Essays on Libor Manipulation

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1.1 Introduction

The London Interbank Offered Rate (Libor) is a set of benchmark interest rates, intended to reflect the average rate at which banks can borrow unsecured funds from other banks, to which trillions of dollars of financial contracts are explicitly tied.[1] It also serves as a component in many models used to value a wide range of assets not explicitly tied to the rate. The British Bankers Association (BBA), the licensor of the rate, has called it "the most important number in the world." The rate is set each day by taking the truncated average of the reported borrowing costs of a panel of large banks. During the upheaval in financial markets that began around August 2007, the Libor began to diverge from some of its historic relationships causing observers to question its proper functioning and some to suggest manipulation by panel banks as the cause of the malfunction. Subsequent research led to investigations by regulators around the world and, by July of 2012, culminated in admissions of manipulation by Barclays, UBS, and

[1]Partially overlapping panels, administered by the Brittish Bankers' Association, determine rates in 10 different currencies and maturities ranging from overnight to twelve months.

the Royal Bank of Scotland.[2] [3]

Much of the public research and discussion of Libor manipulation to date has focused on panel bank incentives, particularly at the height of the crisis, to intentionally report interbank funding costs below actual costs in order to burnish the markets' perception of their riskiness.[4] The primary focus of this paper is another source of manipulation incentives: Panel bank portfolio exposure to the Libor. As revealed in the July 2012 Barclay's admission of manipulation, released as part of a settlement with U.S. and U.K. regulators, individual traders from that bank (and others) had occasionally contacted colleagues responsible for quote submission to request a submission favorable to their trading positions and these requests were often accommodated.

In this paper, we formulate tests of such portfolio driven manipulation based on a simple model of bank quote submissions. In the model, bank profits depend on the actual fix of the rate but they face misreporting costs that are increasing as the reported cost diverges further from the truth. We interpret the dependence of profits on the rate itself as the bank's (or one of a bank's traders) portfolio incentives and the misreporting costs as detection costs. The model predicts, in the presence of this type of misreporting incentive, a particular form of "bunching" in the intraday distribution of Libor quotes. The prediction is due to the form of the rate setting mechanism, which takes the average of the interquartile quotes submitted by the panel banks. If a given bank wants to change the overall Libor (as opposed to simply reporting costs) and it has a good forecast of the location of the pivotal quotes-those quotes above or below which the quote will not participate in the average-its own quotes will tend to bunch around these pivotal quotes. Outside these pivotal quotes its marginal impact on the rate, and thus the marginal profit of misreporting, goes to zero while its marginal misreporting cost increases.

[2] An earlier version of this paper, that predates these investigations and contains additional analysis, is available on the authors' webpage. In February 2012 it was announced that UBS had admitted to manipulating the Yen Libor, while Barclays has admitted to manipulating the Dollar Libor.

[3] Most of these investigations are ongoing as of the writing of this draft.

[4] See Mollenkamp and Whitehouse (2008) *Wall Street Journal* report for an early, influential argument along these lines.

In our empirical analysis, we aim to statistically distinguish "too much" bunching of a given bank's quotes around the pivotal quotes, relative to a plausible joint distribution of true borrowing costs. Without much a priori information on the joint distribution of actual borrowing costs this is challenging as different such distributions can display an arbitrarily high degree of bunching of individual quotes around a given rank quote. To address this our testing strategy compares the amount of bunching around pivotal quotes in the actual cross sectional distribution of quotes with that of a plausible benchmark distribution estimated by fitting a vector autoregression model to the vector of quotes. The primary assumption embodied by this specification is that in the long run, bank borrowing costs should be correlated through similarities in the banks themselves. It is natural to think, for example, that U.S. based banks should have positively correlated costs or that all banks with large retail operations should have correlated interbank borrowing costs. The crucial contrast here is that the benchmark distribution rules out long run relationships between a bank's borrowing costs and the borrowing cost of a day's fourth, or any other, rank bank. This exclusion is our source of identification.

The specific predictions of our model allow us to argue we distinguish portfolio driven incentives from other sources of manipulation incentives and from generic market frictions, unrelated to manipulation, that may cause divergence between Libor rates and other, comparable rates. In the reputational theory of misreporting, for example, each bank should only care about the markets' perception of its own individual quote not on the overall fix of the rate. Though these market perceptions themselves may depend on an individual bank's position relative to other banks, there is no reason to think the market should condition this perception on a bank's position relative to the pivotal quotes specifically. The welfare and legal ramifications of Libor manipulation may depend crucially on distinguishing these sources of misreporting incentives. While manipulation driven by reputation concerns allow for a maintaining-stability-in-the-public-interest type justification, portfolio related incentives allow for no such rationalization; to the extent that manipulation helped a bank's bottom line they must have hurt another party's. Distinguishing these sources is also important in determining an appropriate policy fix of the problem. For example, one suggested fix has been making individual submissions anonymous. This makes sense in the presence of reputation related incentives, however, in the presence of trading related incentives such a change could exacerbate the problem by decreasing detection costs.

Despite the “smoking gun” evidence of portfolio-driven manipulation turned up in regulatory investigations, our results are not of just academic interest. The general picture of manipulation, painted by colorful emails discovered and testimony given, is one of the infrequent and idiosyncratic behavior of a few traders at a few banks. Our results, by their very strength, suggest otherwise. The nature of our tests are such that their power will depend on the prevalence of portfolio driven manipulation. We find strong statistical evidence of the bunching pattern predicted by our model even while taking pains to attribute the observed variation in quotes to plausible variation in costs. Moreover, we find evidence of manipulation in the more recent past, even as the turmoil of financial crisis had receded somewhat. Our tests of manipulation are also able to pick up smaller deviations than those based on no-arbitrage arguments which, by their nature, are too coarse to detect deviations as small as one basis point or less.

This paper is related to a long literature that attempts to detect hidden corruption and conspiracies by using forensic methods based on economic models of cheating. The contexts for these studies is diverse, ranging from sport (Wolfers (2006)), to standardized testing in schools (Jacob and Levitt (2003)) to international development (Olken and Barron (2009)) and politics (Ferraz and Finan (2008)). Zitzewitz (2012) surveys the broad literature on these forensic methods. Harrington (2005), Porter (2005) and Abrantes-Metz and Bajari (2010) survey a long literature specifically on detecting price fixing cartels.

Though Snider and Youle (2010) was the first academic paper to explore the implications of and evidence for portfolio-driven manipulation, there has also been some other academic work relating to detecting manipulation of the Libor specifically. The study of Abrantes-Metz et al. (2012) is the first such work of this kind to our knowledge and preceded the original version of this paper. The authors apply a screen for collusion developed by Abrantes-Metz et al. (2006), finding suspicious patterns. Abrantes-Metz et al. (2011) apply a test based on Benford’s Law, a statistical regularity in the distribution of digits in data sets, to Libor submissions and again find highly irregular patterns.

The rest of the paper proceeds as follows: Section 2 discusses the history of the Libor, its recent strange behavior, and the most recent findings of regulatory investigations. Section 3 lays out a simple model of portfolio driven manipulation and performs some numerical experiments that motivate our tests. Section 4 examines the empirical evidence, develops tests for our theory, and describes our results. Section 5 concludes.

1.2 Libor

Libor is intended to represent the rate at which banks in London offer unsecured Eurodollar deposits. Eurodollars are simply dollar deposits held outside the U.S. and thus outside the U.S. regulatory and Federal Reserve system. The rates and basic rate setting process emerged in the 1980's in response to the rise of derivatives market and the subsequent demand for standardized, uniform Eurodollar rates to write into these contracts (Stigum and Creszensi (2007) ch. 7). The usage and importance of the rate grew with derivatives market and, by 2007, over $300 trillion worth of contracts explicitly referenced it. They have also become ubiquitous benchmark rates used for the valuation of a wide range of assets that are not explicitly tied to Libor.

In their *Wall Street Journal* article, Mollenkamp and Whitehouse (2008) brought public attention to the strange behavior of the rates during the financial crisis. Among other evidence, they showed panel bank rate submissions were out of line with what one would expect from credit default swap (CDS) spreads, essentially the premia on insuring against individual firm default risk, of those banks. If bank dollar borrowing costs were entirely driven by default risk, these premia should be tightly correlated with rate submissions. Indeed, in a frictionless world no arbitrage conditions suggest a bank's borrowing cost should be very close to the risk free rate plus that bank's CDS spread. In Snider and Youle (2010), we document additionally, at the bank level, within bank *changes* in CDS spreads have had little explanatory power in determining rate submissions or a bank's rank in the panel.

Further strange behavior can be seen in Libor's divergence from similar rates. The

Eurodollar bid rate is an aggregation of actual bids by market makers in the Eurodollar market. Prior to August 2007, the Eurodollar bid rate and Libor behaved as we might expect a bid-ask spread to behave; Libor submissions are a bank's perceived ask rate they would face in the Eurodollar market. Figure 1 shows the spread between Libor and Eurodollar bid rate from January 2005 to July 2012. Banks submitted quotes over the pre-August 2007 period ranged between 6 and 12 basis points above the Eurodollar bid rate. Around August 2007, bank quotes and the resulting Libor fixing fell below the Eurodollar bid rate. As shown in the figure, Libor rates remained well below Eurodollar bid rate, 10-40 basis points, until late summer of 2011 when the spread climbed sharply and again became positive in early 2012. Incidentally, this sharp rise in the spread toward the end of the sample period was preceded by an announcement that UBS was cooperating with antitrust enforcers, making the graph suggestive of cartel breakdown episodes.

Kuo et al. (2012) compare Libor submissions with bank bids in the Federal Reserve Term Auction Facility (TAF) and inferred term borrowing costs derived from FedWire, the reporting system for actual interbank transactions within the Federal Reserve system (See Kuo et al. (2013) for a description). They find Libor submissions were 10-30 basis points lower than the comparison rates in the immediate aftermath of the Bear Stearns and Lehman Failures. Over other periods, however, they find that Libor rates are statistically indistinguishable from the comparison rates.

In recent testimony to the European Parliament Economic and Monetary Affairs Committee on Libor reform, CFTC Chairman Gary Gensler provides a thorough discussion and graphical review of the suspicious patterns in Libor based on the logic of no-arbitrage and similar arguments (Gensler (2012)). Notable in the presentation of these results is that the anomalous behavior of Libor rates appear to persist to the present. We omit a full rehash of all this evidence and refer the interested reader to this testimony and the wealth of other sources now available.

The divergence of Libor rates from comparable rates and the apparent violations of no arbitrage conditions suggest some form of malfunction in the determination of these rates. However, many areas of financial markets have seen logical and historic relationships upset since the onset of the financial crisis so simple malfunction does not imply the divergence is due to manipulation. Term, unsecured interbank lending markets experienced dramatic illiquidity problems beginning with the onset of the financial crisis and persisting to the present (Kuo et al. (2013), Afonso et al. (2011), Wheatley (2012)). Liquidity and related issues in comparison markets, e.g. CDS markets, cast further doubt on the reliability of tests based on pre-crisis history or models of frictionless markets. Moreover, even in the best of times, statistical tests of violations of these logical and historic relationships are relatively coarse and unable to distinguish small deviations that we expect the portfolio driven manipulation to create.

1.2.1 Investigations and Admissions

By July of 2012, regulators around the world, spurred by the evidence discussed above, had opened investigations into the Libor submission process of most Dollar Libor panel banks as well as banks in various other currency panels. Most of these investigations are ongoing but in July 2012 the CFTC, Department of Justice, and UK Financial Services Administration had announced they had settled with Barclays over Libor manipulation. The bank agreed to pay a fine totalling over $400 million and also agreed to a public release of findings from the investigation. The findings reveal that both reputation driven and portfolio driven incentives caused upper level bank management, in the former case, and individual traders, in the later case to request particular quotes or a particular direction of quotes from the bank's Libor submitters dating back to at least 2005.

Not surprisingly, the reputation incentive appears to have been at work primarily during the hectic depths of the financial crisis. As the subprime crisis started to heat up in the middle of 2007, Barclays relatively high Libor submissions, in conjunction with the bank's access of the Bank of England Emergency Lending Facility and reports of high exposure to subprime SIVs, began receiving negative press and market reaction. On September 3, 2007, Barclays quotes were 6-9bps above the next highest submission in three Dollar tenors and near the top of the range in most others. A Bloomberg column, published that day, entitled "Barclays Takes a Money Market Beating", discussing the high quotes, ends with the ominous "There's knowledge buried in the price that Barclays is being charged in the money markets. We just don't know what that knowledge is yet" (Gilbert (2007)).

In response to the negative press, senior Barclays management directed submitters to start "keep[ing] their heads below the pararpet", to avoid a negative reaction from the markets (Commission (2012) p.19). For example,

> "On November 29, 2007 the supervisor of the U.S. Dollar Libor submitters convened a telephone discussion with the senior Barclays Treasury managers and the U.S. Dollar Libor submitters. The supervisor said if the submitters submitted the rate for a particular tenor at 5.50, which was the rate they believed to by the appropriate submission, Barclays would be 20 basis points above 'the pack' and 'it's going to cause a shit storm.' The supervisor asked the issue be taken 'upstairs' meaning that it should be discussed among the more senior levels of Barclays management. The most senior Barclays treasury manager agreed that he would do so. For the Libor submission, the group decided to compromise by determining to set at the same level as another bank, a rate of 5.3, which was, again, not the rate the submitters believed to be appropriate for Barclays." (Ibid. p.21)

Barclays management and treasury staff believed they were following the lead of other banks and the market reaction, singling them out, associated with not doing so would be unjustified and that this was leading the overall Libor to remain much lower than actual average costs. In the same November 29, 2007 discussion,

> "the group also discussed their belief that other banks were submitting unrealistically low rates and speculated that other banks were basing submissions on derivatives positions...One of the senior Barclays Treasury managers called a BBA representative and stated that he believed that Libor panel banks, including Barclays, were submitting rates that were too low because they were afraid to 'stick their heads above the parapet' and

> that ‘no one will get out of the pack, the pack sort of stays low.’ ” (Ibid p.21)

As the previous quote also indicates, those in the know suspected manipulation due to trading incentives during the financial crisis. The CFTC order reveals such behavior predated the crisis, going back at least to early 2005, and continued until at least into 2009.[5] Unlike the misreporting for reputation reasons, misreporting for trading reasons seems to have been initiated by individual traders and there is no evidence it was approved by upper level management. Requests from traders usually have come via, often casual and jocular, emails and instant messages mostly asking for changes, both high and low, in the one and three month dollar Libor. For example, a February 1, 2006 message from a Barclays trader in New York to a trader in London read

> “You need to take a look at the reset ladder. We need 3M to stay low for the next 3 sets and then I think we will be completely out of our 3M position. Then its on. [Submitter] has to go crazy with raising 3M Libor.” (Ibid. p.9)

Several communications between the traders and submitters reveal an awareness of the particulars of the rate setting process. Specifically, traders sometimes requested that submitters report rates that would get the submission “kicked out” or “knocked out” of the panel, i.e. a quote outside the interquartile range. For example, a November 22, 2005 message from a senior trader in New York to a Trader in London,

> “WE HAVE TO GET KICKED OUT OF THE FIXINGS TOMORROW!! We need a 4.17 fix in 1m (low fix) We need a 4.41 fix in 3m.” (Ibid p.9)

Several communications also reveal awareness of detection costs in the form of regulator discovery and punishment. For example, a March 13, 2006 email exchange between

[5]Some accounts have Libor manipulation going as far back as the early 1990s. See Douglas Keenan’s July 26, 2012 *Financial Times* op-ed “My Thwarted Attempt to Tell of Libor Shenanigans.”

a Barclay's trader in New York and a Libor submitter,

> Trader: "The big day [has] arrived...My NYK are screaming at me about an unchanged 3m libor. As always any help wd be greatly appreciated. What do you think you'll go for 3m?"
>
> Submitter: "I am going 90 although 91 is what I should be posting."
>
> Trader: "[...] when I retire and write a book about this business your name will be written in golden letters[...]."
>
> Submitter: "I would prefer this [to] not be in any book!" (UK Financial Services Authority Final Notice p.12)

The language and frequency of requests suggests that traders believed their requests would be routinely accommodated by rate submitters. The UK FSA analyzed around 100 email and instant message requests uncovered by their investigation and found that rate submissions were consistent with the requests about 70% of the time. The Barclays communications also implicated at least four other banks, as yet unnamed, for cooperating with the requests of Barclay's traders. This, along with the fact that the Barclay's investigation found evidence of many attempts to influence submissions to the Euribor panel, a similar Euro rate with around 40 panelists so no substantial movement could not be accomplished by a single bank, suggests that many banks must have participated in manipulation.

1.3 A Simple Model of Quote Submission

We model the quote submission process as a game played between the Libor panel banks. There are 16 banks indexed by $i = 1, 2, ..., 16$. Each day the banks choose their quotes q_i. Bank i's actual borrowing cost is given by c_i drawn from some joint distribution $H(c_1, c_2, ..., c_{16})$ some part of which may be private information to the bank. We denote the vector of 16 quotes and costs as q and c respectively. The Libor fix is a function of submitted quotes and is given by:

$$L(q) = \frac{1}{8} \sum_{j=1}^{16} \mathbf{1} \left\{ q_j > s^4, q_j \leq s^{12} \right\} q_j$$

Where s^4 is the day's fourth highest, or left, pivotal quote and s^{12} is the days twelfth highest or right pivotal quote.

Banks may have incentive to manipulate the fix because their final payoffs depend on the realization of it. A bank will, however, not want to submit a quote too far from its actual cost because doing so risks detection and punishment. Specifically we model a bank's expected payoff by:

$$\pi_i = E_{q_{-i}}[v_i L(q) - \frac{\delta}{2}(q_i - c_i)^2]$$

Given its information a bank chooses its quote to maximize this expected payoff. The first order condition determining the bank's best response is given by:

$$\frac{v_i}{8\delta} \int \mathbf{1} \left\{ q_i > s^4(q), q_i \leq s^{12}(q) \right\} F_i(dq_{-i}) - (q_i - c\, i) = 0$$

Where F_i is bank i's beliefs about the distribution of other quotes conditional on its information. Letting $G_i(q_i)$ denote bank i's equilibrium beliefs about the probability its quote participates, in the truncated average, the equilibrium relationship between costs and quotes is:

$$q_i = c_i + \frac{v_i}{8\delta} G_i(q_i)$$

When the location of the pivotal quotes are known with certainty, as in the complete information version of the game, the G function is a step function that is one for quotes between the pivotal quotes and zero outside. Figure 1.2 shows a schematic representation of how these manipulation incentives affect the intraday distribution of quotes vis a vis the intraday distribution of costs. In the figure four banks, e, f, h, and j, have incentive to push the rate down. All four banks equate the marginal benefit of skewing their quote, $\frac{v}{8}G(q_i)$, with the marginal cost, $\delta(q_i - c_i)$, where v is negative (i.e. the incentive is to push the rate down). Banks e and f's marginal cost function intersects the marginal benefit function at it's discontinuity, which occurs at the fourth highest quote, d. The quotes of e and f are thus identical to the quote of d and there is bunching at the fourth.

1.3.1 Numerical Experiments

Figure 1.3 shows some results of a numerical experiment and foreshadows our testing approach. In the pictured experiment, we assume that there are 12 banks, named bank 1-bank12, that occasionally attempt to manipulate the rate. Banks 13-16 never attempt to manipulate the rate. Underlying bank costs are drawn from a normal distribution with mean 1 and covariance matrix set to match the empirical covariance matrix of Libor quotes less the daily mean quote over the period January 2005 to July 2012. The strength of manipulation in each period, $\frac{v_{it}}{8\delta}$, are i.i.d draws from a mixture distribution with 4/5 probability of no manipulation, i.e. $\frac{v_{it}}{8\delta} = 0$ for all banks, and 1/5 probability that each of the 12 manipulating banks have incentives drawn uniform on $[-1/24, 0]$. For each of 10000 runs of the model, we calculate equilibrium quotes for the static, complete information game.[6]

The top panel of figure 1.3 shows the distribution of quotes of manipulator bank 1 less the day's fourth lowest among the 15 other banks (blue bars). Also shown is the distribution of bank 1's actual costs minus the fourth lowest actual cost among the 15 other banks (white bars) and the distribution of simulated quotes less the simulated fourth lowest actual cost of the 15 other banks (red bars), where the quotes are simulated from a fitted multivariate normal distribution. The bottom panel of figure 1.3 shows the

[6]There are, in general, multiple equilibria for a given vector of costs. For these experiments we focus on the *maximally distored* equilibrium, the equilbrium with the largest average difference between costs and quotes. In an earlier version we showed that all complete information equilibria display the same type of bunching we focus on here.

same distributions for the non-manipulating bank 16. The pooled empirical distribution of bank 1's normalized quotes displays a large discontinuity at 0 relative to the pooled distribution of bank 16's normalized quotes and the simulated distribution.

