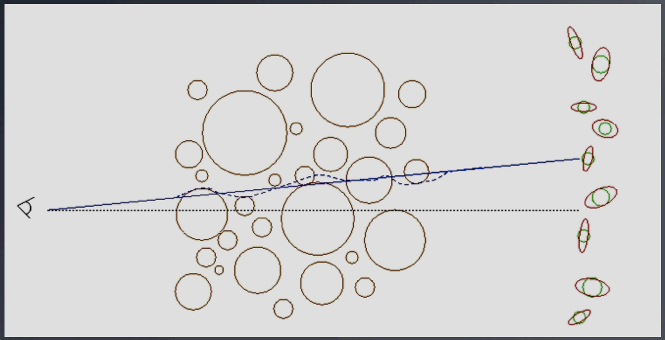
Weak lensing of CMB polarization

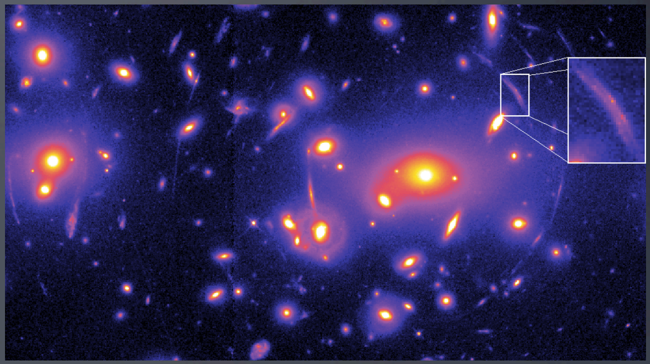
Uros Seljak
UC Berkeley/LBNL
Minneapolis, Jan 15, 20135

Overview

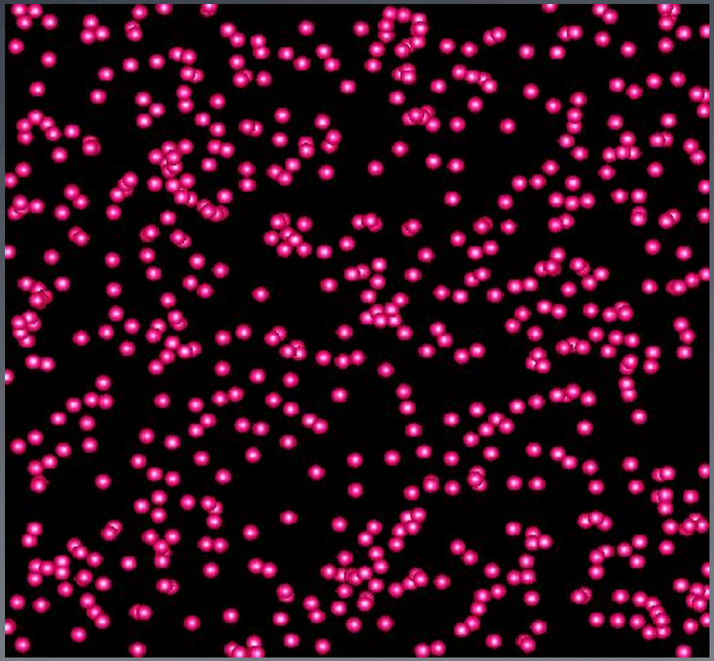
- a review of CMB polarization lensing, and theoretical challenges
- Basics of WL in CMB polarization: the good and the bad
- What can we measure?
- Future of WL CMB

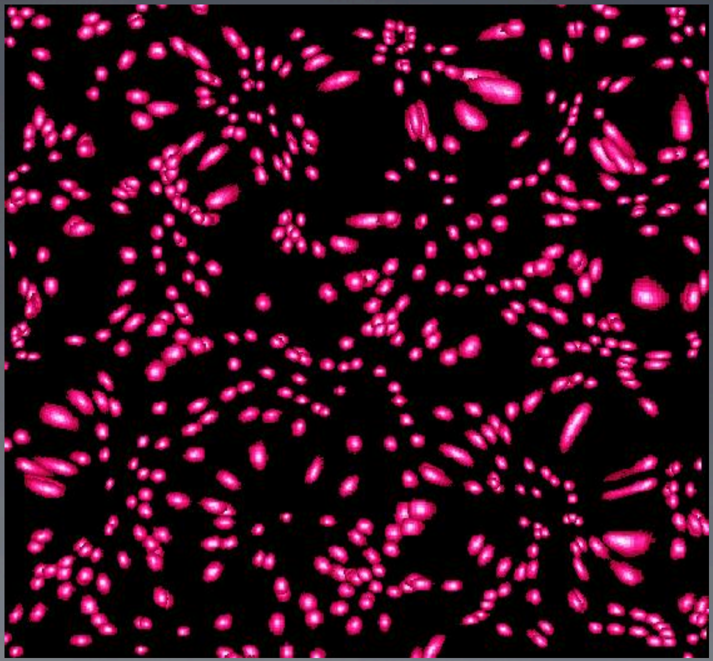
Weak Gravitational Lensing





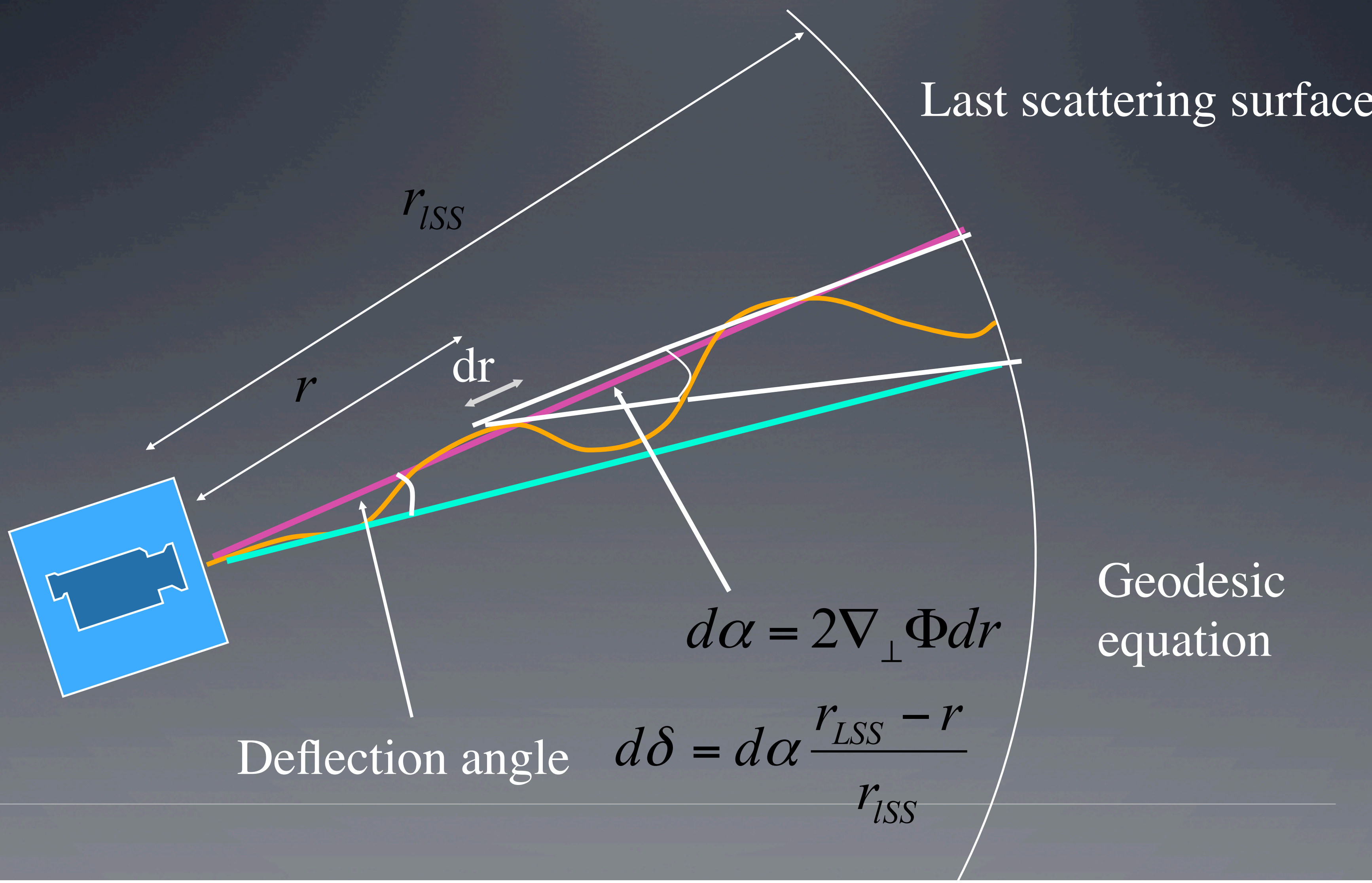
Distortion of background images by foreground matter





