MKC

Finite Element Method Package
of
Learner Programs
(FEMPAC)

UC TR77001

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1. PACKAGE DESCRIPTION

1.1 DESCRIPTION OF AVAILABLE PROGRAMS

This package is built up of two types of programs. The first type is support programs which are intended to facilitate development and display of input data, and display and analysis of results. The second type is application programs, each intended to solve problems in one specific application area. Detailed description of options available, input required, and sample applications are included for each program in the following chapters of this manual.

Support programs include automatic grid generation for programs using the simple three-node linear triangular element. The program has an option for renumbering nodes to minimize storage requirements. Grid plotting on the line printer is also available for programs using the simple linear triangular element with or without automatic grid generation. Result plotting is available for both nodal values and element resultants for pertinent variables within each application program. More detailed description of grid and result plotting are given in Chapters 4 and 5 respectively.

Application programs available and support options available for each are summarized in Table 1.1. More detail on each application program is given in Chapter 3 of this manual.

1.2 FLOW CHART OF PACKAGE

The following flow chart describes the general flow during execution of FEMPAC. Decisions are made by the program based on the Main Control Parameters input by the user. These control parameters are described in Section 1.4.

1.3 DESCRIPTION OF INPUT METHOD - FORMAT FREE INPUT

The main obstacle encountered in applying this package is communicating the correct information to the computer. In computer programming, information is transferred from external devices (the card reader, normally) to internal storage by means of read statements. The transfer of information into the computer is called INPUT, a term that appears frequently in this manual.

There are two possible modes of input available for this package. The first is FORMATTED INPUT. Formatted information is prepared on a card in a format specified by a format statement in the computer program and can not be deviated from. This form of input is not used here, but alternative read statements for its implementation are included in the programs as comments. A specified format makes the preparation of input data an error-prone, space-counting affair. The second mode of input eliminates these problems by making data input format free, hence it is called FORMAT-FREE INPUT. The input values are separated only by a delimiter which normally is a comma or blank space. One disadvantage in using this type of input is the need for extra zero values on a card to terminate data input.

Table 1.1

Programs Available in FEMPAC

Program	Application Prog. No. (NAPPL)	Capabilities	Element Type	Available Support Options		
				Grid Generation	Grid Plotting	Result Plotting
2-D Elasticity	1	Two-dimensional plane stress and plane strain problems with optional thermal stresses.	linear triangle	Yes	Yes	Yes
2-D Heat Transfer	2	Two-dimensional steady state heat transfer with surface convection or specified boundary temperatures and internal heat sources.	linear triangle	Yes	Yes	Yes
2-D Ground Water Flow	3	Two-dimensional steady state field problem with sources and/or sinks, seepage can be included for ground water flow.	linear triangle	Yes	Yes	Yes
Torsion of Non-circular Shaft	4	Calculates shear stresses in non-circular shaft for given torsional input.	linear triangle	Yes	Yes	Yes
2-D Transient Heat Transfer	5	Two-dimensional transient heat transfer with surface convection or specified boundary temperatures and internal heat generation.	linear triangle	Yes	Yes	Yes
Truss	6	Simple trusses.	linear element	No	No	No
Axisymmetric Elasticity	7	Axisymmetric stress and strain problems with optional thermal stresses.	linear triangle	Yes	Yes	Yes
Axisymmetric Heat Transfer	8	Axisymmetric steady state heat transfer with surface convection or specified boundary temperature	linear triangle	Yes	Yes	Yes
Plane Frame	9	Plane frames with end moments	linear beam	No	No	No

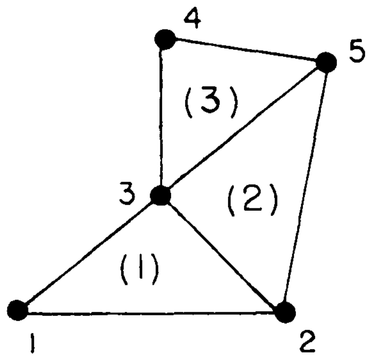
1.6.2 Control Parameters for Element Data

Control parameters for element data for programs using linear triangular elements establishes the size of the problem and the number of material properties sets to be used. The element data control parameters for these cases include, by order of input:

NE - number of elements
NP - number of nodes
NBW - bandwidth* = maximum difference in node numbers within any element + 1.
NMPSET - number of material properties sets to be input

If automatic grid generation is used, these parameters are input to the application program by the grid generation program. If the user inputs the element data, a card including the Control Parameters for Element Data must be input with the parameters listed above.

Sample bandwidth determination



Bandwidth = (5-2) + 1 = 4

1.7 MATERIAL PROPERTIES TABLES

A common procedure has been established for input of material properties for all application programs in this package. A table of sets of material properties is established and stored. Material properties for a region or for an element are then selected by referring to the set number of the desired set of material properties. Table 1.2 shows what properties are included in a material properties sets for each application program. The table is sized for five parameters in each material properties set. Some applications do not require five parameters. Zero values must be used to specify the unused parameters in those material properties sets in application programs not requiring all five parameters. Up to twenty sets of material properties may be input. Each set must be input on a separate card. The number of material properties sets to be input is controlled by the parameter NMPSET which is input either through the automatic grid generation program or within the element data control parameters. Sets are numbered by order of their input.

---

*Note this is bandwidth assuming one degree of freedom (unknown) at a node. Application programs automatically adjust bandwidth for number of unknowns at a node.

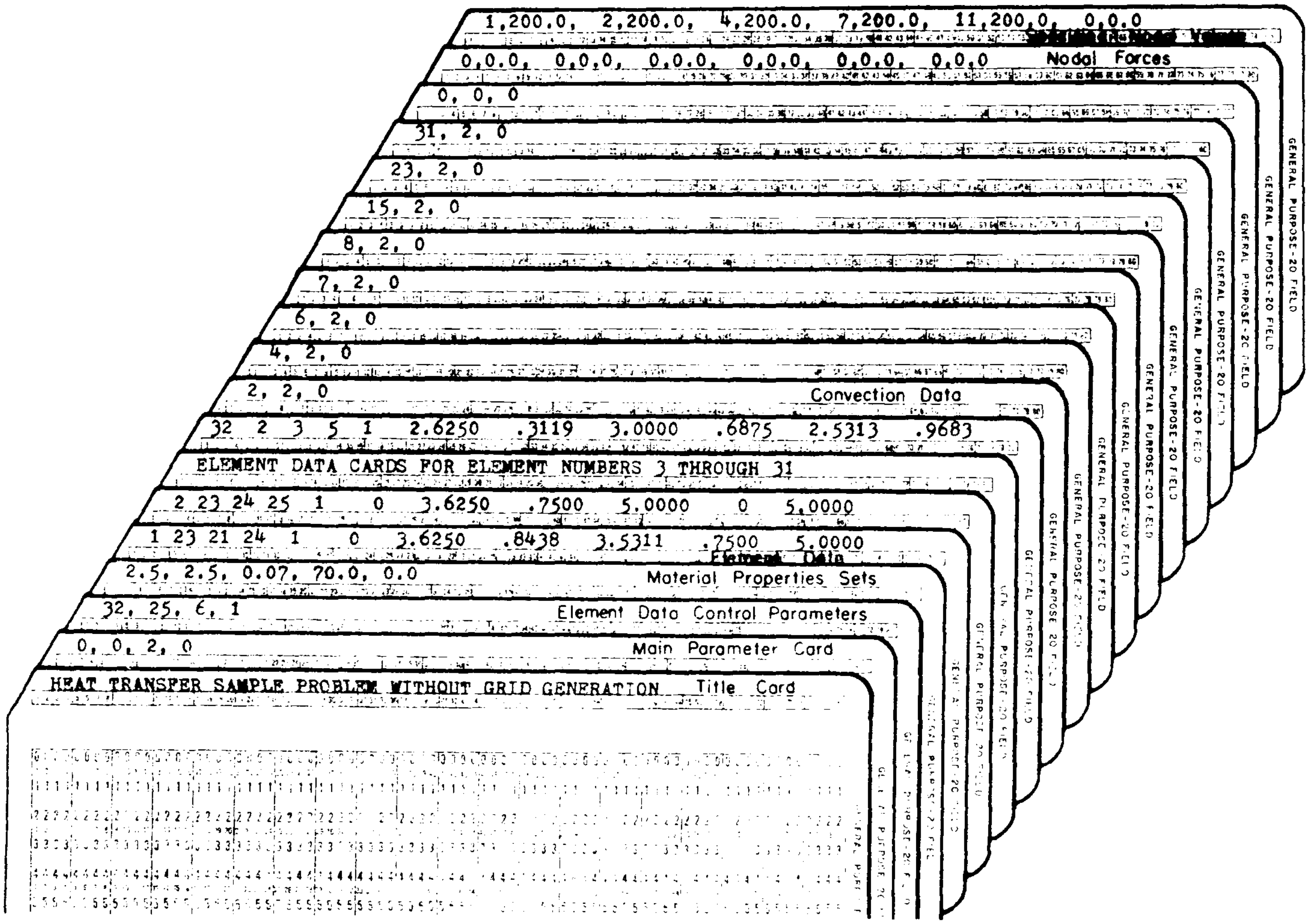
Table 1.2. Material Property Sets

Application Program	Material property (1)	Material property (2)	Material property (3)	Material property (4)	Material property (5)
2-D Elasticity	Young's modulus (EM)	Poisson's ratio (PR)	Coef. of thermal expansion (ALPHA)	Equilibrium temperature of body (TEMP)	Thickness of body (T)
2-D Heat transfer	Conductivity X - direction (KXX)	Conductivity Y - direction (KYY)	Convection coefficient (H)	Fluid temperature (TINF)	N/A (0.0)
Groundwater	Transmissivity X - direction (TXX)	Transmissivity Y - direction (TYY)	N/A (0.0)	N/A (0.0)	N/A (0.0)
Torsion	Shear modules (G)	Shaft length (SL)	Number axis of sym. *2 (PCT)	N/A (0.0)	N/A (0.0)
2-D Transient heat transfer	Conductivity X - direction (KXX)	Conductivity Y - direction (KYY)	Convection coefficient (H)	Fluid temperature (TINF)	Specific heat*density (CRHO)
Simple truss	Elastic modulus (EM)	Coefficient of thermal expansion (ALPHA)	Cross-section area (AREA)	N/A (0.0)	N/A (0.0)
Axisymmetric Elasticity	Young's modulus (EM)	Poisson's ratio (PR)	Coef. of thermal expansion (ALPHA)	Equilibrium temperature of body (TEMP)	N/A 0.0
Axisymmetric Heat Transfer	Conductivity R-direction (KRR)	Conductivity Z-direction (KYY)	Convection coefficient (H)	Fluid temperature (TINF)	N/A 0.0

Plane Frame - See Application Section

```
ELEMENT RESULTANTS

  ELEMENT      GRAD(X)         GRAD(Y)        AVE TEMP
     1      -.39042E+00     -.37570E+01     .18227E+03
     2      -.38803E+00     -.37583E+01     .18054E+03
     3      -.12292E+01     -.37666E+01     .18180E+03
     4      -.11430E+01     -.38050E+01     .18017E+03
     5      -.19859E+01     -.37803E+01     .18081E+03
     6      -.18886E+01     -.38190E+01     .17922E+03
     7      -.26626E+01     -.38023E+01     .17940E+03
     8      -.26275E+01     -.38159E+01     .17776E+03
     9      -.49315E+00     -.51352E+01     .18803E+03
    10      -.53955E+00     -.50972E+01     .18600E+03
    11      -.14407E+01     -.51962E+01     .18787E+03
    12      -.14215E+01     -.52085E+01     .18593E+03
    13      -.22182E+01     -.52769E+01     .18729E+03
    14      -.22239E+01     -.52739E+01     .18533E+03
    15      -.28468E+01     -.53771E+01     .18657E+03
    16      -.29530E+01     -.53281E+01     .18441E+03
    17      -.73732E+00     -.62446E+01     .19375E+03
    18      -.68892E+00     -.63077E+01     .19177E+03
    19      -.17205E+01     -.63140E+01     .19378E+03
    20      -.16908E+01     -.63432E+01     .19194E+03
    21      -.25514E+01     -.64206E+01     .19357E+03
    22      -.25480E+01     -.64231E+01     .19167E+03
    23      -.31668E+01     -.65557E+01     .19353E+03
    24      -.32786E+01     -.64887E+01     .19135E+03
    25      -.11657E+01     -.69735E+01     .19858E+03
    26      -.95899E+00     -.74611E+01     .19703E+03
    27      -.18782E+01     -.71656E+01     .19856E+03
    28      -.19209E+01     -.70867E+01     .19714E+03
    29      -.30870E+01     -.71287E+01     .19854E+03
    30      -.29125E+01     -.73690E+01     .19710E+03
    31      -.48801E+01     -.58664E+01     .19866E+03
    32      -.35497E+01     -.71948E+01     .19719E+03
```

Fig. 3.2.6 Input for Heat Transfer Without Grid Generation



```
1,200.0,  2,200.0,  4,200.0,  7,200.0,  11,200.0,  0,0.0     Nodal Values
0,0.0,  0,0,0,  0,0.0,  0,0.0,  0,0.0,  0,0.0     Nodal Forces
0, 0, 0
31, 2, 0
23, 2, 0
15, 2, 0
8, 2, 0
7, 2, 0
6, 2, 0
4, 2, 0
2, 2, 0     Convection Data
32  2  3  5  1   2.6250   .3119   3.0000   .6875   2.5313   .9683
ELEMENT DATA CARDS FOR ELEMENT NUMBERS 3 THROUGH 31
2 23 24 25  1   0   3.6250   .7500   5.0000   0   5.0000
1 23 21 24  1   0   3.6250   .8438   3.5311   .7500   5.0000
     Element Data
2.5, 2.5, 0.07, 70.0, 0.0     Material Properties Sets
32, 25, 6, 1     Element Data Control Parameters
0, 0, 2, 0     Main Parameter Card
HEAT TRANSFER SAMPLE PROBLEM WITHOUT GRID GENERATION     Title Card
```

3.3 IRROTATIONAL FLOW OF IDEAL FLUIDS

3.3.1 Introduction

This program can be used to analyze two-dimensional irrotational flow of ideal fluids governed by the equation.

$$K_{xx} \frac{\partial^2 \phi}{\partial x^2} + K_{yy} \frac{\partial^2 \phi}{\partial y^2} + Q = 0$$

with boundary conditions

$$K_{xx} \frac{\partial \phi}{\partial x} 1_x + K_{yy} \frac{\partial \phi}{\partial x} 1_y + q = 0 \text{ on } S_1$$

or

$$\phi = \phi_o \text{ on } S_2$$

Different interpretations of the parameters of the equation leads to the following three formulations

1) Groundwater Flow

ϕ - pressure head a) elevation above bottom of aquifer for unconfined flow
b) sum of elevation head and pressure head for confined flow

Q - internal source, discharge and recharge (recharge assumed positive)

q - boundary source, seepage (into region assumed positive)

2) Stream Function

ϕ - stream function value, difference between adjacent lines of constant value yields flow rate
$K_{xx} = K_{yy} = 1$ (By definition)
Q = 0.0 (By definition)
q = 0.0 (By definition)

3) Velocity Function

ϕ - velocity potential; sum of elevation, pressure and velocity heads
$K_{xx} = K_{yy} = 1.0$ (By definition)
Q = 0.0
q - flow across boundary

The program input development description will be given in detail for the groundwater case. A brief discussion of implementation for the other two cases is given in Section 3.3.7. The program used simple linear triangular elements in all cases. Automatic grid generation can be used with this program.

3.3.2 Material Properties Description

The material property sets for this program include, in the following order,

K_{xx} - hydraulic conductivity in the X - direction (= 1 for stream or velocity function applications)

K_{yy} - hydraulic conductivity in the Y - direction (= 1 for stream or velocity function applications)

Dummy Variable 1 - = 0.0
Dummy Variable 2 - = 0.0
Dummy Variable 3 - = 0.0

Any compatible set of units may be used. Up to twenty sets of material properties may be input.

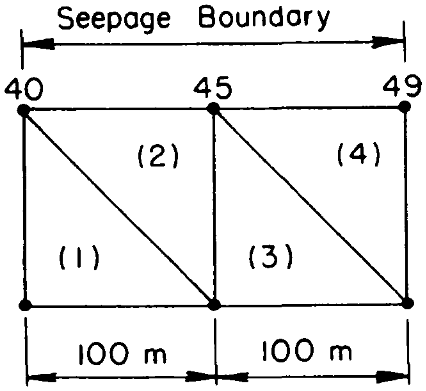
3.3.3 Control Parameter for Application

The control parameter read in during the application program section for irrotational flow is

ITYPE = 1 Groundwater Flow
= 2 Stream Function
= 3 Velocity Function

3.3.4 Seepage Input

Values for seepage into or out of the region under study are input as nodal forces on the boundary nodes. The seepage along the side of the element is calculated and then 1/2 allotted to each node. Seepage into the region is considered positive. Consider the example below:



Assume infiltration rate = 0.30 m^3/day/m into region.

Seepage for each element = 0.30 m^3/day/m*100m

= 30 m^3/day

Seepage by node = 40 15 m^3/day
45 30 m^3/day
49 15 m^3/day

The seepage values would be input as nodal forces at nodes 40, 45, and 49.

3.3.5 Boundary Values

Boundary conditions are input in two sections. First, the nodal forces which for the groundwater case would be nodal values for seepage and pumping (fluid added to region assumed positive). Second, specified nodal values, which would be known nodal potentials.

Nodal forces and nodal values are both input by node number and associated value with six sets per card.[1] Each card must have 6 sets of node number and associated value. If a complete card is not needed, the remaining sets should be specified with zero values. If an even multiple of 6 sets occurs or no values are to be input, a card with 6 sets of zero values (12 zeros) must follow to terminate that section of input data. Nodal forces for the example shown in the previous section would appear as

```
 40,  15.0,    45,   30.0,    49,    15.0    0,0.0,    0,0.0,    0,0.0
```

Consider the case where nodes 12, 15, 19, and 21 are known to have potential values of 100.0, 115.0, 115.0, and 114.0, respectively.
The specified nodal values would appear as

```
 12,  100.0,  15,  115.0,  19,  115.0,  21,  114.0   0,   0.0,   0,   0.0
```

3.3.6 Input Description

Table 3.3.1 shows the inputs required for the two possible cases using this program. The first case, auto-grid case, assumes use of the automatic grid generation program. The second case, non-auto-grid case, assumes the user will input the element data from cards.

