ESSAYS ON NATURAL RESOURCES MANAGEMENT WITH POTENTIAL REGIME SHIFT

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Dedication

To my parents.

Abstract

This dissertation includes three essays on natural resources management with potential regime shift, which is a rapid and persistent change of ecosystem processes leading to decline in the economic value of natural resources. The first essay analyzes the impact of a regime shift that reduces the natural growth of a renewable resource and shows that aggressive management is optimal under reasonable conditions. This is in contrast to the precautionary principle discussed in recent literature of resource economics. The second essay focuses on the allocation of risk of regime shift. It is shown that the regime shift that only threatens a portion of the resource stock causes more aggressive management, and the effect of regime shift changes non-monotonically as the share of threatened stock increases. The third essay considers a duopolistic resource market where the regime shift has asymmetric effects on two Cournot players' private resource stocks. Some examples are used to show that Cournot competition causes distortions that depend on the relative sizes of two Cournot players' stocks and the share of stocks that are under the threat of regime shift. It is found that the largest loss in social welfare occurs in the case where the regime shift affects the entire stock of one Cournot player's stock and has no impact on the other Cournot player's stock.

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Chapter 1

Introduction

Regime shifts are rapid switch of system behaviors caused by changes in feedbacks between system components (e.g., interactions among species or among multiple producers and consumers). Examples of regime shifts in ecological systems include shifts in coral reefs between coral-dominated and algal-dominated systems [25], and in shallow lakes between oligotrophic and eutrophic conditions [54]. In financial markets, where the expectations of other traders influence returns, shifts in investor sentiment lead to shifts between bull and bear markets [3, 53]. Similarly, changes in expectations can lead to shifts between multiple potential equilibria in the entire economy [2, 7]. Regime shifts can also occur in social systems (e.g, consumer fads) and in political systems with changes in governments (literally regime shifts).

Regime shifts are also characterized by hysteresis. Once a regime shift has occurred it may be difficult or impossible to reverse the process and recover the original regime. In ecological systems, an increase in phosphorus inputs into shallow lakes can trigger a shift from oligotrophic to eutrophic conditions but it may take a far greater reduction in phosphorus inputs over an extended period of time to shift the lake back from eutrophic to oligotrophic conditions [56]. Similarly, it may take prolonged good or bad economic news before a sufficient number of people shift expectations and generate a new equilibrium.

When a regime shift occurs in a ecosystem, it can have positive or negative impacts on the benefits that human gain from the services provided by the ecosystem. Of greatest concern from a resource management viewpoint are cases in which a regime shift causes a decline in the economic value of natural resources. For example, the local economy of Newfoundland, Canada relied on the offshore cod fishery for hundreds of years until the middle of the last century when modern fishing vessels became available. Then the local economy entered a boom phase followed by a bust in 1992 caused by the sudden collapse of the cod fish stock. After that, 40,000 people lost their jobs and the local population decreased by 10% in the following decade.

To study the impacts of regime shift on resource management, this dissertation develops three closely related models on the optimal extraction of natural resource and the equilibrium in resource market with imperfect competition.

Chapter 2 analyzes a renewable resource model where a regime shift reduces the natural growth of the renewable resource, and finds that the threat of potential regime shift can make the optimal extraction more aggressive or more precautionary compared to the case without threat of regime shift. If lowering extraction of the resource lowers the risk of a regime shift, the resource manager will take precautionary measures and lower exploitation compared to a case without potential regime shift. Such "risk reduction effect" (as [42]) occurs when the probability of a regime shift is endogenous. Moreover, a regime shift will cause future resource availability to be lower, which lowers post-regime shift harvest and raises post-regime shift marginal utility of harvest. A forward looking manager will take this into account by reducing the initial extraction rate thereby saving more stock to increase future harvests ("consumption smoothing effect"). Finally, a regime shift reduces the resource growth rate and makes saving the resource a poorer investment because it will have a lower rate of return. A forward looking manager will take this effect into account by increasing the current extraction rate and saving less stock for the future ("investment effect").

The investment effect causes optimal management to be more aggressive, but the risk reduction effect and the consumption smoothing effect cause optimal management to be more precautionary. We will see that under certain conditions justified by reasonable parameter values, the investment effect outweighs the other two effects, leading to more aggressive management in the face of a potential regime shift compared to the case without threat of potential regime shift. With aggressive management a potential regime shift will increase current resource exploitation and reduce resource stocks as compared to the case with no potential regime shift. This result is surprising in light of previous literature in which the potential for a regime shift was thought to cause optimal management to be more precautionary ([15], [21], [28], [42] and [61]).

Chapter 3 focuses on the allocation of risk from a regime shift. The risk can vary across different pools of a resource stock. For example, oil extraction could be affected by very different environmental disturbances in Gulf of Mexico, Eastern Siberia or Middle East. In many cases, when one particular resource stock is under the threat of a potential regime shift, perfect substitutes from alternative risk-free stocks in other locations are usually available. In order to focus on the effect purely from the asymmetric allocation of risk, the regime shift is simplified to a negative stock effect that reduces the availability of an exhaustible resource.

A negative stock effect reduces the consumption probability frontier, i.e., the intertemporal budget constraint a resource manager is subject to, and influences the intertemporal decisions on consumption and saving through an income effect and a substitution effect.[1] These two effects work in opposite directions, thus the optimal behavior could be either more conservative or more aggressive as compared to the risk-free case, depending on the relative magnitudes of these two effects. The income effect and substitution effect caused by a symmetric shock offset each other perfectly for the case where preferences have unit elasticity of intertemporal substitution (log utility) and extraction is costless.

However, when the allocation of risk is asymmetric, i.e., only a portion of the entire resource stock is under the threat of potential regime shift, the model shows that optimal management is more aggressive as compared to the risk-free case and to the case where the risk is symmetric. Furthermore, the effect of an asymmetric regime shift changes non-monotonically as the share of risky stock increases. As the share of risky stock decreases from one so the risk becomes asymmetric, the optimal extraction becomes more and more aggressive, and reaches an upper limit at a particular share of the risky stock. Then further decreases of the share of risky stock lowers the optimal extraction. The optimal extraction reduces to the risk-free level, which is identical to the optimal level with symmetric risk, when the share of risky stock reaches zero.

[1] This is similar to the consumption smoothing effect and the investment effect discussed in Chapter 2. Jones and Manuelli [27] also discussed these effects induced by the risk from Ito process in neoclassic growth models.

This model of asymmetrically allocated potential regime shift is extended in Chapter 4 to study an exhaustible resource market with duopolistic competitors. The global exhaustible resource market is usually affected by social and political conditions that are different across resource extraction countries. Extractors in countries where expropriation of private stocks is imminent [26, 32, 34] face additional risk as compared to extractors in countries where private property rights are well established and protected. Furthermore, technological innovation that reduce demand for exhaustible resources in the innovating country could also have unbalanced effect on the economic values of *in situ* stocks between the innovating country and the countries without the new technology [11, 12, 14]. Such risk heterogeneity is characterized by potential regime shift that could be allocated on different competitors' stocks asymmetrically.

Similar to Stiglitz [59], Cournot competition does not distort the Pareto optimal level of extraction of exhaustible resource stocks with symmetrically allocated risk when demand is derived from isoelastic preferences and extraction cost is zero. This is because in dynamic equilibrium the distortion effect of market power canceled out across time, and there is no ability to price discriminate. However, when the risk of potential regime shift is asymmetrically allocated on the stocks of two Cournot players, aggregate extraction in equilibrium would be different from the Pareto optimal outcome in many cases. Strategic behavior could increase or decrease the extraction in Cournot equilibrium as compared to the Pareto optimal level, depending on the relative size of the two players' stocks and the share of resource stock that is under the risk of potential regime shift.

With asymmetric risk, Cournot competition can induce distortions that lead to impacts on extraction and remaining stock, and therefore causes loss in social welfare. In the risk-free case and the case with symmetric risk, welfare loss is zero because the Pareto optimal extraction is achieved in Cournot equilibrium. However, with asymmetrically allocated risk, welfare loss is positive in many cases. When the potential regime shift only affects a portion of one player's stock and has no effect on the stock of the other player, welfare loss starts to increase as the share of the first player's risky stock increases. The largest welfare loss occurs in the fully specialized case where the risk affects the entire stock of the first player and has no impact on the second player's stock. When the risk is further spread out to affect a portion of the second player's stock (and the first player's entire stock), welfare loss starts to decrease as the share of the second player's risky stock continues to increase. In the extreme case where both players' stocks are completely affected by the potential regime shift (symmetric risk), welfare loss reduces to zero.

Chapter 2

The optimal management of renewable resources under the risk of potential regime shift

2.1 Introduction

Complex systems, such as ecosystems or market economies, are characterized by interactions among multiple components (e.g., interactions among species or among multiple producers and consumers). Such complex systems can undergo changes in feedbacks between system components causing rapid and persistent shifts in system behavior. Examples of such regime shifts in ecological systems include shifts in coral reefs between coral-dominated and algal-dominated systems [25], and in shallow lakes between oligotrophic and eutrophic conditions [54]. In financial markets, where the expectations of other traders influence returns, shifts in investor sentiment lead to shifts between bull and bear markets [3, 53]. Similarly, changes in expectations can lead to shifts between multiple potential equilibria in the entire economy [2, 7]. Regime shifts can also occur in social systems (e.g, consumer fads) and in political systems with changes in governments (literally regime shifts).

Regime shifts are characterized not only by relatively rapid shifts in system behavior but also by hysteresis. Once a regime shift has occurred it may be difficult or impossible to reverse the process and recover the original regime. In ecological systems, an increase in phosphorus inputs into shallow lakes can trigger a shift from oligotrophic to eutrophic conditions but it may take a far greater reduction in phosphorus inputs over an extended period of time to shift the lake back from eutrophic to oligotrophic conditions [56]. Similarly, it may take prolonged good or bad economic news before a sufficient number of people shift expectations and generate a new equilibrium.

A regime shift may have either positive or negative effects on welfare. Of greatest concern for management are cases where a regime shift causes a decline in welfare. For example, a shift from coral-dominated to algal-dominated reef may reduce tourism and fish harvests, and a shift from a bull to a bear market can reduce wealth and potentially trigger a financial crisis. In this paper we analyze the case where a potential regime shift results in lower welfare.

Early research on regime shifts in resource economics focused on catastrophic collapse with zero utility post regime shift. Cropper [10] analyzed potential system collapse triggered when the state variable exceeds an uncertain threshold. Reed [44, 45] used the Pontryagin maximum principle to transform Cropper's stochastic problem into a deterministic problem to find an analytical solution. With an exogenous probability of catastrophic collapse, optimal environmental management is more aggressive compared to the case with no threat of collapse. The potential to lose the resource in the future gives more incentive to use the resource in the current period rather than save for the future ("use it or lose it"). When the risk of collapse is endogenous, optimal management is contingent on the shape of the hazard function and the magnitude of disutility induced by the collapse and can be either aggressive or precautionary [9].

More recent research has analyzed the effect of regime shift that reduces stock but not to zero, changes the growth dynamics of the stock, or shifts preferences. This literature has generally found that the threat of regime shift causes optimal management to be more precautionary. Tsur and Zemel [61] studied a model of pollution in which "reversible events" lead to a reduction in post event welfare. They conclude that "reversible events always imply more conservation" (p. 968), which is equivalent in our terms to the threat of potential regime shift always leading to precautionary management. This result follows from the assumption that the hazard rate and the penalty inflicted by the regime shift are non-decreasing functions of the pollution level. Polasky et al. [42] developed unambiguous results for a potential regime shift that reduces the natural growth of a renewable resource. With a linear benefit function, they found that a potential regime shift has no effect on the optimal management when the risk is exogenous, and induces precautionary management when the risk is endogenous. Precautionary management is also found by de Zeeuw and Zemel [15] in the context of pollution control where a regime shift causes a structural change in preferences but does not affect the pollution decay rate. Numerical simulations by Gjerde et al. [21] and Keller et al. [28] in a potential regime shift for climate change also suggest precautionary management.

This paper builds from Polasky et al. [42] in which a regime shift reduces the natural growth of a renewable resource. We use a general utility function rather than a linear function as in Polasky et al. [42]. We use dynamic programming methods to evaluate the changes in value of harvesting the resource caused by biophysical change in the resource growth function. This change in value is captured by a damage function based on the value functions before and after the regime shift. As found in the early literature [9, 61], the shape of the damage function is crucial to determine how a potential regime shift influences optimal management.

In contrast with the previous research in which the the shape of the damage function is given by assumptions [9, 61], we use general forms of utility function, natural growth function and hazard function to study the shape of the damage function analytically. We show that damage function can be either increasing or decreasing in resource stock. Therefore, the threat of potential regime shift can make the management more aggressive (when the damage function increases in stock) or more precautionary (when the damage function decreases in stock) compared to management without threat of regime shift.

We find that a potential regime shift affects optimal management through multiple effects, which have not been fully captured by the existing literature. If lowering exploitation of the resource lowers the risk of a regime shift, the resource manager will take precautionary measures and lower exploitation compared to a case without potential regime shift. We call this effect the "risk reduction effect." The risk reduction effect occurs when the probability of a regime shift is endogenous. The risk reduction effect is the only effect that occurs when the utility function is linear [42].

However, there are two additional effects from a potential regime shift with general forms of utility function and resource growth function. First, a regime shift will cause future resource availability to be lower, which lowers post-regime shift harvest and raises post-regime shift marginal utility of harvest. A forward looking manager will take this into account by reducing the initial exploitation rate thereby saving more stock to increase future harvests ("consumption smoothing effect").

Second, a regime shift reduces the resource growth rate and makes saving the resource a poorer investment because it will have a lower rate of return. A forward looking manager will take this effect into account by increasing the current exploitation rate and saving less stock for the future ("investment effect").

The investment effect causes optimal management to be more aggressive, but the risk reduction effect and the consumption smoothing effect cause optimal management to be more precautionary. We provide a condition under which the investment effect outweighs the other two effects, leading to more aggressive management in the face of a potential regime shift compared to the case without threat of potential regime shift. With aggressive management a potential regime shift will increase current resource exploitation and reduce resource stocks as compared to the case with no potential regime shift. This result is surprising in light of previous literature in which the potential for a regime shift was thought to cause optimal management to be more precautionary.

However, similar results to ours can be found in the consumption-saving problem from standard growth models. An exogenous shock to the general economic condition that reduces the return on capital would lead to lower future income and encourage higher saving to smooth consumption over time. On the other hand, this shock would also cause the return on saving to be lower and discourage saving. Thus the overall effect of a negative economic shock depends on the relative magnitude of these two effects, which are sometimes called income effect and substitution/price effect.

In our model of renewable resource management, we find that aggressive management can occur for reasonable parameter values. We simulate the effect of a potential regime shift on either the carrying capacity or the intrinsic growth rate of the natural growth of a renewable resource. Our simulation shows that a potential regime shift is likely to cause aggressive management if risk endogeneity is suitably small and the elasticity of intertemporal substitution (EIS) is suitably large. Our simulation results also show that the relationship between the effect of a potential regime shift and relative risk aversion can be non-monotonic. It is well known that environmental uncertainty can have complicated effect on the optimal management [40]. Holding the relative risk aversion constant, an increase in the variance of a stochastic shock can induce a non-monotonic response to the hazard [6, 52, 67]. We also find that increasing risk aversion can have non-monotonic impacts on optimal management.

In the next section we build intuition by solving for a two-period model where the potential regime shift may occur after the harvest decision is made in the initial period. In Section 2.3 we consider a discrete time infinite horizon model with a non-zero probability of regime shift in each period. Section 2.4 illustrates the main results using a numerical simulation. Section 2.5 contains concluding comments. All technical proofs are in Section 2.6.

2.2 A two-period example

We start with a two-period model that is relatively simple to analyze but illustrates most of the important results contained in the infinite-horizon model analyzed in Section 3. Consider the optimal management problem for a renewable resource stock whose harvest supplies the only consumption good in the economy. The growth of the renewable resource is characterized by a production function $f : S \times X \to S$, where $s \in S \subset \mathbb{R}_+$ represents resource stock size, and $x \in X \subset \mathbb{R}_+$ represents an aspect of the environmental quality. We assume both S and X are nonempty, convex and compact. We can write the production function as $f = \phi + s$, where ϕ represents natural growth. For example, $\phi = gs(1 - s/K)$ is the standard logistic natural growth function. In this case, x could represent the *intrinsic growth rate* g or the *carrying capacity* K. We assume f satisfies the following assumptions,

Assumption 1a. $f(s, x)$ is twice continuously differentiable on $S \times X$;

Assumption 1b. $f_1(s, x) > 0$ and $f_{11}(s, x) < 0$;[1]

Assumption 1c. $f_2(s, x) > 0$ and $f_{12}(s, x) > 0$;

Assumption 1d. For all $x \in X$, $f(0, x) = 0$, and there exists carrying capacity K such that $s \leq f(s, x) \leq K$ for all $0 \leq s \leq K$, and $f(s, x) < s$ for all $s > K$.

[1] Throughout this paper we will use subscript i to denote the partial derivative of a function to its ith argument, and subscript ij to denote the mixed second order partial derivatives to its ith and jth arguments.

A regime shift is introduced as a decrease of x by $\Delta x > 0$. Then by Assumption 1c, for any given stock level s, a regime shift would reduce the resource production $f(s, x)$ and the gross return rate $R(s, x) = f_1(s, x)$.

Given initial resource stock s_0, the stock after growth is $f(s_0, x)$ in the initial period $t = 0$. The resource manager chooses harvest, $h_0 \in [0, f(s_0\, x)]$, which is nonstorable and consumed immediately to generate utility according to function $u : \mathbb{R}_{++} \to \mathbb{R}$. We assume u satisfies the following assumptions,

Assumption 2a. $u(h)$ is twice continuously differentiable on $\mathbb{R}_{++}$;

Assumption 2b. $u'(h) > 0$ and $u''(h) < 0$;

Assumption 2c. $\lim_{h\to 0} u'(h) = +\infty$ and $\lim_{h\to +\infty} u'(h) = 0$;

Assumption 2d. $-hu''(h)/u'(h) = \gamma > 0$ (constant relative risk aversion).

After h_0 is chosen, the resource stock in the beginning of period $t = 1$ is $s_1 = f(s_0, x) - h_0$. We assume there is a probability of regime shift at $t = 1$ yielding either normal growth, $f(s_1, x)$, or reduced growth $f(s_1, x - \Delta x)$. The probability of regime shift is determined by a hazard function $\lambda : S \to [0, 1]$, which satisfies the following assumptions,

Assumption 3a. $\lambda(s)$ is twice continuously differentiable on S;

Assumption 3b. $\lambda'(s) \leq 0$.

Because $t = 1$ is the last period, with a strictly increasing utility function the resource manager would harvest the entire resource stock after growth, $h_1 = f(s_1, x)$ for normal growth, $\underline{h}_1 = f(s_1, x - \Delta x)$ for reduced growth. The two-period model is summarized in Table 2.1.

Table 2.1: Two-period model

$t = 0$			$t = 1$		
Initial stock	Stock after growth	Harvest	Initial stock	Stock after growth = Harvest	Probability
s_0	$f(s_0, x)$	h_0	$s_1 = f(s_0, x) - h_0$	$f(s_1, x) = h_1$	$1 - \lambda(s_1)$: no shift
				$f(s_1, x - \Delta x) = \underline{h}_1$	$\lambda(s_1)$: shift occurs

The renewable resource management problem considered here has the same structure as the standard growth model, where consumption and capital stock correspond to h and s in our model. A regime shift of the renewable resource growth in our model is similar to a decline in productivity of capital in the standard growth model.

The resource manager maximizes the net present value of utility from harvest where period 1 values are discounted by β. Then define $\overline{X} = \{(x, \Delta x) \in \mathbb{R}^2_+ \,|\, x \in X, x - \Delta x \in X\}$, for any $(x, \Delta x) \in \overline{X}$ and $s_0 \in S$, the resource manager solves the following optimization problem

$$w(s_0) = \max\{u(h_0) + \beta[(1 - \lambda(s_1))u(h_1) + \lambda(s_1)u(\underline{h}_1)]\}$$

by choosing h_0. This problem is equivalent to

$$w(s_0) = \max_{s_1}\{u(f(s_0, x) - s_1) + \beta u(f(s_1, x) - d(s_1))\} \tag{2.1}$$

where

$$d(s_1) = \lambda(s_1)[u(f(s_1, x)) - u(f(s_1, x - \Delta x))]. \tag{2.2}$$

Note that d defined by (2.2) represents the damage to the net present value caused by the risk of potential regime shift. Solving (2.1) the optimal resource stock at the beginning of period 1 in the case with a potential regime shift, s_1^{RS}, satisfies the following first order condition

$$u'(f(s_0, x) - s_1^{RS}) = \beta f_1(s_1^{RS}, x)u'(f(s_1^{RS}, x)) - \beta d'(s_1^{RS}). \tag{2.3}$$

If there is no risk of potential regime shift, or $\lambda(\cdot) = 0$, the optimal resource stock s_1^* in the risk-free case would satisfy

$$u'(f(s_0, x) - s_1^*) = \beta f_1(s_1^*, x)u'(f(s_1^*, x)). \tag{2.4}$$

Without specific functional forms of f, u and λ we cannot explicitly compare s_1^{RS} with s_1^*. Notice that the left hand side of (2.4) is strictly increasing in s_1 and the right hand side of (2.4) is strictly decreasing in s_1, thus the optimal solution s_1^* is uniquely determined. On the other hand, by Assumption 1-3, we can see that $d'(s)$ in (2.3) converges uniformly to 0 on S as Δx converges to 0.[2] Therefore, for any

[2] Because $f(s, x)$ and $f_1(s, x)$ are continuous and strictly increasing in x, and u and u' are continuous and strictly monotonic, then by Dini's theorem the convergence of $u(f(s, x - \Delta x)) \to u(f(s, x))$, $u'(f(s, x - \Delta x)) \to u'(f(s, x))$ and $f_1(s, x - \Delta x) \to f_1(s, x)$ are all uniform on compact set S as $x - \Delta x \to x$.

s_1^{RS} satisfying (2.3),[3] we know $\lim_{\Delta x \to 0} s_1^{RS} = s_1^*$, because $\lim_{\Delta x \to 0} d'(s) = 0$ and the convergence doesn't depend on s. Based on this result, we can discuss the sign of $s_1^{RS} - s_1^*$ at the margin for small Δx.

We call the management aggressive, unchanged, or precautionary if $s_1^{RS} < s_1^*$, $s_1^{RS} = s_1^*$, or $s_1^{RS} > s_1^*$. By (2.3) and (2.4) these situations correspond to $d'(s_1^{RS}) > 0$, $d'(s_1^{RS}) = 0$ or $d'(s_1^{RS}) < 0$, as summarized in Table 2.2. Intuitively, if the damage of a potential regime shift is increasing in the resource stock ($d'(s) > 0$), a resource manager would increase the harvest and save less stock (aggressive management) to reduce the damage.

Table 2.2: Optimal management in the two-period model

Aggressive	$s_1^{RS} < s_1^*$	$d'(s_1^{RS}) > 0$
Unchanged	$s_1^{RS} = s_1^*$	$d'(s_1^{RS}) = 0$
Precautionary	$s_1^{RS} > s_1^*$	$d'(s_1^{RS}) < 0$

In Section 2.6 we show that

$$\lim_{\Delta x \to 0} \frac{d'(s_1^{RS})}{\Delta x} = \lambda'(s_1^*)u'(h_1^*)\frac{\partial h_1^*}{\partial x} + \lambda(s_1^*)R^* u''(h_1^*)\frac{\partial h_1^*}{\partial x} + \lambda(s_1^*)u'(h_1^*)\frac{\partial R^*}{\partial x} \tag{2.5}$$

where $h_1^* = f(s_1^*, x)$ and $R^* = R(s_1^*, x) = f_1(s_1^*, x)$. Then the management is aggressive at the margin, or $\lim_{\Delta x \to 0} d'(s_1^{RS})/\Delta x > 0$, if and only if the following condition is true

$$\underbrace{\gamma \xi_h}_{\text{Consumption smoothing}} + \underbrace{\theta \xi_h}_{\text{Risk reduction}} < \underbrace{\xi_R}_{\text{Investment}} \tag{2.6}$$

where

$$\xi_h = \frac{\partial h_1^*}{\partial x}\frac{x}{h_1^*} \qquad \xi_R = \frac{\partial R^*}{\partial x}\frac{x}{R^*} \qquad \theta = -\frac{\lambda'(s_1^*)h_1^*}{\lambda(s_1^*)R^*}.$$

If the risk is exogenous ($\lambda' = 0$), then $\theta = 0$, and the optimal management strategy is determined by the relative magnitudes of ξ_R and $\gamma \xi_h$. ξ_R is the elasticity of gross return rate (R^*) with respect to x. The regime shift should it occur will reduce the gross return rate, thus the resource stock will become a poorer investment after the regime shift. The resource manager will take this effect into account by increasing the harvest in the initial period and save less resource stock for the future (the "investment effect").

[3] The arguments here are not sufficient to establish the uniqueness of the solutions to (2.3).

ξ_h is the elasticity of optimal harvest (h_1^*) with respect to x. The regime shift should it occur will also cause lower resource availability, which lowers post-regime shift harvest and raises post-regime shift marginal utility of harvest. The resource manager will take this effect into account by reducing the harvest in the initial period thereby saving more stock to smooth harvests across time. The elasticity of intertemporal substitution (EIS), $1/\gamma$, measures the optimal intertemporal adjustment of consumption in response to the fluctuation of the gross return rate. Thus γ serves as a unit adjustment factor such that $\gamma\xi_h$ and ξ_R are comparable. We call $\gamma\xi_h$ the "consumption smoothing effect". Lower EIS ($1/\gamma$) implies that the fluctuation of the gross return rate has less impact on consumption.

When the risk is endogenous ($\lambda' < 0$), then $\theta > 0$. By lowering harvest the resource manager can lower the risk of regime shift, and thus will tend to make optimal management less aggressive. We call this effect the "risk reduction effect." With a positive risk reduction effect the aggressive management condition (2.6) is less likely to hold, and the optimal management is more likely to be precautionary.

In this two-period model we focused on the optimal resource stock s_1^{RS} under the risk of potential regime shift. We found that s_1^{RS} can be either greater than, equal to, or lower than the risk-free optimal resource stock s_1^*, depending on relative magnitudes of consumption smoothing effect, investment effect, and risk reduction effect. In the next section, we will extend the two-period model to an infinite horizon model, and discuss how a potential regime shift will affect the resource stock in the steady state.

2.3 The infinite horizon model

We begin by briefly reviewing the classical renewable resource management problem in a risk-free environment. Then we introduce the risk of potential regime shift and show what impact it has on optimal management at the margin. We derive a condition under which the pre-shift optimal management is aggressive compared with the risk-free situation.

2.3.1 Classical renewable resource management problem

Given the initial resource stock, s_0, the resource manager chooses the optimal plan of harvests $(h_t)_{t\geq 0}$ to solve the following optimization problem

$$V(s_0) = \max \sum_{t=0}^{\infty} \beta^t u(h_t) \tag{2.7}$$

subject to

$$s_{t+1} = f(s_t,\, x) - h_t \tag{2.8}$$

and nonnegativity constraints.

An equivalent dynamic programming problem is characterized by the following Bellman equation

$$V(s_t,\, x) = \max_{s_{t+1} \in \Gamma(s_t,\, x)} \{u(f(s_t,\, x) - s_{t+1}) + \beta V(s_{t+1},\, x)\} \tag{2.9}$$

where

$$\Gamma(s_t,\, x) = \{s_{t+1} \in S \,|\, 0 \leq s_{t+1} \leq f(s_t,\, x)\}. \tag{2.10}$$

Here we treat x as another state variable. The optimal policy for the Bellman equation is defined as

$$g(s,\, x) = \{y \in \Gamma(s,\, x) \,|\, V(s,\, x) = u(f(s,\, x) - y) + \beta V(y,\, x)\}. \tag{2.11}$$

The first order condition of (2.9) is[4]

$$u'(f(s,\, x) - g(s,\, x)) = \beta V_1(g(s,\, x),\, x). \tag{2.12}$$

Thus $g(s,\, x)$ is chosen such that the marginal value of harvest in current period and shadow value of the resource stock in the next period are equalized. The envelope condition of (2.9) is

$$V_1(s,\, x) = u'(f(s,\, x) - g(s,\, x)) f_1(s,\, x). \tag{2.13}$$

From (2.12) and (2.13) we can solve the Euler equation

$$u'(h_t) = \beta f_1(s_{t+1},\, x) u'(h_{t+1}) \tag{2.14}$$

[4] We assume that $V(s,\, x)$ is continuous and strictly increasing in both arguments, and strictly concave and continuously differentiable in s; and $g(s,\, x)$ is a continuous single-valued function (see Section 5.1 of 60).

The solution of (2.9) is characterized by the Euler equation (2.14) and a transversality condition $\lim_{t\to\infty} \beta^t u'(h_t)s_{t+1} = 0$.

The Euler equation (2.14) shows that the present value of marginal utility is equalized across periods. The Euler equation (2.14) together with the growth equation (2.8) compose a two dimensional dynamic system $(s_t,\, h_t)_{t\geq 0}$, of which a nontrivial steady state $(s^*,\, h^*)$ is uniquely determined by

$$f_1(s^*,\, x) = 1/\beta \tag{2.15}$$

$$h^* = f(s^*,\, x) - s^*. \tag{2.16}$$

Throughout this paper we assume $s^* \in S$ exists.

2.3.2 Renewable resource management with a potential regime shift

Now we introduce a potential regime shift. As in the two-period model, the regime shift is characterized by a decrease of x by $\Delta x > 0$. In the infinite horizon model, we assume that the regime shift can happen at any time $t > 0$ with probability $\lambda(s_t)$, and is irreversible. Given $(s_t,\, x,\, \Delta x) \in S \times \overline{X}$, the Bellman equation under the risk of potential regime shift is

$$\begin{aligned} W(s_t,\, x,\, \Delta x) = \max_{s_{t+1}\in\Gamma(s_t,\, x)} &\{u(f(s_t,\, x) - s_{t+1}) \\ &+ \beta(1-\lambda(s_{t+1}))W(s_{t+1},\, x,\, \Delta x) + \beta\lambda(s_{t+1})V(s_{t+1},\, x-\Delta x)\} \end{aligned} \tag{2.17}$$

where $\Gamma(s_t,\, x)$ is given by (2.10). The optimal policy is defined as

$$\begin{aligned} q(s,\, x,\, \Delta x) = \{y \in \Gamma(s,\, x) \,|\, W(s,\, x,\, \Delta x) &= u(f(s,\, x) - y) \\ &+ \beta(1-\lambda(y))W(y,\, x,\, \Delta x) + \beta\lambda(y)V(y,\, x-\Delta x)\} \end{aligned} \tag{2.18}$$

Intuitively, the net present value derived from the renewable resource under the risk, W, depends on the resource stock s, the level of resource production x, and the magnitude of the potential regime shift Δx. The following result shows that regime shift has a negative impact on the net present value of resource stock.

