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Description and Implementation of a Single Cohort and Lifespan Yield and Mortality Model for Forest Stands in Minnesota

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Recent efforts by Zobel et al. (In prep.) produced a single cohort (stand), lifespan yield model (referred to as the ZEO model) that clarifies the entire development of a stand, from initiation through maximum yield and ultimately death of the entire cohort. The ZEO model formulation extends the stand yield model of Walters and Ek (1993) by adding model components that cause previously monotonically increasing yield to decline to zero at some species specific maximum age. A byproduct of model usage is a quantification of yield lost to mortality and the improvement of long rotation yield estimates. This note provides the model form, calibration results for short- and long-lived forest types, and user friendly implementation instructions.

Background

Base model

The equations proposed by Zobel et al. (In prep.) build upon the yield model forms suggested by Walters and Ek (1993). The original models comprise a system of equations that describe several forest attributes, including basal area (B) (ft^2/ac), average stand dominant height (H) (ft), and volume (V) (ft^3/ac), among others. These equations are given below.

$$B = a_1 S^{a_2} A^{a_3} \qquad (1)$$

$$H = g_1 S^{g_2} \left(1 - \exp(g_3 A)\right)^{g_4 S^{g_5}} \qquad (2)$$

$$V = d_1 B^{d_2} H^{d_2} \qquad (3)$$

where A = stand age (years), S = site index (ft) at base age 50 years, and a_1-a_3, g_1-g_5, and d_1-d_3 are forest type specific parameters (see Table 1).

Lifespan model

Using Eq. 1, Zobel et al. (In prep.) proposed a piecewise function (Eq. 4) that describes three compartments of stand basal area development: (1) rapid early growth and yield accumulation; (2) slowed growth yet continued yield accumulation; (3) mortality exceeding growth and yield reduction. The beginning and end of each phase in the model varies depending on the longevity of the dominant or main species associated with the forest type and other age parameters defined by the user (see below). Basal area estimates from the new, more complete ZEO formulation are then used to compute volume estimates using Eq. 3.

$$B_{ZEO} = \begin{cases} a_1 S^{a_2} A^{a_3} & \text{if } A \le A_{split} \\ B_t\left(1 - \dfrac{\alpha_1}{\alpha_2}\right) + \left(B_{t/2}\right)\left(\dfrac{\alpha_1}{\alpha_2}\right) & \text{if } A_{split} < A \le A_{asymp} \\ B_{max} - \left(B_{max}\left(\dfrac{\alpha_3}{\alpha_4}\right)^s\right) & \text{if } A_{asymp} < A \le A_{max} \end{cases} \qquad (4)$$

where A_{split} = age where ZEO estimates begin to diverge from Walters and Ek (1993) estimates, A_{asymp} = age of ZEO maximum yield, A_{max} = maximum cohort age, and t represents the variable value at time (age) t;

and

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Table 1. Coefficient estimates from Walters and Ek (1993) for the basal area (ft^2/ac), height (ft), and volume (ft^3/ac) models.

	Basal area			Height					Volume		
Forest type	a_1	a_2	a_3	g_1	g_2	g_3	g_4	g_5	d_1	d_2	d_3
Jack pine	1.4621	0.7066	0.3168	1.6330	1.0000	-0.0223	1.2419	0.0000	1.7365	0.8231	0.7910
Red pine	2.3990	0.5913	0.3469	1.8900	1.0000	-0.0198	1.3892	0.0000	1.1605	1.0762	0.6228
White pine	25.7407	0.0000	0.3192	1.9660	1.0000	-0.0240	1.8942	0.0000	1.7791	0.9937	0.6084
Balsam fir	3.4075	0.4692	0.3675	2.0901	0.9296	-0.0280	2.8280	-0.1403	3.4532	0.8316	0.6059
White spruce	25.2054	0.0000	0.2916	10.8738	0.5529	-0.0343	34.6885	-0.6139	6.0401	0.8994	0.3755
Black spruce	3.7512	0.4272	0.3799	1.7620	1.0000	-0.0201	1.2307	0.0000	0.7835	0.7496	1.0422
Northern white-cedar	11.3711	0.2333	0.3602	1.9730	1.0000	-0.0154	1.0885	0.0000	2.0023	0.9375	0.6356
Tamarack	8.5456	0.1579	0.3258	1.5470	1.0000	-0.0225	1.1129	0.0000	1.0345	0.7820	0.9352
Oak-hickory	3.0558	0.5309	0.3024	4.5470	0.8170	-0.0128	2.1089	-0.1770	3.5800	1.0163	0.4209
Elm-ash-soft maple	4/7759	0.3450	0.3696	6.1308	0.6904	-0.0195	10.1563	-0.5330	3.9236	0.9791	0.4230
Maple-birch	2.8016	0.5690	0.2912	6.0522	0.6768	-0.0217	15.4232	-0.6354	2.9549	0.9825	0.5978
Aspen	0/6036	0.7735	0.4459	11.4804	0.5039	-0.0281	105.9678	-1.0590	3.1206	0.9241	0.5449
Paper birch	2.1002	0/6092	0.3215	2.4321	0.9207	-0.0168	1.5297	-0.1042	2.7305	0.8924	0.6047
Balsam poplar	0.4097	0.8309	0.4764	11.48.04	0.5039	-0.0281	105.9678	-1.0590	2.6950	0.9042	0.5988