In our empirical analysis our testing procedure is guided by these experiments with the model. Namely, we test whether the pooled distribution of actual, normalized quotes has more mass around 0 than that of a reference distribution. We also test for the presence of a discontinuity at 0 in the distribution of normalized quotes relative to a reference distribution.

A priori, it is likely our tests will be prone to power and size issues. Clearly power will be affected by not only sample size but also by the strength of manipulation incentives since these will affect how frequently the optimal misreported quote will be identical to one of the pivots. Type I errors are also an issue because whenever a non-manipulating bank receives a cost draw that puts it in a pivotal position, manipulating banks will push their own quotes toward the non-manipulator causing the non-manipulator's normalized quote to, itself, be close to 0. Intuitively, even if we were willing to assume the cost distribution was perfectly smooth and exact ties a zero probability occurrence, in observing two banks tied at the fourth lowest we would not be able to say which bank was manipulating or if both were. We explore these issues by performing a series of monte carlo experiments, mimicking our empirical tests, on simulated data generated by our simple model.

Table 1.1 shows the results of these experiments. Each entry in the table reports a summary statistic for the distribution of one sided t-test p-values obtained from simulating the model 1000 times for each associated parameterization. The hypothes tested is that the number of actual quotes, normalized by subtracting the day's fourth lowest quote, falling in the bin 1 basis point below the fourth lowest ($[-.01, 0)$) is less then the simulated number of normalized quotes falling into this bin.[7] The rows in the table

[7] We have run similar tests for an "Above" hypothesis that the number of normalized actual quotes falling in the bin 1bp above the fourth lowest is greater than the simulated number, and also a "Diff" hypothesis that the difference in the number of actual normalized quotes falling above and below is greater than the difference in the number of simulated normalized quotes.

report the average p-value, fraction of tests rejected at the 5% level, and the fraction of tests rejected at the 1% level and these are shown for a manipulator (Bank 1) and non-manipulator (Bank 16) for each of the parameterizations.

For each, simulation we maintain the assumption that there are 12 potential manipulators and bank costs are drawn from a joint normal distribution with mean 1 and covariance matrix equal to the empirical covariance matrix of 3 month Libor quotes less the daily mean quote.[8] Across experiments we vary the sample size and two parameters controlling the frequency of manipulation and the strength of manipulation incentives. The "Fraction of manipulating days" parameter determines the fraction of days on which there is potentially any manipulation so a parameter value of .33 means that on 2/3 of days no banks have any manipulation incentives ($\frac{v_i}{8\delta} = 0, \forall i$). On days in which manipulation is possible the strength of manipulation incentives are determined by the "Distribution of Incentives" parameter. On these days each of the 12 banks receives an incentive, $\frac{v_i}{8\delta}$, drawn iid across banks and days, from a mixture distribution with a 50% probability of getting a 0 draw and 50% probability of getting a draw from the uniform $[-x, 0]$ distribution, where x is either 1/8, 1/24, or 1/40.

A first observation about the table is that each of the tests, evidently, allow us to distinguish the manipulating bank from the non-manipulator. On average, the distribution of manipulator quotes will have less mass, relative to the comparison distribution, just below the pivotal quote. A manipulator will also have more mass just above and a greater difference in the mass just above and just below. The "Above" and "Diff" tests appear to do a much better job both of identifying the manipulator and distinguishing the manipulator from the non-manipulator than does the "Below" test. However, unlike the former two the apparent ability of the "Below" test to contrast the two types

They give similar results

[8] Assuming that fewer banks are potential manipulators makes it easier to distiguish the manipulator from the non-manipulator since there are more cost events that lead to a non-manipulator being bunched at the lower pivot. For example, with only one manipulator a non-manipulating bank will only be bunched at the lower pivot in the event that the non-manipulator receives the fourth highest cost draw and the cost and incentives draw of the manipulator causes it to misreport at the same level as the non-manipulator.

improves, in the sense that the probability of incorrectly rejecting the null decreases for the manipulator while the probability of correctly rejecting the null for the manipulator improves for the "Below" test, whereas the other two tests increase the probability of correctly rejecting for the manipulator but also increase the probability of incorrectly rejecting for the non-manipulator.

Since our cost parameterization comes directly from the data, the table is also informative about the relationship between test statistics and the underlying frequency and intensity of manipulation. When manipulation is less frequent and/or incentives are weaker, the tests in summarized in the table have low power, only rejecting the null of no manipulation at the 5% level in 50-60% of those samples with 150 or 250 observations for the bottom rows of the table where the frequency and strength of manipulation is lowest. The simulated libor in this scenario is, on average .22 bp lower than what would prevail with honest reporting. By contrast, in the parameterization in the top rows of the table, where the null is correctly rejected for a manipulating bank at the 5% level 97-100% of the time, the average realized libor is .83 bp lower than what would prevail with honest reporting. These magnitudes suggest two things. First, our tests are able to detect deviations that are relatively small, when compared to day to day changes in quotes for instance, on average. Second, they suggest a ballpark lower bound on the frequency and intensity of manipulation incentives that we might infer from the strong rejection evidence we find our empirical analysis.

1.4 Data and Empirical Evidence

Our empirical analysis utilizes only data on bank rate submissions. The quotes of each panel bank on every business day from January 1, 2005 to July 1, 2012 were collected from a Bloomberg terminal. We focus on 3 month Dollar Libor submissions over the period ending February 1, 2011 when the panel increased to 21 members. From January 1, 2005 to February 1, 2011 the panel consisted the same 16 members with the exception of one change occurring in February 2009 when Societe General replaced HBOS following the absorption of HBOS by Lloyd's. Motivated by a visual examination of the Eurodollar bid rate-Libor spread shown in Figure 1.1 we split the sample up into 6 periods and perform our analysis on the full sample as well as on each period individually.

Table 1.2 shows some summary statistics for this sample over the various periods. On average, quotes are tightly clustered with an interquartile range of deviations from the median quote ranging from one basis point below to two basis points below. Similarly the interquartile range, the difference between the upper and lower pivotal quotes is quite narrow. Overall, the average size of the range is 7.9 bps, though there is a good deal of variation across our periods, with the range varying from 3.4 bps in the first year and a half of the sample to 48bps in the period containing the Lehman failure. Also notable is that, while the ranking of banks in the panel tends to be persistent, there is still considerable variation in relative ranks over time with the daily standard deviation of a bank's rank from its average rank at 4.39. Moreover, nearly all banks occupy almost all ranks over a sufficiently long horizon. Significant variation in these relative quotes will be important for our testing strategy below.

As noted by Gensler (2012), one of the puzzling features of bank quote behavior is the lack of day to day movement in the submissions. Across all periods and all banks, over 40% of observations show no change from the previous day's quote in spite of significant day to day changes in related rates. An interesting regime change seems to appear in the last 15 months of the summarized sample the number of such zeros jumps to 62% of observations. The lack of comovement of quotes with underlying "cost drivers" (as well as the lack of much movement at all) is the logic behind the collusion tests examined in Abrates Metz et. al (2012).

1.4.1 Empirical Approach

Our model predicts that when banks have direct incentive to manipulate rates, as opposed to misreporting for other reasons, e.g. reputation, their quotes will bunch around the pivotal quotes. Without placing restrictions on the joint distribution of bank quotes over time, obviously any distribution of quotes can be rationalized as truthful by some joint distribution of underlying costs. However, since different distributions will naturally display different degrees of bunching around the twelfth and fourth order statistics, any test will be sensitive to these restrictions. In trying to balance these trade-offs, we start by assuming latent underlying borrowing costs follow a vector autoregressive process.

$$c_t = \beta_0 + \sum \Gamma_\tau c_{t-\tau} + \varepsilon_t$$

Where $\varepsilon_t \sim N(0, \Sigma)$.The VAR specification is a reasonably flexible way to describe time series relationships, however, there are two main restrictions embodied by this assumption. First, we assume that, in the long run, bank borrowing costs are correlated through similarities in the banks themselves. It is natural to think, for example, that U.S. based banks should have positively correlated costs or that all banks with large retail operations should be correlated. The crucial contrast here is that we rule out long run relationships between a bank's borrowing costs and the borrowing cost of the fourth, or any other, rank bank. Second, is the assumptions that innovations are joint normally distributed. While the parametric restriction is necessary given the high dimension of the vector process, it is also undesirable. In the implementation of our tests, this is not directly an issue since, as discussed below, we work with fitted quotes.

Under a null of truthful reporting we can estimate this process using observed quotes. Due to cointegration and the high dimension of the vector process our preferred specification is a two lag, rank two, vector error correction model.

$$\Delta q_t = \Pi q_t + \sum \Lambda_\tau \Delta q_{t-\tau} + \varepsilon_t$$

With these estimates at hand we can examine how differences in the fitted or simulated versus actual distribution of quotes support our theory of manipulation (as opposed to simple misspecification of the cost process). Essentially, our testing strategy is to look for statistically and economically significant differences in the distribution of prediction errors conditional on the position of pivotal quotes. Economically significant, here, means consistent with our model, which predicts a particular form of bunching and not others that might predict clustering of quotes together.[9] Identification comes from transitory changes in the relative bank ranks driven by actual idiosyncratic cost shocks or changes in misreporting incentives. For example, suppose JP Morgan and Citigroup are on average the fourth and fifth ranked banks and their quotes are highly correlated. If neither bank faces idiosyncratic shocks that drive them up or down in relative rank then, in the intraday distribution of quotes, both banks will be bunched at the fourth highest. The fitted model would reflect this and the fitted and simulated quotes of the two banks will also be bunched. If, on the other hand, occasional shocks shuffle JP Morgan out of the fourth rank and Citigroup's submissions continue to bunch with the new occupant of the fourth spot, this will lead to bunching in the actual distribution but not the fitted and simulated distributions.

It is important to note that, if manipulation is present, our model will be contaminated even if our cost specification is correct. Thus, if the null of truthful reporting is false our comparison distribution should be expected to, itself, bunch more around the pivotal quotes than the actual cost distribution as in figure 1.3. How much more will depend on the degree of contamination; how many and how often banks are manipulating. Even if banks are constantly manipulating, however, the contaminated model will not display the predicted discontinuity in the distribution at the pivotal quotes. For this reason, we focus most of our attention on this discontinuity.

1.4.2 Results

Figure 1.4 (Figure 1.5) shows the pooled distribution of quotes of all banks normalized by subtracting the fourth (12th) highest quote of the 15 *other* bank quotes over various time periods. The bottom half of each panel show the fitted versions of the same

[9]Banks may cluster together if they all have incentives to simply not stick their "heads above the parapet" as ordered by one Barclays executive in reference to the bank's rate submission.

normalized quotes.[10] We use fitted quotes for our comparison distribution rather than simulating innovations and adding them to the fitted quotes because we worry about non-normality of the quotes. In particular, the large number of no-change observations suggests that the fitted quotes may be a better choice. We have performed the same analysis using simulated quotes as well and it only strengthens the results.

A couple of features of these figures stand out. First, to a striking degree the distributions resemble the shape predicted by our model for both the upper and lower pivot normalizations. Second, the distribution of normalized fitted quotes also displays a good deal of bunching around the pivotal quotes, demonstrating the importance of developing our benchmark comparison distribution. To statistically verify this graphical story we implement some simple statistical tests, motivated by the numerical experiments with the model. Namely, we test for a discontinuity in the quote distribution at the pivotal quote. A natural approach for such a discontinuity test is suggested by McCrary (2008). Unfortunately rounding of quotes combined with the small scale make the required smoothing impossible so instead we simply compare the histogram bin size of a small interval, $[0, b)$ ($(0, b]$), above the quote minus the fourth (twelfth) highest to the bin size of a small interval, $[-b, 0)$ ($(-b, 0]$), below the quote minus the fourth (twelfth) highest. Since 65% of quotes are rounded to the nearest basis point, an additional 25% are rounded to the half basis point, and most of the rest are rounded to the quarter basis point, our preferred window size is one basis point ($b = .01$) but we report many of our results for the half ($b = .005$) and two ($b = .02$) basis point levels as well.

Tables 1.3 show the results of our "Lower" bunching tests for all banks pooled together at various window widths.[11] The table confirms the graphical evidence. For almost all periods and window widths each of our three bunching tests are significant at the 1% level for the lower pivot normalization. The only exception is in the final period from October 2009 to January 2011 with 1bp window width. Here, the probability of a

[10]That is, for each bank, we subtract the fourth (12th) highest of the 15 other *fitted* quotes from its own fitted quote.

[11]Bank level histograms are available on the authors' website. "The Fix is In: Additional Figures"

normalized quote falling in the bin just below zero is almost identical for the fitted and actual distributions. The prevalence of zero-change days, no doubt, contributes to an overall similarity in the fitted (zero innovation vector) and actual quotes.

For the whole basis point windows, notably, the total mass in the windows around zero are similar for the fitted and actual distributions. Examining the data a bit more closely shows why this is the case. A huge fraction of quotes predicted to fall into the bin just below (just above in the case of the upper pivotal quote normalization) zero, fall into the just above (just below) bin. This demonstrates the mechanics of our tests using the possibly contaminated estimates as a benchmark distribution. If the comparison distribution were the actual distribution of costs we would expect to see quotes moving from bins further above (below) the pivots to bins closer to the pivots. Our tests are instead exploiting the change in the shape of the distributions at the pivotal quotes. Table 3c-d delve into the tests with two way tables showing the joint distribution of fitted and actual normalized quotes pooled over all banks and periods. In the analysis of individual banks, almost all bank-periods that fail our test have this same pattern.

1.4.3 The Timing of Bunching and Collusion

Reports from ongoing Libor investigations and media coverage have indicated likely collusion among panel banks in manipulating rates. The basic implications, in terms of the shape of the intraday distribution of quotes, from our model are unchanged in the presence of collusion. When banks collude, however, the scope for manipulation is much greater and thus has serious implications for the magnitude of manipulation and resultant welfare effects. When a bank acts unilaterally, their ability to distort the rate down, relative to the rate that would prevail from honest reporting, is bounded by 1/8 times the difference between its true cost and its submission. At the other extreme, five or more banks acting in concert move the rate as far as they desired, though in a collusive equilibrium of our model they will not choose to do so due to the convex misreporting costs. To explore collusion in light of our model we simply extend our bunching tests to look at the correlation between banks of bunching behavior over time.

We leave a fuller development of tests for collusion to future work.

Figures 1.6 and 1.7 show the results of redoing our main bunching analysis using a rolling window rather than pooling within discrete periods for 3M and 6M tenors for the dollar Libor. Specifically we calculate kernel smoothed frequency of quotes falling into either the 1bp above the pivot or 1bp below the pivot

$$
\begin{aligned}
Y_{i,t}^{a,4} &= \frac{\sum_{\tau=1}^{T} K(\frac{t-\tau}{h}) 1\{0 \leq q_{it} - s^4 < b\}}{\sum_{\tau=1}^{T} K(\frac{t-\tau}{h})} \\
Y_{i,t}^{b,4} &= \frac{\sum_{\tau=1}^{T} K(\frac{t-\tau}{h}) 1\{-b \leq q_{it} - s^4 < 0\}}{\sum_{\tau=1}^{T} K(\frac{t-\tau}{h})}
\end{aligned}
$$

Where we use a simple, triangular kernel with 10 day bandwidth for K. We calculate the corresponding smoothed measure for quotes normalized by the twelfth highest quote and compare these with the fitted versions of the same.

There is no obviously strong pattern of correlation in bunching behavior in these measures between banks, though visually there appears to be a loose correlation in the timing of bunching episodes across all banks. The graphs also support the general observation that bunching declines in the periods immediately following the failure of Lehman Brothers and the depths of the financial crisis, incidentally the time when it is most likely that reputation driven manipulation was occurring. This is especially true of the upward manipulation tests.

The picture painted here generally corroborates the discussion above. While there appears to be some correlation across banks in their bunching behavior, the correlations are as consistent with the existence of common underlying drivers of manipulation, as in the example of future positions, as they are with an explicit conspiracy. Moreover, since different sets of banks appear to be pushing in opposite directions at the same time and these sets don't appear to be stable, the evidence is not suggestive of any specific set of banks participating in a grand cartel. In general, the data appear consistent with uncoordinated episodic manipulation, which may have involved occasional cooperation among multiple banks.

1.5 Conclusion

Over the past 30 years most corners of financial markets have come to rely on Libor as an essential gauge of the health of money markets and the direct and indirect implications thereof. Such heavy dependence has made the recent revelations of widespread manipulation of these rates shocking to the point of crisis. Concerns about manipulation were originally focused on the most tumultuous period of the financial crisis, when, it was suggested, banks may have been understating their borrowing costs in order to avoid negative market (over)reaction. While such a suggestion was disconcerting, market observers could take solace in the fact that the problems with the rate were confined to times when nothing seemed to be working properly and in the fact that misreporting banks may have been doing a public service by helping avoid further panic. Recently, however, investigations by regulators have uncovered evidence of manipulation driven by bank trading positions with exposure to Libor.

In this paper we have developed tests for portfolio driven manipulation based on a model of Libor panel bank survey submissions. The model predicts that the intraday distribution of panel bank quotes will bunch around the fourth (twelfth) highest quote in the presence of incentives to push the rate down (up). Our simple tests are designed to deal with a couple of the most important empirical challenges presented by the submissions data and alternative indicators of manipulation. Since we do not have a strong prior on the form of the joint distribution of interbank borrowing costs, we develop a flexible benchmark distribution with which to compare the actual quote distribution. Our benchmark distribution is constructed by estimating a VAR model of bank quotes, which imposes that long run cost correlations are related to similarities between the banks themselves but unrelated to the rank of any bank per se. We also take steps to ensure our testing procedure is robust to the rounding and infrequent quote changes found in the data.

Going to the data, we find strong evidence of the type of bunching predicted by our model. Concerns about false negatives and false positives associated with our tests notwithstanding, the bunching evidence is especially strong in the early periods of our sample, with almost every bank individually failing our bunching tests at a very high level of significance. Aspects of our findings are consistent with accounts of collusive behavior, however, they are also consistent with common underlying sources of manipulation incentives such as futures reset dates. Also, consistent with publicly available accounts of manipulation, our evidence suggests that coordination between particular banks was, if anything, on an episode by episode basis as opposed to a more centralized, overarching conspiracy.

One limitation of our analysis is that it requires pooling of quotes over time, making pinpointing specific, suspicious observations difficult. However, we are able to perform our tests at the bank level and at more coarse time breakdown enabling the tests to inform a coherent narrative. Another limitation of our study is that we have not done much to quantify the degree of manipulation. The best we can offer is evidence from numerical simulations of our model. Viewed as back of the envelope calculations, these results suggest test rejections at the level we observe, indicate frequent manipulation and strong incentives with an *average* deviation of observed Libor rates from actual rates over .5bp, which amounts to over a trillion dollars of contract mispricing on aggregate.

Our analysis has several implications for the effective reform of Libor, several of which have already been adopted by the Wheatly commission. One of these is the desirability of putting more banks on the panel. As the number of banks increases, the influence of any one bank on the overall rate diminishes and thus so do the incentives for misreporting. Our model also suggest increased regulatory oversight and audited submission rules would also be desirable as these increase misreporting costs. The Wheatly Commission opted not to adopt the change, suggested by some, to make submissions anonymous instead embargoing quote data for 60 days after submission. Our results suggest this is likely a sensible middle ground. Total anonymity might decrease misreporting costs for panel banks. On the other hand, total visibility increases the likelihood of tacit collusion.