Proposition 1. For any $(s, x, \Delta x) \in S \times \overline{X}$,

$$V(s, x - \Delta x) < W(s, x, \Delta x) < V(s, x).$$

Proof. See Section 2.6. $\square$

The Bellman equation (2.17) can be rewritten as

$$W(s_t, x, \Delta x) = \max_{s_{t+1} \in \Gamma(s_t, x)} \{u(f(s_t, x) - s_{t+1}) + \beta W(s_{t+1}, x, \Delta x) - \beta D(s_{t+1}, x, \Delta x)\} \quad (2.19)$$

where

$$D(s_{t+1}, x, \Delta x) = \lambda(s_{t+1})[W(s_{t+1}, x, \Delta x) - V(s_{t+1}, x - \Delta x)]. \quad (2.20)$$

By Proposition 1 we know that $D(s, x, \Delta x) > 0$ for any $(s, x, \Delta x) \in S \times \overline{X}$. Thus, similar to the two-period model, the risk of potential regime shift reduces net present value by an amount βD.

In Section 2.6 we show that $W(\cdot, x, \Delta x)$ is continuously differentiable under fairly general conditions for the hazard function λ. Then solving (2.19) the first order condition is

$$u'(f(s, x) - q(s, x, \Delta x)) = \beta W_1(q(s, x, \Delta x), x, \Delta x) - \beta D_1(q(s, x, \Delta x), x, \Delta x) \quad (2.21)$$

and the envelope condition is

$$W_1(s, x, \Delta x) = u'(f(s, x) - q(s, x, \Delta x)) f_1(s, x). \quad (2.22)$$

Then by (2.21) and (2.22), for any $(x, \Delta x) \in \overline{X}$, a steady state resource stock s^{RS} under the risk of potential regime shift, if it exists, is determined by

$$f_1(s^{RS}, x) = \frac{1}{\beta} + \frac{D_1(s^{RS}, x, \Delta x)}{u'(f(s^{RS}, x) - s^{RS})} \quad (2.23)$$

2.3.3 Effect of potential regime shift on optimal management

We are interested in how s^{RS} determined by (2.23) compares to s^* determined by (2.15).

Proposition 2. For any $(x,\, \Delta x) \in \overline{X}$, if s^{RS} exists, then

$$s^{RS} \gtreqless s^* \Leftrightarrow D_1(s^{RS},\, x,\, \Delta x) \lesseqgtr 0.$$

Proof. Because $u' > 0$, by (2.15) and (2.23) the desired result follows immediately from the concavity of f in s. □

Proposition 2 shows that the effect of the risk of a potential regime shift is determined by the properties of the damage function D defined by (2.20). If the damage of a potential regime shift is decreasing in the resource stock ($D_1 < 0$), a resource manager would reduce the harvest to increase the resource stock in the steady state (precautionary management). This result is found in prior paper [61].

However, if the damage of a potential regime shift is increasing in the resource stock ($D_1 > 0$), a resource manager would maintain lower resource stock in the steady state (aggressive management) to reduce the damage. We now decompose D_1 to provide more intuition on when $D_1 > 0$, and then derive a condition under which optimal management is aggressive.

From (2.20)

$$\begin{aligned} D_1(s,\, x,\, \Delta x) = \lambda'(s)[W(s,\, x,\, \Delta x) - V(s,\, x - \Delta x)] \\ + \lambda(s)[W_1(s,\, x,\, \Delta x) - V_1(s,\, x - \Delta x)]. \end{aligned} \tag{2.24}$$

We know from Proposition 1 that $W(s,\, x,\, \Delta x) > V(s,\, x - \Delta x)$. With endogenous risk ($\lambda' < 0$), the first term on the right hand side of (2.24) is negative. Endogenous risk gives the resource manager an incentive to increase the resource stock in order to lower the probability of regime shift, the "risk reduction effect."

The second term on the right hand side of (2.24) depends on the difference between shadow prices of resource stocks in the pre-shift risky and post-shift risk-free environments. In Polasky et al. [42] the shadow value of stock is always equal to prices, which is constant. Therefore, shadow values are equal before and after the regime shift ($W_1 = V_1$), which leaves only the first term. This is why in their model there no effect of the potential regime shift when risk is exogenous ($\lambda' = 0$) and management is always precautionary when risk is endogenous ($\lambda' < 0$).

In our model, the shadow values differ pre- vs. post-regime shift. By the envelope conditions of W and V, we know the shadow values are $W_1(s,\, x,\, \Delta x) = u'(h_w) f_1(s,\, x)$ and $V_1(s,\, x - \Delta x) = u'(h_v) f_1(s,\, x - \Delta x)$, where $h_w = f(s,\, x) - q(s,\, x,\, \Delta x)$ and $h_v = f(s,\, x - \Delta x) - g(s,\, x - \Delta x)$ are the harvests in corresponding cases. In our model, a regime shift has opposite effects on marginal utility u' and gross return rate f_1.

As in the two-period model, the regime shift will cause lower resource availability, $f(s,\, x) > f(s,\, x - \Delta x)$, thus tending to reduce harvests after the regime shift and raise post-shift marginal utility. With the potential for regime shift, the resource manager will reduce current harvest in order to have more stock in the future when marginal utility is higher (the "consumption smoothing effect"). However, because a regime shift also results in a lower gross return rate to resource stock, $f_1(s,\, x) > f_1(s,\, x - \Delta x)$, the resource manager tends to increase current harvest in order to reduce the potential loss of value of the resource stock caused by the regime shift (the "investment effect"). Depending on which of these effects dominates, we can have either $W_1(s,\, x,\, \Delta x) > V_1(s,\, x - \Delta x)$ or the reverse. In the following discussion we provide a condition under which the investment effect will outweigh the risk reduction and consumption smoothing effects so that the pre-shift management is aggressive ($s^{RS} < s^*$).

In the risk-free case, steady state resource stock s^* is

$$s^*(x) = \{y \in S \,|\, y = g(y,\, x)\}$$

and by (2.15) we know $s^*(x)$ is a single-valued continuously differentiable function. Similarly, under the risk of potential regime shift, a steady state resource stock s^{RS}, if it exists, satisfies

$$s^{RS}(s,\, \Delta x) = \{y \in S \,|\, y = q(y,\, x,\, \Delta x)\}.$$

In general the existence and uniqueness of s^{RS} is not guaranteed by (2.23). The following proposition shows that the correspondence $s^{RS}(x,\, \Delta x)$ is nonempty when Δx is sufficiently small, and converges to s^* when Δx converges to 0.

Lemma 3. For any $x \in X$, there exists $\delta > 0$ such that for any $|\Delta x| < \delta$, $s^{RS}(x,\, \Delta x)$ is nonempty and

$$\lim_{\Delta x \to 0} s^{RS}(x,\, \Delta x) = s^*(x).$$

Proof. See Section 2.6. □

As Lemma 3 shows, the effect of a potential regime shift on steady state resource stock is continuous. Let $h^* = f(s^*, x) - s^*$ denote the steady state harvest and $R^* = f_1(s^*, x)$ denote the steady state gross return rate in the risk-free case, the following proposition provides a condition under which the optimal management is aggressive under the risk of a potential regime shift.

Proposition 4. (Aggressive management condition) There exists $\delta > 0$ such that $\Delta x \in (0, \delta)$ implies $s^{RS} < s^*$, if the following condition is true

$$\underbrace{\gamma\xi_h^*}_{\text{Consumption smoothing}} + \underbrace{\Theta\xi_h^*}_{\text{Risk reduction}} < \underbrace{\frac{R^*}{R^* - \kappa}\xi_R^*}_{\text{Investment}} \tag{2.25}$$

where

$$\xi_h^* = \frac{\partial h^*}{\partial x}\frac{x}{h^*} \qquad \xi_R^* = \frac{\partial R^*}{\partial x}\frac{x}{R^*} \qquad \Theta = -\frac{\lambda'(s^*)h^*}{\lambda(s^*)(R^* - 1 + \lambda(s^*))}$$

and $\kappa \in (0, 1)$ is the eigenvalue of dynamic system (2.8) and (2.14) close to the risk-free steady state (s^*, h^*), given as the following

$$\kappa = 1 + 0.5\left(1/\beta - 1 - \beta\Omega - \sqrt{(1/\beta - 1 - \beta\Omega)^2 - 4\beta\Omega}\right)$$

where

$$\Omega = h^* f_{11}(s^*, x)/\gamma < 0.$$

Proof. See Section 2.6. □

The right hand side of (2.25) is the investment effect, which is analogous to the right hand side in condition (2.6) for the two-period model. In the infinite horizon model there is an additional term $R^*/(R^* - \kappa)$. Because κ is the eigenvalue of dynamic system (2.8) and (2.14) close to the risk-free steady state, in the optimal solution the evolution of the resource stock is approximated by $s_{t+1} - s^* = \kappa(s_t - s^*)$. This means the effect through the adjustment of resource stock (the investment effect) accumulates over an infinite future. In the two-period model, the entire resource will be harvested at $t = 1$. If we consider the two-period harvesting plan as the solution of a dynamic system, the resource stock converges to 0 instantaneously after $t = 1$ without any preservation for

$t \geq 2$, or equivalently $\kappa = 0$. Thus the right hand side of (2.25) reduces to ξ_R^* as (2.6) in the two-period model.

The left hand side of (2.25) consists of the consumption smoothing effect and the risk reduction effect. When the risk is exogenous ($\lambda' = 0$), then $\Theta = 0$. Thus, similar to the two-period model, with exogenous risk, the consumption smoothing effect $\gamma\xi_h^*$ is the only effect that provides an incentive to be more precautionary in harvesting the resource stock. When the risk is endogenous ($\lambda' < 0$), the resource manager tends to be more precautionary because of the risk reduction effect $\Theta\xi_h^*$.

It is also interesting to note how optimal management changes with γ. For sufficiently small γ and exogenous risk ($\lambda' = 0$), optimal management will be aggressive. For the investment effect $\xi_R^* R^*/(R^* - \kappa)$, notice that steady state (s^*, h^*) is determined by (2.15) and (2.16), thus R^* and ξ_R^* are independent of γ. But we can show that

$$\frac{d\kappa}{d\Omega} = \frac{\beta}{2}\left(\frac{1+\beta-\beta^2\Omega}{\sqrt{(1+\beta-\beta^2\Omega)^2 - 4\beta}} - 1\right) > 0$$

and

$$\lim_{\Omega\to-\infty} \kappa = 0 \qquad \lim_{\Omega\to 0} \kappa = 1.$$

Because $\Omega < 0$, we find that κ is increasing in γ and $\kappa \in (0, 1)$. Therefore, γ affects the investment effect only through κ, and the investment effect is bounded below by ξ_R^* when γ is small. As γ increases, the investment effect also increases, but because $\kappa < 1$, the investment effect is bounded above by $\xi_R^* R^*/(R^* - 1)$.

For the consumption smoothing effect $\gamma\xi_h^*$, because ξ_h^* is independent of γ, $\gamma\xi_h^*$ would be very small if γ is close to 0. Thus, with exogenous risk, sufficiently small γ implies aggressive management because the investment effect is bounded below.[5] However, when the risk is endogenous, the optimal management could be precautionary even when γ is close to 0. Notice that both Θ and ξ_h^* are independent of γ. Thus, if $-\lambda'$ is suitably large, the risk reduction effect $\Theta\xi_h^*$ would dominate the investment effect which is bounded above.

The consumption smoothing effect $\gamma\xi_h^*$ increases proportionally with γ, but the investment effect is bounded above. Thus, even with exogenous risk, optimal management

[5] This does not mean that with exogenous risk and $\gamma = 0$ the optimal management is aggressive, because in this case κ is not well-defined. Polasky et al. [42] showed that when the utility function is linear ($\gamma = 0$) and risk is exogenous that a potential regime shift has no effect on optimal management.

will be precautionary when γ is sufficiently large, because the consumption smoothing effect will dominate the investment effect.

2.4 Examples: the potential regime shift on carrying capacity and intrinsic growth rate

In this section we illustrate the main model results using a standard constant relative risk aversion utility function

$$u(h) = \frac{h^{1-\gamma}}{1-\gamma}$$

and a resource production function $f = \phi + s$ where

$$\phi = gs\left(1 - \frac{s}{K}\right)$$

is the standard logistic natural growth function with carrying capacity K and intrinsic growth rate g. A regime shift can be characterized by either a reduction of K by ΔK, or a reduction of g by Δg.

By equation (2.15), the risk-free steady state resource stock is

$$s^* = K\left(\frac{\beta(1+g)-1}{2\beta g}\right). \tag{2.26}$$

We assume $\beta(1+g) > 1$ to ensure that a strictly positive risk-free steady state resource stock s^* exists.

We use the following hazard function

$$\lambda(s) = \frac{2\overline{\lambda}}{1+\exp(\eta(s/s^*-1))}.$$

Clearly $\lambda(s^*) = \overline{\lambda}$ and $\lambda'(s^*) = -0.5\overline{\lambda}\eta/s^*$. Given $\overline{\lambda} \in (0,\,1)$, the risk is exogenous when $\eta = 0$ and endogenous when $\eta > 0$.

We are interested in characterizing how the steady state resource stock is affected by the regime shift. Based on (2.26), the steady state harvest is $h^* = f(s^*,\,x) - s^*$ and the steady state gross return rate is $R^* = f_1(s^*,\,x)$. Using definitions given in (2.25) and evaluating a potential regime shift in carrying capacity K or intrinsic growth rate

g, we find that

$$(\xi_R^*/\xi_h^*)|_{x=K} = 1 - \beta + \beta g \tag{2.27}$$

$$(\xi_R^*/\xi_h^*)|_{x=g} = 1 - \beta. \tag{2.28}$$

Because $\beta g > 0$, the investment effect is relatively stronger for the regime shift on K. Thus, it is more likely that a regime shift will cause aggressive management when carrying capacity K is reduced compared to the case with a reduction in intrinsic growth rate g.

We set discount factor $\beta = 0.98$. The estimates of EIS $(1/\gamma)$ vary considerably in recent literature, and we consider γ in the range (0, 3).[6] We normalize carrying capacity at $K = 100$, and set the intrinsic growth rate g at 0.05, 0.2, or 0.5.[7] To characterize the probability of a potential regime shift, we set $\overline{\lambda} = 0.1$, so when $s = s^*$ the probability of regime shift is 10%. The risk endogeneity value, η, is set at 0, 1, 2, 3, or 4. These parameter values are summarized in Table 2.3.

Table 2.3: Key parameter values

Preference		Resource production		Hazard function	
β	γ	K	g	$\overline{\lambda}$	η
0.98	0 – 3	100	0.05, 0.2, 0.5	0.1	0, 1, 2, 3, 4

2.4.1 The effect of potential regime shift on steady state

We assume the regime shift will cause a 5% reduction on either K or g. We numerically solve the Bellman equations (2.9) and (2.17).[8] Figure 2.1 shows simulation results for a

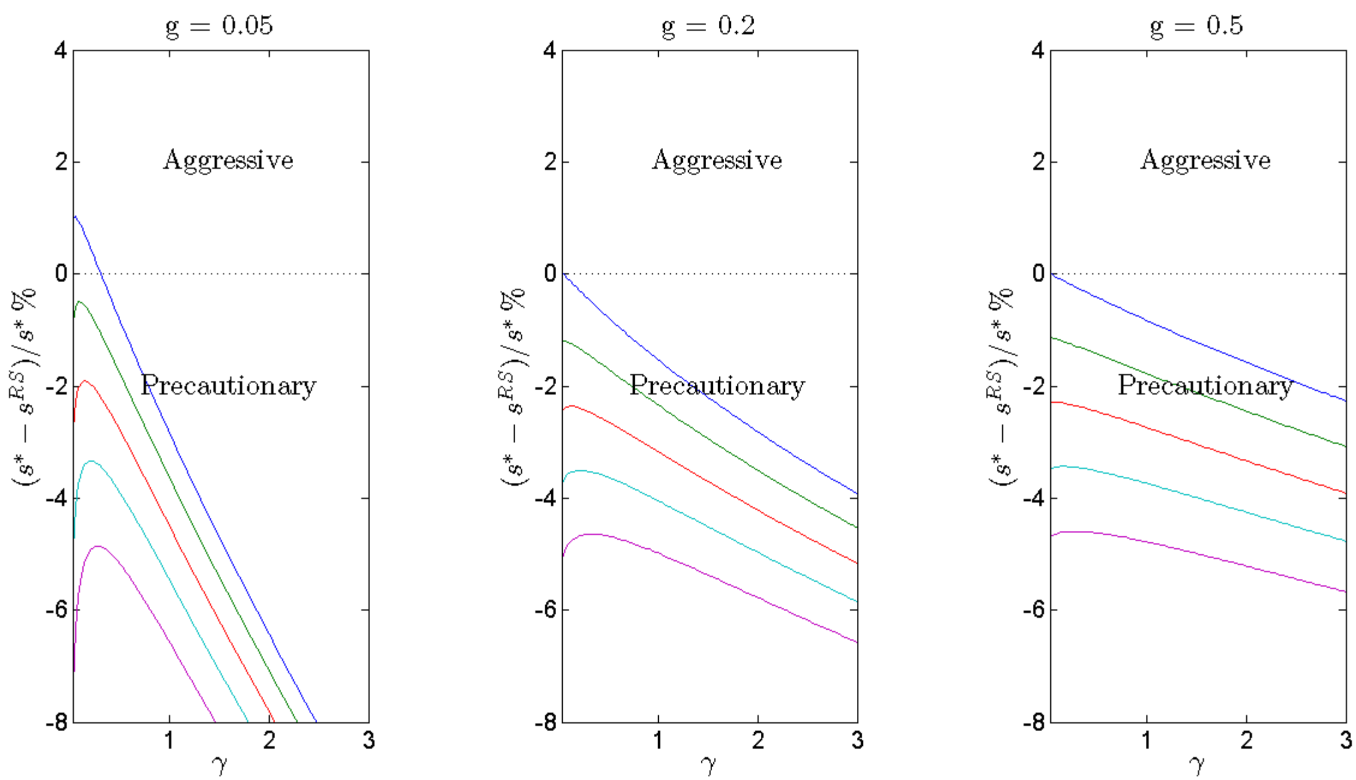
[6] Mulligan [36] used U.S. national account data and estimated that EIS ranges from 0.49 to 2.05 with different specifications. Vissing-Jorgensen [65] estimates EIS using micro data from the U.S. Consumer Expenditure Survey (CEX), and found that EIS is around 0.3-0.4 for stockholders and 0.8-1 for bondholders. Gruber [22] also used the CEX data but considered the variation across individuals in the capital income tax rate. His estimates of EIS vary from 1.54 to 2.36.

[7] The intrinsic growth rate g of marine fish species varies from 0.025 to 0.75 [8]. Musick [37] suggested that $g \geq 0.5$ implies that a fish species has high resilience to fishing pressure, and $g \leq 0.05$ implies low resilience to fishing pressure.

[8] To numerically solve the Bellman equations, we use the COMPECON toolbox developed by Miranda and Fackler [35]. The solver "dpsolve" is modified to incorporate state-dependent discount rate. The codes are available upon request.

potential regime shift on g, and Figure 2.2 shows simulation results for a potential regime shift on K. Both figures plot the percentage change of steady state stock $(s^* - s^{RS})/s^*\%$ against γ, for η =0, 1, 2, 3 and 4 and g = 0.05, 0.2 and 0.5. Values above 0 indicate aggressive management while values below 0 indicate precautionary management.

Figure 2.1: Change of the steady state resource stock with a potential regime shift on g



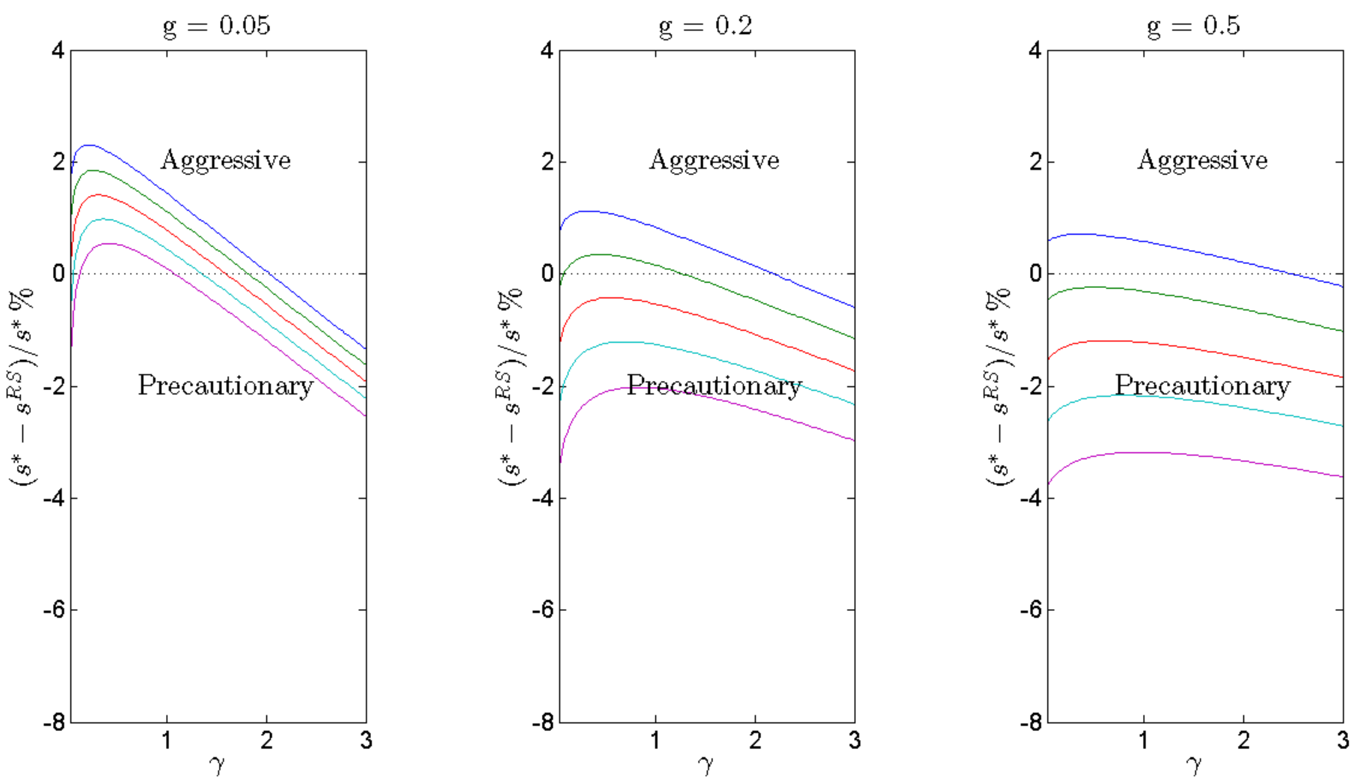
From top to bottom: $\eta = 0, 1, 2, 3, 4$.

The simulations show that higher values of η (greater endogeneity of risk) shift the curve downward, making optimal management more precautionary. As predicted by (2.27) and (2.28), the regime shift on K causes aggressive management more often than regime shift on g (compare Figure 2.2 with Figure 2.1). This result occurs because, *ceteris paribus*, the regime shift on K has a relatively stronger investment effect.

For the regime shift on g, Figure 2.1 shows that optimal management is mostly precautionary. However, when the intrinsic growth rate is very low ($g = 0.05$), aggressive management is optimal with exogenous risk and when γ is lower than approximately 0.4.

For the regime shift on K, Figure 2.2 shows that aggressive management is possible for many reasonable combinations of γ, g and η. For $\gamma < 2$, the simulations show that optimal management is aggressive with exogenous risk and for combinations of low value of g and η. However, when γ is sufficiently high, optimal management is always precautionary.

Figure 2.2: Change of the steady state resource stock with a potential regime shift on K



From top to bottom: $\eta = 0, 1, 2, 3, 4$.

Figure 2.1 and 2.2 also shows that the effect of a potential regime shift can change non-monotonically as γ increases. When γ is very small, the consumption smoothing effect is close to 0. Thus, if the risk reduction effect is stronger than the investment effect, the optimal management is precautionary. As γ increases, both the consumption smoothing effect and the investment effect increase, but the risk reduction effect is independent of γ. Because the investment effect increases relatively faster than the consumption smoothing effect in the beginning, optimal management becomes more aggressive as γ increases. However, the investment effect is bounded above but the consumption smoothing effect increases proportionally in γ. Thus, when γ is sufficiently large, the consumption smoothing effect will become dominant and optimal management will become more precautionary as γ increases.

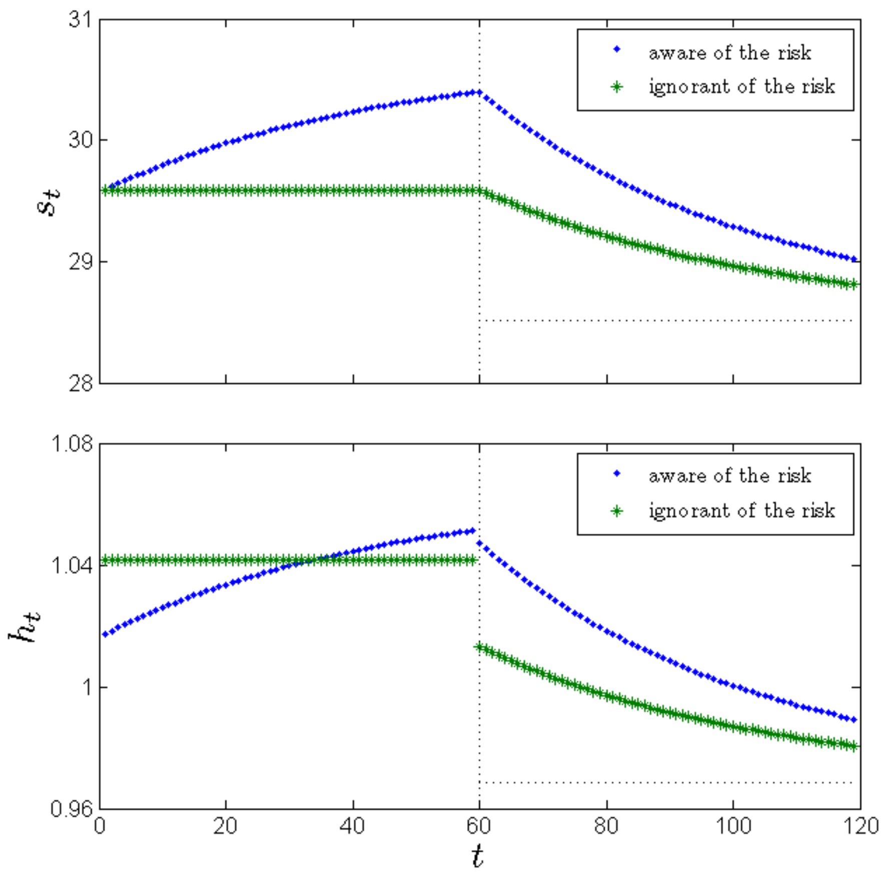
2.4.2 The effect of potential regime shift on transition path

We also simulate the optimal transition path of stock and harvest (s_t, h_t) under the risk of a potential regime shift, and compare it with the optimal path in the risk-free case. We assume that the initial resource stock equals the risk-free steady state s^*. In our simulation the resource manager knows the hazard function, but does not know the exact time of the regime shift. To illustrate the effect of a potential regime shift, we present a transition path for the case where the regime shift occurs at $t = 60$. Because the date of the regime shift is unknown to the resource manager, the optimal transition path before the regime shift is independent of the time of its occurrence.

In the simulation we set $g = 0.05$, $\gamma = 1$ and $\eta = 1$. The values of the other parameters are the same as given previously. With these parameter values, a regime shift on g causes precautionary management and a regime shift on K causes aggressive management.

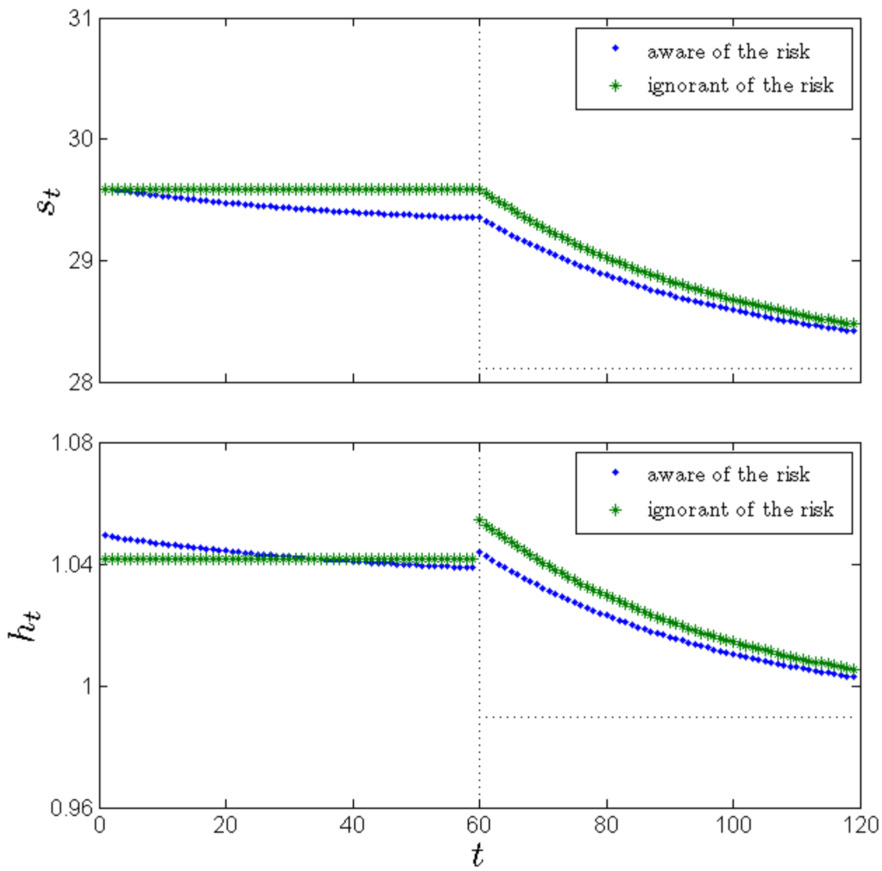
The transition paths on stock and harvest with a potential regime shift on intrinsic growth rate g are plotted in Figure 2.3. If the resource manager is ignorant of the risk and follows the "risk-free" path (asterisks), the resource stock would stay at s^* and harvest would equal $h^* = f(s^*, x) - s^*$ in each period. At $t = 60$, the intrinsic growth rate falls so that the new risk-free steady state resource stock and harvest are lower (illustrated by the horizontal dotted line after $t = 60$). After the regime shift, the optimal management would follow the standard risk-free path that converges to the lower steady state asymptotically. From the transition path of h_t we can see that if the manager is ignorant of the regime shift and follows the "risk-free" path, h_t would jump downward at $t = 60$ in order to get on the path that converges to the lower steady state. In contrast, a resource manager who anticipates a potential regime shift that will result in lower stock growth and lower future harvests has an incentive to build up the resource stock before the regime shift. Precautionary management results in a smoother harvest transition path at $t = 60$, higher harvests and stock levels following the regime shift.