3.3.7 Stream Function and Velocity Function Problems

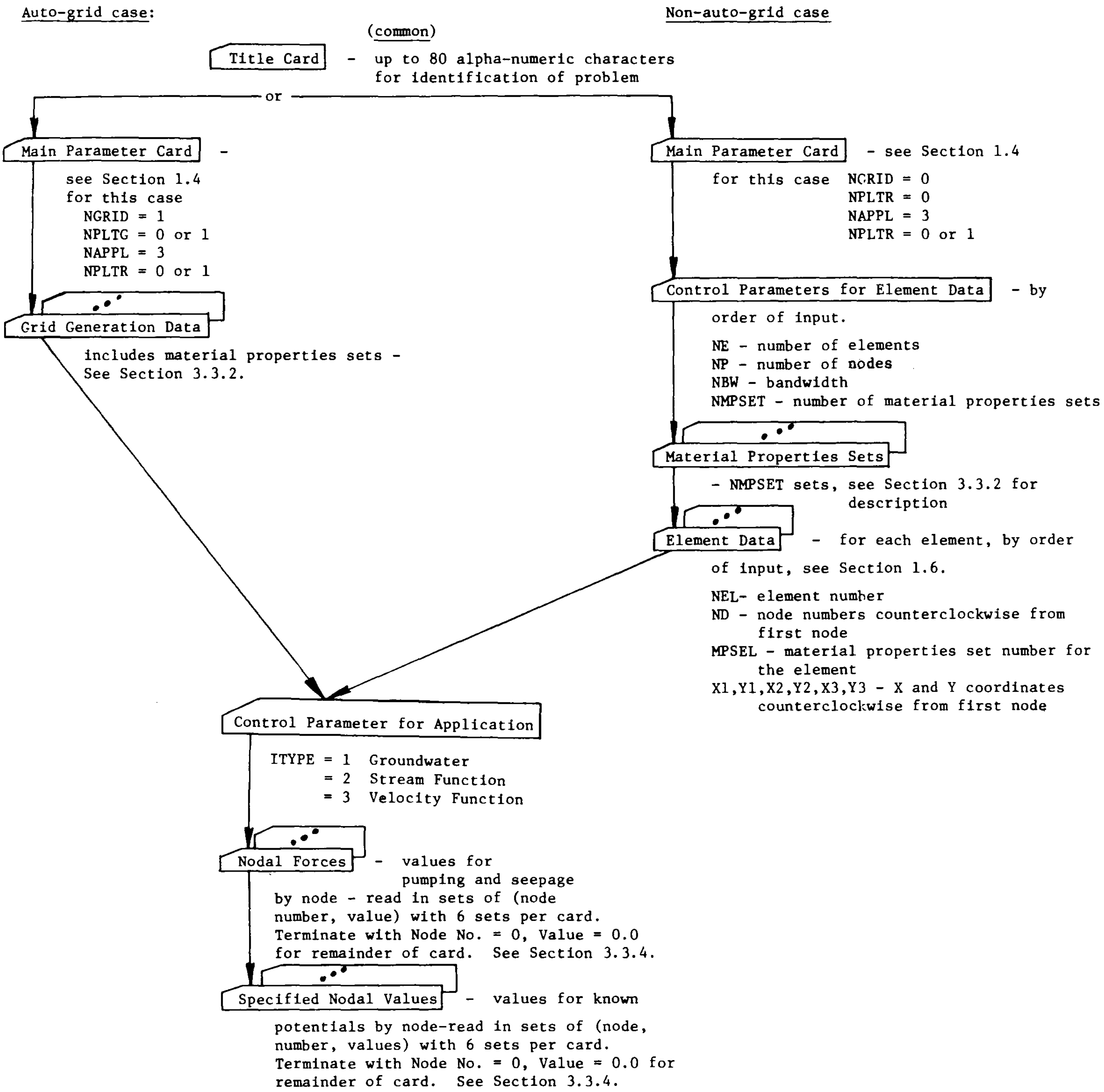
The stream function application is generated by 1) setting the conductivities equal to one, 2) specifying no nodal forces (12 zeros), and 3) specifying fixed nodal values on the no-flow boundaries as known stream function values. The program will then solve for stream function values at the nodes and velocities for each element.

The velocity function approach is generated by 1) setting the conductivities equal to one, 2) inputting nodal forces for flow across a boundary (calculated like seepage) at the appropriate nodes, and 3) specifying one nodal value. Specifying one nodal value fixes the reference for the system, a zero value may be used. The program then solves for the velocity function values at the nodes and velocities for each element.

---

[1] A preliminary run of auto-grid generation may be necessary to determine nodes for boundary conditions.

Table 3.3.1 Program Input for Irrotational Flow



Auto-grid case:

Non-auto-grid case

(common)

Title Card - up to 80 alpha-numeric characters for identification of problem

or

Main Parameter Card - see Section 1.4 for this case
NGRID = 1
NPLTG = 0 or 1
NAPPL = 3
NPLTR = 0 or 1

Grid Generation Data includes material properties sets - See Section 3.3.2.

Main Parameter Card - see Section 1.4 for this case
NGRID = 0
NPLTR = 0
NAPPL = 3
NPLTR = 0 or 1

Control Parameters for Element Data - by order of input.

NE - number of elements
NP - number of nodes
NBW - bandwidth
NMPSET - number of material properties sets

Material Properties Sets - NMPSET sets, see Section 3.3.2 for description

Element Data - for each element, by order of input, see Section 1.6.

NEL- element number
ND - node numbers counterclockwise from first node
MPSEL - material properties set number for the element
X1,Y1,X2,Y2,X3,Y3 - X and Y coordinates counterclockwise from first node

Control Parameter for Application

ITYPE = 1 Groundwater
= 2 Stream Function
= 3 Velocity Function

Nodal Forces - values for pumping and seepage by node - read in sets of (node number, value) with 6 sets per card. Terminate with Node No. = 0, Value = 0.0 for remainder of card. See Section 3.3.4.

Specified Nodal Values - values for known potentials by node-read in sets of (node, number, values) with 6 sets per card. Terminate with Node No. = 0, Value = 0.0 for remainder of card. See Section 3.3.4.

3.3.8 Sample Problem

Figure 3.3.1 shows a region for study. Water infiltrates into the region from a stream on two sides. Heads are known at a number of locations on the boundary of the region. Linear interpolation will be used to obtain nodal values on the sections of the boundary with given potentials. Figure 3.3.2 shows the region divided for automatic grid generation. Figure 3.3.3 shows the input using automatic grid generation. Figure 3.3.4 shows the grid generated. Figure 3.3.5 shows the output of the program.

Figure 3.3.6 shows the input for the same grid as shown in Figure 3.3.4 but not using automatic grid generation. Figure 3.3.7 shows the output of the program.

Fig. 3.3.1 Sample Problem for Groundwater Flow

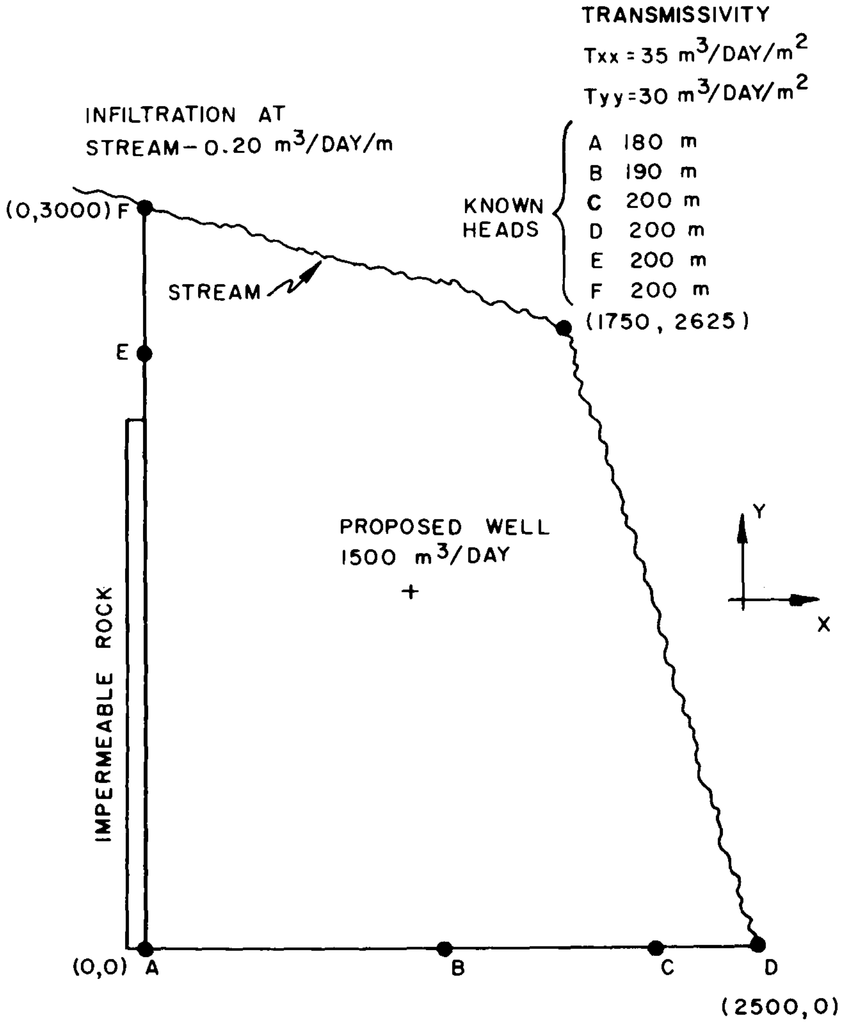


Fig. 3.3.2 Region for Grid Generation

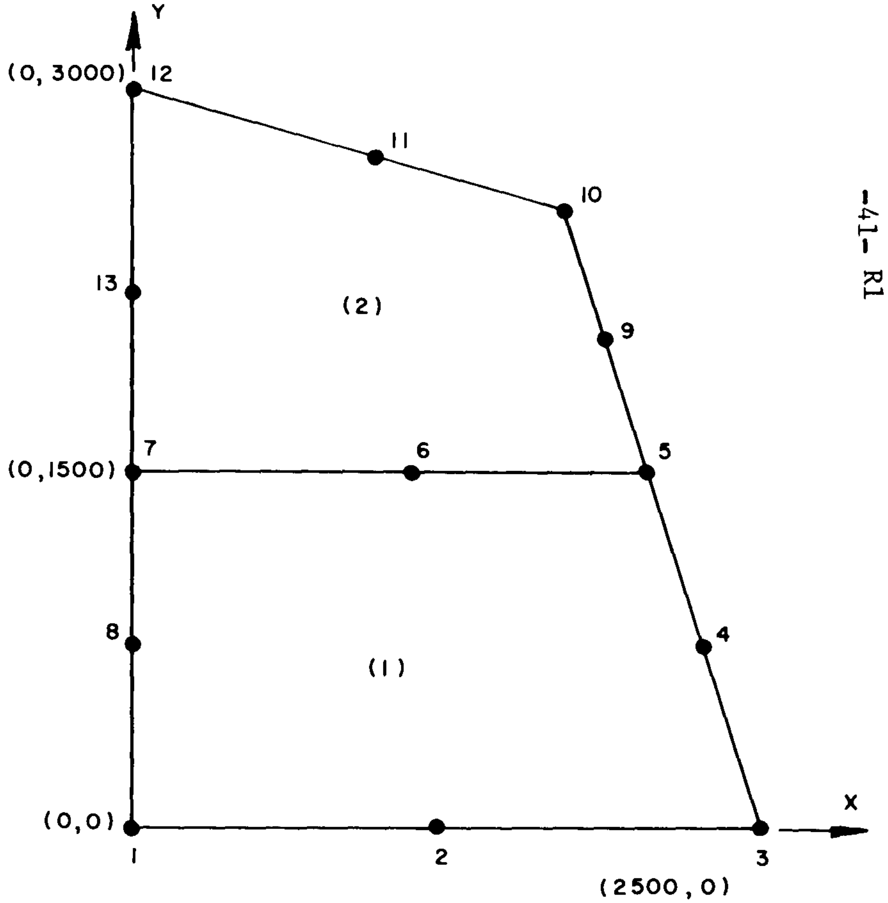
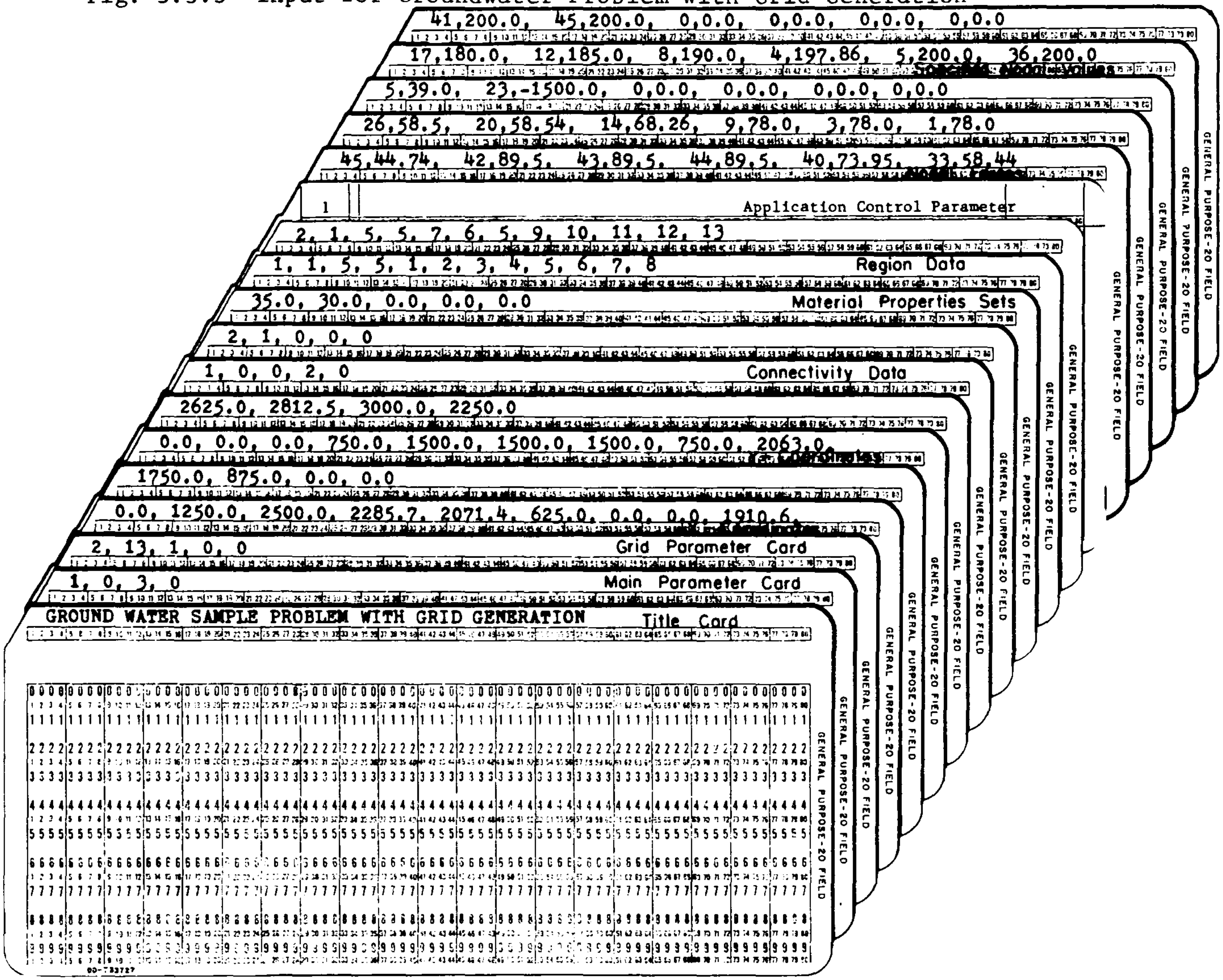


