Essays on Sovereign Default

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Dedication

To my parents, Ulrich and Eva.

Abstract

This thesis consists of three separate chapters. In the first chapter, I review the literature on sovereign debt crises. In the second chapter, I analyze the role nominal debt plays in sovereign debt crises, and in particular default and inflation policies. Using bond-level data on government borrowing, I document that nominal obligations are a large fraction of government debt in emerging market countries. I then show that default and inflation rates vary systematically with debt denomination: high nominal debt shares are associated with low inflation and default rates in these countries. I build a monetary model of sovereign debt with lack of commitment, in which differences in debt denomination generate this pattern, and the government inflates more when debt is real. Issuing real instead of nominal debt has two effects in the model. On the one hand, real debt reduces the incentive to create costly inflation because the value of the debt is fixed in real terms. It thus helps mitigate the commitment problem. On the other hand, because the commitment problem is less severe, real debt facilitates more debt accumulation over time, causing the government to resort to the printing press after all to finance the debt burden. In a calibrated version of the model this second effect dominates: As in the data, inflation and default rates are higher on average when debt is real instead of nominal. Default risk helps generate large differences in inflation and default rates across debt regimes as the government optimally inflates in order to avoid default.

In the third chapter, I study incomplete debt relief in sovereign debt crises. I show that, in the data, sovereign defaults typically do not result in a full debt write-down. On the contrary, creditors recover on average more than half of their investment. I then build a model of sovereign default and incomplete debt relief to study the causes and consequences of incomplete debt relief. In the model, the degree of debt relief directly affects default incentives via bond prices. In particular, a high debt recovery rate - equivalently, little debt relief - reduces recovery risk to investors and tends to offset the effects of default risk. In equilibrium, incomplete debt relief lowers spreads and increases debt-to-output ratios and welfare. Default rates are non-monotonically related to debt relief and lowest for intermediate, but relatively low degrees of debt relief. I use the model to analyze the trade-off between long renegotiations and low debt relief and show that the latter is a more effective tool for achieving low equilibrium default rates and high welfare. Finally, the model predicts that countercyclical recovery rates are not welfare-improving.

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Chapter 1

Critical Review

The key distinguishing feature of sovereign borrowing is the lack of incentives for the borrowing country to repay its creditors. Sovereign borrowing unlike other types of borrowing involves no recourse for lenders by the very nature of sovereignty. International lending is typically unsecured, international courts do not have the legal authority to seize assets, and there are no international bankruptcy laws. As a result, theoretical approaches to the topic have in common a focus on modeling environments with limited commitment, and dealing with the question of how debt is sustainable in equilibrium. Absent an enforcement mechanism, the government cannot commit is tempted to default on any outstanding debt. It is then not clear why creditors in turn would ever lend in the first place in such a setting. The benchmark quantitative sovereign default theory that I focus on in this chapter rationalizes borrowing as a consequence of impatience by the government and default as an opportunity to create state contingency in otherwise incomplete markets. Default is an infrequent event and equilibrium debt levels are positive in the theory because default is costly in different ways.

While early theoretical studies were successful at accounting for some of the high level empirical regularities - such as equilibrium default, countercyclical interest rates - they did abstract from many other features of default and an active literature has developed that is filling these gaps. Aspects that have received particular attention include theories of the maturity structure of sovereign debt and its implications for default, debt renegotiations and debt relief, the link between sovereign risk and other aspects of fiscal policy and the real side of the economy, and the links of sovereign debt crises to other crises. These theoretical advances have been accompanied by empirical studies documenting novel facts about sovereign crises and calling attention to their heterogeneity and previously unexamined details. This includes moving from treating default as a binary indicator to measuring partial default, studying maturity and denomination of debt, determinants of haircuts and other aspects of sovereign debt renegotiations, as well as the relation of sovereign borrowing to the business cycle.

In the following, I will review this literature on sovereign debt and default, starting with empirics before moving on to the theoretical literature and discussing how the two match up.

1.1 Empirical Literature

In this section I will review aspects of empirical research on sovereign debt and default. I will first discuss measurement and definitional issues before reviewing empirical regularities that the literature has established. The two are closely linked and important progress has been made in recent years by looking at sovereign debt markets and crises in a more disaggregated manner and focusing on more careful measurement.

1.1.1 Concepts and Measurement

1.1.1.1 Types of Government Liabilities

What do we mean when we talk about sovereign debt? Most studies are concerned with external debt. This can be taken to mean debt issued in foreign rather than domestic currency, debt held abroad rather than at home, or debt issued under a foreign jurisdiction. In practice with debt being bonded and freely tradable, keeping track of the residence or nationality of the holder of the debt is difficult, so studies have more often used denomination or jurisdiction when defining external sovereign debt. We can furthermore distinguish between debt issued by the national government as opposed to local or regional governments or net of cross-holdings by the central bank or other government branches. Sovereign debt today most often comes in the form of bonds issued that can be traded on secondary markets. Other forms of sovereign debt include bank loans and lending by international institutions like the IMF - both of these are more difficult to value at market prices but studies like Tomz (2007) document bond finance outstripping bank loans for the past two decades, and according to the WDI, while concessional debt is a substantial fraction of external debt for some countries, it is below 3% of GDP every year since 1970 (GDP-weighted average across countries). Within the domain of bonded debt, there are many different kinds of bonds that can be issued. Even though the majority of sovereign bonds are plain vanilla type bonds - pay at maturity, simple deterministic coupon structure - there are exceptions, for example inflation-indexed debt, hybrid bonds, callable bonds and variable rate bonds.

The link between external debt and debt that is in foreign currency and held abroad is becoming increasingly weak in the data. Traditionally when talking about external debt studies tend to consider exclusively foreign currency denominated debt issued on foreign markets, assumed to be held by foreign investors. There is evidence that these distinctions are beginning to blur: Local currency debt is increasingly held by foreigners (Du and Schreger (2013)) and foreign currency debt is issued at home (Chapter 2).

1.1.1.2 Valuation of Liabilities

How should government debt obligations be valued to construct a measure of a sovereign's indebtedness? Most sources, including for example the World Bank, use the face value to arrive at a measure of the stock of sovereign debt - that is the undiscounted sum of future principal payments. This has the advantage of being simple to compute but there are drawbacks. For example, the measure does not take into account coupon payments, which make up a substantial fraction of bond payments in emerging markets and Latin America in particular - a bond with no coupon payments is treated as the same obligation as a bond with the identical principal amount that in addition makes coupon payments regularly before maturity. In addition, it does not discount payments to be made in the future - the consol whose face value is due in an infinite number of period receives as much weight when calculating the stock of debt as a payment that is due tomorrow.

Possible discounts for future payment streams are market rates, or alternatively fixed interest rates that capture at least some of the opportunity cost of holding the given debt instrument. In practice, bond markets except for large issuers are often too illiquid to estimate yield curves and hence market rates, but certainly for developed countries and large emerging market issuers these are readily available from public sources.

Examples where valuation questions are relevant include measuring the maturity and currency composition of government debt. Both of these involve decisions on the best measure of debt to use - face value, market value, including or excluding coupons, converting future payment streams from one currency to another. To take the example of the maturity of debt, a common measure to use is Macaulay duration - a cash-flow weighted average of the dates of future cash flows, where the discount used is typically a constant market yield, so debt that is highly discounted will receive a lower weight. Absent new debt issuances, during a crisis when yield curves are likely to be flatter than during normal times, short debt would be given less weight on account of lower market values in calculating duration. Similarly, for nominal debt or debt denominated in other currencies, inflation expectations and expected exchange rate movements affect the valuation of the debt and should factor into the valuation of the debt - but are hard to come by in many cases where financial markets are not liquid enough and forward exchange rates or measures of inflation expectations not readily available.[1]

It is worth pointing out that which measure of indebtedness is the appropriate one depends often on the context and if the goal is to match a model to the data consistency in measurement across model and data are important.

1.1.1.3 Defining Default

What does it mean for a government to default on its debt? Defining what constitutes default is not uncontroversial. One common measure is a binary indicator as in Beers and Chambers (2006) by Standard and Poor's, for example. The ratings agency lists default episodes back to 1824. They define a default as "the failure to meet a principal or interest payment on the due date (or within the specified grace period)". A default is defined as resolved when "no further near-term resolution of creditors' claims is likely". When payments are rescheduled, they are deemed a default in case the rescheduling is at less favorable terms to the creditors than the original arrangement. Both of these are difficult to quantify precisely, as we will see in more detail below - the likelihood of no further

[1] See more in Arellano and Ramanarayanan (2012), and Chapter 2

resolutions, and measuring whether reschedulings are at less favorable terms. Moreover, this binary definition of default lumps together potentially very different situations - for example, Argentina suspending payments on over 75% of its outstanding external debt, compared with a rescheduling by a few months of interest payments on just one oil warrant as in Venezuela in 2005. Recently there have been studies that to use arrears as a less coarse measure of credit history; examples include Benczur and Ilut (2011), Arellano et al. (2013) and De Paoli et al. (2009).

1.1.2 Empirical Regularities

Having described the main definitional and conceptual issues surrounding sovereign debt and default, I will now outline empirical regularities regarding sovereign debt and default that have been documented in the literature.

1.1.2.1 Default Frequency

Defaults occur with regularity throughout history. Tomz and Wright (2008) study sovereign crises for 176 sovereign entities going back to 1820. The unconditional default probability in this sample - number of country-year pairs in default relative to total number of country-year pairs - was 1.7%. Conditioning on defaulters or restricting the sample to post-1980 raises this number to 3% and 3.8% respectively. Given the discussion of the previous section on measurement and definition of default episodes, Tomz and Wright (2008)'s caution that a more robust measure of default frequency should be used to calibrate models - they suggest the fraction of time spent in default, which is 18% in their sample.[2]

An alternative is to abandon the simple binary default structure and focus on arrears instead, as mentioned above (for example in Arellano et al. (2013)). These measures typically identify similar episodes but tend to differ on precise start and end dates of default episodes. They have the advantage of taking a more disaggregated look at what payments a sovereign refuses to make, given the heterogeneity in debt instruments that countries typically issue (nominal or real, long or short debt, held abroad or domestically, to name but a few examples). On the other hand, legal provisions in bond contracts where

[2] Other references include Reinhart and Rogoff (2009) and Sturzenegger and Zettelmeyer (2007).

default on one bond triggers default on all others are increasingly common, so it remains to be seen if partial default becomes a less relevant concept over time.

1.1.2.2 Cyclicality of Sovereign Defaults

Whether defaults happen during deep recessions (rather than causing them - see the previous section) remains controversial in the empirical literature. Tomz and Wright (2007) find that default does appear to be associated with weak outcomes on the real side of the economy, but perhaps only modestly so. In a sample of 175 countries between 1820 and 2005, defaults occurred in periods where output was below its HP-trend only 60% of the time. The average deviation of output from trend at the start of a default episode was 1.6%. Output tended to recover during default episodes rather than deteriorate further.

De Paoli et al. (2009) on the other hand, in 39 defaults from 1970 to 2000, find large output losses on the order of 5% per year during defaults, including during the first year of a default. Like Tomz and Wright (2007) they use HP-filtered data to construct counterfactual potential output, but they also run regressions to control for several other factors, unlike Tomz and Wright (2007) who focus on unconditional correlations. Defaults in De Paoli et al. (2009) are defined as a threshold on arrears rather than a contractual contract breach (which should reduce the number of defaults they find, and/or shorten the time of a default episode). De Paoli et al. (2009) highlight that twin crises are associated with larger output declines than idiosyncratic default episodes. They also find that output losses increase the longer a country takes to restructure their debts and exit default.[3]

Yeyati and Panizza (2011) using quarterly data for 39 emerging market countries between 1970 and 2005 find large recessions (GDP 3.4% below trend) just prior to a default. They argue that quarterly data is the more suitable frequency at which to analyze data to pick up sharp drops in output surrounding default episodes and resolve some of the identification problem - did a country default because output was low, or was output low because the country defaulted? This is discussed further in section 1.1.2.5.

[3] This latter regularity is also documented in Benjamin and Wright (2009).

1.1.2.3 Haircuts and Debt Relief

Failure to pay on time - a default - does not mean that the payment or at least part of it will never be made. In fact, despite lack of formal bankruptcy proceedings and enforcement mechanisms, creditors to sovereigns recover on average more than half their investment after a default. The flip side of this is that default does not actually reduce a borrower's indebtedness very much on average. Several recent papers have contributed to establishing empirical facts on creditor losses or "haircuts" after sovereign defaults, including Cruces and Trebesch (2011), Benjamin and Wright (2009) and Sturzenegger and Zettelmeyer (2005).

The main results from these empirical studies are that haircuts are high on average (37% in the Cruces and Trebesch (2011) sample) and vary widely. Moreover, renegotiation length is correlated with high haircuts, as is the cost of borrowing post-renegotiation. Cruces and Trebesch (2011) also document that higher haircuts are associated with a lower likelihood of regaining market access after a restructuring.[4] It is to the best of my knowledge an open an little researched question empirically whether debt relief is welfare improving (for theoretical results on this see Chapter 3 and Section 1.2).[5]

Again, there are measurement issues involved: Benjamin and Wright (2009) base their measure on WDI data which only considers face value reductions as haircuts, but not maturity extensions like exchanging old instruments for new ones with later maturity dates. In Sturzenegger and Zettelmeyer (2005)'s preferred measure they instead calculate haircuts as the difference in net present values between the original and the restructured debt. This is harder to compute because it requires calculating prices of both the defaulted and newly restructured debt at times of distress when markets are often illiquid and price quotes not necessarily readily available. Cruces and Trebesch (2011) use the same measure of haircuts but for a larger set of default episodes - the entire universe of sovereign debt restructurings between 1970 and 2000, that is 182 restructurings by 68 separate countries. Where comparisons between Benjamin and Wright (2009) and Cruces and Trebesch (2011) are available, the measures are not close, so again, it is important to pick the appropriate measure for the research purpose at hand and be sure empirical measures and model

[4] Note that they define market access as borrowing after a successful restructuring of debt, not as market access during a default episode. See more on this in subsection 1.1.2.4.

[5] There is a preliminary working paper by Wright et al. (2013) which finds welfare losses due to debt relief.

counterparts are consistent (see Chapter 3).

1.1.2.4 Market Access

Several studies have examined whether and if so for how long defaulters are excluded from borrowing again following a default - a quantity restriction rather than higher borrowing costs. Some studies take “market access” to mean time until renegotiations are completed successfully. Some take it to mean time until a defaulter can borrow again - which is a difficult question to answer for obvious endogeneity reasons.

Benjamin and Wright (2009) who belong to the first category of papers find that for their sample of 90 defaults from 1989 to 2005 renegotiations took on average 8 years. The Tomz and Wright (2007) sample leads to similar numbers. Gelos et al. (2011) define market access as borrowing that leads to increased debt (in terms of face values) and find, for defaults from 1980 to 2000, that it takes a much shorter amount of time after a default until market access is restored - 4.7 years on average, and since 1990 even just 2.9 years.[6] Cruces and Trebesch (2011) define market access as positive net transfers *post-renegotiation* and find that it takes 5.1 years to regain market access.

Measurement differences and questions of causality notwithstanding, regularities that emerge from the literature are that there is at least some loss of market access following a default, the length of exclusion is positively correlated with haircuts, and has fallen since the 1980s.

1.1.2.5 Other Costs of Default

Aside from loss of capital market access and higher borrowing costs, many studies have examined whether there are other costs of default.

Drops in GDP is one candidate that seems difficult to pin down. Borensztein and Panizza (2008) and Yeyati and Panizza (2011) for example have difficulty establishing causality running from default to recessions and instead find evidence that recessions tend to precede defaults and that output recovers in the course of default episodes. Clearly this

[6] They exclude unresolved defaults like Argentina’s 2001 default. They measure market access as an increase in debt in order to exclude cases where a country is unable to borrow but is able to rollover existing debts.

needs to be taken with a grain of salt given the inherent endogeneity problem (see also subsection 1.1.2.2).

Trade costs are another candidate where the evidence is inconclusive. Studies have been able to identify falls in trade flows and trade credit. Rose (2005) finds in a sample of 200 countries between 1948 and 1997 that Paris Club debt renegotiations are associated with drops in bilateral trade of around 8% per year. Borensztein and Panizza (2010) using industry level data show that especially exporters are affected by these trade disruptions. Borensztein and Panizza (2008) and Tomz and Wright (2013) argue that there is limited evidence of direct trade embargoes following defaults, but Borensztein and Panizza (2008) find evidence for trade credit disruptions arising from a spillover channel: Trade credit becomes more expensive because private sector credit worthiness falls along with that of the government. Arteta and Hale (2008) find similar evidence. There is no consensus however on the exact channel through which default affects either trade flows or trade credit.

Cole and Kehoe (1998) in a theoretical contribution suggest reputational spillovers from the sovereign's willingness to repay to other areas of international relations. There is relatively little empirical work on this but in one such study Tomz and Wright (2008) find that sovereign default and other types of expropriation do not generally coincide, which they say is at odds with the spillover hypothesis. Political costs of default more generally are relatively under-researched empirically.[7] Borensztein and Panizza (2008) note that there is anecdotal evidence that defaults make political survival less likely, and that there are reasons to suspect delay in initiating default on account of this. They argue that this is one channel by which defaults could be more costly than otherwise the case.

1.1.2.6 Debt Maturity and Default

The maturity of debt varies significantly across countries and time, and how it changes with risk of default depends on how it is measured. According to Tomz and Wright (2008) in a sample of 137 low and middle income countries in the year 2000 the contractual maturity (the payment date on any outstanding debt furthest into the future) ranged from 10 to

[7] There is a large literature on costs of political instability, including debt accumulation and default, e.g. Easterly and Levine (1997).

40 years, whereas duration (the cash-flow weighted average of the dates of future cash flows, discounted at a constant market yield) assuming a discount of 5% ranged from 3.4 to 14.2 years. Duration is shorter because of discounting, and because for emerging market countries a relatively large part of obligations tends to come in the form of coupon payments rather than the principal. Arellano and Ramanarayanan (2012) document that for four major emerging market borrowers - Mexico, Russia, Brazil and Argentina - between 1996 and 2011, average duration for each country was between 6 and 7 years, maturity several years longer at 9 to 12.

In terms of the relation of maturity to other aspects of sovereign debt crises, Arellano and Ramanarayanan (2012) in their sample, in times of high spreads, duration shortens whereas there is no clear pattern across countries regarding maturity. Broner et al. (2007) show that countries are less likely to issue debt when spreads are high, and that the maturity of issues shortens when term premia are high, that is when long rates are higher than short rates.

1.1.2.7 Joint Crises

External debt defaults often do not occur in isolation, but instead jointly with other crises - sudden stops, domestic debt defaults (explicit) or inflation episodes (implicit default), banking crises, currency crises, or political crises.

Reinhart and Rogoff (2011b) document using a long historical time series going back as far as 1800 that external debt crises are frequently accompanied and in fact often preceded by banking crises. De Paoli et al. (2009) confirm this using a shorter sample post-1970 and different default definition, and consider currency crisis in addition to banking and sovereign. They show that 75% of sovereign crises coincide with a currency crisis, and 67% with a banking crisis. Almost 50% of crises are "triple crises", and the authors estimate that these twin or triple crises are more costly than sovereign defaults by themselves. Arellano and Kocherlakota (2008) document that sovereign and private borrowing costs co-move, and confirm the Reinhart and Rogoff (2011b) result that banking and sovereign crises occur together more often than not, using a sample of emerging and middle income countries between 1976 and 2012. Reinhart and Rogoff (2011b) also show that sovereign defaults occur in "bouts" in the sense that there are periods throughout history when a large fraction of sovereign debt issuers was in default simultaneously.

Reinhart and Rogoff (2011a) build a database of public debt together with external debt, and show that external default often coincides with inflation or explicit domestic default. Related to this, in Chapter 2 of this thesis I show that public debt is predominantly nominal and its prevalence inversely related to inflation. Claessens et al. (2007) investigate the determinants of the denomination of debt and find that political stability and rule of law tend to go hand in hand with higher nominal debt shares. There is evidence that domestic debt is becoming increasingly important as an investment class for international investors (Du and Schreger (2013)) the implications and determinants of which are relatively little researched empirically.

Sudden stops are another type of crisis that are highly correlated with sovereign defaults but there is little formal empirical work on this in the literature. More generally, investigating the links between sovereign default and other types of crises is an interesting area of research.

1.2 Theoretical Literature

1.2.1 Early Qualitative and Limited Enforcement Literature

Early studies of sovereign debt and default have focused on the conceptual issues of why sovereigns repay their debt in the absence of legal enforcement, and consequently how debt can be sustainable in equilibrium. Some of the seminal contributions include Eaton and Gersovitz (1981), Grossman and Van Huyck (1988) and Bulow and Rogoff (1989). Eaton and Gersovitz (1981) focus on non-legal costs of default that can deter a sovereign borrower from defaulting - not extending new credit in particular as a form of retaliation. Bulow and Rogoff (1989) in a well known critique of this line of argument show that such a form of threatened retaliation does not sustain debt in equilibrium if the borrowers still has access to savings instruments. Grossman and Van Huyck (1988), in an idea related to Zame (1993), explore the idea of default as a way of introducing state contingencies into otherwise noncontingent debt contracts and distinguish between default for such insurance reasons ("excusable default") and default that serves no such purpose and was not prices into bonds ("inexcusable default").

More recently, an active strand of the literature builds on models of limited enforcement and complete asset markets, as in Kehoe and Levine (1993) and Thomas and Worrall (1988), to study sovereign debt and default. This literature focuses largely on qualitative theoretical contributions and is not the main focus of this chapter or thesis, so I will only give a brief overview in what follows. See Aguiar and Amador (2015) for a recent detailed review of this area of research. Sovereign governments in these studies are assumed to have limited commitment, meaning that the borrower at any point can change his mind and walk away from a debt contract. The conditions under which he would do so depend on the value of the contract as well as the outside option which is typically modeled as the value of autarky. In a sustainable non-autarkic equilibrium the borrower is incentivized to participate in the asset market rather than choose the outside option. This means that allocations are such that, at every point in time, in every state of the world, staying in the contract is never strictly worse than the outside option. This amounts to a limit on the level of debt sustainable in equilibrium under standard assumption on utility functions which imply that the value of staying in the contract decreases monotonically with debt. In terms of the allocations that are part of this equilibrium, the borrower receives higher consumption in good states of the world than he would in an equilibrium with full commitment where the outside option does not place additional constraints on the equilibrium. Intuitively, in good states of the world risk sharing dictates that he make payments and consume less than the endowment. In order to ensure that leaving the contract is not too appealing, the equilibrium features higher consumption than the case under full commitment. Conversely, in bad states of the world, the borrower receives lower transfers than under full commitment.

Default in this framework can mean one of two things - state contingent payouts or choosing the outside option - but neither type has clear data counterparts. Thinking of default as the outside option is difficult because of the implication that it only occurs off-equilibrium and serves the purpose of sustaining the non-autarkic equilibrium. The alternative are state contingent payments as in Grossman and Van Huyck (1988)'s "excusable" defaults.[8]

[8] There are papers that extend this notion of default. Aguiar and Amador (2011) for example explain how unobservable variation in the outside option generates market incompleteness and thus equilibrium default in the sense of the outside option being chosen in equilibrium. See also Hopenhayn and Werning

In terms of results, this area of research yields (i) that limited commitment impedes risk sharing and thus reduces welfare, (ii) that harsher punishments or equivalently lower values of the outside option are welfare improving because they make deviating less appealing to the borrower, that (iii) limited commitment provides an incentive to save, or equivalently to postpone consumption (see for example Perri (2008) and Aguiar and Amador (2011). The latter is intuitive because a binding participation constraint reduces the marginal value of borrowing today, or equivalently borrowing less can help relax the participation constraint and associated impeded risk sharing. It is therefore related to the result that high initial levels of debt yield allocations further away from the optimal allocations with commitment. If the borrowing country is not more impatient than the market, this will eventually lead him to save and achieve the unconstrained full commitment optimal allocation; otherwise there are perpetual cycles in consumption allocations and implied debt/ net exports. In the quantitative models I discuss below, this incentive to borrowing less is typically counteracted by incentives to borrow because of impatience.