Figure 2.3: Transition paths of (s_t, h_t) under the risk of a potential regime shift on g



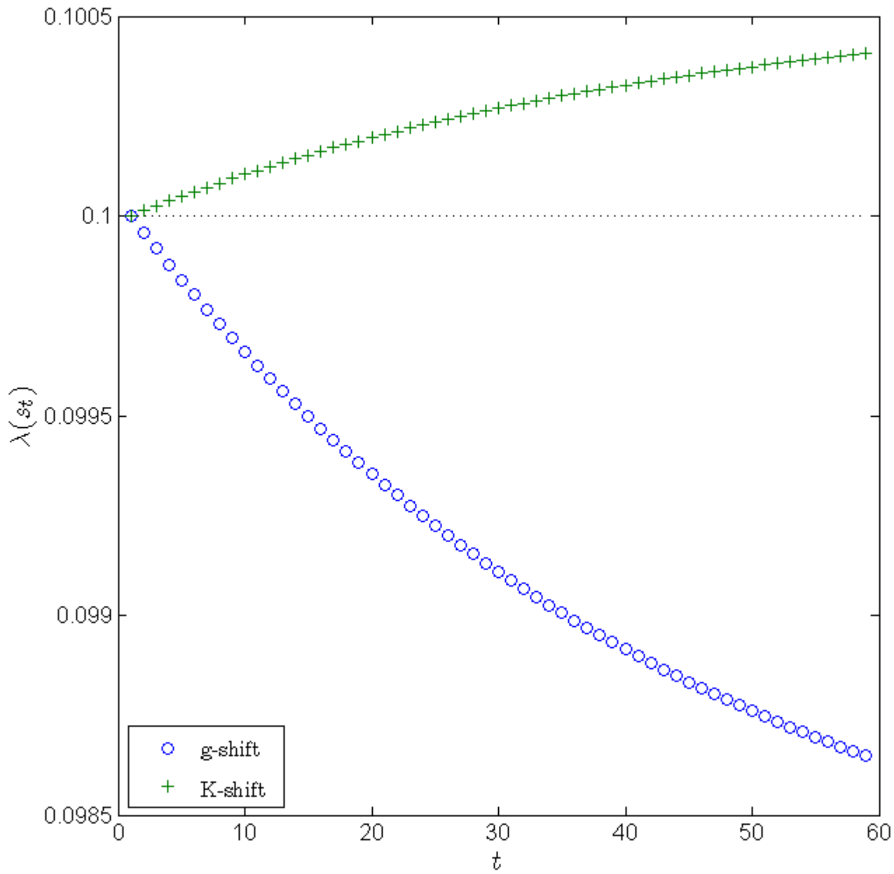
The transition paths on stock and harvest with a potential regime shift on carrying capacity K are plotted in Figure 2.4. The investment effect is relatively more important in the regime shift on K than on g. Because of the decline in the gross return of the stock post regime shift, the optimal management plan following a regime shift is to quickly reduce stock level towards the new lower steady state stock level. There are relatively large losses from having stock levels that differ from the post-regime desired steady state stock. In the case when the resource manager is ignorant of the potential regime shift, the harvest level post-regime shift would actually jump up to a higher level in order to more quickly draw down stock towards the new optimal steady state level. A resource manager that anticipates a potential regime shift will draw down stock prior to a regime shift (aggressive management). Therefore, the gap between the stock size when the regime shift occurs and the desired post-regime shift steady state stock will be smaller. The upward jump in harvest rates will be smaller resulting in a smoother harvest transition path when a regime shift occurs. Were the time of the regime shift known ahead of time, the resource stock would be managed in a way to eliminate the jump in harvest and have a completely smooth transition path. Without prior knowledge of the date when a potential regime shift will occur, aggressive management will reduce but not eliminate the upward jump in harvest when the regime shift occurs.

Figure 2.4: Transition paths of (s_t, h_t) under the risk of a potential regime shift on K



With endogenous risk, changes in stock size will change the probability of regime shift. Figure 2.5 shows the hazard rate of the potential regime shift through time for $t < 60$ as a function of the stock size ($\lambda(s_t)$) for the cases shown in Figures 2.3 and 2.4. If the resource manager is ignorant of the potential for regime shift and follows the "risk-free" path, the hazard rate would be constant at $\lambda(s^*) = \overline{\lambda} = 0.1$. If the regime shift is on the carrying capacity K, the resource manager who is aware of this risk would harvest the resource aggressively, and this leads to higher hazard rate. On the other hand, when the regime shift is on the intrinsic growth rate g, the optimal management is precautionary and this would reduce the the hazard rate along the transition path.

Figure 2.5: Hazard rate of a potential regime shift



2.5 Conclusion

In this paper we showed that the risk of a potential regime shift could cause the optimal management of renewable resources to be precautionary, unchanged, or aggressive as compared to the risk-free case without regime shift. Our results contrast with those of other recent papers on regime shifts in systems dynamics that show that a regime shift will cause management to be more precautionary [15, 42]. Our model includes non-linear benefits functions (varying marginal benefits) and a regime shift on resource growth function, which introduce additional effects into the model. Our model shows that the optimal management depends on the relative magnitudes of three effects: the risk reduction effect, the consumption smoothing effect, and the investment effect. When the risk of a regime shift declines with an increase in resource stock, the resource manager will have an incentive to lower harvest and increase stock thereby reducing the risk of regime shift (risk reduction effect). A regime shift results in lower resource availability and thus reduces harvest and raises the marginal utility. A forward-looking resource manager will take this effect into account by reducing initial harvest thereby saving more stock to smooth harvest rates through time (consumption smoothing effect). However, a regime shift also lowers the gross return rate thus saving the resource stock becomes a poorer investment. A forward looking resource manager will take this into account by increasing initial harvest and saving less stock for the future (the investment effect). Both the risk reduction effect and the consumption smoothing effect cause management to be more precautionary. The investment effect has the opposite impact and causes management to be more aggressive. We characterize conditions under which the investment effect outweighs the other two effects so that a potential regime shift will cause management to be more aggressive overall as compared to the risk-free case.

In a numerical simulation with constant relative risk aversion utility and logistic growth, we showed that a potential regime shift can generate more aggressive management for reasonable parameter values. In particular, we showed that when the regime shift reduces the carrying capacity of the renewable resource then the optimal management tends to be more aggressive with a potential regime shift. In contrast, management tends to be more precautionary with a reduction in the intrinsic growth rate.

While the model incorporates a number of effects, we readily acknowledge that there are additional interesting issues that we have not considered. For example, we considered a regime shift that affects the growth function but not one that directly affects the utility function, or has a simultaneous effect on both. We have relied on a relatively simple hazard function rather than explicitly considering the underlying mechanisms that may govern regime shifts. There may be additional management approaches besides simply managing the level of stock that can affect the likelihood of regime shift, such as reduction of pollution or investments in environmental improvements. In addition, it may be possible to learn about the probability of crossing a threshold or to get early warning signals of impending regime shift [5, 55]. Finally, we assume that there is a single potential regime shift and that this regime shift is irreversible. In reality there may be many potential regimes with the potential to flip back and forth among regimes. Considerations of this sort would complicate the analysis but would not necessarily add insight into optimal management.

2.6 Technical proof

Derivation of Equations (2.5) and (2.6). By (2.2) we can show that

$$\begin{aligned}
d'(s_1^{RS}) &= \lambda'(s_1^{RS})[u(f(s_1^{RS}, x)) - u(f(s_1^{RS}, x-\Delta x))] \\
&\quad + \lambda(s_1^{RS})[u'(f(s_1^{RS}, x))f_1(s_1^{RS}, x) \\
&\quad - u'(f(s_1^{RS}, x-\Delta x))f_1(s_1^{RS}, x-\Delta x)] \\
&= \lambda'(s_1^{RS})[u(f(s_1^{RS}, x)) - u(f(s_1^{RS}, x-\Delta x))] \\
&\quad + \lambda(s_1^{RS})f_1(s_1^{RS}, x)[u'(f(s_1^{RS}, x)) - u'(f(s_1^{RS}, x-\Delta x))] \\
&\quad + \lambda(s_1^{RS})u'(f(s_1^{RS}, x-\Delta x))[f_1(s_1^{RS}, x) - f_1(s_1^{RS}, x-\Delta x)].
\end{aligned}$$

Because $\lim_{\Delta x\to 0} s_1^{RS} = s_1^*$, for the three terms on the right hand side, we have that

$$\begin{aligned}
\lim_{\Delta x\to 0}\frac{u(f(s_1^{RS}, x)) - u(f(s_1^{RS}, x-\Delta x))}{\Delta x} &= u'(f(s_1^*, x))\frac{\partial f(s_1^*, x)}{\partial x} \\
&= u'(h_1^*)\frac{\partial h_1^*}{\partial x}; \\
\lim_{\Delta x\to 0}\frac{u'(f(s_1^{RS}, x)) - u'(f(s_1^{RS}, x-\Delta x))}{\Delta x} &= u''(f(s_1^*, x))\frac{\partial f(s_1^*, x)}{\partial x} \\
&= u''(h_1^*)\frac{\partial h_1^*}{\partial x}; \\
\lim_{\Delta x\to 0}\frac{f_1(s_1^{RS}, x) - f_1(s_1^{RS}, x-\Delta x)}{\Delta x} &= \frac{\partial R^*}{\partial x}.
\end{aligned}$$

Then (2.5) is solved as

$$\lim_{\Delta x\to 0}\frac{d'(s_1^{RS})}{\Delta x} = \lambda'(s_1^*)u'(h_1^*)\frac{\partial h_1^*}{\partial x} + \lambda(s_1^*)R^*u''(h_1^*)\frac{\partial h_1^*}{\partial x} + \lambda(s_1^*)u'(h_1^*)\frac{\partial R^*}{\partial x}.$$

From (2.5) we can rearrange terms to find that

$$\lim_{\Delta x \to 0} \frac{d'(s_1^{RS})}{\Delta x} = \frac{u'(h_1^*)\lambda(s_1^*)R^*}{x} \left(\frac{\lambda'(s_1^*)h_1^*}{\lambda(s_1^*)R^*} \frac{x}{h_1^*} \frac{\partial h_1^*}{\partial x} - \gamma \frac{x}{h_1^*} \frac{\partial h_1^*}{\partial x} + \frac{x}{R^*} \frac{\partial R^*}{\partial x} \right)$$
$$= \frac{u'(h_1^*)\lambda(s_1^*)R^*}{x} (-\theta\xi_h - \gamma\xi_h + \xi_R).$$

Because $u'(h_1^*)\lambda(s_1^*)R^*/x > 0$, aggressive management ($d' > 0$) implies $\gamma\xi_h + \theta\xi_h < \xi_R$. □

Proof of Proposition 1. We first prove the second inequality of this proposition. Define an operator T as

$$(Tw)(s,\, x,\, \Delta x) = \max_{y \in \Gamma(s,\, x)} \{u(f(s,\, x) - y) + \beta(1 - \lambda(y))w(y,\, x,\, \Delta x) + \beta\lambda(y)V(y,\, x - \Delta x)\}. \quad (2.29)$$

As noted in the classical model, V is continuous and strictly increasing in both arguments. Then following standard arguments in literature (Theorem 4.6 of Stokey et al. 60), T has a unique fixed point

$$W(s,\, x,\, \Delta x) = \lim_{n \to \infty} (T^n w)(s,\, x,\, \Delta x) \quad (2.30)$$

and $W(s,\, x,\, \Delta x)$ is continuous, strictly increasing in s and x, and strictly decreasing in Δx, as inherited from u and V. Also by (2.9), we can verify that $(TV)(s,\, x - \Delta x) = V(s,\, x - \Delta x)$ given $\Delta x = 0$. Because the fixed point is unique for any $(s,\, x,\, \Delta x) \in S \times \overline{X}$, we know that $W(s,\, x,\, 0) = V(s,\, x)$. Then $W(s,\, x,\, \Delta x) < V(s,\, x)$ is established because $W(s,\, x,\, \Delta x)$ is continuous and strictly decreasing in Δx.

To prove the first inequality of this proposition, define another operator L as

$$(Lw)(s,\, x,\, \Delta x) = \max_{y \in \Gamma(s,\, x - \Delta x)} \{u(f(s,\, x - \Delta x) - y) + \beta(1 - \lambda(y))w(y,\, x,\, \Delta x) + \beta\lambda(y)V(y,\, x - \Delta x)\}. \quad (2.31)$$

The standard arguments from Stokey et al. [60] ensure that L has a unique fixed point, and according to (2.9) we can verify that $V(s,\, x - \Delta x) = (LV)(s,\, x - \Delta x)$ for any $(s,\, x,\, \Delta x) \in S \times \overline{X}$, or equivalently

$$V(s,\, x - \Delta x) = \lim_{n \to \infty} (L^n w)(s,\, x,\, \Delta x). \quad (2.32)$$

By (2.10) $\Gamma(s, x - \Delta x) \subset \Gamma(s, x)$, then (2.29) and (2.31) imply that for any w it is true that $Tw > Lw$. Thus

$$\lim_{n\to\infty}(T^n w)(s, x, \Delta x) > \lim_{n\to\infty}(L^n w)(s, x, \Delta x)$$

and the desired result follows from (2.30) and (2.32). □

Continuous differentiability of $W(\cdot, x, \Delta x)$. Although the continuity and monotonicity of W are established in the proof of Proposition 1 with standard assumptions, to further establish the concavity and differentiability of $W(\cdot, x, \Delta x)$, we need stronger assumption on the hazard function λ.

Assumption 3c. Given any twice differentiable function w which is weakly concave on S, $|\lambda'(s)| + |\lambda''(s)|$ is bounded above by a function $M_w(s) : S \to \mathbb{R}_+$, such that $w_\lambda(s) = (1-\lambda(s))w(s)+\lambda(s)V(s, x-\Delta x)$ is strictly concave in s for arbitrary $(x, \Delta x) \in \overline{X}$.

The intuition behind Assumption 3c is as the following. Let V' and V'' denote the first and second derivative[9] of V with respect to s. For any $s \in S$, we know $w''_\lambda < 0$ if and only if

$$(1-\lambda)w'' + \lambda V'' < 2\lambda'(w' - V') + \lambda''(w - V).$$

Given that w and V are twice differentiable on compact set S, we know $w - V$ and $w' - V'$ are bounded. Because $w'' \leq 0$, and $V'' < 0$ as noted in the classical model, the last equation holds when M_w is sufficiently small. When w is close to V and w' is close to V', then the function M_w can be large.

With Assumption 3c, we know that the weak concavity of any function $w(\cdot, x, \Delta x)$ on compact set S is inherited by $(Tw)(\cdot, x, \Delta x)$ (defined in the proof of Proposition 1) as strict concavity, thus the fixed point $W(\cdot, x, \Delta x)$ is strictly concave. Then, for any $(x, \Delta x) \in \overline{X}$ consider an arbitrary s_0 in the interior of S, because u satisfies the Inada condition, we know $q(s_0, x, \Delta x) \in int\Gamma(s_0, x)$. Thus according to the envelope theorem of Benveniste and Scheinkman [4] (or Theorem 4.11 of Stokey et al. 60), $W(\cdot, x, \Delta x)$ is continuously differentiable on the interior of S. □

[9] Santos [50, 51] established the C^2 differentiability of the value function under standard assumptions which are satisfied in our model.

Proof of Lemma 3. We first show that the policy function $q(s,\, x,\, \Delta x)$ converges to $g(s,\, x)$ independent of s as Δx converges to 0. Then use this result to prove the convergence of s^{RS} to s^*.

From the proof of Proposition 1 we know $W(s,\, x,\, 0) = V(s,\, x)$, then according to (2.11) and (2.18) $q(s,\, x,\, 0) = g(s,\, x)$. Thus we only need to show $q(s,\, x,\, \Delta x) \to q(s,\, x,\, 0)$ uniformly. Because $V(s,\, x)$ is continuous on $S \times X$ and strictly increasing in x, by Dini's theorem $V(\cdot,\, x - \Delta x) \to V(\cdot,\, x)$ uniformly as $\Delta x \to 0$, thus Proposition 1 ensures $W(\cdot,\, x,\, \Delta x) \to V(\cdot,\, x)$ uniformly. With the uniform convergence of value functions, we can use Theorem 3.8 of Stokey et al. [60] to establish the uniform convergence of policy function and only check the assumptions of the theorem.

Consider a sequence $\{x_n\}$ which converges from below to x. Define functions ω_n and ω on $S \times \Gamma(s,\, x)$ as

$$\begin{aligned}\omega_n(s,\, y) &= u(f(s,\, x) - y) + \beta(1 - \lambda(y))W(y,\, x,\, x - x_n) + \beta\lambda(y)V(y,\, x_n);\\ \omega(s,\, y) &= u(f(s,\, x) - y) + \beta(1 - \lambda(y))W(y,\, x,\, 0) + \beta\lambda(y)V(y,\, x).\end{aligned}$$

We know $\omega_n(s,\, y) \to \omega(s,\, y)$ uniformly because of the uniform convergence of value functions. By definition (2.18)

$$\begin{aligned}q(s,\, x,\, x - x_n) &= \{y \in \Gamma(s,\, x) \,|\, \omega_n(s,\, y) = W(s,\, x,\, x - x_n)\};\\ q(s,\, x,\, 0) &= \{y \in \Gamma(s,\, x) \,|\, \omega(s,\, y) = W(s,\, x,\, 0)\}.\end{aligned}$$

Then by Theorem 3.8 of Stokey et al. [60], $q(s,\, x,\, x - x_n) \to q(s,\, x,\, 0)$ uniformly if $\Gamma(s,\, x)$ is nonempty, compact, convex and continuous at any $(s,\, x) \in S \times X$, and $\omega_n(s,\, y)$ and $\omega(s,\, y)$ are continuous on $S \times \Gamma(s,\, x)$ and strictly concave in y. The assumptions on Γ are satisfied by definition (2.10). The continuity of ω_n and ω is ensured by the continuity of W and V. And Assumption 3c guarantees that ω_n and ω are strictly concave in y. Thus the desired assumptions are satisfied, and the uniform convergence of policy functions is established.

Now we prove the convergence of s^{RS} to s^*. First notice that (2.12) ensures that $g(s,\, x)$ is strictly increasing in s. This implies that the convergence to s^* from any $s_0 \in S$ is monotonic. Then consider arbitrary $\varepsilon > 0$ and $N_\varepsilon(s^*) = (s^* - \varepsilon,\, s^* + \varepsilon)$, for any s^L and s^H in $N_\varepsilon(s^*)$ and $s^L < s^* < s^H$, we have

$$s^L < g(s^L,\, x) < s^* < g(s^H,\, x) < s^H.$$

For all $s \notin N_\varepsilon(s^*)$, because $q(s, x, \Delta x) \to g(s, x)$ uniformly as $\Delta x \to 0$, we can pick some $\delta_1 > 0$ such that for any $|\Delta x| < \delta_1$, $|g(s, x) - q(s, x, \Delta x)| < \min\{|g(s^L, x) - s^L|, |g(s^H, x) - s^H|\}$. Then the following is true

$$
\begin{aligned}
&|q(s, x, \Delta x) - s| \\
\geq &|g(s, x) - s| - |g(s, x) - q(s, x, \Delta x)| \\
\geq &\min\{|g(s^L, x) - s^L|, |g(s^H, x) - s^H|\} - |g(s, x) - q(s, x, \Delta x)| \\
> &0
\end{aligned}
$$

Thus for all $s \notin N_\varepsilon(s^*)$, there exists $\delta_1 > 0$ such that $|\Delta x| < \delta_1$ implies $|q(s, x, \Delta x) - s| > 0$.

On the other hand, we can pick $\delta_2 > 0$ such that for any $|\Delta x| < \delta_2$, the following is true

$$s^L < q(s^L, x, \Delta x) < s^* < q(s^H, x, \Delta x) < s^H.$$

Thus

$$q(s^L, x, \Delta x) - s^L > 0 > q(s^H, x, \Delta x) - s^H.$$

Assumption 3c ensures that $W(\cdot, x, \Delta x)$ is strictly concave. Then applying the theorem of the maximum we know $q(s, x, \Delta x) - s$ is continuous in s. Then there exists some $s^{RS} \in (s^L, s^H) \subset N_\varepsilon(s^*)$ such that $q(s^{RS}, x, \Delta x) - s^{RS} = 0$.

Therefore, given $x \in X$, for any $\varepsilon > 0$, there exists some $\delta = \min\{\delta_1, \delta_2\} > 0$ such that $|\Delta x| < \delta$ implies $s^{RS}(x, \Delta x) \in N_\varepsilon(s^*(x))$ and there is no $s \notin N_\varepsilon(s^*(x))$ satisfies $q(s, x, \Delta x) = s$. This establishes the convergence of $s^{RS}(x, \Delta x)$ to $s^*(x)$. □

Proof of Proposition 4. We want to find the condition under which for any $x \in X$ it is true that $\lim_{\Delta x \to 0}(s^*(x) - s^{RS}(x, \Delta x))/\Delta x > 0$. By Lemma 2, this statement is equivalent to

$$\lim_{\Delta x \to 0} \frac{D_1(s^{RS}(x, \Delta x), x, \Delta x)}{\Delta x} > 0$$

or

$$
\begin{aligned}
\lambda'(s^*) \lim_{\Delta x \to 0} &\frac{W(s^{RS}, x, \Delta x) - V(s^{RS}, x - \Delta x)}{\Delta x} \\
&+ \lambda(s^*) \lim_{\Delta x \to 0} \frac{W_1(s^{RS}, x, \Delta x) - V_1(s^{RS}, x - \Delta x)}{\Delta x} > 0. \quad (2.33)
\end{aligned}
$$

To simplify the notation, we use s^* to denote $s^*(x)$ and s^{RS} to denote $s^{RS}(x,\,\Delta x)$ unless otherwise noted. The rest of this proof is organized in steps.

Step 1. We first consider the first term on the left hand side of (2.33). By (2.19) $W(\cdot,\,x,\,\Delta x)$ evaluated at steady state s^{RS} is

$$W(s^{RS},\,x,\,\Delta x) = \frac{u(f(s^{RS},\,x) - s^{RS}) - \beta D(s^{RS},\,x,\,\Delta x)}{1-\beta}.$$

Let $\Delta s^{RS} = g(s^{RS},\,x-\Delta x) - s^{RS}$, by Lemma 3 we know $\lim_{\Delta x\to 0} \Delta s^{RS} = 0$. Then use condition (2.12) we can show that

$$\begin{aligned}
&V(s^{RS}+\Delta s^{RS},\,x-\Delta x) - V(s^{RS},\,x-\Delta x)\\
=&V_1(s^{RS}+\Delta s^{RS},\,x-\Delta x)\Delta s^{RS} + \mathrm{o}(\Delta s^{RS})\\
=&\frac{u'(f(s^{RS},\,x-\Delta x) - g(s^{RS},\,x-\Delta x))\Delta s^{RS}}{\beta} + \mathrm{o}(\Delta s^{RS}).
\end{aligned}$$

Also by (2.9),

$$\begin{aligned}
&V(s^{RS},\,x-\Delta x)\\
=&\frac{u(f(s^{RS},\,x-\Delta x) - g(s^{RS},\,x-\Delta x))}{1-\beta}\\
&+\frac{\beta[V(s^{RS}+\Delta s^{RS},\,x-\Delta x) - V(s^{RS},\,x-\Delta x)]}{1-\beta}\\
=&\frac{u(f(s^{RS},\,x-\Delta x) - g(s^{RS},\,x-\Delta x))}{1-\beta}\\
&+\frac{u'(f(s^{RS},\,x-\Delta x) - g(s^{RS},\,x-\Delta x))\Delta s^{RS}}{1-\beta} + \mathrm{o}(\Delta s^{RS}).
\end{aligned}$$

Then

$$\begin{aligned}
&(1-\beta)\lim_{\Delta x\to 0}\frac{W(s^{RS},\,x,\,\Delta x) - V(s^{RS},\,x-\Delta x)}{\Delta x}\\
=&\lim_{\Delta x\to 0}\frac{u(f(s^{RS},\,x) - s^{RS}) - u(f(s^{RS},\,x-\Delta x) - g(s^{RS},\,x-\Delta x))}{\Delta x}\\
&-\lim_{\Delta x\to 0}\frac{u'(f(s^{RS},\,x-\Delta x) - g(s^{RS},\,x-\Delta x))\Delta s^{RS}}{\Delta x}\\
&-\beta\lim_{\Delta x\to 0}\frac{D(s^{RS},\,x,\,\Delta x)}{\Delta x}.
\end{aligned}\tag{2.34}$$

We calculate each term on the right hand side of (2.34) separately

$$\lim_{\Delta x \to 0} \frac{u(f(s^{RS},\, x) - s^{RS}) - u(f(s^{RS},\, x - \Delta x) - g(s^{RS},\, x - \Delta x))}{\Delta x} \\ = u'(h^*) \left(f_2(s^*,\, x) + \lim_{\Delta x \to 0} \frac{\Delta s^{RS}}{\Delta x} \right);$$

$$\lim_{\Delta x \to 0} \frac{u'(f(s^{RS},\, x - \Delta x) - g(s^{RS},\, x - \Delta x))\Delta s^{RS}}{\Delta x} = u'(h^*) \lim_{\Delta x \to 0} \frac{\Delta s^{RS}}{\Delta x};$$

$$\lim_{\Delta x \to 0} \frac{D(s^{RS},\, x,\, \Delta x)}{\Delta x} = \lambda(s^*) \lim_{\Delta x \to 0} \frac{W(s^{RS},\, x,\, \Delta x) - V(s^{RS},\, x - \Delta x)}{\Delta x}.$$

Substituting last three equations into (2.34) and rearranging terms

$$\lim_{\Delta x \to 0} \frac{W(s^{RS},\, x,\, \Delta x) - V(s^{RS},\, x - \Delta x)}{\Delta x} = \frac{u'(h^*) f_2(s^*,\, x)}{1 - \beta + \beta\lambda(s^*)}. \tag{2.35}$$

Step 2. Now we consider the second term on the left hand side of (2.33). By (2.22)

$$W_1(s^{RS},\, x,\, \Delta x) = u'(f(s^{RS},\, x) - s^{RS}) f_1(s^{RS},\, x).$$

Also by (2.13)

$$V_1(s^{RS},\, x - \Delta x) = u'(f(s^{RS},\, x - \Delta x) - g(s^{RS},\, x - \Delta x)) f_1(s^{RS},\, x - \Delta x).$$

Then

$$\lim_{\Delta x \to 0} \frac{W_1(s^{RS},\, x,\, \Delta x) - V_1(s^{RS},\, x - \Delta x)}{\Delta x} \\ = u'(h^*) f_{12}(s^*,\, x) + f_1(s^*,\, x) u''(h^*) \left(f_2(s^*,\, x) + \lim_{\Delta x \to 0} \frac{\Delta s^{RS}}{\Delta x} \right). \tag{2.36}$$

Equation (2.15) implies that s^* is continuously differentiable in x, and by Assumption 1b and 1c

$$\frac{\mathrm{d}s^*}{\mathrm{d}x} = -\frac{f_{12}(s^*,\, x)}{f_{11}(s^*,\, x)} > 0. \tag{2.37}$$

Let $\kappa(x - \Delta x)$ be defined with x replaced by $x - \Delta x$. The differentiability of s^* in x and C^2 differentiability of f ensure that $\lim_{\Delta x \to 0} \kappa(x - \Delta x) = \kappa(x) = \kappa$. Consider the risk-free steady state s^* evaluated at $x - \Delta x$ and the risk-free policy function g evaluated at $(s^{RS}(x,\, \Delta x),\, x - \Delta x)$, we have

$$g(s^{RS}(s,\, \Delta x),\, x - \Delta x) - s^*(x - \Delta x) = \kappa(x - \Delta x)(s^{RS}(s,\, \Delta x) - s^*(x - \Delta x)).$$

Then

$$
\begin{aligned}
& \lim_{\Delta x \to 0} \frac{\Delta s^{RS}}{\Delta x} \\
= \; & \lim_{\Delta x \to 0} \frac{g(s^{RS}(s,\, \Delta x),\, x - \Delta x) - s^{RS}(s,\, \Delta x)}{\Delta x} \\
= \; & \lim_{\Delta x \to 0} \frac{(1 - \kappa(x - \Delta x))(s^*(x - \Delta x) - s^{RS}(x,\, \Delta x))}{\Delta x} \\
= \; & (1 - \kappa) \left(\lim_{\Delta x \to 0} \frac{s^*(x) - s^{RS}(x - \Delta x)}{\Delta x} - \lim_{\Delta x \to 0} \frac{s^*(x) - s^*(x - \Delta x)}{\Delta x} \right) \\
= \; & (1 - \kappa) \left(\lim_{\Delta x \to 0} \frac{s^*(x) - s^{RS}(x - \Delta x)}{\Delta x} - \frac{\mathrm{d}s^*}{\mathrm{d}x} \right).
\end{aligned}
$$

Then substituting the last equation into (2.36) and using (2.15), (2.37), $\Omega = h^* f_{11}(s^*,\, x)/\gamma$ and $\gamma = -hu''(h)/h'(h)$, we can show that

$$
\begin{aligned}
& \lim_{\Delta x \to 0} \frac{W_1(s^{RS},\, x,\, \Delta x) - V_1(s^{RS},\, x - \Delta x)}{\Delta x} \\
& \qquad = \frac{u''(h^*)}{\beta} \left(f_2(s^*,\, x) - (1 - \kappa - \beta\Omega) \frac{\mathrm{d}s^*}{\mathrm{d}x} + (1 - \kappa) \lim_{\Delta x \to 0} \frac{s^* - s^{RS}}{\Delta x} \right). \quad (2.38)
\end{aligned}
$$

Step 3. By (2.15) and (2.23)

$$
\lim_{\Delta x \to 0} \frac{f_1(s^*,\, x) - f_1(s^{RS},\, x)}{\Delta x} = -\frac{1}{u'(h^*)} \lim_{\Delta x \to 0} \frac{D_1(s^{RS},\, x,\, \Delta x)}{\Delta x}.
$$

Then

$$
\begin{aligned}
& f_{11}(s^*,\, x) \lim_{\Delta x \to 0} \frac{s^* - s^{RS}}{\Delta x} \\
= \; & -\frac{\lambda'(s^*)}{u'(h^*)} \lim_{\Delta x \to 0} \frac{W(s^{RS},\, x,\, \Delta x) - V(s^{RS},\, x - \Delta x)}{\Delta x} \\
& -\frac{\lambda(s^*)}{u'(h^*)} \lim_{\Delta x \to 0} \frac{W_1(s^{RS},\, x,\, \Delta x) - V_1(s^{RS},\, x - \Delta x)}{\Delta x}.
\end{aligned}
$$

Substituting (2.35) and (2.38) into the last equation we have

$$
-\left(1 - \kappa - \frac{\beta\Omega}{\lambda(s^*)}\right) \lim_{\Delta x \to 0} \frac{s^* - s^{RS}}{\Delta x} = \left(1 + \frac{\Theta}{\gamma}\right) f_2(s^*,\, x) - (1 - \kappa - \beta\Omega) \frac{\mathrm{d}s^*}{\mathrm{d}x}
$$

where

$$
\Theta = -\frac{\lambda'(s^*) h^*}{\lambda(s^*)(R^* - 1 + \lambda(s^*)} \qquad R^* = f_1(s^*,\, x) = \frac{1}{\beta}.
$$

Because $\Omega < 0$ and $0 < \kappa < 1$, then $1 - \kappa - \beta\Omega/\lambda(s^*) > 0$. If a regime shift causes aggressive pre-shift management at the margin, it is true that $\lim_{\Delta x \to 0}(s^* - s^{RS})/\Delta x > 0$, or equivalently

$$\left(1 + \frac{\Theta}{\gamma}\right) f_2(s^*,\, x) < (1 - \kappa - \beta\Omega)\frac{\mathrm{d}s^*}{\mathrm{d}x}.$$

Step 4. Linearizing (2.8) and (2.14) around $(s^*,\, h^*)$, the system dynamics are determined by the eigenvalues and eigenvectors of matrix $[(1/\beta,\, -1),\, (\Omega,\, 1 - \beta\Omega)]$, and the eigenvalues κ satisfies

$$(1/\beta - \kappa)(1 - \beta\Omega - \kappa) + \Omega = 0 \tag{2.39}$$

Then by (2.37), (2.39) and $\Omega = h^* f_{11}(s^*,\, x)/\gamma$, we find

$$(1 - \kappa - \beta\Omega)\frac{\mathrm{d}s^*}{\mathrm{d}x} = \frac{h^* f_{12}(s^*,\, x)}{(R^* - \kappa)\gamma}.$$

Then we know

$$(\gamma + \Theta)\frac{f_2(s^*,\, x)}{h^*} < \frac{R^*}{R^* - \kappa}\frac{f_{12}(s^*,\, x)}{R^*}.$$

Define

$$\begin{aligned}
\xi_h^* &= \frac{\partial h^*}{\partial x}\frac{x}{h^*} = \frac{\partial(f(s^*,\, x) - s^*)}{\partial x}\frac{x}{h^*} = \frac{f_2(s^*,\, x)x}{h^*} \\
\xi_R^* &= \frac{\partial R^*}{\partial x}\frac{x}{h^*} = \frac{\partial f_1(s^*,\, x)}{\partial x}\frac{x}{R^*} = \frac{f_{12}(s^*,\, x)x}{R^*}
\end{aligned}$$

The aggressive management condition follows immediately. Solving (2.39) for κ, the desired form is the solution with absolute value less than 1. $\square$

Chapter 3

The optimal extraction of exhaustible resources with asymmetrically allocated shocks

3.1 Introduction

Exhaustible natural resources can undergo shocks that lead to rapid changes in stocks. For example, discovery of new oil fields increase oil reserves, while massive oil spills (e.g., Kuwait 1991, Gulf of Mexico 2010) reduce oil reserves. The risk of a stock can vary across different pools of a resource stock. When one particular resource stock is under the risk of a potential negative shock, perfect substitutes from alternative risk-free stocks in other locations are usually available. This paper analyze how asymmetric shocks across different pools affect the optimal extraction of an exhaustible resource.