Fig. 3.3.3 Input for Groundwater Problem with Grid Generation



41,200.0, 45,200.0, 0,0.0, 0,0.0, 0,0.0, 0,0.0

17,180.0, 12,185.0, 8,190.0, 4,197.86, 5,200.0, 36,200.0 Specified Nodal Values

5,39.0, 23,-1500.0, 0,0.0, 0,0.0, 0,0.0, 0,0.0

26,58.5, 20,58.54, 14,68.26, 9,78.0, 3,78.0, 1,78.0

45,44.74, 42,89.5, 43,89.5, 44,89.5, 40,73.95, 33,58.44

1 Application Control Parameter

2, 1, 5, 5, 7, 6, 5, 9, 10, 11, 12, 13

1, 1, 5, 5, 1, 2, 3, 4, 5, 6, 7, 8 Region Data

35.0, 30.0, 0.0, 0.0, 0.0 Material Properties Sets

2, 1, 0, 0, 0

1, 0, 0, 2, 0 Connectivity Data

2625.0, 2812.5, 3000.0, 2250.0

0.0, 0.0, 0.0, 750.0, 1500.0, 1500.0, 1500.0, 750.0, 2063.0,

1750.0, 875.0, 0.0, 0.0

0.0, 1250.0, 2500.0, 2285.7, 2071.4, 625.0, 0.0, 0.0, 1910.6,

2, 13, 1, 0, 0 Grid Parameter Card

1, 0, 3, 0 Main Parameter Card

GROUND WATER SAMPLE PROBLEM WITH GRID GENERATION Title Card

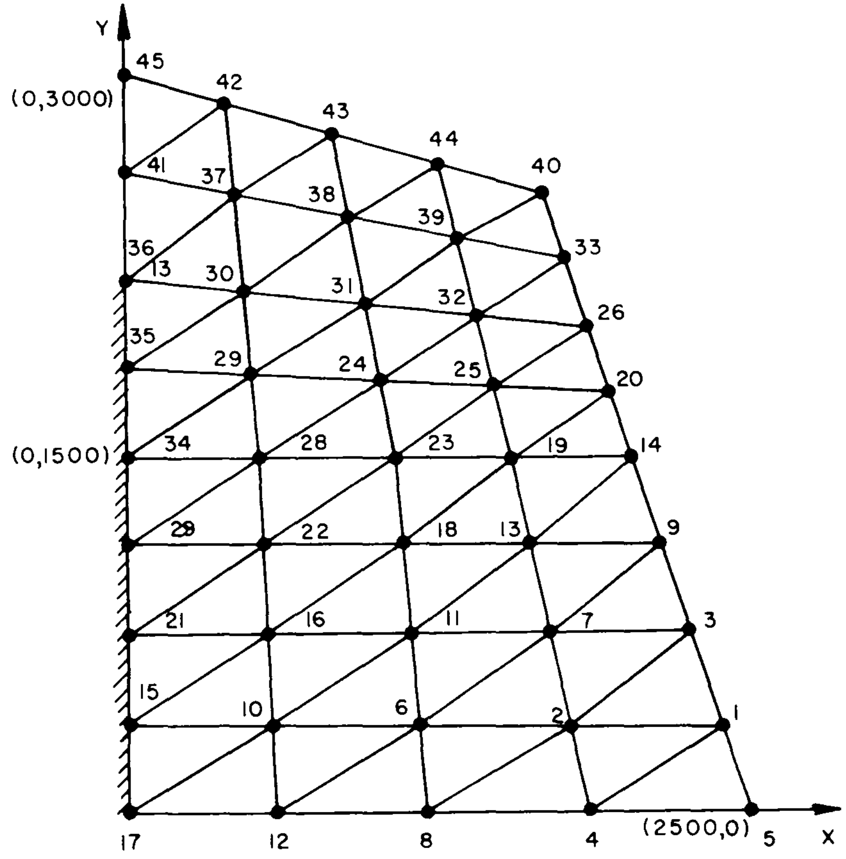
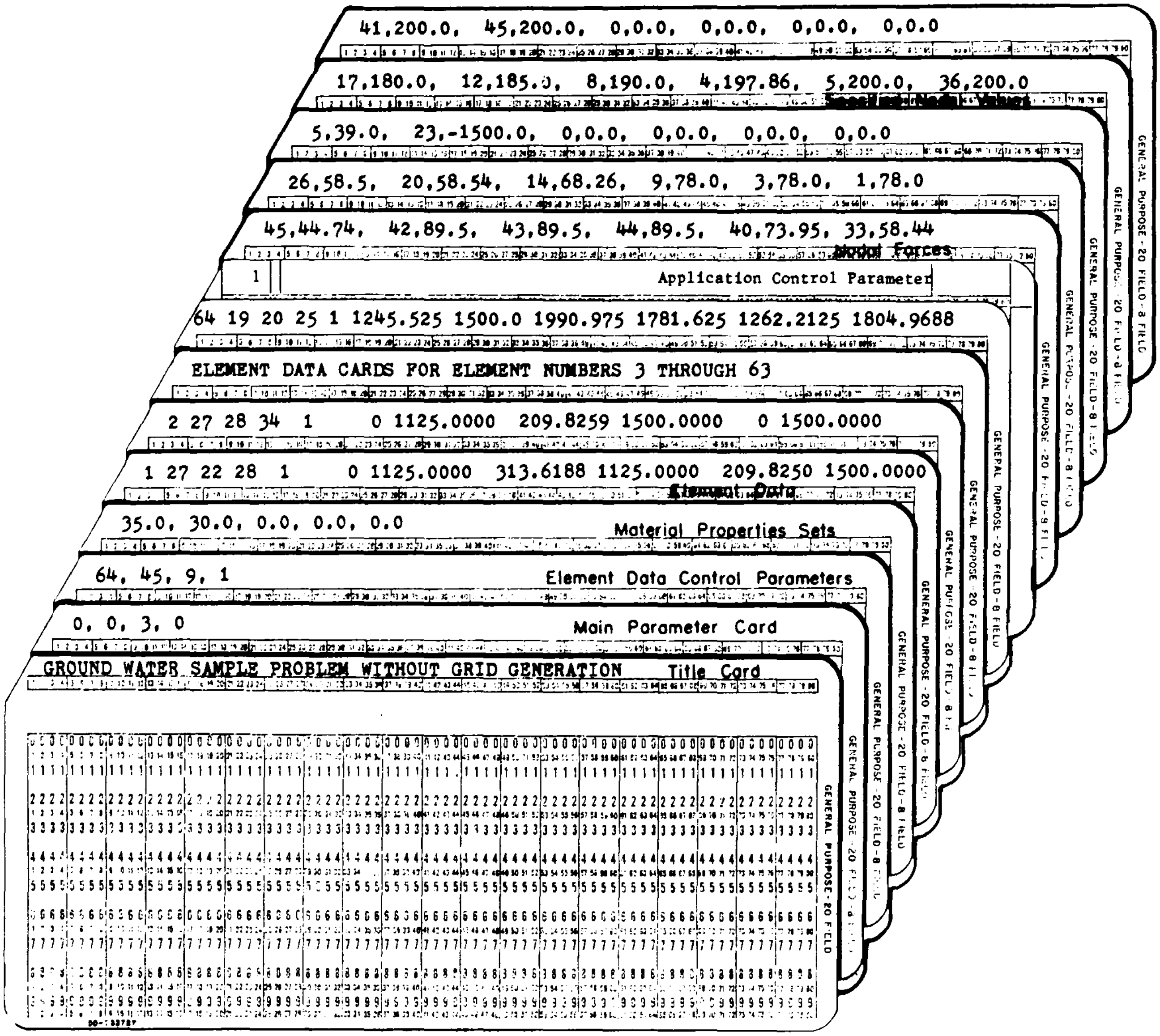


Fig. 3.3.4 Grid Generated Groundwater Example

```
BOUNDARY VALUES

NODAL  FORCES
 45    .44740E+02   42    .89500E+02   43    .89500E+02   44    .89500E+02   40    .73950E+02   33    .58440E+02
 26    .58500E+02   20    .58540E+02   14    .68260E+02    9    .78000E+02    3    .78000E+02    1    .78000E+02
  5    .39000E+02   23   -.15000E+04



PRESCRIBED NODAL  VALUES
 17    .18000E+03   12    .18500E+03    8    .19000E+03    4    .19786E+03    5    .20000E+03   36    .20000E+03
 41    .20000E+03   45    .20000E+03



NODAL  VALUES
  1    .19860E+03    2    .19382E+03    3    .19654E+03    4    .19786E+03    5    .20000E+03    6    .18779E+03
  7    .19086E+03    8    .19000E+03    9    .19485E+03   10    .18380E+03   11    .18488E+03   12    .18500E+03
 13    .18831E+03   14    .19395E+03   15    .18179E+03   16    .18202E+03   17    .18000E+03   18    .18021E+03
 19    .18643E+03   20    .19407E+03   21    .18134E+03   22    .17943E+03   23    .16957E+03   24    .18235E+03
 25    .18875E+03   26    .19494E+03   27    .17985E+03   28    .17904E+03   29    .18538E+03   30    .19333E+03
 31    .18989E+03   32    .19174E+03   33    .19629E+03   34    .17998E+03   35    .18713E+03   36    .20000E+03
 37    .19685E+03   38    .19434E+03   39    .19454E+03   40    .19775E+03   41    .20000E+03   42    .19917E+03
 43    .19716E+03   44    .19682E+03   45    .20000E+03

ELEMENT VELOCITY COMPONENTS

    ELEMENT        VEL(X)          VEL(Y)
         1       .46946E-01      .42475E-01
         2       .15711E+00     -.10358E-01
         3      -.58564E-01      .83084E+00
         4       .79857E+00      .22079E+00
         5      -.45614E+00     -.13340E-01
         6      -.95121E+00      .51203E+00
         7      -.29485E+00     -.42912E-03
         8      -.31874E+00      .36081E-01
         9      -.56860E-01      .19381E+00
        10       .46946E-01      .11940E+00
        11      -.19203E+00      .30440E+00
        12      -.58564E-01      .19340E+00
        13      -.33629E+00      .82581E-01
        14      -.45614E+00      .21007E+00
        15      -.27416E+00      .68246E-01
        16      -.29485E+00      .97486E-01
        17      -.13464E+00      .10995E+00
        18      -.56860E-01      .35734E-01
        19      -.24404E+00      .14582E+00
        20      -.19203E+00      .96334E-01
        21      -.33838E+00      .11510E+00
        22      -.33629E+00      .11288E+00
        23      -.24764E+00      .10377E+00
        24      -.27416E+00      .13820E+00
        25      -.28000E+00      .29929E-01
        26      -.13464E+00     -.14324E+00
        27      -.28000E+00      .76988E-01
        28      -.24404E+00      .38460E-01
        29      -.44016E+00      .16493E+00
        30      -.33838E+00      .56140E-01
        31      -.11984E+00      .82916E-01
        32      -.24764E+00      .23418E+00
        33       .24017E+00     -.23125E+00
        34       .66731E-01               0
        35       .14941E+00     -.28199E+00
        36       .10734E+00     -.21281E+00
        37      -.51751E-01     -.22221E+00
        38      -.35611E-01     -.25175E+00
        39      -.14416E+00     -.19099E+00
        40      -.12926E+00     -.21854E+00
        41       .65583E+00     -.39159E+00
        42       .28995E+00               0
        43       .23932E+00     -.33378E+00
        44       .12187E+00     -.42714E+00
        45      -.14992E+00     -.26878E+00
        46      -.80879E-01     -.39403E+00
        47      -.19405E+00     -.19184E+00
        48      -.15604E+00     -.26849E+00
        49       .15756E+00     -.69939E+00
        50       .54831E+00     -.10296E+01
        51       .20703E+00     -.70625E+00
        52       .18983E+00     -.72056E+00
        53      -.41948E+00     -.62112E+00
        54      -.15177E+00     -.28673E+00
        55      -.26121E+00     -.15693E+00
        56      -.20204E+00     -.28437E+00
        57       .15711E+00     -.57193E+00
        58       .17127E+00     -.56514E+00
        59       .79857E+00     -.65214E+00
        60       .17531E+00     -.11964E+01
        61      -.95121E+00     -.10127E+01
        62      -.39998E+00     -.20958E+00
        63      -.31874E+00     -.90455E-01
        64      -.26342E+00     -.21598E+00
```

Fig. 3.3.6 Input for Groundwater Example Without Grid Generation



```
41,200.0, 45,200.0, 0,0.0, 0,0.0, 0,0.0, 0,0.0
17,180.0, 12,185.0, 8,190.0, 4,197.86, 5,200.0, 36,200.0
5,39.0, 23,-1500.0, 0,0.0, 0,0.0, 0,0.0, 0,0.0
26,58.5, 20,58.54, 14,68.26, 9,78.0, 3,78.0, 1,78.0
45,44.74, 42,89.5, 43,89.5, 44,89.5, 40,73.95, 33,58.44
1
64 19 20 25 1 1245.525 1500.0 1990.975 1781.625 1262.2125 1804.9688
ELEMENT DATA CARDS FOR ELEMENT NUMBERS 3 THROUGH 63
2 27 28 34  1      0 1125.0000  209.8259 1500.0000    0 1500.0000
1 27 22 28  1      0 1125.0000  313.6188 1125.0000  209.8250 1500.0000
35.0, 30.0, 0.0, 0.0, 0.0
64, 45, 9, 1
0, 0, 3, 0
GROUND WATER SAMPLE PROBLEM WITHOUT GRID GENERATION
```

Table 3.6.1 Program Input for Truss

Title Card - up to 80 alpha-numeric characters for identification of problem.

Main Parameter Card - See Section 1.4 for this case NGRID = 0
NPLTG = 0
NAPPL = 6
NPLTR = 0

Control Parameters for Elements Data - by order of input, see Section 3.6.4.

NE - number of elements
NP - number of nodes
NBW - bandwidth
MNPSET - number of material properties sets.

Material Properties Sets - NMPSET sets, see Section 3.6.2 for description.

Element Data - for each element, by order of input, see Section 3.6.5..

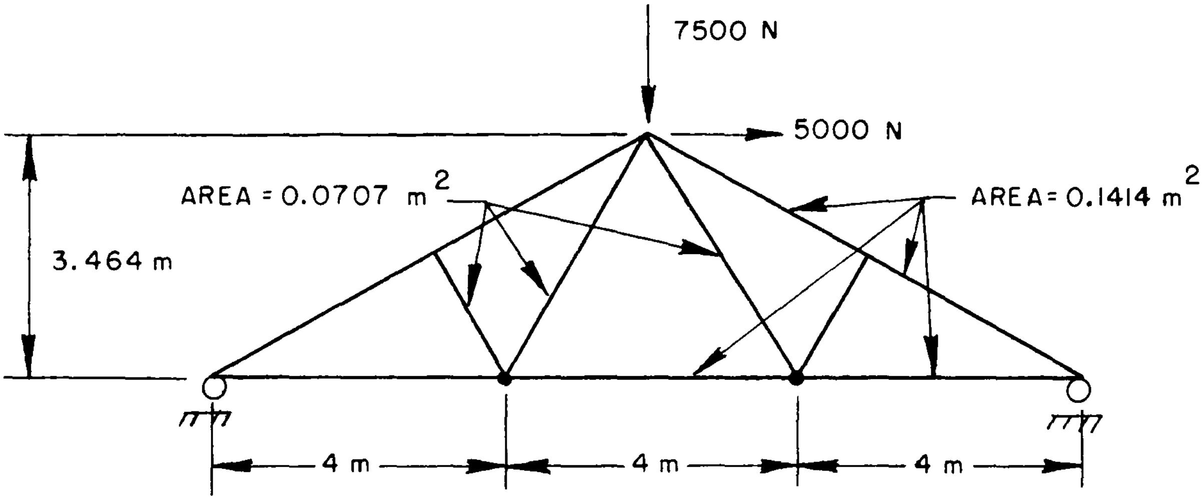
NEL - element number
ND - node numbers
MPSEL - material properties set number for the element
DT - increase in temperature of element
X1,Y1,X2,Y2 - X and Y coordinates for element nodes in same order as node numbers were input.

Nodal Forces - read sets of global degree of freedom associated with force and value, with 6 sets per card - terminate with global d.o.f. = 0, value = 0.0 for remainder of card. See Section 3.6.6.

Specified Nodal Values - read sets of global degree of freedom associated with displacement and value of displacement with 6 sets per card - terminate with global d.o.f. = 0, value = 0.0 for remainder of card.

3.6.8 Sample Problem

Consider the truss problem diagrammed below. The truss is assumed to be fixed at both ends.



E = 20 x 10^{10} N/m^2 COEF. OF THERMAL EXPANSION = 0.0

The following figure shows the truss with elements and nodes labeled for input.

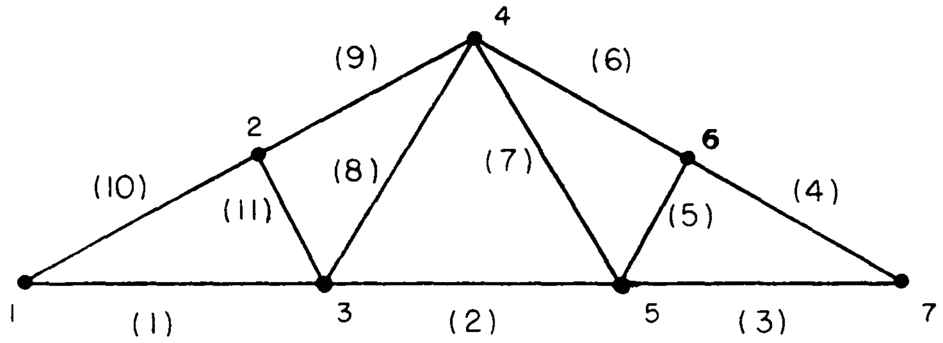
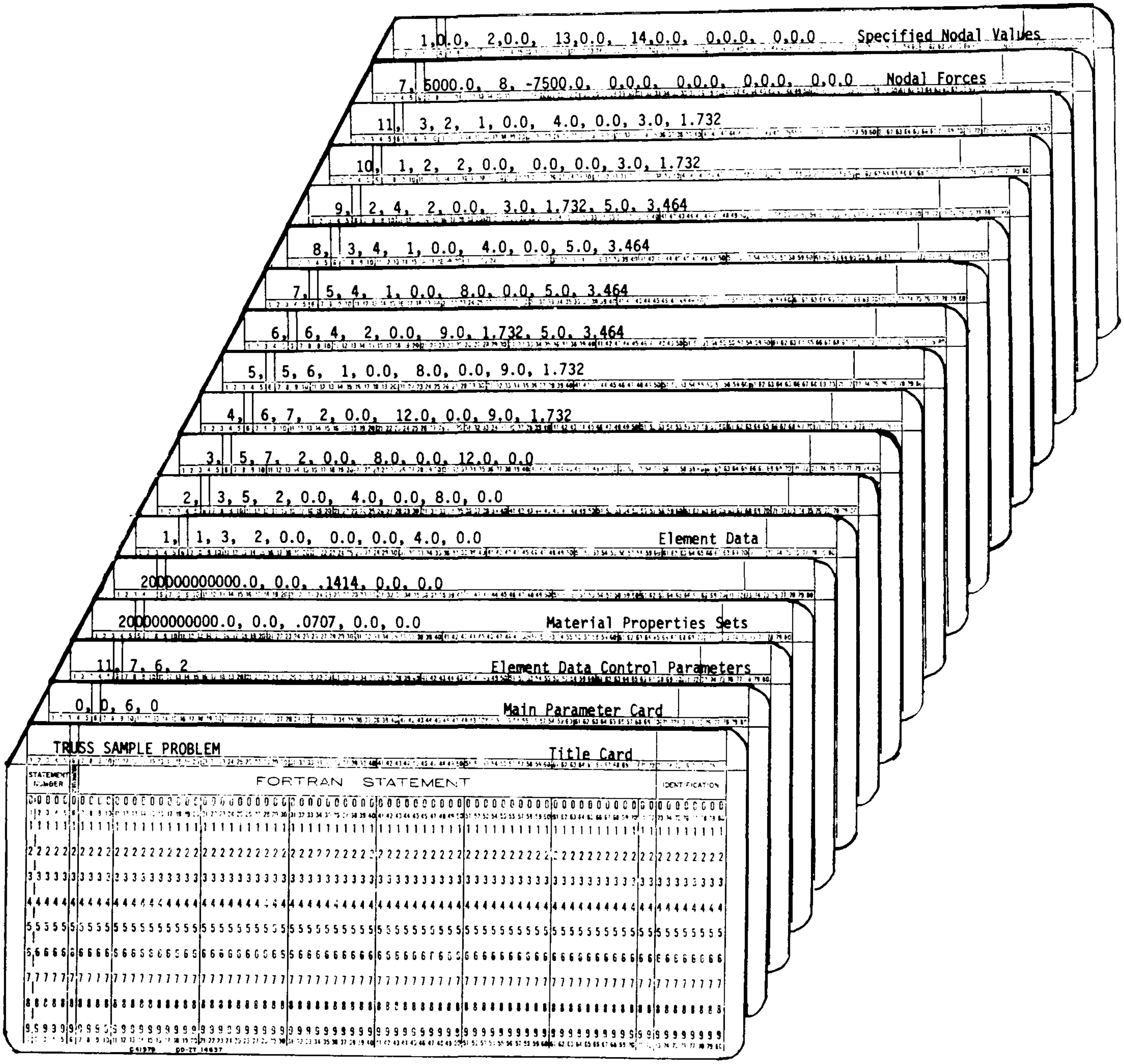


Figure 3.6.1 shows the input for this problem. Note that units must be consistent (i.e., if material properties are in inches, nodal distance must be in inches).