B_{ZEO} = basal area estimate (ft^2/ac) for the proposed model; B_t, $B_{t/2}$, and B_{max} = basal area estimates from Walters and Ek (1993) at age t, $(A_t + A_{split})/2$, and $(A_{asymp} + A_{split})/2$, respectively;

and

$\alpha_1 = A_t - A_{split}$, $\alpha_2 = A_{asymp} - A_{split}$, $\alpha_3 = A_t - A_{asymp}$, $\alpha_4 = A_{max} - A_{asymp}$, and s = α parameter that controls the rate of stand decline.

This model form provides considerable flexibility to the user, as adjusting A_{split}, A_{asymp}, A_{max}, and s allow for model adaptation to a variety of stand decline patterns. In addition, Zobel et al. (In prep.) proposed default values for shorter- and longer-lived species (and their associated forest types) (Table 2). These values provide a starting point for further calibration to specific stand conditions.

Table 2. Model default values from Zobel et al. (In prep.) for A_{split}, A_{asymp}, A_{max}, and the cohort declining rate (s), along with the species used to assign values to a forest type. The types were loosely aggregated by long-lived (first 11) and short-lived (last three) species.

		Zobel et al. (In prep.) default values			
Forest type	**Associated species**	A_{split}	A_{asymp}	A_{max}	s
Jack pine	Jack pine	45	75	150	
Red pine	Red pine	75	124	250	
White pine	White pine	60	150	450	
Balsam fir	Balsam fir	35	75	200	
White spruce	White spruce	40	90	250	4
Black spruce	Black spruce	80	130	250	
Northern white-cedar	Northern white-cedar	80	160	400	
Tamarack	Tamarack	60	100	200	
Oak-hickory	Various	40	120	400	
Elm-ash-soft maple	black ash	40	100	300	
Maple-birch	Sugar maple	140	220	400	
Aspen	Quaking aspen	55	75	100	
Paper birch	Paper birch	62	90	140	3
Balsam poplar	Balsam poplar	60	90	150	

Mortality model

In addition to providing traditional yield estimates, Zobel et al. (In prep.) proposed a mortality yield model that quantifies the effect of cohort aging (Eq. 5). Background mortality is also included through a rate supplied by the user.

$$V_M = \frac{V_{ME} + (m-1)V_{ZEO}}{1-m} \tag{5}$$

where V_M = volume (ft^3/ac) of mortality, V_{WE} and V_{ZEO} = estimated volume (ft3/ac) yield from Walters and Ek (1993) and Zobel et al. (In prep.), respectively, and m = the specified background mortality rate.

Implementation

Using the ZEO methodology involves several sequential steps:

1. Define A_{split}, A_{asymp}, A_{max}, and s for the cohort.
2. Project stand basal area and volume yield using Walters and Ek (1993) (Eq. 1-3).
3. Project stand basal area and volume yield using ZEO methodology (Eq. 4).
4. Specify the background mortality rate and calculate mortality yield (Eq. 5).

