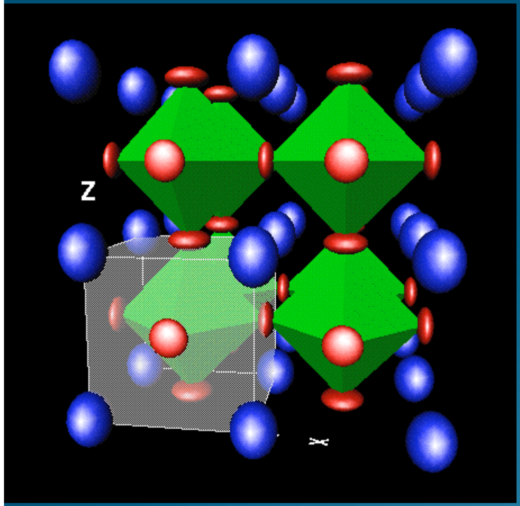
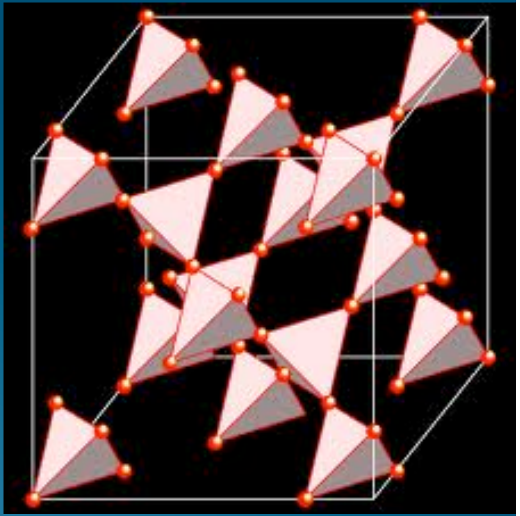
Topological Phases in Complex Oxide Interfaces and Heterostructures

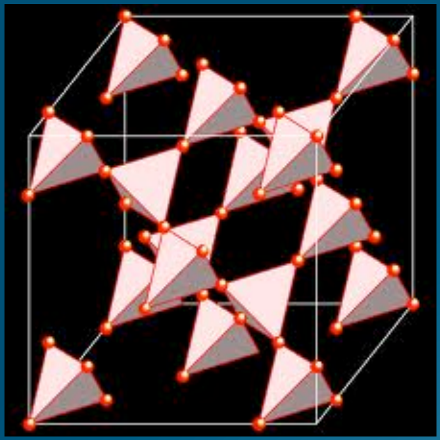


Gregory A. Fiete
University of Texas at Austin

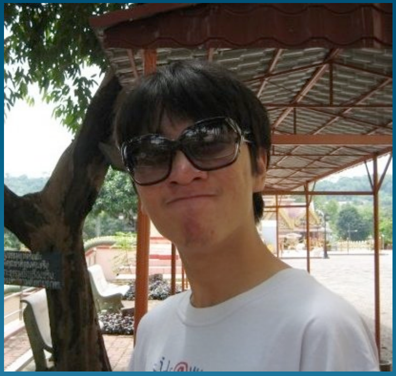


Ruegg, Fiete *Phys. Rev B (Rapid)* **84**, 201103 (2011)
Ruegg, Mitra, Demkov, Fiete *Phys. Rev. B* **85**, 245131 (2012)
Hu, Ruegg, Fiete *Phys. Rev. B* **86**, 235141 (2012) [Editor's Suggestion]
Ruegg, Mitra, Demkov, Fiete *Phys. Rev. B* **88**, 115146 (2013)

Topological Phases in *Bulk* Pyrochlore Oxides: TCI, TCMI, and TI* (and TCI*)



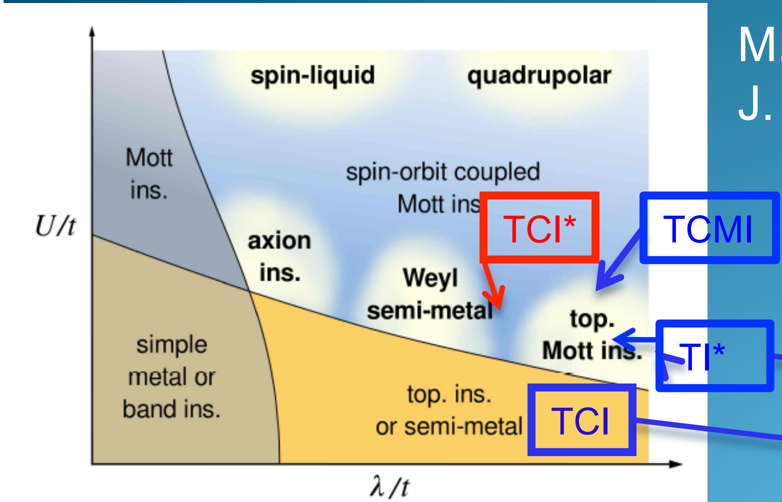
Mehdi Kargarian



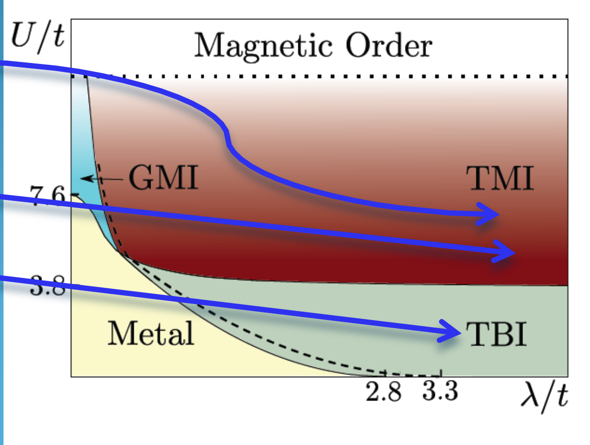
Victor Chua

Joseph Maciejko

M. Kargarian and GAF, *Phys. Rev. Lett*. (2013)
J. Maciejko, V. Chua, and GAF *Phys. Rev. Lett*. (2014)



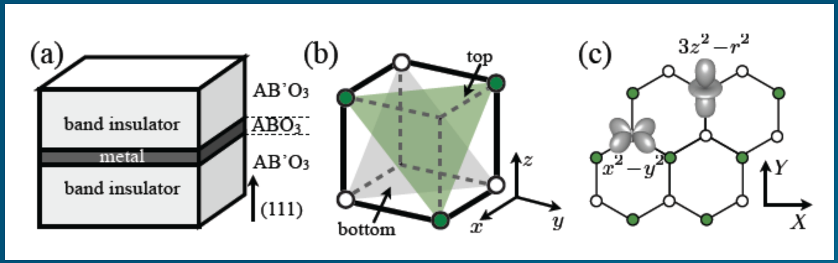
W. Witczak-Krempa, G. Chen, Y.-B. Kim, L. Balents 2014

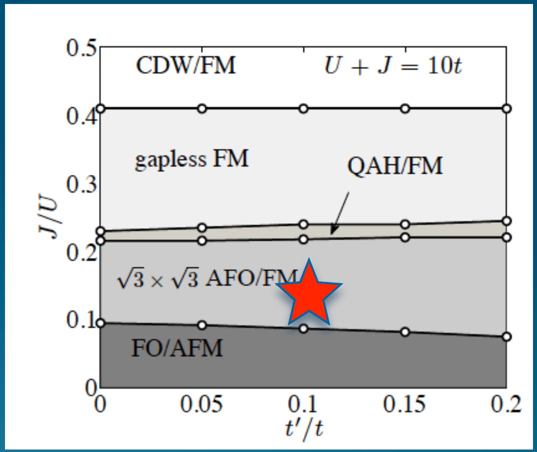


Pesin & Balents
Nat. Phys. 2010

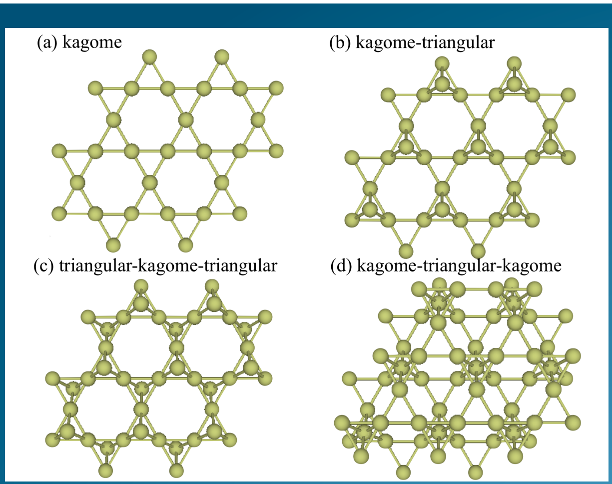
Topological Phases in Complex Oxide Heterostructures

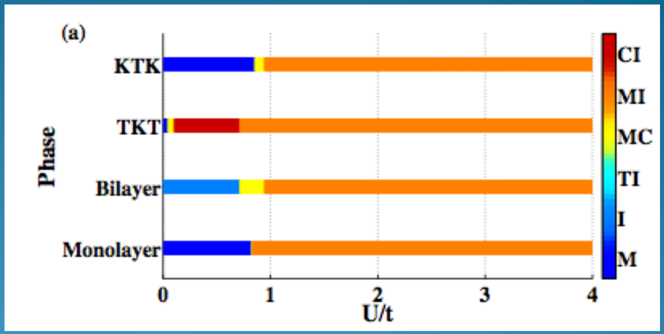
$LaAlO_3/LaNiO_3/LaAlO_3$






Andreas Ruegg






Xiang Hu


Chandrima Mitra

Ruegg, Fiete *Phys. Rev B (Rapid)* **84**, 201103 (2011)
Ruegg, Mitra, Demkov, Fiete *Phys. Rev. B* **85**, 245131 (2012)
Hu, Ruegg, Fiete *Phys. Rev. B* **86**, 235141 (2012) [Editor's Suggestion]
Ruegg, Mitra, Demkov, Fiete *Phys. Rev. B* **88**, 115146 (2013)

Outline

- Brief review of topological insulators:
  -Experiment and Theory

- ABO_3 and $A_2M_2O_7$ interfaces along [111]:
  -Time-reversal invariant TI
  -Time-reversal breaking QAH

- Promising experimental platforms for realizing interacting topological phases: oxides and “engineered structures” derived from them.

ARPES Identified Topological Insulators (3-d):
Bi_2Se_3, Bi_2Te_3, $TlBiSe_2$, Bi_2Te_2Se

- Bi_2Se_3

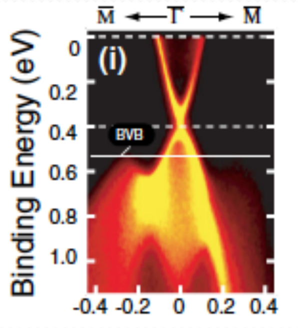
  Xia *et al*. Nat. Phys. **5**, 398 (2009)

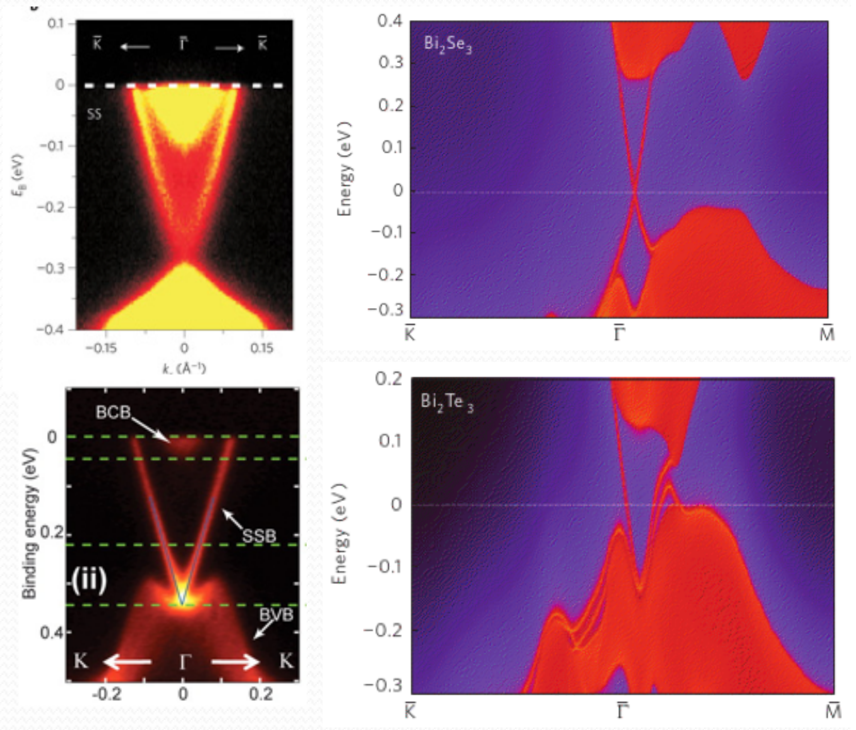
- Bi_2Te_3

  Chen *et al*. Science **325**, 178 (2009)

- $TlBiSe_2$

  Sato *et al*. PRL **105**, 136802 (2010)
  Kuroda *et al*. PRL **105**, 146801 (2010)





