

Essays in Macroeconomics

A DISSERTATION
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

Bernabe Lopez Martin

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Doctor of Philosophy

Timothy J. Kehoe, Advisor

June, 2014

© Bernabe Lopez Martin 2014
ALL RIGHTS RESERVED

Acknowledgements

I would like to thank my advisor Timothy J. Kehoe for his guidance and encouragement throughout my years of graduate studies as well as the members of my dissertation committee Cristina Arellano, Fabrizio Perri, Terry Roe and Kei-Mu Yi. I would also like to thank Naoki Takayama for his patience and enthusiasm. I benefitted from thoughtful feedback from participants at various seminars and conferences and the Workshop in Trade and Development at the University of Minnesota. I gratefully acknowledge the financial support provided by the University of Minnesota Doctoral Dissertation Fellowship, the Drs. Mary and Robert Litterman Fellowship and the Federal Reserve Bank of Minneapolis. The essays in this dissertation also benefitted from the access to data provided by the Economic Research Forum (Egypt), DANE (Colombia) and INEGI (Mexico).

Dedication

To Mariana, my family and friends who made this possible.

Abstract

This dissertation is composed of three separate essays.

In the first essay of the thesis I develop a quantitative framework of firm dynamics where the size of the informal sector is determined by financial constraints and the burden of government taxation. Informal sector firms do not pay taxes but have no access to external finance and face sector specific costs that limit their optimal scale of production. For taxes and financial constraints parameterized for a country like Egypt, I find losses in total factor productivity of over 28%, with drops in the wage level of 68% and in output per worker of 60% relative to the benchmark economy that resembles the US. I then evaluate the impact of three types of policies. I find that the effects of a complete elimination of formal sector registration costs are small (in contrast with theoretical work that ignores financial constraints and the informal sector). I evaluate the introduction of a stochastic tax-enforcement technology that detects informal sector firms and forces them to register with a probability that depends on the scale of production. The effects of this policy are negative: wages and output per worker are lower as well as aggregate TFP, in spite of the fact that the size of the informal sector is reduced. Improving the access to credit for formal sector firms increases wages, aggregate TFP and output per worker while reducing the size of the informal sector.

In my second essay I study the consequences of low financial development in an environment where firms can invest to increase productivity. Lower financial development (i.e., lower access to credit) increases the dispersion of the marginal productivity of capital across firms (misallocation), therefore lowering aggregate production efficiency. However, models of firm dynamics with financial constraints generate modest losses due to misallocation relative to those found in the empirical literature. For my version of this benchmark I compute losses of 3.8% of aggregate TFP for Colombia and 7.3% for Mexico. I revisit these results by constructing a quantitative model of firm dynamics with endogenous accumulation of firm productivity. In this model, financial constraints diminish the incentives of firms to invest in increasing firm productivity, reducing firm

productivity growth. Additionally, for the firms that make investments to increase productivity, financial constraints become more persistent (compared to a model where firm productivity is purely stochastic). This channel amplifies the losses from misallocation to 15.8% for Colombia and 14.7% for Mexico. The model can partially account for the lower life-cycle productivity growth of firms in economies with underdeveloped financial markets and is consistent with more persistent constraints (measured through capital/output ratios) for the most productive units in a panel of manufacturing establishments for Colombia.

In the last essay of this thesis, Naoki Takayama and I study the long-run consequences of recessions for young individuals and the impact of government taxation. Recessions generate large increases in youth unemployment rates and young unemployed workers suffer significant losses in terms of the expected present discounted value of their labor earnings. We build a life cycle model with on-the-job human capital accumulation and aggregate and idiosyncratic productivity shocks (extended to consider ex-ante heterogeneous workers). The unemployment rate for young workers is higher and we find an important quantitative impact of the tax-wedge (consistent with cross-country empirical estimates): in countries where the tax-wedge is higher, unemployment rates are amplified, particularly for young workers. We compute the long-term earnings losses of individuals that lose their job in different states of the economy and find that losses are bigger: (1) in worse aggregate states of the economy, (2) for younger individuals, (3) in economies with a higher tax wedge, (4) for ex-ante lower ability individuals.

Contents

Acknowledgements	i
Dedication	ii
Abstract	iii
Contents	v
List of Tables	ix
List of Figures	xi
1 Informal Sector Misallocation	1
1.1 Introduction	1
1.2 Relation to the Literature	3
1.3 Empirical Analysis	5
1.3.1 Distribution of Employment and Firms	6
1.3.2 The Informal Sector	7
1.3.3 Firm Level Data and Evidence	8
1.4 Model	11
1.4.1 Production Technology	12
1.4.2 Workers	13
1.4.3 Formal Sector Entrepreneurs	14
1.4.4 Informal Sector Entrepreneurs	15
1.4.5 Aggregation	17

1.4.6	Equilibrium	18
1.5	Firm Dynamics and Misallocation	18
1.6	Parameters & Taxes	21
1.6.1	Predetermined and Benchmark Parameters	21
1.6.2	Institutional and Country-Specific Parameters	23
1.7	Quantitative Analysis	24
1.7.1	Baseline Results	25
1.7.2	Robustness Exercise	26
1.8	Policy & Reforms	26
1.8.1	Elimination of Registration Costs	27
1.8.2	Financial Sector Reform	29
1.8.3	Government Enforcement of Registration	29
1.9	Conclusion	30
2	From Firm Productivity Dynamics to Aggregate Efficiency	32
2.1	Introduction	32
2.2	Relation to the Literature	36
2.3	Empirical Motivation	38
2.3.1	Data Description	38
2.3.2	Distribution of Employment and Establishments	39
2.3.3	Establishment Life-Cycle Dynamics	40
2.4	Stylized Model of Innovation and Financial Constraints	43
2.5	Quantitative Model	46
2.5.1	Preferences	48
2.5.2	Production Technology	48
2.5.3	Innovation Technology	49
2.5.4	Workers	50
2.5.5	Formal Sector Entrepreneurs	51
2.5.6	Financial Markets	52
2.5.7	Informal Sector Entrepreneurs	53
2.5.8	Equilibrium	54
2.6	Baseline Parameters	55

2.6.1	Common Parameters Across Countries	55
2.6.2	Country Specific and Institutional Parameters	57
2.7	Quantitative Analysis	58
2.7.1	Main Results	58
2.7.2	Firm Life-Cycle Productivity and Employment Growth	62
2.7.3	Firm Dynamics in the Model and Data	67
2.7.4	Sensitivity Analysis	68
2.8	Conclusions	68
3	The Blighted Youth: An International Comparison of Life-Cycle Un-	
	employment	70
3.1	Introduction	70
3.2	The Long-Term Impact of Recessions on Youth	73
3.3	Policies and Labor Markets	77
3.4	Baseline Environment	79
3.4.1	Demographics	79
3.4.2	Preferences and Technology	80
3.4.3	Markets	80
3.4.4	Timing	81
3.4.5	Value of the Worker	82
3.4.6	Value of the Firm	83
3.4.7	Determination of Wages	84
3.4.8	New Vacancies and Free Entry Condition	85
3.5	Block Recursive Equilibrium	85
3.6	Quantitative Analysis	86
3.6.1	Baseline Parameters and Function Specifications	86
3.6.2	Business Cycle Simulations	88
3.6.3	Simulations of a Recession	89
3.6.4	Losses due to Unemployment in Present Discounted Values	92
3.7	Final Comments	92
	Bibliography	94

Appendices	109
A Appendix to Chapter 1	110
A.1 Size Distribution of Firms	110
A.2 Tax Structure	111
A.3 Robustness Exercise	113
A.4 Algorithm Outline	113
B Appendix to Chapter 2	115
B.1 Output-Capital Ratios and Financial Constraints	115
B.2 Size Distribution of Establishments	115
B.3 Dispersion in Output-Capital Ratios in Colombia	117
B.4 Micro-Enterprizes in Mexico	117
C Appendix to Chapter 3	119
C.1 Block Recursive Equilibrium	119
C.2 Introducing the Tax Wedge	129
C.3 Expected Present Discounted Value of Earnings	131

List of Tables

1.1	Size Distribution of Firms	6
1.2	Probit Estimates of Informality	10
1.3	Regressions of Capital Labor Ratios (OLS)	11
1.4	Predetermined and Benchmark Parameters	22
1.5	US Distribution of Employment and Firms by Size Class	23
1.6	Institutional and Country-Specific Parameters	24
1.7	Baseline Results	25
1.8	Robustness Results	26
1.9	Reforms	28
2.1	Size Distribution of Establishments	39
2.2	The Life-Cycle of Establishments: Employment	41
2.3	The Life-Cycle of Establishments: Productivity	42
2.4	Predetermined Parameters	55
2.5	Calibrated Parameters - US Moments	56
2.6	Institutional/Country Specific Parameters	57
2.7	General Results	60
2.8	Misallocation	61
2.9	Baseline Results: Firm Knowledge Capital and Size	63
2.10	Regressions with Simulated Model and Data Firms	67
2.11	Regressions with Simulated Model and Data Firms	68
3.1	The Long-Term Impact of Recessions on Youth	74
3.2	Unemployment Rate Equations	77
3.3	Parameters	87
3.4	Business Cycle Simulations: No Heterogeneity	89

3.5	Business Cycle Simulations: With Heterogeneity	89
A.1	Distribution of Enterprises and Employment by Size Class	111
A.2	Tax Structures	112
A.3	Distribution of Employment and Firms by Size Class	113
B.1	Distribution of Establishments and Employment	116
B.2	Dispersion in Output-Capital Ratios	117

List of Figures

1.1	Occupation Choice	20
2.1	Employment over Firm Life-Cycle (Model)	62
2.2	Occupation Choice (Mexico/Colombia)	63
2.3	$\log(\text{ability} \cdot \text{knowledge})$ - Mexico	65
2.4	$\log(\text{ability} \cdot \text{knowledge})$ - US	65
2.5	$\log(\text{knowledge})$ - Mexico (formal sector)	66
2.6	$\log(\text{knowledge})$ - US (formal sector)	66
3.1	Simulation of a Recession	91
3.2	Losses due to Unemployment	93

Chapter 1

Informal Sector Misallocation

1.1 Introduction

In many developing countries, like Turkey and Egypt, the informal economy accounts for over 30% of non-agricultural employment¹ and well above 30% of GDP.² The informal sector has long been associated with financial underdevelopment and the excessive burden of taxes and regulation, and attributed significant losses in terms of aggregate productivity, capital accumulation and output. This paper has two objectives: (1) to quantify the losses associated with the existence of a large informal sector, (2) to evaluate policies intended to ameliorate these losses.

I build a model of entrepreneurship and stochastic firm dynamics based upon the frameworks of occupational choice and industry equilibrium of Lucas (1978) and Hopenhayn (1992). Individuals differ in their ability to operate a decreasing returns to scale technology but are homogeneous as workers. More able entrepreneurs set-up firms and decide whether to belong to the formal or informal sector (an early example of self-selection in a static environment is Rauch, 1991). The trade-off is the following: firms

¹Statistics of informal sector employment are discussed in Section 1.2.

²Schneider & Enste (2000) describe nine widely applied methodologies of estimating the size of the shadow economy, highlighting their respective advantages and weaknesses. Data based on labor force and micro-business surveys (as the ones used here) are generally preferred. The shadow economy includes all market-based legal production of goods and services that are concealed from public authorities to avoid taxation, social security contributions and compliance with regulation in general, while pure household production, voluntary nonprofit (social) services and criminal activities are excluded.

in the informal sector avoid taxation and the costs of registration and face an additional cost of production that represents the costs of not having access to enforcement of commercial contracts, demands from corrupt government officials, difficulties in reaching customers, etc. (De Soto, 1989; Perry et al., 2007). This cost limits the optimal scale of firms in the informal sector. The size of the informal sector is determined by financial constraints and the burden of government taxation.

Financial constraints have a leading role in the model. All firms have to make input payments at the beginning of each period, before production takes place. These payments have to be financed and entrepreneurs face collateral constraints. Informal sector entrepreneurs have no access to external finance.³ The entrepreneur may choose to start-off in the informal sector and later transition to the formal sector depending on his assets and ability⁴. In equilibrium the size of the informal sector depends on the burden of taxes and access to credit in the formal sector. Intuitively, lower access to credit in the formal sector reduces the demand for labor and the wage level. This leads to individuals with lower entrepreneurial ability to set-up firms in the informal sector.

The quantitative exercise consists in comparing a benchmark model economy calibrated to be consistent with well functioning financial markets and a small informal sector (the US) with economies with underdeveloped financial markets and medium/large informal sectors (Turkey and Egypt). I find that the frictions considered go a long way in accounting for the differences in aggregate total factor productivity (TFP), the average size of firms and output per capita. Then I evaluate the impact of three different policies: (1) the complete elimination of the costs of registration in the formal sector, (2) an improvement of access to credit in the formal sector, (3) the introduction of a tax-enforcement technology where informal sector firms may be detected with a probability that is increasing in the scale of production, and then are forced to register and

³The difficulties for informal sector firms to collateralize their assets were already stressed in the work of De Soto (1989). In the same manner, Straub (2005), Catão et al. (2009) and Perry et al. (2007) emphasize the requirements of financial institutions such as credible documentation of physical location and pledgeable assets of the firms, their financial statements, etc. which, because of their nature, are not feasible for informal sector firms.

⁴Alternatively, Levenson & Maloney (1998) analyze the dynamic nature of informality in a model where entrepreneurial ability is initially unknown, surviving and more able entrepreneurs transition to the formal sector as they learn their ability.

enter the formal sector. I find that the impact of a policy that eliminates registration costs is small relative to the observed differences in aggregate variables between Egypt and the US. This is consistent with country level case studies that analyze this type of reforms. The results from the tax-enforcement technology are negative: wages, TFP and output per worker are reduced even as the size of the informal sector falls. Increasing the access to credit is key in reducing the size of the informal sector and improving aggregate outcomes.

The rest of the paper is organized as follows. In Section 1.2 I overview the related literature. In Section 1.3 I compare the size distribution of firms, define the informal sector and provide estimates of its size in Turkey and Egypt and exploit firm level data to document empirical regularities of informal sector firms compared to those in the formal sector. The model is presented in S.1.4. A brief characterization of the equilibrium and sources of misallocation are discussed in S.1.5. Calibration and institutional parameters are discussed in S.1.6. In S.1.7 I present the main quantitative results and conduct and discuss policy experiments in S.1.8. Section 1.9 concludes.

1.2 Relation to the Literature

This paper builds upon several strands of the development literature. A brief overview follows.

Misallocation. This paper is related to a literature that underscores the macroeconomic implications of distortions to the allocation of resources across firms, typically focusing on aggregate TFP and output losses, capital accumulation and the size and productivity distribution of firms. Hsieh & Klenow (2009) find that gaps in marginal products of labor and capital across plants can explain a large part of the differences in TFP between China and India compared to the US. Busso et al. (2012) perform a similar empirical exercise for Mexico analyzing informal and formal sector firms. Restuccia & Rogerson (2008) analyze the potential quantitative effects of idiosyncratic tax schemes,

suggesting the importance of evaluating specific distortions. Accordingly, Guner, Ventura & Xu (2008) and Garcia-Santana & Pijoan-Mas (2011) study policies that impose restrictions on the size of firms. Barseghyan & DiCecio (2011) assess the role of entry costs, including regulatory/legal fees for registering firms as well as non-regulatory sunk costs. Financial frictions have also been extensively studied. For recent quantitative examples see Buera, Kaboski & Shin (2011) who analyze endogenous collateral constraints and Greenwood, Sanchez & Wang (2013) who focus on the role of costly state verification, among many others⁵. Midrigan & Xu (2013) challenge the view that financial constraints can represent a quantitatively relevant source of misallocation, arguing that firms can accumulate assets to escape this restriction reducing their impact on aggregate efficiency.

Informal Sector. There is a sizable literature that analyzes the determinants of the size of the informal sector. Many results are now standard:⁶ the size of the informal sector decreases as credit availability improves in the formal sector (Straub, 2005; Antunes & Cavalcanti, 2007; Quintin, 2008; Catão et al., 2009; D’Erasmus & Moscoso-Boedo, 2012); the size of the informal sector increases with labor-market restrictions, heavier regulation of entry and the tax burden of the formal sector and decreases with enforcement of legal obligations (Djankov et al., 2002; Ihrig & Moe, 2004; Antunes & Cavalcanti, 2007; Perry et al., 2007; Prado, 2011; Leal Ordoñez, 2013). At the firm level, compliance with regulation is associated with better access to external finance and informal sector firms are found to be less capital and skilled-labor intensive, less productive, smaller and younger (Levenson & Maloney, 1998; Amaral & Quintin, 2006; Perry et al., 2007; La Porta & Shleifer, 2008; Pratap & Quintin, 2008; World Bank, 2010; de Paula & Scheinkman, 2011; Busso, Fazio & Levy, 2012).

⁵The growth of smaller firms is particularly constrained by the underdevelopment of the financial system. This mechanism has been found to be empirically more robust than other obstacles to firm-growth such as inefficient regulation and taxation, inadequate enforcement of property rights, political instability, poor provision of infrastructure, etc. (Beck & Demirgüç-Kunt, 2006).

⁶Schneider & Enste (2000), Tybout (2000) and Perry et al. (2007) offer extensive sets of references related to the informal sector. The literature also analyzes other aspects of the informal sector related to inequality, labor market segmentation, human capital accumulation, the consequences of trade reform, optimal audit policies, corruption and rent-seeking bureaucracies. These topics are outside the scope of this project. De Soto (1989), a classic reference in the literature, already emphasized the impact of an overburdening regulatory system, weak property rights enforcements and lack of access to external finance in the informal sector.

These two strands of literature are linked. Relatively recent papers have analyzed the informal sector in models that incorporate firm dynamics. Amaral & Quintin (2006) and Antunes & Cavalcanti (2007) combine occupational choice with credit constraints to quantify the role of this friction taking into account the role of the informal sector. Quintin (2008) finds that lax tax enforcement alone does not suffice to generate a large informal sector and contractual imperfections are needed. D’Erasmus & Moscoso-Boedo (2012) develop a model of firm dynamics with imperfect debt enforcement and also evaluate the impact on misallocation, capital accumulation and the size distribution of firms.

The present paper is perhaps most closely related to D’Erasmus & Moscoso-Boedo (2012). As in their model I incorporate the effect of entry costs, taxes and financial frictions (bankruptcy efficiency in their case, collateral constraints in my model). They find that these frictions can explain a drop in total factor productivity of up to 25%. The effect of each friction is analyzed and they find that entry costs account for 3/4 of TFP differences across countries. In contrast to their work, in my model a larger informal sector is associated with lower wages. This provides an incentive for individuals with lower entrepreneurial ability to set up firms, an important mechanism in my model.

Different authors have considered the role of tax enforcement: Fortin, Marceau & Savard (1997), de Paula & Scheinkman (2011), Prado (2011), Leal Ordoñez (2013). It is not surprising that this type of policy has received attention in policy and corporate circles (UK National Audit Office, 2008; McKinsey Global Institute, 2006). I extend the consideration of government tax enforcement to a setting with firm dynamics.

1.3 Empirical Analysis

In this section I document the key empirical facts that motivate this paper. First I compare the distribution of employment and firms by size class for the US, Turkey and Egypt. In Turkey and Egypt a very large share of the non-agricultural private labor force belongs to very small firms (less than 10 workers), which is typically observed for

developing economies. Next, I define informal sector employment and provide measures of its size in these countries. This information will guide the calibration of the model. Then, using micro-level databases for Turkey, Egypt and Morocco, I document how the formal/informal status of the firm is related to education and experience of the manager, size and age of the firm, capital/labor ratios and revenue per worker, the ratio of skilled workers in the firm, while controlling for other variables that are standard in the literature.⁷

1.3.1 Distribution of Employment and Firms

In the US firms with less than 10 workers, represent 74.5% of the total number of firms and account for 12% of employment.⁸ For Turkey and Egypt the share of firms with less than 10 workers is approximately 95% and account for 67% and 77% of employment, respectively.

Table 1.1: Size Distribution of Firms

dist. of employment	<10	10-49	≥50
US	11.8	19.1	69.1
Turkey	67.0	16.0	17.0
Egypt	77.3	10.3	12.4
dist. of firms	<10	10-49	≥50
US	74.5	20.5	5.0
Turkey	95.0	3.2	1.8
Egypt	95.7	4.0	0.3

Recent work by Garcia-Santana & Ramos (2013) documents, at a cross-country level, a significant relationship between average firm size and productivity both at the micro and aggregate levels. Unsurprisingly, it is widely accepted that the preponderance of smaller firms in developing countries is related to the informal sector (see Tybout, 2000).

⁷Examples in this literature are: Levenson & Maloney (1998) for Mexico, de Paula & Scheinkman (2011) for Brazil, World Bank (2010) for Turkey (for this country I use the same dataset) and La Porta & Shleifer (2008) using cross-country firm level data.

⁸See the Appendix for a description of the sources for Table 1.1.

1.3.2 The Informal Sector

In this section the concept of informal sector is defined and estimates of its size are provided for the countries of interest. As emphasized in the literature, informality encompasses different phenomena. In developed economies, informality is generally associated with tax evasion and undeclared labor in registered firms. In emerging economies it is typically associated with small unregistered firms that avoid all or most forms of taxation. I make an important distinction here, following ILO (2012):

Employment in the informal sector is an enterprise-based concept and covers persons working in units that have informal characteristics in relation to, e.g., the legal status, registration, size, the registration of the employees, their bookkeeping practices, etc. Informal employment is a job-based concept and encompasses those persons whose main jobs lack basic social or legal protections or employment benefits and may be found in the formal sector, informal sector or households.

Informal employment can include workers in larger firms with relatively better access to finance that comply with most of their legal obligations. The focus of this paper is on the informal sector. In Turkey, 32% of employment in non-agricultural activities was unregistered (informal) in 2003 (this statistic is 52% for total employment). I compute informal sector employment as the sum of unregistered employers, unregistered self-employed and unpaid unregistered family members and casual employees, which amounted to 21.2% of non-agricultural private employment⁹ in 2003 (TurkStat and ILO Labor Statistics). Estimates of informal sector employment in the US are not available¹⁰, illegal foreign workers represent 3.5% of the workforce (OECD, 2004). However, illegal immigrant work is a different concept from informal sector employment.

⁹In 2007, for example, 26% of employers were unregistered. This is not a large informal sector compared to other developing economies. See ILO (2012) for a sample of countries where data for both informal employment and informal sector employment is available. For comparison, in India the figures are 83% and 67% respectively in non-agricultural activities, in Mexico: 54% and 34%.

¹⁰Neither the BLS nor the ILO have data on informal employment for the US, where it is generally associated to work by illegal migrants (OECD, 2004). The share of the labor force without pension contributions was 7.8% in 2003 (World Development Indicators).

In Egypt, the share of informal wage workers in the private non-agricultural sector is 81.5% (Abdelhamid & El Mahdi, 2003). Wahba (2009) finds¹¹ that informal employment represented 76% of total private, non-agricultural waged employment in 2006 (80.5% in 1998). Of total micro and small enterprises, 82% are informal (El Mahdi, 2002; Ministry of Foreign Trade, 2003). Since small and micro enterprises contribute to 77% of the jobs in the non-agricultural private sector¹², we can reach a preliminary estimation that informal sector employment represents 63% of total non-agricultural private employment. In the baseline calibration of the model I conservatively target an informal sector that accounts for 50% of the labor force.

1.3.3 Firm Level Data and Evidence

The firm level data was collected by teams supervised by Dr. Alia El Mahdi (Egypt), Dr. Bachir Hamdouch (Morocco) and Dr. Sems Ozar (Turkey), as part of a project conducted by the Economic Research Forum (ERF): Promoting Competitiveness in Micro and Small Enterprises in the MENA Region (Middle East and Northern Africa). A detailed description of the dataset and methodology for Turkey is found in Ozar (2006). The database¹³ includes, for each country, information on approximately 5,000 micro and small enterprises (less than 50 workers) in urban areas and covers all sectors except agriculture.

Among the main activities at the four digit level (ISIC, 3rd Revision) in Turkey we

¹¹Data from the Egypt Labor Market Surveys, carried out by the Economic Research Forum in cooperation with CAPMAS. An even more conservative estimate for informal sector employment can be computed considering only informal (no contract, no social security): self-employed, employers, unpaid family workers, casual workers. This results in approximately 40% of the labor force 1998.

¹²In Egypt, to acquire a business licence it takes 372 days and 127 administrative steps passed before 50 public entities. Dissolution and settlement procedures take 25 bureaucratic steps during 244 days and a cost equivalent to over 20 monthly salaries of a worker. A bankruptcy process takes 53 bureaucratic steps, 653 days and the equivalent of well over 50 monthly salaries of a worker (Abdelhamid & El Mahdi, 2003). Given these obstacles, the size of the informal sector is hardly surprising.

¹³In the case of Turkey, the survey was designed so that the weighted results capture the actual distribution of micro-small enterprises across sectors of activity, size, location and gender. The sampling was national in coverage and chosen by stratified, multi-stage systematic sampling method by TurkStat. On the basis of pre-test surveys and assessment of field experience, questions judged to be inaccurate were modified or excluded. Several questions were identified to ensure the consistency among the responses of the interviewees and participation in the survey was voluntary.

observe: maintenance and repair of motor vehicles (5020, 14.9% of observations), other retail in specialized stores (5239, 8.6%), restaurants, bars and canteens (5520, 7.7%), retail sale of textiles, clothing, footwear and leather goods (5232, 6.7%), retail sale in non-specialized stores with food, beverages or tobacco predominating (5211, 5.3%). Two manufacturing activities enter the top ten sectors at the four digit levels: manufacturing of wearing apparel, except fur apparel (1810, 2.6%) and manufacture of furniture (3610, 2.6%). In Egypt, retail sale of food, beverage and tobacco (5220, 26.9%) represents a larger share of total observations and the top manufacturing industry is that of other fabricated metal products (2899, 3.7%) at the seventh position, otherwise the group of top ten activities is unchanged. In Morocco activities are classified in 44 categories so that they are not directly comparable. Nevertheless, the main sectors are broadly unchanged.

The status of a business, formal/informal, is defined according to compliance with legal requirements: a formal firm has a licence, business registration and registration with tax authorities and social security subscription.¹⁴ With this baseline definition the share of informal firms is 36%, 71% and 24% for Egypt, Morocco and Turkey, respectively (in this database). Different (more lax) definitions of informality were used as a robustness check on the regressions below, with very similar results. The different definitions of informal status are highly correlated as firms decide simultaneously on the compliance with respect to different obligations (see for example Perry et al. Ch. 5, 2007).

Table 1.2 shows the probit estimates for informality (statistically insignificant variables are excluded depending on the country). The probability of informality status is decreasing in education and work experience of the manager, age of the firm, the number of workers, years of current management, the ratio of skilled workers, revenue

¹⁴For the ERF Egypt database, in particular, registration certificates were verified during the survey, the results are similar across countries. In all cases surveys were voluntary, strictly confidential and conducted by non-government organizations. As would be expected, there is evidence of very high correlation between a firm's registration status, its tax and social security contributions compliance in other countries, as in Catão et al. (2009), for example.

per worker, and whether the manager is the owner of the firm. The probability of informality status is increasing in the number of relatives of the manager working in the firm and whether the entrepreneur was born in a rural area. In the case of Egypt, the skill ratio is computed as male skilled workers over the total number of workers.

Table 1.2: Probit Estimates of Informality

database	Turkey	Egypt	Morocco
workers	-0.053***	-0.199***	-0.080***
working experience	-0.011***	-0.003*	-0.004**
years of educ. manager	-0.039***	-0.041***	-0.024***
firm age	-0.025***	-0.080***	-0.016***
firm age squared	0.001***	0.001***	0.001***
years current management	-0.007**	—	—
revenue/workers	-0.004***	-0.002*	-0.009***
skilled workers/total	-0.201**	-0.216**	—
owner is manager	-0.146**	—	—
number of relatives work	0.091***	0.098***	—
born rural	0.238***	0.304***	—
industry controls	2 dig.	2 dig.	44 categ.
geographic controls	—	govern.	—
pseudo R2	0.10	0.19	0.13
n. observations	3821	4707	5051
Statistical significance: *** 1%, ** 5%, * 10%.			

Table 1.3 shows the results of OLS regression with $\log(\text{capital-labor})$ ratios as the dependent variable. Capital is defined as the book value of structures, buildings, tools, equipment and inventories. Again, different definitions of informality were used with similar results: for example defining informality simply as firms not registered with the tax administration implied even larger coefficients of informality on the regressions of the capital labor ratios. Regressions on revenue per worker as the dependent variable also showed clear significant and negative coefficients of informality.¹⁵

¹⁵C.I. Jones offers a discussion of the challenges in estimating productivity, even though the literature has found that revenue per worker and true measures of productivity tend to be correlated (in La Porta & Shleifer, 2008). The general lack of book-keeping, recalling errors, volatility of production and

Table 1.3: Regressions of Capital Labor Ratios (OLS)

database	Turkey	Egypt	Morocco
informal	-0.235***	-0.187***	-0.091**
working experience	0.006**	0.004***	0.012***
years of educ. manager	0.056***	0.037***	0.058***
years current management	0.010***	—	—
firm age	0.012***	—	—
skilled workers/total	0.265***	—	0.165***
log(revenue/workers)	0.431***	0.537***	0.769***
born rural	-0.110**	—	-0.136***
constant	13.57***	6.40***	7.82***
industry controls	3 dig.	3 dig.	43 categ.
R2	0.34	0.30	0.46
n. observations	3788	4799	5048

Statistical significance: *** 1%, ** 5%, * 10%.

1.4 Model

The model builds upon the frameworks of occupational choice and industry equilibrium of Lucas (1978) and Hopenhayn (1992). There is a continuum of individuals that face the decision of being workers or entrepreneurs in every period. These individuals differ in their ability as entrepreneurs but are homogeneous in terms of their productivity as workers.

Entrepreneurs have access to a decreasing returns to scale technology and have the option of conducting their business in the informal or formal sectors. The trade-off for this decision is as follows: firms in the informal sector do not pay taxes and avoid the fixed cost of registration and initial minimum capital requirement. However, the volume

fungibility of production inputs with household production make the estimation of productivity of small informal sector firms particularly difficult. Regardless of this difficulty, crude measures of productivity such as revenue per worker are negatively correlated with informality. The total factor productivity (TFP) gap between formal and informal sector firms is approximately 150 percent for manufacturing and 140 percent for services in Turkey (World Bank, 2010). As stressed in the literature, these differences can be due to self-selection in addition to intrinsic characteristics of informal and formal sector firms.

of their output determines the probability of detection by the authorities. If detected, informal sector entrepreneurs have to pay the fixed cost of registration and enter the formal sector, or lose their assets and become workers. There is a convex cost specific to production in the informal sector, motivation for which is provided below. Formal sector entrepreneurs have access to external finance as determined by collateral constraints.