Stochasticity and uncertainty in exhaustible resource models have been broadly discussed in existing literature.[1] Kemp [29], Loury [33] and Heal [24] analyzed the optimal depletion of an exhaustible resource with unknown initial reserve. In their model the chance of exhaustion, an extreme case of negative stock effect that wipes out

[1] Weinstein and Zeckhauser [66] considered the case where future demand is uncertain. Gilbert [20] studied uncertain resource reserve and the learning process that updates the subjective distribution of the reserve. Gaudet and Lasserre [19] analyzed uncertain extraction cost following a Markov process. Uncertainties of demand, price, and reserve characterized by Ito process are analyzed by Pindyck [38] and Pindyck [39].

the entire resource stock and ceases the extraction flow, is always positive along the time path of management. In contrast, Deshmukh and Pliska [16], Arrow and Chang [1], Quyen [43] and Polasky [41] considered the case where costly exploration leads to discrete increments of the exhaustible resource reserves. Dasgupta and Heal [11], Davison [14] and Dasgupta and Stiglitz [12] considered the case where at a stochastic time a new technology is invented. The new technology is independent of the exhaustible resource, thus induces a regime switch after which the essential exhaustible resource becomes inessential in the economy. This causes a sudden decline in the economic value of the resource stock.

As a departure from previous research where the shock affects the entire resource stock symmetrically, we consider asymmetrically allocated shocks, i.e., only a portion of the resource stock is under the threat of a potential shock. In order to focus on the effect of asymmetric allocation of the risk, we assume that a shock only reduces the remaining exhaustible resource stock subject to shock by a given fraction.

A negative stock effect reduces the intertemporal budget constraint a resource manager is subject to, and influences the intertemporal decisions on consumption and saving through an income effect and a substitution effect.[2] These two effects work in opposite directions, thus the optimal behavior could be either more conservative or more aggressive as compared to the risk-free case, depending on the relative magnitudes of these two effects. In our model, the income effect and substitution effect caused by a symmetric shock offset each other perfectly for the case where the preferences have unit elasticity of intertemporal substitution (log utility) and extraction is costless.

Because the extraction cost is zero for all resource stocks, when a resource manager owns risky and risk-free stocks simultaneously, it is always optimal to exhaust the risky stock before extracting the risk-free stock.[3] Under this pattern of optimal management, we show that when utility is log, the asymmetrical allocation of shock induces aggressive management as compared to the risk-free case and to the case where the

[2] Ren and Polasky [48] characterized these effects caused by a sudden shift of the natural growth function in a renewable resource model. Jones and Manuelli [27] also discussed these effects induced by the risk from Ito process in the neoclassic growth models.

[3] When both cost heterogeneity and risk heterogeneity exist, the order of using multiple resource stocks depends on the trade-off between cost minimization and risk aversion, as discussed by Slade [58] and Gaudet and Lasserre [19].

shock is symmetric. Furthermore, the effect of asymmetric shock influences optimal extraction non-monotonically as the share of risky stock varies. In our model a potential shock does not change the optimal extraction rate when the risk is allocated symmetrically and the utility function is in the log form. However, as the share of risky stock decreases from one so the risk becomes asymmetric, the optimal extraction becomes more and more aggressive, and reaches an upper limit at a particular share of the risky stock. Then further decrease of the share of risky stock lowers the optimal extraction. The optimal extraction decreases to the risk-free level, which is identical to the optimal level with symmetric risk, when the share of risky stock reaches zero.

In the next section we explain the basic intuition of the results with a simple two-period model where a shock occurs deterministically in the second period. In Section 3.3 we show that the results in the two-period model extend naturally into the continuous-time infinite horizon model where the occurrence of a shock is stochastic. We solve the optimal extraction rate analytically using dynamic programming methods. The corresponding transition path derived from infinite horizon model is discussed in Section 3.4. Section 3.5 concludes. All technical proofs are in Section 3.6.

3.2 Two-period model with deterministic shock

We start with a two-period model where the shock occurs deterministically in the second period. This two-period model is simple to analyze and illustrates most of the intuition in the infinite horizon where the occurrence of shock is stochastic.

Consider an exhaustible resource stock $k > 0$. Let k^s denote the portion of k which will be affected by the shock in the second period. A resource manager considers how to use k to maximize the present value of utility from consumption over two periods. The extraction in the first period is denoted by c, which is consumed to generate utility according to

$$u(c) = \frac{c^{1-\gamma} - 1}{1 - \gamma} \tag{3.1}$$

where $\gamma > 0$. In the beginning of the second period a shock occurs deterministically. We consider symmetric shock ($k^s = k$) and asymmetric shock ($k^s < k$) separately.

3.2.1 Symmetric shock

A symmetric shock in the second period reduces the remaining resource stock from $k - c$ to $x(k - c)$ where $x \in [0, 1]$. Because the utility function is strictly increasing, the resource manager extracts the leftover $x(k - c)$ completely. Then the resource manager solves the following problem in the first period

$$\max_c \{u(c) + \beta u[x(k - c)]\} \tag{3.2}$$

where $\beta \in (0, 1)$ is the discount factor.

The solution to this problem satisfies the following first order condition

$$c^{-\gamma} = \beta x[x(k - c)]^{-\gamma}, \tag{3.3}$$

and we can solve that optimal first period consumption is

$$c^{SS} = \frac{k}{1 + \beta^{1/\gamma} x^{1/\gamma - 1}} \tag{3.4}$$

where SS denotes "symmetric shock". If there is no shock, or $x = 1$, the first order condition reduces to

$$c^{-\gamma} = \beta(k - c)^{-\gamma}. \tag{3.5}$$

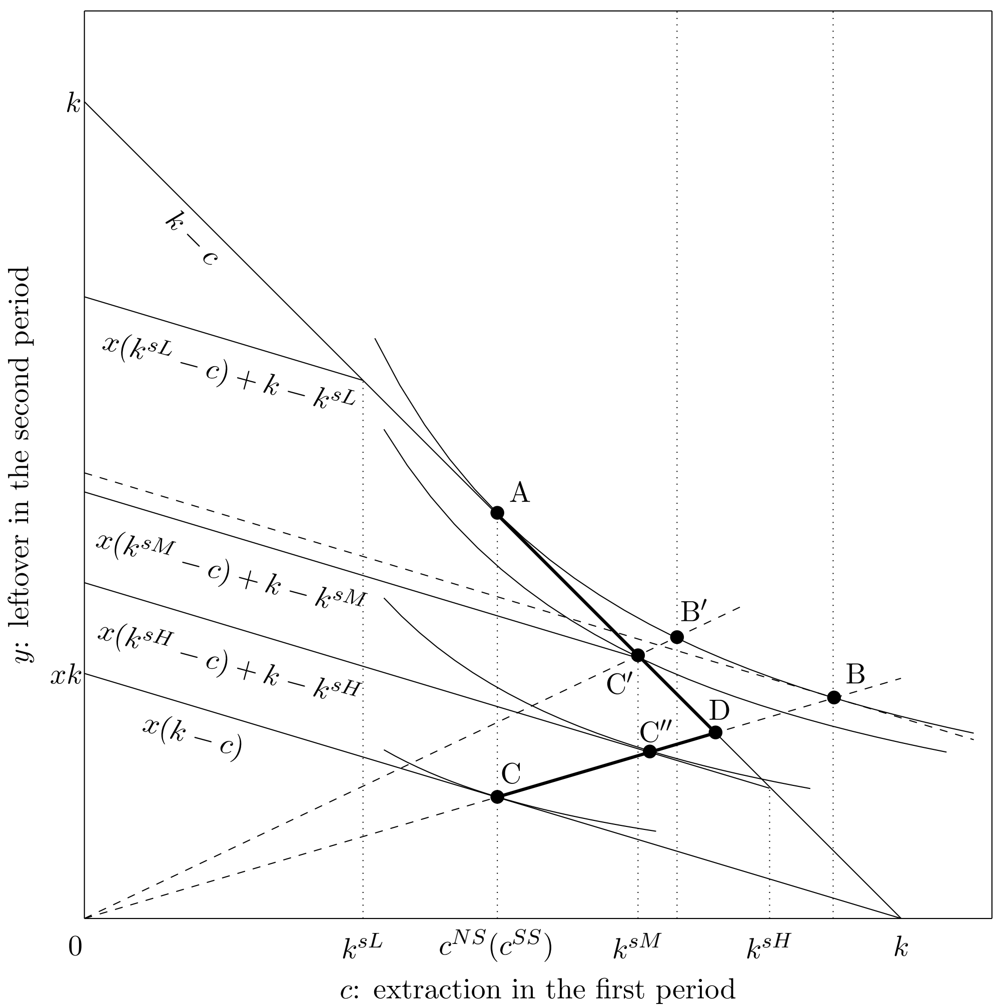
In this case, optimal first period consumption is

$$c^{NS} = \frac{k}{1 + \beta^{1/\gamma}} \tag{3.6}$$

where NS denotes "no shock".

The comparison between c^{SS} and c^{NS} is illustrated in Figure 3.1 with a special case where $\gamma = 1$. As shown in Figure 3.1, the symmetric shock reduces the budget constraint from $y = k - c$ to $y = x(k - c)$, and moves the equilibrium from point A (no shock) to point C (symmetric shock).

Figure 3.1: Equilibria with an asymmetric shock in the two-period model



The horizontal distance from point A to point B is the standard substitution effect. The shock lowers the marginal utility (shadow price) in the second period and encourages the manager to transfer second period extraction to the first period. The horizontal distance from point B to point C is the standard income effect. The shock lowers the manager's available resource stock (overall income) and encourages the manager to reduce the extraction in both periods. This is because that the resource manager with a strictly concave utility function wants to smooth consumption over time.

Substitution effect increases the first period extraction and income effect decreases the first period extraction. By (3.4) and (3.6) we can see that $c^{SS} = c^{NS}$ if and only if $\gamma = 1$, as the case shown in Figure 3.1 where the substitution effect and the income effect fully offset each other. However, when $\gamma \neq 1$, (3.4) and (3.6) imply that $c^{SS} > c^{NS}$ if and only if $x^{1-1/\gamma} > 1$ or $\gamma < 1$. In this case the substitution effect dominates the income effect. When $\gamma > 1$, the comparison of two effects reverses and we know $c^{SS} < c^{NS}$. In sum, a symmetric shock that occurs deterministically in the second period affects optimal extraction in the first period as follows

$$c^{SS} \gtreqless c^{NS} \Leftrightarrow \gamma \lesseqgtr 1. \tag{3.7}$$

This result is familiar in the literature of dynamic models [27, 48]. In the following we will depart from this standard result to see how an asymmetric shock affects the efficient extraction plan differently.

3.2.2 Asymmetric shock

In this case the initial stock k comprises two parts k^s and k^n, and $\tau = k^s/k < 1$. In the beginning of the second period, the shock only reduces the remaining stock of k^s if it is still positive, and has no impact on the remaining stock of k^n. In Section 3.6 we show that the resource manager solves the following problem

$$\max_c \left\{ u(c) + \beta \begin{cases} u[x(k^s - c) + k^n] & c < k^s \\ u(k^s + k^n - c) & c \geq k^s \end{cases} \right\}. \tag{3.8}$$

Intuitively, the resource manager does not extract k^n in the first period as long as k^s is still available. This is because the consumption goods extracted from k^s and k^n are perfect substitutes, but extracting k^s before k^n leads to smaller resource stock that is under threat of shock in the second period. If k^s is not exhausted in the first period ($c < k^s$), the available stock after the second period shock becomes $y = x(k^s - c) + k^n$. However, if k^s is exhausted in the first period ($c \geq k^s$), the shock would not occur in the second period, and the available stock in the second period is simply $y = k^s + k^n - c$. Thus the asymmetric shock reduces the budget constraint only for $c < k^s$. As shown in Figure 3.1, reduced budget constraint is a broken line with a kink at $c = k^s$. Figure 3.1

presents three cases with different sizes of k^s relative to k: high (k^{sH}), medium (k^{sM}) and low (k^{sL}).

For the case where the shock affects a relatively large share of resource stock (k^{sH}), Figure 3.1 shows that the asymmetric regime shift moves the equilibrium from point A to point C″. In this case k^{sH} is larger than the optimal extraction in the first period, which is the interior solution to the first case of (3.8) and the corresponding first order condition is

$$c^{-\gamma} = \beta x[x(k^s - c) + k^n]^{-\gamma}. \tag{3.9}$$

We can solve this equation to find

$$c^{AS} = \left(\frac{1-\tau+\tau x}{x}\right)\frac{k}{1+\beta^{1/\gamma}x^{1/\gamma-1}} = \left(\frac{1-\tau+\tau x}{x}\right)c^{SS} > c^{SS},$$

where AS denotes "asymmetric shock". Note that $c^{AS} < k^s = \tau k$ implies $\tau > 1/(1+\beta^{1/\gamma}x^{1/\gamma})$.

Comparing point C″ with point C in Figure 3.1, it is clear that the asymmetric shock increases first period extraction as compared to the symmetric shock case. To understand this result, first note that the asymmetric shock and the symmetric shock cause the same substitution effect, as measured by the horizontal distance from point A to point B. This occurs because the rate of substitution (slope of indifference curve) is the same at point C″ and point C. However, the asymmetric shock causes a smaller income effect as compared to the symmetric shock. We can see that point B is closer to point C″ than to point C on the same income expansion path. Therefore, when the shock is asymmetric and k^s is not exhausted in the first period, optimal management under the threat of asymmetric shock is more aggressive as compared to the case where the shock is symmetric.

When k^s is as low as k^{sL}, Figure 3.1 shows that asymmetric shock breaks the budget constraint, but does not change the equilibrium point A. This is the second case of the optimization problem (3.8), where the portion of resource stock that would be affected by the shock is exhausted before it occurs. Then according to (3.5) and (3.6) we know that the interior solution of this case satisfies

$$c^{AS} = \frac{k}{1+\beta^{1/\gamma}} = c^{NS},$$

and $c^{AS} \geq k^s = \tau k$ implies $\tau \leq 1/(1+\beta^{1/\gamma})$.

When $1/(1+\beta^{1/\gamma}) < \tau \leq 1/(1+\beta^{1/\gamma}x^{1/\gamma})$, $k^s = \tau k$ is at the medium level k^{sM}. As shown in Figure 3.1, the asymmetric shocks moves the equilibrium to point C′, the kink point on the budget constraint where

$$c^{AS} = k^s = \tau k.$$

This is the solution to the second case of (3.8) where the constraint $c \geq k^s$ is binding. In this case the income effect is the horizontal distance along the income expansion path from point B′ to point C′, and substitution effect is the horizontal distance from point A to point B′. This result occurs because the rate of substitution is constant on the same income expansion path, for indifference curves derived from isoelastic utility.

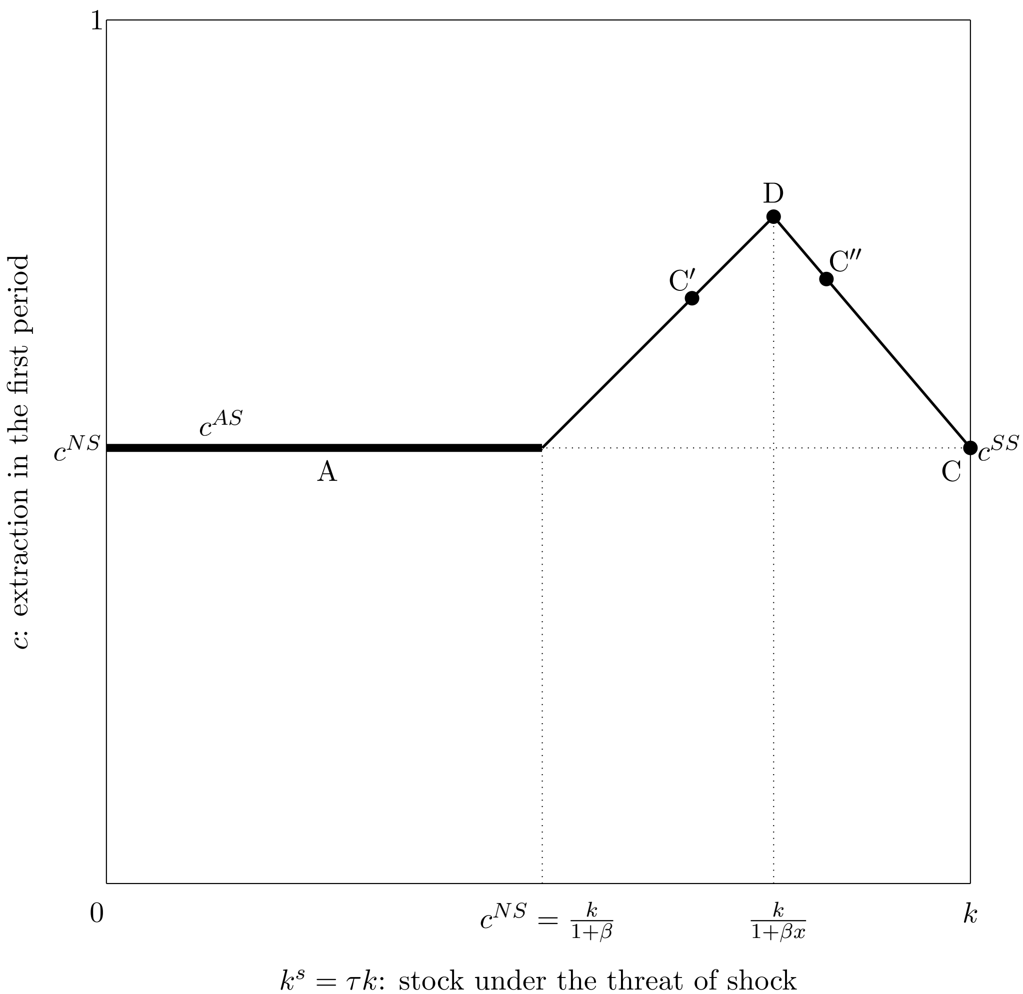
In sum, the solution of (3.8) depends on (k, τ) as the follows

$$c^{AS} = \begin{cases} \dfrac{k}{1+\beta^{1/\gamma}} & \tau \in \left[0, \dfrac{1}{1+\beta^{1/\gamma}}\right) \\ \tau k & \tau \in \left[\dfrac{1}{1+\beta^{1/\gamma}}, \dfrac{1}{1+\beta^{1/\gamma}x^{1/\gamma}}\right) \\ \left(\dfrac{1-\tau+x\tau}{x}\right)\dfrac{k}{1+\beta^{1/\gamma}x^{1/\gamma-1}} & \tau \in \left[\dfrac{1}{1+\beta^{1/\gamma}x^{1/\gamma}}, 1\right] \end{cases}. \tag{3.10}$$

Holding k constant, (3.10) is a continuous and piecewise linear function in τ and plotted in Figure 3.2 for the special case where $\gamma = 1$. This is equivalent to Figure 3.1 where the same result is plotted on the (c, y) plane.

Figure 3.2 shows that when k^s is less than $c^{NS} = k/(1+\beta)$, c^{AS} is constant at c^{NS} as k^s increases, same as equilibrium A in Figure 3.1. When k^s becomes the same as or larger than c^{NS}, c^{AS} starts to increase in k^s. This result occurs because the substitution effect (A-B′) increases faster than the income effect (B′-C′), as in equilibrium C′ in Figure 3.1. In this case the manager wants to exhaust k^s in order to avoid the shock in the second period. When k^s is sufficiently large, exhausting k^s in the first period becomes inefficient and c^{AS} starts to decrease in k^s. This is because the substitution effect is constant at the upper bound (A-B) but the income effect (B-C″) still increases in k^s, as in equilibrium C″ in Figure 3.1.

Figure 3.2: Effect of an asymmetric shock in the two-period model



It is interesting to note in Figure 3.2 that, holding k constant, the first period optimal extraction c^{AS} varies non-monotonically in k^s. This non-monotonicity occurs because that the magnitudes of substitution effect and income effect change differently as k^s increases.

In this section we used a two-period model to analyze how the impact of an asymmetric shock on the optimal management of exhaustible resource is different from that of a symmetric shock. We have seen that the asymmetric shock affects optimal management through affecting the substitution effect and the income effect. The relative magnitude of the two effects changes as the share of resource stock that is under the threat of shock increases, leading to non-monotonic change of the overall effect. In the next section we consider a continuous-time infinite horizon model where the occurrence of shock is stochastic. We will see that the results in the two-period model extends naturally into the infinite horizon case.

3.3 Infinite horizon model with stochastic shock

For a continuous-time dynamic programming framework, the risk-free value of k is captured by a value function V and is characterized by the following problem

$$\rho V(k) = \max_{c}\{u(c) - V'(k)c\}. \tag{3.11}$$

where $\rho = -\log \beta$.

The solution to this problem satisfies

$$c^{NS} = \frac{\rho k}{\gamma}, \tag{3.12}$$

and the corresponding value function is

$$V(k) = \left(\frac{\gamma}{\rho}\right)^{\gamma} \frac{k^{1-\gamma}}{1-\gamma} - \frac{1}{\rho(1-\gamma)}. \tag{3.13}$$

In the following we will analyze how a shock that occurs stochastically changes these results. As in the two-period model, we consider a symmetric shock and a asymmetric shock separately.

3.3.1 Symmetric shock

We assume that at any time $t \geq 0$, there is a potential shock that reduces the entire resource stock symmetrically from k_t to xk_t. Given a constant hazard rate $\lambda \in [0, 1]$, the cumulative probability of the shock occurring at $t \geq 0$ is

$$\Lambda(t) = 1 - e^{-\lambda t}.$$

Once the shock occurs, the dynamic system enters the risk-free phase.

Given initial stock k, a resource manager solves the following problem for an infinitesimally short period Δt

$$W(k) = e^{-\lambda\Delta t}\max_c\{\Delta t u(c) + e^{-\rho\Delta t}W(k-\Delta t c)\} + (1-e^{-\lambda\Delta t})V(xk)$$

where W denotes the expected value of k under the risk of a potential symmetric shock and V is given as (3.13). In the limiting case $\Delta t \to 0$ we have

$$\rho W(k) = \max_c\{u(c) - W'(k)c\} - \lambda[W(k) - V(xk)]. \tag{3.14}$$

Comparing (3.14) with the risk-free problem (3.11), the potential symmetric shock causes a damage $\lambda[W(k) - V(xk)]$ on the value of resource stock k. The following result compares the solution of (3.14), denoted by c^{SS}, and c^{NS} given by (3.12).

Proposition 5. In the continuous-time infinite horizon model, a potential symmetric shock affects the optimal management of an exhaustible resource as following

$$c^{SS} \gtreqless c^{NS} \Leftrightarrow \gamma \lesseqgtr 1.$$

Proof. We assume that $W(k) = Ak^{1-\gamma}/(1-\gamma) + B$. Then the first order condition of (3.14) is

$$c^{-\gamma} = W'(k) = Ak^{-\gamma}. \tag{3.15}$$

This yields

$$c^{SS} = A^{-1/\gamma}k \tag{3.16}$$

and it is clear that $A > 0$. Substituting (3.13), (3.15) and the assumed form of W into (3.14) we find

$$(\rho+\lambda)A\frac{k^{1-\gamma}}{1-\gamma} + (\rho+\lambda)B = \left[\gamma A^{1-\frac{1}{\gamma}} + \lambda x^{1-\gamma}\left(\frac{\gamma}{\rho}\right)^{\gamma}\right]\frac{k^{1-\gamma}}{1-\gamma} - \frac{\rho+\lambda}{\rho(1-\gamma)}.$$

Then $B = -1/[\rho(1-\gamma)]$ and A is determined independently of the state variable k by the following equation

$$\lambda\left[A - x^{1-\gamma}\left(\frac{\gamma}{\rho}\right)^{\gamma}\right] = \rho A^{1-\frac{1}{\gamma}}\left(\frac{\gamma}{\rho} - A^{\frac{1}{\gamma}}\right). \tag{3.17}$$

This implies

$$\left(\frac{\gamma}{\rho}\right)^{\gamma} \gtreqless A \gtreqless x^{1-\gamma}\left(\frac{\gamma}{\rho}\right)^{\gamma}.$$

Because $x < 1$, the following is true

$$A^{-\frac{1}{\gamma}} \gtreqless \frac{\rho}{\gamma} \Leftrightarrow \gamma \lesseqgtr 1.$$

Then the desired result follows by comparing (3.12) and (3.16). $\square$

Proposition 5 shows that the effect of a deterministic symmetric shock in the two-period model, as described by (3.7), extends naturally into the continuous-time infinite horizon model where the symmetric shock occurs stochastically. The risk of potential symmetric shock causes a substitution effect that increases the optimal extraction and an income effect that decreases the optimal extraction. When $\gamma < 1$, the substitution effect dominates the income effect and optimal extraction is higher under the risk of a shock as compared to the risk-free case. When $\gamma = 1$, the substitution and income effects fully offset each other and the risk of potential symmetric shock would not change the optimal extraction. When $\gamma > 1$, the income effect dominates the substitution effect and optimal extraction is lower under the risk of shock as compared to the risk-free case.

3.3.2 Asymmetric shock

Although the risk of potential symmetric shock does not change the optimal management when $\gamma = 1$, this is not true if the shock only affects a portion of the resource stock. Now we consider the case of asymmetric shock where the initial stock k comprises two parts k^s and k^n. At any time $t \geq 0$, a potential shock reduces the remaining stock k^s_t to xk^s_t, $x \in [0, 1]$, with constant hazard rate λ, and has no impact on k^n.

As in the two-period model, the resource manager does not extract k^n as long as k^s is still available. We skip the technical proof and use this result directly to set up the resource manager's problem. Later we will see that the shadow price of the leftover of k^s is always lower than the shadow price of k^n. Thus it is cheaper to use k^s before k^n. Intuitively, the consumption goods extracted from k^s and k^n are perfect substitutes but extracting k^s first leads to smaller resource stock that is under the risk of potential shock.

For an infinitesimally short period Δt the resource manager solves

$$W(k^s, k^n) = e^{-\lambda \Delta t} \max_c \{\Delta t u(c) + e^{-\rho \Delta t} W(k^s - \Delta t c, k^n)\} \\ + (1 - e^{-\lambda \Delta t}) V(xk^s + k^n).$$

Because sufficiently small Δt ensures that $\Delta tc < k^s$ for any $c \in \mathbb{R}_{++}$, we do not need to consider the case in the two-period model where k^s is exhausted. In the limiting case $\Delta x \to 0$ we find

$$\rho W(k^s,\, k^n) = \max_c \{u(c) - W_{k^s}(k^s,\, k^n)c\} - \lambda[W(k^s,\, k^n) - V(xk^s + k^n)].^4 \tag{3.18}$$

Similar to (3.14), the potential shock on k^s causes a damage $\lambda[W(k^s,\, k^n) - V(xk^s + k^n)]$ on the net present value generated from k^s and k^n. Also it is straightforward to see that (3.18) reduces to (3.14) when $k^s = k$ and $k^n = 0$, or equivalently $\tau = k^s/k = 1$.

