

Essays in International Economics

A DISSERTATION

**SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA**

BY

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**IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Doctor of Philosophy**

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July, 2014

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Acknowledgements

I am grateful to my advisers Timothy Kehoe, Fabrizio Perri, and Kei-Mu Yi for their guidance and support. This dissertation would not exist without their assistance. I would also like to thank Hengjie Ai, Manuel Amador, Cristina Arellano, Doireann Fitzgerald, Si Guo, Jonathan Heathcote, Zhen Huo, Elena Pastorino, Jose-Victor Rios-Rull, Benjamin Malin, Joseph Steinberg, Sewon Hur, Illenin Kondo, Jose Asturias, Simran Sahi and seminar participants at the University of Minnesota, the 2013 Vigo Workshop on Dynamic Macroeconomics, the Federal Reserve Bank of Minneapolis, Simon Fraser University, the Federal Reserve Board, and Singapore Management University.

Dedication

To my loving parents, siblings, and friends - those who stood by me through it all

Abstract

This dissertation consists of three essays. The first essay analyzes the effects of fixed costs in generating country pairs that do not trade or do foreign direct investment, and how incorporating such country pairs changes the welfare gains one computes from policy reform. Despite the enormous growth over the past several decades in global trade and investment, most countries still do not trade or invest with one other. Using a recently commissioned dataset, I document that 80% of bilateral trade and FDI relationships are zeros. I construct a model that rationalizes these zeros and allows bilateral relationships to form (aggregate zero-to-one transitions) following policy reform. Exact equilibria do not generically exist in the resulting multidimensional discrete-choice fixed point problem. I develop an algorithm that computes an approximate equilibrium where (1) countries engage in more than 99% of all profitable bilateral relationships available to them, and (2) where 99% of the bilateral relationships they engage in yield positive profits. Relative to models with no aggregate entry or exit, the gains from openness in the model where zeros matter are higher by 30% on average, with the discrepancy larger for countries in the developing world.

In the second essay, I study the impact of international trade on the rise of the service economy. Services now constitute the majority of both value added and labor in the developed world, and its share is rising still. Trade in services, however, comes nowhere near that level as a fraction of aggregate trade, with few service providers exporting to foreign destinations. Moreover, while productivity growth in services as a whole has lagged behind the rest of the economy, service exporters are more productive, sell more, and hire more workers than their domestic counterparts. I construct a Heckscher-Ohlin model where firms have heterogeneous productivity levels and show that the asymmetric

lowering of trade barriers across sectors can qualitatively account for all these facts. The model predicts that labor in skill-abundant countries should move into services. It also features endogenous selection into export markets, with exporters being more productive, selling more, and making more profits than domestic producers. Furthermore, as barriers to service trade remain high relative to non-services, the positive effect that foreign competition in the model has on sector-level average productivity is weaker, generating slower growth in service productivity. These results are shown to be robust to the introduction of intermediates and capital.

In the third essay, I examine the role of debt auctions on quantitative models of sovereign default. Government bonds with default risk are often sold by auction, whether competitive or discriminatory. In standard models of sovereign default, the pricing protocol stipulates the existence of perfectly informed risk-neutral foreign creditors with flat demand curves that price bonds uniformly so they break even in expected value. In contrast, this paper follows the auction literature in assuming that creditors face downward sloping demand curves and uncertainty over the stop-out price at which bonds are sold. The interaction of this auction component with default risk has a significant impact on both the level of government borrowing and probability of default. Further, the auction mechanism matters: if bonds are sold using a competitive auction, it is optimal for lenders to bid their true valuation; in contrast, agents have an incentive to understate their valuation under a discriminatory auction protocol. Understanding the tradeoffs inherent in the choice between competitive and discriminatory bond auctions in the presence of default are particularly pertinent for countries that have historically been vulnerable to sovereign debt crises.

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Chapter 1

Zeros and the Gains from Openness

1.1 Introduction

Globalization has been one of the most important developments of the past half century. Global merchandise trade as a fraction of total output has risen more than twofold in the last four decades (World Trade Organization database) while global FDI as a fraction of total output has risen more than fivefold in the corresponding period (United Nations Conference on Trade and Development STAT). Yet for all this growth the vast majority of bilateral trade and FDI relationships remain full of zeros, i.e. there are no documented flows from one country to the other. Using data from the recently commissioned Coordinated Investment Survey Database and the Direction of Trade Statistics Database, I document that in the period 2009-2011, more than 80% of bilateral trade-FDI relationships in a global sample of over 100 countries contained at least one zero. This raises two key questions: Why do some countries trade and do FDI with each other, but not with others? And if globalization creates winners and losers because

countries only trade or conduct FDI selectively, what do countries really stand to gain from openness?

To address these questions, I develop a model that has both trade and multinational production (MP) and allows for zeros in both. The basic framework is a multi-country monopolistic competition model, where the number of firms engaging in production is fixed, but the number of firms doing MP and exporting is endogenous. To engage in either MP or exports, firms have to pay both fixed and variable iceberg costs. These costs are country-pair specific and are the main source of heterogeneity in the model. The asymmetry in fixed costs is critical to generating the pattern of zeros observed in the data: firms only enter destinations where profits exceed fixed costs and do not enter otherwise. Given that firms from a given country are homogeneous in productivity, when one firm decides not to enter a particular destination, so do all other firms from the same country, resulting in an aggregate zero in either trade or MP. Exporting firms are distinct from firms that do multinational production, and hence countries could be doing both trade and MP, only trade, only MP or neither - as they do in the data.

The gains from openness I consider are a result of a combination of trade and financial liberalization, where trade liberalization comes about with a fall in iceberg trade costs, and similarly for financial liberalization and iceberg MP costs, holding fixed costs and everything else constant. The presence of multiple discrete choices (trade, not trade) and (MP, not MP) across all country pairs make computation of equilibrium difficult, and exact equilibria where firms enter when it is profitable to do so and not otherwise do not always exist. I develop an approximate equilibrium concept and an algorithm to compute such equilibria. In the baseline parameterization, I compute approximate equilibria that are 99% accurate in that (1) countries engage in more than 99% of all profitable bilateral relationships available to them, and (2) of the bilateral relationships they engage in, more than 99% yield positive profits. To assess the impact of zeros, I

compare the results obtained given the baseline parameterization with aggregate zero-to-one transitions (i.e. the formation of bilateral trade or MP relationships) against an alternative parameterization where such transitions never occur. I find that relative to this alternative parameterization, my model generates welfare gains that are 30% higher for the average country, with the discrepancy larger for countries in the developing world. Decomposing these gains into trade and MP, I find that the contribution of MP is larger for the majority of countries in my sample. These two findings suggest including MP and allowing for aggregate zero-to-one transitions matter for understanding what countries stand to gain from openness.

The gains from openness I consider in this paper have a long history in both trade and FDI literatures. In light of the aforementioned growth in these flows, there has been a resurgence in interest in understanding what drives these gains on both theoretical and empirical fronts. On the trade side, Arkolakis, Costinot and Rodriguez-Clare (2012) show that the gains from trade arising in the context of a wide array of models depend only on two statistics: the import penetration ratio and the trade elasticity. On FDI, McGrattan and Prescott (2010) and McGrattan (2012) find that while the gains from FDI are large, the effects of FDI on growth are harder to ascertain. These papers focus on a single channel, whether trade or FDI; in contrast, my objective is to study both, and in particular, to account for the zeros observed in their bilateral flows and the effects zeros turning to non-zeros have on welfare. Ramondo and Rodriguez Clare (2013) build on the seminal work of Eaton and Kortum (2002) and study the gains from openness in a model where countries have a motive to engage in both trade and MP, while abstracting from the zeros in both types of flows. I view my work as complementary to theirs. Earlier work on the interaction between trade and FDI is vast: Costinot and Rodriguez-Clare (2013) and Antras and Yeaple (2013) in their surveys detail the evolution

of this work.¹ More recently, Irarrazabal, Moxnes and Opromolla (2012), building on Eaton, Kortum and Kramarz (2011), account for intrafirm trade using detailed data on exporting and multinational firms from Norway. In a similar vein, Tintelnot (2013) studies global export platforms and estimates his model using German firm-level data; his subsequent analysis on the effects of liberalization are confined to a sample of 12 European and North American countries, where aggregate zeros are minimal. These firm- and industry-level studies typically focus on firms from a particular country or industries from the group of developed economies; my study, on the other hand, focuses on the variation in entry and sales patterns across countries, with particular emphasis on the zeros in the bilateral relationships between countries big and small.

The preponderance of zeros in the data has naturally piqued the interest of researchers in the past. However, most of the analysis has been empirical in nature and focus on the econometric issues that arise in the estimation of gravity-type equations given such nonlinearities. Helpman, Melitz, and Rubinstein (2008) employ an instrumental variables approach that allows them to demonstrate the significance of the inclusion of firm-level heterogeneity in the estimation of a gravity-type model. Santos Silva and Tenreyro (2006) emphasizes the difference between gravity estimates obtained from a poisson pseudo-maximum likelihood estimator and those obtained using ordinary least squares. In contrast, because my goal is to quantify the gains from openness in a world with zeros, I perform my analysis within the context of a general equilibrium model so as to be able to conduct policy experiments.²

¹ e.g. Helpman, Melitz, and Yeaple (2004), Ramondo, Rappoport, and Ruhl (2012), Markusen (1984), Brainard (1997), Rob and Vettas (2003), Fillat and Garetto (2010), Keller and Yeaple (2009), Carr, Markusen and Maskus (2001), Grossman, Helpman and Szeidl (2006), Alfaro and Charlton (2009), Fajgelbaum, Grossman, and Helpman (2013), Bernard, Jensen and Schott (2009)

² Another related strand of the literature looks at the different specifications for the gravity equation that account for the dispersion in (positive) trade flows between countries. Anderson and van-Wincoop (2003) provide a recent treatment of the symmetric case where they show that the gravity equation has theoretical underpinnings when one accounts for the multilateral resistance terms that represent average trade barriers. There are two alternative asymmetric specifications: one with importer effects as in Eaton and Kortum (2002) and another with exporter effects as in Waugh (2008). Here I show

My model builds on the seminal work of Krugman (1979), which Arkolakis, Demidova, Klenow and Rodriguez Clare (2008) extend by allowing for endogenous entry; here I adopt exogenous entry where the measure of potential firms is fixed a la Chaney (2008). In contrast to Chaney (2008), however, I parameterize the fixed costs so that zero-to-one transitions following policy reform are observed in equilibrium. This results in a multidimensional discrete-choice fixed point problem, that analogous to pure strategy Nash equilibria, does not always admit an exact solution. In the spirit of Krusell and Smith (1998), I develop an algorithm that computes an approximate equilibrium. In this environment, the analogue of the average capital stock are cutoff rules that differ by country-pair type: for any destination country, firms from source countries with higher productivity will enter before lower productivity firms, and conversely, firms from source countries with lower productivity exit before higher productivity firms. This parameterization of the policy functions give rise to price indices and profits that when taken as given by agents, results in agents making the right entry decisions over 99% of the time. Along this dimension my paper is closest to Ruhl (2008), who also computes an approximate equilibrium albeit in the context of a very different environment with aggregate uncertainty. One takeaway from my paper is that computation of such approximate equilibria does not require the presence of aggregate uncertainty; the paradigm of using decision rules that are approximately correct can be used to solve other types of problems that require agents to make decisions conditional on aggregate objects that depend on the decisions of a large distribution of heterogeneous agents.

There has been renewed interest in the new varieties that arise as a result of trade, with its concomitant effects on welfare. Kehoe and Ruhl (2013) find that increased trade in the set of least-traded goods accounts for a significant fraction of trade growth

that Waugh's results are robust to the larger sample that I consider and further provide an extension to multinational production by applying the same methodology to show that source country effects get the model closest to matching the correlation between prices and income across countries observed in the data.

following trade liberalization and similar structural breaks.³ Building on the work of Alessandria, Kaboski and Midrigan (2010), Hornok and Koren (2012) show that administrative trade costs associated with shipping goods across borders result in lumpiness that reduces welfare as shipments do not necessarily coincide with the preferred timing of agents' consumption. Armenter and Koren (2013) propose a statistical model with balls and bins to account for the large number of zeros in international shipments when they are mapped against product categories. Eaton, Kortum and Sotelo (2012) show that the standard heterogeneous-firm model can be modified to generate an integer number of firms and as a result account well for the zeros in bilateral trade data. Relative to these papers, my work focuses on the aggregate (i.e. country-level) zeros in both trade and multinational production. The message that zeros matter for welfare remains.⁴

The next section presents the key empirical facts. Section 3 presents the model I construct to accounts for these facts. The quantitative analysis is discussed in Section 4, and the main results are shown in Section 5. The last section concludes. Table and figures can be found in a separate section following the main references. All proofs are relegated to Appendix A. Appendix B contains a description of the algorithm. A two-country example of an approximate equilibrium is presented in Appendix C.

³ Arkolakis (2010) constructs a model where trade liberalization results in a large increase in the trade of goods with previously low volumes of trade. Evenett and Venables (2002) and Hummels and Klenow (2005) find evidence of the importance of the extensive margin for trade growth. Feenstra (1994) and Broda and Weinstein (2006) find that ignoring the extensive margin from additional varieties results in prices that are too high and welfare that is too low.

⁴ di-Giovanni and Levchenko (2013) show that for a trade model with firm-level heterogeneity, zeros at the firm-level matter little for welfare if the firm-size distribution follows Zipf's Law. To rationalize these seemingly opposing results, it suffices to note that in their case, policy would also have a big impact if it affected the infra-marginal firms (which it would in my world with aggregate zeros) instead of simply affecting lower-productivity firms who are at the cutoff.

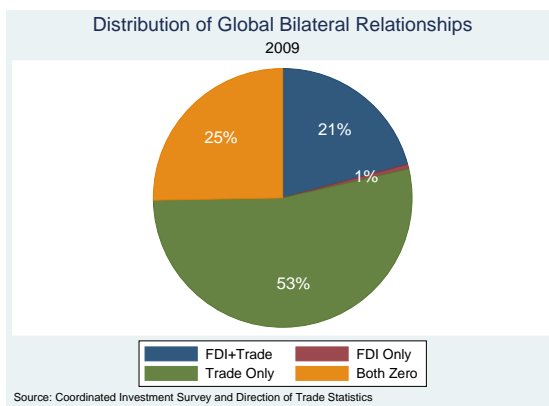
1.2 Zeros in the Data

To construct the database of global bilateral trade and FDI flows, I merge data from two main sources: the Coordinated Investment Survey Database and the Direction of Trade Statistics. The Coordinated Investment Survey Database is a recent initiative by the IMF that was commissioned for the purpose of reconciling differences between the reported bilateral FDI flows by reporter and partner countries; this is my main source for bilateral FDI data, and it runs from 2009-2011. Due to the lag in reporting national data to the international organization performing the survey, the 2009 vintage of the Coordinated Investment Survey Database is most complete and will be the focus of this study. Statistics for 2010 and 2011 are very similar and will not be considered in this paper. The bilateral trade data come from the Direction of Trade Statistics Database. Merging these two datasets, I obtain bilateral trade and FDI flows data for over 100 countries.

Given that bilateral trade and multinational production can either be positive or zero, there are four cases in total. The case where both are positive, the case where both are zero, and the cases where only one is positive and not the other. All four cases are observed in the data. Figure 1 documents that of the bilateral pairs in my sample, 80% contain at least one zero, with only roughly a fifth of all bilateral relationships have both positive trade and FDI flows. Moreover, a quarter of the country pairs do not trade with or do FDI with each other. This shows that despite the remarkable growth in world trade and investment in the past half century, we still live in a world that is nowhere near free trade or investment.

One might think that these zeros are simply a result of a group of countries not trading or investing with one another. Figure 2 shows a scatterplot of the number of

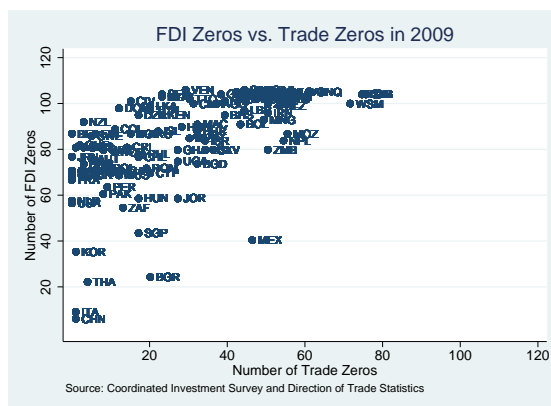
Figure 1.1: The Four Cases



trading partners on the number of FDI partners by reporting country. Clearly, countries that do FDI with more countries also trade with more countries *ceteris paribus*. While there are countries like China and Italy that receive FDI and imports from nearly every country in the sample, there are also countries like New Zealand and Belgium that receive imports from a much larger set of countries than it does FDI. There are naturally countries like Nepal and Zimbabwe that only receive imports and inward FDI from a small subset of countries. In sum, the figure shows that the prevalence of zeros does not just come about because of a certain group of countries, but every country is involved to some extent.

Thus far, the analysis has counted all zeros as equal in the sense that a zero between two big countries is considered the same as a zero between two small countries. In the tables that follow, I weigh the zeros by GDP and consider the sum of the GDPs of the countries that do not trade or do MP with the average reporting country, relative to total world GDP. Reporting countries can be big or small, where big countries have GDP's larger than the sample (world) average, which comes out to roughly 0.5% of

Figure 1.2: Zeros in Trade vs. Zeros in MP



total world GDP. In the entries highlighted in red, I show that the zeros are not simply between small-small country pairs, but often involve at least one big country.

Table 1.1: GDP-Weighted Zeros in Trade

Reporter Type	Big Zeros	Small Zeros	Total Nonzeros
Larger than Average	0.1	0.3	99
Smaller than Average	2.8	1.8	95

Table 1.2: GDP-Weighted Zeros in MP

Reporter Type	Big Zeros	Small Zeros	Total Nonzeros
Larger than Average	18	4.6	77
Smaller than Average	52	6.3	42

Figure 1.3: Zeros by Destination Country (Trade)

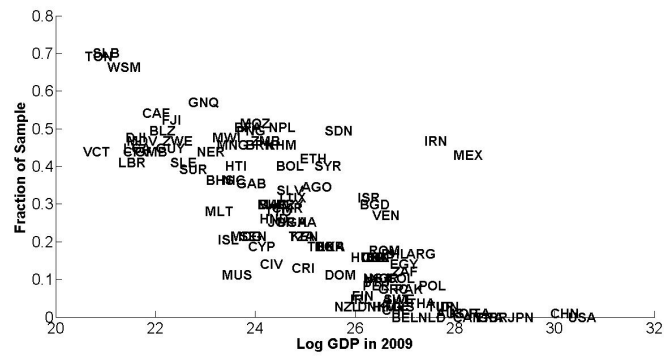


Figure 1.4: Zeros by Destination Country (MP)

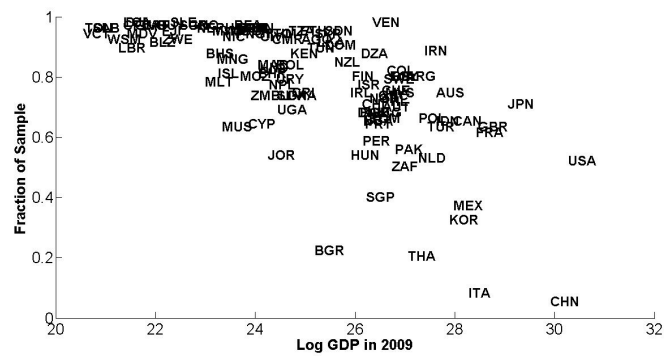


Figure 1.5: GDP-Weighted Zeros by Destination Country (Trade)

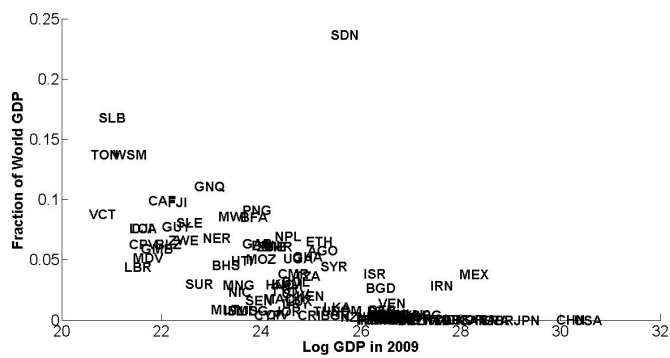
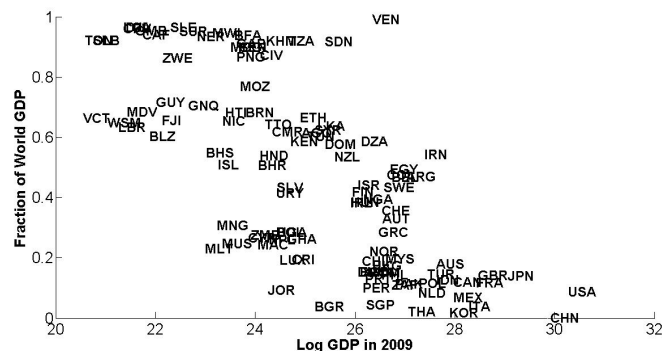


Figure 1.6: GDP-Weighted Zeros by Destination Country (MP)



Look at zeros across different destinations, one also finds significant heterogeneity. For each destination country, Figures 3 and 4 plot the fraction of the sample that said country does not import or receive investment from against GDP. These two figures show that the zeros are not simply an artifact of a subset of countries, but apply to all countries in the sample. For example, less than half the countries in the world invest in the US, even though the US does trade with all the countries in my sample. The number of zeros is negatively correlated with GDP, implying that small countries are less likely recipients of trade and multinational production. Figures 5 and 6 also count the number of zeros by destination by weigh these zeros by GDP. In comparing Figures 3 and 5, we see that while 70% of the world's countries do not export to Tonga or Samoa, these countries only make up about 15% of total world GDP. Figures 4 and 6 paint a different picture. Here, we see that even for countries like Sweden, Egypt or New Zealand, weighing the zeros by GDP does not drastically alter the finding that big countries do not produce there as the zeros for each of these destinations have GDP's that sum up to roughly half of the world's total GDP. In conclusion, zeros are not just between small countries, but often involve big countries as well.

1.3 Model

The model is a monopolistic competition with homogeneous goods setup. There are N countries, and two sets of firms producing differentiated products in each country: a set of firms that produces domestically and exports, and another set that produces domestically and does multinational production. The measure of firms that engage in monopolistic competition is exogenous, and there exists a numeraire good sector as in Chaney (2008). Country i produces w_i units of the freely-traded numeraire good with one unit of labor, and as is standard in this class of models, I only consider equilibria where this good is produced in all countries, in effect pinning down the wage w_i in country i . Both trade and MP are subject to fixed entry and variable iceberg costs that differ both across country pairs and between trade and MP. There is no free entry condition; firms can choose not to enter countries where the fixed costs exceed expected profits. Profits are aggregated into a global fund and distributed proportionally across households. Goods can be produced or traded internationally, and are produced using labor and capital which are mobile within but not across countries.

1.3.1 Consumers

There are $i = 1, 2, \dots, N$ countries and there is a measure L_i of consumers in each country i . Consumers maximize utility obtained from consuming goods in three sectors. Utility from the first sector comes from consumption of the numeraire good. Each of the other two sectors consists of consumption of differentiated goods: goods consumed in one sector can be imported from another country, while the goods consumed in the other sector can be produced by a foreign firm through multinational production. An exogenous fraction $1 - 2\mu$ of income is spent on the numeraire good, leaving a fraction μ to be spent on goods from each of the two differentiated sectors. Preferences are CES over varieties of the differentiated goods with constant elasticity of substitution $\sigma > 1$.

The problem for the representative consumer in country i can then be written as

$$\begin{aligned} & \max_{c_i^0, c_{ij}^M(\omega), c_{ij}^T(\omega)} (1 - 2\mu) \log c_i^0 + \mu \log c_i^T + \mu \log c_i^M \\ & c_i^T = \left(\sum_{j=1}^N \int_{\Omega_i^T} c_{ij}^T(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \\ & c_i^M = \left(\sum_{j=1}^N \int_{\Omega_i^M} c_{ij}^M(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}} \\ & \sum_{j=1}^N \int_{\Omega_i^T} p_{ij}^T(\omega) c_{ij}^T(\omega) d\omega + \sum_{j=1}^N \int_{\Omega_i^M} p_{ij}^M(\omega) c_{ij}^M(\omega) d\omega + p_0 c_i^0 = w_i L_i + r_i K_i + 2w_i L_i \pi \end{aligned}$$

Denote the price indices for trade and MP by P_i^T and P_i^M . These are given by

$$P_i^T = \left[\sum_{j=1}^N \int_{\Omega_i^T} p_{ij}^T(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \quad (1.1)$$

$$P_i^M = \left[\sum_{j=1}^N \int_{\Omega_i^M} p_{ij}^M(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} \quad (1.2)$$

Consumer optimization generates demand functions of the form (with capital-labor ratio $k_i = K_i/L_i$ and capital share α)

$$c_{ij}^M(\omega) = \mu \frac{p_{ij}^M(\omega)^{-\sigma}}{P_i^{M1-\sigma}} Y_i = \mu \frac{p_{ij}^M(\omega)^{-\sigma}}{P_i^{M1-\sigma}} (w_i L_i + r_i K_i + \pi_i) = \mu \frac{p_{ij}^M(\omega)^{-\sigma}}{P_i^{M1-\sigma}} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) \quad (1.3)$$

$$c_{ij}^T(\omega) = \mu \frac{p_{ij}^T(\omega)^{-\sigma}}{P_i^{T1-\sigma}} Y_i = \mu \frac{p_{ij}^T(\omega)^{-\sigma}}{P_i^{T1-\sigma}} (w_i L_i + r_i K_i + \pi_i) = \mu \frac{p_{ij}^T(\omega)^{-\sigma}}{P_i^{M1-\sigma}} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) \quad (1.4)$$

$$c_i^0 = (1 - 2\mu) \frac{Y_i}{p_0} = (1 - 2\mu) \frac{w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right)}{p_0} \quad (1.5)$$

These demand functions are taken as given by individual suppliers whose technology I discuss next.

1.3.2 Technology and Barriers to Trade and MP

Consider the two sectors with differentiated goods. All firms in country j operate a technology with productivity ϕ_j . A firm in the trade sector in country j can access the foreign market i by incurring fixed cost f_{ij}^T ; similarly, a firm belonging to the MP sector in country j can gain access to country i by incurring the fixed cost f_{ij}^M . Exports from j to i are subject to additional variable costs τ_{ij}^T that are of iceberg form. Similarly, there are efficiency losses τ_{ij}^M associated with multinational production in i for firms from j . Labor and capital are required to produce each differentiated good, with Cobb-Douglas production function and capital share α . Firms in the MP sector from any country j then solve N problems, one for each destination i , where they maximize profits given by

$$\begin{aligned}
\pi_{ij}^M &= \max_{p_{ij}^M} \left\{ p_{ij}^M c_{ij}^M - \frac{c_{ij}^M}{\phi_j} \frac{\tau_{ij}^M w_i^{1-\alpha} r_i^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha} - f_{ij}^M, 0 \right\} \\
&= \max_{p_{ij}^M} \left\{ p_{ij}^M c_{ij}^M - c_{ij}^M \frac{\tau_{ij}^M w_i}{\phi_j} \left(\frac{1}{k_i}\right)^\alpha \left(\frac{\alpha}{1-\alpha}\right)^\alpha \frac{1}{(1-\alpha)^{1-\alpha} \alpha^\alpha} - f_{ij}^M, 0 \right\} \\
&= \max_{p_{ij}^M} \left\{ p_{ij}^M c_{ij}^M - c_{ij}^M \frac{\tau_{ij}^M w_i}{\phi_j} \left(\frac{1}{k_i}\right)^\alpha \frac{1}{1-\alpha} - f_{ij}^M, 0 \right\} \\
&= \max_{p_{ij}^M} \left\{ \mu \frac{p_{ij}^{M^{1-\sigma}}}{P_i^{M^{1-\sigma}}} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi\right) - \frac{p_{ij}^{M^{-\sigma}}}{P_i^{M^{1-\sigma}}} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi\right) \frac{\tau_{ij}^M w_i}{\phi_j} \left(\frac{1}{k_i}\right)^\alpha \frac{1}{1-\alpha} - f_{ij}^M, 0 \right\}
\end{aligned}$$

Similarly, firms in the trade sector from any country j then solve N problems, one for each destination i , where they maximize profits given by

$$\begin{aligned}
\pi_{ij}^T &= \max_{p_{ij}^T} \left\{ p_{ij}^T c_{ij}^T - \frac{c_{ij}^T}{\phi_j} \frac{\tau_{ij}^T w_j^{1-\alpha} r_j^\alpha}{(1-\alpha)^{1-\alpha} \alpha^\alpha} - f_{ij}^T, 0 \right\} \\
&= \max_{p_{ij}^T} \left\{ p_{ij}^T c_{ij}^T - c_{ij}^T \frac{\tau_{ij}^T w_j}{\phi_j} \left(\frac{1}{k_j} \right)^\alpha \left(\frac{\alpha}{1-\alpha} \right)^\alpha \frac{1}{(1-\alpha)^{1-\alpha} \alpha^\alpha} - f_{ij}^T, 0 \right\} \\
&= \max_{p_{ij}^T} \left\{ p_{ij}^T c_{ij}^T - c_{ij}^T \frac{\tau_{ij}^T w_j}{\phi_j} \left(\frac{1}{k_j} \right)^\alpha \frac{1}{1-\alpha} - f_{ij}^T, 0 \right\} \\
&= \max_{p_{ij}^T} \left\{ \mu \frac{p_{ij}^{T1-\sigma}}{P_i^{T1-\sigma}} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) - \frac{p_{ij}^{T-\sigma}}{P_i^{T1-\sigma}} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) \frac{\tau_{ij}^T w_j}{\phi_j} \left(\frac{1}{k_j} \right)^\alpha \frac{1}{1-\alpha} - f_{ij}^T, 0 \right\}
\end{aligned}$$

Optimality in such a monopolistic competition setup requires that firms charge the Dixit-Stiglitz markup if it enters. On the other hand, if it does not enter, prices have to tend to infinity to be consistent with zero demand. This yields pricing equations

$$p_{ij}^M(\phi_j) = \begin{cases} \tau_{ij}^M \frac{w_i}{\phi_j} \frac{\sigma}{\sigma-1} \left(\frac{1}{k_i} \right)^\alpha \left(\frac{\alpha}{1-\alpha} \right)^\alpha & \text{if } \pi_{ij}^M > 0 \\ \infty & \text{otherwise} \end{cases} \quad (1.6)$$

$$p_{ij}^T(\phi_j) = \begin{cases} \tau_{ij}^T \frac{w_j}{\phi_j} \frac{\sigma}{\sigma-1} \left(\frac{1}{k_j} \right)^\alpha \left(\frac{\alpha}{1-\alpha} \right)^\alpha & \text{if } \pi_{ij}^T > 0 \\ \infty & \text{otherwise} \end{cases} \quad (1.7)$$

These pricing rules imply that gross profits are proportional to expenditure, and hence the equivalence relations

$$\begin{aligned}
\pi_{ij}^T > 0 &\Leftrightarrow \mu \frac{1}{\sigma} \left(\frac{p_{ij}^T}{P_i^T} \right)^{1-\sigma} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) > f_{ij}^T \\
\pi_{ij}^M > 0 &\Leftrightarrow \mu \frac{1}{\sigma} \left(\frac{p_{ij}^M}{P_i^M} \right)^{1-\sigma} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) > f_{ij}^M
\end{aligned}$$

Firms from j enter country i if profits net of fixed costs are positive. Denote entry by a firm from j in the MP sector in country i by e_{ij}^M and similarly entry into the trade sector by e_{ij}^T . Hence we have the optimal decision rules

$$e_{ij}^M(\phi_j) = \begin{cases} 1 & \text{if } \mu \frac{1}{\sigma} \left(\frac{p_{ij}^M}{P_i^M} \right)^{1-\sigma} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) > f_{ij}^M \\ 0 & \text{otherwise} \end{cases} \quad (1.8)$$

$$e_{ij}^T(\phi_j) = \begin{cases} 1 & \text{if } \mu \frac{1}{\sigma} \left(\frac{p_{ij}^T}{P_i^T} \right)^{1-\sigma} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) > f_{ij}^T \\ 0 & \text{otherwise} \end{cases} \quad (1.9)$$

Note the difference between the wage terms that enter into these pricing relations: the labor cost incurred by the foreign multinational is that of the destination country, while the labor cost incurred by the exporter is that of the source country. This is the primary conceptual difference between trade and FDI in this model.