Table 1.1: Monte Carlo of Test on Model Simulated Data

Distribution of Incentives	Bank 1 (Manipulator) N=150	Bank 1 (Manipulator) N=250	Bank 1 (Manipulator) N = 500	Bank 16 (Non-Manipulator) N=150	Bank 16 (Non-Manipulator) N=250	Bank 16 (Non-Manipulator) N = 500
Uniform[-1/8,0]						
mean p-value	0.26	0.20	0.13	0.49	0.53	0.51
5% significance	40%	45%	60%	16%	10%	11%
1% significance	27%	28%	46%	8%	5%	5%
Uniform[-1/24,0]						
mean p-value	0.23	0.20	0.12	0.39	0.38	0.31
5% significance	42%	46%	60%	21%	22%	29%
1% significance	26%	29%	41%	11%	12%	15%
Uniform[-1/40,0]						
mean p-value	0.34	0.30	0.21	0.43	0.42	0.37
5% significance	29%	34%	42%	20%	17%	23%
1% significance	17%	20%	27%	11%	8%	11%

Data simulated using various parametizations of the complete information static game presented in the paper. Statistics calculated from 10,000 runs of N sample days of simulated data. In each run 12 banks are manipulator banks which means they occasionally have incentives ($v \geq 0$) to manipulate.

Table 1.2: Three Month Dollar Libor Summary Statistics

	Mean	St. Dev	25th Perc.	75th Perc.
All Periods (1,050 trading days)				
Diff. from Day's Median Quote	0.0077	0.0946	-0.0100	0.0200
Daily Interquartile Range	0.0791	0.1332	0.0200	0.0900
Rank Diff. from Bank's Avg. Rank	0.0000	4.3910	-3.4770	3.6420
Change in Quote from Prev. Day	-0.0015	0.0349	0.0000	0.0050
Fraction No Change from Prev. Day	0.4099	0.4918	0.0000	1.0000
1/2005-1/2007 (350 trading days)				
Diff. from Day's Median Quote	-0.0047	0.0282	-0.0050	0.0050
Daily Interquartile Range	0.0344	0.0372	0.0050	0.0650
Rank Diff. from Bank's Avg. Rank	0.0000	4.3007	-3.5941	3.3175
Change in Quote from Prev. Day	0.0027	0.0093	0.0000	0.0100
Fraction No Change from Prev. Day	0.4365	0.4960	0.0000	1.0000
8/2007-8/2008 (216 trading days)				
Diff. from Day's Median Quote	0.0295	0.1486	-0.0200	0.0300
Daily Interquartile Range	0.1684	0.1933	0.0300	0.2100
Rank Diff. from Bank's Avg. Rank	0.0000	4.4254	-3.8066	4.0219
Change in Quote from Prev. Day	-0.0092	0.0479	-0.0200	0.0050
Fraction No Change from Prev. Day	0.2354	0.4243	0.0000	0.0000
9/2008-1/2009 (84 trading days)				
Diff. from Day's Median Quote	0.0786	0.3646	-0.0500	0.1800
Daily Interquartile Range	0.4804	0.3938	0.1300	0.8500
Rank Diff. from Bank's Avg. Rank	0.0000	4.1394	-3.4299	3.2617
Change in Quote from Prev. Day	-0.0116	0.1230	-0.0500	0.0100
Fraction No Change from Prev. Day	0.2307	0.4214	0.0000	0.0000
2/2009-9/2009 (133 trading days)				
Diff. from Day's Median Quote	0.0182	0.0683	-0.0200	0.0600
Daily Interquartile Range	0.1033	0.0500	0.0800	0.1100
Rank Diff. from Bank's Avg. Rank	0.0000	3.3444	-2.0247	2.3929
Change in Quote from Prev. Day	-0.0052	0.0193	-0.0100	0.0000
Fraction No Change from Prev. Day	0.3710	0.4832	0.0000	1.0000
10/2009-1/2011 (267 trading days)				
Diff. from Day's Median Quote	0.0098	0.0391	-0.0100	0.0250
Daily Interquartile Range	0.0507	0.0246	0.0300	0.0650
Rank Diff. from Bank's Avg. Rank	0.0000	2.6163	-1.5223	1.2997
Change in Quote from Prev. Day	0.0000	0.0061	0.0000	0.0000
Fraction No Change from Prev. Day	0.6233	0.4846	0.0000	1.0000

Table 1.3: All Bank Bunching at the 4th Lowest Quote (3M Dollar)

Window	$q - s_4$	All Periods	1/2005-7/2007	8/2007-8/2008	9/2008-1/2009	2/2009-9/2009	10/2009-1/2011
b = .005	Actual	0.029	0.042	0.018	0.005	0.013	0.036
	Simulat.	0.067	0.106	0.046	0.022	0.040	0.059
	p-value	0.000	0.000	0.000	0.000	0.000	0.000
b = .01	Actual	0.071	0.067	0.055	0.023	0.062	0.107
	Simulat.	0.105	0.141	0.083	0.040	0.078	0.108
	p-value	0.000	0.000	0.000	0.000	0.001	0.365
b = .015	Actual	0.089	0.089	0.078	0.032	0.073	0.126
	Simulat.	0.133	0.159	0.113	0.051	0.107	0.152
	p-value	0.000	0.000	0.000	0.000	0.000	0.000
b = .02	Actual	0.117	0.103	0.111	0.053	0.107	0.164
	Simulat.	0.154	0.171	0.136	0.070	0.129	0.184
	p-value	0.000	0.000	0.000	0.001	0.000	0.000

Figure 1.1: Libor - Eurodollar Bid Rate (1/2005-7/2012)

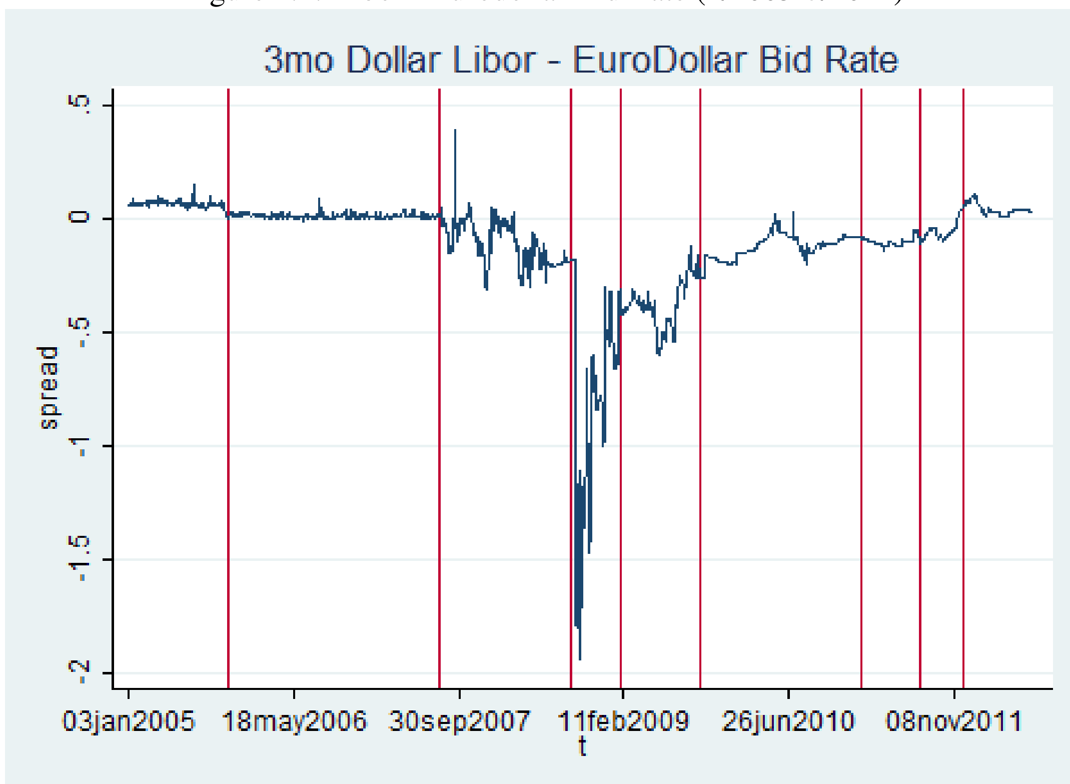


Figure 1.2: Costs vs. Quotes in Model of Manipulation

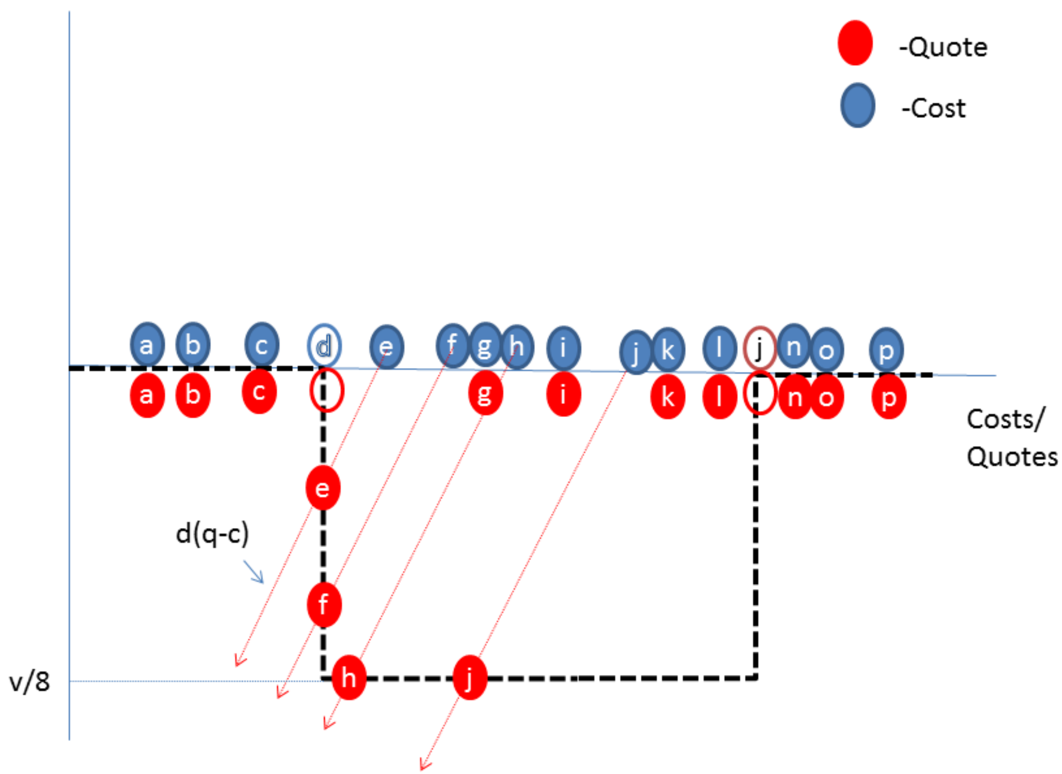


Figure 1.3: Simulated Distribution of Quotes Minus Daily 4th Lowest of 15 Other Banks

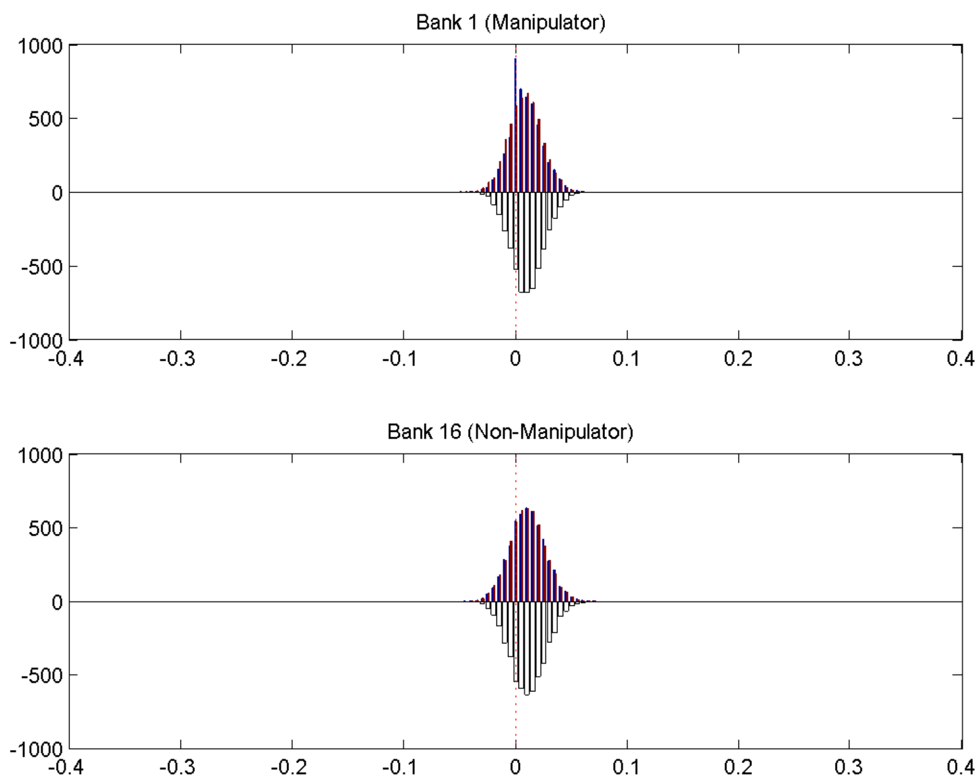


Figure 1.4: All Banks: 3M Bank Quote Minus the 4th Lowest of Fifteen Other Banks

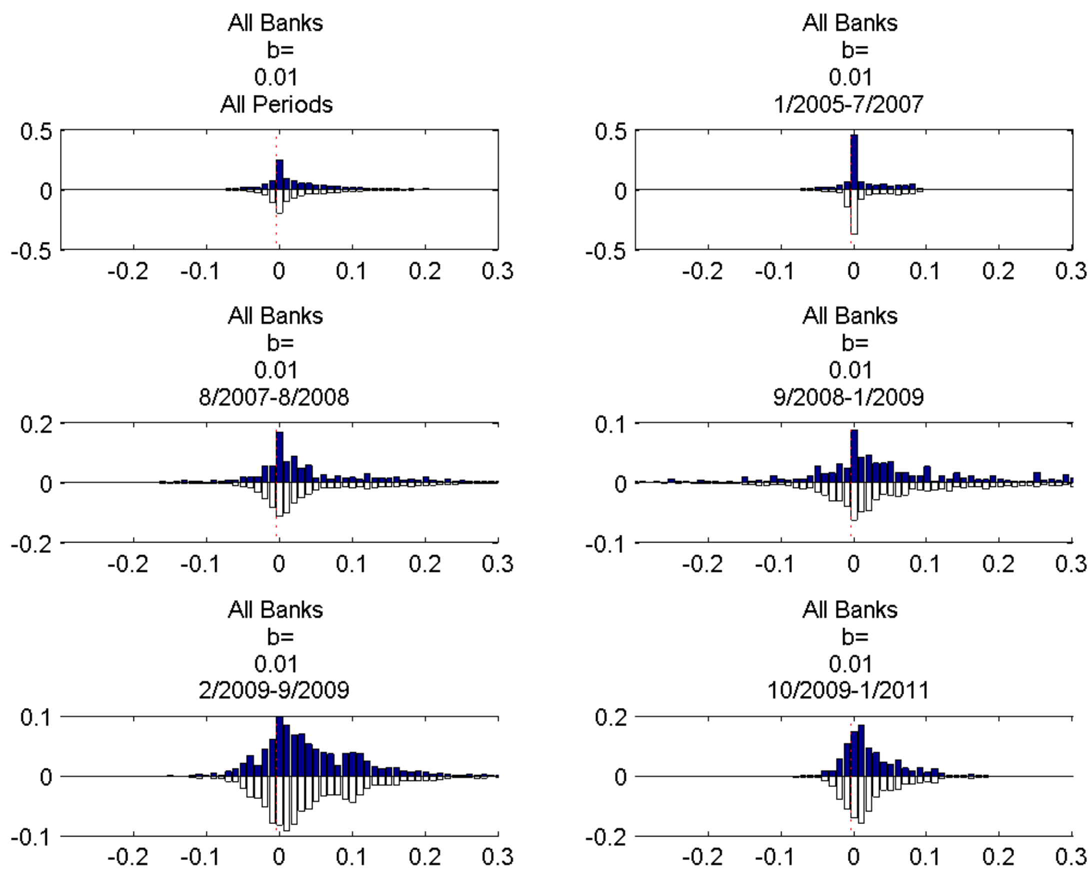


Figure 1.5: All Banks: 3M Bank Quote Minus the 4th Highest of Fifteen Other Banks

