A Firm Foundation: Essays on Firm Choice

A THESIS
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

Enoch Hill

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Doctor of Philosophy

Fabrizio Perri and Timothy J. Kehoe, Advisors

December, 2014

Acknowledgements

I'm deeply grateful to my advisors Fabrizio Perri and Timothy Kehoe for their advice and guidance in completing this dissertation. I'm also grateful to Cristina Arellano, Manuel Amador, and Kei-Mu Yi for their advice and encouragement along the way. This dissertation would not exist without their help and instruction. I am grateful to Terry Roe for his comments as well as for being a member of my dissertation committee. I would like to thank James Schmitz and Elena Pastorino for their valuable feedback. I'm especially grateful to my coauthors David Perez-Reyna and Kai Ding. The long discussions we had together have made me a better economist and human being. I additionally benefitted from conversations with and advice from Mike Maio, Alessandro Peri, Kelvin Wong and Georgios Stefanidis. Many thanks to seminar participants of the Workshop in International Trade and Development at the University of Minnesota as well as participants at the Washington University's 8th Annual Graduate Student Conference and at the XIX Workshop on Dynamic Macroeconomics in Soutomaoir, Spain. Above all, I am indebted to my wife, my parents and my brother for their unconditional support, their love and their wisdom and to my Lord and Savior Jesus Christ for His eternal love and salvation.

Dedication

To my wife Alissa for making this possible.

Abstract

This dissertation consists of three essays.

In the first essay, Kai Ding and I develop a dynamic general equilibrium model in which a change in the importance of firm specific human capital can explain the new pattern in labor productivity as well as partially account for the decrease in the rate of employment recovery (jobless recoveries) observed in the most recent three recessions. Additionally, we present empirical support that the importance of firm specific human capital has in fact increased for recent recessions.

In the second essay, David Perez-Reyna and I incorporate theft in a macroeconomic setting with the goal of understanding the effects of public law enforcement (PLE) on the incarceration rate, aggregate output and average welfare. Our primary finding is that there exists a non-monotonic relation between the level of PLE and all three of these aggregate variables. In particular, for countries with relatively small amounts of PLE, there is an inverse relationship between PLE and both aggregate production and welfare primarily due to an increase in the incarceration rate. However, for countries with higher levels of PLE, the level is positively related to production and welfare and inversely related with the incarceration rate. When applied to a dynamic model, our mechanism can explain why we observe such a large difference in the level of PLE across countries.

In the third essay, David Perez-Reyna and I present a general equilibrium model where heterogeneous consumers endogenously choose whether to become workers, consumers or entrepreneurs in order to analyze how limits on the leverage of banks affect real output. In our model tighter limits on the leverage of banks cause an increase in the spread between the interest rate that banks charge for loans and the interest rate that banks pay for deposits. A higher spread results in two types of distortions: First, firms with the same productivity will have different size. Second, productive firms will cease to exist, while nonproductive ones will enter. These distortions result in lower production.

Contents

List of Tables

List of Figures

Chapter 1

Cautious Hiring: An Explanation for Changes in the Labor Cycle

1.1 Introduction

Even nearly five years after the end of the 2008 recession, unemployment rates continue to remain above levels experienced in the 15 years prior to the recession. This slow rate of recovery in the unemployment rate is a phenomenon shared with the two previous recessions but contrasts sharply with the faster recoveries in unemployment rates prior to the early 1990s. This emergent pattern has commonly been referred to among policymakers, journalists as well as in the economic literature as *jobless recovery*[1]. Jobless recoveries refer to periods following recessions in which rebounds in aggregate output are accompanied by much slower recoveries in aggregate employment[2].

A second empirical fact that has been documented in the literature is that for these same three recessions the previously procyclical pattern in labor productivity has reversed[3]. In fact, for the most recent three recessions the drop in aggregate production is accompanied by an increase in labor productivity. We conjecture that these changes are linked and propose a mechanism that can simultaneously account for both changes. Namely, the increase in relative productivity of experienced to inexperienced workers.

[1]For examples in the literature refer to Gordon (1993), Bernanke (2003), Galí, Smets, and Wouters (2012) and Jaimovich and Siu (2012)

[2]Definition taken from Jaimovich and Siu (2012)

[3]See McGrattan and Prescott (2012) and Galí and van Rens (2010) for examples.

New to our model is the idea that workers develop firm specific human capital, new workers are relatively unproductive compared with experienced workers in the same job. Since this human capital is firm specific, it is not fully reflected in the wage. Consequently, the firm makes an investment in new workers and earns a profit on experienced workers. In this sense, the set of experienced workers with a firm constitute a human capital stock which the firm manages. Hiring new workers lowers the average profits of the firm and makes bankruptcy more likely. Bankruptcy is costly to firms since it causes a loss of their experienced workers. Firms can receive a higher rate of growth only in exchange for a larger probability of default. As a result firms hire cautiously and grow slowly. A key parameter in our model is the relative productivity of experienced workers to inexperienced workers. As this ratio increases, the cost of bankruptcy also increases and firms endogenously choose to grow more slowly.

When the economy is recovering from a recession, startup firms have two opposing forces on labor productivity. On the one hand, average size of startup firms is small due to cautious hiring. Due to decreasing returns to scale in the labor input, this tends to increase the average labor productivity. We interpret this as a larger capital to worker ratio. The second and countering force is that after the recession, startup firms desire to grow which results in a larger proportion of inexperienced workers. These newly hired workers have lower productivity in their new positions than their more experienced counterparts which drives down average labor productivity.

In a fast recovery the later effect outweighs the former causing labor productivity to be procyclical, when a recovery is slower the downward effects of newly hired workers is diluted across time which causes the returns to scale effect to dominate. We use a change in the relative productivity of experienced to inexperienced workers to generate a change in the speed of recovery. While our mechanism does not account for the entirety of the change in average labor productivity over the business cycle between older and more recent recessions, it can account for roughly half of what we observe in the data. Additionally, our benchmark model is able to qualitatively replicate the shift in productivity from being procyclical in earlier recessions and countercyclical in more recent recessions.

Ever since the recession of the early 1990s a number of macroeconomists including Galí, Smets, and Wouters (2012), Gordon (1993), Groshen and Potter (2003), and

Bernanke (2003) have discussed the slower rate of recovery in labor through recent business cycles. Still, no consensus on the cause of the change in the speed of employment recovery has emererged. Our model suggests that one of the reasons for the change in the speed of recovery is explained by a change in the importance of firm specific human capital since the mid 1980s.

We propose that the more technical and job specific the skills demanded by employers are, the greater the relative value of an experienced worker relative to an inexperienced one. Multiple strands in the literature provide support that this parameter has been changing in the last couple of decades. Specifically, the rise of computers has been cited as a substitute for routine labor and a complement for nonroutine cognitive tasks as presented in the empirical work of Autor, Levy, and Murnane (2003). That this phenomena is at the core of the business cycle is further documented in Jaimovich and Siu (2012). Related to this literature is the growing work documenting that employment is becoming increasingly concentrated in the tails of the occupational skill distribution as seen in Acemoglu (1999), Autor and Dorn (2013), Autor, Katz, and Kearney (2006) Goos and Manning (2007) and Goos, Manning, and Salomons (2009).

In addition to changes in the importance of firm specific skills for a job, our mechanism relies on consideration of a firm's labor decision as a form of investment. Our work builds upon the seminal work of Becker (1964) in the sense that the costs and returns of firm-specific training are largely attributed to firms. This gap between the marginal product of labor and wages ties into a vast theoretical literature including Mortensen and Pissarides (1994) and Lazear (1979) as well as a sparser empirical literature on the topic. This arises largely due to the difficulties which lie in determining the marginal contributions of a single worker in the context of a firm with multiple employees working on a common output[4].

The rest of the paper is organized as follows. In Section 1.2 we document the two main features of the more recent recessions. Section 1.3 presents the model and definition of equilibrium. In Section 1.4 we present our steady state results. Section 1.5 calibrates our parameters to the data. In Section 1.6 we present the business cycle properties and section 1.7 concludes.

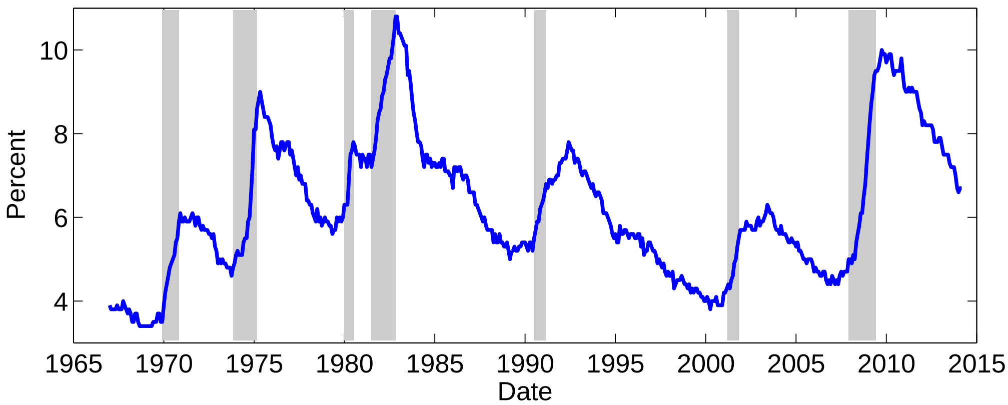
[4]For examples of such papers which attempt to measure gains in marginal output independent of wages see Shaw and Lazear (2008) or Isen (2012).

1.2 Data

Ever since the double dip recession in the early 1980s a number of new patterns emerge in the macroeconomic data, in this section we highlight two of them. Well cited in the literature is that recoveries in unemployment rate have been slower in the recessions in the 1990s onwards when compared with prior recessions. Perhaps even more significant is that these recoveries in unemployment are not only slower, but that they are also slower *relative* to the recovery in GDP. Another way to observe this same fact is to look at the correlation between average labor productivity and GDP over the business cycle. If we measure average labor productivity as real output per hour worked, then if GDP recovers more slowly than labor productivity, recessions will be characterized by a pro-cyclical relation between these two series which is what we observe across business cycles through the "double dip" recession of the early 1980s. Alternatively, if GDP recovers faster than labor productivity, recessions will be characterized by a counter-cyclical relation between these two series which is precisely what we observe in the recessions following 1990.

The unemployment rate between January of 1967 through March of 2014 is displayed in Figure 1.2. As can be observed in the figure, the speed of recovery in the unemployment rate following the three most recent recessions has been significantly slower than in prior recessions.

Figure 1.1: Civilian Unemployment Rate - Jan. 1967-Mar. 2014



Source: Current Population Survey[5]

In order to observe this more clearly, Table 1.1 displays the difference in unemployment between the end of each of the last six recessions and each of the following five years. In recessions prior to the mid 1980s recovery in unemployment following the low point in GDP growth began immediately and continued in the following five years. Contrast that with the most recent three recessions where unemployment has stayed fairly level or even increased in the first two to three years following the recession and even five years after the recession the unemployment rate recovery lags by approximately two percentage points behind the earlier recessions[6].

Table 1.1: Unemployment Recovery

NBER Recession End Date	UR	Dif. in UR Between Recession End and Select Dates: 1 Year	2 Years	3 Years	4 Years	5 Years
Fast Recovery Recessions						
11/1970	5.9	-0.1	0.6	1.1	NA	NA
03/1975	8.6	1	1.2	2.3	2.8	2.3
11/1982	10.8	2.3	3.6	3.8	3.9	5
Average		**1.07**	**1.80**	**2.40**	**3.35**	**3.65**
Slow Recovery Recessions						
03/1991	6.8	-0.9	-0.9	-0.2	0.8	0.8
11/2001	5.5	-0.4	-0.7	-0.2	0.3	0.9
06/2009	9.5	0.1	0.4	1.3	2	NA
Average		**-0.40**	**-0.40**	**0.30**	**1.03**	**0.85**

Source: Current Population Survey and NBER recession dates. Author's calculations

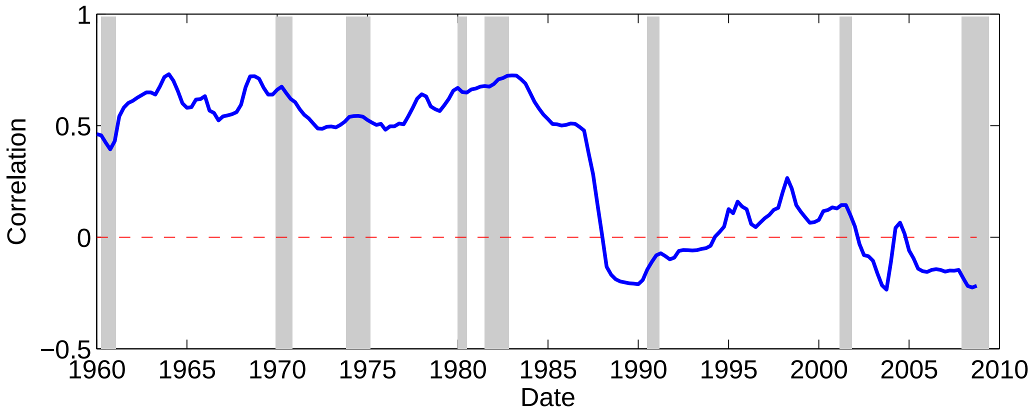
Our second and related fact is that unemployment rate recoveries slowly *relative* to the recovery in GDP. Figure 1.2 displays the 10 year centered moving average correlation between labor productivity and GDP. Each point in the series uses 10 years of seasonally

[5]Data are taken from the Labor Force Statistics of the CPS, downloaded from the BLS website (http://www.bls.gov/data/) on April 10, 2014. As in the footnote on page 3 of Jaimovich and Siu (2012), Employment data at the aggregate and occupational level are available dating back to 1959. However, there are well-documented issues with the early CPS data, especially during the 1961 recession; see, for instance, the 1962 report of the President's Committee to Appraise Employment and Unemployment Statistics entitled "Measuring Employment and Unemployment." The recommendations of this report (commonly referred to as the Gordon report) led to methodological changes adopted by the BLS beginning in 1967 (Stein (1967)). As such, our analysis uses data beginning in July 1967.

[6]The 1980 recession is omitted due to the proximity of the following recession. For the same reason certain dates following the 1971 recession are not included

adjusted HP filtered GDP and real output per hour data surrounding the displayed date to calculate the correlation. Prior to the mid 1980s Labor Productivity was procyclical, however in the time following the mid 1980s labor productivity has been acyclical or even slightly counter cyclical.

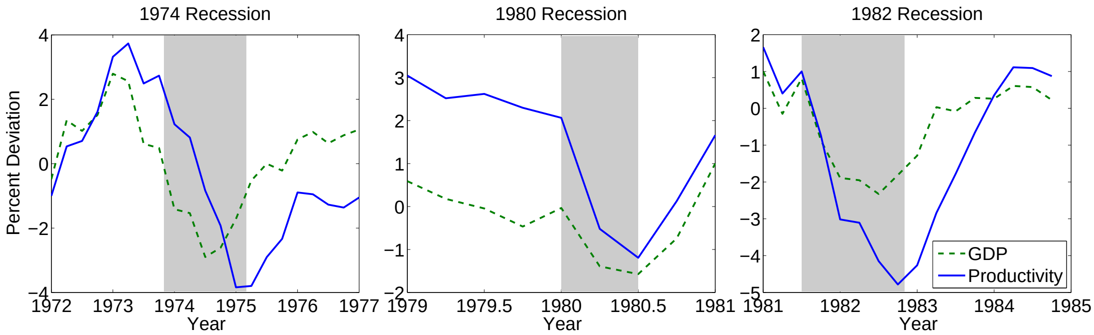
Figure 1.2: 10 Year Centered MA Correlation in Labor Productivity and GDP



Source: Bureau of Labor Statistics and Bureau of Economic Analysis

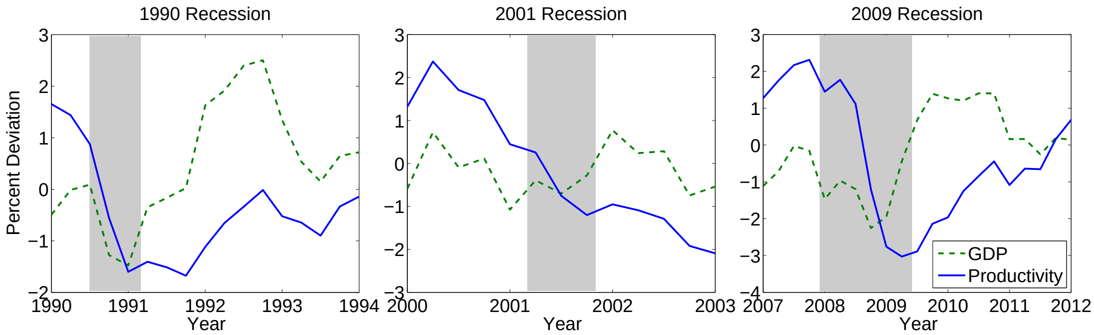
This change has particularly affected the behavior of these series during and immediately following each recession. In the three most recent recessions prior to 1990 (which we will label the "Fast Recoveries,") the two series behaved similarly, dropping with or just before the onset of the recession and recovering fairly quickly following the recession. However in the three recessions following the cycles featuring fast recovery, GDP patterns more or less are similar to the earlier recessions with a slight retardation in the recovery rate whereas labor productivity remains fairly flat leading into the recession, increases rapidly beginning mid recession, and falls a few years after the recession. These features can be observed in Figures 1.3 and 1.4.

Figure 1.3: The "Fast Recovery" Recessions



Source: Bureau of Labor Statistics, Authors' calculations

Figure 1.4: The "Slow Recovery" Recessions



Source: Bureau of Labor Statistics, Authors' calculations

1.3 Environment

There are three types of agents in our model, a representative household, a continuum of heterogeneous firms, and financial intermediaries.

The households have preferences over a single consumption good, and they inelastically supply labor to the firms. The households own the firms and consume the dividends each period.

Firms operate a decreasing returns to scale technology which produces the consumption good using effective labor as the input. This technology is subject to idiosyncratic productivity shocks. Firms have to make their hiring decisions before the realization of the shock.

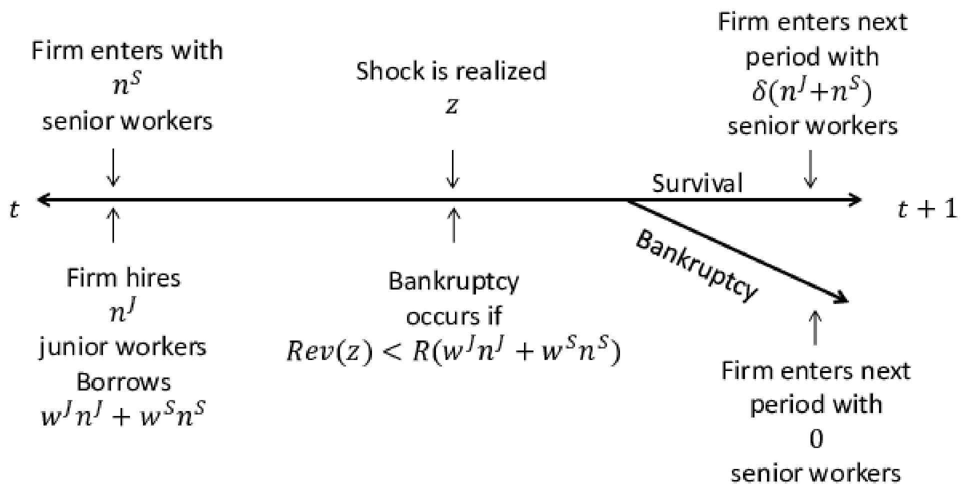
Our model features two types of workers distinguished by their experience levels: junior workers and senior workers. Experience refers to experience *with the specific firm* where a worker is employed. In the first period a worker is at a firm, they are considered junior with a labor productivity of θ^J, meaning that each unit of labor supplied by the junior worker will contribute θ^J units of effective labor. After the initial period, junior workers become senior workers and offer a labor productivity of $\theta^S > \theta^J$. Once a worker becomes senior, he will stay senior forever unless he separates with the firm due to firm bankruptcy or exogenous separation which happens with probability $1 - \delta$.

Firms hire labor and must pay the wage bill before production takes place. The wage payment is financed through an intra-period loan from the financial intermediaries.

The financial intermediaries are risk neutral and offer one-period state-uncontingent loans to firms. Firms default on their debt if current period revenue falls short of the payment due. Upon default, all revenues are seized by the financial intermediaries and the firms' stock of senior workers is lost. The higher the default probability, the higher interest rate the firm is required to pay on their loan.

The timing is as follows. In the beginning of each period, firms make hiring decisions, financial intermediaries lend funds to firms and wages are paid. Next, productivity shocks are realized, output is produced, and revenues are earned. Firms repay loans to financial intermediaries if able and pay out the remaining profits as dividends to the households, otherwise they declare bankruptcy. Firms that have defaulted enter again in the next period with zero senior workers.

Figure 1.5: Timing of the Model



1.3.1 Households

In our model, there is a measure L of households. The households' problem is static and trivial. Each period they supply labor inelastically to the firms and consume the dividends paid out by the firms. The only part of the household problem which will be relevant for the general equilibrium solution will be the stochastic discount factor which firms will use to weight future payouts in various states of the world. Therefore, for simplicity, we model each household as being identical and comprised of the average share of senior, junior and unemployed workers. The households' problem is given by:

$$\max_{\{c_t\}_0^\infty} \sum_{t=0}^{\infty} E\left[\beta^t u(c_t)\right]$$
$$\text{s.t. } c_t \leq w_t^J s_t^J + w_t^S s_t^S + d_t, \text{ for all } t$$

where s_t^J and s_t^S represent the share of junior and senior employed workers in the economy.

The stochastic discount factor of the households is given by:

$$\Lambda_t \equiv \frac{\beta E\left[u^{'}(c_{t+1})\right]}{u^{'}(c_t)}$$

In equilibrium, the resource constraint implies that consumption for each household is equal to the total output of the economy divided by the measure of households:

$$c_t = \frac{Y_t}{L}$$

1.3.2 Firms

Hiring Decision

There is a measure 1 of firms. They each produce a homogenous good according to the production function $y\left(z, n^J, n^S\right) = z\left[\theta^J n^J + \theta^S n^S\right]^{\alpha}$, where $z \sim F$ is an idiosyncratic productivity shock which is iid across firms and time. Firms enter each period with n^S senior workers and decide how many junior workers n^J to hire. Firms finance their wage bill, which can be written as $\left(w^S n^S + w^J n^J\right)$, by taking out an intra-period loan from the financial intermediary. Next they receive their productivity shock z and produce. If the revenue from selling their final good is sufficient to pay back the financial intermediary, firms pay back their debt, distribute the remaining profits as dividends to the households and continue in the next period with $\delta\left(n^J + n^S\right)$ senior workers, where $1-\delta$ is the exogenous separation rate between firms and workers. If the proceeds from output sales are insufficient to pay back financial intermediaries, firms go bankrupt. All the revenue is confiscated by the lenders, and in the next period firms start over with no senior workers.

We assume that at the end of each period firms pay out all profits (if any) as dividends and do not permit firms to retain earnings. There exists a large literature in finance which argues that there are substantial costs of maintaining a large buffer stock. For example Jensen (1986) argued that, in practice, if firms retain a large amount of their earnings in order to build up a buffer, managers use these funds in ways that benefit their private interests rather than the shareholder interests. Since shareholders understand these incentives, they give the managers incentives to pay out funds immediately rather than retain them. We crudely model this effect by preventing firms from retaining any earnings. For brand new start-up firms this assumption makes no difference in the first period but does tighten the constraint in subsequent periods compared to the case where firms are able to retain earnings. Allowing retention of earnings weakens our result quantitatively but not qualitatively.

We normalize the price of final output to be $P = 1$ and measure wage rates w^J, w^S in real output.

Firms' decision solves the following Bellman equation:

$$V(n^S) = \max_{n^J} \left\{ \begin{array}{l} \displaystyle\int_{z^*(n^J,n^S)}^{1} \left[z \left[n^S\theta^S + n^J\theta^J \right]^{\alpha} - \left(w^S n^S + w^J n^J \right) R\left(n^J, n^S \right) \right] f(z)dz \\ +\beta \left[F\left(z^*\left(n^J, n^S \right) \right) V(0) + \left(1 - F\left(z^*\left(n^J, n^S \right) \right) \right) V\left[\delta\left(n^S + n^J \right) \right] \right] \end{array} \right\}$$

where $z^*\left(n^J, n^S\right)$ is the cutoff level of productivity for bankruptcy and $R\left(n^J, n^S\right)$ is the interest rate schedule charged by the financial intermediaries.

Wage Bargaining

In our model, junior and senior workers have different levels of labor productivity due to differences in firm specific human capital. We abstract from general human capital, assuming that wage rates fully compensate workers for any differences in marginal productivity which arise from these differences. In contrast with general human capital, firm specific human capital does not improve the worker's outside option and therefore is not fully reflected in the wage rate. The market for junior workers is competitive and market clearing determines w^J. However, no market exists for senior workers with a specific firm. Therefore, we use a novel approach to Nash Bargaining between senior workers and the firm to determine the wage. This approach will also presents a clean method of empirically identifying $\frac{\theta^S}{\theta^J}$. The details of the bargaining decision are included in Table 1.2.

Table 1.2: Nash Bargaining to Determine Wages

	Match	Separate (Outside Option)
Senior Worker Surplus	w^S	w^J
Firm Surplus	$\frac{\theta^S}{\theta^J} \times w^J - w^S$	0
Total Surplus	$\frac{\theta^S}{\theta^J} \times w^J$	w^J

A senior worker that chooses to stay with the firm receives a wage of w^S whereas if he chooses to look for work elsewhere, the worker looses their firm specific human capital and receives the wage of junior workers w^J. Unlike most matching models, the surplus to the firm is not the lost production of the worker. The firm is able to hire as many junior workers as they would like at wage w^J which can perfectly substitute for the production of the senior worker. Therefore, the surplus derived for a firm employing a senior worker is the savings in wage bill, $\frac{\theta^S}{\theta^J} w^J - w^S$, where the first term is the wage bill required to replace the production from one senior worker by employing $\frac{\theta^S}{\theta^J}$ junior workers.[7] If the firm and worker separate then the firm receives 0 surplus from the relationship since the match no longer exists. For simplicity, we assume there is no commitment between firms and workers and so the bargaining is only based on the static portion of the surplus since the contract will be renegotiated again next period. Given this setup, w^S is defined as:

$$w^S = \arg\max_{w^S} \left(\frac{\theta^S}{\theta^J} \times w^J - w^S \right)^{1-\alpha_{NB}} \left(w^S - w^J \right)^{\alpha_{NB}} \tag{1.1}$$

where α_{NB} is the bargaining weight of workers.

Solving Equation 1.1 gives:

[7] To be exact, this amount is actually an upper bound in the savings for the firm. As long as the bargaining weight of workers $\alpha_{NB} < 1$ the unit cost of effective labor varies between senior and junior workers and therefore affects the decision on the optimal amount of labor to hire. Since this is an overestimate on the surplus derived from the match, this leads to an overestimate on the ratio of the senior wage premium $\frac{w^S}{w^J}$ which causes senior workers to be less valuable to the firm (since they cost more) and consequently lowers the effectiveness of our mechanism. In this sense this assumption works against our result and our resulting estimates serve as a lower bound for the effect this mechanism has on the slow recovery in recent recessions.

$$w^S - w^J = \alpha_{NB} \left(\frac{\theta^S}{\theta^J} \times w^J - w^J \right),$$

which implies

$$\frac{w^S}{w^J} - 1 = \alpha_{NB} \left(\frac{\theta^S}{\theta^J} - 1 \right). \tag{1.2}$$

This equation 1.2 has an intuitive interpretation. Only a portion α_{NB} of the gains in productivity of senior workers relative to junior workers are reflected in their wage. There are two implications from this equation. First, the wage of senior workers relative to junior workers is a proxy for the improvement in productivity for senior workers and therefore, we can use data on changes in the wage bill to give us an indirect measure of the ratio of $\frac{\theta^S}{\theta^J}$. Second, senior workers are more valuable to a firm relative to junior workers.

In equilibrium, since firms compete for workers in the labor market, hiring a junior worker is a costly investment by firms which pays off when the worker becomes senior. In this sense, senior workers are valuable for the firm and should the firm face bankruptcy, the loss of senior workers represent a real cost to the firm. For this reason firms are cautious in their hiring in order to balance growth against the risk of losing their senior worker stock.

1.3.3 Financial Intermediaries

Competitive financial intermediaries make intra-period loans to the firms. Since productivity shocks are i.i.d. across firms, financial intermediaries are not subject to any aggregate revenue uncertainty. They choose an interest rate schedule $R\left(n^J, n^S\right)$ based on the number of senior workers n^S and junior workers n^J at the firm. Interest rate schedule R and cutoff productivity level z^* for bankruptcy are jointly determined by:

$$z^* \left[n^S\theta^S + n^J\theta^J\right]^\alpha = R\left(w^S n^S + w^J n^J\right) \tag{1.3}$$

$$w^S n^S + w^J n^J = \left\{ \begin{array}{l} \left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right) R\left(w^S n^S + w^J n^J\right) \\ + \displaystyle\int_0^{z^*\left(n^J, n^S\right)} z\left[n^S\theta^S + n^J\theta^J\right]^\alpha f(z)dz \end{array} \right\} \tag{1.4}$$

where the Equation 1.3 defines the cutoff level in productivity draws z^* below which firms are unable to repay the loan and Equation 1.4 is the break even condition for risk neutral lenders. We assume that intra-period loans from financial intermediaries are state-uncontingent. Therefore, firms cannot use these loans to hedge against their productivity risks. This lack of insurance arises when there is some asymmetric information between the firm and the financial intermediaries. For example, if the productivity shocks are unobservable by the financial intermediaries.