1.3.3 Profits

As in Chaney, profits made by firms worldwide are pooled into a global mutual fund and redistributed proportionally across households, with the representative household in country i owning $2w_i L_i$ shares (as there is a measure $\theta_i^M = w_i L_i$ firms in country i 's MP sector and similarly a measure $\theta_i^T = w_i L_i$ in its trade sector). Hence profits or dividends per share is given by

$$\pi = \frac{\sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^S}{P_i^S} \right)^{1-\sigma} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) - f_{ij}^S \right] e_{ij}^S}{2 \sum_{i=1}^N w_i L_i} \quad (1.10)$$

1.3.4 Equilibrium

An equilibrium consists of consumption plans c_i^0 , c_{ij}^T for trade and c_{ij}^M for MP, production plans y_i^0 , y_{ij}^T for trade and y_{ij}^M for MP, labor allocations l_i^0 , l_{ij}^T for trade and l_{ij}^M for MP, capital allocations k_{ij}^T for trade and k_{ij}^M for MP, entry decisions e_{ij}^T for trade and e_{ij}^M for MP, pricing decisions p_{ij}^T for trade and p_{ij}^M for MP, price indices P_i^T for trade and P_i^M for MP and profits per share π such that the following conditions hold:

(i) Consumption plans are optimal, and solve the household problem, satisfying (3)-(5).

(ii) Pricing decisions are optimal, and firm charge the Dixit-Stiglitz markup with entry, and prices tend to infinity otherwise: (6)-(7).

(iii) Entry decisions are optimal, and firms only enter markets where profits exceed the fixed entry costs: (8)-(9).

(iv) Production plans are optimal, where output, labor, and capital allocations satisfy

$$y_{ij}^T = \phi_j e_{ij}^T k_{ij}^{T\alpha} l_{ij}^{T1\alpha} \quad (1.11)$$

$$y_{ij}^M = \phi_j e_{ij}^M k_{ij}^{M\alpha} l_{ij}^{M1\alpha} \quad (1.12)$$

$$\frac{r_i k_{ij}^T}{w_i l_{ij}^T} = \frac{r_i k_{ij}^M}{w_i l_{ij}^M} = \frac{\alpha}{1 - \alpha} \quad (1.13)$$

$$y_i^0 = A_i l_i^0 = w_i l_i^0 \quad (1.14)$$

(v) Price indices are consistent with the entry and pricing decisions of firms

$$P_i^M = \left[\sum_{j=1}^N \int_{\Omega_i^M} p_{ij}^M(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} = \left[\sum_{j=1}^N w_j L_j p_{ij}^{M1-\sigma} e_{ij}^M \right]^{\frac{1}{1-\sigma}} \quad (1.15)$$

$$P_i^T = \left[\sum_{j=1}^N \int_{\Omega_i^T} p_{ij}^T(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}} = \left[\sum_{j=1}^N w_j L_j p_{ij}^{T1-\sigma} e_{ij}^T \right]^{\frac{1}{1-\sigma}} \quad (1.16)$$

(vi) Profits or dividend per share are consistent with the entry and pricing decisions of firms: (10).

(vii) All markets clear.

1.4 Quantitative Analysis

In this section, I operationalize the model and use it to illustrate the importance of zeros for the gains from openness that arise from two specific policies: double taxation

treaties and regional trade agreements. First, I discuss the baseline calibration of the model and show that it can match the main features of the data discussed earlier. Next, I describe the policy experiments that I perform to gauge the importance of the aggregate extensive margin. Subsequent to this, I describe the properties of the algorithm that I use to compute the equilibrium following the policy reforms. I close this section with a discussion of the main results.

1.4.1 Benchmark Calibration: With Extensive Margin

I categorize the list of parameters into three groups. The first group consists of parameters common across all countries, as well as the list of country-specific parameters. The second group consists of the country pair-specific iceberg costs, both for trade and MP. Finally, the third group consists of the country pair-specific fixed costs for both trade and MP.

Country-Specific and Common Parameters

A country in the model is defined by a wage, a labor endowment, a capital endowment, and a level of productivity that applies to all its firms. For wages, I extrapolate wage data from the Occupational Wages around the World (OWW) Database constructed by Freeman and Oostendorp (2012). Details for the construction of the wage series can be found in the Appendix. For wages and the other exogenous country-specific parameters given below, values for the US are normalized to 1. Labor endowments are taken from the Penn World Tables (PWT). Capital stocks are constructed using investment and GDP data from the PWT and the perpetual inventory method. Total factor productivity as measured by the Solow residual is taken to be firm-level productivity. The elasticity of substitution σ across differentiated goods in both trade and MP sectors is set to 4. The expenditure share of each of the two differentiated sectors μ is set to 0.25. The

capital share α is set to 0.33.

Country Pair-Specific Iceberg Costs

I consider three possibilities for the specification of the iceberg costs. First is the symmetric case, where the only variables that enter into the estimation of the iceberg costs for trade and MP are distance, border, and language, as in the standard gravity formulation. The second is the asymmetric specification with exporter fixed effects as in Waugh (2008). He shows for trade that this is superior to the other two specifications when he takes his model to the pricing data. The third specification is one with importer fixed effects, as in Eaton and Kortum (2002). The value add in what I am doing as far as this is concerned is that I also consider these different specifications for MP, and do it for a larger sample of countries using updated pricing data from the most recent version of the International Comparison Program (ICP).

I find as does Waugh that the specification with exporter effects does best in accounting for the correlation between tradable prices and income in the data. In addition, the specification with source effects does best in accounting for the correlation between MP prices and income in the data. The variable costs τ_{ij}^T and τ_{ij}^M are then computed using the coefficients obtained from the estimation with exporter and source country effects. The gravity estimates are shown in the next table.

I regress positive trade or MP flows on the standard gravity variables of distance, language and contiguity as well as other control variables. The t-statistics are shown below the regression coefficients. D1 to D6 are distance dummies that represent different intervals, with D1 being the shortest, and D6 the farthest. As expected, these

dummies have negative coefficients, and are strongly statistically significant. Contiguity and common language also have the right signs, in that they are both positive, meaning that sharing a common border and language does increase the volume of trade or multinational production across countries. Finally I also present the estimates for two policy variables: regional trade agreements and double taxation treaties. I find that these variables are statistically significant even at the 1% level and have the right signs. These coefficients can be transformed as in Waugh (2008), and I obtain trade iceberg costs for the OECD that are very similar to Waugh's estimates. The iceberg costs for MP are roughly twice as high as those in trade on average, unsurprisingly higher as MP shares are typically smaller than trade shares in the data.

Country Pair-Specific Fixed Costs

Denote the matrix of trade entry patterns in the data by E^T and similarly the matrix of empirical MP entry patterns by E^M . Given an element e_{ij}^T in E^T , $e_{ij}^T = 1$ means that country i imports from country j in the data and $e_{ij}^T = 0$ means it does not. Given the vector of wages, TFPs, labor endowments, and capital-to-labor ratios, the matrix of iceberg costs and values for the common parameters (σ, α, μ) , we can then construct the prices that firms would charge if they entered each of the N locations according to the Dixit-Stiglitz formula. This yields price matrices P^T and P^M . Given these price and entry matrices, we can construct two matrices of price indices denoted by \mathbf{P}^T and \mathbf{P}^M that would be the price indices firms faced in an equilibrium where entry patterns were exactly as they were in the data. Given the price matrices (P^T, P^M) and matrices of price indices $(\mathbf{P}^T, \mathbf{P}^M)$, we can then write the matrices for bilateral gross profits π^T and π^M with representative elements π_{ij}^T and π_{ij}^M as functions of the global dividend per share π . π_{ij}^T and π_{ij}^M are the profits gross of fixed costs that country j firms in the

Table 1.3: Gravity Estimates for Estimating Iceberg Costs to Trade and MP

	MP	Trade
d1		0.912** (3.29)
d2	-0.338 (-0.93)	
d3	-1.423*** (-4.23)	-0.997*** (-5.47)
d4	-2.727*** (-7.89)	-2.140*** (-12.13)
d5	-2.967*** (-8.81)	-2.481*** (-14.24)
d6	-3.758*** (-10.79)	-3.128*** (-17.74)
contig	0.389 (1.26)	0.590** (2.59)
comlang	0.749*** (4.71)	0.198* (2.52)
dtc	1.436*** (11.91)	
rta		1.036*** (13.12)
cons	-7.923*** (-23.32)	-4.885*** (-28.27)

trade and MP sector can make if it operates in country i . The final general equilibrium object π is then determined given all the aforementioned variables and the fixed cost parameterization specified below.

I parameterize the fixed costs to be functions of the profit and entry matrices. In

particular, for a given positive constant ϵ , I set

$$\begin{aligned}
f_{ij}^T &= \pi_{ij}^T + \epsilon, & \text{if } e_{ij}^T = 0 \\
f_{ij}^T &= \epsilon, & \text{if } e_{ij}^T = 1 \\
f_{ij}^M &= \pi_{ij}^M + \epsilon, & \text{if } e_{ij}^M = 0 \\
f_{ij}^M &= \epsilon, & \text{if } e_{ij}^M = 1
\end{aligned} \tag{1.17}$$

Given that the entry patterns are as they are in the data, the fixed costs are parameterized to guarantee entry where necessary and to suppress it otherwise. With this parameterization of the fixed costs, we can write global dividend per share as a function of ϵ according to (10). Denote this by $\pi(\epsilon)$, to make clear its dependence on the constant ϵ . Then we know that bilateral profits that are functions of the dividend per share, can also be written as functions of ϵ , i.e. $\pi_{ij}^T = \pi_{ij}^T(\pi) = \pi_{ij}^T(\epsilon)$ and $\pi_{ij}^M = \pi_{ij}^M(\pi) = \pi_{ij}^M(\epsilon)$. Then to have an equilibrium where $\pi_{ij}^T(\epsilon) \geq f_{ij}^T = \epsilon$, when $e_{ij}^T = 1$ and $\pi_{ij}^M(\epsilon) \geq f_{ij}^M = \epsilon$, when $e_{ij}^M = 1$, the scalar $\epsilon > 0$ must satisfy the following restrictions:

$$\epsilon \leq \pi_{ij}^T \quad \forall (i, j) \in \Omega^T \tag{1.18}$$

$$\epsilon \leq \pi_{ij}^M \quad \forall (i, j) \in \Omega^M \tag{1.19}$$

where the sets Ω^T and Ω^M contain all the country pairs for which there is entry in trade and MP in the data, i.e.

$$\Omega^T = \{(i, j) : e_{ij}^T = 1\} \quad \Omega^M = \{(i, j) : e_{ij}^M = 1\} \tag{1.20}$$

Define the functions $g_{ij}^T(\epsilon) = \pi_{ij}^T(\epsilon) - \epsilon$ and $g_{ij}^M(\epsilon) = \pi_{ij}^M(\epsilon) - \epsilon$. It is not hard to show that $g_{ij}^T(\epsilon)$ and $g_{ij}^M(\epsilon)$ are strictly decreasing in ϵ , hence there exists $\bar{\epsilon}_{ij}^T > 0$ that solves $g_{ij}^T(\epsilon) = 0$ and similarly $\bar{\epsilon}_{ij}^M > 0$ that solves $g_{ij}^M(\epsilon) = 0$ for each (i, j) pair. Then for any given pair (i, j) such that we need $g_{ij}^T(\epsilon) \geq 0$, we know that as long as $\epsilon \leq \bar{\epsilon}_{ij}^T$, we will have $g_{ij}^T(\epsilon) \geq 0$. Hence, I set

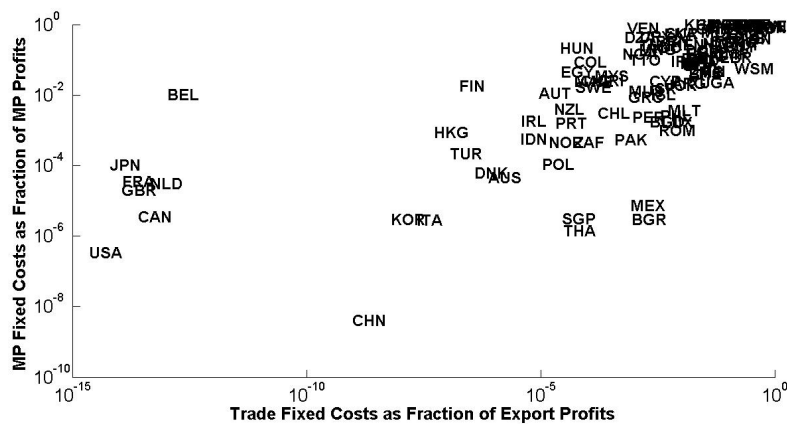
$$\epsilon = \lambda \cdot \min \left\{ \left\{ \bar{\epsilon}_{ij}^T : (i, j) \in \Omega^T \right\} \cup \left\{ \bar{\epsilon}_{ij}^M : (i, j) \in \Omega^M \right\} \right\}, \quad \lambda \in (0, 1) \tag{1.21}$$

Here we need to take the minimum of the ϵ 's that satisfy the individual restrictions because an ϵ that satisfies one condition need not satisfy the other restrictions and the scalar ϵ must be set such that all the restrictions hold at the same time. With this value of ϵ , I verify that in the benchmark equilibrium, profits and fixed costs are such that the model generates entry patterns that match the entry patterns observed in the data one for one.

The next figure shows the average fixed cost as a fraction of profits by destination country for both trade and MP (where the scalar epsilon is the smallest possible). How zeros affect welfare is through fixed costs, because the size of fixed costs is what determines how much countries stand to gain from reform. When fixed costs are really high, there are few zero-to-one transitions, and the effect of having this aggregate extensive margin is minimal. On the other hand, if fixed costs are relatively low, there are more zero-to-one transitions, and the aggregate extensive margin is more important. To understand the size of the fixed costs, I normalize these costs by profits, because this is what matters to firms: how much of its profits have to go towards paying for the initial fixed cost of establishing the trade or multinational relationship. What I present here show are the average fixed costs in the parameterization that maximizes welfare and hence minimizes fixed costs; as a result I obtain fixed costs that are an order of magnitude smaller than those obtained in Alessandria and Choi (2012) who compute fixed costs for US trade in a richer environment with dynamics. The x-axis plots the average fixed cost for trade as a fraction of profit by destination, while the y-axis plots the average fixed cost for multinational production as a fraction of total multinational production profits. Three things can be taken away from this picture. First, the two types of fixed costs are positively correlated: countries with high trade fixed costs tend to have high MP fixed costs as well. This is not surprising as fixed costs are identified by

zeros, and the trade and MP zeros by destination are also positively correlated. Second, though they are positively correlated, the fixed costs for MP are generally higher than the fixed costs for trade. This again relates back to the zeros: there are more MP zeros than trade zeros. Finally, the average fixed cost as a fraction of profits for developing countries are generally bigger than the average fixed costs for developed countries. This reflects the fact that there are still more zeros in developing countries than there are in developed countries.

Figure 1.7: Average Fixed Costs: Identified by Zero Trade and MP

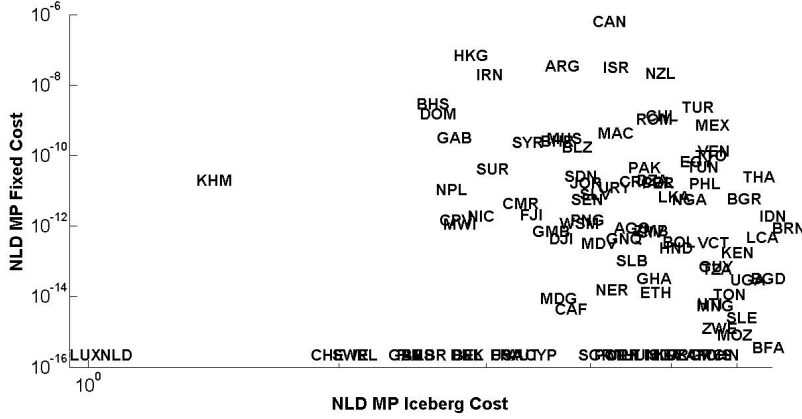


One can go one step further and link the fixed costs obtained in this section with the iceberg costs estimated earlier. To make the connection between iceberg costs and fixed costs, note that they are linked through profits: to the extent that fixed costs have to be higher than profits to suppress entry and profits are negatively correlated with iceberg costs, for the zero pairs, fixed costs and iceberg costs are negatively correlated as the next graph (estimates for Netherlands) shows.

1.4.2 Alternative Parameterization: No Extensive Margin

To make clear the role of the extensive margin, I consider an alternative parameterization of my model where zero-to-one (aggregate entry) and one-to-zero (aggregate exit)

Figure 1.8: Iceberg Variable Costs & Fixed Costs



transitions can never be observed following policy reform that lower the iceberg costs to trade and MP. In order for this to be true, fixed costs have to be sufficiently close to zero for country pairs where positive trade and MP are observed and sufficiently close to infinity for country pairs where zero trade and MP are observed before the reform, as the lemmata and the proposition below show.

Lemma 1. Given $f_{ij}^s = \infty$. for all (i, j) such that $e_{ij}^s = 0$, $s \in \{T, M\}$, there is no aggregate entry following policy reform.

Lemma 2. Given $f_{ij}^s = 0$ for all (i, j) such that $e_{ij}^s = 1$, $s \in \{T, M\}$, then there is no aggregate exit post-reform.

Lemma 3. If there is no aggregate entry and exit following policy reform, dividend per share stays unchanged ($\pi' = \pi$).

Lemmas 1 and 2 suggest that the following alternative parameterization is sufficient

to prevent aggregate entry and exit:

$$\begin{aligned}
 f_{ij}^T &= \infty, & \text{if } e_{ij}^T &= 0 \\
 f_{ij}^T &= 0, & \text{if } e_{ij}^T &= 1 \\
 f_{ij}^M &= \infty, & \text{if } e_{ij}^M &= 0 \\
 f_{ij}^M &= 0, & \text{if } e_{ij}^M &= 1
 \end{aligned} \tag{1.22}$$

This parameterization is not unique. If fixed costs are sufficiently high for pairs that do not trade or do MP initially and sufficiently low for pairs that do, we also obtain the result that zeros before the reform stay zero after the reform and likewise for those that entered before the reform. The following proposition shows that because of Lemma 3, the gains from openness obtained from all these other parameterizations with no aggregate entry and exit coincide with that obtained in the limiting case just presented, so we can compare our benchmark results against this limiting parameterization without loss of generality.

Proposition 1. The welfare gains computed in the limiting parameterization given by $f_{ij}^s = 0$ for all (i, j) such that $e_{ij}^s = 1$, $s \in \{T, M\}$ and $f_{ij}^s = \infty$ for all (i, j) such that $e_{ij}^s = 0$, $s \in \{T, M\}$ coincide with the welfare gains in an alternative parameterization of fixed costs where $f_{ij}^s = \underline{f}$ for all (i, j) such that $e_{ij}^s = 1$, $s \in \{T, M\}$ and $f_{ij}^s = \bar{f}$ for all (i, j) such that $e_{ij}^s = 0$, $s \in \{T, M\}$ where \underline{f} is sufficiently small and \bar{f} is sufficiently large to ensure that there is no aggregate entry or exit post-reform.

1.4.3 Approximate Equilibrium

In what follows, I will consider policy reforms that lower the iceberg costs to trade and MP for a large group of country pairs, holding all other parameters constant. In the alternative parameterization with no room for aggregate entry or exit, computation of the equilibrium is simple as by construction the post-reform entry patterns coincide

with the pre-reform entry patterns - if there were positive trade or MP before, there will be positive flows after the reform, and similarly for the pairs where there none. In the baseline parameterization, however, this is not the case; pairs where there were initially no flows need not remain zero as the fixed cost is not infinitely large, but rather only epsilon larger than what profits would have been in the pre-reform equilibrium, and lower iceberg costs increase profits *ceteris paribus*. Further, if there is aggregate entry, there then can also be aggregate exit, as entry of low cost producers lowers the prevailing price index and reduces demand for existing products. This means we need to compute the entry decision for each firm and each potential destination, giving rise to $N \times N$ ($N = 107$ countries) decisions in the traded sector and another $N \times N$ decisions in the MP sector. For an exact equilibrium to exist, these $2 \times N \times N$ decisions then need to be consistent in the sense that the resulting price indices and dividend per share that results from such decisions are exactly the same set of price indices and dividend per share that firms took as given when making their decisions. Only in certain special regions in the parameter space do such exact equilibria exist. Appendix C presents an example of the different regions in the parameter space in a simple two-country world with trade.

In the general case where an exact equilibrium does not exist, the standard approach of iterating on the general equilibrium objects given by the price indices and the dividend per share fails as cycles result and the algorithm does not converge. To get around this problem, I develop an approximate equilibrium concept and an algorithm that computes such approximate equilibria. In such an equilibrium, the entry decisions of agents need not be perfectly consistent with the price indices and dividend per share that such decisions engender. Positive equilibrium profits net of fixed costs do not automatically imply entry and vice versa. The goal then is to compute an equilibrium that is approximately exact in the sense that false positives and false negatives are kept to a minimum.

Below I formally define the approximate equilibrium concept.

Definition. (Approximate Equilibrium)

An *approximate equilibrium that is $x\%$ accurate* is an equilibrium wherein (1) countries engage in more than $x\%$ of all profitable bilateral relationships available to them, and (2) of the bilateral relationships they engage in, more than $x\%$ yield positive profits.

Given that this study focuses on the effect of aggregate entry and exit on welfare, the accuracy defined in the aforementioned equilibrium concept provides a measure of how consistent aggregate entry and exit decisions are with profits net of fixed costs after reform in the event that an exact equilibrium (equivalently, an approximate equilibrium that is 100% accurate) fails to exist. Rather than iterating in the space of general equilibrium objects, I iterate in decision-rule space. And as I iterate in the space of decision rules, it is important that firms have accurate expectations over the decision rules for the other firms in the world economy. This perception depends on the specification of decision rules that firms take as given; this specification needs to be structured in a way that reflects how different firms behave when faced with the same policy change. Given the parameterization of the fixed costs, the proposition below shows that it is the low cost firms that enter and the high cost firms that exit following policy reform, after controlling for country-pair type. A type is a set of country pairs that are impacted the same way by general equilibrium forces; there are $4N$ for trade and $4N$ for MP, where N is the number of countries. Given this type-specific monotonicity in entry and exit following reform, I specify cutoff rules for the entry decisions and find that the approximate equilibrium computed is accurate 99% of the time. A more detailed description of the algorithm can be found in Appendix B.

Proposition 2. (Type-Specific Cutoffs)

(i) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} = \tau_{ij}^T$ and $e_{ij}^T = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i1} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i1}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i1}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

(ii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} = \tau_{ij}^T$ and $e_{ij}^T = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i2} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i2}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i2}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

(iii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} < \tau_{ij}^T$ and $e_{ij}^T = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i3} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i3}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i3}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

(iv) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} < \tau_{ij}^T$ and $e_{ij}^T = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i4} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i4}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i4}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

(v) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} = \tau_{ij}^M$

and $e_{ij}^M = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change.

Then there exists a cutoff x_{i5} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i5}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i5}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

(vi) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} = \tau_{ij}^M$ and $e_{ij}^M = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change.

Then there exists a cutoff x_{i6} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i6}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i6}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

(vii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} < \tau_{ij}^M$ and $e_{ij}^M = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change.

Then there exists a cutoff x_{i7} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i7}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i7}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

(viii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} < \tau_{ij}^M$ and $e_{ij}^M = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change.

Then there exists a cutoff x_{i8} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i8}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i8}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Corollary 1. (Type-Specific Monotonicity)

(i) Fix destination country i . Consider firms from two countries j and k such that

$\tau_{ij}^{T'} = \tau_{ij}^T$, $\tau_{ik}^{T'} = \tau_{ik}^T$, $e_{ij}^T = 0$, and $e_{ik}^T = 0$. If $c_{ik}^T < c_{ij}^T$, then $e_{ik}^{T'} \geq e_{ij}^{T'}$.

(ii) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{T'} = \tau_{ij}^T$, $\tau_{ik}^{T'} = \tau_{ik}^T$, $e_{ij}^T = 1$, and $e_{ik}^T = 1$. If $c_{ik}^T < c_{ij}^T$, then $e_{ik}^{T'} \geq e_{ij}^{T'}$.

(iii) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{T'} < \tau_{ij}^T$, $\tau_{ik}^{T'} < \tau_{ik}^T$, $e_{ij}^T = 1$, and $e_{ik}^T = 1$. If $c_{ik}^T < c_{ij}^T$, then $e_{ik}^{T'} \geq e_{ij}^{T'}$.

(iv) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{T'} < \tau_{ij}^T$, $\tau_{ik}^{T'} < \tau_{ik}^T$, $e_{ij}^T = 0$, and $e_{ik}^T = 0$. If $c_{ik}^T < c_{ij}^T$, then $e_{ik}^{T'} \geq e_{ij}^{T'}$.

(v) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{M'} = \tau_{ij}^M$, $\tau_{ik}^{M'} = \tau_{ik}^M$, $e_{ij}^M = 0$, and $e_{ik}^M = 0$. If $c_{ik}^M < c_{ij}^M$, then $e_{ik}^{M'} \geq e_{ij}^{M'}$.

(vi) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{M'} = \tau_{ij}^M$, $\tau_{ik}^{M'} = \tau_{ik}^M$, $e_{ij}^M = 1$, and $e_{ik}^M = 1$. If $c_{ik}^M < c_{ij}^M$, then $e_{ik}^{M'} \geq e_{ij}^{M'}$.

(vii) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{M'} < \tau_{ij}^M$, $\tau_{ik}^{M'} < \tau_{ik}^M$, $e_{ij}^M = 1$, and $e_{ik}^M = 1$. If $c_{ik}^M < c_{ij}^M$, then $e_{ik}^{M'} \geq e_{ij}^{M'}$.

(viii) Fix destination country i . Consider firms from two countries j and k such that $\tau_{ij}^{M'} < \tau_{ij}^M$, $\tau_{ik}^{M'} < \tau_{ik}^M$, $e_{ij}^M = 0$, and $e_{ik}^M = 0$. If $c_{ik}^M < c_{ij}^M$, then $e_{ik}^{M'} \geq e_{ij}^{M'}$.

1.4.4 Policy Experiments

I consider the effect of two policy reforms: the formation of regional trade agreements as well as the signing of double taxation treaties globally. Of the 11449 bilateral country pairs in my sample, roughly 20% have double taxation treaties, and similarly, about 20% have regional trade agreements, though the subsamples that have these policies in place do not completely coincide. From my gravity estimation, I find that having a regional trade agreement lowers iceberg trade costs by 32%. By the same token, I find that having a double taxation treaty lowers iceberg MP costs by 38%.

I consider the gains from openness that result from a combination of trade and financial

liberalization. For trade liberalization, I consider the formation of regional trade agreements worldwide: country pairs that are initially not part of a regional trade agreement form a regional trade agreement after the reform. For financial liberalization, I consider the establishment of double taxation treaties: country pairs with no double taxation treaties sign double taxation treaties after reform. I then ask the following questions. What do countries stand to gain from such policies? Are the gains symmetric across countries? Do these gains come primarily through trade or MP? And finally, how do the gains from the model with an extensive margin that allows aggregate entry and exit differ from the gains obtained from the model with no extensive margin?

Trade vs. MP

The total gains from openness are given by

$$\frac{W'_i}{W_i} = \log \frac{\left(\frac{1}{1-\alpha} + 2\pi'\right) P_i^{M\mu} P_i^{T\mu}}{\left(\frac{1}{1-\alpha} + 2\pi\right) P_i^{M'\mu} P_i^{T'\mu}}$$

These gains can come through greater consumption of the varieties in the traded goods sector, the MP sector or the numeraire good sector. The change in welfare due to trade is given by

$$\frac{W_i^{T'}}{W_i^T} = \log \frac{\left(\frac{1}{1-\alpha} + 2\pi'\right)^\mu P_i^{T\mu}}{\left(\frac{1}{1-\alpha} + 2\pi\right)^\mu P_i^{T'\mu}}$$

Similarly, the change in welfare due to MP is given by

$$\frac{W_i^{M'}}{W_i^M} = \log \frac{\left(\frac{1}{1-\alpha} + 2\pi'\right)^\mu P_i^{M\mu}}{\left(\frac{1}{1-\alpha} + 2\pi\right)^\mu P_i^{M'\mu}}$$

Table 3 shows the total gains to trade and financial liberalization decomposed into the trade and MP channels. The countries are ranked in terms of income, and the gains are averaged across the countries belonging to each quartile. Countries in the bottom quartile, for example, gain 6.1% in real income terms on average, with more than half

these gains coming through MP 3.2%, and about a third coming through trade 2.3%. In the baseline parameterization, reforms are welfare-improving on average, but do not affect all countries equally, with more gains accruing to low-income countries. This is not surprising as poor countries gain more from consuming new goods obtained from rich countries than rich countries do from consuming new goods obtained from poor countries. Further, notice the asymmetry in the decomposition of the gains into the trade and MP channels across the different income groups. With the exception of the top quartile where trade generates greater welfare gains than MP, the MP channel dominates. This is unsurprising given that there are more MP zeros than trade zeros in the data, and a lot of these zeros involve rich, highly productive countries (as documented in Section 2). I turn to the role of the extensive margin next.