Time is discrete and the problem of individuals is dynamic: they are permitted to accumulate financial wealth and may find optimal to start their enterprise in the informal sector until they reach a certain level of financial wealth and then register to operate as a formal sector firm (a decision that will also depend on a transitory productivity shock and permanent entrepreneurial ability). The analysis is restricted to the steady state of a small open economy¹⁶ with no aggregate uncertainty.

1.4.1 Production Technology

Firms produce an homogeneous final good that serves as the numeraire. All firms have access to a standard production technology with decreasing returns to scale, but they differ in the entrepreneurial parameter φ , as given by the owner and manager of the firm.¹⁷

$$q = \varphi e^a f(k, l)^\gamma \quad \text{with} \quad f(k, l) = k^\alpha (l + \psi)^{1-\alpha} \quad (1)$$

where k is capital equipment, total labor input is the sum of hired labor $l \geq 0$ and the work of the entrepreneur¹⁸ $\psi \in [0, 1]$, γ is the span-of-control parameter of Lucas

¹⁶Credit conditions in developing countries have been largely associated with exogenous factors such as policy-controlled interest rates, liquidity conditions in international financial markets and intermediation efficiency and market power in the financial sector (Catão et al., 2009). Higher interest rates typically observed in developing countries have been attributed to inefficient and uncompetitive financial markets (Greenwood, Sanchez & Wang, 2013). The impact of higher interest rates will be evaluated as a robustness exercise. Note that Midrigan & Xu (2013) find larger misallocation losses for a closed economy relative to a small open economy in their model.

¹⁷This ability parameter is permanent and not determined by the fact that informal sector firms may operate at a lower scale to avoid detection.

¹⁸This technology accounts for the labor input of managers in micro-firms, a segment that accommodates a large part of the labor force in developing countries. Additionally, low financial assets do not affect the possibility of exploiting this minimum level of labor input.

(1978), which determines the returns to scale (strictly lower than one). In addition to the differences in the permanent ability component, the firm is subject to productivity shocks a , which follow a discrete state Markov process with transition density $\Lambda(a' | a)$, where $e^{\bar{a}} = 1$ results from the unconditional mean of a . This matrix is constructed as a discrete representation of an AR(1) process. Individuals are indexed by state variables $s = (\varphi, a, b)$, where b are financial assets, and $z \in \{w, i, f\}$ denotes whether the individual is a worker or an entrepreneur in the informal or formal sectors, respectively. $M(s, z)$ is the mass of individuals over the state (s, z) .

1.4.2 Workers

Every period a mass of individuals is born¹⁹ and their entrepreneurial ability is drawn from a density distribution $h(\varphi)$. Individuals receive an endowment of financial assets \bar{b} (which can be thought of as bonds). The initial endowment of assets may correspond to the redistribution of the financial wealth of individuals that die in any given period or to an exogenously given (calibrated) value. The initial shock a is drawn from the unconditional distribution derived from $\Lambda(a' | a)$.

Individuals value consumption of the final good through their lifetime²⁰ utility $\mathbb{E}_0 \sum_{t=0}^{\infty} (\beta(1-\delta))^t u(c_t)$, with period utility $u(c) = c^{1-\phi}/(1-\phi)$, discount factor β and probability δ that the individual dies in any period. An individual that dies is immediately replaced by a newly born so that the mass of individuals remains constant.

Individuals choose whether to become workers, offering their labor services inelastically at a wage w per period or they may become entrepreneurs. As workers, individuals are homogeneous and free to move between the informal and formal sectors, thus wages

¹⁹The case of infinitely lived individuals can be accommodated as will be clear below.

²⁰It is standard in the literature to introduce concave utility in the problem of the entrepreneur (as opposed to risk neutral, discounted expected profit evaluations). See, for example, Buera, Kaboski & Shin (2011) and references therein contained. Given the focus of this paper on occupational choice and the problem of small-firm entrepreneurs in developing countries this seems to be the relevant approach. In Egypt approximately 92% of all enterprises have only one proprietor and firms with less than 10 workers account for well over 70% of employment.

are the same for all workers.²¹ The problem of workers amounts to a savings decision, written as the policy function $b' = g_w(\varphi, a, b)$, and their occupation choice:

$$v_w(\varphi, a, b) = \max_{\{c, b' \geq 0\}} u(c) + \beta(1 - \delta) \sum_{\{a'\}} \Lambda(a' | a) v(\varphi, a', b') \quad (2)$$

$$\text{s.t. } c + b' = w + (1 + r)b$$

The occupation choice is:

$$v(\varphi, a, b) = \max\{v_w(\varphi, a, b), v_i(\varphi, a, b), v_f(\varphi, a, b - c_e - r\underline{b})\} \quad (3)$$

where f and i refer to the formal and informal sectors respectively, c_e is the cost of entering the formal sector (there are no fixed costs of entering the informal sector). To register in the formal sector there is a minimum capital requirement condition, $b \geq \underline{b}$. I assume that the worker is able to borrow this amount intra-period to satisfy this requirement, facing only its financial cost.²² We turn next to the problem of the entrepreneurs in the formal and informal sectors.

1.4.3 Formal Sector Entrepreneurs

At the beginning of every period the entrepreneur relinquishes his financial wealth b to a financial intermediary. This deposit earns a net interest rate r . Within the period the entrepreneur is able to collateralize this deposit to make wage payments to hired labor $l(s, f)$ at wage w and obtain capital $k(s, f)$. This collateral constraint restricts the level of inputs used in any given period by $wl + k \leq \lambda_f b$, where λ_f determines the extent to which the formal sector entrepreneur is able to collateralize his financial wealth.²³

²¹The evidence on whether labor markets are segmented between informal and formal sector firms suggests mixed results at best. See the discussions in Maloney (2004), Pratap & Quintin (2008) and Perry et al. (2007, Ch. 3). Wage inequality is outside the scope of this paper so human capital considerations, among others, are ignored.

²²Note that this particular assumption for modeling minimum capital requirements is relatively unburdensome. Barseghyan & DiCecio (2011), for example, include half of the minimum capital requirement in total fixed costs even though, as they stress, these funds might be recoverable. In many countries it is possible to withdraw minimum capital immediately after registration (Djankov, 2009). Results under different specifications will be compared as a robustness exercise.

²³This particular specification of collateral constraint was originally used in Midrigan & Xu (2010).

At the end of the period the entrepreneur makes total factor payments $(1+r)wl$ for hired labor input (including the interest rate cost) and $(r+\nu)k$ (which includes the physical capital depreciation rate ν) and receives $b(1+r)$ from his deposit. We can define intratemporal debt as $d = wl + k - b$, which determines net interest rate payments.²⁴

The entrepreneur also faces an intertemporal decision to save, the solution to which is given by the optimal policy function $b' = g_f(\varphi, a, b) \geq 0$. There are no costs of adjustment for capital, so the endogenous state variable is given by assets b . The dynamic program of this type of entrepreneur is written as:

$$v_f(s) = \max_{\{c,l,k,b' \geq 0\}} u(c) + \beta(1-\delta) \sum_{\{a'\}} \Lambda(a'|a) \max\{v_f(s'), v_w(s')\} \quad (4)$$

$$\text{s.t. } c + b' = (1-\tau)\pi(s, f) + (1+r)b \quad \text{and} \quad wl + k \leq \lambda_f b$$

with firm profits as follows (omitting s on the right hand side):

$$\pi(s, f) = q - (1+r)wl - (r+\nu)k \quad (5)$$

Entrepreneurs in the formal sector face taxes to profits τ .²⁵ Capital depreciates at a rate ν per period. An entrepreneur in the formal sector may choose to become a worker, registration status is lost, but cannot switch directly into the informal sector. The assumption $\beta(1-\delta)(1+r) < 1$ is required to allow the collateral constraints to be quantitatively relevant.

1.4.4 Informal Sector Entrepreneurs

The specification of the collateral constraint for informal sector entrepreneurs is the same as in the formal sector, but the extent to which they are able to collateralize their financial wealth is lower, this is given by $\lambda_i < \lambda_f$ (as a benchmark λ_i equals one).

²⁴As will be clear below, in this setup there is no distinction between renting and owning capital. The budget constraint, ignoring taxes, can also be written as $c + b' = q + (1-\nu)k + (1+r)(b - wl - k)$, where the entrepreneur carries financial wealth across periods.

²⁵An alternative tax structure can be specified as a robustness exercise.

The policy function $b' = g_i(\varphi, a, b) \geq 0$ is the optimal solution to their savings problem. The dynamic program for an informal sector entrepreneur is:

$$v_i(s) = \max_{\{c, l, k, b' \geq 0\}} u(c) + \beta(1 - \delta) \sum_{\{a'\}} \Lambda(a' | a) ((1 - \hat{\kappa}) v(s') + \hat{\kappa} \underline{v}(s'))$$

$$\text{s.t. } c + b' = \pi(s, i) + (1 + r)b \quad \text{and} \quad w l + k \leq \lambda_i b \quad (6)$$

where $\hat{\kappa} = \min\{\kappa q, 1\}$ is the probability of being detected²⁶ (increases continuously with output). When detected, the entrepreneur is given the option to pay $c_e + r \underline{b}$ and register to the formal sector, or give up all assets to start again as a worker. If financial assets b are not enough to cover registration costs, the entrepreneur has no choice but to be a worker once again (an entrepreneur with no financial assets is not able to rent capital for production):

$$\underline{v}(\varphi, a', b') = \max\{v_f(\varphi, a', b' - c_e - r \underline{b}), v_w(\varphi, a', 0)\}$$

Profits for informal sector firms (omitting s on the right hand side) are:

$$\pi(s, i) = q(1 - \xi q) - (1 + r)w l - (r + \nu)k \quad (7)$$

where the cost of producing in the informal sector, in addition to input payments, is given by ξq^2 . This cost²⁷ represents the inability to engage in legal contracts (increasing transaction costs), bribes to corrupt officials, the cost of enforcing their property rights when not protected by the government, worse access to infrastructure facilities and services, lack of a fixed location, etc. (De Soto, 1989; Fortin et al., 1997; Levenson & Maloney, 1998; Straub, 2005; Perry et al., 2007; World Bank, 2010). The specification of this cost implies that it becomes increasingly important with size, an incentive for

²⁶Alternatively, de Paula & Scheinkman (2011) opt for a probability that depends on the capital stock of the firm, equal to zero below a particular threshold, with a discrete jump to one afterwards (in a static framework).

²⁷De Soto (1989) very graphically describes a number of practices followed by informal sector entrepreneurs in Peru to avoid detection and punishment by the authorities as well as other costs of informality: dispersion of employees among a number of smaller and less visible workplaces, avoidance of advertising of goods and services, lack of enforcement of commercial contracts, bribes to corrupt officials (10-15% of gross income compared to 1% paid by formal small business), lack of access to letters of credits or warrants. See also Perry et al. Ch. 5 (2007) for further evidence.

informal sector firms to either operate at a suboptimal scale or to register in the formal sector.

The timing is simple: the entrepreneur first makes production and savings decisions. At the end of the period the firm may be detected by the government, in this case the entrepreneur starts the following period in the formal sector, after paying the costs of registration, or goes back to being a worker. Informal sector firms have the option to register and operate in the formal sector at the beginning of every period after observing their shock a . This decision will depend on the productivity shock, permanent ability and assets of the entrepreneur.

1.4.5 Aggregation

The state space is given by $(\varphi, a, b, z) \in S_\varphi \times S_a \times S_b \times Z$, where $S_\varphi = [1, \infty)$, $S_b = [0, \infty)$, $Z = \{w, i, f\}$, $a \in S_a$ takes on a finite number of values. Let $M : S_\varphi \times S_a \times S_b \times Z \rightarrow \mathbb{R}_+$ denote the measure of individuals over the state space and \bar{M} be the total measure of individuals. A mass of individuals is born every period (equal to the mass of individuals that die) and draw entrepreneurial ability φ from a density function $h(\varphi)$. The labor market clearing condition is given by:

$$\sum_{z \in \{i, f\}} \left(\int l(s, z) M(s, z) ds \right) = \int M(s, w) ds \quad (7)$$

Total output in this economy is:

$$Q = \sum_{z \in \{i, f\}} \left(\int q(s, z) (1 - \xi_z q(s, z)) M(s, z) ds \right) \quad (8)$$

where $\xi_i = \xi$ and $\xi_f = 0$.

Government revenues are destined to projects that do not affect the production technology or utility of individuals.

1.4.6 Equilibrium

Given government policies $(\tau, c_e, \kappa, \underline{b})$ and interest rate r , a small-open economy stationary competitive equilibrium consists of:

- quantities $\{q(s, z)\}_{z \in \{i, f\}}$ and production inputs $\{l(s, z), k(s, z)\}_{z \in \{i, f\}}$,
- savings functions $\{g_z(s)\}_{z \in \{i, f, w\}}$,
- wage w , values $\{v(s), v_i(s), v_f(s), v_w(s)\}$, profits $\{\pi(s, z)\}_{z \in \{i, f\}}$,
- invariant measure $M(s, z)$,

such that:

- workers solve (2), formal sector entrepreneurs solve (4) and informal sector entrepreneurs solve (6),
- market clearing condition for labor (7) holds, the proceeds from taxation are dissipated,
- the measure $M(s, z)$ is consistent with workers and entrepreneurs' policy functions, optimal decision rules and detection probabilities.

1.5 Firm Dynamics and Misallocation

Occupational choice is depicted as in Fig. 1.1 below. This graphs the occupation decision function for a worker:

$$v(\varphi, \bar{a}, b) = \max\{v_w(\varphi, \bar{a}, b), v_i(\varphi, \bar{a}, b), v_f(\varphi, \bar{a}, b - c_e - r\underline{b})\}$$

where the value of the idiosyncratic productivity shock is fixed at its unconditional average $e^{\bar{a}} = 1$. Individuals with relatively low entrepreneurial ability φ and low assets decide to become workers. Those with high enough entrepreneurial ability choose to run a firm depending on the level of assets. Entrepreneurs may opt to start in the informal sector and eventually transition to the formal sector. On average more able entrepreneurs will move earlier to the formal sector or even start with a formal sector firm (this can occur for high idiosyncratic productivity shocks). More productive

entrepreneurs are the ones likely to expand and benefit from a formal-status (see Fajnzylber et al., 2011).

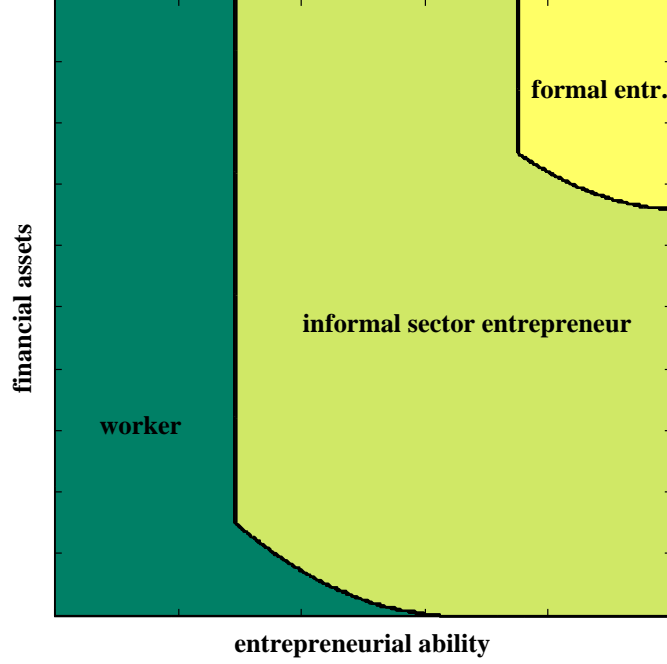
The quantitative exercise will compare a relatively undistorted economy, the US, with two economies with moderate and large informal sectors, Turkey and Egypt respectively. It will be useful however to define misallocation against an efficient benchmark (defined in terms of a social planner's problem). The three economies will exhibit efficiency losses against this benchmark, so that the quantitative results are relevant if the model can account for an important part of the differences in Turkey and Egypt with respect to the US.

Productive efficiency implies equal marginal productivity of both capital and labor across firms. For formal sector firms the first order condition for hired labor is (abstracting from taxes and assuming an interior solution for hired labor $l > 0$):

$$q_l = (1 + r + \mu(s, f)) w$$

where $\mu(s, f)$ is the Lagrange multiplier on the borrowing constraint. This is a static decision as only current period variables are relevant. Collateral constraints can generate dispersion in marginal productivity,²⁸ to the extent that the entrepreneurs are borrowing constrained and there is dispersion in $\mu(s, f)$ (due to different shocks and assets). The argument is similar for capital.

²⁸The discussion of misallocation generated by financial frictions in this section borrows from Midrigan & Xu (2010).

Figure 1.1: Occupation Choice

In the case of informal sector firms, input decisions are distorted by the marginal cost of production ξ as well as by the probability of being detected by the government (when considering tax enforcement). The choices of inputs are no longer static, assuming differentiability for the sake of the argument and omitting unnecessary notation and taxes, the labor input decision would be derived from (for capital the argument is identical):

$$u_c \pi_l + \beta (1 - \delta) \sum_{\{a'\}} \Lambda(a' | a) \hat{\kappa}_l (\underline{v}(s') - v(s')) = \mu(s, i)$$

Government enforcement implies an additional distortion: entrepreneurs limit their output to avoid government detection. Collateral constraints continue to play a role in generating dispersion in the marginal productivity of inputs. The lower bound of labor input for the firm $l + \psi \geq \psi$, with $l \geq 0$, may also imply a deviation from equal marginal productivity for all firms, that may be quantitatively relevant in economies

where a large share of the labor force is self-employed (corner solution). Finally, we need to consider a general equilibrium effect (as in Lucas, 1978): in economies where λ_f is lower there will be lower capital accumulation and lower wages, more people will turn to entrepreneurship (or self employment). In the margin, these entrepreneurs have lower managerial ability.

1.6 Parameters & Taxes

The parameters of the model are divided into three sets. A first set of, mostly standard, parameters is predetermined. Following the literature on misallocation, a second set of parameters is calibrated to match key economic aspects of the US, a relatively undistorted economy. Finally, country specific institutional parameters, related to taxes and registration costs as well as the enforcement of financial contracts are specified.

1.6.1 Predetermined and Benchmark Parameters

The parameters taken from the literature are enumerated in Table 1.4. A period in the model represents a year, $\beta(1-\delta)$ is the effective discount factor (δ is calibrated below), r is the risk free interest rate, ϕ governs the intertemporal elasticity of substitution (Buera et al., 2011), ν is the capital depreciation rate (Barseghyan & DiCecio, 2011; Restuccia & Rogerson, 2008). Production parameters α and γ are also taken from Restuccia & Rogerson (2008). As a benchmark, ψ is set so that the entrepreneur can fully exploit his effective units of labor.²⁹

The idiosyncratic shock a follows an AR(1) process with autocorrelation coefficient ρ with innovations that have a standard deviation of σ . Asker et al. (2012) estimate the productivity process of firms for a large set of emerging economies using different data sets. I take the median of the cross-country estimates for each parameter.³⁰ These

²⁹For an alternative specification, see Gollin (2008).

³⁰This results in an autocorrelation parameter practically equivalent to that of D’Erasmus & Moscoso-Boedo (2012), whereas their standard deviation parameter is 0.23, estimated for the US manufacturing sector.

values are quantitatively critical to evaluate the impact of frictions on aggregate TFP, as established by Asker et al. (2012) and Buera & Shin (2011). Further discussion and an analysis of the impact of alternative parameterizations is carried out below.

Table 1.4: Predetermined and Benchmark Parameters

par.	value	description
α	1/3	income share of capital
γ	0.85	span-of-control (decr. returns to scale)
ψ	1.00	entrepreneurs' labor input
ν	0.08	capital depreciation rate
ϕ	1.50	intertemporal elasticity of substitution
$\beta(1 - \delta)$	0.92	effective discount factor
r	0.04	risk free interest rate
ρ	0.85	autocorrelation coefficient
σ	0.38	standard deviation of shocks
δ	0.085	exogenous annual exit rate
μ_h	-1.795	ability dist. (scaled log-normal)
σ_h	0.83	ability dist. (scaled log-normal)

We turn next to the calibrated parameters in Table 1.4. The exogenous annual exit rate for establishments δ is set to match a total exit rate of 0.10 (Restuccia & Rogerson, 2008). In the model, the total exit rate is determined by the endogenous exit of firms in addition to the stochastic exit shock given by δ . The literature has additionally considered the role of per-period fixed costs of production which affects endogenous exit of firms.

The individual's permanent entrepreneurial ability is drawn from a log-normal distribution $h(\varphi)$ with parameters μ_h and σ_h . The distribution is scaled by setting the minimum value of φ equal to one.³¹ These parameters target the US distribution of employment by firm size³² (Helfand et al., 2007), see Table 1.5.

³¹This is convenient for comparisons with the Pareto distribution (not reported).

³²The Appendix compares the size distribution of firms according to different sources.

Table 1.5: US Distribution of Employment and Firms by Size Class

% employment	1-9	10-19	20-49	50-99	100-499	500-999	1,000+
US - data [†]	11.45	7.70	11.08	8.43	17.89	6.91	36.55
US - model	10.02	8.55	9.42	8.86	18.04	8.19	36.92
% firms	1-9	10-19	20-49	50-99	100-499	500-999	1,000+
US - data	74.27	12.56	8.06	2.69	2.00	0.22	0.20
US - model	66.89	17.18	8.99	3.75	2.51	0.35	0.33

[†]Helfand et al. (2007), average 1995-2000.

1.6.2 Institutional and Country-Specific Parameters

The data for the size of the informal sector was discussed previously. The informal sector includes the workers as well as the entrepreneurs in the informal sector as a share of the labor force. Parameter ξ is set conservatively to match a 1/2 share of employment in the informal sector in Egypt and κ is initially set to zero (the role of enforcement will be discussed in additional policy exercises). For Turkey, ξ targets an informal sector share of employment of 0.2. For the benchmark US economy, we consider no informal sector.

The collateral constraint for the formal sector λ_f targets the ratios of loans by financial institutions to the private sector plus the private bond market capitalization over GDP (data is from Beck, Demirgüç-Kunt & Levine, 2009), which is a typical target in the literature (see for example Quintin, 2008). This ratio is 2.3, 0.15 and 0.32 for the US, Turkey and Egypt, respectively. For the informal sector, λ_i is initially set so that entrepreneurs must self-finance their activities (see, among many others, El-Mahdi, 2002; Amaral & Quintin, 2006; Perry et al., 2007).

Registration costs c_e are from Djankov et al. (2002). They compute the direct official costs of procedures plus the monetized value of the entrepreneur's time (as a fraction of GDP per capita in 1999) associated with meeting legal requirements that a start-up must bear in order to operate legally. Minimum capital requirements are from

the World Bank's Doing Business Report (2004), also expressed in terms of GDP per capita.³³

Table 1.6: Institutional and Country-Specific Parameters

par.	US	Turkey	Egypt	type of tax
λ_f	45	1.16	1.50	collateral formal sector
λ_i	–	1.00	1.00	collateral informal sector
ξ	–	0.1110	0.0034	informal sector specific cost
τ	0.46	0.53	0.54	total tax rate as % of profits
c_e	0.02	0.37	1.17	registration costs plus time value [‡]
\underline{b}	0.00	0.13	7.88	minimum capital requirements [‡]

[‡]expressed in terms of GDP per capita.

We consider a simple tax structure where τ (computed by the World Bank Doing Business Survey) includes taxes paid by a standardized limited liability company expressed as a share of commercial profits (after accounting for allowable deductions and exemptions). The taxes are measured at all levels of government and include profit or corporate income tax, social security contributions and labor taxes paid by the employer, property taxes, dividend and capital gains tax, etc. Taxes withheld (sales tax or value added tax) but not paid by the company are excluded. A more complete tax structure is discussed in the Appendix.

1.7 Quantitative Analysis

In this section I discuss the baseline quantitative results. I then conduct a robustness exercise with respect to the autocorrelation parameter of the idiosyncratic productivity process, which has been shown to be important for the impact of financial frictions on misallocation as discussed below. I also discuss further robustness exercises to be

³³Barseghyan & DiCecio (2011) consider a broader definition of entry barriers which include legal fees of construction permits, utility connections and inspections associated with building a physical location in which to operate a firm as well as minimum capital requirements. In this sense, the figures used here may be considered a lower bound of these costs.

conducted in the future.

1.7.1 Baseline Results

Table 1.7 shows the baseline quantitative results. Total factor productivity is defined as Solow's residual from the equation $Q = AK^\alpha L^{1-\alpha}$, where L is the total workforce. For the benchmark calibration, total TFP drops by 28% in Egypt relative to the US, in the informal sector TFP is 14% lower than in the formal sector. The drop of total TFP in Turkey is 33%, larger than for Egypt due to lower access to credit (the entry cost and the total tax rate are lower than in Egypt). Additionally there are large drops in wages, output per worker and the average size of firms (mostly due to the small size of firms in the informal sector).

Table 1.7: Baseline Results

variable	US	Egypt	Turkey
private credit/total output	2.36	0.31	0.20
share of informal sector labor	n.a.	0.50	0.18
capital/labor total	1.00	0.17	0.10
wage	1.00	0.32	0.24
output per worker total	1.00	0.40	0.31
output per worker informal	n.a.	0.37	0.18
TFP total	1.00	0.72	0.67
TFP informal sector	n.a.	0.66	0.46
TFP formal sector	1.00	0.77	0.72
total capital/output	2.03	0.86	0.65
capital/output formal sector	2.03	0.79	0.63
capital/output informal sector	n.a.	0.94	0.64
avg. firm size total*	40.6	3.6	4.2
avg. firm size informal	n.a.	1.7	1.0
firm exit rate	0.10	0.19	0.18
*average firm size in workers includes entrepreneur.			

1.7.2 Robustness Exercise

I reduce the persistence parameter for the idiosyncratic productivity shock from 0.85 to 0.75. This is within the range of 0.6 to 0.8 suggested by Syverson (2011) for the US. The distribution of entrepreneurial ability is re-calibrated for the US. As discussed in Buera & Shin (2011), with collateral constraints, a reduction in shock persistence leads worse allocation of production factors through entrepreneur's self-financing.

Table 1.8: Robustness Results

variable	US	Egypt
private credit/total output	2.34	0.32
share of informal sector labor	n.a.	0.52
capital/labor total	1.00	0.15
wage	1.00	0.29
output per worker total	1.00	0.36
output per worker informal	n.a.	0.31
TFP total	1.00	0.66
TFP informal sector	n.a.	0.57
TFP formal sector	1.00	0.76
total capital/output	2.01	0.86
capital/output formal sector	2.01	0.72
capital/output informal sector	n.a.	1.03
avg. firm size total*	31.8	2.06
avg. firm size informal	n.a.	1.07
firm exit rate	0.10	0.32

*average firm size in workers includes entrepreneur.

1.8 Policy & Reforms

This section discusses three policy/reform experiments for Egypt (all relative to the baseline calibration): (1) the elimination of registration costs, (2) an improvement in access to credit that increases the ratio of total credit to output from 0.32 to 0.40, (3) government enforcement of registration for informal sector firms.

1.8.1 Elimination of Registration Costs

The simplification of entry regulation has become a popular reform. This has been attributed, at least in part, to the inclusion of business entry indicators in the World Bank's Doing Business Project: in 2003-2008 193 reforms took place in 116 countries (Djankov, 2009). These reforms include standardizing documents and reducing minimum capital requirements, registration costs or bureaucratic steps required to complete the registration process. Empirical cross-country studies suggest that reducing registrations costs can have a large impact on productivity (see the discussion in Djankov, 2009). In the theoretical literature, Antunes & Cavalcanti (2007) find that for a developing country like Peru, the size of the informal sector is equally accounted for by low financial contract enforcement and high regulation costs. Barseghyan & DiCecio (2011), in a model without financial frictions or the informal sector, find that countries in the lowest decile of the entry costs distribution have 1.32 to 1.45 times higher TFP and 1.52 to 1.75 times higher output per worker than countries in the highest decile. Empirical country-case studies, on the other hand, have found a modest impact resulting from reforms that reduce the cost of entering the formal sector.

Several studies have exploited micro-level data to analyze the impact of reducing registration costs for specific countries. Fajnzylber et al. (2011) and Monteiro et al. (2012) evaluate the impact of a program of bureaucracy simplification and tax reduction for micro-enterprises in Brazil, the SIMPLES program. Monteiro et al. (2013) document an increase of 13 percentage points in formal licensing among retail firms created after the program. They find heterogeneous impact across industries, given that requirements to enter and operate in the formal sector differ according to economic activity. More specifically, they find no impact on construction, services and manufacturing sectors. Fajnzylber et al. (2011), in addition to a modest impact on registration, document that firms born under the SIMPLES program show higher levels of revenue and profits, suggesting the adoption of technologies and lines of business that are more permanent, capital intensive and of a larger scale, as measured by number of employees. Kaplan et al. (2011) estimate the effect of simplification of firm registration procedures on business start-ups in Mexico. They suggest that attention in business deregulation may have been overemphasized given the small increase in new start-ups and conclude

that the small benefits of being formal may explain the low impact of the implemented reforms. Bruhn (2012) also examines the case of Mexico and estimates a small impact on registration.

The third column in Table 1.9 shows the results for Egypt, of a complete elimination of registration costs: $c_e + r\bar{b} = 0$. Employment in the informal sector falls by 1 percentage point, the impact on productivity, the average size of firms and capital accumulation are negligible or non-existent.

Table 1.9: Reforms

variable	baseline		reforms		
	US	Egypt	Egypt ¹	Egypt ²	Egypt ³
private credit/total output	2.36	0.32	0.32	0.40	0.35
share of informal sector labor	n.a.	0.50	0.49	0.46	0.48
capital/labor total	1.00	0.17	0.17	0.18	0.13
wage	1.00	0.32	0.33	0.34	0.28
output per worker total	1.00	0.40	0.41	0.42	0.36
output per worker informal	n.a.	0.37	0.38	0.39	0.33
TFP total	1.00	0.72	0.72	0.73	0.69
TFP informal sector	n.a.	0.66	0.67	0.69	0.67
TFP formal sector	1.00	0.77	0.78	0.77	0.73
total capital/output	2.03	0.86	0.87	0.88	0.74
capital/output formal sector	2.03	0.79	0.79	0.85	0.79
capital/output informal sector	n.a.	0.94	0.93	0.92	0.70
avg. firm size total*	40.6	3.6	3.6	3.8	3.2
avg. firm size informal	n.a.	1.7	1.7	1.7	1.3
firm exit rate	0.10	0.19	0.18	0.18	0.20

*average firm size in workers includes entrepreneur.