1.3.4 Recursive Stationary Equilibrium

Definition 1. *A recursive stationary equilibrium consists of a value function V, a policy function $n^J(n^S)$, prices $\left\{w^J, w^S, R(n^J, n^S)\right\}$, a measure of firms $g(n^S)$, and output Y such that:*

1. Given $w^H, w^S, R\left(n^J, n^S\right)$: $n^J(n^S)$ solves firm's Bellman equation with V as a solution

$$V(n^S) =$$
$$\max_{n^J} \left\{ \begin{array}{l} \displaystyle\int_{z^*(n^J, n^S)}^1 \left[z\left[n^S\theta^S + n^J\theta^J\right]^\alpha - \left(w^S n^S + w^J n^J\right) R\left(n^J, n^S\right)\right] f(z)dz \\ +\beta\left[F\left(z^*\left(n^J, n^S\right)\right) V(0) + \left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right) V\left[\delta\left(n^S + n^J\right)\right]\right] \end{array} \right\}$$

where

$$z^* \left[n^S\theta^S + n^J\theta^J\right]^\alpha = R\left(n^J, n^S\right)\left(w^S n^S + w^J n^J\right)$$

2. Financial intermediaries break even with $R\left(n^J, n^S\right)$

$$\left(w^S n^S + w^J n^J\right) = \left\{ \begin{array}{c} \left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right) R\left(w^S n^S + w^J n^J\right) \\ + \int_0^{z^*(n^J, n^S)} z\left[n^S \theta^S + n^J \theta^J\right]^\alpha f(z) dz \end{array} \right\}$$

3. Final good market clears

$$Y = \int_z \int_{n^S} z\left[n^J(n^S)\theta^J + n^S \theta^S\right]^\alpha g(n^S) dn^S dF(z)$$

4. Labor market clears

$$L = \int_{n^S} \left[n^J(n^S) + n^S\right] g(n^S) dn^S$$

1.4 Stationary Equilibrium

To illustrate the intuition of our model, we use the following metaphor.

In our model, senior workers are more productive than junior workers. Since the skills accumulated by senior workers are firm specific, a senior worker's outside option is no different than a junior worker and the surplus of these skills are split via bargaining between the senior worker and the firm. In this sense, senior workers are a valuable resources for firms. Due to wages being set through market clearing in the labor market, the wage rate for junior workers is above their expected productivity and in this sense junior workers are like an investment where they bring in a negative expected return in the initial period and the positive payoff only comes in the future once the junior worker transitions into a senior worker.[8] The investment in junior workers is risky because junior workers lower the average current period revenue of the firm and hence increase the probability that the firm goes bankrupt.

A firm enters a period with a stock of senior workers which produce a positive expected cash flow for the firm. It uses these senior workers to compensate for the

[8]Hiring a junior workers is costly in the period of hiring because the expected productivity of the worker is lower than his wage. However, when the junior worker turns senior in the next period, his expected productivity is higher than his wage and hence the worker contributes to the firms expected profitability.

expected losses from hiring junior workers. Those junior workers which are retained convert into senior workers in the following period. However, the risky nature of hiring junior workers make the firm cautious in hiring them too quickly. The more senior workers a firm has, the less susceptible the firm is to the risk of hiring junior workers, and hence the more a firm would optimally choose to hire. As a result, a start-up firm will hire fewer junior workers relative to a firm with a large stock of senior workers. As firms gradually accumulate enough senior workers, they will reach a satiation point due to the decreasing returns to scale technology. At this point, firms hire just enough junior workers each period to compensate for the exogenous separation of senior workers. This generates a life-cycle growth path of a firm: slowest when first started, gradually increasing over age, and eventually reaching the optimal size.

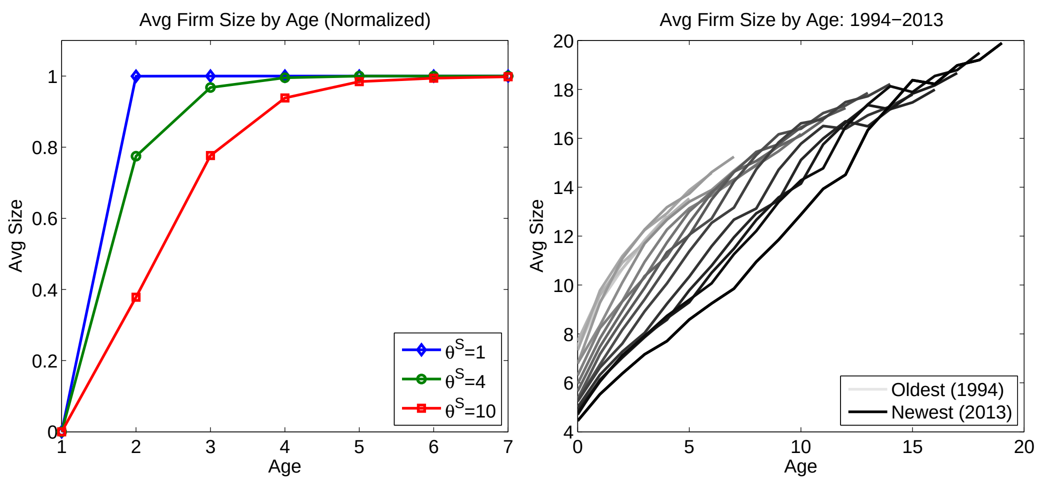
Refer to Figure 1.6 for sample policy functions with varying levels of senior worker human capital θ^S. In the case where senior and junior workers are equally productive ($\theta^S = \theta^J$), the optimal size of a firm is reached in the initial period no matter how many senior workers the firm starts with. This is because junior workers are as productive as senior workers and no longer incur an initial investment upon hiring (nor do senior worker offer a positive expected return in this setting.) In this case the problem is static and there is no state variable. When we introduce a difference in the productivity levels between senior and junior workers ($\theta^S > \theta^J$), optimal size of a firm is gradually reached as the firm balances growth against risk of bankruptcy.

Figure 1.6: Hiring Policy for Firms of Different Sizes

In terms of the probability of default, the more senior workers a firm has, the higher the average current period revenue, and hence the less likely to default. This generates a downward sloping default probability with respect to firm size which is qualitatively consistent with the data.

Next consider a comparative statics experiment where θ^S/θ^J is increased. This change will make junior workers even less productive relative to senior workers. This makes the upfront cost of hiring junior workers higher and the cost of going bankrupt higher.[9] Both of these effects will slow down the hiring of junior workers. As displayed in Figure 1.7 the data displays a pattern that could be explained by an increase in the ratio of θ^S/θ^J in that for recent years, firms are initially smaller and that they grow more slowly.

Figure 1.7: Growth Rate of Firm Size over Age for Different θ^S/θ^J Ratios



Source: Bureau of Labor Statistics, Authors' calculations

[9]The upfront cost is higher because junior workers produces even less relative to their wage. Cost of bankruptcy is higher because senior workers produce even more than their wage, and hence are even more painful to lose upon bankruptcy.

1.5 Quantitative Analysis

1.5.1 Determining the relation between θ^J and θ^S

Of crucial importance to our model is the contribution of firm specific human capital to the effectiveness of labor. More specifically that portion of firm specific human capital which is not reflected in the wage. In practice, apart from the wage, disentangling the marginal contribution of a specific worker is quite difficult as production from most firms involves the coordinated effort of multiple individuals completing various tasks both separately and in groups. Still, there do exist some attempts in the literature to identify the gains in worker productivity from experience without relying on the wage data[10].

One attempt at quantifying the gains to experience is Shaw and Lazear (2008), which documents that output increases dramatically in the first year and a half of employment but that pay profiles are much flatter than output profiles over the corresponding period. In order to document the marginal contribution of individual workers, Shaw and Lazear (2008) use data from an industry where output can be quantified and directly linked to an individual worker, namely they study a firm which installs windshields. According to their paper, "A drawback of the data is that it is surely an underestimate of the returns to skill development across occupations." This is due to the relatively easy learning curve involved in windshield installation. Even still, Shaw and Lazear (2008) find that the average output gain over the first 12 months on the job in this industry for workers employed at a constant hourly rate is 62% while the corresponding change in the wage profile for the same period is 0%. We use these findings to make two claims. First the gains in output from tenure, especially at tenure less than one year, are significant. And second, that the gains in output at the beginning of tenure with a firm are largely unreflected in the corresponding wage.

While papers which study the effects of tenure on output for a specific industry are suggestive, we need to calibrate the relative productivity of senior to junior workers across all industries and demonstrate that it has increased over time. As outlined in Section 1.3 we hypothesize that gains in firm specific human capital are split between workers and firms. In order to identify the magnitude of the relative productivity

[10]See Isen (2012) or Shaw and Lazear (2008) for examples

between senior and junior workers, we employ data from the CPS Displaced Workers, Occupational Mobility and Job Tenure supplements. We use the average wage change of workers who have been displaced from a job for economic reasons (layoffs or plant closings) to discipline our selection of θ^S and θ^J.[11] We use this loss in wages for workers who were displaced due to exogenous reasons in order to isolate that portion of the wage which can be attributed towards firm specific human capital.

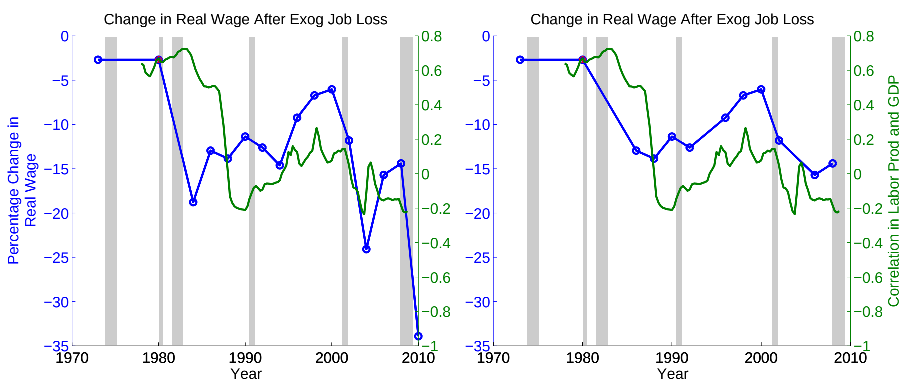
Using data from the displaced worker supplement of the CPS data is advantageous for a number of reasons. In general, it is difficult to disentangle returns to general experience and tenure. The displaced worker supplement provides wage data on specific workers both before and after separation which partially[12] controls for returns to general experience and allows us to isolate returns to tenure. Second, the CPS data permits us to limit the selection effects of separation since the supplement allows us to just use those workers who were separated due to exogenous reasons such as plant closure.

The average change in real wages after an exogenous job loss is displayed in Figure 1.8. The drop in wages following an exogenous separation is affected by the business cycle. Specifically, the first survey conducted immediately following a recession gives a drop in wages larger than the average drop experienced immediately following or preceeding the survey. Panel A of Figure 1.8 includes all data points whereas Panel B of Figure 1.8 excludes the first point following each NBER recession. The series are overlayed with the data from Figure 1.2 which demonstrates a strong correlation between changes in the real wage and patterns in labor productivity over the business cycle providing suggesting evidence linking our mechanism and result.

[11] Similar to the method used by Topel (1991)

[12] Depending on the year of the survey, those interviewed are asked if they were placed in the preceding 1 to 5 years. The fact that general experience tends to increase wages causes our estimate of θ^S/θ^J to be biased downwards both for older and more recent recessions.

Figure 1.8: Evidence for Changes in θ^S/θ^J



Source: Displaced Worker's Supplement (CPS), Authors' calculations

In order to determine the portion of firm specific human capital which is reflected in the wage data, we adopt the wage bargaining parameter used in Hagedorn and Manovskii (2008) of $\alpha^{NB} = 0.05$. The fact that wages are only moderately procyclical pins down the worker's bargaining weight at a relatively low value[13]. Using the drop in wages for displaced workers from the CPS supplement for periods before and after the mid 1980s and a Nash Bargaining Weight for Senior Workers of 0.05 we fit $\theta^S/\theta^J = 2$ for the recessions characterized by fast recoveries in the employment sector[14] and $\theta^S/\theta^J = 4$ for the recessions characterized by slow recoveries in the employment sector[15]. For additional details on the selection of θ^S/θ^J refer to A.1

1.5.2 Parameterization

Our model includes nine parameters in the stationary equilibrium and an additional parameter to determine the magnitude of the shock. The parameters in our model are calibrated using exogenous moments in the data or are selected according to levels standard in the literature. We apply a constant relative risk aversion utility function for households of the form:

[13] Using a low wage bargaining parameter implies productivity gains for senior workers which are in line with the magnitude of the findings of Shaw and Lazear (2008)

[14] Those recessions occurring prior to the mid 1980s

[15] Those recessions occurring post the mid 1980s

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}.$$

Each period in our model represents a year and we select a household discount rate of$\beta = 0.96$ to match an annual interest rate of 4%. We choose a household relative risk aversion of $\gamma = 5$ in line with work in the asset pricing literature and with Kehoe, Midrigan, and Pastorino (2014).

Labor share in the production function is chosen to be $\alpha = 0.6$. Exogenous separation rate between firms and workers is chosen to be $1 - \delta = 0.1$ to match an average employment spell of 2.5 years as in Shimer (2005). We normalize the measure of firms to $M = 1$ and select a total measure of workers in the economy of $L = 16$ to match an average establishment size of 16.5 employees over the past 10 years[16]. The firm specific productivity z is i.i.d. over time following the distribution $f(z) \sim U[0,1]$[17]. Selection of θ^S/θ^J and the Nash Bargaining weight α^{NB} were discussed in the previous section.

To model a recession we introduce a one period, unexpected volatility shock. The shock is a mean preserving spread on the productivity draws of all firms which increases the weight of the tails and causes a larger than expected number of firms to fail. For both the fast recovery recessions and slow recovery recessions we select a shock which induces a 5% drop in output. For additional information on the nature of our volatility shock refer to A.2

[16]Data from Bureau of Labor Statistics, Business Employment Dynamics tables on employment by establishment and number of establishments

[17]Note, we assume z follows a uniform distribution for analytic tractability. Our model result is invariant to the mean of the distribution.

Table 1.3: Parameterization Table

Parameter	Value	Description	Source
β	0.96	Discount Rate	Annual Interest Rate 4%
γ	5	CRRA Parameter	Asset Pricing Literature
α	0.6	Labor Share	60% Labor Share Income
$1-\delta$	0.1	Exogenous Separation Rate	Avg Emp Spell of 2.5 Years
α^{NB}	0.05	NB Weight for Workers	Hagedorn et. al. (2008)
M	1	Total Measure of Firms	Normalization
L	16	Total Measure of Workers	Average Size of Firm: BLS
θ^J	1	Prod of New Hired Workers	Normalization
θ^S_{old}	2	Prod of Exp Workers	CPS Displayed Worker Survey
θ^S_{new}	4	Prod of Exp Workers	CPS Displayed Worker Survey

1.6 Business Cycle Properties

Based on the parameters calibrated in the last section, we now compare the business cycle properties of the unemployment rate and average labor productivity generated by our model to those in the data.

Specifically, we compare our model predictions to the data for the three largest recessions as measured by the change in the unemployment rate: the 1981 recession, the 1973 recession, and the 2008 recession.

Our model generates recovery in unemployment rates that are of similar speed with those in the data. It also matches the procyclical pattern of average labor productivity for the 1973 and 1981 recessions, and the countercyclical pattern in average labor productivity for the 2008 recession observed in the data.

1.6.1 Model Transition Problem

Our model focuses on the recovery from a recession. For simplicity we model a recession as a one-time, unexpected increase to the cross-sectional variance of productivity shock z.[18] Then we look at how the economy recovers from the recession. We assume that

[18]For our mechanism to work, we only need more firms to go bankruptcy during the recession than in the stationary equilibrium. The cause of the recession is unimportant for purposes of our analysis.

real wage is sticky during the period of recovery as observed in the data.[19]

Let the shock happen in period $t = 0$. We assume that it takes T periods for the economy to get back to the stationary equilibrium after the shock. The Bellman equation for individual firms is given by:

$$V_t(n^S) =$$
$$\max_{n^J} \left\{ \begin{array}{c} \int_{z^*(n^J,n^S)}^{1} \left\{ z \left[n^J\theta^J + n^S\theta^S \right]^{\alpha} - \left[w^S n^S + w^J n^J \right] R\left(n^J, n^S\right) \right\} f(z)\, dz \\ +\Lambda_{t,t+1} \left[F\left(z^*\left(n^J, n^S\right)\right) V_{t+1}(0) + \left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right) V_{t+1}\left(\delta\left(n^S + n^J\right)\right) \right] \end{array} \right\}$$

where $\Lambda_{t,t+1}$ is the stochastic discount factor by the household between period t and period $t+1$. Since wages are assumed to be sticky, the cutoff level for productivity $z^*\left(n^S\right)$ for bankruptcy and the interest rate schedule $R\left(n^S\right)$ charged by financial intermediaries remain the same as they are in the stationary equilibrium.

Value functions during the transition are indexed by time t. This is because the distribution of firms $g_t\left(n^S\right)$ is changing over time which will affect the stochastic discount factors $\Lambda_{t,t+1}$.

1.6.2 Algorithm for Solving Transition Dynamics

Our transition problem features heterogeneous firms. Aggregation doesn't hold because of the decreasing returns to scale production technology. At each point in time, we have to keep track of the distribution of firms because it affects the total output in the economy, which in turn affects the stochastic discount factor used by firms to value payoffs in different states.

We solve for the transition using the following algorithm. Instead of iterating on a sequence of value functions, we guess and iterate on a sequence of discounted value functions.

[19]Since our model does not include a search element, there would be no unemployment in our model without the use of some form of wage rigidity. In the data wage data is only weakly procyclical.

1. Guess a sequence of discounted value function $\{DV_t : t = 1, ..., T\}$ defined by

$$DV_{t+1}\left(n^S\right) \equiv \frac{u'\left(C_{t+1}\right)}{u'\left(C_t\right)} V_{t+1}\left(n^S\right). \tag{1.5}$$

With DV defined in Equation 1.5, we can rewrite the firm's Bellman equation as follows:

$$V_t(n^S) = \max_{n^J} \left\{ \begin{array}{c} \int_{z^*(n^J,n^S)}^{1} \left\{ z\left[n^J\theta^J + n^S\theta^S\right]^{\alpha} - \right. \\ \left. \left[w^S n^S + w^J n^J\right] R\left(n^S, n^J\right) \right\} f(z)\, dz \\ +\beta\left[F\left(z^*\left(n^J, n^S\right)\right) DV_{t+1}(0) + \right. \\ \left. \left(1 - F\left(z^*\left(n^J, n^S\right)\right)\right) DV_{t+1}\left(\delta\left(n^S + n^J\right)\right)\right] \end{array} \right\}. \tag{1.6}$$

Note that DV_{t+1} is relevant for the period t hiring decision of firms.

2. For each t, given DV_{t+1}, we solve (1.6). Let $n_t^J\left(n^S\right)$ be the hiring policy function, and $\tilde{V}_t$ the updated value function. The policy function $n_t^J\left(n^S\right)$ allows us to calculate the distribution of firms in the period $t+1$:

$$g_{t+1}(n) = \int_{n^S} \mathbf{1}\left\{\delta\left(n^S + n_t^J\left(n^S\right)\right) = n\right\} g_t\left(n^S\right)\left(1 - F\left(z^*\left(n^S\right)\right)\right) dn^S.$$

In order to update the discounted value function, we also need to calculate the stochastic discount factor, hence output in the economy:

$$C_t = Y_t = \int_{n^S}\int_{z^*(n^J,n^S)}^{1} z\left[n^J h_t^J\left(n^S\right) + n^S\theta^S\right]^{\alpha} f(z)\, dz g_t\left(n^S\right) dn^S$$

3. Based on the updated value function and the consumption level calculated in Step 2, update the sequence of discounted value functions

$$\tilde{DV}_{t+1} = \frac{u'\left(C_{t+1}\right)}{u'\left(C_t\right)} \tilde{V}_{t+1}\left(n^S\right).$$

4. Repeat Steps 1-3 using the updated discounted value functions.

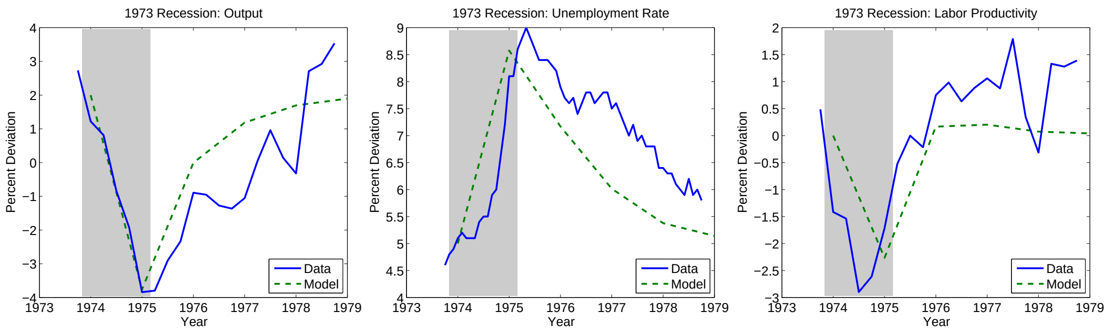
5. Repeat until the discounted value functions all converge

$$\sup_{1 \le t \le T} \sup_{n^S} |DV_t - \tilde{DV}_t| < \text{tolerence}.$$

1.6.3 Comparing Model Predictions to the Data

Using the parameters a and θ^S calibrated in the last section, we compute the evolution of output, the unemployment rate, and average labor productivity during the transition.

Figure 1.9: 1973 Recession: Output, Unemp. Rate, and Avg. Labor Prod.



Source: Bureau of Labor Statistics, Authors' calculations

Figure 1.10: 1982 Recession: Output, Unemp. Rate, and Avg. Labor Prod.

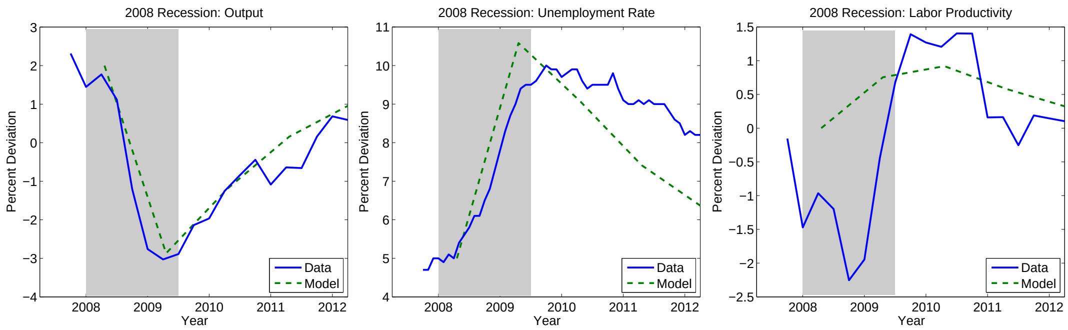
Source: Bureau of Labor Statistics, Authors' calculations

In Figure 1.9 and Figure 1.10, we compare the time series of output, the unemployment rate, and average labor productivity between our model and the data for older recessions (the 1973 recession and the 1982 recession). Note that our model uses relatively few parameters to match the data. We do not change any of the model parameters when matching the 1973 and 1982 recessions and so the model output is identical in both cases.

For both recessions, our model does a reasonable job at approximating the rate of recovery in output given that the only moment we target is the initial drop in output. As for the pattern in unemployment, we can see from the middle panel of Figure 1.9 that the unemployment rate went down by 1.6% (from 9% to 7.4%) within the first year of recovery in the data. In our simulation, the unemployment rate goes down by 1.7%. From the middle panel of Figure 1.10, we see that following the 1982 recession, the unemployment rate went down by 2.3% (from 10.8% to 8.5%) within the first year of recovery while in our simulation it goes down by 1.7%. Overall, we overestimate the speed of recovery in the 1973 recession and underestimate the speed of recovery in the 1982 recession which is as good as possible given that we are only using one set of parameters for both recessions and no parameters are used to target this rate of recovery.

From the right panel of Figure 1.9 and Figure 1.10, we see that following both the 1973 and 1982 recessions, average labor productivity dropped with output. Our model delivers this procyclicality of labor productivity. The intuition for this is as follows. In the 1973 and 1982 recessions, the difference between the productivity levels of senior and junior workers was relatively small. Therefore, start-up firms grew relatively fast. This fast growth in employment results in firms spending little time on the productive portion of the decreasing returns to scale technology. However, our model suggests that average labor productivity in the economy was reduced following the recessions by the lower productivity of junior workers.

Figure 1.11: 2008 Recession: Output, Unemp. Rate, and Avg. Labor Prod.



Source: Bureau of Labor Statistics, Authors' calculations

The only difference in the parameterization between the older recessions and the 2008 recession which generates the model data is a change in θ^S and the severity of the shock to idiosyncratic productivity draws. Again the model does a fairly good job at predicting the rate of recovery in output. In the unemployment path following the 2008 recession, the unemployment rate went down by 0.5% within the first year of recovery in the data. Our model predicts a 1.51% drop (middle panel of Figure 1.11). Our model predicts a slower rate of recovery in unemployment as a result of the change in θ^S/θ^J and though it only explains a portion of the reduction in the rate of unemployment recovery, we are not using any parameters to target either the rise in unemployment or the rate of recovery. Of course there are other effects such as financial frictions and increased specificity of labor which are discussed in the literature and are also contributing to the reduction in the rate of unemployment recovery. These effects are outside the scope of this paper.

In the 2008 recession, average labor productivity went up while output dropped. Our model also delivers this counter cyclical pattern in labor productivity (right panel of Figure 1.11). Intuitively, in 2008, the difference between the productivity of senior and junior workers is relatively large. Therefore, start-up firms grow slowly due to fear of losing their senior workers upon bankruptcy. This slow growth in employment allows firms to take advantage of the highly productive portion of the decreasing returns to scale technology. Under our calibration, this effects dominates the counter effect due to unproductive junior workers which tend to drag down labor productivity. As a result, average labor productivity rises.

1.7 Conclusion

In this paper we document two changes in the pattern of business cycles starting from the 1990 recession. First, the speed of the recovery in unemployment rates has slowed significantly. Second, labor productivity has switched from being procyclical to countercyclical. We present a model which explains both facts through the variation of a single parameter, namely the relative productivity of senior to junior workers.

In our model, when a recession occurs, many firms go bankrupt and need to start up again. This causes two opposing forces on average labor productivity. First, start-up firms employ a disproportionately large number of junior workers resulting in a decrease of average labor productivity. Second, new firms are relatively small and produce on a more productive portion of their decreasing returns to scale technology. These two forces result in a non-linear relationship between the cyclicality of labor productivity and the relative productivity of senior to junior workers.

In older recessions the difference between the productivity of senior and junior workers is low so that new firms tend to grow quickly. This causes an influx of junior workers during periods immediately following a recession. The fast rate of growth for new firms lessens the effects of producing on a more productive portion of the decreasing returns to scale technology. In total, the downward forces on average labor productivity dominates the upward force and results in the procyclicality of labor productivity observed in older recessions.

As the productivity of senior relative to junior workers increases, firms become more cautious (slower) in hiring in order to avoid losing their stock of senior workers upon bankruptcy. This is because the stock of senior workers becomes more valuable to a firm (and the resulting investment required to develop a junior worker into a senior worker increases). This slow hiring results in firms spending more time on the relatively productive portion of their decreasing returns to scale technology, and hence tends to drive up average labor productivity. Additionally, the hiring is spread out over time reducing the intensity of the downward effects on labor productivity of junior workers. In this case, the upward force on average labor productivity dominates the downward force and results in the countercyclicality of labor productivity observed in more recent recessions.

Chapter 2

Public Law Enforcement: More Is not Always Better

2.1 Introduction

Property rights, the ability of firms and consumers to own capital and other resources, are essential to almost every economic model. However, for the most part these rights are taken as given. A walk through the streets in an urban area of virtually any developing country reveals that, in practice, this is not the case. Private security guards, metal bars and large locks are commonplace to counteract theft. To date, the economic literature on theft is primarily limited to the microeconomic arena even though it is an important topic for policymakers. In this paper we incorporate theft, private security and public law enforcement (PLE) in a general equilibrium framework with the goal of understanding the effects of PLE on the incarceration rate, aggregate output and average welfare. Our primary finding is that there exists a non-monotonic relation between the level of PLE and all three of these aggregate variables. In particular, for countries with relatively small levels of PLE, there is an inverse relationship between PLE and both aggregate production and welfare primarily due to an increase in the incarceration rate. However, for countries with higher levels of PLE, the level is positively related to production and welfare and inversely related with the incarceration rate. We also find that private security exhibits a negative externality and is used as a substitute for PLE, which results in an overuse of private security, particularly in economies with a low level of PLE.