1.2.2 Quantitative Sovereign Default Literature

There is a large and growing area of research concerned with the quantitative implications of sovereign debt theories. Chapters 2 and 3 of this thesis follow this literature. It differs from the papers discussed previously in that it considers a more restrictive environment in order to be able to take the models closer to the data - incomplete markets instead of an Arrow-Debreu world,[9] default defined as less than full face value repayment on outstanding debt - and a more restrictive equilibrium concept - Markov perfect equilibria as in Klein et al. (2008) rather than sustainable equilibria as in Chari and Kehoe (1990).[10]

The assumption of incomplete markets delivers equilibrium default in the intuitive sense of the word: In bad states of the world the borrower may prefer to break the contract to increase consumption today rather than pay its debt in full. Default introduces some state contingency. Seminal papers that fit into this literature conceptually are Zame (1993) and Eaton and Gersovitz (1981), and the earliest papers I am aware of that compute

(2008).

[9] There are papers, like Dovis (2013) in the context of sovereign default, that show that incomplete markets can be justified as an implementation of the constrained efficient allocation.

[10] Regarding differences in the solution concept, there are remarkably few attempts to directly compare these equilibrium concepts. One exception is Chang (1998).

models numerically and try to match them to the data are Arellano (2008) and Aguiar and Gopinath (2006).

1.2.3 A Benchmark Model

The simplest benchmark version of a quantitative sovereign default model is an infinite horizon small open exchange economy that receives a an endowment stream $y \in Y$ and is populated by a representative household and a benevolent government that maximizes discounted household lifetime utility. The household consumes a single non-storable consumption good c and receives transfer payments from the government. It does not have access to financial markets. The government has access to non-contingent one-period bonds $b \in B$ that are denominated in units of the consumption good and can be sold to international competitive risk-neutral investors. The government chooses each period whether to default or repay and, conditional on repayment, how much to borrow. The resource constraint in this environment is given by $c + qb' = y + b$. The government's problem can be written recursively as

$$\begin{aligned} V^o(b,y) &= \max_{d \in \{0,1\}} (1-d)V^r(b,y) + dV^d(y) \\ V^r(b,y) &= \max_{b'} u(y + b - q(b',y)b') + \beta \int_{y'} V^o(b',y')dF(y',y) \\ V^d(y) &= u(y^d) + \beta \int_{y'} \left(\eta V^o(0,y') + (1-\eta)V^d(y')\right) dF(y',y) \end{aligned} \tag{1.1}$$

with equilibrium policy functions $\tilde{b}(b,y) = b'$ and $\tilde{d}(b,y,) = d$. Bond prices reflect repayment probabilities and are given by

$$q(b',y) = \frac{1}{1+r} \int_{y'} (1 - \tilde{d}(b',y')dF(y',y) \tag{1.2}$$

A Markov Perfect Equilibrium of the economy are value functions V^o, V^r, V^d, policy functions $\tilde{b}, \tilde{d}$ and a price q that solve the government's problem (1.1) and satisfy (1.2).

The key assumption again is lack of commitment: The government is unable to commit to its borrowing or repayment policies. This introduces a time consistency problem. In order to borrow a large amount of resources and increase consumption today, the government would like to increase the bond price as much as possible by credibly promising not to default. But at the beginning of the next period, the inelastically supplied outstanding stock of debt means that it is tempting for the government not to repay and instead default. The recursive equilibrium concept employed yields a solution that embodies the lack of commitment but is time consistent. In this equilibrium, the government can condition its policies on the current state only and take into account how its actions will affect the future state. But it cannot, for example, take into account how the history of its past policy choices affect the state it is faced with today.

Key properties of benchmark model are (see Arellano (2008) for proofs and details):

- Default incentives are increasing debt.
- Bond prices are decreasing in borrowing.
- Default incentives are decreasing in the endowment if the endowment is iid. This is a result of the concavity of utility and the fact that if default risk is positive, the government on net makes payments on its debt rather than increasing borrowing (equivalently, net exports are positive, the economy experiences capital outflows). If no amount of borrowing can increase consumption, default is relatively appealing. This is increasingly the case for lower levels of income because of concavity of the utility function.
- Interest rates are countercyclical. This is the case because demand for debt is higher in recessions as the government wants to smooth consumption over time. It is helpful to think about this in terms of the partial derivative of the bond price function with respect to the endowment: $\frac{\partial[q(b'(b,y),y)]}{\partial y} = \frac{\partial q}{\partial b'}\frac{\partial b'}{\partial y} + \frac{\partial q}{\partial y} \geq 0$. Default incentives are increasing in debt, so the bond price is decreasing in debt. Since demand for debt is decreasing in the endowment, the bond price overall rises with the endowment. Note that this is true regardless of whether the second term is zero (in the iid case) or not.
- The trade balance is countercyclical provided shocks are persistent. It is procyclical otherwise. This property, just like the previous one, is not a theorem but a quantitative property which in principle depends on parameters. To see why this is the

case, consider the following. The trade balance is countercyclical if the country raises more bond revenue qb' in good times. Then the trade balance falls when output is high. It is helpful to think about the effect of the endowment on bond revenue in terms of the elasticity of the bond price, $e_{b'}$ and e_y:

$$\begin{aligned}\frac{\partial[q(b'(b,y),y)b'(b,y)]}{\partial y} &= \left(\frac{\partial q}{\partial b'}\frac{\partial b'}{\partial y}+\frac{\partial q}{\partial y}\right)b'+\frac{\partial b'}{\partial y}q \\ &= \frac{\partial b'}{\partial y}q\left(1+\frac{\partial q}{b'}\frac{b'}{q}\right)+\frac{\partial q}{\partial y}\frac{y}{q}\frac{b'}{y} \\ &= \frac{\partial b'}{\partial y}q\left(1-e_{b'}\right)+e_y\frac{b'}{y}\end{aligned}$$

If output is iid, the second term is zero since the bond price is not a function of current output. In this case, if the bond price is inelastic ($e_{b'}$ is sufficiently small in absolute terms), bond revenue is increasing in the endowment, and for a given level of debt the trade balance rises with the endowment. Intuitively, demand for debt is higher in recessions and provided the bond price is inelastic enough, that means that you successfully generate more bond revenue, and thus a lower trade balance, in recessions. If the endowment process is persistent on the other hand, the elasticity of the bond price with respect to current output e_y is also important. In particular, if the bond price is sufficiently elastic with respect to output, it can make the trade balance countercyclical. Intuitively, in a recession the probability of being in one again tomorrow is high, and the default probability is higher in recessions. This can reduce the bond price sufficiently to lower bond revenues (reduce capital inflows) and increase net exports in recessions. With persistent shocks, therefore, the trade balance is more likely to be countercyclical in the benchmark model.

What generates equilibrium default in the model? If both value of default and repayment are shifted equally by changes in output, then it is only borrowing that determines default. Optimally the government will be able to avoid it and there will be no equilibrium default. Arellano (2008), to counter this, introduces an asymmetric default cost that makes the bond price relatively less sensitive to borrowing than to endowment fluctuations. In good times the value of default moves less with output than the value of repayment, and moves one for one only at low output levels. As a result, recessions can bring about defaults:

the value of repayment falls by more when you enter a recession than the value of default. Aguiar and Gopinath (2006) generate higher default probabilities by making output more variable, specifically by adding a shock to its trend. See Aguiar and Amador (2015) for how this generates a less elastic bond price with respect to borrowing such that recessions can push the economy into default.

In terms of quantitative results, the model underpredicts the level of spreads and debt, and overpredicts the volatility of spreads. Moreover it can clearly not capture many of the facts discussed in the empirical section - haircuts, partial default, currency and maturity composition of debt name a few. I will discuss key ingredients to generating these predictions, and fixing the shortcomings, next.

1.2.4 Model Predictions and Features

1.2.4.1 Haircuts, Debt Relief and Market Access

The baseline model makes strong assumption regarding the default process: All debt is written off and the sovereign lives in autarky for at least some time after default. This is clearly at odds with the data and there are a number of studies that investigate the consequences of relaxing these assumptions.

In terms of haircuts, Yue (2010) introduces a very simple bargaining protocol that determines what fraction of defaulted debt to repay. Haircuts are tightly linked to one parameter - bargaining power in a one-shot Nash bargaining game. The bargaining game in the model starts after the default decision and takes place prior to re-entry to markets. As a result, the government still re-enters capital markets with no debt, contrary to what we observe in the data. Chapter 3 of this thesis incorporates haircuts in a model with long term debt and bond prices that reflect repayment probabilities inclusive of expected haircuts, both in good credit standing and during a default episodes. I show that harsher punishments in the sense of lower expected debt relief are welfare improving.

Arellano et al. (2013) in a recent working paper model partial default and borrowing while carrying debt in arrears. An ad hoc cost is what delivers partial default as a relatively infrequent event. Borrowing during or after defaults when the country carries debt in arrears is more expensive, as in the data.

The duration of renegotiation is treated in a highly stylized manner in the benchmark model and is more difficult to generalize than haircuts. If given the choice for when to renegotiate the government will almost immediately choose to do so as there are limited benefits to waiting in the benchmark model.

Benjamin and Wright (2009) build a stopping time model to generate observed renegotiation durations together with haircuts, and their positive correlation. The basic intuition for delay in renegotiations and haircuts in their paper is that it pays to wait to renegotiate until the economy is in a boom because that reduces future default risk. The Benjamin and Wright (2009) model assumes no market access during debt renegotiations and calibrates renegotiation duration to a relatively long 7.4 years (compared to the data, see section (1.1)).

Bai and Zhang (2012) provide a theory for why bond restructuring are completed more quickly than bank loan renegotiations.

1.2.4.2 Output Costs of Default

As established by the early qualitative literature on sovereign debt and default, costs of default are essential, conceptually, to rationalize equilibrium sovereign borrowing. Costs of default in the benchmark model are twofold: Direct output costs and exclusion costs. First, output is exogenously reduced in default, $y^d \leq y$. Second, the country is excluded from borrowing again in the period of the default and re-enters capital markets with some constant probability each period thereafter. Conceptually, only one of these is necessary, but it turns out that quantitatively the precise nature especially of the direct output cost is crucial (see section (1.2.3)) in order to make the bond price sufficiently inelastic and generate an area of the debt state space with positive but finite default probabilities.

It turns out that quantitatively the asymmetric cost assumed by Arellano (2008) is more successful at generating a less elastic bond price than the higher variance of output assumed by Aguiar and Gopinath (2006), and therefore higher equilibrium default frequencies and spreads. In their benchmark model, they still only obtain default rates of 0.08% with a very low discount factor of 0.8, and need to introduce bailouts - risk free loans up to a specified upper threshold which effectively subsidize default - to generate higher default rates with lower impatience. This upper threshold introduces the asymmetry that in the Arellano (2008) specification achieved the flattening of the bond price schedule. Chatterjee and Eyigungor (2012) generalize Arellano (2008)'s default cost further and investigate its numerical effects on the level and volatility of spreads (in the context of a paper with a different primary focus - discussed further in section 1.2.4.3). They show that the asymmetry plays an important role in delivering not just higher, but also more volatile spreads.

The reduced form direct default cost is often rationalized as simple way of capturing disruptions to the domestic economy in the wake of a default, say via the banking sector or through impaired trade relations. Mendoza and Yue (2012) build an extension of the benchmark model that micro-founds this. The channel in their model is via trade in intermediate inputs that is financed by working capital loans and disrupted when the sovereign defaults. As a result, domestic production falls and the model endogenously generates recessions during default crises - and in particular default costs that are higher in booms. The authors broadly match facts on defaults and output co-movement with their endogenous default cost model. In their benchmark parameterization, the model slightly overstates the extent to which defaults go hand in hand with bad GDP outcomes. Defaults happen in recessions roughly 80% of the time compared with 60% in the data, and 20% of defaults are associated with severe recessions (2 standard deviations, that is at least 9.2% below trend), compared with 32% in their model. This is an improvement over the benchmark model: Tomz and Wright (2007) show using the Aguiar and Gopinath (2006) model that with transitory shocks all defaults occur with output below trend, and with permanent shocks no less than 85% do.

Quantitative studies vary dramatically in terms of their calibrations of the time a borrower is excluded from financial markets. Partly this is due to the fact that the relatively limited number of empirical studies that exist differ widely in their estimates (see section 1.1) and often do not measure the model counterpart. In most models exclusion is complete, so the data counterpart should be an estimate that attempts to measure full exclusion from financial markets.

One interesting aspect of default costs is that the theoretical studies rely on vastly different off-equilibrium default costs. Chatterjee and Eyigungor (2012), for example, calibrate their model such that a default at mean output levels costs around 5% of output, with at most 20% if default occurs in the best states of the world and 0% for the lowest endowment realizations. The loss is at least 5% in the worst state of the world in Hatchondo and

Martinez (2009) and reaches 67% for the highest output states. This implies that output is reduced in default to a level lower than the worst realization in good credit standing. Of course this does not mean that the realized default costs in the sense of how far output is below trend are widely off the mark, but it does mean that the slightly different models and different calibrations rely on very different off-equilibrium forces.

A final interesting note on the cost of default is that conceptually, as noted in Perri (2008), it is possible that in incomplete market models like the benchmark model, harsher penalties could be detrimental to welfare. The benefit of default in generating some state contingencies in payouts has to be balanced against the cost of making it too appealing and thus debt unsustainable in equilibrium. In practice, it appears the the market incompleteness is not sufficient to make weaker penalties welfare improving. In numerical applications weak penalties consistently lead to high default rates, low equilibrium debt levels and highly volatile consumption (see for example chapter (3). The flip side of this is that policies like bailouts and debt relief are always a bad idea in terms of long run outcomes because they make default easier and the government that cannot commit gives in to the "temptation". In some sense this reflects the finding that the difference between complete and incomplete markets is relatively small in other environments (e.g. Heathcote and Perri (2002)). To my knowledge there is little existing research in the sovereign default literature on what, if anything, might break this prediction.[11]

1.2.4.3 Asset Market Structure

The benchmark model assumes a very simple asset market structure: The only asset that the government has access to is a claim that pays one unit of consumption next period. Naturally it therefore cannot match facts about the maturity and composition along other dimensions of sovereign debt. Subsequent studies have relaxed various aspects of the asset market structure of the benchmark model which has yielded improvements in the performance of the model, as well as important insights into the role that different aspects of financial markets play for sovereign debt crises.

One modification that has been explored in some detail is to allow for bonds of longer

[11] One exception is hyperbolic discounting - motivated for example by political reasons as in Aguiar and Amador (2011).

maturities, for example in Chatterjee and Eyigungor (2012), Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012) and Bi (2006). These bonds are modeled as either perpetuities with geometrically decaying coupon payments (Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012)) or bonds that mature with a constant probability each period (Chatterjee and Eyigungor (2012)). Bi (2006) models explicitly one and two period bonds. The other frameworks have the advantage of modeling an arbitrary maturity without adding state variables. The papers in this strand of the literature show that longer maturity bonds allow the model to match higher debt levels and spreads as well as more volatile spreads and thus bring it closer to the data.

What changes in terms of the mechanics and economics of the model when debt is long term? Mechanically, the bond price does not just reflect repayment probabilities in the next period, but also in future periods. More borrowing today lowers the bond price both because it increases the risk of default tomorrow as in the benchmark model, and because it creates expectations of higher future borrowing and hence higher future default. Debt dilution is one feature that studies of long term debt have focused on - the government by issuing debt now has an incentive to dilute existing debt and create a capital loss for existing bondholders, that is a transfer of resources to the borrower.

Arellano and Ramanarayanan (2012) study the implications that these price movements have for bonds of short and long maturity. Short bonds have an advantage in that there is no future risk of dilution - loosely speaking the incentive for the government not to repay is higher. Long bonds on the other hand have a hedging benefit - if the long bond price falls in bad times, then the government faces lower payments on existing debt. In terms of which effect is more important, Chatterjee and Eyigungor (2012) show that the government prefers to hold short term debt. In other words, the hedging benefit is not large enough, and it is cheaper to eliminate dilution risk for the government. Adding rollover risk, or in other words an additional benefit to holding long term debt, can switch this welfare result.

There are some measurement caveat to these findings. First, it is not clear to what extent the shortcoming of not being able to match spreads was there to begin with. The early quantitative studies as well as the extensions to long maturity debt typically use JP Morgan's EMBI spreads to calibrate their models. These spreads are based on yields of long term bonds of around 10 years. So given that the model bond is a one period instrument whereas the data measure is a long term interest rate, perhaps at least some of the difference between data and model generated spread can be accounted for by term premia.

In terms of debt levels and the improved ability of the long term debt model to match empirically observed debt stocks, a similar criticism applies to at least some of the papers that model long term debt. Hatchondo and Martinez (2009) measure debt stocks as the sum of future principal and coupon payments, discounted at the risk free rate. This is not what most empirical measures do, certainly not standard ones that these papers use to calibrate their models - they neither discount nor include coupon payments. The model in Chatterjee and Eyigungor (2012) is calibrated consistently because their bond measure excludes coupon payments and they do not discount.

Measuring duration is related to this. In the Chatterjee and Eyigungor (2012) formulation of the asset market structure, duration is a constant parameter. With decaying coupons, the duration of a bond is measured as the weighted sum of future payment dates where the weights are given by the fraction of the bond's value paid on that date. This measure of duration mechanically shortens even in the absence of changes in borrowing when default risk is high and bond prices low. In Arellano and Ramanarayanan (2012) with an explicit portfolio choice, they are able to show that the *share* of short debt, measured at market value, increases when bond prices fall. So despite the fact that short bonds become relatively more expensive (the spread curve flattening or inverting), the share of short term borrowing increases.[12]

There are many other extension of the asset market structure that are being investigated in the literature and an exhaustive list would go beyond the scope of this chapter. But to discuss a few - allowing the government to save has been studied in Bianchi et al. (2012) with the goal of explaining reserve accumulation as a way of avoiding default.

Nominal debt is analyzed in Chapter 2 of this thesis. There are earlier qualitative studies on this - see the literature section of the chapter. The main early conclusion emerging from this study is that nominal debt is welfare improving and allows the government to borrow more cheaply - even though it introduces an additional temptation to renege on

[12] They use WDI data to calibrate the debt levels and the model underpredicts the overall levels. The WDI data is face value data whereas in the model they use market values.

promised payments through implicit default, i.e. inflation, this is dominated by the additional flexibility that nominal debt provides.

There are studies that relax the assumption that households have no access to asset markets. Kim and Zhang (2012) study an externality that arises if borrowing is decentralized and done by households but default centralized and a choice of the sovereign, for example. The interaction of sovereign and private borrowing seems like an interesting area of future research.

1.2.4.4 Joint Crises

One relatively underexplored aspect of sovereign default is that in the data it frequently occurs in combination with other crises - banking crises, currency crises, episodes of high inflation, crises in other countries, political upheaval, sudden stops.

Arellano and Kocherlakota (2008) build a model in which private sector defaults on bank loans force the government to default externally in order to make up the revenue shortfall from lower tax revenues. Liquidation constraints like the one in this model provide one rationale for observing banking crisis prior to, or contemporaneously with, sovereign crises.

Arellano and Bai (2013) investigate spillovers and contagion in sovereign debt crises. In their model default in one country can trigger default in another country if they are linked through a common lender. Default by one country increases default probabilities for two reasons. First, lenders make losses when one country defaults, which increases borrowing cost for other countries and hence, all else equal makes, default more attractive. Second, default becomes less costly because borrowers have more bargaining power in renegotiations with the common lender if there are many of them. One implication of the model is that a country may default purely because its neighbor is defaulting rather than for “fundamental” reasons like low productivity.

A large literature going back to Cole and Kehoe (2000) studies self-fulfilling defaults where if debt is high enough an exogenous shock can trigger a stop in lending by foreigners (“rollover risk”) and thus a default. Their original paper is more stylized than the papers in this quantitative literature, but Chatterjee and Eyigungor (2012) for example add the possibility of self-fulfilling default to explain the long observed debt maturity, as do Bianchi et al. (2012) to explain the holding of reserves.

A number of papers have explored theoretically the link between debt crises and other aspects of fiscal policy and politics more generally. Cuadra et al. (2010) rationalize procyclical government spending in a model of sovereign default. Cuadra and Sapriza (2008) introduce political turnover risk as an additional source of default risk. D'Erasmo (2011) adds unobservable types of governments and applies this to explain delays in debt renegotiations. In Chapter 2 of this thesis I study monetary policy and its interaction with default.

1.3 Conclusion

In this chapter I have reviewed the literature on sovereign debt and default. Recent advances on both the empirical and theoretical side have focused on bringing models and data closer together by taking a more disaggregated, detailed look at sovereign debt crises. The following two chapters fit into this strand of work by analyzing two underexplored aspects of sovereign defaults: Incomplete debt write off, and the denomination of government debt.

Chapter 2

Inflation, Default, and the Denomination of Sovereign Debt

2.1 Introduction

Emerging market governments actively manage the denomination of their sovereign debt. Brazil has a declared target of 30-35% inflation indexed debt, South Africa aims for 70% nominal debt, citing the composition of debt as "one of the major risk concerns".[1] At the same time, these countries experience debt crises with default and high inflation episodes. The denomination of government debt determines how countries handle such crises: When debt is nominal, the government can reduce its debt burden through inflation or outright default, whereas with real debt, the government can only default on the debt. Theoretical studies on emerging markets debt crises, however, have largely ignored the denomination of debt. This paper studies the effect of debt denomination on inflation, default and debt crises when governments lack commitment to future policies. The main finding is that nominal debt provides incentives for governments to induce paths of lower average inflation, and to default less frequently, because this allows governments to get better terms on their sovereign debt. Issuing real debt does not reduce the government's incentives to reduce inflation sufficiently.

[1] Sources: "Optimal Federal Public Debt Composition: Definition of a Long-Term Benchmark" by the Brazilian Treasury, 2011. "Debt Management Report 2011/2012" by the South African Treasury.

I document that, empirically, emerging market countries borrow largely in nominal terms, that is in terms such that domestic inflation affects the debt burden. I construct measures of government debt stocks based on Bloomberg bond level data for 27 countries that comprise over 80% of all emerging market countries in terms of GDP and all major issuers in emerging sovereign bond markets, including Brazil, Mexico, China and India. I find that on average in this sample over the past two decades, 75% of sovereign bond debt is nominal. Real and foreign currency emerging market government debt is the exception rather than the rule. I then show that inflation and default rates vary systematically with the share of debt that is in nominal terms. Emerging market countries with high shares of nominal debt tend to experience lower than average inflation and default rates. Average inflation in nominal debt countries was around 5%, but as high as 20% annually for real debt issuers. The numbers are similar for default rates.

The paper then builds a dynamic monetary model of sovereign debt with endogenous default that can rationalize this pattern. I consider two environments, one with nominal and one with real debt, to analyze the effects that debt denomination has on inflation and default incentives. On the one hand, denomination affects how strong incentives are to inflate today. Inflation is more useful when debt is nominal instead of real because it generates seigniorage revenue and erodes the real value of the debt. It is less useful when debt is real since the value of the debt is fixed. On the other hand, the denomination affects borrowing decisions. The government in the model takes into account that more borrowing today increases incentives for future governments to inflate or default. It does so via bond prices, which reflect expected inflation and default. More borrowing will lower bond prices and hence revenues today to the extent that the borrowing creates expectations of future inflation and default. When debt is nominal, borrowing is more restrictive since bond prices fall when inflation expectations rise. When debt is real, bond prices carry no inflation premia so the government can issue bonds without directly reducing revenues through higher inflation premia. It is thus able to borrow more and may inflate after all for sufficiently high debt levels or default risk. The paper finds that in the long run, this second effect dominates and that an economy with real debt has on average higher inflation and default rates.

The paper focuses on an optimal policy problem without commitment. In the model, a benevolent government decides on inflation, default, and new borrowing to finance a stochastic stream of government expenditures and to service the debt. Households provide labor, consume cash and credit goods and lend to the government. Inflation is costly because it distorts cash relative to credit good consumption due to a cash in advance constraint. Default incurs an exogenous resource cost and temporary exclusion from credit markets. As in any Markov environment without commitment, the government takes as given the optimal policy functions of future governments and internalizes the households equilibrium conditions. Importantly, the price of bonds compensates households for future inflation or default risk.