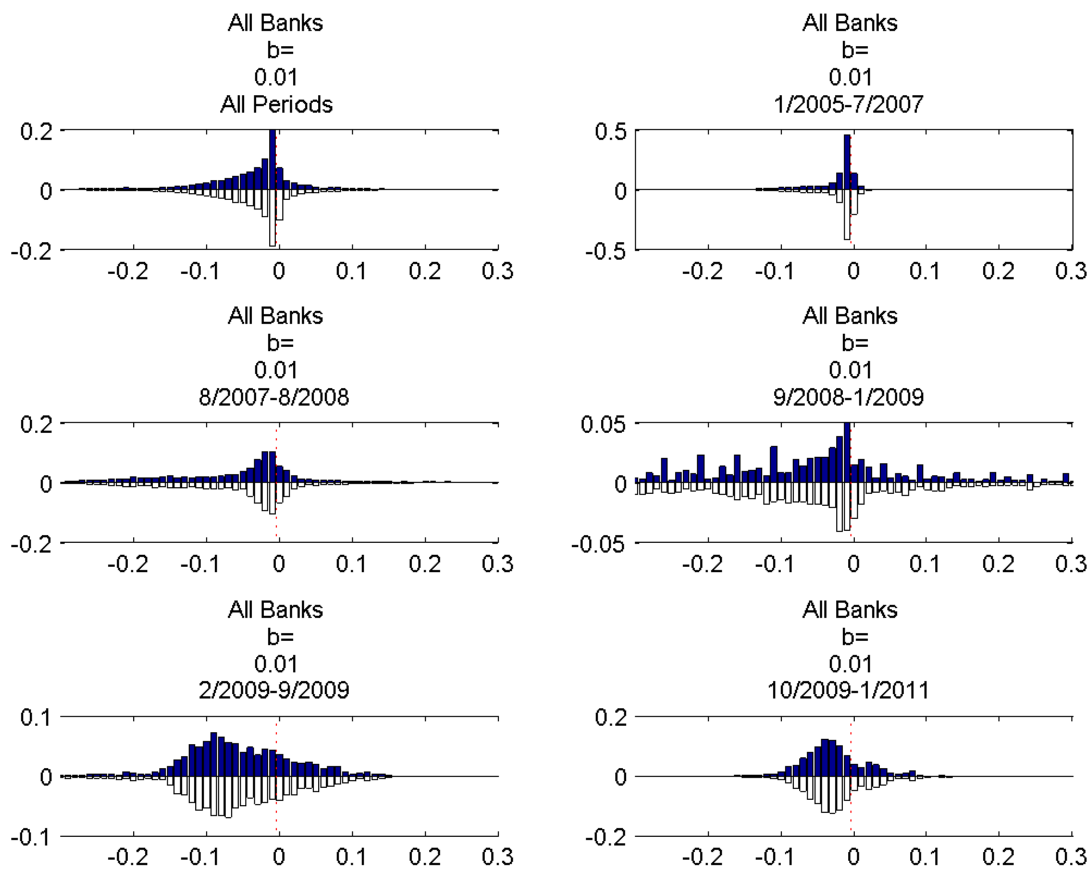


Figure 1.6: All Banks: 3M Dollar Rolling Bunching Test

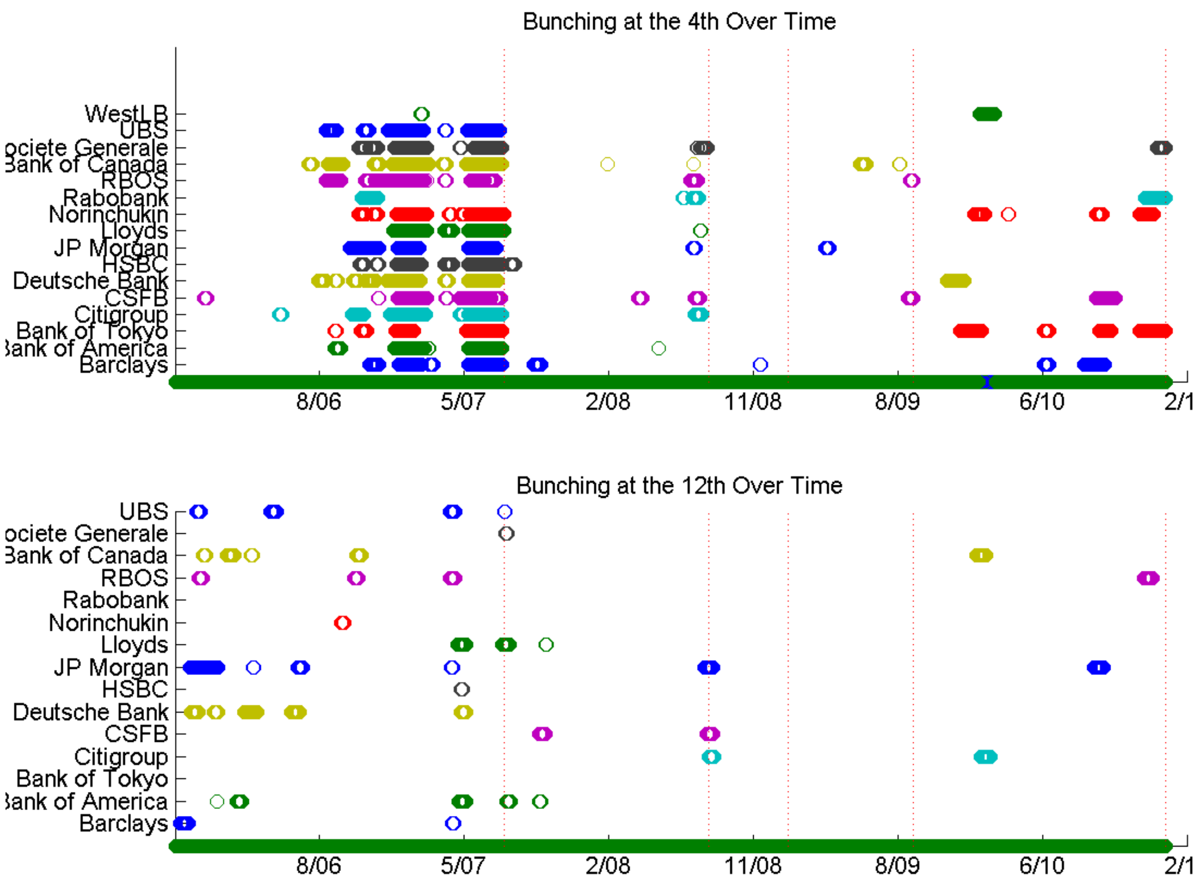
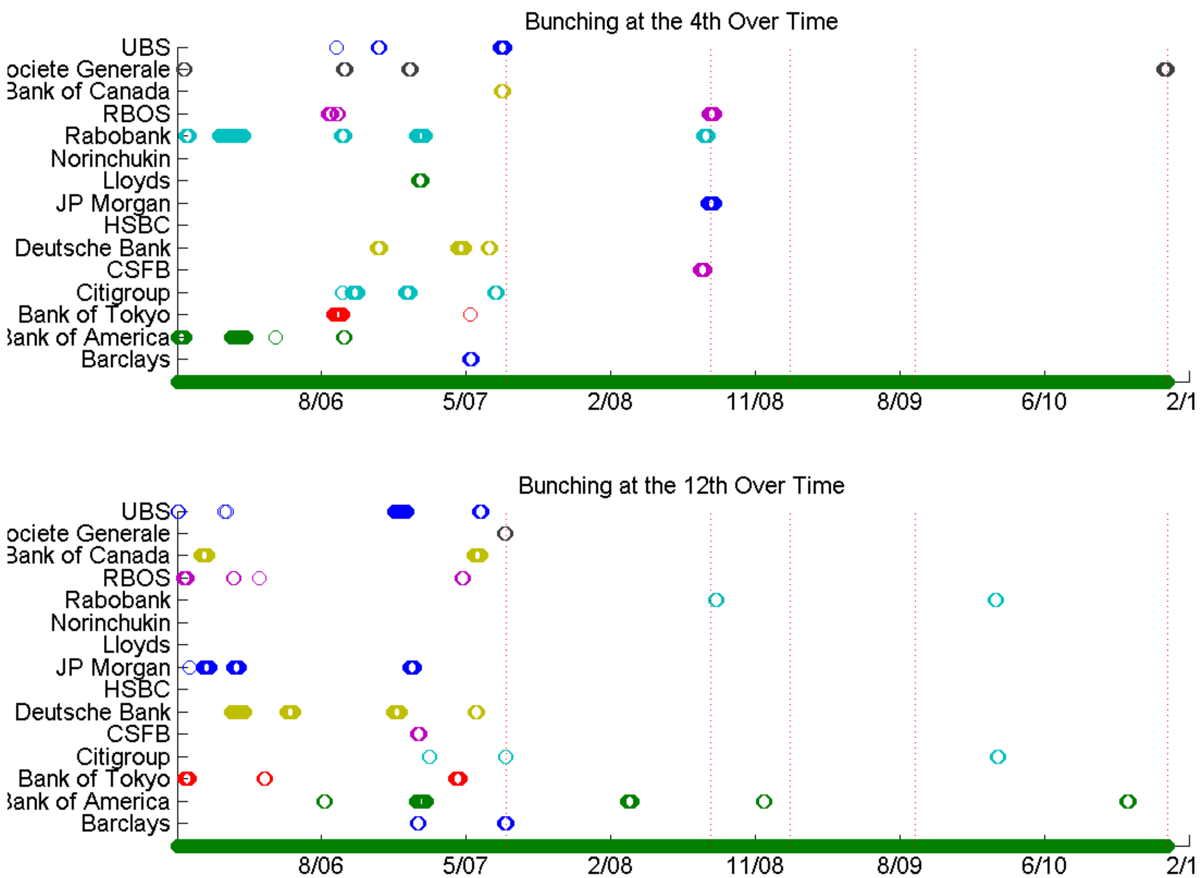


Figure 1.7: All Banks: 6M Dollar Rolling Bunching Test



Chapter 2

Impact and Consquences

2.1 Introduction

> *Trader:* [Would] be nice if you could put 0.90% for 1mth cheers.
> *Quote Submitter:* Sure no prob. I'll probably get a few phone calls but no worries mate!
> *Trader:* If you may get a few phone calls then put 0.88% then.
> *Quote Submitter:* Don't worry mate – there's bigger crooks in the market than us guys!
>
> —Rabobank, internal discussion[1]

The Libor was recently subjected to one of the largest instances of market manipulation in history. Hundreds of trillions of dollars worth of financial contracts were manipulated by large banks, who have since been fined billions of dollars by regulators from four countries.[2] Investigations are ongoing and many class action lawsuits are underway due to the huge volume of contracts adversely affected.[3]

The banks under investigation had simple incentives: they owned contracts whose payouts were functions of the Libor. While recent regulatory investigations have revealed that these portfolio incentives did in fact lead to manipulation, it is not known how this affected the overall Libor rate. It is possible the Libor remained largely unchanged throughout this episode, either due to an inability of banks to consistently execute manipulative intent, or thanks to varying portfolio incentives across banks. On the other hand, manipulation may have caused a systemic distortion, which would undermine the

[1] From the Department of Justice's investigation of Rabobank, see (DOJ (2013))

[2] Wheatley (2012) estimates $300 trillion worth of contracts directly reference the Libor when determining interest rate payments. The regulatory bodies of the USA, UK, Switzerland, and the Netherlands have variously fined Barclays, UBS, the Royal Bank of Scotland and Rabobank.

[3] The city of Baltimore, New Britain Firefighter's and Police Benefit Fund, Atlantic trading USA, and Community Bank&Trust are some of the representative members of the four classes pursuing Libor related damages, according to Perkins Coie (`http://www.perkinscoie.com/libor_faqs/`). See Dodd (2010) for a description of the losses incurred by municipalities.

continued value of the Libor as a benchmark. Any persistent distortion may have affected the allocation of funding in this period, as an estimated \$10 trillion of syndicated loans use the Libor as the variable interest rate (Wheatley (2012)).

In this paper, I quantify the degree to which manipulation distorted the Libor between 2005 and 2009. To do so, I estimate a strategic model where the Libor is formed each day in a noncooperative game of incomplete information. The strategic interaction between banks is generated by the aggregation mechanism of the Libor survey. Of the sixteen quoted rates, only the middle eight quotes are used in the resulting average which determines the benchmark. The four highest and lowest quotes are discarded. This means, if a bank was a manipulator, it would need to forecast the quotes of its peers in order to gauge its marginal ability to influence the overall Libor rate. Variation in this marginal ability across banks and trading days allows me to recover each bank's average portfolio exposure to the Libor and, consequently, what they would have quoted had they had no such exposure.[4]

I find that the Libor was largely accurate prior to the financial crisis starting in late 2007, but was since distorted downwards by eight basis points. This is substantial given the volume of contracts affected and that manipulators routinely made large gains from single basis point changes.[5] I calculate that U.S municipalities, which held \$500 billion worth of interest rate swaps in 2010, would have lost \$455 million from this eight basis point shift over my sample period.[6]

[4]There are many other benchmark interest rates similar to the Libor, including the Tokyo-based Tibor, the Mumbai-based Mibor, the many Euribor, and others. Since these are also typically calculated using truncated averages, the strategy I employ in this paper could be used to study the manipulation of these benchmarks as well.

[5]This is a common theme in the regulatory investigations. See for example FCA (2012a): "Barclays' Derivatives Traders knew on any particular day what their books' exposure to a one basis point (0.01%) movement in Libor or Euribor was." A Barclays trader said to the quote submitter, "We have about 80 [billion] fixing for the desk and each [basis point] lower in the fix is a huge help for us." (Ibid.) See also FCA (2012b) and Snider and Youle (2012) for more examples.

[6]See Preston (2012) and Dodd (2010) for background on municipal ownership of interest-rate swaps.

The transition between an accurate pre-crisis Libor and a distorted post-crisis Libor was driven by sharp changes in volatility and heterogeneity across banks. Prior to the financial crisis, banks had very similar risk characteristics and typically submitted identical or near-identical quotes to the survey. If the other fifteen banks are all submitting the same, correct rate, what could a potential manipulator achieve by submitting something different? Once the crisis began, however, banks' faced newly heterogeneous risks and submitted a broader range of quotes to the survey. This generated a larger interquartile range between the fifth and twelfth highest submissions which gave potential manipulators room in which to work.

With these results in mind, I compare the performance of counterfactual Libor aggregation mechanisms in the presence of active manipulators. This contribution is particularly timely as regulators are currently considering how best to reform to the Libor to safeguard it against future manipulation.[7] In particular, the Financial Conduct Authority (FCA) is considering increasing the size of the Libor panel, anonymizing quotes for three months, and tying quotes to underlying transactions as much as possible.[8] While the FCA also considered changing the current mechanism used to calculate the Libor from the underlying quotes, they concluded this would not improve the Libor's accuracy.

I find, on the contrary, that changing the current mechanism for calculating the Libor can make it considerably less vulnerable to manipulation. In particular, changing the Libor to use the median quote removes virtually all of its systematic downwards bias in the sample period I examine. My results differ from those of the FCA analysis because they assume submitted quotes would not change even if the method used to calculate the Libor were changed. This runs contrary to the idea that manipulators take into account the aggregation mechanism when they strategically submit their quotes. Documents revealed by the investigations show manipulators had a very keen awareness of the exact mechanism.[9] In my counterfactual analysis, manipulators are perfectly aware of

[7]The Libor and other benchmarks are becoming regulated for the first time.

[8]See Wheatley (2012) for a comprehensive review of the FCA proposals.

[9]See, for example, FCA (2012a)

the aggregation mechanism when they submit their quotes.

The counterfactual performance of the median is driven by the difficultly for manipulators to accurately forecast the location of the median on any given day. Even if a manipulator were able to correctly guess the median, they would not be able to skew their quote very far before they were no longer the median submission. In essence, using the median is similar to narrowing the interquartile range. There could also be a feedback effect. If the median mechanism causes most other banks to skew less, manipulators may skew less as they update their median forecasts.

For some values of the model parameters, however, the median can actually perform worse. The is because the distribution of private information plays an important role in my model's equilibrium. The performance of the median, relative to the interquartile range, depends on this distribution as well as the incentives for banks to manipulate. In particular, under certain conditions, a manipulator may be more or less certain they will be the median quote. In this case, the median mechanism would give them much greater power in determining the final rate, which lead the manipulator to skew more than they would otherwise.[10] This ambiguity is why I must bring the model to the data.

I model banks in the Libor panel as playing a noncooperative game of incomplete information. Each bank has a true interbank borrowing cost which depends on publicly observed covariates, as well as an idiosyncratic shock which is private information. Each bank is therefore uncertain of the quotes of the other banks when submitting its own quote. Manipulators are concerned with the quotes of the others because they want to forecast the interquartile range within which they can affect the Libor. Banks that aren't manipulators, however, are not concerned with forecasting the quotes of their peers.

My game is estimated in two steps. First, I nonparametrically estimate each bank's marginal impact on the expected Libor. This marginal impact is an equilibrium object

[10] Diehl (2013) shows the relative performance of the median-quote Libor is ambiguous in a complete information version of the game introduced in Snider and Youle (2012). This ambiguity persists in my current, incomplete information game.

which depends upon the strategies being played by the other banks. In the second step, I form moments from the model's first order conditions and use the submitted quotes and the results from the first step to estimate the game's parameters, which include banks' incentives to manipulate. From this, I construct a "manipulation free" Libor by calculating what banks would have quoted had they had no such incentives.

It is important to note that portfolio exposure to the Libor was not the only reason banks submitted misleading quotes. The other reason was reputational. Each bank's quote is publicly revealed after the Libor is computed. Whenever a bank submits a relatively high quote, thereby admitting to a high cost of borrowing funds from its peers, other market participants might infer something is amiss with that bank. In the run-prone environment of the recent financial crisis, it is unsurprising that banks wished to avoid this negative attention. Indeed, regulators have uncovered many documents expressing banks' desires to avoid being seen as lacking creditworthiness.[11]

I do not attempt to meaningfully capture these reputational incentives for banks to submit misleading quotes. Instead, I control for them with a flexible specification of fixed effects. I use bank-quarter effects and use the within-bank, within-quarter variation in marginal impacts upon the Libor to identify my model. Reputational effects are implicitly incorporated into my counterfactual analysis and the manipulation-free quotes I produce. I do not recover the "correct" Libor, only a Libor free of portfolio-driven manipulation.[12] These reputational incentives to misreport will be reduced by the FCA's new policy of anonymizing individual quotes fir three months. This anonymity, however, will exacerbate banks' portfolio-driven incentives to misreport by reducing the ability of other market participants to examine and monitor the submitted quotes.

Snider and Youle (2012) use a similar model to motivate a test for Libor manipulation. My approach differs from theirs by assuming banks play a game of incomplete information. This relatively minor modeling difference leads to a completely different empirical strategy. Incomplete information creates smoothness in banks' profit functions and allows the derivation of a system of necessary first order conditions. Estimating

[11] Ibid.

[12] These reputational incentives likely would have pushed the Libor downwards even further.

these first order conditions lets me measure the *size* of banks' long term average exposures to the Libor. Knowing this size allows me quantify the extent of the Libor's distortion and examine the accuracy of counterfactual aggregation mechanisms. I am unable, however, to capture manipulation that occurs at a high frequency, which was an important part of the Libor's recent manipulation, and for which Snider and Youle (2012)'s test is better suited to detect.

This paper is related to the recent literature on estimating games that occur in financial markets. Cassola et al. (2013) measures banks' demand for funding through a structural auction model of the EONIA funds service. They use observed bids and the structure of the auction to recover banks' valuations. In my model, I use observed quotes and the structure of the Libor mechanism to recover banks' portfolio exposures. Guerre et al. (2009) use an exclusion restriction in an auction setting to separately identify bidders' risk aversion from their distribution of valuations. I also employ an exclusion restriction to separately identify manipulators' portfolio exposures from their unobserved interbank shocks.

The rest of the paper is structured as follows. Section 2 describes the history of the Libor and the recent manipulation scandal. Section 3 describes my data. Section 4 introduces the strategic model of manipulation. Section 5 describes my estimation procedure and results. Section 6 discusses my results. Section 7 introduces an algorithm to compute the Bayes-Nash equilibrium and compares counterfactual Libor mechanisms. Section 8 concludes.

2.2 History of the Libor

The London Interbank Offered Rate (Libor) is a benchmark interest rate that has grown to become a central institution in financial markets.[13] An estimated $300 trillion of

[13]The Libor is quoted in many different maturities and currencies. In this paper I focus on the three month dollar Libor which, along with the six month Libor, is most commonly used by financial contracts.

contracts use the Libor to determine their obligated interest payments. Many syndicated loans, adjustable rate mortgages, student loans, and other financial products routinely depend upon the Libor in this fashion. An even larger array of interest rate derivatives, such as forwards, futures, and swaps, depend directly upon the Libor.

The Libor is calculated by a daily survey managed by the British Banker's Association (BBA).[14] A fixed panel of sixteen large banks are asked,

> *"At what rate could you borrow funds, were you to do so by asking for and then accepting interbank offers in a reasonable market size just prior to eleven a.m. London time?"*

The four lowest and four highest rate quotes are then discarded, with the average of the remaining middle eight quotes forming the day's Libor. I assume in what follows that, for each bank there is a more or less correct answer to this question, and that they are aware of it. These banks are in constant communication with brokers and one another, making this an easy question for them to resolve.

The modern Libor was established in the 1980s to provide a standardized interest rate benchmark. The growth of the Libor's general use was facilitated by the growth of trading in interest rate derivatives. The Libor was adopted early on in the markets for interest rate derivatives and experienced lock-in due to network effects. It is easier to write and trade contracts using a well established as opposedto obscure benchmarks. Accordingly, the Libor had the distinct advantage of being introduced in the 1980s, well before current rival benchmarks defined using more recent money market interest rates, such as the going rate for repurchase agreements.

While an enormous number of contracts reference the Libor, the London interbank market, for which the Libor is supposed to represent the average price of funds, is small. This market was large in the 1980s, but has been largely been replaced by overnight and

[14] In the period I examine, the BBA was in charge of overseeing the Libor and monitoring submission accuracy. The BBA is an industry group composed largely of the banks sitting on the Libor panel. The Financial Services Authority (FSA) has since required the BBA transfer the governance of the Libor to another private institution.

collateralized forms of lending. Repurchase agreements, commercial paper, overnight federal funds, and other close money market substitutes are now the primary vehicles through which banks and other financial institutions exchange funds. This has made it increasingly difficult to verify submitted Libor quotes to actual interbank trades as such trades are increasingly uncommon.

Despite the recent manipulation, the Libor is still widely used, and will remain largely unchanged in the medium term. The FCA ultimately decided to reform the Libor rather than replace it with another measure. Such a switch could trigger a wave of lawsuits and costly renegotiation of legacy Libor-referencing contracts, and is opposed by the vast majority of Libor stakeholders (Wheatley (2012)). In the long run, however, it is entirely possible that the market will move towards a new benchmark that is less vulnerable to manipulation. In the short run, regulators and market participants are eager to restore the Libor's credibility as a benchmark interest rate.

2.3 Data

I collect the daily Libor quotes for each of the panel banks from a Bloomberg Terminal. I focus on the period beginning October 4th, 2005 and ending October 28th, 2009 for a total of 1,009 trading days. This period includes the movement in the Libor from its high level prior to the onset of the financial crisis in the second quarter of 2007, to its low level in the third quarter of 2009. It also contains some of the most volatile periods of the financial crisis as well as vocal concerns over Libor manipulation in the media.

Table 2.1 shows the raw quote data for the sixteen panel banks for the week beginning on Monday, December 12th, 2007. Several features are worthy of note. First, the Libor is calculated using the middle eight quotes; the four highest and lowest quotes are discarded. Second, banks typically quote close together. Third, banks' quotes are persistant; banks do not typically quote far from their previous values. The within and between bank quote variance, however, alters dramatically over my sample period, as the market moves from calm to stressful periods.

The interquartile range of quotes varies considerably over my sample as shown in figure 2.1. Before the crisis, banks submitted quotes very close to one another.[15] Once the crisis began, however, banks began to change their quotes considerably from day to day, and quotes become different from bank to bank. A clear example of the difference in pre- and post-crisis behavior can be seen in table 2.2. On January 2, 2007, fifteen of the sixteen banks submitted identical quotes and the interquartile range was zero. On October 21, 2008, the sixteen banks submitted eleven unique quotes and the interquartile range was fifteen basis points.

In figure 2.2, I show the submitted Libor quotes relative to the day's average, overlaid with the banks' CDS quotes, a measure of their credit risk, relative to the day's average. Prior to the crisis, there was very little difference between banks in both the CDS spreads and the quotes they submitted to the survey. Once the crisis began, however, there was substantial across-bank heterogeneity both in their CDS spreads and their attendant quotes.