Figure 3.6.2 shows the output of the truss program for this problem.

Figure 3.6.1 Input for Truss Example Problem



```
1,0.0, 2,0.0, 13,0.0, 14,0.0, 0,0.0, 0,0.0      Specified Nodal Values
7, 5000.0, 8, -7500.0, 0,0.0, 0,0.0, 0,0.0, 0,0.0      Nodal Forces
11, 3, 2, 1, 0.0, 4.0, 0.0, 3.0, 1.732
10, 1, 2, 2, 0.0, 0.0, 0.0, 3.0, 1.732
9, 2, 4, 2, 0.0, 3.0, 1.732, 5.0, 3.464
8, 3, 4, 1, 0.0, 4.0, 0.0, 5.0, 3.464
7, 5, 4, 1, 0.0, 8.0, 0.0, 5.0, 3.464
6, 6, 4, 2, 0.0, 9.0, 1.732, 5.0, 3.464
5, 5, 6, 1, 0.0, 8.0, 0.0, 9.0, 1.732
4, 6, 7, 2, 0.0, 12.0, 0.0, 9.0, 1.732
3, 5, 7, 2, 0.0, 8.0, 0.0, 12.0, 0.0
2, 3, 5, 2, 0.0, 4.0, 0.0, 8.0, 0.0
1, 1, 3, 2, 0.0, 0.0, 0.0, 4.0, 0.0      Element Data
200000000000.0, 0.0, .1414, 0.0, 0.0
200000000000.0, 0.0, .0707, 0.0, 0.0      Material Properties Sets
11, 7, 6, 2      Element Data Control Parameters
0, 0, 6, 0      Main Parameter Card
TRUSS SAMPLE PROBLEM      Title Card
```

Figure 3.6.2 Output for Truss Example Problem

```
 TRUSS SAMPLE PROBLEM



APPLICATION PROGRAM TRUSS WILL BE SOLVED



MATERIAL PROPERTIES

SET      MAT PROP 1       MAT PROP 2     MAT PROP 3

            EM              ALPHA          AREA
  1      .2000E+12         0          .7070E-01
  2      .2000E+12         0          .1414E+00



ELEMENT DATA

                          MATERIAL
 NEL    NODE NUMBERS   SET          DT                   X(1)         Y(1)        X(2)        Y(2)
   1     1    3        2                     0              0            0      4.0000           0
   2     3    5        2                     0         4.0000            0      8.0000           0
   3     5    7        2                     0         8.0000            0     12.0000           0
   4     7    6        2                     0        12.0000            0      9.0000      1.7320
   5     5    6        1                     0         8.0000            0      9.0000      1.7320
   6     6    4        2                     0         9.0000       1.7320      5.0000      3.4640
   7     5    4        1                     0         8.0000            0      5.0000      3.4640
   8     3    4        1                     0         4.0000            0      5.0000      3.4640
   9     2    4        2                     0         3.0000       1.7320      5.0000      3.4640
  10     1    2        2                     0              0            0      3.0000      1.7320
  11     3    2        1                     0         4.0000            0      3.0000      1.7320


              NUMBER OF ELEMENTS = NE =    11


              NUMBER OF NODES = NP =     7


              BANDWIDTH = NBW =      6
BOUNDARY VALUES

     NODAL FORCES
  7    .50000E+04     8    -.75000E+04




     PRESCRIBED NODAL  VALUES
  1          0         2          0        13          0        14          0




NODAL VALUES
  1         0        2         0        3    .11884E-05    4   -.34949E-05    5    .14198E-06    6   -.39147E-05
  7    .72500E-06    8   -.38131E-05    9    .16426E-06   10   -.37805E-05   11   -.55206E-06   12   -.31947E-05
 13         0       14         0
ELEMENT FORCES AND STRESSES


    ELEMENT

       1            FORCE      .100382E+04             7          FORCE      -.120868E+04
                    STRESS     .709913E+04                        STRESS     -.170959E+05

       2            FORCE      .157493E+03             8          FORCE       .101713E+04
                    STRESS     .111381E+04                        STRESS      .143865E+05

       3            FORCE     -.116131E+04             9          FORCE      -.597103E+04
                    STRESS    -.821294E+04                        STRESS     -.422279E+05

       4            FORCE     -.913687E+04            10          FORCE      -.586346E+04
                    STRESS    -.646172E+05                        STRESS     -.414672E+05

       5            FORCE      .105502E+04            11          FORCE      -.112841E+04
                    STRESS     .149225E+05                        STRESS     -.159605E+05

       6            FORCE     -.919760E+04
                    STRESS    -.650467E+05
```

3.7 AXISYMMETRIC ELASTICITY

3.7.1 Introduction

This program analyzes axisymmetric bodies subjected to boundary forces and/or displacements. Thermal stresses may be included by inputting temperature values for the elements. Automatic grid generation can also be used with this program.

3.7.2 Material Properties Description

The material properties sets for this program include, in the following order:

EM - Elastic Modulus
PR - Poisson's ratio
ALPHA - Coefficient of thermal expansion
TEMP - Initial steady-state temperature for body
1 Dummy Variable = 0.0

Any compatible set of units may be used. Up to twenty sets of material properties sets may be input.

3.7.3 Control Parameters for Program

Control parameter read in during application program section for axisymmetric elasticity is:

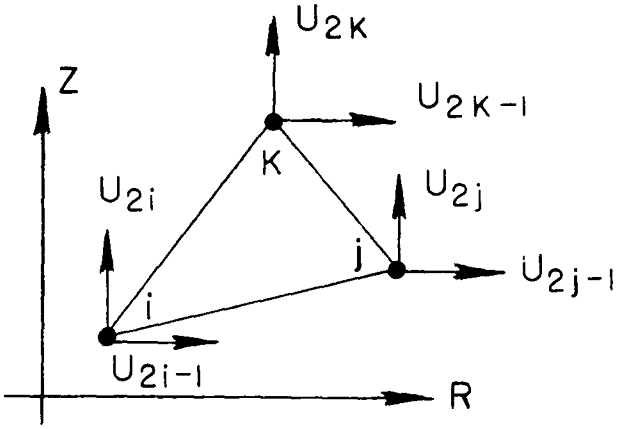
ITEMP - Control parameter for element temperature input.
  =0 -no temperature input
  =1 -temperature input for each element. See Section 3.7.4.

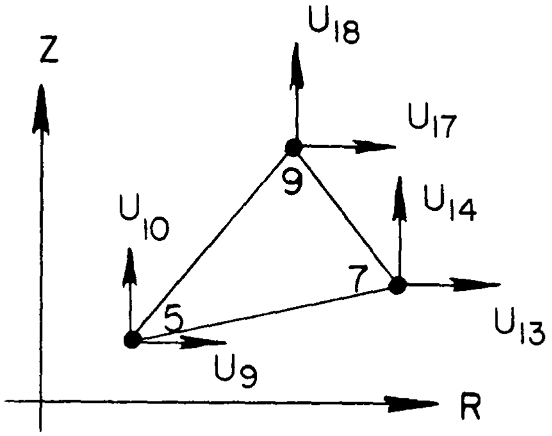
3.7.4 Element Temperatures

Average temperature for each element may be input in order to calculate thermal stresses. This requires specifying the control parameter ITEMP = 1 and inputting of the average element temperatures ordered by element number. The temperature difference between the initial steady-state temperature (TEMP) and the average element temperature is used to calculate the thermal stress.

3.7.5 Vector Components

Axisymmetric elasticity, like two dimensional elasticity, is complicated by having a vector unknown, displacement. This displacement is expressed as the (vector) sum of two other displacements, one in the R-direction and one in the Z-direction. The figures below demonstrate how these components are expressed. Each node has two degrees of freedom (global degrees of freedom). The Z-component is expressed as two times the node number and the R-component is expressed as two times the node number minus 1.



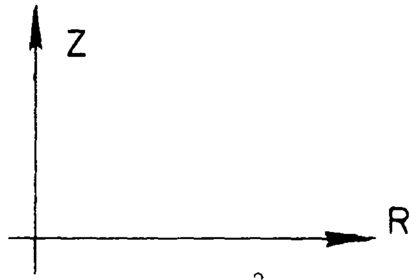


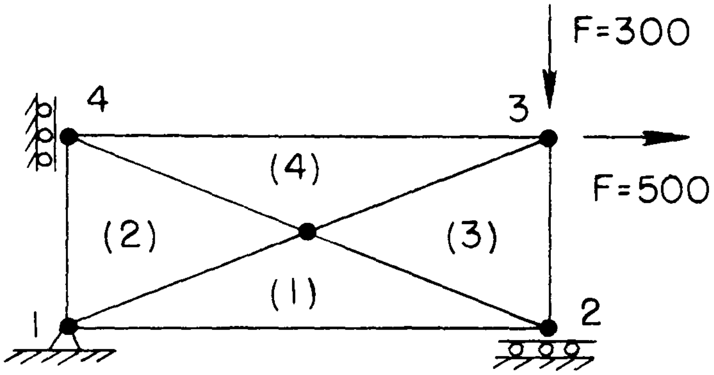
Both the known nodal displacements and any force values are keyed to the appropriate global degree of freedom. See Section 3.7.6.

3.7.6 Boundary Values

Boundary conditions are input in two sections. The first nodal forces; the second is known displacements. Both are input by the global degrees of freedom at each node and the value of the vector component parallel to the direction of that degree of freedom.[1] Each card must have six sets of global d.o.f. and associated values. If a complete card is not needed, the remaining sets must be specified with zero values. If an even multiple of six sets occurs, or no values are to be input, a card with six sets of zeros (12 zeros) must follow to terminate that section of data input.

For the simple region below





The nodal forces[2] would be

6, -300.0, 5. 500.0, 0,0.0, 0,0.0, 0,0.0, 0,0.0

and the known nodal values (displacements):

1, 0.0, 2, 0.0, 4, 0.0, 7,0.0, 0,0.0, 0,0.0

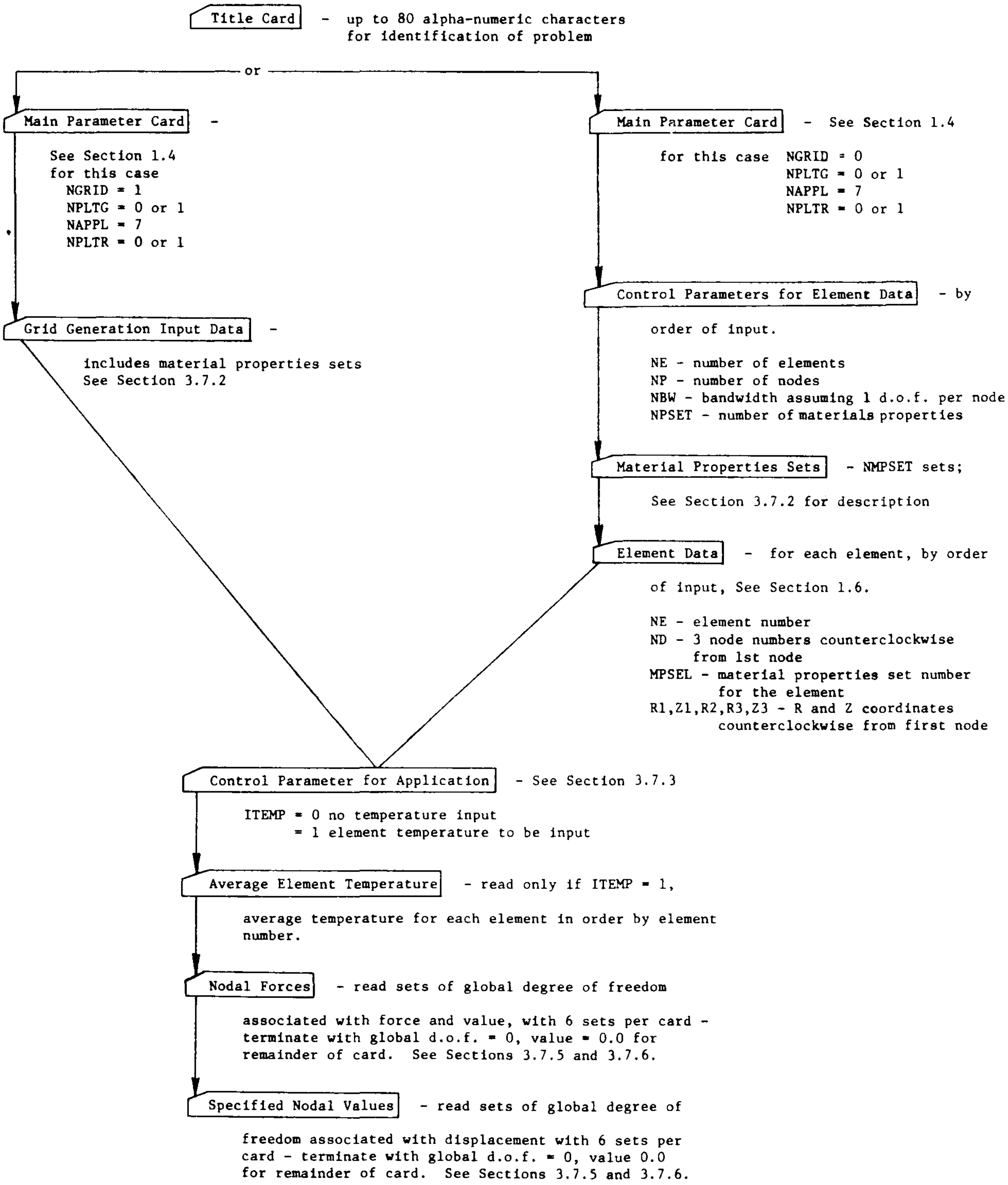
Note that the sign convention assumes forces and displacements in the positive direction are positive, and the opposite are negative.

---

[1] Global degrees of freedom at each node refers to nodes after grid generation when automatic grid generation is used. A preliminary run of auto grid generation may be necessary to determine nodes for boundary conditions.

[2] For a detailed explaination of calculating nodal forces from stress distributions, the user is referred to Segerlind, 1976. Applied Finite Element Analysis, Chapter 12.

Table 3.7.1 Program Input For Axisymmetric Elasticity



Auto-grid case

Non-auto grid case

(common)

Title Card - up to 80 alpha-numeric characters for identification of problem

Main Parameter Card - See Section 1.4 for this case
NGRID = 1
NPLTG = 0 or 1
NAPPL = 7
NPLTR = 0 or 1

Grid Generation Input Data - includes material properties sets See Section 3.7.2

Main Parameter Card - See Section 1.4 for this case
NGRID = 0
NPLTG = 0 or 1
NAPPL = 7
NPLTR = 0 or 1

Control Parameters for Element Data - by order of input.

NE - number of elements
NP - number of nodes
NBW - bandwidth assuming 1 d.o.f. per node
NPSET - number of materials properties

Material Properties Sets - NMPSET sets;

See Section 3.7.2 for description

Element Data - for each element, by order of input, See Section 1.6.

NE - element number
ND - 3 node numbers counterclockwise from 1st node
MPSEL - material properties set number for the element
R1,Z1,R2,R3,Z3 - R and Z coordinates counterclockwise from first node

Control Parameter for Application - See Section 3.7.3

ITEMP = 0 no temperature input
= 1 element temperature to be input

Average Element Temperature - read only if ITEMP = 1,

average temperature for each element in order by element number.

Nodal Forces - read sets of global degree of freedom associated with force and value, with 6 sets per card - terminate with global d.o.f. = 0, value = 0.0 for remainder of card. See Sections 3.7.5 and 3.7.6.

Specified Nodal Values - read sets of global degree of freedom associated with displacement with 6 sets per card - terminate with global d.o.f. = 0, value 0.0 for remainder of card. See Sections 3.7.5 and 3.7.6.

3.7.7 Input Description

Table 3.7.1 shows the inputs required for two possible cases using axisymmetric elasticity. The first assumes the use of automatic grid generation, while the second assumes the user will input the element data from cards.

3.7.8 Sample Problem

Figure 3.7.1 shows a column and footing that will be used as an example for an axisymmetric elasticity problem. The wooden column is subjected to a pressure from above and it is desired to obtain stress-strain results. Using summetry, Fig. 3.7.2 shows the actual problem to be subdivided for automatic grid generation. Fig. 3.7.3 demonstrates the necessary input for the automatic grid generation solution to the problem. Fig. 3.7.4 shows the consequential output. Figure 3.7.5 shows the necessary input not using automatic grid generation.