Unlensed **Lensed**

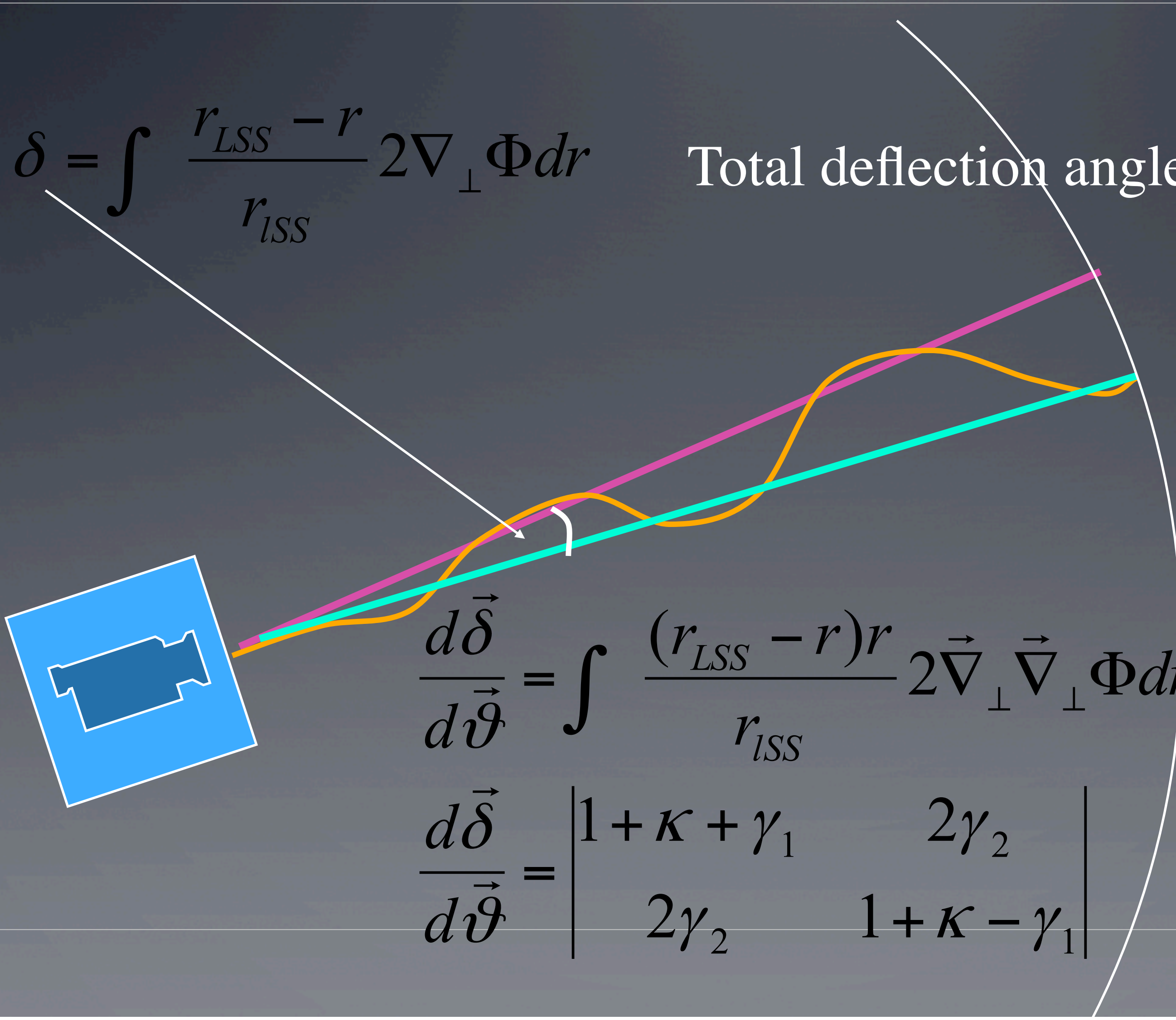
Gravitational lensing of CMB



Deflection angle and shear matrix

$$\delta = \int \frac{r_{LSS} - r}{r_{lSS}} 2\nabla_\perp \Phi dr$$

Total deflection angle



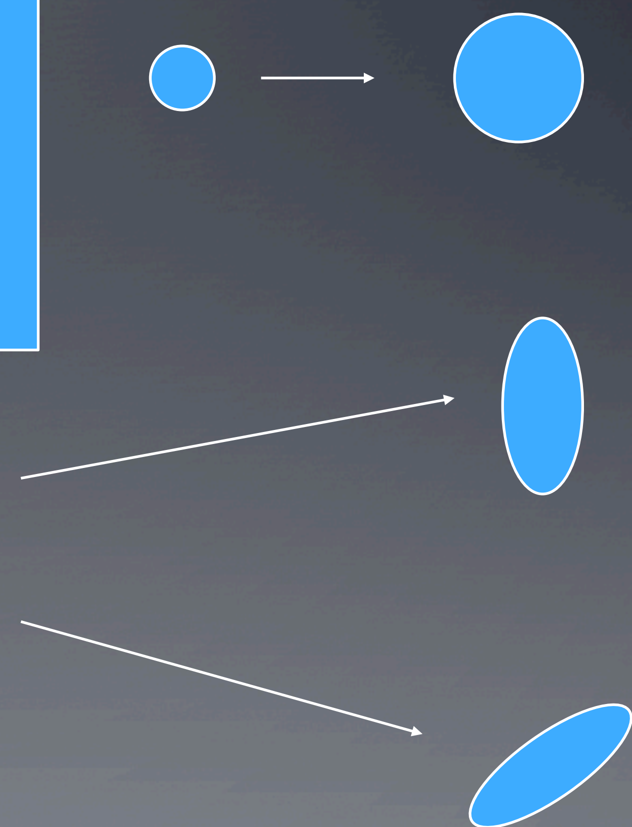
$$\frac{d\vec{\delta}}{d\vec{\vartheta}} = \int \frac{(r_{LSS} - r)r}{r_{lSS}} 2\vec{\nabla}_\perp \vec{\nabla}_\perp \Phi dr$$

$$\frac{d\vec{\delta}}{d\vec{\vartheta}} = \begin{vmatrix} 1+\kappa+\gamma_1 & 2\gamma_2 \\ 2\gamma_2 & 1+\kappa-\gamma_1 \end{vmatrix}$$

Convergence and shear

convergence

$$\kappa = \int \frac{(r_{LSS} - r)r}{r_{lSS}} \vec{\nabla}^2 \Phi dr = \frac{3}{2}\Omega_m H_0^2 \int \frac{(r_{LSS} - r)r}{r_{lSS}} dr \frac{\delta}{a}$$



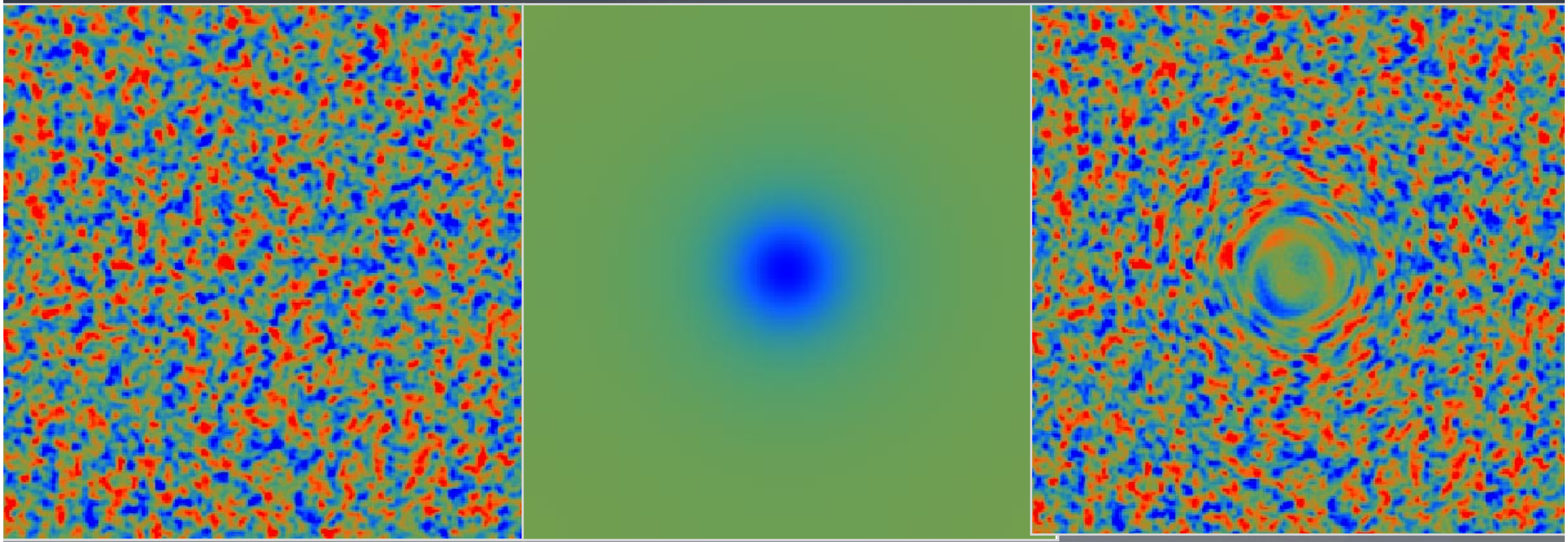
Convergence shear relation in Fourier space

$$\gamma_1(\vec{l}\,) = \kappa(\vec{l}\,)\cos 2\varphi_l$$
$$\gamma_2(\vec{l}\,) = \kappa(\vec{l}\,)\sin 2\varphi_l$$

Effect of gravitational lensing on CMB

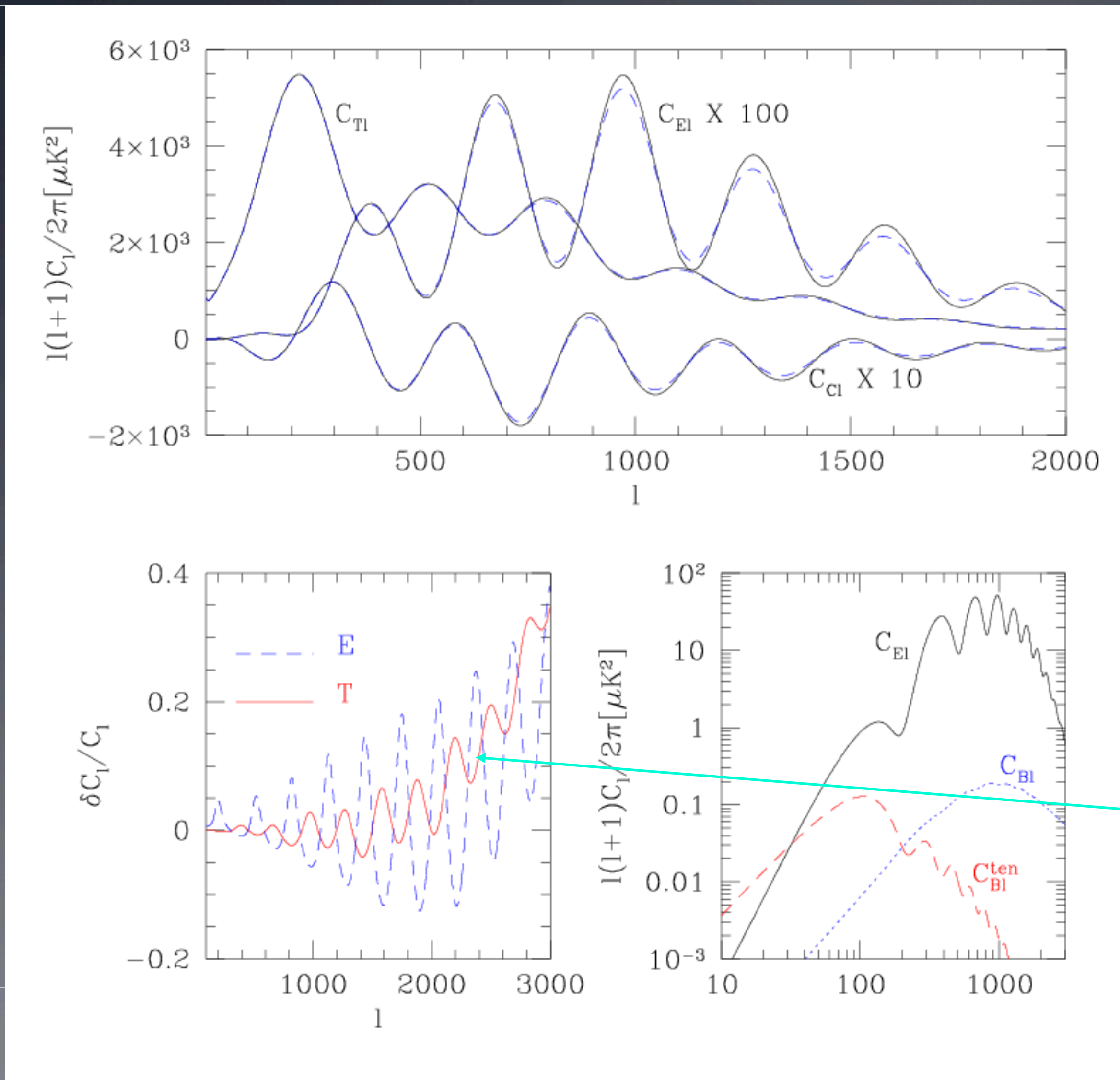
$$T_{lensed}(\vec{\mathbf{n}}) = T_{unlensed}(\vec{\mathbf{n}} + \vec{\mathbf{d}}) \qquad \vec{\mathbf{d}} = -2\vec{\nabla}\nabla^{-2}\kappa$$

- Here κ is the convergence and is a projection of the matter density perturbation.