¹elimination of registration cost ($c_e + r\bar{b} = 0$).

²improvement in access to credit ($\lambda_f = 1.75$).

³increase in govt. enforcement ($\kappa = 0.0002$).

1.8.2 Financial Sector Reform

The exercise consists in increasing the access to credit for formal sector firms so that private credit over total output increases from 0.32 to 0.40, which implies incrementing $\lambda_f = 1.75$. The wage level increases by 6% relative to the benchmark result and output per worker increases by 5%. The size of the informal sector drops from 50% of workers to 46%.

1.8.3 Government Enforcement of Registration

We now turn to government enforcement of registration. One example of a technology that can lead to an increase in enforcement is the use of data matching techniques applied in developed economies (UK National Audit Office, 2008): tax records are matched with external sources of information to identify taxpayers with income streams that have not been declared. Simple inspections and audits are more direct methods of enforcement. In developed economies sanctions range from financial penalties, closure of business, initiation of bankruptcy, criminal prosecution, etc. There are also significant differences in the effectiveness of the judicial systems across countries in terms of prosecuting evaders³⁴.

The role of government enforcement is discussed in policy circles (World Bank, 2010) as well as in the private sector. A study by McKinsey Global Institute (2006) proposes that better enforcement of tax and business regulation would allow to reduce taxes encouraging more companies to join the formal economy in developing countries³⁵. In the UK there are 1.6 tax employees per 1,000 of population, compared to less than 0.03 for countries like Turkey, India and Brazil. Fines for unreported value added taxes can be as high as 3 times the amount of tax evaded and/or up to 6 months in prison in the UK. In Turkey, fines for valued added tax evasion are negligible.

There is, however, little work on understanding the impact of increased enforcement

³⁴See Tax Administration in OECD and Selected Non-OECD Countries: Comparative Information Series.

³⁵In this study the reduction of registration costs is also proposed.

in developing economies. Almeida & Carneiro (2009) study firm level data for Brazil, and find that stricter enforcement of labor regulation constrains firm size and leads to higher unemployment. In general it is not clear that the optimum size of the informal sector is zero. Prado (2011) builds a model where the determinants of the size of the informal sector are taxation, government enforcement and formal sector entry costs. He finds that stricter enforcement does not always imply an economic improvement; under particular contexts countries can benefit from a larger informal sector. Leal-Ordoñez (2013) finds that better enforcement increases output when the informal sector is small but the opposite happens when it is large. I extend this exercise to an environment with firm dynamics and financial constraints.

In the exercise conducted here, we consider a probability of detection by the government that is increasing in output of the informal sector firm³⁶. In the case of detection, the entrepreneur is forced to pay the registration cost and enter the formal sector in the following period. If the entrepreneur does not have sufficient wealth to pay this registration cost, all financial assets are lost and enters the next period as a worker.

The results from this exercise show that although this policy can be successful in reducing the size of the informal sector, it does so at the expense of creating a new distortion in the economy with a large negative impact. The size of the informal sector falls from 50 to 48% of the total workforce, the wage falls by 12.5% and aggregate output per worker is reduced by 10% while aggregate TFP drops by 4%.

1.9 Conclusion

The size of the informal sector is determined by the access to credit in the formal sector, the burden of taxes and registration costs. I find that for a country like Egypt³⁷, where

³⁶I assume that enforcement is costless. Given that my results are that enforcement is negative in terms of welfare and productivity, considering the additional costs of implementing an enforcement technology would increase the losses. Considering a detection technology that depends only on capital would distort the capital/labor ratio and potentially increase misallocation in the informal sector.

³⁷The informal sector is large in many developing economies (ILO, 2012) such as, for example, Pakistan (73%), Philippines (72%), India (68%), Colombia (52%), Peru (49%).

the informal sector accounts for a sizable part of the workforce, aggregate TFP losses associated with these frictions are larger than 28% and wages can be approximately 70% lower than in a financially developed economy like the US.

Given the productivity losses associated with the informal sector, the amount of effort dedicated to policies intended to ameliorate these losses is not surprising. The simplification of entry regulation has become a popular reform: in the period 2003-2008 193 reforms took place in 116 countries (Djankov, 2009). According to the World Bank's Doing Business Survey, Egypt was a top reformer for 2006-2007, this was the second time in 5 years that Egypt was among the top reformers in the world. The reforms included simplifications in the process of starting a business, increased access to credit, easing of the tax burden and minimum capital requirements and registrations costs. I find that the impact of completely eliminating registration costs is small (relative to the gap in different variables between the US and Egypt). This is consistent with empirical country-case studies that exploit firm-level data to evaluate the consequences of these reforms.

I also evaluate the effects of increasing enforcement of registration for informal sector firms. In this case, I find that the consequences are negative: aggregate TFP drops, as well as wages and output per capital. This can explain why countries don't pursue this type of strategy, in spite of suggestions in policy circles and the corporate sector³⁸. From this exercises we can conclude that the only relevant strategy to reduce the size of the informal sector is improving the business environment in the formal sector.

I have abstracted from the interaction with technology accumulation³⁹ and human capital. Additionally, given that different industries are more dependent on external finance⁴⁰, there could be important interactions between the economic structure in terms of different industries and the informal sector.

³⁸Note however that, from this type of policy, formal sector firms would benefit from lower wages.

³⁹I explore the role of the endogenous accumulation of firm productivity in Chapter 2 of this thesis.

⁴⁰Catão et al. (2009) quantify the impact of financial deepening on employment formalization rates in Brazil, finding that results that are more pronounced in industries with higher financial dependence.

Chapter 2

From Firm Productivity Dynamics to Aggregate Efficiency

2.1 Introduction

Differences in total factor productivity (TFP) largely account for cross country differences in output per capita (Parente & Prescott, 2000; Caselli, 2005): controlling for different amounts of production inputs such as labor, human and physical capital, some countries are able to produce more goods and services than others. The empirical literature has documented that misallocation in developing economies can explain an important part of cross-country differences in TFP: in these economies there is more dispersion in the marginal productivity of inputs of production than in more advanced economies (all related literature is discussed below). This means that with the same total amount of inputs of production we could increase output by shifting resources from firms with low marginal productivity to those with higher marginal productivity.

The underdevelopment of financial markets has naturally been proposed as a source of misallocation: the lack of access to credit constrains firms from reaching the optimal level of capital. This implies that constrained firms with different levels of financial assets will have different marginal productivity of capital. However, recent work by Midrigan & Xu (2013) has pointed out that the misallocation losses generated by financial underdevelopment are modest in a quantitative calibrated framework. They find

that in a country with no credit markets the losses are approximately 5% of TFP in their benchmark specification.¹

I revisit this result by considering a model with financial constraints where firms are able to invest every period in order to increase their productivity (knowledge capital). I find that this channel amplifies the effect of financial constraints on misallocation. For Mexico, the model without endogenous firm-productivity accumulation generates misallocation losses of 7.3% in TFP, but this increases to 14.7% in a model with endogenous firm-productivity accumulation.

The amplification result is derived in part from a stronger covariance between firm productivity (which is endogenous in the model with knowledge capital) and output capital ratios (higher for more constrained firms). In the model without knowledge capital, due to the fact that shocks are purely mean-reverting, a firm that is highly constrained in one period is likely to be less constrained in the following period. In a model with knowledge capital, a firm that is constrained in one period can again be highly constrained in the following period if the endogenous productivity component increases.²

Financial constraints also have dynamic consequences by affecting firm productivity accumulation: constrained entrepreneurs invest less in knowledge capital (firm productivity), establishing a link between firm productivity dynamics and aggregate production efficiency.³ Previous work in development and financial frictions has mostly ignored the role of endogenous productivity growth of the firm or considered a one-time technological adoption choice (an exception is the recent work of Cole, Greenwood & Sanchez, 2012). The empirical literature has already stressed the relative lack of growth of firms in

¹To be more specific, they show that although financial constraints can have important quantitative effects on TFP, the impact is not generated through the misallocation channel. They also consider a one-time technology adoption decision without uncertainty, which can increase misallocation to 6.3% of TFP.

²In a different context, the role of the correlation of distortions with firm productivity was emphasized by Restuccia & Rogerson (2008). Hopenhayn (2012) discusses the specific conditions under which correlated distortions can generate a larger impact on TFP.

³The theoretical and empirical literature on the relationship between financial development and economic growth underscores the role of better functioning financial systems in easing the external financing constraints that impede firm growth and innovation (Beck & Demirgüç-Kunt, 2006) and aggregate growth (Levine, 2005).

developing countries (for example Hsieh & Klenow, 2012). The model with endogenous productivity accumulation can partially account for the lower life-cycle productivity growth of firms in an economy with underdeveloped financial markets.

How do financial constraints affect firm productivity growth? Investment in innovation is a costly and uncertain enterprise. As the capacity to obtain external funds is diminished, resources allocated to this effort will be reduced due to different mechanisms at work. First, the return of this investment in the case of success may be diminished by the inability to quickly increase production capacity if the credit necessary to do so is scarce (i.e., if entrepreneurs cannot rent the optimal level of physical capital due to insufficient collateral). Second, financial constraints reduce profits obtained by entrepreneurs and therefore the amount of assets they are able to accumulate in every period. This will affect the amount of resources invested in new technologies.⁴

The empirical literature finds that innovation increases productivity and is therefore crucial for firm performance, whereas the lack of access to external finance constraints innovation and firm productivity growth and therefore reduces aggregate production efficiency.⁵ Several studies have exploited cross-country firm-level data to analyze the role of financial constraints in determining innovation. Gorodnichenko & Schnitzer (2013) use the Business Environment and Enterprise Performance Surveys (BEEPS), covering a wide array of sectors and countries with direct measures of innovation and financial constraints. They conclude that financial constraints restrain the ability of domestically owned firms to innovate and thus to reach the technological frontier. Financial constraints are most detrimental for smaller and younger firms. They also document that financial restrictions at the firm level are strongly negatively correlated with aggregate measures of productivity as well as firm level TFP. In related work, Ayyagari, Demirgüç-Kunt & Maksimovic (2007), also find evidence that access to finance is an

⁴A simple stylized model will illustrate these mechanisms.

⁵For survey-type treatments of the evidence of the positive impact of innovation on firm productivity and size-growth see Hall (2011) and Hall, Mairesse & Mohnen (2010). Although not without challenges, in this empirical literature innovation includes expenditures related to the introduction of new production processes, design and technical specifications, the implementation of new or significantly improved products (goods or services), new organizational methods in business practices, adoption and adaptation of existing technologies (Hall, 2011; Gorodnichenko & Schnitzer, 2013).

important determinant of innovation within a firm. Dabla-Norris, Kersting & Verdier (2010) find that innovation has a stronger effect on productivity in financially developed countries.

Additionally, financially underdeveloped economies will be characterized by a lower average ability of entrepreneurs, many of which have relatively low prospects of generating productivity growth through innovation. This logic does not assume that the pool of entrepreneurs is worse in some countries than others, but rather, due to the lower demand for workers and the lower wages they receive, a larger mass of individuals opt to set up firms or become self-employed. These individuals, in the margin, tend to have lower ability to manage a firm and less incentives to invest in increasing firm productivity. This is exacerbated in developing countries by the existence of a large informal sector. This refers to entrepreneurs that do not register their firm in order to evade their tax obligations but have no access to credit (formal credit requires documentation). In developing economies a large part of the labor force belongs to small-scale, low-productivity and low-growth firms in this sector.⁶ The informal sector needs to be included in a quantitative framework to avoid possible bias in the results. For example, establishment-level data covers formal sector firms (and this is the focus of the quantitative results from the model). However, entrepreneurs may start their operations in the informal sector until a sufficient amount of assets is obtained to outgrow financial constraints and are able to reach an optimal level of production in the formal sector.

The rest of the paper is organized as follows: Section 2.2 (S.2.2) overviews the related literature, S.2.3 provides the empirical motivation of this paper, S.2.4. discusses a stylized model of innovation and financial constraints, S.2.5 presents the quantitative framework, S.2.6 discusses the calibration of the model, S.2.7 presents the main results of the model, S.2.8 concludes.

⁶In the quantitative framework the informal sector is modeled to account for these facts, but all TFP and misallocation computations refer to formal sector firms. Employment in the informal sector is an enterprise-based concept and covers persons working in units that have informal characteristics in relation to, e.g., the legal status, registration, size, the registration of the employees, their bookkeeping practices, etc. (ILO).

2.2 Relation to the Literature

This paper builds upon several strands of the development and macroeconomics literature, a brief overview follows.

Misallocation. This literature refers to the large dispersion of marginal productivity of inputs observed across firms in a developing country, within narrowly defined industries. This finding suggests that great gains in aggregate output can potentially be generated by shifting production inputs from low marginal productivity firms to those with higher productivity. Hsieh & Klenow (2009), for example, find that gaps in the marginal products of labor and capital across plants can explain a large part of the differences in manufacturing TFP between China and India compared to the US. Busso, Fazio & Levy (2012) and Hsieh & Klenow (2012) perform similar empirical exercises for Mexico and Mexico-India, respectively. Busso, Madrigal & Pages (2012) compile evidence for 10 Latin American economies.

In terms of theoretical work, Restuccia & Rogerson (2008) analyze the potential quantitative effects of idiosyncratic tax schemes, suggesting the importance of evaluating specific distortions that affect the allocation of resources across firms. Financial frictions have been extensively studied in the development and firm dynamics literature; for recent quantitative examples see Amaral & Quintin (2010), Buera, Kaboski & Shin (2011), Arellano, Bai & Zhang (2012), Greenwood, Sanchez & Wang (2013) and Steinberg (2013), among many others. My main contribution relative to this literature is to analyze the implications of considering the life-cycle productivity growth of firms and its interaction with financial frictions.

Bhattacharya, Guner & Ventura (2013) introduce non-stochastic accumulation of managerial skills (which determines firm productivity), while Gabler & Poschke (2013) allow firms to allocate resources to probabilistic experimentation, which in the case of success can lead to an increase in firm productivity. These authors evaluate the effects of distortions in the form of idiosyncratic taxes along the lines of Restuccia & Rogerson (2008). They show that assuming an exogenous distribution of firm productivity can

lead to the underestimation of the consequences of distortions that affect the allocation of resources across production units.

Knowledge Capital. Different theories have linked the life-cycle growth of firms (or establishments) to the stochastic accumulation of knowledge specific to the production unit (Klette & Kortum, 2004; Atkeson & Kehoe, 2005). The theoretical framework analyzed here builds on that research. Doraszelski & Jaumandreu (2013) show that by introducing uncertainty in innovation they “(...) allow shocks to accumulate over time, even firms with the same time path of R&D expenditures may not have the same productivity.” They evaluate their model relative to the non-stochastic model of knowledge capital using a panel of manufacturing firms and find that a stochastic framework is favored by the data.

There is also research that emphasizes the role of intangible capital incorporated in the macro-neoclassical framework, such as Parente & Prescott (2000), McGrattan & Prescott (2005), Corrado, Hulten & Sichel (2009). This literature stresses the need to consider investment in intangibles such as software, R&D, as well as investments in building organizations to address many relevant questions in macroeconomics. Total business investment in intangibles has been found to be the dominant source of growth in labor productivity for the US (Corrado et al., 2009).

Informal Sector. There is a literature that analyzes the determinants of the size of the informal sector and its impact on aggregate outcomes as TFP and the size and productivity distribution of firms. Several results can be considered standard: the size of the informal sector decreases as the enforcement of financial contracts improves in the formal sector (Quintin, 2008; D’Erasmus & Moscoso-Boedo, 2012), increases with labor-market restrictions, heavier regulation of entry (for registered firms) and the tax burden of the formal sector and decreases with enforcement of legal obligations (Djankov et al., 2002; Perry et al., 2007; Leal Ordoñez, 2013). At the firm level, compliance with regulation is associated with better access to external finance, while informal sector firms are found to be less capital and skilled-labor intensive, less productive, smaller and younger (Amaral & Quintin, 2006; Perry et al., 2007; Busso et al., 2012). For a

more thorough discussion of the informal sector, see Chapter 1 of this thesis.

2.3 Empirical Motivation

After a brief description of the data, this section documents the empirical evidence that motivates this study.⁷ Relative to what has been documented for the US we can summarize the empirical facts for Colombia and Mexico as follows: (1) establishments grow less in terms of employment and productivity, (2) there is a larger share of employment in smaller establishments, (3) small establishments in the informal sector account for a large share of employment.

2.3.1 Data Description

For Colombia, the data is from the Annual Manufacturers Survey (AMS) for the period 1982-1998, constructed through a project of technical cooperation between the national statistics agency (DANE) and J. Haltiwanger (see Eslava et al., 2004). The AMS consists of an unbalanced panel of plants⁸ with more than 10 employees or sales above a certain limit (approximately 35 thousand US dollars in 1998). The data-set includes information for each plant on output value and prices, input costs and prices, energy consumption in units and prices, number of production and non-production workers, book value of equipment and structures and four digits industry classification codes (CIIU). The AMS underwent changes in methodology of sampling and identification of plants, the creation of longitudinal linkages was necessary to consolidate plant identifiers through three different periods: 1982-1991, a transition period in 1991-1993 and

⁷Due to data availability and for better comparability, both with the literature and across the data-sets utilized here, we restrict our attention to manufacturing establishments. In Mexico, a very small share of firms has more than one establishment: 2.5% out of approximately 3.6 million firms (Busso et al., 2012). Additionally, it has been found that productivity at an establishment is positively related to the productivity of the firm to which it belongs (Bartelsman & Doms, 2000): well-run firms will be able to transfer technology, production methods, product designs and training across their production units. A large part of the literature uses the establishment as the unit of analysis. This approach is, at least in part, driven by data availability (Syverson, 2011).

⁸This data-set is also used in the cross-country study of firm dynamics by Bartelsman et al. (2009). Camacho & Conover (2010) analyze the dispersion of firm productivity applying the methodology developed by Hsieh & Klenow (2009).

1991-1998. Plant-level TFP was generated through the estimation of a capital-labor-materials-energy production function (for details see Eslava et al., 2004).

For Mexico, the data is from the Economic Census 2009 conducted by the national statistics institute (INEGI). The census captures private establishments with a fixed location in urban areas and includes information on sales, workers, value added, value of fixed capital and labor remunerations, among other variables. It covered a total of 17.6 million workers in 3.6 million establishments in manufacturing, retail and wholesale and services (the figures for manufacturing are 4.6 and 0.4 million, respectively). In Mexico, total urban private employment reaches 33 million workers, the majority of those not captured by the Census belong to the informal sector and firms with less than 5 workers (Busso, Fazio & Levy, 2012). In spite of this limitation it is considered the most comprehensive in Latin America (see Busso, Fazio & Levy, 2012; Hsieh & Klenow, 2012).

2.3.2 Distribution of Employment and Establishments

Cross-country data shows that average size of both firms and establishments increase with income per capita and aggregate productivity⁹ (Tybout, 2000; Alfaro, Charlton & Kanczuk, 2009; Garcia-Santana & Ramos, 2013).

Table 2.1: Size Distribution of Establishments

USA ¹	<5	5-19	20-99	100-499	≥500
establishments %	40.47	28.53	22.10	7.75	1.14
employment %	1.90	7.05	23.34	37.36	30.35
Mexico ²	≤5	6-20	21-100	101-500	>500
establishments %	84.18	11.49	2.85	1.12	0.36
employment %	17.47	9.94	11.81	23.58	37.20

¹Cole, Greenwood & Sanchez (2012), ²Census INEGI (2009).

⁹This is not without limitations. As is well known, the statistical under-representation of small firms (typically in the informal sector) in developing economies leads to understating the actual differences. An exception has been documented in European transition economies: plants with less than 20 employees account for a relatively small share of employment. This reflects the presence of large (formerly or still) state-owned firms inherited from the central planning period (Bartelsman et al., 2009).

According to the Economic Census of Mexico of 2009, 27.4% of employment and 95.7% of establishments were accounted for by production units with less than or equal to 20 workers (Table 2.1). For the US, units with less than 20 workers account for 9% and 69% of employment and establishments respectively. These differences are maintained across different broad industry categories: manufacturing, retail and services (see the appendix for the size distribution by broad sectors for both establishments and employment).

These numbers are likely to understate the real differences due to the under-representation of small firms (Busso, Madrigal & Pages, 2012), particularly those in the informal sector.¹⁰ Leal Ordoñez (2013), using micro-enterprise and census data, estimates that the informal sector accounts for 44-50% of employment.

In the case of Colombia, data is available for manufacturing establishments with over 10 workers. In 1998, the share of employment in firms with more than 500 workers is 25.6% (Camacho & Conover, 2010), while in the US, the equivalent figure (considering manufacturing firms with over 10 workers) is 31.7%. The informal sector in Colombia accounts for 52% of non-agricultural employment (ILO Statistics).

2.3.3 Establishment Life-Cycle Dynamics

This subsection documents the life-cycle growth of manufacturing firms in the US, Colombia and Mexico.¹¹ In the US most firms are born small: approximately 96.2

¹⁰In the Appendix I provide a description of the main activities in the informal sector. It is worth noting that approximately 10% of the self-employed and micro-firm entrepreneurs were in that status due to loss of their previous job or because they could not find alternative employment (National Micro-Enterprise Survey of Mexico ENAMIN-2010), thus for the vast majority of the self-employed and micro-firm entrepreneurs this is a (self-reported) voluntary status.

¹¹Although international comparisons of firm data require caution and sometimes remain difficult to interpret, there is evidence of significant cross-country differences in firm-dynamics and post-entry performance. Differences in firm size are largely driven by within-sector differences and not by the sectoral composition of the economy (Bartelsman, Haltiwanger & Scarpetta, 2009). For example, four-digits industry effects account for less than ten percent of cross-section heterogeneity in output, employment and productivity growth rates across establishments (Foster et al., 2001).

percent of firms that are 0-1 years have less than 20 workers.¹² Younger/smaller firms have higher exit rates, but those that survive tend to grow faster than older/larger firms (Klette & Kortum, 2004).

Table 2.2: The Life-Cycle of Establishments: Employment

relative size	USA ¹		Colombia ³		Mexico ¹
	surv.	all	surv.	all	all
age 5-9/age 1-4	1.6	2.0	1.5	1.4	1.4
age 10-14/age 1-4	2.0	3.0	1.7	1.7	1.5
age 15-19/age 1-4	2.3	4.1	1.8*	2.1*	1.6

growth in %	USA ²		Colombia ³		Mexico
	surv.		surv.	all	–
age 6/age 1	106.1		62.7	50.2	–
age 8/age 1	135.2		84.4	73.5	–
age 10/age 1	154.8		104.8	101.1	–

Source: ¹Hsieh & Klenow (2012), ²Audretsch (1995),
³computed w/AMS-DANE (1982-1998), *age 15-16 only.

Audretsch (1995) computes the average employment growth rates for 11,154 manufacturing firms established in 1976 for up to 10 years, we reproduce the results for the growth rates of surviving firms (data is from the Small Business Data Base of the US Small Business Administration). Hsieh & Klenow (2012) impute the life cycle from the employment growth from 1992-1997, comparing the average size of establishments within a given cohort grouped into five-year age bins (reproduced in Table 2.2).

The life-cycle growth of establishments for Colombia is computed using the AMS panel database for the period 1982-1998. We are able to follow establishments up to age 16 (we cannot impute the age of establishments born in 1982 or earlier). For the growth rate of all establishments in the lower panel first I compute, in every year, the average

¹²This is the average for the period 2000-2005. This group of firms (age 0-1 with less than 20 workers) accounted for an average 13.2 percent of total job creation in the same period compared to 8.4 percent for larger firms of age 0-1 (source: Business Dynamics Statistics, Census Bureau). New large firms are partly associated with new U.S. affiliates of foreign-owned firms.

size of all establishments of a particular age. Then I calculate the growth rate of this average for each cohort. Finally, for each age I take the median across cohorts. For example, to calculate growth at age 4, I have 13 observations representing cohorts of establishments born between 1983 and 1995. There is variation across cohorts, taking the average across cohorts instead of the median results in slightly lower life-cycle growth. To compute the growth of survivors in the lower panel, I compute the growth of each individual establishment at each age, I take the average of establishment growth within a cohort and then the median across cohorts (Table 2.2). This procedure is equivalent to the one in Audretsch (1995) but repeated for different cohorts.

Table 2.3: The Life-Cycle of Establishments: Productivity

relative avg.	USA ¹	Colombia ²		Mexico	
	all	surv.	all	all ³	all ¹
age 5-9/age 1-4	1.6	1.0	1.0	1.4	1.5
age 10-14/age 1-4	2.1	1.2	1.1	1.5	1.6
age 15-19/age 1-4	2.8	1.3*	1.2*	1.5	1.6

Source: ¹Hsieh & Klenow (2012),
²computed w/AMS-DANE (1982-1998), *age 15-16 only,
³computed w/INEGI Census (2009), cross section.

For the upper panel of Table 2.2, first I take the average of all establishments age 1-4 for each given year. To calculate the relative size at age 5-9, I can start in 1991 (the first year where we have establishments of age 9) resulting in 8 observations, for age 10-14 we can start in 1996, resulting in 3 observations. This procedure is comparable to that in Hsieh & Klenow (2012) but repeated for different cohorts. For Colombia, Table 2.3 uses firm TFP computed by Eslava et al. (2004).

For Mexico, we have data available from the 2009 Economic Census (a cross section). Hsieh & Klenow (2012) are able to use the Census data for 1999, 2004, 2009, which allows them to track cohorts for up to 10 years. It is not possible to compute statistics for survivors since there is no information to link establishments across time. We replicate their results for the life-cycle growth in terms of employment. For firm

level TFP, in the case of Mexico, we compare their results with estimates from the cross section (Table 2.3).

2.4 Stylized Model of Innovation and Financial Constraints

In this section a stylized two-period model is presented to highlight the interaction between financial constraints and innovation along the intensive and extensive margins.¹³ The intensive margin considers how financial constraints affect innovation for a firm with a given productivity level. The extensive margin refers to the possibility that general equilibrium effects may lead to changes in the composition of firms.

In the first period no production takes place, the entrepreneur is endowed with financial assets $b > 0$ which can be allocated to consumption c in the first period, to savings b' for the second period (in this section we assume there is no interest rate on savings), or invested in the innovation good x . In this set-up, innovation investment is fully financed with internal funds (evidence supporting this assumption is discussed below).

In the second period knowledge capital can take low and high levels, $n \in \{\underline{n}, \bar{n}\}$ respectively, determining the production possibilities. There is a stochastic innovation technology that determines the probability $P(\bar{n} | x) \in [0, 1]$ depending on the amount invested in the innovation good x . This function is increasing and concave in x (decreasing marginal returns).

Production takes place in the second period. At this point the entrepreneur needs to rent capital k at a cost equal to the interest rate r plus the physical depreciation rate of capital δ . The rental of capital is subject to an exogenous collateral constraint $k \leq \psi b'$, where $\psi \geq 1$ is a parameter that determines the ability to collateralize financial assets. After production takes place, consumption for the entrepreneur results from profits of

¹³Alternative prototypical models of this interaction, with a complementary focus on liquidity shocks and the cost of external finance, are discussed in Gorodnichenko & Schnitzer (2013).

the firm and savings. Consumption is valued in both periods through a standard utility function $u(c)$ and discounted in the second period by β . The production technology is given by $n^{1-\nu}k^\nu$ with $\nu \in (0, 1)$.

In the second period, the profits of the firm given knowledge capital n and assets b' are:

$$\pi(n, b') = \max_{\{k\}} n^{1-\nu}k^\nu - (r + \delta)k \quad \text{s.t.} \quad k \leq \psi b'$$

The intertemporal problem of the entrepreneur is to select consumption, savings and investment in innovation to maximize expected discounted utility:

$$\max_{\{x, c, b' \geq 0\}} u(c) + \beta \sum_{\{n'\}} P(n' | x) u(\pi(n', b') + b') \quad \text{s.t.} \quad c + x + b' = b$$

The intertemporal optimality equation for assets b' is given by:

$$u_c(c) = \beta \sum_{\{n'\}} P(n' | x) u_{c'}(\pi(n', b') + b') (\pi_{b'}(n', b') + 1)$$

Where u_c refers to marginal utility and $\pi_{b'}$ is the derivative of profits in the second period with respect to assets. This derivative will be positive when the collateral constraint is binding. In addition to the standard consumption smoothing motive for savings, there is an incentive to save to relax the collateral constraint in the second period.

Consider the function $P(\bar{n} | x) = \zeta x^\lambda$ with parameters $\lambda \in (0, 1)$ and $\zeta > 0$, the intertemporal optimality equation for innovation investment x is (in an interior solution):

$$u_c(c) = \beta \zeta \lambda x^{\lambda-1} (u(\pi(\bar{n}, b') + b') - u(\pi(\underline{n}, b') + b'))$$

We are interested in understanding how financial constraints affect investment in innovation. The left hand side on the intertemporal optimality condition of innovation investment does not depend directly on ψ . The derivative of the right hand side, defining

$\Delta u = u(\pi(\bar{n}, b') + b') - u(\pi(\underline{n}, b') + b')$ is:

$$\frac{\partial \Delta u}{\partial \psi} = u_{c'}(\pi(\bar{n}, b') + b') \frac{\partial \pi(\bar{n}, b')}{\partial \psi} - u_{c'}(\pi(\underline{n}, b') + b') \frac{\partial \pi(\underline{n}, b')}{\partial \psi}$$

where it is always the case that:

$$\frac{\partial \pi(\bar{n}, b')}{\partial \psi} \geq \frac{\partial \pi(\underline{n}, b')}{\partial \psi}$$

with strict inequality when the collateral constraint is binding (it can be binding either in both states or in the high knowledge capital state). If the constraint is only binding in the high knowledge capital state, then $\partial \Delta u / \partial \psi > 0$: this implies that relaxing the collateral constraint promotes innovation investment.

With logarithmic utility it can be proven that $\partial \Delta u / \partial \psi > 0$, which again implies that relaxing the collateral constraint promotes innovation investment. The same result holds with $u(c) = c$, risk neutral preferences.¹⁴ With preferences $u(c) = c^{1-\sigma} / (1-\sigma)$, under some parameterizations (in particular relatively high σ), it is possible for innovation investment to be decreasing in ψ . Innovation investment also depends positively on initial assets. Note that in a dynamic model financial wealth is an endogenous state variable and the entrepreneur may outgrow the collateral constraints by saving.