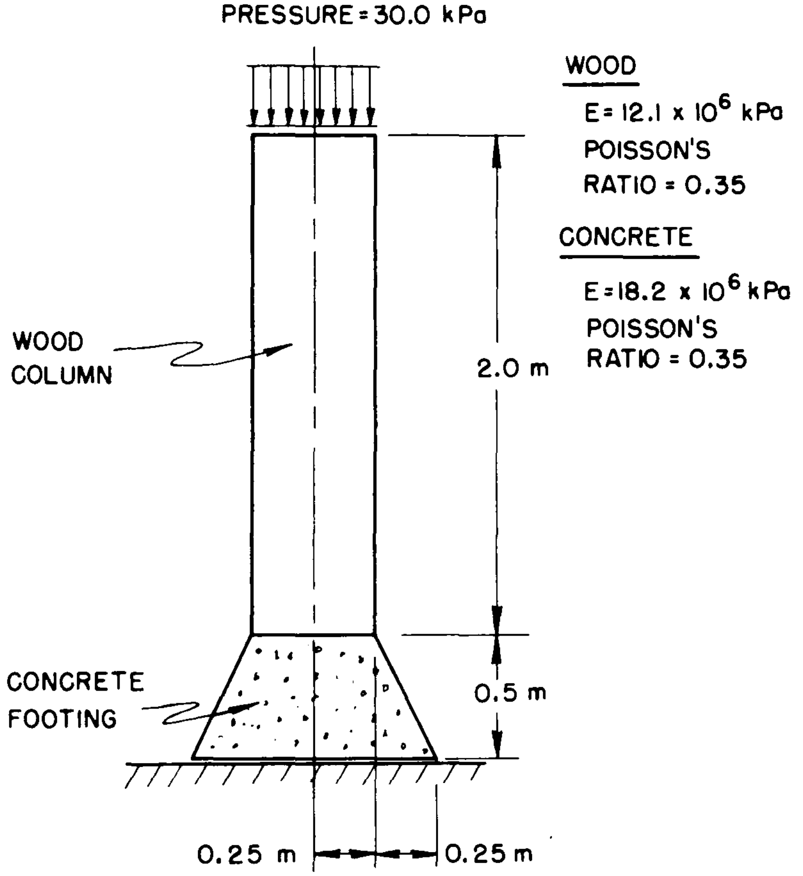


Figure 3.7.1 Example Problem

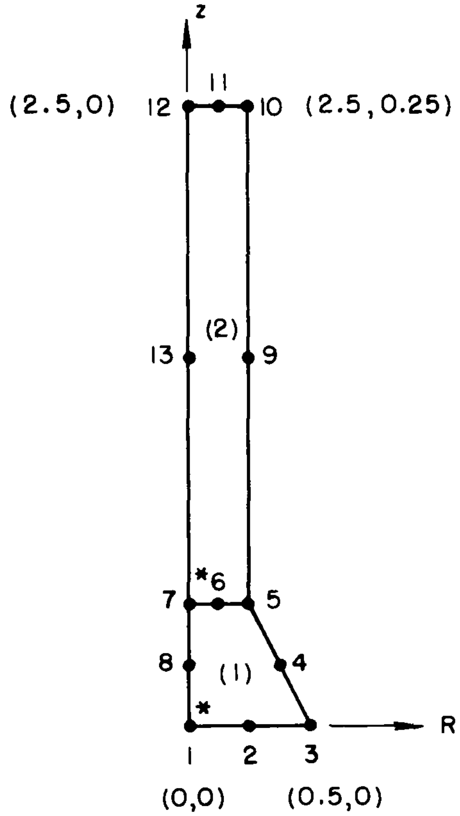


Figure 3.7.2 Regions for Grid Generation

Figure 3.7.3 Input for Axisymmetric Elasticity with Grid Generation

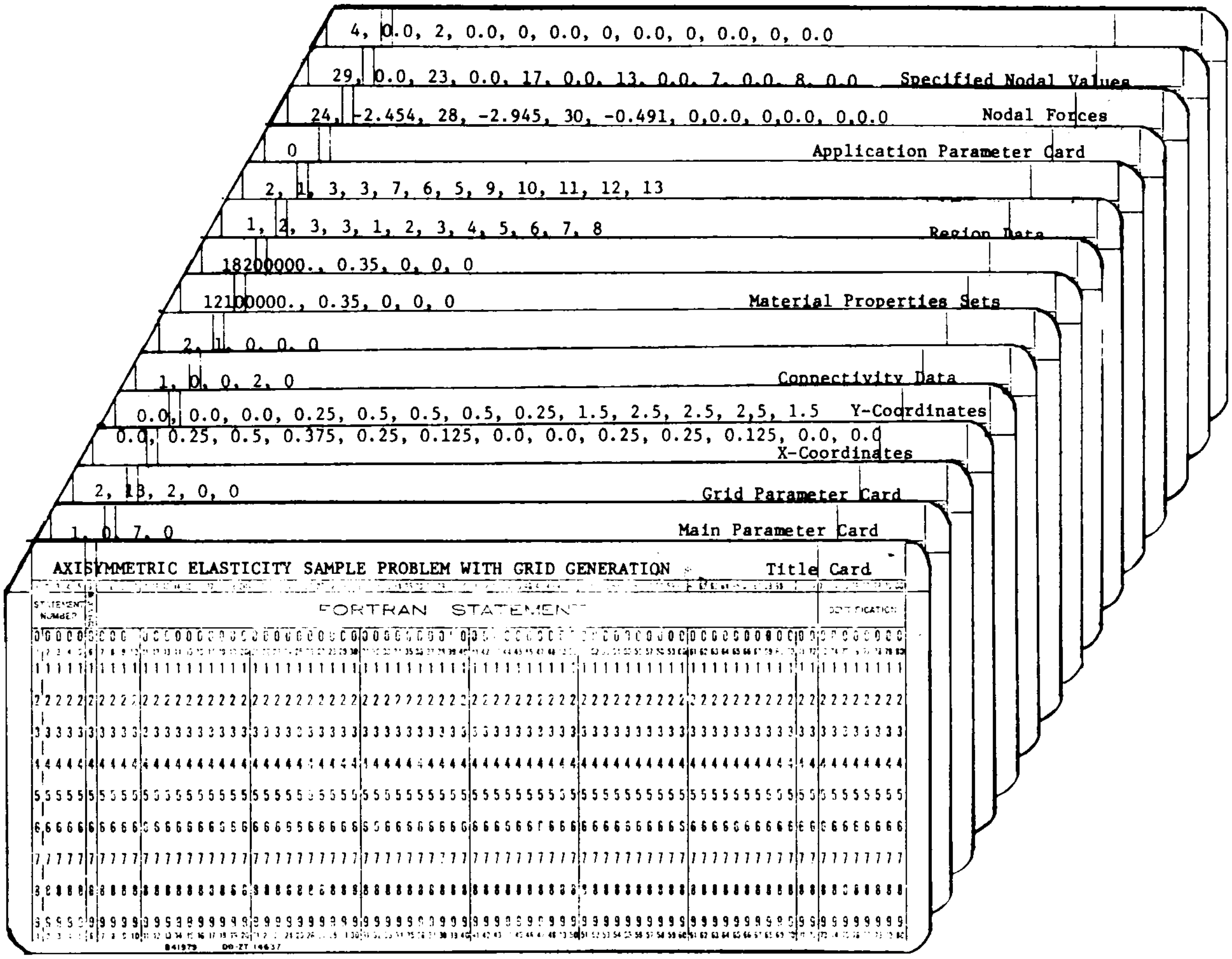

```
4, 0.0, 2, 0.0, 0, 0.0, 0, 0.0, 0, 0.0, 0, 0.0
29, 0.0, 23, 0.0, 17, 0.0, 13, 0.0, 7, 0.0, 8, 0.0
24, -2.454, 28, -2.945, 30, -0.491, 0,0.0, 0,0.0, 0,0.0
0
2, 1, 3, 3, 7, 6, 5, 9, 10, 11, 12, 13
1, 2, 3, 3, 1, 2, 3, 4, 5, 6, 7, 8
18200000., 0.35, 0, 0, 0
12100000., 0.35, 0, 0, 0
2, 1, 0, 0, 0
1, 0, 0, 2, 0
0.0, 0.0, 0.0, 0.25, 0.5, 0.5, 0.5, 0.25, 1.5, 2.5, 2.5, 2,5, 1.5
0.0, 0.25, 0.5, 0.375, 0.25, 0.125, 0.0, 0.0, 0.25, 0.25, 0.125, 0.0, 0.0
2, 13, 2, 0, 0
1, 0, 7, 0
AXISYMMETRIC ELASTICITY SAMPLE PROBLEM WITH GRID GENERATION
```

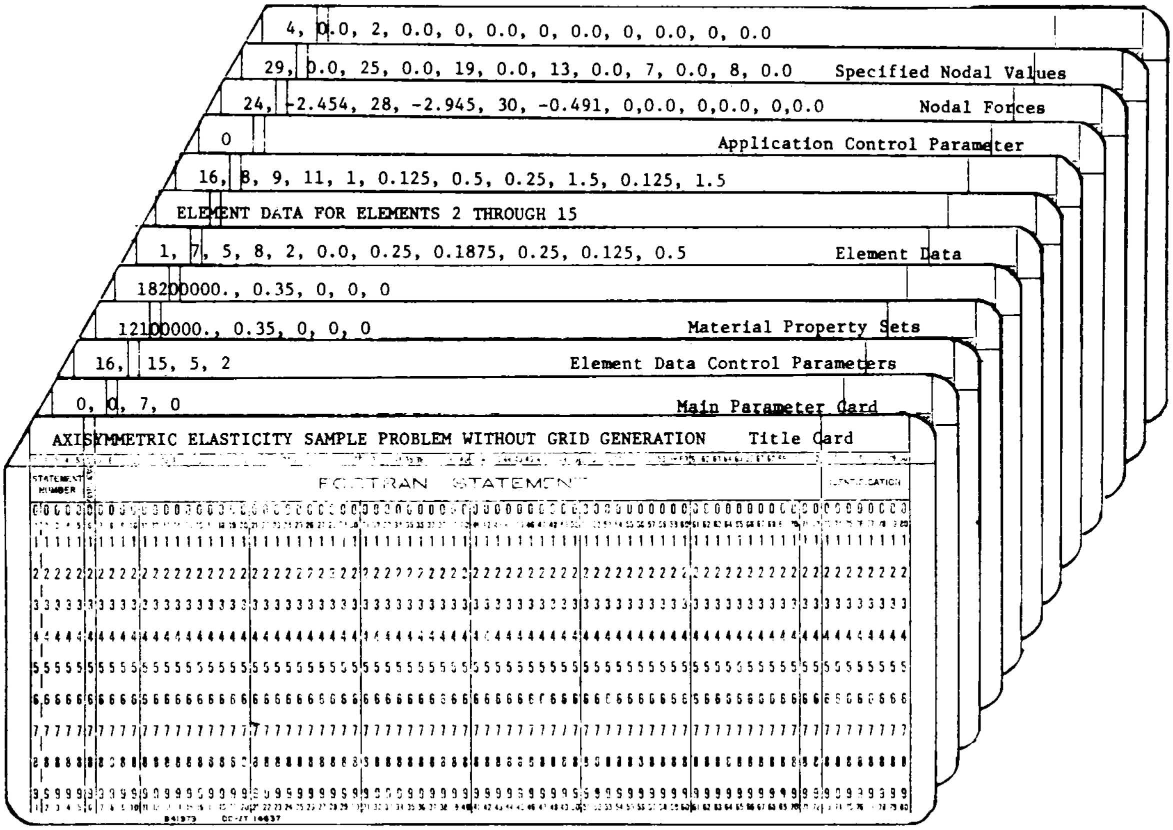
Figure 3.7.4 Output for Axisymmetric Elasticity Example

```
AXISYMMETRIC ELASTICITY SAMPLE PROBLEM WITH GRID GENERATION


THE GRID WILL BE AUTOMATICALLY GENERATED

APPLICATION PROGRAM AXISYMMETRIC ELASTICITY WILL BE SOLVED

GRID RELABELED TO MINIMIZE BANDWIDTH


GLOBAL COORDINATES
NUMBER      R COORD      Z COORD

   1            0            0
   2          .25            0
   3          .50            0
   4          .38          .25
   5          .25          .50
   6          .13          .50
   7            0          .50
   8            0          .25
   9          .25         1.50
  10          .25         2.50
  11          .13         2.50
  12            0         2.50
  13            0         1.50
```

```
CONNECTIVITY DATA

                                   SIDE
REGION                  1       2       3       4

   1                    0       0       2       0
   2                    1       0       0       0


MATERIAL PROPERTIES

SET      MAT PROP 1         MAT PROP 2      MAT PROP 3      MAT PROP 4      MAT PROP 5

             EM                 PR            ALPHA            TEMP              T

  1      .1210E+08        .3500E+00           0                0                0
  2      .1820E+06        .3500E+00           0                0                0


REGION DATA

                                     MATERIAL
REGION              ROWS    COLS       SET                 INPUT REGION NODE NO.

  1                   3       3         2         1     2     3     4     5     6     7     8
  2                   3       3         1         7     6     5     9    10    11    12    13

    NODE NUMBERS
OF SUBDIVIDED REGION

      1
           10     8     6
            7     5     3
            4     2     1


      2
           15    14    12
           13    11     9
           10     8     6


ELEMENT DATA, REGION     1      MAT. PROP. SET =    2

  NEL    NODE NUMBERS            R(1)          Z(1)          R(2)          Z(2)          R(3)          Z(3)
   1      7     5     8            0          .2500         .1875         .2500         .1250         .5000
   2      7     6    10            0          .2500         .1250         .5000             0         .5000
   3      5     3     6        .1875          .2500         .3750         .2500         .2500         .5000
   4      5     6     8        .1875          .2500         .2500         .5000         .1250         .5000
   5      4     2     5            0              0         .2500             0         .1875         .2500
   6      4     5     7            0              0         .1875         .2500             0         .2500
   7      2     1     3        .2500              0         .5000             0         .3750         .2500
   8      2     3     5        .2500              0         .3750         .2500         .1875         .2500


ELEMENT DATA, REGION     2      MAT. PROP. SET =    1

  NEL    NODE NUMBERS            R(1)          Z(1)          R(2)          Z(2)          R(3)          Z(3)
   9     13    11    14            0         1.5000         .1250        1.5000         .1250        2.5000
  10     13    14    15            0         1.5000         .1250        2.5000             0        2.5000
  11     11     9    12        .1250         1.5000         .2500        1.5000         .2500        2.5000
  12     11    12    14        .1250         1.5000         .2500        2.5000         .1250        2.5000
  13     10     8    11            0          .5000         .1250         .5000         .1250        1.5000
  14     10    11    13            0          .5000         .1250        1.5000             0        1.5000
  15      8     6     9        .1250          .5000         .2500         .5000         .2500        1.5000
  16      8     9    11        .1250          .5000         .2500        1.5000         .1250        1.5000


                 NUMBER OF ELEMENTS = NE =    16


                 NUMBER OF NODES = NP =    15


                 BANDWIDTH = NBW =      4
```

```
BOUNDARY VALUES

    NODAL FORCES
 24   -.24540E+01   28   -.29450E+01   30   -.49100E+00



    PRESCRIBED NODAL  VALUES
 29          0     23          0     17          0     13          0      7          0      8          0
  4          0      2          0



NODAL VALUES
  1    .10056E-06    2          0     3    .70392E-07    4          0      5    .10459E-06    6   -.11876E-06
  7          0       8          0     9    .55156E-07   10   -.18362E-06   11    .21223E-06   12   -.44205E-06
 13          0      14   -.17605E-06  15    .13792E-06   16   -.49880E-06   17          0     18   -.24774E-05
 19    .86452E-07   20   -.54545E-06  21   -.19410E-07   22   -.24834E-05   23          0     24   -.40308E-05
 25   -.23936E-07   26   -.24822E-05  27    .74665E-08   28   -.40225E-05   29          0     30   -.40175E-05

ELEMENT STRAINS AND STRESSES

                                                                                                          ANGLE
ELEMENT      ERR        EZZ        ETT        GRZ        SRR        SZZ        STT        TRZ        S1         S2          TMAX    (DEGREES)

      1    .294E-06 -.127E-05  .618E-06  .364E-06 -.168E+01 -.228E+02  .269E+01  .246E+01 -.140E+01 -.231E+02  .108E+02  .834E+02
      2    .412E-06 -.148E-05  .179E-05  .719E-06  .170E+02 -.845E+01  .357E+02  .485E+01  .179E+02 -.934E+01  .136E+02  .796E+02
      3    .264E-06 -.112E-05  .458E-06  .908E-06 -.272E+01 -.214E+02 -.992E-01  .612E+01 -.887E+00 -.232E+02  .112E+02  .734E+02
      4    .594E-06 -.115E-05  .721E-06  .934E-06  .107E+02 -.128E+02  .124E+02  .629E+01  .122E+02 -.144E+02  .133E+02  .759E+02
      5    .282E-06 -.734E-06  .287E-06  .945E-08  .119E+01 -.125E+02  .126E+01  .637E-01  .119E+01 -.125E+02  .685E+01  .897E+02
      6    .294E-06 -.704E-06  .294E-06 -.404E-07  .214E+01 -.113E+02  .214E+01 -.272E+00  .215E+01 -.113E+02  .674E+01 -.888E+02
      7    .121E-06 -.475E-06  .245E-06  .765E-07 -.937E-01 -.812E+01  .158E+01  .515E+00 -.608E-01 -.816E+01  .405E+01  .863E+02
      8    .264E-06 -.648E-06  .283E-06  .351E-06  .196E+01 -.103E+02  .223E+01  .237E+01  .240E+01 -.108E+02  .658E+01  .795E+02
      9    .362E-07 -.154E-05 -.144E-06  .173E-07 -.169E+02 -.310E+02 -.185E+02  .776E-01 -.169E+02 -.310E+02  .706E+01  .897E+02
     10    .597E-07 -.154E-05 -.132E-06 -.156E-07 -.163E+02 -.306E+02 -.180E+02 -.701E-01 -.163E+02 -.306E+02  .715E+01 -.897E+02
     11    .155E-06 -.155E-05 -.311E-07  .460E-07 -.136E+02 -.289E+02 -.152E+02  .215E+00 -.135E+02 -.289E+02  .766E+01  .892E+02
     12   -.597E-07 -.154E-05 -.239E-07 -.394E-07 -.175E+02 -.308E+02 -.172E+02 -.176E+00 -.175E+02 -.308E+02  .663E+01 -.892E+02
     13    .412E-06 -.198E-05  .820E-06  .216E-06 -.418E+01 -.257E+02 -.526E+00  .968E+00 -.414E+01 -.257E+02  .108E+02  .874E+02
     14    .362E-07 -.194E-05  .345E-06 -.120E-06 -.159E+02 -.336E+02 -.132E+02 -.538E+00 -.159E+02 -.336E+02  .886E+01 -.883E+02
     15    .594E-06 -.204E-05  .560E-06  .242E-06 -.388E+01 -.275E+02 -.419E+01  .108E+01 -.383E+01 -.275E+02  .118E+02  .874E+02
     16    .155E-06 -.198E-05  .237E-06 -.109E-06 -.153E+02 -.344E+02 -.145E+02 -.490E+00 -.152E+02 -.345E+02  .960E+01 -.885E+02
```

Figure 3.7.5 Input for Axisymmetric Example Without Grid Generation



```
4, 0.0, 2, 0.0, 0, 0.0, 0, 0.0, 0, 0.0, 0, 0.0
29, 0.0, 25, 0.0, 19, 0.0, 13, 0.0, 7, 0.0, 8, 0.0        Specified Nodal Values
24, -2.454, 28, -2.945, 30, -0.491, 0,0.0, 0,0.0, 0,0.0          Nodal Forces
0                                                   Application Control Parameter
16, 8, 9, 11, 1, 0.125, 0.5, 0.25, 1.5, 0.125, 1.5
ELEMENT DATA FOR ELEMENTS 2 THROUGH 15
1, 7, 5, 8, 2, 0.0, 0.25, 0.1875, 0.25, 0.125, 0.5              Element Data
18200000., 0.35, 0, 0, 0
12100000., 0.35, 0, 0, 0                               Material Property Sets
16, 15, 5, 2                                  Element Data Control Parameters
0, 0, 7, 0                                                Main Parameter Card
AXISYMMETRIC ELASTICITY SAMPLE PROBLEM WITHOUT GRID GENERATION    Title Card
```

3.8 AXISYMMETRIC HEAT TRANSFER

3.8.1 Introduction

This program calculates the temperature distribution in axisymmetric bodies subjected to either prescribed boundary temperatures or internal heat sources or sinks. The body may have surface convection. The program uses only simple linear triangular elements and automatic grid generation may be used.

