Internet Computer Coaches For Introductory Physics Problem Solving

A DISSERTATION
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

Qing Xu Ryan

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Kenneth J. Heller

December 2013

Acknowledgements

First, I am grateful to my adviser Ken Heller whose insight and constructive criticism not only strengthened this research study but also prepared me for future academic pursuits. I am also grateful to Leon Hsu, who not only guided me stepping into the PER community, but also gave me a lot of help and made a lot of contributions to this project. This document also benefited greatly from the feedback of the other committee members: Charles Campbell and Yuichi Kubota. I am particularly thankful for Andrew Mason, Bijaya Aryal and Evan Frodermann who shared a lot of the research work. Particular thanks to Bijaya who scored thousands of student solutions with me. Lots of thanks to Evan Frodermann, who did a lot of analysis of the usage data collected from Spring 2013 and also wrote macros to deal with large data set and made my life much easier. I am very thankful for Jennifer Docktor, who gave me a lot of help when I first stepped into the Physics education research group and continued to answer my questions later on. I would like to thank Erik Hoover, the main programmer who helped build up the computer module prototypes and all the undergraduate programmers. I am also grateful to Anne Loyle-Langholz, Jia-Ling Lin, Allan Jackson, Paul Knutson, Brita Nellermoe, Dick Schwarz, Mandy (Pihlaja) Straub, Jie Yang for useful office and seminar conversations. I would also like to acknowledge all the volunteers who participated in the research project. Finally, I would like to acknowledge my family and friends for helping me get through rough patches along the road.

Dedication

This dissertation is dedicated to my mom and dad: Weizhi Xia & Xiaoping Xu for their love, endless support and encouragement.

Abstract

The ability to solve problems in a variety of contexts is becoming increasingly important in our rapidly changing technological society. Problem-solving is a complex process that is important for everyday life and crucial for learning physics. Although there is a great deal of effort to improve student problem solving skills throughout the educational system, national studies have shown that the majority of students emerge from such courses having made little progress toward developing good problem-solving skills. The Physics Education Research Group at the University of Minnesota has been developing Internet computer coaches to help students become more expert-like problem solvers. During the Fall 2011 and Spring 2013 semesters, the coaches were introduced into large sections (200+ students) of the calculus based introductory mechanics course at the University of Minnesota. This dissertation, will address the research background of the project, including the pedagogical design of the coaches and the assessment of problem solving. The methodological framework of conducting experiments will be explained. The data collected from the large-scale experimental studies will be discussed from the following aspects: the usage and usability of these coaches; the usefulness perceived by students; and the usefulness measured by final exam and problem solving rubric. It will also address the implications drawn from this study, including using this data to direct future coach design and difficulties in conducting authentic assessment of problem-solving.

Table of Contents

List of Tables

List of Figure

CHAPTER 1: INTRODUCTION

1.1 Research Motivation

The ability to solve problems in a variety of contexts is becoming increasingly important in our rapidly changing technological society. We solve problems every day, from prioritizing our daily schedule to making arrangements for a major family event or planning a family trip. Sometimes we are faced with more complicated problems to solve such as: what career path to take and how to get the prerequisite experience in order to get a certain job and how to prepare for an interview, etc. Sometimes we can make these decisions very quickly but sometimes it does take some deliberated thought to solve these problems. Having a good problem solving strategy not only helps us cope with these daily tasks more efficiently, but also provides a better solution.

Problem-solving skills are also very valuable in today's society (Jonassen, 2007) (Martinez, 1998). The employers also put a strong emphasis on problem-solving ability for the potential work force. In 1991 the U.S. Secretary of Labor and the Secretary's Commission on Achieving Necessary Skills (SCANS, 1991) published a report that identified *Thinking Skills* as foundational skills for all competent workers, including creative thinking, decision making, problem solving, visualizing, knowing how to learn, and reasoning.

In order to prepare our students to meet employer demands, problem-solving is often recognized as an important goal for undergraduate education. A list of undergraduate learning outcomes at the University of Minnesota identifies seven such goals, the first of which is "At the time of receiving a bachelor's degree, students will demonstrate the ability to identify, define, and solve problems" (Carney, 2006). In particular, good problem-solving skills are critically important for scientists and engineering majors and we want our students to be able to use these skills to create new knowledge and to apply existing knowledge to the real world. Because an introductory course in physics is a prerequisite for study in nearly all science and engineering fields, it is an ideal venue for teaching problem-solving.

Nationally conducted studies have shown that the majority of students emerge from such courses having made little progress toward developing good problem-solving skills (Reif, 1981). One obstacle to students' learning effective problem-solving strategies is the difficulty and expense of providing good coaching, i.e., supplying students with an environment where they receive guidance and feedback while they solve problems. Even where coaching is built into an introductory physics class, students usually have less than one hour each week where they practice solving problems in an environment where they are coached. Generally, most students don't have this coaching session in a traditional lecture based class.

One way to give students more access to proper coaching on problem solving at a reasonable low cost is to utilize computer coaches. These online programs have the advantages of being non-judgmental, available at the students' convenience at any time and can be repeated as many times as the student desires. At this time, there haven't been such computer programs that provide targeted coaching to students throughout the entire semester in introductory physics. These coaches should coach students on the general decision-making skills integral to expert problem-solving and are different from the existing web-based homework systems/ learning platforms such as LON-CAPA[1], WebAssign[2], Mastering Physics[3], Tycho[4], etc..

There are coaching programs such as Andes[5] that emphasize artificial intelligence design. These programs take a lot of programming expertise and effort which could be difficult to adapt to different teaching preferences. We want to build a computer coach program that aims at giving effective coaching through its pedagogical design without resorting to complicated artificial intelligence.

1.2 Research Questions

The primary questions to this dissertation include:

1. Will a significant population of students choose to use the computer coaches and what are the characteristics of the students who do and don't use the?
2. Do students perceive the coaches as useful?
3. To what extent do the computer coaches enhance students' physics problem-solving skills?

The results of this study will help inform the design of the future problem-solving coaches for introductory physics and influence the techniques for evaluating problem-solving.

1 LON-CAPA(1992). CampusSource. Retrieved 2007-11-30

[2] Guernsey, Lisa (1999-02-12). Textbooks and Tests That Talk Back. *The Chronicle of Higher Education*. Retrieved 2007-06-28.

[3] http://www.masteringphysics.com/

4 http://research.physics.illinois.edu/per/Tycho.html

[5] Schulze, K.G., Shelby, R.N., Treacy, D.J., Wintersgill, M.C., VanLehn, K., Gertner, A. (2000). Andes: An intelligent tutor for classical physics. *The Journal of Electronic Publishing*, University of Michigan Press, Ann Arbor, MI, 6:1, http://www.press.umich.edu/jep/06-01/schulze.html

1.3 Overview of the Dissertation

Chapter 1 outlines the motivation and need for developing computer coaches for introductory physics, lists the research questions addressed in this dissertation, and previews each chapter of the dissertation.

Chapter 2 begins with the definition of problem-solving, and introduces relevant theoretical background for the coaches' pedagogical design which includes: cognitive apprenticeship, reciprocal teaching, expert and novice problem-solving in physics, problem-solving framework, and the use of context-rich problems. The chapter also talks about the software (technical background) that was used in the coach design. A review of research on computer tutors: intelligent tutoring system to modern intelligent tutors will be given. This chapter also distinguishes these computer coaches from other existing computer programs (web-based homework systems / learning platforms & Tutors with Artificial intelligence). It concludes with a review on the problem-solving assessment tool and introduces the particular problem-solving rubric that was used in this study.

Chapter 3 addresses the methodology framework for this study and the practical difficulties of conducting such an experimental study.

Chapter 4 describes the experimental environment of each study in this dissertation and the data analysis for that study.

Chapter 5 summarizes the results of this study and its implications, as well as future research directions.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

Although the description of the important features of problem-solving differs in the literature, there is an agreement that solving problems is a process of making decisions. This chapter briefly reviews the theoretical background and research that are relevant to the design of the coaches. Research studies in computer coach programs are reviewed in this chapter as well as pedagogical and technical design of the computer coaches. The final section of this literature review summarizes some studies on problem-solving assessment instruments in physics and introduces the rubric used in assessing problem-solving in this project. For a more comprehensive review of problem-solving research, see Mayer (1992); Ormrod (2004); Hsu, Brewe, Foster, & Harper, (2004); and Maloney (1994).

2.2 Definitions

"Problem-solving" is a common human process. Every day people confront problems such as extracting a broken light bulb from a socket, deciding which grocery item is a better buy, planning a family vacation, or deciding whom to vote for in a presidential election (Novick & Bassok, 2005). But to try to improve people's problem-solving, we first have to recognize what constitutes a problem. According to Newell (Newell, 1972), a person is confronted with a problem when he wants something and does not know immediately what series of actions he can perform to get it. A problem arises when someone has a goal but does not know how to reach that goal. (Novick & Bassok, 2005). Martinez (Martinez, 1998) defines problem-solving as a process of moving toward a goal when the path to that goal is uncertain. The U.S. Department of

Labor SCANS (Secretary of Labor and members of the secretary's Commission on Achieving Necessary Skills) document describes a person who engages in problem-solving as someone who: "Recognizes that a problem exists (i.e., there is a discrepancy between what is and what should or could be), identifies possible reasons for the discrepancy, and devises and implements a plan of action to resolve it. The person evaluates and monitors their progress, and revises the plan as indicated by findings." (SCANS 1991).

Problem-solving depends on the solver's experience and perception of the task (Martinez 1998). Problem solvers at widely different levels of ability may have quite different interpretations of the problem (Newell & Simon, 1972, p.94). Interpreting a problem often involves decomposing or reducing it into a set of easier-to-solve sub-problems (Newell & Simon, 1972, p.94). As Newell and Simon pointed out, one universal method of solving problems is reducing them to previously solved sub-problems and simply remembering the answers. For example, the second time one has to remove a broken light bulb from a socket, the solution likely can be retrieved from memory (James Holyoak 2005). What is considered a problem for one person may be a well-trodden path, an exercise, for another person (Woods 2000). For example, multiplying 8x7 might be a problem for an 8-year-old but not for readers of this dissertation.

2.3 Expert-Novice Differences in Physics Problem-Solving

Likewise, physics problems also depend on someone's experience and levels of ability. There have been many studies done on the differences between the physics experts and novices in terms of their problem-solving. There were two different traditions: one is to look at the differences in domain knowledge (representation) between experts and novices; the other is to look at the differences in generating solutions. We will introduce these two types of research briefly.

2.3.1 Representation - Domain Knowledge: Experts vs. novices

An example of research into problem representation, Chi gave a set of written problems to physics novices and experts and they were asked group them based on their similarity. Based on how different people carried out these problem categorization tasks, Chi (Chi, 1981) proposed that experts tend to look at the deep structure of the problem, classifying problems by fundamental principles that might be used to solve them such as “Newton’s 2nd law” or “Conservation of Energy”; while novices focused on superficial features of the specific situation that generated the problem, such as those involving an “incline plane” or a “pulley”.

The evidence presented by Hardiman, P.T., Dufresne, R., & Mestre, J.P., (1989) further supported this idea. In one experiment, they compared experts (Ph.D. physicists and advanced physics graduate students) to novices (undergraduate students who had completed the first semester physics course in classical mechanics and received a grade of B or better). They gave a similarity judgment task where a model problem and two comparison problems are presented and the subject must decide which of the comparison problems would be solved most similarly to the model problem. They found that experts predominantly relied on the problem’s deep structures in deciding on the similarity of solution. Some novices, on the other hand, relied predominantly on surface features while others made greater use of principles. This latter set of novices tended to categorize problems similarly to experts, as well as score higher on solving four long single principle physics problems.

2.3.2 Process of Generating Solutions: Experts vs. novices

Other researchers focused on the process differences between experts and novices while solving problems. Reif & Heller (1982) examined the process of solving a problem and broke it into three phases: the description phase, the search for a solution phase; and the assessment of the solution phase. They, and others, found that experts usually construct a *qualitative analysis* of a problem, before writing down quantitative relationships (Chi et al., 1981; Larkin, 1979; Larkin et al., 1980a; Larkin & Reif, 1979). Experts construct a "low-detailed qualitative physical description" (Larkin, 1979) after the initial sketch to make certain there were no inherent difficulties their approach.

Ferguson-Hessler, M.G.M., and Jong, T. (1990) took a different approach to see if there is any difference in the study process between the good or bad students who were categorized based on their previous exam results in electricity and magnetism. 21 students from the Eindhoven University of Technology studied 10-page text on a physics subject, reporting at regular intervals on their study processes. Protocols of five good performers and five poor performers were analyzed. Each statement was classified into 1 of 32 different study processes, and the type of knowledge involved was determined: declarative, procedural, or situational. They found that good students applied what they called deeper processing and less what they called superficial processing than poor students. Their indication of 'superficial processing' included actions such as reading the text or comparing symbols in the text or figures. Their definition of 'deep processing' had two categories: integrating & connecting. Integrating is bringing structure into the new knowledge such as distinguishing major points from side issues (indicated by underlining or boxing an important formula or definition); and connecting is relating knowledge from the text with previous knowledge already present (indicated by thinking of examples from experience) (Ferguson-Hessler & Jong, 1990).

2.3.2 Problem Solving Framework

These two differences in experts and novices are not completely separate. The organization of one's knowledge affects one's problem-solving process and the process one uses to solve problems affects how one organizes knowledge. In the domain of physics problem-solving, the process of moving toward expertise involves development in both these areas (Gick, 1986; Chi, Glaser & Rees, 1982; Dreyfus & Dreyfus, 1986; Elio & Scharf, 1990). Broadly speaking, a difference between an expert and a novice is that a novice believes that each problem has a specific recipe of actions for solving it while an expert has a general decision-making process whose outcome is a set of actions that lead to a solution.

The following subsections introduce some research on general problem-solving frameworks in the domain of machine learning and mathematics, and then addresses its application in physics; how do we teach novices to solve physics problems like experts?

2.3.2.1 General Problem Solver

Heavily influenced by the information-processing approach to cognitive psychology, and by work in computer science on artificially intelligence, Newell and Simon and their colleagues constructed the General Problem Solver (GPS) – a computer program that modeled human problem solving (Ernst & Newell, 1969; Newell & Simon, 1972; Novick & Bassok, 2005). It was intended to work as a universal problem-solver machine. GPS obtained the name of "general problem solver" because it was the first problem-solving program to separate in a clean way a task-independent part of the system containing general problem-solving mechanisms from a part of the system containing knowledge of the task environment (Newell & Simon, 1972). It emphasized the process of generating a solution as distinct from the domain knowledge necessary for that solution (Novick & Bassok, 2005).

2.3.2.2 How to Solve it?

George Polya was a Hungarian Jewish mathematician and noted for his work in heuristics and mathematics education. In the book "how to solve it" (Polya, 1945) the author said when he was a student himself, one question that disturbed him again and again: "Yes, the solution seems to work, it appears to be correct; but how is it possible to invent such a solution? Yes, this experiment seems to work, this appears to be a fact; but how can people discover such facts? And how could I invent or discover such things by myself?" He said trying to understand not only the solution but also the motives and procedures to that solution, and trying to explain the motives and procedures to others, he was finally led to write his book.

In this book, there were four steps outlined in the context of mathematics for a problem-solver. First, the solver has to understand the problem. Second, they need to find the connection between the data and the unknown. They may be obligated to consider auxiliary problems if an immediate connection cannot be found. They should eventually obtain a plan of the solution. Third, they carry out the plan. Fourth, they examine the solution obtained.

2.3.2.3 Minnesota Problem Solving Framework

Most of the problem-solving frameworks are based on the strategy developed by Polya (1945); the Minnesota problem-solving framework (Heller & Heller, 1995 and Heller & Heller, 2000) is a typical example. Since real problem solving is rarely linear, it is meant only to outline the basic stages through which a solver might loop multiple times, and not to imply that problem solving can be reduced to a linear algorithmic process.

The Minnesota problem-solving framework (Heller & Heller, 1995 and Heller & Heller, 2000) is a 5 part framework which is summarized as:

1. Focus the problem

   Draw a picture illustrating the situation

   Determine the question to be answered

   Choose which physics principle(s) to use

2. Describe the physics

   Draw physics diagrams

   Determine target quantity(ies)

Write down quantitative relationships

3. Plan the solution

    Select equation containing the target quantity

    Identify other unknowns in equation

    Solve a sub-problem to find each unknown

    Check units

4. Execute the plan

    Calculate values of target quantity(ies)

5. Evaluate the answer

    Check if answer is properly stated

    Check if answer is unreasonable

    Check if answer is complete

For problem solving experts, they usually follow such a systematic framework when solving a problem. So the question remains; "how do we make novices more expert like?". Research shows through targeted efforts, we could use curricular interventions designed to help students become better problem solvers and become more like the experts (Maloney, 1994). But in order for the intervention to work, we have to teach such a problem-solving framework explicitly to our students (novices).

2.4 Theoretical Background of the Computer Coaches

The following subsections give some general introduction about the theoretical background of the computer coaches and how the computer coaches utilize these theories in its pedagogical design.

2.4.1 Cognitive Apprenticeship

Researchers have shown that it is possible, through targeted efforts, to improve students' problem-solving skills (see Maloney (1994) and Hsu, Brewe, Foster & Harper (2004) for overviews). The common thread running through these efforts is that they are all explicitly or implicitly based on the cognitive apprenticeship model (Collins, Brown & Newman, 1989; Heller, Foster & Heller, 1997).

Cognitive apprenticeship is a theory of the process where a master of a skill teaches that skill to an apprentice. In ancient times, teaching and learning were accomplished through apprenticeship: people taught their children how to speak, grow crops, craft cabinets, or tailor clothes by showing them how and by helping them do it. Apprenticeship was the vehicle for transmitting the knowledge required for expert practice in fields from painting and sculpting to medicine and law. It was the natural way to learn. In modern times, apprenticeship has largely been replaced by formal schooling, except in children's learning of language, in some aspects of graduate education, and in on-the-job training. Collins, Brown, and Holum (Collins, Brown, and Holum, 1991) proposed an alternative model of instruction that is accessible within the framework of the typical American classroom. It is a model of instruction that goes back to apprenticeship but incorporates elements of schooling. They call this model "cognitive apprenticeship" (Collins, Brown, and Newman, 1989).

Within this theory, the necessary functions of teaching incorporate the actions of modeling, coaching, and fading. These actions are supported by temporary instructional tools called scaffolding. Essentially, modeling is showing students precisely the expert-like behavior and knowledge desired by making visible all of the intellectual processes in decision making. Coaching is giving students real-time feedback as they attempt the task by following the model in their own way. Fading is giving students the opportunity to do the task themselves with reduced guidance. Scaffolding is temporary support, or "training wheels," that are removed as students become more proficient. All of these actions take place in what is called the environment of expert practice, where tasks include a meaningful context that have a meaningful outcome. Thus, each task includes a motivation that is comprehensible to the student, a context that can be connected to the experiences of the student, and an outcome that satisfies the motivation for the task (Brown, Collins, Duguid, 1989).

2.4.2 Context-Rich Problems

As mentioned previously, a very important element in the cognitive apprenticeship pedagogy is the environment of expert practice. The apprentices (students) must be put into a situation that is concrete enough for them to imagine and provides a motivation to arrive at a solution. In teaching physics problem-solving, it is important that the students perceive what they are instructed to do for their learning has something related to the real-world and/or their lives. One possible way to provide context could be to assign students to do a real world project but it is complicated and time consuming. Another way to provide context is to design problems that provide students with the environment of expert practice without too much of an implementation effort. Such problems must relate to students' personal experiences and provide them with a motivation of solving that problem.

This type of problem, known as a Context-rich problem, was developed by the PER research group at the University of Minnesota to aid students in learning both physics and problem-solving (Heller & Hollabaugh, 1992). Specifically, these problems are designed to (1) be challenging enough that students need to use an expert-like problem-solving framework to reach a solution, (2) require students to make decisions on how to proceed with the solution, (3) have a context and motivation that appear authentic to students, (4) require students to visualize the situation, and (5) be mathematically straight-forward to solve in several steps from basic principles.

2.4.3 Reciprocal Teaching

Reciprocal teaching is an instructional method consistent with cognitive apprenticeship in which students and teachers take turns playing the role of the teacher. It was initially designed to improve students' reading comprehension.

In reciprocal teaching, as developed by Palincsar and Brown (1984) to aid students who possess grade-level skills in letter-sound correspondence but are unable to construct meaning from the texts they decode. Students read a passage of expository material, paragraph by paragraph. During the reading they learn and practice four reading comprehension strategies: generating questions, summarizing, attempting to clarify word meanings or confusing text, and predicting what might appear in the next paragraph. During the early stages of reciprocal teaching, the teacher assumes the major responsibility for instruction by explicitly modeling the process of using these strategies on a selection of text. After the teacher has modeled, the students practice the strategies on the next section of text, and the teacher supports each student's participation through specific feedback, additional modeling, coaching, hints, and explanation.

2.4.4 Pedagogical Design of the Computer Coaches

How does the design of the computer coaches fit into these learning theories and pedagogies? As discussed previously, the pedagogical design of the computer coaches is based on cognitive apprenticeship. As with any expert human coach, the computer coaches incorporate all of the cognitive apprenticeship modalities of modeling, coaching, and fading. In particular, the computer coaches rely on the instructional strategies of reciprocal teaching (Palincsar & Brown, 1984) and learning from well-studied examples (Zhu & Simon, 1987). To accomplish this we use two types of coaches that employ extensive scaffolding. A third type of coach emphasizes the fading part of the cognitive apprenticeship paradigm. We also used all context-rich problems in the computer coaches to provide the environment of expert practice and every problem was solved by using the five-step problem solving framework where all the decision making process was made very explicit to the students.

In a type 1 coach, “Computer coaches the student,” the computer uses the procedural knowledge elaborated through our task analyses to model an organized decision-making framework for solving physics problems, making the numerous automated decisions of an expert visible. The student is asked to make those decisions, often choosing from among distractors based on known student difficulties. The computer assesses each decision, helps the student diagnose any errors, and guides the student to make corrections, if necessary, before moving on to the next decision in the process. Branching allows the student to follow potentially fruitful, though not necessarily optimal, solution paths. The current feedback is meant to encourage the student to obtain help from other sources, but it is also possible to insert additional instruction within the software.

In a type 2 coach, “Student coaches the computer,” the roles are reversed. The student chooses the decisions to be made, assesses the computer’s decision, and makes any necessary corrections. Because some of the computer’s responses are designed to reflect common student behavior, this coach also gives students practice in the important problem-solving processes of debugging. The computer also acts in an oversight mode, assessing the student’s responses and giving feedback. Again, it is possible to insert additional instruction if deemed useful.

Both the type 1 and type 2 coaches model the entire problem-solving framework. However, students must develop their own framework and be able to solve problems without the scaffolding provided by those coaches. In the type 3 coaches, “Student works independently, computer gives feedback,” the computer presents a problem to the student, who is asked to solve it on paper without any help and then enter an answer. The coach does not assume that a correct answer means that the student has a correct solution, but asks follow up questions to verify the correctness of the solution process. If the student answers a follow-up question incorrectly, the software jumps to the appropriate part of a type 1 coach. If the student cannot get an answer, they can ask for help. They are then asked to select which part of the problem-solving framework is causing them difficulty. The coach asks questions to determine if this is indeed the point of difficulty and selects an appropriate help methodology similar to those found in the type 1 coaches. After providing help, the coach asks the student to resume solving the problem on his or her own. Operationally, the coaches are much like the "Choose your own adventure" books (Montgomery, Peguy & Cannella, 2005) in the sense that the program operates like a flowchart (with loops) with responses determined by a student's input.

Screenshots from all three types of coaches are included in Appendix 1. Working prototypes of each type of coach can be found on our website at http://groups.physics.umn.edu/physed/prototypes.html.

2.4.5 Advantages of the Computer Coaches

The computer coaches were built under the cognitive apprenticeship model. One significant difficulty with implementing cognitive apprenticeship curricula designed to improve students' problem-solving skills is that opportunities for students to receive coaching are, at best, limited. Without coaching, students often revert to weak novice procedures rather than use the expert-like frameworks they are taught (Larkin, McDermott, Simon & Simon, 1980). One approach to increasing the availability of effective coaching is the creation of computer coaches, software delivered via the internet that can provide students with individualized guidance and feedback. Computer coaches have a number of advantages. A computer coach is available when a student desires coaching, even in the middle of the night. A computer coach is very patient and can be seen as less judgmental than a human tutor. Unlike human tutors, computer coaches become more economical while remaining equally effective as they serve more students. Computer coaches cost very little to maintain once created and hosted on a webserver. Finally, computer coaches provide reproducible instruction that can be improved incrementally and systematically by input from the user community.