The primary mechanism which is responsible for this result is relatively straightforward. For countries with low levels of PLE, very few criminals are actually caught. As this level increases, so does the incarceration rate, which removes agents from the labor force. Additionally, there is a general equilibrium effect which lowers the relative income of the non-incarcerated agents through the increased burden of supporting those who are incarcerated. This essentially decreases the deterrence of imprisonment and incents additional agents to become thieves. As the level of PLE increases, the probability of getting caught rises, which deters agents from stealing. At some point the increase in the percent of thieves being caught is outweighed by the deterrence effect on the quantity of individuals choosing to steal. At this point, the incarceration rate begins to decrease in the level of PLE. This non monotonicity in the incarceration rate, which is consistent with the data, is the primary driver behind the additional non monotonicities in aggregate output and welfare. To quantify these effects using our benchmark model, if the level of PLE in Guatemala increased to the level in Mexico, we would predict a *decrease* in aggregate production of 0.33%. Again using our benchmark model, if the level of PLE in Mexico improved to the level of that in the United States, we would predict an *increase* in aggregate production of 2.58%.

For countries with a low level of PLE, we observe a high level of substitution between PLE and expenditures on private security which dampens the effect PLE has on the overall level of theft. Further, not only do these firms substitute private security for PLE, we find that economies with low levels of PLE tend to hire more private security than is socially efficient. If we restrict firms such that they are only permitted to hire a fraction of the private security that they would otherwise find individually optimal, aggregate production is higher than if firms were unrestricted in their private security decisions. The reason for this is that by restricting how much firms can spend on private security, firms end up hiring more workers which produce the final good even though a larger portion of what is produced is stolen.[1]

For high enough levels of PLE, we find that marginal increases in the level of PLE provide significant gains to aggregate production and increase the labor force. As the

[1]In our model theft has a contemporaneous distortion on firms' optimal decision. For an analysis on possible inter-period distortions that crime has on production see Arias, Ibáñez, and Zambrano (2014).

probability of getting caught rises, agents are deterred from stealing and at some point the drop in theft becomes larger than the increase in those thieves who are caught. The reduction in incarceration rates augments the total labor force which increases total production. Additionally, reduction in theft from firms lowers the distortionary wedge between the marginal product of labor and the wage rate which raises both the efficiency and average size of firms. Finally, the increase in the wage rate and the reduction in the burden of the incarcerated on the non-incarcerated increases the actual cost of getting caught and puts further downward pressure on the theft rate.

When we extend our model to a dynamic setting, our results provide an explanation for the large differences we observe in PLE across countries. Since the marginal effects of changes in PLE are very different depending on the current level of PLE, if there are transition costs in modifying this level, countries with low current levels of PLE may initially experience a reduction in welfare and production before seeing improvements in response to increases in PLE. This implies that countries which are sufficiently impatient would prefer to remain in a state of low PLE rather than face the transition path to a high level of PLE.

Data which exists for private security consistently reveals that the correlation between private security expenditures and theft is positive. We match this fact. In our model this relation is caused by PLE which both deters theft and serves as a substitute for private security. In this sense, we make the same empirical observation as North (1968) in that economies where firms have lower private security expenditures are also economies with less theft and often higher production.

In order to direct and validate the way we model the decision of thieves in when and how much to steal, and the way we model private security, we adopt two strategies. First we incorporate existing findings on theft in the psychology and sociology literature. Second we allow agents to vary across two dimensions: in aversion to theft and in level of ability as in the Lucas (1978) span of control model. Granting variation across these dimensions gives insight into micro decisions of agents and across firms. We validate our modeling of theft and private security by matching these patterns to the data.

Heterogenous modeling of agents also gives further insights. Specifically, agents with lower ability earn less which decreases their cost of being caught and increases the likelihood they engage in theft. Second, the distortion from theft across firms is not uniform. Firms managed by entrepreneurs with higher ability afford larger amounts of private security which reduces the wedge between the marginal productivity of labor and the wage rate. This mechanism causes the dispersion across firm size to be increasing in the rate of theft.

As far as we are aware, we present the first general equilibrium model incorporating theft, private security and PLE. However, our work contributes and builds upon a vast theoretical and empirical literature.

Our paper continues in the spirit of the seminal work by Becker (1968). In our model consumers analyze the costs and benefits of committing a crime and make a rational decision of whether to engage in criminal activity. Perhaps the model most similar to ours is the one in Fender (1999) which includes many of the same elements and some of the same results. In that model, corruptible agents choose between work and theft and there is consideration of the level of enforcement which is very similar to our notion of PLE. In line with this paper, we observe similar relationships between the level of enforcement, the number of criminals and the number punished. In contrast to Fender (1999) and Burdett, Lagos, and Wright (2003), our model allows thieves to both work and steal, we include a notion of firms, agents are heterogenous in ability and we incorporate general equilibrium effects. This allows us to match micro data in order to validate our macro results.

Our findings are consistent with the findings in the empirical paper by Buonanno, Drago, Galbiati, and Zanella (2011) and Ibáñez, Rodríguez, and Zarruk (2013). Their work suggests that increases in the incarceration rate deters crime. In our model we support that this effect holds, but the general equilibrium effects can cause pressure on crime (specifically theft) in both directions. Due to the current absence of dynamics in our model, we are unable to address the (largely empirical) literature on the deterrents of the effects of prison on recidivism.[2]

Our paper is also related to the existing literature relating trust, extortion, distortion and firm size. We observe a similar pattern in distribution of firm size due to increases in theft as Ranasinghe (2012) observes from increases in extortion in the sense that higher levels increase dispersion of firm size. Our effects differ slightly in that all firms are smaller than they would be in the absence of theft but the distortion is greatest for

[2] See Drago, Galbiati, and Vertova (2009) for an example.

the smallest firms. Finally, consistent with Grobovšek (2014), we find that increased levels of theft among workers constrains firm size.

The rest of the paper is organized as follows. Section 2.2 presents an empirical motivation for our model and the main mechanism in it. Section 2.3 outlines the model. Section 2.4 presents the primary results. Section 2.5 considers the model in a dynamic setting. Section 2.6 concludes.

2.2 Data and Empirical Motivation

We first use data to validate the main mechanism in our model; namely, the non-monotonic relationship between the incarceration rate and the level of public law enforcement. For this, we make use of the World Bank *Worldwide Governance Indicators* (WGI). The WGI are six governance indicators based on the views and experiences of citizens from over 200 countries compiled using 32 different data sources. In general the sources are of four different types: Surveys on households and firms; commercial business information providers; non-government organizations; and public sector organizations. The WGI are the result of rescaling such sources using a components model, in order to get indicators for each of the countries considered.[3] We use the percentile ranking of the Rule of Law Index from these indicators to proxy for the level of public law enforcement, since it is the indicator that is most consistent with our notion of PLE in the model.[4]

Additionally, we rely on incarceration rates compiled by the International Centre for Prison Studies in their *World Prison Brief* (WPB). The WPB provides data collected from national sources on information about prison systems from over 200 countries. We define the incarceration rate of a country as the prison population per 100,000 inhabitants in a given country.

We first construct an unbalanced panel using the Rule of Law index, RoL, and the incarceration rate, IR, for 202 countries. Data for Rule of Law is available biennually

[3] See Kaufmann, Kraay, and Mastruzzi (2010) for a description of the methodology used to calculate these indicators.

[4] According to the World Bank definition, "The Rule of Law index captures perceptions of the extent to which agents have confidence in and abide by the rules of society, and in particular the quality of contract enforcement, property rights, the police, and the courts, as well as the likelihood of crime and violence".

for most countries starting in 1996 and is available annually from 2002. Availability of incarceration rates, on the other hand, is not homogeneous across countries. We have two to seven observations for each country with a median of six. We estimate the model specified in (2.1).

$$IR_{i,t} = \alpha_i + \beta_1 RoL_{i,t} + \beta_2 RoL_{i,t}^2 + \varepsilon_{i,t}, \tag{2.1}$$

where α_i denotes a country fixed effect. We find a significant non-monotonic relationship between Rule of Law and incarceration rate. In fact, the estimated coefficients imply that there is a hump in the relationship between these two variables: Around the 60-th percentile, further increases in Rule of Law are related with decreases in the incarceration rate. The estimation results are shown in Table 2.1.

Table 2.1: Incarceration as a function of Rule of Law

	Dependant variable is Incarceration Rate
Rule of Law	2.108^{***}
	(0.575)
Rule of Law^2	-0.0171^{***}
	(0.006)
Observations	789
R-squared	0.931

Country fixed-effects

***: Significant at 1%.

Source: World Bank and International Center for Prison Studies. Authors' calculations.

In order to obtain a better understanding of this relationship we look at two particular countries: Colombia and Estonia. We observe that the Rule of Law for both countries improved from 1998 to 2010. However, incarceration rates increased in Colombia, while they decreased in Estonia. A main difference between the two countries was that the Rule of Law in Colombia was relatively low in 1998, while for Estonia it was high. This behavior of incarceration rates is consistent with the predictions of our model.

Colombia had suffered from a civil conflict since the 1960's, but the conflict got worse during the 1990's. In 1998 Colombia had a Rule of Law index of 24.4. One year later Colombia began receiving military aid from the U.S. with the aim of ending the civil conflict.[5] By 2010, the Rule of Law index in Colombia reached 44.5. In the same lapse of time its incarceration rate went from 114 to 181. On the other hand, Estonia had a Rule of Law index of 67.5 in 1998. In 2002 the EU summit in Copenhagen formally invited Estonia to join the EU and a year later Estonians voted to join. In 2010 the Rule of Law index had grown to 83.9. Meanwhile the incarceration rate decreased from 342 in 1998 to 266 in 2010. These two examples are consistent with the patterns we observe more broadly across countries. Namely that incarceration rates are upside down U-shaped in relation to the Rule of Law index.

We now analyze the relationship between Rule of Law and incarceration rates across countries. We use the 2010 Rule of Law and the most recent information available on WGI for the incarceration rate. We also observe a hump-shaped relationship between Rule of Law and incarceration rates in this case, which suggests that the relationship holds both across countries and time. For low levels of public law enforcement, we observe that increases in this level are accompanied by an increase in the incarceration rate. However, once the level of public law enforcement reaches a certain point, further increases are actually related to lower incarceration rates. Table 2.2 shows both a linear and quadratic fit when regressing the incarceration rate on the Rule of Law. We observe that incarceration rates appear to be increasing in the Rule of Law, although after including country controls, this relationship largely disappears. When we add a quadratic Rule of Law term, the coefficient on the first term becomes much larger and increases in significance. Additionally the goodness of fit is significantly improved. Both of these support the use of a quadratic term and that the hump shape in incarceration rates is a better fit for the data.

[5]According to a report from the U.S. Government Accountability Office, as of 2008, U.S. State and Defense departments had provided nearly $5 billion to Colombian military and National Police.

Table 2.2: Incarceration as a function of Rule of Law

	Dependant variable is Incarceration Rate			
	(1)	(2)	(3)	(4)
Constant	106.9***	956.3***	−133.2**	657***
	(24.77)	(173.9)	(62.02)	(205.1)
Rule of Law	1.294***	−0.435	11.71***	6.950**
	(0.454)	(0.773)	(2.523)	(2.909)
Rule of Law^2			−0.097***	−0.074***
			(0.023)	(0.028)
Controls	No	Yes	No	Yes
Observations	209	157	209	157
R-squared	0.038	0.241	0.113	0.274

For incarceration rate we use the most recent information available on www.prisonstudies.org
Rule of Law refers to the Rule of Law index of 2010.
Controls include GDP, mortality rates of children under 5, and life expectancy at birth all 2011
OLS estimation. Standard errors are in parenthesis.
: Significant at 5%, *: Significant at 1%.
Source: World Bank and International Center for Prison Studies. Authors' calculations.

In addition to our macro data, we turn to the World Bank *Enterprise Surveys* to provide us with micro evidence to support our modeling of theft and private security. The surveys are conducted at the firm level using a representative sample of an economy's private sector. The World Bank selected firms for the *Enterprise Surveys* using stratified random sampling. All members of the population have the same probability of being sampled and no weighting of the observations is necessary. However, only firms with 5 or more employees are targeted for an interview and organizations with 100% government ownership are ineligible to participate.[6] Surveys occur face-to-face with business owners and top managers. These surveys have been conducted every year from 2006 to 2011. Nonetheless, in any single country there have been a maximum of two surveys and the vast majority of countries have been surveyed a single time. The final dataset used in this paper includes 130 country-years and averages 373 firms interviewed per country-year combination for a total of 48,436 observations. There are 111

[6]The sample *targets* firms they believe to have 5 or more employees, however some firms are observed to have less than 5 upon conducting the interview.

unique countries where surveys have been conducted. Questions are both qualitative and quantitative in nature. Qualitative questions ask perception of certain business obstacles (e.g. "Do you perceive crime, theft and disorder as a major constraint?"). Quantitative questions of particular relevance request the number of employees, the annual revenue, the amount of annual losses due to theft as well as annual private security expenditures.

Summary statistics are included in Table 2.3. Theft is identified as a "major" constraint by over a quarter of all firms interviewed. Additionally, even though only roughly a quarter of firms directly experienced theft in the year of interview, almost two thirds of firms have positive expenditures on private security. The average security expenditures for firms purchasing private security was 2.6% of total revenues. Firms which experienced theft had an average loss equivalent to 3.8% of their total revenues.[7]

Table 2.3: Summary Statistics

Share of firms that perceive theft as a major constraint	27.8%
Share of firms that had positive expenditures on private security	63.9%
Average private security expenditures*	$59,931
Average private security expenditures over revenue**	2.6%
Firms that experienced theft	24.7%
Average level of theft*	$18,786
Average theft over revenue***	3.8%

*: Levels were converted to 2000 US dollars.

**: Conditional on having positive private security expenditures.

***: Conditional on having experienced theft.

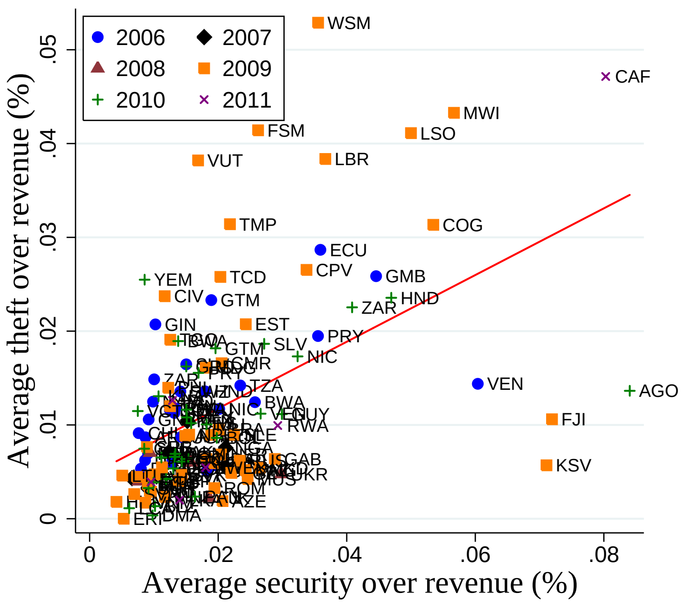
Source: The World Bank (2012). Authors' calculations.

We now make a number of motivating observations where we highlight how the level of public law enforcement is important in determining theft and private security choices in equilibrium. Figure 2.1 shows average experienced theft to average private security expenditures at the country level. We observe that average theft is positively correlated with average private security expenditures and the relationship is significant at the 1% level. We assume that all else constant, theft should decrease in security

[7] While theft accounts for 2.6% of total revenues, this translates to a larger portion of total profits and therefore a larger portion of GDP

expenditures. However, both theft and security decisions are endogenous to the environment. Therefore the observed positive correlation is not causal but is indicative of some tertiary effect. We posit that one of the key drivers of this relationship is public law enforcement. First, as seen in Figure 2.2a, security is decreasing in the country's Rule of Law index which we use as a proxy for public law enforcement. Second, theft is also decreasing in public law enforcement as seen in Figure 2.2b. This additional information seems to support private security being an imperfect substitute for public law enforcement and that a firm's equilibrium private security decision does not fully compensate for the lack of a strong public law enforcement presence.

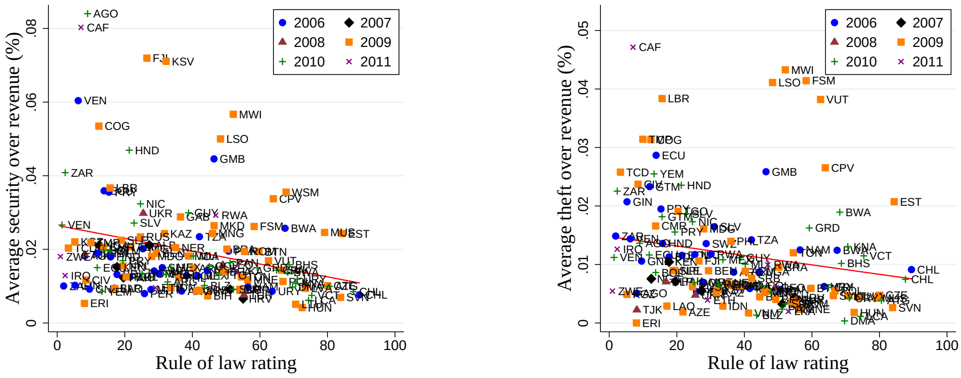
Figure 2.1: Theft over Revenue vs Security Expenditures over Revenue



Source: The World Bank (2012). Authors' calculations.

Figure 2.2: Security Expenditures over Revenue and Theft over Revenue vs Rule of Law

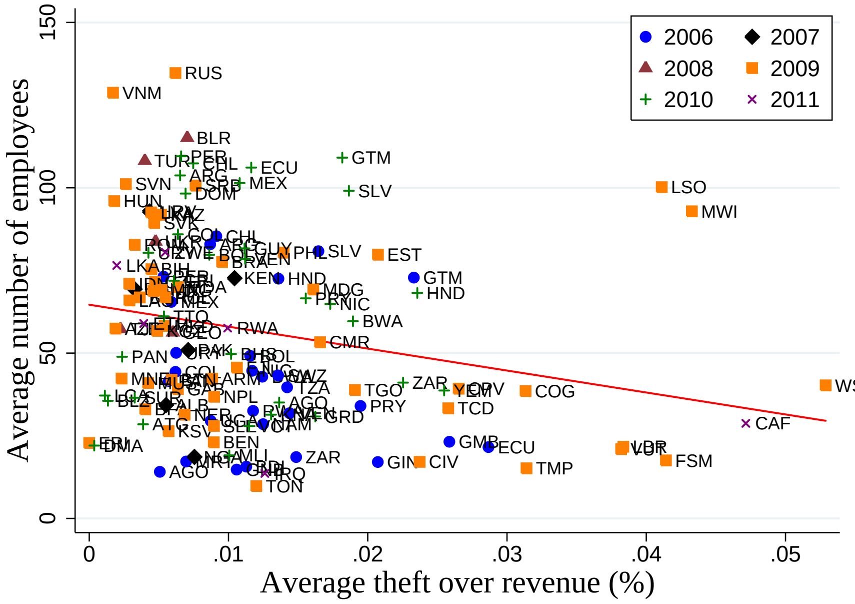
(a) Sec to Rev vs Rule of Law **(b) Theft to Rev vs Rule of Law**



Source: The World Bank (2012). Authors' calculations.

Our final motivating observation is that we observe average firm size to be inversely correlated with average theft, and this relation is also significant at the 1% level. Figure 2.3 shows this relation. A similar observation was made by Grobovšek (2014).

Figure 2.3: Average Size of Firm vs Average Theft over Revenue



Source: The World Bank (2012). Authors' calculations.

To conclude, we find that incarceration is non-monotonic in the Rule of Law. In particular, it is upside down U-shaped, and this relationship holds both across time and across countries. At the micro-level we find suggestive evidence that the level of public law enforcement is important in the determination of equilibrium theft and private security choices.

2.3 Environment

Our model is constructed in the spirit of Lucas (1978). Each consumer makes two choices: whether to become a thief or not, and whether to become an entrepreneur or a worker. Consider a particular consumer. If she chooses to become a thief, she optimally chooses how much to steal from firms, taking as given how much security is hired by each firm. However, she faces an exogenous probability of getting caught and losing what she stole as well as the ability to work or manage a firm. If the consumer decides to become an entrepreneur, she takes into account how much theft she faces and chooses how much security to hire, in addition to choosing the optimal size of her firm. If she becomes a worker, then she works in firms in exchange for a wage.

2.3.1 Consumers

In this economy there is a unit mass continuum of risk neutral consumers, each endowed with a skill level and an aversion to stealing. Consider a consumer with skill level z and aversion to stealing parameter θ. She maximizes her utility, given by (2.2).

$$u(z,\theta) = \max\left\{u_T(z,\theta), u_{NT}(z)\right\}, \tag{2.2}$$

where

$$\begin{aligned} u_T(z,\theta) &= (1-\lambda)\left[\max\left\{w,\pi(z)\right\} + \Pi_T - \Upsilon\right] + \lambda\underline{c} - \theta \\ u_{NT}(z) &= \max\left\{w,\pi(z)\right\} - \Upsilon. \end{aligned}$$

That is, she decides whether to become a thief and get utility $u_T(z,\theta)$ or not become a thief and get $u_{NT}(z)$. In the former case, the consumer steals from firms to get an extra income of Π_T. She gets away with stealing with an exogenous probability $1-\lambda$. With probability λ the consumer gets caught and loses all sources of income. Instead she receives consumption $\underline{c}$. We interpret $\lambda \in [0,1]$ as the level of public law enforcement and we interpret a thief being caught as implying that she goes to jail. In this way, if a consumer is caught, she neither works nor becomes an entrepreneur. Finally, Υ is a lump sum tax applied to consumers that do not go to jail which finances $\underline{c}$ for those in jail. That is,

$$\Upsilon = \frac{\underline{c}\lambda M_T}{1-\lambda M_T}, \tag{2.3}$$

where M_T denotes the mass of consumers who become thieves.

Regardless of the decision to become a thief, the consumer also decides whether to work for a wage w or become an entrepreneur and receive the profits $\pi(z)$ from the firm she manages. If she becomes an entrepreneur her income will depend on her skill z. If the consumer decides not to become a thief, she receives the income either from working or from being an entrepreneur, minus the lump sum tax.

We assume that θ and z are drawn from independent distributions. We will denote by $F(\cdot)$ and $G(\cdot)$ the cumulative distribution functions of θ and z, respectively. A consumer's decision is characterized by z and θ, so we will denote each agent by the realizations of these random variables.

2.3.2 Firms, theft and private security

Consider an entrepreneur with skill level z. She maximizes the profits from the firm she manages, which we will denote as firm z, by hiring workers l_y. The firms produce using a decreasing returns to scale function, zl_y^α, where $\alpha \in (0,1)$. From what firm z produces, $(1-\lambda)M_T\tau$ gets stolen, where M_T denotes the measure of consumers that become thieves and τ is how much each thief decides to steal from firm z.[8] Finally, firm z hires l_s security guards to diminish theft. All firms produce the same final good and we normalize the price of this good to 1. To summarize, firm z solves problem (2.4).

$$\pi(z) \equiv \max_{l_y \geq 0, l_s \geq 0} zl_y^\alpha - wl_y - wl_s - (1-\lambda)M_T\tau(z). \tag{2.4}$$

In order to determine how many security guards are hired, we assume firm z understands the thieves' problem. Agents that choose to steal attempt to steal some amount from all firms and choose an optimal theft intensity from each firm. Consider the problem of a consumer that becomes a thief and decides to steal from firm z. The income derived in stealing from firm z is given by

$$\pi_T(z) \equiv \max_{\tau \geq 0} \tau - C_\tau(z), \tag{2.5}$$

where $C_\tau(z)$ denotes the cost born by those who steal from firm z. We make three assumptions regarding this cost. First, $C_\tau(z)$ is increasing and convex in the amount stolen. The more that is stolen, the greater the transportation costs, storage costs, etc. Additionally, without convexity, thieves would always attempt to steal everything possible or steal nothing at all which does not hold true empirically. Moreover, the thieves' problem does not solely consider the financial costs but also the utility costs of time and anguish involved in planning and carrying out an operation. It is reasonable

[8] Since agents are risk neutral, we are able to abstract from which firms are stolen from and only care about the expected level of theft.

that the cost of theft in terms of planning, stress, and time grows exponentially from stealing a pack of gum to stealing everything in a store.

Our second assumption is that security affects the choice of theft by making it more costly to steal. The presence of a security guard in the firm causes more planning and time in order to be able to steal. This is consistent with what is found by Kraut (1976) where the risk associated with stealing is perceived as a deterrent.

Finally, we assume that the cost of stealing is decreasing in the amount produced by the firm. This accounts for the fact that if a firm is bigger, then there are more things to steal, and so stealing the same amount as from a smaller firm is easier. This is consistent with the results reported by Smigel (1956), who finds that people are more likely to steal from big firms than from small firms.

We assume that $C_\tau(z) \equiv \frac{\phi(l_s(z))}{l_y(z)} \frac{\tau^2}{2}$, where $\phi(l_s) \equiv \left(\frac{\alpha}{1-\alpha} l_s\right)^{\frac{1-\alpha}{\alpha}}$ denotes the amount of provided by hiring l_s guards for a given level of M_T and is strictly increasing and concave in l_s. The solution to (2.5) is

$$\tau(z) = \frac{l_y(z)}{\phi(l_s(z))}. \tag{2.6}$$

Then $\pi_T(z) = \frac{1}{2}\tau(z)$ and the aggregate income received from stealing Π_T is given by

$$\Pi_T \equiv \int_{(z,\theta)\in E} \pi_T(z) dF(\theta) dG(z) - \lambda \int_{(z,\theta)\in E \bigcap T} \pi_T(z) dF(\theta) dG(z), \tag{2.7}$$

where E and T denote the set of consumers that become entrepreneurs and the set of consumers that become thieves, respectively. That is,

$$E \equiv \{(z,\theta) : \pi(z) \geq w\} \tag{2.8}$$

$$T \equiv \{(z,\theta) : u_T(z,\theta) > u_{NT}(z)\}. \tag{2.9}$$

We abuse notation and also refer to E as the set of z for which consumers become entrepreneurs. The specific use of E will be clear from context.

The second term in (2.7) is due to the fact that thieves do not get income when stealing from the firms managed by entrepreneurs that are thieves and get caught.

Recall that a fraction λ of the total entrepreneurs that become thieves go to jail and are unable to manage firms.

2.3.3 Micro Support for Modeling Theft and Private Security

We use the existing literature as well as qualitative patterns in micro-data to motivate our modeling methods. Specifically we make four observations using data from the *Enterprise Surveys* (See Table 2.4). First, both the absolute level of theft and private security expenditures are increasing across firm size. This is consistent with Smigel (1956).

Table 2.4: Results in Theft and Security Across Firms

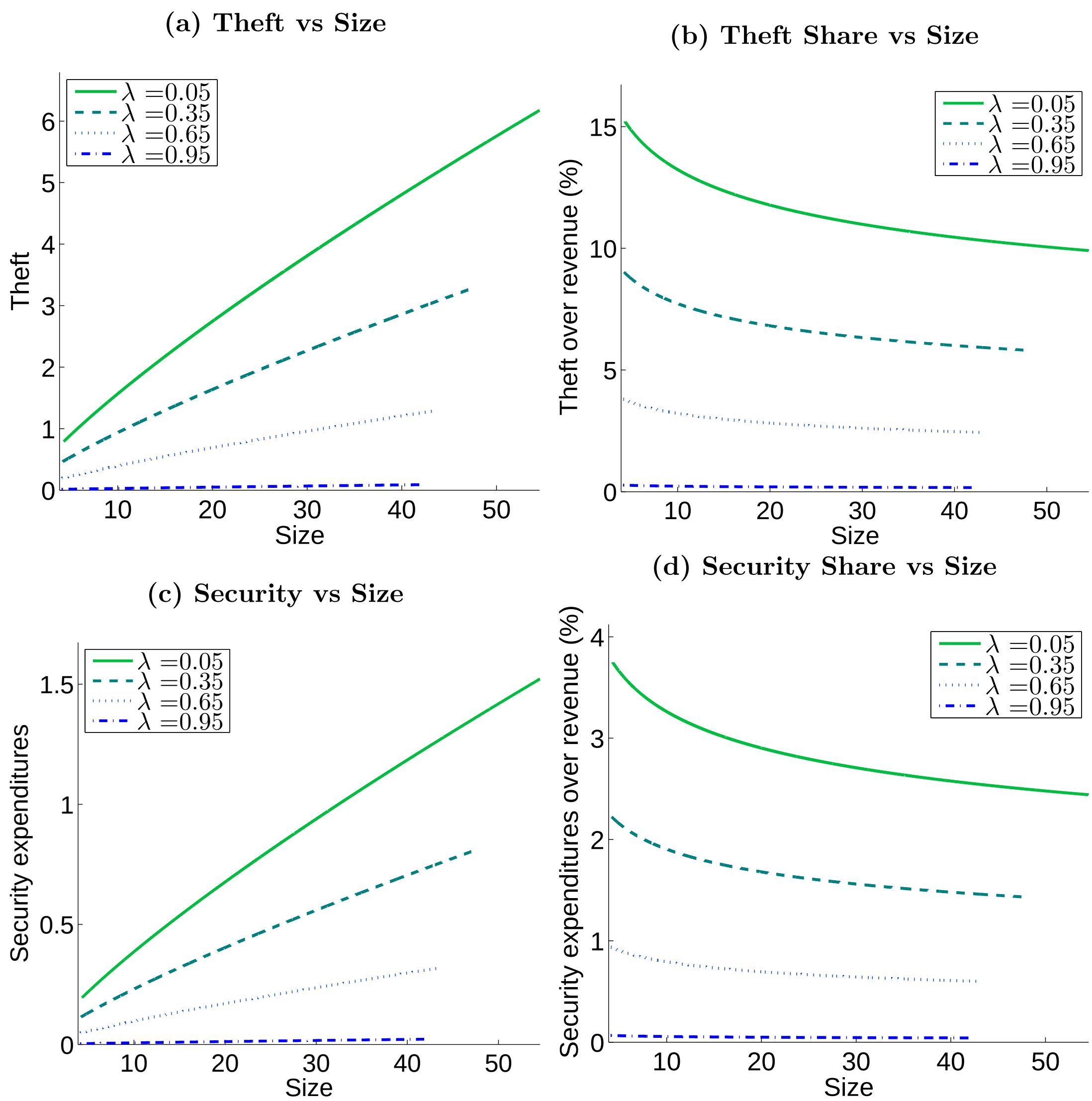
Dependent Variable: Independent Variable	Theft (1)	$\frac{\text{Theft}}{\text{Revenue}}$ (2)	Security (3)	$\frac{\text{Security}}{\text{Revenue}}$ (4)	$\frac{\text{Security}}{\text{Revenue}}$ (5)	$\frac{\text{Security}>0}{\text{Revenue}}$ (6)
Size (Labor)	224.14^{***}		613.99^{***}			
	(12.45)		(42.86)			
Size ($\log$ Labor)		-0.002^{***}		0.001^{***}	0.004^{***}	-0.003^{***}
		(0.000)		(0.000)	(0.001)	(0.000)
Size ($\log \text{Labor}^2$)					-0.000^{**}	
					(0.000)	
Observations	48,299	48,299	48,299	48,299	48,299	30,838
Country-Year Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes

OLS estimation. Standard errors are in parenthesis.
***: Significant at 1%. **: Significant at 5%. *: Significant at 10%.