The Libor is one of many money market rates which govern short-to-medium-term lending between financial institutions. Other money market rates include certificate of deposit rates, repurchase agreement rates, and commercial paper rates. These rates are generally close substitutes and typically co-move with each other and the Libor. Including these variables allows me to control for the secular change in interest rates that occurs in response to monetary policy undertaken by the Federal Reserve. To that end, I use data from the H.15 interest rate series on market rates for commercial paper. I also collect daily data on the federal funds effective rate, which is the overnight analogue of the interbank market.[16]

Table 2.3 shows the frequencies that each bank is either below, inside, or above the day's interquartile mean of quotes. There are systematic differences between banks,

[15] This tight clustering of quotes was noted early on in the suspicion of the Libor by Abrantes-Metz et al. (2012).

[16] There are differences in the banking holidays between London and New York. Consequently, the federal funds effective rate is not always computed every day the Libor is computed, and vice versa. This generates missing values in my data. Whenever a value is unobserved in this fashion, I impute the price to be the last quoted price.

but no bank is either always within or always outside the middle eight. Some banks, however, spend far more time on one side of the quartile than the other. For example, 52.8% of Norinchukin's quotes were above the interquartile range while only 0.7% were below. This is important for the identification of my model, which is discussed further in section five.

2.4 Model of Strategic Manipulation

I model the Libor panel as a noncooperative game of incomplete information played by the banks in the survey. Banks on the panel consist of a true interbank borrowing cost, which depends on publically observed covariates, and a private idiosyncratic shock. Banks submit quotes simultaneously and are uncertain of the other banks' quotes due to the others' private information. Manipulators, therefore, seek to forecast the interquartile range within which they can impact the Libor.

Banks are not necessarily cooperative nor antagonistic. Their attitude towards their peers depends upon their private information and their incentives to manipulate the Libor. In particular, I allow banks to prefer a high Libor, a low Libor, or to be indifferent. Given the flexibility of interest rate derivatives and the secrecy of banks' net exposures, it is *a priori* as likely that banks would "short" the Libor as the opposite.

2.4.1 Setup

Let $i = 1, ..., 16$ index the sixteen banks on the Libor panel. Let $t = 1, ..., T$ index the trading days in my sample. A bank's observed covariates are denoted x_{it}, with $x_t = (x_{1t}, x_{2t}, ..., x_{16t})$. A bank's submitted quote is q_{it}, with $q_t = (q_{1t}, ..., q_{16t})$. The day's Libor is the interquartile mean of the quotes, given by:

$$L(q_t) = \frac{1}{8} \sum_{i=1}^{16} \mathbf{1}\{q_{it} \in IQR(q_t)\} q_{it}$$

where $IQR(q_t)$ is the interquartile range of the submitted quotes q_t.[17]

Banks play a static game of incomplete information. They each receive a private, independent, and idiosyncratic cost shock ϵ_{it}. These covariates include general market information, in the form of prices in related substitute money markets, and include bank specific information, in the form of credit risk measured by the price of default insurance.[18] The residual ϵ_{it} is the unobserved component of interbank borrowing costs,

$$c_{it} = x_{it}\beta_i + \epsilon_{it} \tag{2.1}$$

Each bank's unobserved borrowing cost ϵ_{it} is distributed according to $F_{\epsilon_{it}|x_t}$. While I assume that the cost shock is mean independent of the unobservables, i.e. that $\mathrm{E}[\epsilon_{it}|x_t] = 0$, the distribution of the cost shock can depend upon the observables x_t in its higher moments. This allows costs to be heteroskedastic, which is important given the time-varying volatility evident in my data. The interbank borrowing costs themselves remain correlated through their dependence upon the correlated observable variables $(x_{1t}, x_{2t}, ..., x_{16t})$. Since banks are entangled in the same system of supply and demand of interbank loans and typically share similar risk characteristics, correlated costs are likely.

Banks have strategic incentives to manipulate the Libor stemming from their portfolios. They also don't want to be seen as manipulating it. I model their payoff function as being additive in these two incentives. Bank i's realized profits, given its characteristics and the quotes of the others, is given by:

$$\pi_i(q_{it}, q_{-it}, x_{it}, \epsilon_{it}) = v_i(x_{it})L(q_{it}, q_{-it}) - \delta(q_{it} - x_{it}\beta_i - \epsilon_{it})^2 + \psi_i q_{it} \tag{2.2}$$

The term $v_i(x_{it})$ represents bank i's portfolio exposure to the Libor, which can depend

[17] Note that while Libor is increasing and Lipschitz continuous, it is neither continuously differentiable, supermodular, nor concave. Thus the first order conditions are not sufficient for optimality without further assumptions.

[18] I use daily one year senior CDS spreads at the bank level as the bank-specific credit risk measure.

upon its characteristics.[19] The scalar δ represents the bank's concern over the consequences of misreporting their true interbank borrowing costs. This is because banks are monitored by regulators, other banks, and market participants. They also face the possibility of being probed by regulators who then levy steep fines upon them for manipulation. I do not attempt to model this underlying monitoring. Instead I use a simple quadratic functional form to approximate banks desire to quote near their costs, a desire that was routinely expressed in documents uncovered by investigators.

The term ψ_i captures bank's reputational concerns over their quotes. This stems from the fact that banks' quotes are publicly revealed later in the day. If a bank were to submit a relatively high rate quote it might signal to other market participants that it is not as creditworthy as its peers. This reputational concern might differ across banks and across market conditions. Thus I allow ψ_i to change from quarter to quarter, treating it as a bank-quarter fixed effect in estimation.

The long-run average portfolio exposures $v = (v_1, v_2, ..., v_{16})$ are assumed to be common knowledge. The observed covariates x_t and the parameters $(\delta, \beta_1, ..., \beta_{16}, \psi_1, ..., \psi_{16})$ are also common knowledge. Finally, the distributions of the unobserved shocks F are also common knowledge. Each bank knows these variables and parameters as well as their own private cost shock ϵ_{it}. The only variables bank i is uncertain of when submitting its quotes are the indiosyncratic cost shocks ϵ_{-it} of the other fifteen banks.

I assume that banks play pure strategies.[20] Bank i's strategy at t, denoted ϕ_{it}, maps its information (x_t, ϵ_{it}) into its quote q_{it} such that $\phi_{it}(x_t, \epsilon_{it}) = q_{it}$. A bank's expected

[19] I can't recover each bank's exposure to the Libor at a daily frequency. In principle, there is nothing stopping a trader from setting up a position one day and then rewinding it and setting up the opposite position the next. The test by Snider and Youle (2012) is better suited to detect manipulation driven by such behavior. Since I am trying to recover longer term incentives to skew the Libor, I aim to capture each bank's average exposure to the Libor over my sample period.

[20] The Single Crossing Condition as defined in Athey (2001) is satisfied and therefore a monotone pure strategy equilibrium exists.

profit, when the other banks are playing ϕ_{-it}, is given by,

$$\Pi_i(q_{it}, x_t, \epsilon_{it}; \phi_{-it}) = \int_{\epsilon_{-it}} \pi_i(q_{it}, \phi_{-it}(x_t, \epsilon_{-it}), x_{it}, \epsilon_{it}) dF_{\epsilon_{-it}|x_t} \tag{2.3}$$

Banks are assumed to be playing a pure strategy Bayesian-Nash equilibrium.

Definition: A (pure strategy) **Bayesian-Nash Equilibrium** is a vector of strategies ϕ_t such that, for every bank i and information set (x_t, ϵ_{it}),

$$\phi_{it}(x_t, \epsilon_{it}) \in \text{argmax}_q \Pi_i(q, x_t, \epsilon_{it}; \phi_{-it})$$

For what follows, it is convenient to define an expected Libor (given ϕ_{-it}):

$$\mathcal{L}_i(q_{it}, x_t; \phi_{-it}) \equiv \int_{\epsilon_{-it}} L(q_{it}, \phi_{-it}(x_t, \epsilon_{-it})) dF_{\epsilon_{-it}|x_t} \tag{2.4}$$

Banks are only concerned with forecasting the quotes of their peers insofar as they alter the expected Libor. This allows me to rewrite the expected profit equation (2.3) as,

$$\Pi_i(q_{it}, x_t, \epsilon_{it}; \phi_{-it}) = v_i(x_{it})\mathcal{L}_i(q_{it}, x_t; \phi_{-it}) - \delta(q_{it} - x_{it}\beta_i - \epsilon_{it})^2 + \psi_i q_{it} \tag{2.5}$$

Taking the first order condition and rearranging,

$$q_{it} = \frac{v_i(x_{it})}{2\delta} \frac{\partial}{\partial q_{it}} \mathcal{L}_i(q_{it}, x_t; \phi_{-it}) + \frac{\psi_i}{2\delta} + x_{it}\beta + \epsilon_{it} \tag{2.6}$$

This equation has an intuitive interpretation. Banks quote their costs with a skew term similar to bidders in an auction model.[21] The degree to which they skew is proportional to the ratio of their manipulation ($v_i(x_{it})$) and truth telling (δ) incentives, as well as their marginal impact on the expected Libor. They also skew proportional to their reputational incentives (ψ_i).

[21] For examples, see Krishna (2009)

2.4.2 Multiplicity

My model admits multiple equilibria.[22] This stems from the kinks in the Libor function L which allows banks to coordinate on the aggressiveness of their skewing. Consider a case where banks have identical incentives. If the other banks skew aggressively in bank A's favored direction, bank A also has a wider latitude to skew and will do so. If they don't skew aggressively, bank A is more likely to be excluded from the middle eight by skewing aggressively and will skew less. If the cost distribution has finite support, one can prove the existence of multiple equilibria constructively.

These equilibria, however, are bounded within a set range. This is because the derivative of the expected Libor is always bounded between zero and one eighth, irrespective of the strategies being played by the other banks. This then implies the best responses are bounded as a consequence of the first order condition, as shown in equation 2.6. Intuitively, these bounds come from two facts. First, no manipulator would ever skew the Libor in a direction opposite of their incentives. Second, no manipulator would ever skew the Libor more than it would if only its quote were used to compute the Libor. I use these bounds when confronting this multiplicity in my counterfactual analysis.

This multiplicity, however, poses no problem for my ability to estimate the model, so long as the equilibrium selection mechanism depends only on the observables. This is because an analogue of the first order condition, which averages across the equilibria and is in terms of objects observable to the econometrician (up to the parameters), is a logical implication of the model. I pursue this idea in the remainder of this section.

Suppose for a given x_t there is a finite number $e = 1, ..., E$ of equilibria. The econometrician does not know which equilibrium e is being played on any given day. Let $\Lambda^e(x_t)$ be the probability equilibrium e occurs according to some underlying and unknown selection mechanism. Let ϕ_t^e be the strategy profile corresponding to equilibrium e. The econometrician does not observe $\mathcal{L}_{it}(q_{it}, x_t; \phi_{-it}^e)$ directly, but only the following weighted average across equilibria,

[22] In the complete information version of my model, studied in Snider and Youle (2012) and Diehl (2013), there is also a considerable multiplicity of equilibria.

$$\mathcal{L}_{it}(q_{it}, x_t) \equiv \sum_{e=1}^{E} \Lambda^e(x_t)\mathcal{L}_{it}(q_{it}, x_t; \phi^e_{-it}) \tag{2.7}$$

Similarly, the econometrician does not observe $\frac{\partial}{\partial q_{it}}\mathcal{L}_{it}(q_{it}, x_t; \phi^e_{-it})$ directly, but only the following weighted average across the equilibria,

$$\frac{\partial}{\partial q_{it}}\mathcal{L}_{it}(q_{it}, x_t) = \frac{\partial}{\partial q_{it}}\sum_{e=1}^{E} \Lambda^e(x_t)\mathcal{L}_{it}(q_{it}, x_t; \phi^e_{-it}) = \sum_{e=1}^{E} \Lambda^e(x_t)\frac{\partial}{\partial q_{it}}\mathcal{L}_{it}(q_{it}, x_t; \phi^e_{-it}) \tag{2.8}$$

Finally, the econometrician does not observe quotes conditional on an equilibrium e but only the unconditional quotes,

$$\mathrm{E}[q_{it}|x_t] = \sum_{e=1}^{E} \Lambda^e(x_t)\int_{\epsilon_{it}} \phi^e_{it}(x_t, \epsilon_{it})dF_{\epsilon_{it}} \tag{2.9}$$

However, because both the first order conditions and differentation are linear, a version of the first order condition is implied by the model,

$$\mathrm{E}[q_{it}|x_t] = \sum_{e=1}^{E} \Lambda^e(x_t)\int_{\epsilon_{it}} \phi^e_{it}(x_t, \epsilon_{it})dF_{\epsilon_{it}} \tag{2.10}$$

$$= \sum_{e=1}^{E} \Lambda^e(x_t)\int_{\epsilon_{it}} \left(\frac{v_i(x_t)}{2\delta}\frac{\partial}{\partial q_{it}}\mathcal{L}_{it}(q_{it}, x_t; \phi^e_{-it}) + \frac{\psi_i}{2\delta} + x_{it}\beta_i + \epsilon_{it}\right)dF(\epsilon_{it}) \tag{2.11}$$

$$= \sum_{e=1}^{E} \Lambda^e(x_t)\left(\frac{v_i(x_t)}{2\delta}\frac{\partial}{\partial q_{it}}\mathcal{L}_{it}(q_{it}, x_t; \phi^e_{-it}) + \frac{\psi_i}{2\delta} + x_{it}\beta_i\right) \tag{2.12}$$

$$= \frac{v_i(x_t)}{2\delta}\sum_{e=1}^{E} \Lambda^e(x_t)\frac{\partial}{\partial q}\mathcal{L}_{it}(q_{it}, x_t; \phi^e_{-it}) + \frac{\psi_i}{2\delta} + x_{it}\beta_i \tag{2.13}$$

$$= \frac{v_i(x_t)}{2\delta}\frac{\partial}{\partial q_{it}}\mathcal{L}_i(q_{it}, x_t) + \frac{\psi_i}{2\delta} + x_{it}\beta_i \tag{2.14}$$

Where the equation (2.12) used the first order condition equation (2.6), and equation

(2.14) used equation (2.8). The model then implies the following relationship,

$$\mathrm{E}[q_{it}|x_t] = \frac{v_i(x_t)}{2\delta}\frac{\partial}{\partial q_{it}}\mathcal{L}_{it}(q_{it}, x_t) + \frac{\psi_i}{2\delta} + x_{it}\beta_i \tag{2.15}$$

Which is in terms of objects that are observable (up to the parameters) by the econometrician, and only requires the assumption that the selection mechanism Λ depends only on the observables x_t. This assumption is also used in De Paula and Tang (2012) who develop a formal test for multiple equilibria in entry games. It also means that I need not assume the same equilibrium is being played across different trading days. Relaxing this assumption, which is common in the structural estimation of games, is possible due to the linearity of my first order conditions and the fact that they hold in every equilibrium.

Things are greatly aided by the fact that the Libor-setting game satisfies Milgrom and Shannon (

2.5 Estimation

The challenge posed by the possibility of multiple equilbria drives my empirical strategy. A common technique for estimating non-cooperative games involves the explicit computation of equilibria while searching over the parameter space to minimize an empirical criterion function.[23] Applying that technique in this setting would require me to make strong assumptions about the form of the equilibrium selection function Λ, as well as the functional form of the distribution of the unobservables $F_{\epsilon_{it}|x_t}$.[24] Searching over the parameter space is computationally expensive and would require either simulation methods or the construction of a likelihood function – both complicated by the presence of multiple equilibria.

[23]Many papers use variations of this approach. For an example, see Seim (2006) who circumvents the multiplicity problem by showing the existence of a unique equilibrium under certain conditions.

[24]An alternative is to use a bounds approach as in Ciliberto and Tamer (2009).

To avoid these issues, I implement a two-step estimator.[25] In the first step, I non-parametrically estimate the equilibrium derivative of the expected Libor $\mathcal{L}$ with respect to a bank's quote. I use local linear kernel regression, using the Libor fixes, quotes, and observables from the data. In the second step, I estimate the banks' necessary first order conditions. Specifically, I estimate equation 2.6 using this estimated derivative as a covariate. This allows me to recover each bank's long run average exposure to the Libor $v_i(x_{it})$ as a parameter and its interbank shock ϵ_{it} as a residual.[26]

In the second stage, I use exclusion restrictions to generate additional moment conditions which are necessary to pin down the parameters of my model. This indirect, revealed preference approach is necessary as banks carefully guard the secrecy of their portfolio exposures to the Libor. However, as a robustness check, I compare the results of my estimates with the information on bank portfolio positions revealed in the Call Reports.

2.5.1 Nonparametric First Step

The first step in my estimation procedure is to nonparametrically estimate the derivative of the expected Libor $\frac{\partial}{\partial q_{it}}\mathcal{L}_{it}(q_{it}, x_t)$. I do this by using the definition of a derivative and nonparametrically estimating the expected Libor itself at two nearby points.

The procedure begins by considering how the realized Libor L would have changed had a given bank i submitted a slightly higher quote. I compute,

$$L_{it}^{\Delta} \equiv L(q_{it} + \Delta, q_{-it}) \tag{2.16}$$

for each bank i and day t. This measures the resulting Libor had bank i quoted $q_{it} + \Delta$

[25] My estimator is loosely inspired by Bajari et al. (2007), Pakes et al. (2007), Aguirregabiria and Mira (2007), and Pesendorfer and Schmidt-Dengler (2008). Unlike these approaches, I do not need to assume the same equilibrium is being played across the data – only that the equilibrium selection mechanism is reasonably well behaved.

[26] As the first step is nonparametric and our second parametric, this procedure is a semiparametric regression as discussed in Robinson (1988)

instead of q_{it}. Note that I could use these calculations to compute the realized derivative for each bank as $(L_{it}^{\Delta} - L_t)/\Delta$. For a Δ small enough, this realized derivative will always be zero or one eighth. Either $q_{it} + \Delta$ is included in the interquartile range or it isn't. This is what would be relevant if banks are playing a game of complete information.

However, banks are not perfectly aware of where the pivotal cutoff points of the interquartile range will be located. This is because they are uncertain about the shocks ϵ_{-it}, and hence the resulting quotes, of their peers. This creates a smooth range over where their marginal impact on the expected Libor can lie. This marginal impact remains bounded between zero and one eighth, but can also lie somewhere in between.

Recall that,

$$\mathcal{L}_{it}(q_{it}, x_t) = \sum_{e=1}^{E} \Lambda^e(x_t) \int_{\epsilon_{-it}} L(q_{it}, \phi_{-it}^{e}(x_t, \epsilon_{-it})) dF_{\epsilon_{-it}|x_t} = \mathrm{E}\big[L_t|q_{it}, x_{it}\big] \tag{2.17}$$

where L_t is the realization of the day's Libor, which depends upon the underlying equilibrium, quotes, and cost shocks. Recovering $\mathrm{E}[L_t|q_{it}, x_t]$, however, involves no more than estimating a conditional expectation, as I observe (L_t, q_{it}, x_t) directly in the data. Similarly, by computing L_{it}^{Δ} as above, using the mechanical rule for the Libor mechanism and the observed quotes, I have the following,

$$\mathcal{L}_{it}(q_{it} + \Delta, x_t) = \sum_{e=1}^{E} \Lambda^e(x_t) \int_{\epsilon_{-it}} L(q_{it} + \Delta, \phi_{-it}^{e}(x_t, \epsilon_{-it})) dF_{\epsilon_{-it}|x_t} = \mathrm{E}\big[L_{it}^{\Delta}|q_{it}, x_{it}\big] \tag{2.18}$$

and again, as $(L_{it}^{\Delta}, q_{it}, x_t)$ are all observed, recovering $\mathrm{E}\big[L_{it}^{\Delta}|q_{it}, x_{it}\big]$ involves nothing more than estimating a conditional expectation.

I estimate both $\mathrm{E}\big[L_t|q_{it}, x_{it}\big]$ and $\mathrm{E}\big[L_{it}^{\Delta}|q_{it}, x_{it}\big]$ nonparametrically, using local linear regression.[27] This is a kernel regression – essentially a locally weighted average of the Libors, where the observables (q_{it}, x_t) are used to define what "local" is.