Table 1.4: Percentage Gain in Real GDP After Reform: Trade vs MP

Percentiles	Overall Gains	Trade Gains	MP Gains
00-25	6.1	2.3	3.2
26-50	6.2	2.4	3.3
51-75	5.1	2.3	2.3
76-100	3.3	2.2	0.7
Total	5.2	2.3	2.4

Benchmark vs. Model with No Extensive Margin

In the parameterization section, I discuss two versions of the model: one with a role for the aggregate extensive margin, and one without. Table 4 shows the results of running the same reforms on these two different environments. The model with aggregate entry and exit estimates greater gains from openness than the model with no zero-to-one transitions on average: 5.2% vs. 4.0% following trade and financial liberalization. This comparison between averages does not apply across the board. The underestimation

of the welfare gains from trade and financial liberalization is more significant for the bottom quartiles: whereas countries in the top quartile gain roughly 3% (3.3% or 2.6%) in both environments (with or without zero-to-one transitions), countries in the bottom quartile gain only 4.7% instead of 6.1% when one does not allow for aggregate entry and exit. Apart from overestimating the gains, there is another distinction between the two versions: reforms that lower iceberg costs worldwide unambiguously result in welfare gains in the model with no extensive margin, in contrast to the possible losses following reform in the benchmark model. In the model with no extensive margin, no additional fixed costs are incurred following reform as there is no aggregate entry or exit, and income rises and price indices fall, so welfare has to rise. By contrast, the benchmark model allows for aggregate entry and exit, with potentially higher price indices and lower income given the resources lost in paying for the additional fixed costs. To the extent that previously consumed goods are highly valued, and the fixed costs incurred in the formation of new bilateral relationships are substantial, reforms can result in lower welfare. This does not occur in this particular experiment, but does occur in other experiments (available upon request) where the model with no extensive margin not only overestimates the gains following reform, it predicts positive gains when entry and exit would imply losses. For both its effects on the absolute value of the gains as well as its sign, the aggregate extensive margin is quantitatively important for measuring the gains from openness.

1.5 Conclusion

In this paper, I document the prevalence of zeros in trade and multinational production in the data and study the effect of incorporating the extensive margin that arises from these zeros on the gains from openness. I find that relative to models where zeros matter, models with no aggregate entry or exit underestimates the gains from openness by 30%

Table 1.5: Percentage Gain in Real GDP After Reform: The Role of Zeros

Percentiles	Pure Intensive Margin Model	Benchmark Model
00-25	4.7	6.1
26-50	4.8	6.2
51-75	3.9	5.1
76-100	2.6	3.3
Total	4.0	5.2

on average, with the discrepancy larger for countries in the developing world. There has been growing interest in zeros at the firm- and goods-level; it would be interesting to see how the welfare impact of zeros at these different levels of aggregation differ and how these differences can be reconciled within a unifying framework. By the same token, there has been research that focuses on more complex interactions between trade and multinational production (e.g. intra-firm imports, export-platform multinational production), albeit focusing on firms from a particular country or a smaller group of countries where zeros do not occur. It would be instructive to see what gains result in the context of a model that has both aggregate zero-to-one transitions and direct (not general equilibrium or policy-generated) interactions between trade and multinational production. I leave both these extensions for future research.

Chapter 2

International Trade and the Rise of the Service Economy

2.1 Introduction

Why does labor move into the service sector as countries get richer? The two main explanations that have been propounded thus far are an income elasticity for services that is greater than one (e.g. Kongsamut, Rebelo, and Xie 2001), or aggregate sectoral goods that are complementary (elasticity of substitution less than one) coupled with an exogenously slow-growing service sector (e.g. Ngai and Pissarides 2008). While the former may be intuitively appealing, it is the latter which has been borne out in the data (Duarte and Restuccia, 2008). In this paper, I propose a new mechanism (asymmetrically lower trade barriers) by which labor moves into the service sector and the service sector productivity endogenously grows less over time compared to the rest of the economy.

To help motivate the theory, I document three empirical facts regarding the service

sector. First, I show that services are skill-intensive relative to the rest of the economy. This ties in nicely with the second observation that countries who were skill-abundant fifty years ago continue to be skill-abundant today. The two observations go hand in hand because these skill-abundant countries are the most highly developed countries for which services comprise a bigger portion of total value added. The third piece of evidence that I present relates to the tradability of services: relative to their share of aggregate value added or labor, services make up a much smaller portion of total trade. This suggests that the barriers to service trade are much greater than the barriers to merchandise trade.

How, then, can one make sense of the growing share of services in overall output and labor in light of these three observations? I see the growth in the labor share of services as the result of the opening to trade of the world's most skill-abundant countries. In the Heckscher-Ohlin model with heterogeneous firms that I construct, as trade barriers fall, the return to skilled labor in skill-abundant countries rise and this generates the movement of workers into the skill-intensive sector (services) of the skill-abundant country (the developed world). With the Melitz-Ottaviano (2008) setup that I use to deal with firm-level idiosyncratic productivity, my model also yields predictions that are consistent with firm-level evidence on trade in services presented in Breinlich and Criscuolo (2011): there is selection into exporting markets, and exporters hire more workers, sell more, and are more productive. These features of the data also imply that to model trade in services, firm-level heterogeneity is a must. Further, since the evidence indicates that the trade barriers to services remain high relative to those in the rest of the economy, productivity in that sector will be lower, consistent with Duarte and Restuccia (2008), where they show that low productivity in services account for most of the recent productivity slowdown in the developed world.

It is clear that this paper is closely linked to the literature on structural change, and in this literature, I take my contribution to be establishing the link between trade and the service economy. While there have been papers documenting the rise of the service economy (see Buera and Kaboski, 2011 and the references therein), thus far there has been none documenting how this rise coincides with the opening to trade of countries in which this rise has been observed (countries have to be in the advanced stages of structural transformation for services to be definitively rising over time; manufacturing could be growing instead for countries only beginning the transformation process). On trade and structural change, most research has been directed at accounting for the hump-shaped pattern of labor in manufacturing. Yi and Zhang (2011), for instance, demonstrate that in a multi-sector, Eaton-Kortum world a hump shape is possible even when the growth rate in manufacturing is the highest. Matsuyama (2009) also uses a Ricardian framework to show that high productivity growth in manufacturing need not translate to a falling manufacturing employment share. Coleman (2007), on the other hand, employs a multi-country Heckscher-Ohlin-Ricardo framework to analyze the effects of large emerging countries on world welfare. The first two papers have little to say about differences in factor abundance and intensities across sectors and how these can translate to wage differentials across sectors and slower productivity growth. The last paper has services as a non-traded sector and does not feature firm-level heterogeneity; it is thus unable to account for lower productivity growth in services that is a key feature of the data.

In terms of the modeling environment, there has been a recent resurgence of interest in Heckscher-Ohlin models, in particular those pertaining to trade and growth. Bajona and Kehoe (2006, 2010) and Caliendo (2011) have recently studied the dynamics of

Heckscher-Ohlin models and showed that patterns of specialization in such models are hard to characterize but have rich implications for both prices and quantities that help to bridge the gap between Heckscher-Ohlin models and data. While these models are a step forward from the static analysis of endowment-driven comparative advantage, they are not able to account for intra-industry trade. In contrast, models that incorporate heterogeneity in firm-level productivity produce intra-industry trade, and for certain parameterizations can deliver sharper predictions of industry-level trade that are consistent with the data. Eaton and Kortum (2002, 2005), Melitz (2003) and Chaney (2008) are prominent examples of such models, where the extensive margin is shown to be particularly important. Closer to this paper, Bernard, Redding and Schott (2007) study a two-sector static model that have both the aforementioned features: inter-industry trade as driven by differences in factor endowments across countries, and intra-industry trade as driven by differences in firm-level productivity. However, these authors do not present any evidence that suggests what the sectors in their model represent and embed the Melitz (2003) model (as opposed to Melitz-Ottaviano in my setup) in the standard Heckscher-Ohlin framework. This is important because the mechanism through which average sector-level productivity rises is fundamentally different in these two environments: in Melitz (2003), increased factor market competition (exporters demanding more workers driving up wages) forces out the least productive firms, in contrast to Melitz and Ottaviano (2008), where labor supply is perfectly elastic to differentiated firms while import competition shifts up the price elasticity of demand for all firms, causing the least productive ones to drop out. The basic Melitz (2003) framework also fails to produce endogenously different markups across firms, which is a key feature of both Melitz and Ottaviano (2008) and the data. Finally, my research is also related to the literature on the effects of globalization on the macroeconomy, where the closest paper is Burstein and Vogel (2011). Embedding Eaton-Kortum (2002) into a multi-sector

Heckscher-Ohlin model, they show that globalization is important in quantitatively accounting for the rise of the skill premium. Here I am more interested in the effects of globalization on the service sector.

The paper is organized as follows. Section 2 presents the empirical evidence relevant to the mechanism. The baseline model is described in section 3, which also contains all the main propositions. I extend the model to allow for intermediate goods and capital in Section 4 and show that the main results continue to hold. The last section concludes. All tables and graphs can be found in the Appendix.

2.2 Motivating Evidence

I present three pieces of empirical evidence. The first relates to the relative stability of skill abundance over time. My preferred measure of skill abundance is the fraction of the population aged 15 and older with completed higher education, and I find that countries that have more college graduates 50 years ago continue to have more highly educated workers on average. Further, this positive correlation is robust across decades and is presented in Table 1. Figure 1 shows this positive relationship in its weakest form between 1960 and 2010. Even then the correlation coefficient is 0.77. The source of the data is the Cohen and Soto Education Database (2007).

The second piece of evidence relates to the tradability of services. While more than half of the labor force in most developed countries are now employed in the services, service trade still makes up a much smaller fraction of overall trade. Figure 3 shows the evolution of labor shares in the service sector for the OECD and GGDC countries in the last thirty years. This monotonic increase is well documented in the structural change literature and goes back to the 1960s. Trade data for services are obtained as a residual between total trade data obtained from World Bank and goods trade data obtained

from UN Comtrade. This disparity between the labor and trade share of services is displayed in Figure 2. Note that all 30 countries for which we have data lie below the 45 degree line. I interpret this to mean the barriers to trade in services are significantly higher than barriers to trade in non-services.

The last piece of evidence concerns the skill intensity of the service sector relative to non-services. Given that my measure of skill in this paper is completed higher education, I use data from the March Current Population Survey (CPS) to see whether there are better-educated workers in the service sector relative to the rest of the economy. Table 2 shows that for both average years of education and percentage that complete higher education, service workers score systematically higher than those in non-services in the last 40 years. I take this to be evidence that services are more skill-intensive than the rest of the economy. Snapshots in time of skill intensity of the various subsectors that make up services and non-services are provided in Tables 3 and 4. While there is significant heterogeneity across subsectors within these broader categories, it is clear that service workers on average are better-educated than the workers in the rest of the economy. The (standard) sector classification used is presented in Table 5.

To summarize, I make the following observations regarding the data.

Observation 2.1. *The relative levels of skill abundance across countries have been stable over time.*

Observation 2.2. *Barriers to trade in services are higher than in the rest of the economy.*

Observation 2.3. *In the United States, services are skill-intensive relative to non-services.*

There are two other aggregate-level observations that I will try to address using this framework, the first of which concerns the skill premium and the second, sector-level

productivity in the services. It is well-known that the skill premium as measured by relative wages of college graduates to the rest of the workforce has been rising over time. Moreover, productivity growth in services is lower on average than that of the rest of the economy (Duarte and Restuccia, 2010).

The observations I have highlighted relate to the service sector as a whole. They are instructive for the two main points that I want to make, i.e. movement of labor into services in the developed world and lower productivity in the services. They paint an incomplete picture, however, given that there is significant (firm-level) heterogeneity within both the service and non-service sectors. Much work has been done documenting the properties of firms that engage in goods trade (the latter): see Bernard and Jensen (1995, 1999), Bernard et al. (2007), Wagner (2007), and Greenaway and Kneller (2007). There has been, however, little research documenting the activities of firms involved in service trade, with the exception of the recent contribution by Breinlich and Criscuolo (2011). Using two firm-level datasets on service exports and imports in the UK from 2000 to 2005, these authors show that service exporters are similar to goods exporters in many ways: exporters sell more, hire more workers, and are more productive than non-exporters; also, relatively few firms export and within exporters, only a few firms export many products to many destinations. Given the similarities between service and goods exporters, and the fact that one of the foundations (Melitz and Ottaviano, 2008) on which I build my model can account well for the trends observed in goods exporters, my model will generate implications consistent with the data along this dimension as well.

2.3 Model

The basic setup is a combination of Melitz-Ottaviano (2008) and Heckscher-Ohlin. There are two sectors, one more skill-intensive than the other. There are two factors, skilled and unskilled labor. Both types of labor are specialized in the sense that skilled workers cannot do things that unskilled workers can do, while unskilled labor also cannot be used to substitute for skilled labor. Consider first two countries, one skilled-labor abundant the other unskilled-labor abundant. Within each sector, there is a continuum of differentiated goods which is subject to costly trade, and an outside good that is always produced (thus restricting the parameter space) and freely traded across countries. Factor price equalization is assumed throughout (yet another constraint on the parameter space). Technology is linear, and initially I consider a variant of the model where factors are sector-specific, i.e. the skill-intensive sector uses only skilled labor while the other sector uses only unskilled labor. This setup is reminiscent of Ventura (1997), and it eases the exposition considerably. The main results still go through in more general setups, including those with intermediate and capital goods, as I show later. For now, this assumption guarantees that with the skilled wage normalized to one, the price of the freely traded outside good in the skilled sector is one, while that in the unskilled sector is simply the unskilled wage w_U .

2.3.1 Autarky

Households in each country solve the following problem

$$\begin{aligned} \max \quad & a \log(q_{s,0}^c + \alpha \int q_s^c(i) di) - \frac{1}{2} \gamma \int [q_s^c(i)]^2 di - \frac{1}{2} \eta \left[\int q_s^c(i) di \right]^2 \\ & + (1-a) \log(q_{u,0}^c + \alpha \int q_u^c(i) di) - \frac{1}{2} \gamma \int [q_u^c(i)]^2 di - \frac{1}{2} \eta \left[\int q_u^c(i) di \right]^2 \\ \text{s.t.} \quad & q_{s,0}^c + \int p_s(i) q_s^c(i) di + w_U q_{u,0}^c + \int p_l(i) q_u^c(i) di \leq M \end{aligned}$$

Here a is the income share of the skill-intensive sector, $q_{s,0}^c$ is an individual consumer's demand for the outside good in the skill-intensive sector, and $q_{u,0}^c$ is the corresponding demand for the outside good in the other sector. Individual demand for differentiated goods (varieties) in sector j is denoted by $q_j^c(i)$. We denote income by M . Utility over the differentiated goods in each sector is defined as in Melitz-Ottaviano (2008): α and η control the demand for the differentiated goods relative to the outside good while γ controls the amount of disutility an individual gets from variation in his consumption of the different varieties.

The solution to the problem above are demand functions of the form

$$\begin{aligned} q_s^c(i) &= \frac{1}{\gamma} \left[\alpha - p_s(i) - \frac{\eta N_s (\alpha - \bar{p}_s)}{\gamma + \eta N_s} \right] \\ q_u^c(i) &= \frac{1}{\gamma} \left[\alpha - \frac{p_s(i)}{w_U} - \frac{\eta N_u (\alpha - \frac{\bar{p}_u}{w_U})}{\gamma + \eta N_u} \right] \end{aligned}$$

where $p_j(i)$ denotes that price for the differentiated variety i in sector j , N_j is the number of monopolistically competitive sellers (firms) in sector j , and $\bar{p}_j = \int p_j(i) di$ is the average price of a variety in sector j . Note that demand for individual varieties within sector can be zero, as marginal utilities for all goods are bounded. In particular, when the price of a variety in sector i exceeds $p_{i,max}$, demand for that variety goes to

zero. From the demand functions we get that $p_{i,max}$ is

$$c_{SD} = p_{s,max} = \frac{\gamma\alpha}{\gamma + \eta N_s} + \frac{\eta N_s}{\gamma + \eta N_s} \bar{p}_s$$

$$c_{UD} = p_{u,max} = \frac{\gamma\alpha}{\gamma + \eta N_u} + \frac{\eta N_u}{\gamma + \eta N_u} \frac{\bar{p}_u}{w_U}$$

Given these cutoff points, I can write the (aggregate) demand equations for a country with population (total workforce) L as

$$q_s(i) = q_s^c(i) = \frac{L}{\gamma} [c_{SD} - p_s(i)], \quad q_U(i) = q_U^c(i) = \frac{L}{\gamma} [c_{UD} - \frac{p_u(i)}{w_U}]$$

The price of the differentiated good is obtained by solving the firm's problem. Firms draw idiosyncratic unit cost (inverse productivity) c from a distribution with cdf $G(c)$ and operate linear technology, hence solving the problem given by

$$\max_c p_s(c)q_s(c) - cq_s(c) \quad \Rightarrow \quad q_s(c) - \frac{L}{\gamma} p_s(c) - \frac{cL}{\gamma} = 0$$

$$\max_c p_u(c)q_U(c) - w_U c q_U(c) \quad \Rightarrow \quad q_U(c) - \frac{L}{\gamma} \frac{p_u(c)}{w_U} - \frac{cL}{\gamma} = 0$$

Taking demand functions and factor prices as given, firms maximizing profits set the optimal pricing rule

$$p_s(c) = \frac{1}{2}(c_{SD} + c), \quad p_u(c) = \frac{1}{2}(c_{UD} + c)w_U$$

Given these pricing functions, the maximum price obtained earlier also pins down the cutoff productivity below which firms that produce will encounter zero demand. The inverse of these cutoff levels of productivity are denoted c_{UD} and c_{SD} , consistent with notation in Melitz-Ottaviano (2008). As will be seen shortly, this serves as a sufficient statistic for describing other measures of firm performance at the aggregate level that are often reported in the data: markup, output, sales, and profits. For a firm with

inverse productivity c , the (sector-specific) performance measures are given by

Markup

$$\mu_s(c) = \frac{1}{2}(c_{SD} - c), \quad \mu_U(c) = \frac{1}{2}(c_{UD} - c)w_U$$

Output

$$q_s(c) = \frac{L}{2\gamma}(c_{SD} - c), \quad q_U(c) = \frac{L}{2\gamma}(c_{UD} - c)$$

Sales

$$r_s(c) = \frac{L}{4\gamma}(c_{SD}^2 - c^2), \quad r_U(c) = \frac{L}{4\gamma}(c_{UD}^2 - c^2)w_U$$

Profits

$$\pi_s(c) = \frac{L}{4\gamma}(c_{SD} - c)^2, \quad \pi_U(c) = \frac{L}{4\gamma}(c_{UD} - c)^2w_U$$

Firms with lower unit costs (higher productivity) charge lower prices, but do not completely bear the cost differential in that they charge higher markups as well. Their lower prices enable these firms to make higher profits from increased sales.

It is worth noting that in this environment, as in Melitz-Ottaviano (2008), markups vary across firms, with higher productivity firms charging higher markups as we see in the data. This is in contrast to Melitz (2003) or models with monopolistic competition but homogeneous goods (e.g. Krugman 1980 or Helpman and Krugman 1987), where the markup is tied to the elasticity of substitution across goods. Note that given our assumptions on preferences and technologies, differences across sectors are only manifested in the wage term that enters nominal variables like profits, markups and sales. In particular, output for two firms that belong to different sectors but have the same productivity will be the same in this economy.

To get analytical expressions for these variables and the cutoff level of productivity,

assume that firm-level productivity draws follow a Pareto distribution with lower productivity bound $\frac{1}{c_M}$ and shape parameter $k \geq 1$, i.e. $G(c) = (\frac{c}{c_M})^k$, $c \in [0, c_M]$. Then the free entry condition yields the cutoff level for each sector

$$\int \pi_s(c) dG(c) = f_{e,s} \quad \Rightarrow \quad c_{SD} = \left[\frac{2\gamma f_{e,s} c_m^K (k+1)(k+2)}{L} \right]^{\frac{1}{k+2}}$$

$$\int \pi_U(c) dG(c) = w_U f_{e,U} \quad \Rightarrow \quad c_{UD} = \left[\frac{2\gamma f_{e,U} c_m^K (k+1)(k+2)}{L} \right]^{\frac{1}{k+2}}$$

Given these cutoff values and the assumptions on the productivity distribution, I can write the performance measures for the average firm, which I do only for the skilled-labor sector for the sake of brevity (expressions for the other sector are analogous).

With the average level for any variable x_j given by $\bar{x}_j = \frac{\int_0^{c_{jD}} x_j(c) dc}{G(c_{jD})}$ we have

$$\begin{aligned} \bar{c}_S &= \frac{k}{k+1} c_{SD}, & \bar{q}_S &= \frac{L}{2\gamma} \frac{1}{k+1} c_{SD} = \frac{(k+2)(c_M)^k}{(c_{SD})^{k+1}} f_{e,s} \\ \bar{p}_S &= \frac{2k+1}{2k+2} c_{SD}, & \bar{r}_S &= \frac{L}{2\gamma} \frac{1}{k+2} (c_{SD})^2 = \frac{(k+1)(c_M)^k}{(c_{SD})^k} f_{e,s} \\ \bar{\mu}_S &= \frac{1}{2} \frac{1}{k+1} c_{SD}, & \bar{\pi}_S &= f_{e,s} \frac{(c_M)^k}{(c_{SD})^k} \end{aligned}$$

The cutoff c_{iD} completely summarizes the distribution of prices as well as other performance measures. One other variable critical in developing intuition for the mechanism behind this model is competition. Competition in this world is the number of firms N_i competing in sector i , where N_i is given by

$$N_s = \frac{2\gamma}{\eta} \frac{\alpha - c_{SD}}{c_{SD} - \bar{c}_S}, \quad N_u = \frac{2\gamma}{\eta} \frac{\alpha - c_{UD}}{c_{UD} - \bar{c}_U}$$

Welfare, as measured by indirect utility V , is then

$$V = a \log\left(S + \frac{\alpha - c_{SD}}{2\eta} \left[\alpha - \frac{k+1}{k+2} c_{SD}\right]\right) + (1-a) \log\left(w_U U + \frac{\alpha - c_{UD}}{2\eta} \left[\alpha - \frac{k+1}{k+2} c_{UD}\right]\right)$$

Where the relative wage is determined by income shares and factor endowments

$$\frac{w_U U}{S} = \frac{1-a}{a} \Rightarrow w_U = \frac{1-a}{a} \frac{S}{U}$$

Clearly, welfare increases with decreases in the cutoffs c_{SD} and c_{UD} , as these induce increases in product variety N_S and N_U and decreases in the average price p_S and p_U in each sector.

2.3.2 Free Trade

Free trade in this environment with factor price equalization allows me to characterize equilibrium using the integrated equilibrium approach as in Dixit and Norman (1980). This implies the equilibrium relationships obtained above continue to hold, albeit for the world economy as a whole. I begin with a statement about factor returns.

Claim 2.4. *The return to the skilled labor relative to unskilled labor is lower in the skill-abundant country and rises in the movement from autarky to free trade.*

Proof. Since

$$\frac{U^1}{S^1} < \frac{U^1 + U^2}{S^1 + S^2} < \frac{U^2}{S^2}$$

hence

$$\begin{aligned} \frac{a}{1-a} \frac{U^1}{S^1} &< \frac{a}{1-a} \frac{U^1 + U^2}{S^1 + S^2} < \frac{a}{1-a} \frac{U^2}{S^2} \\ \Leftrightarrow \frac{1}{w_U^{1,A}} &< \frac{1}{w_U^{FT}} < \frac{1}{w_U^{2,A}} \end{aligned}$$

□

As in Heckscher Ohlin, the autarky return to the abundant factor is lower and the opening of both countries to free trade raises the return to these factors when factor price equalization holds. In the world considered above, there is no intersectoral labor reallocation following the opening to trade because factors are sector-specific. But in

a more realistic setting where both factors are needed in the production of goods in either sector, the change in factor returns will induce movements across and within sectors. Within sector movements in this case reflect the change in the cutoff value: the greater competition brought out by the opening of trade lowers the cutoff inverse productivity level at which firms can sell positive amounts of their goods and induces labor reallocation from low productivity to high productivity firms within each sector. The proof for the change in cutoff levels is straightforward as demonstrated in the claim below.

Claim 2.5. *The cutoff levels c_{SD} and c_{UD} for inverse productivity in each country falls from autarky to free trade.*

Proof. Given that

$$c_{SD} = \left[\frac{2\gamma f_{e,s} c_m^K (k+1)(k+2)}{L} \right]^{\frac{1}{k+2}}, \quad c_{UD} = \left[\frac{2\gamma f_{e,U} c_m^K (k+1)(k+2)}{L} \right]^{\frac{1}{k+2}}$$

Under autarky,

$$L^A = L^i = S^i + U^i$$

With free trade

$$L^{FT} = \sum_{i=1}^2 L^i = \sum_{i=1}^2 S^i + U^i$$

Hence for any sector i in country j , where $i \in \{S, U\}$, $j \in \{1, 2\}$, we have

$$c_{iD}^{FT} = c_{iD}^{j,FT} < c_{iD}^{j,A}$$

□

Note that this contrasts with the result obtained in Bernard, Redding and Schott (2007) where sectoral cutoffs are unchanged when moving from free trade to autarky. Here the intuition is that with free trade, domestic producers face greater competition resulting in the exit of lower productivity firms in both sectors. The next claim shows clearly the effects of trade on competition.

Claim 2.6. *Competition in both sectors is higher (i.e. there are more firms in equilibrium) in free trade than in autarky for both countries.*

Proof. By the claim above, $\forall i \in S, U$ and $j \in 1, 2$

$$c_{iD}^{FT} = c_{iD}^{j,FT} < c_{iD}^{j,A}$$

Now note that

$$N_s = \frac{2\gamma}{\eta} \frac{\alpha - c_{SD}}{c_{SD} - \bar{c}_S}, \quad N_u = \frac{2\gamma}{\eta} \frac{\alpha - c_{UD}}{c_{UD} - \bar{c}_U}$$

Since N_i is decreasing in c_{iD} for $i = S, U$, we must have $\forall i \in S, U$ and $j \in 1, 2$

$$N_{iD}^{FT} = N_{iD}^{j,FT} > N_{iD}^{j,A}$$

□

Building on this result, I can complete the basic intuition by discussing the effect greater competition has on the firm-level performance measures. Higher competition forces the average firm to charge a lower markup but greater demand in turn enables them to sell more, get higher revenue and earn greater profits. Hitherto what I've discussed is not fundamentally different than the mechanism in Melitz-Ottaviano (2008). The twist in our setup lies in the fact that the statements on sales, revenue and profit above are now only true for the comparative advantage sector in the skill-abundant country (conversely for the other country); in the comparative disadvantage sector, the return to the factor used intensively falls following the move to free trade so that the aforementioned nominal variables for the average firm no longer respond monotonically to the change in cutoffs as discussed. This is summarized in the next claim.

Claim 2.7. *Average firm-level productivity and output is higher and markup lower in both sectors in free trade than in autarky. In the skill-abundant country, the sales (revenue) and profit of the average firm are higher in the comparative advantage sector under free trade.*

Proof. From above

$$\begin{aligned}\bar{c}_S &= \frac{k}{k+1}c_{SD}, & \bar{q}_S &= \frac{L}{2\gamma} \frac{1}{k+1}c_{SD} = \frac{(k+2)(c_M)^k}{(c_{SD})^{k+1}}f_{e,s} \\ \bar{p}_S &= \frac{2k+1}{2k+2}c_{SD}, & \bar{r}_S &= \frac{L}{2\gamma} \frac{1}{k+2}(c_{SD})^2 = \frac{(k+1)(c_M)^k}{(c_{SD})^k}f_{e,s} \\ \bar{\mu}_S &= \frac{1}{2} \frac{1}{k+1}c_{SD}, & \bar{\pi}_S &= f_{e,s} \frac{(c_M)^k}{(c_{SD})^k}\end{aligned}$$

To complete the argument, I write out the equivalent expressions for the unskilled-labor intensive sector:

$$\begin{aligned}\bar{c}_U &= \frac{k}{k+1}c_{UD}, & \bar{q}_U &= \frac{L}{2\gamma} \frac{1}{k+1}c_{UD} = \frac{(k+2)(c_M)^k}{(c_{UD})^{k+1}}f_{e,u} \\ \bar{p}_U &= \frac{2k+1}{2k+2}c_{UD}w_U, & \bar{r}_U &= \frac{L}{2\gamma} \frac{1}{k+2}(c_{UD})^2w_U = \frac{(k+1)(c_M)^k}{(c_{UD})^k}f_{e,u}w_U \\ \bar{\mu}_U &= \frac{1}{2} \frac{1}{k+1}c_{UD}w_U, & \bar{\pi}_U &= f_{e,u} \frac{(c_M)^k}{(c_{UD})^k}w_U\end{aligned}$$

The claim then follows immediately by noting that $\forall i \in \{S, U\}$ and $j \in \{1, 2\}$

$$c_{iD}^{FT} = c_{iD}^{j,FT} < c_{iD}^{j,A}$$

and

$$\frac{1}{w_U^{1,A}} < \frac{1}{w_U^{FT}} < \frac{1}{w_U^{2,A}}$$

□

Now consider the skill premium. Consider a set of M countries that are opening up to trade. Label the countries in order of decreasing skill abundance, i.e.

$$\frac{S^1}{U^1} > \frac{S^2}{U^2} > \dots > \frac{S^M}{U^M}$$

Then the skill premium in the skill-abundant countries rises as these countries open up to trade in turn, as proved in the next claim. This is qualitatively consistent with the fact that the skill premium has risen over the last few decades even as (tariff and non-tariff) trade barriers have fallen and countries with ever lower levels of skill and development have opened up to trade. This is where Observation 1 comes in: it says that the ranking of countries in terms of skill abundance has remained fairly stable over time. However, as is true of most Heckscher-Ohlin models, the counterfactual implication that the skill premium decreases in the skill-scarce country is also true in this environment; a different mechanism (e.g. skill-biased technical change) would have to be introduced to remedy this shortcoming of the model.

Claim 2.8. *The skill premium in the skill-abundant country rises as countries sequentially open up to trade, the sequence in decreasing levels of skill abundance.*

Proof. The claim follows from Claim 4, the sequence of inequalities above, and the simple observation that

$$\begin{aligned} \frac{U^1}{S^1} &< \frac{U^1 + U^2}{S^1 + S^2} < \frac{U^1 + U^2 + U^3}{S^1 + S^2 + S^3} < \dots < \frac{\sum_{j=1}^M U^j}{\sum_{j=1}^M S^j} \\ \Leftrightarrow \frac{1}{w_U^1} &< \frac{1}{w_U^{1,2}} < \frac{1}{w_U^{1,2,3}} < \dots < \frac{1}{w_U^{1,2,\dots,M}} \end{aligned}$$

where $w_U^{1,2,\dots,j}$ here denotes the return to unskilled labor in a world where only the countries $\{1, 2, \dots, j\}$ have opened up to trade. \square

As mentioned above, there is no margin for inter-sectoral labor reallocations in the Ventura-like model I've been analyzing so far because factors are sector-specific. However, there are still intra-sectoral reallocations following the opening to free trade and these are interesting in their own right. I will discuss the more general setup with inter- as well as intra-sectoral reallocation shortly but focus on this simplified version to isolate the effect of trade on labor reallocation within sectors.

Claim 2.9. *Labor moves from low to high productivity firms within sectors. Some firms exit after the move to free trade; other firms survive but hire fewer workers; still other firms thrive and increase labor demand and output.*

Proof. The first statement is easy to show. Denote $c_{iD}^{j,A}$ to be the cutoff value in country j sector i under autarky, and c_{iD}^{FT} the similar cutoff under free trade. I know from above that $c_{iD}^{j,A} > c_{iD}^{FT}$ hence all firms with inverse productivity $c \in (c_{iD}^{FT}, c_{iD}^{j,A})$ in country j sector i exit (i.e. $q_i^j(c) = l_i^j(c) = 0$) during free trade.