To solve (3.18) analytically we set $\gamma = 1$, so the utility function is $u(c) = \log c$. In this case, Proposition 5 shows that a potential shock that affects the entire stock k symmetrically would not change the optimal extraction. However, this is not true when the potential shock is asymmetric, as shown by the following result.

Proposition 6. When $\gamma = 1$ the solution of (3.18) satisfies

$$c^{AS} = \frac{1 - (1-x)\left(\dfrac{1-\tau}{x\tau + 1 - \tau}\right)^{1+\lambda/\rho}}{1 - (1-x)\left(\dfrac{1-\tau}{x\tau + 1 - \tau}\right)} \rho k, \tag{3.19}$$

and the corresponding value function is

$$W(k^s,\, k^n) = \frac{1}{\rho + \lambda} \log\left[(xk^s + k^n)^{1+\lambda/\rho} - (1-x)(k^n)^{1+\lambda/\rho}\right] - \frac{\log x}{\rho + \lambda} + \frac{\log \rho - 1}{\rho}. \tag{3.20}$$

Proof. See Section 3.6. □

Using (3.20) it is straightforward to show that $W_{k^s} < W_{k^n}$ as long as $k^s > 0$. Thus the shadow price of k^s is always lower and it is cheaper to use k^s before k^n. This result confirms that the resource manager's problem is in the form of (3.18).

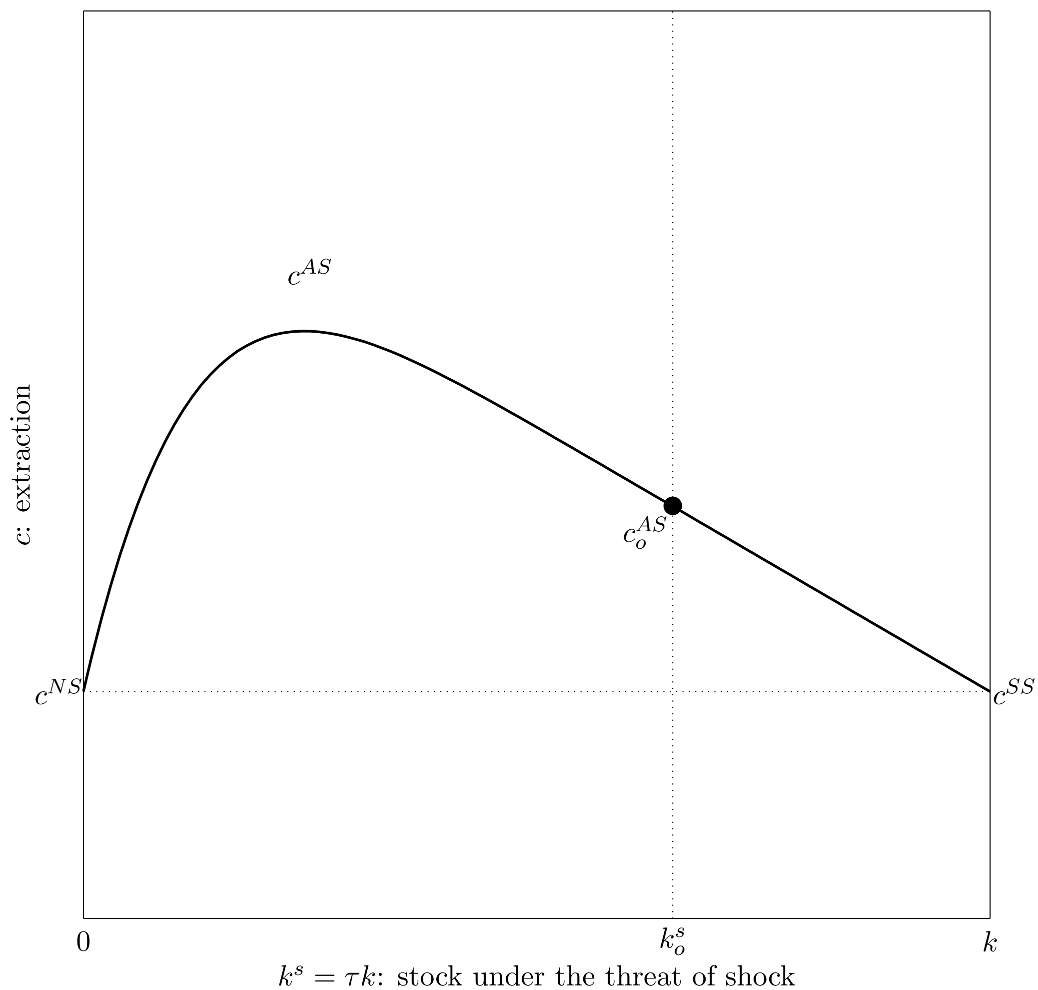
From (3.19) we can see that with a potential asymmetric shock the optimal extraction c^{AR} reduces to the risk-free level $c^{NS} = \rho k$ as (3.12) with $\gamma = 1$ under either of the following three conditions which ensure a risk-free environment: i) zero shock $x = 1$; ii)

[4] Throughout this paper we use subscript to denote the partial derivative to the corresponding argument.

zero hazard rate $\lambda = 0$; or iii) no risky stock $\tau = 0$. It is also interesting to note that when $\tau = 1$, c^{AS} reduces to the optimal extraction with symmetric risk $c^{SS} = \rho k$, which by Proposition 5 is identical to the risk-free level c^{NS} when $\gamma = 1$.

Figure 3.3 plots c^{AS} given by (3.19) holding k constant. Similar to the two-period model, the change of c^{AS} is non-monotonic as the risky stock k^s increases from 0 to k. When the share of k^s is relatively small, the threat of the asymmetric shock encourages the resource manager to increase extraction to draw down k^s in order to avoid the damage when the shock occurs, and an increase of the share of k^s leads to higher c^{AS}. This result occurs because the asymmetric shock causes a substitution effect that increases extraction and an income effect that decreases extraction, and as the share of k^s increases, the substitution effect grows faster than the income effect. However, when the share of k^s is relatively large, rapidly depleting k^s becomes inefficient as this reduces future stock availability and leads to very high marginal utility. It is then optimal to reduce extraction to prepare for the future when the shock potentially causes large reduction of the resource stock. In this case the optimal extraction c^{AS} decreases as the share of k^s increases. This result occurs because the income effect grows faster than the substitution effect.

Figure 3.3: Effect of an asymmetric shock in the infinite horizon model



3.4 Transition path

The transition path with initial condition $K_0 = (k,\, k^s)$ is given by

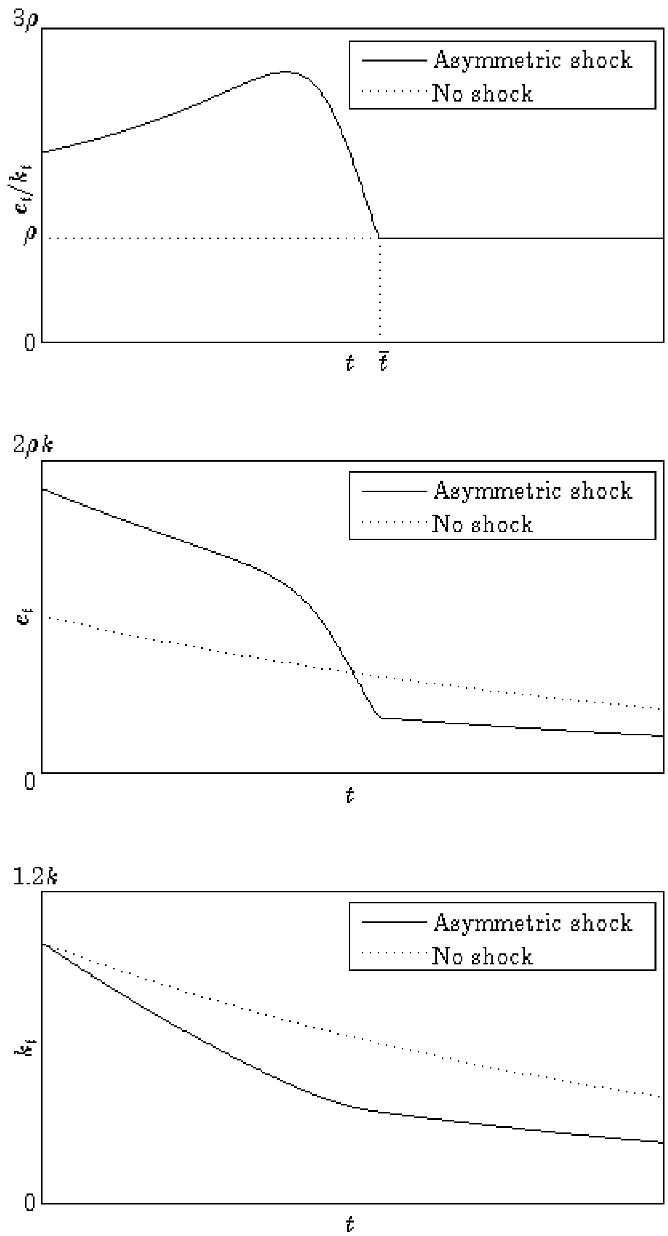
$$\begin{aligned}(\mathbb{C},\, \mathbb{K}) =& (c_t,\, K_t)_{t\geq 0} \\ & c_t \text{ is determined by (3.19)} \\ & \mathrm{d}K_t = \begin{pmatrix} \mathrm{d}k_t \\ \mathrm{d}k_t^s \end{pmatrix} = -\begin{pmatrix} c_t \\ c_t \times \mathrm{I}\{k_t^s > 0\} \end{pmatrix},\end{aligned} \tag{3.21}$$

where I is an indicator function equal to 1 if the condition in the brackets is true and 0 otherwise.

Figure 3.4 compares the transition path (3.21) with the standard risk-free transition path. In the risk-free case, $c_t/k_t = \rho$ is constant over time, as shown by the dashed line in the top chart. The standard risk-free transition paths of c_t and k_t are also plotted as dashed lines in the middle and bottom charts where both c_t and k_t decrease at constant rate ρ and converge to 0 asymptotically.

With an asymmetric shock that potentially affects $k^s = k_o^s < k$ as shown in Figure 3.3, c_t/k_t is not constant over time. This result occurs because the manager only uses k^s and does not extract k^n as long as k^s is available, leading to decreasing $\tau_t = k_t^s/k_t$ overtime. This management strategy, as reflected by Figure 3.3, would cause c_t/k_t to increase initially and then decrease to ρ when k^s is exhausted. Because c_t/k_t is not constant under the risk of an asymmetric shock, the rate of decrease of c_t and k_t also vary along the transition path. The solid lines in Figure 3.4 show these results.

Figure 3.4: Effect of an asymmetric shock on the optimal transition path



The top panel in Figure 3.4 plots the transition path of c_t/k_t starting from the initial condition (k, k_o^s). We can see that c_t/k_t first increases then decreases to the risk-free level ρ at time $\bar{t}$ when k^s is exhausted. After that, the transition path follows the risk-free constant rate ρ. The transition path of c_t is shown in the middle panel. Figure 3.3 shows that the asymmetric risk increases the initial extraction to c_o^{AS} to be higher than the risk-free level $c^{NS} = \rho k$. From Figure 3.4 we see that under the asymmetric risk c_t is higher than the risk-free level over an extended period. When c_t/k_t starts to decrease, c_t itself falls rapidly. Having higher c_t in the initial stage of the transition path leads to faster decline of k_t compared to the risk-free case, as shown in the bottom panel of Figure 3.4. When k_o^s is exhausted, the transition path of c_t and k_t converge to the standard risk-free path. Because k_t is more aggressively extracted in the risky phase, c_t and k_c are persistently lower in the risk-free phase as compared to the case where the asymmetric risk does not exist initially.

3.5 Conclusion

In this paper we analyzed the optimal management of an exhaustible resource under the risk of an asymmetric shock, and compared the results with the risk-free case and the case where the shock is symmetric. We first showed that the effect of a symmetric shock depends on the value of relative risk aversion (γ) of the utility function. The risk of shock causes a substitution effect that increases the optimal extraction and an income effect that decreases the optimal extraction. If the shock is symmetric, the substitution effect is stronger (weaker) than the income effect when $\gamma < (>)1$, leading to more (less) aggressive management of the exhaustible resource as compared to the risk-free case. When $\gamma = 1$, the risk of symmetric shock does not change the optimal extraction from the risk-free level.

However, this is not true if the shock is asymmetric and only affects a portion of the entire exhaustible resource stock. When the shock is asymmetric, we show that optimal management requires a sequential extraction plan that first exhausts the stock under the threat of shock. We used a two-period deterministic model with a shock occurring in the second period to show how the optimal extraction differs with an asymmetric shock compared to a symmetric shock. We also found that the optimal extraction changes non-monotonically in the share of the stock that is under the threat of the shock. This is because the relative magnitude of the substitution effect and the income effect changes with the share of stock exposed to risk.

We extended the two-period deterministic model into an infinite horizon stochastic model where the occurrence of the shock is determined by a constant hazard rate. We showed that the non-monotonic change of optimal extraction in the share of stock exposed to risk extends naturally into the infinite horizon model, and used this result to discuss the corresponding transition path. We showed that with a potential asymmetric shock, the optimal extraction is higher than the risk-free case initially, leading to faster decline of the resource stock. However, when the risky stock is exhausted and transition path enters the risk-free phase, the resource stock and optimal extraction are lower than the case where the risk of asymmetric shock does not exist in the beginning.

We acknowledge that the models in this paper have very simple structure and neglect several interesting real world complications. Firstly, we ignored growth of natural resources. Our benchmark result on the symmetric shock is a simplified version of Ren and Polasky [48] who studied the optimal management of a renewable resource. Future research could include generalizing our result on the asymmetric shock to study its effect on the renewable resource management. In addition, we simplified the shock to be a single-time proportional stock effect. There are more complicated risk structures including time-varying risks and multiple potential shocks. Finally, we focused on the effect of potential shock on optimal management rather than the equilibrium in decentralized market. It would be especially interesting to see if and how the existence of symmetric and asymmetric shocks would change market equilibrium extraction.

3.6 Technical proof

Derivation of maximization problem (3.8). Suppose the resource manager extracts c^s from k^s and c^n from k^n. The the maximization problem is

$$\max_{c^s, c^n}\{u(c^s + c^n) + \beta u[x(k^s - c^s) + (k^n - c^n)]\} \tag{3.22}$$

subject to $0 \leq c^s \leq k^s$ and $0 \leq c^n \leq k^n$. The optimal solution must satisfy following first order conditions

$$u'(c^s + c^n) + p^s = \beta x u'[x(k^s - c^s) + (k^n - c^n)] + q^s \tag{3.23}$$

$$u'(c^s + c^n) + p^n = \beta u'[x(k^s - c^s) + (k^n - c^n)] + q^n \tag{3.24}$$

and the following feasibility and complementary slackness conditions

$$c^s \geq 0 \qquad c^s p^s = 0 \qquad p^s \geq 0 \tag{3.25}$$

$$c^n \geq 0 \qquad c^n p^n = 0 \qquad p^n \geq 0 \tag{3.26}$$

$$c^s \leq k^s \qquad (k^s - c^s) q^s = 0 \qquad q^s \geq 0 \tag{3.27}$$

$$c^n \leq k^n \qquad (k^n - c^n) q^n = 0 \qquad q^n \geq 0. \tag{3.28}$$

Notice that (3.23) and (3.24) implies that

$$p^s - q^s < p^n - q^n. \tag{3.29}$$

We consider three cases: $c^s = 0$, $0 < c^s < k^s$ and $c^s = k^s$.

If $c^s = 0$, (3.27) implies $q^s = 0$. According to (3.29) we know $p^n > q^n \geq 0$. Then (3.26) implies $c^n = 0$ and (3.28) implies $q^n = 0$. This is impossible because both (3.23) and (3.24) are violated when $c^s = 0$ and $c^n = 0$. So we must have $c^s > 0$.

If $0 < c^s < k^s$, (3.25) implies $p^s = 0$ and (3.27) implies $q^s = 0$. According to (3.29) we know $p^n > q^n \geq 0$. Then (3.26) implies $c^n = 0$.

If $c^s = k^s$, (3.25) implies $p^s = 0$. Notice that in this case it is impossible that $c^n = k^n$. Otherwise (3.26) implies $p^n = 0$ and both (3.23) and (3.24) are violated. Then it is must true that $c^n < k^n$ and (3.28) implies $q^n = 0$. Thus $p^n > -q^s$ by (3.29) and in this case (3.26) implies both $c^n = 0$ and $c^n > 0$ are possible.

In sum, define $c = c^s + c^n$, the first case implies $c > 0$; the second case implies k^n is not extracted ($c^n = 0$) if $c < k^s$; and the third case implies k^n is possible to be extracted ($c^n \geq 0$) if k^s is exhausted ($c^s = k^s$). Applying these results to (3.22) we find that the resource manager's problem is equivalent to (3.8). □

Proof of Proposition 6. Given the extraction rate c, if the shock does not occur in an infinitesimally short period Δt, the total resource stock becomes $k - \Delta tc$ and the share of risky stock becomes $(k^s - \Delta tc)/(k - \Delta tc) = (\tau k - \Delta tc)/(k - \Delta tc)$. However, if the shock occurs before Δt, the total stock is $xk^s + k^n = x\tau k + (1-\tau)k$. Then the resource manager solves

$$W(k,\, \tau) = e^{-\lambda\Delta t} \max_c \left\{ \Delta t u(c) + e^{-\rho\Delta t} W\left(k - \Delta tc,\, \frac{\tau k - \Delta tc}{k - \Delta tc}\right)\right\} + (1 - e^{-\lambda\Delta t})V[(x\tau + 1 - \tau)k].$$

In the limiting case $\Delta t \to 0$ we have

$$(\rho + \lambda)W(k,\, \tau) = \max_c \left\{ u(c) - \left[W_k(k,\, \tau) + \frac{(1-\tau)}{k} W_\tau(k,\, \tau)\right] c \right\} + \lambda V[(x\tau + 1 - \tau)k]. \quad (3.30)$$

This is equivalent to resource manager's problem (3.18).

We use the log utility function, and make a guess that

$$W(k,\, \tau) = A\log[g(\tau)k] + B. \quad (3.31)$$

Then the first order condition is

$$\frac{1}{c^{AS}} = \frac{A}{k}\left[1 + (1-\tau)\frac{g'(\tau)}{g(\tau)}\right]. \quad (3.32)$$

Substituting into (3.30) we can solve that

$$(\rho+\lambda)A\log k + (\rho+\lambda)A\log g(\tau) + (\rho+\lambda)B = \log k - \log A - \log\left[1 + (1-\tau)\frac{g'(\tau)}{g(\tau)}\right] - 1 + \frac{\lambda}{\rho}\log k + \frac{\lambda}{\rho}\log(x\tau + 1 - \tau) + \frac{\lambda(\log\rho - 1)}{\rho}.$$

The right hand side uses the fact that when $\gamma = 1$ the risk-free value function $V(k)$ in the form of (3.13) reduces to

$$V(k) = \frac{1}{\rho}\log k + \frac{\log\rho - 1}{\rho}.$$

Then the method of undetermined coefficient implies that $A = 1/\rho$, $B = (\log\rho - 1)/\rho$ and

$$\frac{\rho+\lambda}{\rho}\log g(\tau) = -\log\left[1 + (1-\tau)\frac{g'(\tau)}{g(\tau)}\right] + \frac{\lambda}{\rho}\log(x\tau + 1 - \tau).$$

This yields the following differential equation

$$g(\tau)^{1+\lambda/\rho} + (1-\tau)g'(\tau)g(\tau)^{\lambda/\rho} = (x\tau + 1 - \tau)^{\lambda/\rho}.$$

Standard solution methods yield

$$g(\tau) = \left[\frac{1}{x}(x\tau + 1 - \tau)^{1+\lambda/\rho} + \overline{C}(1-\tau)^{1+\lambda/\rho}\right]^{\rho/(\rho+\lambda)}$$

where $\overline{C}$ is a constant. We set $g(0) = 1$ so $W(k, 0) = V(k)$. In other words, the value function is in the risk-free form when the share of risky stock is 0. Then $\overline{C} = 1 - 1/x$ and

$$g(\tau) = \left[\frac{(x\tau + 1 - \tau)^{1+\lambda/\rho} - (1-x)(1-\tau)^{1+\lambda/\rho}}{x}\right]^{\rho/(\rho+\lambda)} \tag{3.33}$$

Then Equation (3.19) follows from substituting (3.33) into (3.32) and Equation (3.20) follows from substituting (3.33) into (3.31). $\square$

Chapter 4

Cournot competition in an exhaustible resource market with potential shocks to resource stocks

4.1 Introduction

In imperfectly competitive exhaustible resource markets, competitors may bear heterogeneous risks. Environmental conditions vary by location subjecting different resource stocks to different risks. For example, oil extraction could be affected by very different environmental disturbances in the Gulf of Mexico, Eastern Siberia or the Middle East. Resource markets are also affected by social and political conditions that are different across resource extraction countries. Extractors in countries where expropriation of private stocks is imminent [26, 32, 34] face additional risk as compared to extractors in countries where private property rights are well established and protected. Furthermore, technological innovation could also have unbalanced effect on the economic values of *in situ* stocks between the innovating country and countries without the new technology [11, 12, 14]. This paper considers a simple version of risk heterogeneity. The risk in our model is from a potential shock that causes a reduction in an exhaustible resource stock. We introduce heterogeneity by assuming that this shock could affect different competitors' stocks asymmetrically.

Following Weinstein and Zeckhauser [66], Stiglitz [59], Dasgupta and Heal [13] and Eswaran and Lewis [17], we study a Cournot duopoly model where each Cournot player owns a private resource stock.[1] Similar to Stiglitz [59], Cournot competition in our model does not distort the Pareto optimal level of extraction in the risk-free environment when demand is derived from isoelastic preferences and extraction cost is zero. Cournot players, unlike price takers, have some market power and make their extraction decisions based on quantity dependent marginal revenue instead of the exogenously given price. With isoelastic demand and zero extraction cost, marginal revenue is proportional to price and the proportion only depends on the constant demand elasticity. Therefore, in dynamic equilibrium the distortion effect of market power cancels out and Cournot equilibrium coincides with the Pareto optimal outcome.

When the risk of shock is introduced, our model shows that the coincidence between Cournot equilibrium and Pareto optimal solution still holds if both Cournot players' stocks are all affected by the shock, i.e., symmetric shock in our model. However, when the risk of shock is asymmetrically allocated on the stocks of the two Cournot players, aggregate extraction in equilibrium will generally differ from the Pareto optimal outcome. We find that strategic behaviors could increase or decrease the extraction in Cournot equilibrium as compared to the Pareto optimal level, depending on the relative size of the two players' stocks and the share of resource stock that is under the risk of shock.

With an asymmetric shock, Cournot competition can cause distortions that leads to persistent impacts on the transition path of extraction and remaining stock with resulting losses in social welfare. In the risk-free case and the case with symmetric shock, welfare loss is zero because the Pareto optimal extraction is achieved in Cournot equilibrium. However, when the potential shock only affects a portion of one player's stock and has no effect on the stock of the other player, welfare losses increase as the share of the stock subject to risk increases. In our model the risky stock is extracted first[2] and exhausted in limited time. Because the first player takes a longer time to

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[1] Thus our model is different from the common property games such as Khalatbari [30], Levhari and Mirman [31], Sinn [57], Reinganum and Stokey [46], Van der Ploeg [62], and others.

[2] This result occurs because only risk heterogeneity exists in our model. Slade [58] and Gaudet and

exhaust risky stock when its share is larger, there is a larger effect on the transition path which leads to higher welfare losses. The largest welfare loss occurs in the completely asymmetric risk case where the shock affects the entire stock of the first player and has no impact on the second player's stock. This result occurs because the risky stock will never be exhausted by the first player and thus the whole transition path is under the risk of asymmetric shock with consequent distortions. When the risk of shock is further spread out to affect a portion of the second player's stock (and the first player's entire stock), welfare loss starts to decrease as the share of risky stock of the second player increases. This result occurs because in our example the distortion on extraction is very small when the second player has a large share of risky stock. Thus the distortion on transition path is small over prolonged time until the second player significantly reduces the share of risky stock when the distortion on extraction becomes large. This leads to lower welfare loss because the larger distortion in the further future is heavily discounted. When the second player initially has a relatively small share of risky stock, the distortion is much larger for the entire transition path, leading to larger welfare losses. In the extreme case where both players' stocks are completely affected by the risk, welfare loss reduces to zero as is the case with a symmetric shock.

We analyze the Cournot model using a continuous time dynamic programming framework. Thus this paper is among the literature of exhaustible resource oligopolistic models with time consistent subgame perfect equilibria.[3]

The rest of the paper is organized as follows. In the next section we build intuition using a simple two-period model with deterministic shock that occurs in the second period. In Section 4.3 we extend the model into an infinite horizon continuous time stochastic model. We introduce stochasticity by assuming that the shock occurs potentially with constant hazard rate after the the initial time. We will see that results in the deterministic two-period model extend naturally into the infinite horizon stochastic model. We discuss the long-term impact of the risk of shock on transition path and social welfare in Section 4.4. Section 4.5 concludes. All technical proofs are in Section 4.6.

Lasserre [19] showed that with both risk heterogeneity and cost heterogeneity, the order of extracting multiple resource stocks depends on the trade-off between risk aversion and cost minimization.

[3] More discussion of the methodologies and equilibrium concepts in resources models can be found in Eswaran and Lewis [17], Van Long et al. [63] and Gaudet [18] .

4.2 Two-period model with deterministic shock

We now consider a two-period Cournot model with two players. Suppose a representative consumer has the following utility function

$$u(c) = \frac{c^{1-\gamma} - 1}{1-\gamma}. \tag{4.1}$$

Given this utility function, the demand function has the following form

$$p = c^{-\gamma} = (c_1 + c_2)^{-\gamma},$$

where p is the price, c is the aggregate quantity and c_1 and c_2 are individual extraction strategies of the two Cournot players. We assume that $\gamma \in (0, 1]$. In other words, the demand for exhaustible resource has at least unitary elasticity ($1/\gamma \geq 1$).[4]

We consider the case where each player $i = 1, 2$ owns a private exhaustible resource stock k_i and pays no cost in extracting it. Player i maximizes total revenue over two periods by choosing the amount of extraction c_i in the first period, taking the strategy of the other player $j \neq i$ (c_j) as given. In the second period, a shock would occur deterministically, thus affects each player's equilibrium strategy in the first period. In the following we discuss the case with a symmetric shock and an asymmetric shock separately.

4.2.1 Symmetric shock

With a symmetric shock, we assume that in the second period the shock reduces the leftover of each player i's stock $k_i - c_i$ to $x(k_i - c_i)$ where $x \in [0, 1]$. Then player i who takes c_j as given solves the following problem

$$\max_{c_i} \left\{ \frac{c_i}{(c_i + c_j)^\gamma} + \beta \frac{x(k_i - c_i)}{[x(k_i - c_i) + x(k_j - c_j)]^\gamma} \right\}$$

where $\beta \in (0, 1)$ is the discount factor. Under the assumption that $\gamma \leq 1$, both terms in the object function are strictly concave in the choice variable c_i.

[4] The elasticity of long-run demand for exhaustible resource such as oil could be high as with technological that makes the exhaustible resource become more economically available. See Hamilton [23] for more discussion.

The first order conditions of two players are

$$\text{Player 1} \qquad \frac{(1-\gamma)c_1 + c_2}{(c_1+c_2)^{\gamma+1}} = \beta x^{1-\gamma}\frac{(1-\gamma)(k_1-c_1)+k_2-c_2}{(k_1-c_1+k_2-c_2)^{\gamma+1}} \tag{4.2}$$

$$\text{Player 2} \qquad \frac{c_1 + (1-\gamma)c_2}{(c_1+c_2)^{\gamma+1}} = \beta x^{1-\gamma}\frac{k_1-c_1+(1-\gamma)(k_2-c_2)}{(k_1-c_1+k_2-c_2)^{\gamma+1}}. \tag{4.3}$$

Adding (4.2) and (4.3) we find

$$(2-\gamma)(c_1+c_2)^{-\gamma} = (2-\gamma)\beta x^{1-\gamma}(k_1-c_1+k_2-c_2)^{-\gamma}. \tag{4.4}$$

Because (4.2) and (4.3) imply that $c_1/k_1 = c_2/k_2$, for each player i the following is true

$$(2-\gamma)c_i^{-\gamma} = (2-\gamma)\beta x^{1-\gamma}(k_i-c_i)^{-\gamma}. \tag{4.5}$$

Thus, although each player i's strategy is the best response to the competitor's extraction behavior, it is independent of the competitor's resource stock.

Equilibrium extraction

From (4.5) we can solve player i's equilibrium extraction

$$\hat{c}_i^{SS} = \frac{k_i}{1+\beta^{1/\gamma}x^{1/\gamma-1}} \tag{4.6}$$

where "SS" denotes "symmetric shock" and the hat denotes Cournot equilibrium. Adding across $i = 1, 2$ we find the aggregate extraction in Cournot equilibrium is

$$\hat{c}^{SS} = \frac{k}{1+\beta^{1/\gamma}x^{1/\gamma-1}} \tag{4.7}$$

where $\hat{c}^{SS} = \hat{c}_1^{SS} + \hat{c}_2^{SS}$ and $k = k_1 + k_2$. If there is no shock, or $x = 1$, (4.6) and (4.7) become

$$\hat{c}_i^{NS} = \frac{k_i}{1+\beta^{1/\gamma}} \tag{4.8}$$

and

$$\hat{c}^{NS} = \frac{k}{1+\beta^{1/\gamma}} \tag{4.9}$$

where "NS" denotes "no shock", and similar as before, $\hat{c}^{NS} = \hat{c}_1^{NS} + \hat{c}_2^{NS}$.