Suppose now that there are individuals with heterogeneous entrepreneurial ability, which affects the production technology of the firm they manage. The production function is $(\varphi n)^{1-\nu} k^\nu$ where φ , the entrepreneurial ability, varies across individuals. To isolate the role of the extensive margin consider a risk neutral utility function and no collateral constraint. The static profit maximization problem is given by:

$$\pi(\varphi n) = \max_{\{k\}} (\varphi n)^{1-\nu} k^\nu - (r + \delta) k$$

¹⁴Caggese (2012) finds that uncertainty (as measured by the volatility in the sectoral profits-assets ratio in a panel of manufacturing firms) reduces the innovation investment of entrepreneurial firms (financially more undiversified) but not the innovation of non-entrepreneurial firms. These results correspond to innovation related to the production of new products, which is linked to increased uncertainty for the firm. The negative impact of uncertainty on innovation is larger for less diversified firms.

The inter-temporal problem, simplified to isolate the role of the extensive margin, is now given by:

$$\max_{\{x\}} -x + \beta \sum_{\{n'\}} P(n' | x) \pi(\varphi n')$$

With a small amount of algebra it can be shown that the optimal first order condition for innovation investment in an interior solution is:

$$x^{1-\lambda} = \beta \zeta \lambda \varphi (\bar{n} - \underline{n}) (1 - \nu) (\nu / (r + \delta))^{\nu / (1-\nu)}$$

This condition implies that x is increasing in φ when $\nu < 1$. In the quantitative model financial constraints lower the demand for labor resulting in lower wages. This leads to individuals with lower entrepreneurial ability φ to set-up a firm.

2.5 Quantitative Model

The model builds upon the frameworks of occupational choice and heterogeneous entrepreneurial ability¹⁵ of Lucas (1978) and industry dynamics of Hopenhayn (1992). There is a continuum of individuals who possess heterogeneous innate entrepreneurial ability and every period decide whether to be workers or establish a firm and become entrepreneurs. The operations of the firm are subject to transitory stochastic shocks which are observed at the beginning of each period, before production and occupation decisions are made. All individuals earn the same wage as workers, since there is no heterogeneity in their effective units of labor and workers are perfectly mobile.¹⁶

The firm is a storehouse of information (Prescott & Visscher, 1980; Atkeson & Kehoe, 2005), or knowledge capital (Klette & Kortum, 2004; Corrado et al. 2009).

¹⁵Differences in management quality are an important determinant of productivity differences across firms (Bartelsman & Doms, 2000; Foster et al., 2001; Syverson, 2011).

¹⁶The evidence on whether labor markets are segmented across informal and formal sector firms suggests mixed results at best, see the discussion in Perry et al. (2007, Ch. 3 and 4).

Entrepreneurs in the formal sector can, while the firm is in operation, allocate resources to investment in technology through a controlled stochastic process. Innovation is an uncertain enterprise, as in Klette & Kortum (2004) and Atkeson & Burstein (2010): entrepreneurs decide every period the amount of resources devoted to improving firm productivity, which determines the probability of an increase in firm productivity. Knowledge capital summarizes the history of past investment and innovation success of the firm.¹⁷

The entrepreneur, who is both owner and manager of the firm, can opt to conduct operations in the formal or informal sector. The trade-off is the following: formal sector firms have to pay an initial registration cost and taxes but they have better access to external finance. Informal sector firms do not pay taxes or the initial registration cost, but have no access to external finance and cannot accumulate knowledge capital. Additionally informal sector firms face a specific convex cost of production. This cost represents the inability to engage in legal contracts, the cost of enforcing property rights when not protected by the government and worse access to infrastructure facilities and services, etc. (Perry et al., 2007). Entrepreneurs may first establish their firm in the informal sector and later transition to the formal sector but a formal sector entrepreneur may not switch directly into the informal sector.

We can start introducing notation by letting $s = (\varphi, n, a, b)$, where φ is the individual's permanent entrepreneurial ability, a is a transitory productivity shock, b are financial assets and n is knowledge capital. Additionally $z \in \{i, f, w\}$ denotes whether the individual is an entrepreneur in the informal or formal sectors or a worker, respectively.

¹⁷Klette & Kortum (2004) extend the endogenous growth literature by introducing research in incumbent firms. Aghion, Howitt & Mayer-Foulkes (2005) is a relatively recent example of the endogenous growth literature, where the impact of financial frictions on economic growth is assessed.

2.5.1 Preferences

Time is discrete and a period, indexed by t , represents a year. Individuals value the consumption of the final good, denoted c_t , through lifetime and intratemporal preferences represented as follows:

$$U = \mathbb{E} \left[\sum_{t=0}^{\infty} (\beta(1-\mu))^t u(c_t) \right] \quad \text{and} \quad u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma} \quad (1)$$

where β is the discount factor, σ is the coefficient that governs risk aversion. The probability that an individual dies in every period is μ , so that the effective discount factor is $\beta(1-\mu)$. When an individual dies, his assets disappear and he is immediately replaced by another individual with the same entrepreneurial ability φ so that the mass of individuals and their distribution over ability is constant (the rest of the initial state variables are specified below).

2.5.2 Production Technology

In this economy production of the final good is carried out by single establishment firms and each firm is managed by its owner/founder. Individuals possess innate and permanent entrepreneurial ability φ received according to a distribution $h(\varphi)$. Their operations are subject to productivity shocks a that follow an AR(1) process discretized in a Markov matrix denoted $\Lambda(a' | a)$. Additionally, entrepreneurs are able to accumulate knowledge capital denominated n (through a process described below) and have access to a decreasing returns to scale production technology (in terms of capital and labor) that is common across sectors:

$$q = e^a (\varphi n)^{1-\nu} f(k, l)^\nu \quad \text{with} \quad f(k, l) = k^\alpha l^{1-\alpha} \quad (2)$$

where k is capital and l is labor used in production. Following Lucas (1978), we call $\nu < 1$ the span-of-control parameter that determines the decreasing returns to scale (with respect to capital and labor) in the production technology.

2.5.3 Innovation Technology

Entrepreneurs can invest in the innovation good x to increase the stock of knowledge capital.¹⁸ This good has price p_x .¹⁹ Three outcomes are possible every period, depending on the amount of investment in the innovation good in the previous period: knowledge capital may increase by a proportion Δ , it may remain constant, or decrease by Δ . Knowledge capital is defined on the grid $\{\underline{n}, \underline{n}(1 + \Delta), \underline{n}(1 + \Delta)^2, \dots, \bar{n}\}$, where \underline{n} and \bar{n} are the lowest and highest possible levels of knowledge capital, respectively.²⁰

The probability of a successful outcome is given by:

$$P(n' = n(1 + \Delta) | n, x) = \zeta (x/n)^\lambda (1 - \varepsilon) \quad (3a)$$

subject to the following conditions:

$$\zeta (x/n)^\lambda \in [0, 1] \quad \text{and} \quad \{\lambda, \varepsilon\} \in [0, 1) \quad (3b)$$

There are diminishing returns to innovation investment x . Fixing a probability of success in innovation, $P(n(1 + \Delta) | s, x)$, the necessary investment in innovation goods x to increase the size of the firm by a fixed percentage is proportional²¹ to knowledge capital n . The probability of the worst outcome:

$$P(n' = n/(1 + \Delta) | n, x) = \varepsilon \quad (3c)$$

This shock represents negative events not influenced by firm decisions (obsolescence of products, loss of markets to the competition, etc.). Alternatively, labor turnover may lead to organizational forgetting (Benkard, 2000). Knowledge capital summarizes the

¹⁸The stochastic innovation process specified in this section builds on those considered by Klette & Kortum (2004), Atkeson & Burstein (2010).

¹⁹We will initially consider the case where the innovation good is produced with the final good and set $p_x = 1$. This can be extended to consider that innovation requires labor.

²⁰The model can be extended to consider unbounded knowledge capital, which would require additional conditions to guarantee a well defined dynamic program and convergence in the stationary distribution (see Atkeson & Burstein, 2010).

²¹It can easily be verified that optimal labor and capital inputs, output and profit are proportional to knowledge capital n under the production function previously specified in the case of no financial restrictions and the unconditional mean value of the stochastic shock, $e^{\bar{a}} = 1$.

history of investment and success in innovations and governs the size of the firm (as in Klette & Kortum, 2004). Furthermore, it is lost when the firm closes (regardless of whether exit is due to an exogenous exit shock or the entrepreneur finds it optimal to close the firm). Knowledge capital is assumed to be completely firm-specific and there is no market for its trade.

2.5.4 Workers

The problem of the worker amounts to a savings decision (borrowing is not allowed) and determining the conditions under which he will prefer to establish a firm:

$$v_w(s) = \max_{\{b' \geq 0\}} u(c) + \beta(1 - \mu) \sum_{\{a'\}} \Lambda(a' | a) v(s') \quad (4)$$

s.t. $c + b' = w + (1 + r)b$ and $n = \underline{n}$

At the beginning of each period, after observing the transitory productivity shock a , workers face their occupational choice:

$$v(s) = \max\{v_i(s), v_f(\varphi, \underline{n}, a, b - c_e), v_w(s)\} \quad (5)$$

The worker is free to continue in the labor market and earn wage w every period, become an informal sector entrepreneur (represented by the value v_i), or a formal sector entrepreneur, which requires paying the fixed registration cost c_e (this value is represented by v_f).

Whenever individuals re-enter the labor market, their knowledge capital is reset to \underline{n} , this underscores the interpretation that it represents an intangible asset embedded in the firm. Workers are not able to invest in innovation. We abstract from labor-income risk.²² All new-born individuals receive an entrepreneurial ability φ from the distribution $h(\varphi)$, transitory shock a from its unconditional distribution and initial assets \underline{b} (set

²²Labor income risk is an important factor in models of interest rate determination, we will abstract from a complete model of the interest rate and set it exogenously (a standard small-open economy assumption).

equal to zero in the baseline model).

Occupational choice depends on the ability of the individual as an entrepreneur but also on financial wealth, necessary to register the firm or to reach a profitable scale when financial constraints are present. Figure 2.2 depicts the optimal occupation choice (5) of a worker for a fixed level of productivity shock a , in an economy with a large informal sector such as Mexico or Colombia. The graph has entrepreneurial ability φ on the x-axis and b/w (financial assets normalized by the wage) on the y-axis.

2.5.5 Formal Sector Entrepreneurs

Given the choice of labor and capital input, profits for a formal sector entrepreneur are given by:

$$\pi(s, f) = q - (\delta + r)k - wl$$

To register in the formal sector, entrepreneurs have to pay a fixed cost c_e . Once in the formal sector, the entrepreneur may go back to being a worker and c_e is lost. Additionally, the entrepreneur cannot transition directly to the informal sector, at least one period has to be dedicated in the labor market:

$$\begin{aligned} v_f(s) = \max_{\{l, k, x, b' \geq 0\}} & u(c) + \beta(1 - \mu) \sum_{\{a', n'\}} \Lambda(a' | a) P(n' | n, x) \max\{v_w(s'), v_f(s')\} \\ \text{s.t. } & c + b' = (1 - \tau)(\pi(s, f) - p_x x) + (1 + r)b \quad \text{and} \quad k \leq \bar{k}(s, f) \end{aligned} \quad (6)$$

where τ are taxes to profits. The entrepreneur is able to invest in the knowledge capital of the firm as long as the formal firm is active, but is lost if the individual decides to return to the labor market. The choice of capital input is restricted by an endogenous collateral constraint, to which we turn shortly. The firm dies with the entrepreneur.

2.5.6 Financial Markets

In specifying the endogenous collateral constraints I follow Amaral & Quintin (2010) and Buera et al. (2011). At the beginning of a period the entrepreneur makes a deposit b and rents capital k from a financial intermediary. At the end of the period, the entrepreneur receives his deposit earning interest rate r and pays the cost of capital rental at the total rate of $r + \delta$. Borrowing and capital rental are realized within a given period and the assets of the individual are restricted to be positive $b \geq 0$ in all periods.

Entrepreneurs may renege on financial contracts after production has taken place. If this occurs, the entrepreneur keeps a fraction $(1 - \psi)$ of the un-depreciated capital and the revenue net of labor and tax payments.²³ The punishment for default is the loss of the financial assets deposited with the financial intermediary b . Entrepreneurs regain access to financial markets in the following period without additional costs. This implies that a static condition determines enforceable allocations, allowing for the consideration of financial constraints in a tractable manner. In this setup parameter ψ indexes enforcement of financial contracts in the economy, which encompasses economies with no credit $\psi = 0$ and perfect credit markets $\psi = 1$.

The analysis is restricted to financial contracts that are incentive-compatible, there is no default in equilibrium. Effectively, imperfect enforcement of financial contracts determines an upper bound $\bar{k}(s, f)$ on the amount of capital that entrepreneurs are able to borrow.

In the case of no-default the entrepreneur receives profits net of taxes, plus interest rate income from financial assets:

$$\max_{\{l\}} (1 - \tau) (q - w l - (r + \delta) k - p_x x) + (1 + r) b \quad (7)$$

²³I assume that the entrepreneur cannot avoid paying taxes in the event of default.

In the case of default the entrepreneur would receive (off-equilibrium):

$$\max_{\{l\}} (1 - \psi) ((1 - \tau) (q - w l) + (1 - \delta) k) - (1 - \tau) p_x x \quad (8)$$

Capital rental is said to be enforceable if and only if it satisfies (7) \geq (8). Note that equation (8) is specified so that investment in innovation $p_x x$ does not distort the bound of enforceable capital.²⁴

The borrowing limit is increasing in financial wealth since the loss of collateral is greater in the case of default. It is also increasing in productivity and entrepreneurial ability, as only a share of output is kept in the case of default (see Amaral & Quintin, 2010; Buera et al., 2011).

2.5.7 Informal Sector Entrepreneurs

Informal sector entrepreneurs do not pay taxes but have no access to external finance. In addition, there is a sector specific marginal cost that is increasing in output, determined by parameter ξ . Profits for the informal sector firm are:

$$\pi(s, i) = q(1 - \xi q) - (r + \delta)k - w l$$

The problem of the informal sector entrepreneur is:

$$\begin{aligned} v_i(s) &= \max_{\{l, k, b' \geq 0\}} u(c) + \beta(1 - \mu) \sum_{\{a'\}} \Lambda(a' | a) v(s') \\ \text{s.t. } & c + b' = \pi(s, i) + (1 + r)b \quad \text{and} \quad k \leq b \end{aligned} \quad (9)$$

²⁴The following timing assumptions within a period imply that investment in innovation does not affect $\bar{k}(s, z)$: (1) entrepreneur observes shocks and rents capital, (2) production takes place, (3) capital is returned to the intermediary and financial assets are returned to the entrepreneur, (4) investment in the innovation good is decided. Innovation is financed with internal funds as it is subject to asymmetric information problems and cannot be easily collateralized (see Hall & Lerner, 2010; Gorodnichenko & Schnitzer, 2013).

and face the same occupational decision as workers (with $n = \underline{n}$):

$$v(s) = \max\{v_i(s), v_f(\varphi, \underline{n}, a, b - c_e), v_w(s)\}$$

The convex marginal cost specific to the production technology of informal sector firms makes it increasingly costly for larger firms to remain informal and is therefore a key determinant of the size of this sector and the size of firms in the sector. The literature has documented the worse access of informal sector firms to different types of public services and enforcement of property rights and the fact that informal sector firms are relatively small and unproductive.

2.5.8 Equilibrium

The state space is given by (φ, n, a, b, z) , we previously defined $s = (\varphi, n, a, b)$ and $z \in \{i, f, w\}$. Given taxes and registration costs (τ, c_e) and interest rate r , a small-open economy stationary competitive equilibrium consists of:

- optimal quantities $\{q(s, z)\}_{z \in \{i, f\}}$, production inputs $\{l(s, z), k(s, z)\}_{z \in \{i, f\}}$,
- savings policy functions $\{b'(s, z)\}_{z \in \{i, f, w\}}$,
- policy function of investment in the innovation good $\{x(s, f)\}$,
- wage w , values $\{v(s), v_f(s), v_i(s), v_w(s)\}$, profits $\{\pi(s, z)\}_{z \in \{i, f\}}$,
- invariant measure $M(s, z)$ of individuals over the state space,

such that:

- workers solve (4), formal sector entrepreneurs solve (6), informal sectors entrepreneurs solve (9),
- market clearing condition in the labor market holds (entrepreneurs/managers plus workers equals the total mass of individuals), government revenues are dissipated,
- measure $M(s, z)$ is consistent with individuals' policy functions and optimal decision rules.

2.6 Baseline Parameters

The model parameters are divided into three groups: (1) a group of standard parameters taken from the literature, (2) a second group of parameters that are set to match key features of the US economy, (3) a group of country-specific and institutional parameters. Parameters in groups (1) and (2) are common for all countries in the model.

2.6.1 Common Parameters Across Countries

The interest rate r is set to 0.04 (Amaral & Quintin, 2010). The span-of-control parameter ν equal to 0.85 is taken from Atkeson & Kehoe (2005). The effective discount factor $\beta(1-\mu)$ of 0.92 is from Buera et al. (2011). We consider the case $\sigma \rightarrow 1$, log-preferences as in Midrigan & Xu (2013). Parameters α of 1/3 and δ equal to 0.08 are standard in the literature.

Table 2.4: Predetermined Parameters

parameter	value	description
$\beta(1-\mu)$	0.92	effective discount factor
σ	$\rightarrow 1$	risk aversion
r	0.04	interest rate (open economy)
ν	0.85	span-of-control
α	1/3	income share of capital
δ	0.08	capital depreciation rate
ρ	0.50	autocorrelation coefficient
σ_ε	0.40	standard deviation of shocks

For the parameters ρ and σ_ε that govern the idiosyncratic productivity process I take the mid-range of the values estimated by Abraham & White (2006) for a plant-level data-set that covers the manufacturing sector in the US for the period 1976-1999. The standard deviation is approximately equal to the median of the firm-level cross country estimates by Asker et al. (2012).

We now turn to the calibrated parameters in Table 2.5. The exogenous exit rate μ is set to match a total firm exit rate of 0.10. In the model the total exit rate equals the sum of the rate of entrepreneurs deciding to close their firms and the exogenous exit rate. Entrepreneurial ability is distributed according to a discrete Pareto distribution (truncated, with 15 possible values), its parameter is set to match the average size of firms in the US in the period 1995-2005 (Helfand et al., 2007).

Table 2.5: Calibrated Parameters - US Moments

parameter	par.	value
exogenous exit rate	μ	0.08
Pareto dist. (truncated, discrete, scaled)	$h(\varphi)$	0.72
innovation technology - level	ζ	25
innovation technology - curvature	λ	0.69
prob. <u>down</u> negative shock	ε	0.15
size innovation steps	Δ	0.36
target	target	model
firm exit rate	0.10	0.10
average firm size	22.2	22.0
average size age 20 – 24/avg. size age < 5	5.3	5.2
average size age 15 – 19/avg. size age < 5	4.1	3.9
average size age 10 – 14/avg. size age < 5	3.0	2.5
average size age 5 – 10/avg. size age < 5	2.0	1.4

The technology accumulation parameters target the life-cycle growth of firms in terms of labor as in Hsieh & Klenow (2012). For example, I target the average size of firms that are 15-19 years relative to firms that are younger than 5 years old for US manufacturing firms. With these parameters, the model underestimates the growth of firms, in particular at the earlier stages. Additionally, Midrigan & Xu (2013) find that for Korea (a developed economy), the ratio of total investment in intangibles over value added is 0.046 for a data-set of manufacturing firms. This value, however, is 0.01 in my model.

2.6.2 Country Specific and Institutional Parameters

Next, we need to specify parameters that are country specific or determined by institutions. The registration cost is from Djankov et al. (2002): it represents the cost of obtaining legal status to operate a firm, expressed as a share of per capita GDP in 1999. It includes all identifiable official expenses (fees, costs of procedures and forms, fiscal stamps, legal and notary charges, etc.) as well as the monetized value of the entrepreneurs time. The time of the entrepreneur is valued as the product of time required for registration and per capita GDP in 1999 expressed in per business day terms. Ignoring the time value component, the cost is 0.57 in terms of GDP per capita for Mexico and 0.15 for Colombia.

Table 2.6: Institutional/Country Specific Parameters

description	par.	US	Mex.	Col.
total tax rate (% profits)*	τ	0.46	0.55	0.74
registration cost formal sector*	c_e	0.02	0.83	0.34
collateral constraint	ψ	1.00	0.25	0.34
informal sector convex cost	ξ	1.00	0.01	0.02
targets	par.	US	Mex.	Col.
private credit/output (formal sector)	ψ	2.3	0.2	0.2
% share of informal sector labor	ξ	0	46	49

*Source: World Bank & Djankov et al. (2002).
Reg. cost in terms of GDP per capita.

Parameter ψ determines financial development. As is standard in the literature, to set its value I target the ratio of private credit provided by financial institutions and private bond markets over GDP (Beck et al., 2009). For Colombia and Mexico the target corresponds to the middle of the period of the AMS-DANE data-set and for the formal sector following Midrigan & Xu (2013). The value for the US results in an economy with perfect financial markets (the average of the ratio for the 10 years between 1992-2001 is 2.3 which covers the period of the data used to impute firm life-cycle growth in Hsieh & Klenow, 2012).²⁵

²⁵Note that the amplification of misallocation refers to a comparison within a country keeping the level of financial development fixed and not a cross-country comparison. These exercises are discussed

The parameter that determines the convex marginal cost specific to the informal sector ξ affects the optimum production scale of informal sector firms. The target is the share of employment in the informal sector, equal to 0.45 for Mexico and 0.50 for Colombia. A lower value of ξ is necessary for Mexico relative to Colombia, since taxes are much higher in the latter case.

The tax rate τ , taken from the World Bank Doing Business Survey, is a measure of the total amount of taxes and mandatory contributions expressed as a share of commercial profits for a standardized business (after accounting for allowable deductions and exemptions). This measure considers taxes at all levels of government and includes the profit or corporate income tax, social security contributions, labor taxes paid by the employer and dividend taxes, among others. Taxes withheld (such as the personal income tax) or collected and remitted to tax authorities (such as value added taxes, sales taxes) are excluded. This measure simplifies a more complex tax structure that would distort capital labor ratios in the model.

2.7 Quantitative Analysis

In this section, the main quantitative results of the paper are presented and discussed.

2.7.1 Main Results

The main result of this paper is that misallocation losses in a model of financial constraints are amplified when we introduce endogenous firm-productivity accumulation. For exposition, we can equivalently define the potential gains from eliminating the dispersion across firms in the marginal productivity of capital. The focus is on the formal sector to avoid concerns related to measurement in the informal sector. Let J be the set of firms producing in the formal sector. It can be shown that TFP* in the case of

below.

no financial constraints is:²⁶

$$TFP^* = \left[\sum_{\{j \in J\}} (e_j^a)^{\frac{1}{1-\nu}} (\varphi_j n_j) \right]^{1-\nu} \quad (10)$$

With financial constraints the marginal productivity of capital, and therefore the output-capital ratios, vary across firms and aggregate TFP is:

$$TFP = \frac{\left[\sum_{\{j \in J\}} (e_j^a)^{\frac{1}{1-\nu}} (\varphi_j n_j) (q_j/k_j)^{\frac{-\alpha\nu}{1-\nu}} \right]^{1-(1-\alpha)\nu}}{\left[\sum_{\{j \in J\}} (e_j^a)^{\frac{1}{1-\nu}} (\varphi_j n_j) (q_j/k_j)^{\frac{(1-\alpha)\nu-1}{1-\nu}} \right]^{\alpha\nu}} \quad (11)$$

An efficient allocation implies equalizing the marginal product of capital and therefore the average product as well. The gains from eliminating misallocation in the model are computed as $TFP^*/TFP - 1$, with the following interpretation: keeping the set of firms and their productivity constant, this number represents the gains of eliminating differences in the marginal product of capital across firms. This exercise is analogous to the empirical studies in Hsieh & Klenow (2009) and Busso, Madrigal & Pages (2012).²⁷ Note that this is different from the comparison of aggregate TFP across countries, which I label the potential (total) TFP gain in Table 2.7. The latter comparison takes into account the fact that financial frictions also affect the distribution of firm productivity. For example, aggregate TFP is 19% lower in Mexico (formal sector) compared to the US.

The results show that the potential misallocation gains are 14.7% and 15.8% for Mexico and Colombia in the model with endogenous firm productivity accumulation (Table 2.7). I also solve the model without knowledge capital accumulation, equivalent to setting $\Delta = 0$. It is not necessary to change the parameters of financial development ψ and taxes on profits τ . However, ξ needs to be modified keeping the same target of the size of the informal sector for each country, while c_e is changed to target its value relative to output per capita (neither one of these parameters enters TFP or TFP^* directly). Potential misallocation gains in this model are 7.3% and 3.8% for Mexico and

²⁶See Midrigan & Xu (2013) and Buera et al. (2011).

²⁷In the model presented here, as in many models of financial frictions, the only source of dispersion in output-capital ratios is financial underdevelopment.

Colombia, respectively. The larger amplification for Colombia reflects the higher level of taxes in that country.

Table 2.7: General Results

variable*	US	Mex.	Col.
potential misallocation gain	–	14.7%	15.8%
potential (total) TFP gain	–	19.0%	23.4%
output per capita total	1.00	0.44	0.39
output per capita formal	1.00	0.47	0.42
output per capita informal	–	0.39	0.36
wage	1.00	0.47	0.43
capital/output total	2.34	0.58	0.56
total exit rate	0.10	0.13	0.13
total average firm size	22.0	6.3	5.8

*TFP and misallocation refer to the formal sector.

To decompose misallocation gains first define the following variables:

$$X = (e^a)^{1/1-\nu} (\varphi n) \quad Y = \left(\frac{q}{k}\right)^{\frac{-\alpha\nu}{1-\nu}} \quad Z = \left(\frac{q}{k}\right)^{\frac{(1-\alpha)\nu-1}{1-\nu}}$$

Without financial constraints, the unconstrained equivalents of Y and Z (derived from the first order conditions of the static profit maximization problem of the firm) are:

$$Y^* = \left(\frac{r + \delta}{\alpha\nu}\right)^{\frac{-\alpha\nu}{1-\nu}} \quad Z^* = \left(\frac{r + \delta}{\alpha\nu}\right)^{\frac{(1-\alpha)\nu-1}{1-\nu}}$$

We can rewrite TFP in the model with financial constraints in terms of covariances and averages:

$$TFP = J^{1-\nu} \frac{[\sigma(X, Y) + \mathbb{E}(X) \mathbb{E}(Y)]^{1-(1-\alpha)\nu}}{[\sigma(X, Z) + \mathbb{E}(X) \mathbb{E}(Z)]^{\alpha\nu}} \quad (12)$$

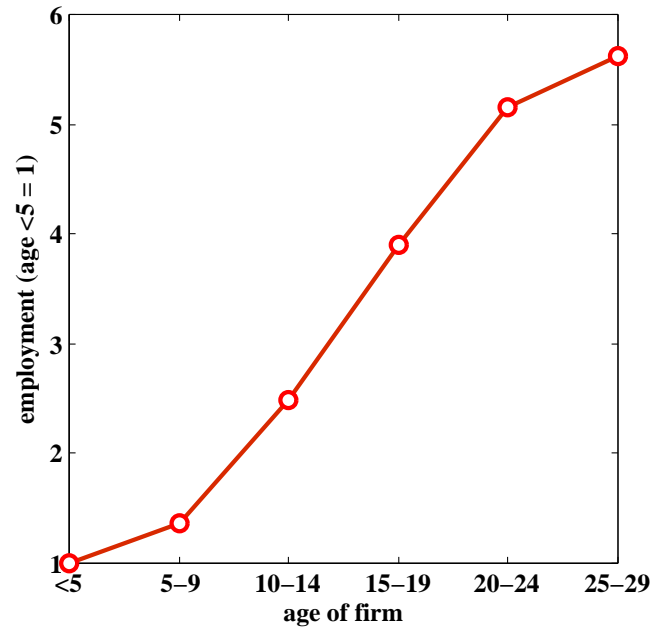
We can now decompose potential misallocation gains into two steps:

- (1) Set Y and Z equal to its optimal unconstrained levels Y^* and Z^* . For Colombia, for example, this step generates a gain of 3.2% in the model without knowledge capital and 9.6% in the model with knowledge capital.
- (2) Eliminate the covariances by setting $\sigma(X, Y) = \sigma(X, Z) = 0$. For Colombia, this step generates a gain of only 0.6% in the model without knowledge capital and 6.1% in the model with knowledge capital, given that $\sigma(X, Y)$ and $\sigma(X, Z)$ are more negative in the latter model. For Mexico, this step generates a gain of 2.1% in the model without knowledge capital and 6.9% in the model with knowledge capital.

Table 2.8: Misallocation

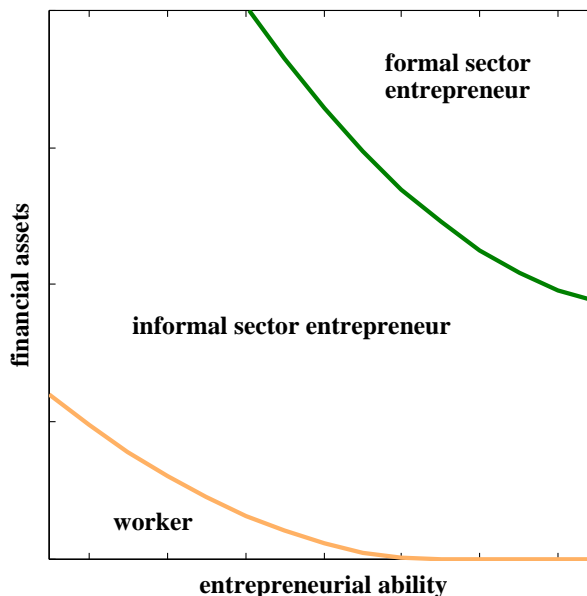
misallocation %	Mex.	Col.
knowledge capital	14.7	15.8
no knowledge capital	7.3	3.8

The covariance terms reflect the fact that it is not only the variance in the marginal-productivity of capital that determines misallocation, but it is also important which firms are constrained. This is related to the discussion of the role of the correlation between firm productivity and distortions in Restuccia & Rogerson (2008) and Hopenhayn (2012). In the model without knowledge capital, due to the fact that shocks are mean-reverting, a firm that is highly constrained in one period is likely to be less constrained in the following period. In a model with knowledge capital, a firm that is very constrained in one period can again be very constrained in the following period if the endogenous productivity component increases (this is further discussed below).