2.5 Technical Design of Computer Coaches

The human interface to the computer coaches is accomplished using a web-delivered graphical user interface (GUI). We chose Adobe® Flash® [6], to be the fundamental environment for the GUI because (1) it is the product of a large and established company, (2) modules can be delivered easily via the web and run on all major operating systems, (3) it allows for incorporation of graphics, sound, and video, and (4) it is object-oriented, which facilitates user-initiated changes. The interface code is a hybrid of two languanges, XML and ActionScript. The XML regulates the interfacing components, such as the placement of panels and labels, while ActionScript handles the interface logic. For example, if a page has a group of radio buttons, the ActionScript code determines what action should be taken based on the user's selection.

Our software operates on the web from any common browser which has Flash® Player capability. We use only interfaces common to the web, such as radio buttons, checkboxes, boxes in which numbers can be entered, clicking and dragging, etc. Every screen of the coach consists of an index bar on the left showing the progress through the problem solving framework and the number of errors made in progressing through previous steps, a central work space where the current part of the problem solution is

[6] http://www.adobe.com/

constructed, the current state of the problem solution on the right, feedback on the bottom, and access to previous steps in the problem solving processes below the feedback. To give students time to consider their decisions and the computer's feedback, the coach requires that the student click a "Continue" button before moving to the next screen.

For the current version (version 1) of these computer coaches, researchers and instructors create new coaches by (1) duplicating an existing coach folder and (2) editing the XMLs for the texts and modify all the pictures in Flash®. For future design, it has already been considered to improve the current interface. Because instructors have different instructional needs and constraints, an easy-to-use web-based GUI is needed to allow the instructors constructing and modifying coaches within a short time. Allowing for this flexibility requires revising the current underlying software structure. The designing process will be discussed more in chapter 5.

2.6 Other Computer Related Systems

2.6.1 Intelligent Tutoring Systems

The idea of creating a computer coach is not new. "Intelligent tutoring systems" (ITS) have been created to try to help students learn a multitude of subjects, with mathematics and computer programming (Anderson et. al. 1995) being the most well-known examples. ITS is a broad term, encompassing any computer program that contains some intelligence and can be used in learning pedagogies (Freedman, 2000). Some systems involve using an artificial intelligence (AI) which are complex, represent an enormous investment of time, effort, and expertise, and are not widely used. Some of the modern intelligent tutors have the intelligence built in by using researched knowledge of student learning, expert behavior, and effective pedagogy.

Andes is an example of AI tutor. It is an ambitious computer tutorial system for problem solving that was constructed at the University of Pittsburgh and by the Navy. The Andes tutor incorporates an artificial intelligence system that attempts to determine the user's mental state and offers guidance and feedback. However, the Andes interface, rather than emphasizing the use of an expert-like decision-making framework based on general physics principles, encourages an equation-driven approach to solving physics problems that is consistent with and can reinforce novice problem-solving tendencies. Also the assessments of the Andes system have thus far been limited to assessing only the correctness of answers to problems and the presence of particular artifacts such as a diagram or a definition of variables in students' written problem solutions (VanLehn et al., 2005).

The modern intelligent tutors often focus more on the intelligence put in by researchers' knowledge and expertise. The design of our coaches belongs to the latter. The intelligence built into our computer coaches is provided by our research knowledge about problem-solving. They are built around knowledge of student learning, expert behavior, and effective pedagogy. They also incorporate research specific to education in the discipline, in this case physics, especially to model likely student difficulties. They recognize the complexity of human learning in that they deliver different modes of coaching and are constructed to integrate with other effective modes of teaching found in the classroom.

Another example of these computer coaches is the Personal Assistant for Learning (PAL) (Reif & Scott, 1999) developed by Reif and Scott at Carnegie Mellon University 1990's. It was designed to address only a subset of the mechanics part of a course, the application of Newton's motion law when solving quantitative physics problems. The "intelligence" in the PAL system was provided by the software designers using their knowledge of common student difficulties in that domain and a cognitive analysis of problem-solving. A small research study in which students' solutions were examined for the correctness of their diagrams and correctness and consistency of their equations with the diagrams found that students using the PALs performed significantly better than those who did not (Scott, 2001). These coaches were only available for Newton'2nd Law, however, and cannot be implemented for one entire semester.

2.6.2 Online Homework Systems

The computer coaches are distinguishable from other online homework systems. There are numerous web-based homework systems available for introductory physics, including LON-CAPA[1], WebAssign[2], Mastering Physics[3], and Tycho[4]. However, because none of them provide targeted coaching on the general decision-making skills integral to expert problem solving, they cannot reasonably be expected to improve students' fundamental outlook regarding problem-solving. Systems such as LON-CAPA[1] and WebAssign[2] primarily check a student's answer and give very limited feedback, typically whether or not a student's answer is correct and perhaps a problem-specific hint. Others such as Tycho[4] and Mastering Physics[3] can model an organized problem-solving strategy in specific tutorial problems but do not emphasize the general decisions involved in all problem-solving and cannot provide interactive coaching that allows for multiple correct solution paths.

Assessments of these systems have been based on the correctness of students' answers, the time it takes students to complete a problem, number of hints requested, or the number of incorrect responses before entering a correct response (Warnakulasooriya, Palazzo & Pritchard, 2007; Lee, et al., 2008), but the correlation between such measures and the progression to expert problem-solving skills is unknown. As can be seen, no existing system has yet satisfactorily met the challenges of creating a generally useful and usable problem-solving coach.

2.7 Engineering Design Process

To be useful, computer coaches must satisfy the requirements of multiple stakeholders, including students, instructors, and administrators. These stakeholders all have their own conception of a computer coach and occasionally these ideas conflict with what is necessary to address cognitive issues in learning. Thus, a useful coach must balance the need to incorporate features that are consistent with good pedagogy, with the need to incorporate features that enable it to be perceived as useful by its users and adopters. Furthermore, the designing process for the computer coaches, like any other engineering design, is an iterative process which adds to the complications.

The engineering design process is the formulation of a plan to help an engineer build a product with a specified performance goal. This process involves a number of steps and parts of the process may need to be repeated many times before production of a final product can begin. The engineering design process is a multi-step process including the research, conceptualization, feasibility assessment, establishing design requirements, preliminary design, detailed design, production planning and tool design, and finally production (Ertas and Jones, 1996).

The design process for the computer coaches was also an iterative process. First, a set of prototypes needed to be built, and experimental studies needed to be constructed to assess the effectiveness of these coaches. The results collected from the studies guided further educational experiment design as well as the further development of these computer coaches. Several cycles of implementation, assessment, and development were necessary to achieve a useful and effective software framework.

2.8 Problem Solving Assessment

2.8.1 Assessment Tool Literature

Having an appropriate assessment tool is very important to measure effective pedagogies and learning outcomes. Currently, there is no single, standard measure to quantitatively assess problem-solving (Adams & Wieman, 2006). In most introductory physics courses, students' problem solutions on homework or exams are given a score based on the correctness of the algebraic or numerical solution. A common grading practice in physics involves giving students partial credit for particular characteristics of their written solution, as compared to the ideal solution developed by the instructor. Usually partial credit values are based on the problem features and physics topic, and can vary substantially across different problems (Henderson et al., 2004), and using such scores to assess the quality of problem-solving is problematic. In some instances, instructors award points based on a problem-solving framework that has been modeled for students during the course. This might be too dependent on students implementing certain procedures to indicate the quality of problem-solving.

Research into problem-solving has used several different means to measure problem-solving performance. One method used by Larkin and Reif (1979) involves measuring the time it takes a problem solver to write down each quantitative expression in their solution, and recording the total time to reach a solution. Some researchers have also investigated problem-solving using think-aloud protocols or interviews, in which students engage in conversation explaining their thought processes as they attempt to solve the problem (van Someren, Barnard, & Sandberg, 1994). A difficulty with these methods is the time involved to prepare and conduct them, the vast amount of data generated from interview transcriptions, and the complicated nature of the data analysis (Harper, 2001). In order to compare problem-solving performance for many students, it is desirable to have a quantitative measure that can be determined relatively quickly. Researchers who have attempted to assess problem solutions on the basis of expert-like characteristics in the written solution include Reif and Heller (1982), Heller, Keith, and Anderson (1992), Huffman (1997), Blue (1997), Foster (2000), Harper (2001), Murthy (2007), and Ogilvie (2007).

2.8.2 Problem Solving Rubric

To build a quantitative instrument to measure of physics problem-solving, the Physics education research group at the University of Minnesota has developed a coding rubric to assess students' written solution based on criteria that describe expert problem-solving characteristics. There have been several versions of this problem-solving rubric and modifications were made based on reliability and validity tests. The most recent version of the rubric was developed by Jennifer Docktor (Docktor, 2009). The rubric considers five general problem-solving processes (Useful Description, Physics Approach, Specific Application of Physics, Mathematical Procedure, Logical Progression), and each category is scored on scale 0to 5.

The rubric is appropriate for assessing written solutions in this study because it is applicable to a range of physics topics and problem features. Details of this problem-solving rubric and the description of each category are given in Appendix 2.

2.8.3 Using Problem Solving Rubric in this Study

Measuring the state of a complex cognitive process in an authentic environment typically uses both qualitative and quantitative techniques combined in a rubric. Such an instrument needs to be sensitive to the development of a general problem-solving process and not sensitive to specialized training for specific behaviors such as drawing a certain kind of diagram, doing mathematics in a certain way, or getting the right answer to a specific problem type. The rubric that was developed is intended to be pedagogy independent and relevant to multiple problem types and topics (Docktor, 2009, Docktor and Heller, 2009). In small scale studies, the rubric was shown to be valid by scoring a range of types of expert and student solutions. Furthermore, assessments of students' problem-solving skills from interviews while solving a problem correlated very strongly with the rubric assessment of their written solutions.

The rubric has been developed under laboratory conditions and tested only in small scale studies. In this study, we will see the actual use of the rubric on a much larger scale implementation in an authentic classroom environment.

2.9 Summary

In this chapter the definitions of problem and the two traditions in problem solving: the differences between experts and novices in representation-domain knowledge and solution generating process were reviewed. The theoretical background for the computer coaches was also discussed. The intelligent tutoring system and existing computer programs (coaching programs and web homework systems) were introduced. The engineering design process was reviewed and the coach design was not a linear process and required several stages of the design/testing/revising cycle. Assessment tools of problem solving such as the problem-solving rubric used in this study was also discussed. Four challenges of doing authentic assessment of any pedagogical intervention were also outlined. The implications from this study in directing both the experimental design as well as future coach design will continue to be discussed in chapter 5.

The next chapter will address the methodological framework of the studies constructed to assess the effectiveness of the coach prototypes (version 1) as well as determining their potential usage and student perception of their utility.

CHAPTER 3: Methodological Framework

3.1 Introduction

This chapter will discuss the methodological challenges in evaluating problem solving in a classroom situation. It will then address the methodology of the three different studies that contribute to the results and discuss the advantages and disadvantages of different methodologies. It describes the procedures for assessing the data in each study.

3.2 Challenges

In any area of research, the most important measurements are often the most difficult. In this case we are trying to investigate a complex cognitive activity: problem-solving in a classroom situation. We face the following challenges: (1) constructing an experiment that measures, controls, or averages over the confounding factors influencing student performance; and (2) using a classroom situation in which improvement is neither blocked nor masked.

Effective problem-solving involves a constellation of cognitive processes that have been assessed in laboratory situations using; for example the think aloud interview techniques or classification tasks (Larkin and Reif, 1979; Chi, Feltovich and Glaser, 1981). However, this complexity makes direct quantitative assessment difficult in an authentic situation. In a classroom setting, any signal may well be buried in noise arising from the very nature of the educational process.

3.2.1 Constructing Experiments

The first challenge is constructing an appropriate experimental study. Doing experiments in education is especially difficult (Bouguen and Gurgand, 2012). Such a study might show progress within a single classroom environment with students receiving different treatments or it could be a comparison between two students in two different classrooms, where one is a treatment group and the other a non-treatment group. To compare two groups, one has to define equal groups (Grubišic, Stankov, Rosic, Zitko, 2009). In our experimental design, we try to achieve the statistical significant equivalence by matching groups on as many variables as possible that could contribute to their performance.

For example, as mentioned by previous researches (Foster, 2000), it is very difficult to statistically control for the Instructor Effect in a university setting. Most professors at a research university teach only one class a term, which prevents having a single instructor teach both a control and an experimental class. And even if an instructor can teach multiple times, it's not appropriate for an instructor to get control and treatment data from two successive years by giving exactly the same exam questions. We can never expect to have a perfectly controlled experiment in education research.

3.2.2 Experimental Environment

The second major challenge is to use classes that neither mask nor block the desired effect. For example, in order to measure a change in students' problem-solving skills, one must ask appropriate exam questions (traditional physics questions often don't evoke the problem-solving process) and reward students for using a strong problem-solving process in their solution (grading cannot simply be based on the final answer nor on the appearance of specific artifacts in the solution such as a particular type of diagram).

3.3 Instructional Settings

This study was conducted in a first semester introductory, calculus-based physics for science & engineering (1301) at the University of Minnesota during multiple Fall and Spring semesters from 2010 to 2013. The course comprises a 50-min lecture 4 times a week, a 2-hr laboratory once a week, and a 50-min discussion once a week. The lectures were taught by physics professors and the laboratory and discussion sessions were taught in smaller sections (15-18 students in each section) by physics teaching assistants. In the fall semesters, there were typically 5 lecture sections of the1301 course with about 200 students enrolled in each section. In the spring semesters, there were typically 2 lecture sections of the 1301 course with about 150 students enrolled in each section. Results from three studies will be presented in this dissertation, including one pilot test and two large-scale studies. The experimental design and constraints of each study will be discussed in detail below.

3.4 Experimental Design of all Studies

3.4.1 Pilot Study

3.4.1.1 Experimental Setup

In Fall 2010, a subset (15) of the computer coaches (8 on Conservation of Energy, 7 on Conservation of Momentum) was given to 20 volunteers from one section of introductory mechanics for science and engineering (PHYS 1301) to be finished in 4 weeks. (Materials for recruiting volunteers and the protocol for the pilot study can be found at Appendix 3.) In Spring 2011, a subset of 23 computer coaches (3 on Kinematics, 4 on Dynamics, 8 on Conservation of Energy, 7 on Conservation of Momentum and 1 on Rotational motion) was given to 9 volunteers from one section of introductory mechanics for science and engineering (PHYS 1301) to be finished in over the semester. (Materials for recruiting volunteers and the protocol for the pilot study same as Appendix 3.)

Information gathered from these pilot studies gave information about the usage of the computer coaches that was used to guide the design of the coaches and the structure of the large scale studies.

3.4.1.2 Methodology

In order to control for as many variables as possible to achieve equal groups, our experimental methodology was to recruit volunteers from a single class to participate in the study, and randomly assign them into treatment group (using the computer coaches) and control group (some other intervention, such as just giving them the same problems from the coaches to work on paper). This would statistically control for background variables including the motivation to participate in such a study because both the treatment and control groups would be selected from volunteers.

In the pilot study, we tested this protocol but found it to have very limited statistical power: we couldn't get enough volunteers and some students dropped from the study during the semester. In Fall 2010, we recruited volunteers by offering 75 dollars for completing 15 computer modules from one section of the introductory physics class and only got about 40 volunteers. Constructing a control and treatment group from the volunteers left only 20 students in each group. In Spring 2011, we only got 9 volunteers. Each of these volunteer groups was further diminished by attrition over the semester. Students get distracted by the multitude of requirements to fulfill for all of their classes so this volunteer work is the first thing they drop when they get behind schedule. For example, in Spring 2011, only 2 students went all the way through to the end of the semester and finished most of the 23 computer modules. The other students dropped out of the study about half way through.

Perhaps the recruitment advertising could be more attractive or the pay to the volunteers needed to be significantly larger to achieve the desired statistics but, in any case, we abandoned this approach for the large scale studies. These difficulties suggest that we needed to provide a stronger motivation for the students to remain in the study.

3.4.2 Fall 2011

3.4.2.1 Experimental Setup

In Fall 2011, the full set of computer coaches was made available to one section (219 students) of an introductory calculus-based mechanics class (PHYS1301, Introductory Physics for Engineering and the Physical Sciences) at the University of Minnesota as one method of satisfying their homework. In this experiment, students were allowed to satisfy their homework requirement either by completing all the computer coaches for a given topic, by submitting a correct answer to the same problems through WebAssign (www.webassign.net) within three attempts, or by a combination of the two methods. Student use of the coaches was monitored by recording their computer keystrokes. During the course, students took four written in-class tests, each with two free- response context-rich problems to solve and a final exam with five more standard problems. We collected the students' written solutions to these 13 problems (See all problems and solutions in Appendix 4).

We also collected written problem solutions from another section (199 students) of the same course taught during the same semester by a different professor. This class did not use computer coaches but did use Learning Assistants (Otero, Finkelstein, Pollock and McCray, 2006) to facilitate small group discussions during lectures which the computer coach class did not. Both sections used the Cooperative Group Problem-Solving pedagogy and emphasized the use of an organized problem-solving framework (Heller and Heller, 1997).

In addition to the students' written problem solutions and keystroke data, we also collected pre- and post-test scores on the Force Concept Inventory (FCI) [8] (Hestenes, Wells and Swackhamer, 1992), a Math diagnostic test (Appendix 7), and the Colorado Learning Attitudes about Science Survey (CLASS)[9] (Adams, Perkins, Podolefsky, Dubson, Finkelstein and Wieman, 2006), as well as scores on the problems as determined by the regular graders for the course, graduate Teaching Assistants (TAs), as part of students' final grade. An end of semester survey was also given to students to get their opinions about using the computer coaches. Two versions of surveys were given at the end of semester emphasizing either the computer coaches or WebAssign (http://www.webassign.net/) (Appendix 5) based on the number of problems students completed (sometimes students attempt a problem without completing it) on the computer coaches.

3.4.2.2 Methodology

By giving these coaches as an online homework option, we managed to get a large number of students using the coaches.

We attempted to address the challenges of controlling the confounding factors and influence on student learning by: (1) making comparisons among students within the class where the coaches were available and (2) by making comparisons between that class and a different lecture section where the coaches were not available. This section was taught during the same semester by a different instructor. The comparisons within the coach class were done between those students who completed more coaches and those who completed fewer. Since both sets of students were in the same class, we control for class environment. In addition, a comparison between the non-coached class and coached class could examine the possible benefits of using the coaches on a much larger sample size. Despite of the variables brought in by different instructors, these two instructors shared of the same teaching philosophy with respect to problem solving and had very similar classroom structures: they both emphasize problem-solving skills and both employed the Minnesota problem-solving framework in their class. Both classes also used context-rich test questions and rewarded students for using a strong problem-solving process in their solution.

There are still issues and difficulties in this methodology. Subsample comparisons within a single class has the advantage of controlling for the class environment but also has the issues of limited statistical power and contamination between groups. Using more than one class addresses both of these issues. However having two classes, even when they are different sections of the same course taught during the same timeframe, brings in confounding parameters such as the effect of the instructors. In addition, the distribution of the student population can be different in the two classes since they are taught at different times during the day so they tend to have different distributions of majors. Even one instructor could teach both sections the populations would be different and the instruction would not be exactly the same. Since the classes would be tested at the time of their class, the instructor couldn't use the same tests.

Another confounding parameter introduced by using more than one class is that the two (multiple) classes have different tests. The structure and composition of the tests reflect the outlook of the instructors and the emphasis of the class. Since the problem-solving evaluation is made on the written work of the students in answering these test questions, this complication makes it difficult to compare students' performances as a function of time (previous research shows that the written solutions can reflect and represent students' problem-solving skills very well: in lab conditions, the written work is consistent with what they say in interviews, and it can be used to represent their problem-solving (Docktor, 2009). It is also true that instructors do not always make up test questions that distinguish between novice to expert-like problem-solving.

The noise introduced by these confounding parameters might be statistically controlled by comparing significantly more than two classes with a correspondingly larger expenditure of analysis effort. For example, having a control group (baseline) that encompasses many classes, all about the same size as the coach group with many professors can statistically average over the noise introduced by having different tests, different professor, etc.

3.4.3 Spring 2013

3.4.3.1 Experimental Setup

In Spring 2013, the coaches were made available in two sections (249 total students with 148/103 in each section) of an introductory calculus-based mechanics class (1301) at the University of Minnesota. The two sections had different lecturers, but both focused on problem-solving facilitated by Cooperative Groups in the labs and discussion sections. Students submitted weekly homework assignments (10% of the course grade) through WebAssign[7] and were allowed 5 tries to earn credit. Roughly one third of these problems were Context-Rich problems on which students could get help from a corresponding computer coach. Students received no direct homework credit for using the coaches. The WebAssign and coached versions of a problem differed only in the symbols and sometimes numbers used to represent quantities in the problem.

During the semester, students took four tests, each with two free-response Context-Rich problems, and a final exam with five Context-Rich problems (See Appendix 6). The five final exam problems were identical for both sections. In addition

[7] http://www.webassign.net/

to the scores and written solutions to these problems, other collected data included pre and post test scores on the Force Concept Inventory[8] (FCI), a Math skills test (Appendix 7), the Colorado Learning Attitudes about Science Survey[9] (CLASS), and a survey about the students' background (Appendix 8). Students' use of the coaches was monitored by recording their keystrokes. They were also surveyed at the middle and the end of the semester about their use of the computer coaches (See surveys at Appendix 9).

3.4.3.2 Methodology

The Fall 2011 study successfully engaged a large number of students using the coaches (almost everybody in the class used the coaches), however, because of the popularity of the coaches, we could not create a clean control group to compare with a treatment group. The intention of the design of the Spring 2013 study was to get both user & non-user population within the coach class.

In the two sections of Spring 2013 1301 class, the students self-select into different user populations that we found could broadly be described as self-confidence (see more detail in section 4.4.1) and we established four characteristics that describes these user populations (section 4.4.1). We found statistical significant equivalent matching groups for these user population from historical baseline based on these characteristics. The baseline was established by sampling from 3145 students during Spring 2009 to Fall 2011.

[8] Hestenes, Wells and Swackhamer, 1992
[9] Adams, Perkins, Podolefsky, Dubson, Finkelstein and Wieman, 2006

3.5 Implications of Methodology

Many educational experiments involve a comparison between a treatment group and a control group. This raises the question of how to choose these two groups. The most accurate way of making these groups would be to hole everything else except for the treatment the same. Since the relevant educational environment is often the classroom, this means dividing the classroom population into subgroups which severely limits the statistical precision of the measurement. In addition, dividing a class in this manner opens the door to contamination between the treatment group and the control group. Moreover, there is the issue of how to choose students for each of the groups. A random selection is difficult to enforce, especially at the university level, and can have ethical difficulties. In addition, a class size is small enough so that randomly selected samples can be different in ways that affect the result. Matching background variables between the two samples further limits a study's statistical power. Recruiting volunteers, whether paid or not, for the treatment sample can bypass the ethical issues but these volunteers are probably different than those who do not volunteer. The selection bias of using volunteers can be addressed by only selecting half of the volunteers for a treatment group and using the other half as a control group. Again this process limits the statistical power of the study.

In Spring 2013, students self-selected into different types of user and non-user groups. Each of these groups was different in background characteristics. All of these students were in the same learning environment but might be expected to perform differently based on their differences in motivation, expectation, and knowledge base.

We overcame this difficulty by directly comparing the performance gap of the user groups within the experimental class with their gap in the control class. The differences in background variables that characterized each group in the experimental class were matched in the control class.

By limiting the control class to one taught during the same semester, we could compare the performance of the different user groups in one class with matched groups in the other on the same final exam. However, the size of the classes (about 200 students) and the number of problems (5 problems on the common final exam) makes it very time consuming to apply the rubric on all of the final exam problems.

To assure a valid problem solving assessment, one should use rubric analysis by trained personal tested for reliability on all the classes. However, the amount of effort required to collect data in this way sacrifices statistics to have more accuracy.