Whether firms produce more or less (and hence hire more or less labor) depends on their idiosyncratic productivity level. The higher a firm's productivity level, the greater the probability it will increase its output in free trade relative to autarky. In particular, I know

$$q_i(c) = \frac{L}{2\gamma}(c_{iD} - c), \quad c_{iD} = \left[\frac{2\gamma f_{e,i} c_m^K (k+1)(k+2)}{L} \right]^{\frac{1}{k+2}}$$

Relative to autarky, in free trade

$$L^{FT} > L^A, \quad \text{but} \quad c_{iD}^{FT} < c_{iD}^A$$

These opposing effects imply that for $q_i^{j,FT}(c) > q_i^{j,A}(c)$ iff

$$c < \tilde{c}_i = \Lambda \frac{L^{FT \frac{k+1}{k+2}} - L^A \frac{k+1}{k+2}}{L^{FT} - L^A}, \quad \Lambda = (2\gamma f_{e,i} c_m^K (k+1)(k+2))^{\frac{1}{k+2}}$$

Hence firms with inverse productivity $c < \tilde{c}_i$ increase production and hire more labor while firms with inverse productivity $c \in (\tilde{c}_i, c_{iD}^{FT})$ cut back and hire less labor. In this way labor is reallocated from less productive to more productive firms. \square

Now that intra-sectoral labor reallocation has been discussed, I turn to the more general case where there is also inter-sectoral labor reallocation. In this case, assume the production function in each sector is of Cobb-Douglas form, i.e. $f(s_i, u_i) = s_i^{\beta_i} u_i^{1-\beta_i}$,

where $1 > \beta_1 > \beta_2 > 0$ so that sector 1 is skill-intensive. Assume country 1 (Home) is skill-abundant. Given these production functions, I know that costs in each sector now take on the form $\kappa_i \frac{w_s^{\beta_i} w_u^{1-\beta_i}}{\beta_i^{\beta_i} (1-\beta_i)^{1-\beta_i}}$, where $\kappa_i = 1$ for the outside good and $\kappa_i = c$ for the differentiated good. Given these assumptions, I now show that both types of labor move into the comparative advantage sector when going from autarky to free trade.

Claim 2.10. *The comparative advantage sector in each country employ a greater share of both types of labor under free trade. Further, the move from autarky to free trade increases the share of both types of labor allocated to the comparative advantage sector.*

Proof. First I derive the wage of unskilled labor relative to skilled labor in this environment with factor price equalization and Cobb-Douglas production technology. Normalize skilled wage $w_S = 1$. Denote global revenues by R . I know that

$$w_S S_i + w_U U_i = R_i \Rightarrow w_S S + w_U U = R, \quad S = \sum_i S_i, \quad U = \sum_i U_i$$

Further, global sector-level labor allocations must satisfy

$$S_i = \frac{\beta_i R_i}{w_S}, \quad U_i = \frac{(1-\beta_i) R_i}{w_U} \Rightarrow S_1 = \frac{\beta_1 R_1}{\beta_1 R_1 + \beta_2 R_2} \Rightarrow S_1 = \frac{\beta_1 a}{\beta_1 a + \beta_2 (1-a)}$$

where the last implication follows from

$$\frac{R_1}{R_2} = \frac{a}{1-a}$$

Then similarly I obtain

$$S_2 = \frac{\beta_2 (1-a)}{\beta_1 a + \beta_2 (1-a)}, \quad U_1 = \frac{(1-\beta_1) a}{(1-\beta_1) a + (1-\beta_2) (1-a)}, \quad U_2 = \frac{(1-\beta_2) (1-a)}{(1-\beta_1) a + (1-\beta_2) (1-a)}$$

Then given that

$$\frac{w_S S_1 + w_U U_1}{w_S S_2 + w_U U_2} = \frac{R_1}{R_2} = \frac{a}{1-a}$$

I have

$$\begin{aligned}
S_1 + w_U U_1 &= \frac{a}{1-a} [S_2 + w_U U_2] \\
\Rightarrow w_U (U_1 - \frac{a}{1-a} U_2) &= \frac{a}{1-a} S_2 - S_1 \\
\Rightarrow w_U &= \frac{(1-\beta_2)(1-a) + (1-\beta_1)a S}{\beta_1 a + \beta_2(1-a)} \bar{U}
\end{aligned}$$

where I used the (global) sectoral labor allocations derived earlier in the last line.

Given this relative wage, note that the country-level sectoral labor allocations $\{S_i^H, U_i^H, S_i^F, U_i^F\}$ also satisfy

$$\frac{S_i^H}{U_i^H} = \frac{S_i^F}{U_i^F} = \frac{w_U}{w_S} \frac{\beta_i}{1-\beta_i} = \frac{\beta_i}{1-\beta_i} w_U$$

where H is used to denote home or country 1 and F foreign or country 2. Hence

$$\begin{aligned}
S_i^H &= U_1^H \frac{\beta_i}{1-\beta_i} w_U \Rightarrow S^H = S_1^H + S_2^H = U_1^H \frac{\beta_1}{1-\beta_1} w_U + U_2^H \frac{\beta_2}{1-\beta_2} w_U \\
\Rightarrow \frac{1}{w_U} S^H &= U_1^H \frac{\beta_1}{1-\beta_1} + (U^H - U_1^H) \frac{\beta_2}{1-\beta_2} \\
\Rightarrow U_1^H &= \frac{\frac{1}{w_U} S^H - \frac{\beta_2}{1-\beta_2} U^H}{\frac{\beta_1}{1-\beta_1} - \frac{\beta_2}{1-\beta_2}}
\end{aligned}$$

Similarly I have (imposing market clearing)

$$\begin{aligned}
U_2^H &= \frac{\frac{\beta_1}{1-\beta_1} U^H - \frac{1}{w_U} S^H}{\frac{\beta_1}{1-\beta_1} - \frac{\beta_2}{1-\beta_2}} \\
S_1^H &= \frac{\frac{\beta_1}{1-\beta_1} \frac{1}{w_U} S^H - \frac{\beta_2}{1-\beta_2} \frac{\beta_1}{1-\beta_1} U^H}{\frac{\beta_1}{1-\beta_1} - \frac{\beta_2}{1-\beta_2}} \\
S_2^H &= \frac{\frac{\beta_2}{1-\beta_2} \frac{\beta_1}{1-\beta_1} U^H - \frac{\beta_2}{1-\beta_2} \frac{1}{w_U} S^H}{\frac{\beta_1}{1-\beta_1} - \frac{\beta_2}{1-\beta_2}}
\end{aligned}$$

Sectoral labor allocations for the foreign country can be found analogously by simply replacing U^H with U^F and S^H with S^F in the equations above.

With the sectoral labor allocations in hand, the rest of the proof is straightforward. First note that as in the case with no sector-specific factors (and no inter-sectoral labor reallocation), the relative return of the abundant factor rises when we move from autarky to free trade:

$$\begin{aligned}
& \frac{(1 - \beta_2)(1 - a) + (1 - \beta_1)a}{\beta_1 a + \beta_2(1 - a)} \frac{S^1}{U^1} \\
& > \frac{(1 - \beta_2)(1 - a) + (1 - \beta_1)a}{\beta_1 a + \beta_2(1 - a)} \frac{S^1 + S^2}{U^1 + U^2} \\
& > \frac{(1 - \beta_2)(1 - a) + (1 - \beta_1)a}{\beta_1 a + \beta_2(1 - a)} \frac{S^2}{U^2} \\
& \Leftrightarrow \frac{1}{w_U^{1,A}} < \frac{1}{w_U^{FT}} < \frac{1}{w_U^{2,A}}
\end{aligned}$$

Then from the sectoral labor allocations just derived, and the rise in the relative return of the abundant factor above, I have that

$$\begin{aligned}
U_1^{H,FT} &> U_1^{H,A}, & U_2^{H,FT} &< U_2^{H,A} \\
S_1^{H,FT} &> S_1^{H,A}, & S_2^{H,FT} &< S_2^{H,A} \\
U_1^{F,FT} &< U_1^{F,A}, & U_2^{F,FT} &> U_2^{F,A} \\
S_1^{F,FT} &< S_1^{F,A}, & S_2^{F,FT} &> S_2^{F,A}
\end{aligned}$$

which proves the second part of the claim. The first part can be seen by noting that given $\beta_1 > \beta_2$, I have

$$\frac{\beta_1}{1 - \beta_1} - \frac{\beta_2}{1 - \beta_2} > 0 > -2 \frac{\beta_1 a + \beta_2(1 - a)}{(1 - \beta_2)(1 - a) + (1 - \beta_1)a}$$

which is equivalent to

$$U_1^{H,A} > U_2^{H,A}$$

Coupled with our observations on how free trade compares to autarky, this yields

$$U_1^{H,FT} > U_1^{H,A} > U_2^{H,A} > U_2^{H,FT}$$

as desired. Similarly for the foreign country. Then since

$$\frac{S_i^H}{U_i^H} = \frac{S_i^F}{U_i^F} = \frac{w_U}{w_S} \frac{\beta_i}{1 - \beta_i}$$

I have

$$S_1^{H,FT} > S_1^{H,A} > S_2^{H,A} > S_2^{H,FT}$$

and the proof is complete. \square

2.3.3 Costly Trade

Thus far, I have allowed either no trade or free trade with no impediments whatsoever. It is instructive to see how the model behaves under these rather extreme assumptions but the data indicate that the costs to trade, while finite, are nonzero as well. This is what I turn to next.

Assume now that to ship 1 unit of sector k good to country j from country i , $\tau_{kj}^i \geq 1$ units have to be produced, where $\tau_{ki}^i = 1$. Without loss of generality denote τ_{kj}^i by $\tau_{k,j} > 1$ when $i \neq j$. It is important that $\tau_{k,j} > 1$ by allowed to vary by sector k , as will be made clear shortly.

The demand functions from the consumer's problem are unchanged, bearing in mind however that the prices (which are set by monopolistically competitive firms) will reflect the costs of moving goods across national borders. These demand equations, taken as given by firms, are

$$q_{s,j}^i = \frac{L^j}{\gamma} \left[\alpha - p_{s,j}^i(c) - \frac{\eta N_S^j (\alpha - \bar{p}_s^j)}{\gamma + \eta N_S^j} \right]$$

$$q_{u,j}^i = \frac{L^j}{\gamma} \left[\alpha - \frac{p_{u,j}^i(c)}{w_U} - \frac{\eta N_U^j (\alpha - \frac{\bar{p}_u^j}{w_U^j})}{\gamma + \eta N_U^j} \right]$$

Now that there are trade costs, the cutoff for inverse productivity are no longer necessarily given by the maximum price that firms can charge before demand for their variety falls to zero. Instead, they vary for exporters and domestic producers and are related by

$$\begin{aligned}\tau_{s,j}c_{jSD}^i &= c_{jSD}^j = p_{jSmax} = \frac{\gamma\alpha}{\gamma + \eta N_S^j} - \frac{\eta N_S^j}{\gamma + \eta N_S^j} \bar{p}_S^j \\ \tau_{u,j}c_{jUD}^i &= c_{jUD}^j = p_{jUmax}w_U = \frac{\gamma\alpha}{\gamma + \eta N_U^j} - \frac{\eta N_U^j}{\gamma + \eta N_U^j} \frac{\bar{p}_U^j}{w_U}\end{aligned}$$

Here c_{jSD}^i is the cutoff for firm inverse productivity of country i firms that want to sell a skill-intensive differentiated variety in country j , N_i^j is the total (domestic and foreign) number of sellers in sector i of country j , and \bar{p}_i^j is the average price of a differentiated variety in sector i of country j . Then from the firm's problem, the first order conditions are

$$\begin{aligned}q_{s,j}^i(c) - \frac{L^j}{\gamma} p_{s,j}^i(c) - \tau_{s,j} \frac{cL^j}{\gamma} &= 0 \\ q_{u,j}^i(c) - \frac{L^j}{\gamma} \frac{p_{u,j}^i(c)}{w_U} - \tau_{u,j} \frac{cL^j}{\gamma} &= 0\end{aligned}$$

Combining this with the demand equations yields the pricing rule

$$p_{s,j}^i(c) = \frac{1}{2}\tau_{s,j}(c_{jSD}^i + c), \quad p_{u,j}^i(c) = \frac{1}{2}\tau_{u,j}(c_{jUD}^i + c)w_U$$

Note also that

$$c_{jSD}^i = \frac{c_{SD}^j}{\tau_{s,j}}, \quad c_{jUD}^i = \frac{c_{UD}^j}{\tau_{u,j}}, \quad \forall i \neq j$$

where we have suppressed the destination country subscript for the domestic cutoff (i.e. $c_{jSD}^j = c_{SD}^j$.) This shows that there is selection into export markets. Because of trade barriers, not all producing firms can export, and in particular, only firms with high enough productivity (low enough inverse productivity) will be able to do so. This implies that exporters on average will have higher productivity. This means that foreign competition will be tougher, and low productivity firms will exit. The equilibrium cutoff

(for production) under costly trade is then lower relative to autarky. Mechanically, this can be seen from the free entry condition, which pins down the cutoff level. For country i ,

$$\frac{L^i}{4\gamma} \int_0^{c_{SD}^i} (c_{SD}^i - c)^2 dG(c) + \frac{L^j \tau_{s,j}^2}{4\gamma} \int_0^{c_{jSD}^i} (c_{jSD}^i - c)^2 dG(c) = f_{e,s}$$

$$\Rightarrow L^i (c_{SD}^i)^{k+2} + L^j \rho_S^j (c_{SD}^i)^{k+2} = \phi_S \gamma$$

$$\text{where } \phi_S = 2\gamma (c_M)^k f_{e,s} (k+2), \quad \rho_S^j = (\tau_{s,j})^{-k}$$

Similarly for country j

$$\Rightarrow L^j (c_{SD}^j)^{k+2} + L^i \rho_S^i (c_{SD}^j)^{k+2} = \phi_S \gamma$$

This yields domestic cutoff inverse productivity levels for the skill-intensive sector

$$c_{SD}^i = \left[\frac{\phi_S \gamma (1 - \rho_S^j)}{L^i (1 - \rho_S^i \rho_S^j)} \right]^{\frac{1}{k+2}}, \quad c_{SD}^j = \left[\frac{\phi_S \gamma (1 - \rho_S^i)}{L^j (1 - \rho_S^i \rho_S^j)} \right]^{\frac{1}{k+2}},$$

Likewise, domestic cutoff inverse productivity levels for the other sector are

$$c_{UD}^i = \left[\frac{\phi_U \gamma (1 - \rho_U^j)}{L^i (1 - \rho_U^i \rho_U^j)} \right]^{\frac{1}{k+2}}, \quad c_{UD}^j = \left[\frac{\phi_U \gamma (1 - \rho_U^i)}{L^j (1 - \rho_U^i \rho_U^j)} \right]^{\frac{1}{k+2}},$$

In Melitz-Ottaviano (2008), as the distribution of exporters' delivered cost matches the distribution of domestic firm costs, the distribution of all firm measures in the open economy setting is the same as in autarky. This property carries over to my model. As

such, (country-specific) average firm-level performance measures are as before:

$$\begin{aligned}
\bar{c}_S^i &= \frac{k}{k+1} c_{SD}^i, & \bar{q}_S^i &= \frac{L^i}{2\gamma} \frac{1}{k+1} c_{SD}^i = \frac{(k+2)(c_M)^k}{(c_{SD}^i)^{k+1}} f_{e,s} \\
\bar{p}_S^i &= \frac{2k+1}{2k+2} c_{SD}^i, & \bar{r}_S^i &= \frac{L^i}{2\gamma} \frac{1}{k+2} (c_{SD}^i)^2 = \frac{(k+1)(c_M)^k}{(c_{SD}^i)^k} f_{e,s} \\
\bar{\mu}_S^i &= \frac{1}{2} \frac{1}{k+1} c_{SD}^i, & \bar{\pi}_S^i &= f_{e,s} \frac{(c_M)^k}{(c_{SD}^i)^k} \\
\bar{c}_U^i &= \frac{k}{k+1} c_{UD}^i, & \bar{q}_U^i &= \frac{L^i}{2\gamma} \frac{1}{k+1} c_{UD}^i = \frac{(k+2)(c_M)^k}{(c_{UD}^i)^{k+1}} f_{e,u} \\
\bar{p}_U^i &= \frac{2k+1}{2k+2} c_{UD}^i w_U^i, & \bar{r}_U^i &= \frac{L^i}{2\gamma} \frac{1}{k+2} (c_{UD}^i)^2 w_U^i = \frac{(k+1)(c_M)^k}{(c_{UD}^i)^k} f_{e,u} w_U^i \\
\bar{\mu}_U^i &= \frac{1}{2} \frac{1}{k+1} c_{UD}^i w_U^i, & \bar{\pi}_U &= f_{e,u} \frac{(c_M)^k}{(c_{UD}^i)^k} w_U^i
\end{aligned}$$

We can see from these results that in this setup, as in the data, exporters by being more productive sell more, hire more workers, and make more profits. Now I am ready to prove the main claim in this section, which relates the high barriers to service trade (Observation 2) to lower productivity growth (Duarte and Restuccia, 2010) in the skill-intensive (Observation 3) service sector.

Claim 2.11. *Due to higher trade barriers, the firms in the skill-intensive sector (services) have lower average productivity than firms in the other sector under costly trade. Hence with sector-level cutoffs the same in autarky, this implies lower productivity growth in the skill-intensive sector.*

Proof. For simplicity, trade costs are symmetric bilaterally. By assumption, $\tau_{s,j} > \tau_{u,j}$, $\forall j$. This implies $\rho_S^j < \rho_U^j$, $\forall j$. Then recall that

$$c_{SD}^j = \left[\frac{\phi_S \gamma (1 - \rho_S^j)}{L^j (1 - \rho_S^i \rho_S^j)} \right]^{\frac{1}{k+2}}, \quad c_{UD}^j = \left[\frac{\phi_U \gamma (1 - \rho_U^j)}{L^j (1 - \rho_U^i \rho_U^j)} \right]^{\frac{1}{k+2}}$$

Since $\rho_S^j < \rho_U^j$, $\forall j$, with $f_{e,s} = f_{e,u}$ (so that $\phi_S = \phi_U$), we have

$$c_{SD}^i > c_{UD}^i, \quad \forall i$$

Finally, simply note that average sector-level inverse productivity

$$\bar{c}_U^i = \frac{k}{k+1} c_{UD}^i, \quad \bar{c}_S^i = \frac{k}{k+1} c_{SD}^i$$

is increasing in the inverse productivity cutoff to prove the claim. \square

2.4 Robustness

I show in this section that the main results derived above continue to hold in the presence of intermediate goods and capital for particular classes of production functions. I abstracted from these elements in the earlier discussion as they are not crucial to the mechanism; they are, however, important empirical objects that need to be incorporated if the model is to be taken as a plausible representation of reality.

2.4.1 Intermediates

With skilled and unskilled labor as the only factors of production in the baseline model presented above, technologies take on the form

$$y_{i,0} = s_{i,0}^{\beta_i} u_{i,0}^{1-\beta_i}, \quad y_i(c) = c s_i(c)^{\beta_i} u_i(c)^{1-\beta_i}, \quad i \in \{1, 2\}, \quad 1 > \beta_1 > \beta_2 > 0$$

Now assume instead that each sector's output is produced from labor and intermediate goods, which include the sector's outside good and full range of differentiated goods. The sectoral composite good is then consumed or used as an input into the production of these intermediate goods. With the composite intermediate good produced with the same technology as the composite final good, we have the following technologies for the

sectoral intermediate good

$$M_s = m_{s,0} + \alpha \int m_s(i) di - \frac{1}{2} \gamma \int [m_s(i)]^2 di - \frac{1}{2} \eta \left[\int m_s(i) di \right]^2$$

$$M_u = m_{u,0} + \alpha \int m_u(i) di - \frac{1}{2} \gamma \int [m_u(i)]^2 di - \frac{1}{2} \eta \left[\int m_u(i) di \right]^2$$

where M_i denotes the composite intermediate good produced in sector i , which has $m_{i,0}$ (sector i outside good) and $m_i(c)$ (sector i differentiated good) as inputs. Note that while this class of technologies have not been used extensively in the literature, they exhibit the concavity and diminishing marginal return properties often considered desirable for production functions. The degree of substitutability across inputs is controlled by γ ; for $\gamma = 0$ the inputs are perfect substitutes. With $\eta = 0$, on the other hand, technology is quadratic in the individual varieties. The outside and differentiated goods are now produced with the following technologies

$$y_{i,0} = (s_{i,0}^{\beta_i} u_{i,0}^{1-\beta_i})^\nu M_i^{1-\nu}, \quad y_i(c) = c(s_i(c)^{\beta_i} u_i(c)^{1-\beta_i})^\nu M_i^{1-\nu}, \quad i \in \{1, 2\}$$

One can show in this environment that the price of the composite intermediate good is the same as the price of the sectoral outside good (when all inputs are used in production), which implies that the consumer problem in the Ventura world (where factors are sector-specific) can now be written as

$$\begin{aligned} \max \quad & a \log(q_{s,0}^c + \alpha \int q_s^c(i) di - \frac{1}{2} \gamma \int [q_s^c(i)]^2 di - \frac{1}{2} \eta \left[\int q_s^c(i) di \right]^2) \\ & + (1-a) \log(q_{u,0}^c + \alpha \int q_u^c(i) di - \frac{1}{2} \gamma \int [q_u^c(i)]^2 di - \frac{1}{2} \eta \left[\int q_u^c(i) di \right]^2) \\ \text{s.t.} \quad & q_{s,0}^c + m_{s,0}^c + \int p_s(i) [q_s^c(i) + m_s^c(i)] di \\ & + w_U [q_{u,0}^c + m_{u,0}^c] + \int p_l(i) [q_u^c(i) + m_u^c(i)] di \leq M \end{aligned}$$

As before, the firms will take the consumers' demand as given and set prices to maximize profits. In this world, we obtain for firm with inverse productivity level c

Prices

$$p_s(c) = \frac{1}{2}(c_{SD} + c), \quad p_U(c) = \frac{1}{2}(c_{UD} + c)w_U$$

Markup

$$\mu_s(c) = \frac{1}{2}(c_{SD} - c), \quad \mu_U(c) = \frac{1}{2}(c_{UD} - c)w_U$$

Output

$$y_s(c) = \frac{L}{\gamma}(c_{SD} - c), \quad y_U(c) = \frac{L}{\gamma}(c_{UD} - c)$$

Demand

$$q_s(c) = m_s(c) = \frac{L}{2\gamma}(c_{SD} - c), \quad q_U(c) = m_U(c) = \frac{L}{2\gamma}(c_{UD} - c)$$

Sales

$$r_s(c) = \frac{L}{2\gamma}(c_{SD}^2 - c^2), \quad r_U(c) = \frac{L}{2\gamma}(c_{UD}^2 - c^2)w_U$$

Profits

$$\pi_s(c) = \frac{L}{2\gamma}(c_{SD} - c)^2, \quad \pi_U(c) = \frac{L}{2\gamma}(c_{UD} - c)^2w_U$$

Comparing these expressions with the ones obtained earlier, it is clear that the same pricing rule and markup decision obtains. The difference in this setup is that for any given price, demand is twice as large owing to intermediate (and not just final good) demand. This implies that expected profits are higher, and hence there is greater entry and competition, leading to higher productivity (lower inverse productivity) cutoffs. This can be seen from free entry, which in this case is

$$\int \pi_s(c) dG(c) = f_{e,s} \quad \Rightarrow \quad c_{SD} = \left[\frac{\gamma f_{e,s} c_m^K (k+1)(k+2)}{L} \right]^{\frac{1}{k+2}}$$

$$\int \pi_U(c) dG(c) = w_U f_{e,U} \Rightarrow c_{UD} = \left[\frac{\gamma f_{e,U} c_m^K (k+1)(k+2)}{L} \right]^{\frac{1}{k+2}}$$

With these results, one can go through the proofs to see that all the propositions in the free trade continue to hold in this economy. As for the costly trade case, the only important thing to note is that once again cutoffs are scaled down by the factor $2^{\frac{1}{k+2}}$ (for the same reason as the case above), so that the claim relating higher barriers to service trade leading to relative slowly productivity growth continue to hold. For completeness, I specify the domestic cutoffs in the costly trade world to end this section.

$$\begin{aligned} \frac{L^i}{2\gamma} \int_0^{c_{SD}^i} (c_{SD}^i - c)^2 dG(c) + \frac{L^j \tau_{s,j}^2}{2\gamma} \int_0^{c_{SD}^j} (c_{SD}^j - c)^2 dG(c) &= f_{e,s} \\ \Rightarrow L^i (c_{SD}^i)^{k+2} + L^j \rho_S^j (c_{SD}^j)^{k+2} &= \tilde{\phi}_S \gamma \end{aligned}$$

$$\text{where } \tilde{\phi}_S = \gamma (c_M)^k f_{e,s} (k+2), \quad \rho_S^j = (\tau_{s,j})^{-k}$$

$$\begin{aligned} c_{SD}^i &= \left[\frac{\tilde{\phi}_S \gamma (1 - \rho_S^j)}{L^i (1 - \rho_S^i \rho_S^j)} \right]^{\frac{1}{k+2}} & c_{SD}^j &= \left[\frac{\tilde{\phi}_S \gamma (1 - \rho_S^i)}{L^j (1 - \rho_S^i \rho_S^j)} \right]^{\frac{1}{k+2}} \\ c_{UD}^i &= \left[\frac{\tilde{\phi}_U \gamma (1 - \rho_U^j)}{L^i (1 - \rho_U^i \rho_U^j)} \right]^{\frac{1}{k+2}} & c_{UD}^j &= \left[\frac{\tilde{\phi}_U \gamma (1 - \rho_U^i)}{L^j (1 - \rho_U^i \rho_U^j)} \right]^{\frac{1}{k+2}} \end{aligned}$$

2.4.2 Capital

Now I consider the environment where capital is an input to production and there is capital accumulation over time. Capital is assumed to be mobile across sectors but not across countries. Technology at the firm level in this case will be of the Cobb-Douglas form, while the sectoral investment aggregator will belong to the class of production functions introduced in the intermediates case above. The capital share ν is assumed to be the same across goods, sectors, and countries. The composite investment good is produced with Cobb-Douglas technology having the same income shares as the utility function. Specifically, we have firm-level technologies given by

$$y_{it,0} = (s_{it,0}^{\beta_i} u_{it,0}^{1-\beta_i})^{1-\nu} (k_{it,0})^\nu, \quad y_{it}(c) = c (s_{it}(c)^{\beta_i} u_{it}(c)^{1-\beta_i})^{1-\nu} (k_{it}(c))^\nu, \quad i \in \{1, 2\}$$

Sector-level technologies given by

$$I_{st} = x_{st,0} + \alpha \int x_{st}(i) di - \frac{1}{2} \gamma \int [x_{st}(i)]^2 di - \frac{1}{2} \eta \left[\int x_{st}(i) di \right]^2$$

$$I_{ut} = x_{ut,0} + \alpha \int x_{ut}(i) di - \frac{1}{2} \gamma \int [x_{ut}(i)]^2 di - \frac{1}{2} \eta \left[\int x_{ut}(i) di \right]^2$$

As mentioned before, this class of technologies exhibit the desirable properties of concavity and diminishing marginal return often required of production functions. The extent to which inputs are substitutable is controlled by γ ; for $\gamma = 0$ the inputs are perfect substitutes. With $\eta = 0$, technology is quadratic in the individual varieties. Finally, the aggregate investment technology is given by

$$I_t = a \log I_{st} + (1 - a) \log I_{ut}$$

The law of motion for capital is standard and given by

$$K_{t+1} = I_t + (1 - \delta)K_t$$

The price of the outside good in sector i in the Ventura-variant of this world is $\frac{r_t^\nu w_{it}^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}}$ where r_t is the rental rate of capital and w_{it} is the skill-specific wage as before. Then, we can write the problem of the household as

$$\max \sum_{t=0}^{\infty} \beta^t \left[a \log(q_{st,0}^c + \alpha \int q_{st}^c(i) di - \frac{1}{2} \gamma \int [q_{st}^c(i)]^2 di - \frac{1}{2} \eta \left[\int q_{st}^c(i) di \right]^2) \right. \\ \left. + (1 - a) \log(q_{ut,0}^c + \alpha \int q_{ut}^c(i) di - \frac{1}{2} \gamma \int [q_{ut}^c(i)]^2 di - \frac{1}{2} \eta \left[\int q_{ut}^c(i) di \right]^2) \right]$$

$$\text{s.t.} \quad \frac{r_t^\nu w_{st}^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}} [q_{st,0}^c + x_{st,0}^c] + \int p_{st}(i) [q_{st}^c(i) + x_{st}^c(i)] di \\ + \frac{r_t^\nu w_{ut}^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}} [q_{ut,0}^c + x_{ut,0}^c] + \int p_{ut}(i) [q_{ut}^c(i) + x_{ut}^c(i)] di \leq M_t$$

The key difference in this setup relative to all the models considered thus far is that the household now solves a dynamic problem, with the intertemporal Euler equation characterizing its consumption and investment decisions. What is nice about this dynamic

setup is that it yields demand equations of a similar form to what obtains in the static case, so that with firms solving a static problem, we can write performance measures at any given point in time t for firm with inverse productivity c as

Prices

$$p_{st}(c) = \frac{1}{2}(c_{SDt} + c) \frac{r_t^\nu w_{st}^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}}, \quad p_{ut}(c) = \frac{1}{2}(c_{UDt} + c) \frac{r_t^\nu w_{ut}^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}}$$

Markup

$$\mu_{st}(c) = \frac{1}{2}(c_{SDt} - c) \frac{r_t^\nu w_{st}^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}}, \quad \mu_{ut}(c) = \frac{1}{2}(c_{UDt} - c) \frac{r_t^\nu w_{ut}^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}}$$

Output

$$y_{st}(c) = \frac{L}{\gamma}(c_{SDt} - c), \quad y_{ut}(c) = \frac{L}{\gamma}(c_{UDt} - c)$$

Demand

$$q_{st}(c) = x_{st}(c) = \frac{L}{2\gamma}(c_{SDt} - c), \quad q_{ut}(c) = x_{ut}(c) = \frac{L}{2\gamma}(c_{UDt} - c)$$

Sales

$$r_{st}(c) = \frac{L}{2\gamma}(c_{SDt}^2 - c^2) \frac{r_t^\nu w_{st}^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}}, \quad r_{ut}(c) = \frac{L}{2\gamma}(c_{UDt}^2 - c^2) \frac{r_t^\nu w_{ut}^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}}$$

Profits

$$\pi_{st}(c) = \frac{L}{2\gamma}(c_{SDt} - c)^2 \frac{r_t^\nu w_{st}^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}}, \quad \pi_{ut}(c) = \frac{L}{2\gamma}(c_{UDt} - c)^2 \frac{r_t^\nu w_{ut}^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}}$$

Things are more complicated if firms are allowed to make dynamic decisions; in that case the distribution of firm-level productivities becomes a state variable (e.g. Ruhl 2008) unless one assumes that at any period a firm may decide not to produce any quantity. In that case, the distribution remains the same in each period, as in Chaney (2005), and we see productivity overshooting in the transition between steady states (with different τ 's). These complications are beyond the scope of this paper; we leave them for future research. In our simplified environment, the key equilibrium objects are the productivity cutoffs, and these turn out to be the same as in the model with intermediates. Here

it is investment inputs that are playing the role of intermediate demand, driving up expected profit, competition and lowering the cutoff inverse productivity (relative to the economy with no capital or intermediates). In particular, the cutoffs in autarky or free trade are given by

$$\int \pi_{st}(c)dG(c) = f_{e,s} \frac{r_t^\nu w_{st}^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}} \Rightarrow c_{SDt} = \left[\frac{\gamma f_{e,s} c_m^K (k+1)(k+2)}{L} \right]^{\frac{1}{k+2}}$$

$$\int \pi_{ut}(c)dG(c) = f_{e,U} \frac{r_t^\nu w_{ut}^{1-\nu}}{\nu^\nu (1-\nu)^{1-\nu}} \Rightarrow c_{UDt} = \left[\frac{\gamma f_{e,U} c_m^K (k+1)(k+2)}{L} \right]^{\frac{1}{k+2}}$$

While the domestic cutoffs for costly trade are

$$\frac{L^i}{2\gamma} \int_0^{c_{SDt}^i} (c_{SDt}^i - c)^2 dG(c) + \frac{L^j \tau_{s,j}^2}{2\gamma} \int_0^{c_{SDt}^j} (c_{SDt}^j - c)^2 dG(c) = f_{e,s}$$

$$\Rightarrow L^i (c_{SDt}^i)^{k+2} + L^j \rho_S^j (c_{SDt}^j)^{k+2} = \tilde{\phi}_S \gamma$$

$$\text{where } \tilde{\phi}_S = \gamma (c_M)^k f_{e,s} (k+2), \quad \rho_S^j = (\tau_{s,j})^{-k}$$

$$c_{SDt}^i = \left[\frac{\tilde{\phi}_S \gamma (1 - \rho_S^j)}{L^i (1 - \rho_S^i \rho_S^j)} \right]^{\frac{1}{k+2}} \quad c_{SDt}^j = \left[\frac{\tilde{\phi}_S \gamma (1 - \rho_S^i)}{L^j (1 - \rho_S^i \rho_S^j)} \right]^{\frac{1}{k+2}}$$

$$c_{UDt}^i = \left[\frac{\tilde{\phi}_U \gamma (1 - \rho_U^j)}{L^i (1 - \rho_U^i \rho_U^j)} \right]^{\frac{1}{k+2}} \quad c_{UDt}^j = \left[\frac{\tilde{\phi}_U \gamma (1 - \rho_U^i)}{L^j (1 - \rho_U^i \rho_U^j)} \right]^{\frac{1}{k+2}}$$

Again, one can go through the proofs in Section 3 to verify that the main results still go through and are hence robust to the introduction of capital.