Given estimates of $\widehat{\mathrm{E}}\big[L_t|q_{it}, x_{it}\big]$ and $\widehat{\mathrm{E}}\big[L^{\Delta}_{it}|q_{it}, x_{it}\big]$, I estimate the derivative of the expected Libor. First I use the definition of a derivative,

$$\frac{\partial}{\partial q_{it}}\mathcal{L}_{it}(q_{it}, x_t) = \lim_{\Delta\to 0}\frac{\mathcal{L}_{it}(q_{it}+\Delta, x_t) - \mathcal{L}_{it}(q_{it}, x_t)}{\Delta}$$

The empirical analogue I use for estimation is then,

$$\widehat{\frac{\partial}{\partial q_{it}}\mathcal{L}}_{it}(q_{it}, x_t) = \frac{\widehat{\mathcal{L}_{it}}(q_{it}+\Delta, x_t) - \widehat{\mathcal{L}}_{it}(q_{it}, x_t)}{\Delta} = \frac{\widehat{\mathrm{E}}\big[L^{\Delta}_{it}|q_{it}, x_{it}\big] - \widehat{\mathrm{E}}\big[L_t|q_{it}, x_{it}\big]}{\Delta}$$

I estimate this partial derivative for a fixed Δ, stipulating that $\Delta \to 0$ as $T \to \infty$ but that $\Delta T \to \infty$ for consistency. In practice, I set the value for Δ to be a quarter of a basis point.[28]

2.5.2 Parametric Second Step

In the second step I estimate the first order conditions (2.15) using the Generalized Method of Moments (GMM), plugging in the derivative recovered in the first step .

Plugging in our first stage estimates into equation (2.15) produces the following estimating equation:

$$\mathrm{E}\big[q_{it} - \frac{v_i(x_t)}{2\delta}\widehat{\frac{\partial}{\partial q_{it}}}\mathcal{L}_{it}(q_{it}, x_t) + \frac{\psi_i}{2\delta} + x_{it}\beta_i \mid x_t\big] = 0 \tag{2.19}$$

[27] For background on local linear regression and its properties, see Fan and Gijbels (1996).

[28] Banks submit quotes in smaller increments very rarely. While this may suggest a discrete choice modeling approach, a discretization that would surround all the quotes banks actually submit around the day's Libor, up to quarter basis point increments, would involve over a hundred bins. Given there are sixteen banks, this means an action space of at least one hundred to the sixteenth power, which generates matrices far beyond my ability to invert.

where I have implicitly assumed that the measurement error in the first step is mean independent of x_t.[29]

From the structure of equation (2.19) we might be content that the model is identified. However, the situation is more pessimistic for a similar environment examined by Guerre et al. (2009), who examine a first price auction where bidders have an unknown (to the econometrician) distribution of valuations for a good, as well as an unknown (again, to the econometrician) risk aversion parameter. This is largely analogous to my environment, where I have an unknown distribution of interbank borrowing shocks and an unknown portfolio exposure parameter. They find that, without further assumptions, their model is nonidentified.

This makes it likely that the identification coming from equation (2.19) is primarily due to my parametric assumptions when specifying bank profits. A more general misreporting cost function, rather than my current quadratic form, will likely make the model nonidentified per Guerre et al. (2009). As I view my quadratic form as a first approximation to some potentially more complex, unknown, form, I pursue the solution of Guerre et al. (2009), which is to use an exclusion restriction. In particular, they assume that the number of bidders in the auction is unrelated to the distribution of bidder valuations. Seeing how observed quotes vary as the number of competitors varies is what enables them to separately identify the risk aversion parameters from the distribution of valuations.

I invoke a related exclusion restriction. However, as I always have the same number of banks in my sample, I use a different set of variables which are unrelated to banks current, unobserved borrowing shocks. I use lagged quotes of the other banks as such a variable, because they are useful for forecasting their current quotes. The logic is that, from a manipulator's point of view, it is not directly concerned with the lagged quotes of its peers, but they do help it predict what the others will quote today and, hence, what its marginal impact on the Libor will be. Specifically, I use the lagged interquartile range of the quotes as an excluded variable,

[29] It is possible that first step measurement error leads to bias in the second step, a regular concern when using two-step estimators to structurally estimate games.

$$\mathrm{E}[\epsilon_{it}|IQR_{t-1}] = 0 \tag{2.20}$$

which I use to form the following unconditional moment conditions for each bank:

$$\mathrm{E}\big[x_{it}\big(q_{it} - \frac{v_i(x_t)}{2\delta}\widehat{\frac{\partial}{\partial q_{it}}}\mathcal{L}_{it}(q_{it}, x_t) + \frac{\psi_i}{2\delta} + x_{it}\beta_i\big)\big] = 0$$
$$\mathrm{E}\big[IQR_{t-1}\big(q_{it} - \frac{v_i(x_t)}{2\delta}\widehat{\frac{\partial}{\partial q_{it}}}\mathcal{L}_{it}(q_{it}, x_t) + \frac{\psi_i}{2\delta} + x_{it}\beta_i\big)\big] = 0$$

I use GMM with these moment conditions where ψ_i is treated as a bank-quarter fixed effect to estimate my parameters.

As my specification of bank profits is homogeneous of degree one in $(v_i(x_i), \delta, \psi_i)$, it is only identified up to scale. This is typical when studying revealed preferences, where it is standard to normalize a utility or cost parameter, or the utility from receiving an outside option. While I can't interpret my parameters in terms of dollars, I can use them predict bank behavior, which is what I need to measure the performance of counterfactual Libor mechanisms.

To see how I identify bank's incentives to manipulate, consider a different world where the Libor was not calculated by an interquartile mean, but instead was a simple average. In that case, $\frac{\partial}{\partial q_{it}}\mathcal{L}_{it}(q_{it}, x_t)$ would always be one sixteenth for every i, t, and (q_{it}, x_t). In this case it would be impossible to recover the average portfolio exposure of a bank to the Libor. The derivative would be collinear with the reputational fixed effect ψ_i. It is only by seeing how a bank's incentives to skew the Libor evolves over time, as they move in and out of the middle eight of the quotes of the other banks, we see variation in their marginal ability to influence the Libor.

The primary assumptions I use to identify my model are (i) the specification of bank profits, (ii) the mean independence of the unobservable cost shock from the observables and (iii) the mean independence of the unobservable cost shock from the lagged interquartile range. Mean independence is weaker than assuming full independence, which would not be appropriate setting given the considerable time-varying heteroskedasticity evident in the data. In this sense, my approach shares a similar feature with the econometric models of time-varying volatility, such as multivariate GARCH models, which are commonly used to model interest rates and other financial variables.[30]

2.6 Results

I present the results of the first step of my estimation procedure in a series of figures. Figure 2.3 shows the marginal impact of all possible quotes for the Bank of Tokyo Mitsubishi on two separate trading days. The blue line corresponds to the realized derivative $(L_{it}^{\Delta} - L_t)/\Delta$, which would obtain if the Bank of Tokyo knew exactly what the day's interquartile range would be.[31] The red line is the estimated derivative $\widehat{\frac{\partial}{\partial q_{it}} \mathcal{L}}_{it}(q_{it}, x_t)$ resulting from my specification of incomplete information. Uncertainty about the private information, and hence the quotes, of a banks' peers smooths out the distribution of its marginal influence on the expected Libor through its uncertainty over the exact location of the pivotal points.

In figures 2.4, 2.5 and 2.6 I present the first stage estimates for three banks along with the relative locations of their quotes compared to the interquartile range of the quotes submitted by their peers. These three figures show the marginal impacts for the quotes banks chose to submit. I also show the marginal impacts for all of the possible quotes a bank could have chosen to submit through time in figure 2.8. Here we can see the tight and short distribution of marginal impacts before the financial crisis, which then grows in breadth and height as the crisis progresses, and then spikes around the default of Lehman brothers. The marginal impacts shown in figure 2.3 are time slices of this

[30] See Bauwens et al. (2006) for a survey of these influential time series models.

[31] This corresponds to the assumption in Snider and Youle (2012), and generates a "bunching" prediction they use to develop a test for strategic manipulation.

contour map for two different days.

The second stage parameter estimates are shown in table 2.4. Most of the banks wish to push the Libor downwards, but not all. Thus the Libor is the result of a "tug of war" in which some banks wish to skew it upwards, and others wish to skew it downwards. Given banks' radically different locations, primary currencies of operation, and business models, this may be expected. The winning side ultimately depends upon the distribution of bank borrowing costs.

I am unable to estimate the first order conditions for the Bank of Tokyo, Societe Generale, HBOS, Norinchukin and WestLB because they do not have CDS spreads for enough of the trading days over the period I consider. In my counterfactual analysis I assume they are truth tellers. Only Societe Generale has been indicted for manipulating the Libor, and the others are not known to have any current ongoing investigations.

Bank of America, Deutschbank and UBS have the smallest incentives to skew. Barclays, the first bank to admit to manipulation, has the largest incentives, although the estimate is not statistically significant. This may be due to a change in behavior over the sample, as seems to be suggested by its quoting behavior in 2.5 and the regulatory investigations. These investigations describe Barclays as moving from portfolio-driven manipulation to a strategy designed to avoid damaging their reputation. See Snider and Youle (2012) for a discussion of Barclays.

2.6.1 Recovering the Manipulation-Free Libor

I have estimated banks' incentives to misreport $\beta \equiv v_i(x_i)/(2\delta)$. Now I construct a "manipulation-free" Libor which would have occurred had their been no incentives to manipulate the Libor by any bank. This is not the "correct" Libor because I have not meaningfully recovered banks' reputational reasons to misreport. Nevertheless, this remains an interesting object to study. Manipulation driven misreporting is probably the more likely source of future Libor distortion, as quotes will soon be anonymous – a policy change that will mitigate banks' concerns about their reputations.

These counterfactual manipulation-free quotes $\widehat{q_t}$ are related to the observed quotes as follows:

$$\widehat{q_{it}} \equiv q_{it} - \widehat{\beta \frac{\partial}{\partial q_{it}}} \mathcal{L}_i(q_{it}, x_t) \tag{2.21}$$

The attendant manipulation-free Libor is defined from these quotes:

$$\widehat{L_t} \equiv L(\widehat{q_t}) \tag{2.22}$$

The difference between this recovered Libor and the actual Libor, $\hat{L} - L$, as well as a 90% confidence interval using the standard errors I compute in my estimate of the incentives β, is shown in figure 2.8. The Libor was largely accurate prior to the financial crisis. Afterwards, however, it diverged downwards as the recovered Libor becomes greater than the actual Libor. Eventually this leads to a nearly eight basis point difference at the end of my sample. This is well after the main storm of the crisis had subsided and the Libor was at a very low level.

The difference between these Libors over time is driven by the evolution of marginal impacts across banks. Generally speaking, most banks want to push it down and marginal impacts rise during the crisis. There was more ability for manipulators to alter the rate as the interquartile range of quotes increased during the financial crisis. This initial increased dispersion was likely driven by the underlying dispersion in interbank borrowing costs and greater market volatility in this period. In more peaceful times, however, the truncation mechanism of the Libor works well and greatly diminishes banks abilities to misreport when their costs are tightly aligned.

2.7 Alternative Libor Aggregation

The FCA has considered alternative ways to form the Libor from the underlying quotes from the panel banks.[32] In particular, they examined alternative benchmarks caluclated by the average and the median of submitted quotes, as well as a random quote selection.

[32] Wheatley (2012)

Their analysis, however, was limited. They took the submitted quotes as given, and examined how the Libor would have looked if it were computed from these quotes in ways other than the interquartile mean. This approach ignores any behavioral responses due to an alteration of the underlying mechanism.

In this section, I analyze these alternatives by recomputing the Bayesian-Nash equilibria under alternative mechanisms.[33] This approach takes the observable characteristics and estimated portfolio incentives of banks as given and allows behavior to vary based on the aggregation method used. I describe the process for calculating the Bayesian-Nash equilibria of my model in detail.

2.7.1 Calculating Bayes-Nash Equilibria

As discussed above, the details of my model and environment lead to a number of computational barriers. The first challenge is the high dimensionality of my game. The second is the very real possibility of multiple equilibria. My solution involves an algorithm similar to an EM-algorithm in maximum likelihood estimation, avoids discretizing the state space, and can compute the bounds on the resulting Libors across the set of possible equilibria.

I avoid the curse of dimensionality by exploiting the limited form of strategic interaction in my model. A bank is only concerned about the behavior of the other banks insofar as they alter its ability to manipulate the overall Libor. More specifically, the marginal impact on the expected Libor is a sufficient statistic for the action profile of the other banks when considering a given bank's payoffs. This naturally leads to an algorithm which simulates this marginal impact. My algorithm alternates between iterating on bank's first order conditions given their beliefs and then updating their beliefs about

[33] I don't independently consider the random and the mean aggregation mechanisms because my banks are risk neutral. Under random selection, where each bank has an equal probability of being selected, leads to identical incentives for banks to submitting misleading quotes in my model, and would generate identical behavior. The random aggregation mechanism, however, would produce a more volatile Libor.

their marginal impact given the current strategies played by the other banks stored in memory. Rather than having to invert large matrices, my computational procedure allows me to iterate on the first order conditions and then check that they are a global optimum. Such a check is necessary because the Libor function is not concave and the first order conditions are not sufficient for optimality.[34]

While I face a potentially large set of equilibria, the set is meaningfully bounded. This is because the best response of every bank is bounded, and the bounds do not depend on the strategy profiles of the other banks. Intuitively this is because banks face increasing convex costs for misreporting and their marginal impact on the expected Libor is bounded between zero and one eighth. No bank would skew more than they would if there impact on the marginal Libor was always one eighth, and no bank would skew in a direction contrary to these costs. These bounds on the best responses bound the possible equilibria, and are common to the environment of Snider and Youle (2012) where they are studied further.[35]

My approach is to initialize the values in my algorithm in such a way as to find the extremal equilibria. The first minimizes the resulting equilibrium Libor and the latter maximizes it. This then bounds the Libors resulting from all of the other equilibria in my model. I find these extremal equilibria by weighting the banks that want to skew it in that direction a maximal amount, and weighting the banks that wish to skew the opposite direction a minimal amount. This leads the updating procedure to settle on

[34]This approach to computing the equilibrium is somewhat similar to computational methods developed for different environments. Krusell and Smith (1998) develops an algorithm to compute macroeconomic models in which agents need to forecast the evolution of the wealth distribution in order to forecast prices and make saving decisions. They find that the first moment of this distribution is a sufficent statistic for households to make nearly optimal decisions. My algorithm to compute the Libor is also inspired by the algorithms of Pakes and McGuire (2001) and Fershtman and Pakes (2012).

[35]It is possible that the multiplicity in my environment is reduced due to the incomplete information version of my game. Bajari et al. (2010) find that an incomplete information version of an entry game has considerably less equilibria as the number of players grow than when information is complete. By the time they reach numbers of players near those in my game, the set of possible equilibria has reduced drastically. Nevertheless, I aspire to be robust to multiple equilibria in my counterfactual analysis by using these bounds.

the extremal equilibrium. In the lowest Libor equilibrium banks which "short" the Libor skew as aggressively as is consistent with individual rationality. Banks which are "long" in the Libor do the same in the highest Libor equilibrium.

2.7.2 Iterative Algorithm

I repeatedly iterate upon the necessary first order conditions and use the resulting quotes to update equilibrium beliefs until those beliefs converge. The pseudocode for my algorithm is as follows. For each period t,

1. Initialize the beliefs G^0 banks have about their equilibrium marginal impacts on the expected Libor.

2. Begin an inner loop (iterating on m):

   (a) Simulate S draws of c_t^s using a calibrated cost process and the realized history of costs $\{c_\tau, \sigma_\tau\}_{\tau=1}^t$.

   (b) Generate the corresponding optimal quotes for each bank, using the necessary first order condition and the current value of G:

   $$q_{it}^s = \frac{v_i(x_i)}{2\delta} G_i^{m-1} + c_{it}^s$$

   (c) Update the value of G^m using the simulated using fixed weights $w \in (0, 1)$ according to:

   $$G_i^m = wG_i^{m-1} + (1-w)\frac{\sum_{s=1}^S L(q_{it}^s + \Delta, q_{-it}^s) - L(q_{it}^s, q_{-it}^s)}{\Delta}$$

   (d) Continue until G^m converges.

3. Compute q_t using the converged G^m and do a grid search to check the quotes are truly optimal as the first order conditions are merely necessary. If they not

optimal, I begin the process again for a different initialization of G^0.

How G^0 is initialized will typically determine the resulting equilibrium. The choice of weights w govern the speed and reliability of the convergence of beliefs. I am able to compute lowest-Libor equilibria by initializing those banks incentives to push the Libor down with large G_i^0's and those with incentives to push the Libor up with G_i^0's of zero. Every bank with $V_i < 0$ will skew their quotes downwards as much as possible. The other banks won't skew at all. This will likely not be consistent with equilibrium and banks will revise their beliefs and consequently skew less. Over time this will converge to the lowest possible Libor value that is consistent with individual rationality. I perform the converse exercise to compute the highest-Libor equilibrium.

2.7.3 Alternative Libor Aggregation Mechanisms

My counterfactual results are presented in table 2.5. The mean has the largest systematic bias, the median has the least, and the interquartile range is intermediate. The poor performance of the mean is due to the fact that manipulators are always able to skew irrespective of the other banks on the panel. Under the mean, there is no limit to skewing aggressively, as there is no relationship between how far a bank skews from its peers and its marginal impact. With the interquartile range the marginal impact of a bank's quote on the expected Libor is decreasing the more extreme the quote is, relative to the others. This is especially so for the median. This is likely the reason why the median performs so much better in the Bayes-Nash equilibria I compute.

An additional reason for the excellent accuracy of the median may be from a strategic complementarity. I find that most banks wish to push the Libor downwards. If other banks skew aggressively downwards, you are more willing to skew also, as this increases the marginal impact of your rate when moving downwards. Under a median, all banks skew less because their incentives are lessened, which then has knock on affects due to this complementarity.

In table 2.5 I also calculate the yearly losses accruing to U.S. Municipalities under these alternative mechanisms. This is based on the estimate that municipalities held \$500 billion notional value of interest rate swaps during this period, in order to hedge the floating rate municipal bonds, whose rates are determined in a competitive market. It is also important to note that, as far as swaps market is concerned, this is a small amount. Banks on the Libor panel routinely have trillions of dollars of notional value interest rate swaps, as shown in their call reports. Nevertheless, these municipalities net exposures' are clear and they are a prominent class of institutions that suffered from a depressed Libor.

2.8 Conclusion

The main problem with the Libor is that the interbank market in which it is determined has become increasingly small. If instead the interbank market were thick and competitive, banks would not have the ability to modify interest rates in a transactions-defined Libor, nor would they have wide latitude to use expert judgment in a survey-defined Libor. Regulators could also easily compare banks submitted quotes with actual transactions. Unfortunately, the interbank market has been largely replaced by overnight and collateralized forms of financing, and is unlikely to ever return to its prior, active status.

The Libor, on the other hand, will remain an important benchmark for third party contracts for the medium term. In the long run, however, it is entirely possible the market will substitute away from contracts defined on the Libor towards close money market substitutes, such as rates for repurchase agreements or certificates of deposit. In the short run, many institutions which hold large portfolios of Libor denominated contracts are eager to restore the credibility of the Libor(Wheatley (2012)).

I also suggest the Libor mechanism be modified to use only the median of the submitted quotes. I find this can increase the accuracy of the Libor by over 70% in equilibrium. This is because the largest manipulators are able to manipulate less on average, and by smaller amounts.

Table 2.1: Submitted Quotes (3M USD Libor; Week of 12/17/2007)

	Monday	Tuesday	Wednesday	Thursday	Friday
Barclays	5.03	5.03	5.00	4.99	4.95
HBOS	4.98	4.98	4.91	4.95	4.95
Deutsche Bank	4.97	4.96	4.93	4.90	4.85
Norinchukin	4.97	4.94	4.92	4.90	4.88
Bank of Tokyo	**4.95**	4.94	4.91	4.89	4.87
WestLB	**4.95**	4.94	4.92	4.89	4.87
RBC	**4.95**	4.93	4.91	4.88	4.85
Bank of America	**4.94**	4.93	4.92	4.88	4.86
UBS	**4.94**	4.91	4.90	4.88	4.86
Citigroup	**4.94**	4.91	4.90	4.89	4.85
Credit Suisse	**4.93**	4.93	4.91	4.81	$4.87\frac{1}{2}$
Lloyds	**4.93**	4.92	4.92	4.85	4.85
HSBC	4.92	4.91	4.90	4.87	4.83
Rabobank	4.92	4.87	4.87	4.80	4.78
RBOS	4.91	4.90	4.90	4.85	4.80
J.P. Morgan	4.90	4.87	4.89	4.86	4.84
Libor	$4.94\frac{1}{8}$	$4.92\frac{5}{8}$	4.91	$4.88\frac{3}{8}$	$4.85\frac{3}{4}$

Snapshot of the raw quote data for the sixteen banks which compose the three month U.S. dollar Libor which, along with the six month, is the most important for third party dollar contracts. The Libor is calculated using the average of the middle eight submitted quotes on each day. The bolded quotes are those which determine Monday's Libor rate.