3.8.2 Material Properties Description

The material properties sets for this program include, in the following order:

KRR - Thermal conductivity in the R-direction
KZZ - Thermal conductivity in the Z-direction
H - Surface convection coefficient
TINF - Fluid temperature at a distance from the convection surfaces
1 Dummy Variable = 0.0

Any compatible sets of units may be used. Up to twenty sets of material properties may be input.

3.8.3 Convection Data

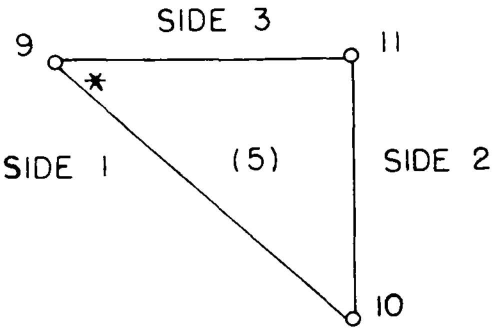
Data for sides of elements that are convective surfaces may be input. The element number and number of the side or sides having convection must be input for each element, in order, for the elements with convection. Sides are numbered 1, 2, and 3, counterclockwise from the first input node for the element.

A maximum of two sides may have convection, therefore, the data is input as:

element no., 1st side for convection, 2nd side for convection

If only one side has convection, a zero value is entered in the second side for convection location.

Assuming that element 5 shown below has surface convection on side 2 and side 3,



* designates first input node

the convection data appears as:

5, 2, 3

This data set must be terminated with a card with element number and side numbers equal to zero, i.e.;

0, 0, 0

IF NO CONVECTION OCCURS for the problem, a card with zero values,

0, 0, 0 must be input for the convection data.

Convection data must be ordered by element numbers.

3.8.4 Boundary Values

Boundary conditions are input in two sections. First, the nodal forces which in this case would be line heat sources or sinks (heat added considered positive). Second, specified nodal values which are known nodal temperatures are input.

Nodal force and nodal values are both input in sets of node number and associated value with six sets per card.[1] If a complete card is not needed, the remaining sets should be specified with zero values. If an even multiple of 6 sets occurs or if no values are to be input, a card with 6 sets of zero values (12 zeros) must follow to terminate that section of input data.

Consider the following example: if the temperature at nodes 1, 6, 9, and 10 were all 20 degrees, the specified nodal values would read,

1, 20.0, 6, 20.0, 9, 20.0, 10, 20.0, 0, 0.0, 0, 0.0

3.8.5 Input Description

Table 3.8.1 shows the inputs required for the two possible cases using Axisymmetric Heat Transfer. The first case, auto-grid, assumes the use of the automatic grid generation program. The second case, non-auto grid, assumes the user will input the element data from cards. The element data must be ordered by element number for this program.

---

[1] A preliminary run of the auto-grid generation may be necessary for convection data and boundary conditions.

3.8.6 Sample Problem

As an example, consider the short section of steam pipe shown in Figure 3.8.1. It is desired to obtain the temperature profile for the critical radius of insulation. The problem can be set up with two different material properties sets, one for the steel pipe and one for the asbestos insulation, as shown in Figure 3.8.2. Figure 3.8.3 shows the proper input for automatic grid generation. Figure 3.8.4 shows the corresponding grid that is generated, and Figure 3.8.5 shows the output. Figure 3.8.6 shows the proper input for the same grid of Figure 3.8.4 but not using automatic grid generation.

Fig. 3.8.1 Sample Problem for Axisymmetric Heat Transfer

T_i = 200.0 C

TINF = 20.0 C

r_o = 4 cm.

r_1 = 6 cm.

r_2 = 12 cm.

KRR(1) = KZZ(1) = 0.48 $\frac{W}{cm\ K}$

KRR(2) = KZZ(2) = 0.002 $\frac{W}{cm\ K}$

H(1) = 0.0

H(2) = 0.025 $\frac{W}{cm^2 K}$

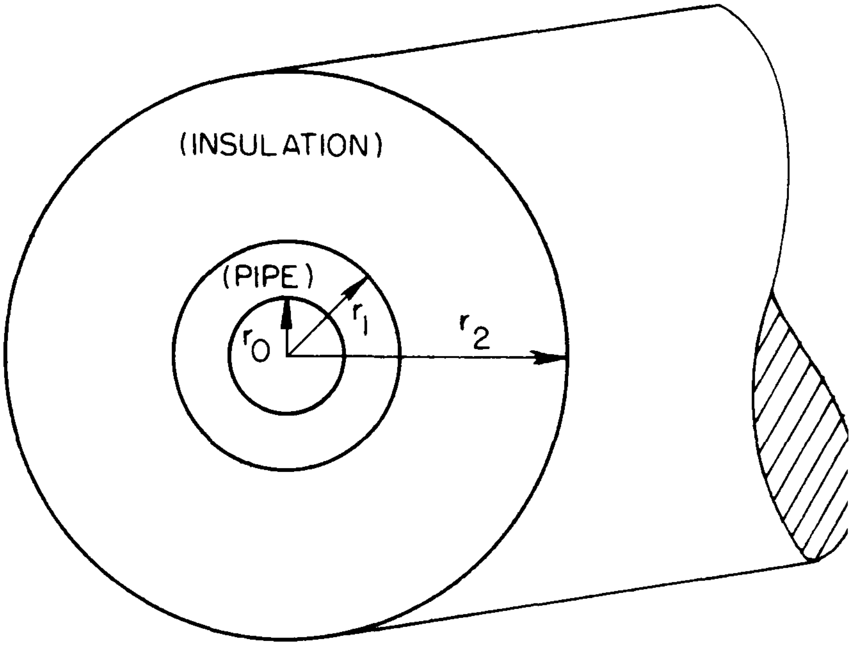


Figure 3.8.2 Region for Automatic Grid Generation

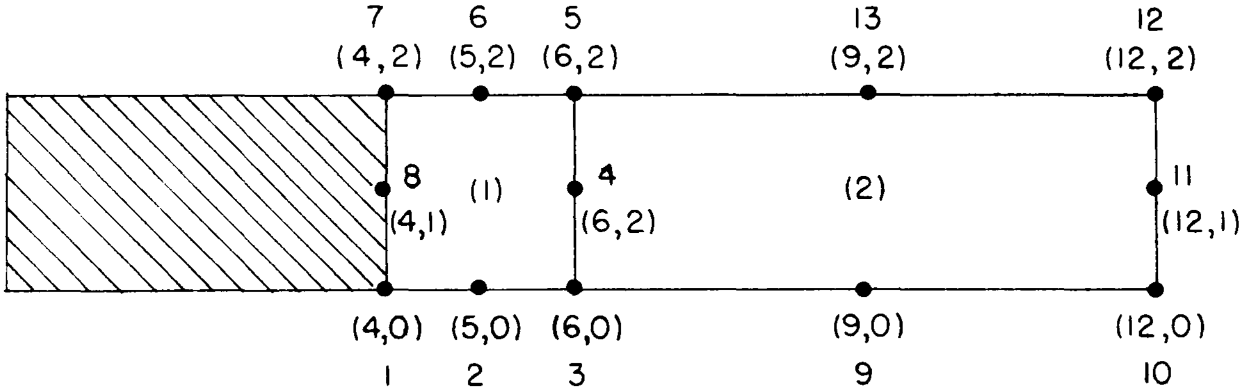
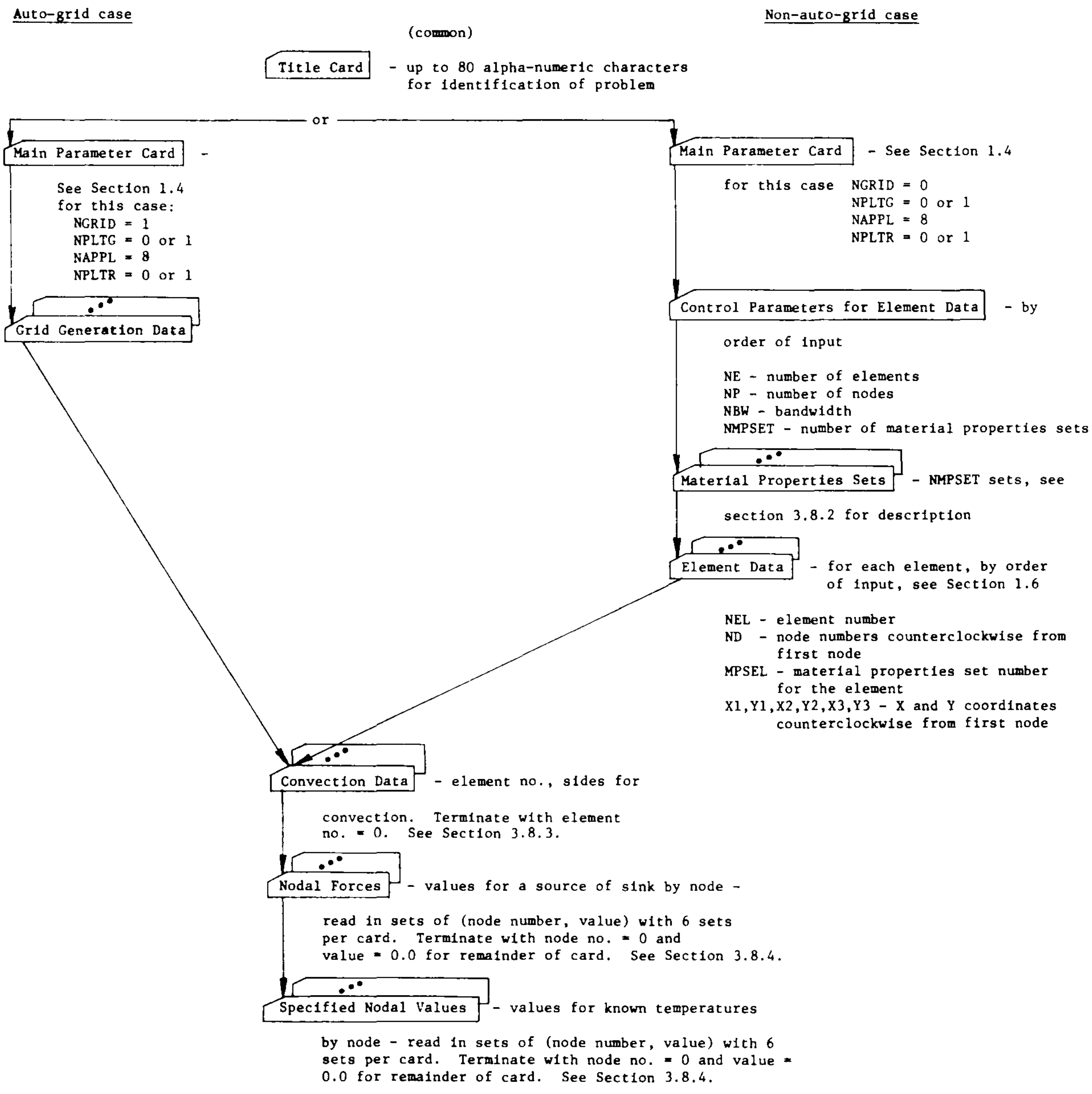


Table 3.8.1 Program Input for Axisymmetric Heat Transfer



Auto-grid case

(common)

Non-auto-grid case

Title Card - up to 80 alpha-numeric characters for identification of problem

or

Auto-grid case

Main Parameter Card - See Section 1.4 for this case:
NGRID = 1
NPLTG = 0 or 1
NAPPL = 8
NPLTR = 0 or 1

Grid Generation Data

Non-auto-grid case

Main Parameter Card - See Section 1.4 for this case
NGRID = 0
NPLTG = 0 or 1
NAPPL = 8
NPLTR = 0 or 1

Control Parameters for Element Data - by order of input

NE - number of elements
NP - number of nodes
NBW - bandwidth
NMPSET - number of material properties sets

Material Properties Sets - NMPSET sets, see section 3.8.2 for description

Element Data - for each element, by order of input, see Section 1.6

NEL - element number
ND - node numbers counterclockwise from first node
MPSEL - material properties set number for the element
X1,Y1,X2,Y2,X3,Y3 - X and Y coordinates counterclockwise from first node

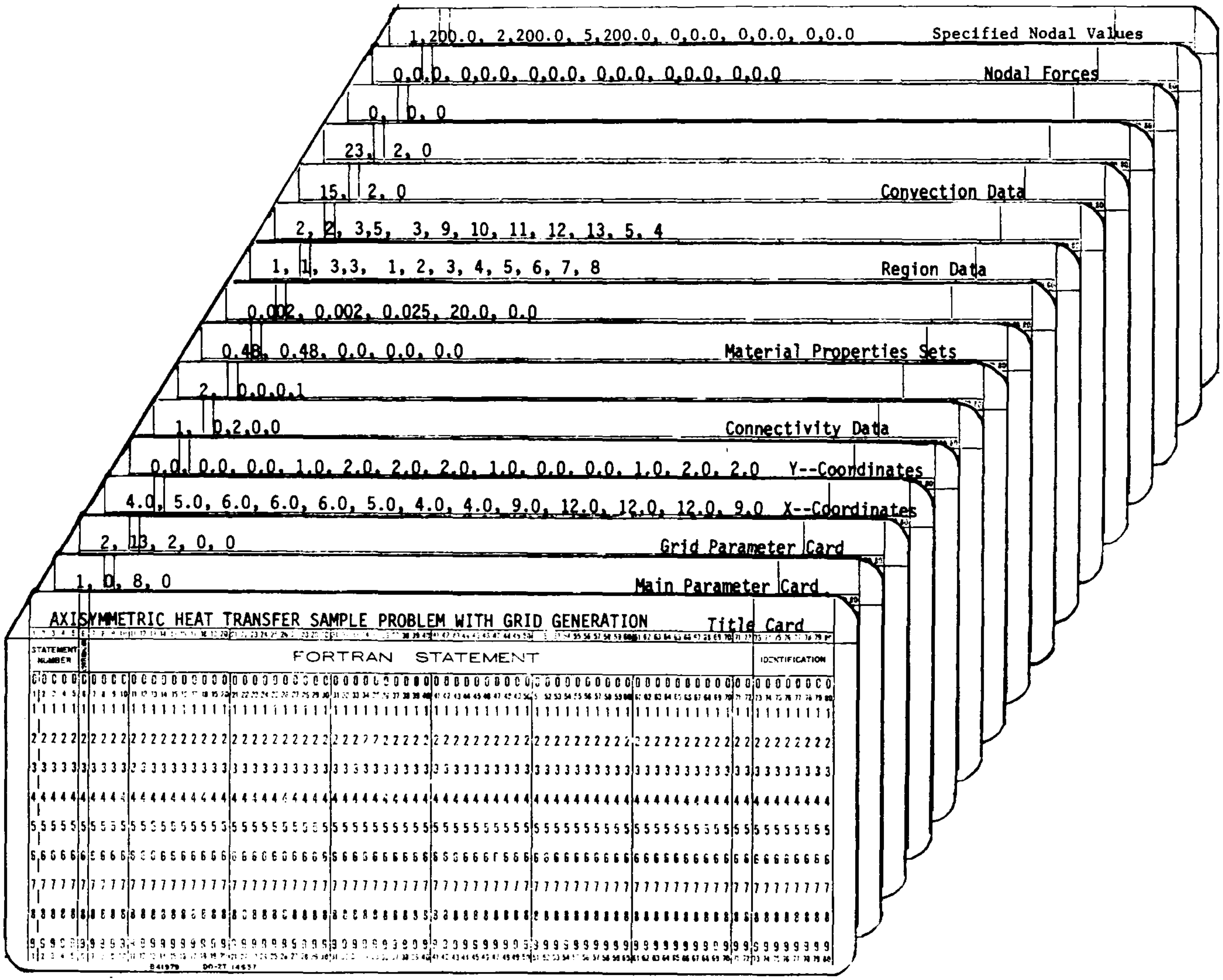
(common)

Convection Data - element no., sides for convection. Terminate with element no. = 0. See Section 3.8.3.

Nodal Forces - values for a source of sink by node - read in sets of (node number, value) with 6 sets per card. Terminate with node no. = 0 and value = 0.0 for remainder of card. See Section 3.8.4.

Specified Nodal Values - values for known temperatures by node - read in sets of (node number, value) with 6 sets per card. Terminate with node no. = 0 and value = 0.0 for remainder of card. See Section 3.8.4.