Okamoto and Hu 2002

Lensing effect on CMB power spectra

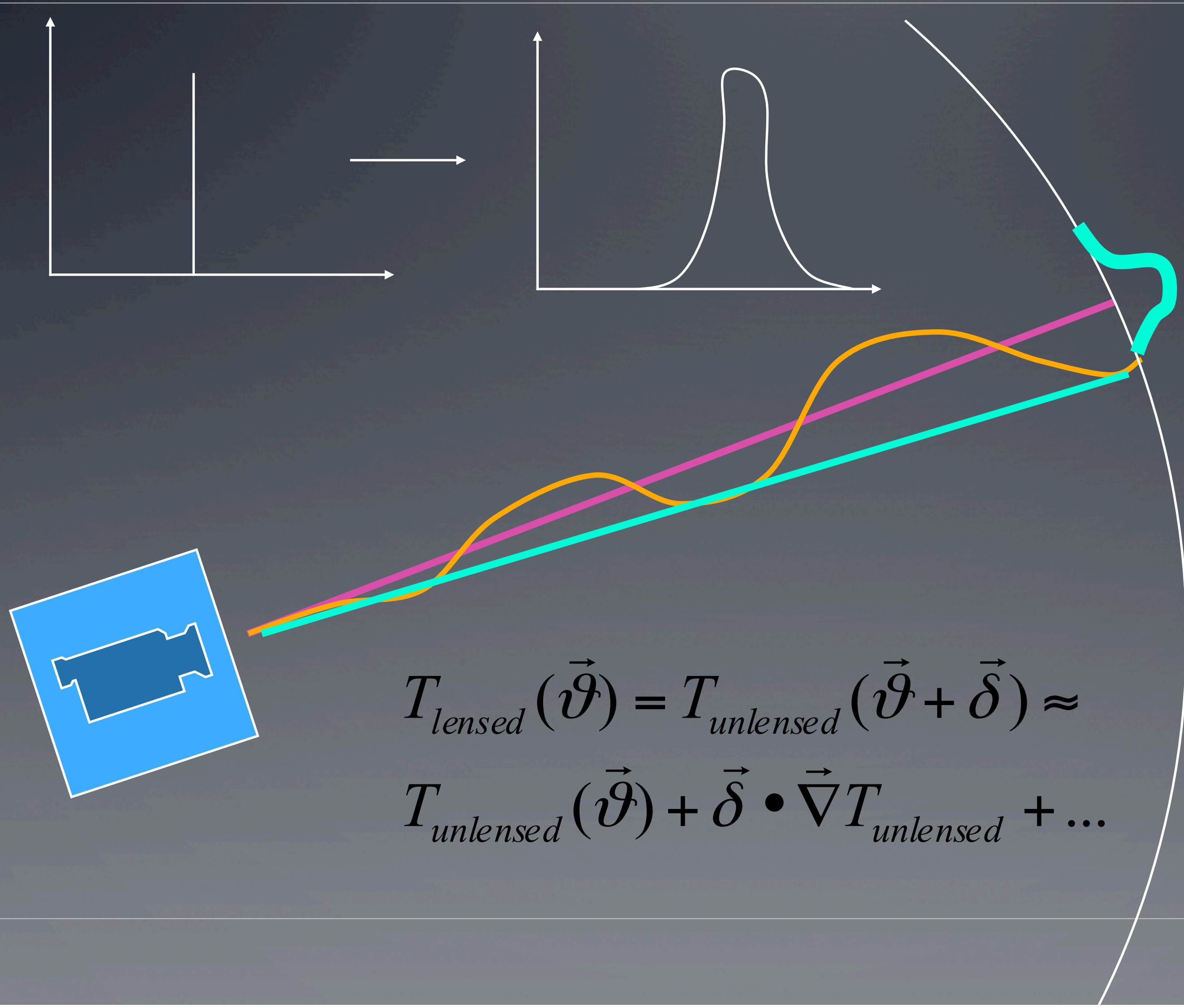


Smoothing and power transfer

US 1995

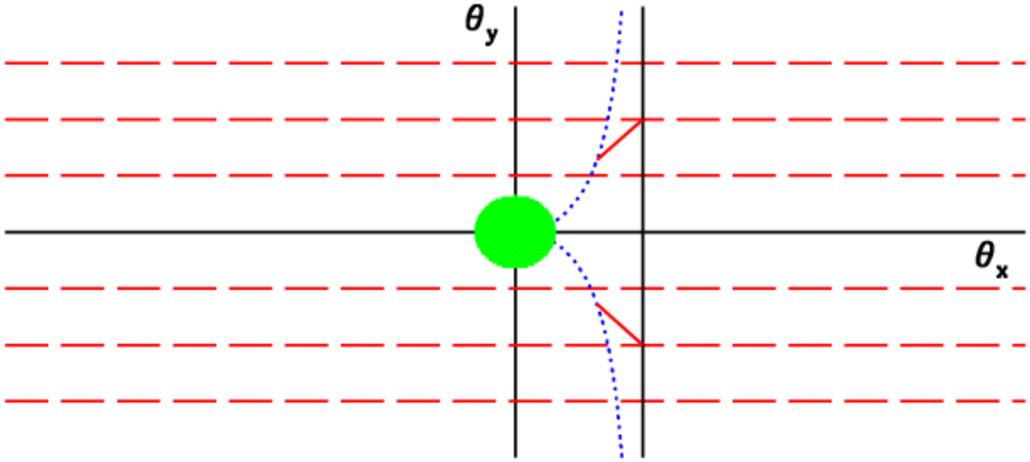
Zaldarriaga and US 1998

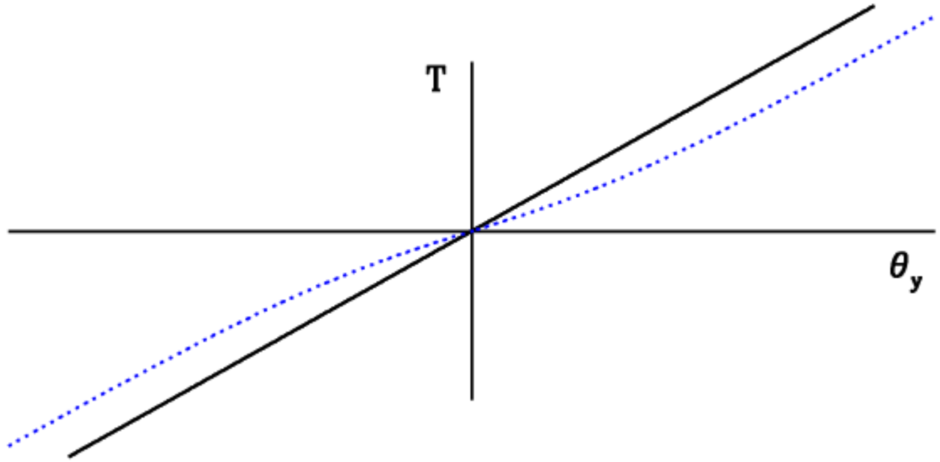
Lensing effect on CMB: smothing of peaks



$$T_{lensed}(\vec{\vartheta}) = T_{unlensed}(\vec{\vartheta} + \vec{\delta}) \approx$$
$$T_{unlensed}(\vec{\vartheta}) + \vec{\delta} \bullet \vec{\nabla} T_{unlensed} + \ldots$$

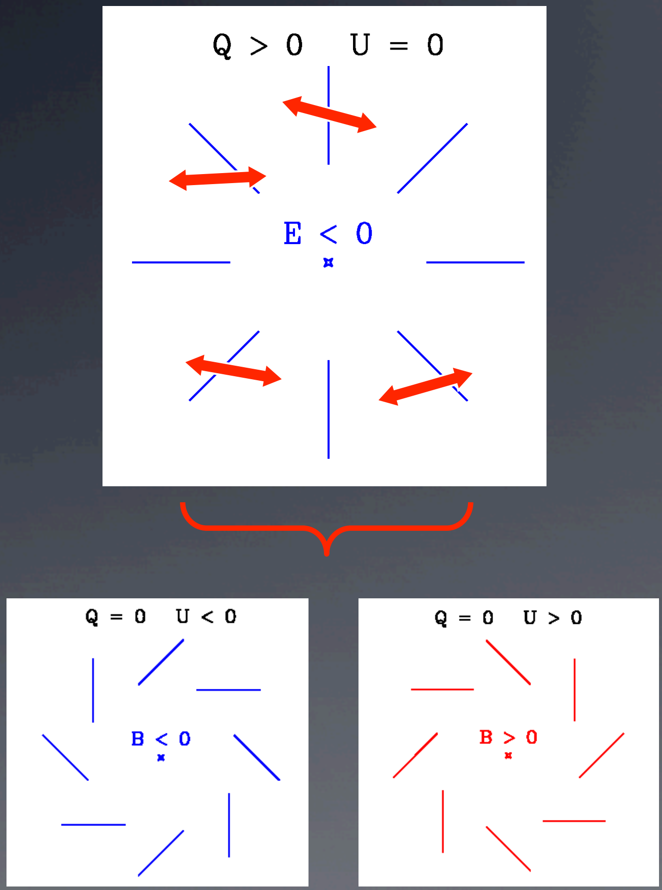
Lensing effect on CMB at high l: power transfer

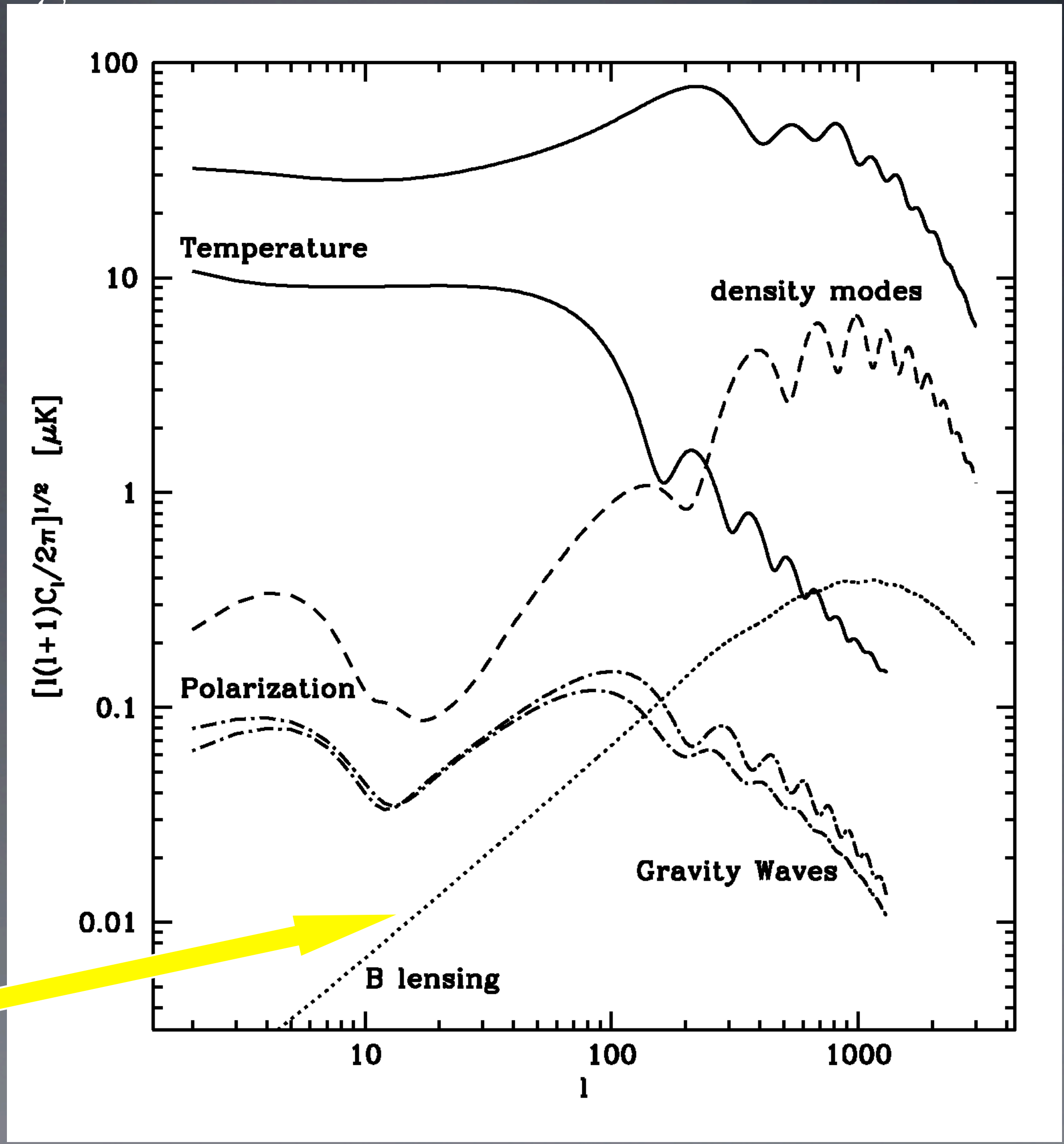




Gravitational Lensing of B polarization

Zaldarriaga & US 1998





Gravitational lensing is a contaminant to B polarization

Can we remove it?