Borrowing in the model is driven by the government's lack of commitment. Bonds provide a lump sum means of raising revenue because bond issuance does not distort private sector allocations. Ex post, however, outstanding debt creates an incentive for the government to inflate and default in order to relax the budget constraint. Incentives to inflate and default are increasing in the level of debt, which is reflected in bond price functions being decreasing in the level of borrowing. Since the government takes into account the effect of borrowing on bond prices, this therefore limits equilibrium borrowing and debt levels.

Debt denomination in the model affects the extent to which the government uses inflation, default or bonds to finance its expenditures. With nominal debt, it uses inflation more readily since it can both raise seigniorage revenue and devalue the debt. Households holding bonds need to be compensated for expected inflation, which lowers the price of nominal bonds and curbs equilibrium borrowing by the government. Real debt provides less of an incentive for the government to inflate since it only generates seigniorage revenue. Real bond prices in turn only need to compensate households for default risk so the government can rely more heavily on bond finance than seigniorage. It will resort to inflationary finance only if the debt burden that needs to be financed is too high, bond revenues are too low because of default risk or it does not want to default instead.

In a simplified version of the model I characterize the trade off between inflation and default in partial equilibrium. I show that, for a given constant level of debt that is being rolled over each period, the government inflates and defaults relatively more when this debt is real than when it nominal to satisfy its budget constraint and maintain the optimal mix of inflation and default taxes. The intuition is that, when moving from a nominal debt to a real debt economy, higher default incentives hurt bond prices and thus revenue, while lower inflation incentives provide no offsetting boost to revenues via bond prices. On net, the government would be left with too little revenue and must raise additional revenue either via inflation, or borrowing which is associated with more inflation and default, in order to make up the shortfall. This simplified partial equilibrium version can feature large differences in inflation and default across the two economies even for the same level of debt.

I evaluate the quantitative effect of debt denomination on inflation and default in a stochastic general equilibrium version of the model that includes cash and credit consumption goods as well as labor income taxes. The nominal debt economy is calibrated to match the observed inflation and default rates in nominal debt issuing countries, and compared to an otherwise identical real debt economy. I find that in the real debt world the government optimally chooses higher inflation and default rates, as in the data. Quantitatively, the model captures the difference in inflation across the two debt regimes, and generates about two thirds of the difference in default rates.

The dynamics of the model shed light on the role of debt denomination in shaping equilibrium outcomes. For low levels of debt, a real debt regime is better able to contain inflation and default probabilities. This reflects the fact that incentives to inflate today are higher with nominal debt: it generates seigniorage and devalues the debt. For higher levels of debt, however, the picture switches. As the real debt economy enters the region of the state space where default risk is positive but finite, bond revenues fall since bond prices reflect the default risk, and the government begins to raise revenue through seigniorage. For the calibrated default cost that generates observed default frequencies in a nominal debt world, default risk is relatively sensitive to debt levels in the real debt economy beyond a certain level of debt. Inflation rates in the real debt economy match this rapid rise as the country substitutes bond finance for inflationary finance. The properties of the shock process together with the default cost ensure that the government in the real debt economy frequently visits this region of the state space where default risk and hence inflation are higher than in the nominal debt economy.

The model captures the co-movement that is observed in the data between inflation and default - inflation tends to rise in the run up to default episodes. Simulated seigniorage and bond revenues as a percentage of GDP are realistic. In terms of debt levels, the real debt economy features higher debt to GDP ratios than the nominal debt economy, but the differences are modest compared to the differences in inflation and default rates. Default risk is important in generating the large differences in inflation rates that we see in the data. Absent default risk, the government in a real debt world accumulates substantially more debt than its nominal bond counterpart, and inflates at modestly higher rates, driven by the higher debt burden that it needs to finance. Issuing nominal debt is welfare improving, with small but positive lifetime consumption equivalent welfare gains of around 0.12%. These gains are the result of both lower average inflation and default costs, as well as lower volatility of allocations.

The paper highlights the importance of the connection between lack of commitment to monetary and fiscal policy. Real debt in the framework presented here does not remove the incentive to inflate. The paper emphasizes that real debt leads to worse outcomes in terms of countries' ability to manage debt crises. When the government cannot commit to either inflation or default, addressing the commitment problem on the fiscal front by issuing real debt can exacerbate the inflationary commitment problem.

Related Literature

The model is a monetary version of quantitative sovereign default models as in Arellano (2008). It shares with this literature that default is modeled as endogenous and dependent on fundamentals, and that governments lack commitment. I introduce costs of inflation and default in standard ways. Inflation is costly because of a cash in advance constraint on consumption as in Lucas and Stokey (1987) and Svensson (1985). Default is costly because it incurs a cost in terms of resources, akin to output costs used in many sovereign default studies.

The paper differs from existing papers in two key dimensions. First, it focuses on the difference between expropriation through inflation versus outright default as qualitatively different phenomena. Other studies restrict attention to inflation when analyzing the role of debt denomination (for example Diaz-Gimenez et al. (2008)), while the sovereign default literature predominantly assumes real, foreign currency external debt. Second, the paper distinguishes between the cost of inflation and debt denomination. In particular, even when bonds are real there is still an incentive to inflate in my framework. This corresponds to an economy where the government issues indexed debt but still has control over its own currency. Issuing real debt is not equated to dollarization or joining a monetary union.

Two papers that are closely related to mine are Martin (2009) and Diaz-Gimenez et al. (2008). The former studies the determination of nominal public debt levels in a setting without commitment. His application focuses on war finance in advanced economies. He does not consider real debt or default. The latter analyzes monetary policy under different debt denominations in an economy as in Nicolini (1998). The authors find that the welfare effect of nominal versus indexed debt are in general ambiguous and show how they depend on parameters, specifically the intertemporal elasticity of substitution. Both papers model money demand as arising from a cash in advance constraint on consumption, and both address lack of commitment on the part of the government, as does this paper. Neither considers the interaction of monetary policy with default which is a key focus here.

Domestic or nominal debt and self-fulfilling sovereign debt crises are the topic of a number of recent papers, including Aguiar et al. (2013), Lorenzoni and Werning (2013), Da-Rocha et al. (2013) and Araujo et al. (2013). They focus on self-fulfilling, expectations driven debt crises as in Calvo (1988) and Cole and Kehoe (2000) whereas I consider default driven by weak fundamentals. Another important difference is that these papers compare economies without any role for domestic monetary policy - a currency union of dollarization - with economies with nominal debt and monetary policy. I focus on an environment where money always plays a role and the country has control over its monetary policy; the issue of debt denomination is distinct from the choices of whether to relinquish control of the printing press. Other related papers that study sovereign default and foreign currency debt are Arellano and Heathcote (2010) in a model of dollarization and limited enforcement, and Gumus (2013) in a two-sector model and bonds that are either denominated in terms of tradables or nontradables. The latter paper finds in a result resembling the one in this paper that debt whose repayment value fluctuates with the state of the world (nontradable debt that is not subject to exchange rate fluctuations) yields better outcomes in terms of default rates and welfare.

Less closely related in terms of modeling approach, but related in terms of topic are numerous papers that address the benefits and costs of indexed versus nominal debt, including Missale (1997), Bohn (1990)who discusses the benefits of nominal debt in terms of making returns state contingent, Barro (1997) and Alfaro and Kanczuk (2010) who argue in favor of indexed debt (without considering explicit default as an option for the government. There is a large literature that explores optimal taxation, including through inflation and default, under full commitment, including in Chari and Kehoe (1999). The computation of Markov equilibria in macroeconomic dynamic models was first developed by Klein et al. (2008).

On the empirical side, there are a number of papers that study the currency composition of sovereign debt, as well as the connection between domestic default and inflation. Reinhart and Rogoff (2011a) focus on domestic debt and default over a long period of time. They document that inflation and default episodes tend to occur together, that domestic default, even though less prevalent than external default, does occur with some frequency. Claessens et al. (2007) explore empirically the determinants of local currency debt as well as debt shares using non-publicly available data from the BIS. Cowan et al. (2006)construct a detailed debt database with a focus on Latin American countries, Guscina and Jeanne (2006) analyze debt composition for a subset of the countries I consider.

2.2 Nominal Government Debt in the Data

In this section I will first document that the majority of emerging market government bond debt is denominated in nominal, local currencies, and second show that in these economies high nominal debt shares tend to be associated with low inflation and default rates.

2.2.1 Bond Data

I construct local and foreign currency government bond debt estimates for a range of emerging market countries using Bloomberg bond-level data. The data set contains all sovereign bond issues that were outstanding at some point between January 1, 1990 and December 31, 2012. For each issue, it includes information on the face value, the currency, the coupon structure, the maturity and issue date.

The countries I consider are a broad set of 27 emerging market countries, as classified

Table 2.1: Emerging bond market characteristics in 2012

	All	LatAm	Asia	Europe	Africa
Local currency	85.44	72.45	96.28	77.11	91.55
Domestic market	84.60	72.42	96.04	72.45	94.46
Zero coupon	18.05	28.05	11.83	12.38	35.63
Fixed rate	64.36	47.40	71.93	71.67	64.27
Pay at maturity	93.78	82.22	99.72	94.91	100.00
Inflation indexed	4.93	10.51	0.47	7.10	5.27
Defaulted/ Restructured	1.31	4.54	0.01	0.02	0.00

Percent. Bonds are valued using discounted US dollar face values. Percentages for regions are out of region totals.

by the IMF in 2012. They account for just over 80% of total emerging market GDP on average over the last two decades and and include the top 20 largest emerging market bond issuers.[2] The countries are not restricted to a particular continent and include Eastern European, Latin American, Asian and African economies. I include Korea which is classified as advanced according to the IMF. The set contains both defaulters and non-defaulters.[3]

To calculate debt stocks for each of these countries, I restrict attention to bonds with a simple payout structure. I only include bonds whose face value is paid back at maturity ("bullet" bonds rather than callable, sinkable or hybrid bonds with equity-like structure), and that have deterministic coupon payments (zero, fixed rate or step rate coupons). I also exclude defaulted bonds.

This is not restrictive as the vast majority of bonds do in fact have such a simple payout structure, as shown in Table (2.1). The table breaks the the data set down by bond characteristics at the end of 2012.[4] It shows that across regions the majority of debt

[2] The top 5 as of 2012 are Brazil, China, India, Korea and Mexico.

[3] See a full list in the appendix. For some countries the time series starts later than 1990 because there are no bonds in the data set early in the sample. See precise start dates in the appendix.

[4] These statistics are based on bonds valued as the discounted sum of their face value. I use this simple measure because it is easy to construct for all bonds, including the ones that I do not include in the calculation of the debt stock later on. It abstracts from coupon payments and assume that bonds that can be redeemed before maturity are never expected to be. Note that many measures of government debt

is in local currencies, issued on domestic markets, with either fixed or zero coupons and payable at maturity. Less than 5% of al debt is indexed, with the highest share in Brazil. Defaulted (and not restructured) bonds are a small fraction of the total.

2.2.2 Debt Stocks

I construct debt stock estimates as the sum of discounted future payments over all outstanding bonds. I first calculate the value $b_{n,t}$ of each bond issue n at time t, as

$$b_{n,t} = \sum_{s=0}^{S} \frac{C_{n,s} + P_{n,s}}{(1+r)^s}$$

where $C_{n,s}$ and $P_{n,s}$ are coupon and principal payments, respectively, at time $t+s$, and S is the number of periods to maturity. I discount future payments at the constant annual interest rate $r = 4\%$.

If the bond is denominated in a currency other than US dollars, I convert the future payment stream using the exchange rate at time t. This means that the value of a bond can change over its lifetime not just because coupon payments are made, but also because of revaluation due to exchange rate movements. The principal and coupon values of indexed bonds are adjusted by multiplying by the so-called index factor - the change in the CPI between the issuance date and time t. The CPI is available monthly for most countries so I interpolate between dates.

The stock of bond debt B_t is then calculated as the sum over all bonds outstanding at time t:

$$\begin{aligned} B_t &= \sum_{n=1}^{N} b_{n,t} \\ &= \sum_{n=1}^{N} \sum_{s=0}^{\bar{S}} \frac{C_{n,s} + P_{n,s}}{(1+r)^s} \end{aligned}$$

where N are the number of bond issues outstanding at time t, and $\bar{S}$ is the maximum number of periods left to maturity for all bonds. The frequency I choose is daily.

stocks do in fact not include any coupon payments, including the external debt estimates by the World Bank WDI.

Figure 2.1: Aggregate bond debt as a fraction of aggregate GDP

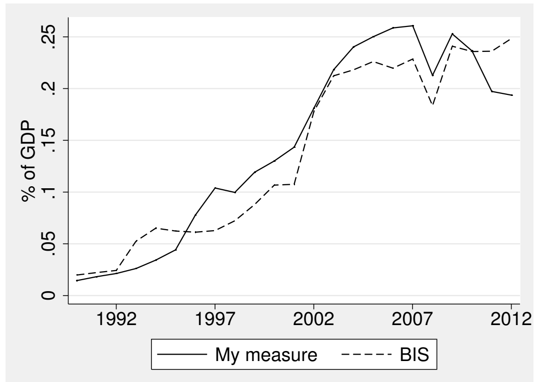


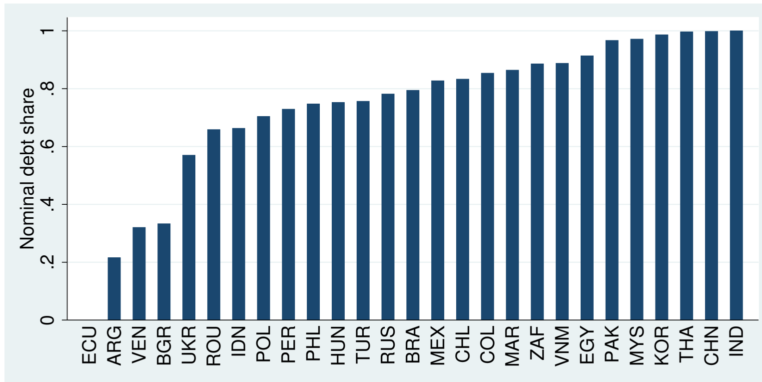
Figure (2.1) plots the aggregate debt stock for all countries in my sample over time as a fraction of aggregate GDP. This amounted to around 30% in 2012. In the Appendix, Figure (2.1) shows that total bond debt issued in my sample of emerging market countries was worth around 4 trillion current US$ in 2012 - much smaller than in advanced economies but growing over time.

In order to assess the accuracy of my estimates I compare them to aggregate data available from the BIS debt securities database. This database records quarterly general government debt securities outstanding for both domestic and international bonds from 1993Q3. The series for domestic securities is less complete - there are data for 16 of my countries and the series start late for some of them.

The sum of domestic and international debt from the BIS is plotted in the Figures alongside my estimates. The two series co-move closely and the levels match up well. The uptick and subsequent drop around 1998/1999 in my series reflects the Russian default and increases in mainly local currency debt - one reason why this is not captured by the BIS is that the series for domestic debt does not start until 2005 for Russia.

The hump-shaped path of debt-to-GDP ratios in the early 2000s is driven by Latin American countries. All of them except Ecuador increased their debt stocks rapidly at the beginning of the millennium from very low levels, and experienced a sharp drop at the onset of the financial crisis, relative to GDP. Most countries saw their bond debt-to-GDP

Figure 2.2: Nominal debt shares in 2012



ratios fall around the time of the financial crisis in 2009.[5]

It is worth noting that the clear upward trend in bond debt levels is not exclusively driven by the move towards bond markets and away from bank debt finance. Gross general government debt according to the IMF WEO shows a similar pattern as the bond debt measure shown here. On average over time, bond debt according to both my and the BIS measure constitutes around half of gross general government debt (which includes in addition liabilities such as social security and pensions), and this fraction actually falls slightly over time.

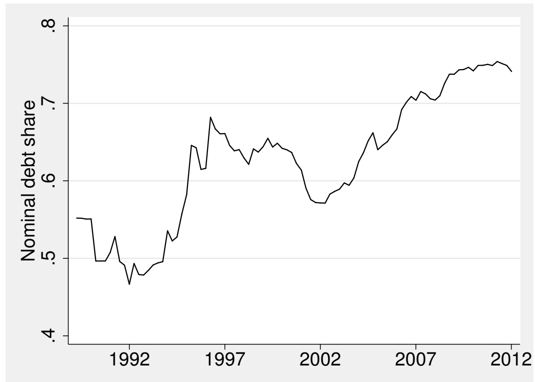
2.2.3 Nominal Debt Shares

I now turn to the question of how much emerging markets rely on foreign currency compared to local currency debt. I construct measures of the debt stock as above, separately for local currency and foreign currency denominated bonds. Figure (2.2) plots the resulting local currency debt share for each country at the end of 2012.

The Figure shows that the majority of emerging market debt is in local currencies. The unweighted average across countries in 2012 is 75%, a GDP-weighted average is slightly higher. All countries except four have local currency shares above 40%. Ecuador has a share of zero since it is dollarized, Argentina, Venezuela and Bulgaria are the other three

[5] The gap towards the end of the sample between my and the BIS measure is driven by my estimates of Brazil's debt stock being too low especially for the last five years of the sample. I am working on improving this.

Figure 2.3: Nominal debt share over time, simple average across countries



countries that rely mostly on foreign currency denominated debt. At the other end of the spectrum, India has not issued a single foreign currency bond (in 2012 or the entire sample). Thailand, Korea, Malaysia, China and Pakistan all have local currency shares in excess of 95%.

Over time the share of debt issued in local currencies has increased substantially. This is shown in Figure (2.3) where we can see that the average local currency share stood at around 50% in 1990. The GDP-weighted average is lower at around 20% as the two largest countries in the sample in the early 1990s (in terms of constant US$ GDP), Mexico and Brazil, issued exclusively foreign currency debt.

Latin American countries generally have seen the largest shifts over time from foreign to local currency. In Asia local currency was more prevalent even early in the sample. Some countries have consistently issued mostly local currency debt: India, Korea, Malaysia, Pakistan, South Africa and Morocco.

I cross-checked my results against data from the IMF's Public Sector Debt (PSD) database as far as possible. Data on the currency composition of public debt is available in the PSD for only 7 of my countries and a short sample. For Brazil, Indonesia, Peru, the Philippines and Poland the series start in 2010, for Mexico in 2005 and only for Hungary do they go back to 1999. Where available, the local currency shares from this data source match mine well.

2.2.4 Nominal Debt, Inflation, and Default Rates

Next I document the correlation between high nominal debt shares and low inflation and default rates.

High inflation and default were widespread in the countries I am considering for at least part of the sample. Only 9 countries never defaulted between 1990 and 2012, with default defined by Standard & Poor's sovereign ratings: China, Colombia, Egypt, Hungary, India, Korea, Malaysia and Thailand and Turkey. Turkey's sovereign crisis in 2001 crisis is not classified as a default by S&P.

Inflation was very high in many of the countries in the sample: 11 of them, all in Latin America or Europe, recorded annual CPI inflation of at least 100% at some point.[6] Turkey, Venezuela, Ecuador and Romania had the highest median inflation over the whole period, all over 20%. Median inflation was the lowest in Morocco, Malaysia, Thailand and Korea with under 4%. Peru's median is actually also just below 4%, mainly because its hyperinflation ended just after the beginning of my sample.

I pool the data across countries and time, and split the observations into deciles by nominal debt share. I then compute average inflation and default rates for each decile. For inflation I exclude episodes of inflation that exceed 100% annually. Including these would make the results only stronger since the hyperinflation episodes are concentrated in the lower deciles of local currency shares.

The left panel of Figure (2.4) shows the resulting graph for inflation. There is a clear negative relationship between nominal debt shares and inflation. For the observations with the highest 10% of nominal debt shares, inflation was on average 8% annually. The lowest 10% in terms of nominal debt shares saw prices increase at a rate of 25%. The right panel of Figure (2.4) shows the analogous graph for default rates.[7] As in the case of inflation, nominal debt debt shares and default rates are negatively correlated, with default rates for the highest nominal debt decile of around 7% compared with 27% for the set with the lowest local share of nominal debt.[8]

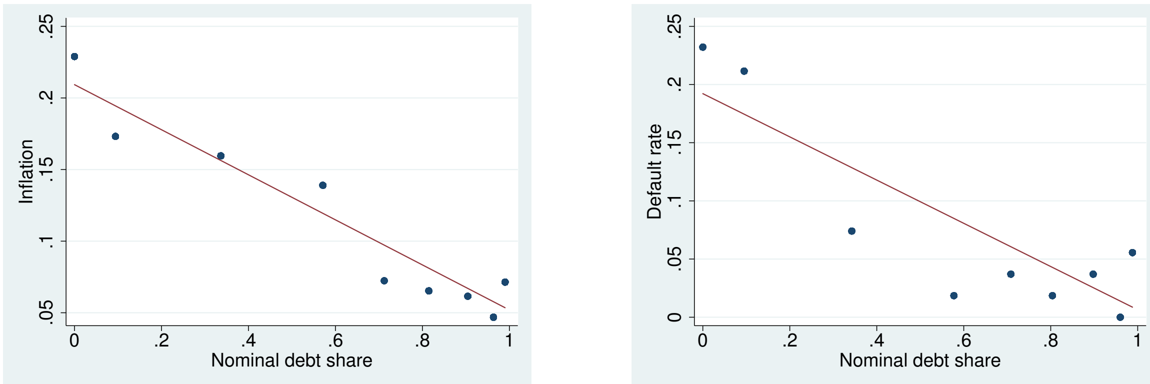
---

[6] Argentina, Brazil, Bulgaria, Ecuador, Peru, Poland, Romania, Russia, Turkey, Venezuela and the Ukraine.

[7] Default rates are defined as the total number of default states relative to the total number of states per decile. A default state is a country/year pair in which the country was in default - say Argentina in 2003. All states are all country/year pairs that fall in the given decile.

[8] See the Appendix for unpooled, raw scatter plots of inflation against nominal debt shares. Raw scatter

Figure 2.4: Inflation rates and default against deciles of nominal debt shares



Pooled data. X-axis: Nominal debt share deciles, 642 obs per decile.

Y-axis left panel: Average inflation per decile. Y-axis right panel: #default obs / #obs per decile.

The correlations from both previous graphs are significant and robust to several modifications, including using median instead of mean inflation, period averages or period end inflation rates, and computing the deciles using local currency shares weighted by GDP. In addition, the Appendix contains the results from pooled and panel regressions of nominal debt shares on inflation and multinomial logistic regressions on default that control for a variety of factors. These controls include GDP, GDP per capita, debt levels, reserves, an index of democratic institutions, exchange rate regimes, independence of monetary authorities and inflation targeting dummies. Even after controlling for these factors, nominal debt shares are found to have a significant effect on inflation and the probability of default.

2.3 Model

In this section I will present a dynamic monetary model of sovereign borrowing without commitment, and use it to study the interaction between inflation, default and the denomination of debt. I will first discuss an economy where the government issues only nominal bonds, and then show what changes when instead it sells claims that are indexed to the price level.

plot of default are not informative obviously since it is a binary variable.

Environment Time is discrete and infinite. The economy is populated by a representative agent with preferences over consumption and leisure, and a benevolent government that faces stochastic exogenous public consumption expenditures. Asset markets are incomplete with only money and one-period noncontingent bonds. The government has the monopoly over printing money and issuing bonds, and it lacks commitment to inflation, default and borrowing policies. When bonds are nominal it can devalue its debt through inflation, whereas real bonds are indexed to the price level. It can default on bonds of either denomination at any point. Inflation is costly because agents are subject to a cash-in-advance constraint on a subset of their consumption purchases, and default incurs a resource cost and temporary exclusion from credit markets.[9]

Households The representative agent maximizes the expected discounted lifetime utility from consumption of cash and credit goods, and leisure. He enters every period with nominal money balances $\bar{m}_t$ and, if the government is not in default, sovereign bonds $\bar{B}_t$. Households split into a shopper who makes consumption purchases and a producer who transforms labor into output with a linear technology. He receives labor income net of taxes. I assume that the shopper must purchase the cash good c_{1t} using money balances that they hold at the start of the period

$$\bar{p}_t c_{1t} \leq \bar{m}_t \tag{2.1}$$

The can use bond holdings and labor income to finance credit good consumption. My timing assumption follows Svensson (1985) and Lucas and Stokey (1987) and implies that unexpected inflation is costly since agents are unable to adjust their balances after uncertainty is resolved. If the cash in advance constraint binds, agents spend all their money on cash consumption goods ((2.1) holds with equality). Higher than expected inflation then reduces the real purchasing power of the money that agents hold to buy c_{1t}.