Table 2.2: Comparison of Pre- and Post-Crisis Interquartile Ranges

Date	1/2/2007		10/21/2008
Barclays	5.36	Barclays	4.10
HBOS	5.36	Credit Suisse	4.00
Deutsche Bank	5.36	Norinchukin	3.95
Norinchukin	5.36	RBOS	3.95
Bank of Tokyo	**5.36**	WestLB	**3.9**
WestLB	**5.36**	Bank of Tokyo	**3.9**
RBC	**5.36**	HBOS	**3.85**
Bank of America	**5.36**	Deutsche Bank	**3.85**
UBS	**5.36**	UBS	**3.85**
Citigroup	**5.36**	RBC	**3.82**
Credit Suisse	**5.36**	Bank of America	**3.75**
Lloyds	**5.36**	HSBC	**3.75**
HSBC	5.36	Lloyds	3.7
Rabobank	5.36	Rabobank	3.6
J.P. Morgan	5.36	J.P. Morgan	3.55
RBOS	$5.35\frac{1}{2}$	Citigroup	3.5
Libor	5.36	**Libor**	$3.83\frac{3}{8}$

A comparison of quotes and interquartile ranges for two different days in the sample period. Before the crisis, banks submitted very similar quotes and the interquartile range was often small, as seen in the left panel. After the crisis, the interquartile range expanded as banks submitted more hetergeneous quotes, as seen on the right.

Table 2.3: Summary Statistics by Bank

	Frequency quote **Below** Interquartile	Frequency quote **Within** Interquartile	Frequency quote **Above** Interquartile
Bank of Tokyo	0.4%	56.7%	42.9%
Bank of America	15.7%	63.4%	20.9%
Barclays	6.1%	60.0%	33.9%
J.P. Morgan	35.9%	58.7%	5.4%
Citigroup	22.5%	76.7%	0.8%
Credit Suisse	2.1%	84.9%	13.0%
DeutschBank	24.5%	63.0%	12.5%
HBOS	6.0%	85.1%	8.9%
HSBC	28.7%	70.4%	0.9%
Lloyds	10.6%	87.7%	1.7%
Norinchukin	0.7%	46.5%	52.8%
Rabobank	27.5%	69.4%	3.1%
RBC	5.3%	87.5%	7.2%
RBOS	33.5%	42.4%	24.1%
UBS	15.9%	77.3%	6.8%
WestLB	2.8%	83.7%	13.5%

The frequencies that the banks in the Libor panel participate below, within, and above the interquartile range of the day's submitted quotes. My identification strategy involves exploiting the variation in this probability of participation over time, so it is comforting to see there are no banks which either always or never participate.

Table 2.4: Estimating Necessary First Order Conditions; Second Stage

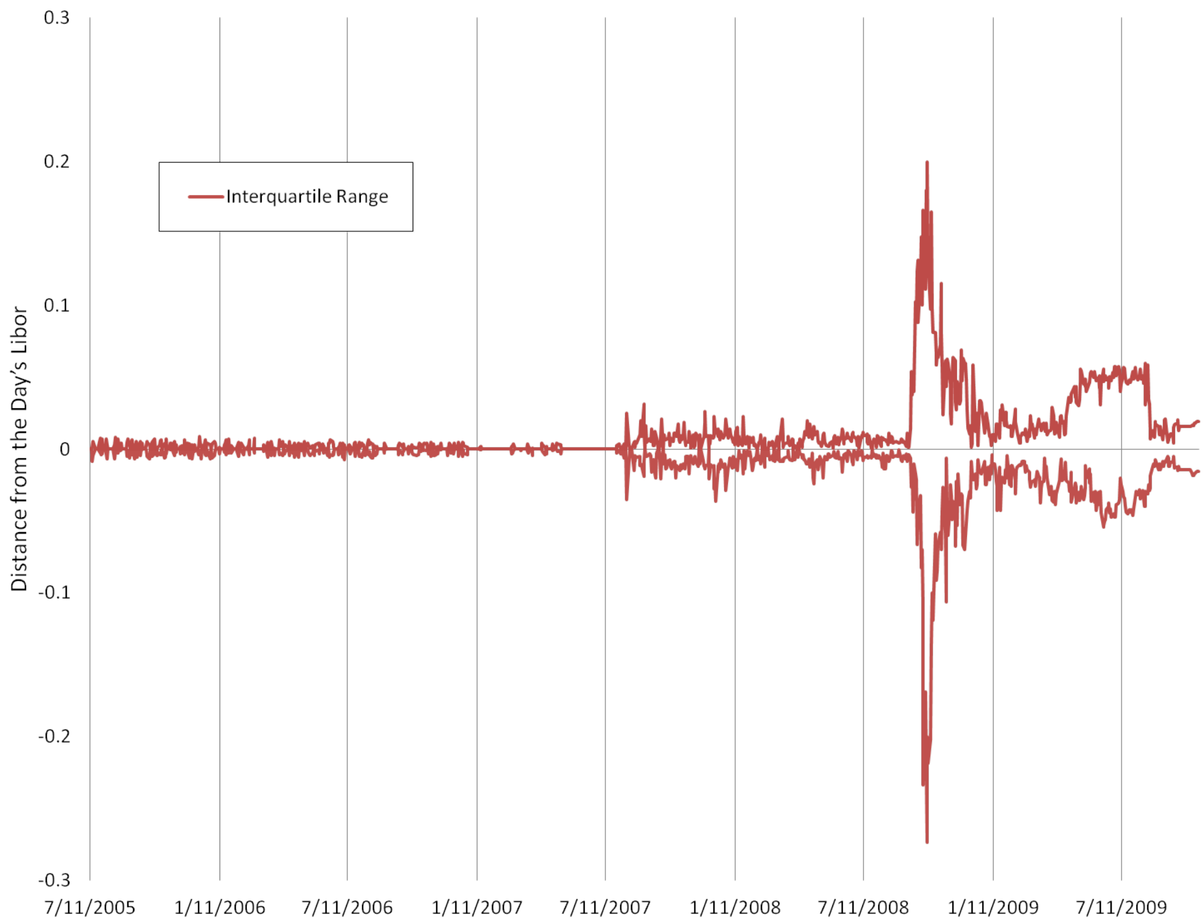
	(Newey-West Standard Errors) N=1025	Point Estimates ($\frac{V_i}{2\delta}$)	Average Skew (bp)
1	Bank of Tokyo-Mitsubishi	-	-
2	Bank of America	-0.66	-3.04
		(0.26)	(1.19)
3	Barclays	-5.29	-13.47
		(9.62)	(24.47)
4	J.P.Morgan	-3.52	-7.90
		(1.25)	(2.81)
5	Citigroup	-1.05	-6.19
		(0.35)	(2.04)
6	Credit Suisse	0.91	5.34
		(0.34)	(2.00)
7	Deutschbank	0.34	1.51
		(0.17)	(0.75)
8	Societe Generale/HBOS	-	-
9	HSBC	-0.96	-4.34
		(0.30)	(1.33)
10	Lloyds	-1.28	-8.41
		(0.56)	(3.68)
11	Norinchukin[1]	-	-
12	Rabobank	-0.82	-4.00
		(0.95)	(4.63)
13	Royal Bank of Canada[2]	-	-
14	Royal Bank of Scotland	0.70	2.15
		(0.26)	(0.80)
15	UBS	0.38	2.36
		(0.09)	(0.58)
16	WestLB[3]	-	-
	Fixed Effects;	Bank-Quarter Effects	
	Additional Controls:	1 Year Senior CDS Spreads by bank Commerical Paper Rate Fed Funds Effective Rate	

Table 2.5: Alternative Libor Mechanisms (Q3 2009)

	Average of Middle Eight	Random or Mean	Median
Average distortion from counterfactual Libor	-3.0bp	-4.7bp	0.0bp
Variance	0.57	0.77	0.53
Sum of squared errors from counterfactual Libor	6.8bp	17.1bp	1.3bp
Loss to U.S. Municipalities (per year)	$150 mill.	$235 mill.	$0

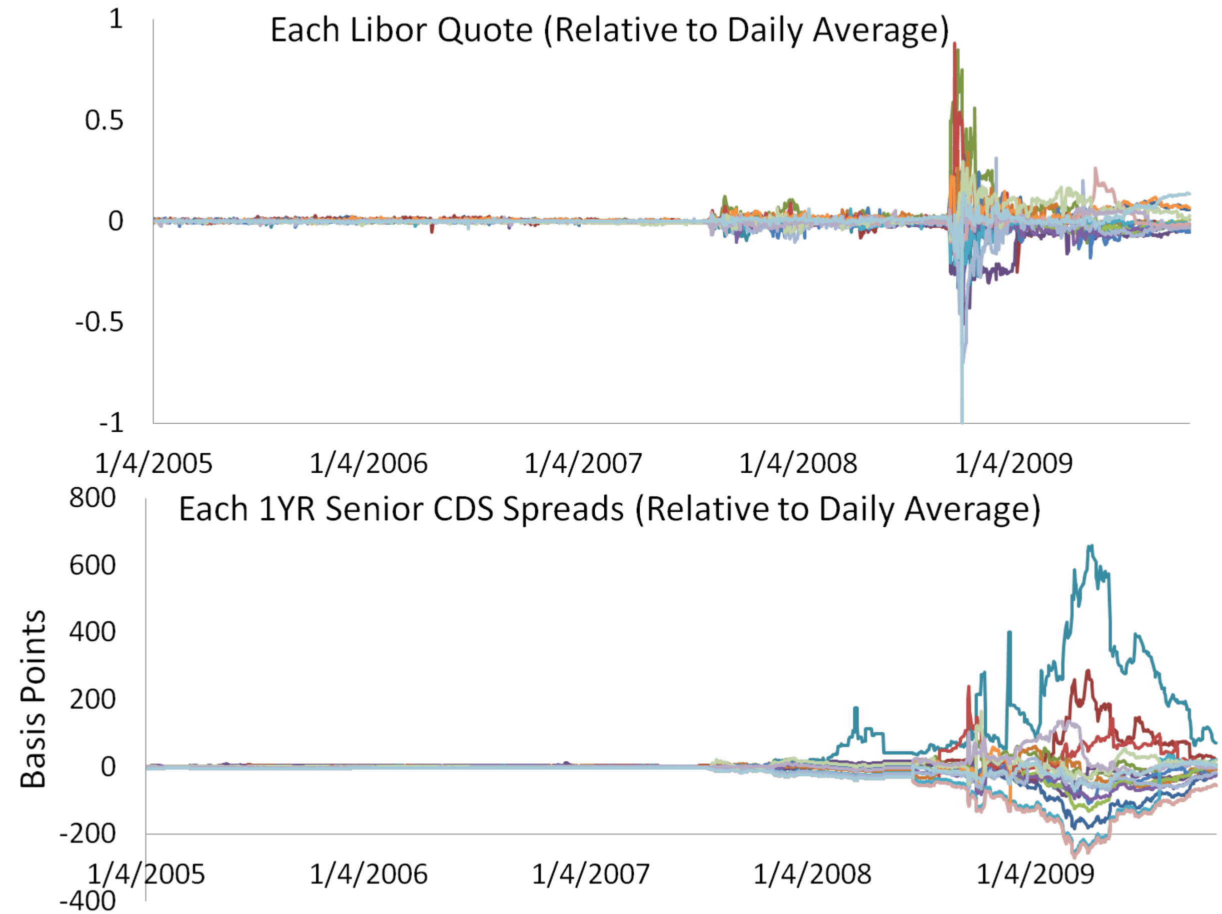
The counterfactual analysis of alternative Libor aggregation mechanisms. A Bayes-Nash equilibria is computed where manipulators have rational expectations and full awareness of the exact mechanism.

Figure 2.1: Interquartile Range of Quotes



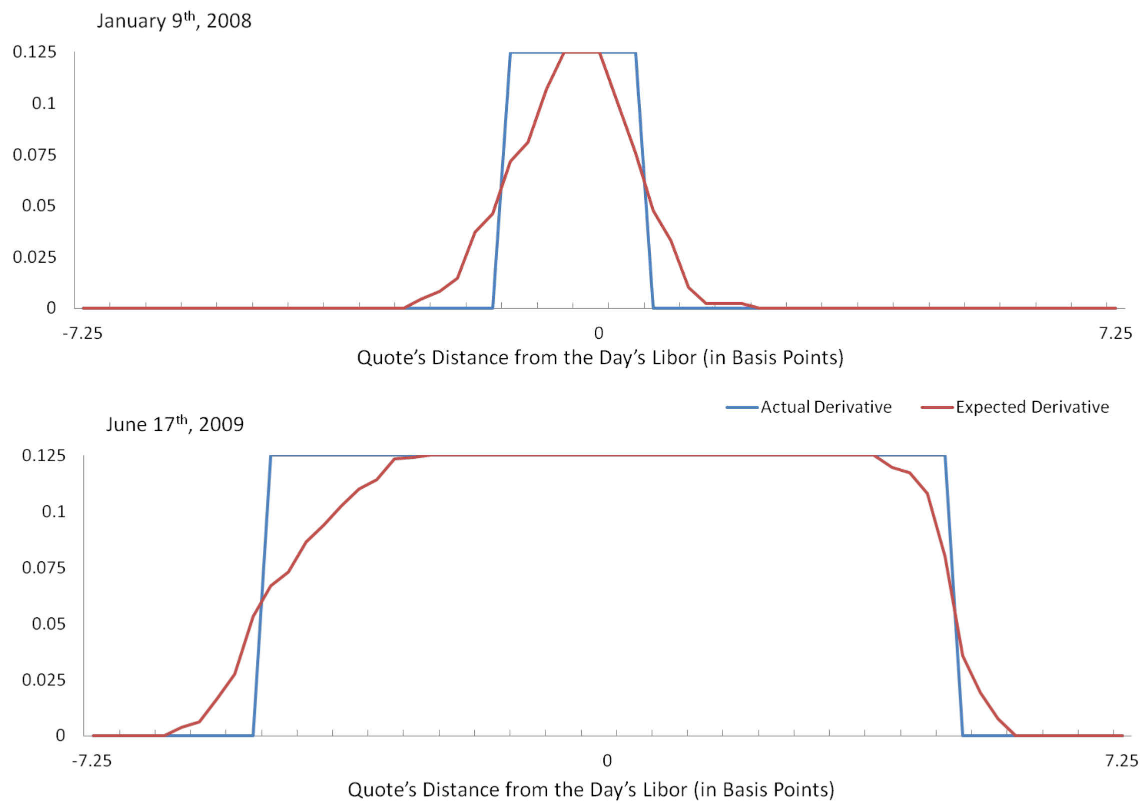
The interquartile range of the quotes submitted by the banks on the Libor panel, relative to the day's Libor fix. The quotes are tightly packed together prior to the financial crisis, but then disperse.

Figure 2.2: Quotes and Credit Risk



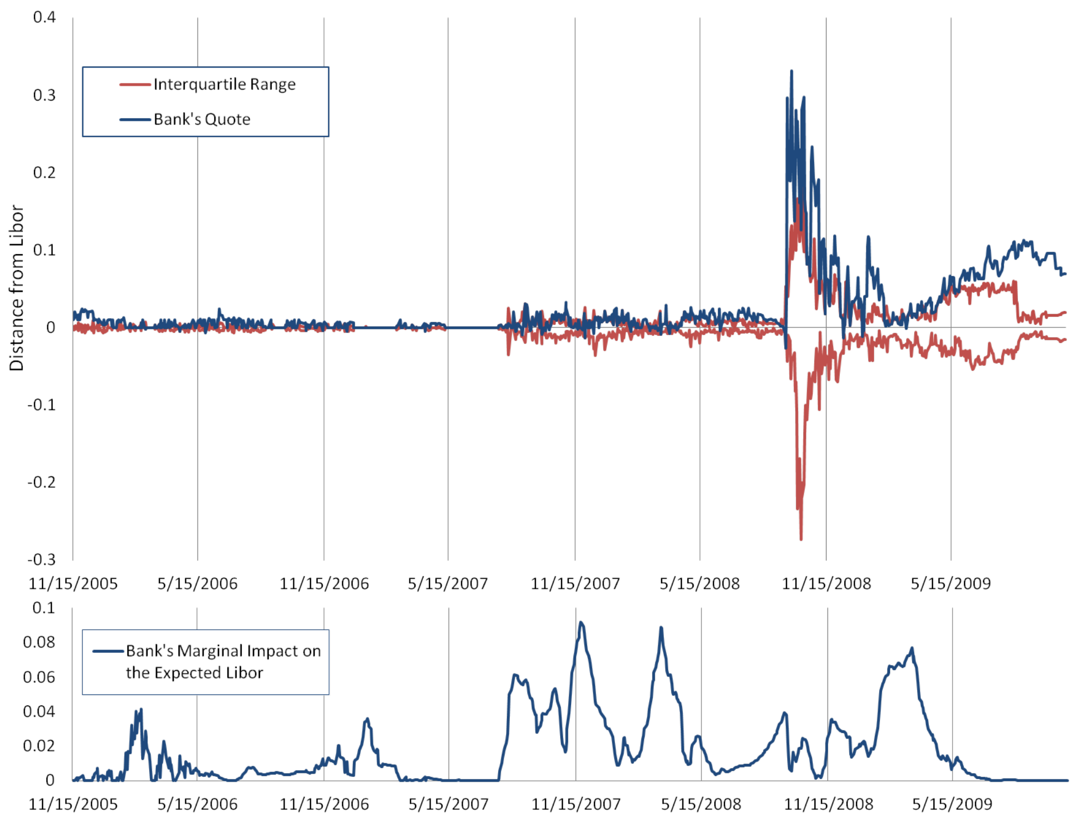
The top graph shows the sixteen submitted quotes over time, relative to the daily average of the quotes. The bottom graph shows the corresponding sixteen one year senior CDS spreads, a measure of credit risk, relative to the daily average.