Given the strong correlation between the rubric scores and the grades assigned by the TAs at our institution and assuming variations in grading can be controlled for by averaging over large data sets, we decided to use regular problem grading to assess problem solving skills. Tests on subsets of the data showed that this is valid as long as there are enough problems are graded within a given exam. In our studies, at least 3 problems each graded by a different grader are necessary to achieve a high correlation between the total problem grade and a rubric score. It is important to emphasize that all TAs at the University of Minnesota receive training on educational principles and techniques, including grading during their first year. At a different institution without such TA support, one would have to test whether the rubric scores of trained personal and the regular grading of tests has a high enough correlation for the grading to be meaningful for a problem solving analysis.

Even with trained TAs where a correlation with rubric scoring has been established, the absolute value of grades can reflect the practice individual instructors and graders. How do we normalize these different classes? One way is to normalize the average grading scores in a class to the historical average if the historical data pool is large enough and the behavior of the students and instructors is not believed to have significantly changed.

3.6 Rubric Application – Rubric Training sets

The problem-solving rubric was developed to assess students' physics problem-solving within five categories. For research purposes, an assessment such as a rubric must have a high inter-rater reliability. We have developed training materials (Appendix 10) to help novices learn to use the rubric.

3.6.1 Calibration

The first part of the training of a novice rater is a calibration session with an experienced rater. The calibration training set used in this study consists of 8 problems from the Spring 2011 class (2 problem from each quiz on the same topic, 4 quizzes total) with 5-10 student solutions (these solutions usually covers good, medium, bad solutions) for each problem. After completing the rubric training materials (Appendix 10), raters start the training process by each scoring a set of student solutions (5-10) for

1 problem on their own and then discuss their scoring with an experienced rater to reach an agreement. After discussion, they start a second problem and score another 5-10 student solutions followed by discussion. They keep doing this until their agreement before discussion reaches 80-90%. If the original disagreement is high, the raters to go back and rescore another set of solutions for the original problems confirm that they now have the desired rate of agreement for those problems. In this study, two raters scored 11 sets of problems: after the first 8 sets of problems, the raters went back to select different student solutions from the original 3 problems to check that the process had converged.

After discussion, the agreed upon score from both raters was used as the final score for the analysis. In rare cases, if raters are unable to achieve agreement even after discussion, they use the averaged score to be the final score for that solution. All training sets are in Appendix 11.

3.6.2 Inter-rater Reliability

For large data sets, such as the case of the Fall 2011 study where there were over 2600 solutions to be scored, it was not possible for raters to compare the score for every solution. Instead, the only one rater scored a fraction of the solutions of each problem . The raters checked their inter-rater reliability by selecting the same 10 solutions for that problem, and then comparing and discussing those scores. From this discussion process, the raters develop some general guidelines for scoring that particular problem. The process was repeated for each new problem.

CHAPTER 4: Experimental Setup and Data Analysis

4.1 Introduction

The set of computer coaches used in this study consists of 35 problems, spanning 6 topics of introductory mechanics (6 on Kinematics, 6 on Dynamics, 8 on Conservation of Energy, 8 on Conservation of Momentum, 5 on Static Equilibrium and Rotational motion, and 2 on Oscillations). We used these coaches in large-scale classroom testing in Fall 2011 and Spring 2013. Pilot tests were run in Fall 2010 and Spring 2011 with a smaller set of computer coaches used by a few students. In this chapter, we will give the results of our measurement of the usage and usability of these coaches, their usefulness as perceived by students, and their effectiveness as measured by problem-solving improvement.

4.2 Usability & Perceived Usefulness

4.2.1 Completion Time

The average time to complete one of the most complicated coach problems was 31 minutes as measured by the key stroke logs of students both in Fall 2011 and Spring 2013). 67% of the students in Spring 2013 and 56% of the students in Fall 2011 who completed this problem spent equal or below the average time. When the completion time is more than an hour, the keystroke data show that the student took several small breaks (10-30 minutes) or a long break (more than 1 hour) while logged on to the coach. We deleted entries with breaks of more than 1 hour when computing average completion time. Taking long breaks is rare: none of the students in Spring 2013 and only 5% of the students in Fall 2011 took more than an hour long break. Students tend to stay on task when working through a coach.

4.2.2 Students' Feedback

When students were given a choice between using a commercial on-line homework system (WebAssign) or these coaches to fulfill a homework assignment, an overwhelming proportion of students chose to use the coaches. In Fall 2011, out of the 219 students, the average number of coaches attempted was 28 and completed was 21.9 out of a maximum of 35. Only 20 students completed fewer than 10 coaches. In Spring 2011, when only entering the correct answer in the on-line homework system could fulfill the homework assignment, 71% of the students attempted more than 20% of the coaches for the first seven weeks of the course. This usage data shows that most students find the computer coaches an attractive course supplement.

All students were given an online survey about the course at the end of the Fall 2011 semester. Students who used more than 10 computer coaches were asked to complete a survey with coach-related questions and the others were asked to complete a survey with WebAssign related questions (Appendix 5). Students answered a set of questions using a 5-point Likert scale. The results for all of the survey questions are in Appendix 11. In Tables 1 and 2 below, we give the responses to survey statements about the students perceptions of the coaches.   In these tables, responses 1 and 2 (agree/strongly agree) are combined as are 4 and 5 (disagree/strongly disagree).

Survey statement (FALL 2011)	Strongly agree /Agree	Neither	Strongly disagree /Disagree
When using the computer coaches, it was usually clear how to proceed.	90% ± 3%	8% ± 2%	2% ± 1%
The computer coaches helped my conceptual knowledge of physics.	81% ± 3%	14% ± 3%	5% ± 2%
The computer coaches did not help improve my problem solving.	8% ± 2%	13% ± 3%	80% ± 3%
The computer coaches helped me identify what I needed to get help with from other sources.	74% ± 3%	21% ± 4%	5% ± 2%

Table 1: Students' responses to survey statements about the computer coaches usability and utility.

FALL 2011	Type 1	Type 2	Type 3
Which type of computer coach did you find most (least) useful at the beginning of the course?	63% ± 4% (5% ± 2%)	23% ± 4% (24% ± 4%)	14% ± 3% (71% ± 4%)
Which type of computer coach did you find most (least) useful at the end of the course?	29% ± 4% (23% ± 4%)	33% ± 4% (29% ± 4%)	38% ± 4% (48% ± 4%)

Table 2: Students' responses about the usefulness of different types of computer coaches

The standard error was computed by using: $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, where $\hat{p}$ is the sample proportion, n is the sample size (Rumsey, 2011).

Rank the components of the physics class in order from most useful (1) to least useful (18) to your learning. Do not use any ties. If you did not use a particular component, then omit it from your ranking. (FALL 2011)	Percentage of students ranking this component	Average ranking (lower is better)
Lectures	100%	3.8±0.3
Computer coaches	99%	3.9±0.2
Practice quizzes	96%	5.1±0.3
Discussion sections	100%	6.0±0.4
In-class clicker questions	98%	6.1±0.3
Out-of class studying with other students	74%	7.3±0.5

In-class quizzes	99%	7.9±0.4
Your own lecture notes	91%	8.4±0.4
Posted lecture notes	73%	8.6±0.4
WebAssign	87%	9.2±0.4
Labs	97%	9.3±0.4
Reading in textbook	88%	9.9±0.4
Out-of-class discussions with professor	47%	10.2±0.7
Writing lab reports	92%	10.4±0.4
Problems in textbook	78%	11.2±0.4
TA help in the tutor room	48%	11.2±0.7
Supp. Text (Competent Problem Solver)	44%	11.8±0.7
Out-of class discussion with your TA	48%	12.6±0.7

Table 3: Students' responses to the forced ranking survey question

According to the survey, 81% of the students agreed that the coaches helped with their conceptual knowledge of physics, and 80% of the students think the coaches helped improving their problem solving skills. In the same survey, students were asked to rank the components of the class based on their perceived usefulness to learning. The results are given in Table 3. In the forced ranking question, the computer coaches were essentially tied for the most useful component of the class with lectures.

4.2.3 Summary

An overwhelming proportion of students chose to use the coaches when given a choice of completing homework using coaches or a common commercial online homework program. Based on the students' survey responses, we conclude that the interface of the coaches was clear and self-explanatory. The step-by-step Type I coach was judged the most useful at the beginning of the course. All three types of coaches were judged equally useful at the end of the course. Students ranked the coaches among the most useful elements of the course for their learning. Students perceived the coaches as useful in improving their conceptual understanding and problem solving skills in physics.

4.3 Rubric Assessment in Fall 2011

The actual usefulness of the computer coaches in problem solving was measured by applying a rubric to students' written problem solutions. The rubric was developed and tested for reliability, validity, and utility for a wide variety of problem solving styles at the University of Minnesota (Docktor, 2009) and includes five categories: (1) representing problem information in a Useful Description (UD), (2) selecting appropriate physics principles (Physics Approach, or PA), (3) applying physics principles to the specific conditions in the problem (Specific Application of Physics or SAP), (4) using appropriate Mathematical Procedures (MP), and (5) the overall communication of an organized reasoning pattern (Logical Progression or LP). Each category is scored on scale of 0-5 (with 5 being the most expert-like), or N/A in cases where the category is not applicable to that particular problem solution. Rubric training and inter-rater reliability check of this process is in Appendix 12.

4.3.1 Within Class Comparisons

In Fall 2011, almost all the students used the coaches but we did attempt to separate the class by the fraction actually completed. We looked at the performance between the frequent completers (completing an average of 33 coaches) and the less frequent completers (completing an average of 12 coaches). Because students attempt a problem without completing the coach, the difference between actual attempts of the two groups is even smaller: the average number of coaches attempted is 34 (out of 35) for the FC group and 21 (out of 35) for the LC group.

To try to control for differences in students, we created pretest-matched subgroups from the FC and LC groups (see Table 4). Although we obtained groups with nearly identical pretest scores (pre FCI, Math and CLASS scores) as well as a closer match with regard to gender (5 female, 19 male in the FC group; 3 female, 21 male in the LC group), the statistical power of the measurement was limited because of the small number of students in the LC group. There was no significant difference between the two matched groups in terms of their performances on post instruction diagnostic exams, quiz scores and rubric scores (Appendix 13). We are not surprised by this result that there isn't a large difference in their problem solving because both groups were heavy users of the coaches.

Matched groups	Top 24 (frequent completers) N = 24	Bottom 24(less frequent completers) N = 24
Gender balance	5 F, 19 M (21% F)	3 F, 21 M (13% F)
FCI pre	59.9% ± 4.0%	59.9% ± 4.1%
Math pre	66.2% ± 4.1%	66.2% ± 3.7%
CLASS pre	64.6% ± 3.4%	64.6% ± 3.2%
avg. # coaches completed	32.8 (30-35)	11.4 (5-15)
avg. # coaches attempted	33.9 (30-35)	21.2 (7-32)

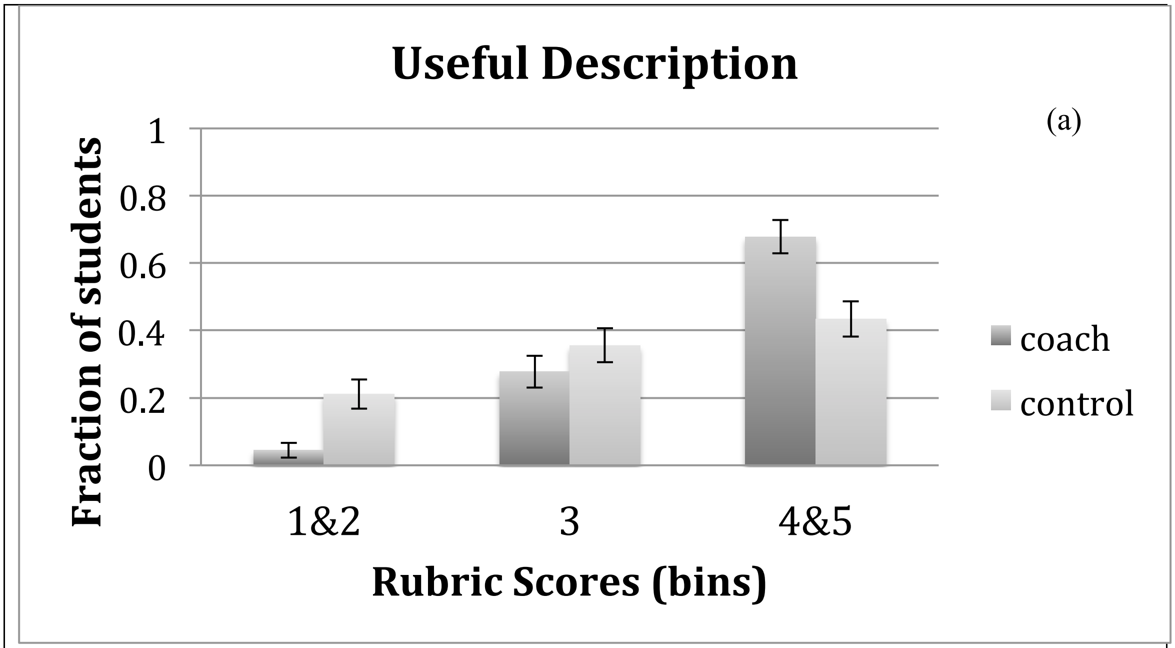
Table 4: Matched FC and LC subgroups: FCI, Math, CLASS scores and gender ratio

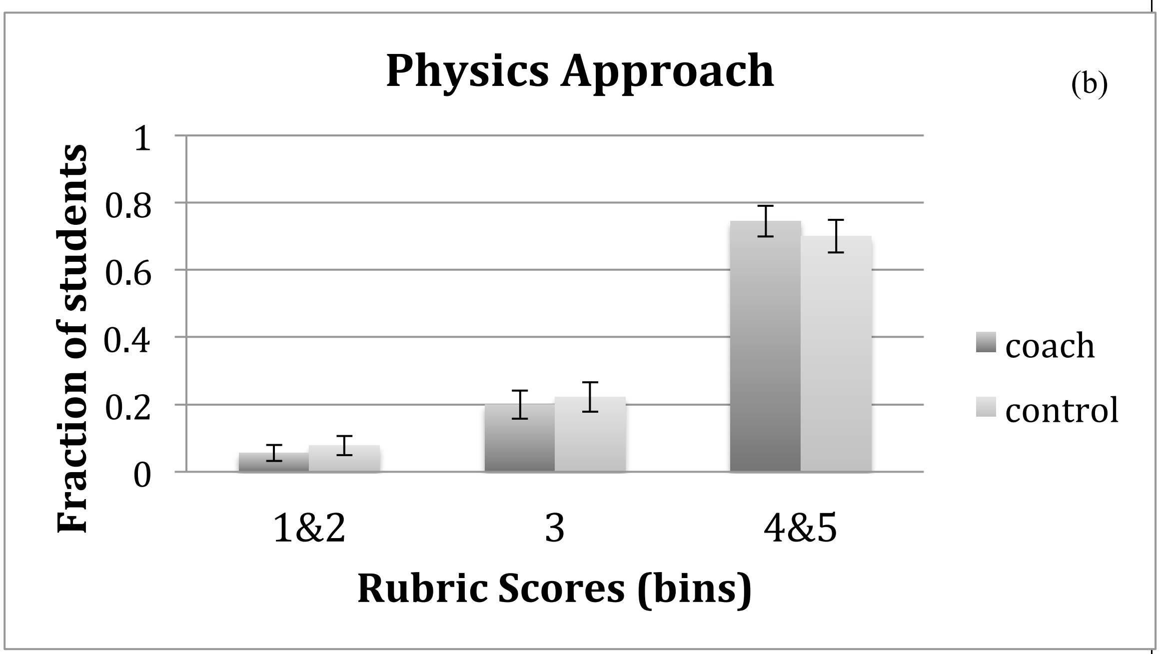
4.3.2 Between Class Comparisons

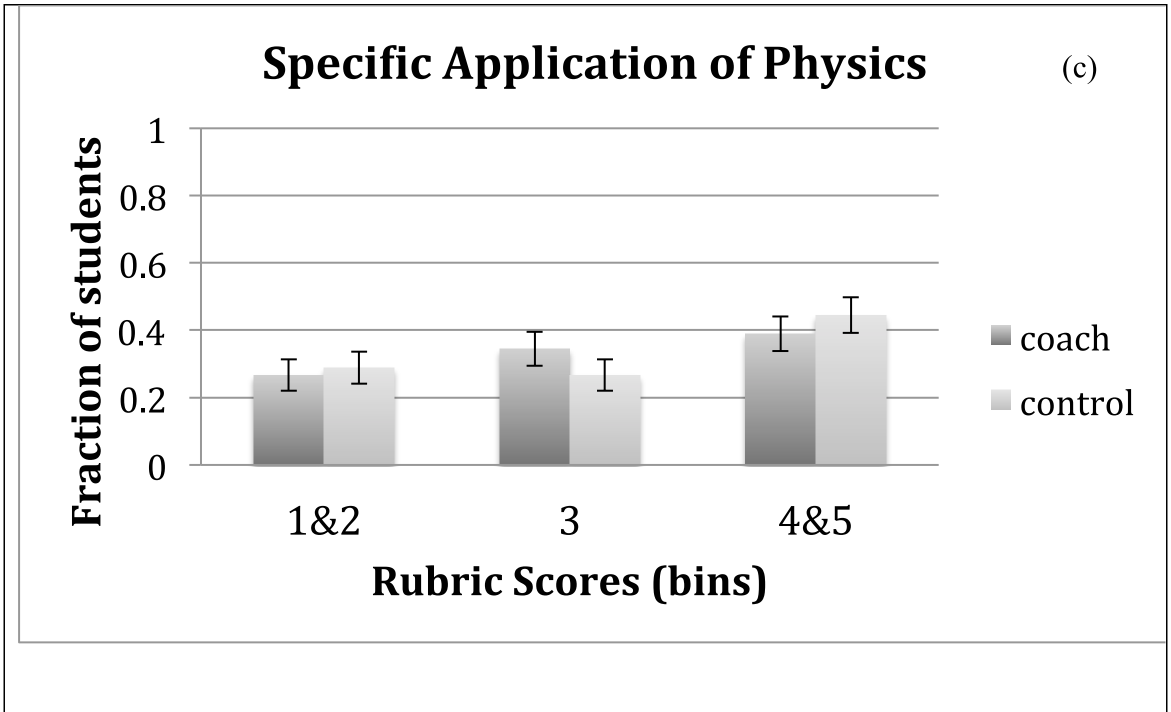
To reduce the effects of learning contamination among students, 99 students from each of two different lecture sections of the same course taught during the same semester were selected to form two matched groups based on the FCI, Math and CLASS pre test scores. In the group from the class using the computer coaches, there were about an equal number of FC and LC students (28 FC, 30 LC), as well as 41 students from neither of those two groups in the sample. The gender ratio was nearly equal in two matched groups (27 females in the group from the class using coaches and 28 females in the group from the other class). Because the two classes used different quiz problems but the same final exam, rubric performance comparisons were made only using the final exam.

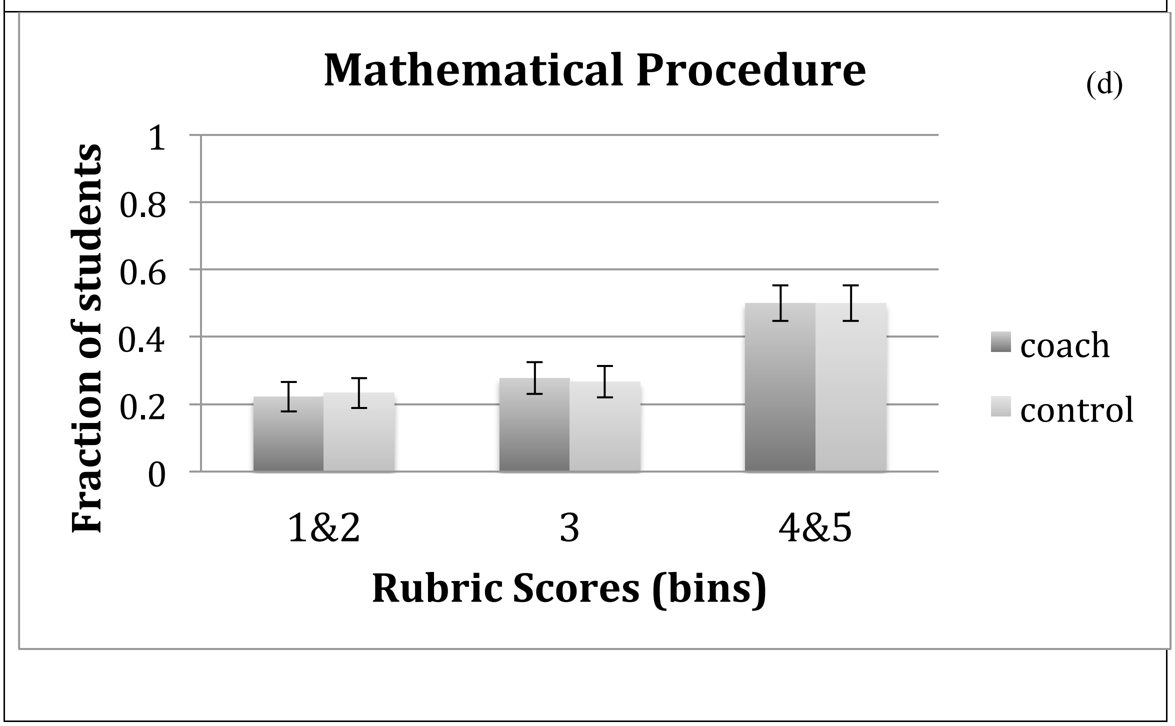
We also looked at the distributions of the scores for each category. Because there are no students who scored 0, we dropped that bin and grouped the rubric scores ranging from 0 to 5 into three bins, 1 or 2 (low), 3 (middle), and 4 or 5 (high). Figure 9(a-e) shows the fraction of students in each class with scores in each bin for all five categories for final exam. Because we are trying to measure a general problem solving ability, the scores on the five problems have been combined into a single score.

The error bars on the graphs were computed using: $\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, where $\hat{p}$ is the sample proportion (the fraction of students in each bin), n is the sample size: since there are 90 students in each group, n=90. Note that the unit of measure used is the person not the number of problems.


Useful Description
(a)
Fraction of students
1
0.8
0.6
0.4
0.2
0
1&2
3
4&5
Rubric Scores (bins)
coach
control


Physics Approach
(b)
Fraction of students
1
0.8
0.6
0.4
0.2
0
1&2
3
4&5
Rubric Scores (bins)
coach
control


Specific Application of Physics
(c)
Fraction of students
1
0.8
0.6
0.4
0.2
0
1&2
3
4&5
Rubric Scores (bins)
coach
control


Mathematical Procedure
(d)
Fraction of students
1
0.8
0.6
0.4
0.2
0
1&2
3
4&5
Rubric Scores (bins)
coach
control

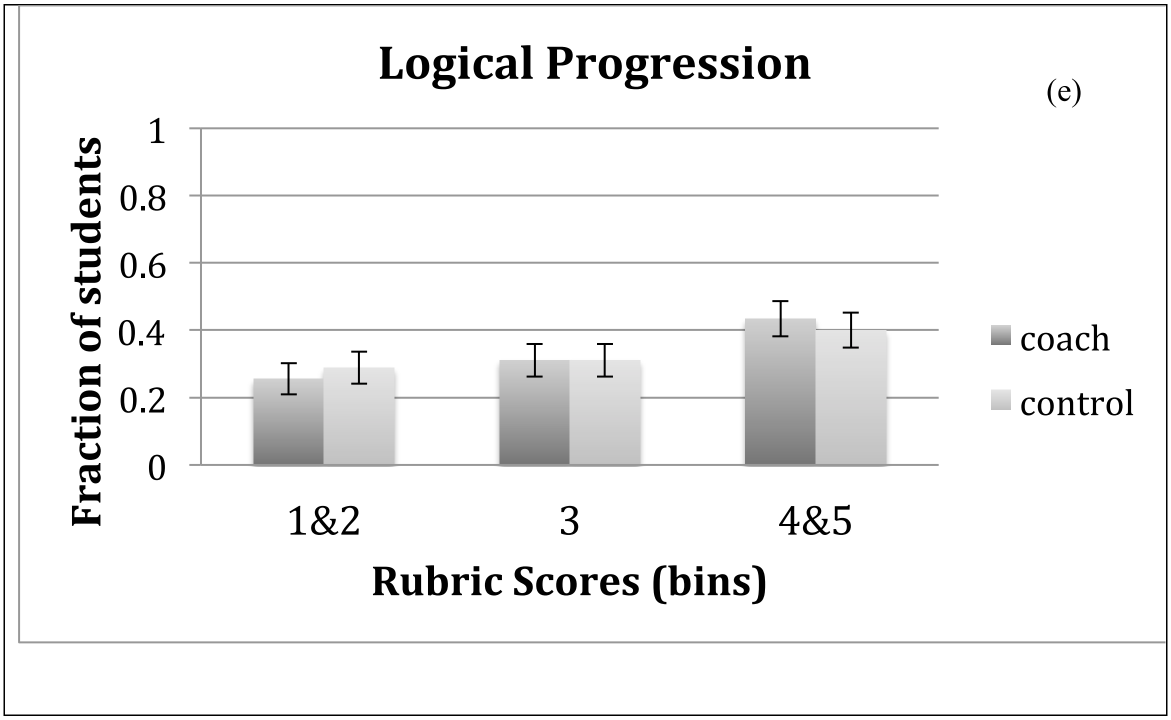


Figure 1 (a-e): Comparison of the percentage of students in each rubric bin for each of the five rubric categories on final exam between the coach and control class

Since the current coaches do not address mathematical procedures in any significant manner. No difference is to be expected for that category. In the other 4 categories, the coach class has fewer low scores in every category. It also has more high scores in 3 out of the 4 categories. This pattern, while suggestive, is not statistically significant except for the Useful Description category.