Next we analyze these same variables as a share of revenue. While theft is increasing in the size of firm, theft as a percentage of revenue is decreasing in the size of firm. The relation with private security expenditures is slightly more complicated. When we regress private security expenditures as a share of revenues we find that the share is increasing slowly in the size of firm. However, when we add a quadratic term on size to the regression we find a hump shape, with private security share increasing for small firms and decreasing for larger firms. Another level of analysis reveals the cause for this hump. The probability that a firm purchases private security is increasing in size. However, given a firm purchases private security (column 6 of Table 2.4), the share of revenue spent on private security is decreasing in the size of firm.

These micro patterns were used to guide our modeling of private security and theft. To the extent possible, given the level of heterogeneity used in our model, we match these patterns for a large range of λ. Figure 2.4 matches qualitatively the results we report in Table 2.4. In particular, Figure 2.4d matches the data in column 6 of Table 2.4. Due to the level of heterogeneity it is not within the scope of our model to exactly match the hump shape found in the data.

Figure 2.4: Matching Micro Patterns



2.3.4 Equilibrium

An equilibrium in this economy is allocations $\{\tau(z), l_y(z), l_s(z)\}_{z\in E}$, wages w, and sets E and T such that

1. $\tau(z)$ satisfies (2.6) for all $z \in E$; and $l_y(z)$ and $l_s(z)$ solve (2.10);

2. E and T satisfy (2.8) and (2.9);

3. the labor market clears:

$$\int_{(z,\theta)\in E} (l_y(z) + l_s(z))\, dF(\theta)dG(z) - \lambda \int_{(z,\theta)\in E\bigcap T} (l_y(z) + l_s(z))\, dF(\theta)dG(z)$$
$$= \int_{(z,\theta)\in E^c} dF(\theta)dG(z) - \lambda \int_{(z,\theta)\in E^c\bigcap T} dF(\theta)dG(z);$$

4. and the good market clears:

$$\begin{aligned} Y &\equiv \int_{(z,\theta)\in E} z l_y(z)^\alpha dF(\theta)dG(z) - \lambda \int_{(z,\theta)\in E\bigcap T} z l_y(z)^\alpha dF(\theta)dG(z) \\ &= \int_{(z,\theta)\in E} \left[w\left(l_y(z) + l_s(z)\right) + \pi(z) + (1-\lambda)\tau(z)M_T\right] dF(\theta)dG(z) \\ &\quad -\lambda \int_{(z,\theta)\in E\bigcap T} \left[w\left(l_y(z) + l_s(z)\right) + \pi(z) + (1-\lambda)\tau(z)M_T\right] dF(\theta)dG(z), \end{aligned}$$

where

$$M_T \equiv \int_{(z,\theta)\in T} dF(\theta)dG(z).$$

2.3.5 Characterization of the equilibrium

Lemma 1 characterizes E and T in terms of equilibrium prices and consumer choices.

Lemma 1.

$$\begin{aligned} E &= \left\{(z,\theta) : z \geq z^E\right\} \\ T &= \left\{(z,\theta) : z < z^E \text{ and } \theta < \theta^W\right\} \bigcup \left\{(z,\theta) : z \geq z^E \text{ and } \theta < \theta^E(z)\right\}, \end{aligned}$$

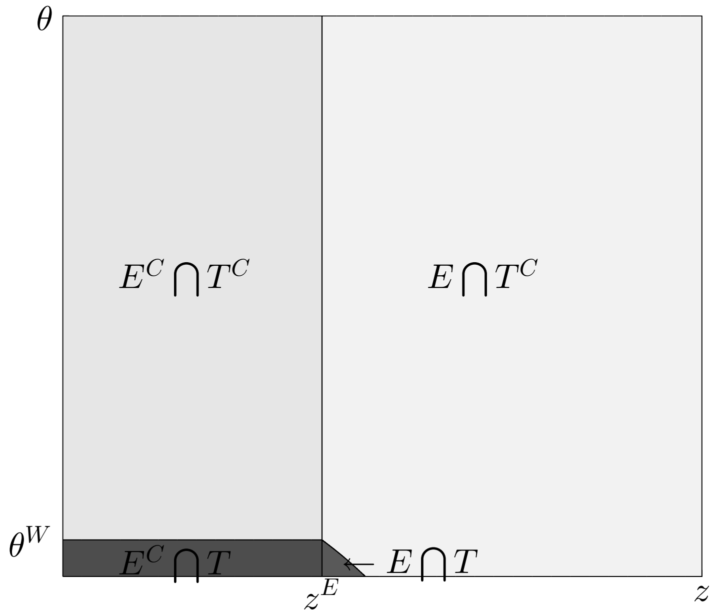
where z^E *is the unique value of* z *such that* $\pi(z^E) = w$ *and*

$$\theta^W \equiv \frac{\lambda \underline{c}}{1 - \lambda M_T} + (1 - \lambda)\Pi_T - \lambda w$$
$$\theta^E(z) \equiv \frac{\lambda \underline{c}}{1 - \lambda M_T} + (1 - \lambda)\Pi_T - \lambda \pi(z).$$

Proof. See B.1.1. □

From Lemma 1 we see that thieves are those agents who have the lowest levels of skill and the lowest aversion to stealing. Since income is increasing in skill, we also observe that those with the smallest incomes are the most likely to become thieves. We note that these results are consistent with both the theoretical and empirical existing literature.[9] Figure 2.5 shows E and T across skill (x-axis) and aversion to stealing (y-axis).

Figure 2.5: E and T across Skill and Aversion to Stealing



As mentioned, firm z knows (2.6). Therefore the firm's problem can be written as

[9]For example see Freeman (1999).

stated in (2.10).

$$\pi(z) = \max_{l_y \geq 0, l_s \geq 0} z l_y^{\alpha} - \left(\frac{(1-\lambda) M_T}{\phi\left(l_s\right)} + w \right) l_y - w l_s. \tag{2.10}$$

Lemma 2 characterizes the demand for labor and security given wages w, as well as firm's profits and how much is stolen from each firm.

Lemma 2. *Assume* $\alpha > 0.5$. *Then*

$$l_y(z) = \left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda) M_T}{w} \right)^{\alpha} \right)^{\frac{1}{1-\alpha}}$$

$$l_s(z) = \frac{1-\alpha}{\alpha} \left(\frac{(1-\lambda) M_T}{w} \right)^{\alpha} \left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda) M_T}{w} \right)^{\alpha} \right)^{\frac{\alpha}{1-\alpha}}$$

$$\pi(z) = \frac{1-\alpha}{\alpha} w \left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda) M_T}{w} \right)^{\alpha} \right)^{\frac{1}{1-\alpha}}$$

$$\tau(z) = \left(\frac{w}{(1-\lambda) M_T} \right)^{1-\alpha} \left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda) M_T}{w} \right)^{\alpha} \right)^{\frac{\alpha}{1-\alpha}}.$$

Proof. See B.1.2. □

Now, notice from (2.10) that the production function satisfies Inada conditions, so $l_y(z) > 0$ for every firm. We assume that $\alpha > 0.5$ so that $\phi(\cdot)$ is strictly concave and first order conditions with respect to l_s are also sufficient. Using the fact that the solution is always interior, taking first order conditions of (2.10) with respect to l_y yields

$$w + \frac{(1-\lambda) M_T}{\phi\left(l_s(z)\right)} = \alpha z l_y^{\alpha - 1}. \tag{2.11}$$

In the absence of theft (i.e. $M_T = 0$) (2.11) reduces to $w = \alpha z l_y^{\alpha-1}$, or $l_y = \left(\frac{\alpha z}{w}\right)^{\frac{1}{1-\alpha}}$. We observe that theft creates a wedge which causes the marginal productivity of labor to be greater than w by a factor of $\frac{(1-\lambda) M_T}{\phi(l_s(z))}$. Observe that the wedge is increasing in the measure of thieves and decreasing in both a higher level of public law enforcement and private security. As a consequence of theft, firms are smaller in equilibrium than in the absence of theft:

$$\left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda)M_T}{w}\right)^{\alpha}\right)^{\frac{1}{1-\alpha}} < \left(\frac{\alpha z}{w}\right)^{\frac{1}{1-\alpha}}.$$

Corollary 1 shows that the ratio of theft experienced by a firm to private security expenditures is constant and greater than 1.

Corollary 1. *The ratio of theft experienced by a firm,* $(1-\lambda)M_T\tau(z)$*, and private security expenditures,* $wl_s(z)$*, is constant and equal to* $\frac{\alpha}{1-\alpha}$*.*

Proposition 1 shows that in equilibrium every firm finds it optimal to hire security.

Proposition 1. *Assume* $\alpha > 0.5$*. Then* $l_s(z) > 0$ *for all* $z \geq z^E$*.*

Proof. By definition z^E is such that $\pi(z^E) = w$. From the expression for $\pi(z)$ in Lemma (2),

$$z^E = \frac{w^{1-\alpha}}{\alpha}\left((1-\lambda)M_T\right)^{\alpha} + \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} w$$

and $l_s(z) > 0$ if and only if $z > \frac{w^{1-\alpha}}{\alpha}\left((1-\lambda)M_T\right)^{\alpha}$. Since $z^E > \frac{w^{1-\alpha}}{\alpha}\left((1-\lambda)M_T\right)^{\alpha}$, the result follows. □

2.4 Results

2.4.1 Parameterization

Our current parametrization is chosen such that reasonable parameter values are able to give results which qualitatively match the micro and macro patterns we observe in the data. Table 2.5 shows the values of the parameters that we use and the moments we target.

We calibrate preference and technology parameters to match key aspects of the US economy. Our model economy consists of eight parameters. In accordance with Buera, Kaboski, and Shin (2011), we assume that entrepreneurial ability follows a Pareto distribution with shape parameter ν and scale parameter $\underline{z}$. Since Buera, Kaboski, and Shin (2011) also fit their model to the US economy, we adopt $\nu = 4.84$ from their paper. We set the nominal Pareto scale parameter $\underline{z}$ at 1 for simplicity. The distribution for preference on theft is assumed to be uniformly distributed and is characterized by $\underline{\theta}$ and $\overline{\theta}$. We calibrate these parameters to fall within a reasonable range given annual reported property crimes and the percentage of US citizens with a criminal record.[10]

We calibrate $\underline{c}$, public expenditure on the incarcerated, by matching the costs per prisoner relative to average income.[11] Parameter α is the returns to scale of the production function. We choose α to target an effective return to scale α of 0.85, which is commonly used in the literature.[12]

Finally, we choose λ, the level of public law enforcement, and the extra degree of freedom we have from the distribution function on θ to match inventory shrinkage and loss prevention expenditures as a percentage of revenue from retail firms, as reported by the 2011 National Retail Security Survey.

Table 2.5: Calibration - Parametrization

Moment	Data	Model	Parameter
Consumption Expenditure per Criminal	0.37	0.37	$\underline{c} = 0.36$
Loss Prevention Expenditures	0.35%	0.35%	$\lambda = 0.82$
Criminal Record	3.1%-27.8%	5.00%	$\overline{\theta} = 4.00$
Inventory Shrinkage	1.42%	1.42%	$\underline{\theta} = -0.69$
Returns to Scale	0.85	0.80	$\alpha = 0.80$
Pareto Shape Parameter	4.84	4.84	$\nu = 4.84$

2.4.2 Macro Results

The primary result of our paper is that changes to the level of public law enforcement have different effects depending on the current *level* of public law enforcement. In Figure 2.6 we observe that for low levels of public law enforcement, increases to this level can actually *decrease* the amount of total production. Small increases in the level of public law enforcement cause a decrease in GDP for those countries with low levels of public law enforcement. Countries with higher levels of public law enforcement demonstrate a

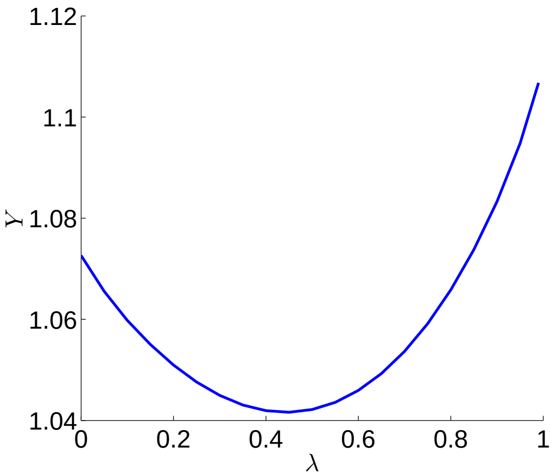
[10]The National Employment Law Project estimates that 27.8% of US adults have a criminal record. On the other hand the FBIs UCR Program reports a property crime rate of 3.1% in 2009.

[11]According to the Bureau of Justice Statistics, as cited in the report "Public Safety, Public Spending" prepared by the Public Safety Performance Project, the marginal cost per prisoner was \$13,797 in 2005. On the other hand the Social Security Administration reports an Average Wage Index in 2005 of \$36,953. We choose $\underline{c}$ so that $\frac{c}{w} = \frac{\$13,797}{\$36,953} = .37$.

[12]See Khan and Thomas (2013) or Ranasinghe (2012) for other papers using a similar number.

positive correlation between per capita GDP and the level of public law enforcement. Finally, we note that the effect of public law enforcement on aggregate production can be as large as 6.02%.

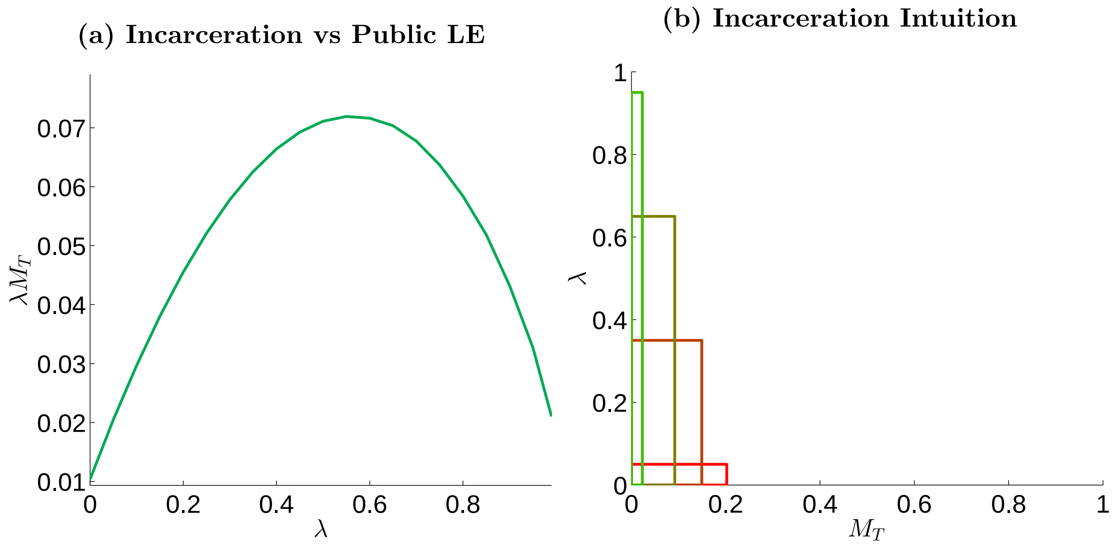
Figure 2.6: Total Production vs Public LE



We explain the primary mechanism for this result with Figure 2.7a. Recall from Table 2.2 the non-monotonicity in the incarceration rate. Our model produces the same pattern as we vary the level of public law enforcement holding all other parameters from the benchmark model fixed. These result can be explained rather intuitively. If we think of the incarceration rate as a rectangle with the vertical axis representing the level of public law enforcement λ, which in our model also represents the percentage of thieves who are caught, and the horizontal axis as the measure of people who steal M_T, then the incarceration rate is simply the area of this rectangle. In the benchmark model we observe that $\frac{\partial M_T}{\partial \lambda} < 0$. At some point the decrease in the measure of thieves outweighs the increase in the percentage of thieves who are caught. This concept is visually represented in Figure 2.7b.[13]

[13] While the levels shown in this figure are quite large relative to incarceration rates observed in the United States, the idea is that increasing public law enforcement causes workers to be removed (or possibly misallocated) from the labor force. Multiple studies have been conducted to review the measure of people in the United States who have a criminal record. This number consistently comes out between one-quarter and one-third of the population. A recent survey from the Society of Human Resources Management shows that 92% of their members perform criminal background checks on some or all job candidates (The Society of Human Resources Management is the largest association of human resources

Figure 2.7: Incarceration



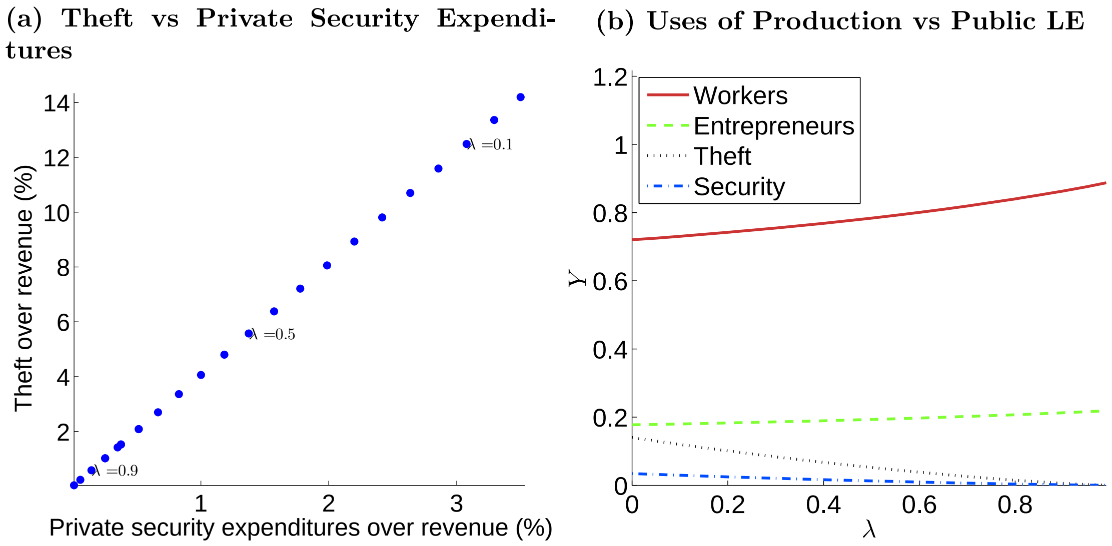
In Figure 2.8a we calculate the equilibrium average theft and average private security expenditures across economies that only differ in their level of public law enforcement (λ). We observe a positive correlation between these two measures.[14] Figure 2.8b splits production into four categories, two of which are security and theft. The security line represents the total cost in final good paid to private security workers. The theft line represents the total value of goods stolen. When we look at these two variables across the level of public law enforcement we see that they match Figures 2.2a and 2.2b. While private security expenditures directly reduce theft, in equilibrium firms hire more private security *and* more agents choose to engage in theft when there is less public law enforcement. In this sense, public law enforcement directly reduces theft, but it also crowds out private security expenditures, which indirectly puts an upward pressure on theft. If policymakers fail to consider this indirect effect, they are likely to overestimate the benefits from public law enforcement.

---

personnel. The survey can be found in *Background Checking: Conducting Criminal Background Checks* (Jan. 22, 2010)). A number of articles, including *65 Million "Need Not Apply"*, put out by the National Employment Law Project, argue that having a criminal background can severely limit job opportunity. While our model is binary in whether an agent is able to work or not, we believe that the actual effect of public law enforcement observed in our model is consistent with what is observed in data.

[14] In our model both measures are perfectly correlated, as proven in the Corollary to Lemma 2.

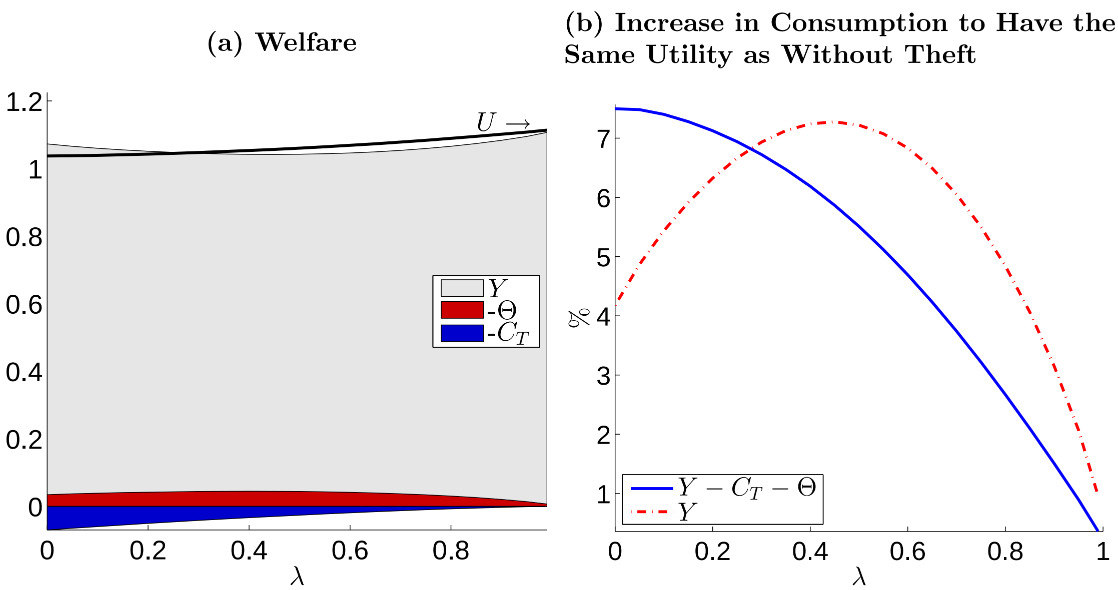
Figure 2.8: Macro Results



2.4.3 Implications on Welfare

Turning to welfare, we observe a similar pattern to that of production relative to the level of public law enforcement. For smaller levels of public law enforcement, increases in the level actually reduce total welfare as seen in Figure 2.9a. Nonetheless, the range of values for which welfare is decreasing is smaller than for production.

Figure 2.9: Effects on Welfare



Explaining in detail this result requires analyzing the expression for welfare. Equation (2.12) shows that welfare is given by the production of the economy minus the cost incurred by thieves when stealing, C_T, and the aversion to steal, Θ. These two extra terms explain why welfare and production are not the same.

$$U \equiv \int_{(z,\theta)} u(z,\theta) dF(\theta) dG(z) = Y - C_T - \Theta, \tag{2.12}$$

where

$$C_T \equiv (1-\lambda) M_T \int_{z \geq z^E} (1 - \lambda F(\theta^E(z))) \frac{a\phi\left(l_s(z)\,|M_T\right)}{z l_y(z)^{\alpha_T}} \frac{\tau(z)^2}{2} dG(z)$$
$$\Theta \equiv \left(\int_{z<z^E} \int_{\underline{\theta}}^{\theta^W} + \int_{z \geq z^E} \int_{\underline{\theta}}^{\theta^E(z)} \right) \theta dF(\theta) dG(z).$$

Next, we analyze the effect of theft on welfare by calculating the extra consumption that consumers in our model require in order to be indifferent to an economy without theft. The economy without theft that we consider is characterized in (B.5) of B.2.

Since welfare includes non-pecuniary costs in utility due to theft and aversion to steal, we analyze how much production needs to increase both including and excluding C_T and Θ. That is, let Y^{NT} denote an economy where there is no theft. The solid line in Figure 2.9b shows $\frac{Y^{NT}+C_T+\Theta}{Y} - 1$ and the dotted line shows $\frac{Y^{NT}}{Y} - 1$. From this figure we can conclude that the effect of theft on welfare is considerable. For some values of λ consumption has to increase by over 7% in order to have the same utility as in an economy without theft. Notice that considering both the costs of theft and the aversion to stealing lowers the amount by which consumption has to be increased for most values of λ since the parametrization shown in Section 2.4.1 implies that it is mostly consumers with negative values of θ who become thieves in equilibrium (i.e. those who get positive utility from the act of stealing.)

2.4.4 Negative Externality of Private Security

We now calculate the negative externality that is caused by hiring private security. For this, we consider an alternative equilibrium where firms are not allowed to hire as much private security as they find optimal. That is, let $l^*(z)$ be the optimal private security hired by firm z; i.e. the value of l_s that is a solution to (2.10). We consider an equilibrium where firm z can only hire $\widehat{l_s}(z) \equiv Sec \times l^*(z)$, for $Sec \in (0,1)$; that is, an equilibrium where firms can only hire a fraction of the security that they find optimal. We keep all parameters of the model as in Table 2.5.

Figure 2.10: Production as a function of λ

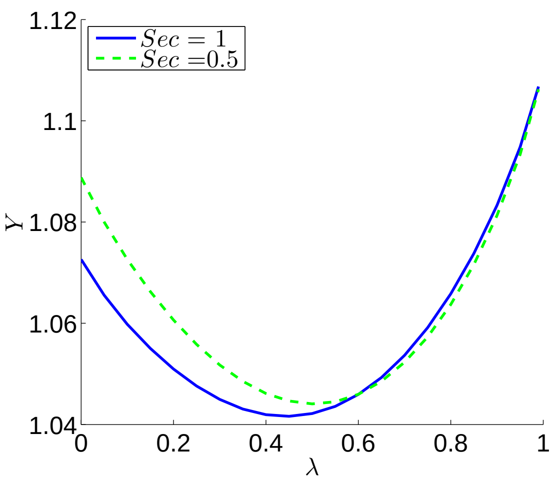


Figure 2.10 contrasts the production in equilibrium for $Sec = 1$ (i.e. the benchmark equilibrium) and for $Sec = 0.5$ (an equilibrium where firms can only hire half as much security as they otherwise find optimal). For low values of λ, restricting private security can increase production up to 1.5%. Private security helps diminish the wedge caused by theft, as seen in (2.11). Nonetheless, when $Sec = 1$, workers that could be hired to produce are hired as private security guards. When λ is low, private security causes a negative externality: workers that are hired as security guards could be hired to produce the final good. Since lower λ implies a higher percentage of revenue spent on security (see Figure 2.8a), the effect of reducing security is much higher for lower λ.

2.4.5 Sensitivity Analysis

We conduct a sensitivity analysis on the remaining variables to better understand how they affect the model. An important parameter in our model is $\underline{c}$. This parameter represents the consumption level received by agents who engage in theft and are caught. As $\underline{c}$ increases, the possibility of getting caught becomes less of a deterrent. Additionally, the burden borne by those who are not caught increases, reducing the value of not engaging in theft and adding incentives towards becoming a thief. As $\underline{c}$ increases, production, the average size of firms and utility all monotonically decrease, and M_T, the measure of people who become thieves, monotonically increases in $\underline{c}$. The implication is that, if you do not care strongly about very negative outcomes for those who are caught stealing, the best policy is to implement very harsh penalties. A potential reason to avoid harsh penalties is concern for the innocent and the costly as well as potentially inaccurate verification of guilt. This is currently outside the scope of this model.