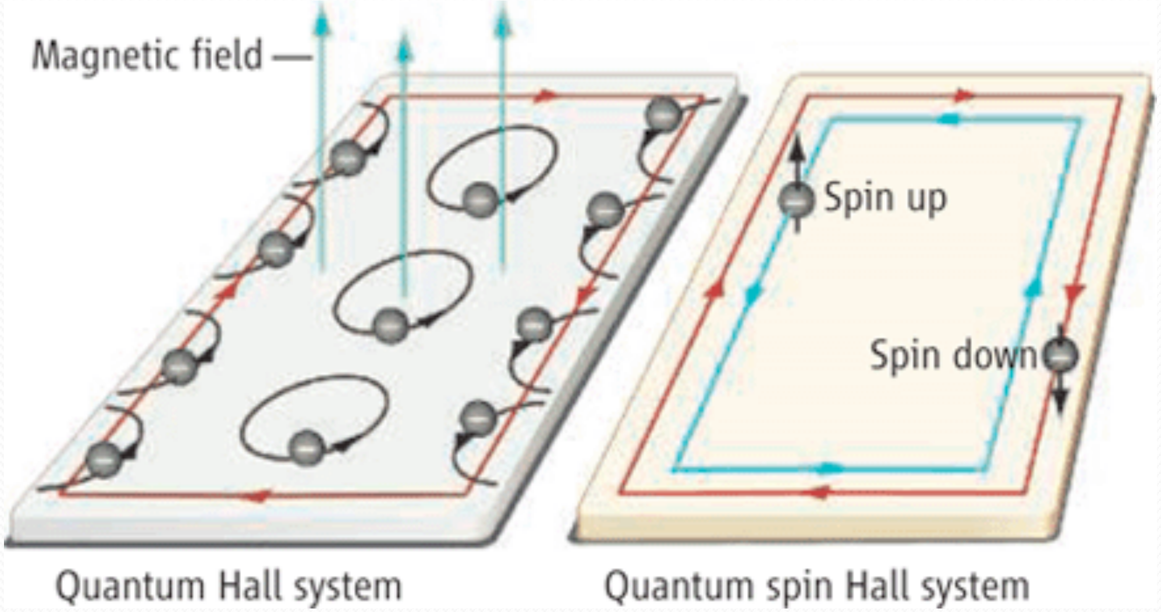
DFT: Zhang *et al.* Nat. Phys. **5**, 438 (2009)

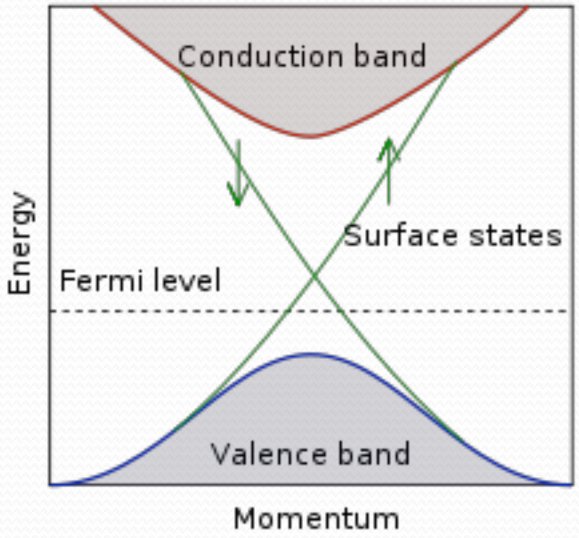
- Bi_2Te_2Se: Single Dirac cone & most insulating bulk

  Z. Ren *et al.* PRB **82**, 241306 (2010)

  (>70% surface conductivity)

Quantum spin Hall systems are time-reversal invariant cousins of Quantum Hall systems driven by spin-orbit coupling





QAH if B=0

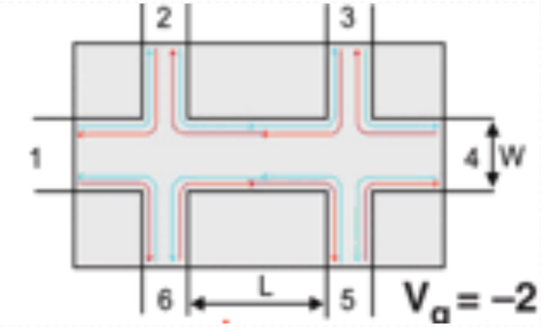
B=0

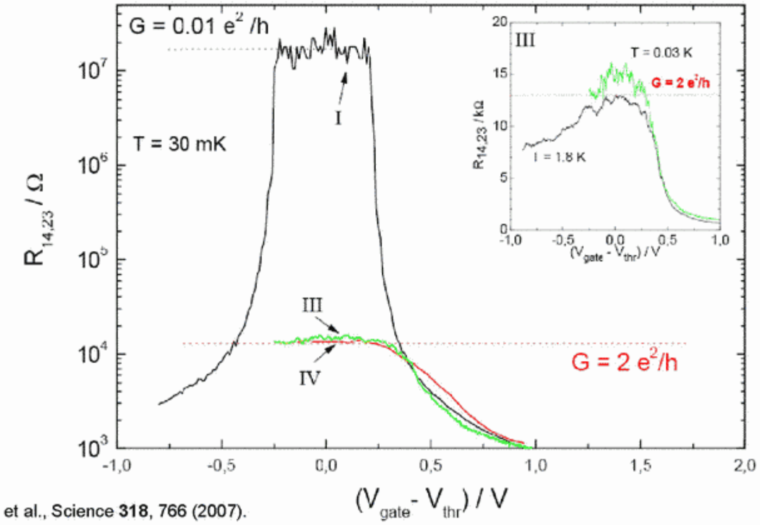
Common feature is *dissipationless edge transport*: simplest way to experimentally identify phases

Transport Identified Topological Phases (2-d):

- HgCdTe quantum wells (TI)

M. Konig *et al*. Science **318**, 766 (2007)





M. König et al., Science **318**, 766 (2007).

- Width-independent conductance indicates transport along edge
- QAH state can be shown by measuring quantized Hall conductance in B=0

Experimentally observed in Cr-(Bi,Sb)$_2$Te$_3$: C.-Z. Chang,…,Q.-K. Xue *Science* (2013)

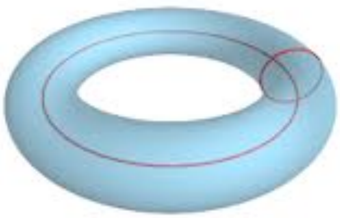
Topological Classification of 2-d Systems: Chern Number and Z_2 invariant

- Z invariant, Chern number (IQHE and QAH):

$$\nu = \frac{1}{2\pi}\int_{BZ} dk_x dk_y (\vec{\nabla}_{\mathbf{k}} \times \vec{A})_z$$

$$\vec{A} = -i\langle u_{\mathbf{k}}|\vec{\nabla}_{\mathbf{k}}|u_{\mathbf{k}}\rangle$$

Two-dimensional momentum space is topologically a torus:



$|u_{\mathbf{k}}\rangle$ is the Bloch function (periodic part of wavefunction)

- Z_2 invariant (TI):

$$\boxed{(-1)^{\nu} = \prod_i \delta_i}$$

$$\delta_i = \prod_{m=1}^{N} \xi_{2m}(\Gamma_i)$$

Z_2 invariant

$$\xi_m(\Gamma_i)$$

TRI momentum

Parity eigenvalue

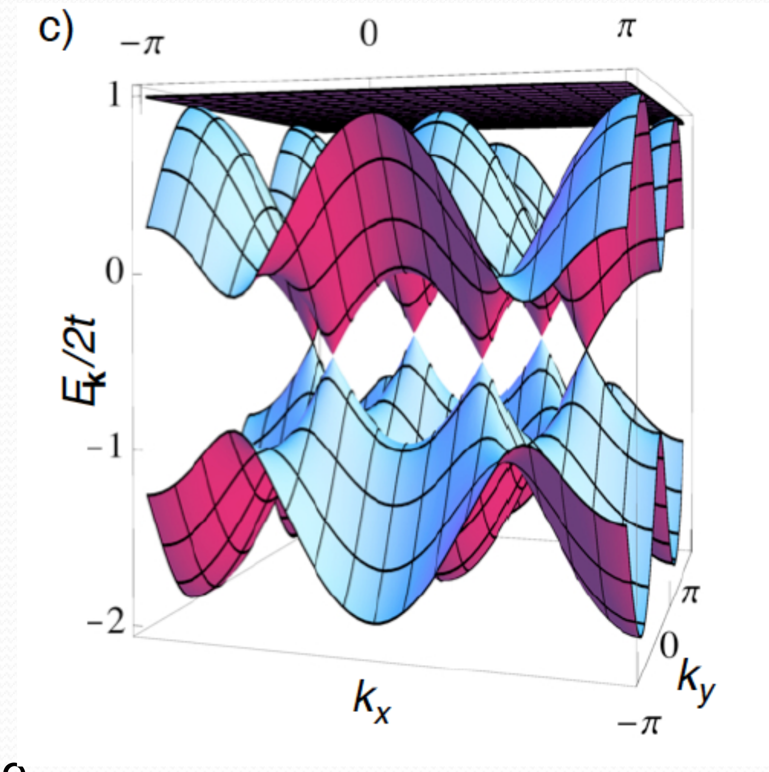
(Inversion symmetry)
Fu and Kane, PRL 2007

Modern formulas, G(k,0)
Z. Wang & S.-C. Zhang
PRX (2012)

Interactions in Two-Dimensional Systems: Add interactions to non-interacting band structure

- Dirac band touching points vs. quadratic band touching points (QBTP):
- Dirac points perturbatively stable against interactions
- QBTP perturbatively unstable to interactions

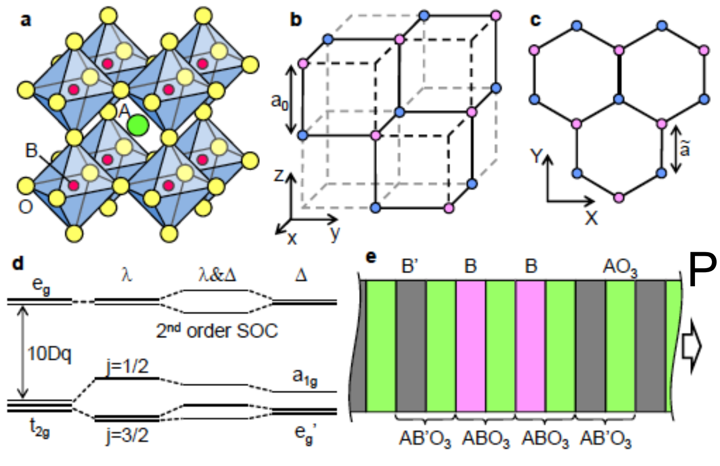
kagome band structure



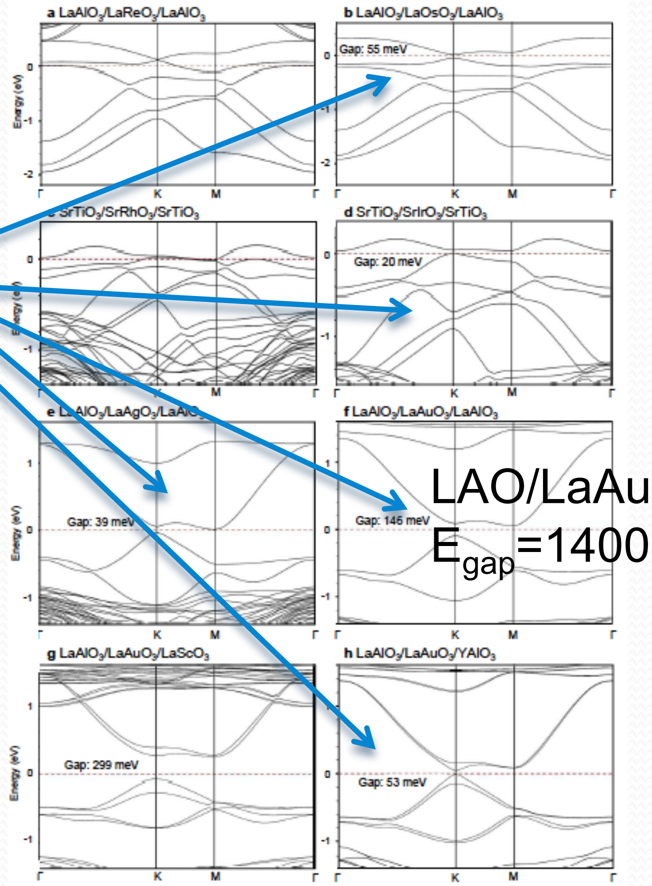
Sun, Yao, Fradkin, and Kivelson, PRL 2009

An interesting new direction for topological phases: oxide heterostructures along [111]

D. Xiao, W. Zhu, Y. Ran, N. Nagaosa, and S. Okamoto Nat. Comm. (2011)

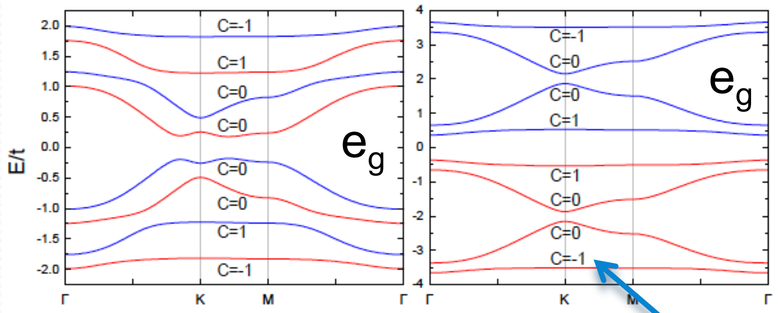


Potential TI



LAO/LaAuO$_3$/LAO

E_{gap}=1400 K



B=0.3t

B=2t

Potential fractional quantum Hall states

(Local) Interaction-driven topological phases in bilayer systems along [111]: LAO/LaNiO$_3$/LAO