	Useful Description	Physics Approach	Specific Application	Mathematical Procedure	Logical Progression
Bin 1&2	p(z = 4.74) <0.0001	p(z = 0.85) = 0.20	p(z = 0.47) = 0.32	p(z = 0.25) = 0.40	p(z = 0.71) = 0.24
Bin 4&5	p(z = 4.67) < 0.0001	p(z = 0.94) = 0.17	p(z = -1.07) = 0.14	p(z = 0) = 0.50	p(z = 0.64) = 0.26

Table 5: p-value of the z-score for the coach and control class on each of the rubric category

Even accounting for a possible selection bias because one of the possible four categories is significant, the coach class performed significantly better on the Useful Description category.

In Appendix 14 we gave another graphical representation of the rubric scores, with the average rubric score of each category compared between the two classes for the five problems.

4.3.3 Rubric Score & Grades Correlation

To assess the problem solving performance of a large number of students, it is useful to use the scores that teaching assistants (TAs) assign to each problem in the normal course of grading. As a test of the validity of these test scores, the sum of rubric scores from all five categories was compared to the grades assigned to the problems by the TAs. It is important to note that the grading of the problems by the TAs was completely independent of the rubric scoring and that the criteria for each were not necessarily related. However, as one would hope, the correlation between the TA score and the summed rubric score was very high, ranging from 0.79 to 0.85 for each problem in the final exam of Fall 2011(Correlation graphs in Appendix 15). The overall correlation for all problems is 0.88 for both the coach class and control class.

Problem 3 of the final exam in the non-coach class did not correlate well (correlation coefficient 0.50) with the rubric score. On careful examination of that problem, it was judged that it was not well graded. See specific examples in Appendix 16. For analysis purposes , we excluded problem 3 from both classes.

4.3.4 FCI

Other than evaluating students' problem solving skills, we also examined the post instruction FCI scores as a measure of whether the coaches influenced conceptual understanding. The coach class's FCI post scores were significantly higher than the control class. The normalized gain <g> of FCI for the coach class was 0.51±0.05, and the control class was 0.40±0.06. <g>is the course average normalized gain, defined as the actual average gain, %<Gain>, divided by the maximum possible actual average gain, %<Gain>max:

<g> = (%<posttest> – %<pretest>) / (100 – %<pretest>) (Hake, 2002)

		FCI pre	FCI post	<g>	Math pre	Math post	CLASS pre	CLASS post
99 matched students	Coach	57.9% ±2.0%	81.2% ±1.4%	0.55 ±0.06	62.9% ±1.7%	73.5% ±1.5%	65.4% ±1.6%	61.2% ±1.8%
	Control	57.9% ±2.0%	75.2% ±1.6%	0.41 ±0.06	62.7% ±1.6%	69.6% ±1.5%	65.4% ±1.6%	56.2% ±1.9%
The entire class	Coach n=159	56.3% ±1.6%	78.8% ±1.2%	0.51 ±0.05	63.5% ±1.4%	73.8% ±1.3%	64.1% ±1.3%	60.5% ±1.4%
	Control n=103	57.7% ±2.1%	74.8% ±1.7%	0.40 ±0.06	62.2% ±1.6%	69.2% ±1.5%	64.8% ±1.6%	57.7% ±1.9%

Table 6: Diagnostic exam scores for the coach class and the control class

Interestingly, the coached class also had higher Math gain and less CLASS loss (CLASS post score is usually lower than the pre score) even though the coaches weren't designed to help their math skill or affect one's learning attitude. The p-value for the t-test on the 99 matched students from the two classes are: 0.06 for Math post and CLASS post which just fall beyond the significant level (0.05).

4.4 User Characteristics

In chapter 1, we asked this research question: Will students use the computer coaches and what are the characteristics of the students who do and don't?

As mentioned previously, when associating homework credit with using the coaches, essentially the entire class chose to use the coaches in Fall 2011. In spring 2013, there was no direct grade benefit for the students' use of the coaches, but of 71% of the students used them for the first half of the course (total coach usage by week 7—Dynamics). By the end of the course, this number had decreased to 20%. The variation of natural coach usage when there was no direct incentive, allowed us to divide the students into groups based on the frequency with which they used the coaches. We then studied each group to determine if there was some set of characteristics that made them more likely to be in one usage group rather than another. Perceived usefulness by each different user population was examined. We could also determine the benefit, if any, of the coaches for each group.

4.4.1 User Groups: Heavy (H), Medium (M), and Light (L) Users

In any course, some students will tend to use resources such as the computer coaches and others find them unnecessary or incompatible with their personal preferences. To design effective coaches, one needs to know the relative sizes and characteristics of each group.

In Spring 2013, we were able to form three groups of students from the 249 total students (70% male (m), 30% female (f)) in the two sections offering the coaches: an L (light/non- user) group of 72 students (85% male, 15% female) who used 0–20% of the coaches[10], an M (medium-user) group of 38 students (55% m, 45% f) who used 40–60%, and an H (heavy-user) group of 49 students (65% m, 35% f) who used 80–100%. We excluded the in between students (20%-40% and 40%-60%) to get non-overlapping populations.

We hypothesized that these usage groups might be different in their gender make up, and their confidence level toward the course. To determine this we used a background survey filled out by the students during the first week of class. In addition to other questions such as their intended major and previous physics background, the questionnaire asks them the grade they expect for the course and the number of hours per week they expect to work on the course. We used the last two questions as an indication of their confidence level. The gender and confidence characteristics for each usage group are given in tables 7 and 8.

One difference among the three groups is in their gender. The proportion of females in the L group is about half that of the class as a whole. This is consistent with research that females with the same performance as males are more willing to seek assistance (Addis and Mahalik, 2003). This is also consistent with the results seen in Fall 2011(Appendix 17). Another difference among the groups is their expectation of

[10] Although there were 35 coaches, only 29 total coaches were considered for the data analysis because a database error made it impossible to track the usage of the first 6 coaches.

the effort required for the class. Students in the L group expected to spend less time studying and to earn a higher grade in the class than students in the H group. No student expected to receive a grade less than a B.

The three groups also differed in their physics preparation as measured by their scores on standardized assessments. Table 7 shows the pretest scores of the three groups by gender. The number (N) differs from those from the entire class because only students who took all three pre-tests are included. A higher FCI pre-test score is correlated with lower use of the computer coaches. There is some indication that this may also be true for the Math skill tests. One might infer that the more poorly prepared students recognize this and choose to use easily accessible help.

From this data, we infer that students in the L group have high confidence in their ability to perform well. Students in the M group similarly expect to do well, but also expect to spend more time doing so. Students in the H group expect to spend more time and are less confident of their success.

Test	L (N=48)		M (N=27)		H (N=35)	
	male	female	male	female	male	female
	85%	15%	67%	33%	66%	34%
FCI	58%±3%	59%±11%	53%±5%	42%±7%	46%±3%	31%±3%
MATH	58%±3%	66%±8%	53%±4%	61%±7%	54%±4%	45%±4%
CLASS	62%±3%	55%±6%	66%±3%	66%±5%	65%±3%	56%±4%

Table 7: Differences in background of the three groups: FCI, Math, CLASS scores and gender ratio. Because FCI scores have been shown to be gender dependent[11], we show the scores by gender.

[11] Docktor, J. & Heller, K. (2008). Gender differences in both force concept inventory and introductory physics performance. In C. Henderson, M. Sabella, & L. Hsu (Eds.) Proceedings of the 2008 Physics Education Research Conference. AIP Conference Proceedings Vol. 1064, pp. 15-18.

	N	Weekly study time (hours)			Expected grade	
		≤5	6-10	≤5	A	B
L	48	25%±3%	46%±4%	29%±3%	71%±3%	29%±3%
M	27	4%±1%	59%±5%	37%±4%	70%±4%	30%±4%
H	35	8%±1%	63%±4%	29%±3%	40%±4%	60%±4%

Table 8: Expectations toward the class of the three user groups

4.4.2 Pattern of Usage by Groups

To see how the different user groups used the coaches, we looked at the pattern of usage by groups (H, M, L) from two aspects: total coach use with time and their way of doing homework.

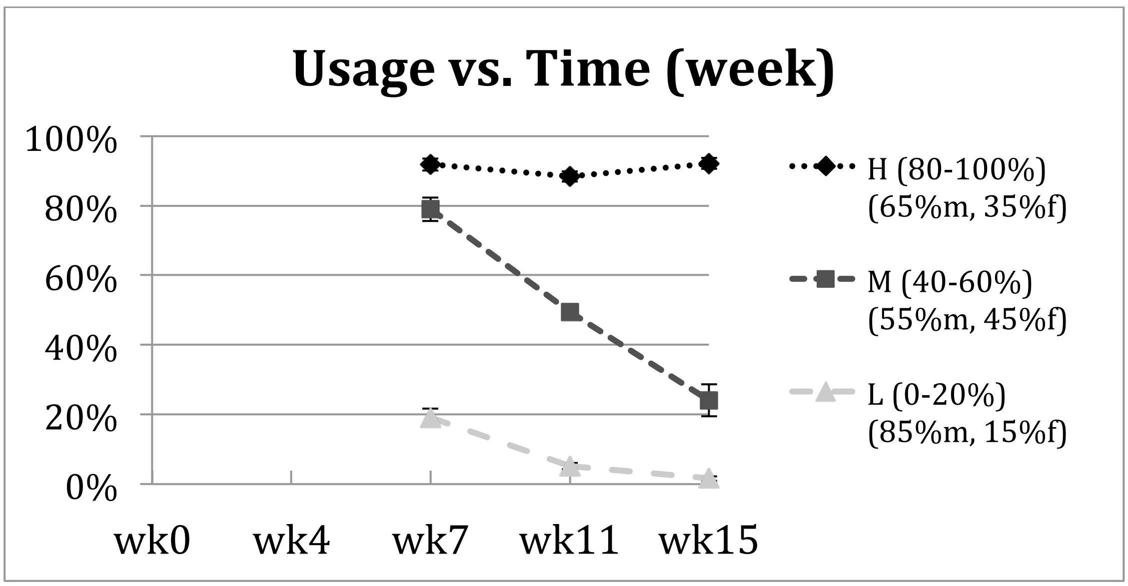


Figure 2: Fraction of coaches used by students in each group associated with each test. The tests were given at the end of week 4, 7, 11, and 15. The lines are drawn to guide the eye.

Figure 2 above shows the fraction of the coaches used by each group preceding each class test. The L group used only 20% of the coaches associated with the topic of the second midterm before the second midterm. Their usage then dropped.[12]

[12] Data on coaches used before the first midterm is not available because of a database error.

In contrast, students in the H group used the coaches consistently throughout the semester. The M group started out using 80% of the coaches but their usage dropped steadily to 20% by the end of the semester.

One possible explanation for the behavior of the M group is that they became more confident problem solvers, and believed that they no longer need the coaches. A second is that the M group decided that the coaches were no longer useful or valuable.

4.4.2.1 Approach to Using Coaches

On the end-of-semester survey in Spring 2013, students were asked to select one of several choices describing how they used the coaches, or to write their own answer. Question: Which of the following statements best describes how you completed the homework in this course?

Which of the following statements best describe how you completed the homework in this course?	L (n=58)	M (n=27)	H (n=43)
I tried to solve the problems on my own and used WebAssign to check the answer. I did not use the computer coaches regularly.	19.0%	0%	0%
I tried to solve the problems on my own and used WebAssign to check the answer. I only used the computer coaches when I ran out of WebAssign submissions.	17.2%	14.8%	4.7%
I tried to solve the problems on my own and used WebAssign to check the answer. I used the computer coaches to see another way to solve the problem.	5.2%	7.4%	2.3%
I tried to solve the problems on my own and used the computer coaches to check my solution method.	3.4%	3.7%	14.0%
I tried to solve the problems on my own and used the computer coaches to help if I got stuck.	48.3%	70.4%	41.9%

I worked through the computer coaches before trying to solve the problems on my own.	0.0%	3.7%	37.2%
I typically did not do the homework.	5.2%	0%	0%
I only used the coaches to study for quizzes.	0%	0%	0%
Others	1.7%	0%	0%

Table 9: Students' responses to survey question in Spring 2013

The most popular choice was "I tried to solve the problems on my own and used the computer coaches for help if I got stuck," selected by 42% of the H group, 70% of the M group and 48% of the L group. The next most popular selection "I worked through the computer coaches before trying to solve the problems on my own," also differed depending on the group. While only 4% of the M group and 3% of the L group selected this option, 37% of students in the H group did so.

Ideally, as students become more competent as well as confident problem solvers, one might expect to see a decrease in the use of the coaches. The fact that the H group not only continued to use almost all the coaches but also that 37% responded that they "…worked through the computer coaches before trying to solve the problems on my own," indicates that some mechanism might be necessary to wean these students from the detailed help provided by the coaches. Discussions of how this information impacts the next iteration of coach design will be in chapter 5.

4.4.3 Perceived Usefulness by Groups

4.4.3.1 Students' Ranking of all Class Components

In Spring 2013, students were asked to rank the components of the physics class in the order from the most useful (10) to the least useful (1). No ties were allowed. The average ranking of each group is below (the higher the number is, the more useful it is ranked). The order of the components listed in the table is the order that was given to students.

Rank the components of the physics class in order from most useful (10) to least useful (1) to your learning. Do not use any ties. (Spring 2013)	H (heavy users)	M (medium users	L (light users)
Lectures	7.23±0.68	7.47±0.93	8.26±0.33
Discussion sections	6.51±0.65	5.85±0.93	7.13±0.41
Labs	5.13±0.49	5.55±0.63	6.10±0.46
Computer coaches	7.02±0.51	7.23±0.52	4.88±0.49
Textbooks	5.87±0.67	4.92±0.92	5.06±0.61
Tutor room	4.34±0.60	3.78±0.83	4.24±0.64
Doing homework	8.14±0.44	7.24±0.60	6.75±0.52
Clicker questions	6.20±0.54	6.66±0.86	6.53±0.45
The competent problem solver (a book)	3.74±0.67	4.68±1.04	3.92±0.58
Feedback from WebAssign	3.58±0.56	3.96±0.79	4.04±0.54

Table 10: Students' responses to the forced ranking survey question in Spring 2013

The computer coaches were ranked by both the H and M groups to be among the top 3 useful components (Table 8), ahead of other course components such as the textbook (5.40±0.76, H&M combined), labs (5.34±0.54, H&M combined), and problem-solving discussion sections (6.18±0.76, H&M combined). All groups ranked the lectures, homework, and discussion sections in the top four components. Even the L group ranked the coaches above the tutor room, where help was available from physics TAs, and the feedback from WebAssign.

4.4.3.2 Problem solving and Conceptual Understanding

In Spring 2013, students were also asked to respond to several statements on a 5-point Likert scale, A-Strongly Disagree, B-Disagree, C-Neither Agree or Disagree, D-

Agree, E-Strongly Agree. The responses of each user group to two statements on the end-semester survey are shown below:

"The computer coaches did not help improve my problem solving in this class".

	Strongly Agree/Agree	Neither	Strongly Disagree/Disagree
H(N=58)	21% ± 2%	12% ± 1%	67% ± 3%
M(N=27)	11% ± 2%	15% ± 2%	74% ± 3%
L(N=43)	34% ± 3%	24% ± 3%	41% ± 4%

Table 11: Students' responses to the end-semester survey statement about the effectiveness of the coaches for improving problem solving.

"The computer coaches helped improve my conceptual knowledge of physics".

	Strongly Agree/Agree	Neither	Strongly Disagree/Disagree
H(N=58)	70% ± 3%	12% ± 1%	19% ± 2%
M(N=27)	63% ± 4%	33% ± 4%	4% ± 1%
L(N=43)	45% ± 4%	28% ± 4%	28% ± 4%

Table 12: Students' responses to the end-semester survey statement about the effectiveness of the coaches in improving conceptual understanding.

Both the heavy and medium user groups perceived that the computer coach helped them improve their problem solving and their conceptual understanding. Interestingly, even though the L group didn't use the computer coaches much, over 40% believed they were useful both in improving problem solving and learning concepts.

4.5 Performance by User Groups in Spring 2013

4.5.1 Final Exams

We compared the performance of the three groups on the final exam, Table 13. There was no significant difference between the H, M and L groups, despite of their differences in Pre FCI, and other background characteristics that would have predicted a worse outcome for those in the H group.

Final Exam	male	female	Total
L (N=48)	68.7% ± 3.3%	79.7% ± 5.7%	70.3% ± 3.0%
M (N=27)	64.1% ± 3.7%	70.2% ± 5.9%	66.1% ± 3.1%
H (N=35)	71.7% ± 2.2%	66.5% ± 5.3%	69.9% ± 2.3%

Table 13: Scores on the final exam problems (%) for the three groups

In order to test the hypothesis that the H group would have performed worse without the coaches, we formed three matched groups (L_match, M_match, and H_match), selected from 3145 students of previous semesters (from Spring 2008 to Fall 2011) where no coaches were used, by pair matching students in each user group to those from the previous classes. We then used those baseline groups to predict how the L, M, and H students in Spring 2013 would have compared on their final exam problems without the coaches.

Students from the three matched groups were pair-matched on the variables that distinguished the groups: Gender, pre FCI, Expected grade, Time expect to study. For each student in the experimental group (Spring 2013), we found 4 matches from the baseline data. 85% of the matched students are a perfect match (all variable is matched perfectly), for the other 15%, we allowed an FCI match within +/- 2 points (out of 30 points), and the Expected study time choice 4 (10-15 hours /week) equivalent to choice 5 (more than 15 hours/week). As a test of unintended selection bias, we varied the selection process by changing the class selected first for matching and the order of matching Fall and Spring classes. We also made sure that each class used for matching has the same percentage of students in the categories that characterized H, M, L users.

The difference on final exam performance between the experimental groups and the baseline groups with the same background characteristics is significant. In Table 14, the the H_match group in the baseline performed 10% lower on the final exam than the L_match group (p=0.92 that they are the same). However, in the experimental class of spring 2013, there is no statistically significant difference between these two groups. This indicates the coaches have helped the less-prepared students improve to the level of their classmates (p <0.01 that the difference is the same as the baseline classes).

Final Exam	Baseline (classes from Spring 2008 to Fall 2011)	Using Coaches (Spring 2013)
L_match	71.9% ± 1.4%	70.3% ± 3.0%
M_match	68.2% ± 1.9%	66.1% ± 3.1%
H_match	61.4% ± 1.6%	69.9% ± 2.3%

Table 14: Final exam scores (%) of the three matched groups in the baseline classes compared to those in the coach class.

4.5.2 Force Concept Inventory

Other than problem solving gains reflected by the final exam score, we also looked at conceptual gains measured by FCI. The absolute FCI gains for the H and M groups were markedly higher than that of the L group. To know what this means, you need to give the actual pre FCI score for each group. There was no significant difference between gains for the male (m) or female (f) students.

Diagnostic Exams	L			M			H		
Numbers in %	total	m	f	total	m	f	total	m	f
FCI gain	12.1	11.8	13.9	19.4	19.0	20.4	20.8	20.3	21.7
Math gain	-2.7	-1.7	-8.3	3.9	5.4	0.8	4.9	6.1	2.5
CLASS gain	-4.5	-4.2	-6.0	-7.3	-4.3	-13.2	-1.9	-3.1	0.5

Table 15: Absolute gain on the diagnostic exams of the three usage groups in the coach class.

4.6 Performance by Groups of Matching Characteristics in Fall 2011

By comparing the differences in final exam performance among the L,M,H user groups and groups with the same background characterizes from classes where no coaches were used, we determined that the coaches helped the less-prepared students (H group: heavy users) improve to the level of their classmates. To determine if they would have helped the non-users (L group) had they used these coaches, we examined the Fall 2011 data where the entire class chose to use the coaches.

So we pair matched each student in the L, M, H group (Spring 2013 where coaches were implemented as having no direct credit for the class) with students in the Fall 2011 (where essentially the entire class used the coaches) based on the variables that characterized L, M, and H users: Gender, pre FCI, Expected grade, Time expected to study (First we find all the perfect matches, and then allow the FCI score to be adjusted up to +/- 3points to exhaust all matches). In this case, the L_match group in the Fall 2011 coach class represented those students who normally not have used the coaches but did use them because of the class credit incentives. The actual coach usage of these hypothesized L/M/H groups is given in table below.

Hypothesized User Groups	L_MATCH (n=39)	M_MATCH (n=20)	H_MATCH (n=21)
# of coaches completed (%)	23±2 (66%)	23±2 (65%)	26±1 (74%)
# of coaches attempted (%)	28±1(80%)	30±1 (85%)	31±1 (89%)

Table 16: The actual coach usage of the hypothesized user groups: L,M,H matched groups in Fall 2011 coach class.

We selected the same L/M/H matches from the other section in Fall 2011 in which coaches were not used (same as the control class used previously for rubric analysis) to compare to the hypothesized L/M/H users in Fall 2011 coach class. We chose this section instead of using historical baseline data for two reasons: (1) this section was taught at the same time which makes it a better control than history data; (2) we have rubric data for this section.

As indicated previously, the final exam problem grade (assigned by a TA) was highly correlated with the rubric score (all five categories added) except problem 3 that was independently determined to be not well graded. We eliminated this problem from the analysis. To correct for any grading bias, we used the rubric scores to provide a correction to the score of every problem on the final exam. To see the relationship between rubric score and exam score for all problems, please refer to Appendix 15.

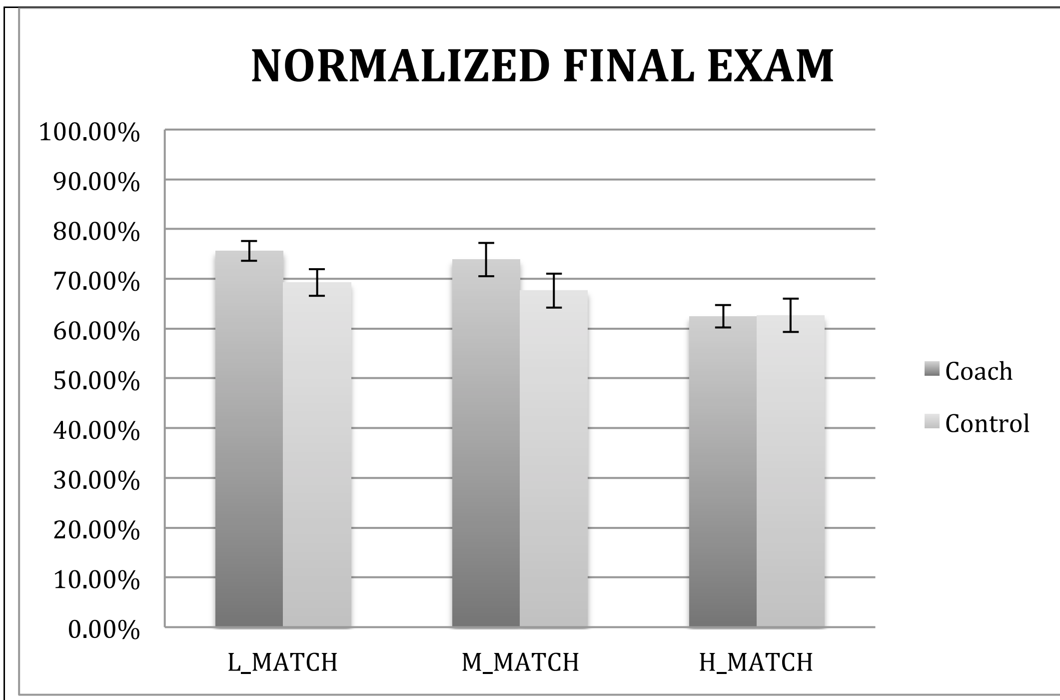


Figure 3: The L, M, H matched groups in Fall 2011 between the coach and control class: the final exam problem solving score is based on rubric to TA grading correlation.