The distribution of θ represents the distribution of the moral fibre of the agents in our model. Apart from matching moments in data, it is difficult to know a proper strategy for determining what this distribution should look like. However, we are able to see how changing the distribution affects the results. Conceptually there are two important components of the distribution of θ which affect theft in our model. First, the measure of people who steal is determined by the measure of people below the cutoffs θ^W or $\theta^E(z)$ in the distribution of θ.[15] Second, the sensitivity of the model to changes in various other parameters depends on the density of the distribution over θ

[15]See Lemma 1 for a characterization of these cutoffs.

at the aforementioned cutoffs.

We make the following observations. First, the model is more sensitive to changes in $\underline{\theta}$ than to changes in $\overline{\theta}$. This is because changes in $\underline{\theta}$ directly affect the measure of people who prefer to steal whereas changes in $\overline{\theta}$ affect the density of people in the range of those who prefer to steal. Lowering $\overline{\theta}$ increases the density of people in the range of those who prefer to steal and vice versa. Second, for the most part the effects of the distribution of θ on equilibrium moments are rather intuitive. The only unusual result is that total welfare is not monotonic in $\underline{\theta}$, but this is easily explained. In all real measures, lowering $\underline{\theta}$ makes the economy worse off; however, recall that θ is the measure of aversion to theft which factors directly into utility. Negative θ's can be interpreted as a rush or pleasure from stealing. As we increase the pleasure from stealing two things happen: The measure of people who steal increases and the extra utility those agents receive from stealing increases. If we make the aversion to theft negative enough, the overall utility can actually begin to decrease in $\underline{\theta}$.

2.4.6 Extensions

We now consider two extensions to the model: First we consider a model where theft causes a destruction of goods. That is, for every unit of good that is stolen, only a fraction δ can be consumed by thieves. In particular we replace thieves' problem (2.5) by (2.13).

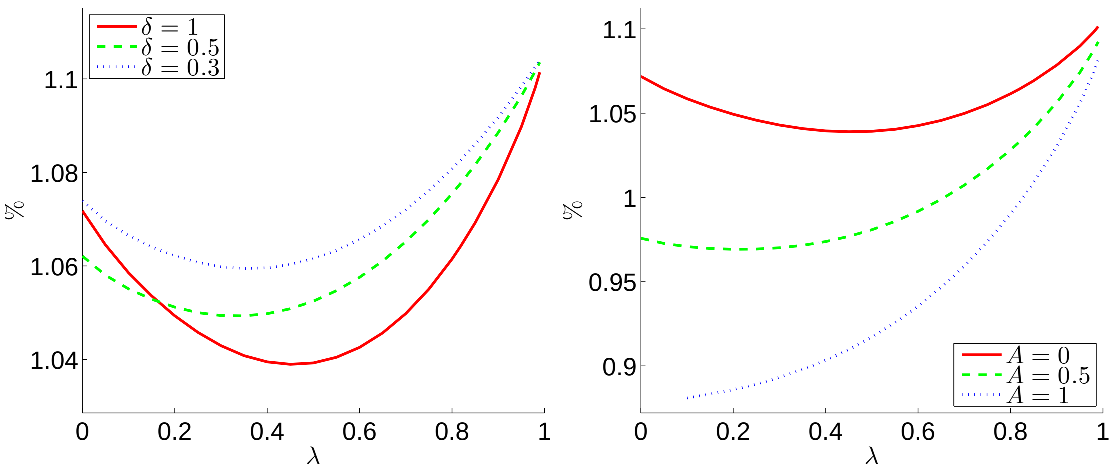
$$\pi_T(z) \equiv \max_{\tau \geq 0} \delta\tau - C_\tau(z). \tag{2.13}$$

Our benchmark model is given by $\delta = 1$ and we consider economies where we change the value of δ. Values such that $\delta < 1$ might act as a deterrent for thieves, since their return for stealing is decreased. However, it might also be the case that they might attempt to steal even more in order to achieve the same consumption as they would otherwise get when $\delta = 1$. In equilibrium we observe that for low levels of δ it is the first effect that dominates. For intermediate levels of δ, it depends on the level of public law enforcement: In economies with low λ theft increases, reducing total production in equilibrium.

In general, $\delta < 1$ causes production to be less sensitive to the level of public law enforcement. Moreover, for low values of δ production is higher across all levels of public law enforcement, relative to the case when $\delta = 1$. See Figure 2.11a.

Figure 2.11: Production for various levels of δ and A

(a) Production for different values of δ **(b) Production for different values of A**



We also consider a model where theft causes labor to be less productive. We can rationalize this by assuming that entrepreneurs know that their employees might be stealing from the firm they are working at, or by assuming that employees who steal work less time, since they devote some time on their job to stealing. We model this feature by replacing (2.4) with (2.14).

$$\pi(z) \equiv \max_{l_y \geq 0, l_s \geq 0} z\left((1 - AM_T)l_y\right)^{\alpha} - wl_y - wl_s - (1 - \lambda)M_T\tau(z), \qquad (2.14)$$

for $A \in [0, 1]$. Our benchmark model is given by $A = 0$ and in this section we consider economies where $A > 0$. The fact that workers are less productive when there is theft causes production to be lower when $A > 0$ than in our benchmark model for all levels of public law enforcement (See Figure 2.11b). Additionally, production becomes more sensitive to the level of λ. In particular, for lower levels of public law enforcement, an increase in the level causes higher labor productivity. For values of A that are high enough, this increment in labor productivity counteracts the removal of agents from the labor force, thus making production increasing in λ for all levels of λ. In summary, we find that while these extensions offer quantitive differences they do not affect our primary qualitative results.

2.5 Extending to a Dynamic Framework

In this section, we extend our static model to a dynamic framework in order to theoretically provide a rationalization for why certain countries persist in a state of low public law enforcement while other countries choose high levels of public law enforcement. To accomplish this we endogenize the level of public law enforcement, λ, by allowing a benevolent government to choose an optimal level of enforcement in each period. In order to simplify the problem, we take our findings from the static model and characterize the policy functions of consumers and firms as reduced form functions of λ.

2.5.1 Dynamic Model

Time is discrete and infinite. There is a unit measure of agents which each produce z units of output per period. M_J represents the fraction of agents currently incarcerated. These agents are unable to produce or steal. Total production each period prior to theft is given by $z(1-M_J)$. M_T continues to represent the measure of agents which choose to attempt theft. The total amount stolen is given by a parameter θ times the proportion choosing to steal. In total, a proportion θM_T of total production is stolen.

The government problem is to choose the level of law enforcement for next period, λ', which incurs a cost given by $\mathcal{C}(\lambda, \lambda')$, a function of the current level and the new level of law enforcement. This gives us the following Bellman equation:

$$V(\lambda, M_J) = \max_{\lambda'} z(1 - M_J)(1 - \theta M_T) - \mathcal{C}(\lambda, \lambda') + \beta V(\lambda', M_J') \tag{2.15}$$

We assume that the government has solved the agent's and firm's problems and therefore model the proportion of thieves M_T and the transition function of the proportion of agents incarcerated M_J' as reduced form functions of state and choice variables of the government. Specifically, the level of theft is solely a function of the current level of law enforcement, λ, and the incarceration rate next period, M_J', is a function of the current incarceration rate, M_J, the fraction of agents choosing to steal today, M_T, and the current level of law enforcement, λ.

The equilibrium to this problem is a value function and policy function which solve Equation (2.15).

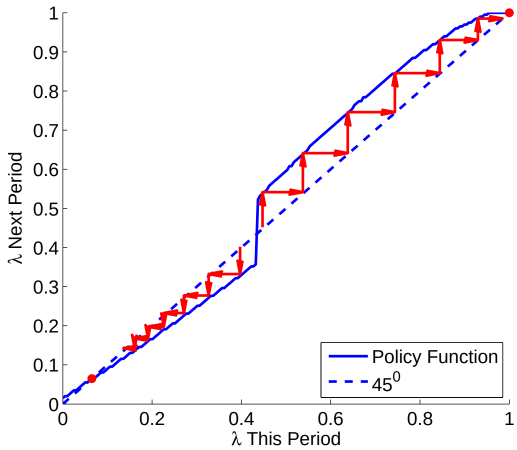
2.5.2 Multiple Steady States

Over a subset of the parameter space we observe a policy function that results in multiple steady states which gives intuition for why we observe countries which persist with a low level of public law enforcement while other countries remain at a high level of public law enforcement. In Figure 2.12 we present sample output for our model for which there exist two stable steady states.[16] The arrows demonstrate how two countries which begin with relatively similar levels of public law enforcement ($\lambda = .4$ and $\lambda = .45$) will diverge over time until they reach the steady states represented by red dots at $\lambda = 0.065$ and $\lambda = 1$.

The intuition for why this occurs is relatively straightforward. The steady state welfare of the $\lambda = 1$ equilibrium is much larger than the equilibrium with a lower value for λ. However, the welfare in between these two steady states is U shaped and the transition cost of moving the level of public law enforcement prevents countries from making large jumps from one point to another. Maintaining a level of law enforcement in the middle of the two steady states is costly and doesn't sufficiently disincent crime but incurs a large cost in lost production due to incarceration. Countries with sufficiently high initial levels of public law enforcement λ will continue investment in additional λ to a level which lowers theft and incarceration rates and obtains the steady state with the high level of welfare. Countries with lower initial levels of public law enforcement will not find it optimal to continue through the transition to a high level of λ going through periods of high incarceration and relatively low deterrence of theft. Instead these countries find it optimal to lower public law enforcement to a relatively inexpensive level. This is represented in Figure 2.12.

[16] In this scenario we used functional forms $M_T = 1-\lambda$, $M'_J = \lambda \times M_T$, and $\mathcal{C}(\lambda, \lambda') = .5\lambda^2 + 5(\lambda - \lambda')^2$. For parameters we chose to let $\theta = 0.52$, $\beta = .96$, and $z = 1$.

Figure 2.12: Multiple Steady States



2.6 Conclusion

In this paper we propose a model of theft, private security and public law enforcement which matches a number of patterns in the micro data. Theft lowers total production directly and indirectly. First, theft acts as a wedge similar to a tax for firms which causes firms to be inefficiently small since the marginal product of labor is greater than the wage rate in equilibria with positive amounts of theft. Private law enforcement helps decrease this wedge, but in order to do so, some of the labor force is taken away from producing the consumption good and used to provide security.

Perhaps the most surprising result of our model is that total production and welfare are not monotonic in levels of public law enforcement. The interaction of theft and public law enforcement is the source of the indirect mechanism that affects the total level of production in the economy. Public law enforcement can reduce total production and welfare because incarcerated agents are removed from the labor force. However, it also increases the disincentives of theft, which causes a reduction in the measure of agents who choose to become thieves. This, in turn, reduces the measure of agents who are incarcerated. The interaction of these two forces can cause non-monotonic effects on the total level of production and welfare which might explain why we observe such vastly different levels of public law enforcement across countries. A simple example is offered in our dynamic extension. Specifically, countries with low levels of public law enforcement do not have immediate benefits from small increases to the level of public law enforcement. The costly transition to a high level of public law enforcement may be sufficient to deter some countries from ever making the investment.

Chapter 3

Leverage Away Your Wedge: An Analysis of Banks' Impact on Output

3.1 Introduction

The banking sector doesn't produce a tangible product but it is clear that it has a tangible effect on the real economy. In this paper we develop a model which allows us to analyze what these effects are and the channel through which these effects are transmitted. Using our model we find that the leverage of banks has both direct and indirect effects on occupational choice, and also indirectly effects the distribution of firm sizes, and real output. The primary channel through which these effects are transmitted is through the spread between the interest rate that banks charge for loans and the interest rate that banks pay for deposits which we will henceforth refer to as the margin of intermediation. We will consider a model without risk so we will only focus on the downside of having limits on the leverage of banks.

In our model firms need to pay for their workers before they produce, in the spirit of Arellano, Bai, and Kehoe (2012). To do this, they can either use their own assets or can take out loans from the banking sector. The primary result from our model is that as banks become less leveraged (i.e. the ratio between deposits over equity decreases), the resulting margin of intermediation in the general equilibrium increases. This margin is responsible for two types of distortion relative to a model with an unconstrained banking sector: First, firms with the same productivity will hire different amounts of workers depending on the assets of the firm. Second, skilled unwealthy consumers will choose to work rather than become entrepreneurs, while nonskilled wealthy consumers will choose to manage firms. In addition, lower leverage in the banking sector will require more bankers to satisfy the demand for loans and deposits. Each of these factors results in a reduction in real output.

An appealing feature of our model is its clear and intuitive characterization of occupational choice among consumers. We allow heterogeneity in consumers along two dimensions; namely, wealth and skill. Rich unskilled consumers choose to become bankers while unwealthy unskilled consumers become workers and skilled consumers choose to manage firms as entrepreneurs. In the parameterization where banks are infinitely leveraged, real allocations are not dependant upon wealth and the model collapses to the model in Lucas (1978) where wealth only affects consumption but has no effect on occupational choice or real output. In this case skilled consumers choose to become entrepreneurs while unskilled consumers choose to work; and the marginal productivity across firms is constant and firms are perfectly assortative in size along the skill of entrepreneurs.

As the leverage of the banking sector decreases, the margin of intermediation increases. As a consequence, the wealth of consumers begin to have real effects on occupational choices of consumers and the hiring decisions of firms, since this causes the cost of the marginal worker to differ based on the wealth of the entrepreneur. Essentially, wealthy entrepreneurs face a lower marginal cost per worker than unwealthy entrepreneurs, which causes firm size to vary across the wealth of entrepreneurs. For two entrepreneurs with the same skill level, the wealthier entrepreneur will hire more workers than the less wealthy entrepreneur. If the skill of the entrepreneurs is sufficiently small, it is possible that the unwealthy consumer will prefer to become a worker rather than manage a firm and face the higher interest rate on loans required to hire workers. Additionally, it is possible that wealthy consumers, who would have worked in the scenario where banks are infinitely leveraged, are incented to become entrepreneurs due to the reduction in the return of their assets. Even though their skill at managing workers is low relative to the rest of the entrepreneurs, they can obtain a higher return from using their wealth to hire workers rather than investing their assets with banks.

We relate our model to the misallocation literature. Banerjee and Moll (2010), Buera, Kaboski, and Shin (2011) and Midrigan and Xu (2014), among others, analyze the role of misallocation on the productivity of an economy. In these models, misallocation arises mostly due to financial constraints. In our case, it is the leverage of the financial intermediaries that causes the misallocation.

Our model is also related to the literature relating financial development and efficiency with production. Levine (2005) offers a comprehensive literature review of this field. In Greenwood, Sanchez, and Wang (2010) and Greenwood, Sanchez, and Wang (2013) costly state verification causes a difference in the marginal product of capital and its user cost. In our model it is the leverage of the banking sector that causes different consumers to face different interest rates.

Erosa (2001) is probably the most similar model to ours. In his model intermediation costs cause an exogenous margin of intermediation. In our case this margin of intermediation arises endogenously from the leverage of the banking sector. Erosa (2001) also analyzes occupational choice, although in his setup the only heterogeneity of consumers is in age, as his model is dynamic. We explore an additional occupational choice; namely, becoming a banker.

This paper is organized as follows: Section 3.2 presents the model; Section 3.3 characterizes the solution to the model; Section 3.4 presents a benchmark model without limits on the leverage of banks; Section 3.5 shows the main results; Section 3.6 highlights preliminary relations we observe in data. Finally, Section 3.7 concludes.

3.2 Model

We consider a two period model. Consumers are heterogeneously endowed with skill and wealth. At the beginning of the first period they choose whether to become workers, entrepreneurs or bankers. Workers receive their wages in the first period and save to consume in the second period. Entrepreneurs manage firms and need to pay their workers in the first period. Nonetheless, the firms they manage produce in the second period. Therefore they might need to borrow from banks in order to pay their wage bills. Bankers set up a bank in the first period and receive the profits from the bank in the second period. Banks take deposits from consumers and lend to firms. They face an exogenous limit on their leverage.

We assume that there is a unit measure of consumers who maximize utility by choosing to be an entrepreneur, a worker or a banker. Each consumer is endowed with a skill level z and some wealth a. We assume that z and a are drawn from a distribution with positive support that we will denote by $G(z \times a)$. A consumer's decision is characterized by z and a, so we will denote consumers by the realizations of these random variables. Consider consumer (z, a). He solves the following problem

$$u(z,a) = \max_{\mathcal{O}} \left\{ u^{\mathcal{W}}(a), u^{\mathcal{E}}(z,a), u^{\mathcal{B}}(a) \right\}, \tag{3.1}$$

where $u^{\mathcal{W}}(a)$ denotes the utility derived from becoming a worker, $u^{\mathcal{E}}(z,a)$ is the utility from becoming an entrepreneur, and $u^{\mathcal{B}}(a)$ is the utility from becoming a banker.

3.2.1 Workers

Denote the set of workers by $\mathcal{W}$. In period 1 workers use their wages w and wealth to consume and to save, s. In the second period the worker's income is given by the return on savings, r^D. The utility of being a worker with wealth a is given by (3.2).

$$\begin{aligned} u^{\mathcal{W}}(a) &= \max_{s} \ln c_1 + \beta \ln c_2 \\ c_1 &= w + a - s \\ c_2 &= (1 + r^D)s. \end{aligned} \tag{3.2}$$

3.2.2 Entrepreneurs

Denote the set of entrepreneurs by $\mathcal{E}$. In the first period entrepreneurs use their wealth to consume, to pay for the workers l they hire, and they can save or borrow. In the second period entrepreneurs consume the production of the firm. If they borrowed in the first period, they repay their debt at an interest rate of r^L. If they saved, they get a return of r^D on their savings. The utility of being an entrepreneur with skill z and wealth a is given by (3.3).

$$
\begin{aligned}
u^{\mathcal{E}}(z,a) = \max_{s,l} \ln c_1 + \beta \ln c_2 \\
c_1 = a - wl - s \\
c_2 = zl^{\alpha} + (1+r^L)s\mathbf{1}_{\{s<0\}} \\
+ (1+r^D)s\mathbf{1}_{\{s\geq 0\}}.
\end{aligned}
\tag{3.3}
$$

Each entrepreneurs belongs to one of three types: Entrepreneurs that borrow to pay for their workers, $s<0$, entrepreneurs that have enough wealth to pay for workers and deposit the difference, $s>0$, and entrepreneurs that spend all their available wealth to hire workers, $s=0$. We will denote by $\mathcal{E}_L$ the set of entrepreneurs that borrow, by $\mathcal{E}_D$ the set of entrepreneurs that save and by $\mathcal{E}_O$ the rest of entrepreneurs.

3.2.3 Bankers

Denote the set of bankers by $\mathcal{B}$. In the first period bankers consume part of their wealth. The rest of their wealth, s, is used as equity for the bank they manage. In the second period bankers consume the profits from that bank. The utility of a banker with wealth a is given by (3.4).

$$
\begin{aligned}
u^{\mathcal{B}}(a) = \max_{s} \ln c_1 + \beta \ln c_2 \\
c_1 = a - s \\
c_2 = \pi^B(s).
\end{aligned}
\tag{3.4}
$$

The profits of a bank with equity s are given by (3.5)

$$
\begin{aligned}
\pi^B(s) \equiv \max_{L,D} \ (1+r^L)L - (1+r^D)D \\
\text{s. t. } D + s = L \\
\frac{D}{s} \leq \lambda.
\end{aligned}
\tag{3.5}
$$

The first constraint in (3.5) is the balance sheet constraint of the bank: The bank lends its equity and the deposits it takes. The second constraint implies that there is a limit on how many resources a bank can intermediate. Specifically the limit is on how many deposits a bank can take per unit of equity. This limit is exogenous and we denote it by λ.

We now define an equilibrium for this economy:

Definition 2. *An equilibrium for this economy is allocations* $x^{\mathcal{W}}(a) \equiv \{s^{\mathcal{W}}(a)\}$, $x^{\mathcal{E}}(z,a) \equiv \{l(z,a), s^{\mathcal{E}}(z,a)\}$, $x^{\mathcal{B}}(a) \equiv \{s^{\mathcal{B}}(a)\}$ *and* $x^{B}(s) \equiv \left\{L^B(s), D^B(s)\right\}$, *prices* $p \equiv \{w, r^L, r^D\}$ *and sets* $\mathcal{W}$, $\mathcal{B}$ *and* $\mathcal{E}$ *such that*

1. $\mathcal{W}$, $\mathcal{B}$ *and* $\mathcal{E}$ *are such that* $\mathcal{O}(z,a)$ *is a solution to* (3.1) *for all* (z,a)*;*
2. *given* p, $x^{\mathcal{W}}(a)$ *is a solution to* (3.2)*;*
3. *given* p, $x^{\mathcal{E}}(z,a)$ *is a solution to* (3.3)*;*
4. *given* p, $x^{\mathcal{B}}(a)$ *is a solution to* (3.4)*;*
5. *given* p, $x^{B}(s)$ *is a solution to* (3.5)*;*
6. *and markets clear:*
   (a) *Deposits:*
   $$\int_{\mathcal{W}} s^{\mathcal{W}}(a)dG(z \times a) + \int_{\mathcal{E}_D} s^{\mathcal{E}}(z,a)dG(z \times a) = \int_{\mathcal{B}} D^B(s^{\mathcal{B}}(a))dG(z \times a);$$
   (b) *loans:*
   $$\int_{\mathcal{E}_L} s^{\mathcal{E}}(z,a)dG(z \times a) + \int_{\mathcal{B}} L^B(s^{\mathcal{B}}(a))dG(z \times a) = 0;$$
   (c) *labor:*
   $$\int_{\mathcal{W}} dG(z \times a) = \int_{\mathcal{E}} l(z,a)dG(z \times a);$$
   (d) *goods:*
   $$\int c_1(z,a)dG(z \times a) = \int adG(z \times a)$$
   $$\int c_2(z,a)dG(z \times a) = \int_{\mathcal{E}} zl(z,a)^{\alpha}dG(z \times a).$$

3.3 Characterizing the model

We will first prove that in equilibrium the interest rate of loans is greater than the interest rate on deposits. This implies that there is an incentive to manage a bank, rather to deposit in one. Additionally, the profits from banks are linear in the wealth that is used to run them.

Lemma 3. *In equilibrium $r^L \geq r^D > -1$ and $r^L = r^D$ only as $\lambda \to \infty$. Furthermore, the profits of a bank with equity s can be written as $\pi^B(s) = (1 + r^B)s$ with*

$$r^B = r^L + \lambda(r^L - r^D);$$

$r^B \geq r^D$ and $r^B = r^D$ only as $\lambda \to \infty$. Additionally, loan supply and deposit demand are given by

$$\begin{aligned} D^d(s) &= \lambda s \\ L^s(s) &= (1 + \lambda)s. \end{aligned} \tag{3.6}$$

Proof. First notice that if $r^D \leq -1$, then banks will demand an infinite amount of deposits. If $r^L < r^D$ then banks will not supply loans since the cost of deposits is higher than the revenue they can get from lending, so it must be the case that $r^L \geq r^D$. Now, the bank is risk neutral. Therefore the second constraint in (3.5) binds. The supply of loans follows from the balance sheet constraint $(D(s) + s = L(s))$. This proves (3.6).

Plugging (3.6) into the objective function of (3.5) yields $\pi^B(s) = (1 + r^B)s$, where $r^B \equiv r^L + \lambda(r^L - r^D)$. Now, notice that the only way to have a finite r^B as $\lambda \to \infty$ is if $r^L = r^D$. Additionally, r^B can also be written as $r^B = r^D + (1 + \lambda)\left(r^L - r^D\right)$. $r^L \geq r^D$ implies that $r^B \geq r^D$, with equality only as $\lambda \to \infty$. □

Now we will prove that both workers and bankers save in the first period. The reason for this is that these consumers have no source of income in the second period. Lemma 4 characterizes the solution of (3.2) and (3.4).

Lemma 4. *The solution of* (3.2) *is*

$$s^{\mathcal{W}}(a) = \frac{\beta}{1 + \beta}(w + a). \tag{3.7}$$

The solution of (3.4) *is*

$$s^{\mathcal{B}}(a) = \frac{\beta}{1+\beta}a. \tag{3.8}$$

Proof. The first order condition of (3.2) is

$$\frac{1}{w+a-s} = \frac{\beta}{s}.$$

Lemma 3 implies that the first order condition of (3.4) can be written as

$$\frac{1}{a-s} = \frac{\beta}{s}.$$

Solving for s in (3.7) and (3.8) yields the result. $\square$

Finally we will characterize the solution of (3.3). Since entrepreneurs that borrow and entrepreneurs that save face a different interest rate, there will be misallocation: Firms with the same productivity have different sizes depending on the wealth of the entrepreneur that manages them. The misallocation depends on the margin of intermediation.

Lemma 5. *The solution of* (3.3) *is*

$$l(z,a) = \begin{cases} \left(\frac{\alpha z}{(1+r^L)w}\right)^{\frac{1}{1-\alpha}} & \textit{if } a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \\ \frac{\alpha\beta}{1+\alpha\beta}\frac{a}{w} & \textit{if } \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \le a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \\ \left(\frac{\alpha z}{(1+r^D)w}\right)^{\frac{1}{1-\alpha}} & \textit{if } a \ge \delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \end{cases}$$

$$s^{\mathcal{E}}(z,a) = \begin{cases} \frac{\beta}{1+\beta}a - \frac{1+\alpha\beta}{1+\beta}\left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} & \textit{if } a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \\ 0 & \textit{if } \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \le a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \\ \frac{\beta}{1+\beta}a - \frac{1+\alpha\beta}{1+\beta}\left(\frac{z}{1+r^D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} & \textit{if } a \ge \delta_{\mathcal{E}_D,\mathcal{E}_O}(z), \end{cases}$$

where

$$\delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \equiv \frac{1+\alpha\beta}{\beta}\left(\frac{z}{1+r^D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}$$
$$\delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \equiv \frac{1+\alpha\beta}{\beta}\left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

Entrepreneurs with $a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$ *will borrow, entrepreneurs with* $a \geq \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ *will save and entrepreneurs with* $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ *will spend all their available wealth in paying for workers.*

Proof. If $s \neq 0$ the first order conditions of (3.3) can be written as

$$\frac{w}{a-wl-s} = \frac{\beta\alpha z l^{\alpha-1}}{zl^\alpha + (1+r)s}$$
$$\frac{1}{a-wl-s} = \frac{\beta(1+r)}{zl^\alpha + (1+r)s},$$

where $r = r^D$ if $s > 0$ and $r = r^L$ if $s < 0$. If $s = 0$ first order conditions of (3.3) can be written as

$$\frac{w}{a-wl} = \frac{\alpha\beta}{l}.$$

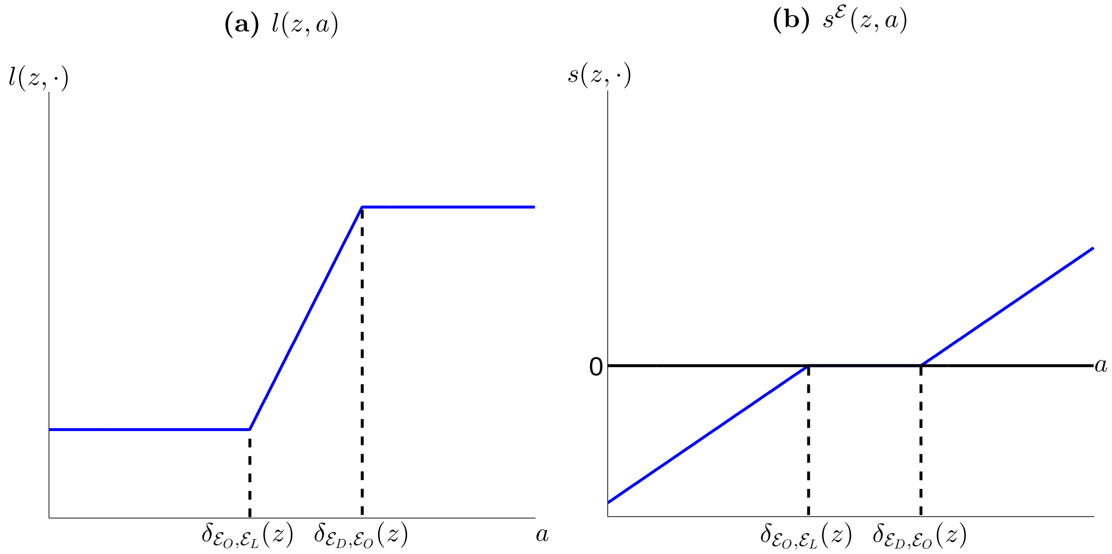
The proof of the Lemma follows from solving for s and l. The expressions for $\delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ and $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$ follow from analyzing when $s > 0$ or $s < 0$. □

For the moment fix w. Then there are two effects on entrepreneurs of having a positive margin of intermediation: The larger this margin is, the bigger the range in firm sizes for consumers with the same skill z across the spectrum of wealth a.

Wealthy entrepreneurs will have enough wealth to pay for their workers and save the difference. Due to this, the marginal cost of an employee will depend on r^D. On the other hand, unwealthy entrepreneurs need to borrow to pay for their workers, so the marginal cost of an employee will depend on r^L. Figure 3.1a highlights this point.

Additionally, a higher margin of intermediation implies that the difference in wealth between an entrepreneur that is able to save and an entrepreneur that borrows is higher. In other words, holding w constant, a higher margin of intermediation implies a larger difference between $\delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ and $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$. As a consequence less entrepreneurs use banks as financial intermediaries, since more entrepreneurs use all their available wealth to hire workers. See Figure3.1b for a graphical representation of this point.