Reconstruction of lensing I:

Zaldarriaga and US 1998

$$\kappa \propto (\nabla_x T)^2 + (\nabla_y T)^2$$

$$\gamma_1 \propto (\nabla_x T)^2 - (\nabla_y T)^2$$

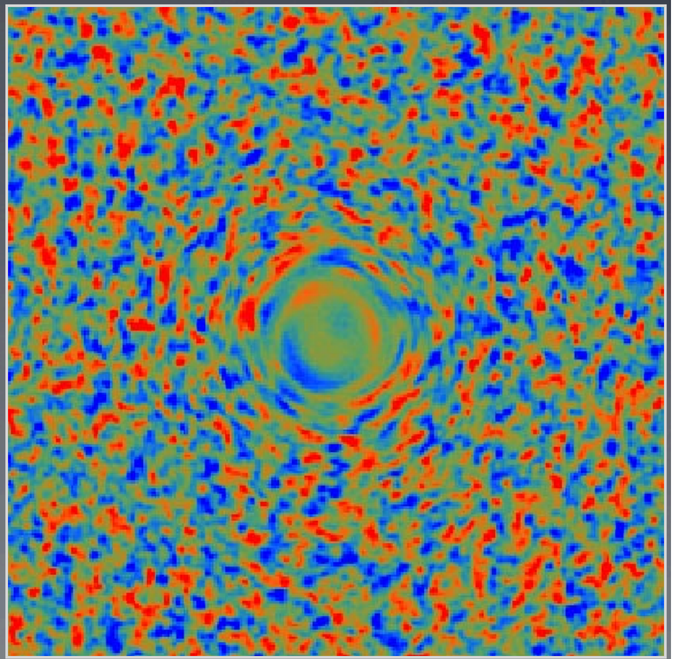
$$\gamma_2 \propto 2(\nabla_x T)(\nabla_y T)$$

Local estimate of typical patch size or shape

Compare to global average

Must be quadratic in T

This is close to optimal for T in Planck, but not for higher resolution experiments



Reconstruction of lensing II:

Hu 2000, Okamoto and Hu 2002

Nonvanishing off-diagonal terms of 2-point correlator give estimator for lensing potential

Estimation procedure identical to C_l estimator: quadratic estimator is the 1st order step in Newton-Raphson's procedure to maximize likelihood function

$$T_{lensed}(\vec{\vartheta}) = T_{unlensed}(\vec{\vartheta} + \vec{\delta}) \approx T_{unlensed}(\vec{\vartheta}) + \vec{\delta} \bullet \vec{\nabla} T_{unlensed} + \ldots$$

$$T_{lensed}(\vec{L}) = T_{unlensed}(\vec{L}) + \sum_{l} T_{unlensed}(\vec{l})(\vec{L} - \vec{l}) \bullet \vec{l}\, \varphi(\vec{L} - \vec{l}) + \ldots$$

$$\vec{\delta}(\vec{l}) = \vec{l}\, \varphi(\vec{l})$$

$$\overleftrightarrow{C} = \left\langle T(\vec{l}) T(\vec{l}\,') \right\rangle = C_l \delta_{ll'} + (\vec{l} - \vec{l}\,')(C_l \vec{l} - C_{l'} \vec{l}\,') \varphi(\vec{l} - \vec{l}\,')$$

$$\varphi(\vec{l}) = \frac{1}{2} F_{ll'}^{-1} (\vec{T} C^{-1} \frac{\partial \overleftrightarrow{C}}{\partial \varphi(\vec{l}\,')} C^{-1} \vec{T})$$

Gravitational lensing in CMB: reconstruction of lensing

$$\kappa \propto (\nabla_x T)^2 + (\nabla_y T)^2$$
$$\gamma_1 \propto (\nabla_x T)^2 - (\nabla_y T)^2$$
$$\gamma_2 \propto 2(\nabla_x T)(\nabla_y T)$$

Local estimate of typical patch size or shape

Compare to global average

Zaldarriaga & US 1998

$$T_{lensed}(\vec{\vartheta}) = T_{unlensed}(\vec{\vartheta}+\vec{\delta}) \approx T_{unlensed}(\vec{\vartheta}) + \vec{\delta}\bullet\vec{\nabla}T_{unlensed} + \ldots$$
$$T_{lensed}(\vec{L}) = T_{unlensed}(\vec{L}) + \sum_{l} T_{unlensed}(\vec{l})(\vec{L}-\vec{l})\bullet\vec{l}\,\varphi(\vec{L}-\vec{l}) + \ldots$$
$$\vec{\delta}(\vec{l}) = \vec{l}\,\varphi(\vec{l})$$
$$\vec{C} = \left\langle T(\vec{l})T(\vec{l}\,')\right\rangle = C_l\delta_{ll'} + (\vec{l}-\vec{l}\,')(C_l\vec{l} - C_{l'}\vec{l}\,')\varphi(\vec{l}-\vec{l}\,')$$
$$\varphi(\vec{l}) = \frac{1}{2}F_{ll'}^{-1}(\vec{T}C^{-1}\frac{\partial\vec{C}}{\partial\varphi(\vec{l}\,')}C^{-1}\vec{T})$$

Optimal quadratic estimator

Okamoto and Hu 2002

Reconstruction of lensing III: ML method in polarization

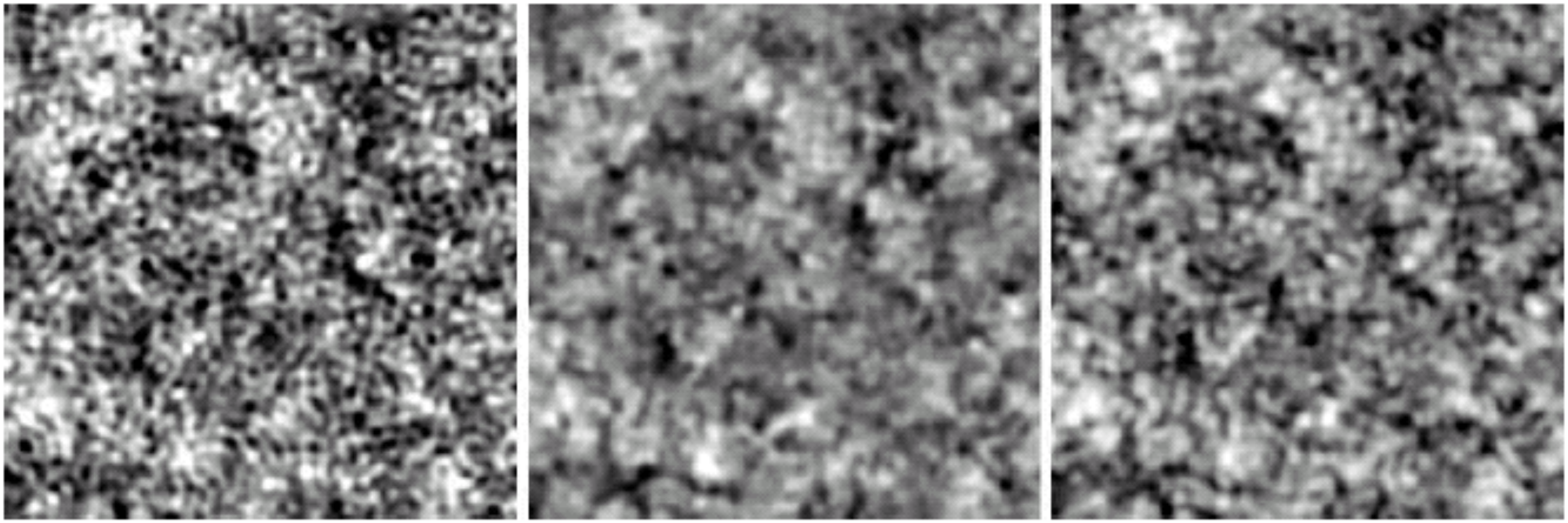
Hirata and US 2003

- For low detector noise main statistical information is provided by B mode polarization:
  → B mode polarization is not present in primary anisotropy (except for non-scalar modes)
  → Therefore with B mode polarization we measure lensing, we are not limited by statistical fluctuations in the primary CMB, rather by noise, systematics, foregrounds, …

- If these issues can be controlled, measuring B mode polarization is the ultimate CMB lensing experiment.