Final Exam	L_MATCH	M_MATCH	H_MATCH
Coach	75.6 % ± 2.0%	73.9% ± 3.4%	62.5% ±2.3%
Control	69.3% ± 2.7%	67.7% ± 3.5%	62.7% ±3.4%
p value (t-test)	$p < 0.05$	$p = 0.06$	$p=0.96$

Table 17: The L, M, H matched groups in Fall 2011 between the coach and control class: final exam & t-test

We can see both the L and M matched groups in the coach class performed better than the control class, with the groups with Light user characteristics most significant ($p <0.05$). The hypothesized L and M groups benefited from using the computer coaches, while maintaining the hierarchy between the groups.

It is interesting that the students with heavy user characteristics (and are indeed heavy users) in Fall 2011 didn't perform significantly better than the control group as they did in Spring 2013. One hypothesis is that in Fall 2011, these students could go get credit for their homework by just completing the coach, while in the Spring 2013 they had to use the results of the coach at least to enter the algebraic answer to the question WebAssign. The algebraic quantity names in the WebAssign were different than in the coaches so the students needed to translate the coach answer. Of course, it could just be a statistical fluctuation because of the small sample size

4.7 Discussion

When faced with a choice of completing homework using coaches or an online homework correcting program (WebAssign), students overwhelmingly chose to use the coaches. When using the coaches received no credit in the course, the less-prepared students, females, and students who expect a lower-letter grade (indication of being less confident) tended to use the coaches more.

Students perceived the coaches as useful in improving their conceptual understanding of physics as well as improving their problem solving skills. Students also agreed that the coaches improved their confidence when working with new problems.

The use of the computer coaches was also shown to improve students' performance on FCI (Force Concept Inventory). When where was no incentive to use the coaches (spring 2013), the students who used the coaches (heavy users) improved their problem solving relative to that predicted given their background. When giving direct homework credit for using the coaches (fall 2011), the students who normally would not have used the coaches (confident students with good course preparation) performed significantly better than the same matching group selected from another section of the course taught during the same semester that did not have coaches available. The rubric category (Useful Description) is shown to have a strong statistically significant improvement ($p<0.0001$) over the control class (fall 2011).

The implications drawn from this study, including using these data to direct future coach design and difficulties in conducting authentic assessment of problem solving, will be discussed in the next chapter.

CHAPTER 5: Conclusions and Implications

5.1 Introduction

To develop instructional tools to help students improve their problem solving requires the assessment of problem solving in a classroom environment and the design of practical experimental situations that allow that assessment to occur. This section summarizes the most significant results from the studies presented in Chapter 4 and how those results address the research questions stated in Chapter 1. In addition, this chapter discusses the implications draw from the study and future research directions. We will also summarize how we have overcome some of the challenges in conducting an authentic assessment of problem solving and point out research directions to further addressing the remaining issues. Finally, we will discuss how the results of this study impact the design of future problem solving coaches.

5.2 Conclusions

In chapter 1, we raised the research question: will students use computer coaches and if so, what are the characteristics of the students who will and won't? We found that when these coaches were put on the same footing as a popular computerized homework system, essentially everybody in the class used them. In Fall 2011, out of the 219 students, the average number of coaches attempted was 28 and completed was 21.9 out of a maximum of 35. Only 20 students completed fewer than 10 coaches.

When the computer coaches are made available to students but the students have no direct incentive to use them, 71% of the students used them for the first half of the course (total coach usage by week 7—Dynamics). By the end of the course, this number had decreased to 20%. The variation of natural coach usage when there was no direct incentive, allowed us to divide the students into groups based on the frequency with which they used the coaches. We found that students who we characterized as less confident, poorer conceptual preparation, students who expected to work more for a lower grade, and tended to use the coaches more.

We could classify students into three groups based on their usage of the coaches. The low use group (L) used only 20% of the coaches associated with the second midterm, approximately half way through the semester. Their usage then dropped. In contrast, students in the high use group (H) used the coaches consistently throughout. A middle use group (M) used 80% at the half way point in the semester but their usage dropped steadily throughout the semester, ending at 20%.

When using the coaches, only 4% of the M group and 3% of the L group worked through the computer coaches before trying to solve the problems on their own while 37% of students in the H group did so. This potentially unhealthy usage pattern has implications for future coach design and will be discussed later in this chapter.

The second research question is if students perceive these coaches as useful. All students perceived the coaches as useful in improving their conceptual understanding of physics as well as improving their problem solving skills. They also agreed that the coaches improve their confidence when working with new problems.

The computer coaches were ranked by both the H and M groups to be among the top 3 useful components of the course (Table 8), ahead of other standard course components such as the textbook, labs, and a tutorial room staffed by teaching assistants. Even the L group ranked the coaches above the tutor room and the feedback from the web based homework system, WebAssign.

The final research question is to what extent does the coaches improve students' problem solving skills. A strong correlation (0.8~0.9) was found between the evaluation of student problem solving by researchers using a rubric to assess expert-like problem solving and final exam scores assigned by TAs. This implies that the TA scoring of student problem solutions can be used to evaluation purposes at least when the TAs have the training and support available at the University of Minnesota. A brief summary of that training and support is given in Appendix 10.

Using the coaches improved the problem solving of both underprepared, low confidence students and well-prepared high confidence students. The low confidence students are naturally heavy users of the coaches. These students improve their problem solving significantly above where it would be in the course without the coaches to equal the performance of well-prepared, confident students who did not use the coaches. This increase in their final exam performance was equivalent to 2/3 of a letter grade. This later group does not naturally use the coaches but when offered a slight incentive will do so. When they then become heavy users of the coaches, their problem solving performance improves above a sample of similar students who do not use the coaches.

5.3 Implications

5.3.1 Rubric in Action

By grouping the quantized scale of rubric bins (0-5), we successfully used the rubric to distinguish the performance between the coach and control class in Fall 2011 on Useful Description Category. The rubric did not detect any statistically significant differences in the other categories, although there is certainly a trend. It is possible that the quantized scale of the rubric (0-5), may amplify skill regression over the short term[13] or may not be sensitive enough to find the small amounts of progress expected on short time scales.

We, and others (Keith and Anderson, 1992) also found when used to track student problem solving progress over the period of one semester, the score reflect the different level of difficulty of different physics topics (Appendix 13). Moreover, because the rubric was developed to distinguish between expert and novice problem solving, its scoring procedures may not be appropriate to track student problem solving progress over timescale as short as one semester where progress is not monotonic (Siegler, 2004). However, the rubric may have enough sensitivity if used over a timescale long enough for student behavior to average over non-monotonic behavior. For example, it is likely that there are qualitatively different stages of problem solving development such as that which has been called competent problem solving (Dreyfus and Dreyfus, 2005). In such hypothetical intermediate stages, students might exhibit

13 R. S. Siegler, J. Cogn. and Develop. 5, 1-10 (2004).

behavior that, while different from novices, is not necessarily more closely aligned with experts.

For future research directions, one should make comparisons between the problem solving behavior of students who used the coaches and those that did not over a long enough timescale to be insensitive to such stages. Reviewing previous work, that scale is likely to be longer than a semester (Siegler, 2004; Heller, Keith and Anderson, 1992). Extending a study to a second semester would require supporting curricular material for a second subdiscipline. For example, in introductory physics, this would require building computer coaches to address electricity and magnetism as well as the first semester mechanics.

5.3.2. Experimental Environment

As mentioned in chapter 3, one needs to use classes as experimental settings that neither mask nor block the desired effect: problem solving in this case. We believe these two challenges have been met in our experiment. However, when trying to measure the effect of a specific treatment (in our case, the use of computer coaches), any signal might be hidden by the simultaneous use of other problem solving pedagogies. For example, such masking was demonstrated in the case of conceptual learning where the combined use of individually effective pedagogies did not show a cumulative gain (Cummings, Thornton and Kuhl, 1999). A test of the effectiveness of the problem solving coaches should be repeated at different institutions that meet two requirements: the course emphasizes problem solving so that the students are motivated to learn it and does not employ another effective pedagogy for teaching problem solving. This means the institution must give students class tests that are not multiple choice and use problems on those tests that evoke problem solving behavior

In summary, in testing the effectiveness of problem solving computer coaches the standard a statistical and procedural difficulty in achieving appropriate measurement discrimination in an environment with a high noise to signal ratio. Improvements in the assessment techniques should be explored such as refining the application of the rubric to increase its sensitivity or finding a robust normalization method when combining multiple classes to increase statistical power. One method of averaging over the inherent noise generated by testing on different topics is to make the measurement over a longer time scale. Another method of increasing the signal to noise ratio might be to make the measurement in another environment that does not use a pedagogy that has already been shown to improve problem solving.

5.3.3 Coach Design

As mentioned in chapter 2, designing a software framework for computer coaches is an iterative process. Several cycles of implementation, assessment, and development will likely be necessary to achieve a useful and effective software framework. This design has to satisfy multiple stakeholders: institutions, instructors and students.

5.3.3.1 Instructors

To produce of the set of prototype coaches used for this dissertation, the decision structure of each coach and its software framework were basically procedural in nature. Providing differing amounts of flexibility was achieved by designing three distinct types of coaches but only one type of coach was available for each problem. The most popular and numerous type of coach (called type 1) had limited flexibility in that it employed branching to follow reasonable student choices in the problem solution but was rigid in the order of decisions that followed those branches. Instructors wishing to modify the coaching pattern or build coaches for different problems needed knowledge of the underlying software language with more significant changes in procedure requiring more sophisticated software knowledge. For example, to make new coaches or make even minor modifications of the existing coaches, one needs to modify the graphics using Flash and the text using XML. It is possible for people with no programming knowledge to make the changes but it takes a significant amount of time. Using undergraduates with no experience in Flash or XML, it took 60 hours to build an initial draft of a new coached problem. For an experienced programmer, it still took about 30 hours/problem. These drafts then need several rounds of revising and editing by instructors before they are ready to be released to students. For instructors to be willing to implement these coaches, the coaches must be able to be more easily modified. This requirement was identified in a workshop on the coaches given to physics instructors.

One of the major improvements for the next round of coaches, now under construction, is ease of modification for instructors. We have developed a new software framework that allows instructors to build a new coach with no knowledge of the underlying software using only a graphical user interface (GUI).

5.3.3.2 Students

One of the goals of this study was to assess students' perceptions of the utility of the computer coaches. In improving the usability of the coaches, one could choose to focus on improving the user experience and effectiveness for students who tend to use the coaches, or on trying to make the coaches more attractive to a larger fraction of students. Because most of the students in the physics course chose to use a substantial fraction of the coaches when the rewards are equal to others in the course, the next iteration will focus on the former, which may, as a byproduct, lead to a larger user base.

The keystroke data shows that the average time to complete a single problem using a coach was less than 31 minutes, comparable to the time spent by students interacting with a human coach in office hours. However, many students thought that the computer coaches took too long. On the mid-semester survey, 49% of the respondents answered "Agree" or "Strongly agree" to the statement "Using the computer coaches for homework made the homework take too long." Furthermore, 37% of the answers to the free- response question "What do you like least about the computer coaches?" mentioned that the computer coaches were either too long or too repetitive. In designing the next iteration of coaches, a major improvement in the software framework will be to allow more flexibility in the student pathway through the coaches. This will allow students, or their instructors, to choose a different grain size for the decisions that students choose to solve the problem, thus addressing both the time and repetition issues.

With this flexibility, the coaches might also engage students with different learning priorities at different times in the course. As students become more competent as well as confident problem solvers, one might wish to make larger jumps in the problem-solving framework. In the next iteration, students would be able to use the same coach differently at the beginning and at the end of the course. This might prove to be more attractive and useful to the M group (moderate users) whose usage decreased over the course of the semester. Meanwhile, students who perceive themselves as more competent in problem solving but could still benefit from using the coaches (the L group (light users) might take advantage of this flexibility .

Ideally, as students become more competent as well as confident problem solvers, one might expect to see a decrease in the use of the coaches. The fact that the H group (heavy users) not only continued to use almost all the coaches but also that a large fraction responded that they "...worked through the computer coaches before trying to solve the problems on my own," indicates that some mechanism is necessary to wean these students from the detailed help provided by the coaches. For these students, the instructor could enforce using a progressively course decision grain size while allowing the student to return to a detailed grain size if really necessary. The next iteration of the coaches should be designed to better address the needs of the heavy and medium user population by having adjustable (by instructors or students) decision grain size. This will allow students to jump to sections of the problem solving framework that address their issues without repeatedly going through coaching they do not need. For the H group, it will encourage bypassing detailed coaching while still allowing them to have step-by-step coaching from the beginning to the end of a problem if desired.

Overall, when designing the next generation of these coaches, having a new GUI and to make the coaches more easily editable by the instructors is a priority. Meanwhile, building more flexibility within the coach could help reduce the perception of the coaches as rigid or repetitive and might engage students with different learning priorities at different times in the course.

A computer coach is limited in its ability to help students identify their difficulties and remediate them and for this reason is not intended to replace a good human coach. However, when developed, computer coaches could provide a helpful approximation of the office hour experience that is available on demand and with whatever repetition is desired by the student. We expect that most students will still need human intervention provided by the instructor and other students to make significant learning gains. Nevertheless, we hope that computer coaches interacting with students on the Internet can be a flexible tool to support the learning of a diverse set of students in the introductory physics course.

REFERENCES

Adams, W.K., & Wieman, C.E. (2006). Problem solving skill evaluation instrument – Validation studies. In L. McCullough, L. Hsu, & P. Heron (Eds.), *AIP Conference Proceedings Vol. 883: 2006 Physics Education Research Conference* (pp. 18-21). Melville, NY: American Institute of Physics.

Anderson, J. R., Corbett, A. T., Koedinger, K. R., and Pelletier, R. (1995). Cognitive tutors: lessons learned, *J. Learn Sci*.4, 167-207.

Blue, J. M. (1997). *Sex differences in physics learning and evaluations in an* 232 *introductory course.* Unpublished doctoral dissertation, University of Minnesota, Twin Cities.

Brown, J. S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32-41.

Carney, A. (2006, October). What do we want our students to learn? *Transform*, *1*, 1-6.

Chi, M.T.H., Feltovich, P. & Glaser, R. (1981). Categorization and representation of physics problems by experts and novices. *Cognitive Science*, 5, 121-152.

Chi, M.T.H., Glaser, R., & Rees, E. (1982). Expertise in problem solving. In R.J. Sternberg (Ed.), *Advances in the psychology of human intelligence vol. 1* (pp. 7-75). Hillsdale, NJ: Lawrence Erlbaum Associates.

Collins, A., Brown, J. S. & Holum, A. (1991). Cognitive Apprenticeship: Making Thinking Visible. *American Educator*, p.6-11, 38-46.

Collins, A., Brown, J.S., and Newman, S.E. (1989). “Cognitive Apprenticeship: Teaching the Crafts of Reading, Writing and Mathematics,” in *Knowing,* *Learning and Instruction*, edited by L. Resnick, Hillsdale, NJ: Erlbaum, pp. 453 494.

Cummings, K., Marx, J., Thornton, R., & Kuhl, D. (1999). *Am.J.Phys*. 67, S38-S44

Docktor, J.L. (2009). Development and validation of a physics problem-solving assessment rubric. *Unpublished doctoral dissertation*, University of Minnesota, Twin Cities.

Docktor, J.L., & Heller, K. (2009). Assessment of Student Problem Solving Processes. *Physics Education Research Conference,* Melville, NY

Dreyfus, H.L. & Dreyfus, S.E. (1986). *Mind over machine*. New York: Free Press.

Dreyfus, H.L. & Dreyfus, S.E., (2005). *Org.Studies* 26(5), 779-792

Duncker. (1935). *Zur psychologie des produktiven denkens*, [Psychology of Productive Thinking]. Springer. OCLC 667728.

Duncker, Karl (1945). *On Problem Solving*. Psychological Monographs **58**. American Psychological Association. OCLC 968793.

Elio, R. & Scharf, P.B. (1990). Modeling novice-to-expert shifts in problem-solving strategy and knowledge organization. *Cognitive Science*, *14*(4), 579-639.

Ernst, G. & Newell, A. (1969). *GPS: A Case Study in Generality and Problem Solving*. New York: Academic Press.

Ertas, A. & Jones, J. (1996). The Engineering Design Process. 2nd ed. New York, N.Y., John Wiley & Sons, Inc.

Ferguson-Hessler, M.G.M., and Jong, T. (1990). Studying Physics Texts: Differences in Study Progresses Between Good and Poor Performances. *Cognition and Instruction***,** 7(1) 41-54.

Foster, T. (2000). *The development of students' problem-solving skills from instruction emphasizing qualitative problem-solving.* Unpublished doctoral dissertation, University of Minnesota, Twin Cities.

Freedman, Reva (2000). What is an Intelligent Tutoring System? *Intelligence* **11** (3): 15–16.

Gick, M.L. (1986). Problem-solving strategies. *Educational Psychologist,* 21(1 & 2), 99-120.

Hardiman, P.T., Dufresne, R., & Mestre, J.P. (1989). The relationship between problem categorization and problem solving among experts and novices. *Memory and Cognition,* 17(5), 627–638.

Harper, K.A. (2001). *Investigating the development of problem solving skills during a freshman physics sequence.* Unpublished doctoral dissertation, The Ohio State University.

Heller, P., Foster, T., & Heller, K. (1997). Cooperative group problem solving laboratories for introductory classes. In Redish, E.F. and Rigden, J.S. (Eds.), *AIP Conference Proceedings Vol. 399: The Changing Role of Physics Departments in Modern Universities, Proceeding of International Conference of Undergraduate Physics Education, Part Two: Sample Classes*, (pp. 913-935). Melville, NY: American Institute of Physics.

Heller, K., & Heller, P (1995), "*The Competent Problem Solver, A Strategy for Solving Problems in Physics*", calculus version, 2nd ed., Minneapolis, MN: McGraw Hill.

Heller, K., & Heller, P. (2000). *Competent Problem Solver – Calculus Version*. New York: McGraw-Hill.

Heller, P. & Hollabaugh, M. (1992). Teaching problem solving through cooperative grouping. Part 2: Designing problems and structuring groups. *Am. J. Phys.* 60(7), 637-644.

Heller, P., Keith, R., and Anderson, S., (1992). "Teaching problem solving through cooperative grouping," *Am. J. Phys*. 60, 627-636.

Henderson, C., Yerushalmi, E., Kuo, V., Heller, P., Heller, K. (2004). Grading student problem solutions: The challenge of sending a consistent message. American *Journal of Physics, 72(2),* 164-169.

Hsu, L., Brewe, E., Foster, T.M., & Harper, K.A. (2004). Resource letter RPS-1: Research in problem solving. *American Journal of Physics*, *72*(9), 1147-1156.

Hsu, L., & Heller, K. (2004). *Proceedings of Physics Education Research Conference,* Sacramento, 790, Melville, NY.

Hsu, L., & Heller, K. (2009). Computer problem-solving coaches. *Proceedings of the National Association for Research in Science Teaching 82nd International Conference*, Garden Grove, CA.

Huffman, D. (1997). Effect of explicit problem solving instruction on high school students' problem-solving performance and conceptual understanding of physics.

Journal of Research in Science Teaching, 34(6), 551-570.

Johnson, M. (2001). Facilitating high quality student practice in introductory physics, *Am. J. Phys*. 69(7): S2-S11.

Jonassen, D.H. (2007). *Learning to solve complex scientific problems*. New York: Lawrence Erlbaum Associates Taylor Francis Group, LLC.

Kal, V. (1988). *Philosophia antiqua: On intuition and discursive reasoning in Aristotle.* Leiden, The Netherlands: E.J. Brill.

Larkin, J. (1979). Processing information for effective problem solving. *Engineering Education*, 70(3), 285-288.

Larkin, J., McDermott, J., Simon, D., & Simon, H. A. (1980). Expert and novice performance in solving physics problems. *Science,* 208, 1335-1342.

Larkin, J.H., McDermott, J., Simon, D.P., & Simon, H.A. (1980a). Expert and novice performance in solving physics problems. *Science*, 208(4450), 1335-1342.

Larkin, J., & Reif, F. (1979). Understanding and teaching problem solving in physics. *European Journal of Science Education*, 1(2), 191-203.

Lear, J. (1980). *Aristotle and logical theory*. London: Cambridge University Press.

Lee, Y.J., Palazzo, D. J., Warnakulasooriya, R., & Pritchard, D. E. (2008). Measuring student learning with item response theory. *Physical Review Special Topics-Physics Education Research, 4,* 010102.

Leonard, W.J., Dufresne, R.J., and Mestre, J.P. (1996). Using qualitative problem-solving strategies to highlight the role of conceptual knowledge in solving problems, *Am. J. Phys*. 64, 1495-1503.

LON-CAPA(1992). *CampusSource*. Retrieved 2007-11-30

Maloney, D. P. (1994). Research on problem solving: Physics. In *D. L. Gabel (Ed.),* *Handbook of research on science teaching and learning* (pp. 327-356). New York: Macmillan.

Mastering Physics. http://www.masteringphysics.com/

Martinez, M. E. (1998). What is problem solving? *Phi Delta Kappan, 79,* 605-609.

Mayer, R.E. (1992). *Thinking, problem solving, cognition* (2nd ed.). New York: W.H. Freeman and Company.

Montgomery, R. A., Peguy, L., & Cannella, M. (2005). The abominable snowman (Choose your own adventure, No. 1) Chooseco: Waitsville, VT.

Murthy, S. (2007). Peer-assessment of homework using rubrics. In L. Hsu, C. Henderson, & L. McCullough (Eds.), *AIP Conference Proceedings: Vol. 951. 2007 Physics Education Research Conference* (pp. 156-159). Melville, NY: American Institute of Physics.

Newell, A., & Simon, H.A. (1972). *Human problem solving*. Englewood Cliffs, NJ: Prentice-Hall, Inc.

Novick & Bassok, (2005). Thinking and reasoning: A reader's guide. In Holyoak, K. J., & Morrison, R. G. (Eds.), *The Cambridge Handbook of Thinking and Reasoning* (pp.1-9). Cambridge, UK: Cambridge University Press.

Ogilvie, C.A. (2007). Moving students from simple to complex problem solving. In D.H. Jonassen (Ed.), *Learning to solve complex scientific problems* (pp. 159-185). New York: Lawrence Erlbaum Associates Taylor Francis Group, LLC.

Ormrod, J.E. (2004). *Human learning* (4th ed.). Upper Saddle River, NJ: Pearson Education, Inc.

Palincsar, A.L., and Brown, A. L. (1984). *Cogn. Instruct*. 1,117-175.

Polya, G. (1945). *How to Solve It*, Garden City, NY: Doubleday.

Reif, F. (1981). Teaching problem solving – A scientific approach. *The Physics Teacher*, *19*(5), 310-316.

Reif, F. (1995). Understanding and Teaching Important Scientific Thought Processes, *Am. J. Phys*. 63, 17-32

Reif, F., & Heller, J.I. (1982). Knowledge structure and problem solving in physics. *Educational Psychologist*, 17(2), 102-127.

Reif, F. and Scott, L. A. (1999). Teaching scientific thinking skills: Students and computers coaching each other, *Am. J. Phys.* 67, 819-831

Rumsey, D. J. (2011) Statistics For Dummies, 2nd Edition

SCANS (1991). *Secretary of Labor and members of the secretary's Commission on Achieving Necessary Skills*, U.S. Department of Labor, p32.

Schulze, K.G., Shelby, R.N., Treacy, D.J., Wintersgill, M.C., VanLehn, K., Gertner, A. (2000). Andes: An intelligent tutor for classical physics. *The Journal of Electronic Publishing*, University of Michigan Press, Ann Arbor, MI, 6:1, http://www.press.umich.edu/jep/06-01/schulze.html

Scott, L. A. (2001). *Design and assessment of an interactive physics tutoring environment.* Unpublished doctoral dissertation, University of Pittsburgh.

Siegler, R.S., (2004). *J. Cogn. And Develop.* 5, 1-10.