2.5 Conclusion

The explosion in global trade has been one of the starkest developments in the last fifty years: this is particularly true of merchandise trade, while service trade has lagged behind. Just as significant has been the structural transformation characterizing the dynamics of growth in the developed world: in particular, the rise of the service economy. This paper draws a connection between these two phenomena by developing a new theory for the structural change in services and presenting supporting evidence regarding

the service sector. I show that in a Heckscher-Ohlin model with heterogeneous firms, the lowering of trade barriers leads to the reallocation of labor into the skill-intensive industry (services) of skill-abundant countries (the developed world). Further, as barriers to trade in the services still remain higher than the rest of the economy, the benefits accruing to service sector-level productivity as a result of foreign competition have only been realized to a limited extent, implying lower productivity growth in the services. The model also features endogenous selection into export markets, with exporters selling more, hiring more, and being more productive on average than domestic producers. These model implications are all borne out in the data, and continue to hold in the presence of intermediate goods and capital. What remains to be seen is how important service trade is in quantitatively accounting for structural change, the skill premium and the productivity slowdown in the developed world; this is something I leave for future research.

% College	edu1960	edu1970	edu1980	edu1990	edu2000	edu2010
edu1960	1.0000					
edu1970	0.9749	1.0000				
edu1980	0.9461	0.9871	1.0000			
edu1990	0.8995	0.9582	0.9837	1.0000		
edu2000	0.7931	0.8567	0.8988	0.9389	1.0000	
edu2010	0.7764	0.8196	0.8546	0.9008	0.9765	1.0000

Table 2.1: Correlation Matrix, % of Population with Completed Higher Education

Figure 2.1: Skill Abundance Over Time

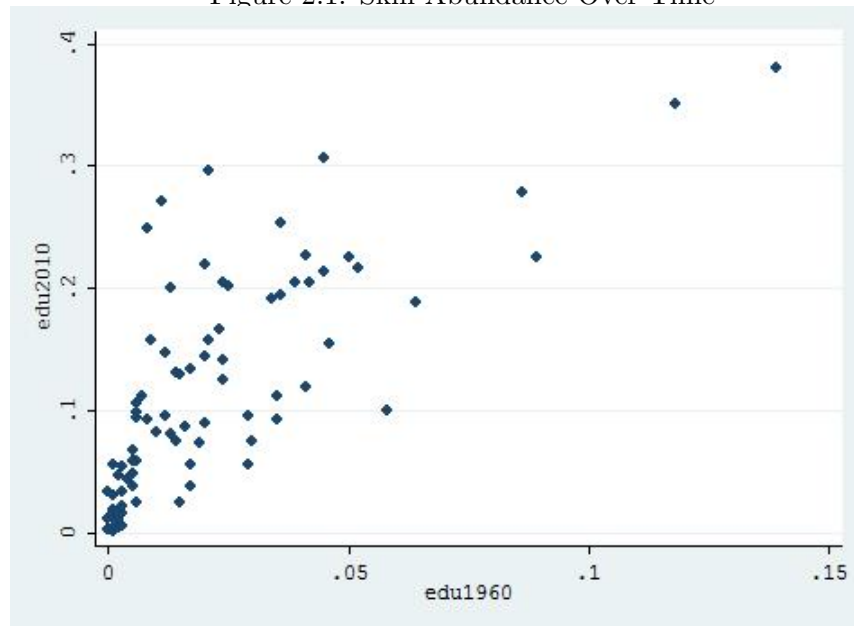
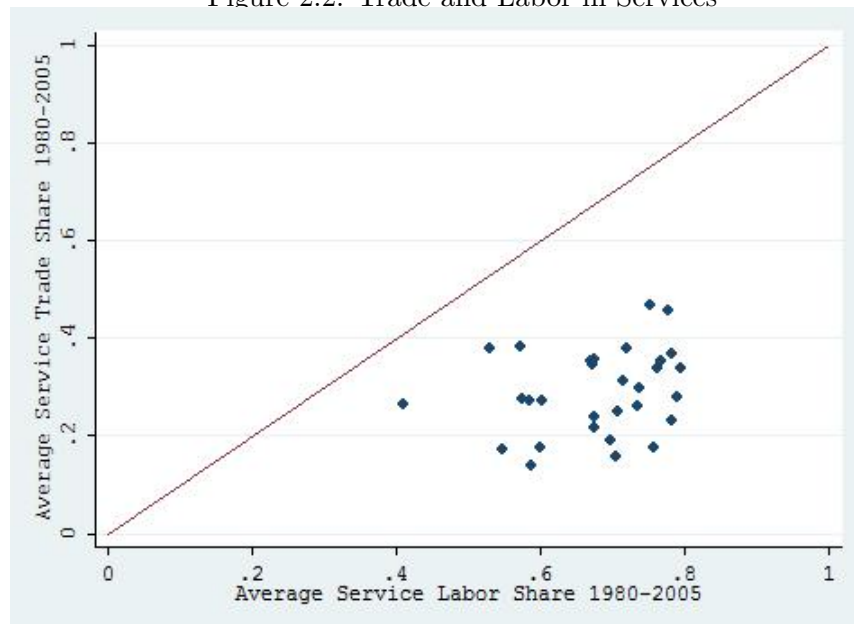


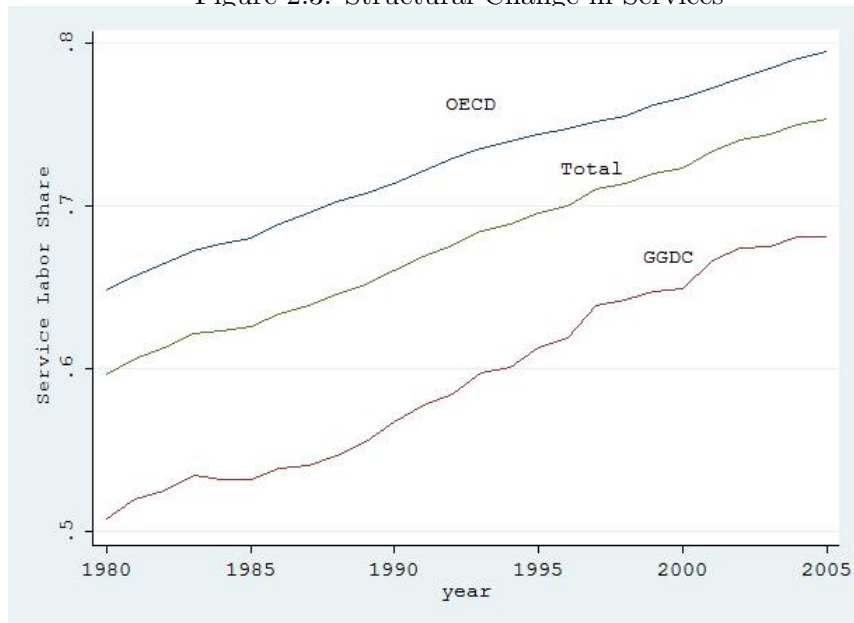
Figure 2.2: Trade and Labor in Services



% Year	NSCollege	SCollege	NSYrsEduc	SYrsEduc
1967	6.72	14.23	10.17	11.06
1968	6.64	14.60	10.20	11.13
1969	7.32	14.76	10.34	11.23
1970	7.57	15.60	10.38	11.32
1971	7.47	16.14	10.42	11.39
1972	7.82	16.74	10.52	11.47
1973	8.33	17.74	10.59	11.57
1974	9.16	18.30	10.70	11.64
1975	9.10	18.64	10.73	11.70
1976	9.88	18.98	10.83	11.74
1977	10.20	18.95	10.87	11.78
1978	10.64	19.70	11.01	11.85
1979	11.00	20.32	11.06	11.91
1980	11.56	20.26	11.09	11.94
1981	12.27	21.03	11.16	11.98
1982	13.45	22.24	11.24	12.07
1983	14.14	22.57	11.31	12.11
1984	14.81	22.73	11.37	12.12
1985	15.10	22.82	11.38	12.15
1986	15.76	23.17	11.44	12.18
1987	16.27	23.60	11.47	12.19
1988	15.85	24.55	11.46	12.24
1989	16.08	24.74	11.49	12.27
1990	16.39	24.72	11.53	12.28
1991	13.54	19.97	12.08	12.88
1992	14.27	20.61	12.15	12.97
1993	14.62	21.04	12.22	13.00
1994	15.24	21.72	12.21	13.05
1995	14.70	22.12	12.22	13.05
1996	15.58	22.08	12.26	13.05
1997	15.72	22.61	12.31	13.07
1998	16.12	23.14	12.35	13.10
1999	16.25	23.12	12.36	13.10
2000	16.60	23.62	12.37	13.11
2001	17.22	24.04	12.39	13.14
2002	17.26	24.28	12.35	13.17
2003	18.49	24.48	12.46	13.18
2004	18.62	24.30	12.46	13.17
2005	18.19	24.76	12.46	13.21
2006	18.23	25.66	12.47	13.24
2007	18.60	26.24	12.51	13.30
2008	18.69	26.40	12.48	13.33
2009	19.18	26.68	12.50	13.36

Table 2.2: Skill Intensity Across Sectors - % College and Average No. of Years Educated

Figure 2.3: Structural Change in Services



Sector	% College	Share of Total Employed
Agriculture, Forestry, Fishing	2.91	4.75
Mining	8.12	0.66
Manufacturing, D	7.74	16.34
Manufacturing, ND	6.76	11.26
Non-Services	6.72	33.01
Transportation	3.71	3.63
Telecommunications	6.29	1.15
Utilities	7.24	1.47
Construction	3.59	5.99
Wholesale Trade	10.27	3.37
Retail Trade	3.89	15.68
Finance, Banks, Real Estate	15.56	4.53
Business Services	11.13	3.07
Personal Services	1.77	6.18
Entertainment, Rec Services	11.94	1.15
Professional Services	38.32	15.39
Public Administration	15.98	5.37
Services	14.23	66.99

Table 2.3: Skill Intensity Across Subsectors in 1968

Sector	% College	Share of Total Employed
Agriculture, Forestry, Fishing	12.16	2.49
Mining	14.04	0.47
Manufacturing, D	20.89	5.99
Manufacturing, ND	21.56	4.06
Non-Services	19.18	13.01
Transportation	13.23	3.12
Telecommunications	32.28	0.75
Utilities	22.94	0.81
Construction	10.52	7.44
Wholesale Trade	28.26	2.47
Retail Trade	13.92	17.79
Finance, Banks, Real Estate	41.20	5.80
Business Services	31.10	7.05
Personal Services	12.85	3.09
Entertainment, Rec Services	27.89	2.60
Professional Services	38.22	23.84
Public Administration	33.70	4.72
Services	26.68	86.99

Table 2.4: Skill Intensity Across Subsectors in 2010

Sector	Code	Subsector
Non-Services	00	Agriculture, Forestry, and Fishing
Non-Services	01	Mining
Services	02	Construction
Non-Services	03	Manufacturing - Durables
Non-Services	04	Manufacturing - Non Durables
Services	05	Transportation
Services	06	Telecommunications
Services	07	Utilities and Sanitary Services
Services	08	Wholesale trade
Services	09	Retail trade
Services	10	Finance, Banking, Real Estate
Services	11	Business and Repair Services
Services	12	Personal Services
Services	13	Entertainment and Recreation Services
Services	14	Professional and Related Services
Services	15	Public Administration

Table 2.5: Sector Classification of March CPS Data

Chapter 3

Bond Auctions and Sovereign Default

3.1 Introduction

The recent wave of sovereign debt crises has reignited interest in the ways in which governments renege on their promise to repay their debt. Government debt is typically issued through bond auctions that are either discriminatory or competitive in nature. Discriminatory auctions specify different prices for different bonds, depending on the order in which the bonds are sold. With a competitive auction, all bonds are sold at the same price, typically the price specified in the highest unsuccessful bid. In this paper, I consider the intersection of both default and these two different auction mechanisms. In particular, I ask the following questions. Do countries default more if they use a discriminatory as opposed to a competitive auction? And how different are the predictions of standard quantitative sovereign default models when bonds are sold by auction?

To address these questions, I develop a framework where the government in a small

open economy deciding on both its debt level and default faces lenders that have downward sloping demand for government bonds. The basic framework is a quantitative sovereign default model where a small open economy issues bonds through an auction to lenders with price-elastic demand curves and are hence price takers. This price-taking assumption is appropriate when there are a large number of bidders and a large number of units for sale, as is true in this environment. I consider two types of auction mechanisms: discriminatory auctions, where successful bidders pay their bid; and competitive auctions, where all bidders pay the market clearing stop-out price. The government issuing the bonds sets a grid of prices and invites bidders to submit demand schedules for these different prices. Aggregating these demands allows the seller to compute the stop-out price at which demand and supply are matched. Because bidders in the discriminatory auction must pay their bids for all units demanded up to the stop-out price, they shade their bids and understate their true willingness to pay. In contrast, bidders in a competitive auction have no such incentive as all units are sold at the stop-out price and hence are willing to bid their true valuation. Given the auction yields and its previous debt, the issuing government can then choose to repay its debt or default and be excluded from financial markets.

Standard quantitative sovereign default models are characterized by incomplete markets where governments only have access to a noncontingent bond that can be defaulted on. Given incompleteness of markets resulting from a single bond, these models typically adopt a pricing protocol that is only one step removed from that prevalent in single-bond economies, where the departure simply accounts for the probability of default. Here I build on this framework by incorporating a price grid and auction markets for this single bond, so while there is one asset type, different units of this asset are sold at different prices, and prices depend on the level of demand independent of default. Introducing default induces two effects. It introduces a positive probability of default, which in turn

affects the amount of bonds that can be sold by the issuing government. For the regions of the parameter space that I have explored, both auction environments yield higher default probabilities and lower borrowing levels than what obtains in the standard environment where bonds are not auctioned away. Further, while competitive auctions yield higher revenues than discriminatory auctions in the multi-unit independent-value environment with risk-neutral buyers considered here when default is not permitted, no such ranking exists for the environment with default: higher revenue from greater borrowing is no longer guaranteed when higher debt can trigger default. That predictions for the model incorporating both default and auction elements differ markedly from both the model with only default and the model with only auctions demonstrate that the interaction between these two elements matter for the results, notwithstanding its empirical relevance.

This paper contributes to the body of work focusing on quantitative models that generate empirically plausible probabilities of default and levels of debt in equilibrium.¹

I follow this literature in assuming that the bond seller is a small open economy that receives a stochastic endowment and decides whether or not to repay its debt in each period, in the spirit of Eaton and Gersovitz (1981). Where I deviate from previous work is in assuming a pricing protocol where the seller presents a set of prices and allows risk-neutral lenders to submit bid functions for these prices which when aggregated then determines the market stop-out price. There are sovereign default papers where countries can issue bonds with multiple prices. These bonds, however, are typically different assets: e.g. long- and short-term bonds as in Arellano and Ramanarayanan (2012) or Hatchondo and Martinez (2009). By contrast, here the same asset can have a different price depending on the order in which it is sold (discriminatory auction) or the total number of units sold (competitive auction) - an artifact of the fact that

¹ see for example, Aguiar and Gopinath (2006), Arellano (2008), D'Erasmus and Mendoza (2013), and Yue (2010).

pricing is determined through a multi-unit auction process. Moreover, because bond pricing differs from that which obtains in the standard environment and also differs across the different auction formats, the auction mechanism chosen also impacts both the probability of default and the level of sustainable debt. This then introduces another element into the academic and policy implications of issuing bonds subject to default: under what conditions is the auction mechanism chosen optimal, and how different would our forecasts for both default probabilities and debt levels be if we assume that bonds are not sold by auction when they really are?

This paper also builds on a large literature studying the optimal auction mechanism for selling government bonds. Using a special survey on treasury auctions covering 48 countries, Brenner, Galai and Sade (2009) document that market-oriented economies using common law tend to use competitive auctions while less market-oriented economies practicing civil law tend to use discriminatory auctions. Not only are both types of auctions utilized in practice, several countries have also switched from one pricing protocol to another. There is also a large theoretical literature examining the revenue-maximizing multi-unit auction mechanism under a wide variety of arrangements.² Ausubel, Cramton, Pycia, Rostek, and Weretka (2014) show that the revenue and efficiency rankings for the two types of auctions are ambiguous in general; only under certain special conditions is one auction necessarily revenue superior to the other. This paper builds on the framework in Nautz (1995), who analyzes optimal bidding for a multi-unit auction with independent values if the seller sets a discrete price grid, by embedding it within a quantitative sovereign default model. While the competitive auction is more efficient and yields greater revenue than the discriminatory auction in both the discrete-price, risk-neutral environment in Nautz (1995) and its continuous-price, risk-averse counterpart in Nautz and Wolfstetter (1997), this is no longer necessarily true in the environment with

² e.g. Wilson (1972), Milgrom and Weber (1982), Back and Zender (1993), Wang and Zender (2002)

default as greater borrowing also raises the probability of default which in turn lowers the revenue of the borrowing country by reducing the price at which its bonds can be sold. Given the prevalence of defaults documented in Reinhart and Rogoff (2009), it is important to account for its effect on bond prices when examining the revenue and efficiency properties of bond auctions, as this paper attempts to do.

The next section presents a simple motivating example. Section 3 presents the model I use to run the quantitative experiments. The quantitative analysis and main results are shown in Section 4. The last section concludes. Table and figures can be found in a separate section following the main references.

3.2 Simple Examples

The benchmark problem for the standard model with default but no auctions is given by

$$\begin{aligned} \max_{d_l, d_h, p} \quad & u(c_1) + \frac{1}{2}u(c_{2h}) + \frac{1}{2}u(c_{2l}) \\ c_1 = & 1 + qp \\ c_{2h} = & \max\{4 - p, 4\theta\} = d_h(4\theta) + (1 - d_h)(4 - p) \\ c_{2l} = & \max\{2 - p, 2\theta\} = d_l(2\theta) + (1 - d_l)(2 - p) \\ q = & \frac{1}{2}(1 - d_h) + \frac{1}{2}(1 - d_l) \end{aligned}$$

As a first step, I now present the problem for the model with auctions but no default. Suppose the seller then sets four prices $q \in \{\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$ in the price grid and the individual lender's demand function is given by $p_i = D(q_i) = 4/3(1 - q_i)$ for each $q_i \in \{\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$ so that market demand is given by $p = D(q) = 4/3(1 - \sum_{q \geq \bar{q}} q)$ where the stop-out price is given by \bar{q} .

There are then four cases to consider. Case 1 where the stop-out price is $1/4$ and

demand is $p^1 = 4/3(1 - 1/4) + 4/3(1 - 2/4) + 4/3(1 - 3/4) + 4/3(1 - 1) = 2$ and the revenue from bonds sold is $\sum^1 qp = q_1p_1 + q_2p_2 + q_3p_3 + q_4p_4 = 1/44/3(1 - 1/4) + 2/4(4/3)(1 - 2/4) + 3/4(4/3)(1 - 3/4) + 1/4(4/3)(1 - 1) = \frac{5}{6}$ so that welfare is given by $W^1 = u(1 + \sum^1 qp) + 0.5u(4 - p^1) + 0.5u(2 - p^1) = u(1\frac{5}{6}) + 0.5u(2) + 0.5u(0)$. Case 2 is where the stop-out price is $2/4$ and revenue from the bonds sold is $\sum^2 qp = q_1p_1 + q_2p_2 + q_3p_3 = 2/4(4/3)(1 - 2/4) + 3/4(4/3)(1 - 3/4) + 1/4(4/3)(1 - 1) = 7/12$ and market demand is $p^2 = 4/3(1 - 2/4) + 4/3(1 - 3/4) + 4/3(1 - 1) = 1$ hence welfare is $W^2 = u(1\frac{7}{12}) + 0.5u(3) + 0.5u(1)$. Case 3 is where the stop-out price is $3/4$ and the revenue is then $\sum^3 qp = 3/4(4/3)(1 - 3/4)$ and demand $p^3 = 4/3(1 - 3/4)$ with welfare being $W^3 = u(1\frac{1}{4}) + 0.5u(3\frac{2}{3}) + 0.5u(1\frac{2}{3})$. The fourth case is identical to autarky in this example because demand when $p = 1$ is zero. The seller then chooses to sell p^i units of the bond where $W^i = \max_k W^k$.

The example above assumes that bidder bid their true valuation in that market demand is simply the sum of individual demand up to the stop-out price. This is relevant to the competitive auction, as it is optimal in this case to bid ones true valuation. In a discriminatory auction, however, bid-shading is optimal, so that lenders' bidding functions will differ from their true demand functions. Consider the following alternative environment. Suppose the seller sets a three-point distribution $q \in \{\frac{1}{3}, \frac{2}{3}, 1\}$ while the expectations for the stop-out price of the price-taking bidders follow a uniform distribution, i.e.

$$F(q_1) = F(\frac{1}{3}) = \frac{1}{3}, \quad F(q_2) = F(\frac{2}{3}) = \frac{2}{3}, \quad F(q_3) = F(1) = 1$$

Then define for each i the indices i^* and i_* that are the minimum and maximum integers such that $B_i = B_{i_*+1} = \dots = B_{i^*}$. Then the optimal bid function $B(p)$ as a function of a bidder's true demand function $D(p)$ is given by

$$B(q_i) = D\left(q_{i^*} + F_{i_*} \frac{q_{i^*} - q_{i_*}}{F_{i^*} - F_{i_*}}\right)$$

where $F(q_{i_*}) = F_{i_*}$.

Now suppose $i = 1$ so that $B_1 > B_2 > B_3 > 0$. Then $i_* = 0$ and $i^* = 1$ so $q_{i^*} - q_{i_*} = \frac{1}{3}$, $F_{i_*} = 0$ and $F_{i^*} - F_{i_*} = \frac{1}{3}$. In this case,

$$B\left(\frac{1}{3}\right) = B(q_1) = D\left(q_{i^*} + F_{i_*} \frac{q_{i^*} - q_{i_*}}{F_{i^*} - F_{i_*}}\right) = D(q_1) = D\left(\frac{1}{3}\right)$$

Next suppose the stop-out price is q_2 , i.e. $i = 2$ so that $B_2 > B_3 > 0 = B_1$. In this case, $i_* = 1$ and $i^* = 2$ so that $q_{i^*} - q_{i_*} = \frac{1}{3}$, $F_{i_*} = \frac{1}{3}$ and $F_{i^*} - F_{i_*} = \frac{1}{3}$. Hence

$$B\left(\frac{2}{3}\right) = B(q_2) = D\left(q_{i^*} + F_{i_*} \frac{q_{i^*} - q_{i_*}}{F_{i^*} - F_{i_*}}\right) = D(q_2 + F_1) = D(1) < D\left(\frac{2}{3}\right) = D(q_2)$$

Note that in contrast to the first case, bid shading occurs here. In fact, bid shading only does not occur for the limiting case where the price is at the lower bound of its support (i.e. there is positive demand at all price levels) as we will see shortly.

To complete the discussion, consider the last case where $i = 3$. In this case $B_3 > 0 = B_1 = B_2$. In this case, $i_* = 2$ and $i^* = 3$ so that $q_{i^*} - q_{i_*} = \frac{1}{3}$, $F_{i_*} = \frac{2}{3}$ and $F_{i^*} - F_{i_*} = \frac{1}{3}$. Hence

$$B(1) = B(q_3) = D\left(q_{i^*} + F_{i_*} \frac{q_{i^*} - q_{i_*}}{F_{i^*} - F_{i_*}}\right) = D(q_3 + F_2) = D\left(\frac{5}{3}\right) < D(1) = D(q_3)$$

Similar to the previous case, bidders understate their own valuation and in this case, the bid shading is even greater than before.

Turning now to the case with both auctions and default. First consider the case where the bonds are sold through a competitive auction. In this case, as above, bidders optimally choose to bid their true valuations, so that bidding functions and demand functions coincide. Let the price grid be given by $q = [q_1, q_2, \dots, q_N]$, where $q_1 < q_2 < \dots < q_N$. Let the demand function for price point q_i be $p_i = D(q_i) = g - hq_i$

for $q_i \leq \frac{q}{h}$, and $p_i = D(q_i) = 0$ for $q_i > \frac{q}{h}$. The problem for the sovereign issuing bonds with stop-out price \bar{q} then becomes

$$\begin{aligned} & \max_{d_l, d_h, p} u(c_1) + \frac{1}{2}u(c_{2h}) + \frac{1}{2}u(c_{2l}) \\ c_1 &= 1 + \bar{q} \sum_{q_i \geq \bar{q}} \left[D\left(\frac{1}{1-\delta}q_i\right) - D\left(\frac{1}{1-\delta}q_{i+1}\right) \right] \\ c_{2h} &= d_h(4\theta) + (1-d_h) \left(4 - \sum_{q_i \geq \bar{q}} \left[D\left(\frac{1}{1-\delta}q_i\right) - D\left(\frac{1}{1-\delta}q_{i+1}\right) \right] \right) \\ c_{2l} &= d_l(2\theta) + (1-d_l) \left(2 - \sum_{q_i \geq \bar{q}} \left[D\left(\frac{1}{1-\delta}q_i\right) - D\left(\frac{1}{1-\delta}q_{i+1}\right) \right] \right) \\ p &= \sum_{q_i \geq \bar{q}} \left[D\left(\frac{1}{1-\delta}q_i\right) - D\left(\frac{1}{1-\delta}q_{i+1}\right) \right] \\ \delta &= \frac{1}{2}d_h + \frac{1}{2}d_l \end{aligned}$$

With a discriminatory auction, bidders shade their bids so that bidding and demand functions no longer coincide. Let the bidding function for price q_i be $b_i = B(q_i) = D\left(\frac{1}{1-\delta} \left\{ q_i^* + F_{i^*} \frac{q_i^* - q_{i^*}}{F_{i^*} - F_{i^*}} \right\}\right)$. The problem for the sovereign then becomes

$$\begin{aligned} & \max_{d_l, d_h, p} u(c_1) + \frac{1}{2}u(c_{2h}) + \frac{1}{2}u(c_{2l}) \\ c_1 &= 1 + \sum_{q_i \geq \bar{q}} q_i [B(q_i) - B(q_{i+1})] \\ c_{2h} &= d_h(4\theta) + (1-d_h) \left(4 - \sum_{q_i \geq \bar{q}} [B(q_i) - B(q_{i+1})] \right) \\ c_{2l} &= d_l(2\theta) + (1-d_l) \left(2 - \sum_{q_i \geq \bar{q}} [B(q_i) - B(q_{i+1})] \right) \\ p &= \sum_{q_i \geq \bar{q}} [B(q_i) - B(q_{i+1})] \\ \delta &= \frac{1}{2}d_h + \frac{1}{2}d_l \end{aligned}$$

Note that the bidding function $B(q)$ here differs from the case without default; in particular, the argument is scaled by the factor $\frac{1}{1-\delta}$ to reflect the fact that the bid goes to zeros (the argument goes to infinity) as the default probability goes to one. Similarly, we could have written the bid function for the competitive auction with default as $\bar{B}(q)$ instead of $D\left(\frac{1}{1-\delta}q\right)$; the choice of writing it out explicitly in terms of the demand function is to contrast it with the case where there is no default, in which case the bid coincides with the demand function $D(q)$. Once again, the difference lies in the presence of the default probability; the higher the probability of default, the lower the corresponding bid (as the argument inside the demand function increases).

3.3 Model

I consider a dynamic model of defaultable debt where debt is sold either through a competitive or discriminatory auction. A small open economy receives a stochastic stream of income, y , which follows a Markov process with support Y and transition function $f(y_t, y_{t+1})$. The economy can sell single-period bonds to risk-neutral lenders through one of the aforementioned auction formats. The economy can default on its debt at any time, as financial contracts are unenforceable. If the economy defaults, it temporarily loses access to international financial markets and suffers output loss every period it is in default.