Figure 2.1: Employment over Firm Life-Cycle (Model)

2.7.2 Firm Life-Cycle Productivity and Employment Growth

In Table 2.9 I compute the life-cycle growth and accumulation of knowledge capital for the three baseline model economies. By age 15, the ratio of $n\{\text{age}=15\}/\underline{n}$ is on average 15.7 in the US, but it is less than half this number for Colombia and Mexico. These differences in endogenous productivity accumulation translate into lower life-cycle growth of firms, as shown in the lower panel.

Figure 2.2: Occupation Choice (Mexico/Colombia)**Table 2.9:** Baseline Results: Firm Knowledge Capital and Size

	USA		Mexico		Colombia	
knowledge cap. ¹	all		formal ²	all	formal ²	all
age 5/age 1	2.8		1.6	1.2	1.5	1.1
age 10/age 1	7.9		3.9	3.5	3.6	2.7
age 15/age 1	15.7		7.0	6.9	7.9	7.4
	USA		Mexico		Colombia	
# workers ³	all		all		all	
age 6-10/age 1-5	1.4		1.2		1.2	
age 11-15/age 1-5	2.5		2.1		1.3	

¹Average across firms of $n\{\text{age}=x\}/\underline{n}$.

²Firms that are formal at age x . ³Includes manager.

Figures 2.3 and 2.4 show the model cross-section of $\log(n\varphi)$ including formal and informal sector firms with respect to age: the x-axis corresponds to the age of the firm and y-axis corresponds to $\log(n\varphi)$. I fit a quadratic polynomial to this relationship,

where the number of simulated firms was increased until the results were unchanged. The range of $\log(n\varphi)$ incorporates an extensive-margin effect: in Mexico managers with lower entrepreneurial ability φ set up firms, specially in the informal sector (these firms are not included in the TFP/misallocation computations). The fitted value of $\log(\varphi n)$ is lower at every age in Mexico. To isolate the life-cycle component of knowledge capital, Figures 2.5 and 2.6 show the model cross-section of $\log(n)$ with respect to age for Mexico and US only for formal sector firms.

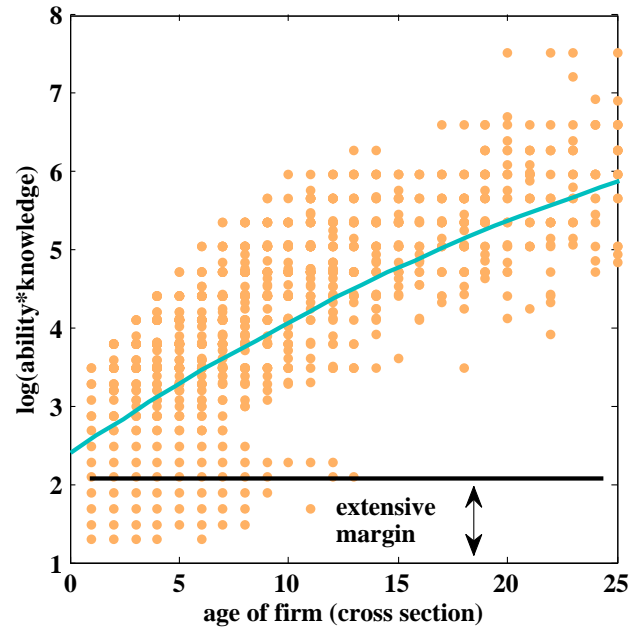
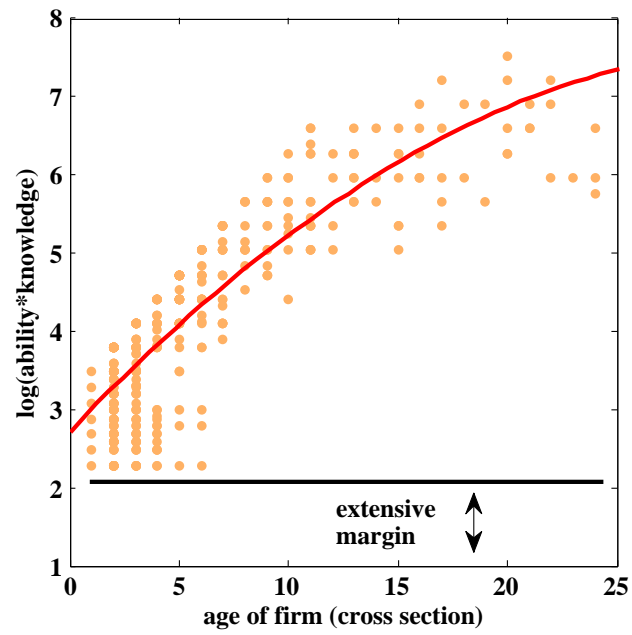
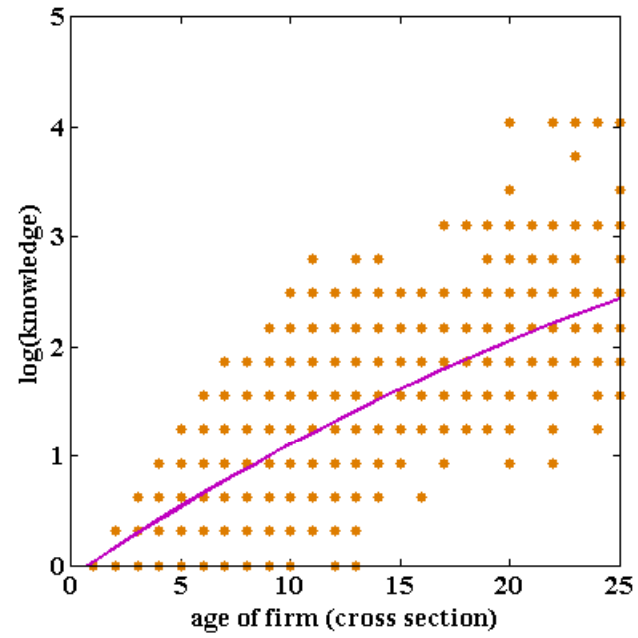
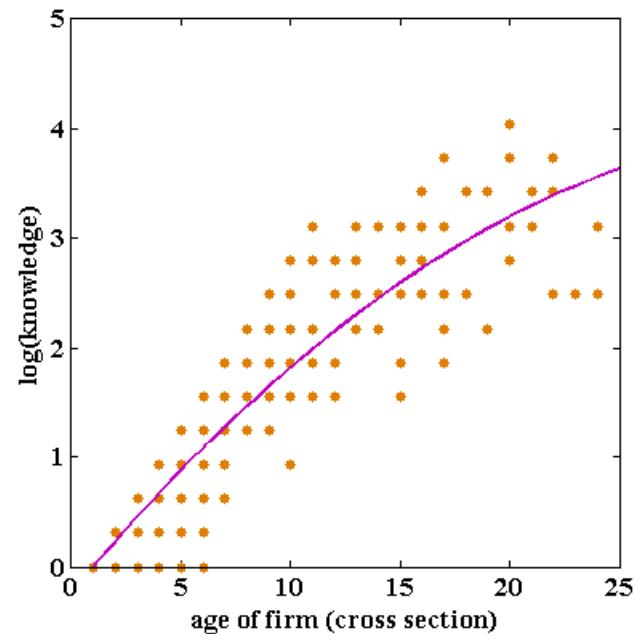
Figure 2.3: $\log(\text{ability} \cdot \text{knowledge})$ - MexicoFigure 2.4: $\log(\text{ability} \cdot \text{knowledge})$ - US

Figure 2.5: $\log(\text{knowledge})$ - Mexico (formal sector)**Figure 2.6:** $\log(\text{knowledge})$ - US (formal sector)

2.7.3 Firm Dynamics in the Model and Data

Table 2.10: Regressions with Simulated Model and Data Firms

explained: output/cap. variables (logs)	data		knowledge		standard	
	(1)	(2)	(1)	(2)	(1)	(2)
lag - output/cap.*firm TFP	-0.02*	–	-0.04	–	-0.43	–
lag - firm TFP	–	-0.06*	–	-1.38	–	-3.38
lag - output/capital ratio	0.58*	0.58*	0.42	0.95	0.76	1.46
year-age controls, firm f. effects	yes	yes	yes	yes	yes	yes
industry controls	yes	yes	no	no	no	no
R ² - within	0.37	0.37	0.49	0.52	0.56	0.64

Statistical significance *1%. (model regressions: all coefficients significant).

As previously discussed, the joint dynamics of output-capital ratios and firm productivity have implications for the impact of financial constraints on misallocation. In the model without knowledge capital, productivity shocks are purely stochastic and mean-reverting. In this case, a firm that is highly constrained in one period is likely to be less constrained in the following period. In a model with knowledge capital, a firm that is constrained in one period can again be highly constrained in the following period if the endogenous productivity component increases. The table below shows that the model with knowledge capital is better able to replicate the dynamics between firm productivity and output-capital ratios estimated from the data.

In terms of firm productivity growth the model with knowledge capital also performs better than the standard model (Table 2.11).²⁸

²⁸This also holds if we consider firm productivity levels instead of firm productivity growth.

Table 2.11: Regressions with Simulated Model and Data Firms

explained: TFP growth variables ¹ (logs)	data		knowledge		standard	
	(1)	(2)	(1)	(2)	(1)	(2)
firm TFP	-0.45*	-0.56*	-1.20	-1.10	-1.87	-1.75
output/capital ratio	0.05*	0.06*	0.27	0.22	0.40	0.34
age of firm	–	-0.03*	–	-0.10	–	-0.11
year controls, firm f. effects	yes	yes	yes	yes	yes	yes
industry controls	yes	yes	no	no	no	no
R ² - within	0.19	0.24	0.33	0.34	0.44	0.45

Statistical significance *1%. (model regressions: all coefficients significant).
¹TFP growth computed between t and $t + 1$, regressors in period t .

2.7.4 Sensitivity Analysis

Introducing knowledge capital can affect the stochastic properties of firm productivity $e^a (\varphi n)^{1-\nu}$ and affect the level of misallocation. The role of persistence in firm productivity is discussed in Buera & Shin (2011). They argue that lower persistence may increase losses from misallocation. I compute the model without knowledge capital with a persistence parameter as low as 0.2 for Mexico: misallocation increases but only to 8.8% (increasing persistence reduces misallocation in my simulations). Although further exercises and alternative calibrations may be conducted, the stochastic properties of the process including knowledge capital do not seem independently responsible for the increase in misallocation: in the baseline calibration of the model with knowledge capital for Mexico, the variance of the marginal productivity of capital is 0.15, close to the lower bound of 0.14 in Midrigan & Xu (2013) and slightly above the 0.12 for my model without knowledge capital.

2.8 Conclusions

The objective of this paper is to contribute to the understanding of the link between firm productivity dynamics and aggregate production efficiency. In particular I focus

on TFP losses attributed to misallocation, which the empirical literature finds to be quantitatively important.

The underdevelopment of financial markets has been proposed as a source of misallocation. However, in a quantitative calibrated model, misallocation losses generated by financial underdevelopment are modest, as pointed out by Midrigan & Xu (2013). I find that considering a model with endogenous firm-productivity accumulation, the misallocation losses are amplified. In the case of Mexico financial constraints generate losses of 7.3% in a model without endogenous firm-productivity and 14.7% in a model with firm-productivity accumulation. This result suggests that the life-cycle accumulation in firm productivity can be important for understanding how financial constraints can generate misallocation. Furthermore, financial constraints affect the distribution of firm productivity and the level of aggregate TFP by distorting the accumulation of productivity at the firm level.

Chapter 3

The Blighted Youth: An International Comparison of Life-Cycle Unemployment

3.1 Introduction

Across advanced OECD economies, youth unemployment rates are typically at least double those of adult unemployment rates and are more sensitive to business cycle fluctuations. Bell & Blanchflower (2011), for example, estimate that youth unemployment rates change 1.79 percentage points for each 1 per cent change in adult rates. The large negative and long-lasting effects on earnings for young workers of entering the labor market in a downturn have been well documented (discussed below). In countries like Spain and Greece, where the unemployment rate for workers of age 15-24 has recently surpassed 50%, the welfare implications are potentially large.

The purpose of our study is to evaluate the costs of recessions and, in particular, the long lasting effects on young workers entering the labor market during a downturn¹. To this effect we build an heterogeneous worker life-cycle model of unemployment with

¹Standard models of search and unemployment, in the Diamond-Mortensen-Pissarides (DMP) tradition, are not able to generate earnings losses comparable to those found in the data (Davis & von Wachter, 2011).

on-the-job human capital accumulation and aggregate productivity shocks². We are interested the quantitative role of the tax-wedge and minimum wages in determining the impact of recessions on unemployment rates across countries.

The quantitative model builds upon the theoretical work of Guido Menzio & Shouyong Shi (2010a, 2010b, 2011). They develop a framework of directed job search and free entry of firms. The main advantage of this framework is the nature of the resulting block recursive equilibrium: value and policy functions of agents are independent of the endogenous distribution of workers across individual states (in our version they will depend on the aggregate state of the economy only through the realization of the aggregate productivity state). This framework is thus particularly useful due to its tractability for analyzing the effect of aggregate productivity shocks on the labor market.

Most closely related to our paper is Menzio et al. (2012). They study a life-cycle model with on-the-job human capital accumulation, search and learning frictions and use this theory to decompose the life-cycle profile of wages, transition rates and productivity into the effects of age variation in work-life expectancy, human capital and match quality.

Our interest is on the interaction of recessions and labor market institutions, the tax-wedge and minimum wages in particular. Therefore we introduce aggregate productivity shocks. To consider labor market institutions we modify the wage determination process from Menzio et al. (2012). In our model the market where a worker decides to search is indexed by ability, age, human capital and the wage paid in the first period. If the match is maintained posterior wages are determined through a Nash-bargaining process.

We also extend the model by introducing heterogeneous ability of workers. Huggett, Ventura & Yaron (2006) exploit a life-cycle model of human capital accumulation to

²We abstract from welfare losses associated with incomplete markets for risk sharing. Rogerson & Schindler (2002) asses the welfare costs of the risk of job displacement, associated with large persistent losses in income. They abstract from heterogeneity in idiosyncratic wage growth (other than that generated by a displacement shock) and from labor market frictions that affect the outcomes experienced by workers.

replicate the age dynamics and cross-sectional properties of the US earnings distribution. They find that differences in the ability to accumulate human capital are essential to reproduce an increase in earnings dispersion over the life cycle and that these differences account for the bulk of the variation in the present value of earnings across agents. Huggett et al. (2011) find that as of age 23, heterogeneity in initial conditions in terms of human capital and learning ability (capacity to accumulate human capital) is the main source of variation in realized lifetime earnings³, as opposed to shocks received over their lifetime. There is evidence from the literature on recessions and youth unemployment that low ability young workers suffer larger losses than higher ability young workers. Furthermore, since these workers will not be able to accumulate human capital, it may affect the persistence of unemployment rates. Workers with less education typically have higher unemployment rates and these rates are more sensitive to business cycle fluctuations.

In our quantitative framework we find that youth unemployment rates are higher and more sensitive to increments in the tax-wedge relative to total unemployment rates. We then simulate one time shocks to aggregate productivity and compare the evolution of unemployment rates in economies with different tax-wedges. We provide estimates of losses in terms of present discounted value of earnings to young workers in different aggregate states of the economy.

The rest of the paper is organized as follows: Section 3.2 discusses the literature on the impact of recessions on young workers entering the labor market, S.3.3 provides a brief overview on the empirical evidence and theoretical work on different institutions and policies and labor market outcomes, S.3.4 and S.3.5 describe our theoretical framework and the nature of the block recursive equilibrium, S.3.6 consists of a discussion of the parameters as well as the quantitative analysis of the model, S.3.7 concludes with final comments.

³Their choice of age is given by the observation that many people will have finished formal schooling. We note that the literature that estimates the impact of youth unemployment on lifetime/long-term earnings considers the range 16/18 to 25 (discussed below). Huggett et al. (2011) calibrate the initial conditions in terms of human capital as exogenous. Admittedly, as those authors recognize, pushing back the age will raise the issue of the importance of one's family or, more broadly, one's environment. We leave that as a topic for further research.

3.2 The Long-Term Impact of Recessions on Youth

Entering the labor market during a recession has a large negative and persistent impact on the labor earnings of the young⁴. Unemployment generates a direct loss of income but there are additional large and long lasting effects that represent costs above the direct cost. The literature is too vast for a complete review, but a set of the main results is presented, with a focus on the evidence for advanced OECD economies⁵.

The general consensus is that an unemployment spell consistently imposes a persistent wage scar upon individuals both in terms of income and posterior unemployment spells (Arumpalam, 2001; Gregg & Tominey, 2005). It is also known that recessions are associated with relatively large increases in unemployment for the young and those with low education (Genda et al., 2010; Bell & Blanchflower, 2011).

Although not focusing on recessions, Kletzer & Fairlie (2003) estimate the long-term costs of job displacement for young adults: five years after a job loss the shortfall in annual earnings is 9% lower for men relative to what would have been expected absent the job loss. For older workers total losses largely represent immediate earnings losses whereas for young workers the loss of opportunities for rapid earnings growth is more important (they find that young displaced workers do not experience a large decline in earnings following displacement). For young workers, substantial costs may be associated with job displacement in the form of missed or delayed opportunities to accumulate human capital.

Kahn (2010) analyzes the labor market experience of those graduating from college

⁴Youth is defined as age over the minimum school-leaving age (typically 16-18 for OECD countries) and less than 25 (Bell & Blanchflower, 2011).

⁵We abstain from comparing these estimations across countries. The wide differences in labor market institutions, educational systems, demographic environments, data availability and applied methodologies make any attempt to compare the estimates an uninteresting exercise (this is a point already made in the literature). A discussion of the statistical techniques employed in the literature is outside the scope of this paper.

as a function of macroeconomic conditions in the US. She estimates an initial wage loss of 6-7% for a 1 percentage point increase in the unemployment rate measure and even 15 years after college graduation the loss is 2.5% and statistically significant.

Table 3.1: The Long-Term Impact of Recessions on Youth

country	earn. loss	period/lag	exercise (<u>shock</u>)
Austria	6.5%	accumulated	1 p.p. unemp. rate increase
Canada	5%	accumulated	5 p.p. unemp. rate increase
Japan	5-7%	12 yrs. later	1 p.p. unemp. rate increase
USA	2.5%	15 yrs. later	1 p.p. unemp. rate increase
Sweden	17%	5 yrs. later	50 days youth unemployment
UK	10%	at age 42	6 months+ of youth unemp.

Source: Brunner & Kuhn (Austria, 2010), Oreopoulos et al. (Canada, 2012), Genda et al. (Japan, 2010), Nordström Skans (Sweden, 2004), Gregg & Tominey (UK, 2005), Kahn (2010, USA).

For Canada, Oreopoulos et al. (2012), also considering those graduating from college, estimate that a rise in unemployment rates by 5 percentage points implies an initial loss in earnings of about 9 percent that halves within 5 years and finally fades to zero after 10 years. The role of heterogeneity is also emphasized: advantaged graduates (at the top of the wage distribution) suffer less as they recover within 2-4 years through a process of mobility towards better firms, while earnings of less advantaged graduates can be permanently affected by cyclical downgrading. The least advantaged suffer a loss of 8 percent of cumulative earnings in their first 10 years, double those of the median graduate. The effects of a recession are strongest for young workers, relative to workers with more experience.

For Austria, Brunner & Kuhn (2010) estimate the effects of labor market entry conditions on wages of males entering the labor market and find that a one percentage point increase in the initial local unemployment rate is associated with an approximate shortfall in lifetime earnings of 6.5 percent (average of the accumulated wage losses within the first 20 years of labor market experience). For Japan, Genda et al. (2010) estimate that a one percentage point rise in the unemployment rate at entry reduces the

likelihood of being employed by 3-4 percentage points for over 12 years. The same event leads to earnings losses of 5-7% for over 12 years for those without college education. Moreover, a recession at the time of entry not only lowers annual earnings but also raises the likelihood of nonemployment and part-time employment for the less educated. A considerable part of the negative on earnings is the effect through lower likelihood of regular stable employment.

For the UK, Gregg & Tominey (2005) estimate the scar from early unemployment to be approximately 10% at age 42 for having over 6 months of youth unemployment if individuals avoid repeat exposure to unemployment. The negative impact is approximately twice as large if the effect on repeated unemployment is taken into account. Early individual unemployment experiences significantly raise the propensity to adult unemployment (see also Gregg, 2001). The role of heterogeneity is emphasized (see also Burgess et al., 2003). The literature in general stresses the importance of heterogeneity⁶ associated with education and ability of the young workers. Individuals securing better qualifications on leaving full-time education are less prone to youth unemployment. This suggests that education can help youths recover from early unemployment but it is not commonly undertaken.

For Sweden, Nordström Skans (2004) estimate that 50 days of unemployment in the year following high school graduation leads to a 3 percentage points higher probability to experience a similar period of unemployment and a decrease in total annual earnings of 17% 5 years later. In Norway, Raaum & Røed (2006) find that individuals who face depressed local labor markets (6% local unemployment rate vs. 1%) when they graduate from secondary education, are subject to relatively high rates of non-employment during their whole prime-age work career.

The severity of long term income losses depend on the business cycle: Davis & von Wachter (2011) estimate that in present value terms men lose on average 1.4 years of pre-displacement earnings if displaced in mass-layoffs events that occur when the national

⁶This is not without econometric challenges: identifying causal effects of past unemployment is a difficult task due to unobserved heterogeneity.

unemployment rate is below 6 percent. This loss doubles to 2.8 years of pre-displacement earnings if the event occurs when the unemployment rate exceeds 8 percent.

A number of theories can potentially predict persistent negative effects from unemployment (see Gregg & Tominey, 2005). An unemployment spell can lead to the depreciation of general skills⁷ and the loss of firm specific skills. For those entering the labor market, time without a job is time forgone in terms of human capital accumulation. Theories of on-the-job search will predict that displacement from a high quality match implies a higher probability of return to a low quality position. Therefore, losses may not solely be due to human capital depreciation. For young workers, mobility plays an important role as it contributes to early wage growth⁸. Theories of screening have also been considered as mechanisms that are able to generate persistent income losses from unemployment. Michaud (2012) provides a theory of information and labor markets with search and matching to account for persistent wage losses of laid-off workers. In this set-up employer uncertainty about unemployed workers' abilities can account for 71% of the long-term wage losses following a lay-off.

We note also that some of these studies estimate the losses suffered by individuals who had unemployment spells, while others refer to losses for those entering the labor market in a downturn but did not experience unemployment. The main channel considered in our theory is through on-the-job human capital accumulation and depreciation. In a recession, youth unemployment rates are higher. Young workers without a job are unable to accumulate human capital, affecting their long term income prospects. Additionally, workers that lose their jobs lose a part of their human capital.

⁷More generally, unemployment spells are also associated with declines in health and general well-being (Bell & Blanchflower, 2011; Davis & von Wachter, 2011).

⁸There is a sizable literature analyzing the sources of life-cycle wage growth. To mention a recent example, Bagger, Fontaine, Postel-Vinay & Robin (2013) construct and estimate an equilibrium job search model with human capital accumulation, employer heterogeneity and individual level shocks. Career wage growth is decomposed into the contributions of human capital and job search: typically considered the two main driving forces of the earnings/experience profile.

3.3 Policies and Labor Markets

This section provides brief overviews of recent empirical evidence and theoretical work, with a focus on OECD economies.⁹

Table 3.2: Unemployment Rate Equations

	total (male)		youth (male)	
tax wedge	0.28***	0.21***	0.45***	0.34***
union density	-0.06***	0.03	-0.11***	0.04
employment protection	-0.55*	-0.50	0.51	-0.38
high corporatism	-1.14***	-1.43***	-1.17	-1.73**
avg. replacement rate	0.14***	0.09***	0.15***	0.08*
output gap	-0.50***	—	-0.98***	—
tfp shock	—	-10.99***	—	-27.44***
terms of trade	—	18.51***	—	33.86***
interest rate	—	0.16***	—	0.26***
labor demand	—	17.60***	—	33.87***
country controls	yes	yes	yes	yes
time controls	yes	yes	yes	yes
R-squared	0.89	0.88	0.90	0.89
n. observations	405	372	404	371

Stat. significance: *** 1%, ** 5%, * 10%.
Source: Bassanini & Duval (2006), World Development Indicators.

The tax wedge is defined as the difference between the gross labor costs to employers and the consumption wage paid to employers, i.e. the wage after deduction of direct and indirect taxes, including payroll taxes, income taxes and consumption taxes (Addison & Teixeira, 2001). Nickell et al. (2005) estimate that a 10 percentage point increase in total employment tax rate leads to approximately a 1 percentage point increase in unemployment in the long run. They also find that changes in labor market institutions

⁹Addison & Teixeira (2001) survey the literature on the labor market consequences of employment protection legislation. They conclude that the preponderance of the studies support the hypothesis that stricter employment protection rules result in lower employment-population ratios. There is, however, no consensus with respect to the effect on unemployment rates. See also Bassanini & Duval (2006) and Nickell et al. (2005). The ambiguous impact of firing costs is also found in the theoretical literature (see Ljungqvist, 2002).

explain approximately 55% of the rise in European unemployment from the 1960s to the first half of the 1990s, much of the remainder being due to the deep recession in the latter period. They estimate that changes in the benefit system and increases in labor taxes contribute the most to the increase of 6.8 percentage points in unemployment in this period: the combination of benefits and taxes are responsible for two thirds of the part of the long-term rise in European unemployment that the considered institutions explain (see also Nickell & Layard, 1999).

In line with these findings, Bassanini & Duval (2006) estimate that changes in labor market institutions can account for two-thirds of non-cyclical unemployment changes in OECD countries. In particular, they estimate that a 10 percentage point reduction in the tax wedge would be associated with a drop in the unemployment rate by 2.8 percentage points. They also conclude that the level and duration of unemployment benefits have a significantly positive impact on unemployment. We re-estimate their specifications using male unemployment rates. For a 10 percent point reduction in the tax wedge the total male unemployment rate increases by 2.1-2.8 percentage points, the youth unemployment rate increases by 3.4-4.5 percentage points (Table 3.2). Youth unemployment rates are more sensitive to different types of macroeconomic shocks.

There is work examining the interaction of labor market institutions with the business cycle. Bernal-Verdugo et al. (2012b). estimate a large negative impact of financial crises on unemployment, finding that the effect is less pronounced in the short-run (generally less than or equal to 2-3 years) but more persistent in countries with more rigid labor market institutions. The effects are larger for youth unemployment. It is often found that unemployment for youth is more sensitive to labor market institutions. Nickell & Layard (1999) argue that minimum wages are likely to have a significant impact on the unemployment rate of low skill and young workers.

In terms of theoretical work, Prescott (2004) uses a neoclassical growth model with a stand-in household to argue that variations in tax rates account for most differences in labor supply in advanced G7 economies. Ohanian, Raffo & Rogerson (2008), also

exploiting a neoclassical growth model, confirm that taxes can account for a large proportion of cross-country differences and variation in hours worked (the product of total civilian employment and annual hours per worker) for a set of OECD countries. Ljungqvist & Sargent (2007) examine the previous analysis and emphasize the role of differences in unemployment benefits rather than tax rates to explain cross-country differences in employment.¹⁰

3.4 Baseline Environment

In this section we describe the theoretical framework without permanent heterogeneous ability to avoid cumbersome notation¹¹. The framework consists of a life-cycle model with on-the-job human capital accumulation. There are frictional labor markets with search and matching. Search is directed and markets are labeled by age of the worker, human capital and the first period payment to the worker. After the first period, wages are determined through a Nash-bargaining process. There are aggregate and idiosyncratic (match-specific) productivity shocks.

3.4.1 Demographics

- There is continuum of workers of measure normalized to one, uniformly distributed across overlapping generations with age $t \in \{1, \dots, T\}$. Each worker is endowed with one indivisible unit of labor.
- The mass of entering (newly born) workers is equal to $1/T$ which equals the mass of retiring/dying workers.
- There is a continuum of firms with positive measure (continuum of potential firms having infinite mass).

¹⁰The role of segmented/two-tier labor markets has also been analyzed, see Blanchard & Landier (2002) and Cahuc & Postel-Vinay (2002).

¹¹Introducing this dimension is straightforward, as it amounts to solving for the block recursive equilibrium for the different types of ex-ante heterogeneous workers. With that extension markets are also indexed by ability of the worker.

3.4.2 Preferences and Technology

- We assume risk neutrality for both workers and firms and a common discount factor $\beta \in (0, 1)$.
- Stochastic shocks: aggregate productivity is denoted $y \in Y$ with AR(1) process $\Lambda(y' | y)$ and match idiosyncratic productivity $z \in Z$ with AR(1) process denoted $\Lambda(z' | z)$, we may also write $s = (z, y)$ and $\Lambda(s' | s)$ as the joint process (this allows for more general joint stochastic processes).
- The human capital of the worker is $h \in \mathbf{R}_+$, which evolves according to the law of motion $h' = h + 1$ for the periods during which the worker is employed and remains fixed when the worker is not employed. There is an initial level of human capital \underline{h} for all newborn workers. When a match between a worker and a firm is destroyed, human capital depreciates to $\mu(h)$, with lower bound at \underline{h} . The upper bound on human capital is \bar{h} , with $\bar{h} \leq T$.
- Production is carried out in a match between a firm and a worker with production technology $f(y, z, h)$.

3.4.3 Markets

- There is continuum of markets labeled by $(w, h, t, y) \in \mathbf{R}_+ \times \mathbf{N}^3$ where firms commit to pay w for the first period of the match to a worker with (h, t) .
- After the initial period the wage is determined through Nash-bargaining when the worker has no alternative job offer. We assume the worker accepts an offer in the market he has chosen to search¹².
- The measure of unemployed workers is written as $u(h, t)$ where $u : \mathbf{N}^2 \rightarrow \mathbf{R}_+$, the measure of employed workers is $e(z, h, t)$ where $e : Z \times \mathbf{N}^2 \rightarrow \mathbf{R}_+$. The aggregate state vector is then $\psi = (u, e, y)$.
- The tightness for a labor market with (w, h, t) is denoted $\theta_t(w, h, \psi)$.

¹²At the bargaining stage the idiosyncratic productivity level available for production for a potential new match is known but not the value that will be effective for production at the existing match in that same period.