Figure 3.1: Entrepreneur with skill z



Corollary 2 shows misallocation in a slightly different way. Let $r(z,a)$ be the marginal return of hiring $l(z,a)$ workers. This return will be decreasing in the wealth of the entrepreneur. $r(z,a)$ is the opportunity cost of using wealth for hiring workers. If an unwealthy entrepreneur uses one extra dollar to hire workers, he is borrowing more and therefore is spending r^L. On the other hand, an extra unit of wealth that a wealthy entrepreneur spends on hiring workers could be used to get a return of r^D if it was used instead to save in a bank.

Corollary 2. *Let* $r(z,a) \equiv \frac{z\alpha l(z,a)^{\alpha-1}}{w} - 1$. *Then*

$$r(z,a) = \begin{cases} r^L & \text{if } a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \\ z\left(\frac{\alpha}{w}\right)^{\alpha}\left(\frac{1+\alpha\beta}{\beta a}\right)^{1-\alpha} - 1 & \text{if } \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \\ r^D & \text{if } a \geq \delta_{\mathcal{E}_D,\mathcal{E}_O}(z). \end{cases}$$

$r(z,a)$ *is continuous and decreasing in* a.

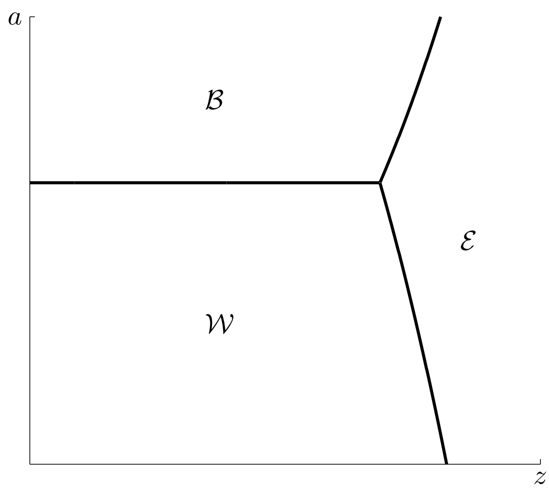
With the results shown in Lemmas 4 and 5 we are able to determine the occupational choice of consumers. In C.1 we determine explicitly the boundaries in skill and wealth that determine the occupational choice of consumers as a function of the prices in this economy. That allows us to fully characterize each consumer, taking prices as given.

From Lemma 5 we conclude that within the set of entrepreneurs, wealthy entrepreneurs (high a) will be able to save and unwealthy entrepreneurs (low a) will need to borrow. In general we find that consumers with low wealth a and low skill z will choose to become workers. The fact that these consumers have low skill makes it better for them to work than to set up a firm. Additionally, their low wealth makes it optimal for them to get an extra source of income in the first period. The only way to do this is by becoming a worker.

On the other hand, consumers with low skill and high wealth will become bankers. Similar to workers, having a low skill level is a deterrent from becoming an entrepreneur. Nonetheless, the high level of wealth makes it better for these consumers to set up a bank, rather than to become workers, since $r^B > r^D$ in equilibrium. Finally, entrepreneurs will be consumers with high skill. As shown in Lemma 5, the level of wealth will affect the size of the firm that they manage.

It is worth mentioning that the occupational choice of consumers depends on their wealth. Figure 3.2 shows graphically the different occupation choices of consumers in (z, a) space.

Figure 3.2: Types of consumers depending on skill and wealth



$\mathcal{B}$: Bankers. $\mathcal{W}$: Workers. $\mathcal{E}$: Entrepreneurs.

3.4 Model with perfectly efficient banking sector

Contrast the model characterized in Section 3.3 with a model where the margin of intermediation is 0. In this model there will only be an interest rate r. As mentioned in Lemma 3, this can be achieved in the limit as λ approaches infinity. Recall from Lemma 3 that in this case $r^B = r^L = r^D$, so only a consumer with infinite wealth will be willing to be a banker and this bank will be infinitely leveraged and have 0 profits. We can interpret this case as a model where consumers do not need a financial intermediary to get wealth from consumers that are willing to save to borrowing entrepreneurs. In this case consumers choose whether to become entrepreneurs or workers. The utility of a consumer endowed with skill level z and wealth a is given by

$$u(z,a) = \max_{\mathcal{O}} \left\{ u^{\mathcal{E}}(z,a), u^{\mathcal{W}}(a) \right\}, \tag{3.9}$$

where

$$\begin{aligned} u^{\mathcal{W}}(a) &= \max_{s} \ln c_1 + \beta \ln c_2 \\ c_1 &= w + a - s \\ c_2 &= (1+r)s. \end{aligned} \tag{3.10}$$

and

$$\begin{aligned} u^{\mathcal{E}}(z,a) &= \max_{s,l} \ln c_1 + \beta \ln c_2 \\ c_1 &= a - wl - s \\ c_2 &= zl^{\alpha} + (1+r)s. \end{aligned} \tag{3.11}$$

In this case, entrepreneurs face the same interest rate, regardless if they borrow or save. Workers, as before, will decide to save since they don't have any source of income in the second period. An equilibrium for this economy is defined as follows.

Definition 3. *An equilibrium for this economy is allocations* $x^{\mathcal{W}}(a) \equiv \{s^{\mathcal{W}}(a)\}$ *and* $x^{\mathcal{E}}(z,a) \equiv \{l(z), s^{\mathcal{E}}(z,a)\}$*, prices* $p \equiv \{w, r\}$ *and sets* $\mathcal{W}$ *and* $\mathcal{E}$ *such that*

1. *$\mathcal{W}$ and $\mathcal{E}$ are such that $\mathcal{O}(z,a)$ is a solution to* (3.9) *for all* (z,a)*;*

2. *given p,* $x^{\mathcal{W}}(a)$ *is a solution to* (3.10)*;*

3. *given p,* $x^{\mathcal{E}}(z,a)$ *is a solution to* (3.11)*;*

4. *and markets clear:*

   (a) *Savings:*
   $$\int_{\mathcal{W}} s^{\mathcal{W}}(a)dG(z\times a) + \int_{\mathcal{E}} s^{\mathcal{E}}(z,a)dG(z\times a) = 0;$$

   (b) *labor:*
   $$\int_{\mathcal{W}} dG(z\times a) = \int_{\mathcal{E}} l(z)dG(z\times a);$$

   (c) *goods:*
   $$\int c_1(z,a)dG(z\times a) = \int adG(z\times a))$$
   $$\int c_2(z,a)dG(z\times a) = \int_{\mathcal{E}} zl(z)^{\alpha}dG(z\times a).$$

Lemmas 6 and 7 characterize the solution to (3.10) and (3.11).

Lemma 6. *The solution of* (3.10) *is*

$$s^{\mathcal{W}}(a) = \frac{\beta}{1+\beta}(w+a).$$

Proof. The first order condition of (3.10) is

$$\frac{1}{w+a-s} = \frac{\beta}{s}.$$

Solving for s yields the result. □

Lemma 7. *The solution of* (3.11) *is*

$$l(z) = \left(\frac{\alpha z}{(1+r)w}\right)^{\frac{1}{1-\alpha}}.$$

$$s^{\mathcal{E}}(z,a) = \frac{\beta}{1+\beta}a - \frac{1+\alpha\beta}{1+\beta}\left(\frac{z}{1+r}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

Let

$$\delta_{\mathcal{E}_D,\mathcal{E}_L}(z) \equiv \frac{1+\alpha\beta}{\beta}\left(\frac{z}{1+r}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

Entrepreneurs will save if $a \geq \delta_{\mathcal{E}_D,\mathcal{E}_L}(z)$ *and borrow otherwise.*

Proof. The first order conditions of (3.11) can be written as

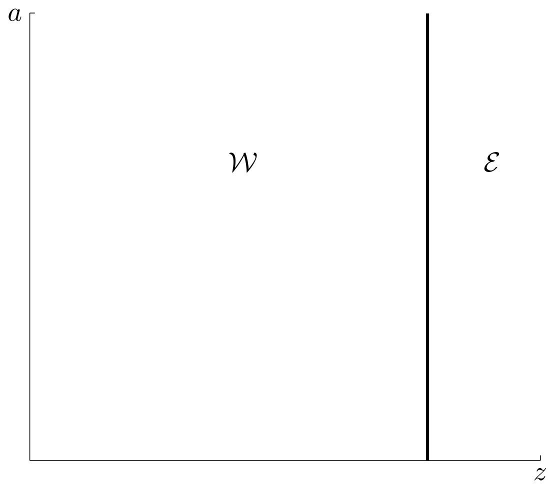
$$\begin{aligned}
\frac{w}{a-wl-s} &= \frac{\beta\alpha z l^{\alpha-1}}{zl^{\alpha}+(1+r)s} \\
\frac{1}{a-wl-s} &= \frac{\beta(1+r)}{zl^{\alpha}+(1+r)s},
\end{aligned}$$

The proof of the Lemma follows from solving for s and l. The expression for $\delta_{\mathcal{E}_D,\mathcal{E}_L}(z)$ follows from analyzing when $s > 0$ or $s < 0$. □

Notice that in this case the size of the firms does not depend on the wealth of the entrepreneur. With the results shown in Lemmas 6 and 7 we are able to determine the occupational choice of consumers. In C.2 we determine the explicit boundaries between the two occupational choices of consumers as functions of w and r.

Since every consumer faces the same interest rate, the boundary that determines the occupational choice between workers and entrepreneurs will not depend on the level of wealth. Similar to Lucas (1978), the occupation choice depends exclusively on skill level. Less skilled consumers will become workers since the consumption they get from setting up a firm would be lower than consumption from working. Figure 3.3 shows a graphical characterization of consumers in the (z, a) space.

Figure 3.3: Types of consumers depending on skill and wealth



$\mathcal{W}$: Workers. $\mathcal{E}$: Entrepreneurs.

We solve this model by stating and solving an equivalent Social Planner Problem (See C.3).

3.5 Results

We first prove a lemma that allows us to characterize labor remuneration in the model. We then provide an example which allows us to get a closed form solution. Finally we show some numerical results to highlight the main results of our model.

3.5.1 Labor remuneration is constant

Lemma 8 proves that labor remuneration in the model is constant.

Lemma 8. *Let A be the total amount of wealth in the economy and denote the mass of workers by $M_{\mathcal{W}}$. Then*

$$wM_{\mathcal{W}} = \alpha\beta A.$$

Proof. See C.4. □

We give an overview of the proof of Lemma 8. Consider first the case where wages are paid in the same period as when production takes place. Given the production function of the firms in our model, it holds that $wM_{\mathcal{W}} = \alpha Y$, where Y denotes total production. Now, in our model wages are paid in period 1, while production takes place in period 2. Therefore the marginal cost of labor depends on interest rates. that is,

$$(1+\tilde{r})wM_{\mathcal{W}} = \alpha Y, \tag{3.12}$$

where $\tilde{r}$ is an average interest rate of the economy.[1] Additionally, in our model total consumption in the first period is given by the total amount of wealth in the economy, A, while total consumption in the second period equals total production, Y. Finally, the fact that consumers have log utility implies the following aggregate Euler equation

$$Y = \beta(1+\tilde{r})A. \tag{3.13}$$

Plugging in (3.12) into (3.13) yields the result in the Lemma. Lemma 8 implies that the effect of productivity affects wages indirectly through the measure of workers. In other words, more productive economies will have higher wages since the measure of workers will be smaller.

3.5.2 Example

We consider a particular distribution that allows us to find a close form solution to the model. Given constraints on parameters, we are able to abstract from changes in occupational choice to focus on the main source of distortion; namely, the difference in size by firms with the same productivity. Finally we are able to derive an analytical solution for total output and show that it is increasing in λ since output decreases with the margin of intermediation. The distribution we consider is specified in Definition 4.

Definition 4. *Let $\widetilde{G}(\cdot)$ be the following distribution on z and a:*

1. *z takes values z_1 and z_2, with weights δ_z and $1-\delta_z$, respectively. We assume $z_1 < z_2$.*
2. *For $z = z_1$: a takes values a_1 and a_2, with weights δ_a and $1-\delta_a$, respectively. We assume $a_1 < a_2$.*

[1]Recall from the corollary to Lemma 5 that each type of entrepreneur faces a different interest rate in this economy.

3. *For $z = z_2$: Continuum of values of a distributed according to a uniform distribution between 0 and $\overline{a}$.*

Proposition 2 states the equilibrium prices and occupational choices in this economy. The specific set of assumptions on parameters, as well as the proof of the proposition, can be found in C.4.

Proposition 2. *Given $\widetilde{G}(\cdot)$,*

$$
\begin{aligned}
w &= \alpha\beta \frac{\delta_a\delta_z a_1 + (1-\delta_a)\delta_z a_2 + (1-\delta_z)\frac{\overline{a}}{2}}{\delta_a\delta_z} \\
1 + r^L &= \frac{z_2}{\beta}(1+\alpha\beta)^{1-\alpha}\left(\frac{\delta_a\delta_z}{\delta_a\delta_z a_1 + (1-\delta_a)\delta_z a_2 + (1-\delta_z)\frac{\overline{a}}{2}}\right)^{\alpha}\left(\frac{1-\delta_z}{2\overline{a}(1+\lambda)(1-\delta_a)\delta_z a_2}\right)^{\frac{1-\alpha}{2}} \\
1 + r^D &= \frac{z_2}{\beta}(1+\alpha\beta)^{1-\alpha}\left(\frac{\delta_a\delta_z}{\delta_a\delta_z a_1 + (1-\delta_a)\delta_z a_2 + (1-\delta_z)\frac{\overline{a}}{2}}\right)^{\alpha} \\
&\times\left(\frac{1}{1-\left(\frac{2}{\overline{a}(1-\delta_z)}\right)^{\frac{1}{2}}\left((\lambda-\alpha\beta)(1-\delta_a)\delta_z a_2 - \alpha\beta(1-\delta_z)\frac{\overline{a}}{2} - (1+\alpha\beta)\delta_a\delta_z a_1\right)^{\frac{1}{2}}}\right)^{1-\alpha}
\end{aligned}
$$

and occupational choices

$$
\begin{aligned}
\mathcal{W} &= (z_1, a_1) \\
\mathcal{B} &= (z_1, a_2) \\
\mathcal{E}_D &= \{(z_2, a) : \delta_{\mathcal{E}_D,\mathcal{E}_O}(z_2) \le a \le \overline{a}\} \\
\mathcal{E}_O &= \{(z_2, a) : \delta_{\mathcal{E}_O,\mathcal{E}_L}(z_2) \le a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z_2)\} \\
\mathcal{E}_L &= \{(z_2, a) : 0 \le a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z_2)\}
\end{aligned}
$$

are an equilibrium in this economy, where $\delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$, and $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$ are as defined in Lemma 5.

Proof. See C.4. □

The fact that the margin of intermediation is decreasing in λ is a corollary to Proposition 2.

Corollary 3. *$r^L - r^D$ is decreasing in λ.*

Additionally, we prove that the differences in sizes for firms decreases as λ increases.

Corollary 4. *The dispersion in firm size is decreasing in λ.*

Proof. The dispersion in sizes of firms is increasing in $r^L - r^D$. □

Furthermore, from Proposition 2 we are able to derive an expression for total output in this economy. Proposition 3 proves that total production is increasing in λ, since it is decreasing in the margin of intermediation.

Proposition 3. *Let*

$$\mathcal{C}_L \equiv \frac{1-\delta_z}{2\overline{a}(1+\lambda)(1-\delta_a)\delta_z a_2}$$

$$\mathcal{C}_D \equiv \frac{1}{1-\left(\frac{2}{\overline{a}(1-\delta_z)}\right)^{\frac{1}{2}}\left((\lambda-\alpha\beta)(1-\delta_a)\delta_z a_2-\alpha\beta(1-\delta_z)\frac{\overline{a}}{2}-(1+\alpha\beta)\delta_a\delta_z a_1\right)^{\frac{1}{2}}}.$$

Then total output in this economy is equal to

$$Y = z_2\frac{1-\delta_z}{\overline{a}}\left(\frac{1}{1+\alpha\beta}\right)^{\alpha}\left(\frac{M_{\mathcal{W}}}{A}\right)^{\alpha}\left[\overline{a}\left(\frac{1}{\mathcal{C}_D}\right)^{\alpha}-\frac{\alpha}{1+\alpha}\left[\left(\frac{1}{\mathcal{C}_D}\right)^{1+\alpha}-\left(\frac{1}{\mathcal{C}_L}\right)^{\frac{1+\alpha}{2}}\right]\right]$$

Furthermore, an increase in λ decreases the last term in Y, which is a function of $r^L - r^D$.

Proof. See C.4. □

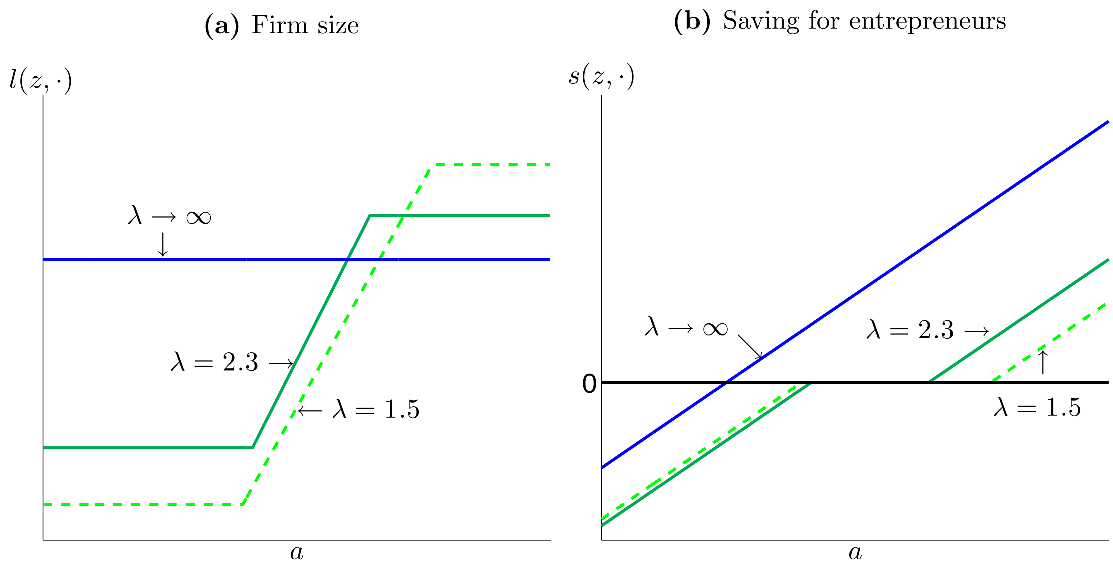
3.5.3 Numerical solutions

We now solve the model numerically by assuming that z and a are drawn from independent uniform distributions. We also set $\alpha = 0.7$ and $\beta = 0.96$.[2] We analyze what happens as λ increases. We find that $r^L - r^D$ is decreasing in λ. Additional to the effect this has on the dispersion on the sizes of firms with the same productivity, we also find that there is an effect on occupational choice; namely, we observe that unskilled consumers who choose to become entrepreneurs in economies with low levels of λ, will choose to become workers as λ increases. Also, skilled consumers who became workers for low levels of λ, choose to manage firms for higher limits on the leverage of banks.

[2] The results we show hold qualitatively for various parameterizations.

As λ increases, the margin of intermediation goes down, which implies lower misallocation: The heterogeneity in size among firms with same productivity decreases. Figure 3.4 shows what happens to misallocation as λ increases. Figure 3.4a shows the effect on firm size for an entrepreneur under different values of λ. As $\lambda \to \infty$ misallocation disappears. Firms with the same productivity will have the same size, as the continuous line shows. For finite values of λ, there will be misallocation.

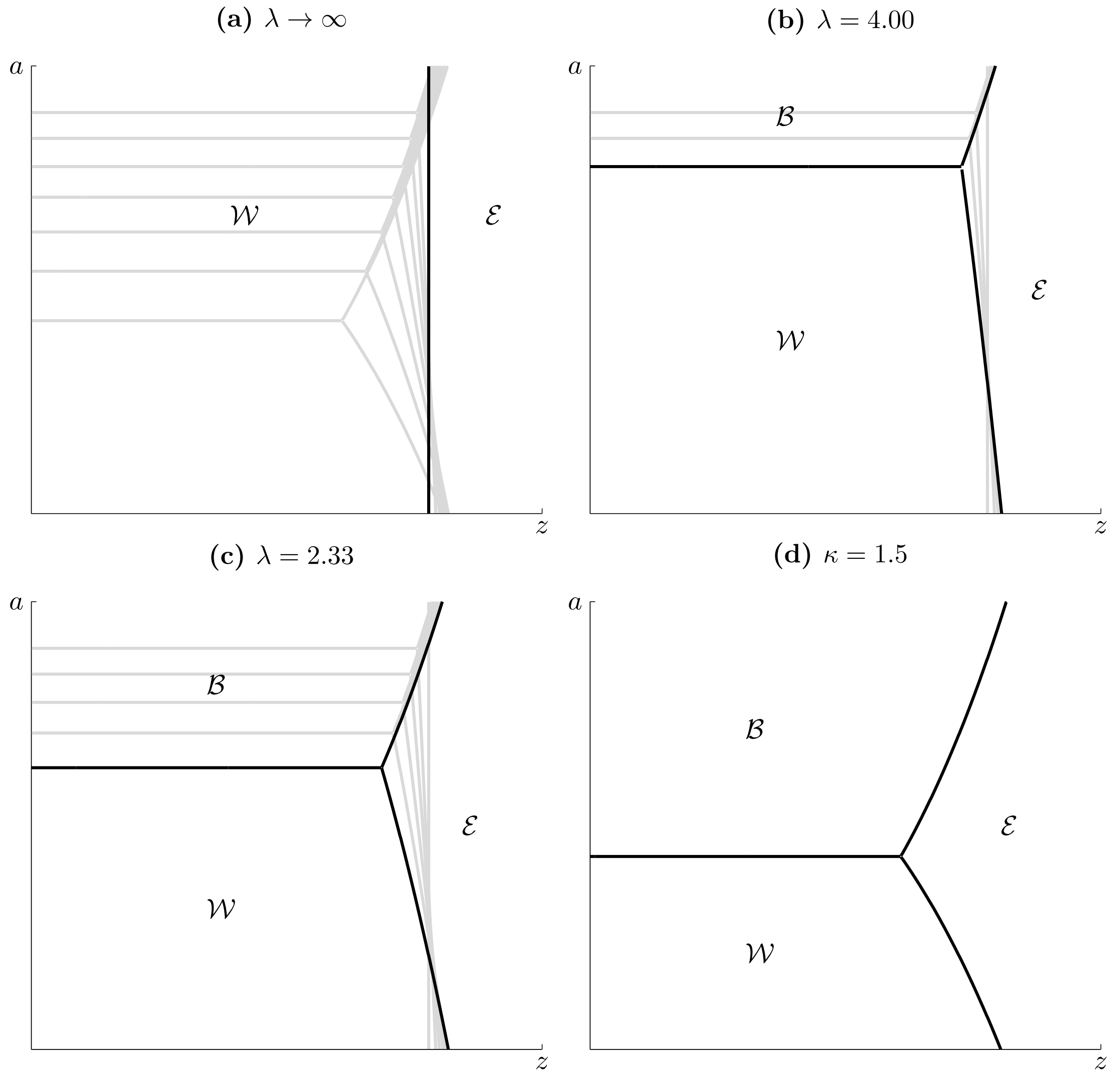
Figure 3.4: Entrepreneur with skill z



Now, recall that the kinks in firm size correspond to values of a such that $a = \delta_{\mathcal{E}_O,\mathcal{E}_L}$ and $a = \delta_{\mathcal{E}_D,\mathcal{E}_O}$. A higher value of λ implies a smaller difference between $\delta_{\mathcal{E}_O,\mathcal{E}_L}$ and $\delta_{\mathcal{E}_D,\mathcal{E}_O}$. A consequence of a higher λ is that more entrepreneurs use banks as financial intermediaries, since less entrepreneurs use all their available wealth to hire workers. Figure 3.4b highlights this point: higher levels of λ imply less entrepreneurs that neither borrow nor save.

Furthermore, as λ increases another distortion diminishes: Skilled consumers choose to manage firms, while unskilled consumers become workers. That is, consumers that choose to become entrepreneurs when $\lambda < \infty$, choose to become workers when $\lambda \to \infty$. Similarly, consumers that choose to become workers when $\lambda < \infty$, decide to become entrepreneurs when $\lambda \to \infty$.

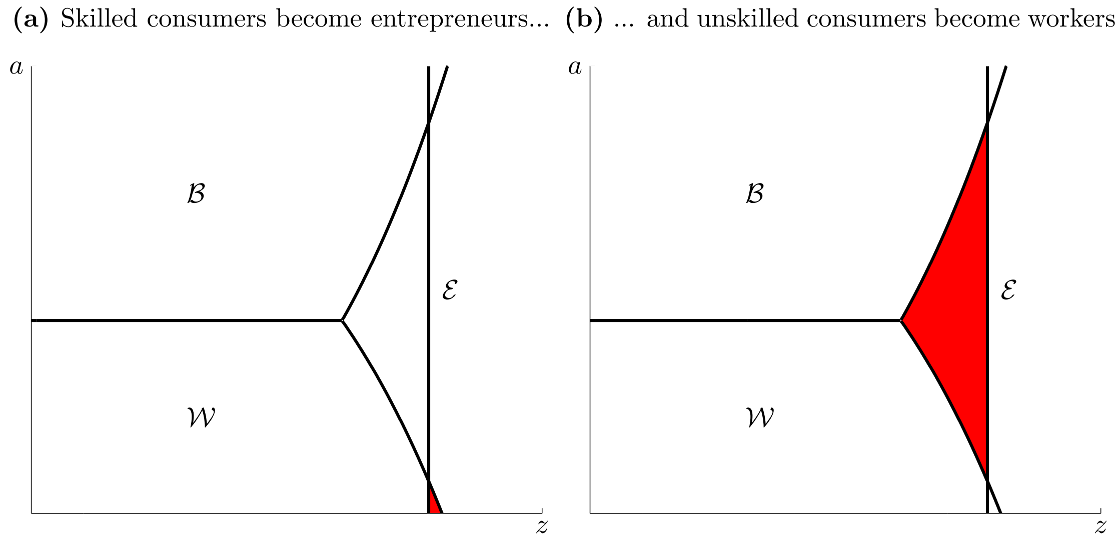
Figure 3.5: Boundaries for different values of λ



$\mathcal{B}$: Bankers. $\mathcal{W}$: Workers. $\mathcal{E}$: Entrepreneurs.

To understand the source of this distortion, Figure 3.6 shows the occupational choices of consumers in the (z, a) space for different values of λ. As λ increases the threshold in wealth above which consumers prefer to become bankers over workers goes up. That is, less workers become bankers. The main reason for why this occurs is that the spread between r^B and r^D decreases, which is largely a consequence of the decrease in the margin of intermediation.

Figure 3.6: Changes in occupational choice



$\mathcal{B}$: Bankers. $\mathcal{W}$: Workers. $\mathcal{E}$: Entrepreneurs.

More importantly, as λ increases, the slope of the the boundary that determines the occupation choice between workers and entrepreneurs increases. That is, the decision between becoming a worker or an entrepreneur becomes less dependent on the level of wealth of the consumer than on its skill. Given that the margin of intermediation is decreasing in λ, Lemma 9 shows this by proving that in an economy with $\lambda < \infty$ an unwealthy consumer needs to have a higher skill than a wealthy consumer in order to become an entrepreneur. Furthermore, this difference is increasing in the margin of intermediation. We prove the lemma for a particular case in which the minimum value that a attains is 0. Nonetheless the result holds for any general distribution with positive support.

Lemma 9. *A consumer with wealth above* $a_1 = \frac{1+\alpha\beta}{\beta(1-\alpha)}w$ *will become an entrepreneur as long as* $z \geq z_1$ *where*

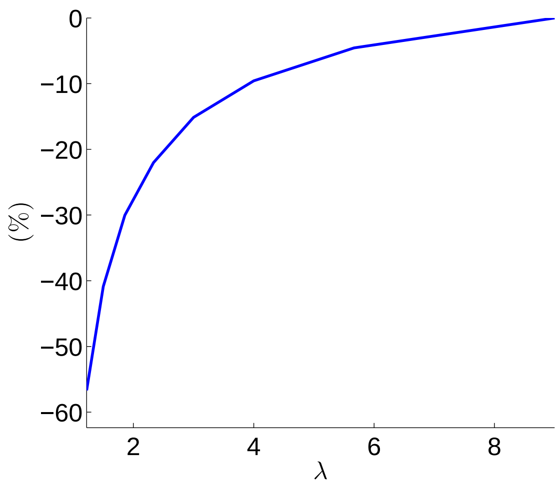
$$z_1 \equiv \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} w(1+r^D).$$

A consumer with no wealth ($a = 0$) will become an entrepreneur as long as $z \geq z_2$ where

$$z_2 \equiv \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} w(1+r^L) \left(\frac{1+r^D}{1+r^L}\right)^{\frac{\beta(1-\alpha)}{1+\beta}}.$$

$z_2 > z_1$ as long as $r^L - r^D > 0$. Furthermore, $z_2 - z_1$ is decreasing in λ.