Utility for the representative agent in the small open economy (henceforth "the seller") comes from smoothing consumption over time, with preferences given by

$$E \sum_{t=0}^{\infty} \beta^t u(c_t)$$

The seller sets a grid of prices $q = [q_1, q_2, \dots, q_N]$ where $q_1 < q_2 < \dots < q_N$. He invites bidders to submit bid functions which we denote

$$B(q_1) \geq B(q_2) \geq \dots \geq B(q_N)$$

where $B(q_i)$ gives the number of bonds the bidder would like to buy at price q_i . If it defaults, it operates in autarky. If it does not default, the sovereign faces the following budget constraint at time t if it operates a discriminatory auction:

$$c_t + \sum_{q_{i,t+1} \geq \bar{q}_{t+1}} q_{i,t+1} [B(q_{i,t+1}) - B(q_{i+1,t+1})] = y_t + \sum_{q_{i,t} \geq \bar{q}_t} [B(q_{i,t}) - B(q_{i+1,t})]$$

if instead it operates a competitive auction, the budget constraint becomes

$$c_t + \bar{q}_{t+1} \sum_{q_{i,t+1} \geq \bar{q}_{t+1}} [B(q_{i,t+1}) - B(q_{i+1,t+1})] = y_t + \sum_{q_{i,t} \geq \bar{q}_t} [B(q_{i,t}) - B(q_{i+1,t})]$$

The bid functions need not coincide with his true willingness to pay, which we denote with demand functions $D(q_i)$ for price q_i . Denote the inverse demand function by $Z(q) = D^{-1}(q)$. Aggregating the bid functions allows the seller to compute the stop-out price, where bond supply and demand coincide. Denote the probability that the stop-out price is below q_i by $F(q_i)$, where F reflects the bidder's expectations over the distribution of the stop-out price. There is a large number of bidders so every single bidder behaves as if their actions have no impact over the price distribution, so that they take prices as given. Bidders, which are the international lenders in this case, are assumed to be risk-neutral.

3.3.1 Discriminatory Auctions

$$\begin{aligned} \max_{B_1, B_2, \dots, B_N} \sum_{i=1}^N [F_i - F_{i-1}] \sum_{j=i}^N A_j \\ \text{where } A_i = \frac{1 - \delta}{1 + r} \int_{B_{i+1}}^{B_i} Z(s) ds - q_i (B_i - B_{i+1}) \\ B_i \geq B_{i+1}, \quad \forall i \end{aligned}$$

The nature of demand being monotonic and the constraints being convex ensure that first order conditions are both necessary and sufficient for an optimum. Then define the

indices i^* and i_* that are the minimum and maximum integers such that $B_i = B_{i_*+1} = \dots = B_{i^*}$. The interpretation is that q_{i^*} is the highest price below q_i at which the monotonicity requirement $B_j \geq B_{j+1}$ is not binding. And similarly, q_{i_*} is the lowest price not below q_i at which the monotonicity requirement $B_j \geq B_{j+1}$ is not binding. If the monotonicity constraint does not bind at q_i , then $q_{i_*} = q_{i-1}$ and $q_{i^*} = q_i$.

Proposition 1. *The optimal bidding function for the discriminatory auction with default is given by*

$$B(q_i) = D \left(\frac{1+r}{1-\delta} \left\{ q_{i^*} + F_{i_*} \frac{q_{i^*} - q_{i_*}}{F_{i^*} - F_{i_*}} \right\} \right)$$

Proof. Form the Lagrangian

$$L(B_1, B_2, \dots, B_N, \lambda_1, \dots, \lambda_N) = \sum_{i=1}^N F_i A_i + \sum_{i=1}^N \lambda_i (B_i - B_{i+1})$$

From which I obtain the optimality conditions:

$$\begin{aligned} F(q_i) \left[\frac{1-\delta}{1+r} Z(B_i) - q_i \right] + F(q_{i-1}) \left[q_{i-1} - \frac{1-\delta}{1+r} Z(B_i) \right] + \lambda_i - \lambda_{i-1} &= 0 \\ B_i \geq B_{i+1} \text{ and } \lambda_i [B_i - B_{i+1}] &= 0 \\ \lambda_i &\geq 0 \end{aligned}$$

Then define as above q_{i^*} is the highest price below q_i at which the monotonicity requirement $B_j \geq B_{j+1}$ is not binding. And similarly, q_{i_*} is the lowest price not below q_i at which the monotonicity requirement $B_j \geq B_{j+1}$ is not binding. This implies $\lambda_{i_*} = \lambda_{i^*} = 0$ and $B_i = B_{i_*+1} = \dots = B_{i^*}$.

Assume that $B_1 > 0$. The bidder does not participate in the auction $B_1 = 0$ if and only if he is certain his reservation price lies above the stop-out price of the auction ($Z(0) \leq q_1$). I assume that lenders stand ready to make at least one bid, which in this case corresponds to the lowest price on the grid.

Next assume that there is an index $i \in (1, \dots, N)$ such that $F(q_i) = F(q_{i-1})$. Then the initial first order condition implies that

$$0 > q_{i-1} - q_i = \frac{\lambda_{i-1} - \lambda_i}{F(q_i)} \geq \frac{-\lambda_i}{F(q_i)}$$

Therefore $\lambda_i > 0$ and hence $B_i = B_{i+1}$. This means the bidder places a zero bid at price q_i . This is possible given that bidding at the higher price q_i instead of at q_{i-1} is unnecessary if it does not increase the bidder's chance of winning.

Summing the first of the optimality conditions from $i_* + 1$ to i^* yields

$$F(q_{i_*}) \left[\frac{1 - \delta}{1 + r} Z(B_i) - q_{i_*} \right] = F(q_{i^*}) \left[\frac{1 - \delta}{1 + r} Z(B_i) - q_{i^*} \right]$$

which means one cannot gain from reshuffling demand from p_{i_*} to p_{i^*} . Further, $\lambda_{i^*} = 0$ implies $F(q_{i^*}) > F(q_{i_*})$. Therefore solving the previous equation for $Z(B_i)$ yields

$$\frac{1 - \delta}{1 + r} Z(B_i) = \frac{F(q_{i^*})q_{i^*} - F(q_{i_*})q_{i_*}}{F(q_{i^*}) - F(q_{i_*})} = q_{i^*} + F(q_{i_*}) \frac{q_{i^*} - q_{i_*}}{F(q_{i^*}) - F(q_{i_*})}$$

Thus

$$B_i = D \left(\frac{1 + r}{1 - \delta} \left\{ q_{i^*} + F(q_{i_*}) \frac{q_{i^*} - q_{i_*}}{F(q_{i^*}) - F(q_{i_*})} \right\} \right)$$

□

3.3.2 Competitive Auctions

In contrast to a discriminatory auction, bidders pay the stop-out price for every unit they receive in a competitive auction. Thus the problem becomes

$$\max_{B_1, B_2, \dots, B_N} \sum_{i=1}^N [F_i - F_{i-1}] \left(\int_0^{B_i} \frac{1 - \delta}{1 + r} Z(s) ds - q_i B_i \right)$$

subject to $B_i \geq B_{i+1}, \quad \forall i$

This turns out to have a much simpler solution. The optimal bid in this environment is given in the next proposition.

Proposition 2. *The optimal bidding function for the competitive auction with default is given by*

$$B(q_i) = D \left(\frac{1+r}{1-\delta} q_i \right)$$

Proof. Monotonicity of demand guarantees that the maximization problem is well-behaved, so that the first-order conditions are necessary and sufficient for an optimum. These conditions are given by

$$(F_i - F_{i-1}) \left(\frac{1-\delta}{1+r} Z(B_i) - q_i \right) = 0, \quad \forall i = 1, \dots, N$$

Hence

$$\frac{1-\delta}{1+r} Z(B_i) = q_i \Rightarrow B(q_i) = D \left(\frac{1+r}{1-\delta} q_i \right)$$

as desired. □

3.3.3 Recursive Formulation

I focus on Markov equilibria and write down the sovereign lender's problem as a dynamic program. The problem has two states - the stock of debt b_t and income of the economy y_t . At any given state, the value of the option to default is given by

$$v^o(b, y) = \max_{c,d} \{v^c(b, y), v^d(y)\}$$

where $v^c(b, y)$ is the value of continuing in the contract and not defaulting and $v^d(y)$ is the value function under default.

I assume that when default occurs, the sovereign defaults on all debt, and there is no seniority or partial default. When the borrower defaults, income falls to $y^d = \theta y$, $\theta < 1$

and temporarily operates in autarky; with probability λ the sovereign regains access to international financial markets the following period so that the value of default is then

$$v^d(y) = u(\theta y^d) + \beta \int_{y'} [\lambda v^o(0, y') + (1 - \lambda)v^d(y')] f(y, y') dy'$$

When the borrower to continue in the contract, his value is given by

$$v^c(b, y) = \max_{b', y} \left\{ u(c) + \beta \int_y v^o(b', y') f(y, y') dy' \right\}$$

subject to the constraints

$$c = y - \sum_{q'_i \geq \bar{q}'} [B(q'_i) - B(q'_{i+1})] + \sum_{q \geq \bar{q}} [B(q_i) - B(q_{i+1})] \quad \text{if discriminatory}$$

$$c = y - \bar{q}' \sum_{q'_i \geq \bar{q}'} [B(q'_i) - B(q'_{i+1})] + \sum_{q \geq \bar{q}} [B(q_i) - B(q_{i+1})] \quad \text{if competitive}$$

$$b = \sum_{q \geq \bar{q}} B(q), \quad b' = \sum_{q' \geq \bar{q}'} B(q')$$

Note that the current state b determines the stop-out price \bar{q} above which all prices on the price grid have positive demand. Similarly, the choice of b' determines the stop-out price next period \bar{q}' above which all points on the price grid will be equilibrium auction prices. The bidding function $B(q)$ entering the constraints depends on the auction mechanism used. With a competitive auction, the bid function for price point q_i is given by

$$B(q_i) = D \left(\frac{1+r}{1-\delta} q_i \right)$$

as derived earlier. By contrast, the bid function for price point q_i under a discriminatory auction is given by

$$B(q_i) = D \left(\frac{1+r}{1-\delta} \left\{ q_{i^*} + F_{i_*} \frac{q_{i^*} - q_{i_*}}{F_{i^*} - F_{i_*}} \right\} \right)$$

where the indices i^* and i_* are the minimum and maximum integers such that $B_i = B_{i_*+1} = \dots = B_{i^*}$ and F is the lender's expected distribution over the stop-out price.

Consistency requires that the default probability δ satisfies

$$\delta(b', y) = \int_{y'} d(b', y') f(y, y') dy'$$

where the default policy function satisfies

$$d(b, y) = \begin{cases} 1 & \text{if } v^c(b, y) \geq v^d(y) \\ 0 & \text{if } v^c(b, y) < v^d(y) \end{cases}$$

3.4 Quantitative Analysis

I now solve the model numerically to evaluate how its quantitative predictions differ not only across the two different auction formats, but also relative to the standard model with no auction. In this draft, I will simply perform a numerical simulation with the proper calibration to be done in subsequent revisions. The main goal of the exercise is to see how introducing the auction component affects average debt levels and the probability of default.

3.4.1 Parameterization

The period utility function for the sovereign borrower is assumed to be of CRRA form $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. The risk aversion coefficient σ is set to two, following Arellano and Ramanarayanan (2012) who take their value from business cycle studies. Income can take only one of two states $y \in \{y_l, y_h\}$. The transition probability is given by

$$\Pi = \begin{bmatrix} \pi_l & 1 - \pi_l \\ 1 - \pi_h & \pi_h \end{bmatrix}$$

I find that the value of autarky depends on the levels of β , the coefficient of risk aversion σ , transition probabilities π_h and π_l is low, and the difference between y_h and y_l . The value of autarky in turn affects the threshold level of current bond holdings below which

the country will default. To make default more likely, values for these parameters are assigned the values listed in Table 1 below.

Description	Parameter	Value
Risk aversion	σ	2
Patience	β	0.5
Interest rate	r	0.01
Transition probability	π_h	0.7
Transition probability	π_l	0.5
Income level	y_h	1
Income level	y_l	0.01
Demand slope	h	0.08
Demand intercept	g	1
Reentry probability	λ	0
Output cost	θ	1

Table 3.1: Default Model Parameterization

The asset space spans the interval $[-3, 0]$, with 401 grid points in total. The price grid covers the range $\frac{1}{1+r}[0.95, 1]$. I simulate the model 100 times for 10000 periods and discard the first 500 periods to reduce dependence on initial conditions. I then perform two experiments, applying the same path of shocks in each case. In the first experiment, I compare the performance of the standard default model with no auctions against a model where default is not allowed. This is to establish a benchmark against which the models with both auctions and default can be compared against, and to isolate the effects of allowing for (only) default in this model of sovereign debt. The second experiment then compares the results from (1) the standard model with no auction, (2) the model with a discriminatory auction, and (3) the model with a competitive auction. The parameter values chosen above guarantee that default is observed during the simulated run for each of these models.

3.4.2 Default vs. No Default

Table 2 presents some sample statistics comparing the overall performance of two models, one where default is allowed and another where it is not, and bonds are not sold via auction. It compares these two models along the key dimensions of consumption and debt, as well as the differences in default probabilities and bond prices. I take the average across all simulations for assets b and consumption c and find that the no-default model can sustain significantly higher levels of borrowing (0.3861 vs 0.0741) which in turn translate to far less variable consumption paths (0.0969 vs 0.2109). Given my parameterization, I also find that default occurs 2.34% of the time in this standard default model with no auction which then translates to a slightly lower average bond price (0.9896 vs 0.9901), relative to what the price would be without default.

Variable	No Default	Default
mean(b)	-0.3861	-0.0741
mean(c)	0.6233	0.6261
var(b)	0.0888	0.0455
var(c)	0.0969	0.2109
q	0.9901	0.9896
default	0	2.34

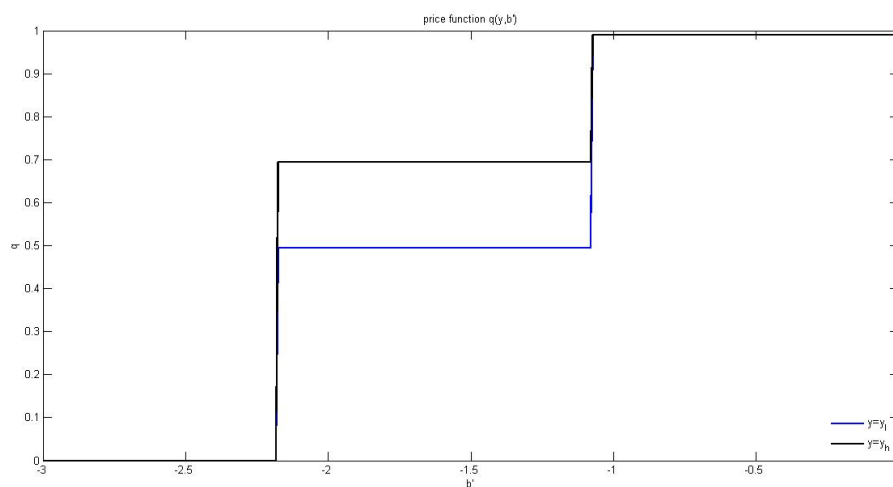
Table 3.2: Model Statistics (Default vs No Default)

The bond price q faced by a small open economy in the case where there is no default is the inverse of the risk-free rate. The path of q for the default case is lower when the default rate is positive. With only two states for income, there are two levels of assets B_l and $B_h > B_l$, such that the country defaults no matter what the income shocks is if $b < B_l$, defaults for low income shocks if $b \in [B_l, B_h]$ (hence repay with probability $1 - \pi_l$ if $y = y_l$ and π_h if $y = y_h$), and does not default for any income shock if $b > B_h$.

Correspondingly, the price function (shown in Figure 1) is

$$q(b', y) = \begin{cases} 0 & b' < B_l, y \in \{y_l, y_h\} \\ \frac{1-\pi_l}{1+r} & b' \in [B_l, B_h], y = y_l \\ \frac{\pi_h}{1+r} & b' \in [B_l, B_h], y = y_h \\ \frac{1}{1+r} & b' > B_h, y \in \{y_l, y_h\} \end{cases}$$

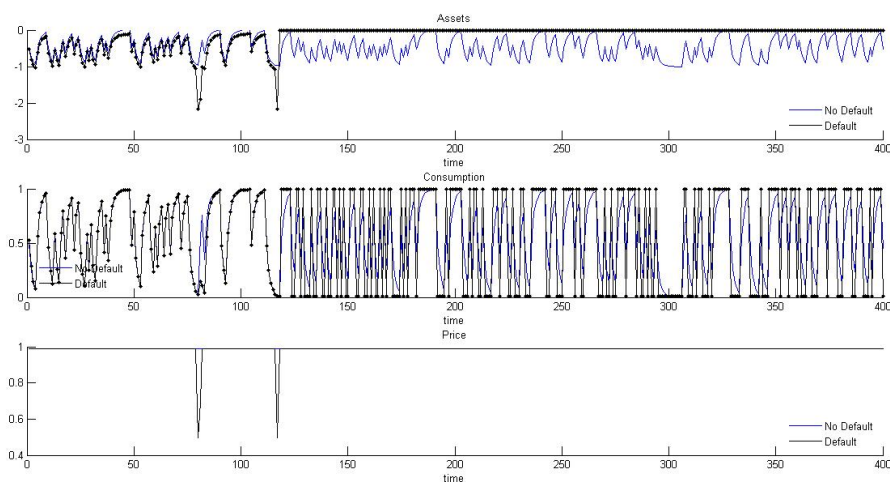
Figure 3.1: Bond Pricing: Default vs No Default



The first 400 periods of one of the simulations run is shown in Figure 2 below. Observe that consumption is less volatile for the model without default as consumers can borrow more in states where the income realization and asset holdings are low in order to smooth consumption over time. After the default, there is no more smoothing so the relative volatility is higher if we take the whole path compared to taking the path only up to the default date. The figure below shows one particular path, where the country defaults after 120 periods and consumes its income afterwards (lives in autarky). The country that would like to borrow $b' \in [B_l, B_h]$ poses a risk to the lenders - if tomorrow's income shock is low it defaults. The probability that the country gets

y_l tomorrow is π_l if $y = y_l$ today and $1 - \pi_h$ if $y = y_h$ today. For the sample path above there are periods where price is $(1 - \pi_l)/(1 + r)$ since the country is in low states and wants to borrow b' in the default region. In the first case the next period's state is high so the country does not default, the second time a default occurs.

Figure 3.2: Simulation: Default vs No Default



3.4.3 Default with No Auction vs. Default with Auction

In contrast to the previous subsection, here I focus on the case where default is always allowed but bonds can be sold via auction. In particular, I consider three different environments: the standard environment as in Arellano (2008) where there is no auction, an environment where bonds are sold through a competitive auction, and another environment where bonds are sold through a discriminatory auction. Table 3 presents summary statistics comparing these models along the key dimensions of consumption and debt and default probabilities. I take the average across all simulations for assets b and consumption c and find that the standard no-auction default model can sustain higher

levels of borrowing than the default models with auctions (0.0741 vs 0.0441 or 0.0240) and borrowing in the competitive auction default model is higher than that sustained in the discriminatory auction default model. In terms of welfare, consumption is least volatile in the standard no-auction model and most volatile in the discriminatory auction default model. I find that default occurs most often in the discriminatory auction model, with default probabilities (4.70% on average) twice as high as that observed in the standard no-auction default model (2.34%), where default probabilities are lowest. This is consistent with the level of sustainable debt being lowest in the discriminatory auction default, and highest in the standard no-auction default model.

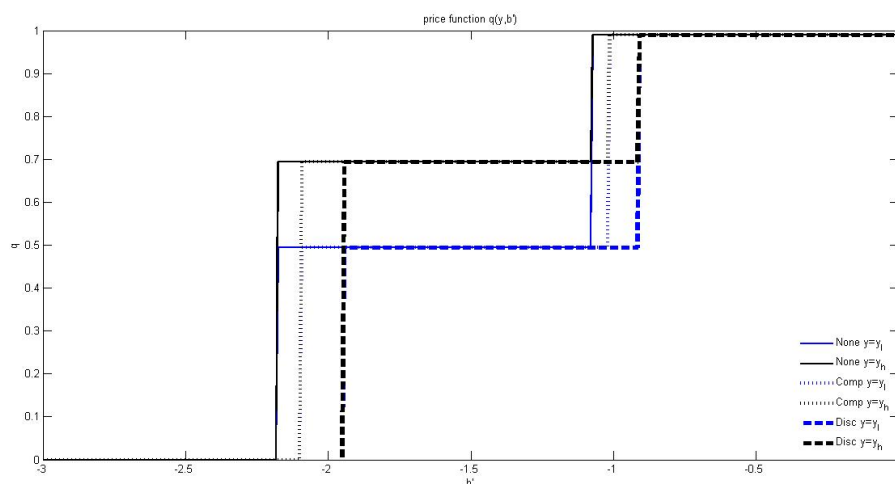
Variable	No Auction	Competitive Auction	Discriminatory Auction
mean(a)	-0.0741	-0.0441	-0.0240
mean(c)	0.6261	0.6260	0.6253
var(a)	0.0455	0.0297	0.0180
var(c)	0.2109	0.2163	0.2209
default	2.34	3.23	4.70

Table 3.3: Model Statistics (Auctions Under Default)

As argued earlier, with only two states for income, there are two levels of assets B_l and $B_h > B_l$, such that the country defaults no matter what the income shocks is if $b < B_l$, defaults for low income shocks if $b \in [B_l, B_h]$ (hence repay with probability $1 - \pi_l$ if $y = y_l$ and π_h if $y = y_h$), and does not default for any income shock if $b > B_h$. The key difference with the introduction of the various auction mechanisms is that it alters the bond pricing protocol, where instead of a single price, the same asset can be sold to different sellers at different prices (discriminatory auction) or be sold at a different price if a different amount is sold (competitive auction). Given my parameterization, the revenues from the discriminatory auction fall short of the revenues from the competitive auction, which in turn are lower than that of standard model with

no auction. The sovereign then defaults in more states of the world in the discriminatory auction world relative to the competitive auction environment, with the lowest number of default states occurring in the world with no auction (the standard setup). This explains the finding shown in Figure 3 that $B_l^n < B_l^c < B_l^d$ and $B_h^n < B_h^c < B_h^d$, which in turn translates to the ranking of sustainable debt levels documented earlier. (Note that prices with auction will not simply be a single point but will be a grid of various price points; the single-line diagrams used in the figure are simplified to ease exposition.)

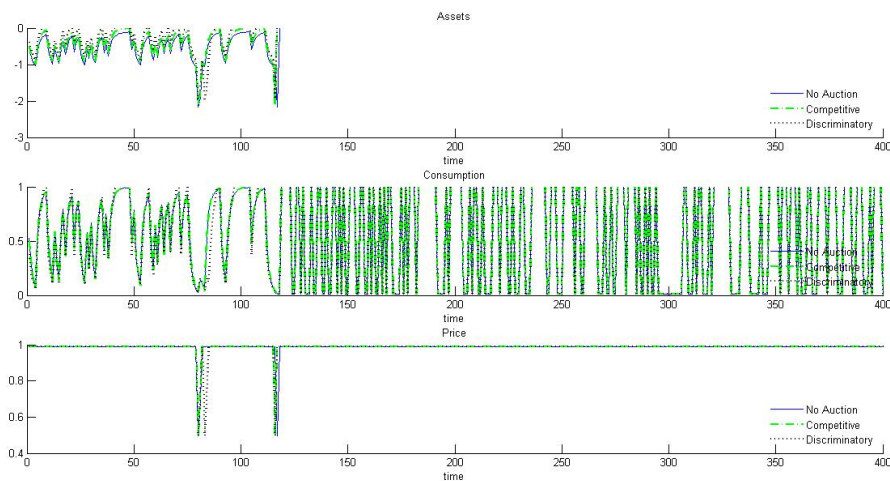
Figure 3.3: Bong Pricing: Auctions under Default



The first 400 periods of one of the simulations run is shown in Figure 4 below. In contrast to the first experiment (no-default vs. default), here it is not obvious that consumption in one environment is more variable than another. We do see, however, that after default occurs around period 120 that consumption simply tracks output (autarky) and there is far more variability post-default than pre-default. Notice also that in this particular simulation, while default occurs at the same time for all three

models, this need not be the case and there are time paths where default occurs in one of the two auction models but not in the standard model (hence the higher average default probabilities recorded earlier).

Figure 3.4: Simulation: Auctions under Default



Further, even though default occurs at the same time for all three models, the evolution of assets and consumption differ across these different environments. Note in particular that the discriminatory model has a double-dip around period 80 while only a single-dip is observed in the other two environments. This implies that while $b' \in [B_l^d, B_h^n]$ the first time we observe a dip in the asset price so that all three prices fall to $(1 - \pi_l)/(1 + r)$, $b' \in [B_h^c, B_h^n]$ the second time a dip occurs, so that the bond price falls only in the discriminatory auction case as agents will not default on such levels of debt in the other two environment (see Figure 4 for an intuitive explanation). In these periods where prices are lower because of default risk, the country is in the low state and wants to borrow b' in the default region. In the first few instances the next period's state is high so the country does not default, but at period 120 the period next state is

low and a default occurs.

3.5 Conclusion

In this paper, I develop a quantitative sovereign default model where bonds are sold via auction, whether discriminatory or competitive. I find that relative to the standard default model with no auctions, the default models where bonds are auctioned off predict higher default probabilities and lower debt levels. Relative to the discriminatory auction, I also find that bonds are more valuable under the competitive auction protocol, leading to smoother consumption, higher sustainable levels of debt, and lower default probabilities. Brenner, Galai and Sade (2009) document that both types of auctions are used in practice, and find that market-oriented economies using common law tend to use competitive auctions while less market-oriented economies practicing civil law tend to use discriminatory auctions. It would be instructive to take the competitive auction default model to Argentinian data to see how different the results are when one allows for a competitive auction (the empirically relevant case) relative to the case where bonds are not sold via auction (the standard model). Ecuador, on the other hand, uses a discriminatory auction to sell debt: in this case, the relevant comparison would be between the discriminatory auction default model and the standard model with no auction. There have been also cases where countries have switched from one auction to another (e.g. the US switch to a competitive auction in the 1990s); in this case, the relevant experiment would compare the results from the two auction models. I leave these extensions for future research.

Chapter 4

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Appendix A

Appendix

A.1 Proofs

Lemma 1. Given $f_{ij}^s = \infty$ for all (i, j) such that $e_{ij}^s = 0$, $s \in \{T, M\}$, there is no aggregate entry following policy reform.

Proof. Entry following reform requires that there exists (i, j) such that $e_{ij}^s = 0$ and $e_{ij}^{s'} = 1$ for $s \in \{T, M\}$. But this means that $\pi_{ij}^s < f_{ij}^s$ and $\pi_{ij}^{s'} \geq f_{ij}^s$. This cannot be true given that $f_{ij}^s = \infty$ and $\pi_{ij}^{s'} < \infty$, $\pi_{ij}^s < \infty$.

Lemma 2. Given $f_{ij}^s = 0$ for all (i, j) such that $e_{ij}^s = 1$, $s \in \{T, M\}$, then there is no aggregate exit post-reform.

Proof. Exit following reform requires that there exists (i, j) such that $e_{ij}^s = 1$ and $e_{ij}^{s'} = 0$ for $s \in \{T, M\}$. But this means that $\pi_{ij}^s \geq f_{ij}^s$ and $\pi_{ij}^{s'} < f_{ij}^s$. This cannot be true given that $f_{ij}^s = 0$ and $\pi_{ij}^{s'} > 0$, $\pi_{ij}^s > 0$.

Lemma 3. If there is no aggregate entry and exit following policy reform, dividend per share stays unchanged ($\pi' = \pi$).

Proof. Recall the expression for dividend per share

$$\pi = \frac{\sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^S}{P_i^S} \right)^{1-\sigma} w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) - f_{ij}^S \right] e_{ij}^S}{2 \sum_{i=1}^N w_i L_i}$$

Rearranging to isolate the terms that are functions of π to yield

$$\begin{aligned} & 2 \left\{ \sum_{i=1}^N w_i L_i - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^S}{P_i^S} \right)^{1-\sigma} w_i L_i \right] e_{ij}^S \right\} \pi \\ &= \frac{1}{1-\alpha} \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^S}{P_i^S} \right)^{1-\sigma} w_i L_i \right] e_{ij}^S - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^S \end{aligned}$$

Similarly, post-reform we have

$$\begin{aligned} & 2 \left\{ \sum_{i=1}^N w_i L_i - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^{S'}}{P_i^{S'}} \right)^{1-\sigma} w_i L_i \right] e_{ij}^{S'} \right\} \pi' \\ &= \frac{1}{1-\alpha} \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^{S'}}{P_i^{S'}} \right)^{1-\sigma} w_i L_i \right] e_{ij}^{S'} - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j f_{ij}^{S'} e_{ij}^{S'} \end{aligned}$$

To see that $\pi = \pi'$, it suffices to note that for any $S \in \{T, M\}, i = 1, 2, \dots, N$ we have

$$\begin{aligned} \sum_{j=1}^N w_j L_j \left[p_{ij}^{S'} \right]^{1-\sigma} e_{ij}^{S'} &= P_i^{S'1-\sigma} \\ \sum_{j=1}^N w_j L_j \left[p_{ij}^S \right]^{1-\sigma} e_{ij}^S &= P_i^{S1-\sigma} \end{aligned}$$

and because there is no aggregate entry or exit, $e_{ij}^{S'} = e_{ij}^S, \forall (S, i, j)$, so that

$$\sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^{S'} = \sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^S$$

Hence

$$\begin{aligned} \pi &= \frac{\frac{1}{1-\alpha} \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^S}{P_i^S} \right)^{1-\sigma} w_i L_i \right] e_{ij}^S - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^S}{2 \left\{ \sum_{i=1}^N w_i L_i - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^S}{P_i^S} \right)^{1-\sigma} w_i L_i \right] e_{ij}^S \right\}} \\ &= \frac{\frac{1}{1-\alpha} \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^{S'}}{P_i^{S'}} \right)^{1-\sigma} w_i L_i \right] e_{ij}^{S'} - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j f_{ij}^S e_{ij}^{S'}}{2 \left\{ \sum_{i=1}^N w_i L_i - \sum_{i=1}^N \sum_{j=1}^N \sum_{s=M,T} w_j L_j \left[\mu \frac{1}{\sigma} \left(\frac{p_{ij}^{S'}}{P_i^{S'}} \right)^{1-\sigma} w_i L_i \right] e_{ij}^{S'} \right\}} = \pi' \end{aligned}$$

Proposition 1. The welfare gains computed in the limiting parameterization given by $f_{ij}^s = 0$ for all (i, j) such that $e_{ij}^s = 1$, $s \in \{T, M\}$ and $f_{ij}^s = \infty$ for all (i, j) such that $e_{ij}^s = 0$, $s \in \{T, M\}$ coincide with the welfare gains in an alternative parameterization of fixed costs where $f_{ij}^s = \underline{f}$ for all (i, j) such that $e_{ij}^s = 1$, $s \in \{T, M\}$ and $f_{ij}^s = \bar{f}$ for all (i, j) such that $e_{ij}^s = 0$, $s \in \{T, M\}$ where \underline{f} is sufficiently small and \bar{f} is sufficiently large to ensure that there is no aggregate entry or exit post-reform.