Tycho. http://research.physics.illinois.edu/per/Tycho.html

van Someren, M.W., Barnard, Y.F., & Sandberg, J.A.C. (1994). *The think aloud method: A practical guide to modeling cognitive processes.* San Diego, CA:

Academic Press Inc.

VanLehn, K., Lynch, C., Taylor, L., Weinstein, A., Shelby, R., Schulze, K., Treacy, D., and Wintersgill, M. (2002), in *Intelligent Tutoring Systems: 6th International Conference,* edited by S. A. Cerri, G. Gouarderes, and F. Paraguacu, Berlin: Springer, pp. 367-376.

VanLehn, K., Lynch, C., Schulze, K., Shapiro, J.A., Shelby, R., Taylor, L., Treacy, D., Weinstein, A., and Wintersgill, M. (2005). The ANDES physics tutoring system: Lessons learned. *International Journal of Artificial Intelligence and Education*, 15(3), 147-204.

Warnakulasooriya, R., Palazzo, D. J., & Pritchard, D. E. (2007). Time to completion of web- based physics problems with tutoring. *Journal of the Experimental Analysis of Behavior, 88,* 103-113.

Guernsey, L.(1999). Textbooks and Tests That Talk Back. *The Chronicle of Higher Education*. Retrieved 2007-06-28.

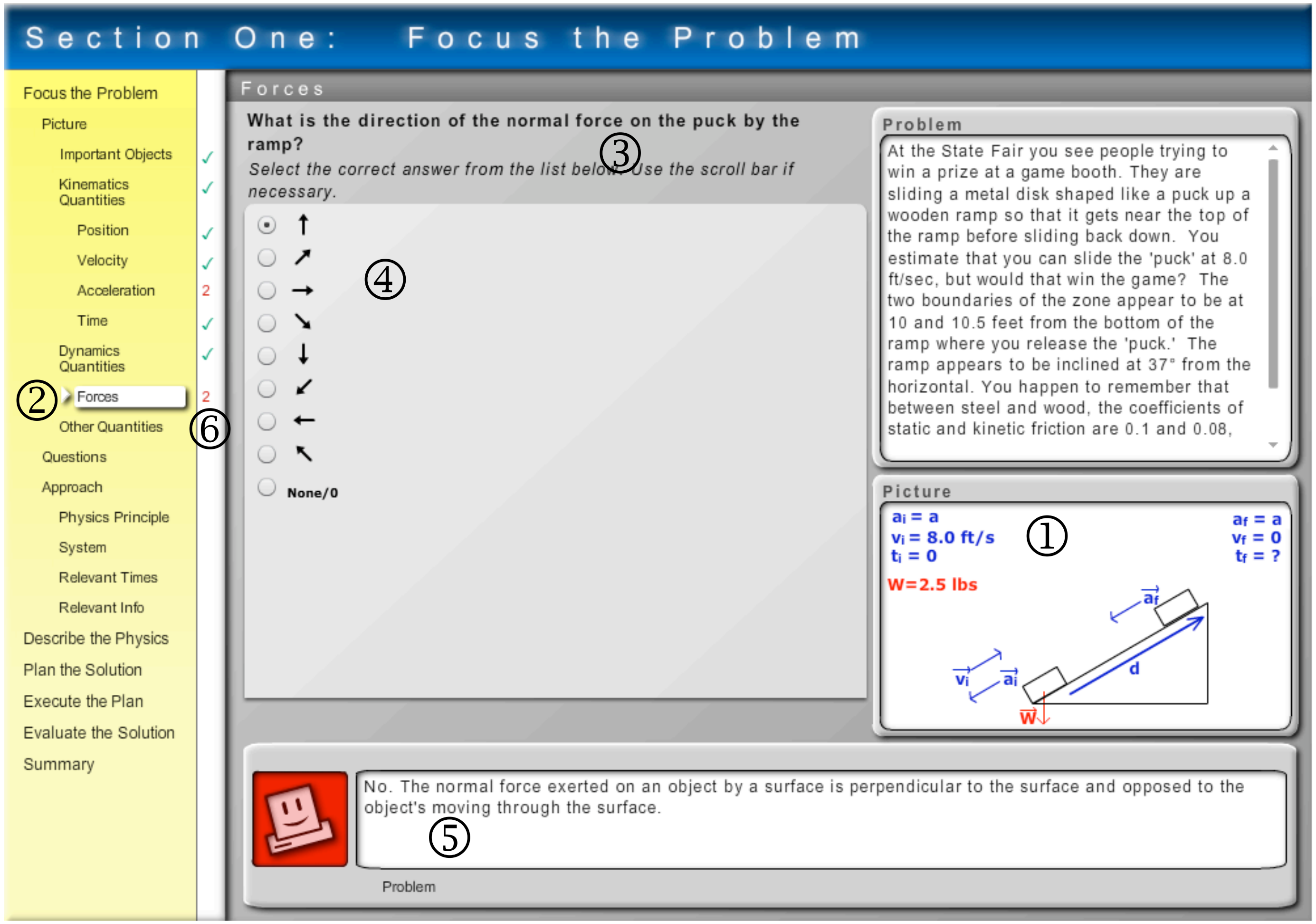
Woods, D.R. (2000). An evidence-based strategy for problem solving. *Journal of Engineering Education, 89(4),* 443-459.

Yerushalmi, E., Henderson, C., Heller, K., Heller, P., & Kuo, V. (2007). Physics faculty beliefs and values about the teaching and learning of problem solving part I: Mapping the common core, *Physical Review Special Topics: Physics Education Research, 3,* 020109.

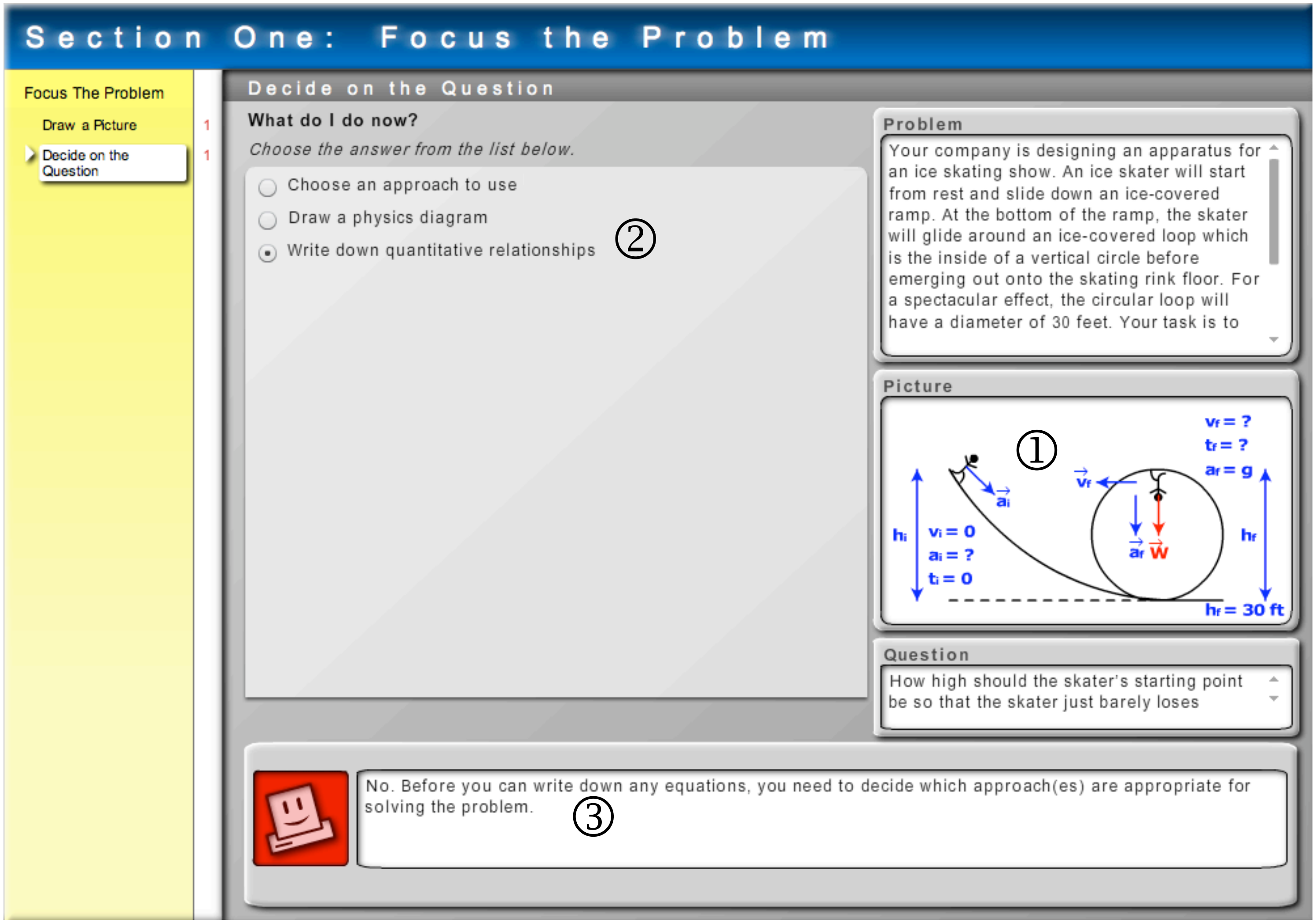
Zhu, X. & Simon, H. A. (1987). Learning mathematics from examples and by doing. *Cognition and Instruction, 4,* 137-166.

APPENDICES

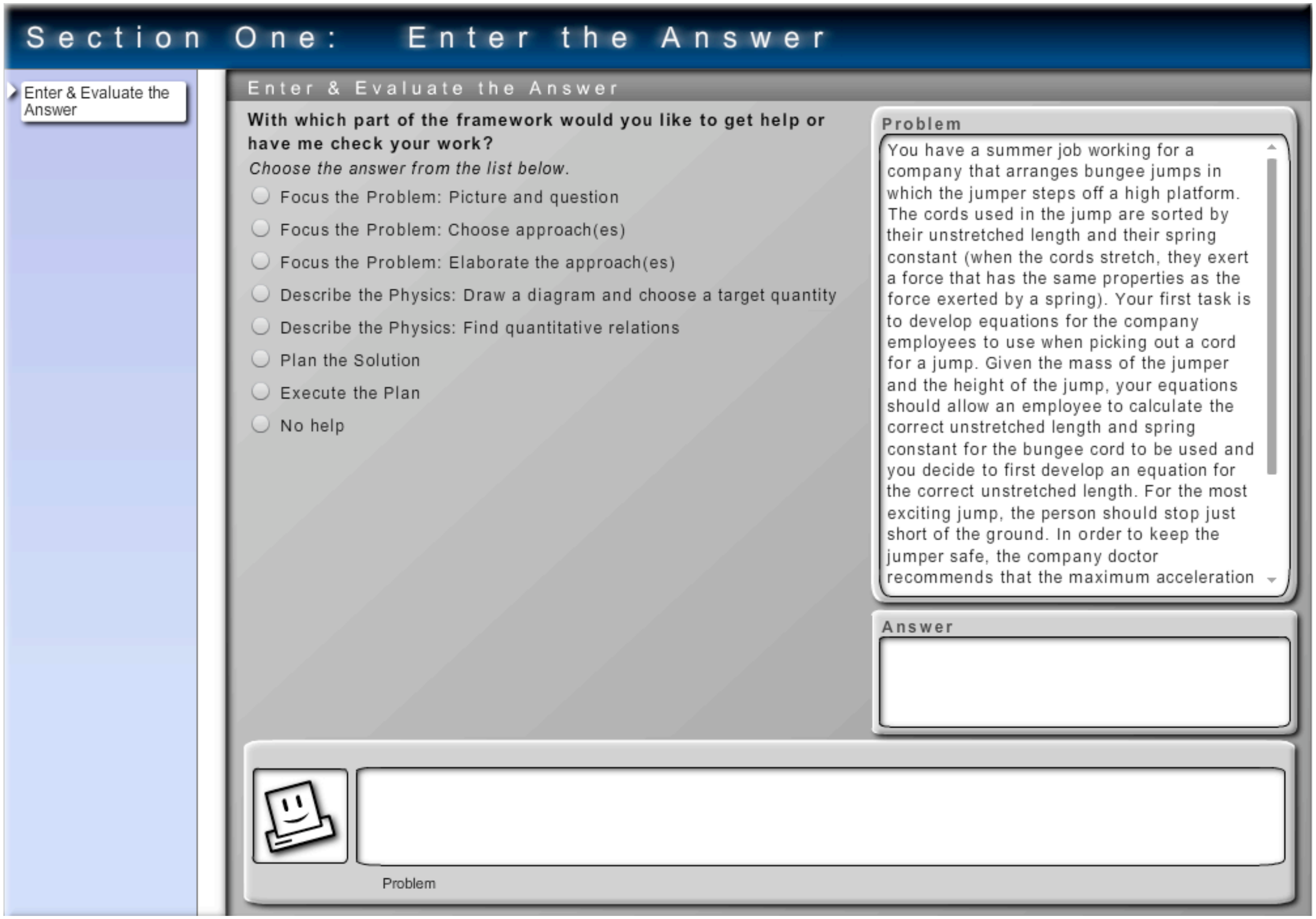
Appendix 1. Screen Shots of Computer Coaches (3 types)



Screenshot from a type 1 coach (computer coaches student). The display shows a partially completed picture①. The computer has decided on a step② and asks the student to specify the direction of a force③. The student's decision on this step④ is incorrect, and the computer provides feedback⑤. A red number to the right of each step ⑥ indicates the number of incorrect responses the student made during the implementation of that step, while a checkmark indicates that the step was implemented correctly the first time.



Screenshot from a type 2 module (student coaches computer). The display shows a completed picture①. The student, acting as a coach, has decided on a step for the computer, in its role as a student, to do②, but it is not an appropriate step at this point. The computer, in its oversight role, gives the student feedback③.



Screenshot from a type 3 module (student works independently, computer gives feedback). If the student gets stuck solving a problem or enters an incorrect answer, the computer asks the student to decide where in the problem solving process the difficulty might occur.

Appendix 2. Problem Solving Rubric

Problem-Solving Assessment Rubric (Version 4) (Docktor 2009, pg92)

	5	4	3	2	1	0	NA(problem)	NA(Solver)
USEFUL DESCRIPTION	**The description is useful, appropriate, and complete.**	**The description is useful but contains minor omissions or errors.**	**Parts of the description are not useful, missing, and/or contain errors.**	**Most of the description is not useful, missing, and/or contains errors.**	**The entire description is not useful and/or contains errors.**	**The solution does not include a description and it is necessary for this problem /solver.**	**A description is not necessary for this problem.**	**A description is not necessary for this solver.**
PHYSICS APPROACH	**The physics approach is appropriate and complete.**	**The physics approach contains minor omissions or errors.**	**Some concepts and principles of the physics approach are missing and/or inappropriate.**	**Most of the physics approach is missing and/or inappropriate.**	**All of the chosen concepts and principles are inappropriate.**	**The solution does not indicate an approach, and it is necessary for this problem/ solver.**	**(i.e., it is given in the problem statement)**	**An explicit physics approach is not necessary for this solver.**
SPECIFIC APPLICATION OF PHYSICS	**The specific application of physics is appropriate and complete.**	**The specific application of physics contains minor omissions or errors.**	**Parts of the specific application of physics are missing and/or contain errors.**	**Most of the specific application of physics is missing and/or contains errors.**	**The entire specific application is inappropriate and/or contains errors.**	**The solution does not indicate an application of physics and it is necessary.**	**An explicit physics approach is not necessary for this problem. (i.e., it is given in the problem)**	**Specific application of physics is not necessary for this solver.**
MATHE-MATICAL PROCEDURES	**The mathematical procedures are appropriate and complete.**	**Appropriate mathematical procedures are used with minor omissions or errors.**	**Parts of the mathematical procedures are missing and/or contain errors.**	**Most of the mathematical procedures are missing and/or contain errors.**	**All mathematical procedures are inappropriate and/or contain errors.**	**There is no evidence of mathematical procedures, and they are necessary.**	**Specific application of physics is not necessary for this problem.**	**Mathematical procedures are not necessary for this solver.**
LOGICAL PROGRESSION	**The entire problem solution is clear, focused, and logically connected.**	**The solution is clear and focused with minor inconsistencies**	**Parts of the solution are unclear, unfocused, and/or inconsistent.**	**Most of the solution parts are unclear, unfocused, and/or inconsistent.**	**The entire solution is unclear, unfocused, and/or inconsistent.**	**There is no evidence of logical progression, and it is necessary.**	**Mathematical procedures are not necessary for this problem or are very simple.**	**Logical progression is not necessary for this solver.**

Category Descriptions (Docktor 2009, pg93):

Useful Description assesses a solver's skill at organizing information from the problem statement into an appropriate and useful representation that summarizes essential information symbolically and visually. The description is considered "useful" if it guides further steps in the solution process. A *problem description* could include restating known and unknown information, assigning appropriate symbols for quantities, stating a goal or target quantity, a visualization (sketch or picture), stating qualitative expectations, an abstracted physics diagram (force, energy, motion, momentum, ray, etc.), drawing a graph, stating a coordinate system, and choosing a system.

Physics Approach assesses a solver's skill at selecting appropriate physics concepts and principle(s) to use in solving the problem. Here the term *concept* is defined to be a general physics idea, such as the basic concept of "vector" or specific concepts of "momentum" and "average velocity". The term *principle* is defined to be a fundamental physics rule or law used to describe objects and their interactions, such as the law of conservation of energy, Newton's second law, or Ohm's law.

Specific Application of Physics assesses a solver's skill at applying the physics concepts and principles from their selected approach to the specific conditions in the problem. If necessary, the solver has set up specific equations for the problem that are consistent with the chosen approach. A *specific application of physics* could include a statement of definitions, relationships between the defined quantities, initial conditions, and assumptions or constraints in the problem (i.e., friction negligible, massless spring, massless pulley, inextensible string, etc.)

Mathematical Procedures assesses a solver's skill at following appropriate and correct mathematical rules and procedures during the solution execution. The term *mathematical procedures* refers to techniques that are employed to solve for target quantities from specific equations of physics, such as isolate and reduce strategies from algebra, substitution, use of the quadratic formula, or matrix operations. The term *mathematical rules* refers to conventions from mathematics, such as appropriate use of parentheses, square roots, and trigonometric identities. If the course instructor or researcher using the rubric expects a symbolic answer prior to numerical calculations, this could be considered an appropriate mathematical procedure.

Logical Progression assesses the solver's skills at communicating reasoning, staying focused toward a goal, and evaluating the solution for consistency (implicitly or explicitly). It checks whether the entire problem solution is clear, focused, and organized logically. The term logical means that the solution is coherent (the solution order and solver's reasoning can be understood from what is written), internally consistent (parts do not contradict), and externally consistent (agrees with physics expectations).

Appendix 3. Consent Form for Volunteers

CONSENT FORM
Using Computers as Personal Problem Solving Coaches

You are invited to be in a research study of physics problem solving. You were selected as a possible participant because you are enrolled in an introductory physics course at the University of Minnesota and you volunteered. We ask that you read this form and ask any questions you may have before agreeing to be in the study. This study is being conducted by: Leon Hsu, Department of Postsecondary Teaching and Learning; Ken Heller, Department of Physics; Andrew Mason, Department of Physics & Department of Postsecondary Teaching and Learning; and Qing Xu, Department of Physics.

Background Information
The purpose of this study is to investigate the problem solving processes used by students in introductory physics courses. This information will be used to develop problem solving instruction and assessment materials.

Procedures:
If you agree to be in this study, we would ask you to do the following things:
1. Attempt to solve physics problems printed on a worksheet. Try to talk out loud while you are solving the problem. During this time your actions will be videotaped and your voice will be recorded.
2. The investigator may ask you questions during the interview process. The whole interview lasts about one hour.

Risks and Benefits of being in the Study
The study has no appreciable risks. We hope that you will acquire additional practice solving physics problems similar to those in your physics course.

Compensation:
If you complete the procedures listed above, you will receive payment of $25 upon completion of the problem-solving interview.

Confidentiality:
The records of this study will be kept private. In any sort of report we might publish or presentation we might make, we will not include any information that will make it possible to identify a subject. Research records will be stored securely and only researchers will have access to the records. Video and audio tapes will only be accessible to the researchers and will be destroyed three years after the completion of the study.

Voluntary Nature of the Study:
Participation in this study is voluntary. Your decision whether or not to participate will not affect your current or future relations with the University of Minnesota or the Department of Physics. If you decide to participate, you are free to not answer any

question or withdraw at any time without affecting those relationships.

Contacts and Questions:
The researchers conducting this study are: Leon Hsu, Ken Heller, Andrew Mason and Qing Xu. You may ask any questions you have now. If you have questions later, **you are encouraged** to contact Leon Hsu at 250B Burton Hall, 612-625-3472, lhsu@umn.edu.

If you have any questions or concerns regarding this study and would like to talk to someone other than the researcher(s), **you are encouraged** to contact the Research Subjects' Advocate Line, D528 Mayo, 420 Delaware St. Southeast, Minneapolis, Minnesota 55455; (612) 625-1650.

You will be given a copy of this information to keep for your records.

Statement of Consent:
I have read the above information. I have asked questions and have received answers. I consent to participate in the study.

Signature:_________________________________ Date: __________________

Signature of Investigator:____________________ Date: __________________

IRB Code #0903S60722
Version Date: Oct 6, 2010

Appendix 4. All Fall 2011 Problems and Solutions

Quiz 1

Quiz 1 9/23/11

Instructions:
1. Put your name and student ID# on the scantron sheet.
2. Answer the 5 multiple choice questions on the scantron sheet.
3. Put your problem solutions on the paper provided.
4. Fill out the information at the top of <u>every</u> sheet that you use.

Ground rules:
1. This is a closed book, closed notes quiz. Calculators are permitted.
2. Calculations **MUST** begin with one of the mathematical relationships on the back of this sheet.

Grading rubric:
Your solution should include:
- A clear re-description of the problem that helps you to solve it.
- An explicit statement of the physics principles and assumptions used in solving the problem.
- Mathematics must begin with one of the fundamental relationships on the back of this page.
- The solution should follow a logical progression and be easily followed by a grader.
- Numbers not plugged in until the very end.
- The answer should be explicitly evaluated for obvious signs of incorrectness.

Possibly helpful framework:
Focus the Problem
Picture and given information
Question
Approach

Describe the Physics
Diagram(s) and define quantities
Target quantity(ies)
Quantitative Relationships

Plan the Solution
Plan the mathematics

Execute the Plan
Calculate target quantities

Evaluate the Answer
Properly stated?
Not unreasonable?
Complete?

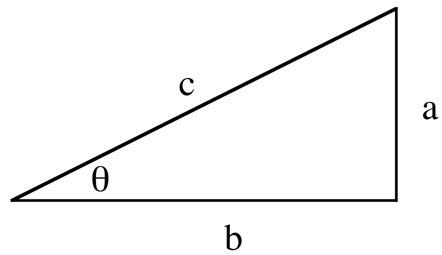
Common conversion factors

1 mile = 5280 ft.

1 m = 3.28 ft.