Lensing with CMB polarization

INPUT QUADRATIC ITERATIVE



(Hirata & US 2003)

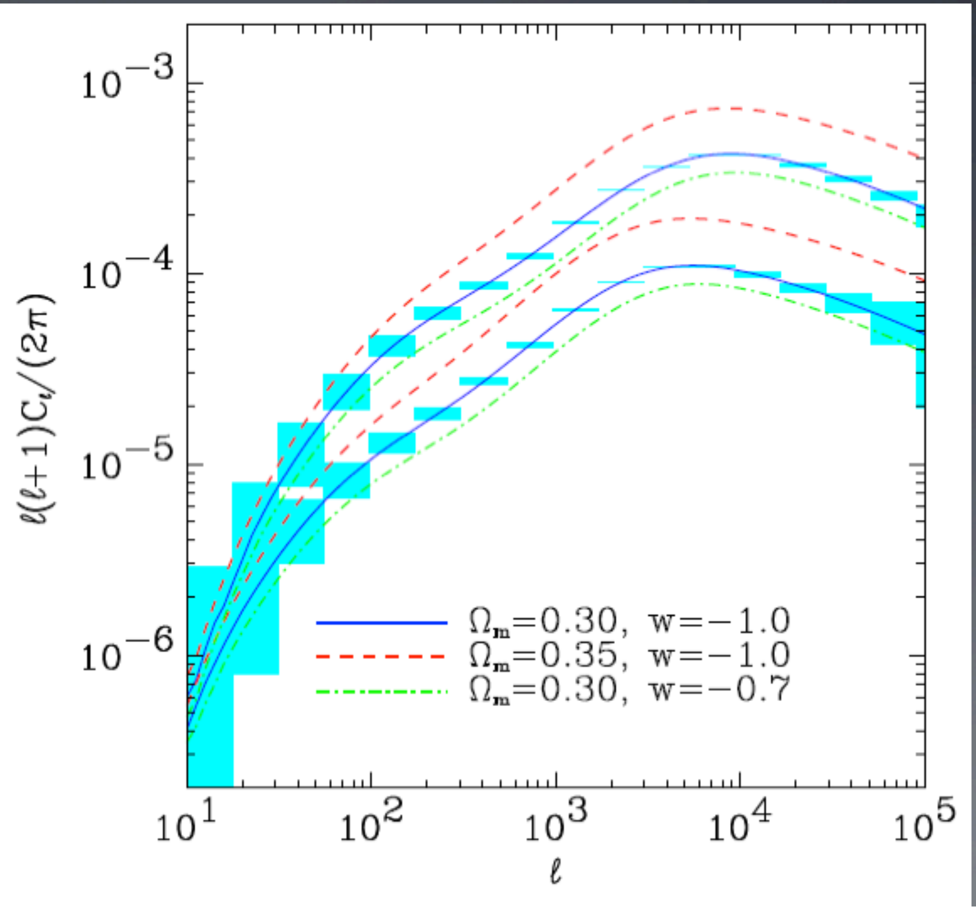
- 1.4 μK arcmin noise
- 4 arcmin beam
- 8.5x8.5 degrees
- Convergence scale -0.12 to +0.12
- S/N>1 on each mode out to L=1000.

Convergence power spectrum

$$C_l^\kappa = \frac{9}{4}\Omega_0^2 \int_0^{w_s} \mathrm{d}w \frac{g^2(w)}{a^2(w)} P_{3D}\left(\frac{l}{f_K(w)}; w\right) \times \frac{f_K(w_s - w) f_K(w)}{f_K(w_s)}.$$

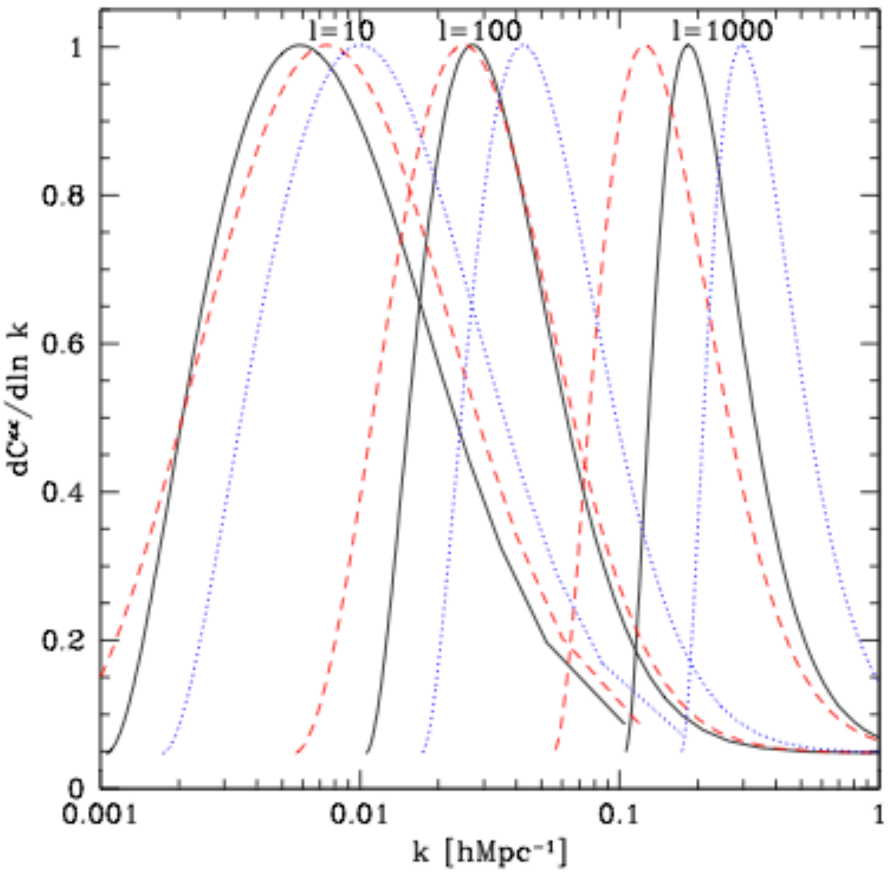
Refregier et al SNAP3

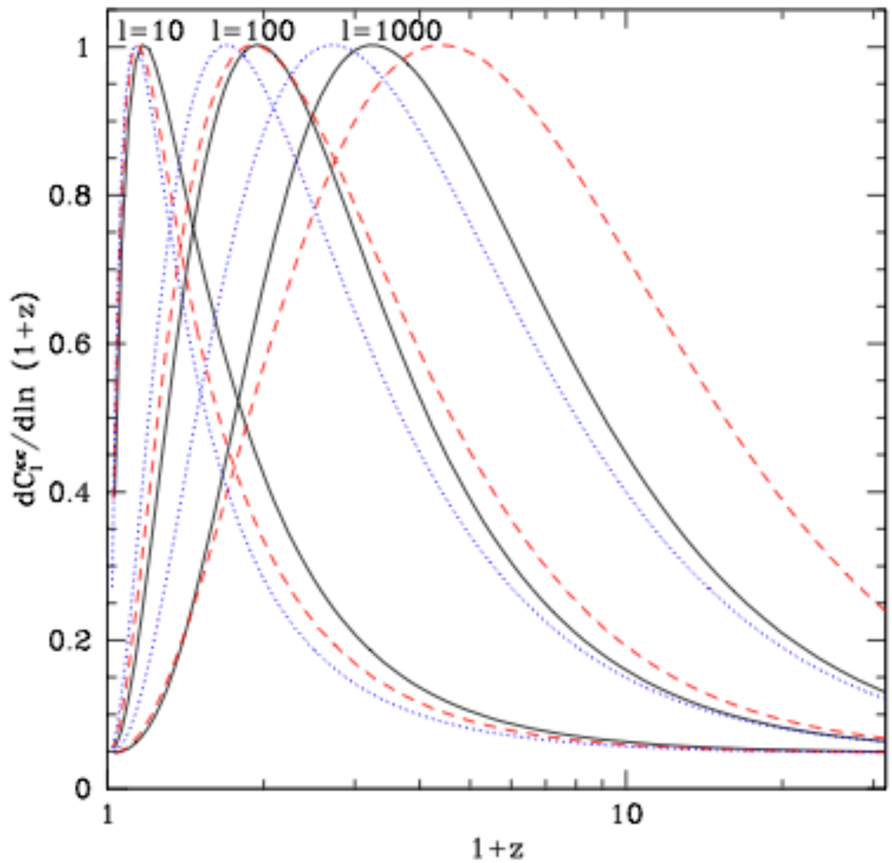
- Just a projection of total matter P(k)
- Need P(k) for dark matter: use N-body simulations
- Sensitive to many cosmological parameters



redshift and k dependence

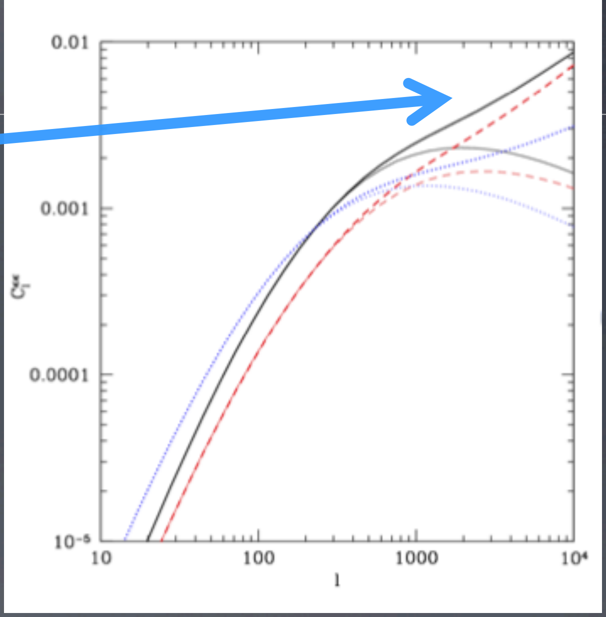
Zaldarriaga & US 1999

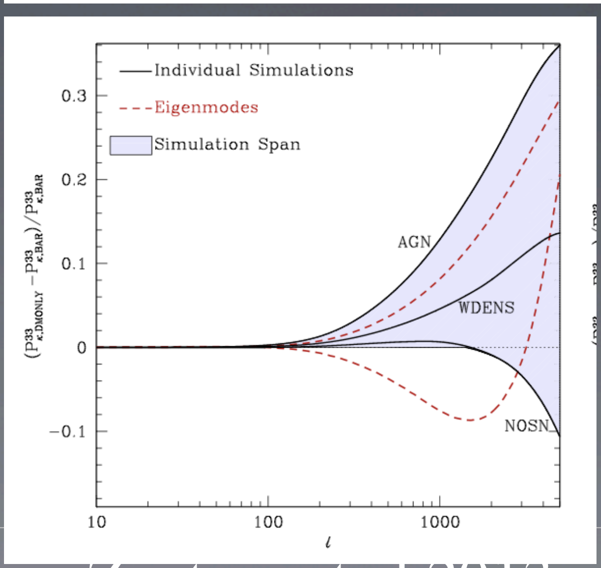




Theoretical challenges

- Nonlinear effects: need DM nonlinearities
- Baryonic effects: baryons redistribute dark matter inside halos: compress (cooling) or expand (AGN feedback)?
- Challenge: small scale baryonic physics effects can be projected to low l for nearby halos
- Solution: marginalize over the uncertainties of physics inside the halos, while keeping the information of the halo mass (Mohammed & US 2014)
- Application to CMB lensing in progress