K.-Y. Yang *et al*. PRB Rapid (2011); Ruegg and Fiete PRB Rapid (2011)

$$H = H_0 + H_{\rm int} + H_{\rm V} + H_{\rm JT}$$

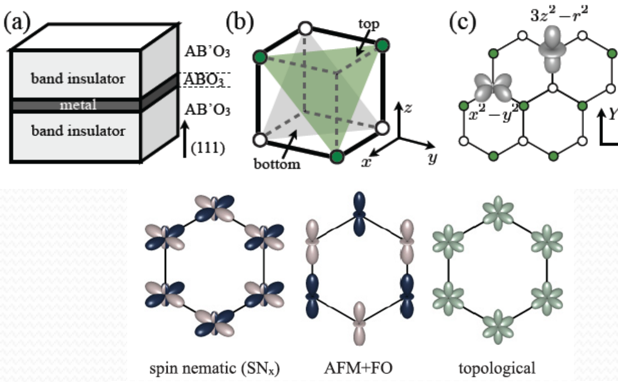
$$H_{\rm int} = \sum_{\boldsymbol r}\Big[U\sum_{\alpha} n_{r\alpha\uparrow}n_{r\alpha\downarrow} + (U'-J)\sum_{\alpha>\beta,\sigma} n_{r\alpha\sigma}n_{r\beta\sigma} + U'\sum_{\alpha\neq\beta} n_{r\alpha\uparrow}n_{r\beta\downarrow} + J\sum_{\alpha\neq\beta} d^{\dagger}_{r\alpha\uparrow}d_{r\beta\uparrow}d^{\dagger}_{r\beta\downarrow}d_{r\alpha\downarrow} + I\sum_{\alpha\neq\beta} d^{\dagger}_{r\alpha\uparrow}d_{r\beta\uparrow}d^{\dagger}_{r\alpha\downarrow}d_{r\beta\downarrow}\Big].$$

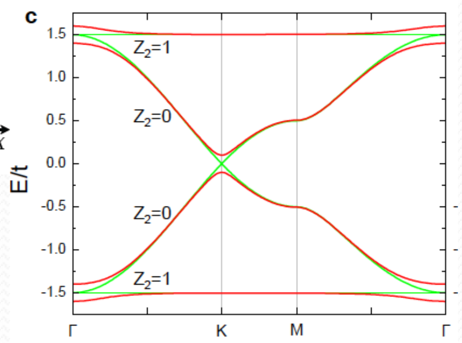
U=U'+2J
I=J

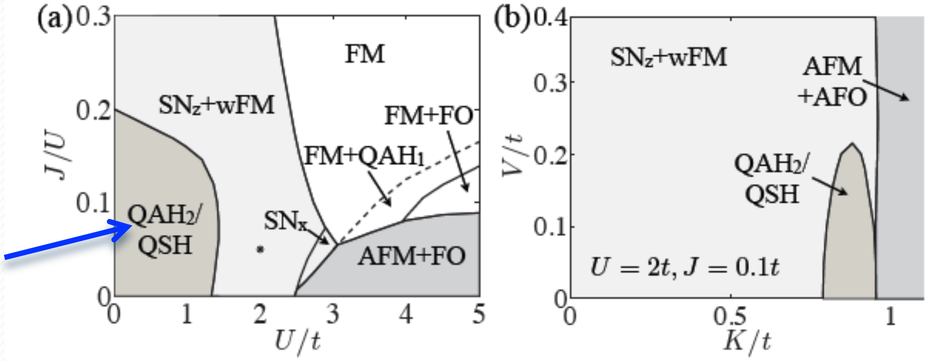
$$H_{\rm V} = \frac{V}{2}\sum_{\mathbf{k},\sigma\alpha}\left[d^{\dagger}_{1\alpha\sigma}d_{1\alpha\sigma} - d^{\dagger}_{2\alpha\sigma}d_{2\alpha\sigma}\right]$$

$$H_{\rm JT} = K\sum_{\langle i,j\rangle}\tau_i^l\tau_j^l$$

$$E_g \propto \frac{t^2}{U-3J}e^{-6\pi t/(U-3J)}$$



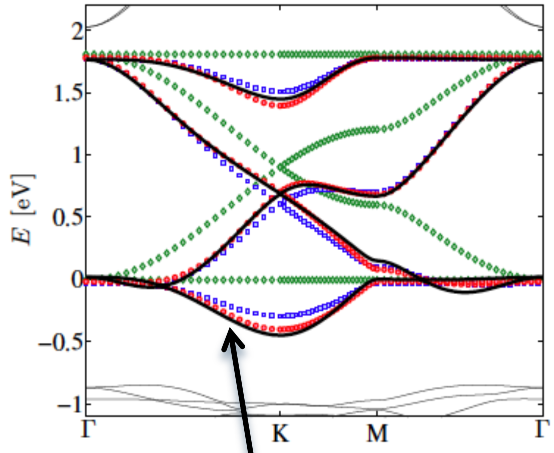




Generation of topological phases from purely local interactions even with zero spin-orbit coupling.

Extended Phase Diagram for LaNiO$_3$ Bilayer

Ruegg, Mitra, Demkov, Fiete PRB 85, 245131 (2012)



1st neighbor

2nd neighbor

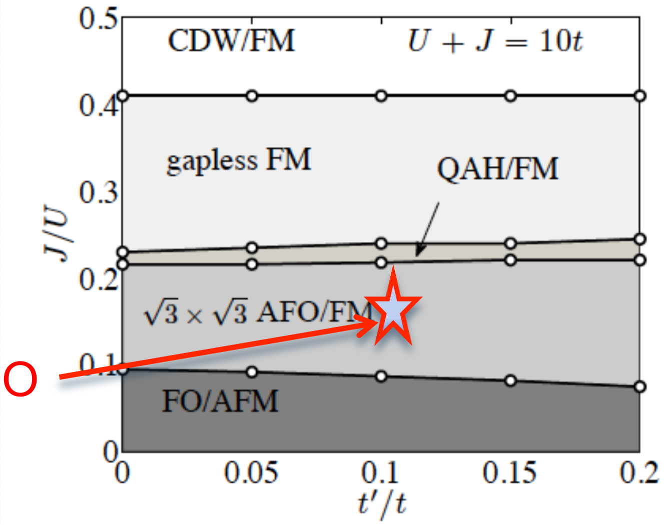
fFt	t (eV)	t' (eV)	Δ (eV)	t_δ (eV)	E_F (eV)
A	0.603	0	0	0	−0.701
B	0.600	0.058	0	0	−0.693
C	0.598	0.062	−0.023	0	−0.693
D	0.598	0.062	−0.023	−0.007	−0.693

DFT result (LDA)

$$H_{\text{int}} = \sum_r \Big[U \sum_\alpha n_{r\alpha\uparrow} n_{r\alpha\downarrow} + (U' - J) \sum_{\alpha>\beta,\sigma} n_{r\alpha\sigma} n_{r\beta\sigma} + U' \sum_{\alpha\neq\beta} n_{r\alpha\uparrow} n_{r\beta\downarrow} + J \sum_{\alpha\neq\beta} d^\dagger_{r\alpha\uparrow} d_{r\beta\uparrow} d^\dagger_{r\beta\downarrow} d_{r\alpha\downarrow} + I \sum_{\alpha\neq\beta} d^\dagger_{r\alpha\uparrow} d_{r\beta\uparrow} d^\dagger_{r\alpha\downarrow} d_{r\beta\downarrow} \Big].$$

U=U'+2J
I=J

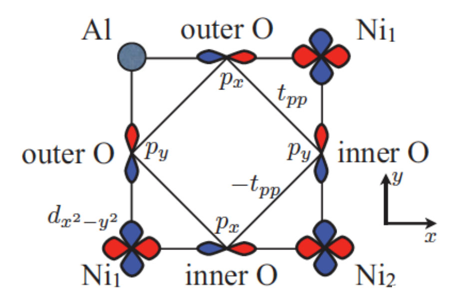
(LNO



Charge-transfer physics in LaNiO$_3$ Bilayer (Explicitly retain O *p*-orbitals)

Ruegg, Mitra, Demkov, Fiete PRB 85, 245131 (2012)

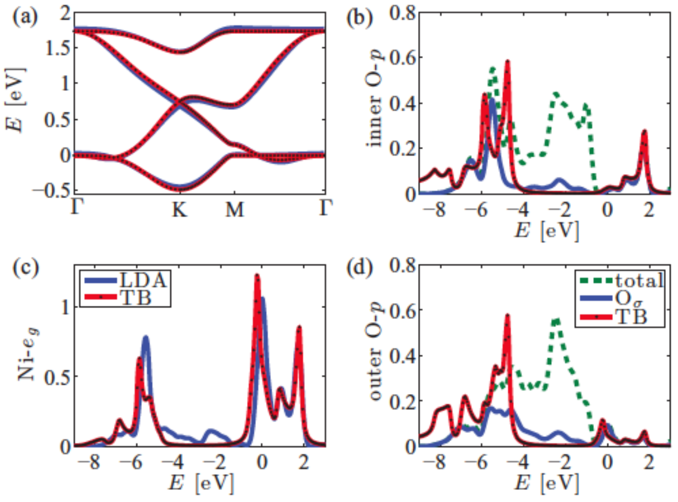
$$H = \sum_{i\sigma} \epsilon_{pi} p^\dagger_{i\sigma} p_{i\sigma} + \sum_{i\alpha\sigma} \epsilon_d d^\dagger_{i\alpha\sigma} d_{i\alpha\sigma} + H_p + H_{\mathrm{hyb}} + \overbrace{H_{\mathrm{int}} + H_{\mathrm{DC}}}^{\text{Mean-field (HF)}}$$

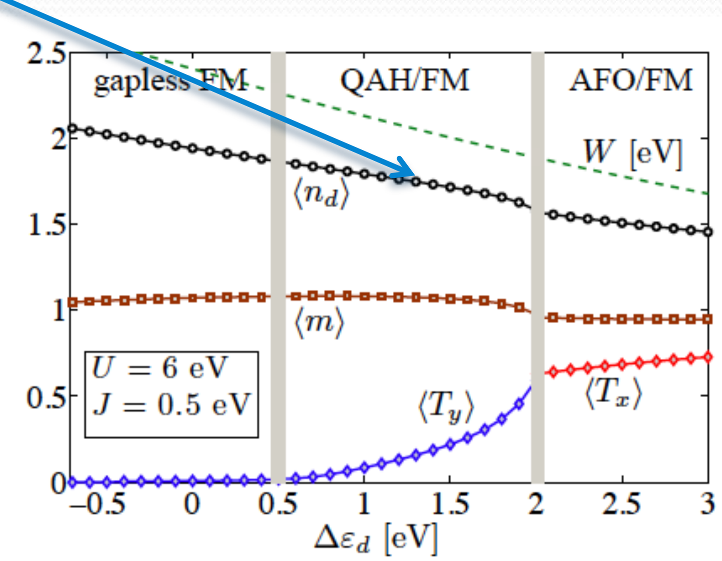


$$H_{\mathrm{hyb}} = \sum_{\langle i,j\rangle} (V^{\alpha}_{ij} p^\dagger_{i\sigma} d_{j\alpha\sigma} + \mathrm{H.c})$$

Charge-transfer physics

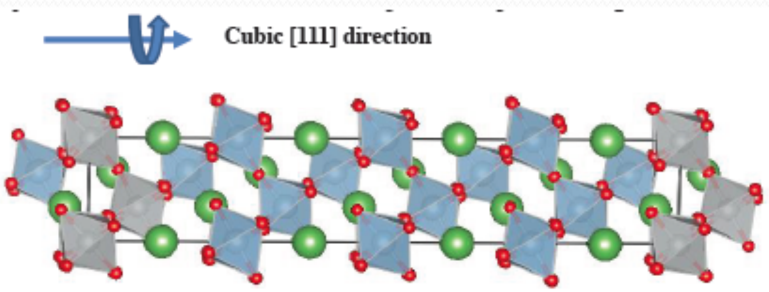
Related work:
Andy Millis Group
(explicit *p*-orbital)



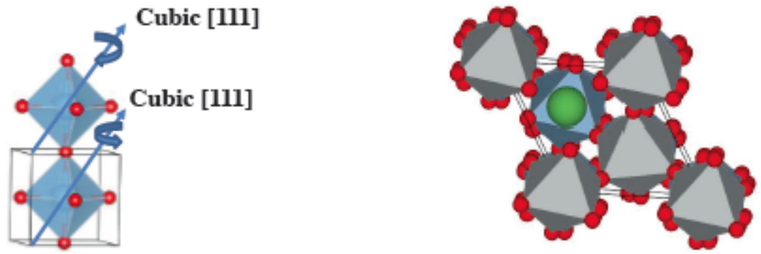


Lattice Strain Effects: Internal

Ruegg, Mitra, Demkov, Fiete PRB (2013)