Comparing (4.6) and (4.8) we find

$$\hat{c}_i^{SS} \geq \hat{c}_i^{NS} \Leftrightarrow \gamma \leq 1 \quad \text{for} \quad i = 1, 2, \tag{4.10}$$

and by (4.7) and (4.9)

$$\hat{c}^{SS} \geq \hat{c}^{NS} \Leftrightarrow \gamma \leq 1. \tag{4.11}$$

The second period shock has competing effects through quantity and price. It reduces the second period sales of each player, but also increases the market price due to lower aggregate supply. According to (4.10) and (4.11), when $\gamma < 1$, the threat of symmetric shock encourages both players to increase the extraction in the first period, leading to higher aggregate extraction. This result occurs because, by (4.2) and (4.3), the symmetric shock lowers the marginal revenue of both player in the second period when $\gamma < 1$. In this case, sales decrease more than price rises in the second period. Thus each Cournot player can increase total revenue by shifting extraction from the second period to the first period. However, when $\gamma = 1$, the quantity effect and price effect fully offset each other and symmetric shock has no impact on equilibrium extraction at either individual level or aggregate level.

Social welfare

Given aggregate resource stock $k = k_1 + k_2$, the social welfare is measured by the net present value of representative consumer's utility over two periods and maximized as the following

$$\max_{c}\{u(c) + \beta u[x(k-c)]\}. \tag{4.12}$$

The first order condition to this problem is

$$c^{-\gamma} = \beta x^{1-\gamma}(k-c)^{-\gamma}. \tag{4.13}$$

This yields the Pareto optimal extraction

$$c^{SS} = \frac{k}{1 + \beta^{1/\gamma} x^{1/\gamma - 1}}. \tag{4.14}$$

When there is no shock, or $x = 1$, the Pareto optimal extraction becomes

$$c^{NS} = \frac{k}{1 + \beta^{1/\gamma}}. \tag{4.15}$$

Note that the Pareto optimal extraction rates given by (4.14) and (4.15) are not different from the aggregate extractions in Cournot equilibrium given by (4.7) and (4.9). As pointed out by Stiglitz [59], with isoelastic market demand and zero extraction cost, monopolistic power has no distorting impact on the Pareto optimal extraction plan because the monopolist's marginal revenue is proportional to the market price (marginal utility) and the proportion is constant over time. So the distortion between marginal revenue and market price within each period cancels out across periods. This result is also true in the Cournot duopolistic model. The aggregate marginal revenue of Cournot competitors, captured by (4.4), is proportional to the representative consumer's marginal utility in (4.13), and the proportion is $2 - \gamma$ in both periods.

Because there is no distortion in the quantity of aggregate extraction, Cournot competition causes no welfare loss with symmetric shock or without shock. However, with an asymmetric shock, Cournot competition will distort the Pareto optimal solution in many cases, as illustrated in the following examples.

4.2.2 Asymmetric shock

When the shock is asymmetric, we assume that it only affects a certain part of the entire resource stock. We consider two examples. In the first example, the shock affects a portion of player 1's stock and has no impact on player 2's stock. In the second example, the shock affects all of player 1's stock and a portion of player 2's stock.

Example 1

We assume that player 1's stock k_1 is comprised of two parts, k_1^s and k_1^n. In the second period, the shock reduces the leftover of k_1^s if it is still available and has no impact on the leftover of k_1^n. We also assume that the shock does not affect the leftover of player 2's stock k_2 in the second period. Player 1 would not extract k_1^n as long as k_1^s is available. This occurs because extractions from k_1^s and k_1^n are perfect substitutes and extracting k_1^s before k_1^n leads to smaller resource stock that is under the threat of shock. A proof of this argument is similar to Section 2.1 of Ren and Polasky [47]. Thus, if k_1^s is not exhausted in the first period, in the second period the total stock of player 1 is $x(k_1^s - c_1) + k_1^n$. If k_1^s is exhausted and k_1^n is extracted in the first period, player 1 has $k_1^s + k_1^n - c_1 = k_1 - c_1$ in the second period.

To simplify the analysis we set $\gamma = 1$, then player 1 solves the following problem to maximize the total revenue over two periods

$$\max_{c_1} \left\{ \frac{c_1}{c_1 + c_2} + \beta \begin{cases} \dfrac{x(k_1^s - c_1) + k_1^n}{x(k_1^s - c_1) + k_1^n + k_2 - c_2} & c_1 < k_1^s \\ \dfrac{k_1 - c_1}{k_1 - c_1 + k_2 - c_2} & c_1 \geq k_1^s \end{cases} \right\}, \tag{4.16}$$

and the similar problem for player 2 is

$$\max_{c_2} \left\{ \frac{c_2}{c_1 + c_2} + \beta \begin{cases} \dfrac{k_2 - c_2}{x(k_1^s - c_1) + k_1^n + k_2 - c_2} & c_1 < k_1^s \\ \dfrac{k_2 - c_2}{k_1 - c_1 + k_2 - c_2} & c_1 \geq k_1^s \end{cases} \right\}. \tag{4.17}$$

Let

$$\tau_1 = k_1^s / k_1 \qquad \text{and} \qquad \theta_1 = 1 - \tau_1 + x\tau_1. \tag{4.18}$$

We can solve for the Cournot equilibrium $(\hat{c}_1^{AS},\, \hat{c}_2^{AS})$, where "AS" denotes "asymmetric shock", which depends on $(k_1,\, \tau_1,\, k_2)$

$$(\hat{c}_1^{AS},\, \hat{c}_2^{AS}) = \begin{cases} \left(\dfrac{k_1}{1+\beta},\, \dfrac{k_2}{1+\beta} \right) & \tau_1 \in \left[0,\, \dfrac{1}{1+\beta}\right) \\ \left(\tau_1 k_1,\, \dfrac{k_1 + k_2}{1 + \sqrt{\beta\,(1/\tau_1 - 1)}} - \tau_1 k_1 \right) & \tau_1 \in \left[\dfrac{1}{1+\beta},\, \overline{\tau}_1\right) \\ \left(\dfrac{\theta_1 k_1}{x + \beta \left(\dfrac{\theta_1 k_1 + x k_2}{\theta_1 k_1 + k_2} \right)^2},\, \dfrac{x k_2}{x + \beta \left(\dfrac{\theta_1 k_1 + x k_2}{\theta_1 k_1 + k_2} \right)^2} \right) & \tau_1 \in [\overline{\tau}_1,\, 1] \end{cases} \tag{4.19}$$

where $\overline{\tau}_1$ is determined by

$$\frac{\overline{\tau}_1}{1 - \overline{\tau}_1} \left(\frac{1 - \overline{\tau}_1 + x\overline{\tau}_1 + x k_2/k_1}{1 - \overline{\tau}_1 + x\overline{\tau}_1 + k_2/k_1} \right)^2 = \frac{1}{\beta}.$$

It can be verified that $1/(1+\beta) < \overline{\tau}_1 < 1$. We will discuss this solution after solving the second example.

Example 2

In this example the allocation of the shock affects the entire stock of the first player, k_1, and a part of the stock of the second player, k_2. Specifically, we assume that k_2 is comprised of two parts, k_2^s and k_2^n. In the second period, the shock reduces the leftover of k_2^s if it is still available and has no impact on the leftover of k_2^n. Thus, if k_2^s is not exhausted in the first period, the total stock of player 2 is $x(k_2^s - c_2) + k_2^n$ in the second period. If k_2^s is exhausted and k_2^n is extracted in the first period, player 2 has $k_2^s + k_2^n - c_2 = k_2 - c_2$ in the second period. We also assume that the shock will reduce the leftover of player 1's stock k_1.

In this example player 1 solves

$$\max_{c_1} \left\{ \frac{c_1}{c_1 + c_2} + \beta \begin{cases} \dfrac{x(k_1 - c_1)}{x(k_1 - c_1) + x(k_2^s - c_2) + k_2^n} & c_2 < k_2^s \\ \dfrac{x(k_1 - c_1)}{x(k_1 - c_1) + k_2 - c_2} & c_2 \geq k_2^s \end{cases} \right\}, \tag{4.20}$$

and player 2 solves

$$\max_{c_2} \left\{ \frac{c_2}{c_1 + c_2} + \beta \begin{cases} \dfrac{x(k_2^s - c_2) + k_2^n}{x(k_1 - c_1) + x(k_2^s - c_2) + k_2^n} & c_2 < k_2^s \\ \dfrac{k_2 - c_2}{x(k_1 - c_1) + k_2 - c_2} & c_2 \geq k_2^s \end{cases} \right\}. \tag{4.21}$$

Let

$$\tau_2 = k_2^s / k_2 \qquad \text{and} \qquad \theta_2 = 1 - \tau_2 + x\tau_2, \tag{4.22}$$

we solve for Cournot equilibrium $(\hat{c}_1^{AS},\, \hat{c}_2^{AS})$, which depends on $(k_1,\, k_2,\, \tau_2)$

$$(\hat{c}_1^{AS},\, \hat{c}_2^{AS}) = \begin{cases} \left(\dfrac{k_1}{1 + \beta x \left(\dfrac{k_1 + k_2}{x k_1 + k_2} \right)^2},\, \dfrac{k_2}{1 + \beta x \left(\dfrac{k_1 + k_2}{x k_1 + k_2} \right)^2} \right) & \tau_2 \in \left[0,\, \dfrac{1}{1 + \beta x \left(\dfrac{k_1 + k_2}{x k_1 + k_2} \right)^2} \right) \\ \left(\dfrac{x k_1 + \theta_2 k_2}{x + \sqrt{\beta x \,(1/\tau_2 - 1)}} - \tau_2 k_2,\, \tau_2 k_2 \right) & \tau_2 \in \left[\dfrac{1}{1 + \beta x \left(\dfrac{k_1 + k_2}{x k_1 + k_2} \right)^2},\, \dfrac{1}{1 + \beta x} \right) \\ \left(\dfrac{k_1}{1 + \beta},\, \dfrac{\theta_2 k_2}{x(1 + \beta)} \right) & \tau_2 \in \left[\dfrac{1}{1 + \beta x},\, 1 \right] \end{cases} \tag{4.23}$$

Distortion in extraction

Cournot equilibria with an asymmetric shock (4.19) and (4.23) distort the Pareto optimal extraction that maximizes the representative consumer's total utility in two periods. As discussed by Ren and Polasky [47], when $\gamma = 1$, the Pareto optimal extraction plan solves

$$\max_c \left\{ \log c + \beta \begin{cases} \log[x(k^s - c) + k^n] & c < k^s \\ \log(k^s + k^n - c) & c \geq k^s \end{cases} \right\}. \tag{4.24}$$

where $k^s = k_1^s$ in the first example, $k^s = k_1 + k_2^s$ in the second example and $k^n = k_1 + k_2 - k^s = k - k^s$. Then the Pareto optimal extraction is

$$c^{AS} = \begin{cases} \dfrac{k}{1+\beta} & \tau \in \left[0, \dfrac{1}{1+\beta}\right) \\ \tau k & \tau \in \left[\dfrac{1}{1+\beta}, \dfrac{1}{1+\beta x}\right) \\ \dfrac{\theta k}{x(1+\beta)} & \tau \in \left[\dfrac{1}{1+\beta x}, 1\right] \end{cases} \tag{4.25}$$

where

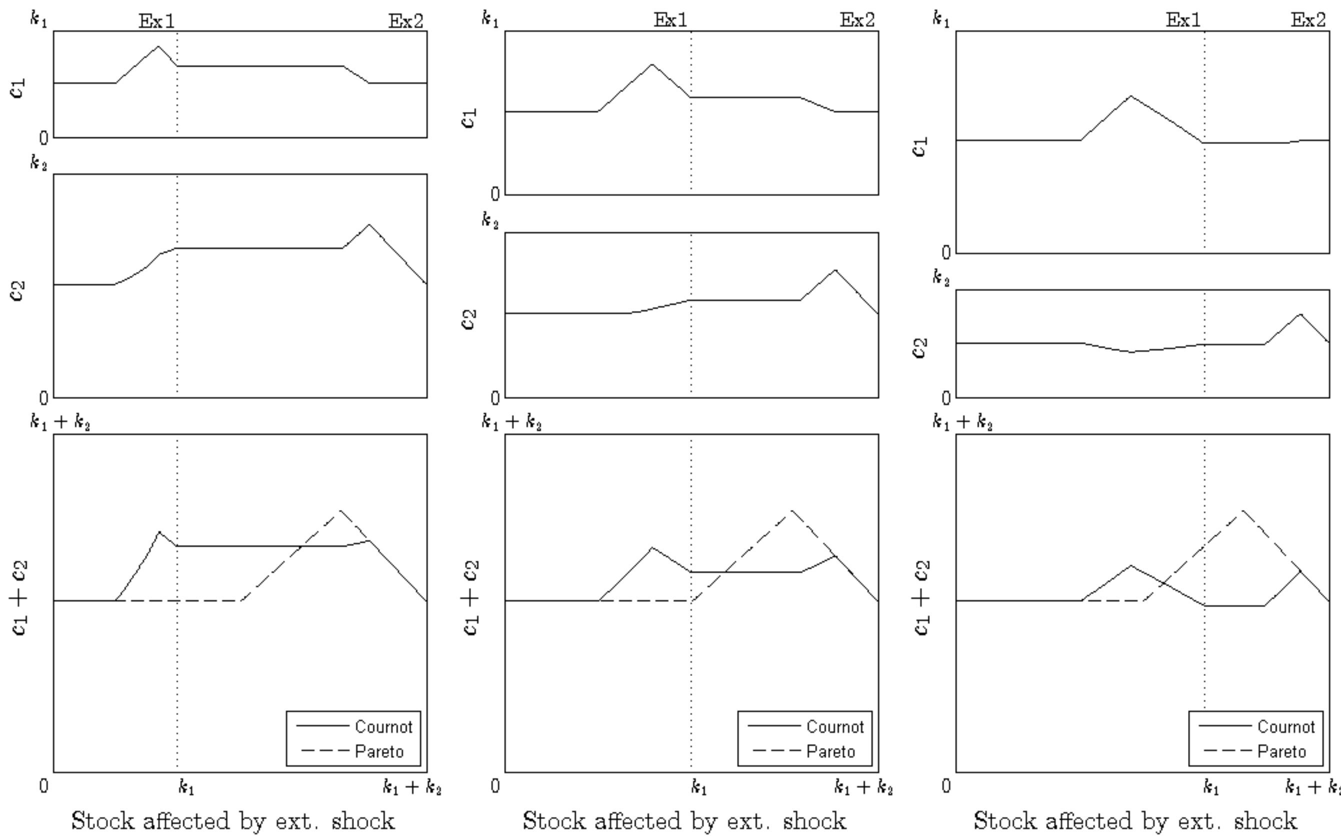
$$\tau = k^s/k \qquad \text{and} \qquad \theta = 1 - \tau + x\tau. \tag{4.26}$$

The Pareto optimal extraction follows a sequential plan where k^s is extracted before k^n because a planner would like to reduce the size of the resource stock that will be affected by the shock in the second period. When $\tau < 1/(1+\beta)$, optimal extraction is larger than k^s and is not affected by the shock. This is the interior solution of the second case of (4.24). When $1/(1+\beta) \leq \tau < 1/(1+\beta x)$, it is optimal to exhaust k^s exactly. This is the corner solution of the second case of (4.24) where the optimal extraction is constrained by the shock. When $\tau > 1/(1+\beta x)$, exhausting k^s becomes inefficient. This is the first case of (4.24) where the shock will occur in the second period because the leftover of k^s is positive. Figure 4.1 shows how the quantity of Pareto optimal extraction (4.25) is distorted in the two examples of Cournot competition given $k = 1$, $\beta = 0.98$ and $x = 0.3$.[5]

[5] These parameter values are used throughout this paper. The following qualitative results are robust with variation in these parameter values.

Figure 4.1: Extraction rate in the two-period model: Cournot equilibrium vs. Pareto optimal

(a) $k_1/k_2 = 1/2$ (b) $k_1/k_2 = 1$ (c) $k_1/k_2 = 2$



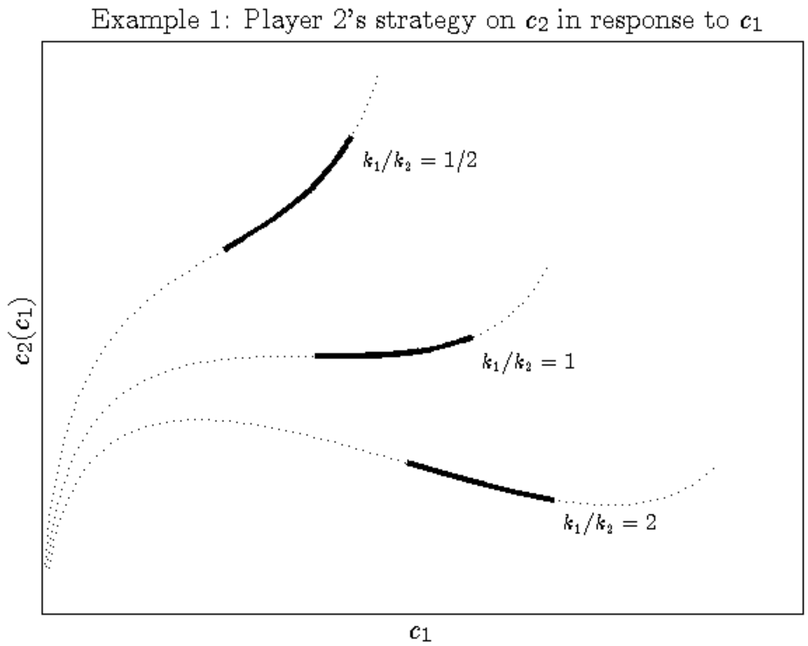
The Cournot equilibrium strategies of two players are plotted in the top and middle panels of Figure 4.1 against k_1^s. The left, middle and right panels of Figure 4.1 show results for $k_1/k_2 = 1/2$, 1, 2, respectively. The aggregate extraction in Cournot equilibrium is plotted in the bottom panel as a solid line. The dashed line in the bottom panel is the Pareto optimal extraction given by (4.25), which is independent of the value of k_1/k_2.

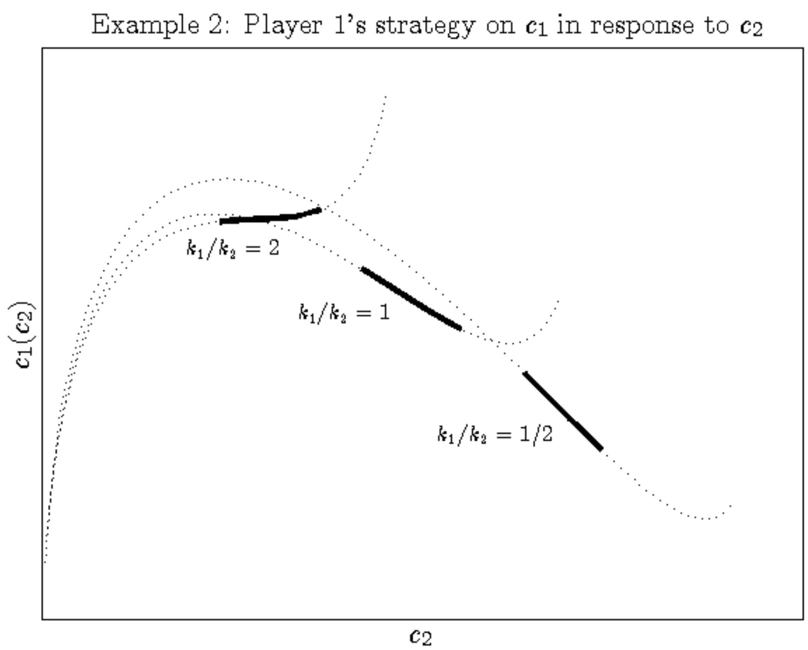
According to (4.19), when $\tau_1 = k_1^s/k_1 < 1/(1+\beta)$, k_1^s is smaller than player 1's equilibrium extraction $k_1/(1+\beta)$ and is exhausted in the first period. In this case the shock has no impact on Cournot equilibrium. The top and middle panels of Figure 4.1 show that equilibrium strategies of both players do not change as the size of k_1^s increases. In the bottom chart of each panel we also see that the aggregate extraction in Cournot equilibrium (solid line) equals to the Pareto optimal solution (dashed line). This is consistent with (4.9) and (4.15) that market power has no distorting effect when the shock does not affect the Cournot equilibrium.

When $1/(1+\beta) \le \tau_1 < \overline{\tau}_1$, (4.19) shows that player 1's strategy is binding at $k_1^s = \tau_1 k_1$. This result is the corner solution of the second case of player 1's problem (4.16). In this case player 1 would like to exhaust k_1^s in the first period in order to avoid the shock in the second period. Given player 1's strategy, player 2's best response function, solved from the second case of (4.17), is plotted as dashed lines in the top chart of Figure 4.2 for $k_1/k_2 = 1/2$, 1 or 2. The solid part of each dashed line is corresponding to the second case of (4.19) where player 1's strategy is binding at k_1^s. We see that when k_1 is relatively small as compared to k_2 ($k_1/k_2 = 1/2$ or 1), it is optimal for player 2 to extract more in response to player 1's binding strategy which is increasing in k_1^s. However, when k_1 is relatively large ($k_1/k_2 = 2$), player 2's best response is to extract less when player 1 increases extraction.[6] These results are consistent with the top and middle panels in Figure 4.1.

[6] In our model where the demand function has constant unitary elasticity, the best response function is non-monotonic and the slope depends on the relative size of the competitors. This is different from the result when the demand function is concave, or at least not "too" convex, as discussed by Varian [64]. With a linear demand, i.e., $p = q - c_1 - c_2$, the best response function is always decreasing.

Figure 4.2: Best response to the competitor's binding behavior in the two-period model





The bottom panels of Figure 4.1 show that Cournot competition begins to distort the Pareto optimal solution when player 1's strategy is binding. By (4.19) the aggregate extraction in Cournot equilibrium starts to increase when τ_1 becomes greater than $1/(1+\beta)$, but by the first case of (4.25) the Pareto optimal extraction is still constant.[7]

Note that besides the distortion in quantity, there is also the distortion in the order of extracting different types of stocks. In the Pareto optimal solution, k_2 is not extracted as long as k_1^s is still available. However, in the Cournot model, player 2's extraction from k_2 is simultaneous with player 1's extraction from k_1^s.

When $\tau_1 \geq \overline{\tau}_1$, k_1^s is sufficiently large that in Cournot equilibrium it is not exhausted by player 1 in the first period (if the condition is satisfied strictly). In this case the shock would occur in the second period. Using (4.18) and (4.19) we can show that given k_1 and k_2, $\hat{c}_1^{AS}$ is decreasing in k_1^s and $\hat{c}_2^{AS}$ is increasing in k_1^s. At the aggregate level, $\hat{c}_1^{AS} + \hat{c}_2^{AS}$ is decreasing in k_1^s and the Pareto optimal solution is distorted. Figure 4.1 shows that the aggregate extraction in Cournot equilibrium could be either higher or lower as compared to the Pareto optimal solution, depending on the value of k_1^s/k_1 and k_1/k_2.

For the second example, as shown by (4.23), when $\tau_2 = k_2^s/k_2 < 1/(1 + \beta x(k_1 + k_2)^2/(xk_1 + k_2)^2)$, player 2's equilibrium extraction in the first period is larger than k_2^s, thus the shock only affects player 1's stock in the second period. In this case the equilibrium strategies of both players do not change as the size of k_2^s increases. Because player 2 extracts positive amounts from k_2^n which is not affected by the shock, while player 1 only extract a portion of k_1 that is under the threat of the shock, the Pareto optimal sequential extraction plan is distorted. The Pareto optimal quantity is also distorted as shown in the bottom charts of Figure 4.1, and we can see that the direction of the distortion depends on the value of k_2^s/k_2 and k_1/k_2.

As τ_2 increases, player 2's strategy is binding at $k_2^s = \tau_2 k_2$ when $1/(1 + \beta x(k_1 + k_2)^2/(xk_1 + k_2)^2) \leq \tau_2 < 1/(1 + \beta x)$. This is the corner solution of the second case of player 2's problem (4.21). Player 1's best response function solved from the second case of (4.20) is plotted as dashed lines in the bottom chart of Figure 4.2 for $k_1/k_2 = 1/2$, 1 or 2, and the solid part of each dashed line is corresponding to player 2's binding

[7] By (4.18) and (4.26), $\tau = k_1\tau_1/(k_1 + k_2)$. We can verify that when $\tau_1 = 1/(1 + \beta)$, the condition for the first case of (4.25), $\tau < 1/(1 + \beta)$, is still satisfied.

strategy. When k_2 is relatively large as compared to k_1 ($k_1/k_2 = 1/2$ or 1), player 1 would extract less in respond to player 2's binding strategy which is increasing in k_2^s. However, when k_2 is relatively small ($k_1/k_2 = 2$), it is optimal for player 1 to extract more as player 2's extraction increases. These results are consistent with the top and middle panels in Figure 4.1. As shown by the bottom panels of Figure 4.1, the aggregate extraction in Cournot equilibrium distorts the Pareto optimal solution.

When $\tau_2 \geq 1/(1+\beta x)$, k_2^s is sufficiently large and in equilibrium player 2 would not exhaust k_2^s in the first period (if the condition is satisfied strictly). In the second period, the shock will affect the leftovers of k_1 and k_2^s. According to (4.22) and (4.23), given k_1 and k_2, player 2's equilibrium extraction $\hat{c}_2^{AS}$ is decreasing in k_2^s and player 1's equilibrium extraction $\hat{c}_1^{AS}$ is constant. It is interesting to note that the aggregate extraction $\hat{c}_1^{AS} + \hat{c}_2^{AS}$ in this case is the same as the Pareto optimal solution. This is shown in Figure 4.1 and can be verified by comparing (4.23) with (4.25).[8]

The solutions of the two examples, as plotted in Figure 4.1, show that the Pareto optimal extraction is distorted when $\tau_1 > 1/(1+\beta)$ and $\tau_2 < 1/(1+\beta x)$. According to (4.18), (4.22) and (4.26), this is equivalent to

$$\frac{k_1}{(1+\beta)(k_1+k_2)} < \tau < \frac{k_1 + k_2/(1+\beta x)}{k_1+k_2}. \tag{4.27}$$

Depending on the values of τ and k_1/k_2, aggregate extraction in Cournot equilibrium could be either higher or lower as compared to the Pareto optimal quantity.

Welfare loss

We have seen that Pareto optimal extraction is distorted when (4.27) is true. We now discuss the loss in social welfare caused by the distortion. Substituting (4.25) into (4.24) we can solve for the maximized welfare in the Pareto optimal case

$$\Pi = \begin{cases} (1+\beta)\log k + \log \dfrac{\beta^\beta}{(1+\beta)^{1+\beta}} & \tau \in \left[0, \dfrac{1}{1+\beta}\right) \\ (1+\beta)\log k + \log[\tau(1-\tau)^\beta] & \tau \in \left[\dfrac{1}{1+\beta}, \dfrac{1}{1+\beta x}\right) \\ (1+\beta)\log k + \log \dfrac{\theta^{1+\beta}\beta^\beta}{x(1+\beta)^{1+\beta}} & \tau \in \left[\dfrac{1}{1+\beta x}, 1\right] \end{cases}. \tag{4.28}$$

[8] By (4.22) and (4.26), $\tau = (k_1 + k_2\tau_2)/(k_1+k_2)$. We can verify that the condition for the third case of (4.25), $\tau < 1/(1+\beta x)$, is satisfied when $\tau_2 = 1/(1+\beta x)$.

For the Cournot equilibria in the two examples, the welfare is measured as follows

$$\hat{\Pi} = \log(\hat{c}_1^{AS} + \hat{c}_2^{AS}) + \beta \begin{cases} \log(k_1^s + k_1^n - \hat{c}_1^{AS} + k_2 - \hat{c}_2^{AS}) & k_1^s \leq \hat{c}_1^{AS} \\ \log[x(k_1^s - \hat{c}_1^{AS}) + k_1^n + k_2 - \hat{c}_2^{AS}] & k_1^s > \hat{c}_1^{AS} \\ \log[x(k_1 - \hat{c}_1^{AS}) + k_2^s + k_2^n - \hat{c}_2^{AS}] & k_2^s \leq \hat{c}_2^{AS} \\ \log[x(k_1 - \hat{c}_1^{AS}) + x(k_2^s - \hat{c}_2^{AS}) + k_2^n] & k_2^s > \hat{c}_2^{AS} \end{cases}.$$

Then by (4.19) the welfare corresponding to the first example is

$$\hat{\Pi} =$$
$$\begin{cases} (1+\beta)\log(k_1+k_2) + \log \dfrac{\beta^\beta}{(1+\beta)^{1+\beta}} & \tau_1 \in \left[0,\, \dfrac{1}{1+\beta}\right) \\ (1+\beta)\log(k_1+k_2) + \log \dfrac{\sqrt{\beta(1/\tau_1 - 1)}^{\beta}}{(1+\sqrt{\beta(1/\tau_1-1)})^{1+\beta}} & \tau_1 \in \left[\dfrac{1}{1+\beta},\, \overline{\tau}_1\right) \\ (1+\beta)\log(\theta_1 k_1 + x k_2) + \log \dfrac{\beta^\beta \left(\dfrac{\theta_1 k_1 + x k_2}{\theta_1 k_1 + k_2}\right)^\beta}{\left[x + \beta\left(\dfrac{\theta_1 k_1 + x k_2}{\theta_1 k_1 + k_2}\right)^2\right]^{1+\beta}} & \tau_1 \in [\overline{\tau}_1,\, 1] \end{cases} \tag{4.29}$$

and using (4.23) the welfare corresponding to the second example is

$$\hat{\Pi} =$$
$$\begin{cases} (1+\beta)\log(k_1+k_2) + \log \dfrac{\beta^\beta x^\beta \left(\dfrac{k_1+k_2}{x k_1 + k_2}\right)^\beta}{\left[1 + \beta x \left(\dfrac{k_1+k_2}{x k_1 + k_2}\right)^2\right]^{1+\beta}} & \tau_2 \in \left[0,\, \dfrac{1}{1+\beta x\left(\dfrac{k_1+k_2}{x k_1+k_2}\right)^2}\right) \\ (1+\beta)\log(x k_1 + \theta_2 k_2) + \log \dfrac{\sqrt{\beta x(1/\tau_2 - 1)}^{\beta}}{(x+\sqrt{\beta x(1/\tau_2-1)})^{1+\beta}} & \tau_2 \in \left[\dfrac{1}{1+\beta x\left(\dfrac{k_1+k_2}{x k_1+k_2}\right)^2},\, \dfrac{1}{1+\beta x}\right) \\ (1+\beta)\log(x k_1 + \theta_2 k_2) + \log \dfrac{\beta^\beta}{x(1+\beta)^{1+\beta}} & \tau_2 \in \left[\dfrac{1}{1+\beta x},\, 1\right] \end{cases} \tag{4.30}$$

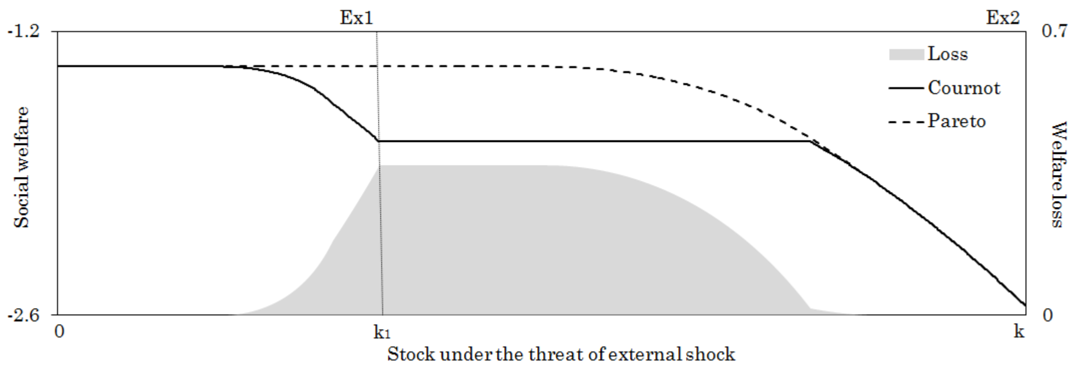
Figure 4.3 compares Π, the maximized social welfare from the Pareto optimal solution given by (4.28), with $\hat{\Pi}$, the social welfare from the Cournot equilibrium given by

(4.29) and (4.30), for $k_1/k_2 = 1/2$, 1 or 2. The social welfare from the Pareto optimal extraction, plotted by the dashed lines, is independent of k_1/k_2 ans is thus the same in all three panels of Figure 4.3. We can see that the Pareto welfare is constant when $\tau < 1/(1+\beta)$ because the optimal extraction involves extracting all stock exposed to shock in the first period. When τ becomes larger, optimal extraction is affected by the shock and the Pareto welfare starts to decrease as τ rises.