Securities markets opens and households decide how to allocate receipts from cash god sales and invoices from credit good sales between money $\bar{m}_{t+1}$ and bonds $\bar{B}_{t+1}$ to carry into the next period. In equilibrium prices will adjust such that money and bond markets clear and households hold exactly as much money and bonds as the government issues.

[9] I abstract from endogenous renegotiation and partial default in this paper.

The household therefore faces the budget constraint

$$\bar{p}_t c_{1t} + \bar{p}_t c_{2t} + \bar{m}_{t+1} + q_{nt}\bar{B}_{t+1} = (1-\tau)\bar{p}_t n_t + \bar{m}_t + \bar{B}_t$$

if the government is not in default and

$$\bar{p}_t c_{1t} + \bar{p}_t c_{2t} + \bar{m}_{t+1} = (1-\tau)\bar{p}_t n_t + \bar{m}_t$$

if it has defaulted. The labor tax rate τ is fixed exogenously. I introduce it since labor income taxes are an important source of government revenue empirically, but their determination is not the focus of this paper.

Following Cooley and Hansen (1991), in order to make the problem stationary I divide all nominal variables by the aggregate money supply, that is $x \equiv \frac{\bar{x}}{\bar{M}}$, and define the money growth rate as $\mu_t \equiv \frac{\bar{M}_{t+1}}{\bar{M}_t} - 1$. Note that $\frac{\bar{x}_{t+1}}{\bar{M}_t} = \frac{\bar{x}_{t+1}}{\bar{M}_{t+1}}\frac{\bar{M}_{t+1}}{\bar{M}_t} = \frac{\bar{x}_{t+1}}{\bar{M}_{t+1}}(1+\mu_t)$. With this normalization, the households solves the problem

$$\max_{\{c_{1t}, c_{2t}, n_t, m_{t+1}, B_{t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{1t}, c_{2t}, 1-n_t)$$

subject to

$$p_t c_{1t} + p_t c_{2t} + (1+\mu_t)(m_{t+1} + q_{nt}B_{t+1}) = (1-\tau)p_t n_t + m_t + B_t$$

if the government is not in default,

$$p_t c_{1t} + p_t c_{2t} + (1+\mu_t)m_{t+1} = (1-\tau)p_t n_t + m_t$$

if it is in default, and the cash in advance constraint

$$p_t c_{1t} \leq m_t$$

as well as a nonnegativity constraint on money balances. I do not impose the constraint that the agent cannot be in debt ($\bar{B}_t < 0$) but in a distortionary equilibrium that features positive inflation and default rates they will lend to the government rather than the other way around. The utility function satisfies the standard properties.

Competitive Equilibrium and Asset Prices We can use the first order conditions of the household problem to characterize the competitive equilibrium in this economy. The cash-in-advance constraint implies

$$u_{1t} - u_{2t} \geq 0 \tag{2.2}$$

and as already mentioned in a competitive equilibrium where the cash in advance constraint binds such that $u_{1t} - u_{2t} > 0$, we have

$$c_{1t} = \frac{m_t}{p_t}$$

Labor is pinned down by

$$u_{lt} = (1 - \tau) u_{2t}$$

We can derive expressions for asset prices from the household's problem. The first order conditions to his problem give rise to an equation for the money growth rate

$$\mu_t = \beta \mathbb{E}_t \left[\frac{u_{1,t+1}}{u_{2,t}} \frac{p_t}{p_{t+1}} \right] - 1 \tag{2.3}$$

Define consumer price inflation as $\pi_{t+1} \equiv \frac{p_{t+1}}{p_t} - 1$ and note that consumer price inflation and the money growth rate are related through

$$1 + \bar{\pi}_{t+1} = (1 + \pi_{t+1})(1 + \mu_t)$$

so we can alternatively express equation (2.3) as an equation for the price of money

$$1 = \beta \mathbb{E}_t \left[\frac{u_{1,t+1}}{u_{2,t}} \frac{1}{1 + \bar{\pi}_{t+1}} \right] \tag{2.4}$$

From equations (2.3), (2.4) and (2.2) we see that in a nonstochastic steady state monetary policy follows the Friedman rule - negative money growth and inflation at the rate of time preference - only if the cash in advance constraint does not bind. Outside the steady state, higher expected future inflation implies higher expected marginal utility of the cash good: If inflation is high, agents are cash-strapped and value cash good consumption relatively more.

Denote the states of the world in which the government chooses to default tomorrow by δ_{t+1}. The household problem implies the following expression for the bond price:

$$q_{nt} = \beta \mathbb{E}_t \left[\frac{u_{2,t+1}}{u_{2,t}} \frac{1}{1+\bar{\pi}_{t+1}} (1-\delta_{t+1}) \right] \tag{2.5}$$

The price of nominal bonds reflects default, inflation and risk premia. The higher the risk of default δ_{t+1} and future inflation $\bar{\pi}_{t+1}$, the lower the price at which the bond sells today - agents demand a higher return to be compensated for the risk. The ratio of marginal utilities $\frac{u_{2,t+1}}{u_{2t}}$ reflects movements in the risk free rate. If this ratio is high, agents want to shift consumption from today to tomorrow by saving in bonds, driving up their price.

Government The government is benevolent and wants to finance exogenous public expenditures in the least distortionary way. It can print money, issue one-period noncontingent bonds, raise taxes, and default. It makes its decisions before the goods and securities markets for households open but after the shock has been realized. The government budget constraint if it repays is given by

$$\bar{M}_t + \bar{B}_t + \bar{p}_t g_t = \tau \bar{p}_t n_t + \bar{M}_{t+1} + q_{nt} \bar{B}_{t+1}$$

whereas in default they need to finance all of their expenditures with labor tax revenues and by printing money

$$\bar{M}_t + \bar{p}_t g_t^d = \tau \bar{p}_t n_t + \bar{M}_{t+1}$$

with $g^d \geq g$.[10] As in the household case, I normalize to get

$$1 + B_t + p_t g_t = \tau p_t n_t + (1+\mu_t)(1 + q_{nt} B_{t+1})$$

and

$$1 + p_t g_t^d = \tau p_t n_t + (1+\mu_t)$$

[10] Whether it is households or the government paying the cost is not crucial.

in repayment and default respectively.

I assume that the government is unable to commit to its policies and analyze Markov perfect equilibria of the economy throughout the paper. Thus I assume that the government, when making its decisions, can only condition them on current fundamentals - debt B_t and the shock z_t - and take as given the policies implemented by future governments as well as the competitive equilibrium. In particular, it is unable to take into account that its policies today affected yesterday's outcomes. To make this concrete, the government would like to be able to promise to only inflate today and never again, but it cannot credibly commit to doing that. The government tomorrow only considers current and future tradeoffs and ignores promises made yesterday. The government today therefore knows and must take into account that inflating today will make it more expensive for the government tomorrow to borrow. Mechanically this can be seen from expressions (2.4) and (2.5), which enter the government budget constraint today but are functions of future inflation and default policies.

Real Debt In an economy where bonds are real instead of nominal, their value is fixed in units of the consumption good. Denote a claim to one unit of consumption by $b = \frac{\bar{B}}{\bar{p}}$ and its price by q_r. Then the household faces the following budget constraint if the government does not default

$$\bar{p}_t c_{1t} + \bar{p}_t c_{2t} + \bar{m}_{t+1} + \bar{p}_t q_{rt} b_{t+1} = (1-\tau)\bar{p}_t n_t + \bar{m}_t + \bar{p}_t b_t$$

and the government budget constraint is

$$\bar{M}_t + \bar{p}_t \bar{b}_t + \bar{p}_t g_t = \tau \bar{p}_t n_t + \bar{M}_{t+1} + \bar{p} q_{rt} b_{t+1}$$

There are two main difference between issuing real compared to nominal debt. The first is that the value of outstanding liabilities is fixed in terms of consumption units for real debt. This can be seen by comparing the budget constraints of households and government in the two economies. An increase in the price level does not devalue the debt - the government cannot inflate it away.

The other difference arises from bond prices. We can derive bond prices in the same way as for the nominal debt economy to obtain

$$q_{rt} = \beta \mathbb{E}_t \left[\frac{u_{2,t+1}}{u_{2,t}} (1 - \delta_{t+1}) \right] \tag{2.6}$$

Compare this with (2.5): Because real debt cannot be inflated away, it does not carry an inflation premium and future inflation does not lower revenue from selling bonds today.

It is useful to note that bond prices can alternatively be written as follows, using the expression for the money growth rate (2.3) and inflation (2.4) which are the same in either economy:

$$q_{nt} = \mathbb{E}_t \left[\frac{u_{2,t+1}}{u_{1,t+1}} (1 - \delta_{t+1}) \right] \tag{2.7}$$

$$q_{rt} = \mathbb{E}_t \left[\frac{u_{2,t+1}}{u_{1,t+1}} (1 + \bar{\pi}_{t+1})(1 - \delta_{t+1}) \right] \tag{2.8}$$

This way of writing bond prices highlights the link between money and bonds as alternative assets in the model. For both real and nominal debt we see that the more the cash in advance constraint is expected to bind, that is the lower $\frac{u_{2,t+1}}{u_{1,t+1}}$, the lower bond prices are today. The reason is that money is in higher demand. Households would prefer to hold more money to avoid being short of cash in the next period.

In addition we can see that real bond prices are *higher* the higher inflation is tomorrow. This may seem counterintuitive, but is in fact also related to money and bonds being alternative assets. When inflation is expected to be high, real bonds are a more appealing investment than money since households know it cannot be devalued. This drives up bond prices relative to the price of money.

It is important to remember then that even if debt is real, the bond price is not independent of inflation because households are the holders of both money and bonds in the economy. The net effect of higher inflation on real bond prices is ambiguous: On the one hand, agents want to hold more money to avoid the cash in advance constraint binding which drives bond prices down, on the other, inflation makes real bonds the more attractive investment which drives their price up. In a nominal debt economy, bond prices unambiguously fall with higher inflation.

I will discuss the implications of this in the next sections after defining equilibria of both economies.

2.3.1 Recursive Equilibrium

I will state the problem recursively in order to define an equilibrium. The state of the nominal debt economy is B, the bond to money ratio, and the shock to government expenditure z. Assume $B \in \mathbb{B} \subset \mathbb{R}_+$ and $z \in \mathbb{Z} \subset \mathbb{R}$. In an equilibrium the government maximizes the representative household's utility subject to the government's budget constraint and the competitive equilibrium conditions. I am going to let B', p and d be the government's choice variables. The cash in advance constraint links prices to cash consumption, and the first order conditions pin down equilibrium money growth residually.[11] The commitment problem, as discussed above, means that borrowing today affects future price and default policies and that the government recognizes this, that is $p' = P(B', z')$ and $d' = D(B', z')$. Denote the exogenous probability of re-entering capital markets after a default by η.

Nominal Debt The government's option value of default is then given by

$$V(B, z) = \max \left\{ V^r(B, z), dV^d(z) \right\} \tag{2.9}$$

where the value of repayment is

$$V^r(B, z) \quad = \quad \max_{p, B'} u\left(c_1, c_2, 1 - n\right) + \beta \int_{z'} V(B', z') dFz' \tag{2.10}$$

[11] Alternatively and equivalently, think about the economy starting the period with a fixed money stock. Then the price level is pinned down by the cash in advance constraint during the goods market, and the difference between start and end of period money stocks pin down the money growth rate. The government decides on the money stock next period which shapes expectations for the price level tomorrow (see equation 2.4), and similarly for borrowing levels which affect default expectations.

subject to

$$\begin{aligned}
1+B+pg(z) &= \tau pn + \left[1+\mu(p,B',z)\right]\left[1+q_n(p,B',z)B'\right] && (2.11)\\
c_1+c_2+g(z) &= n && (2.12)\\
c_1 &= \frac{1}{p} && (2.13)\\
u_c-u_l &\geq 0 && (2.14)\\
u_l &= (1-\tau)u_2 && (2.15)\\
\mu(p,B',z) &= \beta\int_{z'}\left[\frac{u_1'(P^r(B',z'),z')}{u_2(p,z)}\frac{p}{P^r(B',z')}\right]dF(z',z)-1 && (2.16)\\
q_n(p,B',z) &= \beta\int_{z'}\left[\frac{u_2'(P^r(B',z'),z')}{u_2(p,z)}\frac{p}{P^r(B',z')}(1-D(B',z'))\right]dF(z',z) && (2.17)
\end{aligned}$$

that is, the budget constraint, the resource constraint, the cash in advance constraint, the intratemporal competitive equilibrium condition pinning down labor, the cash in advance inequality condition from the household problem, and expressions for the money growth rate and bond prices.

In default the value is

$$V^d(z) = \max_p u\left(c_1,c_2,1-n\right) + \beta\int_{z'}\left[\eta V(0,z') + (1-\eta)V^d(z')\right]dFz' \qquad (2.18)$$

subject to the analogous conditions

$$\begin{aligned}
1+pg^d(z) &= \tau pn + [1+\mu(p,0,z)] && (2.19)\\
c_1+c_2+g^d(z) &= n && (2.20)\\
c_1 &= \frac{1}{p} && (2.21)\\
u_l &= (1-\tau)u_2 && (2.22)\\
u_c-u_l &\geq 0 && (2.23)\\
\mu(p,B',z) &= \beta\int_{z'}\left[\frac{u_1'(P^d(z'),z')}{u_2(p,z)}\frac{p}{P^d(z')}\right]dF(z',z)-1 && (2.24)
\end{aligned}$$

with $g^d = g + h(g), h(g) \geq 0$. Note that both in repayment and default money growth rates depend on future prices, but the relevant price functions are different - in repayment, future price levels depend on the level of borrowing. In default - as I will show below - they reflect the probability of re-entering capital markets with zero debt. We can define an equilibrium price function $P(B,z)$ as

$$P(B,z) = \mathbb{1}\left(V^r(B,z) \geq V^d(z)\right) P^r(B,z) + \mathbb{1}\left(V^r(B,z) < V^d(z)\right) P^d(z) \tag{2.25}$$

and for default

$$D(B,z) = \mathbb{1}\left(V^r(B,z) < V^d(z)\right) \tag{2.26}$$

Definition 1. *Let* $V : \mathbb{B} \times \mathbb{Z} \to \mathbb{R}, P : \mathbb{B} \times \mathbb{Z} \to \mathbb{R}_{++}, H : \mathbb{B} \times \mathbb{Z} \to \mathbb{B}$ *and* $D : \mathbb{B} \times \mathbb{Z} \to \{0,1\}$*. A Markov perfect equilibrium of the nominal debt economy are functions* V, P, D, H *as well as* $c_1(B,z), c_2(B,z), n(B,z)$ *and prices* $\mu(P,H,z)$ *and* $q_n(P,H,z)$ *that solve the government's problem (2.9) through (2.26) and where* $B' = H(B)$*.*

Real Debt Analogously to the nominal debt economy, the recursive problem of the government in a real debt economy is the following: Its option value to default is given by

$$V(b,z) = \max\left\{V^r(b,z), dV^d(z)\right\} \tag{2.27}$$

where the value of repayment is

$$V^r(b,z) = \max_{p,b'} u\left(c_1, c_2, 1-n\right) + \beta \int_{z'} V(b',z') dFz' \tag{2.28}$$

subject to

$$
\begin{aligned}
1 + pb + pg(z) &= \tau pn + \left[1 + \mu(p, b', z)\right] + q_r(p, B', z)pb' && (2.29)\\
c_1 + c_2 + g(z) &= n && (2.30)\\
c_1 &= \frac{1}{p} && (2.31)\\
u_l &= (1-\tau)u_2 && (2.32)\\
u_1 - u_2 &\geq 0 && (2.33)\\
\mu(p, b', z) &= \beta \int_{z'} \left[\frac{u_1'(P^r(b', z'), z')}{u_2(p, z)} \frac{p}{P^r(b', z')}\right] dF(z', z) - 1 && (2.34)\\
q_r(p, b', z) &= \beta \int_{z'} \left[\frac{u_2'(P^r(b', z'), z')}{u_2(p, z)} (1 - D(b', z'))\right] dF(z', z) && (2.35)
\end{aligned}
$$

and the value of default is

$$
V^d(z) = \max_p u\left(c_1, c_2, 1-n\right) + \beta \int_{z'} \left[\eta V(0, z') + (1-\eta)V^d(z')\right] dFz' \tag{2.36}
$$

subject to

$$
\begin{aligned}
1 + pg^d(z) &= \tau pn + \left[1 + \mu(p, B', z)\right] && (2.37)\\
c_1 + c_2 + g^d(z) &= n && (2.38)\\
c_1 &= \frac{1}{p} && (2.39)\\
u_l &= (1-\tau)u_2 && (2.40)\\
u_1 - u_2 &\geq 0 && (2.41)\\
\mu(p, B', z) &= \beta \int_{z'} \left[\frac{u_1'(P^d(z'), z')}{u_2(p, z)} \frac{p}{P^d(, z')}\right] dF(z', z) - 1 && (2.42)
\end{aligned}
$$

with $g^d = g + h(g), h(g) \geq 0$, and P and D satisfy

$$
P(B, z) = \mathbb{1}\left(V^r(B, z) \geq V^d(z)\right) P^r(B, z) + \mathbb{1}\left(V^r(B, z) < V^d(z)\right) P^d(z) \tag{2.43}
$$

$$
D(B, z) = \mathbb{1}\left(V^r(B, z) < V^d(z)\right) \tag{2.44}
$$

Definition 2. *Let* $V : \mathbb{B}\times\mathbb{Z} \to \mathbb{R}, P : \mathbb{B}\times\mathbb{Z} \to \mathbb{R}_{++}, H : \mathbb{B}\times\mathbb{Z} \to \mathbb{B}$ *and* $D : \mathbb{B}\times\mathbb{Z} \to \{0, 1\}$*. A Markov perfect equilibrium of the real debt economy are functions* V, P, D, H *as well as* $c_1(B, z), c_2(B, z), n(B, z)$ *and prices* $\mu(P, H, z)$ *and* $q_n(P, H, z)$ *that solve the government's problem (2.27) through (2.44) and where* $b' = H(b)$*.*

Note that we can use the competitive equilibrium conditions for prices as well as the resource constraint to express the government's problem purely in terms of the state, choice for borrowing as well as current and future prices and default decisions (see the Appendix).

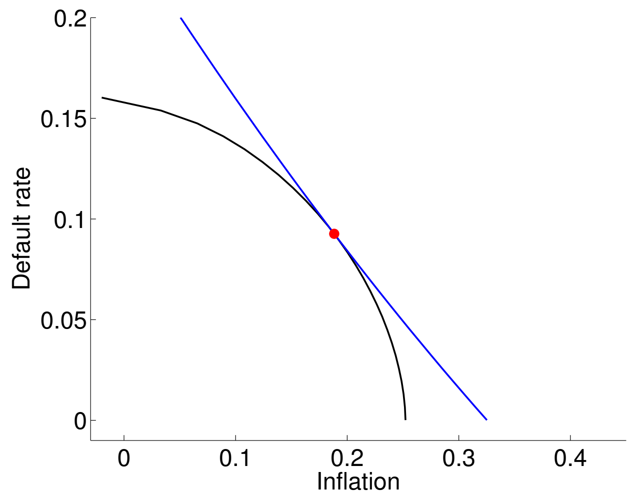
2.4 Inflation, Default and Borrowing in Equilibrium

In this section I analyze the channels through which the denomination of bonds affects the equilibrium of this model. We will see that the government in general prefers to spread the costs of inflation and default and thus uses both. Real debt on the one hand makes inflation less appealing because the debt burden is fixed in real terms and cannot be devalued. But on the other hand the government is less worried about causing inflation in the future because expected inflation does not depress bond prices when debt is real. In addition, the equilibrium level of borrowing affects the degree to which the government chooses to repudiate and monetize its liabilities. If the government borrows more then this adds to upward pressure on prices and default risk.

In order to draw out the main forces at work in the model I use a simplified version in this section. The key simplification is that I assume default is a continuous variable with the government choosing a default rate $d \in [0, 1]$ each period. Defaulting incurs costs $t(d), t_d > 0, t_{dd} \geq 0$. There is no exclusion after default. With this assumption I can use first order conditions at an interior solution to the problem and characterize tradeoffs more precisely. I also abstract from uncertainty, labor income taxes and credit consumption goods. See the Appendix for a full definition of this simplified model.

Consider a nominal debt economy first. We can rewrite the government budget constraint purely in terms of allocation by using the competitive equilibrium conditions to substitute out prices. Then the constraints on the government's problem can be written

Figure 2.5: Static tradeoff between inflation and default



For a given B and income (borrowing B' with associated seigniorage and bond revenue)

as[12]

$$G^n(B,p,d,B',P(B'),D(B')) \equiv -u_2\left(\frac{1+(1-d)B}{p}+g+t(d)\right)+\beta\left(\frac{u_1'+u_2'(1-D(B'))B'}{P(B')}\right)=0 \tag{2.45}$$

and

$$F(p,d) \equiv u_1(p,d)-u_2(p,d) \geq 0$$

The first order conditions are

$$\begin{aligned} -\frac{u_c-u_l}{p^2}+\lambda G_p+\gamma F_p &= 0 \\ -u_l T_d+\lambda G_d+\gamma F_d &= 0 \\ \lambda\left(G_{b'}+G_P\frac{\partial P(B')}{\partial B'}+G_D\frac{\partial D(B')}{\partial B'}\right)+\beta\lambda' G_b' &= 0 \end{aligned} \tag{2.46}$$

At an interior solution where $F(p,d) > 0$ such that the cash in advance constraint is binding, the ratio of the first two conditions characterize the intratemporal optimal tradeoff

[12] See the Appendix for a full derivation

between inflation and default

$$\frac{u_1 - u_2}{p^2(u_2 t_d)} = \frac{G_p^n}{G_d^n} \tag{2.47}$$

where

$$G_p^n = \frac{\partial G^n}{\partial p} = u_2 \frac{(1+(1-d)B)}{p^2} + (u_{12} - u_{22}) \frac{1}{p^2} \left[\frac{(1+(1-d)B)}{p} + g + t(d) \right] \tag{2.48}$$

$$G_d^n = \frac{\partial G^n}{\partial d} = u_2 \frac{B}{p} - u_2 t_d + u_{22} t_d \left[\frac{(1+(1-d)B)}{p} + g + t(d) \right] \tag{2.49}$$

The left hand side of (2.47) is the relative cost of inflating compared to defaulting. The government is willing to trade off a fall in prices that reduces distortions by $\frac{u_1 - u_2}{p^2}$ for an increase in default that creates distortions $u_2 t_d$. Figure (2.5) illustrates this. It shows an "indifference curve" for default and inflation rates.[13] Along the curve, utility is held constant. The marginal rate of substitution between default and inflation, in other words the slope of this indifference curve, is given by the left hand side of equation (2.47).

The right hand side of equation (2.47) represents the relative benefit to the government of devaluing through inflation compared to default in terms of relaxing its budget constraint. For a given level of income (the right hand side of (2.45)) and debt B, the picture plots the pairs of default and inflation rates that satisfy the government budget constraint. The slope of this line is equal to the negative of the right hand side of (2.47). At an optimum the government picks the lowest default and inflation pair that is feasible, at the point of tangency, the red dot in the picture.

The previous discussion focused purely on the intratemporal decision, holding borrowing and revenue fixed, but there are of course dynamic effects. It is not possible to solve analytically for the policy function for debt or the steady state, but we can use the intertemporal first order condition of the government's problem to gain insight into what drives equilibrium borrowing and how it affects the tradeoff between inflation and default. The intertemporal Euler condition for the government's problem, equation(2.46), describes the tradeoffs involved. It states that the government at an optimum equates the marginal benefit of borrowing today with the cost of repaying tomorrow. Borrowing more today

[13] The figure plots the tradeoffs in terms of inflation rather than price levels for expositional clarity. Analogous figures in terms of the price level look the same. See the Appendix for functional forms and parameters used in this example.

lowers bond revenues but increases seigniorage revenue, which is reflected in the terms $G_P \frac{\partial P(B')}{\partial B'}$ and $G_D \frac{\partial D(B')}{\partial B'}$. The faster the sum of these terms falls with the level of debt, the less debt the government will accumulate in equilibrium. Note that we can derive analytical expressions for G_P and G_D but not for the partials with respect to the equilibrium policy functions, $\frac{\partial P(B')}{\partial B'}$ and $\frac{\partial D(B')}{\partial B'}$.[14] Numerically both prices and default rates are increasing in debt levels, such that higher equilibrium debt levels translate to upward pressure on inflation and default.