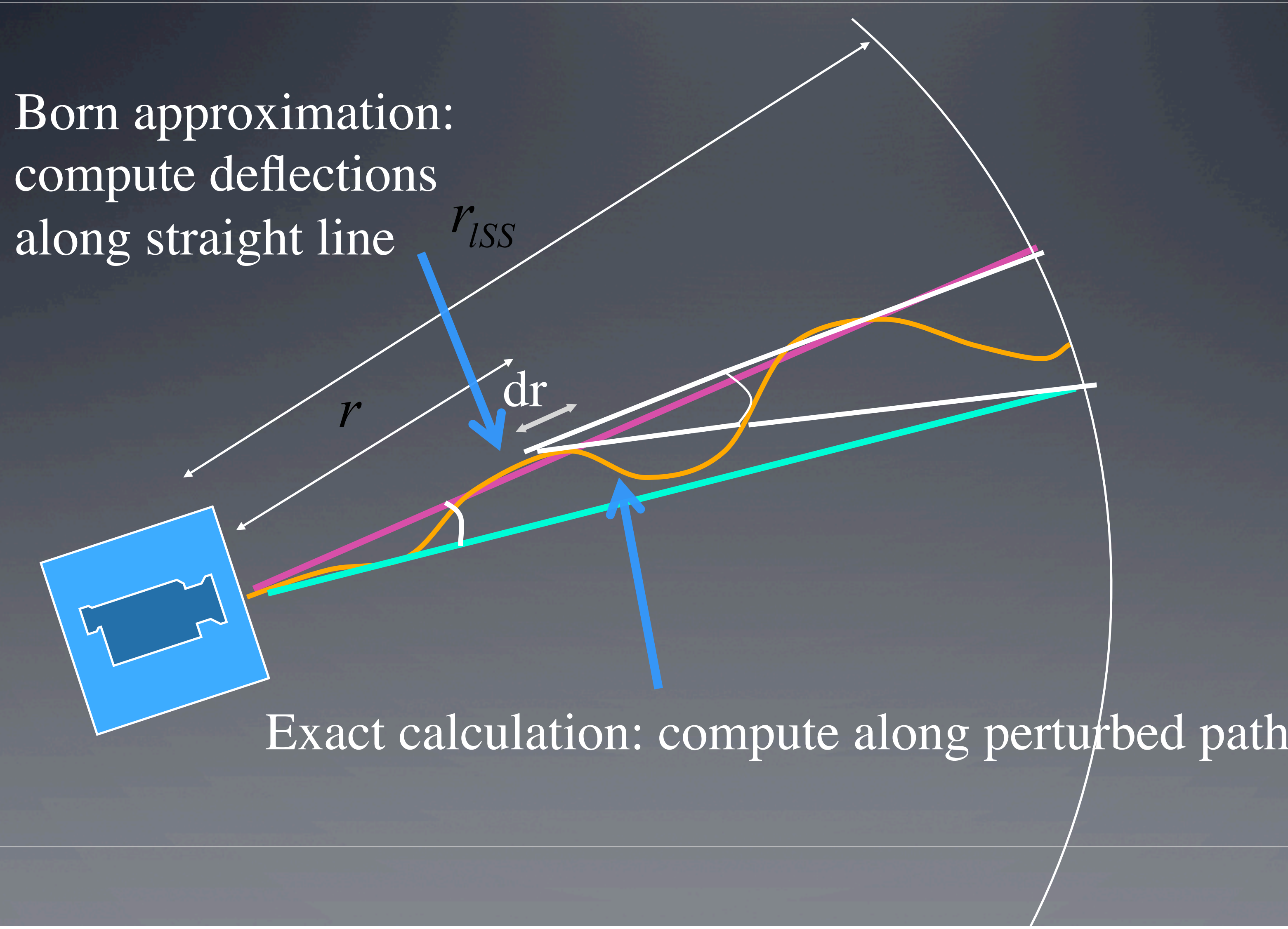


Zentner et al 2013

Higher order correlations

- There are several biases in CMB lensing, at various levels of $C_l^{\kappa\kappa}$: $N^{(0)}$, $N^{(1)}$, $N^{(2)}$
- So far everyone assumed lensing potential is gaussian
- At 2nd order there will be terms like κ^2
- $<\kappa^2\kappa>$=bispectrum of κ, leading to $N^{(3/2)}$ term
- Higher order correlations need to be included (work in progress): expected to be small, but possibly percent levels
- We already need this now: Planck data are at 40 sigma: σ_8 at 1.25% level (fixing all other parameters)
- Future data: 200++ sigma: theorists have some work to do to keep up with the experiments!

Multiple deflections



Multiple deflections: perturbative approach

- There are multiple deflections that lead to corrections beyond the "Born approximation" (Hirata & US 2003, Krause & Hirata 2010):

$$\frac{\mathrm{d}}{\mathrm{d}\chi}\left(\frac{\mathrm{d}\mathbf{n}}{\mathrm{d}\chi}\chi\right) = -2\frac{\partial\Phi(\mathbf{x}(\mathbf{n},\chi);z(\chi))}{\partial\mathbf{n}}\chi$$

$$\begin{aligned} n_i(z_s) &= n_{0i} + d_i^{(1)}(z_s) + d_i^{(2)}(z_s) + d_i^{(3)}(z_s) \\ &= n_{0i} + d^{(1)}(z_s) - 2\int_0^{\chi_s} \mathrm{d}\chi W(\chi,\chi_s)\chi^2\Phi_{,ia}(\chi)d_a^{(1)}(\chi) - 2\int_0^{\chi_s}\mathrm{d}\chi W(\chi,\chi_s)\chi^2\left(\frac{1}{2}\chi\Phi_{,iab}(\chi)d_a^{(1)}(\chi)d_b^{(1)}(\chi) + \Phi_{,ia}(\chi)d_a^{(2)}(\chi)\right)\end{aligned}$$

$$\mathbf{A}(\mathbf{n}_0, z_s) = \frac{\partial\mathbf{n}(z_s)}{\partial\mathbf{n}_0} = \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2-\omega \\ -\gamma_2+\omega & 1-\kappa+\gamma_1 \end{pmatrix}$$

$$\kappa = \tfrac{1}{2}(\psi_{11}+\psi_{22})$$

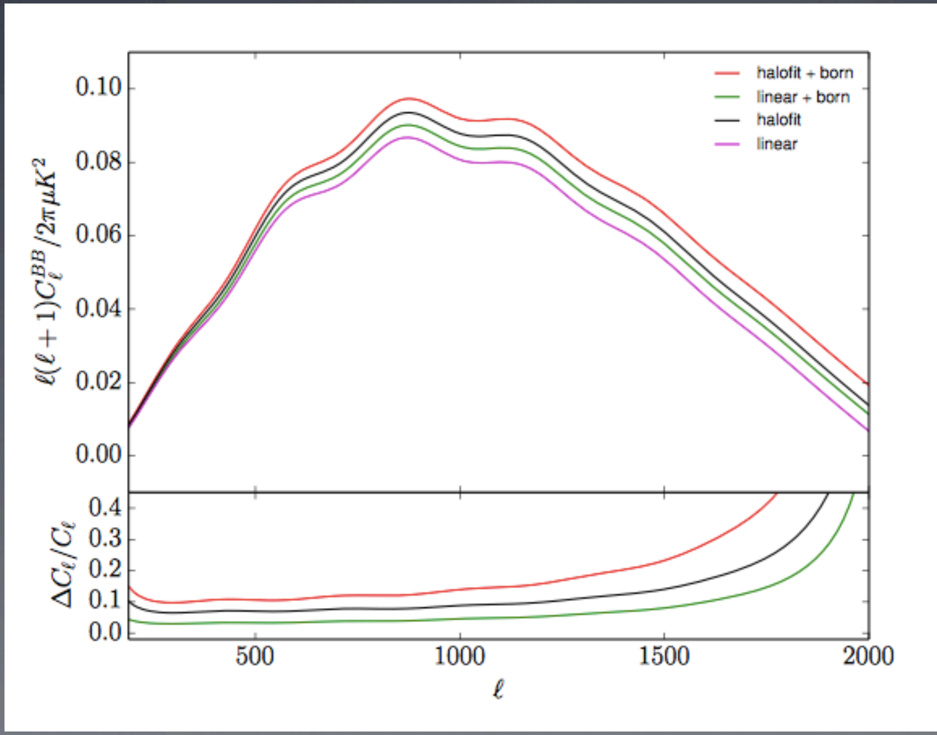
$$\omega = \tfrac{1}{2}(\psi_{12}-\psi_{21})$$

$$\psi_{ij}(\mathbf{n}_0, z_s) = \psi_{ij}^{(1)}(\mathbf{n}_0, z_s) + \psi_{ij}^{(2)}(\mathbf{n}_0, z_s) + \psi_{ij}^{(3A)}(\mathbf{n}_0, z_s) + \psi_{ij}^{(3B)}(\mathbf{n}_0, z_s) + \psi_{ij}^{(3C)}(\mathbf{n}_0, z_s)$$

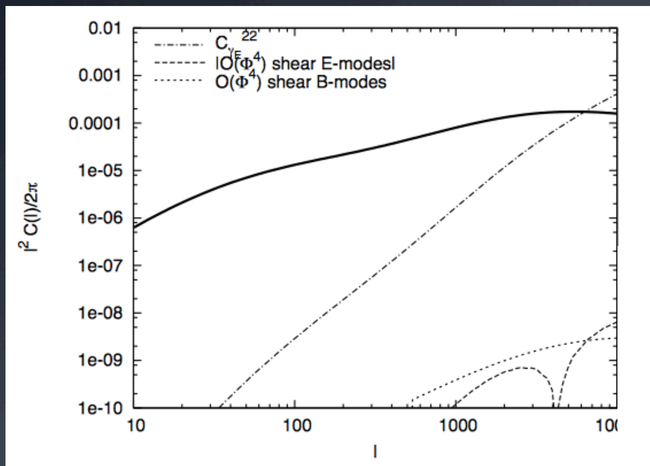
For ω need to go to 2^{nd} order (22 only), for κ to 3^{rd} order: 13+22

Multiple deflections: important for C_l^{BB}?

- Recent claim by Hagstotz, Schafer, Merkel (2014) that this is a very large effect on $C_l^{\kappa\kappa}$ and on C_l^{BB}

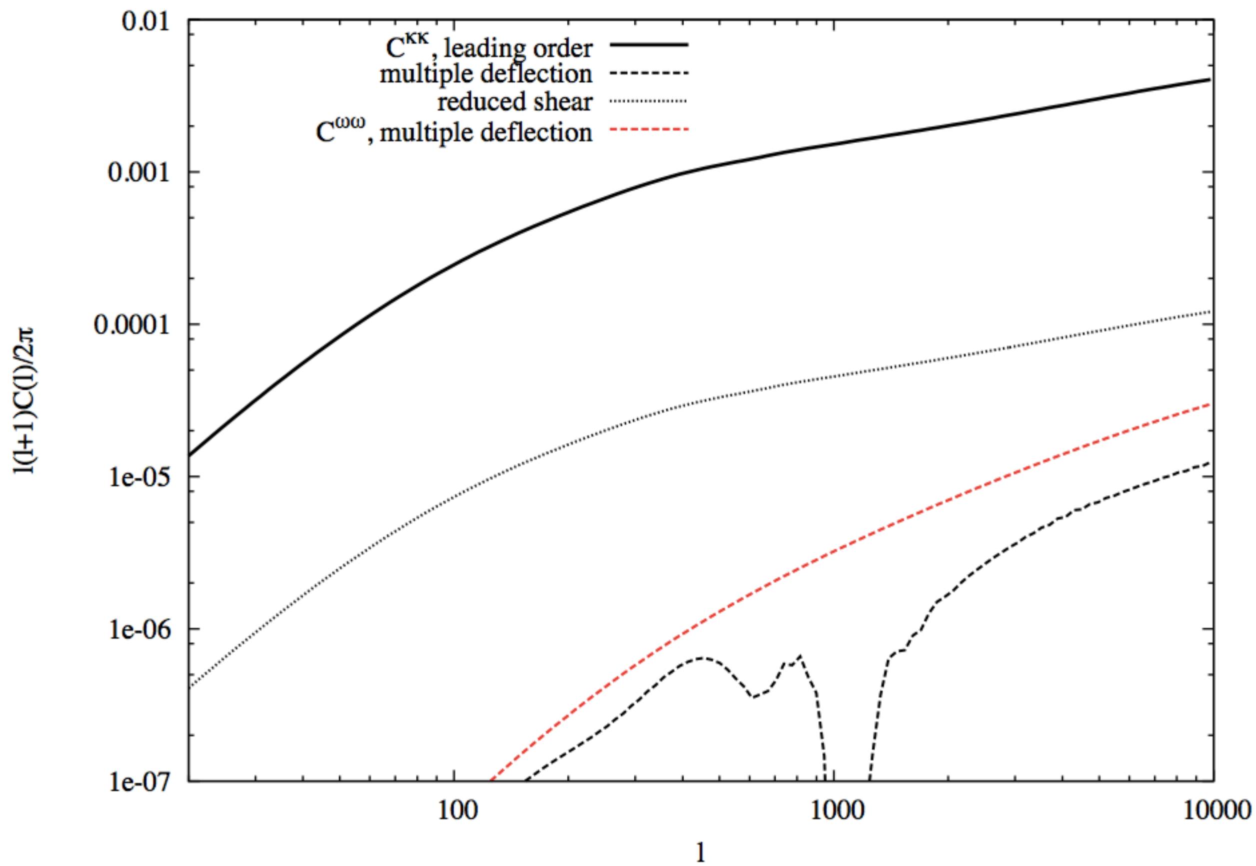


Strong cancellations of 22 and 13



Krause & Hirata 2010

Cancellation missed?