Useful Mathematical Relationships

For a right triangle:



$$\sin\theta = \frac{a}{c}, \quad \cos\theta = \frac{b}{c}, \quad \tan\theta = \frac{a}{b}$$

$$a^2 + b^2 = c^2, \quad \sin^2\theta + \cos^2\theta = 1$$

For a circle: $C = 2\pi R, \quad A = \pi R^2$

For a sphere: $A = 4\pi R^2, \quad V = \frac{4}{3}\pi R^3$

If $Ax^2 + Bx + C = 0$, then $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$$\int ax^n \, dx = \frac{ax^{n+1}}{n+1} + C \qquad \int \left(f[x] + g[x]\right)dx = \int f[x]dx + \int g[x]dx$$

Fundamental definitions

$$\vec{v} = \frac{d\vec{r}}{dt} \quad \left(v_x = \frac{dx}{dt}\right), \qquad \vec{a} = \frac{d\vec{v}}{dt} \quad \left(a_x = \frac{dv_x}{dt}\right)$$

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} \quad \left(v_{avg,x} = \frac{\Delta x}{\Delta t}\right), \qquad \vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} \quad \left(a_{avg,x} = \frac{\Delta v_x}{\Delta t}\right)$$

$$speed_{avg} = \frac{\text{distance traveled}}{\Delta t}$$

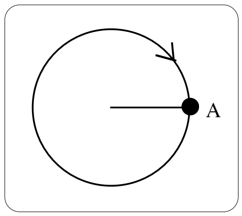
Under certain conditions:

$$\Delta x = \frac{1}{2}a_x\left(\Delta t\right)^2 + v_{ix}\,\Delta t \qquad \Delta v_x = a_x\,\Delta t$$

$$a = \frac{v^2}{R}$$

Multiple choice questions (5 points each)

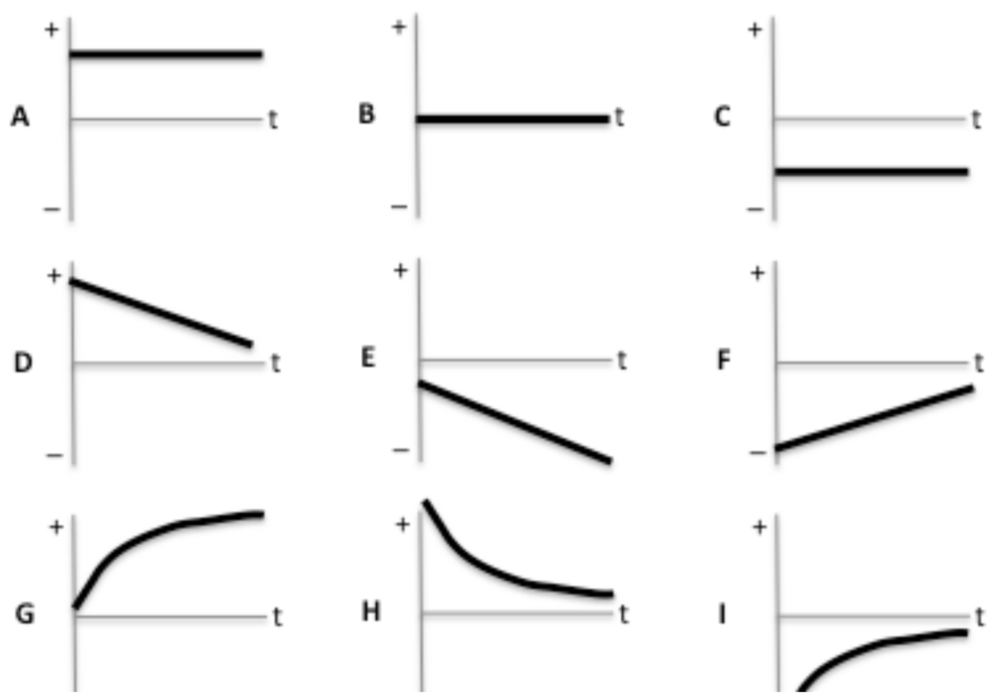
1. A ball attached to the end of a string is swung clockwise in a vertical circle (the plane of the circle is vertical). At point A, the ball is speeding up. At that point, in which direction is the ball's acceleration?



A. downward
B. between down and left
C. left
D. between down and right
E. None of the above

2. In lab problem 1.1, you investigated position v. time, instantaneous speed v. time, and instantaneous acceleration v. time graphs for an object moving at constant speed. Which of the following choices below represents the position v. time, instantaneous speed v. time, and instantaneous acceleration vs. time graphs (in that order) for an object moving at a speed that <u>decreases at a steady rate</u>? The letter refer to the graphs below.

A. G, D, F
B. G, D, E
C. I, D, C
D. H, E, E
E. None of the above



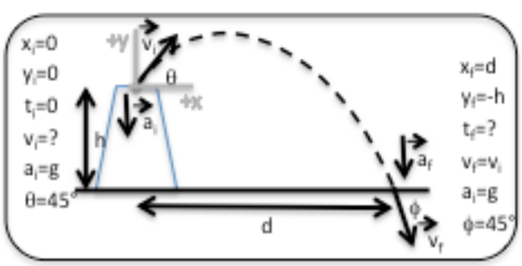
3. Suppose that a ball is launched downward with an initial speed of 2 m/s. Based on your results from lab problem 2.2 (or your other knowledge), the acceleration of that ball after being launched is

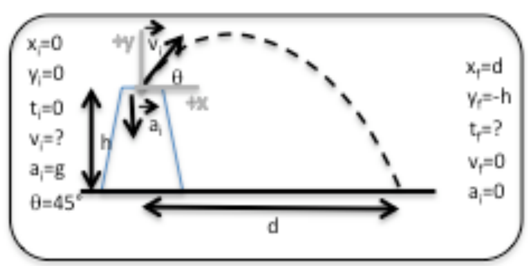
A. 4.9 m/s^2
B. 7.8 m/s^2
C. 11.8 m/s^2
D. 19.6 m/s^2
E. None of the above

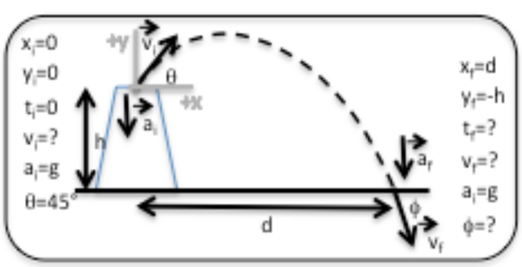
4. A ball is tossed straight upward and then falls back down. Which of the following combinations correctly describe the direction of its velocity and acceleration during its round trip (from going up after leaving the thrower's hand to just before it is caught by the thrower?

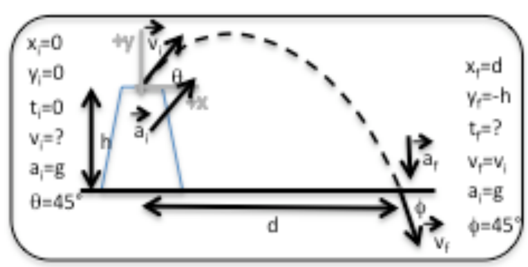
Choice	While moving upward	At the highest point	While moving downward
A	Velocity up Acceleration down	Velocity zero Acceleration down	Velocity down Acceleration down
B	Velocity up Acceleration up	Velocity zero Acceleration down	Velocity down Acceleration down
C	Velocity up Acceleration up	Velocity zero Acceleration zero	Velocity down Acceleration down
D	Velocity up Acceleration down	Velocity zero Acceleration zero	Velocity down Acceleration up
E	None of the above		

5. A student is trying to solve the problem "A rock is ejected from a volcano at an angle of 45°. How far will the rock land from the volcano as a function of the initial speed of the rock?" The student decides to draw a picure of the situation with the rock just after it is ejected from the volcano and just before it lands on the ground. Which of the following pictures is correct and will be the most helpful to the student in solving

A


B


C


D


the problem?

E. None of the above pictures are correct

Problems (25 points each)

1. As the stunt coordinator on a movie set, it is your job to arrange a scene in which a stunt double steps off a bridge and lands onto some mattresses in the back of a large truck that is driving under the bridge. You find that the bridge is 50 feet above the ground. The mattresses in the bed of the truck are about 4 feet above the ground. The truck will be driving toward the bridge at a steady speed of 20 miles per hour. To carry out the stunt safely, you decide to calculate where to place a traffic cone by the side of the road so that when the truck passes the cone, the stunt double will step off the bridge and land safely in the back of the truck.

2. You are helping a friend with an enormous art project. Your friend wants to use an air cannon to shoot fist-sized balls of paint onto a mural on the side of a building. When the balls strike the mural, they break and produce a large splatter of paint. Your friend wants the balls to be traveling horizontally when they impact in order to produce the most symmetric splatters and asks you to find a formula for the angle above the horizontal at which to aim the cannon in order for the paint ball to hit the mural at a given height above the ground. The specifications of the air cannon say that the launch speed of the balls of paint is 13 m/s.

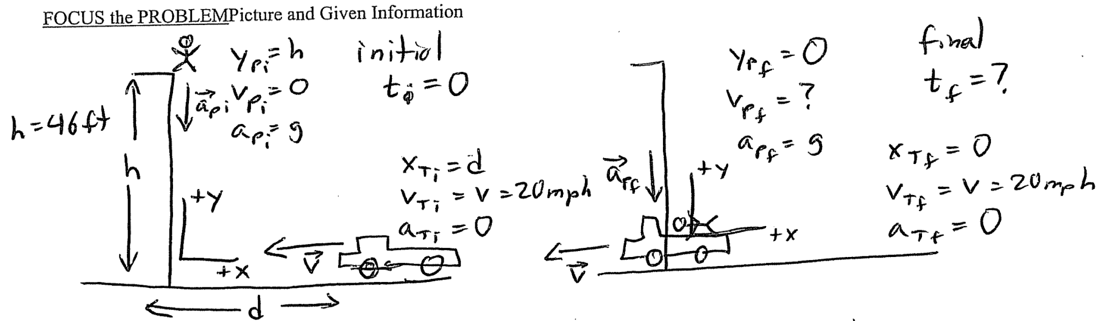
Answers to Multiple Choice Questions

1. **B**
2. **C**
3. **E**
4. **A**
5. **C**

Solutions to Long Problems:

Problem #1

FOCUS the PROBLEM Picture and Given Information



Question(s)

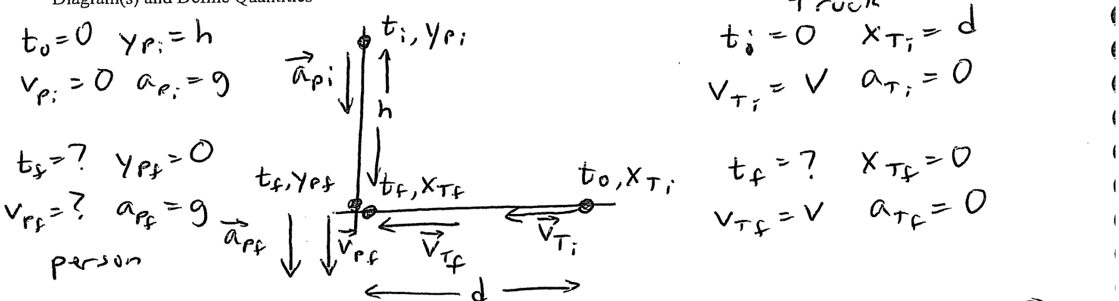
How far away from bridge should truck be when person steps off of bridge?

Approach

Use kinematics
System: Truck
System: Person
Assume air resistance negligible

t_i: just after person steps off bridge.
t_f: just before person lands on mattresses.

DESCRIBE the PHYSICS
Diagram(s) and Define Quantities



Target Quantity(ies) d

Quantitative Relationships Const accel, so $a = \frac{dv}{dt} \rightarrow$ ① $\Delta v = a\Delta t$; $v = \frac{dx}{dt} \rightarrow$ ② $\Delta x = \frac{1}{2}a\Delta t^2 + v_i\Delta t$

Apply to person

① $-v_{P_f} - 0 = (-g)t_f \Rightarrow \boxed{v_{P_f} = g\,t_f}$

② $0 - h = \frac{1}{2}(-g)(t_f)^2 + 0 \Rightarrow \boxed{h = \frac{1}{2}g\,t_f^2}$

Apply to truck

$v - v = 0$ true, but not useful.

$0 - d = 0 + (-v)\,t_f$

$\Rightarrow \boxed{d = v\,t_f}$

PLAN the SOLUTION
Construct Specific Equations

Find d

$d = v t_f$

Find t_f

$h = \frac{1}{2} g t_f^2$

$t_f = \sqrt{\frac{2h}{g}}$

$$d = v\sqrt{\frac{2h}{g}}$$

Check units:

$$d = \left[\frac{m}{s}\right]\sqrt{\frac{[m]}{[m/s^2]}} = \left[\frac{m}{s}\right][s] = m$$

Units OK.

Check Units

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$d = (20\,mph)\sqrt{\frac{2(46\,ft.)}{(9.8\,m/s^2)}}$$

Don't forget to convert units!

$$d = 49.6\ ft.$$

EVALUATE the ANSWER
Is Answer Properly Stated?

d has units of length

The magnitude of d is not unreasonable.

Is Answer Unreasonable?

Functional dependence:

$h\uparrow, d\uparrow$ Higher bridge = place cone further away OK.

$v\uparrow, d\uparrow$ Truck drives faster, put cone further away. OK

Is Answer Complete?

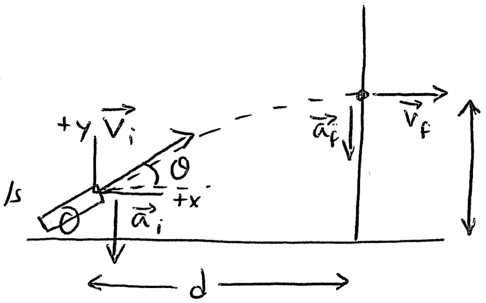
The traffic cone should be placed 49.6 ft before the bridge.

(extra space if needed)

Problem #2

FOCUS the PROBLEM Picture and Given Information

$t_i = 0$
$x_i = 0$
$y_i = 0$
$v_i = v = 13\,m/s$
$a_i = g$



$t_f = ?$
$x_f = d$
$y_f = h$
$v_f = ?$
$a_f = g$

Unknowns
t_f, v_f, θ, d
(h is treated as a known because it is given)

Question(s)

At what angle should the cannon be aimed so that the point ball strikes the wall horizontally at height h,

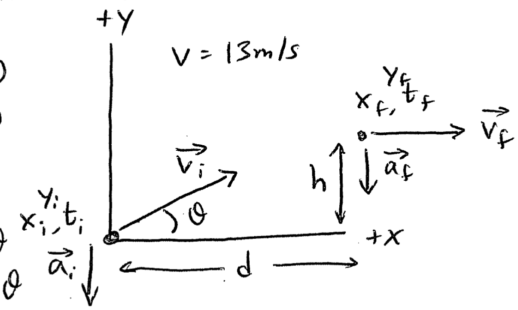
Approach

Assumptions:
Ignore air resistance
Ignore height of cannon or measure impact spot height from cannon muzzle.

Use kinematics
System: point ball
t_i: just after ball leaves cannon
t_f: just before ball impacts wall.

DESCRIBE the PHYSICS
Diagram(s) and Define Quantities

$t_i = 0$
$x_i = 0$
$y_i = 0$
$v_i = v$
$a_i = g$
$v_{ix} = v\cos\theta$
$v_{iy} = v\sin\theta$



$t_f = ?$
$x_f = d$
$y_f = h$
$v_{fx} = v_f = ?$
$v_{fy} = 0$
$a_f = 0$

Target Quantity(ies) θ

Quantitative Relationships Constant accel, so $\Delta v = a\Delta t$, $\Delta x = \frac{1}{2}a_x \Delta t^2 + v_{ix}\Delta t$

x direction: $v_f - v\cos\theta = 0$

$$d = \frac{1}{2}(0)(t_f)^2 + v\cos\theta\, t_f$$

y - direction: $0 - v\sin\theta = -g\,t_f$

$$h = \frac{1}{2}(-g)t_f^2 + v\sin\theta\, t_f$$

PLAN the SOLUTION
Construct Specific Equations

Bad choice. No way to find d.

Find θ

~~$\Delta x = v\cos\theta\, t_f$~~ $v\sin\theta = g t_f$

Find t_f

$$h = -\frac{1}{2} g t_f^2 + v\sin\theta\, t_f$$

$$\frac{1}{2} g t_f^2 - v\sin\theta\, t_f + h = 0$$

$$t_f = \frac{v\sin\theta \pm \sqrt{v^2\sin^2\theta - 4\left(\frac{1}{2}g\right)(h)}}{g}$$

$$t_f = \frac{v\sin\theta \pm \sqrt{v^2\sin^2\theta - 2gh}}{g}$$

$$v\sin\theta = g t_f$$

$$v\sin\theta = v\sin\theta \pm \sqrt{v^2\sin^2\theta - 2gh}$$

$$0 = \pm\sqrt{v^2\sin^2\theta - 2gh}$$

$$v^2\sin^2\theta = 2gh$$

$$\sin\theta = \frac{\sqrt{2gh}}{v}$$

$$\theta = \sin^{-1}\left[\frac{\sqrt{2gh}}{v}\right]$$

Check Units

$\frac{\sqrt{2gh}}{v}$ should be unitless.

$$\frac{\sqrt{[m/s^2][m]}}{[m/s]} = 1 \quad \text{OK.}$$

EXECUTE the PLAN
Calculate Target Quantity(ies)

We have no number for h, so leave as

$$\theta = \sin^{-1}\left[\frac{\sqrt{2gh}}{v}\right] \qquad v = 13\ m/s$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes, θ is an inverse trig function. $\sqrt{2gh}/v$ is unitless.

Is Answer Unreasonable?

Max height is $\sqrt{2gh}/v = 1$ (shoot straight up)

As $v\uparrow$, $\theta\downarrow$ (but will need to move cannon further from wall)

$h\uparrow$, $\theta\uparrow$ (bigger angle for higher height)

Is Answer Complete?

$g\uparrow$, $\theta\uparrow$ (stronger gravity, bigger angle)

Yes, we have a formula for the angle.

(extra space if needed)

More checks.
Limiting cases.
If $h \to 0$, $\theta \to 0$

Quiz 2 10/14/11

Instructions:

1. Put your name and student ID# on the scantron sheet.
2. Answer the 5 multiple choice questions on the scantron sheet.
3. Put your problem solutions on the paper provided.
4. Start each problem on a new sheet.
5. Fill out the information at the top of every sheet that you use.

Ground rules:

1. This is a closed book, closed notes quiz. Calculators are permitted.
2. Calculations **MUST** begin with one of the mathematical relationships on the back of this sheet.

Grading rubric:

Your solution should include:

- A clear re-description (such as a picture or diagram) of the problem that helps you to solve it.
- An explicit statement of the physics principles and assumptions used in solving the problem.
- Mathematics must begin with one of the fundamental relationships on the back of this page.
- The solution should follow a logical progression and be easily followed by a grader.
- Numbers not plugged in until the very end.
- The answer should be explicitly evaluated for obvious signs of incorrectness.

Possibly helpful framework:

Focus the Problem
    Picture and given information
    Question
    Approach

Describe the Physics
    Diagram(s) and define quantities
    Target quantity(ies)
    Quantitative Relationships

Plan the Solution
    Plan the mathematics

Execute the Plan
    Calculate target quantities

Evaluate the Answer
    Properly stated?
    Not unreasonable?
    Complete?

Common conversion factors and constants

1 mile = 5280 ft.

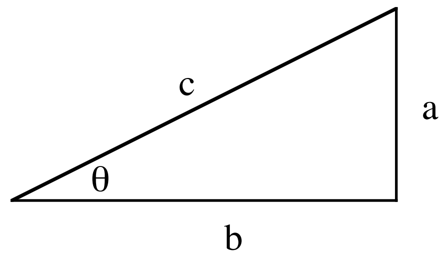
$g = 9.8\ \text{m/s}^2 = 32\ \text{ft/s}^2$

1 m = 3.28 ft.

9.8 N = 2.2 lbs.

Useful Mathematical Relationships

For a right triangle:



$$\sin\theta = \frac{a}{c}, \quad \cos\theta = \frac{b}{c}, \quad \tan\theta = \frac{a}{b}$$

$$a^2 + b^2 = c^2, \quad \sin^2\theta + \cos^2\theta = 1$$

For a circle: $C = 2\pi R, \quad A = \pi R^2$

For a sphere: $A = 4\pi R^2, \quad V = \frac{4}{3}\pi R^3$

If $Ax^2 + Bx + C = 0$, then $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$$\int ax^n\, dx = \frac{ax^{n+1}}{n+1} + C$$

$$\int \left(f[x] + g[x]\right) dx = \int f[x]\,dx + \int g[x]\,dx$$

Fundamental definitions

$$\vec{v} = \frac{d\vec{r}}{dt} \quad \left(v_x = \frac{dx}{dt}\right), \qquad \vec{a} = \frac{d\vec{v}}{dt} \quad \left(a_x = \frac{dv_x}{dt}\right)$$

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} \quad \left(v_{avg,x} = \frac{\Delta x}{\Delta t}\right), \qquad \vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} \quad \left(a_{avg,x} = \frac{\Delta v_x}{\Delta t}\right)$$

$$speed_{avg} = \frac{\text{distance traveled}}{\Delta t}$$

$$\vec{F}_{net} = \Sigma F = m\vec{a}$$

Under certain conditions:

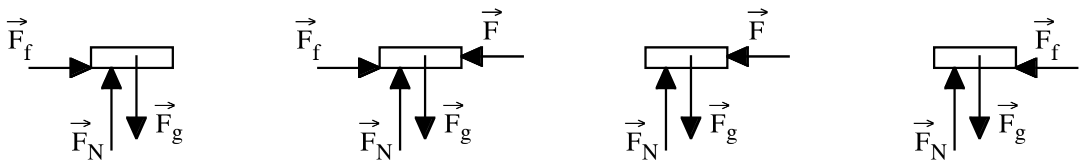
$$\Delta x = \frac{1}{2} a_x (\Delta t)^2 + v_{ix}\,\Delta t \qquad \Delta v_x = a_x\,\Delta t \qquad a = \frac{v^2}{R}$$

$F = \mu_k F_N$ $F \le \mu_s F_N$ $W = mg$ $F = k\,\Delta x$

Multiple choice questions (5 points each)

1. I give a book a quick shove to the **<u>left</u>** across the lecture bench. Which free-body diagram best represents the forces on the book as it slides on the bench after losing

A. B. C. D.

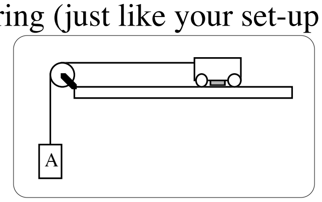


contact with my hand? Ignore air drag.

2. For no particular reason, a chair is thrown through a window. Which of the following is true?
   A. The force on the chair by the window is larger than the force on the window by the chair.
   B. The force on the chair by the window is smaller than the force on the window by the chair.
   C. The force on the chair by the window has the same magnitude as the force on the window by the chair.
   D. At first, the force on the chair by the window has the same magnitude as the force on the window by the chair. After the window breaks, the force on the window by the chair is larger than the force on the chair by the window.
   E. None of the above are true.

3. On level ground, the gravitational force on a parked car and the normal force on it have the same magnitude because
   A. the gravitational and normal forces are mutual (action/reaction) forces.
   B. the net force on the car is zero.
   C. both A and B.
   D. neither A nor B.

4. The picture at right shows a cart on a track attached to a string (just like your set-up in lab). In addition, there is a pad on the bottom of the cart that rubs against the track, providing a friction force. When a mass of 0.20 kg is hung from the string at the location marked A, the cart is pulled along at a constant speed of 1.5 m/s. If a mass of 0.40 kg were hung from the string instead, the cart would be pulled along:

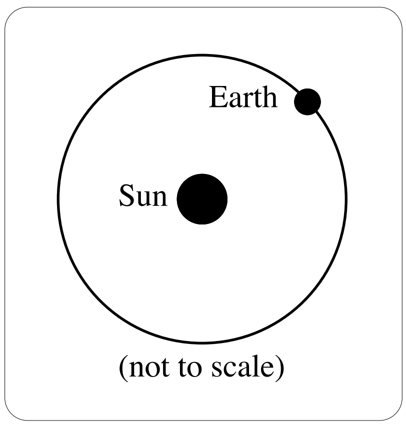


   A. at a constant speed of 3 m/s.
   B. at a constant speed greater than 1.5 m/s, but not necessarily 3 m/s.

C. with a continuously increasing speed.
D. for a while at a constant speed greater than 1.5 m/s, then with an increasing speed.
E. for a while with an increasing speed, then with a constant speed thereafter.

5. The earth orbits the sun in a nearly circular orbit. The sun is 300,000 times more massive than the earth. Which force acts on the earth **in addition** to the gravitational force on the Earth by the sun?

   A. A force of motion in the direction of the circular orbit.
   B. A centrifugal force acting outward (away from the sun)
   C. A centripetal force acting inward (toward the sun).
   D. A normal force
   E. There are no additional forces on the Earth.



Problems (25 points each)

1. In a ride found at many carnivals, riders enter a cylindrical room and stand against the walls. The room then spins rapidly, giving the riders a feeling of being pressed against the wall, and finally, the floor drops away, leaving the people pinned against the wall. Because you are in charge of buying the correct type of motor for such a ride, you must first calculate the rotation rate necessary for the room in order for this ride to work. The diameter of the room is 20 feet and the maximum expected weight of a rider will be 300 lbs. A typical coefficient of friction between the walls and clothing is 0.6.