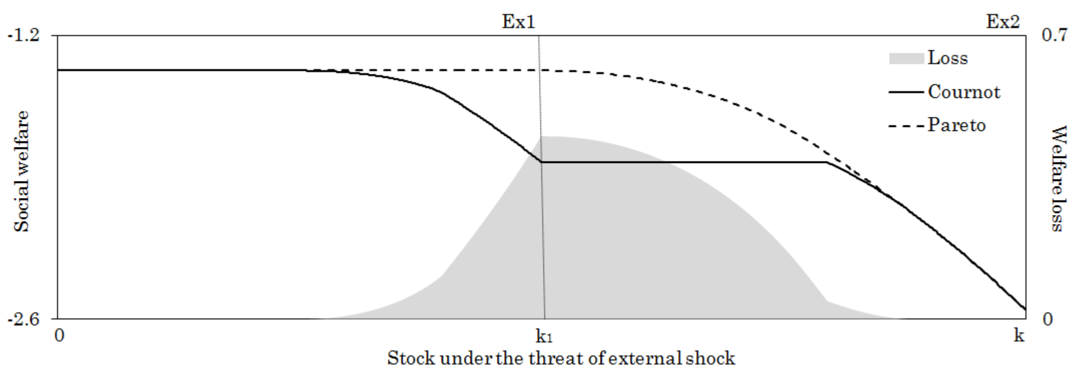
The welfare from Cournot equilibrium is plotted by the solid lines in Figure 4.3. The welfare loss, or the difference between the Pareto welfare and Cournot welfare, is shown by the shaded areas. We can see that Cournot competition causes welfare loss. When τ is sufficiently small such that $\tau_1 < 1/(1+\beta)$ is satisfied, comparing (4.29) with (4.28) we can see that Cournot competition causes no welfare loss in the first example. When τ is sufficiently large such that $\tau_2 > 1/(1+\beta x)$ is satisfied, comparing (4.30) with (4.28) we also see that Cournot competition causes no welfare loss in the second example. These extreme cases are consistent with Figure 4.1. However, between these two extreme cases Cournot competition will distort the aggregate extraction and leads to a loss in social welfare.

Figure 4.3: Welfare loss in the two-period model

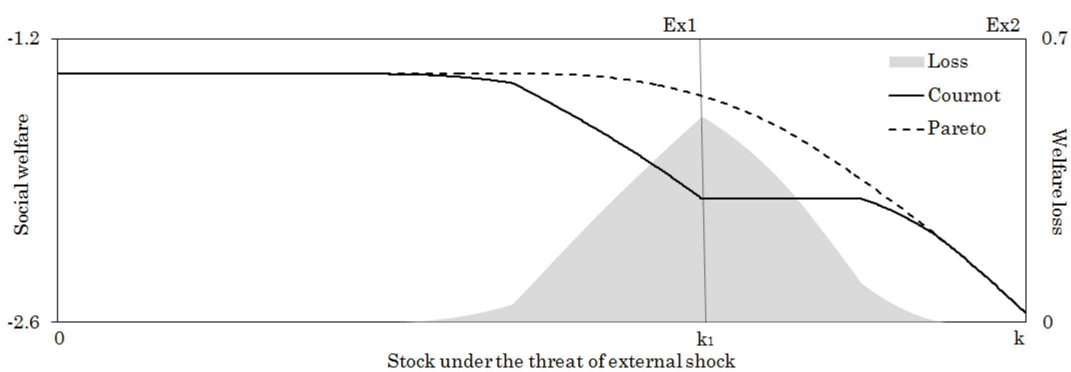
$k_1/k_2 = 1/2$



$k_1/k_2 = 1$



$k_1/k_2 = 2$



In the first example, when $\tau_1 \geq 1/(1+\beta)$, player 1's equilibrium extraction is constrained by the the shock, leading to distortion in aggregate extraction. Figure 4.3 shows that the welfare from Cournot equilibrium starts to decrease in τ_1. However, the welfare in the Pareto optimal solution is still constant because the optimal extraction is not constrained by the shock because $\tau < 1/(1+\beta)$. Therefore, welfare loss starts to increase in τ_1. The welfare loss reaches maximum when $\tau_1 = 1$. As the shock is further spread out to affect player 2's stock, Cournot competition becomes the case in the second example. In the second example, welfare from Cournot equilibrium is constant if $\tau_2 < 1/(1+\beta x(k_1+k_2)^2/(xk_1+k_2)^2)$. In this case player 2's equilibrium extraction is not constrained by the shock. Thus the welfare loss becomes smaller as the welfare from the Pareto solution starts to decrease when the optimal extraction is constrained by the shock. When τ_2 rises to $1/(1+\beta x)$, there is no distortion in extraction and the welfare loss reduces to 0.

In this section we used a two-period model to analyze the impact of a shock on Cournot equilibrium. We found that when the shock affects both Cournot players' stocks symmetrically it does not change the first period equilibrium extraction when demand has constant unitary elasticity.However, when the allocation of the shock is asymmetric, we found that even with unitary elastic demand, Cournot equilibrium changes with the relative size of the Cournot players' stocks and the share of stock that is under the threat of the shock. Also in Cournot equilibrium, both players will extract stocks simultaneously, violating the Pareto optimal sequential plan that requires extracting the threatened stock first. Besides the distortion in the order of extracting different types of stocks, the quantity of extraction is also distorted in many cases. We showed that the distortion in quantity and welfare loss depend on the relative sizes of the Cournot players' stocks and the share of threatened stock. In the next section we will examine these results in an infinite horizon model where the occurrence of the shock is stochastic.

4.3 Infinite horizon model with stochastic shock

In the infinite horizon model each player $i = 1, 2$ owns initial stock k_i. Within a continuous-time risk-free framework player i solves the following problem

$$\hat{V}^i(k_1, k_2) = \max_{c_i} \left\{ \Delta t \frac{c_i}{(c_1 + c_2)^\gamma} + e^{-\rho \Delta t} \hat{V}^i(k_1 - \Delta t c_1, k_2 - \Delta t c_2) \right\}$$

taking the extraction strategy c_j of the other player $j \neq i$ as given. The continuous-time discount factor is given as $\rho = -\log \beta$. In the limiting case $\Delta t \to 0$

$$\rho \hat{V}^i(k_1, k_2) = \max_{c_i} \left\{ \frac{c_i}{(c_1 + c_2)^\gamma} - \hat{V}^i_{k_1}(k_1, k_2) c_1 - \hat{V}^i_{k_2}(k_1, k_2) c_2 \right\}. \tag{4.31}$$

[9]

We can verify that the the equilibrium extraction of player i is

$$\hat{c}_i^{NS} = \frac{\rho k_i}{\gamma} \tag{4.32}$$

and the corresponding value function of is

$$\hat{V}^i(k_1, k_2) = \left(\frac{\gamma}{\rho} \right)^\gamma \frac{k_i}{(k_1 + k_2)^\gamma}. \tag{4.33}$$

Because equilibrium strategies are solved using dynamic programming methods, the solutions are time consistent and subgame perfect.

We now introduce a stochastic shock into the model. We assume that at any time $t \geq 0$, a shock occurs with constant hazard rate $\lambda \in [0, 1]$. Then the cumulative probability of the shock occurring by $t \geq 0$ is

$$\Lambda(t) = 1 - e^{-\lambda t}. \tag{4.34}$$

The shock only occurs once and the dynamic system enters the risk-free phase after the occurrence. As the two-period model, we consider a symmetric shock and an asymmetric shock separately. Our model has complete information so the hazard rate and allocation of the shock is known to both players.

[9] Throughout this paper we use subscript to denote the partial derivative to the corresponding argument.

4.3.1 Symmetric shock

The symmetric shock once it occurs will reduce both players' stocks to a proportion x of the size prior to the occurrence. Then under the threat of an symmetric shock, player i takes the other player's strategy c_j as given and solves

$$\hat{W}^i(k_1,\, k_2) = e^{-\lambda\Delta t} \max_{c_i} \left\{ \Delta t \frac{c_i}{(c_1+c_2)^\gamma} + e^{-\rho\Delta t}\hat{W}^i(k_1 - \Delta t c_1,\, k_2 - \Delta t c_2) \right\} + (1 - e^{-\lambda\Delta t})\hat{V}^i(xk_1,\, xk_2)$$

where $\hat{V}^i$ is given by (4.31). In the limiting case $\Delta t \to 0$

$$\rho\hat{W}^i(k_1,\, k_2) = \max_{c_i} \left\{ \frac{c_i}{(c_1+c_2)^\gamma} - \hat{W}^i_{k_1}(k_1,\, k_2)c_1 - \hat{W}^i_{k_2}(k_1,\, k_2)c_2 \right\} - \lambda[\hat{W}^i(k_1,\, k_2) - \hat{V}^i(xk_1,\, xk_2)]. \quad (4.35)$$

Comparing (4.35) with the risk-free model (4.31), we see that the potential symmetric shock causes a damage $\lambda[\hat{W}^i(k_1,\, k_2) - \hat{V}^i(xk_1,\, xk_2)]$ on player i's value function. Because complete information is assumed in our model, dynamic programming method ensures that the solution of (4.35), denoted by $\hat{c}_i^{SS}$, is a subgame perfect equilibrium.

Equilibrium extraction

The following result compares $\hat{c}_i^{SS}$, solution of (4.35), with the risk-free Cournot equilibrium $\hat{c}_i^{NS}$ given by (4.32).

Proposition 7. In the infinite horizon model, a potential symmetric shock affects the Cournot equilibrium as follows

$$\hat{c}_i^{SS} \geq \hat{c}_i^{NS} \Leftrightarrow \gamma \leq 1 \quad \text{for} \quad i = 1,\, 2.$$

Same result also holds for $\hat{c}^{SS} = \hat{c}_1^{SS} + \hat{c}_2^{SS}$ and $\hat{c}^{NS} = \hat{c}_1^{NS} + \hat{c}_2^{NS}$.

Proof. Assuming $\hat{W}^i(k_1,\, k_2) = Ak_i/(k_1+k_2)^\gamma$, the first order condition of (4.35) is

$$\frac{c_1 + c_2 - \gamma c_i}{(c_1+c_2)^{\gamma+1}} = A\frac{k_1 + k_2 - \gamma k_i}{(k_1+k_2)^{\gamma+1}} \quad \text{for} \quad i = 1,\, 2.$$

This implies that the equilibrium satisfies $\hat{c}_1^{SS}/k_1 = \hat{c}_2^{SS}/k_2$ and

$$\hat{c}_i^{SS} = A^{-1/\gamma}k_i. \quad (4.36)$$

Then substituting (4.33), (4.36), and the assumed form of $\hat{W}^i$ into (4.35) we find A is determined as the following

$$(\rho+\lambda)A = \gamma A^{1-1/\gamma} + \lambda x^{1-\gamma}(\gamma/\rho)^{\gamma}. \tag{4.37}$$

This implies

$$\left(\frac{\gamma}{\rho}\right)^{\gamma} \gtreqless A \gtreqless x^{1-\gamma}\left(\frac{\gamma}{\rho}\right)^{\gamma}.$$

Because $x < 1$, the following is true

$$A^{-\frac{1}{\gamma}} \geq \frac{\rho}{\gamma} \Leftrightarrow \gamma \leq 1.$$

Then the desired result at individual level follows by comparing (4.32) with (4.36) and the desired result at aggregate level is straightforward. □

Proposition 7 shows that the two-period results (4.10) and (4.11) extend naturally to the infinite horizon model with a stochastic shock. The shock potentially reduces the sales of both players, but also increases the market price due to lower aggregate supply. When $\gamma < 1$ sales decrease more than price rises, leading to lower marginal revenue that encourages both players to increase the extraction as compared to the risk-free case. When $\gamma = 1$, two competing effects though quantity and price fully offset each other and the potential symmetric shock has no impact on equilibrium extraction.

Social welfare

The social welfare is measured by the net present value of the representative consumer's utility flow to the infinite future. As discussed by Ren and Polasky [47], given aggregate initial stock $k = k_1 + k_2$ the representative consumer solves

$$\rho W(k) = \max_c \{u(c) - W'(k)c\} - \lambda[W(k) - V(xk)], \tag{4.38}$$

where V is the standard risk-free value function given by

$$\rho V(k) = \max_c \{u(c) - V'(k)c\}. \tag{4.39}$$

The solution of (4.38) satisfies

$$c^{SS} = A^{-1/\gamma}k \tag{4.40}$$

where A is determined exactly the same as in (4.37). In the risk-free case $\lambda = 0$, then by (4.37), (4.40) reduces to the solution of (4.39)

$$c^{NS} = \frac{\rho k}{\gamma}. \tag{4.41}$$

We have the following result.

Proposition 8. In the infinite horizon model, Cournot equilibrium does not distort the Pareto optimal extraction and leads to no welfare loss when there is no shock or the risk of shock is symmetric.

Proof. The desired result in the risk-free case follows from (4.32) and (4.41), and the desired result with symmetric shock follows from (4.36) and (4.40). □

Proposition 8 shows the coincidence between Cournot equilibria and Pareto optimal solutions in the risk-free case and the case where the potential shock is symmetric. In the Cournot model each player i's marginal revenue $[(1-\gamma)c_i + c_j](c_1+c_2)^{-\gamma-1}$ is proportional to the representative consumer's marginal utility (market price) $p = (c_1+c_2)^{-\gamma}$. The equilibrium strategy (4.32) and (4.36) show that each Cournot player i extracts a constant fraction from k_i, implying that the proportion $1-\gamma c_i/(c_1+c_2) = 1-\gamma k_i/(k_1+k_2)$ is constant over time. Therefore, similar to Stiglitz [59], the distortion effect of Cournot competition at each single time point cancels out and there is no loss in social welfare.

However, when the potential shock affects k_1 and k_2 asymmetrically, the coincidence between Cournot equilibria and Pareto optimal solutions fails. In the following we will extend the two examples discussed in the two-period model into infinite horizon to show how an asymmetric stochastic shock changes the Cournot players' strategies so that they do not equal the Pareto optimal solutions.

4.3.2 Asymmetric shock

Example 1

As in the two period model, we first consider the case where player 1's stock k_1 comprises two parts k_1^s and k_1^n and $\tau_1 = k_1^s/k_1$. A potential shock only affects k_1^s and has no impact on k_1^n and player 2's stock k_2. Then given these state variables the value functions of two players can be denoted by $\hat{W}^1(k_1, k_2, \tau_1)$ and $\hat{W}^2(k_1, k_2, \tau_1)$.

We now claim, and verify later, that given $\gamma = 1$, for any $\tau_1 \in [0, 1]$, both $\hat{W}^1(k_1, k_2, \tau_1)$ and $\hat{W}^2(k_1, k_2, \tau_1)$ are homogeneous of degree 0 in (k_1, k_2). Then define

$$\phi = k_1/k_2, \tag{4.42}$$

we know $\hat{W}^1(k_1, k_2, \tau_1) = \hat{W}^1(\phi, 1, \tau_1)$ and $\hat{W}^2(k_1, k_2, \tau_1) = \hat{W}^2(\phi, 1, \tau_1)$, so we only need to use (ϕ, τ_1) as state variables. Then let θ_1 be defined as in (4.18), player i solves

$$\begin{aligned}\hat{W}^i(\phi, \tau_1) = e^{-\lambda\Delta t} \max_{c_i} \left\{ \Delta t \frac{c_i}{c_1 + c_2} + e^{-\rho\Delta t} \hat{W}^i \left(\frac{k_1 - \Delta t c_1}{k_2 - \Delta t c_2}, \frac{\tau_1 k_1 - \Delta t c_1}{k_1 - \Delta t c_1} \right) \right\} \\ + (1 - e^{-\lambda\Delta t}) \hat{V}^i(\theta_1 k_1, k_2)\end{aligned}$$

where $\hat{V}^i$ is given by (4.31). In the limiting case $\Delta t \to 0$ we have

$$\begin{aligned}\rho \hat{W}^i(\phi, \tau_1) = \max_{c_i} \left\{ \frac{c_i}{c_1 + c_2} - \left[\phi \hat{W}^i_\phi + (1 - \tau_1) \hat{W}^i_{\tau_1} \right] \frac{c_1}{k_1} + \phi \hat{W}^i_\phi \frac{c_2}{k_2} \right\} \\ - \lambda [\hat{W}^i(\phi, \tau_1) - \hat{V}^i(\theta_1 k_1, k_2)]. \quad (4.43)\end{aligned}$$

We have the following result.

Proposition 9. The subgame perfect Cournot equilibrium that satisfies the dynamic programming problem (4.43) is

$$(\hat{c}_1^{AS}, \hat{c}_2^{AS}) = \frac{\rho + \lambda}{\rho + \lambda \dfrac{\theta_1}{g_1(\tau_1)} \left[\dfrac{g_1(\tau_1)\phi + 1}{\theta_1 \phi + 1} \right]^2} \times [g_1(\tau_1)\rho k_1, \rho k_2], \tag{4.44}$$

where $g_1(\tau_1)$ is determined by the following differential equation

$$g_1(\tau_1) + (1 - \tau_1) g_1'(\tau_1) = 1 + \frac{\lambda}{\rho} \left[\frac{\theta_1}{g_1(\tau_1)} - x \right] \left[\frac{g_1(\tau_1)\phi + 1}{\theta_1 \phi + 1} \right]^2 \tag{4.45}$$

with boundary condition $g_1(0) = 1$. The value functions of the two players are

$$\begin{aligned}\hat{W}^1(\phi, \tau_1) &= \frac{1}{\rho + \lambda} \left[\frac{g_1(\tau_1)\phi}{g_1(\tau_1)\phi + 1} \right] + \frac{\lambda}{\rho(\rho + \lambda)} \left(\frac{\theta_1 \phi}{\theta_1 \phi + 1} \right), \\ \hat{W}^2(\phi, \tau_1) &= \frac{1}{\rho + \lambda} \left[\frac{1}{g_1(\tau_1)\phi + 1} \right] + \frac{\lambda}{\rho(\rho + \lambda)} \left(\frac{1}{\theta_1 \phi + 1} \right).\end{aligned}$$

Proof. See Section 4.6. □

Note that boundary condition $g_1(0) = 1$ implies that Cournot equilibrium (4.44) reduces to the risk-free form $(\rho k_1,\, \rho k_2)$ when $\tau_1 = 0$. Also, as claimed before, both $\hat{W}^1$ and $\hat{W}^2$ are homogeneous of degree 0 in $(k_1,\, k_2)$ because $\phi = k_1/k_2$. We will discuss these results in Proposition 9 after solving the second example.

Example 2

Now we consider the case where the shock affects the entire stock of player 1, k_1, and a portion k_2^s of player 2's stock k_2. Let τ_2 and θ_2 be defined as (4.22) and ϕ defined as (4.42), player i solves

$$\hat{W}^i(\phi,\, \tau_2) = e^{-\lambda\Delta t} \max_{c_i} \left\{ \Delta t \frac{c_i}{c_1 + c_2} + e^{-\rho\Delta t} \hat{W}^i \left(\frac{k_1 - \Delta t c_1}{k_2 - \Delta t c_2},\, \frac{\tau_2 k_2 - \Delta t c_2}{k_2 - \Delta t c_2} \right) \right\} \\ + (1 - e^{-\lambda\Delta t}) \hat{V}^i(x k_1,\, \theta_2 k_2)$$

where $\hat{V}^i$ is given by (4.31). In the limiting case $\Delta t \to 0$ we have

$$\rho \hat{W}^i(\phi,\, \tau_2) = \max_{c_i} \left\{ \frac{c_i}{c_1 + c_2} - \phi \hat{W}^i_\phi \frac{c_1}{k_1} + \left[\phi \hat{W}^i_\phi - (1 - \tau_2) \hat{W}^i_{\tau_2} \right] \frac{c_2}{k_2} \right\} \\ - \lambda [\hat{W}^i(\phi,\, \tau_2) - \hat{V}^i(x k_1,\, \theta_2 k_2)]. \quad (4.46)$$

We have the following result.

Proposition 10. The subgame perfect Cournot equilibrium that satisfies the dynamic programming problem (4.46) is

$$(\hat{c}_1^{AS},\, \hat{c}_2^{AS}) = \frac{\rho + \lambda}{\rho + \lambda \dfrac{x\theta_2}{g_2(\tau_2)} \left[\dfrac{\phi + g_2(\tau_2)}{x\phi + \theta_2} \right]^2} \times [\rho k_1,\, g_2(\tau_2) \rho k_2], \quad (4.47)$$

where $g_2(\tau_2)$ is determined by the following differential equation

$$g_2(\tau_2) + (1 - \tau_2) g_2'(\tau_2) = 1 + \frac{\lambda x}{\rho} \left[\frac{\theta_2}{g_2(\tau_2)} - x \right] \left[\frac{\phi + g_2(\tau_2)}{x\phi + \theta_2} \right]^2 \quad (4.48)$$

with boundary condition $g_2(0) = 1$. The value functions of the two players are

$$\hat{W}^1(\phi,\, \tau_2) = \frac{1}{\rho + \lambda} \left[\frac{\phi}{\phi + g_2(\tau_2)} \right] + \frac{\lambda}{\rho(\rho + \lambda)} \left(\frac{x\phi}{x\phi + \theta_2} \right),$$

$$\hat{W}^2(\phi,\, \tau_2) = \frac{1}{\rho + \lambda} \left[\frac{g_2(\tau_2)}{\phi + g_2(\tau_2)} \right] + \frac{\lambda}{\rho(\rho + \lambda)} \left(\frac{\theta_2}{x\phi + \theta_2} \right).$$

Proof. See Section 4.6. □

Note that the Cournot model (4.46) with $\tau_2 = 0$ is equivalent to Cournot model (4.43) with $\tau_1 = 1$. In both cases the entire k_1 is under the risk of shock and k_2 is risk-free. Again, as claimed before, both $\hat{W}^1$ and $\hat{W}^2$ are homogeneous of degree 0 in (k_1, k_2) because $\phi = k_1/k_2$.

Distortion in extraction

Unlike the risk-free model (4.31) and the model with symmetric shock (4.35) where the Pareto optimal solution is not distorted, Cournot models with asymmetric shock such as (4.43) and (4.46) have equilibra that are not Pareto optimal. Ren and Polasky [47] discussed the following representative consumer's problem with asymmetric shock

$$\rho W(k^s, k^n) = \max_c\{u(c) - W_{k^s}(k^s, k^n)c\} - \lambda[W(k^s, k^n) - V(xk^s + k^n)], \qquad (4.49)$$

where V is given by (4.39), $k^s = k_1^s$ in the first example, $k^s = k_1 + k_2^s$ in the second period and $k^n = k_1 + k_2 - k^s = k - k^s$. They found that the Pareto optimal solution satisfies

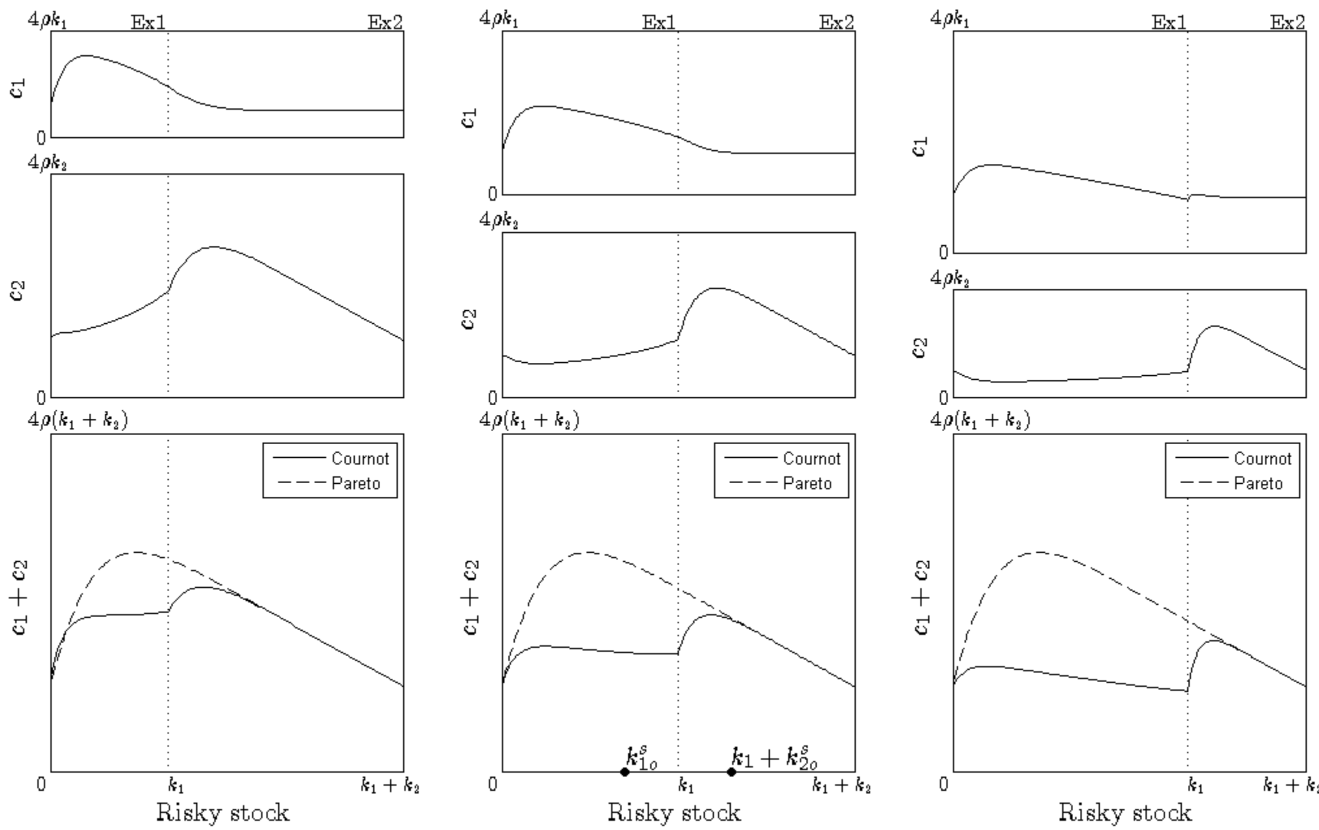
$$c^{AS} = \frac{1 - (1-x)[(1-\tau)/\theta]^{1+\lambda/\rho}}{1 - (1-x)[(1-\tau)/\theta]}\rho k, \qquad (4.50)$$

where τ and θ are given as (4.26). As in the two-period model (4.24), the Pareto optimal solution in the infinite horizon model also follows a sequential plan where k^s is extracted before k^n.

Figure 4.4 plots Cournot equilibria (4.44) and (4.47) of the two examples and compares them with the Pareto solution (4.50) with the same values of k, $\rho(\beta)$ and x used in the two-period model In left, middle and right panels of Figure 4.4 we show results when $\phi = k_1/k_2 = 1/2,\ 1,\ 2$, respectively. Equilibrium (4.44) of the first example is plotted on the left part of each panel against k_1^s and equilibrium (4.47) of the second example is plotted on the right part of each panel against k_2^s. The top and middle panels show the individual strategies. In the bottom panel the solid line is the aggregate extraction in Cournot equilibrium and the dashed line is the corresponding Pareto optimal extraction.

Figure 4.4: Extraction in the infinite horizon model: Cournot equilibrium vs. Pareto optimal

(a) $k_1/k_2 = 1/2$ (b) $k_1/k_2 = 1$ (c) $k_1/k_2 = 2$



From the top and middle panels in Figure 4.4 we can see that the pattern of individual strategies in equilibrium depends on the relative size of the stocks own by two Cournot players. In either of the two examples, when the asymmetric shock causes one player to increase extraction, the competitor who holds sufficiently large stock would extract more, but not otherwise. For instance, example 1 in Panel (a) shows that asymmetric shock causes player 1 to increase extraction when risky stock k_1^s is relatively small. In this case player 2 also increase extraction because k_2 is sufficiently large as compared to k_1 ($k_1/k_2 = 1/2$). However, in Panels (b) and (c) player 2 extracts less when k_2 becomes relatively smaller ($k_1/k_2 = 1$ or 2). Example 2 in Panels (a) and (b) are cases where the asymmetric shock causes player 2 to increase extraction when k_2^s is relatively small. Because k_1 is sufficiently small as compared to k_2 in these cases ($k_1/k_2 = 1/2$ or 1), player 1 extracts less in equilibrium. However, Panel (c) shows that when k_1 becomes sufficiently large ($k_1/k_2 = 2$), player 1 extracts more in response to the increase of player 2's extraction.

The bottom panels in Figure 4.4 shows that the Pareto optimal solution could be distorted by Cournot competition with asymmetric shock. The distortion in quantity is also accompanied by a distortion in the order of extraction. In example 1, player 2 always extracts the risk-free stock k_2 simultaneously with player 1's extraction of the risky stock k_1^s. This violates the Pareto optimal solution that requires exhausting k_1^s before any positive amount of risk-free stock is extracted. In example 2, although both player extract risky stock, the Pareto optimal quantity still could be distorted by Cournot competition. We can see that the direction and scale of the distortion depends on the relative size of stocks owned by two players and the share of the risky stock.