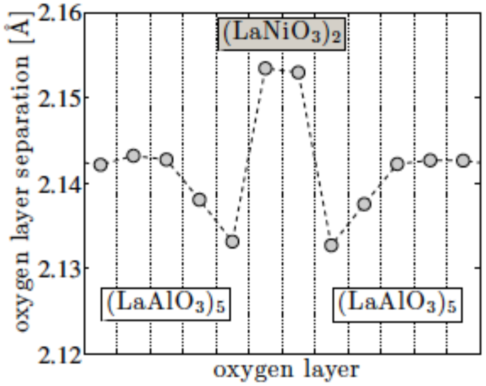


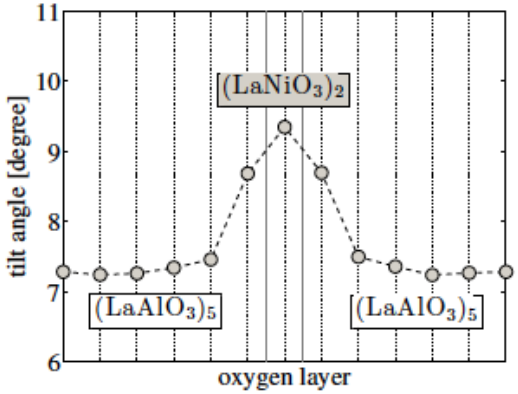
(a) Supercell

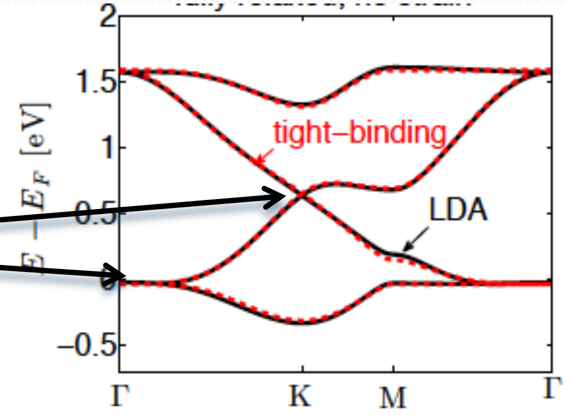


(b) Octahedral tilts

(c) View along [111]







QBCP & Dirac point intact

Internal lattice strain leaves unstrained picture essentially unchanged, *even quantitatively!*

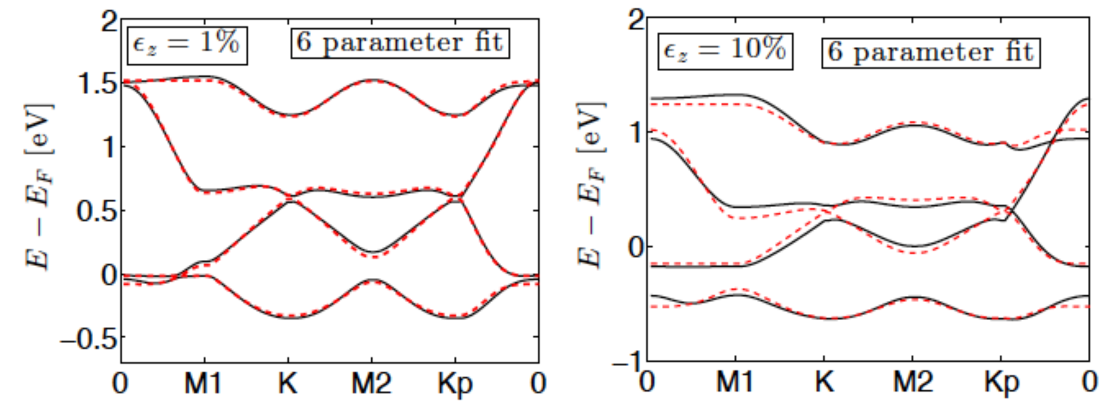
fit	t [eV]	t' [eV]	Δ [eV]	E_F [eV]
unrelaxed	0.598	0.062	-0.023	-0.693
fully relaxed	0.541	0.045	-0.017	-0.641

Lattice Strain Effects: External

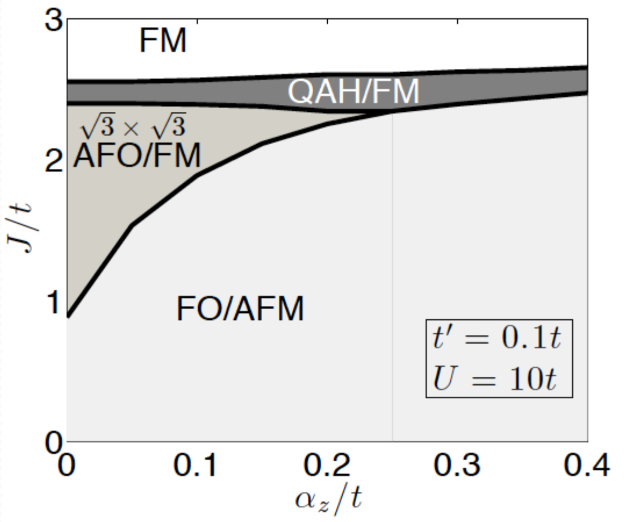
Ruegg, Mitra, Demkov, Fiete *Phys. Rev. B* (2013)

Apply symmetry-breaking strain along the [001] cubic direction:

$$\mathbf{a_1} = a_0(1-\mu x)\mathbf{i},$$
$$\mathbf{a_2} = a_0(1-\mu x)\mathbf{j},$$
$$\mathbf{a_3} = a_0(1+x)\mathbf{k},$$



(a) (b)



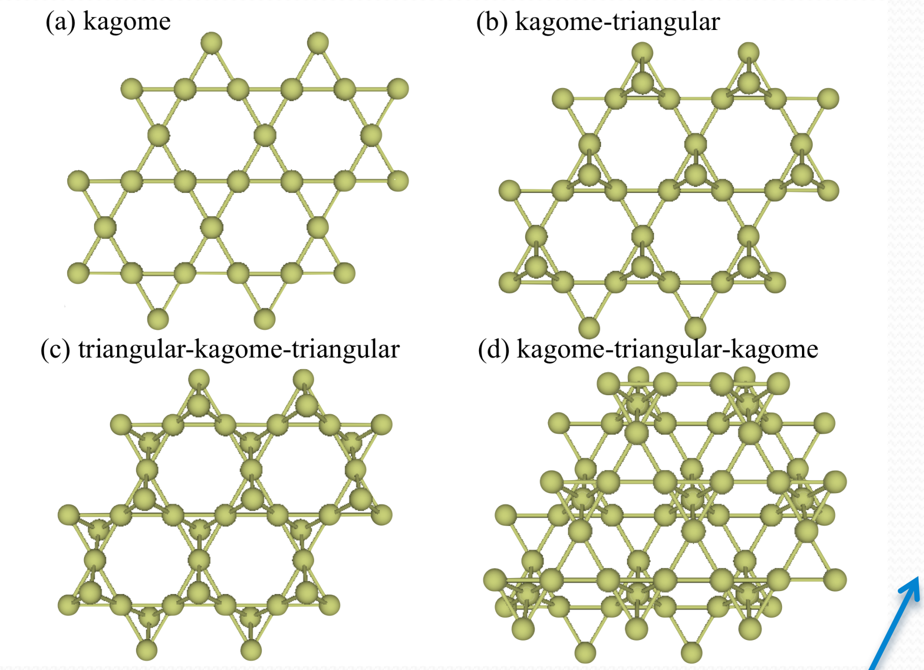
Dominant effect is orbital splitting: $H_z = \alpha_z \sum_{\boldsymbol{r}} \left(n_{\boldsymbol{r},x^2-y^2} - n_{\boldsymbol{r},z^2}\right)$

Externally imposed strain along [001] favors FO/AFM order.
Topological phase unaffected.

Topological phases in single layer, bilayer, and trilayer pyrochlore systems along (111): $A_2M_2O_7$

Hu, Ruegg, Fiete PRB (2012) Editors' Suggestion

B.-J. Yang and N. Nagaosa, arxiv:1403.2207

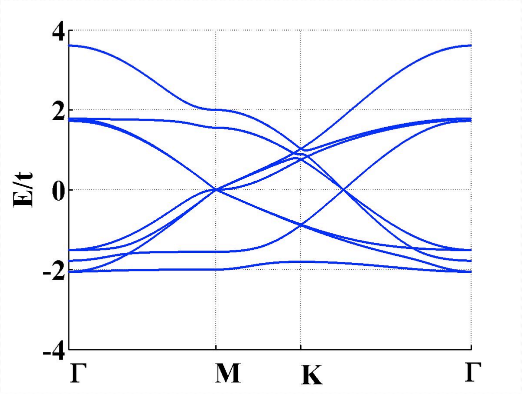


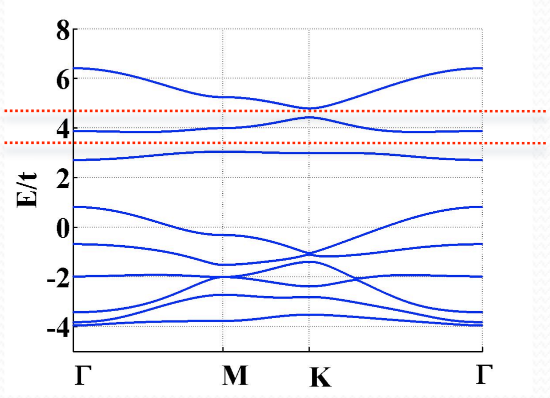
Kagome layer

$$t_\sigma = -1$$

$$U = 0$$

$$\lambda = 0$$





$$\lambda = 4t$$

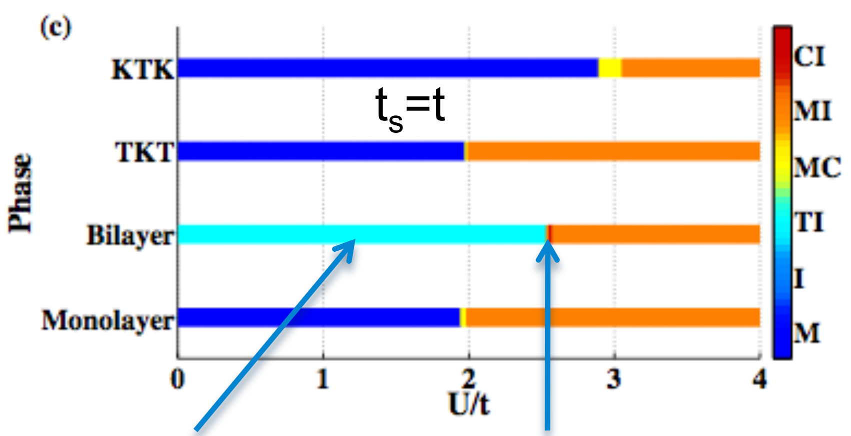
$$H_{SOC} = -\lambda \sum_i \mathbf{l}_i \cdot \mathbf{s}_i$$

Z_2 Topological Insulator fillings

Hartree-Fock Phase diagram for multilayer $A_2Ir_2O_7$

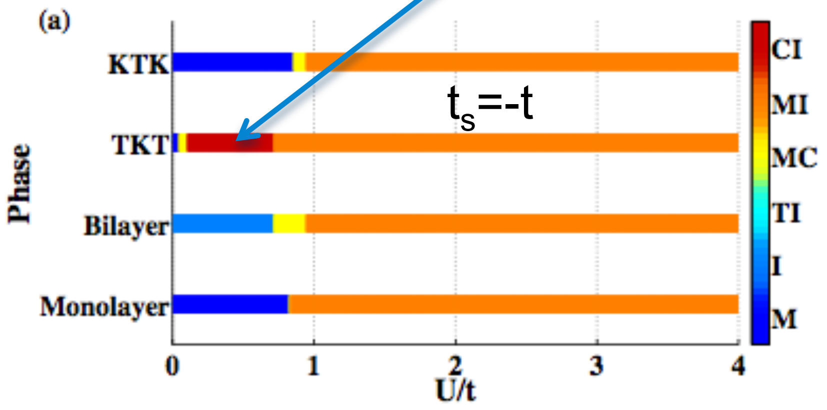
Hu, Ruegg, Fiete, PRB (2012) Editors' Suggestion

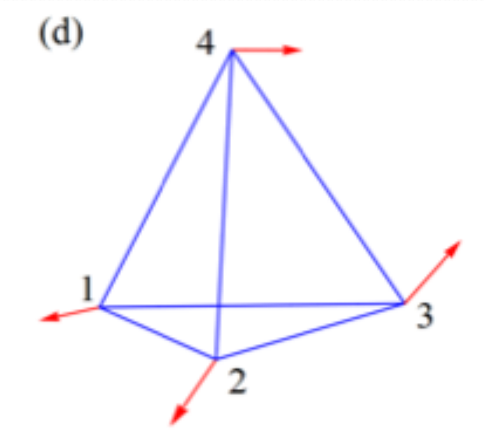
f=1/2, ½-filled j=1/2 band



Z_2 TI

Chern Insulator





Non-coplanar spin order-> Chern number

Summary

- We have presented a number of interacting lattice models that realize topological phases.

- Transition metal oxides (Iridates in particular) appear to be promising venue for a variety of topological phases.

- Heterostructures allow an exceptional range of tunability and may form the basis of device applications of topological phases.

Predictions for topological phases in pyrochlore iridates (partial)

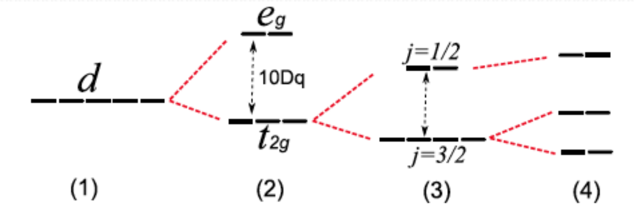
- A_2Ir_2O7 (undistorted): D. Pesin, L. Balents, Nat. Phys. (2010)
  "Topological Mott Insulator" (TMI)
- A_2Ir_2O7 (distorted): B-J. Yang, Y.-B. Kim, PRB (2010)
  Single-particle type TI in non-interacting model
- A_2Ir_2O7 (distorted+interaction): M. Kargarian, J. Wen, GAF PRB (2011)
  "Weak Topological Mott Insulator" (WTMI)
- Y_2Ir_2O7 (magnetically ordered): Wan *et al.* PRB (2011)
  Weyl semi-metal
- A_2Ir_2O7: Go, Witczak-Krempa, Jeon, Park, Y.-B. Kim PRL (2012)
  Axion insulator (found via cellular DMFT)
- A_2Ir_2O7: M. Kargarian, GAF PRL (2013)
  Topological crystalline insulator, TCMI
- A_2Ir_2O7: J. Maciejko, V. Chua, GAF PRL (2014)
  TI*, SM*

Minimal model for $A_2Ir_2O_7$?