3.4.4 Timing

For the rest of this section we assume that a block recursive equilibrium exists and omit ψ from the vector of state variables, the aggregate shock y does remain as a relevant aggregate state variable. The existence of the block recursive equilibrium is proven by construction in the appendix, following the methodology of Menzio et al. (2012) for a life cycle economy with directed search and free-entry of firms (adapted to introduce our extensions). The timing within a period is as follows:

- **Entry-and-Exit of Workers, Aggregate Shock.** At the beginning of the period newly born workers enter the market and workers of age $T + 1$ retire and die. The aggregate productivity shock y is revealed.
- **Search and Matching.** The unemployed workers search for a job with probability λ_u , while employed workers are allowed to search for an alternative job with probability λ_e . A firm opens a vacancy after paying vacancy cost c_v . A worker in market (w, h, t, y) meets a vacancy with probability $p(\theta_t(w, h, y))$ where $p : \mathbf{R}_+ \rightarrow [0, 1]$ is a twice-differentiable, strictly increasing and strictly concave function with $p(0) = 0$ and $p(\infty)=1$. A vacancy in market (w, h, t, y) meets a worker with probability $q(\theta_t(w, h, y))$ where $q : \mathbf{R}_+ \rightarrow [0, 1]$ is a twice-differentiable, strictly decreasing function and with $q(\theta) = p(\theta)/\theta$, $q(0) = 1$ and $q(\infty)=0$.
- **Wage Determination.** When the worker meets a new firm, the firm pays the initial posted w in the first period. If the worker is matched with no alternative job offers, the wage is determined through Nash-bargaining with the current firm. If no agreement is reached, the match is destroyed. The unemployed worker accepts the offer he receives (if any), otherwise produces and consumes b .
- **Production.** The idiosyncratic productivity state at the beginning of the period is z , at the production stage the new idiosyncratic shock z' is revealed (for new matches it is known in advance¹³ and equal to \bar{z}). The match produces $f(y, z', h)$. The accumulation of human capital takes place with production: a matched worker

¹³We may set \bar{z} equal to any value $z \in Z$.

that enters the period and produces with human capital h , is endowed with human capital $h' = h + 1$ immediately after production takes place. The matched worker gets paid and his consumption takes place.

- **Exogenous Separation.** There is a probability δ of a shock that destroys the match (exogenous job destruction).

3.4.5 Value of the Worker

Let's consider in parts the problem of the unemployed worker before the search stage. With probability λ_u the unemployed individual has the possibility of searching, if successful receives wage w^u and enters next period as an employed worker with probability $1 - \delta$. This part of the value of the unemployed worker is given by:

$$\lambda_u p(\theta_t(w^u, h, y)) \left\{ w^u + \beta \sum_{\{y'\}} \Lambda(y' | y) \left((1 - \delta) V_{t+1}(h', \bar{z}, y') + \delta U_{t+1}(\mu(h'), y') \right) \right\}$$

All new matches produce with idiosyncratic productivity value \bar{z} in their first period of existence, it will also be the state value at the beginning of the following period. The unemployed worker may remain unemployed for two reasons: he was not given a chance to search for a job, or was unsuccessful in his search. In this case, he produces and consumes b today and enters the next period with unemployment status, with unchanged level of human capital at age $t + 1$. This component can be written as:

$$(1 - \lambda_u p(\cdot)) \left\{ b + \beta \sum_{\{y'\}} \Lambda(y' | y) U_{t+1}(h, y') \right\}$$

We can write the beginning-of-the-period value of unemployment as:

$$U_t(h, y) = \max_{\{w^u\}} b + \beta \sum_{\{y'\}} \Lambda(y' | y) \left\{ U_{t+1}(h, y') + \lambda_u p(\theta_t(w^u, h, y)) S_t^u(w^u, h, y') \right\}$$

where $S_t^u(w^u, h, y')$ summarizes the gain for the worker of finding a match:

$$S_t^u(w^u, h, y') = \left[\frac{w^u - b}{\beta} + (1 - \delta) V_{t+1}(h', \bar{z}, y') + \delta U_{t+1}(\mu(h'), y') - U_{t+1}(h, y') \right]$$

V_{t+1} is the beginning of the period value of the matched worker (after the aggregate shock is revealed). The policy function of the unemployed worker is $w_t^u(h, y)$.

A matched worker may be given a chance to search for an alternative job offer with probability λ_e , if successful he receives the posted wage of the corresponding labor market and enters the next period as a matched worker with probability $1 - \delta$, this possibility can be written as:

$$\lambda_e p(\theta_t(w^a, h, y)) \left\{ w^a + \beta \sum_{\{y'\}} \Lambda(y' | y) ((1 - \delta) V_{t+1}(h', \bar{z}, y') + \delta U_{t+1}(\mu(h'), y')) \right\}$$

If the currently matched worker does not receive an alternative job offer:

$$(1 - \lambda_e p(\cdot)) V_t^b(h, z, y)$$

where the wage in $V_t^b(h, z, y)$ is determined through Nash-bargaining. The value function is then:

$$V_t(h, z, y) = \lambda_e p(\cdot) \left\{ w^a + \beta \sum_{\{y'\}} \Lambda(y' | y) ((1 - \delta) V_{t+1}(h', \bar{z}, y') + \delta U_{t+1}(\mu(h'), y')) \right\} \\ + (1 - \lambda_e p(\cdot)) V_t^b(h, z, y)$$

The policy function for a matched worker is denoted $w_t^a(h, z, y)$. Before discussing how this value is determined through the bargaining process it will be useful to describe the problem of the firm.

3.4.6 Value of the Firm

At the beginning of the period, when the aggregate shock is revealed, the value of a currently matched firm is $F_t(h, z, y)$. After the search stage there are two possible situations, with probability $\lambda_e p(\theta_t(w^a, h, y))$ the worker has found an alternative job offer and the previous match is destroyed. If the worker has no alternative job offer the new value of the firm is determined at the bargaining stage $F_t^b(h, z, y)$. We can then

write the beginning-of-the-period value of the firm as:

$$F_t(h, z, y) = (1 - \lambda_e p(\theta_t(w^a, h, y))) F_t^b(h, z, y)$$

The value of a newly matched firm is $G_t(w^a, h, \bar{z}, y)$:

$$G_t(w^a, h, \bar{z}, y) = f(y, \bar{z}, h) - w^a + \beta \sum_{\{y'\}} \Lambda(y' | y) F_{t+1}(h', \bar{z}, y'),$$

We turn next to the bargaining stage.

3.4.7 Determination of Wages

If the worker was unsuccessful in obtaining an alternative offer (whether because he did not have the possibility of searching, or was not successful in the search stage), his outside option is (human capital depreciates if the match is destroyed):

$$b + \beta \sum_{\{y'\}} \Lambda(y' | y) U_{t+1}(\mu(h), y')$$

while reaching an agreement with the current employer gives (before z' is revealed):

$$w^b + \beta \sum_{\{s'\}} \Lambda(s' | s) \left\{ (1 - \delta) V_{t+1}(h', z', y') + \delta U_{t+1}(\mu(h'), y') \right\}$$

For the firm, at the bargaining stage the outside value is zero. The value of maintaining the match is:

$$F_t^b(h, z, y) = -w^b + \sum_{\{s'\}} \Lambda(s' | s) \left\{ f(y, z', h) + \beta (1 - \delta) F_{t+1}(h', z', y') \right\}$$

Note that current period production takes place with productivity value z' and y . There is a cutoff function $z_t^b(h, y)$, the lowest level of the idiosyncratic productivity shock such that the joint surplus of the match is non-negative. Given these values, worker and firm bargain over the wage, through a Nash-bargaining process where the worker has bargaining power ξ (see Appendix). Finally $F_{T+1} = 0$, $U_{T+1} = 0$ and $V_{T+1} = 0$.

3.4.8 New Vacancies and Free Entry Condition

To close the model we specify the free entry condition of firms. The cost of a vacancy is c_v , in equilibrium the following condition has to hold:

$$c_v \geq q(\theta_t(w, h, y)) \left\{ f(y, \bar{z}, h) - w + \beta(1 - \delta) \sum_{\{y'\}} \Lambda(y' | y) F_{t+1}(h', z, y') \right\}$$

and $\theta_t(w, h, y) \geq 0$ with complementary slackness.

3.5 Block Recursive Equilibrium

Definition. A *Block Recursive Equilibrium* (BRE) consists of value functions U_t for unemployed workers, V_t for employed workers, F_t for previously matched firms and G_t for newly matched firms, policy functions w_t^u for unemployed workers and w_t^a for employed workers, a bargained wage function w_t^b determined between an employed worker and a firm, a cutoff productivity function z_t^b , and a tightness function θ_t for $t = 1, \dots, T$ such that (i) $U_t, V_t, F_t, G_t, w_t^u, w_t^a, w_t^b, z_t^b$ and θ_t depend on ψ only through y for $t = 1, \dots, T$, (ii) F_t, G_t and θ_t are consistent with the firm's rationality and the free-entry condition for $t = 1, \dots, T$, (iii) U_t and w_t^u solve the unemployed worker's problem for $t = 1, \dots, T$, (iv) V_t and w_t^a solve the employed worker's problem for $t = 1, \dots, T$, and (v) w_t^b and z_t^b solve the bargaining problem between an employed worker and a firm for $t = 1, \dots, T$.

Theorem. A recursive equilibrium exists and is block recursive and unique.

The proof is in the appendix for the case without human capital depreciation (the extension is straightforward). To gain some intuition on this result first consider the assumption of directed search. Markets are indexed by age and human capital of the worker (and ability when this extension is considered). Thus, a firm opening a vacancy in a particular market will know the characteristics of the worker that it will potentially find. If search was not directed, to calculate the expected discounted profits of opening a vacancy the firm would need to know the distribution of workers with different

characteristics (for example, human capital determines in part the productivity of the match).

In the market that the workers searches for a job the number of vacancies will adjust so that the free-entry condition holds for the firms. There are different pairs of first-period wages and market tightness that could deliver zero expected discounted profits for firms. The additional condition that determines this pair in equilibrium is a concave maximization search problem for each particular type of worker. In the last period of the worker, it is straightforward to verify that all value and policy functions as well as bargained wages are independent of the distribution of workers over their individual state variables. By backward induction a block recursive equilibrium can be constructed.

3.6 Quantitative Analysis

In this section we describe the specification employed for the different functions of the model, the calibration strategies and standard parameters employed in the literature. We then simulate the economy to evaluate the role of the tax-wedge on unemployment rates.

3.6.1 Baseline Parameters and Function Specifications

We modify the production function of Menzio et al. (2012), by considering an AR(1) process for idiosyncratic productivity shocks, AR(1) process for aggregate productivity, heterogeneous ability of workers. The production function is:

$$f(a, y, z, h) = a e^{z+y} h^\gamma$$

where a is the permanent ability of the individual, z is the match-idiosyncratic productivity shock, y is the aggregate productivity shock, γ determines curvature with respect to human capital in the production function.

- A time period is one month and $\beta \in [0.996, 0.9967]$ is typically calibrated so that the annual real interest rate is equal to 4-5 percent.
- Bargaining power is equal for firms and workers, $\xi = 1/2$.
- We set the home production parameter b to target a total average unemployment rate of 6.5 percent. The vacancy cost is set to 10.42 times the value of home production following Menzio et al. (2012).
- In the case with no initial heterogeneity $a = 1$, with heterogeneity $a \in \{0.9, 1.0, 1.1\}$ with weights $\{0.25, 0.50, 0.25\}$, respectively. The range of human capital is from $\underline{h} = 1$ to \bar{h} (set to match the peak of mean earnings, Huggett et al., 2006). The function for human capital depreciation is the grid approximation to $\mu(h) = h/(1.08)$.

Table 3.3: Parameters

parameter description	par.	value
discount factor	β	0.996
periods of life	T	40×12
vacancy cost	c_v	$10.42 \times b$
production - curvature	γ	0.06
human capital range	\bar{h}	25×12
human capital depreciation	$\mu(h)$	$h/(1.08)$
exogenous destruction	δ	0.01
unemployed search prob.	λ_u	1.00
on-the-job search prob.	λ_e	0.80
matching function	$p(\theta)$	$\min\{\theta^{1/2}, 1\}$
aggregate process autocorrelation	ρ_y	0.90
aggregate process volatility	σ_y	0.01
idiosyncratic process autocorrelation	ρ_z	0.00
idiosyncratic process volatility	σ_z	0.12
bargaining parameter	ξ	1/2

Parameters related to probabilities and distributions:

- For the aggregate productivity process $\Lambda(y' | y)$, considering a quarterly time period, Den Haan et al. (2000) utilize an autocorrelation¹⁴ of 0.95 and standard deviation 0.007. Pries (2008), in a monthly model, selects the autocorrelation and the standard deviation process to target quarterly US data on real average output per worker in the non-farm business sector, with autocorrelation of 0.878 and standard deviation of 0.02.
- For the idiosyncratic productivity process, Den Haan et al. (2000) consider an iid shock with standard deviation equal to 0.101. Bilts et al. (2011) consider a persistence of idiosyncratic productivity of 0.97 and a standard deviation of 0.13. The initial idiosyncratic productivity level for new matches is set to the highest idiosyncratic productivity level.
- For long-term employment relationships, quarterly US worker separation rates lie in the range 8-10 percent (Den Haan et al., 2000; pg. 490). Den Haan et al. (2000) consider a 10 percent rate of total separation (see also Pries, 2008), with an exogenous separation rate of 0.068. We consider exogenous job destruction and the possibility of endogenous job destruction. In Menzio & Shi (2011) the exogenous destruction rate is $\delta \in \{0.012, 0.026\}$.
- λ_e determines the rate of transition to new jobs, λ_u governs the rate of transition from unemployment to employment. In Menzio & Shi (2011) $\lambda_e \in \{0.735, 0.904\}$.

3.6.2 Business Cycle Simulations

We conduct simulations to learn about the impact of the tax-wedge and the role of heterogeneity in the ability of workers. The benchmark tax wedge is 0.344.¹⁵ In the case of no heterogeneity, for example, increasing the tax wedge by 5 p.p. increases the unemployment rate from 0.065 to 0.079.

¹⁴We apply the Rouwenhorst method of approximating a stationary AR(1) process following Kopecky & Suen (2010), this method is found to be reliable relative to other methods in approximating highly persistent processes.

¹⁵The OECD definition of the total tax wedge: combined central and sub-central government income tax plus employee and employer social security contribution taxes, as a percentage of labour costs defined as gross wage earnings plus employer social security contributions. For the US the total tax wedge is 34.4%.

Table 3.4: Business Cycle Simulations: No Heterogeneity

average	$\tau = 0.344$	$\tau = 0.394$	$\tau = 0.444$
youth unemployment	0.094	0.114	0.164
<u>prime</u> unemployment	0.059	0.071	0.102
total unemployment	0.065	0.079	0.115
volatility	$\tau = 0.344$	$\tau = 0.394$	$\tau = 0.444$
youth unemployment	0.037	0.039	0.039
<u>prime</u> unemployment	0.036	0.048	0.058
total unemployment	0.026	0.041	0.050

The unemployment rate for workers of age 21-25 is always higher¹⁶ compared to the unemployment rate for those aged 35-45 and the total unemployment rate. The tax wedge increases the volatility of unemployment rates in the case of no heterogeneity, but the effect is not monotonic in a version of the model with ex-ante heterogeneity.¹⁷

Table 3.5: Business Cycle Simulations: With Heterogeneity

average	$\tau = 0.344$	$\tau = 0.394$	$\tau = 0.444$
youth unemployment	0.093	0.118	0.343
<u>prime</u> unemployment	0.058	0.073	0.294
total unemployment	0.065	0.082	0.305
volatility	$\tau = 0.344$	$\tau = 0.394$	$\tau = 0.444$
youth unemployment	0.039	0.046	0.016
<u>prime</u> unemployment	0.042	0.052	0.011
total unemployment	0.032	0.045	0.010

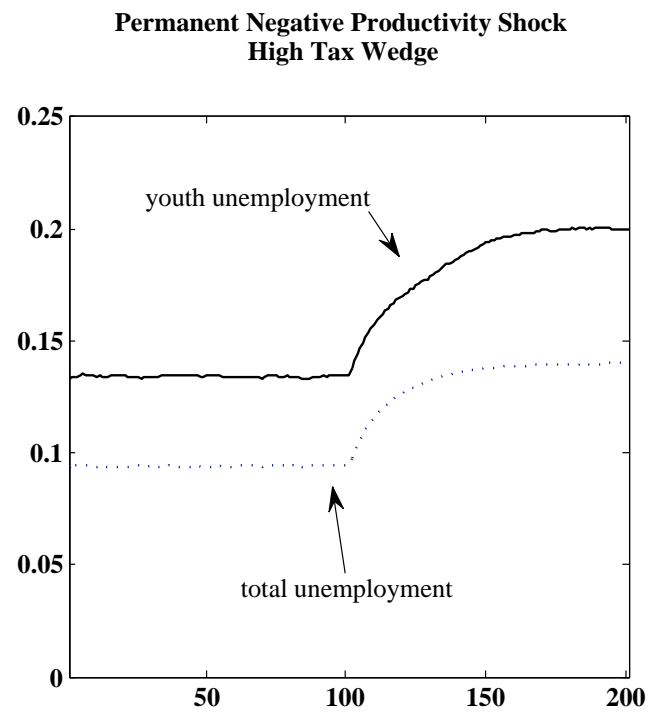
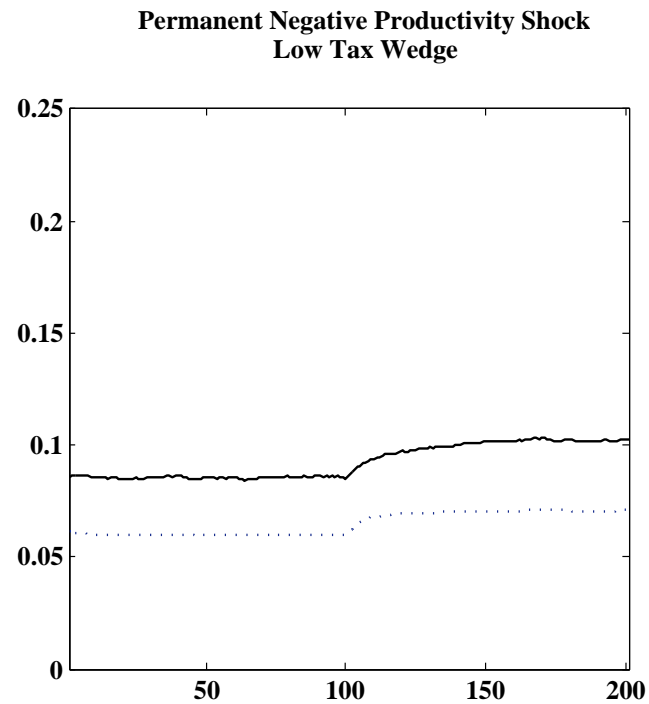
3.6.3 Simulations of a Recession

In this section we simulate two economies, one with a tax wedge of 0.344 and another economy with a tax wedge 10 percentage points higher, equal to 0.444. In both

¹⁶Individuals enter the market unemployed at age 20, we give them one year to find a job (12 opportunities) and consider age 21-25 for youth unemployment.

¹⁷For the computation of volatility, we first take the natural log of the series, then remove the Hodrick-Prescott trend with a filter parameter of 10^5 (see Pries, 2005).

economies the aggregate productivity shock permanently falls from the highest to lowest possible state. In the high tax-wedge economy, the starting difference between youth and total unemployment is larger compared to the low tax-wedge economy and is more amplified by a recession.

Figure 3.1: Simulation of a Recession

3.6.4 Losses due to Unemployment in Present Discounted Values

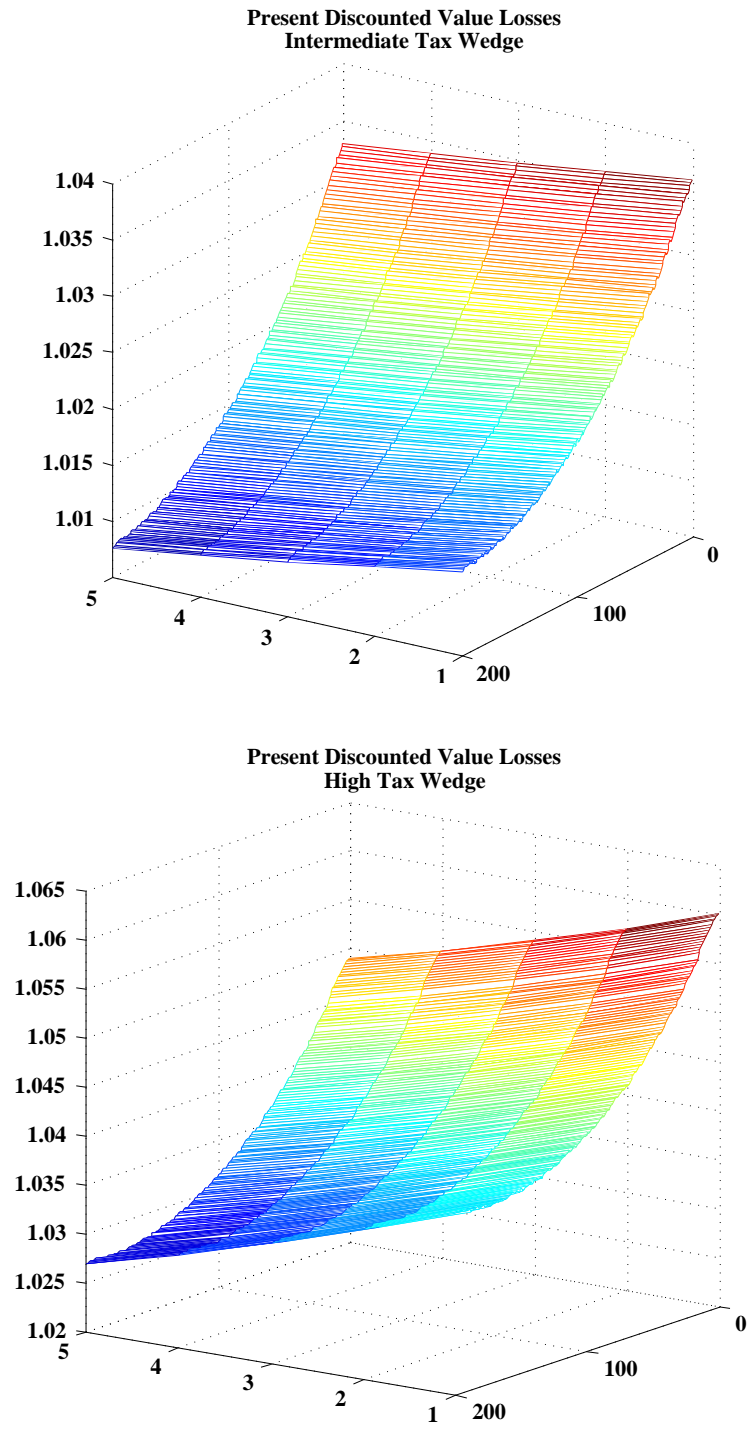
In this final quantitative section we compute the expected present discounted losses in labor earnings caused by unemployment (graphs in the appendix). The graphs show the ratio of the expected present discounted value of labor earnings of an employed worker of a particular age (in months starting at 20 years old), relative to the expected present discounted value of labor earnings for an individual of the same age that lost his job and whose human capital depreciated accordingly.¹⁸ The graphs show this ratio for different ages and in 5 different states of the aggregate productivity (5 is the highest aggregate productivity and 1 is the lowest). The main results are that losses are bigger: (1) in worse aggregate states of the economy, (2) for younger individuals, (3) in the economy with the higher tax wedge. Additionally, in a version of the model with ex-ante heterogeneity, the losses are larger for low ability individuals.

3.7 Final Comments

Recessions generate sizable losses for young workers entering the labor force. Unemployment generates a direct income loss but also a fall in future income attributed to foregone human capital accumulation. We analyze a life-cycle model of workers with heterogeneous ability and on-the-job human capital accumulation and depreciation due to job loss¹⁹. We find that unemployment rates of young workers are typically higher and more sensitive to the tax-wedge, consistent with the empirical estimates. Long-term earnings losses generated by match destruction are bigger: (1) in worse aggregate states of the economy, (2) for younger individuals, (3) in the economy with the higher tax wedge, (4) for ex-ante low ability individuals.

¹⁸See the appendix for a mathematical derivation of the expected present discounted value of labor earnings.

¹⁹This depreciation may be due to firm-specific or occupation-specific human capital. Kambourov & Manovskii (2009) find that when occupational experience is taken into account, tenure in a certain industry or a particular employer has quantitatively little importance in accounting for wages. This evidence could be interpreted as contrary to information theories of wage formation where employer learn about the productivity of their workers. Occupation specific human capital is transferable across employers.

Figure 3.2: Losses due to Unemployment

Bibliography

Abdelhamid, D. and A. El Mahdi (2003). “The Small Business Informality Challenge: Lessons Learned From Country Experiences and The Road Ahead of Egypt,” Economic Research Forum, WP #0324.

Abraham, A. and K. White (2006). “The Dynamics of Plant-Level Productivity in U.S. Manufacturing,” Center of Economic Studies, U.S. Census Bureau, # 06-20.

Addison, J.T. and P. Teixeira (2001). “The Economics of Employment Protection,” IZA Discussion Paper #381.

Aghion, P., P. Howitt and D. Mayer-Foulkes (2005). “The Effect of Financial Development on Convergence: Theory and Evidence,” Quarterly Journal of Economics, 120 (1), pp. 173-222.

Alfaro, L., A. Charlton and F. Kanczuk (2009). “Plant Size Distribution and Cross-Country Income Differences,” NBER International Seminar on Macroeconomics.

Almeida, R. and P. Carneiro (2009). “Enforcement of Labor Regulation and Firm Size,” Journal of Comparative Economics, 37 (1), pp. 28-46.

Amaral, P.S. and E. Quintin (2006). “A Competitive Model of the Informal Sector,” Journal of Monetary Economics, 53 (7), pp. 1541-1553.

Amaral, P.S. and E. Quintin (2010). “Limited Enforcement: Financial Intermediation, and Economic Development: A Quantitative Assessment,” International Economic Review, 51 (3), pp. 785-811.

Antunes, A.R. and T.V.V. Cavalcanti (2007). “Start Up Costs, Limited Enforcement, and the Hidden Economy,” European Economic Review, 51 (1), pp. 203-224.

Arellano, C., Y. Bai and J. Zhang (2012). “Firm Dynamics and Financial Development,” Journal of Monetary Economics, 59 (6), pp. 533-549.

Arulampalam, W. (2001). “Is Unemployment Really Scarring? Effects of Unemployment on Wages,” Economic Journal, 111 (475), pp. F585-F606.

Asker, J., A. Collard-Wexler and J. De Loecker (2012). “Productivity Volatility and the Misallocation of Resources in Developing Economies,” WP.

Atkeson, A. and A.T. Burstein (2010). “Innovation, Firm Dynamics and International Trade,” Journal of Political Economy, 118 (3), pp. 433-484.

Atkeson, A. and P.J. Kehoe (2005). “Modeling and Measuring Organization Capital,” Journal of Political Economy, 113 (5), pp. 1026-1053.

Audretsch, D.B. (1995). “Innovation, Growth and Survival,” International Journal of Industrial Organization, 13 (4), pp. 441-457.

Ayyagari, M., A. Demirgüç-Kunt and V. Maksimovic (2007). “Firm Innovation in Emerging Markets,” World Bank Policy Research WP. # 4157.

Bagger, J. F. Fontaine, F. Postel-Vinay, J.M. Robin (2013). “Tenure, Experience, Human Capital and Wages: A Tractable Equilibrium Search Model of Wage Dynamics,” WP.

Barseghyan, L. and R. DiCecio (2011). “Entry costs, industry structure, and cross-country income and TFP differences,” Journal of Economic Theory, 146 (5), pp. 1828-1851.

Bartelsman, E.J. and M. Doms (2000). “Understanding Productivity: Lessons from Longitudinal Microdata,” Journal of Economic Literature, 38 (3), pp. 569-594.

Bartelsman, E.J., J. Haltiwanger and S. Scarpetta (2009). “Measuring and Analyzing Cross-country Differences in Firm Dynamics,” in T. Dunne, B. Jensen and M.J. Roberts (Eds.), *Producer Dynamics: New Evidence from Micro Data*. University of Chicago Press.

Bassanini, A. and R. Duval (2006). “The Determinants of Unemployment Across OECD Countries: Reassessing the Role of Policies and Institutions,” OECD Economics Studies, 42, pp. 7-86.

Beck, T. and A. Demirgüç-Kunt (2006). “Small and medium-size enterprises: Access to finance as a growth constraint,” Journal of Banking and Finance, 30 (11), pp. 2931-2943.

Beck, T., A. Demirgüç-Kunt and R. Levine (2009). “Financial Institutions and Markets across Countries and Over Time - Data and Analysis,” Policy Research Working Paper #4943, The World Bank.

Bell, D.N.F. and D.G. Blanchflower (2011). “Young people and the Great Recession,” Oxford Review of Economic Policy, 27 (2), pp. 241-267.

Benkard, C.L. (2000). “Learning and Forgetting: The Dynamics of Aircraft Production,” American Economic Review, 90 (4), pp. 1034-1054.

Bernal-Verdugo, L.E., D. Furceri and D. Guillaume (2012a). “Labor Market

Flexibility and Unemployment: New Empirical Evidence of Static and Dynamic Effects,” Comparative Economic Studies, 54 (2), pp. 251-273.

Bernal-Verdugo, L.E., D. Furceri and D. Guillaume (2012b). “Crises, Labor Market Policy, and Unemployment,” IMF Working Paper #12/65.

Bhattacharya, D., N. Guner and G. Ventura (2013). “Distortions, Endogenous Managerial Skills and Productivity Differences,” Review of Economic Dynamics, 16 (1), pp. 11-25.

Bils, M., Y. Chang and S.B. Kim (2011). “Worker Heterogeneity and Endogenous Separations in a Matching Model of Unemployment Fluctuations,” American Economic Journal: Macroeconomics, 3 (1), pp. 128-154.

Blanchard, O. and A. Landier (2002). “The Perverse Effects of Partial Labour Market Reform: Fixed-Term Contracts in France,” Economic Journal, 112 (480), pp. F214-F244.

Bruhn, M. (2012). “A Tale of Two Species: Revisiting The Effect of Registration Reform on Informal Business Owners in Mexico,” The World Bank, WP #5971.

Brunner, B. and A. Kuhn (2010). “The Impact of Labor Market Entry Conditions on Initial Job Assignment, Human Capital Accumulation, and Wages,” Institute for Empirical Research in Economics, University of Zurich, WP. N. 520.

Buera, F., J. Kaboski and Y. Shin (2011). “Finance and Development: A Tale of Two Sectors,” American Economic Review, 101 (5), pp. 1964-2002.

Buera, F.J. and Y. Shin (2011). “Self-insurances vs. self-financing: A welfare analysis of the persistence of shocks,” Journal of Economic Theory, 146 (3), pp. 845-862.