Proof. Denote the equilibrium objects that arise from the alternative parameterization with hats, e.g. \hat{P}_i, \hat{W} . We know given that there is no aggregate entry or exit post-reform, Lemma 3 implies that $\hat{\pi}' = \hat{\pi}$ (even as $\hat{\pi} \neq \pi$). Next note that because $e_{ij}^{S'} = e_{ij}^{S'} = e_{ij}^S$ for all $i, j = 1, 2, \dots, N$ and $S \in \{T, M\}$ we have $P_i^{S'} = \hat{P}_i^{S'}$ for all $i = 1, 2, \dots, N$ and $S \in \{T, M\}$. The normalization stipulates that $P_0 = P'_0 = 1$. Then given that welfare is given by

$$W_i = \Gamma + \log \left\{ w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right) \right\} - (1-2\mu) \log P_0 - \mu \log P_i^M - \mu \log P_i^T = \Gamma \log \left(\frac{w_i L_i \left(\frac{1}{1-\alpha} + 2\pi \right)}{P_0^{1-2\mu} P_i^{M\mu} P_i^{T\mu}} \right)$$

We have that the gains from openness are given by

$$\frac{W'_i}{W_i} = \log \frac{\left(\frac{1}{1-\alpha} + 2\pi' \right) P_i^{M\mu} P_i^{T\mu}}{\left(\frac{1}{1-\alpha} + 2\pi \right) P_i^{M\mu} P_i^{T\mu}}$$

Hence

$$\frac{\hat{W}'_i}{\hat{W}_i} = \log \frac{\left(\frac{1}{1-\alpha} + 2\hat{\pi}'\right) P_i^{\hat{M}^\mu} \hat{P}_i^{\hat{T}^\mu}}{\left(\frac{1}{1-\alpha} + 2\hat{\pi}\right) P_i^{\hat{M}'^\mu} \hat{P}_i^{\hat{T}'^\mu}} = \log \frac{P_i^{\hat{M}^\mu} \hat{P}_i^{\hat{T}^\mu}}{P_i^{\hat{M}'^\mu} \hat{P}_i^{\hat{T}'^\mu}} = \log \frac{P_i^{M^\mu} P_i^{T^\mu}}{P_i^{M'\mu} P_i^{T'\mu}} = \log \frac{\left(\frac{1}{1-\alpha} + 2\pi'\right) P_i^{M^\mu} P_i^{T^\mu}}{\left(\frac{1}{1-\alpha} + 2\pi\right) P_i^{M'\mu} P_i^{T'\mu}} = \frac{W'_i}{W_i}$$

as desired. Q.E.D.

Proposition 2. (i) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} = \tau_{ij}^T$ and $e_{ij}^T = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i1} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i1}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i1}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$ but $c_{ij}^T \geq x_{i1}, c_{ik}^T < x_{i1}$. This pair of inequalities imply that $\pi_{ik}^T > \pi_{ij}^T$. We also know from $e_{ij}^T = 0, e_{ik}^T = 0$ that

$$\begin{aligned} \pi_{ij}^T &< f_{ij}^T = \pi_{ij}^T + \epsilon \\ \pi_{ik}^T &< f_{ik}^T = \pi_{ik}^T + \epsilon \end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\begin{aligned} \frac{\pi_{ik}^{T'}}{\pi_{ik}^T} &= \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \\ \frac{\pi_{ij}^{T'}}{\pi_{ij}^T} &= \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \end{aligned}$$

So that the change in bilateral gross profits (through the price index and income from dividends) due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$, we also know that $\pi_{ij}^{T'} \geq f_{ij}^T$ and $\pi_{ik}^{T'} < f_{ik}^T$. Take the first of these two inequalities. We have

$$\pi_{ij}^{T'} \geq f_{ij}^T \Rightarrow \pi_{ij}^{T'} \geq \pi_{ij}^T + \epsilon \Rightarrow (\delta - 1)\pi_{ij}^T \geq \epsilon \Rightarrow \delta - 1 \geq \frac{\epsilon}{\pi_{ij}^T}$$

Then the last inequality implies

$$(\delta - 1)\pi_{ik}^T \geq \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \Rightarrow \pi_{ik}^{T'} - \pi_{ik}^T \geq \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \Rightarrow \pi_{ik}^{T'} \geq \pi_{ik}^T + \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T}$$

However, combining this with $\pi_{ik}^{T'} < f_{ik}^T = \pi_{ik}^T + \epsilon$ yields

$$\pi_{ik}^T + \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \leq \pi_{ik}^{T'} < f_{ik}^T = \pi_{ik}^T + \epsilon$$

which is a contradiction as $\pi_{ik}^T > \pi_{ij}^T$.

(ii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} = \tau_{ij}^T$ and $e_{ij}^T = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i2} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i2}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i2}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$ but $c_{ij}^T \geq x_{i2}, c_{ik}^T < x_{i2}$. This pair of inequalities imply that $\pi_{ik}^T > \pi_{ij}^T$. We also know from $e_{ij}^T = 1, e_{ik}^T = 1$ that

$$\begin{aligned} \pi_{ij}^T &> f_{ij}^T = \epsilon \\ \pi_{ik}^T &> f_{ik}^T = \epsilon \end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\frac{\pi_{ik}^{T'}}{\pi_{ik}^T} = \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

$$\frac{\pi_{ij}^{T'}}{\pi_{ij}^T} = \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

So that the change in bilateral gross profits (through the price index and income from dividends) due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$, we also know that $\pi_{ij}^{T'} \geq f_{ij}^T$ and $\pi_{ik}^{T'} < f_{ik}^T$. Take the second of these two inequalities. We have

$$\pi_{ik}^{T'} < f_{ik}^T \Rightarrow \pi_{ik}^{T'} < \epsilon \Rightarrow \delta \pi_{ik}^T < \epsilon \Rightarrow \delta < \frac{\epsilon}{\pi_{ik}^T}$$

Then the last inequality implies

$$\delta \pi_{ij}^T < \frac{\epsilon \pi_{ij}^T}{\pi_{ik}^T} \Rightarrow \pi_{ij}^{T'} < \frac{\epsilon \pi_{ij}^T}{\pi_{ik}^T} \Rightarrow \pi_{ij}^{T'} < \epsilon$$

where the last implication comes from $\pi_{ik}^T > \pi_{ij}^T$. This is a contradiction as $\pi_{ij}^{T'} \geq f_{ij}^T = \epsilon$.

(iii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} < \tau_{ij}^T$ and $e_{ij}^T = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i3} for each i such that for firms from countries with cost $c_{ij}^T \geq x_{i3}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i3}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$ but $c_{ij}^T \geq x_{i3}, c_{ik}^T <$

x_{i3} . This pair of inequalities imply that $\pi_{ik}^T > \pi_{ij}^T$. We also know from $e_{ij}^T = 1, e_{ik}^T = 1$ that

$$\begin{aligned}\pi_{ij}^T &> f_{ij}^T = \epsilon \\ \pi_{ik}^T &> f_{ik}^T = \epsilon\end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\begin{aligned}\frac{\pi_{ik}^{T'}}{\pi_{ik}^T} &= \left(\frac{\tau_{ik}^{T'}}{\tau_{ik}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \\ \frac{\pi_{ij}^{T'}}{\pi_{ij}^T} &= \left(\frac{\tau_{ij}^{T'}}{\tau_{ij}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}\end{aligned}$$

Because $\tau_{ik}^{T'}/\tau_{ik}^T = \tau_{ij}^{T'}/\tau_{ij}^T$ the change in bilateral gross profits due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{\tau_{ij}^{T'}}{\tau_{ij}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$, we also know that $\pi_{ij}^{T'} \geq f_{ij}^T$ and $\pi_{ik}^{T'} < f_{ik}^T$. Take the first of these two inequalities. We have

$$\pi_{ik}^{T'} < f_{ik}^T \quad \Rightarrow \quad \pi_{ik}^{T'} < \epsilon \quad \Rightarrow \quad \delta \pi_{ik}^T < \epsilon \quad \Rightarrow \quad \delta < \frac{\epsilon}{\pi_{ik}^T}$$

Then the last inequality implies

$$\delta \pi_{ij}^T < \frac{\epsilon \pi_{ij}^T}{\pi_{ik}^T} \quad \Rightarrow \quad \pi_{ij}^{T'} < \frac{\epsilon \pi_{ij}^T}{\pi_{ik}^T} \quad \Rightarrow \quad \pi_{ij}^{T'} < \epsilon$$

where the last implication comes from $\pi_{ik}^T > \pi_{ij}^T$. This is a contradiction as $\pi_{ij}^{T'} \geq f_{ij}^T = \epsilon$.

(iv) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{T'} < \tau_{ij}^T$ and $e_{ij}^T = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i4} for each i such that for firms from countries with cost

$c_{ij}^T \geq x_{i4}$, the optimal choice is not to enter after the reform ($e_{ij}^{T'} = 0$) while firms from countries with cost $c_{ik}^T < x_{i4}$ choose to enter after the reform ($e_{ik}^{T'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$ but $c_{ij}^T \geq x_{i4}, c_{ik}^T < x_{i4}$. This pair of inequalities imply that $\pi_{ik}^T > \pi_{ij}^T$. We also know from $e_{ij}^{T'} = 0, e_{ik}^{T'} = 0$ that

$$\begin{aligned}\pi_{ij}^T &< f_{ij}^T = \pi_{ij}^T + \epsilon \\ \pi_{ik}^T &< f_{ik}^T = \pi_{ik}^T + \epsilon\end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\begin{aligned}\frac{\pi_{ik}^{T'}}{\pi_{ik}^T} &= \left(\frac{\tau_{ik}^{T'}}{\tau_{ik}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \\ \frac{\pi_{ij}^{T'}}{\pi_{ij}^T} &= \left(\frac{\tau_{ij}^{T'}}{\tau_{ij}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}\end{aligned}$$

Because $\tau_{ik}^{T'}/\tau_{ik}^T = \tau_{ij}^{T'}/\tau_{ij}^T$ the change in bilateral gross profits due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{\tau_{ij}^{T'}}{\tau_{ij}^T}\right)^{1-\sigma} \left(\frac{P_i^{T'}}{P_i^T}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{T'} = 1, e_{ik}^{T'} = 0$, we also know that $\pi_{ij}^{T'} \geq f_{ij}^T$ and $\pi_{ik}^{T'} < f_{ik}^T$. Take the first of these two inequalities. We have

$$\pi_{ij}^{T'} \geq f_{ij}^T \quad \Rightarrow \quad \pi_{ij}^{T'} \geq \pi_{ij}^T + \epsilon \quad \Rightarrow \quad (\delta - 1)\pi_{ij}^T \geq \epsilon \quad \Rightarrow \quad \delta - 1 \geq \frac{\epsilon}{\pi_{ij}^T}$$

Then the last inequality implies

$$(\delta - 1)\pi_{ik}^T \geq \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \quad \Rightarrow \quad \pi_{ik}^{T'} - \pi_{ik}^T \geq \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \quad \Rightarrow \quad \pi_{ik}^{T'} \geq \pi_{ik}^T + \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T}$$

However, combining this with $\pi_{ik}^{T'} < f_{ik}^T = \pi_{ik}^T + \epsilon$ yields

$$\pi_{ik}^T + \frac{\epsilon\pi_{ik}^T}{\pi_{ij}^T} \leq \pi_{ik}^{T'} < f_{ik}^T = \pi_{ik}^T + \epsilon$$

which is a contradiction as $\pi_{ik}^T > \pi_{ij}^T$.

(v) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} = \tau_{ij}^M$ and $e_{ij}^M = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i5} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i5}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i5}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$ but $c_{ij}^M \geq x_{i5}, c_{ik}^M < x_{i5}$. This pair of inequalities imply that $\pi_{ik}^M > \pi_{ij}^M$. We also know from $e_{ij}^M = 0, e_{ik}^M = 0$ that

$$\begin{aligned}\pi_{ij}^M &< f_{ij}^M = \pi_{ij}^M + \epsilon \\ \pi_{ik}^M &< f_{ik}^M = \pi_{ik}^M + \epsilon\end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\begin{aligned}\frac{\pi_{ik}^{M'}}{\pi_{ik}^M} &= \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \\ \frac{\pi_{ij}^{M'}}{\pi_{ij}^M} &= \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}\end{aligned}$$

So that the change in bilateral gross profits (through the price index and income from dividends) due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$, we also know that $\pi_{ij}^{M'} \geq f_{ij}^M$ and $\pi_{ik}^{M'} < f_{ik}^M$. Take the first of these two inequalities. We have

$$\pi_{ij}^{M'} \geq f_{ij}^M \quad \Rightarrow \quad \pi_{ij}^{M'} \geq \pi_{ij}^M + \epsilon \quad \Rightarrow \quad (\delta - 1)\pi_{ij}^M \geq \epsilon \quad \Rightarrow \quad \delta - 1 \geq \frac{\epsilon}{\pi_{ij}^M}$$

Then the last inequality implies

$$(\delta - 1)\pi_{ik}^M \geq \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M} \Rightarrow \pi_{ik}^{M'} - \pi_{ik}^M \geq \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M} \Rightarrow \pi_{ik}^{M'} \geq \pi_{ik}^M + \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M}$$

However, combining this with $\pi_{ik}^{M'} < f_{ik}^M = \pi_{ik}^M + \epsilon$ yields

$$\pi_{ik}^M + \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M} \leq \pi_{ik}^{M'} < f_{ik}^M = \pi_{ik}^M + \epsilon$$

which is a contradiction as $\pi_{ik}^M > \pi_{ij}^M$.

(vi) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} = \tau_{ij}^M$ and $e_{ij}^M = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them are not affected by the policy change. Then there exists a cutoff x_{i6} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i6}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i6}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$ but $c_{ij}^M \geq x_{i6}, c_{ik}^M < x_{i6}$. This pair of inequalities imply that $\pi_{ik}^M > \pi_{ij}^M$. We also know from $e_{ij}^M = 1, e_{ik}^M = 1$ that

$$\pi_{ij}^M > f_{ij}^M = \epsilon$$

$$\pi_{ik}^M > f_{ik}^M = \epsilon$$

where $\epsilon > 0$ is a small positive number. Note that

$$\frac{\pi_{ik}^{M'}}{\pi_{ik}^M} = \left(\frac{P_i^{M'}}{P_i^M} \right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

$$\frac{\pi_{ij}^{M'}}{\pi_{ij}^M} = \left(\frac{P_i^{M'}}{P_i^M} \right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

So that the change in bilateral gross profits (through the price index and income from dividends) due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{P_i^{M'}}{P_i^M} \right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$, we also know that $\pi_{ij}^{M'} \geq f_{ij}^M$ and $\pi_{ik}^{M'} < f_{ik}^M$. Take the second of these two inequalities. We have

$$\pi_{ik}^{M'} < f_{ik}^M \Rightarrow \pi_{ik}^{M'} < \epsilon \Rightarrow \delta \pi_{ik}^M < \epsilon \Rightarrow \delta < \frac{\epsilon}{\pi_{ik}^M}$$

Then the last inequality implies

$$\delta \pi_{ij}^M < \frac{\epsilon \pi_{ij}^M}{\pi_{ik}^M} \Rightarrow \pi_{ij}^{M'} < \frac{\epsilon \pi_{ij}^M}{\pi_{ik}^M} \Rightarrow \pi_{ij}^{M'} < \epsilon$$

where the last implication comes from $\pi_{ik}^{M'} > \pi_{ij}^M$. This is a contradiction as $\pi_{ij}^{M'} \geq f_{ij}^M = \epsilon$.

(vii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} < \tau_{ij}^M$ and $e_{ij}^M = 1$, i.e. all country pairs where the firms from source i enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i7} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i7}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i7}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$ but $c_{ij}^M \geq x_{i7}, c_{ik}^M < x_{i7}$. This pair of inequalities imply that $\pi_{ik}^M > \pi_{ij}^M$. We also know from $e_{ij}^M = 1, e_{ik}^M = 1$ that

$$\pi_{ij}^M > f_{ij}^M = \epsilon$$

$$\pi_{ik}^M > f_{ik}^M = \epsilon$$

where $\epsilon > 0$ is a small positive number. Note that

$$\frac{\pi_{ik}^{M'}}{\pi_{ik}^M} = \left(\frac{\tau_{ik}^{M'}}{\tau_{ik}^M} \right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M} \right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

$$\frac{\pi_{ij}^{M'}}{\pi_{ij}^M} = \left(\frac{\tau_{ij}^{M'}}{\tau_{ij}^M} \right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M} \right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Because $\tau_{ik}^{M'}/\tau_{ik}^M = \tau_{ij}^{M'}/\tau_{ij}^M$ the change in bilateral gross profits due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{\tau_{ij}^{M'}}{\tau_{ij}^M} \right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M} \right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$, we also know that $\pi_{ij}^{M'} \geq f_{ij}^M$ and $\pi_{ik}^{M'} < f_{ik}^M$. Take the first of these two inequalities. We have

$$\pi_{ik}^{M'} < f_{ik}^M \quad \Rightarrow \quad \pi_{ik}^{M'} < \epsilon \quad \Rightarrow \quad \delta \pi_{ik}^M < \epsilon \quad \Rightarrow \quad \delta < \frac{\epsilon}{\pi_{ik}^M}$$

Then the last inequality implies

$$\delta \pi_{ij}^M < \frac{\epsilon \pi_{ij}^M}{\pi_{ik}^M} \quad \Rightarrow \quad \pi_{ij}^{M'} < \frac{\epsilon \pi_{ij}^M}{\pi_{ik}^M} \quad \Rightarrow \quad \pi_{ij}^{M'} < \epsilon$$

where the last implication comes from $\pi_{ik}^M > \pi_{ij}^M$. This is a contradiction as $\pi_{ij}^{M'} \geq f_{ij}^M = \epsilon$.

(viii) Fix destination country i . Consider firms from all countries j such that $\tau_{ij}^{M'} < \tau_{ij}^M$ and $e_{ij}^M = 0$, i.e. all country pairs where the firms from source i do not enter destination j pre-reform, and the iceberg costs between them fall as a result of the policy change. Then there exists a cutoff x_{i8} for each i such that for firms from countries with cost $c_{ij}^M \geq x_{i8}$, the optimal choice is not to enter after the reform ($e_{ij}^{M'} = 0$) while firms from countries with cost $c_{ik}^M < x_{i8}$ choose to enter after the reform ($e_{ik}^{M'} = 1$).

Proof. Suppose not, and there exists j, k such that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$ but $c_{ij}^M \geq x_{i8}, c_{ik}^M <$

x_{i8} . This pair of inequalities imply that $\pi_{ik}^M > \pi_{ij}^M$. We also know from $e_{ij}^M = 0, e_{ik}^M = 0$ that

$$\begin{aligned}\pi_{ij}^M &< f_{ij}^M = \pi_{ij}^M + \epsilon \\ \pi_{ik}^M &< f_{ik}^M = \pi_{ik}^M + \epsilon\end{aligned}$$

where $\epsilon > 0$ is a small positive number. Note that

$$\begin{aligned}\frac{\pi_{ik}^{M'}}{\pi_{ik}^M} &= \left(\frac{\tau_{ik}^{M'}}{\tau_{ik}^M}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i} \\ \frac{\pi_{ij}^{M'}}{\pi_{ij}^M} &= \left(\frac{\tau_{ij}^{M'}}{\tau_{ij}^M}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}\end{aligned}$$

Because $\tau_{ik}^{M'}/\tau_{ik}^M = \tau_{ij}^{M'}/\tau_{ij}^M$ the change in bilateral gross profits due to the reform for both j and k are the same. Denote this change by δ , i.e.

$$\delta = \left(\frac{\tau_{ij}^{M'}}{\tau_{ij}^M}\right)^{1-\sigma} \left(\frac{P_i^{M'}}{P_i^M}\right)^{\sigma-1} \frac{Y_i'}{Y_i}$$

Given that $e_{ij}^{M'} = 1, e_{ik}^{M'} = 0$, we also know that $\pi_{ij}^{M'} \geq f_{ij}^M$ and $\pi_{ik}^{M'} < f_{ik}^M$. Take the first of these two inequalities. We have

$$\pi_{ij}^{M'} \geq f_{ij}^M \quad \Rightarrow \quad \pi_{ij}^{M'} \geq \pi_{ij}^M + \epsilon \quad \Rightarrow \quad (\delta - 1)\pi_{ij}^M \geq \epsilon \quad \Rightarrow \quad \delta - 1 \geq \frac{\epsilon}{\pi_{ij}^M}$$

Then the last inequality implies

$$(\delta - 1)\pi_{ik}^M \geq \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M} \quad \Rightarrow \quad \pi_{ik}^{M'} - \pi_{ik}^M \geq \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M} \quad \Rightarrow \quad \pi_{ik}^{M'} \geq \pi_{ik}^M + \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M}$$

However, combining this with $\pi_{ik}^{M'} < f_{ik}^M = \pi_{ik}^M + \epsilon$ yields

$$\pi_{ik}^M + \frac{\epsilon\pi_{ik}^M}{\pi_{ij}^M} \leq \pi_{ik}^{M'} < f_{ik}^M = \pi_{ik}^M + \epsilon$$

which is a contradiction as $\pi_{ik}^M > \pi_{ij}^M$.

A.2 Computational Algorithm

Step 1: Parameterize the distribution of decision rules e'_{ij} for all i, j as follows:

$$e_{ij}^{T'} = \begin{cases} 0 & \text{if } \tau_{ij}^{T'} = \tau_{ij}^T, e_{ij}^T = 0, c_{ij}^T \geq x'_{i1} \\ 1 & \text{if } \tau_{ij}^{T'} = \tau_{ij}^T, e_{ij}^T = 0, c_{ij}^T < x'_{i1} \\ 0 & \text{if } \tau_{ij}^{T'} = \tau_{ij}^T, e_{ij}^T = 1, c_{ij}^T \geq x'_{i2} \\ 1 & \text{if } \tau_{ij}^{T'} = \tau_{ij}^T, e_{ij}^T = 1, c_{ij}^T < x'_{i2} \\ 0 & \text{if } \tau_{ij}^{T'} < \tau_{ij}^T, e_{ij}^T = 1, c_{ij}^T \geq x'_{i3} \\ 1 & \text{if } \tau_{ij}^{T'} < \tau_{ij}^T, e_{ij}^T = 1, c_{ij}^T < x'_{i3} \\ 0 & \text{if } \tau_{ij}^{T'} < \tau_{ij}^T, e_{ij}^T = 0, c_{ij}^T \geq x'_{i4} \\ 1 & \text{if } \tau_{ij}^{T'} < \tau_{ij}^T, e_{ij}^T = 0, c_{ij}^T < x'_{i4} \end{cases}$$

$$e_{ij}^{M'} = \begin{cases} 0 & \text{if } \tau_{ij}^{M'} = \tau_{ij}^M, e_{ij}^M = 0, c_{ij}^M \geq x'_{i5} \\ 1 & \text{if } \tau_{ij}^{M'} = \tau_{ij}^M, e_{ij}^M = 0, c_{ij}^M < x'_{i5} \\ 0 & \text{if } \tau_{ij}^{M'} = \tau_{ij}^M, e_{ij}^M = 1, c_{ij}^M \geq x'_{i6} \\ 1 & \text{if } \tau_{ij}^{M'} = \tau_{ij}^M, e_{ij}^M = 1, c_{ij}^M < x'_{i6} \\ 0 & \text{if } \tau_{ij}^{M'} < \tau_{ij}^M, e_{ij}^M = 1, c_{ij}^M \geq x'_{i7} \\ 1 & \text{if } \tau_{ij}^{M'} < \tau_{ij}^M, e_{ij}^M = 1, c_{ij}^M < x'_{i7} \\ 0 & \text{if } \tau_{ij}^{M'} < \tau_{ij}^M, e_{ij}^M = 0, c_{ij}^M \geq x'_{i8} \\ 1 & \text{if } \tau_{ij}^{M'} < \tau_{ij}^M, e_{ij}^M = 0, c_{ij}^M < x'_{i8} \end{cases}$$

Note the similarity between these decision rules and the lemmata proved earlier.

Step 2: Guess an initial vector of cutoffs $\{(x_{i1}, x_{i2}, x_{i3}, x_{i4}, x_{i5}, x_{i6}, x_{i7}, x_{i8})\}_{i=1,2,\dots,N}$. Do

this with percentiles of the elements in the $8N$ sets $\{\Omega_{i1}, \Omega_{i2}, \Omega_{i3}, \Omega_{i4}, \Omega_{i5}, \Omega_{i6}, \Omega_{i7}, \Omega_{i8}\}_{i=1,2,\dots,N}$,

defined by

$$\Omega_{i1} = \{j : \tau_{ij}^{T'} = \tau_{ij}^T, e_{ij}^T = 0\}$$

$$\Omega_{i2} = \{j : \tau_{ij}^{T'} = \tau_{ij}^T, e_{ij}^T = 1\}$$

$$\Omega_{i3} = \{j : \tau_{ij}^{T'} < \tau_{ij}^T, e_{ij}^T = 1\}$$

$$\Omega_{i4} = \{j : \tau_{ij}^{T'} < \tau_{ij}^T, e_{ij}^T = 0\}$$

$$\Omega_{i5} = \{j : \tau_{ij}^{M'} = \tau_{ij}^M, e_{ij}^M = 0\}$$

$$\Omega_{i6} = \{j : \tau_{ij}^{M'} = \tau_{ij}^M, e_{ij}^M = 1\}$$

$$\Omega_{i7} = \{j : \tau_{ij}^{M'} < \tau_{ij}^M, e_{ij}^M = 1\}$$

$$\Omega_{i8} = \{j : \tau_{ij}^{M'} < \tau_{ij}^M, e_{ij}^M = 0\}$$

A cutoff x_{ih} is then associated with the percentile $k_{ih} = 100|\hat{\Omega}_{ih}|/|\Omega_{ih}|$ for $h = 1, 2, 3, \dots, 8$, where $\hat{\Omega}_{ih} = \{j : j \in \Omega_{ih}, c_{ij} < x_{ih}\}$.

Step 3: Given the parameterization of the decision rules and guess for the cutoffs, we can then compute what the prices indices P_i^t are for all i as well as the dividend per share π^t . Here the superscript t denotes the t -th iteration.

Step 4: Given the price indices P_i^t and dividend per share π^t , we can compute the decision rule e'_{ij} for firms from each source country j and destination country i . Denote this matrix (i.e. distribution) of decision rules by E . These decision rules then yield price indices P_i^n and dividend per share π^n .

Step 5: Given the price indices P_i^n and dividend per share π^n , we can construct the profits π_{ij}^n that firms from j can make by exporting to i and compare this against the corresponding fixed cost f_{ij} to determine the entry pattern $e'_{ij}{}^n$. Denote the matrix of such entry patterns by E^n . If $E = E^n$ then we are done.

Step 6: If $E \neq E^n$, check if the cutoff rules implied by the decision rules just obtained k_{ih}^n coincide with the guess for the cutoff rules k_{ih}^t . If yes, then terminate; in this case we obtain an approximate equilibrium as no further updating can be done and $E \neq E^n$. If no, then proceed to update the cutoffs according to $k_{ih}^{t+1} = \lambda k_{ih}^n + (1 - \lambda)k_{ih}^t$ where the step size is $\lambda \in (0, 1)$. The cutoff rules implied by the decision rules e'_{ij} are constructed as follows: $k_{ih}^n = 100|\hat{\Omega}_{ih}^n|/|\Omega_{ih}|$ where

$$\hat{\Omega}_{i1}^n = \{j : j \in \Omega_{i1}, e_{ij}^{T'} = 1\}$$

$$\hat{\Omega}_{i2}^n = \{j : j \in \Omega_{i2}, e_{ij}^{T'} = 1\}$$

$$\hat{\Omega}_{i3}^n = \{j : j \in \Omega_{i3}, e_{ij}^{T'} = 1\}$$

$$\hat{\Omega}_{i4}^n = \{j : j \in \Omega_{i4}, e_{ij}^{T'} = 1\}$$

$$\hat{\Omega}_{i5}^n = \{j : j \in \Omega_{i5}, e_{ij}^{M'} = 1\}$$

$$\hat{\Omega}_{i6}^n = \{j : j \in \Omega_{i6}, e_{ij}^{M'} = 1\}$$

$$\hat{\Omega}_{i7}^n = \{j : j \in \Omega_{i7}, e_{ij}^{M'} = 1\}$$

$$\hat{\Omega}_{i8}^n = \{j : j \in \Omega_{i8}, e_{ij}^{M'} = 1\}$$

With the updated cutoff rules, return to Step 3 and iterate until convergence.

A.3 Two-Country Example

A.3.1 Introduction

There are two countries $i = 1, 2$. Each has a measure $\theta_i = w_i L_i$ of firms that are considering which markets to enter. For simplicity consider only an environment with trade (i.e. no multinational production). International trade is subject to both iceberg and fixed costs, which can be asymmetric across country pairs. Consider a policy reform that lowers the iceberg costs for firms from 2 to export to 1 from $\tau_{12} = 2$ to $\tau'_{12} = 1$.

Let $\tau_{21} = \tau'_{21} = 1$. The trade elasticity is $\sigma = 2$ and the expenditure share for the differentiated goods sector is $\mu = 1/2$. The other exogenous parameters are given below

$$w_1 = 1, \quad w_2 = 1, \quad L_1 = 1, \quad L_2 = 1, \quad \phi_1 = 2, \quad \phi_2 = 1$$

This implies

$$p_{11} = p'_{11} = 1, \quad p_{22} = p'_{22} = 2, \quad p_{12} = \frac{\sigma}{\sigma - 1} \frac{w_2 \tau_{12}}{\phi_2} = 4, \quad p_{21} = \frac{\sigma}{\sigma - 1} \frac{w_1 \tau_{21}}{\phi_1} = 1, \quad p'_{12} = \frac{\sigma}{\sigma - 1} \frac{w_2 \tau'_{12}}{\phi_2} =$$

Now we will look at the equilibria that can arise given different values for (f_{12}, f_{21}) , i.e. we will partition the $f_{12} - f_{21}$ space into the regions where the four cases $(e_{12}, e_{21}) \in \{(0, 0), (0, 1), (1, 0), (1, 1)\}$ apply. We will do this both pre-reform and post-reform. Note: In general, one should consider not only four cases, but sixteen cases, as entry decisions e_{11}, e_{22} are also binary choice variables that can be 0 or 1. To ease exposition, I assume throughout this illustration that these decisions are $e_{11} = 1$ and $e_{22} = 1$.

Note that given our assumptions on the values of the parameters, the profit for a firm from j exporting to i denoted by π_{ij} is simply given by

$$\pi_{ij} = \mu \frac{1}{\sigma} \left(\frac{p_{ij}}{P_i} \right)^{1-\sigma} w_i L_i (1 + \pi) = \frac{1}{2} \frac{1}{2} \left(\frac{p_{ij}}{P_i} \right)^{-1} (1 + \pi) = \frac{1}{4} \frac{P_i}{p_{ij}} (1 + \pi)$$

This will be useful for the characterization that follows.