- Key energy scales:

  t~1.5 eV, E_{SOC}~0.5 eV, U~2 eV, $E_{t2g\text{-}eg}$~2 eV,

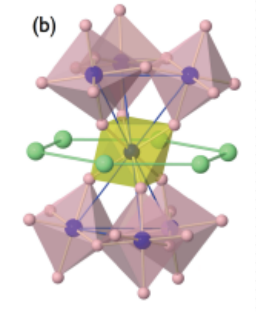
  Δ~0.3-0.5 eV, J_H~0.5 eV



  Moreover, extended 5d orbitals may feel significant crystal fields beyond the local oxygen environment, e.g., Hozio *et al.*, arXiv:1212.4009

  Elements of minimal not entirely clear at present.

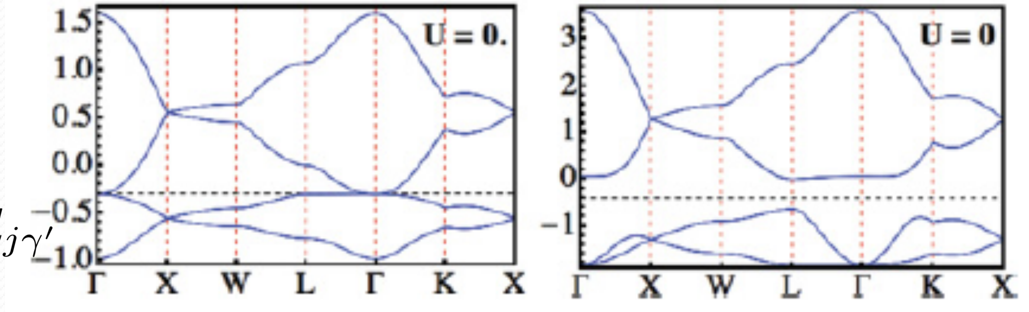
W. Witczak-Krempa, G. Chen, Y.-B. Kim, L. Balents arxiv:1305.2193



- A central issue for topological physics is the j=1/2 band structure, namely order of degeneracies at the Γ point: 2-4-2 vs 4-2-2.

$$H_0 = \sum_{<ij>} (T^{\gamma\gamma'}_{o,ij} + T^{\gamma\gamma'}_{d,ij}) d^\dagger_{i\gamma} d_{j\gamma'}$$

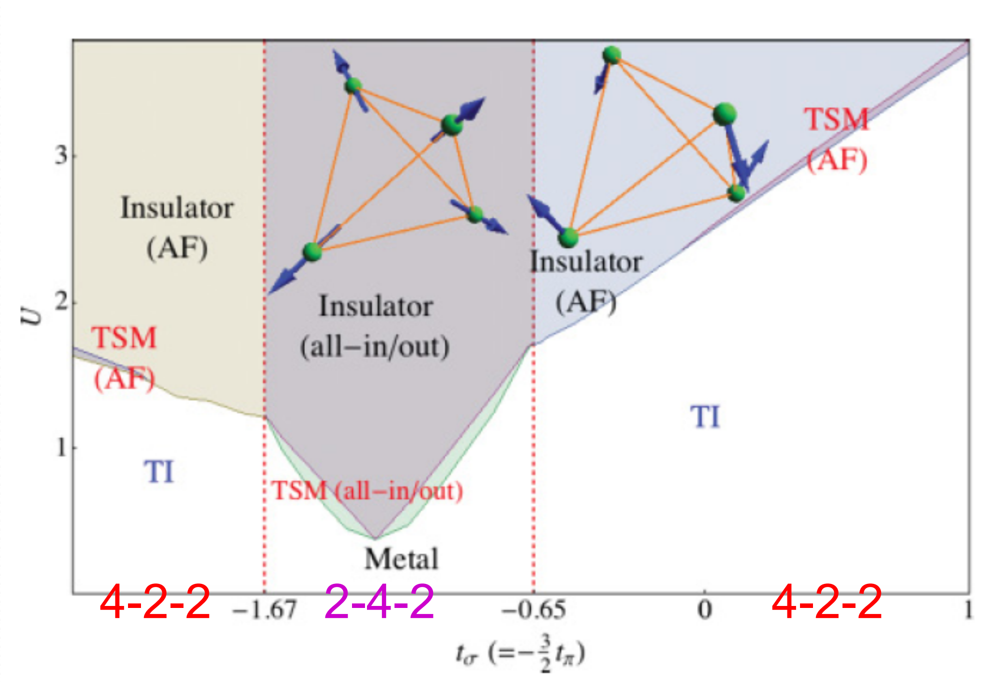
$$= \sum_{<ij>} (t_1 \delta_{\gamma\gamma'} + i t_2 \mathbf{d}_{ij} \cdot \vec{\sigma}_{\gamma\gamma'}) d^\dagger_{i\gamma} d_{j\gamma'}$$



W. Witczak-Krempa and Y.-B. Kim, PRB (2012)

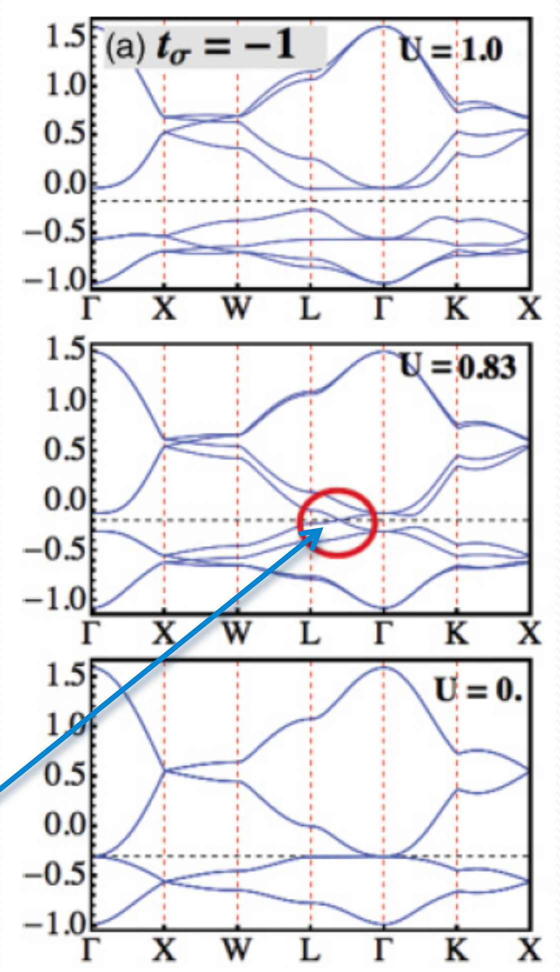
Topological phases in pyrochlore Iridate systems: $A_2Ir_2O_7$
Hartree-Fock Calculations for j=1/2

Witczak-Krempa and Kim, PRB (2012)



$$H = H_{d-d} + H_{d-o-d} + H_U$$

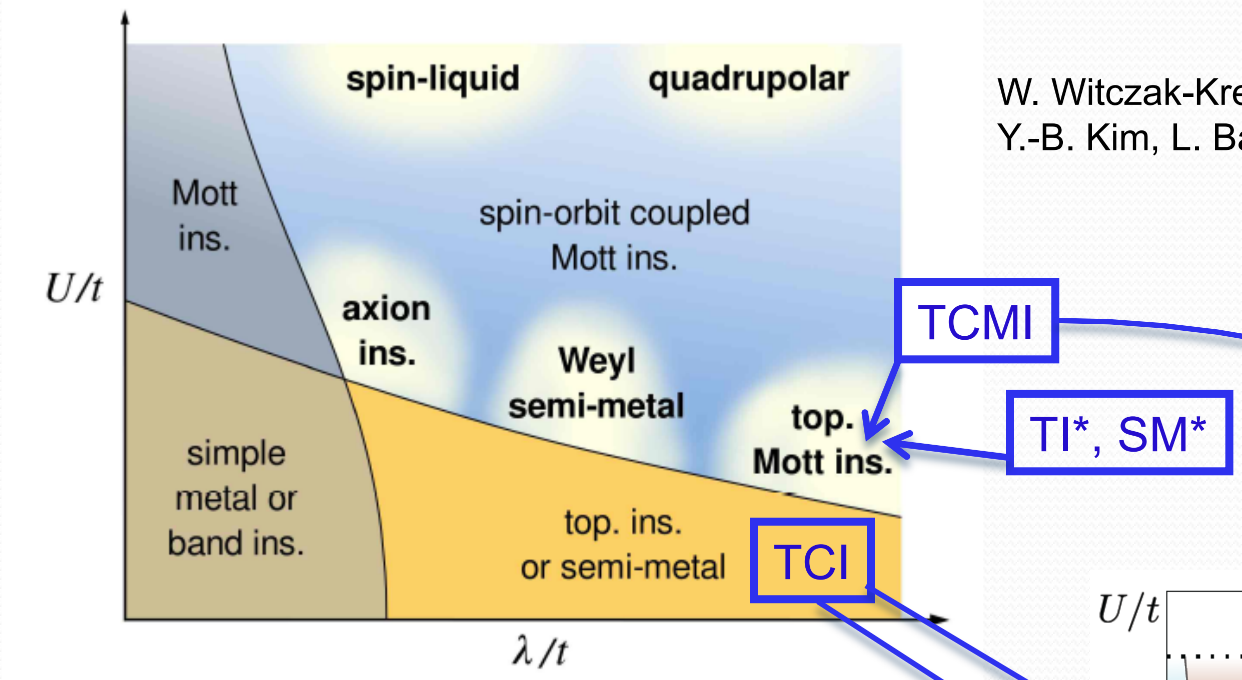
Weyl point



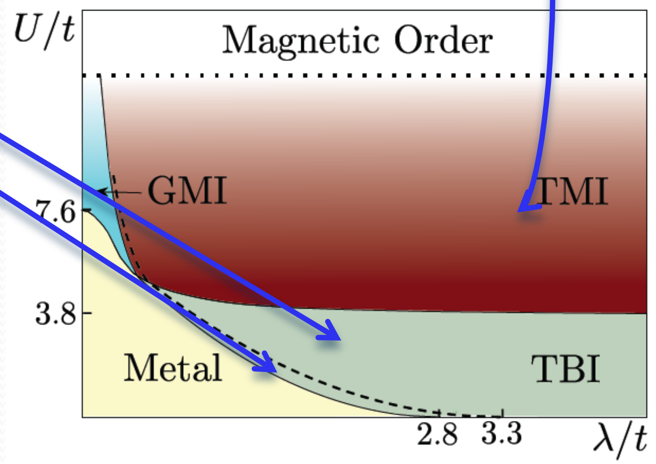
Predictions for topological phases in pyrochlore iridates with Γ-point degeneracies given

- A_2Ir_2O7 (undistorted): D. Pesin, L. Balents, Nat. Phys. (2010)
  "Topological Mott Insulator" (TMI) 4-2-2
- A_2Ir_2O7 (distorted): B-J. Yang, Y.-B. Kim, PRB (2010)
  Single-particle type TI in non-interacting model 4-2-2
- A_2Ir_2O7 (distorted+interaction): M. Kargarian, J. Wen, GAF PRB (2011)
  "Weak Topological Mott Insulator" (WTMI) 4-2-2
- Y_2Ir_2O7 (magnetically ordered): Wan *et al.* PRB (2011)
  Weyl semi-metal 2-4-2 4-2-2
- A_2Ir_2O7: Go, Witczak-Krempa, Jeon, Park, Y.-B. Kim PRL (2012)
  Axion insulator (found via cellular DMFT) 4-2-2
- A_2Ir_2O7: M. Kargarian, GAF PRL (2013)
  Topological crystalline insulator, TCMI 4-2-2
- A_2Ir_2O7: J. Maciejko, V. Chua, GAF arXiv:1307.5566
  TI* 4-2-2, SM* 2-4-2

Schematic Phase Diagram



W. Witczak-Krempa, G. Chen,
Y.-B. Kim, L. Balents arxiv:1305.2193



Pesin, Balents Nat. Phys. (2010)

- A_2Ir_2O7: M. Kargarian, GAF PRL (2013)
  Topological crystalline insulator, TCMI 4-2-2
- A_2Ir_2O7: J. Maciejko, V. Chua,GAF PRL (2014)
  TI* 4-2-2, SM* 2-4-2

Topological Crystalline Insulators: Mirror Chern Number

- Predicted by Liang Fu and collaborators:

  --T. H. Hsieh, H. Lin, J. Liu, W. Duan, A. Bansil, L. Fu, Nat Commun, 3, 982 (2012). SnTe Material class

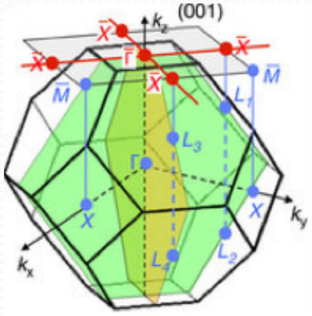
  --Also recent, J. Liu, W. Duan, L. Fu arXiv:1304.0430. IV-VI semiconductors