Figure 2.3: Marginal Impact on Libor (Bank One)



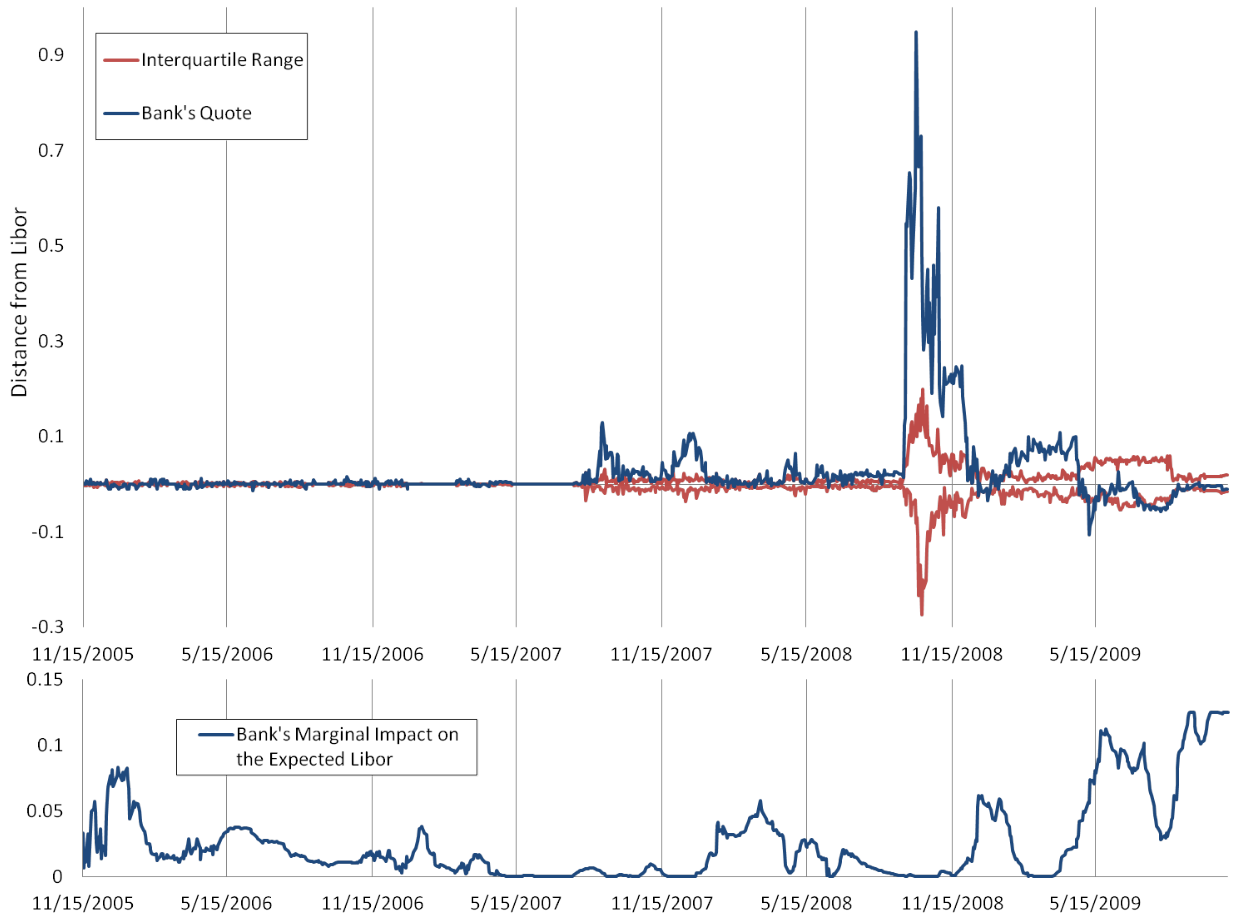
Bank of Tokyo Mitsubishi's marginal ability to influence the Libor based on two separate trading days. The actual derivative would occur in a game of complete information where BoTM knew with certainty what the other banks would quote. The kinks occur at those quotes where BoTM gets kicked out of the interquartile range. With incomplete information, BoTM is uncertain where these kinks will be, and smooths out its marginal ability to influence the rate.

Figure 2.4: First Stage Results for Bank One (Bank of Tokyo Mitsubishi)



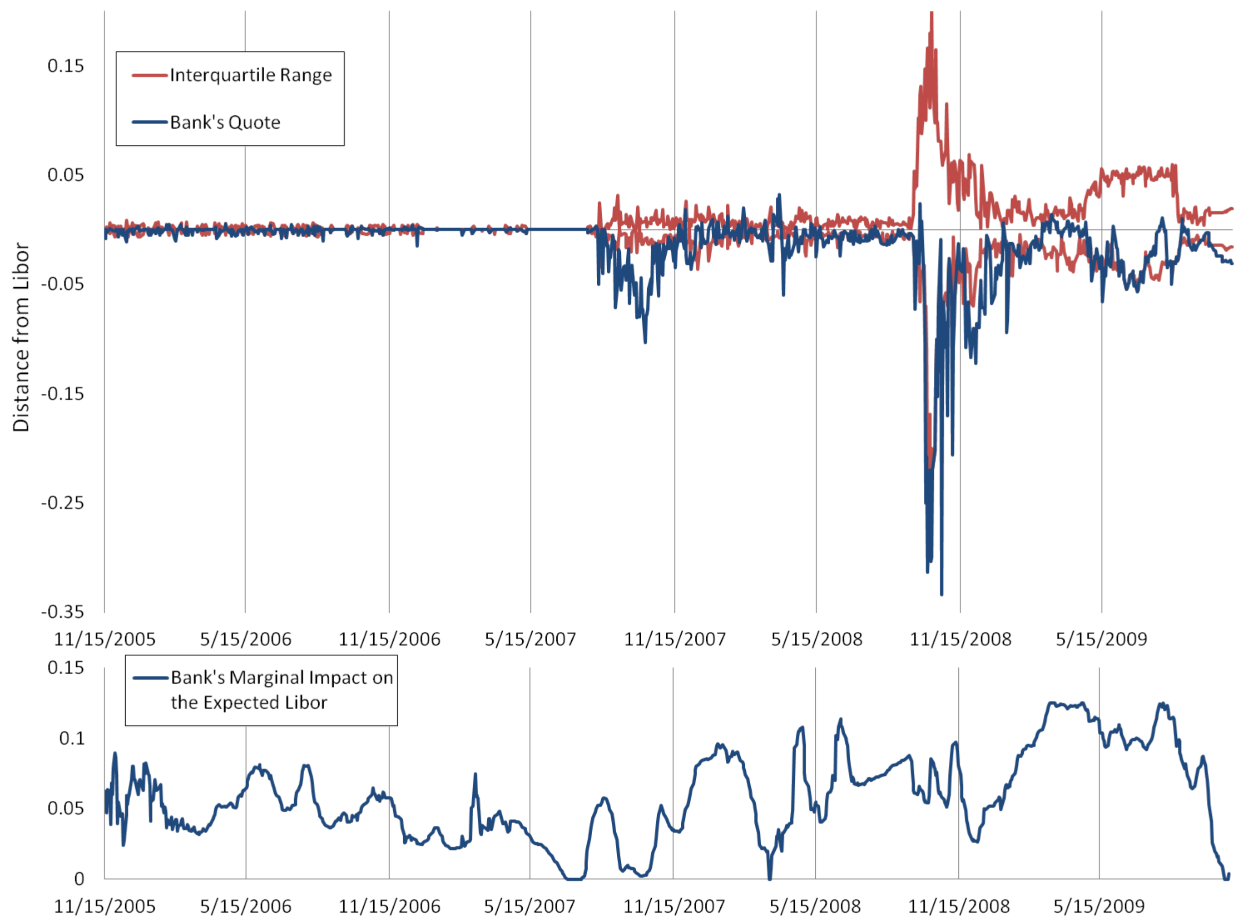
This figure shows two aligned graphs. The top graph shows the quotes of the bank in question relative to the interquartile range of the quotes of its peers, normalized to the day's Libor. The bottom graph shows the results of my first stage estimations, in which I nonparametrically recover the bank's marginal impact upon the Libor. The bank's marginal impact varies over time as it near and further relative to the interquartile range of its peers. Towards the ends of my sample it submits relatively high quotes and has a very low marginal impact on the Libor.

Figure 2.5: First Stage Results for Bank Three (Barclays)



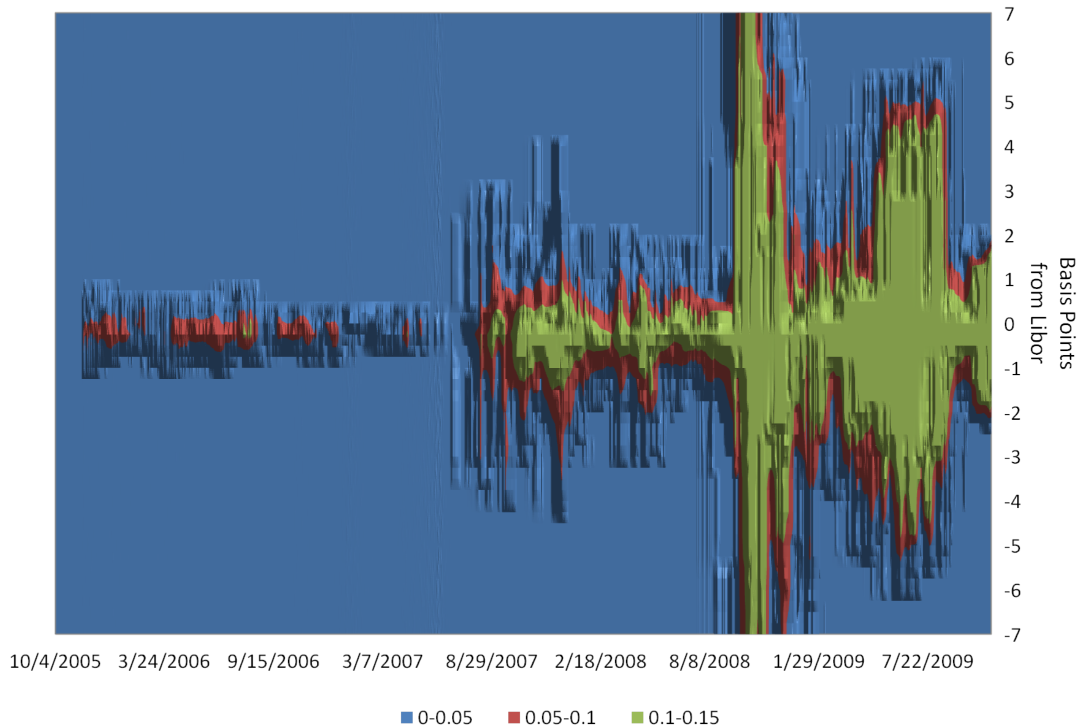
A similar figure to that of 2.4 but now for Barclays. This bank used to submit high quotes relative to the others, and thus had a low marginal impact on the Libor. Its quoting behavior changed in 2009 and moved downwards relative to the quotes of its peers, and typically had a great marginal impact on the Libor.

Figure 2.6: First Stage Results for Bank Five (Citigroup)



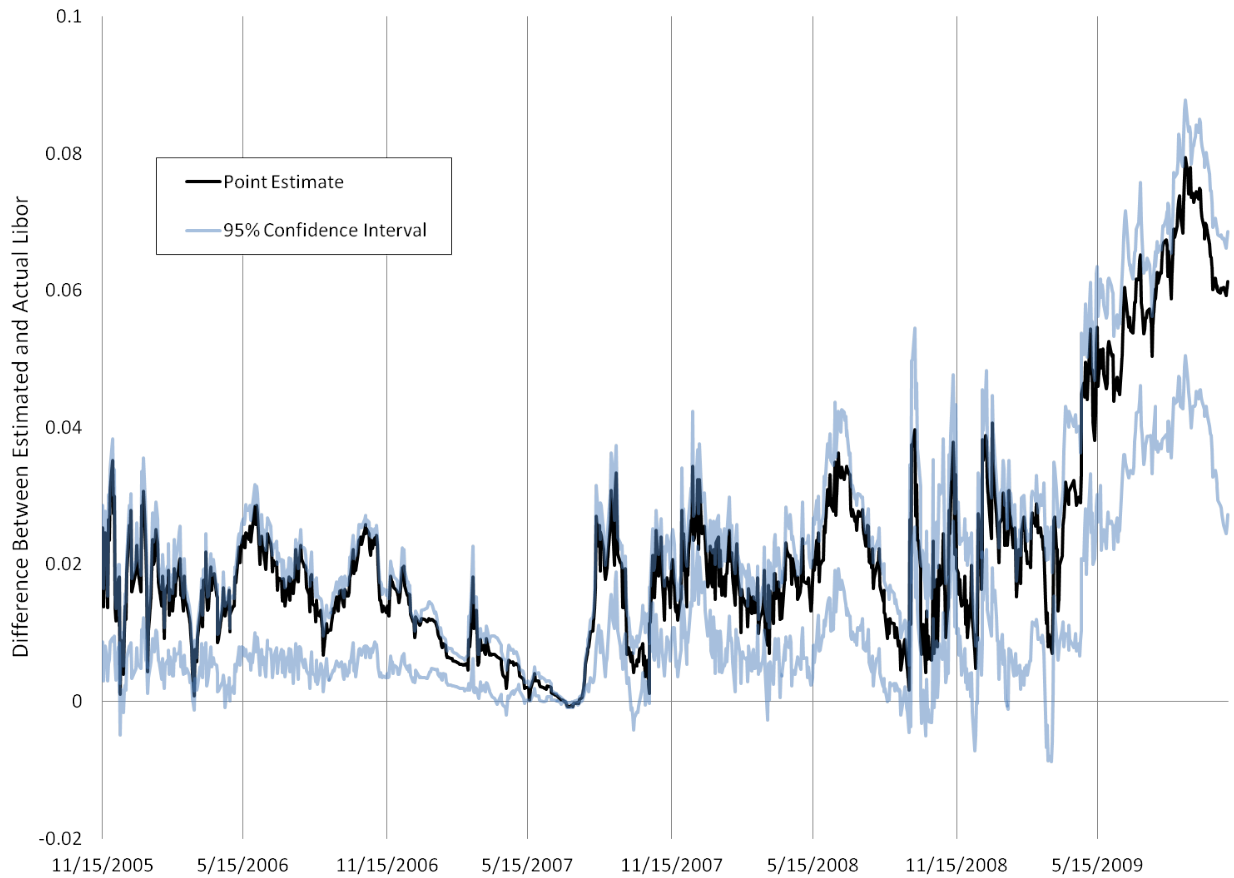
A similar figure to that of 2.4 but now for Citigroup. This bank is positioned relative to the others such that it typically has a high marginal impact on the Libor.

Figure 2.7: First Stage Results for Bank 1 (BoTM): Contour Map



This is a contour map of the first stage nonparametric estimates for bank 1 (Bank of Tokyo Mitsubishi). Results for the other 15 banks are similar. The Y axis is the distance of the bank's submitted quote from the day's realized Libor. The Z-axis, or height, is the estimated derivative of the expected Libor in the bank's quote. The bank has little expected influence on the Libor for extreme quotes, but moreso when quoting near "the middle of the pack."

Figure 2.8: Recovered Libor



The difference between the “manipulation-free” Libor I estimate and the reported Libor. As the series is positive and growing, manipulation distorted the Libor downwards. See section 6 for a discussion.

References

Abrantes-Metz, R. and Bajari, P. (2010). Screens for conspiracies and their multiple applications-extended. *Competition Policy International Journal*, 6(2).

Abrantes-Metz, R., Froeb, L., Geweke, J., and Taylord, C. (2006). A variance screen for collusion. *International Journal of Industrial Organization*, 24(3):467–486.

Abrantes-Metz, R., Judd, G. G., and Villas-Boas, S. B. (2011). Tracking the libor rate. *Applied Economics Letters*, 18(10):893–899.

Abrantes-Metz, R., Kraten, M., Metz, A., and Seow, G. (2012). Libor manipulation? *Journal of Banking and Finance*, 36(1):136–150.

Afonso, G., Kovner, A., and Schoar, A. (2011). Stressed, not frozen: The federal funds market in the financial crisis. *The Journal of Finance*, 66(4):1109–1139.

Aguirregabiria, V. and Mira, P. (2007). Sequential estimation of dynamic discrete games. *Econometrica*, 75(1):1–53.

Athey, S. (2001). Single crossing properties and the existence of pure strategy equilibria in games of incomplete information. *Econometrica*, 69(4):861–889.

Bajari, P., Benkard, L., and Levin, J. (2007). Estimating dynamic models of imperfect competition. *Econometrica*, 75(5):1331–1370.

Bajari, P., Hong, H., and Nekipelov, D. (2010). Computing equilibria in static games of incomplete information using the all-solution homotopy. *Operations Research*, 58.

Bauwens, L., Laurent, S., and Rombouts, J. (2006). Multivariate garch models: a survey. *Journal of applied econometrics*, 21(1):79–109.

Cassola, N., Hortascu, A., and Kastl, J. (2013). The 2007 subprime market crisis through the lens of european central bank auctions for short-term funds. *Econometrica*, 81(4):1309–45.

Ciliberto, F. and Tamer, E. (2009). Market structure and multiple equilibria in airline markets. *Econometrica*, 77(6):1791–1828.

Commission, C. F. T. (2012). Cftc orders barclays to pay $200 million penalty for attempted manipulation of and false reporting concerning libor and euribor benchmark interest rates. Technical report, US Commodity Futures Trading Commission, Washington, DC.

De Paula, A. and Tang, X. (2012). Inference of signs of interaction effects in simultaneous games with incomplete information. *Econometrica*, 80(1):143–172.

Diehl, C. (2013). The libor mechanism and related games. *Unpublished Manuscript*.

Dodd, R. (2010). Municipalities in trouble with derivatives: Muni bombs. *Finance and Development*, 47(2).

DOJ (2013). Press release: Rabobank admits wrongdoing in libor investigation, agrees to pay $325 million criminal penalty. http://www.justice.gov/opa/pr/2013/October/13-crm-1147.html.

Fan, J. and Gijbels, I. (1996). *Local Polynomial Modelling and Its Applications*, volume 66 of *Monographs on Statistics and Applied Probability*. CRC Press.

FCA (2012a). Final notice to Barclays Bank plc. http://www.fsa.gov.uk/static/pubs/final/barclays-jun12.pdf.

FCA (2012b). Final notice to UBS AG. http://www.fsa.gov.uk/static/pubs/final/ubs.pdf.

Ferraz, C. and Finan, F. (2008). Exposing corrupt politicians: The effects of brazil's publicly released audits on electoral outcomes. *Quarterly Journal of Economics*, 123:703–735.

Fershtman, C. and Pakes, A. (2012). Dynamic games with asymmetric information: A framework for empirical work. *The Quarterly Journal of Economics*, 127(4):1611–1661.

Gensler, G. (2012). Remarks of chairman gary gensler. Technical report, European Parliament, Economic and Monetary Affairs Committee, Brussels, Belgium.

Gilbert, M. (2007). Barclays takes a money-market beating. *Bloomberg*.

Guerre, E., Perrigne, I., and Vuong, Q. (2009). Nonparametric identification of risk aversion in firstprice auctions under exclusion restrictions. *Econometrica*, 77(4):1193–1227.

Harrington, J. (2005). Optimal cartel pricing in the presence of an antitrust authority. *International Economic Review*, 46(1):145–169.

Jacob, B. and Levitt, S. (2003). Rotton apples: An investigation of the prevalence and predictors of teacher cheating. *Quarterly Journal of Economics*, 118(3):843–877.

Krishna, V. (2009). *Auction Theory*. Academic Press, 2 edition.

Krusell, P. and Smith, A. (1998). Quantitative macroeconomic models with heterogeneous agents. *Journal of Political Economy*, 106(5):867–896.

Kuo, D., Skeie, D., and Vickery, J. (2012). A comparison of libor to other measures of bank borrowing costs. *Federal Reserve Bank of New York, Working Paper*.

Kuo, D., Skeie, D., Vickery, J., and Youle, T. (2013). Identifying term interbank loans from fedwire payments data. *Federal Reserve Bank of New York, Working Paper*.

McCrary, J. (2008). Testing for manipulation of the running variable in the regression discontinuity design. *Journal of Econometrics*, 142(2):698–714.

Mollenkamp, C. and Whitehouse, M. (2008). Study casts doubt on key rate; wsj analysis suggests bank may have reported flawed interest rate data for libor. *Wall Street Journal*, (A1).

Olken, B. and Barron, P. (2009). The simple economics of extortion: Evidence from trucking in aceh. *Journal of Political Economy*, 117(3):417–452.

Pakes, A. and McGuire, P. (2001). Stochastic algorithms, symmetric markov perfect equilibrium, and the 'curse' of dimensionality. *Econometrica*, 69(5):1261–1281.

Pakes, A., Ostrovsky, M., and Berry, S. (2007). Simple estimators for the parameters of discrete dynamic games (with entry/exit examples). *RAND Journal of Economics*, 38(2):373–399.

Pesendorfer, M. and Schmidt-Dengler, P. (2008). Asymptotic least squares estimators for dynamic games. *The Review of Economic Studies*, 75(3):901–928.

Porter, R. (2005). Detecting collusion. *Review of Industrial Organization*, 26(2):147–167.

Preston, D. (2012). Rigged libor hits states-localities with $6 billion: Muni credit. *Bloomberg*.

Robinson, P. (1988). Root-n-consistent semiparametric regression. *Econometrica: Journal of the Econometric Society*, pages 931–954.

Seim, K. (2006). An empirical model of firm entry with endogenous product-type choices. *RAND Journal of Economics*, 37(3):619–640.

Snider, C. and Youle, T. (2010). Does libor reflect bank borrowing costs? *UCLA Unpublished Working Paper*.

Snider, C. and Youle, T. (2012). The fix is in: Detecting portfolio driven manipulation of the libor? *Available at SSRN 2189015*.

Stigum, M. and Creszensi, A. (2007). *Stigum's Money Market*.

Wheatley, M. (2012). The wheatley review of libor. Technical report, HM Treasury.

Wolfers, J. (2006). Point shaving in ncaa basketball. *American Economic Review*, 96(2):279–283.

Zitzewitz, E. (2012). Forensic economics. *Journal of Economic Literature*, 50(3):731–69.