In terms of Figure (2.5), higher borrowing and debt tend to shift the budget constraint out. The government partly finances a higher debt burden through inflation and default taxes since additional bond sales do not generate sufficient revenue.

2.4.1 Debt Denomination

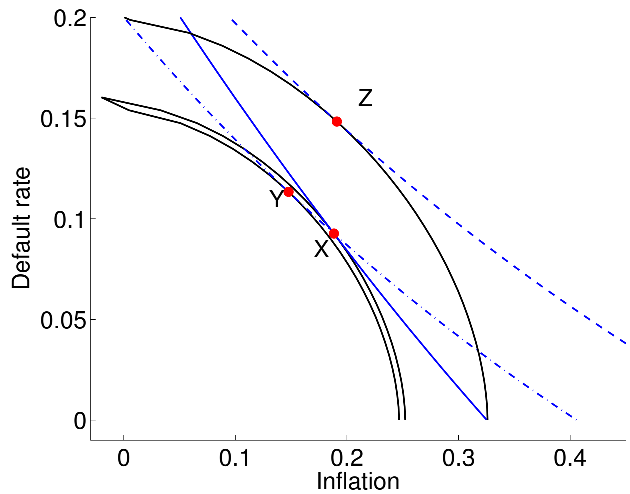
When debt is real, incentives to inflate and default change. The relative cost of inflation compared to default is unaffected by the denomination of the debt since we are in a closed economy and debt does not affect utility directly via a resource constraint. What does change is the relative benefit from inflating, both today and tomorrow.

In terms of the *intra*temporal trade off that the government faces, inflation becomes less appealing when debt is real. We can see this by looking at the budget constraint for real debt

$$G(b, p, d, b', P(b'), D(b')) \equiv -u_2 \left(\frac{1}{p} + (1-d)b + g + t(d) \right) + \beta \left(\frac{u_1'}{P(b')} + u_2'(1 - D(b'))b' \right)$$

[14] An alternative way of expressing the Euler equation without substituting out prices is $\lambda \left(\frac{1+\mu}{p} q_n + \frac{\partial \mu}{\partial B'} \left(\frac{1}{p} + \frac{q_n B'}{p} \right) + \frac{\partial q_n}{\partial B'} \frac{(1+\mu)B'}{p} \right) = \beta \lambda' \frac{1-d'}{p'}$. The first term in brackets represents the direct additional revenue from selling a marginal unit of debt, q_n, in real terms. The second term captures additional seigniorage revenue. An increase in borrowing increases money growth and thus seigniorage revenue. The third term is the effect of increased borrowing on bond prices. Future expected inflation and default drive down bond prices and thus revenue today. The right hand side is the marginal cost of repaying the debt, in real terms.

Figure 2.6: Nominal and real debt: Substitution and income effects on inflation and default



Fixed $\frac{B}{p} = b$. X: Nominal bonds. Z: Real bonds. $X \to Y$: Substitution effect. $Y \to Z$: Income effect.

and comparing

$$\begin{aligned} G_p &= u_2 \frac{1}{p^2} + (u_{12} - u_{22}) \frac{1}{p^2} \left[\frac{1}{p} + (1-d)b + g + t(d) \right] \\ G_d &= u_2 b - u_2 t_d + u_{22} t_d \left[\frac{1}{p} + +(1-d)b + g + t(d) \right] \end{aligned}$$

with the corresponding expressions for nominal debt, equations (2.48) and (2.49). G_d is unchanged, but the second term in (2.48), $u_2 \frac{(1-d)B}{p^2} > 0$ is missing in the analogous expression for real debt. Intuitively, the same drop in inflation now requires a smaller increase in default rates to satisfy the budget constraint, since the lower inflation did not increase the real value of outstanding bonds.

Figure (2.6) plots this along with the previous tradeoff from the nominal debt economy. It shows that the dashed budget constraint for the real debt economy has a flatter slope. Everything else equal, if the government in the nominal debt economy chooses a point like X, the government in the real debt economy with the same real debt burden and revenue would choose point Y - higher default and lower inflation.

How is revenue affected when debt is real instead of nominal? Recall that nominal bond prices in this model incorporate risk, default and inflation premia. In particular, the higher expected default and inflation tomorrow, the lower the bond price today since investors need to be compensated for the risk. This provides incentives for the government today to avoid inducing inflation tomorrow by borrowing more. The payout of real bonds on the other hand is not affected by inflation - there are no inflation premia in these bonds. As a result, the government today has less of a disincentive to cause inflation tomorrow by borrowing.

An alternative way of seeing this is to consider a steady state in which the real value of bond debt is the same across the two economies. Suppose the economy has been a nominal debt economy at a point like X in Figure (2.6), and suppose one morning all bond debt is suddenly indexed. As discussed above, holding revenue constant, the static effect is to increase default rates and lower inflation rates (moving to Y). We held revenue fixed, but the Markov government takes into account future policies which change with the denomination of the debt. In particular, with debt being real, a decrease in inflation rates will not be reflected in bond prices and thus give no boost to revenue, while an increase in default rates still hurts revenues. In terms of the budget constraint, as a result, without adjusting borrowing, revenues are too low. The government must either increase borrowing, or increase distortions today to make up the difference, and move to a point like Z in Figure (2.6). In the Appendix I prove under simplifying assumptions on utility and default costs, inflation and default are higher in a steady state where real debt levels are identical across a real and a nominal debt economy.

In this section we discussed how inflation, default rates and the level of borrowing are determined in the model. The government in general uses both default and inflation. When debt is real instead of nominal, there are two countervailing effects on equilibrium default and inflation: On the one hand, the government is less likely to use inflation because it is less effective at relaxing its budget constraint - real debt cannot be devalued through inflation. On the other hand, precisely because bond revenues are not hurt by future inflation, this encourages debt accumulation and for sufficiently high debt levels and default rates, the government may resort to higher inflation to generate seigniorage revenue after all. The next section explores which of these effects dominates quantitatively.

2.5 Quantitative Exercise

In this section I use the quantitative version of the model to evaluate which of the forces identified in the previous section dominate and to what extent the model can capture the data.

2.5.1 Parameters and Functional Forms

Utility is assumed to exhibit constant elasticity of substitution between cash and credit good consumption, and is separable in leisure:

$$u(c_1, c_2, 1-n) = \frac{\left((\alpha c_1^{\rho} + (1-\alpha)c_2^{\rho})^{\frac{1}{\rho}}\right)^{1-\sigma}}{1-\sigma} + \nu\frac{(1-n)^{1-\theta}}{1-\theta}$$

Government spending in the model is given by $g = Ae^{z}$

$$z_{t+1} = \rho z_t + \epsilon_{t+1}, \epsilon \sim N(0, \sigma_\epsilon)$$

I calibrate the economy to match inflation and default rates in the average nominal debt issuing country in my sample. Average public consumption to GDP ratio in the data is 15%. Given a calibrate value for average hours worked, this pins down the value for A. Instead of estimating the shock process based on the process for government spending for a particular country, I pick typical values for its persistence and volatility that are observed in a range of emerging market countries. I do not calibrate it to a particular time series since my question and calibration target is a cross-sectional observation. The average income tax rate is 15%. The long run growth rate of GDP is around 2% annually, so I choose $\beta = 0.98$ which implies an annual real risk free rate in the model economy of around 2%.

Government expenditures in default are parameterized as

$$\begin{aligned} g^d &= g + h(g) \\ &= g + \max\left\{0, \chi_1\frac{\mathbb{E}[g]}{g} + \chi_2\left(\frac{\mathbb{E}[g]}{g}\right)^2\right\} \end{aligned}$$

Table 2.2: Parameters

Parameter	Value	Description/ Target
β	0.98	Discount factor
τ	0.10	Labor income tax rate
A	0.04	$\mathbb{E}[g]$
ρ	0.95	Persistence of AR(1) of g
σ_ϵ	0.0075	Volatility of AR(1) of of g
η	0.25	Probability of exclusion
ν, θ	1.93,8.51	Leisure level/ curvature
α	0.01	Cash good weight
ρ	-2.89	Cash - credit elasticity
χ_1,χ_2	-0.59,0.66	Default cost
σ	4.39	Risk aversion

My specification implies that default is more expensive when government expenditures are low. This is analogous to papers in the sovereign default literature that assume that the cost of default is higher in good times. Without this assumption the value of default is too sensitive a function of the state and the model cannot generate large regions of the state space in which the country borrows with positive but finite default risk. I choose the probability of re-entry η such that the average default episode takes 4 years.

The remaining parameters are chosen to match moments from the data: Inflation, default rates, the debt-to-GDP ratio and ratio of cash to credit goods in the average nominal debt issuing country in my sample, as well as average hours worked of 0.3. Table (2.2)summarizes the chosen parameter values.

2.5.2 Simulation Results

Table (2.3) presents statistics from the simulated model and compares them to the data. The targeted statistics for the nominal debt economy and the data counterparts are in the

Table 2.3: Data and simulated model statistics

	Nominal		Real	
	Data	Model	Data	Model
Inflation	5.5	6.7	21.1	23.0
Default probability	2.0	2.9	19.8	12.7
Cash-credit good ratio	30.4	30.4	21.9	29.8
Debt-to-GDP ratio	18.0	17.5	12.3	22.0

All statistics are in percent, model statistics are averages over periods of good credit standing, excluding the first 10 periods after the end of a default episode.

first two columns. The third and fourth column contain data and model statistics for real debt issuers.

As in the data, inflation and default rates are higher in the model when debt is real than when it is nominal. In terms of magnitudes the model slightly overpredicts the increase in inflation, and captures around two thirds of the change in default rates across debt regimes. Cash to credit goods ratios do not change much. The model predicts that debt levels in the real debt economy are higher than in the nominal debt economy but the difference is smaller across debt regimes than for inflation and default rates.

Figure 2.7 plots how the government accumulates debt over time in both economies. At a constant mean expenditure shock, the government begins to accumulate debt to finance expenditures. Inflation and default probabilities rise together with debt. In the real debt economy, the country begins to inflate later. At low debt levels expenditures plus debt service are low enough for the country to not resort to inflation. Only at sufficiently high debt levels inflation and default probabilities begin to rise.[15] An important feature of the graph is that inflation and default probabilities rise rapidly in the real debt economy. Given the calibrated default process, default risk rises quickly beyond a given debt threshold and the country counteracts that with equally rapidly rising inflation. Debt accumulation levels off as default risk rises.

[15] I plot realized inflation since that is my data counterpart, unexpected inflation looks qualitatively similar.

Figure 2.7: Debt, default and inflation dynamics in the model economies (percent)

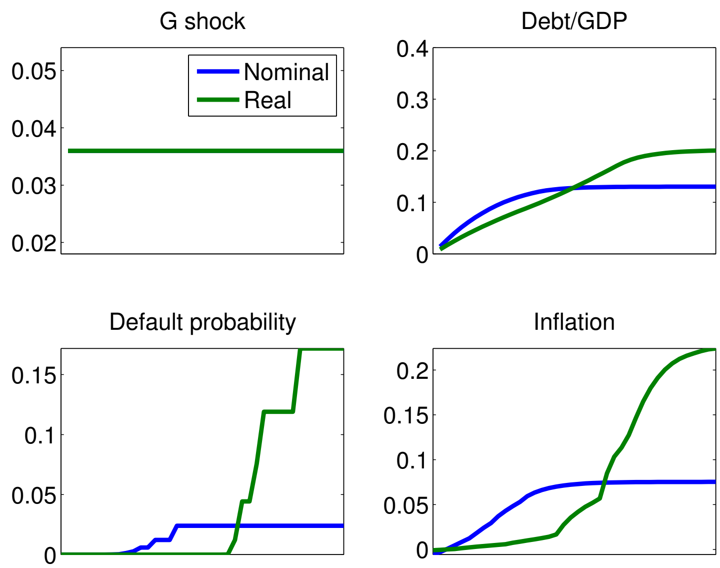


Figure 2.8 shows how seigniorage is used in both economies to offset the reduction in net bond revenues that occurs as bond prices fall. In the nominal debt economy this occurs sooner (at lower debt levels) and less suddenly. Even at low inflation rates bond prices fall with future expected inflation, lowering bond revenues and causing the country to switch to seigniorage. With real debt, the country uses bonds to generate revenue and only switches to seigniorage at higher debt levels.

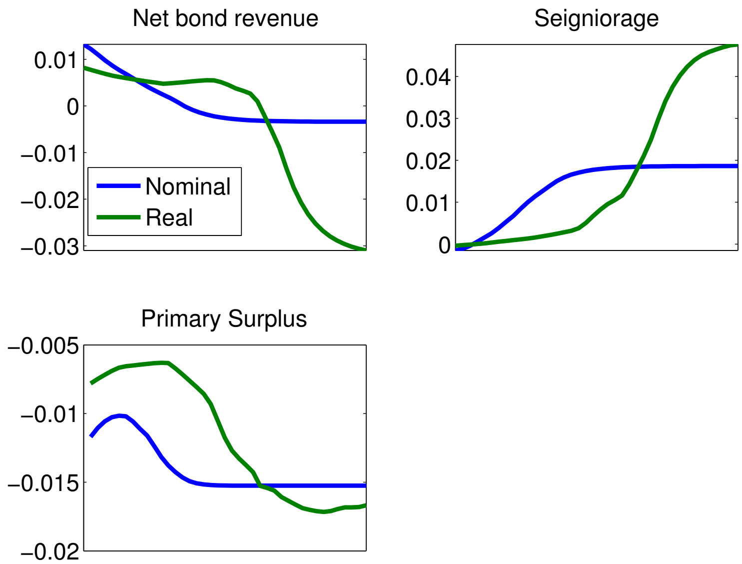
To assess the welfare consequences of debt denomination policies I compute the consumption equivalent welfare gain of living in nominal debt economy. Specifically, I compute the fraction of aggregate lifetime consumption ω that agents born into the in real debt economy with no assets give up to live in nominal debt economy:

$$\mathbb{E}_0 \sum_t \beta^t u(\hat{c}_{n,t}, 1 - \hat{n}_{nt}) \quad = \quad \mathbb{E}_0 \sum_t \beta^t u(\hat{c}_{r,t}(1+\omega), 1 - \hat{n}_{rt})$$

which with power utility is the ω that solves

$$\omega \quad = \quad \left(\frac{\int V_r(0,z) dF(z)}{\int V_n(0,z) dF(z)} \right)^{\frac{1}{1-\sigma}} - 1$$

Figure 2.8: Sources of revenue in the model economies as a percent of GDP



In my benchmark calibration the welfare gain from living in the nominal debt economy is 0.12% of lifetime consumption - small but positive.

In the real debt economy the rise in inflation is closely linked to default risk. In order to understand to what extent debt accumulation compared to default risk drive the result that inflation is higher in a world with real debt I conduct a counterfactual experiment: What happens under both debt regimes when default is prohibitively costly such that it is not observed in equilibrium? Table 2.4 summarizes the main model statistics from this experiment along with the baseline results. It shows that without default risk, debt levels are substantially higher in the real debt economy while the difference across debt regimes in inflation is much smaller than in the baseline. This suggests that default risk is an important factor in generating the observed differences in inflation rates between nominal and real debt issuing countries. Without default risk differences in inflation rates are driven by debt accumulation with both the level and difference across debt regimes being counterfactually high. Default risk brings the model closer to the data.

Table 2.4: Model simulations: Prohibitively costly default

	Baseline		No default	
	Nominal	Real	Nominal	Real
Inflation	6.7	23.0	15.2	28.7
Default probability	2.9	12.7	0.0	0.0
Debt-to-GDP ratio	17.5	22.0	26.0	58.1

All statistics are in percent, model statistics are averages over periods of good credit standing, excluding the first 10 periods after the end of a default episode.

2.6 Conclusion

In this paper I have studied sovereign inflation and default policies, and how these depend on debt denomination. In the data, countries that issue nominal debt tend to achieve lower inflation and default rates. A monetary model of sovereign borrowing and lack of commitment can account for this observation as a result of a change in debt denomination alone. Real debt lowers incentives to inflate today since it cannot be monetized, but it increases incentives to borrow and thus inflate tomorrow since real bond prices carry no inflation premia and inducing future inflation is less costly for the government. The second of these two forces dominates in a quantitative version of the model, leading to higher inflation and default rates, and lower welfare, in a real debt economy compared with an otherwise identical nominal debt economy.

The model highlights that even if bonds are real, money still plays a role and affects borrowing decisions. In particular, issuing real debt does not remove incentives to inflate. In terms of policy, the model suggests that issuing real debt in an attempt to address weak commitment by the policy maker may be short sighted and in fact exacerbate inflationary commitment problems. Considering lack of commitment as the only relevant factor, that is even absent institutional reforms, the model suggests that nominal debt issuance is a desirable policy.

In the paper I restrict attention to a setting where the debt denomination is a primitive and explore the consequences of debt regimes for government debt, default and inflation policies. It would be interesting to extend the paper in this dimension and explore whether and under what conditions the model can replicate the observed portfolios of nominal and real debt. Relatedly, studying the extent to which changes in institutional factors might affect these portfolios and changes in them over time is an interesting avenue of future research.

Chapter 3

Sovereign Default with Incomplete Debt Relief

3.1 Introduction

Sovereign default episodes tend to be associated with less than complete debt relief for the defaulting country. Creditors recover on average more than half their investment, and in around 13% of cases investors get paid back in full after a default.[1] This is an important aspect of default episodes: The degree of debt relief after a default directly affects the default incentives that a country faces. Intuitively, if a sovereign expects to have to repay a large fraction of any debt it defaults on, default becomes a less attractive option.

Studying the implications of incomplete debt relief is therefore important for evaluating the costs and benefits of alternative default restructuring processes. The international community runs several programs explicitly designed to provide debt relief, for example the Heavily Indebted Poor Countries (HIPC) Initiative by the IMF and World Bank, whose aim is "to ensure that no poor country faces a debt burden it cannot manage", or the Paris Club of lenders. In this paper I develop a framework to analyze the motivation underlying these programs. Should debt relief be a priority when the goal is to lessen the costs associated with sovereign debt crises?

[1] For the remainder of this paper I will use the words haircut and debt relief interchangeably. The recovery rate is 1 minus the haircut. High debt relief therefore is equivalent to low a recovery rate. I will define what these measures are in the data and in the model below.

I first document features of sovereign debt default episodes. Recovery rates in the data are positive and range from 10 to 100%. Restructuring negotiations are time-consuming and take on average more than 7 years. There is a link between debt relief and the time it takes to settle a sovereign default: The longer renegotiations take, the larger the eventual haircut for investors tends to be. Interestingly, recovery rates are not strongly related to macroeconomic conditions during default episodes.

I then build a model of sovereign default with long-term bonds and incomplete debt relief to study the the implications of these features of the data. The model is based on Eaton and Gersovitz (1981) and Arellano (2008). As in their studies, there is lack of commitment, markets are incomplete, so default occurs in equilibrium, and default risk is endogenous. The calibrated model matches the Argentinian economy well; it replicates its debt-to-GDP ratio, average spreads, their volatility, observed debt recovery rates, and does well on a number of other emerging markets business cycle dimensions.

In this setting, debt relief turns out not to be welfare-improving in general. The key feature that drives this result is the fact that lower recovery risk now affects bond prices. In the standard model with complete debt write-down, there is a one-to-one link between higher default rates and lower bond prices, as bond prices simply reflect the probability of repayment. In this paper, there is an additional factor: incomplete debt relief. Now the investor can recover some of his investment after a default, which makes defaulted debt valuable. This is reflected in the price of current debt because it is taken into account when borrowing decisions take place.

Non-zero recovery rates therefore drive a wedge between default probabilities and bond prices. They make bond prices less sensitive to pure default risk. Incomplete debt relief in this sense weakens the effects of default risk on spreads. Lower spreads in turn allow the country to extend the region of the asset space in which it can borrow without defaulting, leading to higher average debt-to-GDP ratios and higher ex-ante welfare. The welfare difference are large, more than 4% of lifetime consumption.

I use the model to study the trade-off between length of renegotiations and debt relief. On the one hand, more time spent restructuring raises the costs of default, but on the other hand, higher haircuts make default more attractive since upon re-entry to capital markets the debt burden will be lower. In the model, recovery rates turn out to be the stronger of these two forces in the sense that the prospect of little debt relief is more effective at improving equilibrium outcomes than protracted renegotiations. Renegotiation length matters, but increasingly less so the higher recovery rates become.

Finally, I extend the model to allow for procyclical recovery rates - countries pay back more of what they owe if they can afford to because macroeconomic conditions are good. The model predicts that more lenient restructuring terms lead to worse outcomes in equilibrium. Spreads and default rates are higher, sustainable debt levels and welfare lower.

The results in this paper shed light on features that a financial system designed to mitigate the effects of sovereign defaults should have. They highlight the importance of debt relief in shaping equilibrium outcomes, and to some extent restructuring durations.

Note that I focus throughout on the effects of debt relief rather than attempting to explain the recovery rates that we see in the data. There are studies that deliver the outcomes whose effects I study from micro-foundations, for instance Benjamin and Wright (2009). But for the purposes of this paper, I take observed debt resolution patterns as given and focus on analyzing their implications.

The paper builds on the literature of sovereign default models with incomplete markets.

Arellano (2008) studies business cycles of emerging economies in a setting with short term debt and zero recovery rates. Yue (2010) models renegotiation but imposes re-entry to capital markets with no debt. Benjamin and Wright (2009) build a bargaining model with one-period debt that delivers delays and haircuts observed in the data under certain specifications of the bargaining powers of the parties.

To think about restructuring, we need to consider bond contracts that last longer than one period. There are a number of recent papers that introduce longer term debt. Bi (2006) considers one and two-period bonds with endogenous renegotiation, but no restructuring - her two-period bonds come due at the end of the renegotiation by construction so the sovereign re-enters capital markets with zero debt. Hatchondo and Martinez (2009) as well as Chatterjee and Eyigungor (2012) model long term debt in an otherwise standard default model (no renegotiation, no restructuring).[2] They show that these are better able to simultaneously generate high spreads and debt levels compared to short term debt models.

Arellano and Ramanarayanan (2012) model both short and long term debt to study

[2] Hatchondo et al do not model exclusion, but let the country borrow in the period of the default; note this is not restructuring, the value of the old debt is written off entirely.

the maturity composition and term structure effects of sovereign defaults. They emphasize that long term debt is useful for consumption smoothing, while short term bonds provide higher incentives to repay.

The rest of the paper is structured as follows. Section 3.2 documents empirical regularities of sovereign default episodes and restructuring, Section 3.3 presents the model. I characterize the equilibrium and explain the main mechanism of the model in Section 3.4. For the quantitative exercise of the paper, I calibrate the model in Section 3.5 and present the main results in Section 3.6. I consider extensions to the benchmark model in Section 3.7. Section 3.8 concludes.