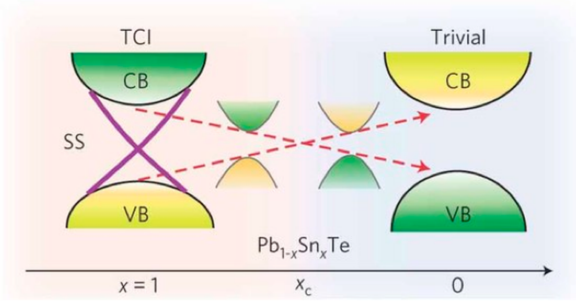
- Realized essentially simultaneously by three groups:

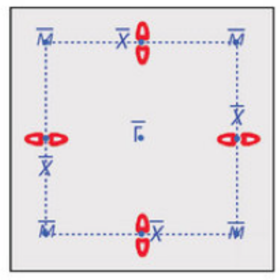
  S.-Y. Xu, … M. Z. Hasan, Nat Commun, 3, 1192 (2012) $Pb_{1-x}Sn_xTe$

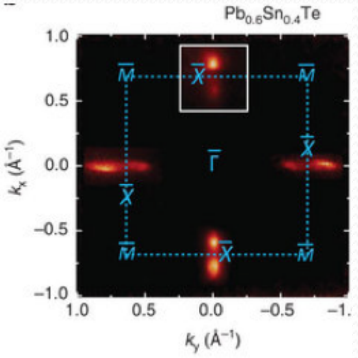
  Y. Tanaka, … Y. Ando, Nat Phys, 8, 800 (2012). SnTe

  P. Dziawa, … T. Story, Nat Mater, 11, 1023 (2012). $Pb_{1-x}Sn_xSe$



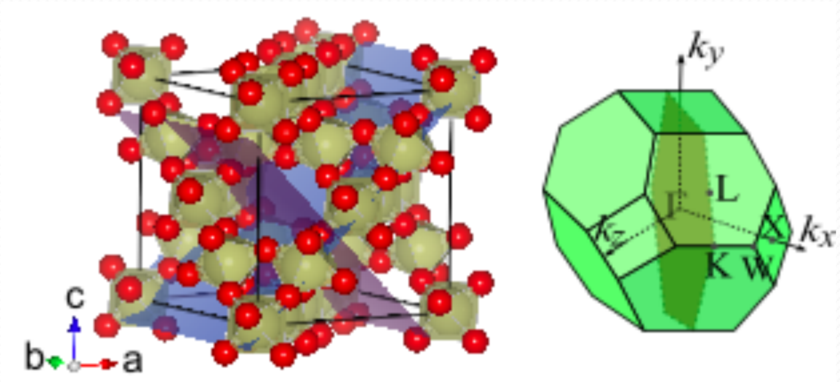






Topological Crystalline Insulator (TCI) in $A_2Ir_2O_7$

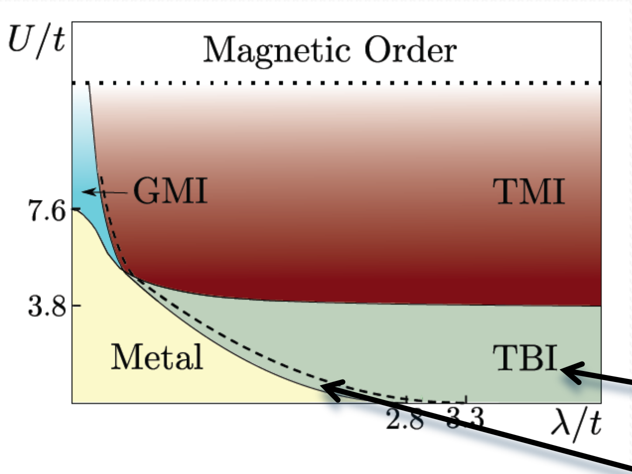
M. Kargarian and GAF PRL (2013)



$$H_0 = \sum_i t_i^{\gamma\gamma'} d_{i\gamma}^\dagger d_{i\gamma'} + \sum_{<ij>} (T_{o,ij}^{\gamma\gamma'} + T_{d,ij}^{\gamma\gamma'}) d_{i\gamma}^\dagger d_{j\gamma'}$$

$$t_i = \varepsilon_d - \lambda l \cdot s$$ 4-2-2

States on the mirror plane can be organized by mirror eigenvalue +/- i



Pesin and Balents (2012)

Mirror Chern number is difference of Chern numbers for each mirror eigenstate:

$$n_M = (n_{+i} - n_{-i})/2$$

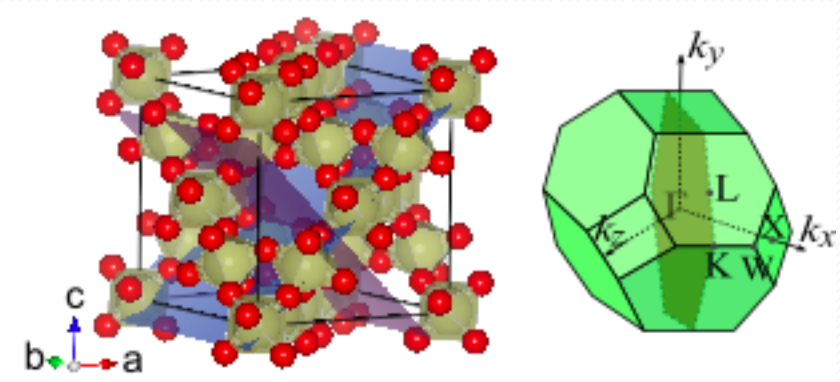
$$n_M = -1 \text{ for } \lambda > \lambda_c$$

$$n_M = +1 \text{ for } \lambda < \lambda_c$$

Teo, Fu, Kane PRB (2007)

Computing TCI invariant for $A_2Ir_2O_7$

M. Kargarian and GAF PRL (2013)



$$H_0 = \sum_i t_i^{\gamma\gamma'} d^\dagger_{i\gamma} d_{i\gamma'} + \sum_{<ij>} (T^{\gamma\gamma'}_{o,ij} + T^{\gamma\gamma'}_{d,ij}) d^\dagger_{i\gamma} d_{j\gamma'}$$

$$t_i = \varepsilon_d - \lambda l \cdot s$$

4-2-2

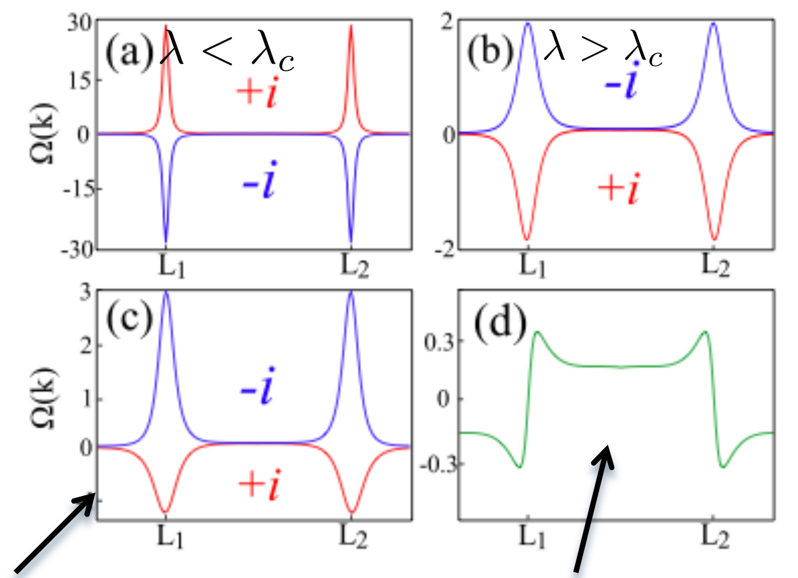
$$n_M = (n_{+i} - n_{-i})/2$$

$$n_M = -1 \text{ for } \lambda > \lambda_c$$

$$n_M = +1 \text{ for } \lambda < \lambda_c$$

$$\Omega(\mathbf{k}) = \nabla \times \mathbf{A}$$

$$\mathbf{A} = i \sum_n \langle u_n(\mathbf{k}) | \nabla | u_n(\mathbf{k}) \rangle$$

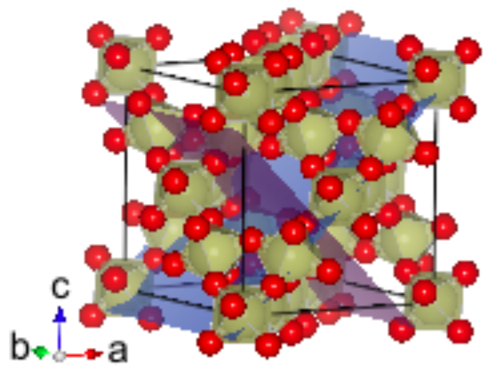


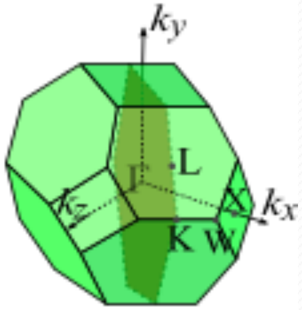
TRS Breaking, Mirror Preserved

TRS Breaking, Mirror Broken

TCI edge states for $A_2Ir_2O_7$: Slab geometry [010] Direction
Mirror Planes: (101) and (10$\underline{1}$)

M. Kargarian and GAF PRL (2013)



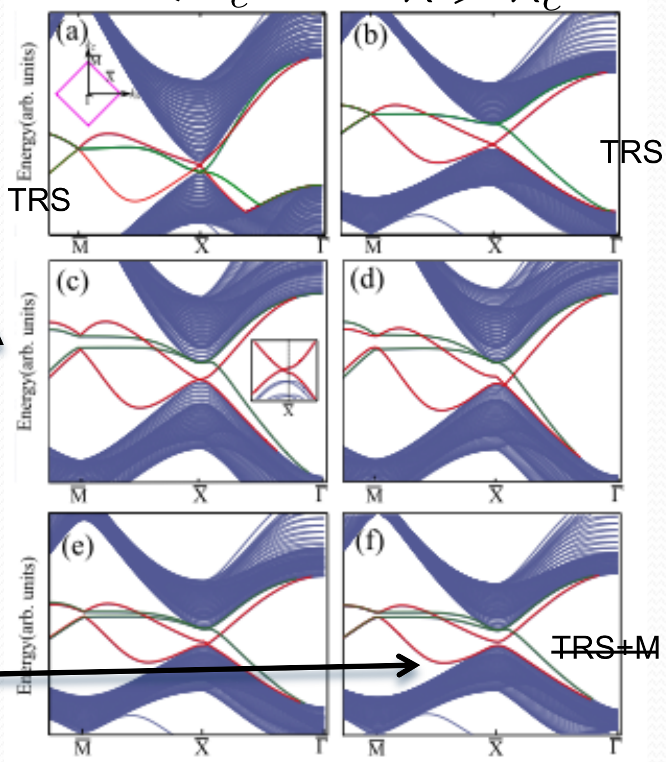


$$H_0 = \sum_i t_i^{\gamma\gamma'} d^\dagger_{i\gamma} d_{i\gamma'} + \sum_{<ij>} (T^{\gamma\gamma'}_{o,ij} + T^{\gamma\gamma'}_{d,ij}) d^\dagger_{i\gamma} d_{j\gamma'}$$

$$t_i = \varepsilon_d - \lambda l \cdot s$$

4-2-2

$\lambda < \lambda_c$ $\lambda > \lambda_c$

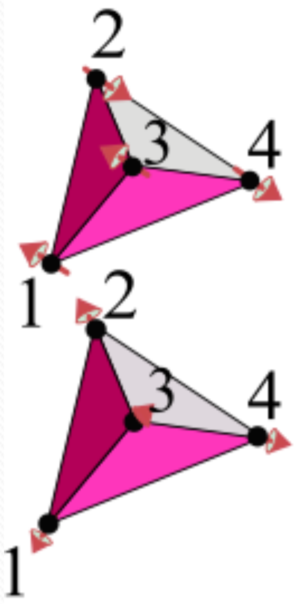


TRS

TRS

~~TRS+M~~

$$H' = \sum_i \mathbf{B}_i \cdot \mathbf{S}_i$$

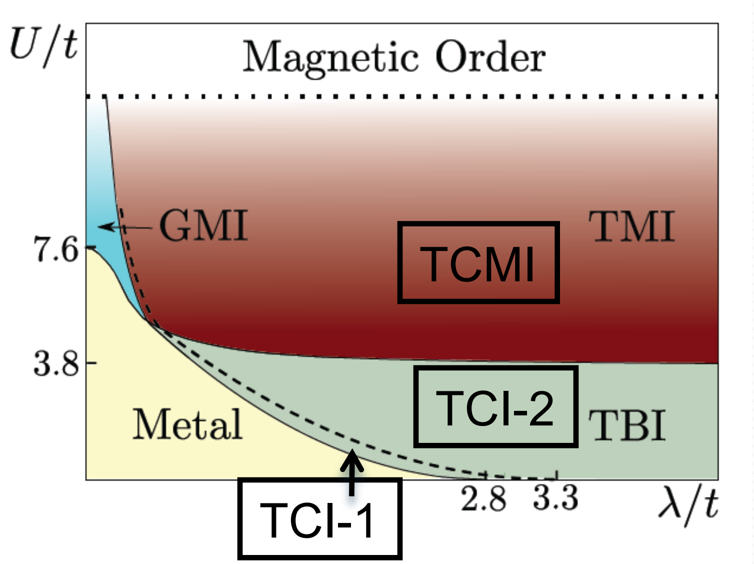


TRS Breaking, Mirror Preserved

TRS Breaking, Mirror Broken

“TCMI” from slave rotors: Refined Phase Diagram

M. Kargarian and GAF PRL (2013)



Pesin and Balents Nat. Phys. (2010)

$$H_0 = \sum_i t_i^{\gamma\gamma'} d^\dagger_{i\gamma} d_{i\gamma'} + \sum_{<ij>} (T^{\gamma\gamma'}_{o,ij} + T^{\gamma\gamma'}_{d,ij}) d^\dagger_{i\gamma} d_{j\gamma'}$$

$$t_i = \varepsilon_d - \lambda l \cdot s$$

4-2-2

$$H_U = U \sum_i \Big(\sum_\gamma n_{i\gamma} - n_d\Big)^2$$

$$d_{j\gamma} = e^{i\theta_j} f_{j\gamma}$$

charge

spin

Find that TMI is also TCMI—a spin liquid with topological band structure protected by both time-reversal and mirror symmetries. TBI is two “flavors” of TCI.