Figure 3.7: Production relative to economy with $\lambda = 9$



Proof. See C.4. □

Figure 3.6 shows the effect of the distortion on occupation choice. Figure 3.6a shows consumers who choose to become entrepreneurs in an economy without limits on leverage ($\lambda \to \infty$) but in the model with $\lambda < \infty$ choose to become workers. That is, without limits on leverage these consumers choose to manage firms since they are skilled consumers. However, these consumers have low wealth. If they chose to become entrepreneurs when $\lambda < \infty$, they would need to finance a large portion of their wage bill by borrowing. Therefore, in an economy with $\lambda < \infty$ they are better off by becoming workers.

On the other hand, Figure 3.6b shows the consumers that would choose to become entrepreneurs in an economy with limits on leverage ($\lambda < \infty$). These consumers are wealthy enough to be able to finance a significant share of their wage bill without needing to borrow. As λ increases, these consumers are better off becoming workers, since they are not skilled enough.

The increased distortions mentioned before cause production to be lower for low levels of λ. Figure 3.7 shows how production is increasing in λ. In this figure we compare the production in an economy with a given level of λ relative to the production of an economy with $\lambda = 9$. With the current parametrization we are able to explain up to 56% differences in production.

We also analyze what happens as we change other parameters in the model. In particular, we analyze what happens as we change the ratio of wealth to skill in the model. That is, let $\overline{a}$ be the supremum of the support of the distribution for wealth and let $\overline{z}$ be the equivalent for skill. We analyze changes in $\frac{\overline{a}}{\overline{z}}$ for a given value of λ and holding other parameters constant. We find that distortions of having limits on leverage decrease: As this ratio increases, consumers are wealthier in the first period, relative to the second period. This implies that interest rates will be lower in equilibrium since there is more wealth that is going to be saved and less will be borrowed. Furthermore, banks become bigger, which causes the margin of intermediation to be lower.

3.6 Data

The objective of this section is to highlight that data supports the mechanism we mention in the model. That is, there is a negative correlation between the margin of intermediation and production. Additionally, we document that the margin of intermediation is negatively correlated with financial inclusion, where financial inclusion is defined as the percentage of firms that rely on banks to finance their working capital. This is consistent with the results in the model since a lower margin of intermediation implies that the measure of firms that borrow from banks increases.

Table 3.1: Production vs margin of intermediation

Dependent Variable: log GDP per capita	
Margin of intermediation	-0.880^{***}
	(0.072)
Number of observations	2,886

OLS estimation. Standard errors are in parenthesis.
***: Significant at 1%.
Country fixed effects.

We use data from the Global Financial Development Database (GFDD). This is an extensive database that includes measures of financial development for over 200 countries from 1960 to 2010.[3] The measures are divided in metrics of depth, access, efficiency and stability of the financial markets in order to analyze the different roles that financial systems play in an economy. See Cihak, Demirguc-Kunt, Feyen, and Levine (2012) for a further description of the database.

Table 3.2: Financial inclusion vs margin of intermediation

Dependent Variable: Financial inclusion	
Margin of intermediation	-1.117^{*}
	(0.599)
Number of observations	134

OLS estimation. Standard errors are in parenthesis.
*: Significant at 10%.
Country fixed effects.

We only consider countries whose population has been above 1,000,000 at some point in time and we estimate the following econometric model:

$$GDP_{i,t} = \beta_1 + \beta_2 MI_{i,t} + \mu_i + \varepsilon_{i,t}.$$

We define $GDP_{i,t}$ as the natural logarithm of GDP per capita in 2000 US dollars. MI is the margin of intermediation, defined in the database as the bank-lending deposit spread. We include country fixed effects to control for other country specific characteristics.

[3]Some variables are available for a shorter span of time.

Table 3.1 shows the results of the estimation. We observe that there is a negative correlation between the margin of intermediation and the GDP per capita. An increase of a percentage point in the margin of intermediation results in a decrease of 0.9% in GDP per capita. This is consistent with the results shown in Erosa (2001) and Greenwood, Sanchez, and Wang (2013).

We then test other implications of the model; namely, as the margin of intermediation decreases more firms rely on the banking sector to finance their wage bill. For this we estimate (3.14):

$$FI_{i,t} = \alpha_1 + \alpha_2 MI_{i,t} + \mu_i + \varepsilon_{i,t}. \tag{3.14}$$

We define FI as the percentage of firms that use banks to finance working capital, which we denote as financial inclusion. Cihak, Demirguc-Kunt, Feyen, and Levine (2012) compiled this measure from the Enterprise Surveys, which are surveys conducted by the World Bank to emerging countries (See The World Bank (2012) for further details). As mentioned in Cihak, Demirguc-Kunt, Feyen, and Levine (2012), financial development is positively correlated with income. Therefore, it is safe to assume that firms that don't rely on banks to finance their working capital are most likely not able to use financial markets to save either. Table 3.2 shows the result of the estimation of (3.14). We document a negative correlation between financial inclusion and the margin of intermediation: a one percent increase in the margin of intermediation reduces the percentage of firms using banks to finance their working capital by 1.1%. This is consistent with the results of our model: as the margin of intermediation decreases, the measure of firms that borrow from banks increases.

3.7 Conclusion

This paper analyzes the relationship between the leverage of banks and real allocations. Economies with banks that have a lower leverage experience a higher margin of intermediation, which affects both the occupational choices of consumers and the distribution of firm sizes across the wealth of individuals. When the margin of intermediation is large, wealthy entrepreneurs can hire workers for a significantly lower cost relative to unwealthy entrepreneurs. This variation in marginal costs of employees translates into heterogeneity in firm sizes across the spectrum of wealth for otherwise identical firms.

Occupational choice is also distorted: consumers with substantial skill but sufficiently small wealth may be dissuaded from managing firms due to the large costs of taking out loans, whereas wealthy but less skilled consumers may manage firms due to their relative advantage in inexpensive financing and the low opportunity cost of using those funds to hire workers rather than depositing. Future work will focus on analyzing the tradeoff of having limits on the leverage of banks in an economy with risk.

Bibliography

ACEMOGLU, D. (1999): "Changes in Unemployment and Wage Inequality: An Alternative Theory and Some Evidence," American Economic Review, 89(5), 1259–1278.

ARELLANO, C., Y. BAI, AND P. KEHOE (2012): "Financial Frictions and Fluctuations in Volatility," Federal Reserve Bank of Minneapolis Research Department Staff Report, 466.

ARIAS, M. A., A. M. IBÁÑEZ, AND A. ZAMBRANO (2014): "Agricultural Production Amid Conflict: The Effects of Shocks, Uncertainty, and Governance of Non-State Armed Actors," DOCUMENTOS CEDE 011005, UNIVERSIDAD DE LOS ANDES-CEDE.

AUTOR, D. H., AND D. DORN (2013): "The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market," American Economic Review, 103(5), 1553–97.

AUTOR, D. H., L. F. KATZ, AND M. S. KEARNEY (2006): "The Polarization of the U.S. Labor Market," American Economic Review, 96(2), 189–194.

AUTOR, D. H., F. LEVY, AND R. J. MURNANE (2003): "The Skill Content Of Recent Technological Change: An Empirical Exploration," The Quarterly Journal of Economics, 118(4), 1279–1333.

BANERJEE, A. V., AND B. MOLL (2010): "Why Does Misallocation Persist?," American Economic Journal: Macroeconomics, 2(1), 189–206.

BECKER, G. S. (1964): Human Capital: A Theoretical and Empirical Analysis with Special Reference to Education, no. beck64-1 in NBER Books. National Bureau of Economic Research, Inc.

——— (1968): "Crime and Punishment: An Economic Approach," Journal of Political Economy, 76, 169.

BERNANKE, B. S. (2003): "The Jobless Recovery," http://www.federalreserve.gov/boarddocs/speeches/2003/200311062/default.htm.

BUERA, F. J., J. P. KABOSKI, AND Y. SHIN (2011): "Finance and Development: A Tale of Two Sectors," American Economic Review, 101(5), 1964–2002.

BUONANNO, P., F. DRAGO, R. GALBIATI, AND G. ZANELLA (2011): "Crime in Europe and the United States: dissecting the "reversal of misfortunes"," Economic Policy, 26(67), 347–385.

BURDETT, K., R. LAGOS, AND R. WRIGHT (2003): "Crime, Inequality, and Unemployment," The American Economic Review, 93(5), pp. 1764–1777.

CIHAK, M., A. DEMIRGUC-KUNT, E. FEYEN, AND R. LEVINE (2012): "Benchmarking financial systems around the world," Policy Research Working Paper Series 6175, The World Bank.

DRAGO, F., R. GALBIATI, AND P. VERTOVA (2009): "The Deterrent Effects of Prison: Evidence from a Natural Experiment," Journal of Political Economy, 117(2), pp. 257–280.

EROSA, A. (2001): "Financial Intermediation and Occupational Choice in Development," Review of Economic Dynamics, 4(2), 303–334.

FENDER, J. (1999): "A general equilibrium model of crime and punishment," Journal of Economic Behavior & Organization, 39(4), 437–453.

FREEMAN, R. B. (1999): "The economics of crime," in Handbook of Labor Economics, ed. by O. Ashenfelter, and D. Card, vol. 3 of Handbook of Labor Economics, chap. 52, pp. 3529–3571. Elsevier.

GALÍ, J., F. SMETS, AND R. WOUTERS (2012): "Slow Recoveries: A Structural Interpretation," Journal of Money, Credit and Banking, 44, 9–30.

Galí, J., and T. van Rens (2010): "The Vanishing Procyclicality of Labor Productivity," IZA Discussion Papers 5099, Institute for the Study of Labor (IZA).

Goos, M., and A. Manning (2007): "Lousy and Lovely Jobs: The Rising Polarization of Work in Britain," The Review of Economics and Statistics, 89(1), 118–133.

Goos, M., A. Manning, and A. Salomons (2009): "Job Polarization in Europe," American Economic Review, 99(2), 58–63.

Gordon, R. J. (1993): "The Jobless Recovery: Does It Signal a New Era of Productivity-led Growth?," Brookings Papers on Economic Activity, 24(1), 271–316.

Greenwood, J., J. Sanchez, and C. Wang (2013): "Quantifying the Impact of Financial Development on Economic Development," Review of Economic Dynamics, 16(1), 194–215.

Greenwood, J., J. M. Sanchez, and C. Wang (2010): "Financing Development: The Role of Information Costs," American Economic Review, 100(4), 1875–91.

Grobovšek, J. (2014): "Management and Aggregate Productivity," Mimeo, University of Edinburgh.

Groshen, E. L., and S. Potter (2003): "Has structural change contributed to a jobless recovery?," Current Issues in Economics and Finance, 9(Aug).

Hagedorn, M., and I. Manovskii (2008): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," American Economic Review, 98(4), 1692–1706.

Ibáñez, A. M., C. Rodríguez, and D. Zarruk (2013): "Crime, Punishment, and Schooling Decisions: Evidence from Colombian Adolescents," IDB Publications 82164, Inter-American Development Bank.

Isen, A. (2012): "Dying to Know: Are Workers Paid Their Marginal Product?," .

Jaimovich, N., and H. E. Siu (2012): "The Trend is the Cycle: Job Polarization and Jobless Recoveries," NBER Working Papers 18334, National Bureau of Economic Research, Inc.

Jensen, M. C. (1986): "Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers," The American Economic Review, 76(2), pp. 323–329.

Kaufmann, D., A. Kraay, and M. Mastruzzi (2010): "The Worldwide Governance Indicators: Methodology and Analytical Issues," Policy Research Working Paper Series 5430, The World Bank.

Kehoe, P., V. Midrigan, and E. Pastorino (2014): "Debt Constraints and Employment," Unpublished.

Khan, A., and J. K. Thomas (2013): "Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity," Working Paper, The Ohio State University.

Kraut, R. E. (1976): "Deterrent and Definitional Influences on Shoplifting," Social Problems, 23(3), pp. 358–368.

Lazear, E. P. (1979): "Why Is There Mandatory Retirement?," Journal of Political Economy, 87(6), pp. 1261–1284.

Levine, R. (2005): "Finance and Growth: Theory and Evidence," in Handbook of Economic Growth, ed. by P. Aghion, and S. Durlauf, vol. 1 of Handbook of Economic Growth, chap. 12, pp. 865–934. Elsevier.

Lucas, Jr., R. E. (1978): "On the Size Distribution of Business Firms," Bell Journal of Economics, 9(2), 508–523.

McGrattan, E. R., and E. C. Prescott (2012): "The Labor Productivity Puzzle," in Government Policies and the Delayed Economic Recovery, ed. by L. E. Ohanian, J. B. Taylor, and I. J. Wright, Book Chapters, chap. 6. Hoover Institution, Stanford University.

Midrigan, V., and D. Y. Xu (2014): "Finance and Misallocation: Evidence from Plant-Level Data," American Economic Review, 104(2), 422–58.

Mortensen, D. T., and C. A. Pissarides (1994): "Job Creation and Job Destruction in the Theory of Unemployment," The Review of Economic Studies, 61(3), pp. 397–415.

North, D. C. (1968): "Sources of Productivity Change in Ocean Shipping, 1600-1850," Journal of Political Economy, 76, 953–970.

Ranasinghe, A. (2012): "Property Rights, Extortion and the Misallocation of Talent," Working paper, University of Manitoba.

Shaw, K., and E. P. Lazear (2008): "Tenure and output," Labour Economics, 15(4), 704–723.

Shimer, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," American Economic Review, 95(1), 25–49.

Smigel, E. O. (1956): "Public Attitudes Toward Stealing as Related to the Size of the Victim Organization," American Sociological Review, 21(3), pp. 320–327.

Stein, R. L. (1967): "New definitions for employment and unemployment.," Employment and Earnings and the Monthly Report on the Labor Force, 14(2), 3–27.

The World Bank (2012): "Enterprise Surveys," .

Topel, R. (1991): "Specific Capital, Mobility, and Wages: Wages Rise with Job Seniority," Journal of Political Economy, 99(1), pp. 145–176.

Appendix A

Appendix to Chapter 1

A.1 Selecting θ^S/θ^J

As outlined in Section 1.5, we use data from the supplemental questions for the CPS to infer the relative productivity of an senior to an junior worker. Unfortunately these supplements are not administered every year and the questions have varied somewhat over time. The first time these supplemental questions were asked was in 1973, questions inquiring about displacement as well as prior and current wages were not asked again until 1984 and have continued to be included bi-annually through 2010.

Following the method used in Topel (1991) we restrict attention to male respondents between the ages of 20 and 60 whose jobs end exogenously. We then deflate nominal wages by the GNP price deflator for consumption expenditure. For these workers we calculate the average change in log weekly wages for the prior and current jobs and use equation 1.2 to calculate the ratio of θ^S/θ^J implied by the data through the lens of our model.

The drop in wages following an exogenous separation is affected by the business cycle. Specifically, the first survey conducted immediately following a recession gives a drop in wages larger than the average drop experienced immediately following or preceding the survey. We therefore calculate the implied θ^S/θ^J both including and excluding these points. For the supplemental survey administered prior to the double dip recession[1] we find an average change in log weekly wages of -2.7% which implies a

[1] Supplemental survey questions which include data regarding wages of the prior and current job were

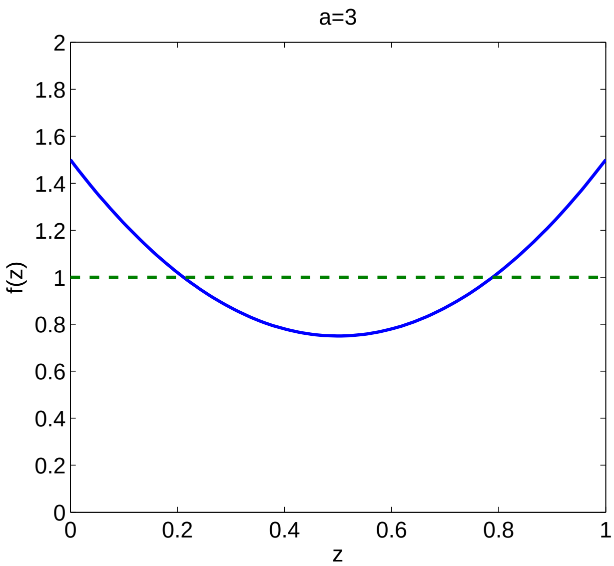
value of θ^S/θ^J of 1.55. When we include the 1984 data point we observe a change in log weekly wages of -10.7% which implies a value of θ^S/θ^J of 3.40. For those surveys administered after the double dip recession we find an average change in log weekly wages of -14.7% which implies a value of θ^S/θ^J of 4.45. When we exclude the post recession data points we observe an average change in log weekly wages of -11.5% which implies a value of θ^S/θ^J of 3.59. In our benchmark model we select an θ^S/θ^J of 2 for the "fast recovery" recessions and 4 for the "slow recovery" recessions.

A.2 Aggregate Uncertainty Shocks

In order to analyze the transition path implied by our model we apply a one period exogenous aggregate uncertainty shock applied to the distribution of the productivity draws. In the steady state the distribution of shocks is represented by the uniform distribution $f(z) = 1$ for $z \in [0, 1]$. To induce a recession we apply a one period shock which is a mean preserving spread on the original productivity draws for firms and is given according to $f(z) = a(z - .5)^2 + (1 - \frac{a}{12})$. This maintains the average productivity across firms while increasing the weight in the tails of the distribution. A visual representation of the uncertainty shock is displayed in Figure A.1.

only included one time before the double dip recession (in 1973). Therefore the value of θ^S/θ^J for the "fast recoveries" is based on the data from this survey.

Figure A.1: Aggregate Uncertainty Shock



Appendix B

Appendix to Chapter 2

B.1 Select Proofs

B.1.1 Proof of Lemma 1

Proof. The production function of every firm satisfies Inada conditions, so $l_y(z) > 0$ for all $z \in E$. Now, the Envelope Theorem, (2.10) and the assumptions on ϕ imply $\pi'(z) = l_y(z)^\alpha \left(1 - \frac{(1-\lambda)M_T}{a\phi(l_s(z)|M_T)}\right) > 0$. Additionally, $\lim_{z\to 0} \pi(z) \leq 0$. Also, $\pi(z) < w$ for all z cannot be an equilibrium since in this case there would be no entrepreneurs.

On the other hand, from (2.9), $(z,\theta) \in T$ if and only if $u_T(z,\theta) > u_{NT}(z)$. From (2.2) and (2.3) we have that $(z,\theta) \in T$ if and only if

$$\frac{\lambda \underline{c}}{1 - \lambda M_T} + (1-\lambda)\Pi_T - \lambda \max\{w, \pi(z)\} > \theta. \tag{B.1}$$

The definition of z^E and (B.1) imply the result. □

B.1.2 Proof of Lemma 2

Since $\alpha > 0.5$, the following first order conditions of (2.10) characterize the solution to this problem:

$$w + \frac{(1-\lambda)M_T}{\phi(l_s)} = \alpha z l_y^{\alpha-1} \tag{B.1}$$

$$w = (1-\lambda)M_T l_y \frac{\phi'(l_s)}{\phi(l_s)^2}. \tag{B.2}$$

Solving for l_y in (B.1) yields

$$l_y = \left(\frac{\alpha z}{w + \frac{(1-\lambda)M_T}{\phi(l_s)}} \right)^{\frac{1}{1-\alpha}}. \tag{B.3}$$

Plugging (B.3) in (B.2) yields

$$w = \left(\frac{\alpha z}{w\phi(l_s) + (1-\lambda)M_T} \right)^{\frac{1}{1-\alpha}} (1-\lambda)M_T \phi'(l_s)\phi(l_s)^{\frac{1}{1-\alpha}-2}.$$

Our assumption that $\phi(l_s) \equiv \left(\frac{\alpha}{1-\alpha} l_s\right)^{\frac{1-\alpha}{\alpha}}$ satisfies

$$\phi'(l_s)\phi(l_s)^{\frac{1}{1-\alpha}-2} = 1,$$

so in equilibrium

$$l_s(z) = \frac{1-\alpha}{\alpha} \left(\frac{(1-\lambda)M_T}{w} \right)^{\alpha} \left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda)M_T}{w} \right)^{\alpha} \right)^{\frac{\alpha}{1-\alpha}}. \tag{B.4}$$

Plugging (B.4) into (B.3) yields

$$l_y(z) = \left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda)M_T}{w} \right)^{\alpha} \right)^{\frac{1}{1-\alpha}}. \tag{B.5}$$

Plugging (B.4) and (B.5) into (2.6) yields

$$\tau(z) = \left(\frac{w}{(1-\lambda)M_T}\right)^{1-\alpha}\left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda)M_T}{w}\right)^{\alpha}\right)^{\frac{\alpha}{1-\alpha}}. \tag{B.6}$$

Finally, $\pi(z)$ results from plugging (B.4) to (B.6) into the objective function of (2.4):

$$\pi(z) = \frac{1-\alpha}{\alpha} w \left(\frac{\alpha z}{w} - \left(\frac{(1-\lambda)M_T}{w}\right)^{\alpha}\right)^{\frac{1}{1-\alpha}}.$$

B.2 Model $\lambda = 1$

Assume $\lambda = 1$. Depending on parameter values, in equilibrium there could be theft. That is, even if thieves cannot consume what they steal, their aversion to becoming thieves, θ, and the consumption they get when they get caught, $\underline{c}$, can be such that some households are better off stealing. If $\lambda = 1$ then Lemma 1 implies

$$\begin{aligned} \theta^W &= \frac{\underline{c}}{1-M_T} - w \\ \theta^E(z) &= \frac{\underline{c}}{1-M_T} - \pi(z). \end{aligned} \tag{B.1}$$

There will be theft in an equilibrium with $\lambda = 1$ as long as $\theta^W \geq \inf_\theta \operatorname{supp}\{F(\theta)\}$. In this case firm z's problem is

$$\pi(z) \equiv \max_{l_y \geq 0, l_s \geq 0} z l_y^\alpha - w l_y - w l_s. \tag{B.2}$$

The solution of (B.2) is

$$\begin{aligned} l_y(z) &= \left(\frac{\alpha z}{w}\right)^{\frac{1}{1-\alpha}} \\ l_s(z) &= 0. \end{aligned} \tag{B.3}$$

Plugging (B.3) into (B.2) we have

$$\pi(z) = (1-\alpha) z^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

Notice that $\pi(z)$ is strictly increasing in z, so there exists a cutoff z^E such that $\pi(z^E) = w$, which implies consumers choose to be workers for $z < z^E$ and decide to be entrepreneurs for $z \geq z^E$ and

$$z^E = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} w.$$

Then the equilibrium in this case is characterized by w and M_T such that

$$\begin{aligned} M_T &= F(\theta^W)G(z^E) + \int_{z \geq z^E} F(\theta^E(z))dG(z) \\ \int_{z \geq z^E} l_y(z)(1 - F(\theta^E(z)))dG(z) &= (1 - F(\theta^W))G(z^E) \\ \theta^W &\geq \inf_{\theta} \operatorname{supp}\{F(\theta)\}. \end{aligned} \tag{B.4}$$

If the first two equations of (B.4) are satisfied, but the third one is not, then we have an economy as in Lucas (1978). That is, there is no theft in equilibrium and consumers choose between being workers or entrepreneurs. Firms' profits are given by (B.2) with $M_T = 0$, so the equilibrium of this economy is characterized by (B.5).

$$\begin{aligned} M_T &= 0 \\ l_y(z) &= \left(\frac{z\alpha}{w}\right)^{\frac{1}{1-\alpha}} \\ z^E &= \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} w \\ \int_{z \geq z^E} l_y(z)dG(z) &= G(z^E). \end{aligned} \tag{B.5}$$

Appendix C

Appendix to Chapter 3

C.1 Characterizing the boundaries that determine the occupational choices

In this section we will characterize the boundaries that determine the occupational choice of consumers given values of w, r^D and r^L. This characterization allows us to analyze how changes in λ affect production. Consumers will make an occupational choice depending on the utility they can achieve from that occupation (see (3.1)). Lemma 10 characterizes the utility that consumers get depending on their occupational choice.

Lemma 10. *The utility of consumer (z,a) is*

$$u(z,a) = \begin{cases} (1+\beta)\ln\frac{a+w}{1+\beta} + \beta\ln\beta(1+r^D) & \text{if } (z,a)\in\mathcal{W} \\ (1+\beta)\ln\frac{a}{1+\beta} + \beta\ln\beta(1+r^B) & \text{if } (z,a)\in\mathcal{B} \\ (1+\beta)\ln\left(\frac{a}{1+\beta} + \frac{1-\alpha}{1+\beta}\left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}\right) + \beta\ln\beta(1+r^L) & \text{if } (z,a)\in\mathcal{E}_L \\ (1+\beta)\ln\left(\frac{a}{1+\beta} + \frac{1-\alpha}{1+\beta}\left(\frac{z}{1+r^D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}\right) + \beta\ln\beta(1+r^D) & \text{if } (z,a)\in\mathcal{E}_D \\ \ln\frac{a}{1+\alpha\beta} + \beta\ln z\left(\frac{\alpha\beta}{1+\alpha\beta}\frac{a}{w}\right)^{\alpha} & \text{if } (z,a)\in\mathcal{E}_O. \end{cases}$$

Proof. In this model $c_2(z,a) = \beta(1+r(z,a))c_1(z,a)$ for all consumers (z,a), where $r(z,a) = r^D$ for $(z,a)\in\mathcal{W}$, $r(z,a) = r^B$ for $(z,a)\in\mathcal{B}$ and it is defined in Corollary 2 of Lemma 5 for $(z,a) \in \mathcal{E}$. Therefore

$$u(z,a) = (1+\beta)\ln c_1(z,a) + \beta \ln \beta(1+r(z,a)).$$

The proof of the lemma follows then from Lemmas 4 and 5. □

We now compare the utility derived from two occupations at a time. Lemmas 11 to 17 show the boundaries that arise from these comparisons. The proof of these lemmas follows from Lemma 10. It is worth noticing that some boundaries only depend on the level of wealth (the choice between being a worker and a banker), or exclusively on the skill level (the choice between being a worker and an entrepreneur that saves). The rest of the boundaries depend on a combination of both wealth and skill level.

Lemma 11. *Let*

$$a_{\mathcal{W},\mathcal{B}} \equiv \frac{w}{\left(\frac{1+r^B}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - 1}.$$

If $a \geq a_{\mathcal{W},\mathcal{B}}$ *then* $(z,a) \notin \mathcal{W}$ *and if* $a < a_{\mathcal{W},\mathcal{B}}$ *then* $(z,a) \notin \mathcal{B}$.

Lemma 12. *Let*

$$z_{\mathcal{W},\mathcal{E}_D} = \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} w(1+r^D).$$

If $z < z_{\mathcal{W},\mathcal{E}_D}$ *then* $(z,a) \notin \mathcal{E}_D$ *and if* $z \geq z_{\mathcal{W},\mathcal{E}_D}$ *then* $(z,a) \notin \mathcal{W}$.

Lemma 13. *Let*

$$\delta_{\mathcal{W},\mathcal{E}_O}(a) = \beta^{1-\alpha}(1+r^D)\left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{a+w}{1+\beta}\right)^{\frac{1+\beta}{\beta}}\left(\frac{1+\alpha\beta}{a}\right)^{\frac{1+\alpha\beta}{\beta}}.$$

If $z < \delta_{\mathcal{W},\mathcal{E}_O}(a)$ *then* $(z,a) \notin \mathcal{E}_O$ *and if* $z \geq \delta_{\mathcal{W},\mathcal{E}_O}(a)$ *then* $(z,a) \notin \mathcal{W}$.

Lemma 14. *Let*

$$\delta_{\mathcal{W},\mathcal{E}_L}(z) = \frac{w - (1-\alpha)\left(\frac{1+r^L}{1+r^D}\right)^{\frac{\beta}{1+\beta}}\left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{1+r^L}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - 1}.$$

If $a < \delta_{\mathcal{W},\mathcal{E}_L}(z)$ *then* $(z,a) \notin \mathcal{E}_L$ *and if* $a \geq \delta_{\mathcal{W},\mathcal{E}_L}(z)$ *then* $(z,a) \notin \mathcal{W}$.

Lemma 15. *Let*

$$\delta_{\mathcal{B},\mathcal{E}_D}(z) = \frac{(1-\alpha)\left(\frac{z}{1+r^D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{1+r^B}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - 1}.$$

If $a \geq \delta_{\mathcal{B},\mathcal{E}_D}(z)$ then $(z,a) \notin \mathcal{E}_D$ and if $a < \delta_{\mathcal{B},\mathcal{E}_D}(z)$ then $(z,a) \notin \mathcal{B}$.

Lemma 16. *Let*

$$\delta_{\mathcal{B},\mathcal{E}_O}(z) = \frac{1}{\beta}\frac{(1+\beta)^{\frac{1+\beta}{\beta(1-\alpha)}}}{(1+\alpha\beta)^{\frac{1+\alpha\beta}{\beta(1-\alpha)}}}\left(\frac{z}{1+r^B}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

If $a \geq \delta_{\mathcal{B},\mathcal{E}_O}(z)$ then $(z,a) \notin \mathcal{E}_O$ and if $a < \delta_{\mathcal{B},\mathcal{E}_O}(z)$ then $(z,a) \notin \mathcal{B}$.

Lemma 17. *Let*

$$\delta_{\mathcal{B},\mathcal{E}_L}(z) = \frac{(1-\alpha)\left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}}{\left(\frac{1+r^B}{1+r^L}\right)^{\frac{\beta}{1+\beta}} - 1}.$$

If $a \geq \delta_{\mathcal{B},\mathcal{E}_L}(z)$ then $(z,a) \notin \mathcal{E}_L$ and if $a < \delta_{\mathcal{B},\mathcal{E}_L}(z)$ then $(z,a) \notin \mathcal{B}$.