Fig. 3.8.3 Input for Heat Transfer Example with Grid Generation



```
1,200.0, 2,200.0, 5,200.0, 0,0.0, 0,0.0, 0,0.0          Specified Nodal Values
0,0.0, 0,0.0, 0,0.0, 0,0.0, 0,0.0, 0,0.0                       Nodal Forces
0, 0, 0
23, 2, 0
15, 2, 0                                               Convection Data
2, 2, 3,5,  3, 9, 10, 11, 12, 13, 5, 4
1, 1, 3,3,  1, 2, 3, 4, 5, 6, 7, 8                     Region Data
0.002, 0.002, 0.025, 20.0, 0.0
0.48, 0.48, 0.0, 0.0, 0.0                   Material Properties Sets
2, 0.0,0.1
1, 0,2.0,0                                  Connectivity Data
0.0, 0.0, 0.0, 1.0, 2.0, 2.0, 2.0, 1.0, 0.0, 0.0, 1.0, 2.0, 2.0  Y--Coordinates
4.0, 5.0, 6.0, 6.0, 6.0, 5.0, 4.0, 4.0, 9.0, 12.0, 12.0, 12.0, 9.0  X--Coordinates
2, 13, 2, 0, 0                              Grid Parameter Card
1, 0, 8, 0                                  Main Parameter Card
AXISYMMETRIC HEAT TRANSFER SAMPLE PROBLEM WITH GRID GENERATION      Title Card
```

Fig. 3.8.4 Grid Generated for the Axisymmetric Heat Transfer Example

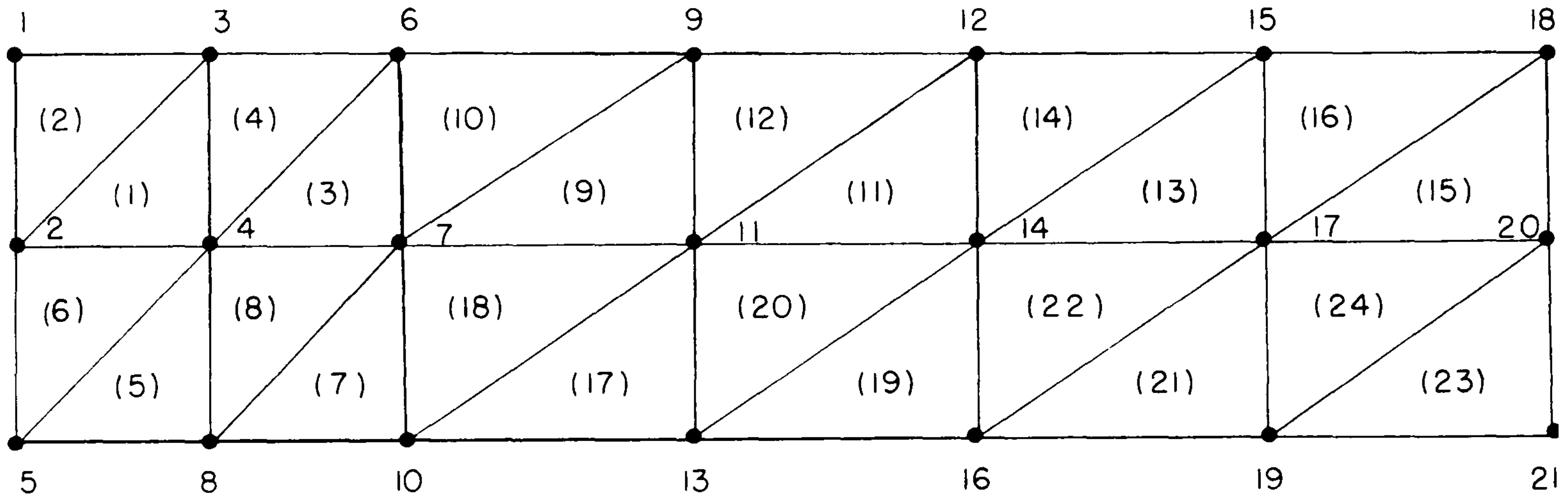
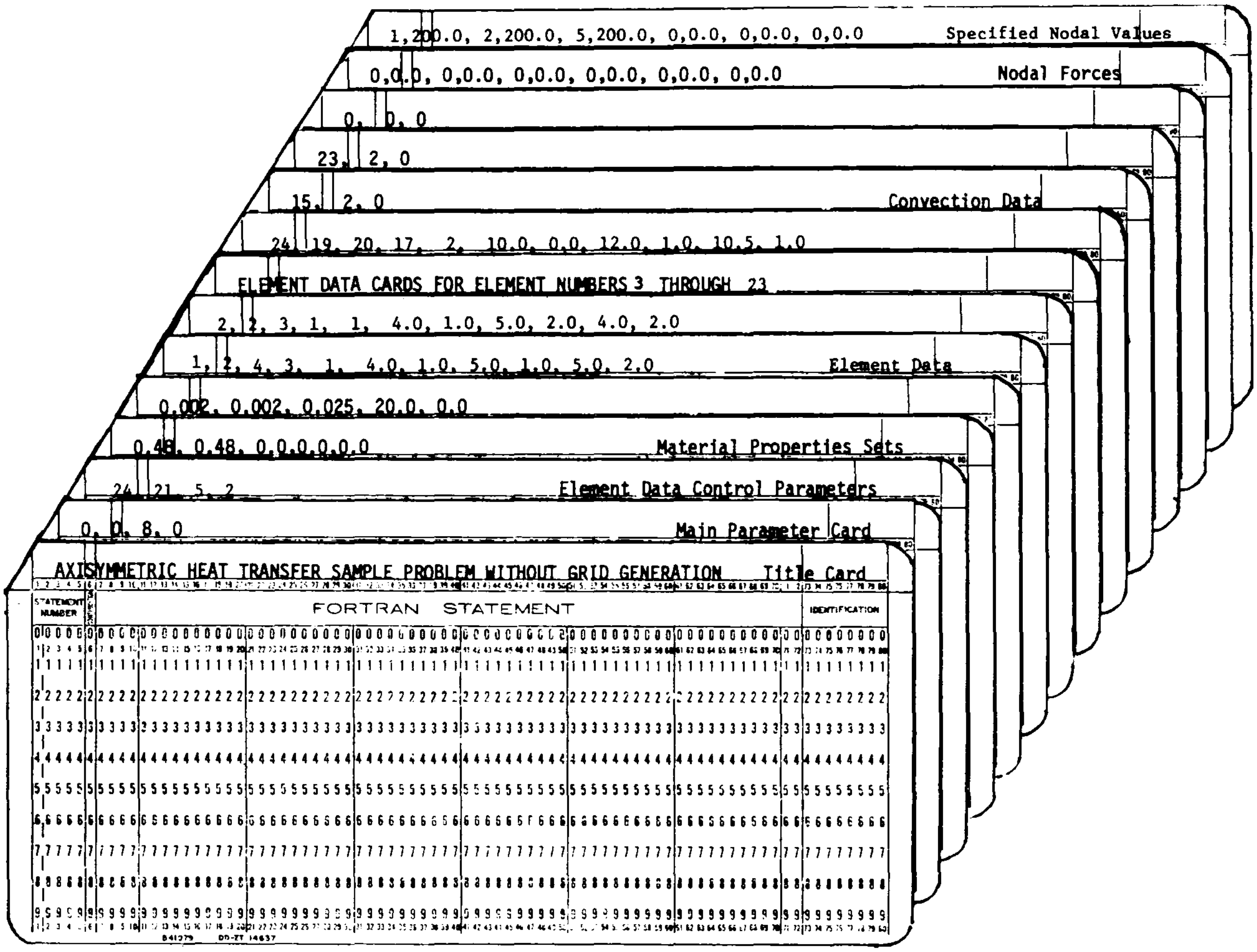


Fig. 3.8.5 Output for Axisymmetric Heat Transfer Example with Grid Generation

```
AXISYMMETRIC HEAT TRANSFER SAMPLE PROBLEM WITH GRID GENERATION



THE GRID WILL BE AUTOMATICALLY GENERATED

APPLICATION PROGRAM AXISYMMETRIC HEAT TRANSFER WILL BE SOLVED

GRID RELABELED TO MINIMIZE BANDWIDTH


GLOBAL COORDINATES

NUMBER     R COORD     Z COORD

   1           0           0
   2        1.00           0
   3        2.00           0
   4        2.00        1.00
   5        2.00        2.00
   6        1.00        2.00
   7           0        2.00
   8           0        1.00
   9        5.00           0
  10        8.00           0
  11        8.00        1.00
  12        8.00        2.00
  13        5.00        2.00


CONNECTIVITY DATA

                              SIDE
REGION                 1      2      3      4

   1                   0      2      0      0
   2                   0      0      0      1


MATERIAL PROPERTIES

SET     MAT PROP 1     MAT PROP 2     MAT PROP 3     MAT PROP 4     MAT PROP 5

           KRR            KZZ             H             TINF           N/A
  1     .4800E+00      .4800E+00        0              0              0
  2     .2000E-02      .2000E-02      .2500E-01      .2000E+02        0


REGION DATA

                                  MATERIAL
REGION              ROWS   COLS     SET                INPUT REGION NODE NO.

   1                  3      3       1         1    2    3    4    5    6    7    8
   2                  3      5       2         3    9   10   11   12   13    5    4

    NODE NUMBERS
OF SUBDIVIDED REGION

        1
              1    3    6
              2    4    7
              5    8   10

        2
              6    9   12   15   18
              7   11   14   17   20
             10   13   16   19   21
```

```
ELEMENT DATA, REGION    1     MAT. PROP. SET =   1

  NEL     NODE NUMBERS          R(1)        Z(1)        R(2)        Z(2)        R(3)        Z(3)
   1     2    4    3              0       1.0000      1.0000      1.0000      1.0000      2.0000
   2     2    3    1              0       1.0000      1.0000      2.0000           0      2.0000
   3     4    7    6         1.0000       1.0000      2.0000      1.0000      2.0000      2.0000
   4     4    6    3         1.0000       1.0000      2.0000      2.0000      1.0000      2.0000
   5     5    8    4              0            0      1.0000           0      1.0000      1.0000
   6     5    4    2              0            0      1.0000      1.0000           0      1.0000
   7     8   10    7         1.0000            0      2.0000           0      2.0000      1.0000
   8     8    7    4         1.0000            0      2.0000      1.0000      1.0000      1.0000


ELEMENT DATA, REGION    2     MAT. PROP. SET =   2

  NEL     NODE NUMBERS          R(1)        Z(1)        R(2)        Z(2)        R(3)        Z(3)
   9     7   11    9         2.0000       1.0000      3.5000      1.0000      3.5000      2.0000
  10     7    9    6         2.0000       1.0000      3.5000      2.0000      2.0000      2.0000
  11    11   14   12         3.5000       1.0000      5.0000      1.0000      5.0000      2.0000
  12    11   12    9         3.5000       1.0000      5.0000      2.0000      3.5000      2.0000
  13    14   17   15         5.0000       1.0000      6.5000      1.0000      6.5000      2.0000
  14    14   15   12         5.0000       1.0000      6.5000      2.0000      5.0000      2.0000
  15    17   20   18         6.5000       1.0000      8.0000      1.0000      8.0000      2.0000
  16    17   18   15         6.5000       1.0000      8.0000      2.0000      6.5000      2.0000
  17    10   13   11         2.0000            0      3.5000           0      3.5000      1.0000
  18    10   11    7         2.0000            0      3.5000      1.0000      2.0000      1.0000
  19    13   16   14         3.5000            0      5.0000           0      5.0000      1.0000
  20    13   14   11         3.5000            0      5.0000      1.0000      3.5000      1.0000
  21    16   19   17         5.0000            0      6.5000           0      6.5000      1.0000
  22    16   17   14         5.0000            0      6.5000      1.0000      5.0000      1.0000
  23    19   21   20         6.5000            0      8.0000           0      8.0000      1.0000
  24    19   20   17         6.5000            0      8.0000      1.0000      6.5000      1.0000


             NUMBER OF ELEMENTS = NE =   24


             NUMBER OF NODES = NP =   21


             BANDWIDTH = NBW =     5
CONVECTION FROM SIDE  2 OF ELEMENT  15
CONVECTION FROM SIDE  2 OF ELEMENT  23

BOUNDARY VALUES

     NODAL FORCES



     PRESCRIBED NODAL  VALUES
  1    .20000E+03    2    .20000E+03    5    .20000E+03



NODAL VALUES
  1    .20000E+03    2    .20000E+03    3    .19887E+03    4    .19891E+03    5    .20000E+03    6    .19853E+03
  7    .19855E+03    8    .19895E+03    9    .12747E+03   10    .19858E+03   11    .12777E+03   12    .81841E+02
 13    .12807E+03   14    .81985E+02   15    .48070E+02   16    .82129E+02   17    .48140E+02   18    .21239E+02
 19    .48212E+02   20    .21297E+02   21    .21356E+02

ELEMENT RESULTANTS

     ELEMENT       GRAD(R)          GRAD(Z)          AVE TEMP
         1       -.10869E+01      -.40263E-01      .19926E+03
         2       -.11272E+01      -.90949E-12      .19962E+03
         3       -.36030E+00      -.22149E-01      .19867E+03
         4       -.34218E+00      -.40263E-01      .19877E+03
         5       -.10496E+01      -.37263E-01      .19929E+03
         6       -.10869E+01           0           .19964E+03
         7       -.37514E+00      -.22424E-01      .19869E+03
         8       -.36030E+00      -.37263E-01      .19881E+03
         9       -.47187E+02      -.30718E+00      .15126E+03
        10       -.47377E+02      -.22149E-01      .17485E+03
        11       -.30526E+02      -.14402E+00      .97199E+02
        12       -.30417E+02      -.30718E+00      .11236E+03
        13       -.22563E+02      -.70811E-01      .59398E+02
        14       -.22514E+02      -.14402E+00      .70632E+02
        15       -.17895E+02      -.58161E-01      .30226E+02
        16       -.17887E+02      -.70811E-01      .39150E+02
        17       -.47001E+02      -.30150E+00      .15147E+03
        18       -.47187E+02      -.22424E-01      .17497E+03
        19       -.30630E+02      -.14442E+00      .97396E+02
        20       -.30526E+02      -.30150E+00      .11261E+03
        21       -.22612E+02      -.71276E-01      .59494E+02
        22       -.22563E+02      -.14442E+00      .70751E+02
        23       -.17904E+02      -.58268E-01      .30288E+02
        24       -.17895E+02      -.71276E-01      .39216E+02
```

Figure 3.8.6 Input for Axisymmetric Heat Transfer Without Grid Generation



```
AXISYMMETRIC HEAT TRANSFER SAMPLE PROBLEM WITHOUT GRID GENERATION
0, 0, 8, 0
24, 21, 5, 2
0.48, 0.48, 0.0,0.0,0.0
0.002, 0.002, 0.025, 20.0, 0.0
1, 2, 4, 3, 1, 4.0, 1.0, 5.0, 1.0, 5.0, 2.0
2, 2, 3, 1, 1, 4.0, 1.0, 5.0, 2.0, 4.0, 2.0
ELEMENT DATA CARDS FOR ELEMENT NUMBERS 3 THROUGH 23
24, 19, 20, 17, 2, 10.0, 0.0, 12.0, 1.0, 10.5, 1.0
15, 2, 0
23, 2, 0
0, 0, 0
0,0.0, 0,0.0, 0,0.0, 0,0.0, 0,0.0, 0,0.0
1,200.0, 2,200.0, 5,200.0, 0,0.0, 0,0.0, 0,0.0
```

3.9 PLANE FRAME

3.9.1 Introduction

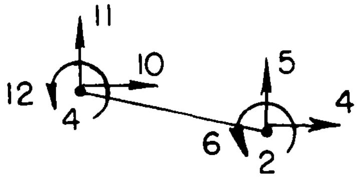
This program will calculate the axial force, shear force, and bending moments in the elements of a simple frame structure. An assemblage of beam elements not restricted to lying along a straight line is referred to as a frame. Common loading states for members considered in this program include bending plus axial loading.

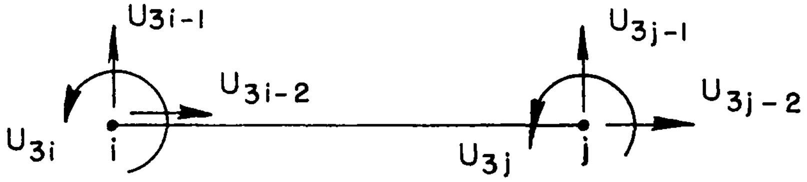
Closed rings and curved frames are investigated by using an idealization consisting of an assemblage of straight-line segments.

This program uses a simple two-node element with six degrees of freedom (See Section 3.9.2). Automatic grid generation, grid plotting, and result plotting are not available for this program. Element data control parameters and element data must be input by the user in the form described in the following sections.

3.9.2 Simple Beam Element

The element type used in this program consists of a 2-node element with six degrees of freedom



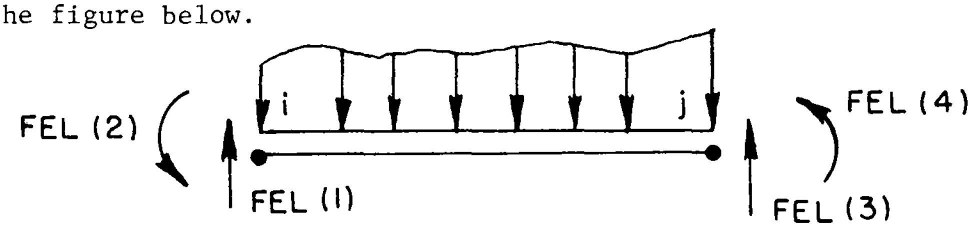


The three degrees of freedom at each node are displacements in the X- and Y-directions, and rotation. Degrees of freedom are calculated from the node number as diagrammed above. Positive moments and rotation are assumed clockwise.

Input of concentrated forces at nodes and known displacements and rotations are input in vector components keyed to the degree of freedom.

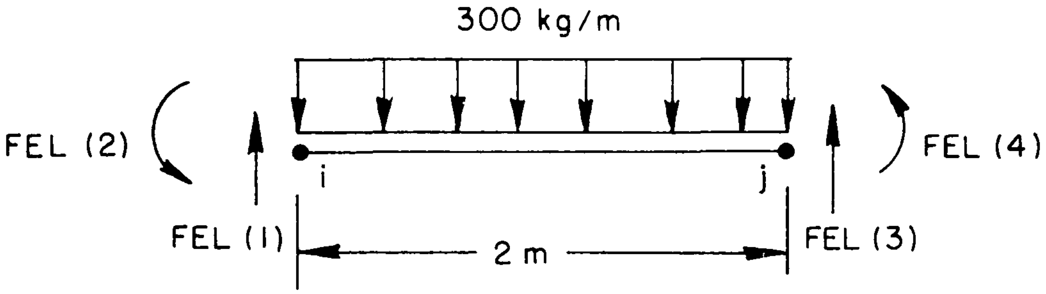
3.9.3 Fixed End Loads

Distributed loads are input as fixed end loads. The fixed end loads are the calculated moment and shear at the ends of the element due to the distributed load assuming zero deflection and rotation at the ends. Positive orientation for the loads and moments relative to the element are shown in the figure below.