Burgess, S., C. Propper, H. Rees and A. Shearer (2003). “The class of 1981:

the effects of early career unemployment on subsequent unemployment experiences,” Labour Economics, 10 (3), pp. 291-309.

Busso, M., M.V. Fazio and S. Levy (2012). “(In)Formal and (Un)Productive: The Productivity Costs of Excessive Informality in Mexico,” Inter-American Development Bank.

Busso, M., L. Madrigal and C. Pagés (2012). “Productivity and Resource Misallocation in Latin America,” Inter-American Development Bank WP-306.

Caggese, A. (2012). “Entrepreneurial Risk, Investment, and Innovation,” Journal of Financial Economics, 106 (2), pp. 287-307.

Cahuc, P. and F. Postel-Vinay (2002). “Temporary jobs, employment protection and labor market performance,” Labour Economics, 9 (1), pp. 63-91.

Camacho, A. and E. Conover (2010). “Misallocation and Productivity in Colombia’s Manufacturing Industries,” Inter-American Development Bank WP-123.

Caselli, F. (2005). “Accounting for Cross-Country Income Differences,” in P. Aghion and S.N. Durlauf (Eds.), *Handbook of Economic Growth*, Vol. 1, Ch. 9. North-Holland, Amsterdam.

Catão, L.A.V., C. Pagés, M.F. Rosales (2009). “Financial Dependence, Formal Credit and Informal Jobs: New Evidence from Brazilian Household Data,” Inter-American Development Bank.

Cole, H.L., J. Greenwood and J.M. Sanchez (2012). “Why Doesn’t Technology Flow from Rich to Poor Countries?” Federal Reserve Bank of St. Louis.

Corrado, C., C. Hulten and D. Sichel (2009). “Intangible Capital and U.S. Economic Growth,” Review of Income and Wealth, 55 (3), pp. 661-685.

D’Erasmus, P.N. and H.J. Moscoso-Boedo (2012). “Financial Structure, Informality and Development,” Journal of Monetary Economics, 59 (3), pp. 286-302.

D’Erasmus, P.N. (2013). “Access to Credit and the Size of the Formal Sector in Brazil,” Inter-American Development Bank, WP #404.

Dabla-Norris, E., E. Kersting and G. Verdier (2010). “Firm Productivity, Innovation and Financial Development,” IMF WP/10/49.

Davis, S.J. and T.M. von Wachter (2011). “Recessions and the Cost of Job Loss,” NBER WP# 17638.

de Paula, A. and J.A. Scheinkman (2011). “The Informal Sector: An Equilibrium Model and Some Empirical Evidence from Brazil,” Review of Income and Wealth, 57, pp. S8-S26.

De Soto, H. (1989). The Other Path: The Invisible Revolution in the Third World. New York, Harper & Row Publishers.

den Haan, W.J., G. Ramey and J. Watson (2000). “Job Destruction and Propagation of Shocks,” American Economic Review, 90 (3), pp. 482-498.

Djankov, S., R. La Porta, F. Lopez de Silanes and A. Shleifer (2002). “The Regulation of Entry,” Quarterly Journal of Economics, CXVII (1), pp. 1-37.

Djankov, S. (2009). “The Regulation of Entry: A Survey,” World Bank Research Observer, 24 (2), pp. 183-203.

Djankov, S., T. Ganser, C. McLiesh, R. Ramalho and A. Shleifer (2010). “The Effect of Corporate Taxes on Investment and Entrepreneurship,” American Economic Journal: Macroeconomics, 2 (3), pp. 31-64.

Doraszelski, U. and J. Jaumandreu (2013). “R&D and Productivity: Estimating Endogenous Productivity,” Review of Economics Studies, forthcoming.

El-Ehwany, N. and M. Metwally (2001). “Labor Market Competitiveness and Flexibility in Egypt,” FEMISE Research Programme.

El-Mahdi, Alia (2002). “Towards Decent Work in the Informal Sector: The Case of Egypt,” Employment Sector, International Labour Office.

Eslava, M., J. Haltiwanger, A. Kugler and M. Kugler (2004). “The effects of structural reforms on productivity and profitability enhancing reallocation: evidence from Colombia,” Journal of Development Economics, 75 (2), pp. 333-371.

Fajnzylber, P., W.F. Maloney and G.V. Montes-Rojas (2011). “Does formality improve micro-firm performance? Evidence from the Brazilian SIMPLES program,” Journal of Development Economics, 94 (2), pp. 262-276.

Fortin, B., N. Marceau and L. Savard (1997). “Taxation, Wage Controls and the Informal Sector,” Journal of Public Economics, 66 (2), pp. 293-312.

Foster, L., J.C. Haltiwanger and C.J. Krizan (2001). “Aggregate Productivity Growth: Lessons from Microeconomic Evidence,” in E.R. Dean, M.J. Harper and C.R. Hulten (Eds.), *New Developments in Productivity Analysis*. University of Chicago Press.

Gabler, A. and M. Poschke (2013). “Experimentation by Firms, Distortions and Aggregate Productivity,” Review of Economic Dynamics, 16 (1), pp. 26-38.

Garcia-Santana, M. and J. Pijoan-Mas (2011). “Small Scale Reservation Laws and the Misallocation of Talent,” CEPR, DP8242.

Garcia-Santana, M. and R. Ramos (2013). “Distortions and the Size Distribution of Plants: Evidence from Cross-Country Data,” WP.

Genda, Y., A. Kondo and S. Ohta (2010). “Long-Term Effects of a Recession at Labor Market Entry in Japan and the United States,” Journal of Human Resources, 45 (1), pp. 157-195.

Gollin, D. (2008). “Nobody’s business but my own: Self-employment and small enterprise in economic development,” Journal of Monetary Economics, 55 (2), pp. 219-233.

Gorodnichenko, Y. and M. Schnitzer (2013). “Financial Constraints and Innovation: Why Poor Countries Don’t Catch Up?” Journal of the European Economic Association (forthcoming).

Greenwood, J., J.M. Sanchez and C. Wang (2013). “Quantifying the Impact of Financial Development on Economic Development,” Review of Economic Dynamics, 16 (1), pp. 194-215.

Gregg, P. and E. Tominey (2005). “The wage scar from male youth unemployment,” Labour Economics, 12 (4), pp. 487-509.

Guner, N., G. Ventura and Y. Xu (2008). “Macroeconomic Implications of Size-Dependent Policies,” Review of Economic Dynamics, 11 (4), pp. 721-744.

Hall, B.H., J. Mairesse and P. Mohnen (2010). “Measuring the Returns to R&D,” Handbook of the Economics of Innovation, B.H. Hall and N. Rosenberg (Eds.), Volume 2, Ch. 22, pg. 1033-1082.

Hall, B.H. and J. Lerner (2010). “The Financing of R&D and Innovation,” Handbooks of the Economics of Innovation, B.H. Hall and N. Rosenberg (Eds.), Volume 1, Ch. 14, pp. 609-639.

- Hall, B.H. (2011).** “Innovation and Productivity,” NBER WP #17178.
- Helfand, J., A. Sadeghi and D. Talan (2007).** “Employment Dynamics: Small and Large Firms Over the Business Cycle,” Monthly Labor Review, March 2007.
- Hopenhayn, H.A. (1992).** “Entry, Exit and Firm Dynamics in Long Run Equilibrium,” Econometrica, 60 (5), pp. 1127-1150.
- Hopenhayn, H.A. (2012).** “On the Measure of Distortions,” WP.
- Hsieh, C.T. and P.J. Klenow (2012).** “The Life Cycle of Plants in India and Mexico,” WP.
- Hsieh, C.T. and P.J. Klenow (2009).** “Misallocation and Manufacturing TFP in China and India,” Quarterly Journal of Economics, 124 (4), pp. 1403-1448.
- Huggett, M., G. Ventura and A. Yaron (2006).** “Human capital and earnings distribution dynamics,” Journal of Monetary Economics, 53 (2), pp. 265-290.
- Huggett, M., G. Ventura and A. Yaron (2011).** “Sources of Lifetime Inequality,” American Economic Review, 101 (7), pp. 2923-2954.
- Ihrig, J. and K.S. Moe (2004).** “Lurking in the Shadows: The Informal Sector and Government Policy,” Journal of Development Economics, 73 (2), pp. 541-557.
- International Labour Office (2012).** Statistical Update on Employment in the Informal Economy, ILO Dept. of Statistics.
- Kahn, L.B. (2010).** “The Long-Term Labor Market Consequences of Graduating from College in a Bad Economy,” Labour Economics, 17 (2), pp. 303-316.
- Kambourov, G. and I. Manovskii (2009).** “Occupational Specificity of Human

Capital,” International Economic Review, 50 (1), pp. 63-115.

Kaplan, D.S., E. Piedra and E. Seira (2011). “Entry regulation and business start-ups: Evidence from Mexico,” Journal of Public Economics, 95 (11-12), pp. 1501-1515.

Kenar, N. (2009). “Informality: Regulations, Institutions and Enforcement,” prepared for Country Economic Memorandum: Informality: Causes, Consequences, Policies (World Bank 2010).

Kheir-El-Din, H., S. Fawzy and Amal Refaat (2001). “Investment Incentives, Marginal Effective Tax Rates, and the Cost of Capital in Egypt,” FEMISE Research Programme.

Klette, J. and S. Kortum (2004). “Innovating Firms and Aggregate Innovation,” Journal of Political Economy, 112 (5), pp. 986-1018.

Kletzer, L.G. and R.W. Fairlie (2003). “The Long-Term Costs of Job Displacement for Young Adult Workers,” Industrial and Labor Relations Review, 56 (4), pp. 682-698.

Kopecky, K.A. and R.M.H. Suen (2010). “Finite state Markov-chain approximations to highly persistent processes,” Review of Economic Dynamics, 13 (3), pp. 701-714.

La Porta, R. and A. Shleifer (2008). “The Unofficial Economy and Economic Development,” Brookings Papers on Economic Activity, 39 (2), pp. 275-363.

Leal Ordoñez, J.C. (2013). “Tax collection, the informal sector, and productivity,” Review of Economic Dynamics, in press.

Levenson, A.R. and W.F. Maloney (1998). “The Informal Sector, Firm Dynamics, and Institutional Participation,” Policy Research Working Paper #1988, The World

Bank.

Levine, R. (2005). “Finance and Growth: Theory and Evidence,” *Handbook of Economic Growth*, Vol. 1A, Ch. 12.

Ljungqvist, L. (2002). “How Do Lay-off Costs Affect Employment?” *Economic Journal*, 112 (482), pp. 829-853.

Ljungqvist, L. and T.J. Sargent (2007). “Do Taxes Explain European Employment? Indivisible Labor, Human Capital, Lotteries, and Savings,” *NBER Macroeconomics Annual 2006*, Vol. 21.

Lucas, R.E. (1978). “On the Size Distribution of Business Firms,” *Bell Journal of Economics*, 9 (2), pp. 508-523.

Maloney, W.F. (2004). “Informality Revisited,” *World Development*, 32 (7), pp. 1159-1178.

McGrattan, E.R. and E.C. Prescott (2005). “The Regulation and Value of US and UK Corporations,” *Review of Economic Studies*, 72 (3), pp. 767-796.

McKinsey Global Institute (2006). *Driving Growth, Breaking Down Barriers to Global Prosperity*. Boston, Harvard Business School Publishing.

Menzio, G. and S. Shi (2010a). “Block recursive equilibria for stochastic models of search on the job,” *Journal of Economic Theory*, 145 (4), 1453-1494.

Menzio, G. and S. Shi (2010b). “Directed Search on the Job, Heterogeneity, and Aggregate Fluctuations,” *American Economic Review: Papers & Proceedings*, 100 (2), pp. 327-332.

- Menzio, G. and S. Shi (2011).** “Efficient Search on the Job and the Business Cycle,” Journal of Political Economy, 119 (3), pp. 468-510.
- Menzio, G., I.A. Telyukova and L. Visschers (2012).** “Directed Search over the Life Cycle,” PIER WP #12-002.
- Michaud, A.M. (2012).** “An Information Theory of Worker Flows and Wage Dispersion,” WP.
- Midrigan, V. and D.Y. Xu (2013).** “Finance and Misallocation: Evidence from Plant-Level Data,” American Economic Review, forthcoming (WP-2010).
- Ministry of Foreign Trade, Egypt (2003).** Profile of Micro, Small and Medium Enterprises in Egypt.
- Monteiro, J.C.M. and J.J. Assunção (2012).** “Coming out of the shadows? Estimating the impact of bureaucracy simplification and tax cut on formality in Brazilian microenterprises,” Journal of Development Economics, 99 (1), pp. 105-115.
- Nickell, S. and R. Layard (1999).** “Labor Market Institutions and Economic Performance,” in O. Ashenfelter and D. Card (Eds.), Handbook of Labor Economics, Ch. 46, pp. 3029-3084.
- Nickell, S., L. Nunziata and W. Ochel (2005).** “Unemployment in the OECD since the 1960s. What Do We Know?” The Economic Journal, 115 (500), pp. 1-27.
- Nishida, M., A. Petrin and T.K. White (2013).** “Are We Undercounting Reallocation’s Contribution to Growth?” WP.
- Nördstrom Skans, O. (2004).** “Scarring effects of the first labour market experience: A sibling based analysis,” Institute for Labour Market Policy Evaluation, WP 2004:14.

- OECD (2001).** “Productivity and Firm Dynamics: Evidence from Microdata,” OECD Economic Outlook, 69, pp. 209-223.
- OECD (2002).** “Small and Medium Enterprise Outlook,” OECD.
- OECD (2004).** “Informal Employment and Promoting the Transition to a Salaried Economy,” OECD Employment Outlook, Ch. 5.
- Ohanian, L., A. Raffo and R. Rogerson (2008).** “Long-term changes in labor supply and taxes: Evidence from OECD countries, 1956-2004,” Journal of Monetary Economics, 55 (8), pp. 1353-1362.
- Oreopoulos, P., T. von Wachter, A. Heisz (2012).** “The Short- and Long-Term Career Effects of Graduating in a Recession,” American Economic Journal: Applied Economics, 4 (1), pp. 1-29.
- Ozar, S. (2006).** “Micro and Small Enterprises in Turkey: Uneasy Development,” Economic Research Forum, Research Report Series #0420.
- Parente, S.L. and E.C. Prescott (2000).** Barriers to Riches. Cambridge, Massachusetts: MIT Press.
- Perry, G.E., W.F. Maloney, O.S. Arias, P. Fajnzylber, A.D. Mason, J. Saavedra-Chanduvi (2007).** Informality, Exit and Exclusion. The World Bank.
- Prado, M. (2011).** “Government Policy in the Formal and Informal Sectors,” European Economic Review, 55 (8), pp. 1120-1136.
- Pratap, S. and E. Quintin (2008).** “The Informal Sector in Developing Countries: Output, Assets and Employment,” in J.B. Davies, Ed., Personal Wealth from a Global Perspective, Oxford University Press.

- Prescott, E.C. (2004).** “Why Do Americans Work So Much More Than Europeans?” Quarterly Review - FRB Minneapolis, 28 (1), pp. 2-13.
- Prescott, E.C. and M. Visscher (1980).** “Organization Capital,” Journal of Political Economy, 88 (3), pp. 446-461.
- Pries, M.J. (2008).** “Worker heterogeneity and labor market volatility in matching models,” Review of Economic Dynamics, 11 (3), pp. 664-678.
- Quintin, E. (2008).** “Contract Enforcement and the Size of the Informal Economy,” Economic Theory, 37 (3), pp. 395-416.
- Raaum, O. and K. Røed (2006).** “Do Business Cycle Conditions at the Time of Labor Market Entry Affect Future Employment Prospects?” Review of Economics and Statistics, 88 (2), pp. 193-210.
- Rauch, J.E. (1991).** “Modelling the Informal Sector Formally,” Journal of Development Economics, 35 (1), pp. 33-47.
- Restuccia, D. and R. Rogerson (2008).** “Policy Distortions and Aggregate Productivity with Heterogeneous Establishments,” Review of Economic Dynamics, 11 (4), pp. 707-720.
- Rogerson, R. and M. Schindler (2002).** “The welfare costs of worker displacement,” Journal of Monetary Economics, 49 (6), pp. 1213-1234.
- Rossi-Hansberg, E. and M.L.J. Wright (2007).** “Establishment Size Dynamics in the Aggregate Economy,” American Economic Review, 97 (5), pp. 1639-1666.
- Schneider, F. and D.H. Enste (2000).** “Shadow Economies: Size, Causes and Consequences,” Journal of Economic Literature, XXXVIII, pp. 77-114.

Steinberg, J. (2013). “Information, Contract Enforcement, and Misallocation,” WP.

Straub, S. (2005). “Informal Sector: The Credit Market Channel,” Journal of Development Economics, 78 (2), pp. 299-321.

Syverson, C. (2011). “What Determines Productivity?” Journal of Economic Literature, 49 (2), pp. 326-365.

Tauchen, G. (1986). “Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions,” Economic Letters, 20 (2), pp. 177-181.

Tybout, J.R. (2000). “Manufacturing Firms in Developing Countries: How Well Do They Do, and Why?” Journal of Economic Literature, XXXVIII, pp. 11-44.

United Kingdom National Audit Office (2008). Comparing how some tax authorities tackle the hidden economy, RAND Europe.

Wahba, J. (2009). “The Impact of Labor Market Reforms on Informality in Egypt,” International Development Research Centre Canada & Population Council, August 2009.

World Bank (2010). “Turkey - Country Economic Memorandum, Informality: Causes, Consequences, Policies,” Report N. 48523-TR, March, 2010.

Appendices

Appendix A

Appendix to Chapter 1

A.1 Size Distribution of Firms

The data for the US distribution of firms is from Helfand et al. (2007), average 1990-2000. These statistics account for 97% of the total number of employees in private industries (code USPRIV, Federal Reserve Economic Data). The size distribution of firms data (establishment data is also available) can be compared to that of Rossi-Hansberg & Wright (2007), taking the average of the years 1990, 1992, 1994, 1995, 1997 and 2000 (includes firms with zero employees), see Table A.1. OECD (2002) also provides information for the US on the distribution of enterprises by size class (Table A.1).

For Turkey, the data is from World Bank (2010) and OECD (2002) (similar numbers are obtained from TurkStat, Household Labour Force Survey). Note that from OECD (2002) the size categories are different for US and Turkey. Data for registered workplaces is available from the Social Security Institution (see Kenar, 2009).

For Egypt, data is from the Central Agency for Public Mobilization and Statistics (CAPMAS), the main statistical agency of the Egyptian government, Establishment Census 1996 (see Ministry of Foreign Trade, 2003). The Egyptian data, for non-agricultural activities, is divided in sizes 1-4, 5-14, 15-49 and 50+ workers (Ministry of Foreign Trade, 2003, Table 1 for establishments and Table 4 for employment). The

dist. of firms by size is: 92.7, 6.12, 0.91, and 0.3% respectively. Figures obtained from the 2006 Economic Census are shown in Table A.1.

Table A.1: Distribution of Enterprises and Employment by Size Class

% enterprises	<u>0-9</u>	<u>10-19</u>	<u>20-99</u>	<u>100-499</u>	<u>500+</u>
US (OECD)	56.8	15.8	20.7	5.2	1.5
US (Rossi-Hansberg & Wright, 2007)	78.7	10.8	8.8	1.4	0.2
US (Helfand et al., 2007)	74.5	12.5	10.6	1.9	0.4
% enterprises	<u>0-9</u>	<u>10-49</u>	<u>50-99</u>	<u>100-499</u>	<u>500+</u>
Turkey (OECD)	95.0	3.2	0.8	0.9	0.2
% enterprises and employment	<u>1-9</u>	<u>10-49</u>	<u>50-249</u>	<u>250-499</u>	<u>500+</u>
Turkey (% of registered enterprises)	85.5	12.5	1.7	0.2	0.1
Turkey (% of registered workers)	28.6	32.5	21.8	7.5	9.6
% establishments and employment	<u>0-1</u>	<u>2-10</u>	<u>11-100</u>	<u>101-1,000</u>	<u>1,000+</u>
Egypt (% of formal sector estab.)	90.34	7.82	1.71	0.13	0.01
Egypt (% of formal sector workers)	51.94	22.35	11.19	9.34	5.18

A.2 Tax Structure

Tax rates in Turkey¹ and Egypt, for the years 2000-2002 were as follows:

- the corporate income tax rate for Turkey was 33% (no special treatment for small-medium enterprises, OECD-2002), in Egypt the standard corporate income tax rate is 40%, reduced to 32% for manufacturing,
- the tax in Turkey for income from capital investments (interests and dividends) is included in the taxable gross income of individuals, the rate varies 0.15-0.35. For Egypt the unified income tax applies to commercial and industrial profits, the range is 20-40%.

¹For Turkey the data is from the OECD Tax Database and KPMG. For Egypt the data is from Kheir-El-Din et al. (2001) and El-Ehwany & Metwally (2001).

Table A.2: Tax Structures

type of tax	US	Turkey	Egypt
corporate income	0.353	0.330	0.32-0.40
corporate distributions (dividends)	0.174	0.15-0.35	0.20-0.40
employer social sec. contribution	0.076	0.215	0.220
labor-tax (Djankov et al., 2010)	0.072	0.229	0.217
total tax % of profits (W.B.)	0.46	0.53	0.54
OECD total tax wedge (2002)	0.301	0.425	–

For the US, taxes on corporate income and corporate distributions respectively, are taken from McGrattan & Prescott (2005). Both are estimated averages for the period 1990-2001. The OECD Tax Database shows 0.35 as the central government corporate income tax rate², for the same period, similar to the one in McGrattan & Prescott (2005).

For both the US and Turkey the marginal rate for employer social security contributions is available. For the US, more specifically, it is the rate for annual gross earnings in the range of \$7-106 thousand from the OECD Tax Database, 2002-2003. For Egypt the rate of total contributions is 36% of the basic salary, 14% are contributions of the employee and 22% is paid by the employer.

Djankov et al. (2010) estimate labor taxes as the sum of all labor-related taxes, including payroll taxes, mandatory social security contributions, mandatory health insurance, mandatory unemployment insurance, and any local contributions that are proportional to payroll or number of employees. These estimations are based on taxes applicable to a standardized domestic enterprise and are expressed as a percentage of pre-tax earnings.

Finally, the OECD tax wedge a measure of the difference between labor costs to the employer and the corresponding net take-home pay of the employee is calculated by expressing the sum of personal income tax, employee plus employer social security contributions together with any payroll tax, minus benefits as a percentage of labor costs.

²The rate is closer to 0.39 when combining the central and sub-central (statutory) rates given by the adjusted, for deductions in respect of sub-central tax, central government rate plus the sub-central rate.

The OECD recognizes that this measure may be less than the true labor costs faced by employers because, for example, employers may also have to make non-tax compulsory payments. In the same way employees take home may be reduced if they have to pay such non-tax payments. The average tax wedge measures identify that part of total labor costs which is taken in tax and social security contributions net of cash benefits.

A.3 Robustness Exercise

The following table documents the distribution of employment and firms by size class obtained by the model when considering an autocorrelation parameter of $\rho = 0.75$ for idiosyncratic shocks (parameters for the distribution of entrepreneurial ability are unchanged from the baseline specification).

Table A.3: Distribution of Employment and Firms by Size Class

% employment	1-9	10-19	20-49	50-99	100-499	500-999	1,000+
US - data [†]	11.45	7.70	11.08	8.43	17.89	6.91	36.55
US - model	14.18	7.74	9.32	8.51	17.64	7.89	34.72
% firms	1-9	10-19	20-49	50-99	100-499	500-999	1,000+
US - data	74.27	12.56	8.06	2.69	2.00	0.22	0.20
US - model	75.12	12.54	6.99	2.84	1.96	0.27	0.27

[†]Helfand et al. (2007), average 1995-2000.

A.4 Algorithm Outline

Given the interest rate and government policies, computing the equilibrium amounts to finding the wage w that clears the labor market. The grid for the ability variable φ consists of 20 points, with weights given by a discretized Log-normal distribution. The Markov matrix $\Lambda(a' | a)$ is constructed following Tauchen (1986), with 9 possible state values. The grid for financial assets b has up to 750 points (as necessary depending on parameters and ability of the entrepreneur). The algorithm can be summarized as follows:

- (1) Guess wage level w .
- (2) Solve for quantities³ $\{q(s, z)\}_{z \in \{i, f\}}$, production inputs $\{l(s, z), k(s, z)\}_{z \in \{i, f\}}$, profits $\{\pi(s, z)\}_{z \in \{i, f\}}$.
- (3) Through value function iteration, until a desired level of precision is reached, obtain policy and value functions $\{g_z(s)\}_{z \in \{i, f, w\}}$, $\{v(s), v_i(s), v_f(s), v_w(s)\}$.
- (4) Run 300 simulations of 25,000 individuals for 200 periods (this requires less memory than fewer simulations with more individuals, it is verified that increasing the number of simulations/individuals/periods does not change the results). The number of individuals with each entrepreneurial ability level is determined by the weights given by the parameterized Log-normal distribution. An individual that dies is replaced by another individual with the same entrepreneurial ability.
- (5) Compute the aggregates using the cross section of the last period of the simulations and check the market clearing condition for labor. Return to step (1) and adjust w and calibrated parameters as necessary.

³In the case of government enforcement (where detection depending on output is considered), output decisions and value functions are solved jointly for informal sector firms.

Appendix B

Appendix to Chapter 2

B.1 Output-Capital Ratios and Financial Constraints

Consider a standard profit maximization problem of a firm with access to a production technology with decreasing returns to scale (as the one in the quantitative framework) and productivity z :

$$\max_{\{k,l\}} z (k^\alpha l^{1-\alpha})^\nu - w l - (r + \delta) k$$

subject to a restriction $k \leq \bar{k}$ (where \bar{k} is derived from a financial constraint). It is straightforward to derive from the first order condition of capital that $\alpha \nu (q/k) = r + \delta + \gamma$ where $\gamma \geq 0$ is a multiplier on the financial constraint.

B.2 Size Distribution of Establishments

The source of data for the US in Table B.1 is Statistics of U.S. Businesses¹ (Census Bureau). Industries are classified according to NAICS:² manufacturing (codes 31-33), retail (44-45), services includes the following 2 digit categories: 48-49 (transportation and warehousing), 51 (information), 52 (finance and insurance), 53 (real estate and

¹<http://www.census.gov/econ/susb/>

²<http://www.census.gov/cgi-bin/sssd/naics/naicsrch?chart=2007>

rental and leasing), 54 (professional, scientific and technical services), 55 (management of companies and enterprises), 56 (administrative and support and waste management and remediation services), 71 (arts, entertainment and recreation), 72 (accommodation and food services), 81 (other services, except public administration). I exclude the following categories: 61 (educational services), 62 (health care and social assistance), 92 (public administration), 11 (agriculture, forestry, fishing, hunting), 21 (mining, quarrying, oil and gas extraction), 22 (utilities), 23 (construction). The group all sectors includes the total of the non-excluded categories, this accounts for a total of 78.5 million workers. Including all the categories accounts for 111.97 million workers (the size distribution including all categories does not show significant changes).

For Mexico, the data in Table B.1 is from Busso, Fazio & Levy (2012), corresponding to the Economic Census 2004. We note that the data understates true differences given the under-representation of small firms in Mexico, in particular those in the informal sector (see Busso, Madrigal & Pages, 2012; Leal Ordoñez, 2013).

Table B.1: Distribution of Establishments and Employment

USA¹	establishments				employment			
	<5	5-9	10-49	≥50	<5	5-9	10-49	≥50
shares in %								
all sectors	47.52	12.56	⇒	39.92	5.32	5.48	⇒	89.20
manufacturing	36.81	15.77	⇒	47.43	1.89	2.90	⇒	95.21
manufacturing ²	40.47	14.05	29.21	16.27	1.90	2.28	15.77	80.06
retail	37.85	12.67	⇒	49.47	5.18	5.97	⇒	88.85
services	50.96	12.29	⇒	36.76	6.06	5.87	⇒	88.06
Mexico³	establishments				employment			
shares in %	≤5	6-10	11-50	>50	≤5	6-10	11-50	>50
all sectors	90.62	4.95	3.49	0.94	35.17	7.50	14.86	42.47
manufacturing	83.42	7.44	6.01	3.13	13.85	4.30	10.28	71.56
retail	93.37	3.67	2.47	0.49	53.58	8.47	15.76	22.19
services	88.64	6.14	4.28	0.95	34.67	9.23	17.83	38.26

¹Statistics of U.S. Businesses (CB), ²Cole, Greenwood & Sanchez (2012),

³Busso, Fazio & Levy (2012).

B.3 Dispersion in Output-Capital Ratios in Colombia

Table B.2 documents the dispersion in the log-output/capital ratios for the AMS database, where capital includes buildings, structures, machinery and equipment. In each year and for each 4 digit industry I take firms with more than 10 workers. Within each industry that has more than 20 observations I compute the variance and the difference in the levels of different percentiles. For each year I then take the average of a particular measure across industries.

Table B.2: Dispersion in Output-Capital Ratios

statistic ¹	1982	1986	1990	1994	1998	82-98 ²
variance	1.7	1.6	1.6	1.7	2.2	1.7
90-10 percentiles	3.1	3.0	3.0	3.2	3.5	3.1
85-15 percentiles	2.4	2.3	2.4	2.6	2.8	2.5
80-20 percentiles	2.0	1.9	1.9	2.0	2.2	2.0
80-50 percentiles	0.9	1.0	1.0	1.0	1.1	1.0
# 4 digit industries	57	60	59	57	54	57
avg. # observations per industry	83	81	84	70	68	78

Source: computed w/AMS-DANE (1982-1998).

¹Average across 4 digit industries, ratios in logs. ²Average 17 years.

B.4 Micro-Enterprizes in Mexico

The National Survey of Micro-Enterprizes (ENAMIN) is conducted every two years and includes data on firms with up to 15 workers in manufacturing, and up to 10 workers in construction, transportation, retail and services. INEGI estimates that approximately 41.6% of the labor force belongs to firms in this scale of production (approximately 18.1 million workers). The data collected by this survey includes information on the manager/owner of the firm: education, experience, time in present position and reasons for setting up a business, among other variables. Regarding the firm itself, the information collected includes: year the business was established, accounting and registry, equipment, expenditures, investment, income, access to finance and number of workers,

among other variables.