3.2 Data

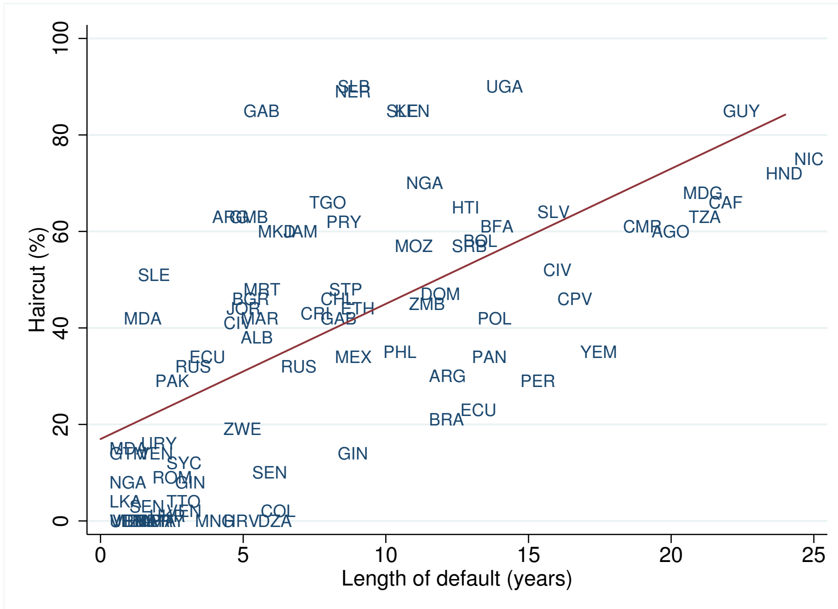
In this section I document features of sovereign defaults for 70 countries and 85 default episodes that started and ended between 1979 and 2005.[3]

I define default as in Benjamin and Wright (2009), which is based on the most widely used source on sovereign defaults published by the ratings agency Standard & Poor's (Beers and Chambers (2006)). S&P define a default to have started in a given year if interest or principal payments are not made within the grace period, or if debts are rescheduled on "less favorable terms than the original contract". A default ends in the year that a settlement occurs and when "no further near-term resolution of creditors claims is likely". Benjamin and Wright (2009) adjust this definition to account for the fact that defining the start of a default when contracts are rescheduled may underestimate the true length of a default. They also use additional sources to determine the month in which a default started and ended.

Haircut estimates are taken from Benjamin and Wright (2009), the most comprehensive source available. Their estimates are based on data from the World Bank Global Development Finance database (GDF hereafter). In particular, they calculate losses as the cumulative sum of interest and principal forgiven, debt reduction and debt buyback throughout a given default episode. They discount the cumulated losses back to the start of the default using a constant 10% interest rate. Haircuts are then given by these discounted

[3] The main sources are Benjamin and Wright (2009), Sturzenegger and Zettelmeyer (2005) and the World Bank Global Development Finance (GDF) database. See the data appendix for details.

Figure 3.1: Haircuts and length of default episodes



Source: Benjamin and Wright (2009)

losses relative to outstanding debt at the start of default.[4]

There are a number of common features across default episodes.

Investors typically lose some, but not all of their investment. Haircuts are on average 38%. The maximum is 90% in the case of Uganda and the Solomon Islands. In 11 out of the 85 default episodes that data is available for investors took no losses.

Restructuring is time-consuming. On average default episodes lasted 7.4 years, the median is slightly lower at 6 years.

The longer a default episode lasts, the higher eventual haircuts tend to be. The relationship between the two is shown in Figure (3.1).[5] It shows that a 1 year increase in

[4] There are at least two alternative measures of haircuts: (1) The market values of new debt relative to the market value of old debt; (2) the market value of new debt relative to the face value of old debt. Both are more difficult to calculate because they require estimates of yields, in the case of (1) on defaulted, frequently non-traded debt. The Benjamin and Wright (2009) measure sidesteps this difficulty while still correlating highly with haircut measures (1) and (2) calculated for a subset of countries in Sturzenegger and Zettelmeyer (2005), and it is much more extensive than any other source.

[5] The discounting used in the construction of the haircut estimates makes a positive correlation between duration and haircut more likely. The correlation is also present in the Sturzenegger and Zettelmeyer (2005)

default episode length is associated with 2.8 percentage point increase in haircuts. This is statistically significant at more than the 1% level.

Cyclical conditions are not clearly related to haircuts. Recovery rates are lower if the country defaults in a year with negative year-on-year output growth. 1 percentage point lower growth rates mean 1 percentage higher haircuts in this case. But defaulting when output is *below trend* is unrelated to haircuts. As for cyclical conditions at the *end* of default episodes, either measure is unrelated to haircuts - it is not the case that countries that emerge from a default episode in good times pay back more.

I now turn to presenting a model that captures these patterns of sovereign default and restructuring episodes.

3.3 Model

Consider a small open economy. Time is discrete and infinite. A benevolent government maximizes the present discounted value of consumption of the domestic residents:

$$\sum_{t=0}^{\infty} \beta^t u(c_t)$$

where $u(\cdot)$ is continuous, strictly increasing and strictly concave and $\beta \in (0,1)$ is the discount factor.

Income is stochastic and given each period by

$$w = y + m$$

Here $y \in Y \subset \mathbb{R}_{++}$ follows a finite-state Markov process and $m \in M = [-\bar{m}, \bar{m}]$ is drawn iid from a continuous distribution with mean zero. I follow Chatterjee and Eyigungor (2012) in introducing the continuous state variable m in order to make robust computation of the model possible. Its variance is set to a small number in the application such that it does not affect the dynamics.[6]

estimates, however, whose measure does not rely on this discounting.

[6] Intuitively, it smooths over discrete jumps in behavior that can occur in response to small changes in prices when the state space is completely discretized. It does so by effectively introducing randomization over choices. Discrete jumps cause convergence problems of the price functions. These problems, while

Financial markets are incomplete. There are non-contingent, long-term bonds b that the government can trade with risk-neutral, competitive international investors in order to smooth household consumption. The government cannot commit to repaying, nor to its future borrowing and default decisions. In choosing new debt issuances, it takes into account that these will affect the price at which it can sell them. When it defaults, the sovereign loses access to capital markets and receives a lower endowment each period it spends in autarky. This endowment loss is meant to represent the cost of default and is standard in the literature. With a fixed probability it restructures its debt and re-enters capital markets with the new debt.

I assume that b takes on values in the finite set $B \subset \mathbb{R}_{--}$.[7] In line with the literature, I focus on Markov perfect equilibria, that is I assume that the government's borrowing and default decisions depend only on payoff-relevant state variables.

3.3.1 Debt Contracts

Long-term debt is modeled as in Chatterjee and Eyigungor (2012). Bond contracts consist of the principals and coupons they pay and mature probabilistically. Each period, with probability λ a contract of size b matures and pays the principal b. If it does not mature, with probability $1 - \lambda$, it pays a coupon zb, where $z \in [0, 1)$. Total principal and coupon payments in each period t on outstanding contracts of size b_t are therefore given by

$$b_t[\lambda + (1 - \lambda)z]$$

This is typically the definition of the stock of debt. Note in a standard model with one-period, zero coupon bonds, $\lambda = z = 0$ and this reduces to b_t.

Debt issuance between period t and $t + 1$ is given by

$$b_{t+1} - (1 - \lambda)b_t$$

The average maturity of a bond contract in this setup is simply the reciprocal of the

present in standard default models, are exacerbated in a setting with long-term debt because prices are recursive. See Chatterjee and Eyigungor (2012) for details.

[7] The fact that it is strictly negative is important to be able to prove existence of a pricing function, but the highest level can be arbitrarily close to zero and the bound will not be binding in simulations.

probability of maturing, $1/\lambda$. The advantage of specifying bond contracts in this way is that it allows for a parsimonious state space. Instead of keeping track of all past issuances not yet matured, the government only needs to keep track of one state variable, b, when it solves its decision problem. This is what I describe next.

3.3.2 Decision Problem

Consider the problem of a government with outstanding contracts $b \in B$ and endowments $(y, m) \in (Y \times M)$ that is in good credit standing.

The government has the option to default with value

$$V(b, y, m) = \max_{d \in \{0,1\}} \left\{ (1-d) V^r(b, y, m) + d V^d(y, -\bar{m}) \right\} \tag{3.1}$$

where $V^r(b, y, m)$ is the value of repaying and $V^d(y, b, -\bar{m})$ is the value of defaulting. For simplicity I assume the country gets the minimum value of the low variance shock m in the case of default.[8]

The value of repaying is

$$V^r(b, y, m) = \max_{b' \in B} u(c) + \beta E_{y',m'|y} \left[V(b', y', m') \right] \tag{3.2}$$

$$c = y + m + [\lambda + (1-\lambda)z]\, b - q(b', y) \left[b' - (1-\lambda) b \right] \tag{3.3}$$

Here b' are new debt contracts issued and $q(b', y)$ is their price. The budget constraint holds with equality and implies that households consume their income, less principal and coupon payments on outstanding bonds, plus revenues from new debt issuances.[9] Next period the government again has the option to default with expected discounted value of $\beta E_{y',m'|y} [V(b', y', m')]$.

The value of defaulting is

$$V^d(y, m) = u(\phi(y) + m) + \beta E_{y',m'|y} \left[(1-\eta) V^d(y', m') + \eta V(\tilde{b}, y', m') \right] \tag{3.4}$$

[8] I do not need it to help the computation of V^d since the value of default is independent of optimal policy functions and prices. Given that the shock has small variance, setting it to any fixed value $\in M$ will simplify the computations and not affect the dynamics.

[9] Note $b < 0$ is debt.

In default, the government cannot borrow and suffers output cost $\phi(y) \leq y$, $\phi'(y) > 0$, so consumption is given by $c = \phi(y) + m$. In the next period, the country will remain in autarky with probability $1 - \eta$. With probability η the sovereign restructures the debt it defaulted on to $\tilde{b}$ and re-enters capital markets with these new obligations, facing again the option to default.

In the benchmark model I assume that $\tilde{b}$ is exogenous and fixed. This is generalized later to allow the level of restructured debt to depend on cyclical conditions at the time of restructuring, $\tilde{b} = H(y')$. Note that I assume full default on all payments due since during autarky neither interest or coupon payments enter the budget constraint. I also do not consider the possibility that restructuring alters the coupon or maturity structure of bonds; z and λ remain fixed parameters.

Denote the optimal policy functions associated with the decision problem in 3.1 through 3.4 by $d = d(b, y, m)$ and $b' = a(b, y, m)$.

3.3.3 Bond Prices and Equilibrium

Under perfect competition and risk-neutrality, international investors earn zero expected profits and payoffs are equal to the probability of repayment. This determines the prices of sovereign bonds in the model.

We can write long-term debt prices recursively as follows. The price of one unit of current, non-defaulted debt is

$$q(b', y) = E_{y',m'|y}\left[(1 - d(b', y', m'))\frac{\lambda + (1-\lambda)\left[z + q(a(b', y', m'), y')\right]}{1+r} + d(b', y', m')\frac{q_D(b', y')}{1+r}\right] \tag{3.5}$$

The first term on the right hand side is the expected payoff from a unit investment if the government chooses not to default. In this case, the investor receives with probability λ the principal and with probability $1 - \lambda$ the coupon payment plus the value of the investment tomorrow. Note that the value tomorrow depends on the optimal policy function $a(b', y', m')$: Lenders in pricing long-term bonds use the policy rule to forecast tomorrow's bond values.

The second term on the right hand side corresponds to the case when the government defaults. The debt becomes delinquent rather than non-existent as in standard models, and therefore has a value, $q_D(b, y)$.

This value of a unit of delinquent debt in turn is given by the following expression:

$$q_D(b, y) = E_{y'|y}\left[(1-\eta)\frac{q_D(b, y')}{1+r} + \eta\frac{\tilde{b}}{b}\frac{q(\tilde{b}, y')}{1+r}\right] \tag{3.6}$$

With probability $1-\eta$ the debt stays delinquent and is carried forward into the next period. With probability η, the debt is restructured and becomes current. In this case I need to scale by $\frac{\tilde{b}}{b}$ since $q(b', y)$ is defined in units of current rather than defaulted debt.

Note that the debt is carried forward without accruing interest. I abstract from this possibility for technical reasons and because as a fraction of total external public debt, interest arrears are small. The median throughout defaults is 3% on average in the set of countries I have data for.[10]

A *Markov perfect equilibrium* then are policy functions $d(b, y, m)$ and $a(b, y, m)$, value functions $V(b, y, m), V^r(b, y, m)$ and $V^d(y, m)$ and prices $q(b, y)$ and $q_D(b, y)$ such that, (i) taking prices as given, the policy and value functions solve the decision problem in (3.1), (3.2) and (3.4), and (ii) bond prices satisfy investors' zero expected profits and reflect optimal default and repayment decisions.

3.4 Properties of the Recursive Equilibrium

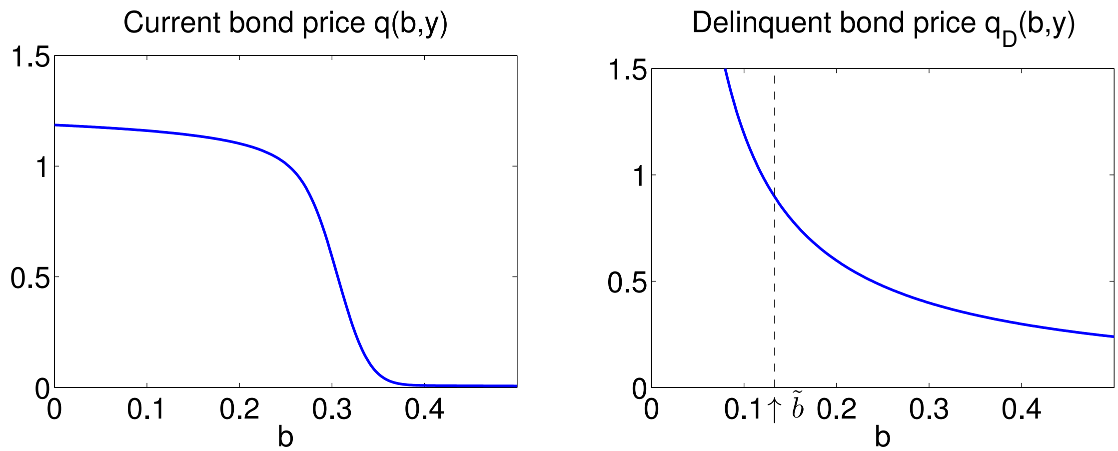
How does the introduction of incomplete debt relief affect the properties of the model? In this section I will show how bond prices, default incentives and spreads are related to recovery rates in the model.

For this, I need a few results that I state here as propositions:

Proposition 1. *The value function $V(b, y, m)$ exists, is strictly decreasing in indebtedness, strictly increasing in m, and continuous in m.*

[10] If I wanted interest to accrue, I would have to choose the rate at which it does - either the debt-specific interest rate $\frac{1}{q_D(b,y')}$ or the risk-free rate $1+r$. The former would make the model intractable. The second runs into the problem of an unbounded state space of defaulted debt - if a constant interest accrued and the country had a string of bad luck and remained in autarky for a very large number of periods, the value of the defaulted debt would not be bounded.

Figure 3.2: Bond price schedules



Debt b is expressed as a fraction of mean annual output. The price schedules are shown for mean output.

Proposition 2. *Default decisions $d(b, y, m)$ are increasing in indebtedness.*

Proposition 3. *If price schedules $q(b', y)$ and $q_D(b, y)$ are decreasing in debt, then borrowing decisions $a(b, y, m)$ are decreasing in debt.*

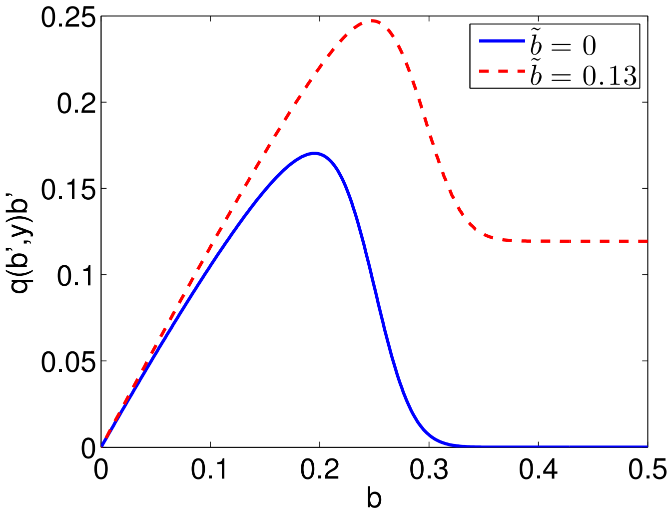
Proposition 4. *There exists price schedules $q(b', y)$ and $q_D(b, y)$ that are decreasing in debt.*

These are straightforward extensions of the results in Chatterjee and Eyigungor (2012). [11]

Figure (3.2) illustrates the result from Proposition (4) by plotting an example of the equilibrium price functions from the quantitative part of the paper. Schedules are shown conditional on repayment and default, respectively. We can see that both price schedules are decreasing in debt. What determines their precise shapes? Notice that the price of current debt is bounded above by the risk-free price which here is given by $\frac{\lambda+(1-\lambda)z}{r+\lambda} \approx 1.3$. For very low levels of debt, default risk is small and the bond price almost risk-free.

[11] Chatterjee and Eyigungor (2012) also prove that default decisions $d(b, y, m)$ are decreasing in m, and borrowing decisions $a(b, y, m)$ are increasing in m. It is also easy to show that in this model, as in other models in this strand of literature, the policy, value and price functions are increasing in the endowment. These results are well-known and the proofs standard and do not change with the introduction of $\tilde{b} \neq 0$, so I omit them here.

Figure 3.3: Market value of new debt issuances
Benchmark model with incomplete debt relief ($\tilde{b} = 0.13$) and full debt relief model ($\tilde{b} = 0$)



Debt issuance b is expressed as a fraction of mean annual output. The market value schedules are shown for mean output and current debt $b = 0$.

The price of defaulted debt therefore becomes very high as debt levels approach zero. Mechanically, the scaling factor in Equation (3.6) dominates.[12]

In fact we can see that the price of defaulted debt exceeds the risk-free price for all levels of debt just below the recovery value $\tilde{b}$ (the dashed vertical line in the graph). This is intuitive: If the country defaults with debt less than $\tilde{b}$, recovery rates $\frac{\tilde{b}}{b}$ will be larger than 1. Suppose there was no time cost to settle the debt and get paid. Then this is a very attractive investment opportunity for creditors - they receive more than they lent out. Hence the price must be higher than the risk-free price to be consistent with zero expected profits. The fact that in the Figure q_D exceeds the risk-free price for levels of debt somewhat below $\tilde{b}$ reflects the fact that the delay in restructuring is not zero.

Bond prices determine the market value of new debt issuances and thus the ability of the country to raise revenue and increase consumption. Figure (3.3) compares market values of new debt issuances for the benchmark model with the standard, full debt relief

[12] But q_D remains bounded since we assumed that b takes values in the finite set $B \subset \mathbb{R}_{--}$

model. I assume for simplicity that current debt $b = 0$, so the expressions for market values are given by the simpler expressions $q(b', y)b'$ and $q_D(\tilde{b}, y)\tilde{b}$ respectively. The Figure shows that introducing incomplete debt relief leads to higher market values throughout that peak at a higher level of debt - in other words, the country can borrow more before prices start falling. Conversely, any desired level of revenue can be achieved with lower debt levels.

Note that the graphs in the Figure are market values, not revenues from new debt issuances. Put differently, the price schedules used to value debt issuances take into account default decisions. This explains why market values are flat for sufficiently high debt levels in the model with incomplete debt relief: Beyond a certain debt level, default is certain (as discussed below) but in the model with incomplete debt relief the price of defaulted debt remains positive. It is never optimal for the country to borrow in this range since it can achieve the equivalent revenue with smaller debt issuance and thus lower future expense.[13]

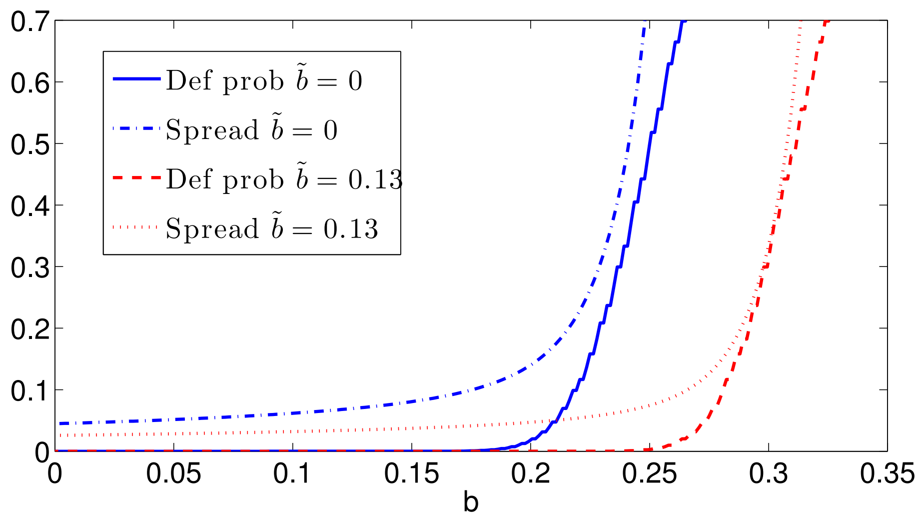
The revenue for the government on the other hand continues to fall in this model as in the standard model, as debt rises since the price of current debt falls to zero for sufficiently high debt levels. This distinction between market values and revenues is absent in the model with full debt relief since the price of defaulted debt there is zero.

Why can the country borrow more when debt relief is incomplete? This result is driven by the fact that defaulted debt is valuable in this model. When the country enters default, prices fall, but not to zero, since creditors will receive at least some of their investment in a settlement. As a result, prices in good credit standing are less sensitive to borrowing levels than in the standard model, too, since they reflect the fact that debt remains valuable even in default. This is clear from Equation (3.5) and illustrated in Figure (3.4). Spreads for the model with non-zero recovery rates rise more slowly with debt than in the case of full debt relief.

Consistent with this, default rates rise more slowly with debt when recovery rates are non-zero. This is also shown in Figure (3.4). Default probabilities in the benchmark model with incomplete debt relief are 0% at debt-to-output of around 0.25, for example, whereas in the model with full debt relief, default rates are 60% at the same level of indebtedness.

[13] We need to ensure that market values are maximized when default probabilities are less than 1. This is satisfied provided restructuring durations are not too short and recovery rates not too high (from Equations (3.5) and (3.6)). As long as restructuring does not happen with probability 1, for example, we can have recovery rates greater than 1. These conditions are are not too restrictive when relating the model to the data, and they are satisfied in the numerical examples in this paper.

Figure 3.4: Default probabilities and spreads
Benchmark model with incomplete debt relief ($\tilde{b} = 0.13$) and full debt relief model ($\tilde{b} = 0$)



Debt b is expressed as a fraction of mean annual output. Default probabilities and spreads are at mean output. Spreads are annual.

In addition, the Figure shows how incomplete debt relief decouples spreads and default rates. When default probabilities are 10% for example, spreads are around 22% in the model with full debt relief, but only 15% in the model with incomplete debt relief. This reflects the fact that default risk is mitigated to some extent when recovery rates are non-zero.

We have seen that bond prices fall in default, but not to zero. What determines how far they fall? Suppose the recovery rate is 1. Then debt is essentially risk-free except for the time it takes to restructure. So the floor of the price will be very close to the risk-free price. As recovery rates fall, delinquent debt prices decrease to reflect the increased recovery risk in addition to the risk of delayed payment. For the benchmark calibration, prices fall to around 0.35 which implies an average annual spread in default and restructuring of 52%.

3.5 Calibration

For the quantitative exercises in the paper, I calibrate the model to Argentina at quarterly frequency. Data on debt relief and default episode duration are available for the last two Argentinian defaults. The first episode took place from 1982 to 1993, the second from 2001 to 2005, with an average duration over the two episodes of 7.4 years. Debt relief was 53% on average.

For the income process, I estimate an AR(1) process of linearly detrended, quarterly real GDP from 1980Q1 to 2001Q4 from the Argentinian Ministry of Finance (MECON),

$$\ln y_t = \rho \ln y_{t-1} + \epsilon_t, \quad \epsilon \sim N(0, \sigma_\epsilon^2)$$

and discretize the estimated process to a 200-state Markov chain. For the transitory shock, I assume that

$$m \sim \text{trunc } N(0, \sigma_m)$$

with truncation points $-\bar{m}$ and $\bar{m}$. I set these points as well as the variance of the shock as in Chatterjee and Eyigungor (2012). The important considerations in choosing them are that (i) the standard deviation of the transitory shock must be sufficiently high to ensure convergence of the pricing function, and (ii) the truncation point must be sufficiently small to ensure that in default and exclusion with the output cost, the household still has positive consumption for each output realization.

I specify the persistent component of output in default and exclusion, following Chatterjee and Eyigungor (2012), as

$$\phi(y) = y - \max(0, d_0 y + d_1 y^2)$$

where in the calibration $d_0 < 0$ and $d_1 > 0$. Note that this implies that the output cost rises more than proportionately with output, as in Arellano (2008), which has the effect of making default relatively more costly in booms than recessions.