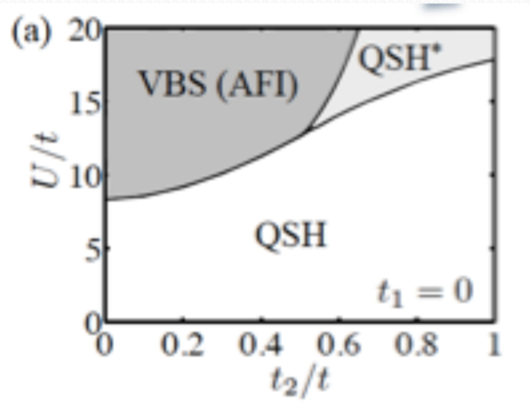
TI* from slave spins: Topological Field Theory & Mean-field Theory

J. Maciejko, V. Chua, GAF PRL (2014)

- Idea is to generalize QSH* phase to three-dimensions

  A. Ruegg and GAF PRL (2012)

$$H = \sum_{rr'} \sum_{\alpha\beta} t^{rr'}_{\alpha\beta} c^{\dagger}_{r\alpha} c_{r'\beta} + \frac{U}{2} \sum_{r} \left(\sum_{\alpha=\uparrow,\downarrow} n_{r\alpha} - 1 \right)^2$$



S. Huber and A. Ruegg PRL (2009)
A. Ruegg, S. Huber, M. Sigrist PRB (2010)

$$\boxed{c_{r\alpha} = f_{r\alpha} \tau^x_r}$$

R. Nandkishore, M. A. Metliski, T. Senthil PRB (2012)
Zhong, Wang, Luo arXiv:1304.2099
-Exactly solvable models

**J. Maciejko and A. Ruegg PRB (2013) CI* Phase

Field Theory for TI*: General

J. Maciejko, V. Chua, GAF PRL (2014)

$$H = \sum_{rr'} \sum_{\alpha\beta} t^{rr'}_{\alpha\beta} c^\dagger_{r\alpha} c_{r'\beta} + \frac{U}{2} \sum_r \left(\sum_{\alpha=\uparrow,\downarrow} n_{r\alpha} - 1 \right)^2$$

$$\boxed{c_{r\alpha} = f_{r\alpha} \tau^x_r}$$

Topological U=0

Local constraint: G_r=1 $\quad G_r = (-1)^{\sum_\alpha f^\dagger_{r\alpha} f_{r\alpha} + \frac{1}{2}(\tau^z_r - 1)}$

Compute: $Z = \mathrm{Tr}(\mathrm{e}^{-\beta \mathrm{H}} \mathrm{P})$ where $P = \prod_r [(1 + G_r)/2]$

Find:

$$Z = \int D\bar{f}_{i\alpha} D f_{i\alpha} \sum_{\{\tau^x_i\}} \sum_{\{\sigma_{ij}\}} e^{-S_{\mathbb{Z}_2}[\bar{f}, f, \tau^x, \sigma]}$$

$$S_{\mathbb{Z}_2} = S_{\tau^x} + S_f + S_B$$

$$S_{\tau^x} = -\kappa \sum_{ij} \tau^x_i \sigma_{ij} \tau^x_j,$$

$$S_f = -\sum_{ij} \sum_{\alpha\beta} t^{ij}_{\alpha\beta} \bar{f}_{i\alpha} \sigma_{ij} f_{j\beta},$$

$$e^{-S_B} = \prod_{i,j=i-\hat{\tau}} \sigma_{ij}$$

Field Theory for TI*: Low-energy

J. Maciejko, V. Chua, GAF PRL (2014)

Write Z_2 gauge theory in terms of U(1) gauge theory: $\sigma_{ij} = e^{ia_{ij}}$ Ukawa, Windey, Guth PRB (1980)

$$S_{U(1)} = S_{\mathbb{Z}_2}[\sigma_{ij} = e^{ia_{ij}}] + S_n \quad \text{where} \quad S_n = -ip\sum_{ij} n_{ij}a_{ij} \qquad \text{p=2}$$

Integrate out gapped slave fermions and focus on ***deconfined*** TI* phase: U(1) gauge field weakly coupled, so lattice unimportant -> take the continuum limit. $n_{ij} \to n_\mu, a_{ij} \to a_\mu$ U(1) gauge invariance requires $\partial_\mu n_\mu = 0 \to n_\mu = \frac{1}{4\pi}\epsilon_{\mu\nu\lambda\rho}\partial_\nu b_{\lambda\rho}$ where $b_{\mu\nu}$ is a compact U(1) 2-form.

$$S_n \to \frac{p}{4\pi}\int d^4x\, \epsilon^{\mu\nu\lambda\rho} b_{\mu\nu}\partial_\lambda a_\rho$$

(3+1)-d level p *BF* term

Cho & Moore Ann. Phys. (2011)
Chan, Hughes, Ryu, Fradkin PRB (2013)

After integrating out slave fermions, $\mathcal{L}_{\mathrm{TI}^*} = \frac{p}{4\pi}\epsilon^{\mu\nu\lambda\rho}b_{\mu\nu}\partial_\lambda(a_\rho - eA_\rho) + \frac{\theta}{32\pi^2}\epsilon^{\mu\nu\lambda\rho}f_{\mu\nu}f_{\lambda\rho}$

Integrate out $b_{\mu\nu}$: $\boxed{\mathcal{L}_{\mathrm{em}} = \frac{\theta e^2}{32\pi^2}\epsilon^{\mu\nu\lambda\rho}F_{\mu\nu}F_{\lambda\rho}}$

Topological degeneracy=8 on T^3
Non-trivial braid angle of $2\pi/p$.

TI-like magneto-electric response, but non-trivial ground state degeneracy and braiding statistics

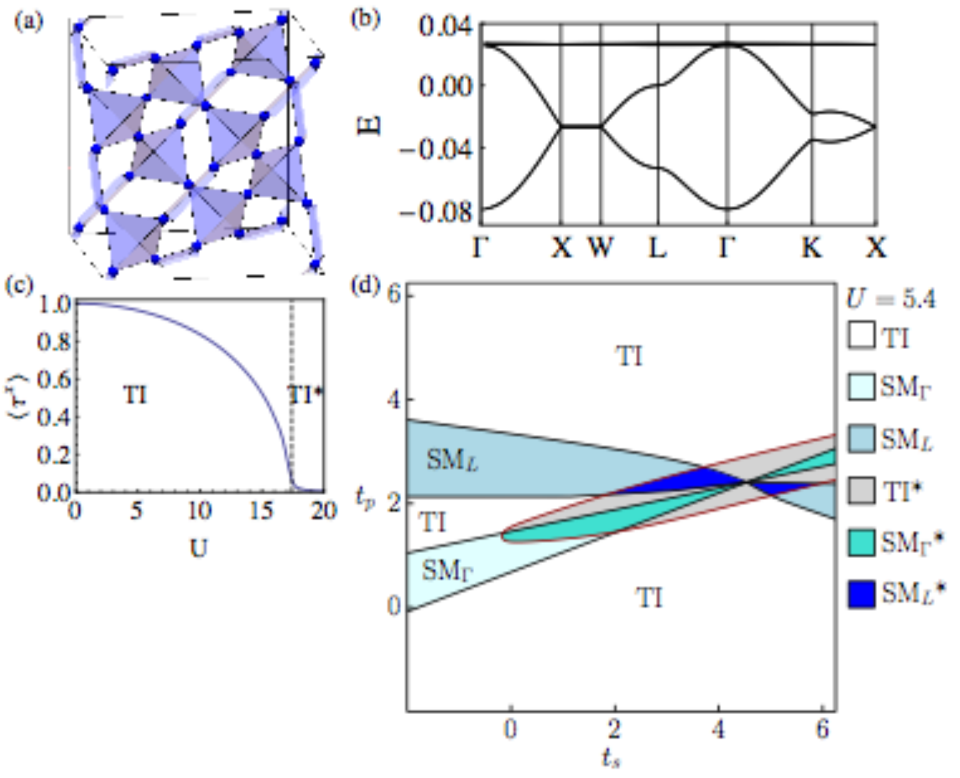
Mean-Field Theory for TI*: Phase Diagram and Braiding

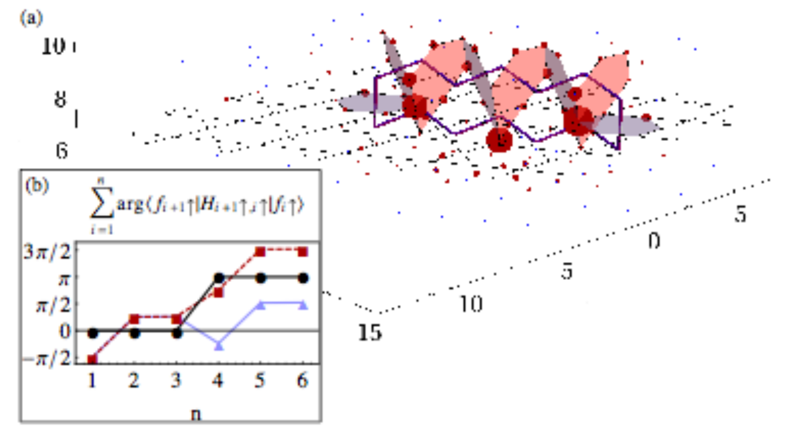
J. Maciejko, V. Chua, GAF PRL (2014)

$$H_{\mathrm{MF}} = \sum_{rr'} \sum_{\alpha\beta} \chi^{rr'}_{\alpha\beta} \sigma_{rr'} f^{\dagger}_{r\alpha} f_{r'\beta} + \sum_{rr'} J_{rr'} \sigma_{rr'} \tau^x_r \tau^x_{r'} + \frac{U}{4} \sum_r (\tau^z_r + 1)$$

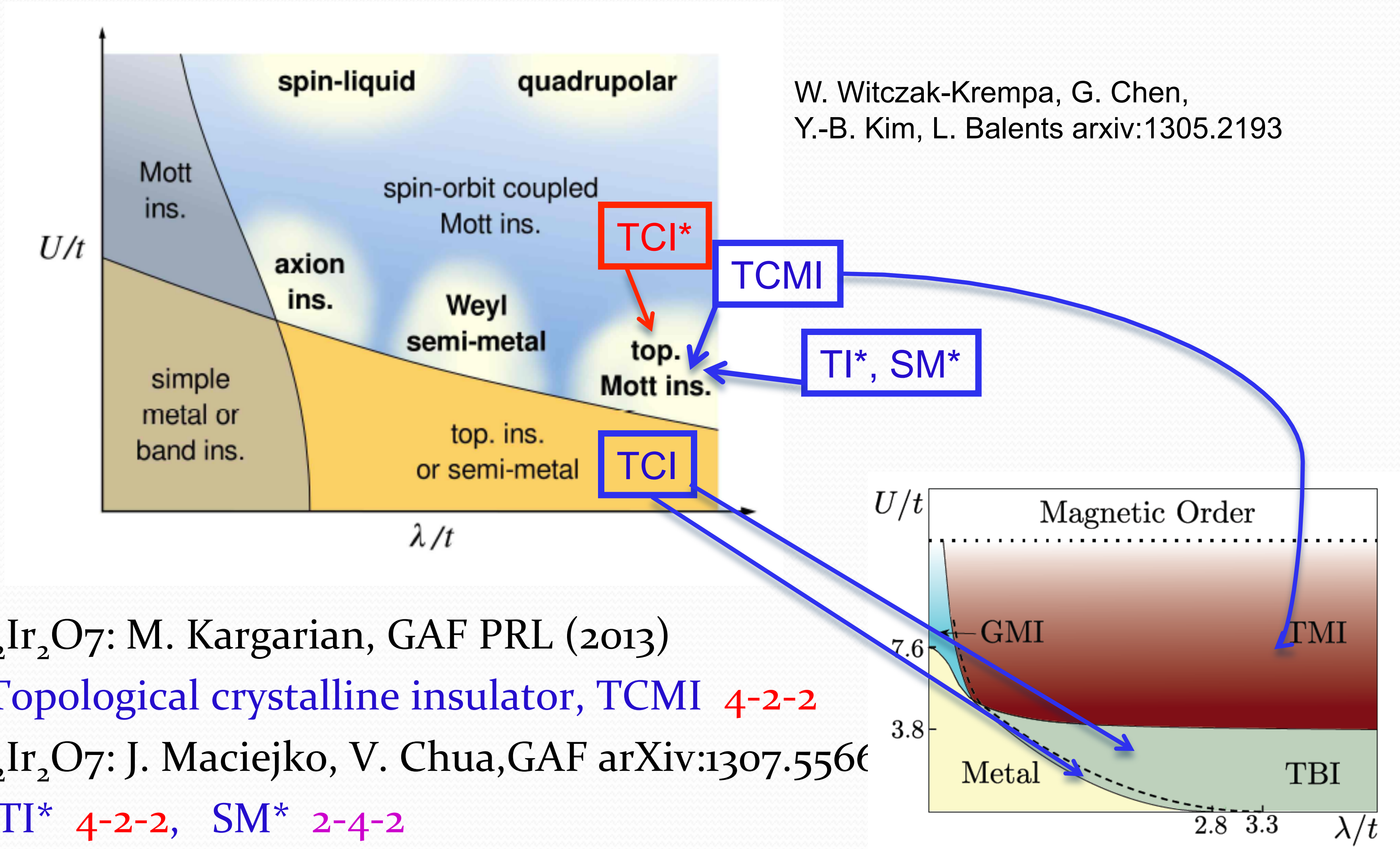
$$\chi^{rr'}_{\alpha\beta} = t^{rr'}_{\alpha\beta} \sigma_{rr'} \langle \tau^x_r \tau^x_{r'} \rangle_{\mathrm{MF}},$$

$$J_{rr'} = \sum_{\alpha\beta} t^{rr'}_{\alpha\beta} \sigma_{rr'} \langle f^{\dagger}_{r\alpha} f_{r'\beta} \rangle_{\mathrm{MF}}$$