4.4 Long-term impact of shock

The distortion from Cournot competition has a persistent impact on the transition path of extraction and resource stock, and therefore leads to a loss in social welfare.

4.4.1 Transition path

We now discuss how the transition path is distorted in the two examples using special cases where the initial stock and risk distributions between two players are as marked on the bottom figure of Panel (b) in Figure 4.4.

The special case of the first example, with initial condition $\hat{K}_0 = (k_1,\, k^s_{1o},\, k_2)$, has transition path given by

$$
\begin{aligned}
(\hat{\mathbb{C}},\, \hat{\mathbb{K}}) =& (\hat{C}_t,\, \hat{K}_t)_{t\geq 0} \\
\hat{C}_t =& \begin{pmatrix} \hat{c}_{1t} \\ \hat{c}_{2t} \end{pmatrix} \text{ is determined by (4.44)} \\
\mathrm{d}\hat{K}_t =& \begin{pmatrix} \mathrm{d}\hat{k}_{1t} \\ \mathrm{d}\hat{k}^s_{1t} \\ \mathrm{d}\hat{k}_{2t} \end{pmatrix} = -\begin{pmatrix} \hat{c}_{1t} \\ \hat{c}_{1t} \times \mathrm{I}\{\hat{k}^s_{1t} > 0\} \\ \hat{c}_{2t} \end{pmatrix},
\end{aligned}
\tag{4.51}
$$

where I is an indicator function equal to 1 if the condition in the brackets is true and 0 otherwise. For the second example, the transition path with initial condition $\hat{K}_0 = (k_1,\, k_2,\, k^s_{2o})$ is given by

$$
\begin{aligned}
(\hat{\mathbb{C}},\, \hat{\mathbb{K}}) =& (\hat{C}_t,\, \hat{K}_t)_{t\geq 0} \\
\hat{C}_t =& \begin{pmatrix} \hat{c}_{1t} \\ \hat{c}_{2t} \end{pmatrix} \text{ is determined by (4.47)} \\
\mathrm{d}\hat{K}_t =& \begin{pmatrix} \mathrm{d}\hat{k}_{1t} \\ \mathrm{d}\hat{k}_{2t} \\ \mathrm{d}\hat{k}^s_{2t} \end{pmatrix} = -\begin{pmatrix} \hat{c}_{1t} \\ \hat{c}_{2t} \\ \hat{c}_{2t} \times \mathrm{I}\{\hat{k}^s_{2t} > 0\} \end{pmatrix}.
\end{aligned}
\tag{4.52}
$$

The transition path that follows Pareto optimal solution, with initial condition $K_0 = (k,\, k^s)$, is given by

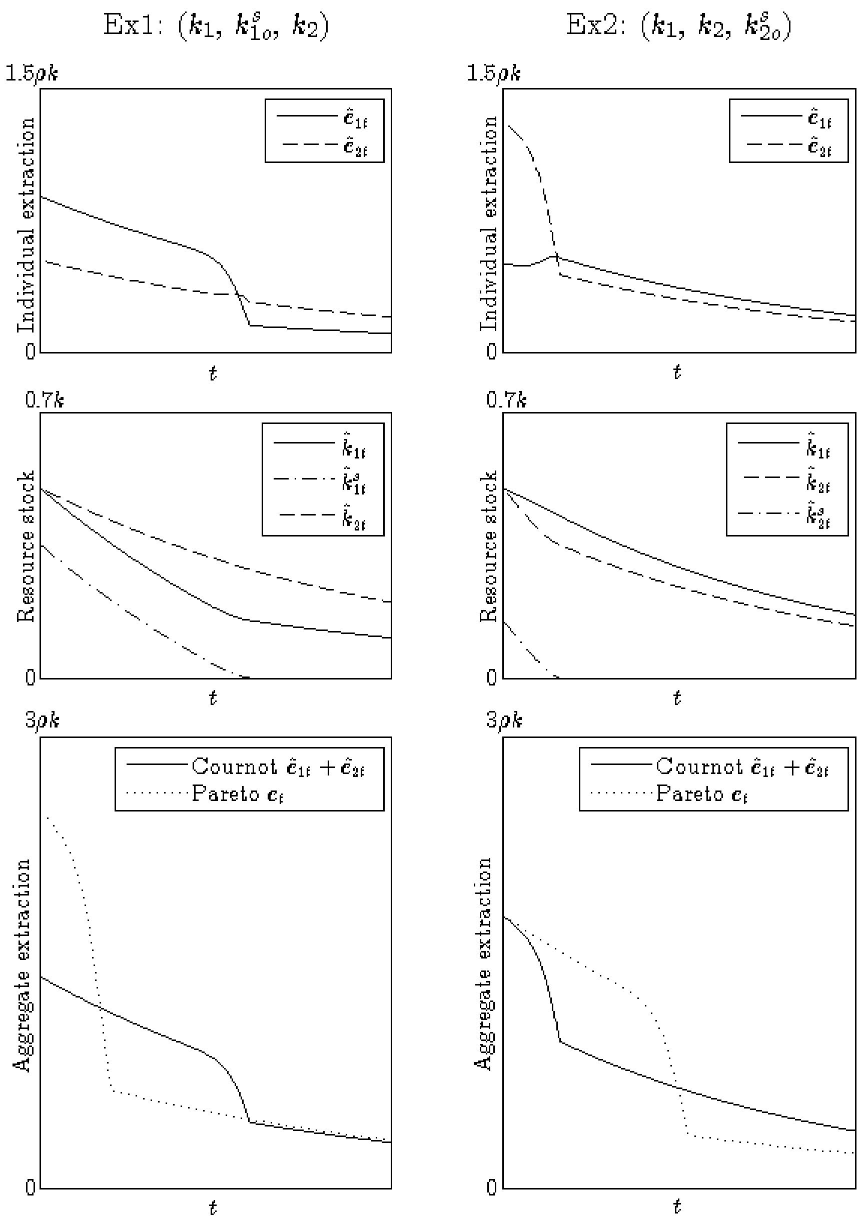
$$
\begin{aligned}
(\mathbb{C},\, \mathbb{K}) =& (c_t,\, K_t)_{t\geq 0} \\
& c_t \text{ is determined by (4.50)} \\
\mathrm{d}K_t =& \begin{pmatrix} \mathrm{d}k_t \\ \mathrm{d}k^s_t \end{pmatrix} = -\begin{pmatrix} c_t \\ c_t \times \mathrm{I}\{k^s_t > 0\} \end{pmatrix}.
\end{aligned}
\tag{4.53}
$$

These transition paths, from either Cournot equilibrium or Pareto solution, are *ex ante* in the sense that the potential asymmetric shock is known to the Cournot players or representative consumer but has not occurred yet. Figure 4.5 compares (4.51) and (4.52) with (4.53). The left panel plots the special case for the first example and the right panel plots the special case for the second example.

The transition path (4.51) in the first example has initial condition (k_1, k_{1o}^s, k_2) where $k_1 = k_2 = k/2$ and k_{1o}^s is marked on Figure 4.4 Panel (b). In this case Figure 4.4 shows that player 1 extracts more than player 2 initially. During the early stage of the transition path, the left panel of Figure 4.5 shows that the asymmetric shock that only affects player 1's risky stock k_{1o}^s causes player 1 to extract more than player 2 (top panel), leading to faster decrease of k_1 than k_2 (middle panel) and decreasing k_1/k_2. When the remainder of k_{1o}^s becomes sufficiently small as compared to k_1, $\hat{c}_1$ decreases much faster than earlier but the decreasing rate of $\hat{c}_2$ changes little. After k_{1o}^s is exhausted, the risk of shock is eliminated and Cournot competition has no distorting effect (Proposition 8). In this risk-free stage k_1 is smaller than k_2 because k_1 is more aggressively extracted in the earlier risky stage, leading to persistently lower $\hat{c}_1$ as compared to $\hat{c}_2$ in the long-run.

The bottom left panel of Figure 4.5 plots the transition path of aggregate extraction in Cournot equilibrium of the first example and the corresponding Pareto optimal extraction path given by (4.53). Because the Pareto path requires higher initial extraction from the risky stock k_{1o}^s than player 1 extracts in Cournot equilibrium, the exhaustion of k_{1o}^s is sooner on the Pareto path. The Pareto optimal extraction, although higher during the initial period, decreases at a faster rate initially and then is below the Cournot extraction path over an extended period of time. When k_{1o}^s is exhausted by player 1, the remainder path of the aggregate extraction in Cournot equilibrium is efficient conditioned on the remaining stock but is persistently lower than the Pareto path because Cournot competition causes more aggressive extraction earlier.

Figure 4.5: Transition path with an asymmetric shock



The right panel of Figure 4.5 plots the transition path (4.52) of the second example with the initial condition (k_1, k_2, k_{2o}^s) as marked on Figure 4.4 Panel (b). In this case the asymmetric shock affects player 1's entire stock k_1 and a portion k_{2o}^s of player 2's stock k_2. The asymmetric risk on k_2 causes player 2 to extract more than player 1 initially (top panel). This leads to faster decrease of k_2 than k_1 (middle panel) and increasing k_1/k_2. As the share of k_{2o}^s in k_2 decreases, player 2's extraction decreases but player 1's extraction increases over time. After k_{2o}^s is exhausted, the transition path enters a fully specialized stage where player 2's stock becomes completely risk-free but player 1's stock is fully risky. In this case, (4.47) with $g_2(0) = 1$ implies that both player extract a same constant fraction from their available stocks. Thus k_1 and k_2 decreases at same rate and k_2 is persistently lower than k_1, leading to persistently lower $\hat{c}_2$ than $\hat{c}_1$ in the long-run.

Comparing the aggregate extraction in the Cournot equilibrium and the corresponding Pareto optimal solution given by (4.53), the bottom right panel of Figure 4.5 shows that the distortion is very limited initially. However, as the remainder of k_{2o}^s in k_2 becomes smaller, Cournot competition causes greater distortion compared to the optimal extraction on the Pareto path. Because the entire k_1 is affected by the potential shock, the risky stock is never exhausted along the Cournot path in the second example. Thus the asymmetric shock affects the entire Cournot equilibrium transition path, which is inefficient even after k_{2o}^s is exhausted by player 2. This causes persistent distortion because on the Pareto path the risky stock $(k_1 + k_{2o}^s)$ would be exhausted in finite time. Figure 4.5 shows that in our special case the Pareto path of extraction is lower than the Cournot path in the long-run. But according to (4.47), the long-run relative levels of Pareto path and Cournot path depend on model parameters.

4.4.2 Welfare loss

The distortion in extraction caused by Cournot competition with potential asymmetric shock leads to a loss in social welfare. Because the occurrence of the shock is stochastic, social welfare is measured as an *ex ante* expectation. This is different from the two-period model where the shock occurs deterministically.

Suppose the occurrence of potential shock is realized at $T \geq 0$, then according to (4.34) the probability of this special case is $\Lambda(T)$. After the occurrence the remainder of the resource stock and corresponding transition path become risk-free, and by Proposition 8 Cournot competition would not cause any loss in social welfare. Then for this special case, the social welfare from Cournot path (4.51) in the first example is

$$\hat{\Pi}(\hat{\mathbb{C}}\,|\,T) = \int_0^T e^{-\rho t}\log(\hat{c}_{1t} + \hat{c}_{2t})\mathrm{d}t + e^{-\rho T} V(\theta_1 \hat{k}_{1T} + \hat{k}_{2T})$$

where V is the risk-free value function given by (4.39) with $\gamma \to 1$ and θ_1 is given by (4.18). Then, as in Reed [44] and Reed [45], the expected social welfare is

$$\begin{aligned}\hat{\Pi}(\hat{\mathbb{C}}) &= \int_0^\infty \hat{\Pi}(\hat{\mathbb{C}}\,|\,T)\mathrm{d}\Lambda(T) \\ &= \int_0^\infty e^{-(\rho+\lambda)t}[\log(\hat{c}_{1t} + \hat{c}_{2t}) + \lambda V(\theta_1 \hat{k}_{1t} + \hat{k}_{2t})]\mathrm{d}t. \end{aligned} \tag{4.54}$$

Similarly, the expected social welfare from Cournot path (4.52) in the second example is

$$\hat{\Pi}(\hat{\mathbb{C}}) = \int_0^\infty e^{-(\rho+\lambda)t}[\log(\hat{c}_{1t} + \hat{c}_{2t}) + \lambda V(x\hat{k}_{1t} + \theta_2 \hat{k}_{2t})]\mathrm{d}t \tag{4.55}$$

where θ_2 is given by (4.22). For the Pareto transition path (4.53) the maximized social welfare is measured by

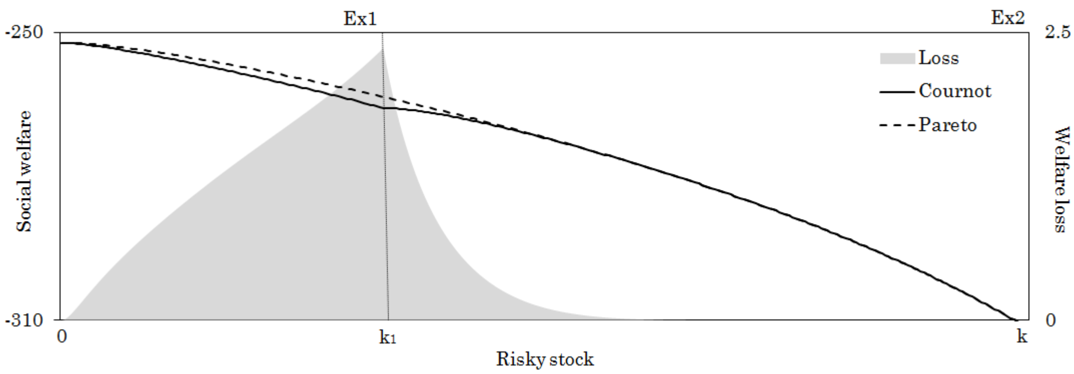
$$\Pi(\mathbb{C}) = \int_0^\infty e^{-(\rho+\lambda)t}[\log c_t + \lambda V(\theta k_t)]\mathrm{d}t \tag{4.56}$$

where θ is given by (4.26). According to the standard argument of principle of optimality, $\Pi(\mathbb{C})$ given by (4.56) is equivalent to $W(k^s,\, k^n)$ given by (4.49).

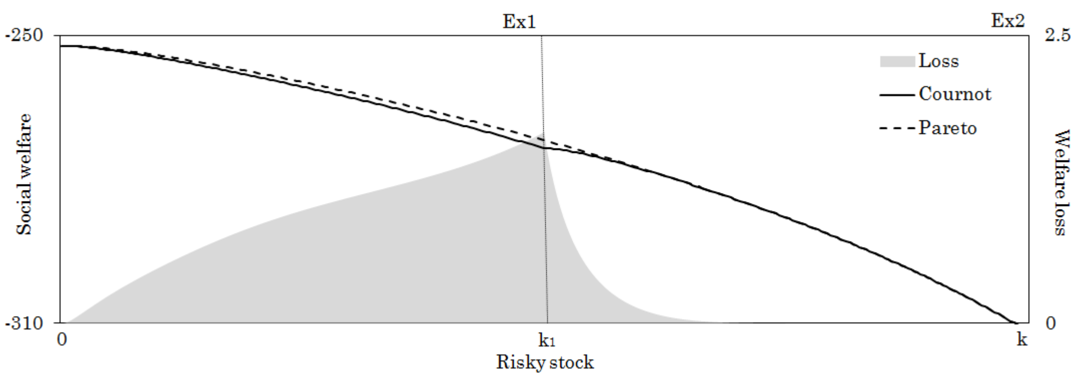
Using welfare measures (4.54) and (4.55) we calculate the social welfare from the Cournot path and compare it with the Pareto optimal solution (4.56) in Figure 4.6. The results are plotted against the share of risky stock based on the parameters used before in each of the three panels for $k_1/k_2 = 1/2$, 1 or 2.

Figure 4.6: Welfare loss in the infinite horizon model

$k_1/k_2 = 1/2$



$k_1/k_2 = 1$



$k_1/k_2 = 2$

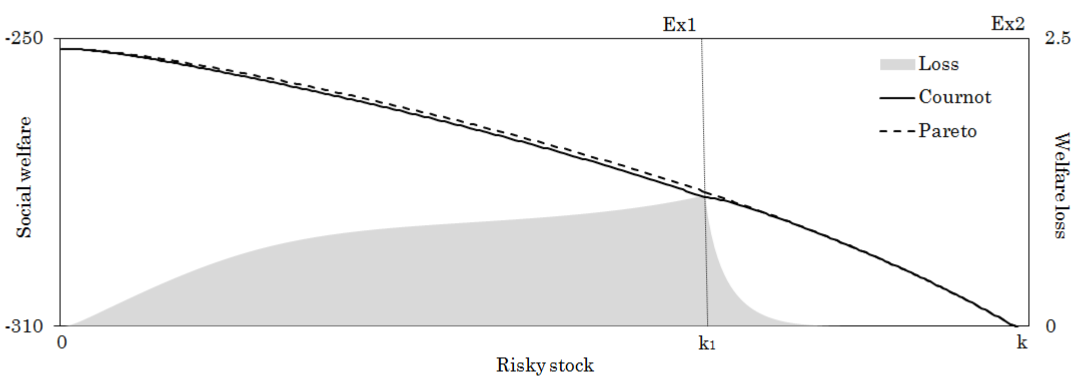


Figure 4.6 shows that, in general an increase in the share of stock under the threat of shock leads to lower welfare. This is true for both Cournot equilibrium and Pareto optimal solution. As predicted by Proposition 8, there is no loss in social welfare in the Cournot equilibrium as compared to the Pareto optimal outcome when the risky stock is 0 (no shock) or the entire k is under the risk (symmetric shock).

However, Cournot competition causes positive welfare losses when the risk only affects a portion but not all of the stock. Given k_1 and k_2, an increase in the share of risky stock k_1^s in k_1 leads to higher welfare loss. This result occurs because player 1 exhausts the risky stock k_1^s in longer time when k_1^s/k_1 is higher, so that there is longer period of time when player 1's extraction is constrained by risk and Cournot equilibrium extraction is therefore distorted. The largest welfare loss occurs when k_1^s increases to k_1. In this case player 1's entire stock is risky thus the risky stock will never be exhausted. In this case the whole transition path is distorted by Cournot competition.

When a positive share of player 2's stock k_2 is also affected by potential shock, the welfare loss decreases as the share of risky stock k_2^s in k_2 increases. This result occurs because, as shown on Figure 4.4, when the the potential shock affects a relatively large share of player 2's stock, the distortion of aggregate extraction is very small. Thus the distortion on transition path is small over prolonged time until player 2 significantly reduces the share of the risky stock when the distortion on extraction becomes large. This leads to lower welfare loss because the larger distortion in the further future is heavily discounted. When player 2 initially has a relatively small share of risky stock, the distortion is much larger for the entire transition path, leading to larger welfare loss.

4.5 Conclusion

We studied the effect of a shock on Cournot competition in an exhaustible resource market. We considered symmetric and asymmetric shocks separately. In the case of a symmetric shock, we showed that Cournot equilibrium coincides with the Pareto optimal outcome with isoelastic demand and zero extraction cost. This result occurs because, similar to the risk-free monopolistic model of Stiglitz [59], Cournot players make extraction decisions based on quantity dependent marginal revenue which is proportional to the market price and the proportion only depends on the constant elasticity of demand. Thus with constant elasticity the distortion effect cancels out across time and the Cournot equilibrium is therefore equivalent to the Pareto optimal decision.Without distortion, the maximum social welfare from the Pareto optimal solution is achieve in Cournot equilibrium.

However, we found that Cournot competition distorts the Pareto optimal solution in many cases when the risk of shock affects the Cournot players' stocks asymmetrically. We used two examples and showed that the direction and scale of distortion in extraction are complicated and depend on the relative size of the stocks owned by two players and the share of stock that is under the risk of shock.

With an asymmetric shock, Cournot competition is distorting and has persistent impact on the transition path and therefore leads to a loss in social welfare. In our examples we showed that when the shock only affects a portion of one player's stock and has no impact on the other player's stock, the welfare loss is increasing in the share of risky stock. This result occurs because it takes a longer time for the first player to exhaust risky stock when its share is larger and eliminate the distortion that occurs while there is an asymmetric shock. However, when the risk of shock also affects a portion of the second player's stock, the welfare loss starts to decrease in the share of risky stock. In this case the distortion is tiny when a relatively large portion of the second player's stock is risky, and not significant until a sufficient amount of the risky stock is extracted. Thus higher share of risky stock causes lower welfare loss because the distortion is small over prolonged time of the initial part of the transition path and the larger distortion in the further future is heavily discounted. In our examples the largest welfare loss occurs in the fully specialized case where the first player owns all the risky stock and second player owns all the risk-free stock. This is because the first player will never exhaust the risky stock and the whole transition path is under the risk of asymmetric shock.

We kept our model lean to derive analytical results that focus on the allocation of risk. Doing so meat neglecting some other interesting issues. First, we assume a simple market structure with two Cournot players where each of them owns a private stock. Thus our model has very limited implication to the common property problem or to the cartel-fringe models.[10] In addition, the risk in our model is characterized by a single-time irreversible shock with constant hazard rate. This simplification ignores some real world complications including time-varying risks and multiple potential shocks. Moreover, our model treats exhaustible resource as the final consumption good thus the consumption and utility flow converge to zero asymptotically. A more sophisticated model that could generate sustainable growth must also include the reproducible capital and consider an exhaustible resource as an intermediate input. These extensions would enhance the model's predictions and could be appropriate directions for future research.

4.6 Technical proof

Proof of Proposition 9. Let $(c_{1t},\, c_{2t})_{t\geq 0}$ denote the subgame perfect Cournot equilibrium that satisfies the dynamic programming problem (4.43), we have

$$W^1(\phi,\,\tau_1) + W^2(\phi,\,\tau_1) = \int_0^\infty e^{-\rho t}\frac{c_{1t}}{c_{1t}+c_{2t}}\mathrm{d}t + \int_0^\infty e^{-\rho t}\frac{c_{2t}}{c_{1t}+c_{2t}}\mathrm{d}t = \frac{1}{\rho}.$$

Thus

$$W^1_\phi + W^2_\phi = 0 \qquad \text{and} \qquad W^1_{\tau_1} + W^2_{\tau_1} = 0.$$

Then the first order conditions of (4.43) imply

$$\text{Player 1} \quad \frac{c_2}{(c_1+c_2)^2} = \frac{\phi W^1_\phi}{k_1} + \frac{(1-\tau_1)W^1_{\tau_1}}{k_1} = -\frac{\phi W^2_\phi}{k_1} - \frac{(1-\tau_1)W^2_{\tau_1}}{k_1} \tag{4.57}$$

$$\text{Player 2} \quad \frac{c_1}{(c_1+c_2)^2} = -\frac{\phi W^2_\phi}{k_2} = \frac{\phi W^1_\phi}{k_2}. \tag{4.58}$$

Substituting (4.57) and (4.58) into (4.43) we find

$$W^1(\phi,\,\tau_1) = \frac{1}{\rho+\lambda}\left(\frac{c_1}{c_1+c_2}\right) + \frac{\lambda}{\rho(\rho+\lambda)}\left(\frac{\theta_1\phi}{\theta_1\phi+1}\right) \tag{4.59}$$

$$W^2(\phi,\,\tau_1) = \frac{1}{\rho+\lambda}\left(\frac{c_2}{c_1+c_2}\right) + \frac{\lambda}{\rho(\rho+\lambda)}\left(\frac{1}{\theta_1\phi+1}\right) \tag{4.60}$$

where the second term on the right hand side of both equations are according to (4.33) with $\gamma = 1$ and (4.42). Taking a guess that

$$\frac{c_1}{c_2} = g_1(\tau_1)\phi, \tag{4.61}$$

[10] This literature is from the pioneering work of Salant [49] and followed by many other authors

we can solve W^1_ϕ, $W^1_{\tau_1}$ and W^2_ϕ from (4.59) and (4.60). Then (4.57) and (4.58) imply

$$\begin{aligned} & \frac{g_1(\tau_1)k_1+k_2}{c_1+c_2} \\ \text{Player 1} \quad &= \frac{g_1(\tau_1)+(1-\tau_1)g'(\tau_1)}{\rho+\lambda}+\frac{\lambda x}{\rho(\rho+\lambda)}\left[\frac{g_1(\tau_1)\phi+1}{\theta_1\phi+1}\right]^2 \\ \text{Player 2} \quad &= \frac{1}{\rho+\lambda}+\frac{\lambda}{\rho(\rho+\lambda)}\frac{\theta_1}{g_1(\tau_1)}\left[\frac{g_1(\tau_1)\phi+1}{\theta_1\phi+1}\right]^2. \end{aligned} \tag{4.62}$$

The second equality of (4.62) yields (4.45) which is a differential equation in τ_1. The boundary condition of $g_1(\tau_1)$ is set as $g_1(0)=1$ to ensure that Cournot equilibrium (4.44) reduces to the risk-free form $(\rho k_1,\, \rho k_2)$ when $\tau_1=0$. Then using (4.61) and (4.62) we can solve (4.44). □

Proof of Proposition 10*.* Similar as the proof of Proposition 9, we have

$$W^1(\phi,\,\tau_2)=\frac{1}{\rho+\lambda}\left(\frac{c_1}{c_1+c_2}\right)+\frac{\lambda}{\rho(\rho+\lambda)}\left(\frac{x\phi}{x\phi+\theta_2}\right) \tag{4.63}$$

$$W^2(\phi,\,\tau_2)=\frac{1}{\rho+\lambda}\left(\frac{c_2}{c_1+c_2}\right)+\frac{\lambda}{\rho(\rho+\lambda)}\left(\frac{\theta_2}{x\phi+\theta_2}\right) \tag{4.64}$$

Taking a guess that

$$\frac{c_1}{c_2}=\frac{\phi}{g_2(\tau_2)}, \tag{4.65}$$

the first order conditions of (4.46) imply

$$\begin{aligned} & \frac{k_1+g_2(\tau_2)k_2}{c_1+c_2} \\ \text{Player 1} \quad &= \frac{1}{\rho+\lambda}+\frac{\lambda x}{\rho(\rho+\lambda)}\frac{\theta_2}{g_2(\tau_2)}\left[\frac{\phi+g_2(\tau_2)}{x\phi+\theta_2}\right]^2 \\ \text{Player 2} \quad &= \frac{g_2(\tau_2)+(1-\tau_2)g'(\tau_2)}{\rho+\lambda}+\frac{\lambda x^2}{\rho(\rho+\lambda)}\left[\frac{\phi+g_2(\tau_2)}{x\phi+\theta_2}\right]^2. \end{aligned} \tag{4.66}$$

The second equality of (4.66) yields (4.48) which is a differential equation in τ_2. To verify the boundary condition of $g_2(\tau_2)$ is $g_2(0)=1$, note that Cournot model (4.46) with $\tau_2=0$ is equivalent to Cournot model (4.43) with $\tau_1=1$. Thus $g_2(0)=g_1(1)$. When $\tau_1=1$, (4.18) shows that $\theta_1=x$. Then given $g_1'(1)\in\mathbb{R}$, using (4.45) we find $g_1(1)=1$ and $[1+g_1(1)\phi]^2+g_1(1)\rho(1+x\phi)^2/(\lambda x)=0$. Both solutions of the quadratic function are negative and therefore dropped. Then using (4.65) and (4.66) we can solve (4.47). □

Chapter 5

Conclusion

This dissertation includes three closely related essays on natural resource management with a potential regime shift. The first essay shows that the risk of a potential regime shift could cause the optimal management of renewable resources to be precautionary, unchanged, or aggressive as compared to the risk-free case. Which of these outcomes occur depends on the relative magnitudes of three competing effects: the risk reduction effect, the consumption smoothing effect, and the investment effect. Both the risk reduction effect and the consumption smoothing effect cause management to be more precautionary. The investment effect has the opposite impact and causes management to be more aggressive. An analytical condition is provided that shows when aggressive management will occur. This result is in contrast with recent papers where a regime shift will cause management to be more precautionary [15, 42]. In a numerical simulation it is shown that a potential regime shift can generate more aggressive management for reasonable parameter values. In particular, a regime shift that lowers the carrying capacity of the renewable resource is more likely to cause aggressive management. In contrast, management tends to be more precautionary with a potential regime shift that lowers the intrinsic growth rate.

The second essay analyzes optimal management under the risk of asymmetric regime shift, which is simplified as a shock that reduces the stock of an exhaustible resource. Without extraction cost, the optimal management under asymmetric shock requires a sequential extraction plan which first exhausts the stock that is under the threat of shock. Under this pattern of management, the optimal extraction rate changes non-monotonically in the share of risky stock, due to the change in the relative magnitude of the substitution effect and the income effect caused by the risk of shock. With log utility, it is shown that when the share of risky stock is relatively low, an increase in the share of risky stock leads to higher extraction. However, when the share of risky stock is sufficiently high, further increase of it provides incentive to extract less. This result is used to analyze the transition path. With the risk of asymmetric shock, the optimal extraction is higher than the risk-free case initially, leading to faster decrease of resource stock and persistently lower extraction in the future.

The third essay applies the asymmetric regime shift model of exhaustible resource in a duopolistic market. In the benchmark model of symmetric risk, Cournot equilibrium coincides with the Pareto optimal outcome for the case with isoelastic demand and zero extraction cost. However, Cournot competition distorts the Pareto optimal solution in many cases when the risk of shock affects the Cournot players' stocks asymmetrically. Some examples are used to show that the direction and scale of distortion could be complicated and depend on the relative size of the stocks owned by two players and the share of stock that is under the risk of shock. Asymmetric risk has impacts on the extraction path and leads to a loss in social welfare. In the case where the shock only affects a portion of one player's stock and has no impact on the other player's stock, welfare loss is increasing in the share of risky stock. The largest welfare loss occurs in the fully specialized case where the first player owns all the risky stock and second player owns all the risk-free stock. When the risk of shock is further spread out to affect a portion of the second player's stock, welfare loss starts to decrease in the share of risky stock.

While this dissertation investigates several important effects of potential regime shift on natural resource management, the results could be expanded in a number of dimensions. For example, regime shifts discussed in this dissertation are all on the supply side and have no direct impact on the utility function and corresponding demand; the risk is charactered by simple reduced form hazard function that has no explanatory power on the underlying mechanisms that govern regime shifts; the occurrence of regime shift is simplified as a single-time irreversible event thus repeating events with the potential to flip back and forth among regimes are ignored. Addressing these limits could be appropriate directions for future research.

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