The remaining subsections illustrate the use of the model to obtain volume and mortality estimates using the short-lived species calibration for an aspen stand 70 years old with site index 65 ft. An additional example shows results for the same stand at 90 years old. Finally, Figure 1 shows the lifespan yield curves for the stand and marks the location of each example estimate.

Example 1

Step 1: A_{split}, A_{asymp}, A_{max}, and s

Projections for the example stand will use the default (Table 2) values, except for maximum age with a value of 100 years and a split age of 50 years..

Step 2: Walters and Ek (1993) yield

$$B = 0.6036\left(65^{0.7735}\right)\left(70^{0.4459}\right) = 101.34\, ft^2\ /\ ac$$

$$H = 11.4804\left(65^{0.5039}\right)\left(1 - \exp\left[-0.0281(70)\right]\right)^{105.9678\left(65^{-1.059}\right)} = 77.53\, ft$$

$$V = 3.1206\left(101.34^{0.9241}\right)\left(77.53^{0.5449}\right) = 2{,}384.25\, ft^3/ac$$

Step 3: ZEO yield

Since cohort age exceeds $A_{max/2}$ but not A_{max} - $A_{max/4}$, we use the second phase equation:

$A_{(t-1)/2}$ = (69+50)/2 = 59.5 years

$B_{(t-1)/2} = 0.6036(65^{0.7735})(59.5^{0.4459}) = 94.25\ ft^2/ac$

α_1 = 70 - 50 = 20 years

α_2 = 25 years

$$B_{ZEO} = 101.34\left(1 - \frac{20}{25}\right) + 94.24\left(\frac{20}{25}\right) = 95.25\, ft^2\ /\ ac$$

$$V_{ZEO} = 3.1206\left(95.67^{0.9241}\right)\left(77.53^{0.5449}\right) = 2{,}260.71\, ft^3\ /\ ac$$

Step 4: Mortality yield

Projections for this cohort will use a background mortality rate of 3%.

$$V_M = \frac{2384.25 + (0.03 - 1)2260.71}{1 - 0.3} = 197.28\, ft^3\ /\ ac$$

Example 2

For the same stand 20 years later (all other variables held constant), the Walters and Ek (1993) equations give the following results:

$B = 113.35\ ft^2/ac$; $H = 84.54$ ft; and $V = 2{,}771.95\ ft^3/ac$

Since cohort age exceeds A_{max} - $A_{max/4}$, we use the third phase equation to compute ZEO yield, with the rate parameter $s = 2$:

$A_{B_{max}}$ = (100 - 26 + 50)/2 = 62 years

B_{max} = 96.00 ft^2/ac

α_3 = 90 - 25 - 50 = 15 years

$$B_{ZEO} = 96.00 - \left(96.00\left[1 - \frac{15}{25}\right]^2\right) = 80.64\, ft^2 / ac$$

V_{ZEO} = 3.1206 ($80.64^{0.9241}$) ($84.54^{0.5449}$) = 2,023.66 ft^3/ac

Finally, the mortality estimate follows:

$$V_M = \frac{2771.95 + (0.03 - 1)2023.66}{1 - 0.03} = 834.02\, ft^2 / ac$$

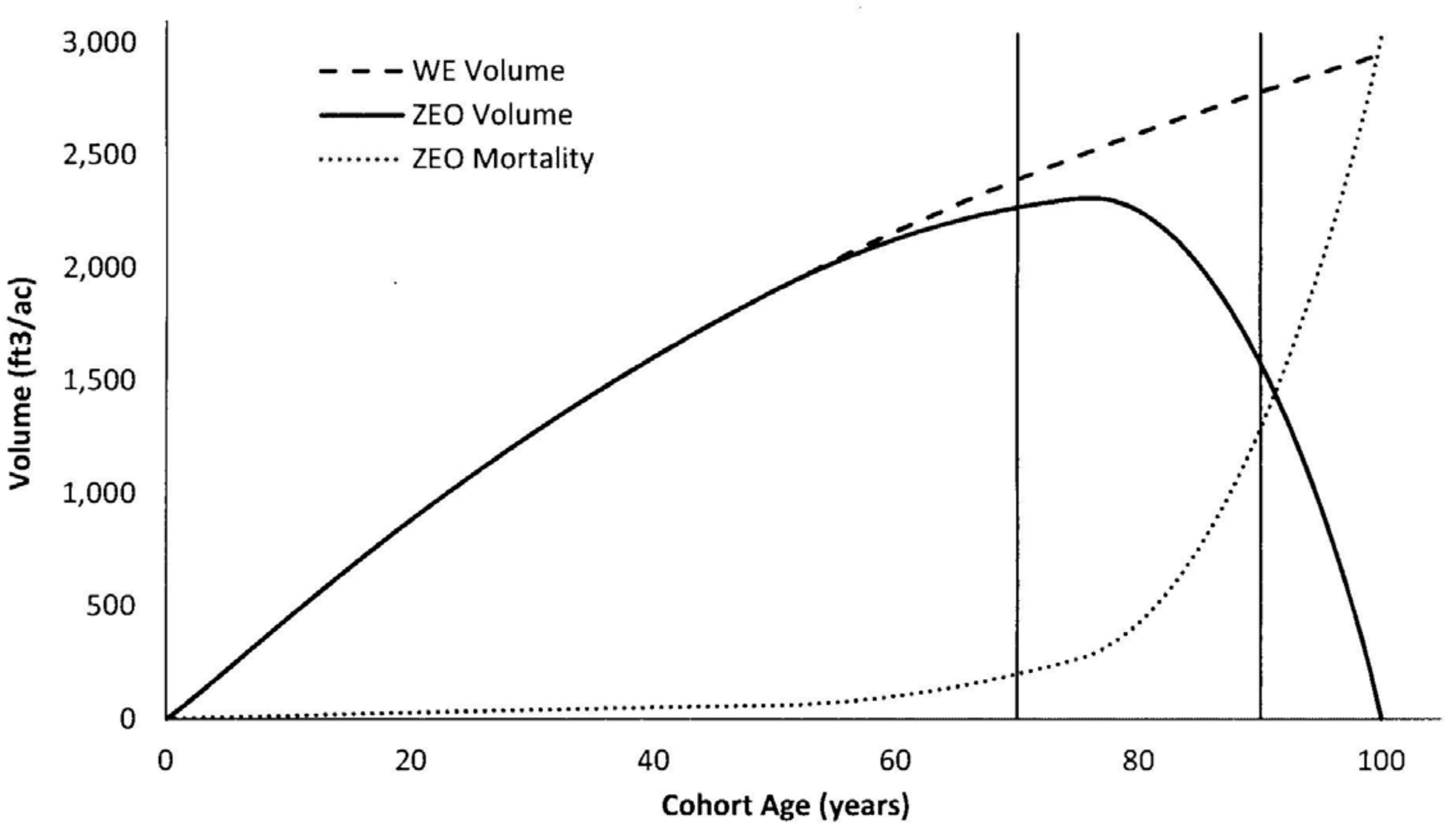

Figure 1. Yield curves for an aspen cohort with maximum age 100 years. The ZEO model used the short-lived default parameter values. For Example 1 and 2, the yield estimates are identified by vertical lines (at 70 and 90 years, respectively).

Discussion

The utility of the ZEO model arises from the fact that the USDA Forest Service, Forest Inventory and Analysis (FIA) dataset (USDA 2012) used by Walters and Ek (1993) lacked representation of older age and deteriorating stand conditions. That was sufficient when the original interest was in managing younger, rather than older stands. However, forest management today often includes stands well beyond the point of maximum economic or biological increment.

The examples here illustrate how to use the ZEO model for the short-lived aspen species. Note that in only 20 years, the total mortality yield nearly quadruples, showing the often rapid decline observed during aspen stand breakup (Fralish 1980; Shields and Bockheim 1981). For long-lived species, model application is similar, but typically with slower accumulations of mortality yield as the cohort is gradually replaced (e.g., succession). The short- and long-lived calibrations may be used interchangeably for a forest type, depending on the unique species/site relationship. A Visual Basic Application has been written to implement the model components and is available at http://iic.umn.edu/zeo. Ultimately, the ZEO approach provides more realistic yield estimates for older cohorts and quantifies the effect on yield of non-traditional management time horizons (e.g., extended rotation forestry) as compared to the earlier Walters and Ek (1993) model.

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