Period utility is standard CRRA given by

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

The risk-free rate is set to 1%, the average real rate of return on 3-month US Treasury bills. Coupon rates and average maturities are based on Broner et al. (2007). The probability of re-entry in the baseline calibration is set to match an average time between default and settlement for Argentina's default episodes.

Finally, the discount factor β, the two parameters of the output cost function, d_0 and d_1, and the recovered debt level $\tilde{b}$ are calibrated to match (i) Argentina's average debt-to-GDP ratio, (ii) average spreads, (iii) their volatility while not in default or exclusion, and (iv) the average recovery rate of defaulted debt.

The data for these targeted statistics are from the World Bank and Global Financial Data. Spreads are EMBI spreads from the Global Financial Database, the series starts in 1993:4. Debt-to-GDP is annual public and publicly guaranteed debt relative to GDP in US$ from the World Bank GDF database for 1993 to 2001.

The GDF database defines the face value of external debt as the sum of all future undiscounted principal payments. Coupon payments are not treated as an obligation until they are past due. The model analogue of this measure, given the structure of debt contracts, is simply $\frac{b}{y}$, and correspondingly $\frac{\tilde{b}}{y}$ for restructured debt. To see this note that the principal due in the future from b units of bonds outstanding today is given by $\lambda b + (1-\lambda)\lambda b + (1-\lambda)^2 \lambda b + ... = \sum_{s=1}^{\infty} \lambda (1-\lambda)^{s-1} b = b$.

Table 3.1: Parameters

Parameter	Description	Target	Value
γ	Risk aversion		2
r	Risk free rate	US T-bill rate	0.01
z	Coupon rate	Broner et al. (2007)	0.03
λ	Debt maturity of $\frac{1}{\lambda}$ quarters	Broner et al. (2007)	0.05
η	Avg. default length of $\frac{1}{\eta}$ quarters	Benjamin and Wright (2009)	0.034
ρ	Persistence of y	Argentinian GDP	0.945
σ_ϵ	Standard deviation of y	Argentinian GDP	0.025
$\bar{m}$	Bounds on m	Argentinian GDP	0.006
σ_m	Standard deviation of m	Argentinian GDP	0.003

What do haircuts correspond to in the model? In the data, the measure I am using is based on the face value of losses, discounted to the time of default. Therefore I use face values in the model as well, instead of market values. The components of haircuts in the data include interest forgiven, principal forgiven and debt stock reduction/ debt buybacks. In the GDF, principal forgiven and debt reduction/buyback together make up on average 83% of these, and in the case of Argentina's defaults even 100%. In the model, I therefore abstract from interest arrears and do not explicitly distinguish between the other components, so all components are captured by $\tilde{b}$. Putting this together, the recovery rate in the model is simply given by $\frac{\tilde{b}}{b}$ and the haircut, or debt relief, is $1 - \frac{\tilde{b}}{b}$.[14]

The baseline parameterization for the pre-set parameters is summarized in Table (3.1). The parameters that are set to match statistics are shown in the results section along with the targeted moments.

The asset space is discretized to a 350-point equally spaced grid. The computational algorithm for a simpler version of the model with long-term debt but without restructuring is described in detail in Chatterjee and Eyigungor (2012) and generalizes to this model, so

[14] Strictly speaking I need to discount since that is what Benjamin and Wright (2009) say they do. They use a constant rate, so this would just scale things. I will do this as a robustness check, but have not in the results I present below.

I will not repeat it here.

3.6 Simulation Results

In this section I discuss the quantitative properties of the model. Table (3.2) summarizes the simulation results. I simulate the model for 5000 periods and report statistics based on periods of good credit standing (that is, I exclude default and exclusion periods).[15] I compare the model-generated statistics to data from Argentina, and to the model with full debt write-down.

The Argentinian business cycle statistics are based on quarterly, seasonally adjusted, real series on GDP, consumption and net exports from the Argentinian Finance Ministry MECON, starting in 1980:1 (net exports start in 1993:1). In the data, as in the model, output and consumption are in logs and linearly detrended, and net exports are expressed as a fraction of output. The measures of debt, spreads and recovery rates are discussed in the calibration section.

The model matches the data well. It replicates key features of emerging market business cycles: Consumption is more volatile than output, the trade balance is volatile and countercyclical, and spreads are countercyclical. The model with zero recovery rates on the other hand generates debt-to-output ratios that are too low and spreads that are too high and volatile. The default rate also increases compared to the benchmark model.[16] We can see that incomplete debt relief allows the country to borrow more and at a lower price.[17]

3.6.1 The Role of Debt Relief

In this subsection I analyze how the model dynamics vary with the degree of debt relief. I solve and simulate the model for a range of values of $\tilde{b}$ and calculate the implied recovery

[15] Figure (B.1) in the Appendix shows a sample path of the simulations. Simulating for longer or repeatedly did not change the results.

[16] Argentina's default frequency considering just the past 30 years that I have debt relief data on is 6.7%. Over the course of its history since independence in 1811, Argentina defaulted 7 times in total, making for a default rate of around 3.5%. I use the latter in the results but do not target either statistic.

[17] The model with full debt write-down also generates a weakly pro-cyclical trade balance in my benchmark calibration. This is very sensitive to the discount factor, with $\beta = 0.95$ for example the correlation becomes negative.

Table 3.2: Simulation results

	Data	Benchmark Model	Model with full debt write-down
Calibrated parameters:			
$\tilde{b}$		0.13	0.00
β		0.94	0.94
d_0		-0.25	-0.25
d_1		0.31	0.31
Targets:			
Mean recovery rate	0.53	0.53	0.00
Mean spread	0.076	0.077	0.118
Std.dev. of spread	0.035	0.037	0.073
Mean debt-to-GDP ratio	0.25	0.25	0.19
Other statistics:			
$\sigma(c)/\sigma(y)$	1.10	1.10	1.08
$\sigma(NX/y)/\sigma(y)$	0.22	0.36	0.47
$\rho(c, y)$	0.98	0.94	0.90
$\rho(NX/y, y)$	-0.64	-0.11	0.07
$\rho(spread, y)$	-0.88	-0.38	-0.41
Mean debt service	0.053	0.078	0.056
Default probability	0.035	0.057	0.071

rates, business cycle statistics as well as welfare. My welfare measure is certainty equivalent consumption, that is the constant level of consumption c such that

$$\frac{c^{1-\sigma}}{(1-\beta)(1-\sigma)} = \sum_y V(b_{min}, y, 0)\Gamma_y \tag{3.7}$$

where the right-hand side is the expected lifetime value under the minimum amount of debt, no iid shock, and the stationary distribution $\Gamma_{y'}$ of the Markov chain for y.

Table (3.3) shows the results for equilibrium recovery rates between 4 and 107%. Figure (3.5) plots the key series - the recovery rate, debt-to-output ratio, default probability and average spread - for the whole range of recovery levels. The coordinates in the figure corresponding to the benchmark model and the model with full debt-relief are marked with red and green dots, respectively.

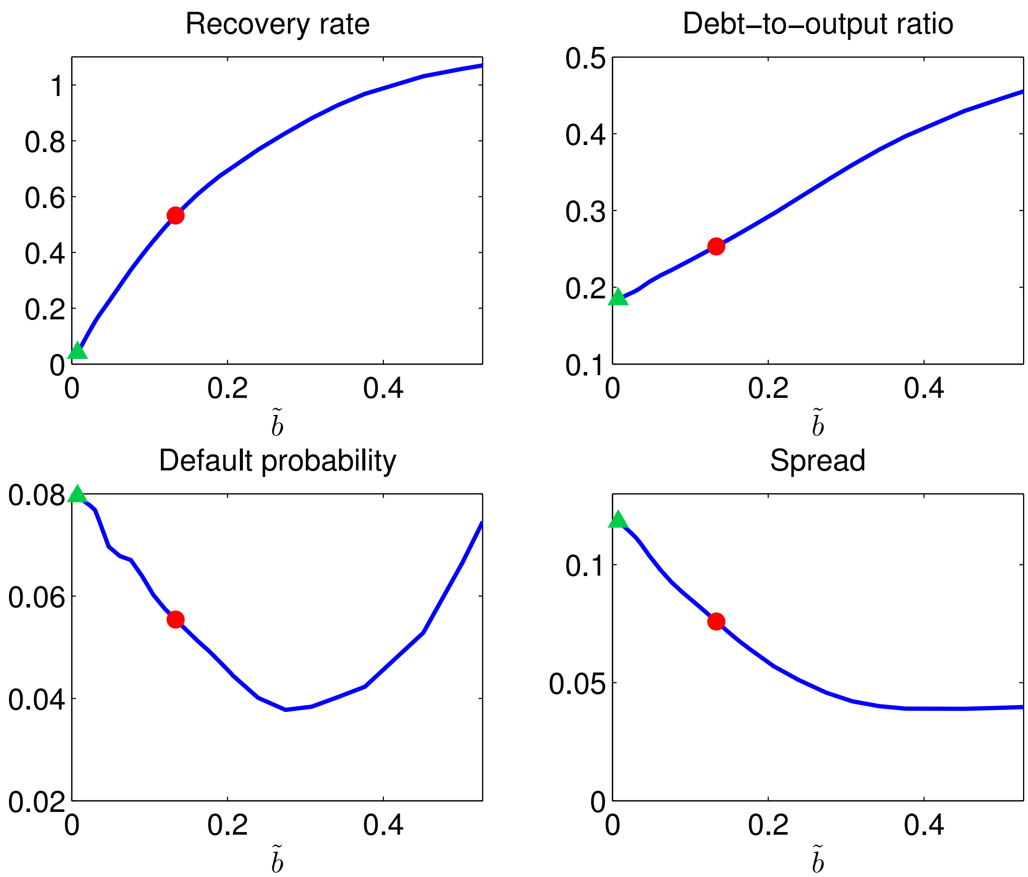
The Figure shows that the degree of debt relief has large effects on the model dynamics. Raising recovery rates from 0 to 1 increases debt ratios by roughly a factor of 2.5. In the model with full debt write-down, the country can only sustain debt of 18% of output, while with recovery rates around 1 this increases to 45%. Average spreads with full debt relief are 3 times as high as spreads with virtually no haircuts: annually 12% compared to 4%.

Default rates, unlike debt ratios and spreads, are non-monotonically related to recovery rates. The minimum of 3.7% occurs at relatively high recovery rates of around 0.9. With no debt relief, countries default 4.5% of the time. It is interesting that default rates at the maximum simulated recovery rate of 1.07 are about as high as at the opposite extreme when there is full debt relief, approximately 8%.

Note that recovery rates, default frequencies and spreads are closely linked. We can see that lower recovery risk offsets some of the effect of default risk, as discussed in the theoretical section above. The guarantee of a haircut less than 100% effectively takes some of the "bite" out of default and drives a wedge between default probabilities and spreads.

The same intuition explains why default rates are non-monotonically related to recovery rates. For recovery rates below 0.9, both higher recovery rates and lower default probabilities act to reduce spreads. For recovery rates above 0.9, however, spreads stop falling as the increase in default rates offsets the effect of higher recovery rates on bond prices. The country effectively borrows with higher risk of default, which is not reflected in bond prices

Figure 3.5: Varying recovery levels



The recovery *level* $\tilde{b}$ on the x-axis is expressed as a fraction of mean annual output. The top left panel shows the recovery *rate*, $\frac{\tilde{b}}{b}$. Spreads are annual. The red dots correspond to the benchmark model, the green triangles to the model with full debt relief.

Table 3.3: Varying the recovery level $\tilde{b}$

	Recovery level $\tilde{b}$				
	0.01	0.05	0.13	0.21	0.53
Mean recovery rate	0.04	0.22	0.53	0.70	1.07
Welfare	1.012	1.015	1.023	1.032	1.054
Mean spread	0.118	0.103	0.076	0.057	0.040
Std.dev. of spread	0.069	0.058	0.037	0.024	0.017
Mean debt-to-output ratio	0.18	0.21	0.25	0.30	0.45
$\sigma(c)/\sigma(y)$	1.08	1.09	1.11	1.12	1.21
$\sigma(NX/y)/\sigma(y)$	0.51	0.43	0.36	0.33	0.40
$\rho(c,y)$	0.89	0.92	0.95	0.96	0.95
$\rho(\frac{NX}{y},y)$	0.09	0.00	-0.12	-0.23	-0.36
$\rho(spread,y)$	-0.37	-0.40	-0.39	-0.35	-0.44
Mean debt service	0.04	0.22	0.53	0.70	1.07
Default probability	0.08	0.07	0.06	0.04	0.07

Welfare is certainty equivalent consumption with $b = m = 0$. The recovery level is debt expressed as a percentage of annual mean output. The recovery rate is $\tilde{b}/b$.

because of the offsetting effect of higher recovery rates. In fact, the lack of increase in bond prices is precisely what enabled the country to sustain revenues from new bond issuances in the first place. Default still happens more frequently with higher levels of debt, because it turns out that cost of defaulting more often are outweighed by the benefit of being able to borrow more.

In terms of welfare, less debt relief is always better in this framework, as shown in row 2 in Table (3.3), and the model suggests that the welfare differences can be large, more than 4% of permanent consumption.

Why is welfare unambiguously higher for higher recovery rates? Note that what I am measuring is ex-ante welfare. The value of default is decreasing in recovery rates, while the value of repaying is increasing in recovery rates for sufficiently low levels of outstanding debt, and including for the minimum level of debt which is what the welfare measure is based on. This drives up ex-ante welfare. Ex-post of course the country does sometimes default and suffer the explicit and implicit costs attached to it.

It is worth noting that the level of recovered debt $\tilde{b}$, which is what is exogenous in my model, cannot be increased indefinitely. Equilibrium recovery rates $\frac{\tilde{b}}{b}$ reach a maximum of 1.07 but then fall back and plateau at 1 for even higher levels of recovered debt. At the same time, default rates keep increasing up to 1. Intuitively, for sufficiently high levels of $\tilde{b}$, a default pushes the country in settlement to a level of debt so high that it is optimal to default again immediately in every state. It will then continually default on this same level of debt and owe it again after settlement. This extreme case is probably not relevant for the analysis here, although it could be interesting. One interpretation is that punishments over and above debt owed, for example in the form of austerity measures that have a similar effect on country's balance sheets, can be counterproductive.

3.7 Extensions

In this section I analyze the interaction between recovery rates and (i) the length of renegotiation and (ii) cyclical conditions at the time of restructuring.

3.7.1 The Trade-off between Restructuring Duration and Debt Relief

In the data, we observe a negative correlation between restructuring duration and recovery rates. In the model, too, there is trade-off between the times it takes to settle a default and the outcome of the default: On the one hand, shorter settlement times make default less costly in terms of output costs and loss of market access. On the other hand, high recovery rates take away from one of the benefits of default, namely debt relief.

We can ask what would have been a restructuring outcome that is welfare-equivalent for Argentina but took less time and involved lower haircuts. Welfare here is defined as in the last section (Equation (3.7)).

Solving and simulate the benchmark model under such a counterfactual restructuring scenario that yields the same level of welfare, I find that Argentina would have been as well off if it had restructured in just 1.8 years and accepted recovery rates of 83%. This corresponds to a 5.6 year reduction in default episode length in exchange for a 26

Table 3.4: Welfare-equivalent counterfactual experiment

	Benchmark Model	Counterfactual
$\tilde{b}$	0.13	0.17
η	0.03	0.13
Welfare	1.023	1.023
Recovery rate	0.53	0.83
Restructuring duration (yrs)	7.40	1.87
Mean spread	0.076	0.050
Std.dev. of spread	0.037	0.015
Mean debt-to-output ratio	0.25	0.21
$\sigma(c)/\sigma(y)$	1.10	1.07
$\sigma(NX/y)/\sigma(y)$	0.36	0.21
$\rho(c, y)$	0.94	0.98
$\rho(NX/y, y)$	-0.11	-0.23
$\rho(spread, y)$	-0.38	-0.28
Mean debt service	0.078	0.065
Default probability	0.057	0.043

percentage point increase in recovery rates. Defaults happen less frequently under the alternative scenario, the government is slightly less heavily indebted and average spreads are lower and less volatile (see Table (3.4).[18] Assuming we are interested in, perhaps, achieving better outcomes for investors, or maintaining lower debt levels or default rates, this scenario seems like the more desirable outcome.

The trade-off is illustrated more generally in Figure (3.6). It plots welfare for models with a range of combinations of restructuring durations η and recovery rates $\frac{\tilde{b}}{b}$. The benchmark model is marked by a red dot.

We can see that, for a given restructuring duration, higher recovery rates are welfare-improving. This is the result discussed in the last section. Conversely, holding recovery rates fixed at a value less than 1, *longer* restructuring durations are better. The intuition is simple: Long restructuring times act as an effective threat. They make default a very unattractive option, and thus, in equilibrium, yield lower default rates, spreads and higher debt levels, and thus higher ex-ante welfare.

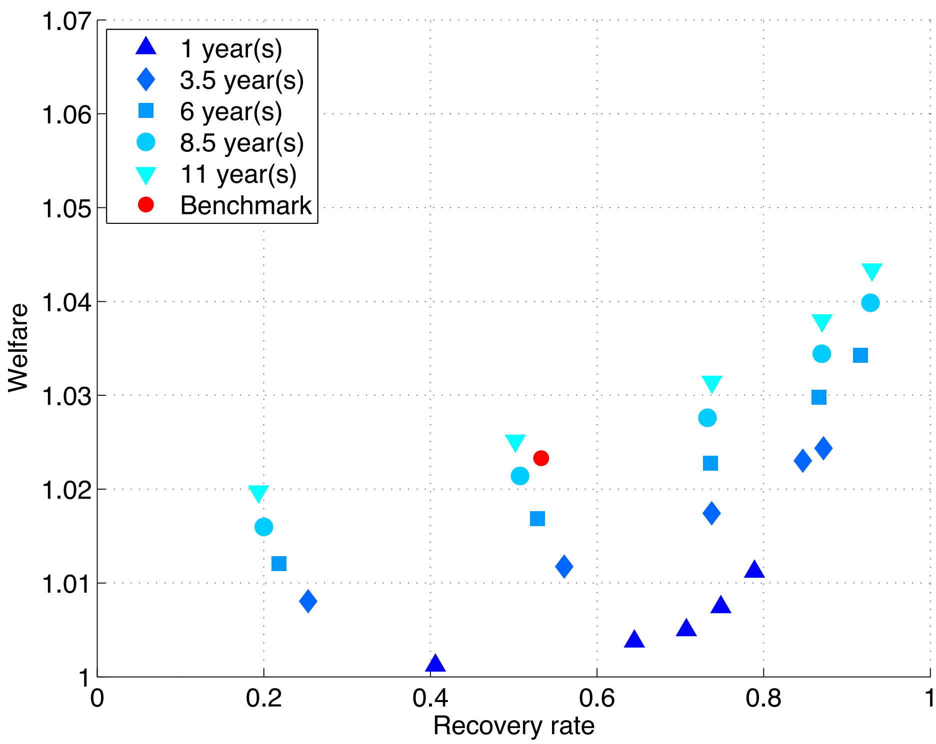
As recovery rates approach 1, however, the country becomes increasingly indifferent between long or short restructuring duration of default. In the extreme, if recovery rates are 1 or higher, the relationship switches and short durations become welfare-improving. This happens because for virtually no debt relief the only relevant consideration is the cost of time spent in autarky. The country has nothing to gain from defaulting in terms of debt relief, so clearly shorter exclusion times are better.

In this framework, therefore, recovery rates are more powerful in affecting equilibrium outcomes than the restructuring duration. Higher recovery rates are strictly better for any given restructuring time, while the choice between long or short renegotiations depends on how high recovery rates are.

This suggests that focusing on enforcing high recovery rates should perhaps be given higher priority than when thinking about desirable features of a robust international financial architecture that aims to mitigate adverse effects of sovereign defaults. Both the prospect of lengthy renegotiations and of little debt relief contribute to better outcomes in this framework, but when facing the choice between them, little debt relief appears to be the more effective tool.

[18] This is not the only possible counterfactual outcome, of course. There are others with smaller reductions in restructuring duration and haircuts - and with larger ones.

Figure 3.6: The trade-off between restructuring duration and debt relief



3.7.2 Cyclical Recovery Rates

In this section, I ask if countries that emerge from default under good macroeconomic conditions should pay back more of what they owe. The assumption that countries that fare poorly should be granted greater debt relief is implicit in many policy discussions. What are the costs of benefits of cyclical recovery rates in the model?

In order to address this question, I extend the benchmark model to allow for recovery levels $\tilde{b}$ to depend on the state y at the time of restructuring. I focus on a simple functional form and analyze varying degrees of cyclicality of recovery rates.

The dynamic program in this case is very similar to the benchmark. The value of the option to default, the value of repaying and the price of current, non-defaulted debt are unchanged from equations (3.1),(3.2) and (3.5) so I will not repeat them here. The value of default changes to reflect the fact that $\tilde{b}$ is now a function of output at the time of re-entering capital markets, $\tilde{b} = H(y)$

$$V^d(y,m) = u(\phi(y) + m) + \beta E_{y',m'|y} \Big[(1-\eta)V^d(b, y', m') + \eta V(H(y'), y', m')\Big] \tag{3.8}$$

Similarly, the value of one unit of defaulted debt is now given by

$$q_D(b,y) = E_{y'|y} \left[(1-\eta)\frac{q_D(b,y')}{1+r} + \eta \frac{\tilde{b}(y')}{b} \frac{q(H(y'),y')}{1+r}\right] \tag{3.9}$$

Note that the level of restructured debt depends on output in the period the country emerges from restructuring. So when making the default decision, and during exclusion, the government forms expectations over the outcome of restructuring rather than knowing it for certain. The definition of equilibrium is unchanged from the baseline model.

I assume that $H(y)$ takes the form

$$\tilde{b} \quad = \quad H(y) = \alpha_0 + \alpha_1 \log y$$

where y is output in log deviations from trend. I solve the model such that the recovery value when output is at trend ($\log y = 0$) is the same as in the benchmark, and for a range of $\alpha_1 \in [-1, 4]$. Negative values of α_1 correspond to procyclical recovered debt levels -

Table 3.5: Cyclical recovery rates

α_1	Recovery rate	Welfare	Mean spread	Std.dev. of spread	Debt	Default prob.
-1	0.46	1.019	0.093	0.050	0.25	0.069
0	0.53	1.023	0.076	0.037	0.25	0.057
1	0.60	1.028	0.058	0.024	0.26	0.045
4	0.83	1.037	0.037	0.011	0.30	0.039

Debt is expressed as a fraction of annual output. $\alpha_0 = -0.133$. $\alpha_1 < 0$ means that debt relief is procyclical.

more debt relief in bad times. For example, $\alpha_1 = -1$ implies that an increase in output 10% above trend is associated with 1 percentage point more recovered debt (recall that $\tilde{b} < 0$ is debt and that it is expressed as a fraction of mean output).[19]

Table (3.5) compares the model outcomes for a range of cyclical recovery rates. Consistent with the earlier findings, procyclical recovery rates are not welfare-improving. It is better not to give the sovereign more lenient restructuring terms in a recession. The intuition is as before: Acyclical or even countercyclical recovery rates increase the ex-ante cost of default and thus result in equilibrium in low default rates, low spreads and high levels of borrowing.

3.8 Conclusion

In this paper I have developed a framework to think about the costs and benefits of debt relief and its interaction with two other important aspects of sovereign default - restructuring length and cyclical conditions. Less debt relief, or equivalently a low haircut for investors, turns out to have positive effects on default rates, spreads and welfare in equilibrium. Incomplete debt relief lowers recovery risk and thus acts to offset the negative effects of pure default risk on spreads and the sovereign's ability to borrow. From the country's perspective, little debt relief is a deterrent that makes default a less attractive option.

[19] In the data, the OLS estimates of the coefficients are $\alpha_0 = -0.57$ (significant at 1% level) and $\alpha_1 = -0.42$ (not statistically significant, p-value 0.3). This reflects that countries in the cross-section exit default with a median debt-to-GDP ratio of 51%, GDP on average 6% below trend, and the level of restructured debt not being significantly cyclical.