The survey provides information of registration of the firm (whether it belongs to the informal sector). Approximately 17.6 thousand of the business managers/owners replied that they had not initiated any formal process of registration with Government authorities in the 2010 survey (6.9 thousand replied that they had). Considering the firms that had not initiated any formal registration process the main activities (accounting for 70% of the group of non-registered firms) were the following: retail of food, beverages and tobacco (code 4611, 11.6%), preparation of food and beverages (7221, 8.6%), intermediation and retail of massive communications media (4690, 7.3%), contractors in construction (2382, 5.7%), food industry (3110, 5.5%), personal services (8121, 4.1%), preparation of food and beverages without fixed location (7222, 3.8%), repairs and maintenance of equipment, machinery, household and personal appliances (8112, 3.6%), retail of food and beverages without fixed location (4612, 3.5%), maintenance of automobiles and trucks (8111, 3.3%), fabrication of clothing (3150, 3%), residential construction (2361, 2.9%), retail of clothing, accessories and footwear (4631, 2.9%), retail of clothing, accessories and footwear without fixed location (4632, 2.4%), manufacturing of textiles except clothing (3140, 2.3%).

Appendix C

Appendix to Chapter 3

C.1 Block Recursive Equilibrium

This appendix proves the existence and uniqueness of the Block Recursive Equilibrium for an economy without human capital depreciation (this extension is straightforward).

Definition. A *Block Recursive Equilibrium* (BRE) consists of value functions U_t for unemployed workers, V_t for employed workers, F_t for previously matched firms and G_t for newly matched firms, policy functions w_t^u for unemployed workers and w_t^a for employed workers, a bargained wage function w_t^b determined between an employed worker and a firm, a cutoff productivity function z_t^b , and a tightness function θ_t for $t = 1, \dots, T$ such that (i) $U_t, V_t, F_t, G_t, w_t^u, w_t^a, w_t^b, z_t^b$ and θ_t depend on ψ only through y for $t = 1, \dots, T$, (ii) F_t, G_t and θ_t are consistent with the firm's rationality and the free-entry condition for $t = 1, \dots, T$, (iii) U_t and w_t^u solve the unemployed worker's problem for $t = 1, \dots, T$, (iv) V_t and w_t^a solve the employed worker's problem for $t = 1, \dots, T$, and (v) w_t^b and z_t^b solve the bargaining problem between an employed worker and a firm for $t = 1, \dots, T$.

Theorem. A recursive equilibrium exists and is block recursive and unique.

Proof. We construct a block recursive equilibrium. Denote a statement " $U_t, V_t, F_t, G_t, w_t^u, w_t^a, w_t^b, z_t^b$ and θ_t are uniquely computed and they depend on ψ only through y

for t'' as (S_t) . We first show that (S_T) holds and then proceed by backward induction.

At age T the value of an unemployed worker with no job offer after the search stage is:

$$U_T^n(h, \psi) = b,$$

and we can write as $U_T^n(h, \psi) = U_T^n(h, y)$.

At the bargaining stage, if an agreement can be reached (the joint surplus is positive), the value of remaining in the current match for a worker without an alternative job offer is given by the bargained wage function:

$$w_T^b(h, z, \psi),$$

while the outside option at this stage is $U_T^n(h, y)$, and the value of the firm (recalling that $F_{T+1} = 0$) is:

$$-w_T^b(h, z, \psi) + \sum_{\{z'\}} \Lambda(z' | z) f(y, z', h),$$

and the outside value of the firm is zero.

Thus, at age T , the bargaining problem for the continuing match is:

$$\max_{\{w^b\}} \left\{ w^b - b \right\}^\xi \left\{ -w^b + \sum_{\{z'\}} \Lambda(z' | z) f(y, z', h) \right\}^{1-\xi},$$

the joint surplus is:

$$-b + \sum_{\{z'\}} \Lambda(z' | z) f(y, z', h).$$

Let the cutoff productivity $z_T^b(h, \psi)$ be the lowest z such that the joint surplus is non-negative. Noting that y is the only necessary component in ψ to determine this

cutoff, $z_T^b(h, \psi) = z_T^b(h, y)$.

If $z \geq z_T^b(h, y)$, the bargaining problem has a unique solution:

$$w_T^b(h, z, \psi) = (1 - \xi) b + \xi \sum_{\{z'\}} \Lambda(z' | z) f(y, z', h),$$

otherwise the bargaining fails and the employed worker and the firm receive the outside value. We can see that $w_T^b(h, z, \psi) = w_T^b(h, z, y)$.

Therefore, at the bargaining stage the employed worker's value is:

$$V_T^b(h, z, \psi) = \begin{cases} w_T^b(h, z, y) & \text{if } z \geq z_T^b(h, y), \\ b & \text{if } z < z_T^b(h, y), \end{cases}$$

and the firm's value is:

$$F_T^b(h, z, \psi) = \begin{cases} -w_T^b(h, z, y) + \sum_{\{z'\}} \Lambda(z' | z) f(y, z', h) & \text{if } z \geq z_T^b(h, y), \\ 0 & \text{if } z < z_T^b(h, y). \end{cases}$$

Noting that the right hand sides of the values do not have ψ except y , we can write $V_T^b(h, z, \psi) = V_T^b(h, z, y)$ and $F_T^b(h, z, \psi) = F_T^b(h, z, y)$.

On the other hand, the value of the worker that has found an alternative job offer is simply the wage posted in the market where he has searched:

$$V_T^a(w^a, h, \bar{z}, \psi) = w^a,$$

and this does not depend on ψ directly, so $V_T^a(w^a, h, \bar{z}, \psi) = V_T^a(w^a, h, \bar{z}, y)$. The value of the newly matched firm is:

$$G_T(w^a, h, \bar{z}, \psi) = f(y, \bar{z}, h) - w^a,$$

and hence $G_T(w^a, h, \bar{z}, \psi) = G_T(w^a, h, \bar{z}, y)$.

Then, the free-entry condition for the firm at this stage is (for a wage w),

$$c_v \geq q(\theta_T(w, h, \psi)) G_T(w, h, \bar{z}, y)$$

and $\theta_T(w, h, \psi) \geq 0$ with complementary slackness. It follows that:

$$\theta_T(w, h, \psi) = \begin{cases} q^{-1} \left(\frac{c_v}{f(y, \bar{z}, h) - w} \right) & \text{if } c_v \leq f(y, \bar{z}, h) - w, \\ 0 & \text{if } c_v > f(y, \bar{z}, h) - w, \end{cases}$$

and hence $\theta_T(w, h, \psi) = \theta_T(w, h, y)$ as the right hand side depends on ψ only through y . Equivalently,

$$\begin{aligned} w &= f(y, \bar{z}, h) - \frac{c_v}{q(\theta_T(w, h, y))} & \text{if } c_v \leq f(y, \bar{z}, h) - w, \\ \theta_T(w, h, y) &= 0 & \text{if } c_v > f(y, \bar{z}, h) - w. \end{aligned}$$

Thus, before the search stage the value of the matched worker is:

$$\begin{aligned} V_T(h, z, \psi) &= \max_{\{w^a\}} \left\{ \lambda_e p(\theta_T(w^a, h, y)) w^a + (1 - \lambda_e p(\theta_T(w^a, h, y))) V_T^b(h, z, y) \right\}, \\ &= \max_{\{w^a\}} \left\{ \lambda_e (-c_v \theta_T(w^a, h, y) + p(\theta_T(w^a, h, y)) (f(y, \bar{z}, h) - V_T^b(h, z, y))) + V_T^b(h, z, y) \right\}, \\ &= \max_{\theta \geq 0} \left\{ \lambda_e (-c_v \theta + p(\theta) (f(y, \bar{z}, h) - V_T^b(h, z, y))) + V_T^b(h, z, y) \right\}, \end{aligned}$$

so if $f(y, \bar{z}, h) \leq V_T^b(h, z, y)$ then the solution is zero, and otherwise the objective function is strictly concave in θ . Thus, this problem has a unique solution $\theta_T^a(h, z, \psi)$. Since the objective function depends on ψ only through y , $\theta_T^a(h, z, \psi) = \theta_T^a(h, z, y)$ and $V_T(h, z, \psi) = V_T(h, z, y)$. Therefore,

$$\begin{aligned} w_T^a(h, z, \psi) &= f(y, \bar{z}, h) - \frac{c_v}{q(\theta_T^a(h, z, y))} & \text{if } \theta_T^a(h, z, y) > 0, \\ w_T^a(h, z, \psi) &\geq f(y, \bar{z}, h) - c_v & \text{if } \theta_T^a(h, z, y) = 0. \end{aligned}$$

Noting the market with $\theta = 0$ is empty, without loss of generality:

$$w_T^a(h, z, \psi) = f(y, \bar{z}, h) - \frac{c_v}{q(\theta_T^a(h, z, y))},$$

and hence $w_T^a(h, z, \psi) = w_T^a(h, z, y)$.

Similarly we have at the beginning of age T value of unemployment:

$$\begin{aligned} U_T(h, \psi) &= \max_{\{w^u\}} \left\{ \lambda_u p(\theta_T(w^u, h, y)) w^u + (1 - \lambda_u p(\theta_T(w^u, h, y))) U_T^n(h, y) \right\}, \\ &= \max_{\{w^u\}} \left\{ \lambda_u (-c_v \theta_T(w^u, h, y) + p(\theta_T(w^u, h, y)) (f(y, \bar{z}, h) - U_T^n(h, y))) + U_T^n(h, y) \right\}, \\ &= \max_{\theta \geq 0} \left\{ \lambda_u (-c_v \theta + p(\theta) (f(y, \bar{z}, h) - U_T^n(h, y))) + U_T^n(h, y) \right\}, \end{aligned}$$

so if $f(y, \bar{z}, h) \leq U_T^n(h, y)$ then the solution is zero, and otherwise the objective function is strictly concave in θ . Thus, this problem has a unique solution $\theta_T^u(h, \psi)$. Since the objective function depends on ψ only through y , $\theta_T^u(h, \psi) = \theta_T^u(h, y)$ and $U_T(h, \psi) = U_T(h, y)$. Therefore, we uniquely specify as:

$$w_T^u(h, \psi) = f(y, \bar{z}, h) - \frac{c_v}{q(\theta_T^u(h, y))},$$

and hence $w_T^u(h, \psi) = w_T^u(h, y)$.

The beginning of age T value of the firm previously matched is

$$F_T(h, z, \psi) = (1 - \lambda_e p(\theta_T(w, h, y))) F_T^b(h, z, y),$$

so $F_T(h, z, \psi) = F_T(h, z, y)$.

Therefore, we can see that (S_T) holds.

We are ready to go back to age $T - 1$. The value of a worker that has not found

a job at the search stage is:

$$U_{T-1}^n(h, \psi) = b + \beta \sum_{\{y'\}} \Lambda(y' | y) U_T(h, y'),$$

so $U_{T-1}^n(h, \psi) = U_{T-1}^n(h, y)$.

At the bargaining stage, if an agreement can be reached through Nash-bargaining, the value for a worker of remaining in the match is:

$$w_{T-1}^b(h, z, \psi) + \beta \sum_{\{s'\}} \Lambda(s' | s) \left\{ (1 - \delta) V_T(h', z', y') + \delta U_T(h', y') \right\},$$

while the outside option at this stage is $U_{T-1}^n(h, y)$. The value of the firm of remaining in the match is:

$$-w_{T-1}^b(h, z, \psi) + \sum_{\{s'\}} \Lambda(s' | s) \left\{ f(y, z', h) + \beta (1 - \delta) F_T(h', z', y') \right\},$$

and the outside value of the firm is fixed at zero.

Thus, at age $T - 1$, the bargaining problem for the continuing match is:

$$\max_{\{w^b\}} \left[w^b + \beta \sum_{\{s'\}} \Lambda(s' | s) \left\{ (1 - \delta) V_T(h', z', y') + \delta U_T(h', y') \right\} - U_{T-1}^n(h, y) \right]^\xi \\ \times \left[-w^b + \sum_{\{s'\}} \Lambda(s' | s) \left\{ f(y, z', h) + \beta (1 - \delta) F_T(h', z', y') \right\} \right]^{1-\xi}$$

and the joint surplus is:

$$-U_{T-1}^n(h, y) + \sum_{\{s'\}} \Lambda(s' | s) \left\{ f(y, z', h) + \beta ((1 - \delta) (V_T(h', z', y') + F_T(h', z', y')) + \delta U_T(h', y')) \right\}.$$

The cutoff productivity $z_{T-1}^b(h, \psi)$ is the lowest z such that the joint surplus is non-negative, and $z_{T-1}^b(h, \psi) = z_{T-1}^b(h, y)$ as above.

If $z \geq z_{T-1}^b(h, y)$, the bargaining problem has a unique solution:

$$w_{T-1}^b(h, z, \psi) = \xi \left\{ \sum_{\{s'\}} \Lambda(s' | s) (f(y, z', h) + \beta(1 - \delta) F_T(h', z', y')) \right\} \\ + (1 - \xi) \left\{ U_{T-1}^n(h, y) - \beta \sum_{\{s'\}} \Lambda(s' | s) ((1 - \delta) V_T(h', z', y') + \delta U_T(h', y')) \right\},$$

otherwise the bargaining fails and the employed worker and the firm receive the outside value. We can see that $w_{T-1}^b(h, z, \psi) = w_{T-1}^b(h, z, y)$.

Therefore, at the bargaining stage the employed worker's value is:

$$V_{T-1}^b(h, z, \psi) = w_{T-1}^b(h, z, y) + \beta \sum_{\{s'\}} \Lambda(s' | s) \left\{ (1 - \delta) V_T(h', z', y') + \delta U_T(h', y') \right\},$$

if $z \geq z_{T-1}^b(h, y)$ and $V_{T-1}^b(h, z, \psi) = U_{T-1}^n(h, y)$ otherwise. The firm's value is:

$$F_{T-1}^b(h, z, \psi) = -w_{T-1}^b(h, z, y) + \sum_{\{s'\}} \Lambda(s' | s) \left\{ f(y, z', h) + \beta(1 - \delta) F_T(h', z', y') \right\},$$

if $z \geq z_{T-1}^b(h, y)$ and $F_{T-1}^b(h, z, \psi) = 0$ otherwise. Thus, we can write $V_{T-1}^b(h, z, \psi) = V_{T-1}^b(h, z, y)$ and $F_{T-1}^b(h, z, \psi) = F_{T-1}^b(h, z, y)$.

On the other hand, the value of the worker that has found an alternative job offer is:

$$V_{T-1}^a(w^a, h, \bar{z}, \psi) = w^a + \beta \sum_{\{y'\}} \Lambda(y' | y) ((1 - \delta) V_T(h', \bar{z}, y') + \delta U_T(h', y')),$$

so $V_{T-1}^a(w^a, h, \bar{z}, \psi) = V_{T-1}^a(w^a, h, \bar{z}, y)$. The value of the newly matched firm is:

$$G_{T-1}(w^a, h, \bar{z}, \psi) = f(y, \bar{z}, h) - w^a + \beta \sum_{\{y'\}} \Lambda(y' | y) F_T(h', \bar{z}, y'),$$

and hence $G_{T-1}(w^a, h, \bar{z}, \psi) = G_{T-1}(w^a, h, \bar{z}, y)$.

Then, the free-entry condition is:

$$c_v \geq q(\theta_{T-1}(w, h, \psi)) G_{T-1}(w, h, \bar{z}, y)$$

and $\theta_{T-1}(w, h, \psi) \geq 0$ with complementary slackness. It follows that:

$$\theta_{T-1}(w, h, \psi) = q^{-1} \left(\frac{c_v}{f(y, \bar{z}, h) - w + \beta \sum_{\{y'\}} \Lambda(y' | y) F_T(h', \bar{z}, y')} \right)$$

if $c_v \leq f(y, \bar{z}, h) - w + \beta \sum_{\{y'\}} \Lambda(y' | y) F_T(h', \bar{z}, y')$, and $\theta_{T-1}(w, h, \psi) = 0$ otherwise, so $\theta_{T-1}(w, h, \psi) = \theta_{T-1}(w, h, y)$ as the right hand side depends on ψ only through y . Equivalently,

$$w = f(y, \bar{z}, h) + \beta \sum_{\{y'\}} \Lambda(y' | y) F_T(h', \bar{z}, y') - \frac{c_v}{q(\theta_{T-1}(w, h, y))},$$

if $c_v \leq f(y, \bar{z}, h) - w + \beta \sum_{\{y'\}} \Lambda(y' | y) F_T(h', \bar{z}, y')$, and $\theta_{T-1}(w, h, \psi) = 0$ otherwise.

Thus, before the search stage the value of the matched worker is:

$$\begin{aligned}
V_{T-1}(h, z, \psi) &= \max_{\{w^a\}} \left\{ \lambda_e p(\theta_{T-1}(w^a, h, y)) [w^a + \beta \sum_{\{y'\}} \Lambda(y' | y) ((1 - \delta)V_T(h', \bar{z}, y') + \delta U_T(h', y'))] \right. \\
&\quad \left. + (1 - \lambda_e p(\theta_{T-1}(w^a, h, y))) V_{T-1}^b(h, z, y) \right\}, \\
&= \max_{\{w^a\}} \left\{ \lambda_e (-c_v \theta_{T-1}(w^a, h, y) + p(\theta_{T-1}(w^a, h, y)) (f(y, \bar{z}, h) \right. \\
&\quad \left. + \beta \sum_{\{y'\}} \Lambda(y' | y) (F_T(h', \bar{z}, y') + (1 - \delta)V_T(h', \bar{z}, y') + \delta U_T(h', y')) - V_{T-1}^b(h, z, y))) \right. \\
&\quad \left. + V_{T-1}^b(h, z, y) \right\}, \\
&= \max_{\theta \geq 0} \left\{ \lambda_e (-c_v \theta + p(\theta) (f(y, \bar{z}, h) + \beta \sum_{\{y'\}} \Lambda(y' | y) (F_T(h', \bar{z}, y') \right. \\
&\quad \left. + (1 - \delta)V_T(h', \bar{z}, y') + \delta U_T(h', y')) - V_{T-1}^b(h, z, y))) + V_{T-1}^b(h, z, y) \right\},
\end{aligned}$$

so if $f(y, \bar{z}, h) + \beta \sum_{\{y'\}} \Lambda(y' | y) (F_T(h', \bar{z}, y') + (1 - \delta)V_T(h', \bar{z}, y') + \delta U_T(h', y')) \leq V_{T-1}^b(h, z, y)$ then the solution is zero, and otherwise the objective function is strictly concave in θ . Thus, this problem has a unique solution $\theta_{T-1}^a(h, z, \psi)$. Then, $\theta_{T-1}^a(h, z, \psi) = \theta_{T-1}^a(h, z, y)$ and $V_{T-1}(h, z, \psi) = V_{T-1}(h, z, y)$ as above. Therefore, we uniquely specify:

$$w_{T-1}^a(h, z, \psi) = f(y, \bar{z}, h) + \beta \sum_{\{y'\}} \Lambda(y' | y) F_T(h', \bar{z}, y') - \frac{c_v}{q(\theta_{T-1}^a(h, z, y))},$$

and hence $w_{T-1}^a(h, z, \psi) = w_{T-1}^a(h, z, y)$.

Similarly we have at the beginning of age $T - 1$ value of unemployment:

$$\begin{aligned}
U_{T-1}(h, \psi) &= \max_{\{w^u\}} \left\{ \lambda_u p(\theta_{T-1}(w^u, h, y)) [w^u + \beta \sum_{\{y'\}} \Lambda(y' | y) ((1 - \delta)V_T(h', \bar{z}, y') + \delta U_T(h', y'))] \right. \\
&\quad \left. + (1 - \lambda_u p(\theta_{T-1}(w^u, h, y))) U_{T-1}^n(h, y) \right\}, \\
&= \max_{\{w^u\}} \left\{ \lambda_u (-c_v \theta_{T-1}(w^u, h, y) + p(\theta_{T-1}(w^u, h, y)) (f(y, \bar{z}, h) \right. \\
&\quad \left. + \beta \sum_{\{y'\}} \Lambda(y' | y) (F_T(h', \bar{z}, y') + (1 - \delta)V_T(h', \bar{z}, y') + \delta U_T(h', y')) - U_{T-1}^n(h, y)) \right. \\
&\quad \left. + U_{T-1}^n(h, y) \right\}, \\
&= \max_{\theta \geq 0} \left\{ \lambda_u (-c_v \theta + p(\theta) (f(y, \bar{z}, h) + \beta \sum_{\{y'\}} \Lambda(y' | y) (F_T(h', \bar{z}, y') \right. \\
&\quad \left. + (1 - \delta)V_T(h', \bar{z}, y') + \delta U_T(h', y')) - U_{T-1}^n(h, y)) \right\} + U_{T-1}^n(h, y),
\end{aligned}$$

so if $f(y, \bar{z}, h) + \beta \sum_{\{y'\}} \Lambda(y' | y) (F_T(h', \bar{z}, y') + (1 - \delta)V_T(h', \bar{z}, y') + \delta U_T(h', y')) \leq U_{T-1}^n(h, y)$ then the solution is zero, and otherwise the objective function is strictly concave in θ . Thus, this problem has a unique solution $\theta_{T-1}^u(h, \psi)$. Then, $\theta_{T-1}^u(h, \psi) = \theta_{T-1}^u(h, y)$ and $U_{T-1}(h, \psi) = U_{T-1}(h, y)$. Therefore, we uniquely specify:

$$w_{T-1}^u(h, \psi) = f(y, \bar{z}, h) + \beta \sum_{\{y'\}} \Lambda(y' | y) F_T(h', \bar{z}, y') - \frac{c_v}{q(\theta_{T-1}^u(h, y))},$$

and hence $w_{T-1}^u(h, \psi) = w_{T-1}^u(h, y)$.

The beginning of age $T - 1$ value of the firm previously matched is:

$$F_{T-1}(h, z, \psi) = (1 - \lambda_e p(\theta_{T-1}(w, h, y))) F_{T-1}^b(h, z, y),$$

so $F_{T-1}(h, z, \psi) = F_{T-1}(h, z, y)$.

Therefore, we can see that (S_T) implies (S_{T-1}) . Hence, by induction, (S_t) holds for $t = 1, \dots, T$, i.e. $U_t, V_t, F_t, G_t, w_t^u, w_t^a, w_t^b, z_t^b$ and θ_t are uniquely computed and they depend on ψ only through y for $t = 1, \dots, T$. ■

C.2 Introducing the Tax Wedge

We introduce the tax wedge into the baseline framework, with a focus on the parts of the model that are modified. With a tax wedge τ , if an agreement can be reached through Nash-bargaining at the bargaining stage, the value for a worker of remaining in the match is (at age $T - 1$):

$$(1 - \tau) w_{T-1}^b(h, z, y) + \beta \sum_{\{s'\}} \Lambda(s' | s) \left\{ (1 - \delta) V_T(h', z', y') + \delta U_T(h', y') \right\},$$

while the outside option at this stage is $U_{T-1}^n(h, y)$. The value of the firm of remaining in the match conserves its form. Thus, at age $T - 1$, the bargaining problem for the continuing match is:

$$\max_{\{w^b\}} \left[-w^b + \sum_{\{s'\}} \Lambda(s' | s) \left\{ f(y, z', h) + \beta (1 - \delta) F_T(h', z', y') \right\} \right]^{1-\xi} \times \\ \left[w^b (1 - \tau) + \beta \sum_{\{s'\}} \Lambda(s' | s) \left\{ (1 - \delta) V_T(h', z', y') + \delta U_T(h', y') \right\} - U_{T-1}^n(h, y) \right]^\xi$$

and $\tau \cdot w^b$ is subtracted from the joint surplus in the original problem.

The cutoff productivity $z_{T-1}^b(h, y)$ is the lowest z **such that the surplus of both firm and worker are non-negative**. If $z \geq z_{T-1}^b(h, y)$, the bargaining problem has a unique solution:

$$w_{T-1}^b(h, z, y) = \xi \left\{ \sum_{\{s'\}} \Lambda(s' | s) (f(y, z', h) + \beta (1 - \delta) F_T(h', z', y')) \right\} \\ + (1 - \xi) (1 - \tau)^{-1} \left\{ U_{T-1}^n(h, y) - \beta \sum_{\{s'\}} \Lambda(s' | s) ((1 - \delta) V_T(h', z', y') + \delta U_T(h', y')) \right\},$$

otherwise the bargaining fails and the employed worker and the firm receive the outside value.

Therefore, at the bargaining stage the employed worker's value is:

$$V_{T-1}^b(h, z, y) = w_{T-1}^b(h, z, y) (1 - \tau) + \beta \sum_{\{s'\}} \Lambda(s' | s) \left\{ (1 - \delta) V_T(h', z', y') + \delta U_T(h', y') \right\},$$

if $z \geq z_{T-1}^b(h, y)$ and $V_{T-1}^b(h, z, y) = U_{T-1}^n(h, y)$ otherwise. The firm's value is:

$$F_{T-1}^b(h, z, y) = -w_{T-1}^b(h, z, y) + \sum_{\{s'\}} \Lambda(s' | s) \left\{ f(y, z', h) + \beta(1 - \delta)F_T(h', z', y') \right\},$$

if $z < z_{T-1}^b(h, y)$ and $F_{T-1}^b(h, z, y) = 0$ otherwise.

On the other hand, the value of the worker that has found an alternative job offer is:

$$V_{T-1}^a(w^a, h, \bar{z}, y) = w^a(1 - \tau) + \beta \sum_{\{y'\}} \Lambda(y' | y) ((1 - \delta)V_T(h', \bar{z}, y') + \delta U_T(h', y')),$$

and the value of the newly matched firm is:

$$G_{T-1}(w^a, h, \bar{z}, y) = f(y, \bar{z}, h) - w^a + \beta \sum_{\{y'\}} \Lambda(y' | y) F_T(h', \bar{z}, y'),$$

Then, the free-entry condition is unchanged:

$$c_v \geq q(\theta_{T-1}(w, h, y)) G_{T-1}(w, h, \bar{z}, y)$$

and $\theta_{T-1}(w, h, y) \geq 0$ with complementary slackness.

Thus, before the search stage the value of the matched worker is:

$$V_{T-1}(h, z, y) = \max_{\{w^a\}} \left\{ \lambda_e p(\theta_{T-1}(\cdot)) \left[w^a(1 - \tau) + \beta \sum_{\{y'\}} \Lambda(\cdot) ((1 - \delta)V_T(h', \bar{z}, y') + \delta U_T(h', y')) \right] \right. \\ \left. + (1 - \lambda_e p(\theta_{T-1}(\cdot))) V_{T-1}^b(h, z, y) \right\}$$

Which can alternatively be written as:

$$V_{T-1}(h, z, y) = \max_{\theta \geq 0} \left\{ \lambda_e (-c_v \theta (1 - \tau) + p(\theta) (f(\cdot) (1 - \tau) + \beta \sum_{\{y'\}} \Lambda(\cdot) (F_T(h', \bar{z}, y') (1 - \tau) \right. \\ \left. + (1 - \delta)V_T(h', \bar{z}, y') + \delta U_T(h', y')) - V_{T-1}^b(h, z, y))) + V_{T-1}^b(h, z, y) \right\}$$

Using the unique solution $\theta_{T-1}^a(h, z, y)$ for the above problem, we can uniquely specify:

$$w_{T-1}^a(h, z, y) = f(y, \bar{z}, h) + \beta \sum_{\{y'\}} \Lambda(y' | y) F_T(h', \bar{z}, y') - \frac{c_v}{q(\theta_{T-1}^a(h, z, y))}$$

Similarly we have at the beginning of age $T - 1$ value of unemployment:

$$\begin{aligned} U_{T-1}(h, y) = & \max_{\{w^u\}} \left\{ \lambda_u p(\theta_{T-1}(\cdot)) [w^u (1 - \tau) + \beta \sum_{\{y'\}} \Lambda(\cdot) ((1 - \delta) V_T(h', \bar{z}, y') + \delta U_T(h', y'))] \right. \\ & \left. + (1 - \lambda_u p(\theta_{T-1}(\cdot))) U_{T-1}^n(h, y) \right\} \end{aligned}$$

Which can alternatively be expressed as:

$$\begin{aligned} U_{T-1}(h, y) = & \max_{\theta \geq 0} \left\{ \lambda_u (-c_v \theta (1 - \tau) + p(\theta) (f(\cdot) (1 - \tau) + \beta \sum_{\{y'\}} \Lambda(y' | y) (F_T(h', \bar{z}, y') (1 - \tau) \right. \\ & \left. + (1 - \delta) V_T(h', \bar{z}, y') + \delta U_T(h', y')) - U_{T-1}^n(h, y))) + U_{T-1}^n(h, y) \right\}, \end{aligned}$$

Therefore, using the unique solution $\theta_{T-1}^u(h, y)$ for the above problem, we uniquely specify:

$$w_{T-1}^u(h, y) = f(y, \bar{z}, h) + \beta \sum_{\{y'\}} \Lambda(y' | y) F_T(h', \bar{z}, y') - \frac{c_v}{q(\theta_{T-1}^u(h, y))}$$

The beginning of age $T - 1$ value of the firm previously matched is

$$F_{T-1}(h, z, y) = (1 - \lambda_e p(\theta_{T-1}(w, h, y))) F_{T-1}^b(h, z, y).$$

C.3 Expected Present Discounted Value of Earnings

We compute the expected present discounted value of earnings, for the case of no taxes. At age T the value of an unemployed worker with no job offer after the search stage is $U_T^n(h, y) = b$. We denote the expected present discounted value of earnings as $\hat{U}_T^n(h, y) = 0$. At the bargaining stage the value of an employed worker is $\hat{V}_T^b(h, z, y) = w_T^b(h, z, y)$ if $z \geq z_T^b(h, y)$ and $\hat{V}_T^b(h, z, y) = 0$ otherwise. The value for a worker that finds an

alternative job offer is $V_T^a(w^a, h, \bar{z}, y) = \hat{V}_T^a(w^a, h, \bar{z}, y) = w^a$. At the search stage the earnings value of the workers is evaluated at the equilibrium $\theta_T^a(h, z, y)$:

$$\hat{V}_T(h, z, y) = \lambda_e(-c_v\theta + p(\theta)(f(y, \bar{z}, h) - \hat{V}_T^b(h, z, y))) + \hat{V}_T^b(h, z, y)$$

The earnings value of the unemployed worker at the search stage is:

$$\hat{U}_T(h, y) = \lambda_u(-c_v\theta + p(\theta)(f(y, \bar{z}, h) - \hat{U}_T^n(h, y))) + \hat{U}_T^n(h, y)$$

evaluated at the equilibrium $\theta_T^u(h, y)$. Going back one period, the earnings value of an unemployed worker is:

$$\hat{U}_{T-1}^n(h, y) = 0 + \beta \sum_{\{y'\}} \Lambda(y' | y) \hat{U}_T(h, y')$$

By backward induction we can compute $\{\hat{V}_t, \hat{V}_t^b, \hat{V}_t^a, \hat{U}_t, \hat{U}_t^n\}$ for all t .