Sign Convention and Subscripts
for Fixed End Loads

The following numerical example will demonstrate calculation of the fixed end loads. Equations for moments and shears may be obtained for beam derivations available in appropriate texts.



$$\text{SHEARS } \frac{\omega L}{2} = \frac{300(2)}{2} = 300$$

$$\text{MOMENTS } \frac{\omega L^2}{12} = \frac{300(4)}{12} = 100$$

FEL(1) = 3000
FEL(3) = 3000

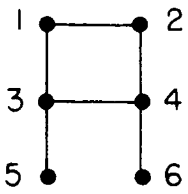
FEL(2) = 100
FEL(4) = -100

3.9.4 Element Data Control Parameters

Element data control parameters establish the size of the problem. Element data control parameters for Plane Frame include, in order of input

NP - number of nodes
NE - number of elements
NBW - bandwidth = (maximum difference in node numbers of any element + 1)*3

Sample bandwidth determination



$$\text{BANDWIDTH} = ((3-1)+1) * 3 = 9$$

3.9.5 Nodal Coordinate Input

X-coordinates of the nodes are input in the order which they have been numbered. Beginning on a new card, Y-coordinates are input in the order which they have been numbered. NP coordinate values should be input for each set of coordinates.

3.9.6 Element Data

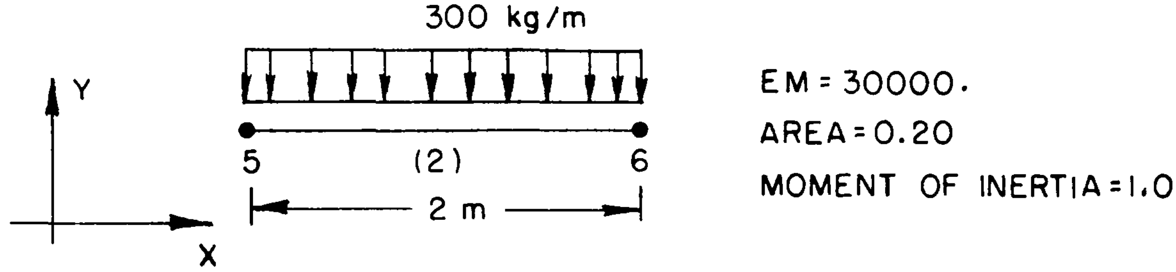
The element data establishes the element number, node numbers, material and cross-sectional properties, and fixed end loads for each element.

Element data for each element are input in the following order

NEL - element number
ND - node numbers; node i, node j
EM - elastic modulus of element
AREA - cross-sectional area of element
RI - moment of inertia of cross-section
FEL( ) - fixed-end loads (See 3.9.3).

Element data must be input with each element starting on a new card.

Sample element data card

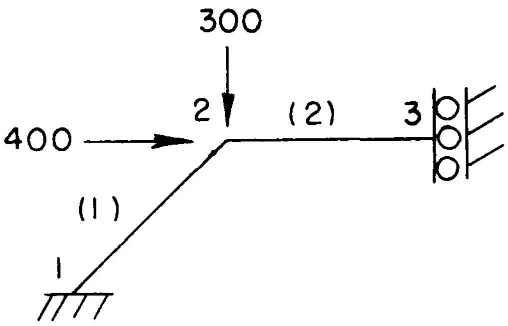


```
NE   ND      EM        AREA   RI          FEL( )
 2, 5, 6, 30000., 0.2 , 1.0 , 300.  , 100., 300. ,- 100.
```

3.9.7 Boundary Values

Boundary values are input in two sections. The first is non-zero nodal forces. The second is known displacements and rotations. Both are input by global degrees of freedom at each node. Values for displacements and forces are divided in vector components parallel to the direction of appropriate degrees of freedom. Each card must have six sets consisting of global degree of freedom and associate value. If a complete card is not needed, the remaining sets should be specified with zero values. If an even multiple of 6 sets occurs or no values are to be input, a card with 6 sets of zeros (12 zeros) must follow to terminate that section of data input.

For the simple frame shown below



the nodal forces would be input as

```
4,- 400., 5,- 300., 0, 0.0 , 0, 0.0, 0, 0.0, 0, 0.0
```

and the known nodal values (displacements and rotations)

```
1, 0.0, 2, 0.0, 3, 0.0 7, 0.0 9, 0.0 0, 0.0
```

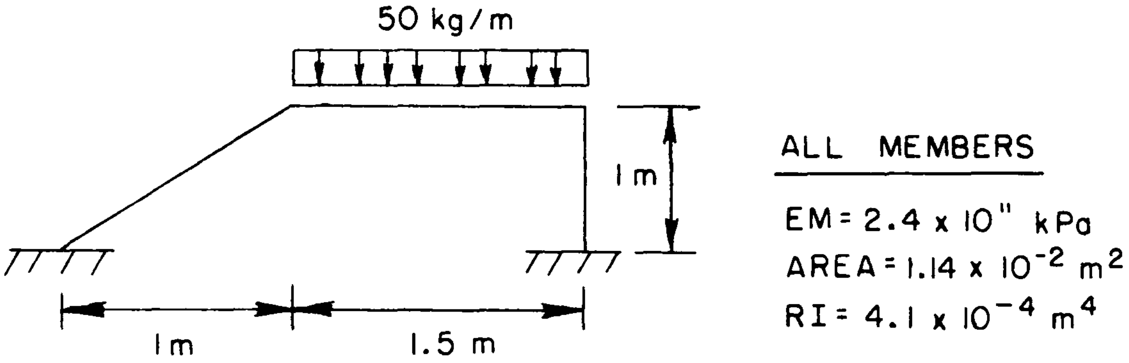
Note sign convention has been established such that forces and displacements in the positive coordinate direction are assumed positive. Those in the opposite direction are assumed negative. Counterclockwise rotations and moments are assumed positive.

3.9.8 Input Description

Table 3.9.1 shows the inputs required for the frame program.

3.9.9 Sample Problem

Consider the simple frame problem diagrammed below. The frame is assumed to be fixed at both ends



The following figure shows the frame with elements and nodes labeled for input.

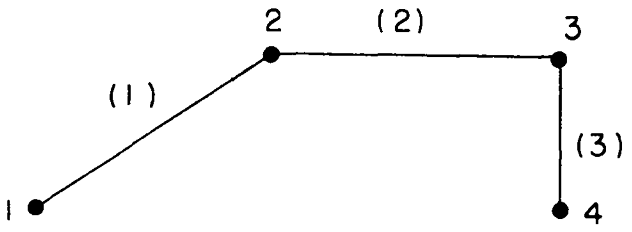


Table 3.9.1 Program Input for Plane Frame

Title Card - up to 80 alpha-numeric characters for identification of problem.

Main Parameter Card - See Section 1.4 for this case NGRID = 0
NPLTG = 0
NAPPL = 9
NPLTR = 0

Control Parameters for Elements Data - by order of input, see Section 3.9.4.

NP - number of nodes
NE - number of elements
NBW - bandwidth

Element Data - for each element, by order of input, see Section 3.9.6.

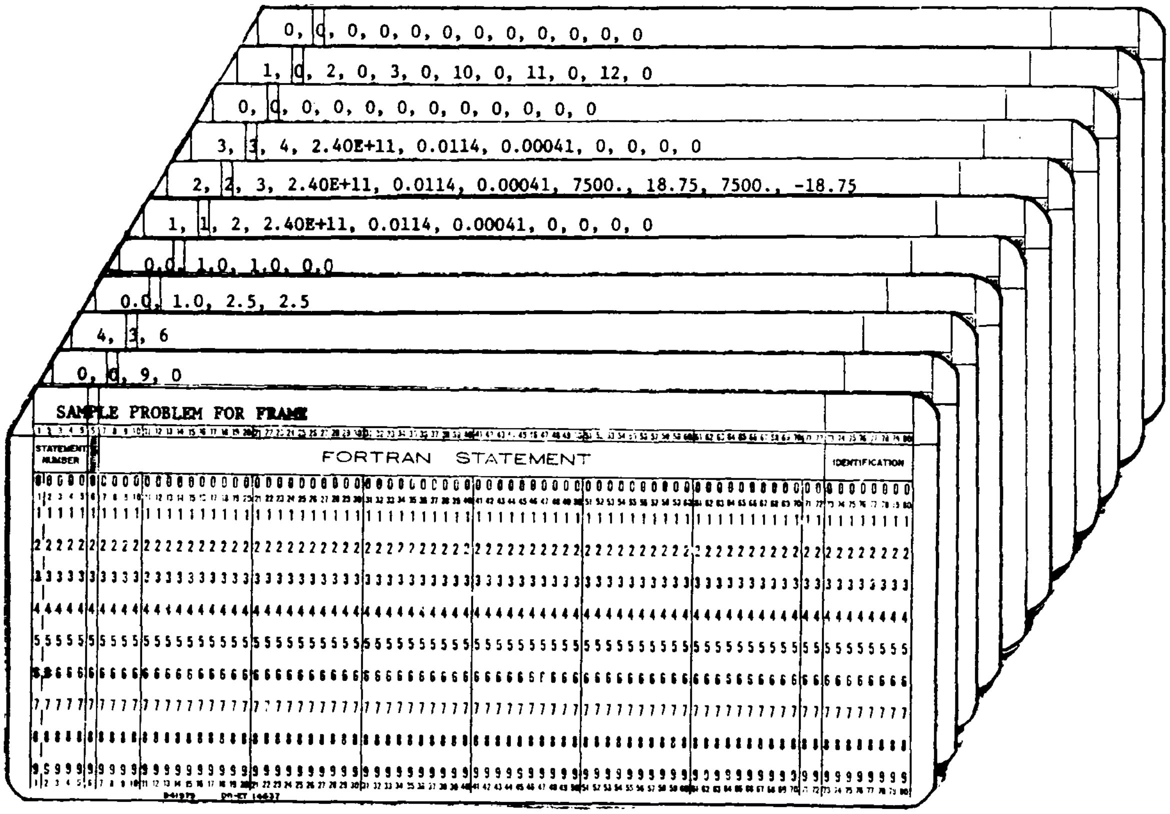
NEL - element number
ND - node number
EM - elastic modulus
AREA - cross-sectional area
RI - moment of inertia
FEL(1) thru FEL(4) - fixed end loads

Nodal Forces - read sets of global degree of freedom associated with force and value, with 6 sets per card - terminate with global d.o.f. = 0, value = 0.0 for remainder of card. See Section 3.9.7.

Specified Nodal Values - read sets of global degree of freedom associated with displacement or rotation, and value of displacement with 6 sets per card - terminate with global d.o.f = 0, value = 0.0 for remainder of card.

Figure 3.9.1 shows the input for this problem. Note the units must be consistent (i.e., if material properties use meters as unit of length, nodal distances must be in meters).

Figure 3.9.2 shows the output of the frame program for this problem.

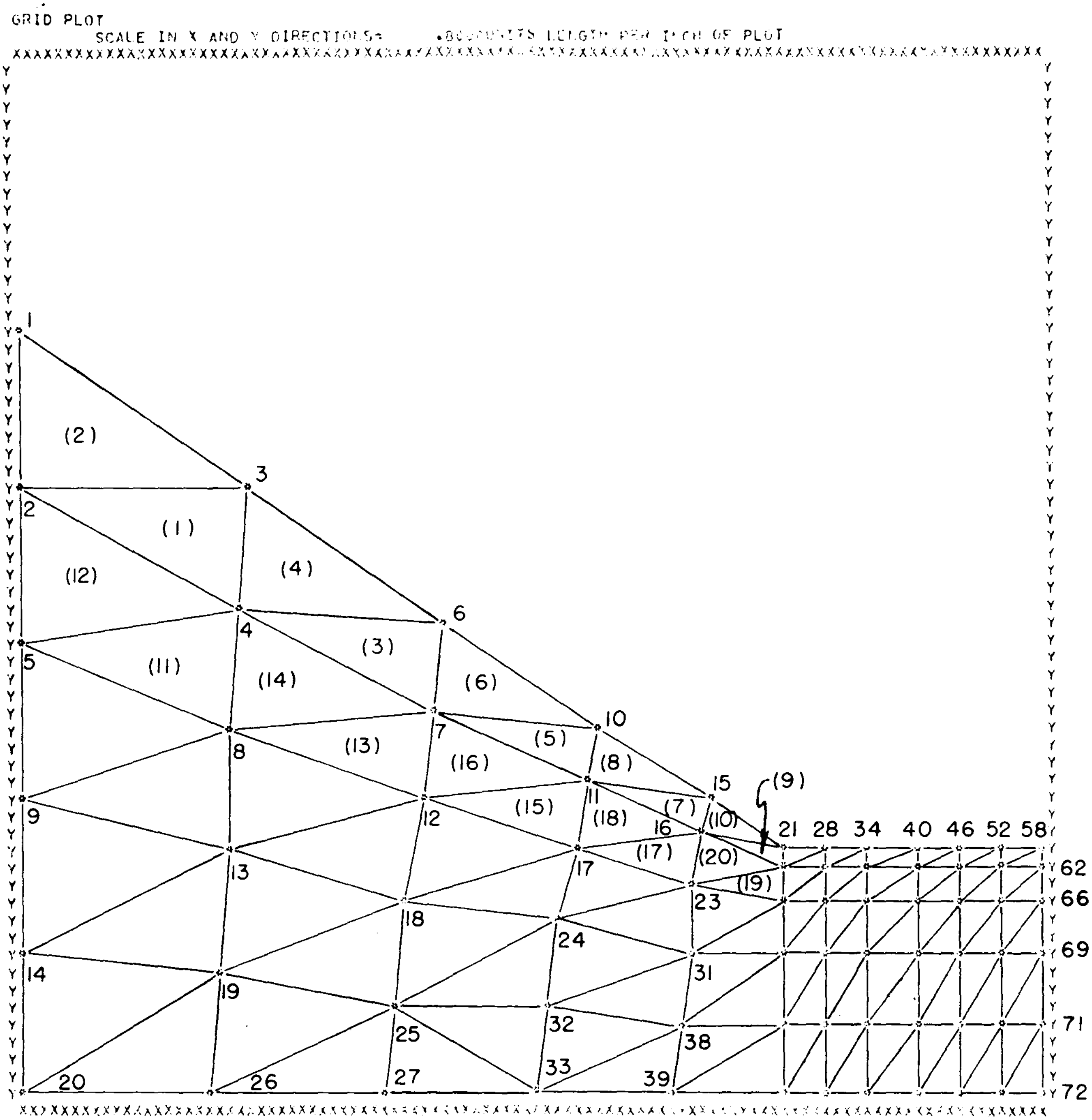


```
SAMPLE PROBLEM FOR FRAME
0, 0, 9, 0
4, 3, 6
0.0, 1.0, 2.5, 2.5
0.0, 1.0, 1.0, 0.0
1, 1, 2, 2.40E+11, 0.0114, 0.00041, 0, 0, 0, 0
2, 2, 3, 2.40E+11, 0.0114, 0.00041, 7500., 18.75, 7500., -18.75
3, 3, 4, 2.40E+11, 0.0114, 0.00041, 0, 0, 0, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
1, 0, 2, 0, 3, 0, 10, 0, 11, 0, 12, 0
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0
```

Figure 3.9.2 Output for Frame Sample Problem

```
SAMPLE PROBLEM FOR FRAME



APPLICATION PROGRAM  PLANE FRAME   WILL BE SOLVED


NODAL COORDINATES
NODE          X              Y
   1          0              0
   2       1.00           1.00
   3       2.50           1.00
   4       2.50              0


ELEMENT DATA
                                                   MOMENT OF                FIXED END LOADS
NEL NODE NUMBERS  ELASTIC MODULUS    AREA           INERTIA           FI         MI          FJ          MJ
  1    1      2     .240E+12      .114E-01     .410E-03            0          0           0           0
  2    2      3     .240E+12      .114E-01     .410E-03       .750E+04   .188E+02   .750E+04  -.188E+02
  3    3      4     .240E+12      .114E-01     .410E-03            0          0           0           0
BOUNDARY VALUES

     NODAL FORCES



     PRESCRIBED NODAL  VALUES
  1         0        2         0         3          0        10          0        11          0        12          0



NODAL VALUES
  1         0        2         0         3          0         4   .30651E-05     5  -.71946E-05     6   .21588E-05
  7   .18022E-05     8  -.34044E-05      9  -.29725E-06    10          0        11          0        12          0


ELEMENT NODAL FORCES


ELEMENT              AXIAL FORCE                    SHEAR FORCE                    BENDING MOMENT
                 I              J              I              J              I              J
   1
          .564910E+04    -.564910E+04    .239139E+04   -.239139E+04   -.184117E+04   -.154076E+04
   2
          .230355E+04    -.230355E+04    .568548E+04    .931452E+04    .154076E+04    .118102E+04
   3
          .931452E+04    -.931452E+04    .230355E+04   -.230355E+04   -.118102E+04   -.112253E+04
```

Figure 4.2 Grid Plot Developed for Boundary Condition Determination



5. RESULT PLOTTING

5.1 PURPOSE

Interpretation and use of the large amount of data which can be produced by a finite element program is often tedious and difficult. The purpose of the result plotting program is to assist in interpretation of pertinent output variables. The program produces line-printer plots of the same scale and size as those produced by the grid plotting program (ten inch square plot).

5.2 REQUIREMENT FOR USE

The user need only specify the parameter NPLTR = 1 on the main control parameter card to obtain result plots. No additional input is required. Table 5.1 below shows the results which will be plotted for each application program.

Table 5.1 Result Plotting

Application Program	NAPPL	Variables Plotted	Nodal or Element Values
2-D Elasticity	1	Stress X-direction	Element
		Stress Y-direction	Element
		Shear Stress	Element
		Maximum Principal Stress	Element
		Minimum Principal Stress	Element
		Principal Shear Stress	Element
2-D Heat Transfer	2	Temperatures	Nodal
2-D Groundwater	3	Potentials	Nodal
		Velocity X-direction	Element
Torsion	4	Shear Stress ZX	Element
		Shear Stress ZY	Element
		Maximum Shear Stress	Element
2-D Transient Heat Transfer	5	Temperatures - each time step at which values are printed.	Nodal
Truss	6	Result plotting not available.	
Axisymmetric Elasticity	7	Stress in R-direction	Element
		Stress in Z-direction	Element
		Tangential Stress	Element
		Shear Stress	Element
		Maximum Normal Stress	Element
		Minimum Normal Stress	Element
		Maximum Shear Stress	Element
Axisymmetric Heat Transfer	8	Temperatures	Nodal
Plane Frame	9	Result plotting not available.	

5.3 PLOTTING PROCEDURE AND SAMPLE PLOT

This program divides the range of the variable to be plotted into ten equal subranges which are labelled 0 through 9.
Subrange values are printed at the right of each plot. Appropriate subrange labels are then plotted at nodal or element centriodal locations. Figure 5.1 shows a sample result plot.