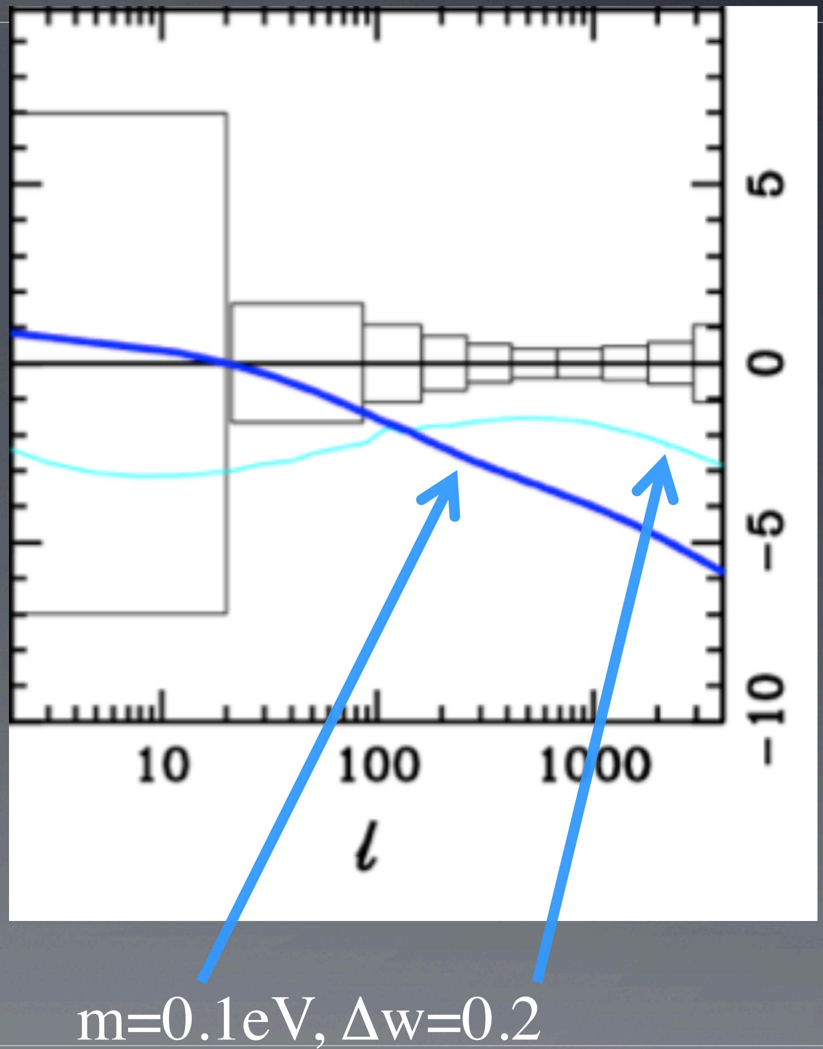


Our calculation suggests the effect is negligible for $C_l^{\kappa\kappa}$ and C_l^{BB}

Krause et al., in prep.

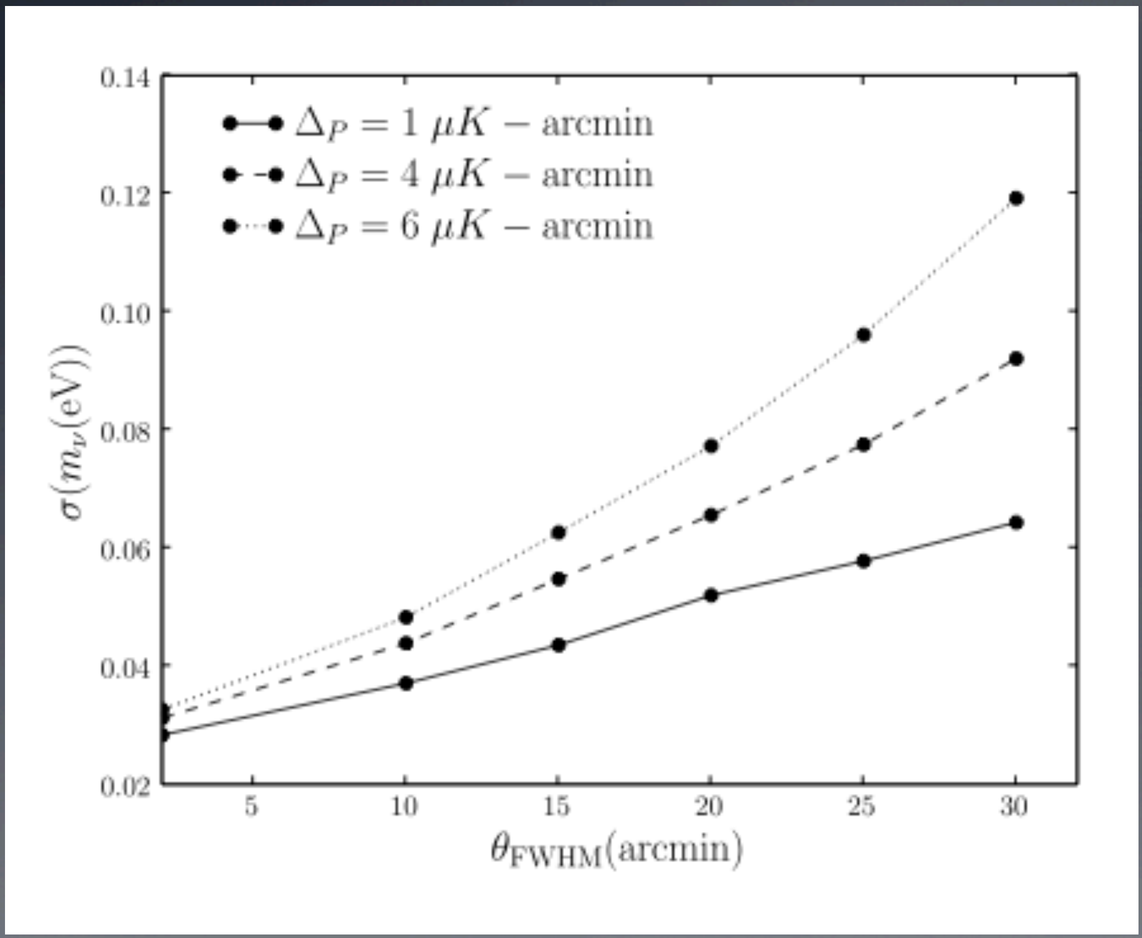
How to measure neutrino mass with CMB?

- Neutrino free streaming inhibits growth of structure on scales smaller than free streaming distance
- If neutrinos have mass they contribute to the total matter density, but since they are not clumped on small scales dark matter growth is suppressed
- Minimum signal at 0.06eV level makes 3% suppression in power, mostly at l>1000



Kaplinghat etal 2003

Predicted limits on neutrino mass



Smith etal 2008

using optimal quadratic estimator, can possibly do better with iterative estimator

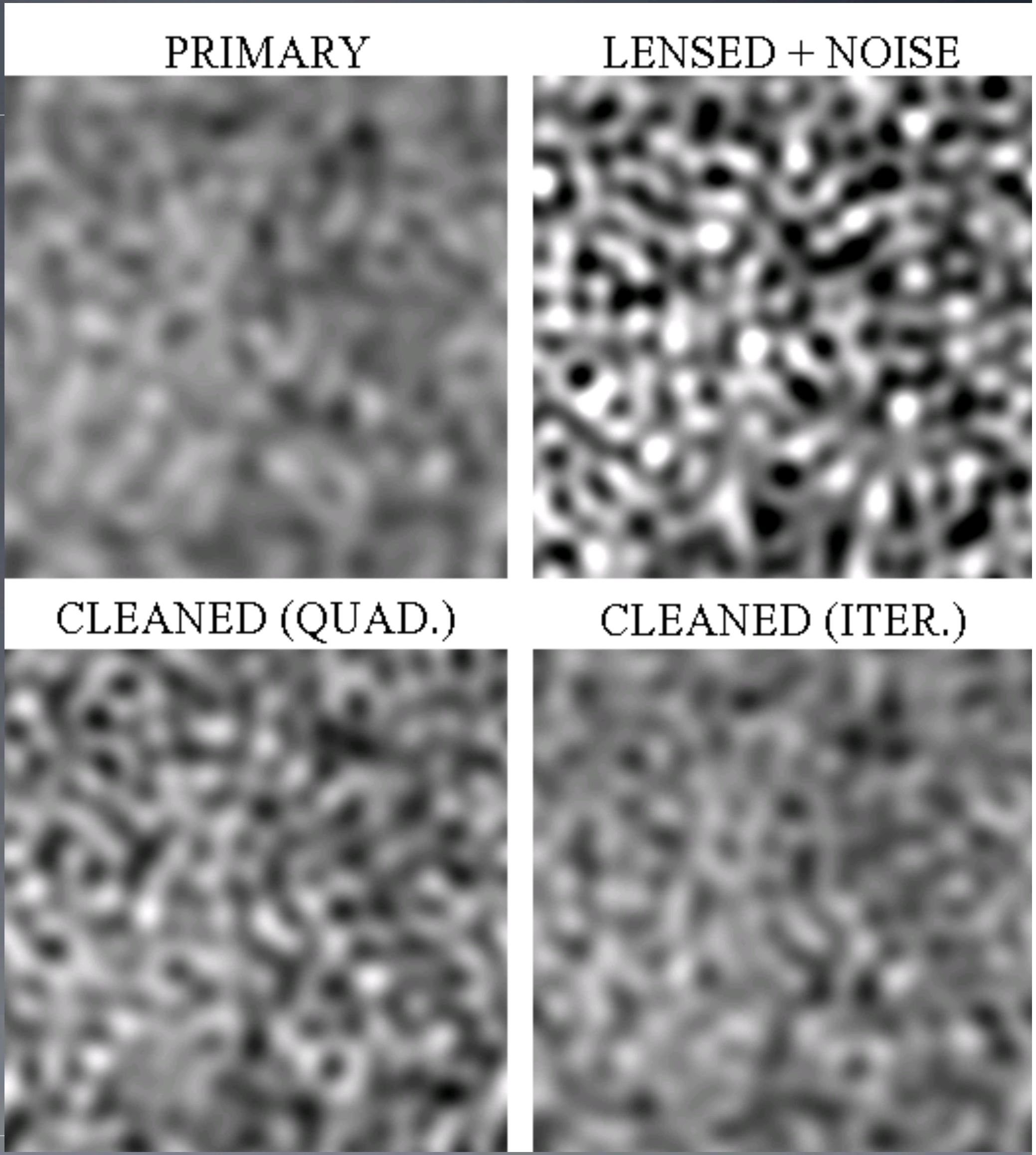
One can reach as low as 0.01-0.02eV on the sum of neutrino mass: guaranteed detection!