This effects turns out to be stronger than the prospect of protracted renegotiations. And consistently with these findings, countercyclical recovery rates are not welfare-improving in the model because they increase equilibrium recovery and default risk by making default relatively more appealing.

The model has implications for policy and in particular the design of an international financial architecture that prevents defaults or lessens its adverse effects. Given the large apparent costs of sovereign defaults in the data, identifying factors that contribute to better sovereign default outcomes is important. The framework in this paper suggests that recovery rates have large effects on default outcomes and influencing them should therefore be a policy priority. Lengthy renegotiations are relevant, but less effective than the prospect of little debt relief. Not allowing for higher haircuts because of weak cyclical conditions would also help equilibrium outcomes.

It would be interesting to extend the current framework to think about other dimensions of debt restructuring, for example changes to maturity and coupon structures, as well as considering the implementation of effective policies by modeling restructuring endogenously. This is left for future work.

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Appendix A

Appendix to Chapter 2

A.1 Equilibrium - Simplified Model

Assume households have preferences over cash and credit good consumption and supply labor inelastically: $U = \sum_t \beta^t u(c_1, c_2)$. Assume the government can choose to default partially every period, where default incurs a resource cost $t(d)$ with $t(0) = 0$ and $t_d > 0$.

Nominal Bonds

The household faces the following constraints

$$pc + (1+\mu)(m' + q_n B') = pn + m + (1-d)B$$

$$pc \leq m$$

The competitive equilibrium first order conditions imply

$$\begin{aligned} u_c - u_l &\geq 0 \\ \mu &= \beta \frac{u_1' p}{u_2 p'} - 1 \\ q_n &= \beta \frac{u_2'}{u_2} \frac{p}{p'(1+\mu)}(1-d') \\ &= \frac{u_2'}{u_1'}(1-d') \end{aligned}$$

Note that $c_1 + c_2 + g + t(d) = n = 1$.

The government faces the budget constraint

$$1 + (1-d)B + pg - pt(d) = (1+\mu)(1 + q_n B')$$

which can be rewritten in terms of allocations as

$$G(B, B', p, P(B'), d, D(B)) \equiv -u_2 \left[\frac{(1 + (1-d)B)}{p} + g + t(d) \right] + \beta \frac{(u_1' + u_2'(1 - D(B'))B')}{P(B')}$$

Definition 3. *Let* $V : \mathbb{B} \to \mathbb{R}, P : \mathbb{B} \to \mathbb{R}_{++}, H : \mathbb{B} \to \mathbb{B}$ *and* $D : \mathbb{B} \to [0,1]$. *A Markov perfect equilibrium of the simplified economy with nominal debt are functions* V, P, D, H *such that*

$$V(B) = \max_{p,d,B'} u\left(\frac{1}{p}, 1 - \left(\frac{1}{p} + g + t(d)\right)\right) + \beta V(B')$$

subject to

$$\begin{aligned} G(B, B', p, P(B'), d, D(B)) &= 0 \\ F(p,d) \equiv u_1(p,d) - u_2(p,d) &\geq 0 \\ B' &= H(B) \end{aligned}$$

Real Bonds

The household faces the following constraints

$$pc + (1+\mu)m' + q_r b' = pn + m + (1-d)b$$

$$pc \leq m$$

The competitive equilibrium first order conditions imply

$$\begin{aligned} u_c - u_l &\geq 0 \\ \mu &= \beta \frac{u_1' p}{u_2 p'} - 1 \\ q_r &= \beta \frac{u_2'}{u_2}(1-d') \\ &= \frac{u_2'}{u_1'}(1-d')\frac{p'(1+\mu)}{p} \end{aligned}$$

Note that $c_1 + c_2 + g + t(d) = n = 1$.

The government faces the budget constraint

$$1 + (1-d)b + pg - pt(d) = (1+\mu) + q_r b'$$

which can be rewritten in terms of allocations as

$$G(b, b', p, P(b'), d, D(b)) \equiv -u_2 \left[\frac{1}{p} + (1-d)b + g + t(d)\right] + \beta \left(\frac{u_1'}{P(b')} + u_2'(1 - D(b'))b'\right)$$

Definition 4. *Let $V : \mathbb{B} \to \mathbb{R}, P : \mathbb{B} \to \mathbb{R}_{++}, H : \mathbb{B} \to \mathbb{B}$ and $D : \mathbb{B} \to [0,1]$. A Markov perfect equilibrium of the simplified economy with real debt are functions V, P, D, H such that*

$$V(b) = \max_{p,d,b'} u\left(\frac{1}{p}, 1 - \left(\frac{1}{p} + g + t(d)\right)\right) + \beta V(b')$$

subject to

$$\begin{aligned} G(b, b', p, P(b'), d, D(b)) &= 0 \\ F(p,d) \equiv u_1(p,d) - u_2(p,d) &\geq 0 \\ b' &= H(b) \end{aligned}$$

First Order Conditions

$$
\begin{aligned}
-\frac{u_c-u_l}{p^2}+\lambda G_p+\gamma F_p &= 0\\
-u_l T_d+\lambda G_d+\gamma F_d &= 0\\
\lambda\left(G_{B'}+G_P\frac{\partial P(B')}{\partial B'}+G_D\frac{\partial D(B')}{\partial B'}\right)+\beta\lambda' G'_B &= 0
\end{aligned}
$$

where for nominal debt

$$
\begin{aligned}
G_p^N &= u_l\frac{(1+(1-d)B)}{p^2}+(u_{lc}-u_{ll})\,\frac{1}{p^2}\left[\frac{(1+(1-d)B)}{p}+g+T(d)\right]\\
G_d^N &= u_l\frac{B}{p}-u_l T_d+u_{ll}T_d\left[\frac{(1+(1-d)B)}{p}+g+T(d)\right]
\end{aligned}
$$

and

$$
\begin{aligned}
G_P &= -\frac{\beta}{P(B')}\left(\frac{u'_c}{P(B')}+\frac{u'_l(1-D(B'))B'}{P(B')}+\frac{u'_{cc}-u'_{cl}}{P(B')^2}+(1-D(B'))\frac{B'}{P(B')}\frac{(u'_{cl}-u'_{ll})}{P(B')}\right)\\
&= -\frac{\beta}{P(B')}\left(\frac{u'_c}{P(B')}+\frac{u'_l(1-D(B'))B'}{P(B')}\right.\\
&\quad+\frac{1}{P(B')}\left[\frac{u'_{cc}}{P(B')}-u'_{ll}(1-D(B'))\frac{B'}{P(B')}\right]\\
&\quad\left.+\frac{u'_{cl}}{P(B')}\left[-\frac{1}{P(B')}+(1-D(B'))\frac{B'}{P(B')}\right]\right)\\
G_D &= -\beta\frac{B'}{P(B')}(u'_l+u'_{ll}t_D(1-D(B')))
\end{aligned}
$$

while for real debt

$$
\begin{aligned}
G_p^R &= \frac{u_l}{p^2}+(u_{lc}-u_{ll})\,\frac{1}{p^2}\left[\frac{1}{p}+(1-d)b+g+T(d)\right]\\
G_d^R &= u_l b-u_l T_d+u_{ll}T_d\left[\frac{1}{p}+(1-d)b+g+T(d)\right]
\end{aligned}
$$

and

$$\begin{aligned}
G_P &= -\frac{\beta}{P(b')}\left(\frac{u'_c}{P(b')} + \frac{u'_{cc} - u'_{cl}}{P(b')^2} + (1 - D(b'))b'\frac{(u'_{cl} - u'_{ll})}{P(b')}\right) \\
&= -\frac{\beta}{P(b')}\left(\frac{u'_c}{P(b')}\right. \\
&\quad +\frac{1}{P(b')}\left[\frac{u'_{cc}}{P(b')} - u'_{ll}(1 - D(b'))b'\right] \\
&\quad \left.+\frac{u'_{cl}}{P(b')}\left[-\frac{1}{P(b')} + (1 - D(b'))b'\right]\right) \\
G_D &= -\beta b'(u'_l + u'_{ll}t_D(1 - D(b')))
\end{aligned}$$

A.2 Equilibrium - Full Model

The government's problem can be expressed purely in terms of allocations by substituting out competitive equilibrium conditions. An alternative equilibrium definition therefore is:

Definition 5. *Let $V : \mathbb{B} \times \mathbb{Z} \to \mathbb{R}, P : \mathbb{B} \times \mathbb{Z} \to \mathbb{R}_{++}, H : \mathbb{B} \times \mathbb{Z} \to \mathbb{B}$ and $D : \mathbb{B} \times \mathbb{Z} \to \{0, 1\}$. Define also $\bar{z}(H(B))$ as the highest $z \in \mathbb{Z}$ for which the government chooses not to default. Then a Markov perfect equilibrium of the economy with nominal debt are functions V, P, D, H and a corresponding $\bar{z}(H(B))$ such that*

$$V(B, z) = \max\left\{V^r(B, z), V^d(z)\right\}$$

where the value of repayment is

$$V^r(B, z) = \max_{p,B'} u\left(\frac{1}{p}, n(p) - \frac{1}{p} - g, 1 - n(p)\right) + \beta\int_{z'} V(B', z')dF(z', z)$$

subject to

$$\begin{aligned}
u_2\left[\frac{(1 + B)}{p} + g - n(p)\right] + u_l n(p) &= \beta\left(\mathbb{E}\left[\zeta_1(B', z')\right] + \mathbb{E}\left[\zeta_{2n}(B', z')\right]B'\right) \\
u_c - u_l &\geq 0 \\
u_l(n(p)) &= (1 - \tau)u_2(p) \\
B' &= H(B)
\end{aligned}$$

where

$$
\begin{aligned}
\mathbb{E}\left[\zeta_1(B', z')\right] &= \int_{z' \leq \bar{z}'} \frac{(u_1^r)'}{P^r(B', z')} f(z', z) dz' + \int_{z' > \bar{z}'} \frac{\left(u_1^d\right)'}{P^d(z')} f(z', z) dz' \qquad \text{(A.1)} \\
\mathbb{E}\left[\zeta_{2n}(B', z')\right] &= \int_{z' \leq \bar{z}'} \frac{(u_2^r)'}{P^r(B', z')} f(z', z) dz' + \int_{z' > \bar{z}'} \frac{\left(u_2^d\right)'}{P^d(z')} f(z', z) dz'
\end{aligned}
$$

and the value of default is

$$
V^d(z) = \max_p u\left(\frac{1}{p}, n(p) - \frac{1}{p} - g^d, 1 - n(p)\right) + \beta \int_{z'} \left[\eta V(0, z') + (1-\eta) V^d(z')\right] dF(z', z)
$$

subject to

$$
\begin{aligned}
u_2\left[\frac{1}{p} + g^d - n(p)\right] + u_l n(p) &= \beta\left(\eta \mathbb{E}\left[\zeta_1(0, z')\right] + (1-\eta)\mathbb{E}\left[\zeta_1^d(z')\right]\right) \\
u_c - u_l &\geq 0 \\
u_l(n(p)) &= (1-\tau) u_2(p) \\
B' &= H(B)
\end{aligned}
$$

where ζ_1 *is defined as in A.1, and if the country stays in default*

$$
\mathbb{E}\left[\zeta_1^d(z')\right] = \int_{z'} \frac{\left(u_1^d\right)'}{P^d(z')} f(z', z) dz'
$$

and P *and* D *satisfy*

$$
P(B, z) = \mathbb{1}\left(V^r(B, z) \geq V^d(z)\right) P^r(B, z) + \mathbb{1}\left(V^r(B, z) < V^d(z)\right) P^d(z)
$$

$$
D(B, z) = \mathbb{1}\left(V^r(B, z) < V^d(z)\right)
$$

Bond prices and money growth rates are given by, respectively

$$
\begin{aligned}
q_n(B', z) &= \frac{\mathbb{E}[\zeta_{2n}(B', z')]}{\mathbb{E}[\zeta_1(B', z')]} \\
q_r(b', z) &= \frac{\mathbb{E}[\zeta_{2r}(b', z')]}{\mathbb{E}[\zeta_1(b', z')]} \frac{(1 + \mu(b', z))}{p} \\
&= \beta \frac{\mathbb{E}[\zeta_{2r}(b', z')]}{u_2} \\
\mu(B', z) &= \beta \frac{\mathbb{E}[\zeta_1(B', z')]}{\frac{u_2}{p}} - 1
\end{aligned}
$$

A.3 Proofs

Claim. Consider the simple model (defined in Appendix A). Suppose utility is logarithmic in consumption and linear in leisure, and suppose default costs are linear $t(d) = \chi_1 d$. Consider a given steady state level of debt that is identical in the real and nominal debt economies: $b_r = b_n \equiv \frac{B}{p_n}$. Then inflation is higher in the nominal debt economy if and only if default rates are higher.

Proof. Suppose utility is log in consumption and linear in leisure: $u = \log c + \alpha l$ where $l = 1 - n$. Bond prices and money growth rates are determined in the competitive equilibrium and given by

$$
\begin{aligned}
\mu &= \beta \frac{u_1' p}{u_2 p'} - 1 \\
&= \beta \frac{p}{\alpha}
\end{aligned}
$$

and

$$
\begin{aligned}
q_n &= \beta \frac{u_2'(1 - d')p}{u_2 p'(1 + \mu)} \\
&= \frac{u_2'(1 - d')}{u_1'} \\
&= \frac{\alpha(1 - d')}{p'}
\end{aligned}
$$

The budget constraint for the government then is

$$\frac{1}{p} + \frac{(1-d)B}{p} + g + t(d) = \frac{1+\mu}{p}\left(1 + q_n B'\right) \tag{A.2}$$

$$\frac{1}{p} + \frac{(1-d)B}{p} + g + t(d) = \frac{\beta}{\alpha}\left(1 + \frac{\alpha(1-d')B'}{p'}\right) \tag{A.3}$$

The other constraint is the intratemporal condition relating default and inflation:

$$\frac{u_1 - u_2}{p^2 u_2 t_d} = \frac{G_p}{G_d}$$

which under our assumptions reduces to

$$\frac{p-\alpha}{\alpha t_d} = \frac{1 + (1-d)B}{-\frac{B}{p} + t_d} \tag{A.4}$$

The analogous equations for a real debt economy are given by

$$\begin{aligned} q_r &= \beta \frac{u_2'(1-d')}{u_2} \\ &= \frac{u_2'(1-d')}{u_1'} \frac{(1+\mu)p'}{p} \\ &= \alpha(1-d')\frac{1+\mu}{p} \end{aligned}$$

$$\begin{aligned} \frac{1}{p} + (1-d)b + g + t(d) &= \frac{1+\mu}{p} + q_r b' \\ \frac{1}{p} + (1-d)b + g + t(d) &= \frac{\beta}{\alpha}\left(1 + \alpha(1-d')b'\right) \end{aligned}$$

and

$$\frac{p-\alpha}{\alpha t_d} = \frac{1}{-b + t_d}$$

Use subscripts to denote objects from the two different types of economy (n for nominal and r for real). Now consider the steady state of these economies with the same real value of debt taken as given, $b_r = b_n \equiv \frac{B}{p_n}$. Then we can solve two equations in two unknowns for equilibrium prices and default rates: For nominal debt

$$\begin{aligned}\frac{1}{p_n} + (1-\beta)(1-d_n)b_n + t(d_n) &= \frac{\beta}{\alpha} - g \\ (p_n - \alpha)(t_{dn} - b_n) &= \alpha t_{dn}(1 + (1-d_n)p_n b_n)\end{aligned}$$

and for real debt

$$\begin{aligned}\frac{1}{p_r} + (1-\beta)(1-d_r)b_r + t(d_r) &= \frac{\beta}{\alpha} - g \\ (p_r - \alpha)(t_{dr} - b_r) &= \alpha t_{dr}\end{aligned}$$

Suppose $p_r < p_n$. Then we must have

$$-d_n b + t(d_n) \quad < \quad -d_r b + t(d_r)$$

from the budget constraint. The net cost from inflating has to be lower in the nominal case. Under the assumption of linear default costs, this implies $d_r < d_n$ provided debt is not too low and the default cost is not too high, that is provided $b - \chi > 0$. This will hold in any equilibrium by the intratemporal FOC. Note that if $b - \chi < 0$, then the left hand side of the intratemporal condition is negative. But the right hand side is strictly positive for $d > 0$. □

More generally, for different functional forms, provided the cost of defaulting is not disproportionately larger than the debt burden, we have that $d_r < d_n$. This is in general true the higher debt, and the lower the default cost.

A.4 Data

A.4.1 Sources and Coverage

All data run from 1990 through 2012 unless otherwise noted. Annual GDP data in current and constant US$ are from the WDI. The constant price series for Argentina ends in 2006. Monthly CPI inflation is from the IFS (except for the index for China which is from Global Financial Data). The series start later than January 1990 for Romania (Oct 1991), Ukraine (Jan 1993), Russia (Jan 1993) and Vietnam (Jan 96). Exchange rate data are from Global Financial Data. The series are daily except for Russia (weekly until Jan 1992) and Romania (monthly until Feb 1990). Bond data are from Bloomberg. The default data are based on Standard & Poor's from 1990 to 2006, and extended thereafter through various other sources (news articles etc.). Quarterly data on aggregate government debt securities are from the BIS debt securities database. They are the sum of domestic and international debt securities; the international series is only available from 1993Q3. Annual general government debt is from the IMF World Economic Outlook.

Table A.1: Countries, regions and first bond data observations in my sample

Latin America		Asia	
Argentina	15 Oct 1992	China	2 Jan 1990
Brazil	2 Jan 1990	Indonesia	30 Jul 1996
Chile	6 Dec 1994	India	2 Jan 1990
Colombia	8 May 1992	Korea	2 Jan 1990
Ecuador	8 May 1997	Malaysia	2 Jan 1990
Mexico	13 Mar 1991	Pakistan	6 Mar 1991
Peru	9 Mar 1998	Philippines	2 Jan 1990
Venezuela	15 Jan 1991	Thailand	2 Jan 1990
		Vietnam	28 Jul 2000

Europe		Africa	
Bulgaria	29 Jan 1998	Egypt	22 May 1997
Hungary	2 Jan 1990	Morocco	18 Mar 1993
Poland	12 Mar 1994	South Africa	2 Jan 1990
Romania	28 May 1996		
Russia	14 May 1993		
Turkey	2 Jan 1990		
Ukraine	21 Mar 1995		

A.4.2 Regression Evidence

Regressions results documenting the negative relationship between nominal debt shares and inflation and default, respectively, are shown in Table (A.2). I estimate a linear model with country fixed effects for inflation rates

$$\log \text{inflation}_{it} = \alpha_i + \beta x_{it} + \epsilon_{it}$$

where x_{it} is the nominal debt share.

For default probabilities I use a simple logit model

$$\text{logit}P(d_{it} = 1|x_{it}, \zeta_i) = \alpha + \beta x_{it}$$

and also estimate two other versions where I allow for either random or fixed effects.

The tables show a significant negative relationship between nominal debt shares and both inflation and default. In terms of magnitude, the left panel shows that issuing nominal instead of real debt increases inflation by 1.5 log points. This corresponds roughly to a change from 5% annual inflation when all debt is nominal to around 20% when it is real. With year fixed effects the effect is somewhat smaller but still significant at 10%. The right panel of the Table shows that the probability of default is 90% lower for a country that issues all nominal instead of all real debt (I report the odds ratio of 0.118 in square brackets). Allowing for country random effects the magnitude of this effect shrinks to 50% but remains significant.

Table A.2: Inflation and default probabilities

	Log inflation	
Nominal debt share	-1.435**	-0.896*
	(0.580)	(0.473)
N	6220	6220
adj. R^2	0.434	0.570
Time FE	No	Yes
Country	FE	FE

		Default	
Nominal debt share	-2.138***	-0.675***	-0.649***
	(0.621)	(0.205)	(0.205)
	[0.118]	[0.509]	[0.523]
N	6418	6418	2663
Country	-	RE	FE

Standard errors in parentheses * p¡0.10 ** p¡0.05 *** p¡0.01 Odds ratios in square brackets

A.4.3 Additional Figures

Figure A.1: Aggregate bond debt in my sample of countries

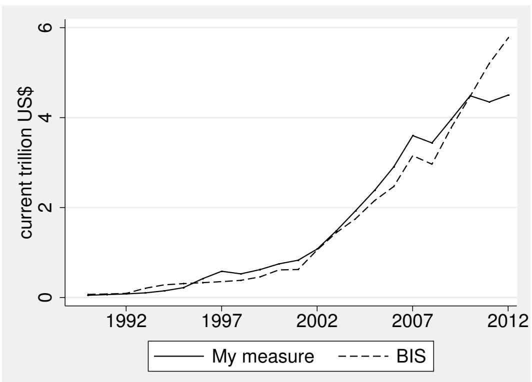
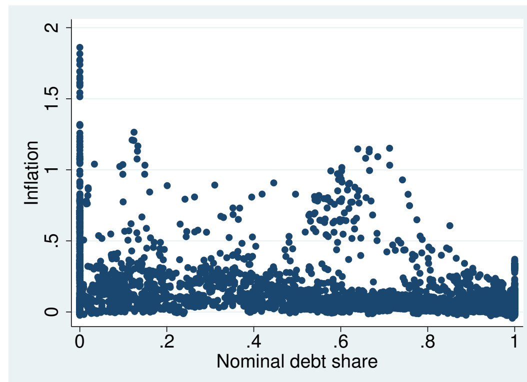


Figure A.2: Raw scatter plot of inflation against nominal debt share



Appendix B

Appendix to Chapter 3

B.1 Data

B.1.1 Cross Section Data

The set of countries I analyze is constrained by the availability of haircut estimates. I rely on Benjamin and Wright (2009), but drop Rwanda, Myanmar and Vietnam from their original sample (the first two were in conflict much of the sample, for the latter data availability during its default episode is limited).

The estimates of restructuring duration are also from their paper.

The remaining data is from the World Bank Global Development Finance (GDF) database:

- Public and publicly guaranteed debt (current US$). Series code: DT.DOD.DPPG.CD
- Debt service on external debt, public and publicly guaranteed (PPG) (TDS, current US$). Series code: DT.TDS.DPPG.CD
- Average maturity on new external debt commitments (years). Series code: DT.MAT.DPPG
- Average interest on new external debt commitments (%). Series code: DT.INR.DPPG
- GDP (current US$): NY.GDP.MKTP.CD
- GDP (constant LCU): NY.GDP.MKTP.KN

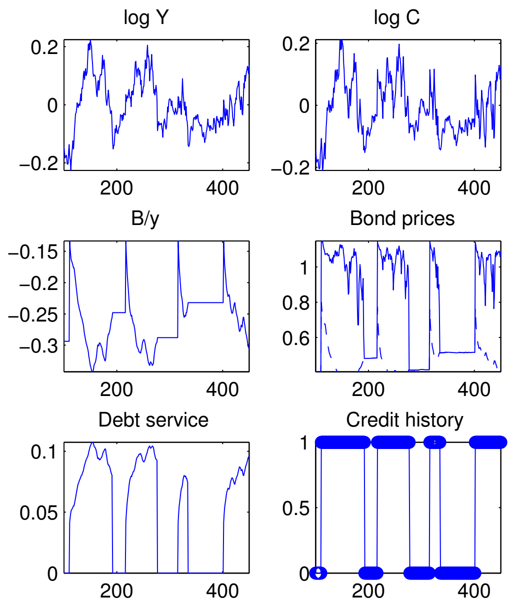
All series are annual from 1970-2010.

B.1.2 Argentinian Business Cycle Data

- Debt and debt service data are from the World Bank GDF as above
- Bond spread data for Argentina are EMBI+ spreads from Global Financial Data. The series runs from 1993-2012, frequency is monthly pre-1998, daily from 1998 on, I take quarter averages. Series code: EMBPARGD
- National accounts data for Argentina are quarterly, seasonally adjusted, real series on GDP, consumption and net exports from the Argentinian Finance Ministry MECON, starting in 1980:1 (net exports start in 1993:1). Output and consumption are in logs and linearly detrended, and net exports are expressed as a fraction of output

B.2 Additional Results - Simulated Paths

Figure B.1: Sample simulation paths for the benchmark model



The dashed line in the panel for bond prices is the price of defaulted debt. Debt service and spreads are expressed in percent annually. Debt is expressed as a fraction of annual output. Credit history = 1 means good credit standing, credit history = 0 means default or exclusion.