Summary

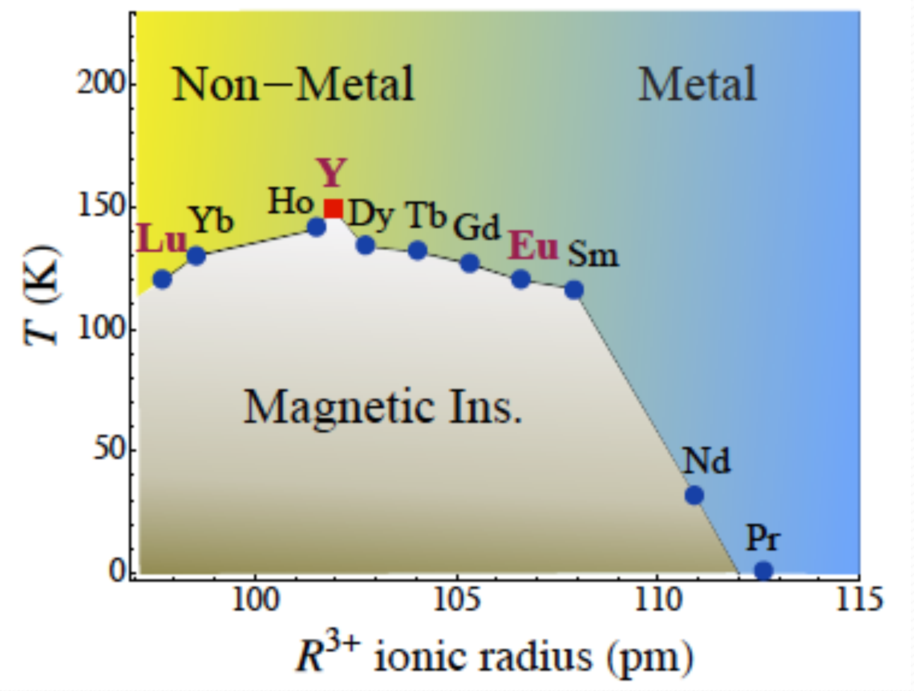


W. Witczak-Krempa, G. Chen,
Y.-B. Kim, L. Balents arxiv:1305.2193

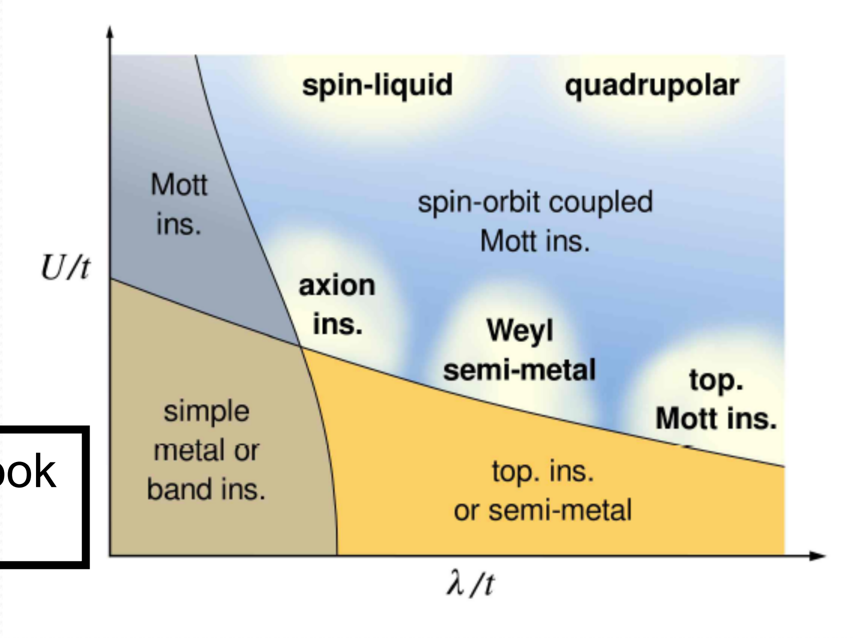
Pesin, Balents Nat. Phys. (2010)

- A_2Ir_2O7: M. Kargarian, GAF PRL (2013)
  Topological crystalline insulator, TCMI 4-2-2
- A_2Ir_2O7: J. Maciejko, V. Chua,GAF arXiv:1307.556
  TI* 4-2-2, SM* 2-4-2

Experimental Phase Diagram for **Bulk** Pyrochlore Iridates

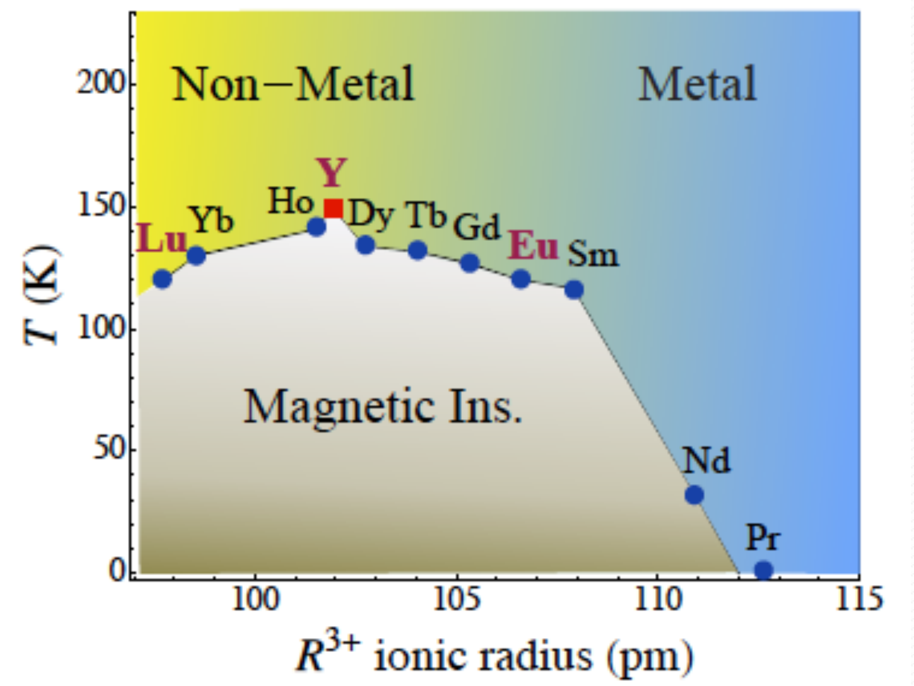


W. Witczak-Krempa, G. Chen,
Y.-B. Kim, L. Balents arxiv:1305.2193

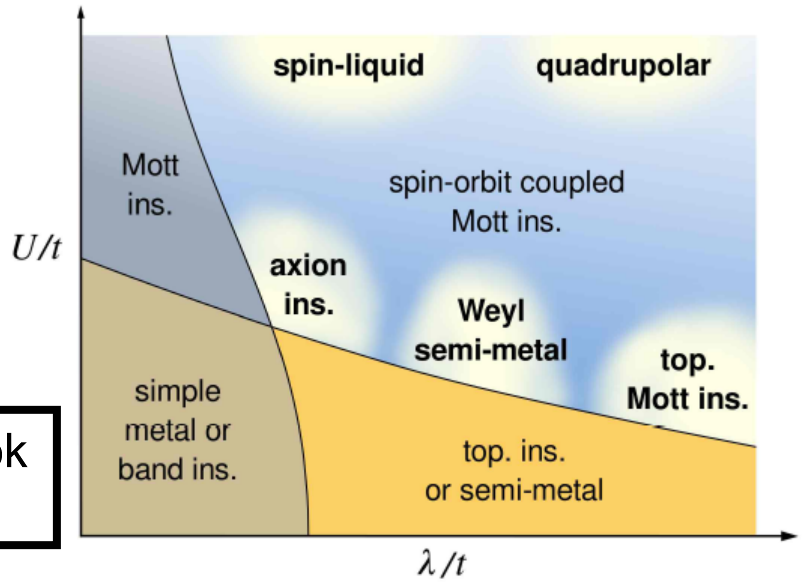


Overall, these particular materials do not look promising for topological phases.

Experimental Phase Diagram for **Bulk** Pyrochlore Iridates



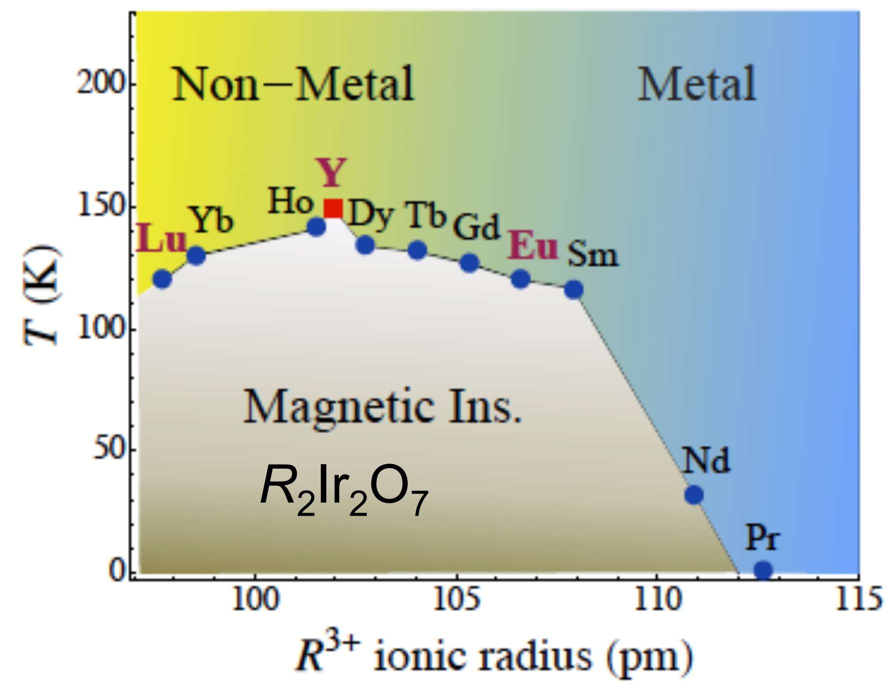
W. Witczak-Krempa, G. Chen,
Y.-B. Kim, L. Balents arxiv:1305.2193



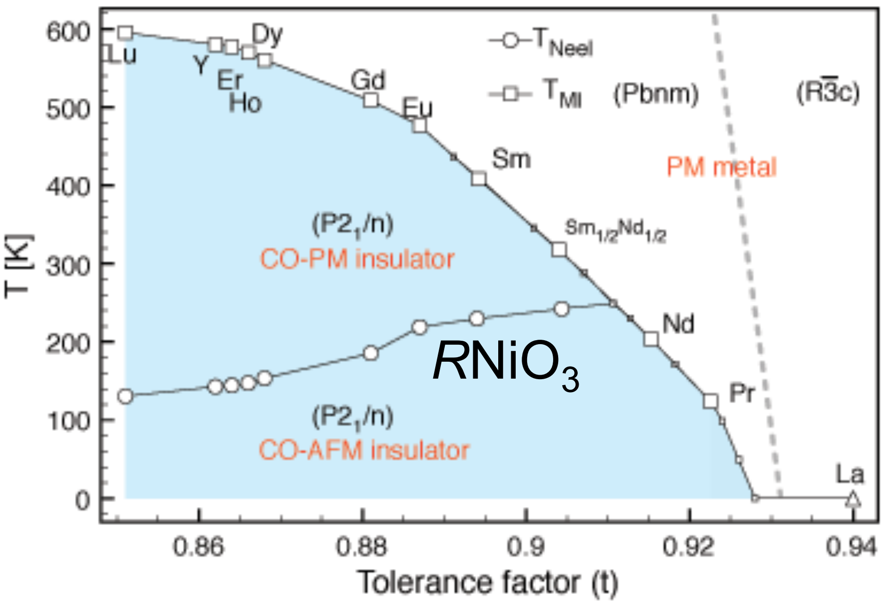
Overall, these particular materials do not look promising for topological phases.

But thin films are a different story!

Experimental Phase Diagram for Bulk Pyrochlore Iridates and Bulk Perovksite Nickelates



W. Witczak-Krempa, G. Chen,
Y.-B. Kim, L. Balents arxiv:1305.2193



From Triscone Group Website

Overall, these particular materials do not look promising for topological phases.

But thin films are a different story!

Experimental Results

- Midday, Meyers, Kareev, Moon, Gray, Moon, Liu, Freeland, Chakhalian *Appl. Phys. Lett*. (2012).