Delensing of GW B modes

• Another possible application of lensing reconstruction techniques is to separate the lensing B polarization from the inflationary gravitational wave contribution

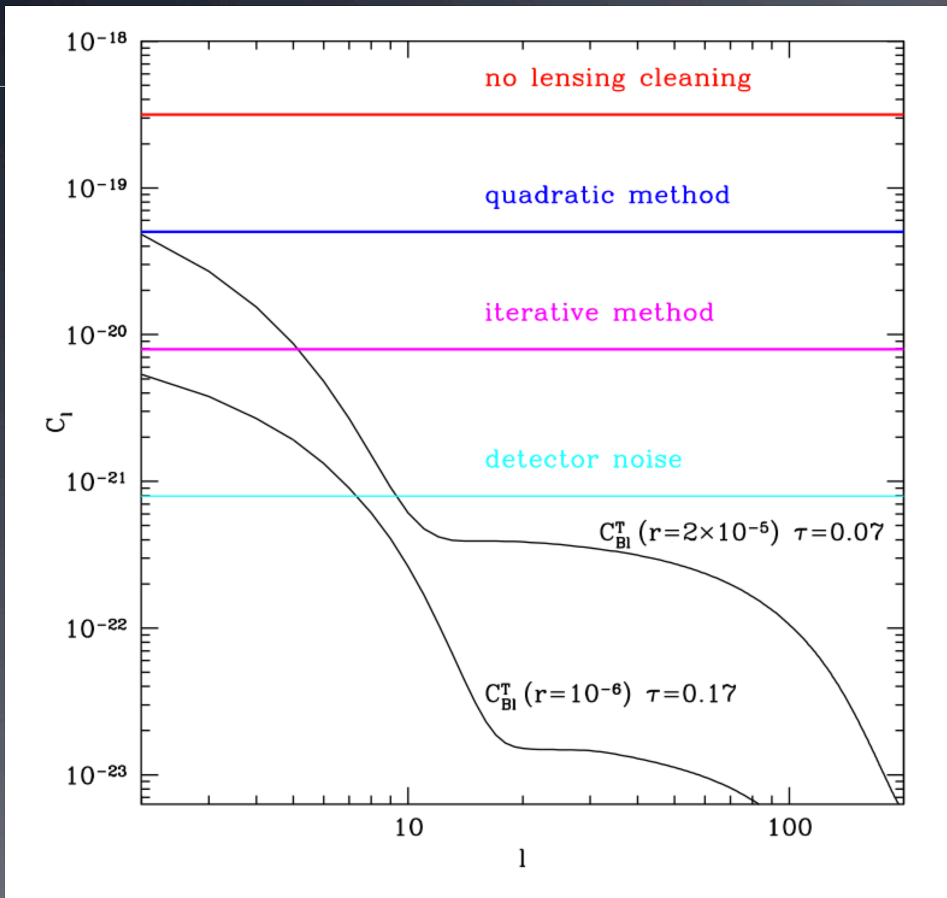
•The same lensing potential that we can reconstruct from B at high l is also responsible for B noise at low l

• Toy simulation at right for 0.5 μK arcmin noise, 4 arcmin beam



US & Hirata (2003)

How well can we delens B polarization?



US & Hirata 2003

For very low detector noise (0.25μKarcmin, 2’ beam) it can be pushed to r=T/S=10^{-5} (on low l reionization peak)

Delensing with CIB: up to 2.5 reduction in power (B. Sherwin)

Beam FWHM	Instrument noise $w_P^{-1/2}$, μK arcmin 6.00	3.00	1.41	1.00	0.50	0.25
Quadratic estimator						
$20'$	8.73	7.13	6.70	6.48	5.71	4.75
$15'$	7.73	5.11	3.92	3.64	3.28	3.06
$10'$	7.49	4.79	3.53	3.22	2.88	2.68
$7'$	7.32	4.59	3.29	2.98	2.62	2.40
$4'$	7.20	4.39	3.02	2.69	2.30	2.09
$2'$	7.11	4.26	2.86	2.53	2.15	1.99
Iterative estimator						
$7'$	7.31	4.45	2.87	2.42	1.80	1.45
$4'$	7.17	4.23	2.56	2.07	1.39	1.00
$2'$	7.09	4.10	2.40	1.91	1.22	0.83

What is the ultimate limit of delensing?

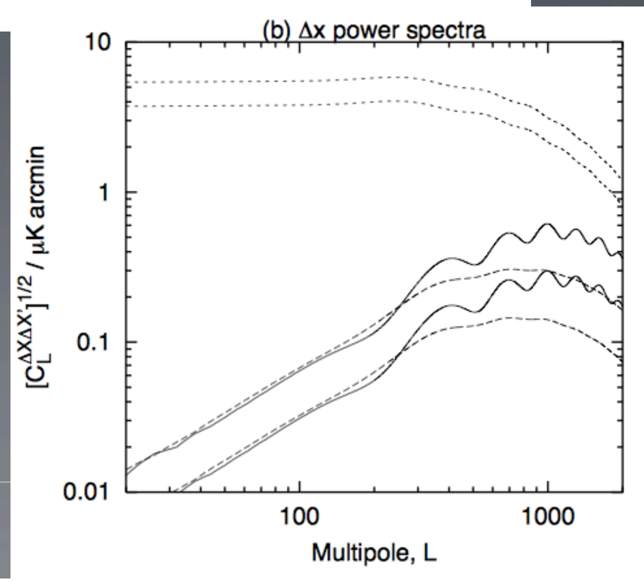
- In the absence of noise ML method simply solves linear system of equations for κ given observed E and B: same number of equations as unknowns allows for perfect delensing

- This is no longer the case in presence of rotation ω

$$\hat{B}_{\mathbf{l}} = \sum_{\mathbf{l}'} \frac{1}{\sqrt{4\pi}} \left(\frac{2}{l'^2}\right) \left[\mathbf{l}' \cdot (\mathbf{l} - \mathbf{l}')\kappa_{\mathbf{l}'} + \star\mathbf{l}' \cdot (\mathbf{l} - \mathbf{l}')\omega_{\mathbf{l}'}\right] E_{\mathbf{l}-\mathbf{l}'} \sin 2\alpha$$

- ω induced noise is of the order of

0.2μKarcmin (Hirata & US 2003)



The era of CMB polarization lensing has begun

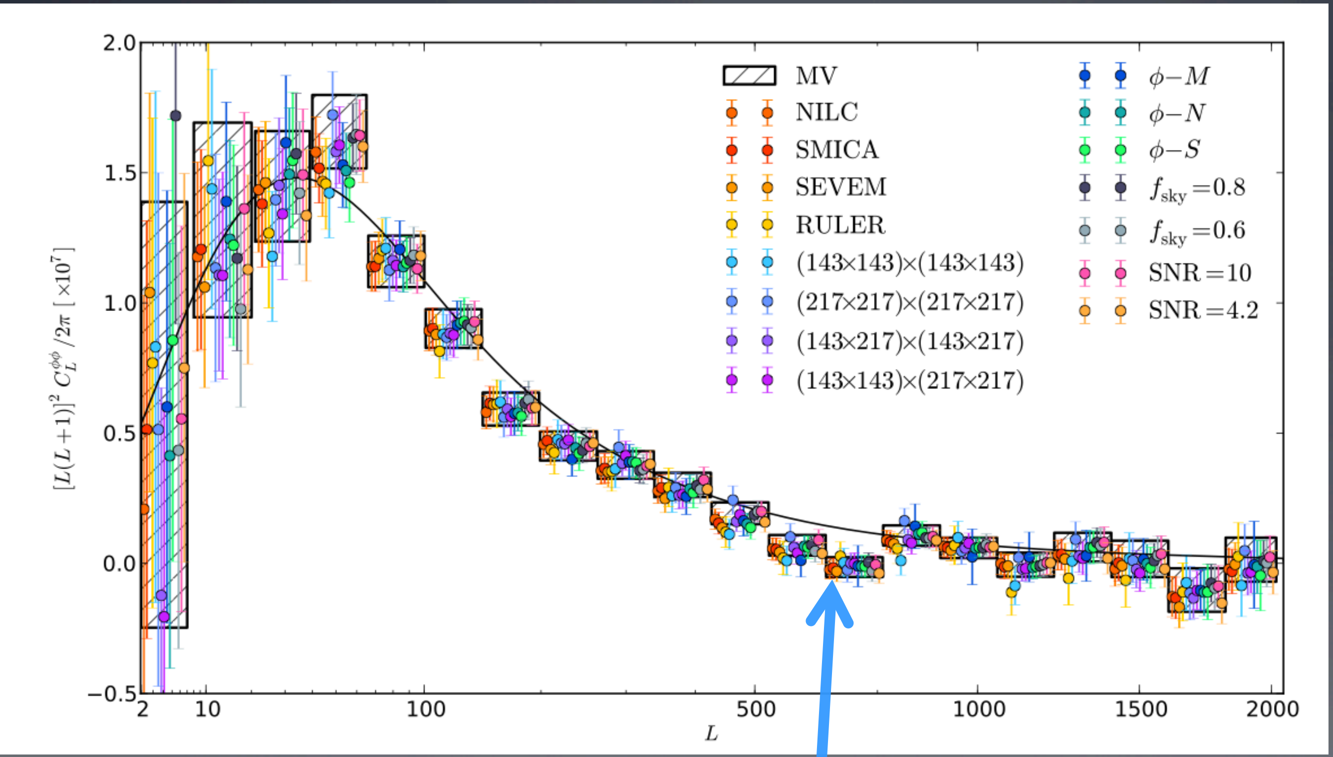
First detection of B lensing using cross-correlation of SPT polarization with Herschel, Hanson et al 2013, direct detections by Polarbear, ACT, SPT and BICEP2

Next generation of ACT/SPT/Polarbear…: 200+ sigma

Open question: how is CMB lensing reconstruction affected by foregrounds?

State of the art in CMB lensing: Planck

40 sigma in Planck 2015



Is this something we do not understand? Can we develop new estimators where biases are smaller?

Calibrating weak lensing surveys with CMB lensing

- One of primary concerns for galaxy weak lensing surveys (DES, LSST, Euclid…) is ellipticity-shear conversion bias and photometric redshift calibration bias, both of which appear as a multiplicative factor
- CMB lensing very clean: does not suffer from these and one can use CMB lensing cross-correlation with galaxy lensing to calibrate the galaxy lensing bias (Vallinotto 2012)
- One can further improve by using cross-correlation of each with a galaxy survey (Das & Spergel 2013)
- The gains can be a factor of 10+: for LSSTxCMBPol from 4% to 0.3% calibration (Vallinotto 2012)
- This would allow LSST to do low redshift dark energy science (e.g lensing tomography) that CMB lensing cannot do

Conclusions

- CMB lensing is powerful and becoming mature
- Lensing potential measures projected matter fluctuations
- none of current LSS probes competitive with Planck lensing
- Data will keep improving in the future: LSS surveys may need to play catchup to CMB polarization experiments
- CMB lensing may be needed to calibrate LSS surveys
- Theory has to catch up with the progress on the data front: baryonic effects, higher order correlations, multiple deflections etc.