A.3.2 Pre-Reform

Case 1: $(e_{12}, e_{21}) = (0, 0)$

In this equilibrium, firms only operate domestically and there is no entry into international markets either way. Hence we have

$$P_1 = p_{11} = 1, \quad P_2 = p_{22} = 2$$

In this case we also have $\pi = \frac{1}{3}$, and

$$\begin{aligned}\pi_{11} &= \frac{1}{4}(1 + \pi) \\ \pi_{12} &= \frac{1}{4} \frac{P_1}{p_{12}}(1 + \pi) = \frac{1}{16}(1 + \pi) = \frac{1}{16} \frac{4}{3} = \frac{1}{12} \leq f_{12} \\ \pi_{21} &= \frac{1}{4} \frac{P_2}{p_{21}}(1 + \pi) = \frac{1}{2}(1 + \pi) = \frac{2}{3} \leq f_{21} \\ \pi_{22} &= \frac{1}{4} \frac{P_2}{p_{22}} = \frac{1}{4}(1 + \pi) = \frac{1}{3}\end{aligned}$$

Case 2: $(e_{12}, e_{21}) = (0, 1)$

In this equilibrium, country 2 firms enter (i.e. exports to) country 1 but not the other way around. Hence we have

$$P_1 = p_{11} = 1, \quad P_2 = [p_{21}^{-1} + p_{22}^{-1}]^{-1} = \frac{2}{3}$$

In this case we have

$$\begin{aligned}\pi_{11} &= \frac{1}{4}(1 + \pi) \\ \pi_{12} &= \frac{1}{16}(1 + \pi) \\ \pi_{21} &= \frac{1}{4} \frac{2/3}{1}(1 + \pi) = \frac{1}{6}(1 + \pi) \\ \pi_{22} &= \frac{1}{4} \frac{2/3}{2} = \frac{1}{12}(1 + \pi)\end{aligned}$$

Hence

$$\pi = \frac{\frac{3+2+1}{12}(1 + \pi) - f_{21}}{2} = \frac{1}{3} - \frac{2}{3}f_{21}$$

It is instructive to note that

$$\begin{aligned}\pi_{\max} &= \frac{1}{3}, \quad f_{21} = 0 \\ \pi_{\min} &= \frac{1}{6}(1 + \pi_{\min}) \Rightarrow \pi_{\min} = \frac{1}{5}, \quad f_{21} = \frac{1}{5}\end{aligned}$$

In general

$$\pi_{21} = \frac{1}{16} \left(\frac{4}{3} - \frac{2}{3}f_{21} \right) = \frac{1}{12} - \frac{1}{24}f_{21} \leq f_{12}, \text{ for } f_{21} \in (0, \frac{1}{5})$$

Case 3: $(e_{12}, e_{21}) = (1, 0)$

In this equilibrium, country 1 firms enter (i.e. exports to) country 2 but not the other way around. Hence we have

$$P_1 = [p_{11}^{-1} + p_{12}^{-1}]^{-1} = \frac{4}{5}, \quad P_2 = p_{22} = 2$$

In this case we have

$$\begin{aligned} \pi_{11} &= \frac{1}{4} \frac{P_1}{p_{11}} (1 + \pi) = \frac{1}{4} \frac{4}{5} (1 + \pi) = \frac{1}{5} (1 + \pi) \\ \pi_{12} &= \frac{1}{4} \frac{P_1}{p_{12}} (1 + \pi) = \frac{1}{4} \frac{4/5}{4} (1 + \pi) = \frac{1}{20} (1 + \pi) \\ \pi_{21} &= \frac{1}{4} \frac{P_2}{p_{21}} (1 + \pi) = \frac{1}{4} \frac{2}{1} (1 + \pi) = \frac{1}{2} (1 + \pi) \\ \pi_{22} &= \frac{1}{4} \frac{P_2}{p_{22}} (1 + \pi) = \frac{1}{4} (1 + \pi) \end{aligned}$$

Hence

$$\pi = \frac{\frac{5+4+1}{20}(1 + \pi) - f_{12}}{2} = \frac{1}{3} - \frac{2}{3}f_{12}$$

It is instructive to note that

$$\begin{aligned} \pi_{\max} &= \frac{1}{3}, \quad f_{12} = 0 \\ \pi_{\min} &= \frac{\frac{5+4}{20}(1 + \pi)}{2} = \frac{9}{40}(1 + \pi_{\min}) \Rightarrow \pi_{\min} = \frac{9}{31}, \quad f_{12} = \frac{2}{31} \end{aligned}$$

In general

$$\pi_{21} = \frac{1}{2} \left(\frac{4}{3} - \frac{2}{3}f_{12} \right) = \frac{2}{3} - \frac{1}{3}f_{12} \leq f_{21}, \text{ for } f_{12} \in \left(0, \frac{2}{13} \right)$$

Case 4: $(e_{12}, e_{21}) = (1, 1)$

In this equilibrium, country 1 firms enter (i.e. exports to) country 2 and vice versa.

Hence we have

$$P_1 = [p_{11}^{-1} + p_{12}^{-1}]^{-1} = \frac{4}{5}, \quad P_2 = [p_{21}^{-1} + p_{22}^{-1}]^{-1} = \frac{2}{3}$$

In this case we have

$$\begin{aligned}\pi_{11} &= \frac{1}{5}(1 + \pi) \\ \pi_{12} &= \frac{1}{20}(1 + \pi) \\ \pi_{21} &= \frac{1}{6}(1 + \pi) \\ \pi_{22} &= \frac{1}{12}(1 + \pi)\end{aligned}$$

Hence

$$\pi = \frac{1}{3} - \frac{2}{3}(f_{12} + f_{21})$$

$$\pi_{\max} = \frac{1}{3}, \quad f_{12} = 0, \quad f_{21} = 0$$

$$\pi_{\min} = \frac{\frac{5+12}{60}(1 + \pi)}{2} = \frac{17}{120}(1 + \pi_{\min}) \Rightarrow \pi_{\min} = \frac{17}{103}, \quad f_{21} = \frac{1}{20} \frac{120}{103} = \frac{6}{103}, \quad f_{12} = \frac{1}{6} \frac{120}{103} = \frac{20}{103}$$

In general

$$\begin{aligned}\pi_{12} &= \frac{1}{20} \left(\frac{4}{3} - \frac{2}{3}f_{12} - \frac{2}{3}f_{21} \right) = \frac{2}{30} - \frac{1}{30}f_{12} - \frac{1}{30}f_{21} \leq f_{12} \\ \pi_{21} &= \frac{1}{6} \left(\frac{4}{3} - \frac{2}{3}f_{12} - \frac{2}{3}f_{21} \right) = \frac{2}{9} - \frac{1}{9}f_{12} - \frac{1}{9}f_{21} \leq f_{21}\end{aligned}$$

This implies

$$\begin{aligned}f_{12} &\leq \frac{30}{31} \left(\frac{2}{30} - \frac{1}{30}f_{21} \right) \\ f_{21} &\leq \frac{9}{10} \left(\frac{2}{9} - \frac{1}{9}f_{12} \right)\end{aligned}$$

Hence the endpoints are $\left\{ \left(\frac{2}{31}, 0 \right), \left(0, \frac{2}{10} \right) \right\}$.

A.3.3 Post-Reform

Case 1: $(e'_{12}, e'_{21}) = (0, 0)$

In this equilibrium, firms only operate domestically and there is no entry into international markets either way. Hence we have

$$P'_1 = p'_{11} = 1, \quad P'_2 = p'_{22} = 2$$

In this case we also have $\pi' = \frac{1}{3}$, and

$$\begin{aligned}\pi'_{11} &= \frac{1}{4}(1 + \pi') \\ \pi'_{12} &= \frac{1}{4} \frac{P'_1}{p'_{12}}(1 + \pi') = \frac{1}{8}(1 + \pi') = \frac{1}{8} \frac{4}{3} = \frac{1}{6} \leq f_{12} \\ \pi'_{21} &= \frac{1}{4} \frac{P'_2}{p'_{21}}(1 + \pi') = \frac{1}{2}(1 + \pi') = \frac{2}{3} \leq f_{21} \\ \pi'_{22} &= \frac{1}{4} \frac{P'_2}{p'_{22}} = \frac{1}{4}(1 + \pi') = \frac{1}{3}\end{aligned}$$

Case 2: $(e'_{12}, e'_{21}) = (0, 1)$

In this equilibrium, country 2 firms enter (i.e. exports to) country 1 but not the other way around. Hence we have

$$P'_1 = p'_{11} = 1, \quad P'_2 = [p'_{21}{}^{-1} + p'_{22}{}^{-1}]^{-1} = \frac{2}{3}$$

In this case we have

$$\begin{aligned}\pi'_{11} &= \frac{1}{4}(1 + \pi') \\ \pi'_{12} &= \frac{1}{8}(1 + \pi') \\ \pi'_{21} &= \frac{1}{6}(1 + \pi') \\ \pi'_{22} &= \frac{1}{12}(1 + \pi')\end{aligned}$$

Hence

$$\pi' = \frac{1}{3} - \frac{2}{3}f_{21}$$

It is instructive to note that

$$\begin{aligned}\pi'_{\max} &= \frac{1}{3}, \quad f_{21} = 0 \\ \pi'_{\min} &= \frac{1}{5}, \quad f_{21} = \frac{1}{5}\end{aligned}$$

In general

$$\pi'_{21} = \frac{1}{8} \left(\frac{4}{3} - \frac{2}{3}f_{21} \right) = \frac{1}{6} - \frac{1}{12}f_{21} \leq f_{12}, \text{ for } f_{21} \in \left(0, \frac{1}{5}\right)$$

Case 3: $(e'_{12}, e'_{21}) = (1, 0)$

In this equilibrium, country 1 firms enter (i.e. exports to) country 2 but not the other way around. Hence we have

$$P'_1 = [p'^{-1}_{11} + p'^{-1}_{12}]^{-1} = \frac{2}{3}, \quad P'_2 = p'_{22} = 2$$

In this case we have

$$\begin{aligned} \pi'_{11} &= \frac{1}{6}(1 + \pi') \\ \pi'_{12} &= \frac{1}{12}(1 + \pi') \\ \pi'_{21} &= \frac{1}{2}(1 + \pi') \\ \pi'_{22} &= \frac{1}{4}(1 + \pi') \end{aligned}$$

Hence

$$\pi' = \frac{\frac{2+1+13}{12}(1 + \pi') - f_{12}}{2} = \frac{1}{3} - \frac{2}{3}f_{12}$$

It is instructive to note that

$$\begin{aligned} \pi'_{\max} &= \frac{1}{3}, \quad f_{12} = 0 \\ \pi'_{\min} &= \frac{5}{24}(1 + \pi'_{\min}) \Rightarrow \pi'_{\min} = \frac{5}{19}, \quad f_{12} = \frac{2}{19} \end{aligned}$$

In general

$$\pi'_{21} = \frac{1}{2} \left(\frac{4}{3} - \frac{2}{3}f_{12} \right) = \frac{2}{3} - \frac{1}{3}f_{12} \leq f_{21}, \text{ for } f_{12} \in \left(0, \frac{2}{19} \right)$$

Case 4: $(e_{12}, e_{21}) = (1, 1)$

In this equilibrium, country 1 firms enter (i.e. exports to) country 2 and vice versa.

Hence we have

$$P'_1 = [p'^{-1}_{11} + p'^{-1}_{12}]^{-1} = \frac{2}{3}, \quad P'_2 = [p'^{-1}_{21} + p'^{-1}_{22}]^{-1} = \frac{2}{3}$$

In this case we have

$$\begin{aligned}\pi'_{11} &= \frac{1}{6}(1 + \pi') \\ \pi'_{12} &= \frac{1}{12}(1 + \pi') \\ \pi'_{21} &= \frac{1}{6}(1 + \pi') \\ \pi'_{22} &= \frac{1}{12}(1 + \pi')\end{aligned}$$

Hence

$$\pi' = \frac{1}{3} - \frac{2}{3}(f_{12} + f_{21})$$

$$\begin{aligned}\pi'_{\max} &= \frac{1}{3}, \quad f_{12} = 0, \quad f_{21} = 0 \\ \pi'_{\min} &= \frac{1}{8}(1 + \pi'_{\min}) \Rightarrow \pi'_{\min} = \frac{1}{7}, \quad f_{21} = \frac{1}{6} \frac{8}{7} = \frac{4}{21}, \quad f_{12} = \frac{1}{12} \frac{8}{7} = \frac{2}{21}\end{aligned}$$

In general

$$\begin{aligned}\pi'_{12} &= \frac{1}{12} \left(\frac{4}{3} - \frac{2}{3}f_{12} - \frac{2}{3}f_{21} \right) = \frac{1}{9} - \frac{1}{18}f_{12} - \frac{1}{18}f_{21} \leq f_{12} \\ \pi'_{21} &= \frac{1}{6} \left(\frac{4}{3} - \frac{2}{3}f_{12} - \frac{2}{3}f_{21} \right) = \frac{2}{9} - \frac{1}{9}f_{12} - \frac{1}{9}f_{21} \leq f_{21}\end{aligned}$$

This implies

$$\begin{aligned}f_{12} &\leq \frac{18}{19} \left(\frac{1}{9} - \frac{1}{18}f_{21} \right) \\ f_{21} &\leq \frac{9}{10} \left(\frac{2}{9} - \frac{1}{9}f_{12} \right)\end{aligned}$$

Hence the endpoints are $\left\{ \left(\frac{2}{19}, 0 \right), \left(0, \frac{2}{10} \right) \right\}$.

A.3.4 Equilibria Pre- and Post-Reform

Given the characterization of the two previous sections, we can partition the $f_{12} - f_{21}$ space into the following regions.

Pre-reform, we have the five regions P_i , $i = 0, 1, \dots, 4$ where P_i corresponds to the region that applies to the i -th case described earlier (e.g. P_2 corresponds to $(e_{12}, e_{21}) = (0, 1)$), and the 0-th case is the region in the parameter space where no equilibria exist.

$$P_1 = \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/12, f_{21} \geq 2/3\}$$

$$P_2 = \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/12 - 1/24f_{12}, f_{21} \leq 1/5\}$$

$$P_3 = \{(f_{12}, f_{21}) \in R_+^2 : f_{21} \geq 2/3 - 1/3f_{12}, f_{12} \leq 2/31\}$$

$$P_4 = \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 30/31(2/30 - 1/30f_{12}), f_{21} \geq 9/10(2/9 - 1/9f_{12})\}$$

$$P_0 = R_+^2 \setminus \{P_1 \cup P_2 \cup P_3 \cup P_4\}$$

Post-reform, the fixed cost space is partitioned into the following regions

$$Q_1 = \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/6, f_{21} \geq 2/3\}$$

$$Q_2 = \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/6 - 1/12f_{12}, f_{21} \leq 1/5\}$$

$$Q_3 = \{(f_{12}, f_{21}) \in R_+^2 : f_{21} \geq 2/3 - 1/3f_{12}, f_{12} \leq 2/19\}$$

$$Q_4 = \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 18/19(1/9 - 1/18f_{12}), f_{21} \geq 9/10(2/9 - 1/9f_{12})\}$$

$$Q_0 = R_+^2 \setminus \{Q_1 \cup Q_2 \cup Q_3 \cup Q_4\}$$

Then combining these two partitions of the fixed-cost space, we get the following regions:

$$A_{11} = P_1 \cap Q_1 = \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/6, f_{21} \geq 2/3\}$$

$$A_{13} = P_1 \cap Q_3 = \{(f_{12}, f_{21}) \in R_+^2 : 1/12 \leq f_{12} \leq 2/19, f_{21} \geq 2/3\}$$

$$A_{33} = P_3 \cap Q_3 = \{(f_{12}, f_{21}) \in R_+^2 : f_{21} \geq 2/3 - 1/3f_{12}, f_{12} \leq 2/31\}$$

$$A_{22} = P_2 \cap Q_2 = \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/6 - 1/12f_{21}, f_{21} \leq 1/5\}$$

$$A_{24} = P_2 \cap Q_4 = \{(f_{12}, f_{21}) \in R_+^2 : f_{12} \geq 1/12 - 1/24f_{21}, f_{21} \leq 1/5\}$$

$$\cap \{(f_{12}, f_{21}) : f_{12} \leq 18/19(1/9 - 1/18f_{21}), f_{21} \leq 9/10(2/9 - 1/9f_{12})\}$$

$$A_{44} = P_4 \cap Q_4 = \{(f_{12}, f_{21}) \in R_+^2 : f_{21} \leq 9/10(2/9 - 1/9f_{12}), f_{12} \geq 30/31(2/30 - 1/30f_{12})\}$$

$$A_{10} = P_1 \cap Q_0 = \{(f_{12}, f_{21}) \in R_+^2 : 2/19 \leq f_{12} \leq 1/6, f_{21} \geq 2/3\}$$

$$A_{20} = P_2 \cap Q_0 = P_2 \setminus \{A_{24} \cup A_{22}\}$$

$$A_{03} = P_0 \cap Q_3 = Q_3 \setminus \{A_{33} \cup A_{13}\}$$

$$A_{04} = P_0 \cap Q_4 = Q_4 \setminus \{A_{44} \cup A_{24}\}$$

$$A_{12} = A_{14} = A_{21} = A_{23} = A_{31} = A_{32} = A_{34} = A_{41} = A_{42} = A_{43} = \{\}$$

$$A_{01} = A_{02} = A_{30} = A_{40} = \{\}$$

$$A_{00} = R_+^2 \setminus \{A_{11} \cup A_{13} \cup A_{33} \cup A_{22} \cup A_{24} \cup A_{44} \cup A_{10} \cup A_{20} \cup A_{03} \cup A_{04}\}$$

Here A_{ij} means that pre-reform, the equilibrium is Case i and post-reform, the equilibrium is Case j for any $(f_{12}, f_{21}) \in A_{ij}$. Note that there are many sets A_{ij} that are empty, in which case no exact equilibria exist, both pre- and post-reform. There are two regions A_{ij} where $i \neq j$ and $i, j \neq 0$: A_{24} and A_{13} . These are interesting because they represent the equilibria where a change in entry patterns following the policy reform. A_{13} means that after lowering the iceberg cost to export from 2 to 1, the entry pattern changed from Case 1, i.e. $(e_{12}, e_{21}) = (0, 0)$, to Case 3, i.e. $(e_{12}, e_{21}) = (1, 0)$, so the policy change induces country 2 to start exporting the differentiated good to 1, even though firms from 1 still do not export to 2. The A_{24} equilibrium is analogous in that

the policy induces firms from 2 to start exporting, but in this case, both before and after the reform, firms from country 1 export to country 2. There are four nonempty sets A_{ii} where there is no change in the entry patterns after the reform, so we can expect the effect of the extensive margin to be minimal. The other non-empty sets A_{ij} have either $i = 0$ or $j = 0$; this means that before or after the reform, equilibria may fail to exist.

A.3.5 The Algorithm at Work: An Approximate Equilibrium

As before, suppose there are two countries $i = 1, 2$. Each has a measure $\theta_i = w_i L_i$ of firms that are considering which markets to enter. For simplicity consider only an environment with trade (i.e. no multinational production). An example allowing for multinational production is available upon request. International trade is subject to both iceberg and fixed costs, which can be asymmetric across country pairs. Consider a policy reform that lowers the iceberg costs for firms from 2 to export to 1 from $\tau_{12} = 2$ to $\tau'_{12} = 1$. Let $\tau_{21} = \tau'_{21} = \tau_{11} = \tau'_{11} = \tau_{22} = \tau'_{22} = 1$. The trade elasticity is $\sigma = 2$ and the expenditure share for the differentiated goods sector is $\mu = 1/2$. The other exogenous parameters are given below

$$w_1 = 1, \quad w_2 = 1, \quad L_1 = 1, \quad L_2 = 1, \quad \phi_1 = 2, \quad \phi_2 = 1$$

This implies

$$p_{11} = p'_{11} = 1, \quad p_{22} = p'_{22} = 2, \quad p_{12} = \frac{\sigma}{\sigma - 1} \frac{w_2 \tau_{12}}{\phi_2} = 4, \quad p_{21} = \frac{\sigma}{\sigma - 1} \frac{w_1 \tau_{21}}{\phi_1} = 1, \quad p'_{12} = \frac{\sigma}{\sigma - 1} \frac{w_2 \tau'_{12}}{\phi_2} =$$

Define

$$\begin{aligned} Q_{11} &= \mu \frac{1}{\sigma} \left(\frac{p_{11}}{P_1} \right)^{1-\sigma} = \mu \frac{1}{\sigma} \left(\frac{P_1}{p_{11}} \right) = \frac{1}{4} \\ Q_{21} &= \mu \frac{1}{\sigma} \left(\frac{p_{21}}{P_2} \right)^{1-\sigma} = \mu \frac{1}{\sigma} \left(\frac{P_2}{p_{21}} \right) = \frac{1}{6} \\ Q_{22} &= \mu \frac{1}{\sigma} \left(\frac{p_{22}}{P_2} \right)^{1-\sigma} = \mu \frac{1}{\sigma} \left(\frac{P_2}{p_{22}} \right) = \frac{1}{12} \end{aligned}$$

Then

$$\pi = \frac{\pi_{11} + \pi_{21} + \pi_{22} - 3\epsilon}{2} = \frac{(\frac{1}{4} + \frac{1}{6} + \frac{1}{12})(1 + \pi) - 3\epsilon}{2} = \frac{1}{3} - 2\epsilon$$

Hence the restrictions are given by

$$\pi_{ij} > \epsilon \Leftrightarrow Q_{ij}(1 + \pi) > \epsilon \Leftrightarrow Q_{ij} \left(\frac{4}{3} - 2\epsilon \right) > \epsilon \Leftrightarrow \epsilon < \frac{\frac{4}{3}Q_{ij}}{1 + 2Q_{ij}}$$

Applied to the three relevant cases, this yields the restrictions

$$\begin{aligned} \epsilon &< \frac{\frac{1}{3}}{1 + \frac{1}{2}} = \frac{2}{9} \\ \epsilon &< \frac{\frac{2}{9}}{1 + \frac{1}{3}} = \frac{1}{6} \\ \epsilon &< \frac{\frac{1}{9}}{1 + \frac{1}{6}} = \frac{2}{21} \end{aligned}$$

Let $\epsilon = \frac{1}{20}$. Then we have $\pi = \frac{1}{3} - \frac{1}{10} = \frac{7}{30}$. Then the baseline parameterization is

$$f_{12} = \pi_{12} + \epsilon = \frac{61}{480}, \quad f_{11} = f_{22} = f_{21} = \epsilon = \frac{1}{20}$$

To show that there is no exact equilibrium after reform, note that either $e'_{12} = 1$ or $e'_{12} = 0$.

Consider first the case with $e'_{12} = 1$. We have $\pi' \in [\frac{1}{3} - \frac{8}{3}\frac{1}{20}, \frac{1}{3}]$, and $P_1 \in \{2, \frac{2}{3}\}$, depending on the entry decisions of other agents. If $P'_1 = 2$ and $e'_{11} = 0$, we must have $\pi'_{11} < f_{11}$ and $\pi'_{12} > f_{12}$. But $\pi'_{11} = \frac{1}{4}\frac{2}{1}(1 + \pi') = \frac{1}{2}(1 + \pi') \geq \frac{1}{2}(\frac{4}{3} - \frac{8}{3}\frac{1}{20}) = \frac{9}{15} > \frac{1}{20} = f_{11}$, a contradiction. If, on the other hand, $P'_1 = \frac{2}{3}$ and $e'_{11} = 1$, we must have $\pi'_{11} > f_{11}$ and $\pi'_{12} > f_{12}$. But in this case we have $\pi'_{12} = \frac{1}{4}\frac{2}{2}(1 + \pi') = \frac{1}{12}(1 + \pi') \leq \frac{1}{12}\frac{4}{3} = \frac{1}{9} < \frac{61}{480} = f_{12}$, a contradiction. Hence there is no exact equilibrium when $e'_{12} = 1$.

Then for exact equilibrium to exist, we must have $e'_{12} = 0$. In this case we need only consider the case when $P' = 1$ and $e'_{11} = 1$ because otherwise $P'_1 = 0$ (when $e'_{11} = 0$)

and demand is not well-defined. If $e'_{12} = 0$, $P' = 1$, and $e'_{11} = 1$, we must have $\pi'_{11} > f_{11}$ and $\pi'_{12} < f_{12}$. But $\pi'_{12} = \frac{1}{4} \frac{1}{2} (1 + \pi') \geq \frac{1}{8} \left(\frac{4}{3} - 2 \frac{1}{20} \right) = \frac{1}{8} \frac{37}{30} = \frac{74}{480} > \frac{61}{480} = f_{12}$, a contradiction. Hence, exact equilibrium does not exist given this parameterization.

With no exact equilibrium, I now proceed to compute an approximate equilibrium.

As in the algorithm, define the sets

$$\begin{aligned}\Omega_{i1} &= \{j : \tau'_{ij} = \tau_{ij}, e_{ij} = 0\}, & \Omega_{i2} &= \{j : \tau'_{ij} = \tau_{ij}, e_{ij} = 1\} \\ \Omega_{i3} &= \{j : \tau'_{ij} < \tau_{ij}, e_{ij} = 0\}, & \Omega_{i4} &= \{j : \tau'_{ij} < \tau_{ij}, e_{ij} = 1\}\end{aligned}$$

There are $4N = 8$ sets. For this example these sets are

$$\begin{aligned}\Omega_{11} &= \{\}, & \Omega_{12} &= \{1\}, & \Omega_{13} &= \{\}, & \Omega_{14} &= \{2\} \\ \Omega_{21} &= \{\}, & \Omega_{22} &= \{1, 2\}, & \Omega_{23} &= \{\}, & \Omega_{24} &= \{\}\end{aligned}$$

Suppose the initial cutoffs are $k_{ij} = 100$, $\forall i = 1, 2$ and $j = 1, 2, 3, 4$. Given the cutoffs we have that the guess for the post-reform entry patterns are given by

$$E = \begin{bmatrix} e'_{11} & e'_{12} \\ e'_{21} & e'_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

This yields $P'_1 = \frac{2}{3} = P'_2$ and $\pi' = \frac{1}{3} - \frac{8}{3}\epsilon = \frac{1}{5}$. This in turn yields

$$\begin{aligned}\pi'_{11} &= \frac{1}{6}(1 + \pi') = \frac{1}{5} > \frac{1}{20} = f_{11} \\ \pi'_{12} &= \frac{1}{12}(1 + \pi') = \frac{1}{10} < \frac{61}{480} = f_{12} \\ \pi'_{21} &= \frac{1}{6}(1 + \pi') = \frac{1}{5} > \frac{1}{20} = f_{21} \\ \pi'_{22} &= \frac{1}{12}(1 + \pi') = \frac{1}{10} > \frac{1}{20} = f_{22}\end{aligned}$$

Hence the equilibrium is only 75% accurate.

There are no updates to the cutoffs of sets that are empty: $k_{11}^{(2)} = k_{13}^{(2)} = k_{21}^{(2)} = k_{23}^{(2)} = k_{24}^{(2)} = 100$ because $\hat{\Omega}_{11}^n = \hat{\Omega}_{13}^n = \hat{\Omega}_{21}^n = \hat{\Omega}_{23}^n = \hat{\Omega}_{24}^n = \{\}$. Further, $\hat{\Omega}_{12}^n = \{1\}$, $\hat{\Omega}_{14}^n = \{\}$, $\hat{\Omega}_{22}^n = \{1, 2\}$ yields $k_{12}^n = 100$, $k_{14}^n = 0$, $k_{22}^n = 100$. Let the Newton step be $\lambda = 0.2$. Then the guess for the next iteration is $k_{12}^{(2)} = \lambda k_{12}^n + (1 - \lambda)k_{12}^{(1)} = 100$, $k_{14}^{(2)} = \lambda k_{14}^n + (1 - \lambda)k_{14}^{(1)} = 80$, $k_{22}^{(2)} = 100$.

Because $k_{12}^{(2)} = 80$, we have

$$E^{(2)} = \begin{bmatrix} e_{11}'^{(2)} & e_{12}'^{(2)} \\ e_{21}'^{(2)} & e_{22}'^{(2)} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Which yields $P_1'^{(2)} = 1$, $P_2'^{(2)} = \frac{2}{3}$ and $\pi' = \frac{7}{30}$. This in turn yields

$$\begin{aligned} \pi'_{11} &= \frac{1}{4}(1 + \pi') = \frac{37}{120} > \frac{1}{20} = f_{11} \\ \pi'_{12} &= \frac{1}{8}(1 + \pi') = \frac{74}{480} > \frac{61}{480} = f_{12} \\ \pi'_{21} &= \frac{1}{6}(1 + \pi') = \frac{37}{180} > \frac{1}{20} = f_{21} \\ \pi'_{22} &= \frac{1}{12}(1 + \pi') = \frac{37}{360} > \frac{1}{20} = f_{22} \end{aligned}$$

As before, the equilibrium is only 75% accurate.

Likewise, there are no updates to the cutoffs of sets that are empty: $k_{11}^{(3)} = k_{13}^{(3)} = k_{21}^{(3)} = k_{23}^{(3)} = k_{24}^{(3)} = 100$ because $\hat{\Omega}_{11}^n = \hat{\Omega}_{13}^n = \hat{\Omega}_{21}^n = \hat{\Omega}_{23}^n = \hat{\Omega}_{24}^n = \{\}$. Further, $\hat{\Omega}_{12}^n = \{1\}$, $\hat{\Omega}_{14}^n = \{2\}$, $\hat{\Omega}_{22}^n = \{1, 2\}$ yields $k_{12}^n = 100$, $k_{14}^n = 100$, $k_{22}^n = 100$. Then the guess for the next iteration is $k_{12}^{(3)} = \lambda k_{12}^n + (1 - \lambda)k_{12}^{(2)} = 100$, $k_{14}^{(3)} = \lambda k_{14}^n + (1 - \lambda)k_{14}^{(2)} = 84$, $k_{22}^{(3)} = 100$.

Because $k_{12}^{(3)} = 84$, we have

$$E^{(3)} = \begin{bmatrix} e_{11}^{(3)} & e_{12}^{(3)} \\ e_{21}^{(3)} & e_{22}^{(3)} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$$

Resulting in the same price indices and dividend per share as the second iteration:

$$P_1^{(3)} = 1, P_2^{(3)} = \frac{2}{3} \text{ and } \pi' = \frac{7}{30}. \text{ Then as above we obtain } k_{12}^{(4)} = 100, k_{14}^{(4)} = \lambda k_{14}^n + (1 - \lambda)k_{14}^{(3)} = 87.2, k_{22}^{(4)} = 100.$$

Iterating in this fashion, we find $k_{14}^{(t)} = \lambda \cdot 100 + (1 - \lambda)k_{14}^{(t-1)}$ which implies

$$\begin{aligned} k_{14}^{(t)} - k_{14}^{(t-1)} &= 0.2(100 - k_{14}^{(t-1)}) \\ \Rightarrow k_{14}^{(t)} &> k_{14}^{(t-1)}, \quad \|k_{14}^{(t+1)} - k_{14}^{(t)}\| < \|k_{14}^{(t)} - k_{14}^{(t-1)}\| \end{aligned}$$

So that $\|k_{14}^{(t)} - k_{14}^{(t-1)}\| \rightarrow 0$ as $t \rightarrow \infty$. Suppose we set the tolerance to be 0.1. Then when does the iteration stop? We have

$$\begin{aligned} k_{14}^{(5)} &= \lambda k_{14}^n + (1 - \lambda)k_{14}^{(4)} = 89.60 \\ k_{14}^{(6)} &= \lambda k_{14}^n + (1 - \lambda)k_{14}^{(5)} = 91.81 \\ k_{14}^{(7)} &= \lambda k_{14}^n + (1 - \lambda)k_{14}^{(6)} = 93.45 \\ &\dots \\ k_{14}^{(19)} &= \lambda k_{14}^n + (1 - \lambda)k_{14}^{(18)} = 99.55 \\ k_{14}^{(20)} &= \lambda k_{14}^n + (1 - \lambda)k_{14}^{(19)} = 99.64 \end{aligned}$$

And because $\|k_{14}^{(20)} - k_{14}^{(19)}\| < 0.1$, the process stops and we settle on an approximate equilibrium that is 75% accurate, which in this case is the highest percentage possible given the number of available country pairs. It is easy to verify that the cutoffs for the other sets remain unchanged in the iterations leading up to algorithmic termination.