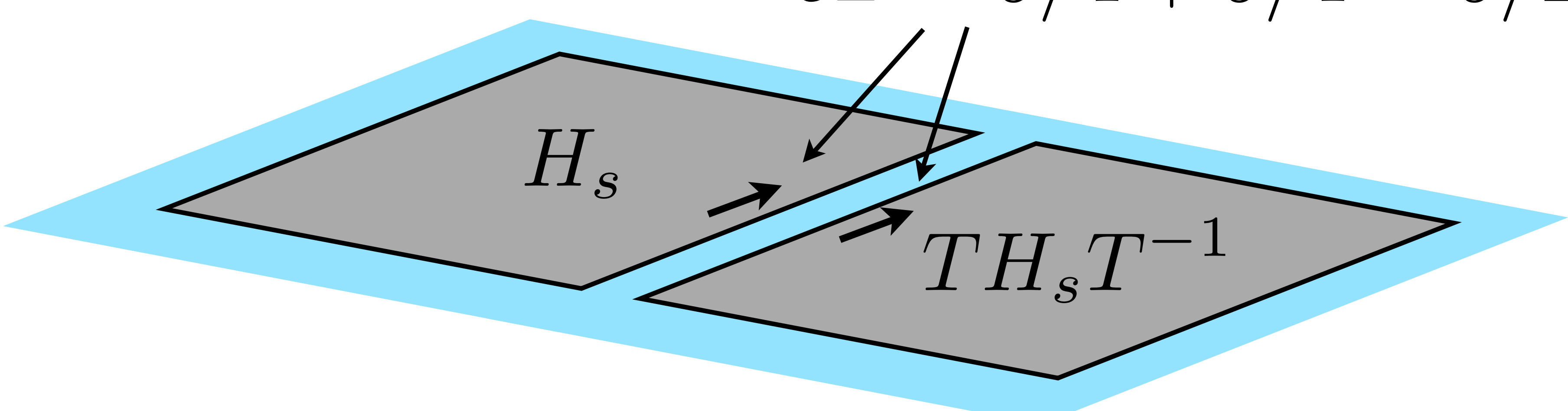
- electron must be Kramers doublet:



decorated Walker-Wang model: bind Haldane chains to e-links

- surface poised between p+ip and p-ip:

$$c_- = 9/4 + 9/4 = 9/2$$



Topological terminations for other ν

with M. Metlitski, X. Chen, A. Vishwanath
(see also Wang, Senthil, arXiv: 1401.1142)

- $\nu = 2$: $\{1, S\} \times \{1, F\} \qquad T: S \leftrightarrow SF$

- all other ν can be obtained by stacking copies of $\nu = 2$:
  - $\nu = 4$: $\{1, S_1\} \times \{1, S_2\} \times \{1, F\}$
  - $\nu = 8$: $\{1, S_1\} \times \{1, S_2\} \times \{1, S_1'\} \times \{1, S_2'\} \times \{1, F\}$

    $S_1 S_2 S_1' S_2'$ is a $T^2 = 1$ boson; condense it and get:

    $\{1, e, m, \varepsilon\} \times \{1, F\}$

    bosonic TSc

  - $\nu = 16$: trivial gapped T-symmetric surface