Additionally to the boundaries shown in the previous lemmas, there are two other boundaries we need to take into account: $\delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ and $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$. As mentioned in Lemma 5,

$$\delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \equiv \frac{1+\alpha\beta}{\beta}\left(\frac{z}{1+r^D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}$$
$$\delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \equiv \frac{1+\alpha\beta}{\beta}\left(\frac{z}{1+r^L}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

Entrepreneurs with $a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$ will borrow, entrepreneurs with $a \geq \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ will save, and entrepreneurs with $\delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ will spend all their available wealth in paying for workers. It is worth mentioning that these boundaries do not depend on utility, but rather on feasibility: If an entrepreneur borrows ($s^{\mathcal{E}}(z,a) < 0$) then this entrepreneur cannot choose to save.

Proposition 4 characterizes the sets of consumers. There are three cases: If the spread between the return of setting up a bank, r^B, and the interest rate on deposits, r^D is sufficiently low, then there will be three types of entrepreneurs. On the other extreme, if the spread between r^B and the interest rate on loans, r^L is sufficiently large, in equilibrium there will only be entrepreneurs that borrow.

Proposition 4. *If in equilibrium* $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} \geq \frac{1+r^B}{1+r^D}$*, then there will be the three types of entrepreneurs. In this case the occupational choice of consumers is characterized by the following sets:*

$$\begin{aligned}
\mathcal{B} &= [\underline{z}, z_{\mathcal{W},\mathcal{E}_D}) \times [a_{\mathcal{W},\mathcal{B}}, \overline{a}] \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{E}_D} \text{ and } a \geq \delta_{\mathcal{B},\mathcal{E}_D}(z)\} \\
\mathcal{W} &= [\underline{z}, z_{\mathcal{W},\mathcal{E}_D}) \times [\underline{a}, a_{\mathcal{W},\mathcal{B}}) \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{E}_D}, a \geq \underline{a}, z < z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L} \text{ and } z < \delta_{\mathcal{W},\mathcal{E}_O}(a)\} \\
&\quad \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}, a \geq \underline{a} \text{ and } a < \delta_{\mathcal{W},\mathcal{E}_L}(z)\} \\
\mathcal{E}_D &= \{(z,a) : z \geq z_{\mathcal{W},\mathcal{E}_D} \text{ and } \delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \leq a < \delta_{\mathcal{B},\mathcal{E}_D}(z)\} \\
\mathcal{E}_O &= \{(z,a) : z \geq \delta_{\mathcal{W},\mathcal{E}_O}(a) \text{ and } \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \leq a < \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)\} \\
\mathcal{E}_L &= \{(z,a) : a \geq \delta_{\mathcal{W},\mathcal{E}_L}(z) \text{ and } \underline{a} \leq a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z)\},
\end{aligned}$$

where

$$z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L} \equiv \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{\beta}{(1+\beta)\left(\frac{1+r^L}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - (1+\alpha\beta)}\right)^{1-\alpha} w(1+r^L).$$

If in equilibrium $\frac{1+r^B}{1+r^L} \leq \left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} < \frac{1+r^B}{1+r^D}$*, then there will be no entrepreneurs that deposit. In this case the occupational choice of consumers is characterized by the following sets:*

$$\begin{aligned}
\mathcal{B} &= [\underline{z}, z_{\mathcal{W},\mathcal{B},\mathcal{E}_O}) \times [a_{\mathcal{W},\mathcal{B}}, \overline{a}] \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{B},\mathcal{E}_O} \text{ and } a \geq \delta_{\mathcal{B},\mathcal{E}_O}(z)\} \\
\mathcal{W} &= [\underline{z}, z_{\mathcal{W},\mathcal{B},\mathcal{E}_O}) \times [\underline{a}, a_{\mathcal{W},\mathcal{B}}) \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{B},\mathcal{E}_O}, a \geq \underline{a}, z < z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L} \text{ and } z < \delta_{\mathcal{W},\mathcal{E}_O}(a)\} \\
&\quad \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}, a \geq \underline{a} \text{ and } a < \delta_{\mathcal{W},\mathcal{E}_L}(z)\} \\
\mathcal{E}_O &= \{(z,a) : z \geq \delta_{\mathcal{W},\mathcal{E}_O}(a) \text{ and } \delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \leq a < \delta_{\mathcal{B},\mathcal{E}_O}(z)\} \\
\mathcal{E}_L &= \{(z,a) : a \geq \delta_{\mathcal{W},\mathcal{E}_L}(z) \text{ and } \underline{a} \leq a < \delta_{\mathcal{E}_O,\mathcal{E}_L}(z)\},
\end{aligned}$$

where

$$z_{\mathcal{W},\mathcal{B},\mathcal{E}_O} \equiv \frac{(1+\alpha\beta)^{\frac{1+\alpha\beta}{\beta}}}{(1+\beta)^{\frac{1+\beta}{\beta}}} \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{\beta}{\left(\frac{1+r^B}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - 1}\right)^{1-\alpha} w(1+r^B).$$

If in equilibrium $\frac{1+r^B}{1+r^L} > \left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}}$*, then there will only be entrepreneurs that borrow. In this case the occupational choice of consumers is characterized by the following sets:*

$$\begin{aligned}
\mathcal{B} &= [\underline{z}, z_{\mathcal{W},\mathcal{B},\mathcal{E}_L}) \times [a_{\mathcal{W},\mathcal{B}}, \overline{a}] \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{B},\mathcal{E}_L} \text{ and } a \geq \delta_{\mathcal{B},\mathcal{E}_L}(z)\} \\
\mathcal{W} &= [\underline{z}, z_{\mathcal{W},\mathcal{B},\mathcal{E}_L}) \times [\underline{a}, a_{\mathcal{W},\mathcal{B}}) \bigcup \{(z,a) : z \geq z_{\mathcal{W},\mathcal{B},\mathcal{E}_L}, a \geq \underline{a} \text{ and } a < \delta_{\mathcal{W},\mathcal{E}_L}(z)\} \\
\mathcal{E}_L &= \{(z,a) : a \geq \delta_{\mathcal{W},\mathcal{E}_L}(z) \text{ and } \underline{a} \leq a < \delta_{\mathcal{B},\mathcal{E}_L}(z)\}.
\end{aligned}$$

where

$$z_{\mathcal{W},\mathcal{B},\mathcal{E}_L} \equiv \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{\left(\frac{1+r^B}{1+r^L}\right)^{\frac{\beta}{1+\beta}} - 1}{\left(\frac{1+r^B}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - 1}\right)^{1-\alpha} w(1+r^L).$$

Proof. The strategy to prove the proposition relies on the fact that, depending on the values of $\frac{1+r^B}{1+r^D}$ and $\frac{1+r^B}{1+r^L}$, some types of entrepreneurs will not exist. This will be a consequence of the slope of some boundaries.

First, recall from Lemma 3 that $r^B > r^L > r^D$ in equilibrium as long as $\lambda < \infty$. Let

$$a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O} = \frac{1+\alpha\beta}{\beta(1-\alpha)} w.$$

Notice that

$$\begin{aligned}
\delta_{\mathcal{B},\mathcal{E}_D}\left(z_{\mathcal{W},\mathcal{E}_D}\right) &= a_{\mathcal{W},\mathcal{B}} \\
\delta_{\mathcal{W},\mathcal{E}_O}\left(a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O}\right) &= z_{\mathcal{W},\mathcal{E}_D} \\
\delta_{\mathcal{E}_D,\mathcal{E}_O}\left(z_{\mathcal{W},\mathcal{E}_D}\right) &= a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O}.
\end{aligned}$$

As long as $a_{\mathcal{W},\mathcal{B}} \geq a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O}$ there will exist entrepreneurs that save in equilibrium. $a_{\mathcal{W},\mathcal{B}} \geq a_{\mathcal{W},\mathcal{E}_D,\mathcal{E}_O}$ and $\delta_{\mathcal{B},\mathcal{E}_D}(z) \geq \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ if and only if $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} \geq \frac{1+r^B}{1+r^D}$. Finally,

$\delta_{\mathcal{E}_O,\mathcal{E}_L}(z) > \delta_{\mathcal{E}_D,\mathcal{E}_O}(z)$ since $r^L > r^D$, so if $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} \geq \frac{1+r^B}{1+r^D}$ there will be entrepreneurs that neither borrow nor deposit.

Now consider the case where there are no entrepreneurs that save in equilibrium. Notice that

$$\delta_{\mathcal{W},\mathcal{E}_O}\left(a_{\mathcal{W},\mathcal{B}}\right) = z_{\mathcal{W},\mathcal{B},\mathcal{E}_O}$$
$$\delta_{\mathcal{B},\mathcal{E}_O}\left(z_{\mathcal{W},\mathcal{B},\mathcal{E}_O}\right) = a_{\mathcal{W},\mathcal{B}}.$$

Let

$$a_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L} \equiv \frac{1+\alpha\beta}{(1+\beta)\left(\frac{1+r^L}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - (1+\alpha\beta)} w.$$

Notice that

$$\delta_{\mathcal{W},\mathcal{E}_O}\left(a_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}\right) = z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}$$
$$\delta_{\mathcal{E}_O,\mathcal{E}_L}\left(z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}\right) = a_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}$$
$$\delta_{\mathcal{W},\mathcal{E}_L}\left(z_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}\right) = a_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L},$$

so as long as $a_{\mathcal{W},\mathcal{B}} \geq a_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}$ there will exist entrepreneurs that neither borrow nor save. $a_{\mathcal{W},\mathcal{B}} \geq a_{\mathcal{W},\mathcal{E}_O,\mathcal{E}_L}$ and $\delta_{\mathcal{B},\mathcal{E}_O}(z) \geq \delta_{\mathcal{E}_O,\mathcal{E}_L}(z)$ if and only if $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} \geq \frac{1+r^B}{1+r^L}$.

Finally notice that

$$\delta_{\mathcal{W},\mathcal{E}_L}\left(z_{\mathcal{W},\mathcal{B},\mathcal{E}_L}\right) = a_{\mathcal{W},\mathcal{B}}$$
$$\delta_{\mathcal{B},\mathcal{E}_L}\left(z_{\mathcal{W},\mathcal{B},\mathcal{E}_L}\right) = a_{\mathcal{W},\mathcal{B}}.$$

If $\left(\frac{1+\beta}{1+\alpha\beta}\right)^{\frac{1+\beta}{\beta}} < \frac{1+r^B}{1+r^L}$ there will only be entrepreneurs that borrow in equilibrium. $\square$

Lemma 18 determines the skill level $z^0_{\mathcal{W},\mathcal{E}_L}$ above which there will be no workers.

Lemma 18. *Let*

$$z^{\underline{a}}_{\mathcal{W},\mathcal{E}_L} = \left(\frac{w - \underline{a}\left(\left(\frac{1+r^L}{1+r^D} \right)^{\frac{\beta}{1+\beta}} - 1 \right)}{1-\alpha} \right)^{1-\alpha} \left(\frac{w}{\alpha} \right)^{\alpha} (1+r^L) \left(\frac{1+r^D}{1+r^L} \right)^{\frac{\beta(1-\alpha)}{1+\beta}}$$

Then

$$\delta_{\mathcal{W},\mathcal{E}_L}\left(z^{\underline{a}}_{\mathcal{W},\mathcal{E}_L} \right) = \underline{a}.$$

C.2 Characterizing the boundaries that determine the occupational choice as $\lambda \to \infty$

In this section we will characterize the boundaries that determine the occupational choice of consumers given values of w and r. Consumers will choose their occupation depending on the utility they can derive from it (see (3.9)). Lemma 19 characterizes the utility derived from each occupation.

Lemma 19. *The utility of consumer* (z, a) *is*

$$u(z,a) = \begin{cases} (1+\beta)\ln \frac{a+w}{1+\beta} + \beta \ln \beta(1+r) & \text{if } (z,a) \in \mathcal{W} \\ (1+\beta)\ln \left(\frac{a}{1+\beta} + \frac{1-\alpha}{1+\beta} \left(\frac{z}{1+r} \right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w} \right)^{\frac{\alpha}{1-\alpha}} \right) + \beta \ln \beta(1+r) & \text{if } (z,a) \in \mathcal{E}. \end{cases}$$

Proof. In this model $c_2(z,a) = \beta(1+r)c_1(z,a)$ for all consumers (z,a). Therefore

$$u(z,a) = (1+\beta)\ln c_1 + \beta \ln \beta(1+r).$$

The proof of the lemma follows from Lemmas 6 and 7. □

We now compare the utility derived from the two occupations. Lemma 20 shows the boundary that arises from this comparison. The proof of this lemma follows from Lemma 19. It is worth noticing that this boundary only depends on the skill level.

Lemma 20. *Let*

$$z_{\mathcal{W},\mathcal{E}} \equiv \left(\frac{1}{1-\alpha} \right)^{1-\alpha} \left(\frac{1}{\alpha} \right)^{\alpha} w(1+r).$$

If $z < z_{\mathcal{W},\mathcal{E}}$ *then* $(z,a) \in \mathcal{W}$ *and if* $z \geq z_{\mathcal{W},\mathcal{E}}$ *then* $(z,a) \in \mathcal{E}$.

Proposition 5 characterizes the sets of consumers.

Proposition 5. *Consumer occupations are characterized by*

$$\mathcal{W} = \{(z,a) : z < z_{\mathcal{W},\mathcal{E}}\}$$
$$\mathcal{E} = \{(z,a) : z \geq z_{\mathcal{W},\mathcal{E}}\}.$$

C.2.1 Solving the model

Let

$$A \equiv \int adG(z \times a)$$

Since for each individual, consumption in the two periods is related by

$$c_2(z,a) = \beta(1+r)c_1(z,a).$$

This must also hold in summation, which implies:

$$\int c_2(z,a)dG(z \times a) = \int \beta(1+r)c_1(z,a)dG(z \times a). \tag{C.1}$$

We use Lemmas 7 and 20 and the market clearing condition for labor to solve for $C_{w,r} \equiv w(1+r)$. This allows us to have an expression for total production. Then we use A and total production to solve for r using (C.1). Finally we solve for w.

C.3 Social Planner Problem

The Social Planner Problem is

$$\max_{l(z),c_1(z,a),c_2(z,a),o} \int \left(v(c_1(z,a) + \beta v(c_2(z,a))\right) dG(z \times a)$$

$$\int c_1(z,a) dG(z \times a) = \int a dG(z \times a)$$

$$\int c_2(z,a) dG(z \times a) = \int_{\mathcal{E}} z l(z)^{\alpha} dG(z \times a)$$

$$\int_{\mathcal{E}} l(z) dG(z \times a) = \int_{\mathcal{W}} dG(z \times a).$$

Notice that aggregate consumption in period 1 is exogenous, so maximizing social welfare implies maximizing production in the second period. Now, the marginal productivity of labor is constant across all firms, but adding an extra entrepreneur implies increasing the marginal productivity of labor, since the average size of a firm decreases. So solving the Social Planner Problem reduces to finding a boundary above which consumers will become entrepreneurs. We know that

$$l(z) = \left(\frac{\alpha z}{MPL}\right)^{\frac{1}{1-\alpha}}.$$

From the labor market clearing condition we have

$$\left(\frac{\alpha}{MPL}\right)^{\frac{1}{1-\alpha}} \int_{z \geq z_{\mathcal{W},\mathcal{E}}} z^{\frac{1}{1-\alpha}} dG(z \times a) = \hat{G}(z_{\mathcal{W},\mathcal{E}}),$$

where

$$\hat{G}(z') \equiv \int_{z<z'} dG(z \times a).$$

Then

$$MPL = \alpha \left(\frac{(1-\hat{G}(z_{\mathcal{W},\mathcal{E}}))\mathbf{E}\left[z^{\frac{1}{1-\alpha}} | z \geq z_{\mathcal{W},\mathcal{E}}\right]}{\hat{G}(z_{\mathcal{W},\mathcal{E}})} \right)^{1-\alpha}.$$

The Social Planner Problem is equivalent to

$$\begin{aligned}\max_{z_{\mathcal{W},\mathcal{E}}} H(z_{\mathcal{W},\mathcal{E}}) \equiv \max_{z_{\mathcal{W},\mathcal{E}}} \int_{\mathcal{E}} z f(l(z)) dG(z \times a) \\ = \max_{z_{\mathcal{W},\mathcal{E}}} \left(\frac{\alpha}{MPL(z_{\mathcal{W},\mathcal{E}})}\right)^{\frac{\alpha}{1-\alpha}} \int_{z \geq z_{\mathcal{W},\mathcal{E}}} z^{\frac{1}{1-\alpha}} dG(z \times a) \\ = \max_{z_{\mathcal{W},\mathcal{E}}} \left(1 - \hat{G}(z_{\mathcal{W},\mathcal{E}})\right)^{1-\alpha} \mathbf{E}\left[z^{\frac{1}{1-\alpha}} | z \geq z_{\mathcal{W},\mathcal{E}}\right]^{1-\alpha} \hat{G}(z_{\mathcal{W},\mathcal{E}})^{\alpha}.\end{aligned}$$

C.4 Select Proofs

C.4.1 Proof of Lemma 8

To prove this Lemma it is sufficient to consider aggregate wealth across the different occupational choices. That is, let $A_{\mathcal{O}}$ denote the aggregate wealth endowed to consumers who choose occupation $\mathcal{O}$, where $\mathcal{O} \in \{\mathcal{W}, \mathcal{B}, \mathcal{E}_L, \mathcal{E}_O, \mathcal{E}_D\}$. Additionally, let

$$\begin{aligned}Z_{\mathcal{E}_D} \equiv \left[\int_{\mathcal{E}_D} z^{\frac{1}{1-\alpha}} dG(z \times a)\right]^{1-\alpha} \\ Z_{\mathcal{E}_L} \equiv \left[\int_{\mathcal{E}_L} z^{\frac{1}{1-\alpha}} dG(z \times a)\right]^{1-\alpha}.\end{aligned}$$

That is, $Z_{\mathcal{E}_D}^{\frac{1}{1-\alpha}}$ ($Z_{\mathcal{E}_L}^{\frac{1}{1-\alpha}}$) is the $\frac{1}{1-\alpha}$-th moment of the skill endowed to the entrepreneurs that are depositors (borrowers). Finally let $M_{\mathcal{W}}$ be the mass of workers:

$$M_{\mathcal{W}} \equiv \int_{\mathcal{W}} dG(z \times a).$$

Then the market clearing condition for deposits can be written as

$$\begin{aligned}\frac{\beta}{1+\beta}(wM_{\mathcal{W}} + A_{\mathcal{W}}) + \frac{\beta}{1+\beta} A_{\mathcal{E}_D} - \frac{1+\alpha\beta}{1+\beta}\left(\frac{Z_{\mathcal{E}_D}}{1+r^D}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} \\ = \lambda \frac{\beta}{1+\beta} A_{\mathcal{B}};\end{aligned} \tag{C.2}$$

for loans as

$$\frac{\beta}{1+\beta}A_{\mathcal{E}_L} - \frac{1+\alpha\beta}{1+\beta}\left(\frac{Z_{\mathcal{E}_L}}{1+r^L}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}} + (1+\lambda)\frac{\beta}{1+\beta}A_{\mathcal{B}} = 0; \tag{C.3}$$

and for labor as

$$M_{\mathcal{W}} = \left(\frac{\alpha Z_{\mathcal{E}_L}}{(1+r^L)w}\right)^{\frac{1}{1-\alpha}} + \frac{\alpha\beta}{1+\alpha\beta}\frac{A_{\mathcal{E}_O}}{w} + \left(\frac{\alpha Z_{\mathcal{E}_D}}{(1+r^D)w}\right)^{\frac{1}{1-\alpha}}. \tag{C.4}$$

(C.2) and (C.3) can rewritten as (C.5) and (C.6), respectively.

$$\left(\frac{\alpha Z_{\mathcal{E}_L}}{(1+r^L)w}\right)^{\frac{1}{1-\alpha}} = \frac{\beta}{1+\alpha\beta}\left(A_{\mathcal{E}_L} + (1+\lambda)A_{\mathcal{B}}\right)\frac{\alpha}{w} \tag{C.5}$$

$$\left(\frac{\alpha Z_{\mathcal{E}_D}}{(1+r^D)w}\right)^{\frac{1}{1-\alpha}} = \frac{\beta}{1+\alpha\beta}\left(wM_{\mathcal{W}} + A_{\mathcal{W}} + A_{\mathcal{E}_D} - \lambda A_{\mathcal{B}}\right)\frac{\alpha}{w}. \tag{C.6}$$

Plugging (C.5) and (C.6) into the labor market clearing condition we get

$$M_{\mathcal{W}} = \frac{\beta}{1+\alpha\beta}\left(A_{\mathcal{E}_L} + (1+\lambda)A_{\mathcal{B}}\right)\frac{\alpha}{w} + \frac{\alpha\beta}{1+\alpha\beta}\frac{A_{\mathcal{E}_O}}{w} + \frac{\beta}{1+\alpha\beta}\left(wM_{\mathcal{W}} + A_{\mathcal{W}} + A_{\mathcal{E}_D} - \lambda A_{\mathcal{B}}\right)\frac{\alpha}{w}. \tag{C.7}$$

The result follows from reorganizing (C.7).

C.4.2 Assumptions on parameters in Section 3.5.2

The following assumptions on parameters guarantee that the prices stated in Proposition 2 are an equilibrium:

$$z_2 > \left(\frac{1}{1-\alpha}\right)^{1-\alpha}\left(\frac{1}{\alpha}\right)^{\alpha} w(1+r^L) > z_1.$$

This assumption guarantees that consumers with $z = z_2$ choose to become entrepreneurs are entrepreneurs while the other consumers don't.

$$a_2 > \frac{w}{\left(\frac{1+r^B}{1+r^D}\right)^{\frac{\beta}{1+\beta}} - 1} > a_1.$$

This assumption guarantees that consumers with $a = a_2$ choose to become bankers and consumers with $a = a_1$ choose to become workers.

$$\overline{a} > \frac{1+\alpha\beta}{\beta}\left(\frac{z_2}{1+r^D}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{w}\right)^{\frac{\alpha}{1-\alpha}}.$$

This assumption guarantees that there are entrepreneurs that deposit.

$$0 < (\lambda-\alpha\beta)(1-\delta_a)\delta_z a_2 - \alpha\beta(1-\delta_z)\frac{\overline{a}}{2} - (1+\alpha\beta)\delta_a\delta_z a_1 < (1-\delta_z)\frac{\overline{a}}{2}.$$

The first inequality is consistent with the existence of consumers other than workers that save; namely, entrepreneurs that deposit. The second inequality is consistent with the existence of consumers that do not save; namely. entrepreneurs that borrow and entrepreneurs that neither borrow nor deposit.

$$\begin{aligned}&(1+\lambda)(1-\delta_a)\delta_z a_2\\ &< \left(\left(\frac{1-\delta_z}{2\overline{a}}\right)^{\frac{1}{2}} - \left((\lambda-\alpha\beta)(1-\delta_a)\delta_z a_2 - \alpha\beta(1-\delta_z)\frac{\overline{a}}{2} - (1+\alpha\beta)\delta_a\delta_z a_1\right)^{\frac{1}{2}}\right)^2.\end{aligned}$$

This final assumption guarantees that $r^L > r^D$, which is necessary to have an equilibrium.

C.4.3 Proof of Proposition 2

The strategy to prove this proposition will be to assume that parameter values are such that an equilibrium exist. Then we show that the conditions on this parameters satisfy what is stated in C.4.2. We will also build on the proof of Lemma 8. Given the definitions in C.4.1, under $\widetilde{G}(\cdot)$ we have

$$
\begin{aligned}
M_{\mathcal{W}} &= \delta_a \delta_z \\
A_{\mathcal{W}} &= a_1 \delta_a \delta_z \\
A_{\mathcal{B}} &= a_2 (1-\delta_a) \delta_z \\
A_{\mathcal{E}_D} &= \frac{1-\delta_z}{\overline{a}} \int_{\delta_{\mathcal{E}_D,\mathcal{E}_O}(z)}^{\overline{a}} a da \\
A_{\mathcal{E}_L} &= \frac{1-\delta_z}{\overline{a}} \int_{0}^{\delta_{\mathcal{E}_O,\mathcal{E}_L}(z)} a da \\
Z_{\mathcal{E}_D} &= z_2 \left(\frac{1-\lambda_z}{\overline{a}} \right)^{1-\alpha} \left(\overline{a} - \delta_{\mathcal{E}_D,\mathcal{E}_O}(z) \right)^{1-\alpha} \\
Z_{\mathcal{E}_L} &= z_2 \left(\frac{1-\lambda_z}{\overline{a}} \right)^{1-\alpha} \left(\delta_{\mathcal{E}_O,\mathcal{E}_L}(z) \right)^{1-\alpha} .
\end{aligned}
\tag{C.8}
$$

Reorganizing (C.5) and (C.6) we get

$$
\begin{aligned}
1 + r^L &= Z_{\mathcal{E}_L} \left(\frac{\alpha}{w} \right)^{\alpha} \left(\frac{1+\alpha\beta}{\beta \left(A_{\mathcal{E}_L} + (1+\lambda) A_{\mathcal{B}} \right)} \right)^{1-\alpha} \\
1 + r^D &= Z_{\mathcal{E}_D} \left(\frac{\alpha}{w} \right)^{\alpha} \left(\frac{1+\alpha\beta}{\beta \left(w M_{\mathcal{W}} + A_{\mathcal{W}} + A_{\mathcal{E}_D} - \lambda A_{\mathcal{B}} \right)} \right)^{1-\alpha} .
\end{aligned}
\tag{C.9}
$$

The result follows form plugging (C.8) into (C.9), and using the result from Lemma 8. The proof that the assumptions stated in C.4.2 are sufficient restrictions on the parameters arises from guaranteeing that there are no negative roots in the resulting r^L and r^D and by using the characterization of the boundaries that determine the occupational choices in the model (See C.1).

C.4.4 Proof of Proposition 3

The proof of this proposition relies on the result stated in Proposition 2. Notice that $M_{\mathcal{W}}$ and A are constant in the model. Then we can write r^L and r^D as

$$
\begin{aligned}
1 + r^L &= \frac{z_2}{\beta} (1+\alpha\beta)^{1-\alpha} \left(\frac{M_{\mathcal{W}}}{A} \right)^{\alpha} \mathcal{C}_L^{\frac{1-\alpha}{2}} \\
1 + r^D &= \frac{z_2}{\beta} (1+\alpha\beta)^{1-\alpha} \left(\frac{M_{\mathcal{W}}}{A} \right)^{\alpha} \mathcal{C}_D^{1-\alpha} .
\end{aligned}
$$

Notice that $r^L - r^D$ is a function of $\mathcal{C}_L - \mathcal{C}_D$. Furthermore, from the corollary to Proposition 2 this difference is decreasing in λ.

Now, total production in this economy is given by the sum of what is produced by each type of entrepreneur. Let $Y_\mathcal{T}$ be total production by entrepreneur of type $\mathcal{T}$, where $\mathcal{T} \in \{L, O, D\}$. Taking into account the results in Proposition 2 and Lemma 5 we have

$$\begin{aligned}
Y_L &= z_2 \frac{1-\delta_z}{\bar{a}} \left(\frac{1}{1+\alpha\beta}\right)^\alpha \left(\frac{M_\mathcal{W}}{A}\right)^\alpha \left(\frac{1}{\mathcal{C}_L}\right)^{\frac{1+\alpha}{2}} \\
Y_O &= z_2 \frac{1-\delta_z}{\bar{a}} \left(\frac{1}{1+\alpha\beta}\right)^\alpha \left(\frac{M_\mathcal{W}}{A}\right)^\alpha \frac{1}{1+\alpha} \left[\left(\frac{1}{\mathcal{C}_D}\right)^{1+\alpha} - \left(\frac{1}{\mathcal{C}_L}\right)^{\frac{1+\alpha}{2}}\right] \\
Y_D &= z_2 \frac{1-\delta_z}{\bar{a}} \left(\frac{1}{1+\alpha\beta}\right)^\alpha \left(\frac{M_\mathcal{W}}{A}\right)^\alpha \left(\bar{a} - \frac{1}{\mathcal{C}_D}\right) \left(\frac{1}{\mathcal{C}_D}\right)^\alpha .
\end{aligned}$$

The result follows from adding $Y_\mathcal{T}$ for $\mathcal{T} \in \{L, O, D\}$.

C.4.5 Proof of Lemma 9

The proof of this lemma relies on what is proven in C.1. First, from Proposition 4 we see that a consumer with skill z_1 and wealth a_1 choose to be an entrepreneur that is able to use its wealth to pay for workers. Additionally, if $\underline{a} = 0$ a consumer with skill z_2 and wealth $a = 0$ chooses to become an entrepreneur that needs to borrow to pay for workers according to Lemma 12 and Proposition 4. Finally,

$$z_2 - z_1 = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^\alpha (1+r^D)^{\frac{\beta(1-\alpha)}{1+\beta}} \left[(1+r^L)^{\frac{1+\alpha\beta}{1+\beta}} - (1+r^D)^{\frac{1+\alpha\beta}{1+\beta}}\right],$$

which is increasing in $r^L - r^D$.