2. While driving in the mountains, you notice that when the freeway goes steeply down hill, there are emergency exits every few miles. These emergency exits are straight dirt ramps which leave the freeway and are sloped uphill. They are designed to stop trucks and cars that lose their brakes on the downhill stretches of the freeway. You are curious, so you stop at the next emergency exit. You estimate that the road rises at an angle of 15° from the horizontal and is about 300 ft. long. What is the maximum speed of a truck that you are sure will be stopped by this ramp, even in the iciest Minnesota driving conditions?

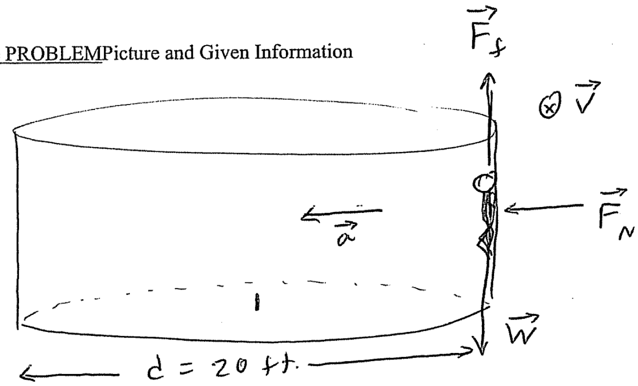
Answers to Multiple Choice Questions

1. **A**
2. **C**
3. **B**
4. **C**
5. **E**

Solutions to Long Problems:

Problem 1

FOCUS the PROBLEM Picture and Given Information



Question(s)

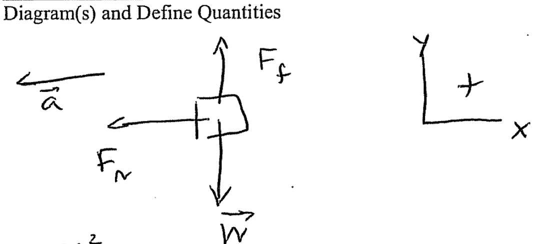
What is the rotation rate necessary (how fast does this have to go) for the ride to work?

Approach

Assumptions
- Neglect air drag.
- constant speed rotation.

Dynamics
system: person
time: while room is spinning

DESCRIBE the PHYSICS
Diagram(s) and Define Quantities



$a = \frac{v^2}{R}$

Target Quantity(ies) Rotation rate (T time to make one rotation)

Quantitative Relationships

$F_{net} = ma$

$x: \; -F_N = m\left(\frac{-v^2}{R}\right) \longrightarrow v = \frac{2\pi R}{T} \longrightarrow F_N = m\frac{4\pi^2 R}{T^2}$

$y: \; F_f - W = m(0) \longrightarrow F_f = \mu_s F_N \longrightarrow \mu_s F_N = W = mg$

PLAN the SOLUTION
Construct Specific Equations

Find T

$$F_N = m\frac{4\pi^2 R}{T^2}$$

Find F_N

$$\mu_s F_N = mg$$

$$F_N = \frac{mg}{\mu_s}$$

$$\frac{mg}{\mu_s} = m\frac{4\pi^2 R}{T^2}$$

$$T = 2\pi\sqrt{\frac{R\mu_s}{g}}$$

Check Units

$$T = \sqrt{\frac{[m]}{[m/s^2]}} = \sqrt{s^2} = s$$

units of time

OK ✓

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$T = 2\pi\sqrt{\frac{(10\text{ ft.})(0.6)}{(32.2\text{ ft/s}^2)}}$$

$$= 2.7\text{ sec.}$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes. Answer ~~be~~ for rotation time (period) is in seconds.

Is Answer Unreasonable?

2.7 sec seems reasonable for a rotation rate (or at least, not too unreasonable)

Is Answer Complete?

The room must spin around once every 2.7 seconds.

(extra space if needed)

Functional dependence check:

$\mu \downarrow$ $T \downarrow$ (less friction means you have to spin faster)

$g \uparrow$ $T \downarrow$ (stronger gravity means you have to spin faster)

PLAN the SOLUTION
Construct Specific Equations

Find v_i

$$d = -\frac{1}{2}at_f^2 + v_i t_f$$

Find t_f

$$v_i = at_f$$

Find a

$$g\sin\theta = a$$

$$v_i = g\sin\theta\, t_f$$

$$t_f = \frac{v_i}{g\sin\theta}$$

$$d = -\frac{1}{2}(g\sin\theta)\left(\frac{v_i}{g\sin\theta}\right)^2 + v_i\left(\frac{v_i}{g\sin\theta}\right)$$

$$d = -\frac{1}{2}\left(\frac{v_i^2}{g\sin\theta}\right) + \frac{v_i^2}{g\sin\theta}$$

$$d = \frac{v_i^2}{2g\sin\theta}$$

$$v_i = \sqrt{2dg\sin\theta}$$

Check Units

$$d = \frac{[m/s]^2}{[m/s^2]} = m$$ units ok ✓

$$v_i = \sqrt{[m][m/s^2]} = \sqrt{\frac{m^2}{s^2}} = \frac{m}{s}$$ ✓

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$v_i = \sqrt{2(300ft)(32ft/s^2)\sin 15^\circ}$$

$$= 70.5 ft/s = 48 mph$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Velocity has correct units.

Is Answer Unreasonable?

No, this is a reasonable speed for a truck. In real life, "friction" from gravel road would play a major part in stopping the truck.

Is Answer Complete?

We have found max speed of vehicle that would be stopped in the "worst case."

(extra space if needed)

$d\uparrow, v_i\uparrow$ Longer ramp can stop a faster truck.

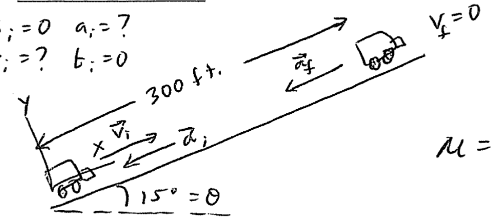
$\theta\uparrow, v_i\uparrow$ higher tilt means you can stop a faster truck.

Problem 2

FOCUS the PROBLEM Picture and Given Information

$x_i = 0$ $a_i = ?$

$v_i = ?$ $t_i = 0$



$x_f = d = 300$ ft. $a_f = a_i$

$v_f = 0$ $t_f = ?$

$\mu = 0$

Origin @ bottom of ramp.

Question(s)

What is the fastest truck that will be stopped by ramp? (max speed)

Approach

Assume

- No friction (icy road)
- Neglect size of truck (measure from back)
- Neglect air drag

Kinematics

System: Truck

Times: Bottom of ramp to top of ramp.

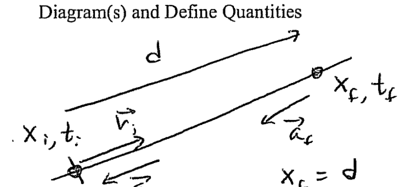
Dynamics

System: Truck

Time: while going up ramp

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities



$x_i = 0$

$v_i = ?$

$a_i = ? = a$

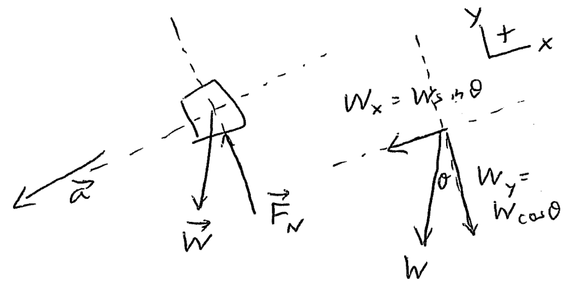
$t_i = 0$

$x_f = d$

$v_f = 0$

$a_f = a_i = a$

$t_f = ?$



Target Quantity(ies) v_i

Quantitative Relationships

$$\Delta v = a \Delta t$$

$$0 - v_i = (-a)(t_f - 0)$$

$$\boxed{v_i = a t_f}$$

$$\Delta x = \frac{1}{2} a_x \Delta t^2 + v_{ix} \Delta t$$

$$d - 0 = \frac{1}{2}(-a)(t_f - 0)^2 + v_i(t_f - 0)$$

$$\boxed{d = -\frac{1}{2} a t_f^2 + v_i t_f}$$

$$F_{net} = ma$$

$$x: \; -W\sin\theta = m(-a)$$

$$y: \; F_N - W\cos\theta = 0$$

$$W = mg$$

$$\boxed{g\sin\theta = a}$$

$$\boxed{F_N = mg\cos\theta}$$

Quiz 3 11/11/11

Instructions:
1. Put your name and student ID# on the scantron sheet.
2. Answer the 5 multiple choice questions on the scantron sheet.
3. Put your problem solutions on the paper provided.
4. Start each problem on a new sheet.
5. Fill out the information at the top of every sheet that you use.

Ground rules:
1. This is a closed book, closed notes quiz. Calculators are permitted.
2. Calculations **MUST** begin with one of the mathematical relationships on the back of this sheet.

Grading rubric:
Your solution should include:
- A clear re-description (such as a picture or diagram) of the problem that helps you to solve it.
- An explicit statement of the physics principles and assumptions used in solving the problem.
- Mathematics must begin with one of the fundamental relationships on the back of this page.
- The solution should follow a logical progression and be easily followed by a grader.
- Numbers not plugged in until the very end.
- The answer should be explicitly evaluated for obvious signs of incorrectness.

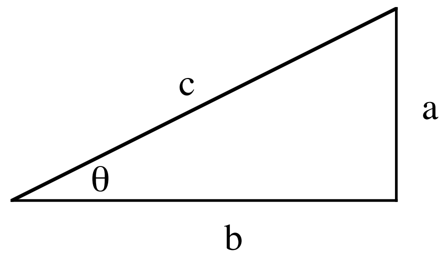
Common conversion factors and constants

1 mile = 5280 ft. | $g = 9.8\ \text{m/s}^2 = 32\ \text{ft./s}^2$

1 m = 3.28 ft. | 9.8 N = 2.2 lbs.

Useful Mathematical Relationships

For a right triangle:



$$\sin\theta = \frac{a}{c},\quad \cos\theta = \frac{b}{c},\quad \tan\theta = \frac{a}{b}$$

$$a^2 + b^2 = c^2,\quad \sin^2\theta + \cos^2\theta = 1$$

For a circle: $C = 2\pi R,\quad A = \pi R^2$

For a sphere: $A = 4\pi R^2,\quad V = \frac{4}{3}\pi R^3$

If $Ax^2 + Bx + C = 0$, then $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$$\int ax^n\, dx = \frac{ax^{n+1}}{n+1} + C \qquad \int \left(f[x] + g[x]\right)dx = \int f[x]dx + \int g[x]dx$$

Fundamental definitions

$$\vec{v} = \frac{d\vec{r}}{dt}\ \left(v_x = \frac{dx}{dt}\right), \qquad \vec{a} = \frac{d\vec{v}}{dt}\ \left(a_x = \frac{dv_x}{dt}\right)$$

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}\ \left(v_{avg,x} = \frac{\Delta x}{\Delta t}\right), \qquad \vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}\ \left(a_{avg,x} = \frac{\Delta v_x}{\Delta t}\right)$$

$$speed_{avg} = \frac{\text{distance traveled}}{\Delta t}$$

$$\vec{F}_{net} = \Sigma F = m\vec{a} \qquad E_f - E_i = E_{in} - E_{out}$$

$$\vec{p}_f - \vec{p}_i = \vec{p}_{in} - \vec{p}_{out}$$

$$E_{in}, E_{out} = \left|\int \vec{F} \bullet d\vec{l}\right| \qquad \vec{p}_{in}, \vec{p}_{out} = \left|\int \vec{F}\, dt\right|$$

Under certain conditions:

$$\Delta x = \frac{1}{2}a_x(\Delta t)^2 + v_{ix}\Delta t \qquad \Delta v_x = a_x\,\Delta t \qquad a = \frac{v^2}{R}$$

$$F = \mu_k F_N$$

$$F \leq \mu_s F_N$$

$$W = mg$$

$$F = k\,\Delta x$$

$$E = \frac{1}{2} m v^2$$

$$PE = mgh$$

$$PE = \frac{1}{2} k \Delta x^2$$

$$\vec{p} = m\vec{v}$$

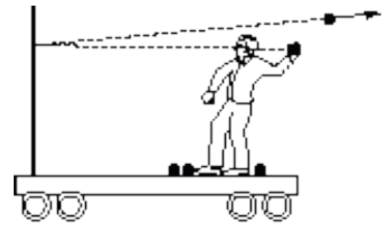
Multiple choice questions (5 points each)

1. A styrofoam ball is dropped from 1 meter above the ground first in the air (case A), and then in an airtight chamber from which all the air has been pumped out (case B). Which of the following statements are true?
   A. Both the work done on the ball by the Earth's gravitational force and the impulse delivered to the ball by the Earth's gravitational force are larger in case A than case B.
   B. Both the work done on the ball by the Earth's gravitational force and the impulse delivered to the ball by the Earth's gravitational force are smaller in case A than case B.
   C. The work done on the ball by the Earth's gravitational force is larger in case A than in case B. The impulse delivered to the ball by the Earth's gravitational force is smaller in case A than in case B.
   D. The work done on the ball by the Earth's gravitational force is smaller in case A than in case B. The impulse delivered to the ball by the Earth's gravitational force is larger in case A than in case B.
   E. None of the above are true.

2. Suppose that 15,000 J are required to accelerate a car from rest to 10 mph. Ignoring air resistance and friction, the amount of energy required to accelerate a car from 10 mph to 20 mph is
   A. 7,500 J
   B. 15,000 J
   C. 30,000 J
   D. 45,000 J
   E. 60,000 J

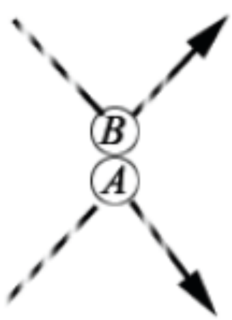
3. Standing with both feet on the ground, I bend my knees and then jump straight into the air. Which of the following statements is true during the time interval of the jump?
   A. The work and impulse on me by the normal force by the ground are both non-zero.
   B. The work and impulse on me by the normal force by the ground are both zero.
   C. The work on me by the normal force by the ground is zero, but the impulse on me by the ground is non-zero.
   D. The work on me by the normal force by the ground is non-zero, but the impulse on me by the ground is zero.
   E. None of the above are true.

4. Standing on a cart with very little friction, you throw a ball at a wall that is mounted on the cart. If the ball bounces straight back as shown in the drawing, what happens to the cart?



   A. The cart moves to the right.
   B. The cart moves to the left.
   C. The cart stays where it is, though it might first move a little to the right, then the left.
   D. There is not enough information to decide.
   E. None of the above is true.

5. Two billiard balls collide as shown in the picture. The balls have the same speeds before and after the collision. Which of the choices below best represents the direction of the impulse on ball B by ball A during the collision?
   A. Up
   B. Down
   C. Right
   D. Between down and right
   E. Between up and right



Problems (25 points each)

1. Because you both like to watch the trains, you and a friend are hanging out at the railroad transfer station sitting on a small railway flatcar (basically a wheeled platform). The discussion turns to physics and you have the following disagreement: your friend thinks that if both of you jump off the back of the flatcar at the same time, the final speed of the flatcar will be the same as if you were to jump off the back one at a time. However, you think that the final speed of the flatcar will be different in the two cases. To resolve the problem, you decide to calculate the final speed of the flatcar in both cases. You and your friend have just about the same mass and you estimate the mass of the flatcar to be about 5 times larger than either of your masses. Also, you find that you can both jump off the back of the flatcar with the same speed relative to the flatcar. To simplify your calculation, you decide to ignore any effects of friction.

2. One summer, you and a friend decide to build a primitive elevator for a tree house that you are constructing for some neighborhood children. The elevator consists of a rope thrown over a branch with a platform tied to either end of the rope. This branch is 4 meters above the ground and you have wrapped it with some smooth plastic to greatly reduce any friction between it and the rope. One platform starts on the ground while the other starts suspended 3 meters above the ground. To operate the elevator, a child sits on the platform on the ground while someone in the tree house places a counterweight on the suspended platform. The platform with the counterweight then descends to the ground, pulling the child up to the tree house in the process. Because the parents are worried about safety, you decide to calculate the weight of the counterweight that should be placed on the suspended platform to lift a 60 lb. child up to the tree house such that the speed of the child will never exceed 2 ft/s. The platforms weigh about 5 lbs. each.

Answers to Multiple Choice Questions

1. **E**
2. **D**
3. **C**
4. **B**
5. **A**

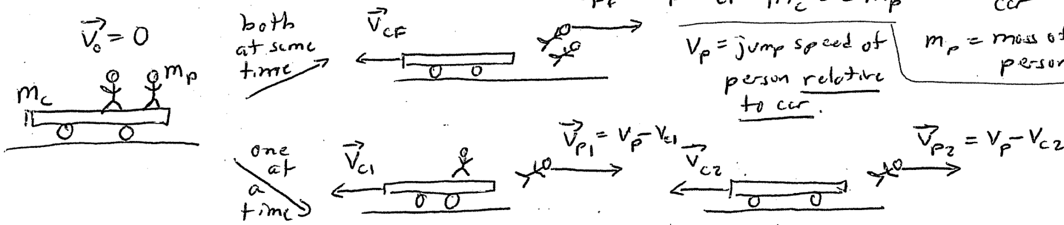
Solutions to Long Problems:

Assumptions: • Neglect air drag and friction

FOCUS the PROBLEM Picture and Given Information

$\vec{V}_{PF} = V_p - V_{CF}$ $m_c = 5m_p$ m_c = mass of car

V_p = jump speed of person relative to car. m_p = mass of person



Since V_p is relative to car, we need to subtract speed of car from V_p to find final speed of jumper relative to the ground.

Question(s)

What is final speed of flatcar if (a) both people jump off together or (b) one person jumps off at a time.

Approach

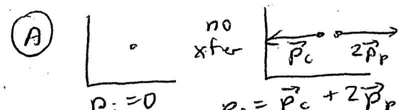
(A) Momentum
System: Car + 2 people
i: before anyone jumps
f: after both jump
$\vec{p}_i = 0$ $\vec{p}_f = \vec{p}_c + 2\vec{p}_p$ $\vec{p}_{in} = \vec{p}_{out} = 0$

(B) Momentum
System: Car + 2 people
i: before anyone jumps
f: after one person jumps
$\vec{p}_i = 0$, $\vec{p}_f = \vec{p}_{c+1person} + \vec{p}_{jumper}$
$\vec{p}_{in} = \vec{p}_{out} = 0$

(C) Momentum
System: Car + 1 person
i: After 1 person jumps
f: After 2nd person jumps
$\vec{p}_i = \vec{p}_{car+1person}$
$\vec{p}_f = \vec{p}_{car} + \vec{p}_{jumper}$
$\vec{p}_{in} = \vec{p}_{out} = 0$

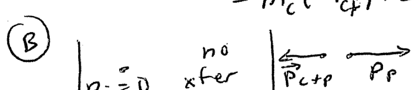
DESCRIBE the PHYSICS
Diagram(s) and Define Quantities

(A)



$p_f = \vec{p}_c + 2\vec{p}_p$
$= m_c(-V_{cf}) + 2m_p(V_p - V_{cf})$

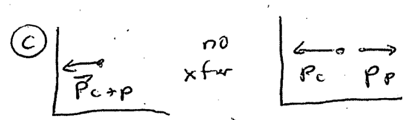
(B)



$p_i = 0$

$p_f = \vec{p}_{c+1person} + \vec{p}_{jumper}$
$= (m_c + m_p)(-V_{c1}) + m_p(V_p - V_{c1})$

(C)



$p_i = (m_c + m_p)(-V_{c1})$

$p_f = m_c(-V_{c2}) + m_p \cdot (V_p - V_{c2})$

Target Quantity(ies)

V_{CF}, V_{c2}

Quantitative Relationships

(A) $-m_c V_{cf} + 2m_p(V_p - V_{CF}) = 0$

(B) $-(m_c + m_p)V_{c1} + m_p(V_p - V_{c1}) = 0$

(C) $-m_c V_{c2} + m_p(V_p - V_{c2}) - (-(m_c + m_p)V_{c1}) = 0$

Additional relation

$m_c = 5m_p$

PLAN the SOLUTION
Construct Specific Equations

Find v_{cF}

$$-m_c v_{cF} + 2m_p(v_p - v_{cF}) = 0$$

$$-m_c v_{cF} + 2m_p v_p - 2m_p v_{cF} = 0$$

$$v_{cF} = \frac{2m_p v_p}{2m_p + m_c} \xrightarrow[m_c = 5m_p]{\text{using}} \frac{2}{7} v_p$$

Find v_{c2}

$$-m_c v_{c2} + m_p(v_p - v_{c2}) + (m_c + m_p)v_{c1} = 0$$

Find v_{c1}

$$-(m_c + m_p)v_{c1} + m_p(v_p - v_{c1}) = 0$$

Find m_c

$$m_c = 5m_p$$

$$-6m_p v_{c1} + m_p v_p - m_p v_{c1} = 0$$

$$v_{c1} = \frac{m_p v_p}{7m_p} = \frac{v_p}{7}$$

$$-5m_p v_{c2} + m_p v_p - m_p v_{c2} + 6m_p\left(\frac{v_p}{7}\right) = 0$$

$$6m_p v_{c2} = m_p v_p \frac{13}{7}$$

$$v_{c2} = \frac{13}{42} v_p$$

Check Units

$v_{cF} = [m/s]$

$v_{c2} = [m/s]$

both v_{c2} and v_{cF} have units of velocity

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$v_{cF} = \frac{2}{7} v_p \qquad v_{c2} = \frac{13}{42} v_p$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes, final velocity of flatcar is in terms of jumping velocity of person.

Is Answer Unreasonable?

No, v_{cF} is smaller than v_p, since $m_c > m_p$.

Is Answer Complete?

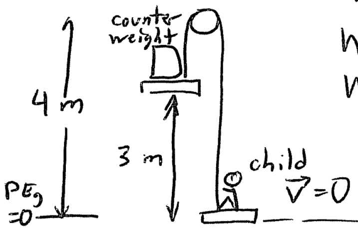
We have found speed of ~~both~~ flatcar in both cases.

(extra space if needed)

We can also check the first result. $v_{cF} = \frac{2m_p}{2m_p + m_c} v_p$.

As mass of flatcar gets bigger, v_{cF} gets smaller (since there's a bigger mass to shove).

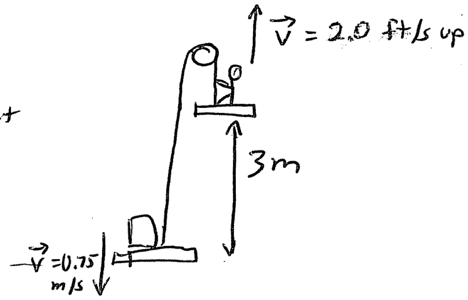
FOCUS the PROBLEM Picture and Given Information



$W_p = 5$ lbs. platform

$W_c = 60$ lbs. child

$W_w = ?$ counterweight



Question(s)

What is the weight of the counterweight to pull the child up so that V_{child} is always less than 2 ft/s?

Approach

Conservation of Energy

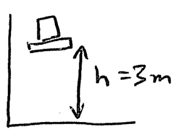
System: 2 platforms, child, counterweight, Earth

E_i = PE_g of counterweight + platform $\qquad E_{in} = E_{out} = 0$

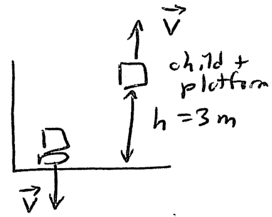
E_f = PE_g of child + platform, KE of 2 platforms, child, counterweights

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities



no xfer



Assumptions

- Neglect friction between branch + rope.
- Neglect mass of rope
- Neglect air drag.

$E_i = W_w h + W_p h$

$E_f = W_c h + W_p h + \frac{1}{2}\left(\frac{W_w}{g}\right)v^2 + \frac{1}{2}\left(2\frac{W_p}{g}\right)v^2 + \frac{1}{2}\left(\frac{W_c}{g}\right)v^2$

Target Quantity(ies)

W_w

Quantitative Relationships

$E_f - E_i = E_{in} - E_{out}$

$\left[(W_c + W_p)h + \frac{1}{2g}(W_w + W_c + 2W_p)v^2\right] - \left[h(W_w + W_p)\right] = 0$

$2 ft/s = 0.6 m/s$

$$\frac{(0.6 m/s)^2}{9.8 m/s^2}$$

PLAN the SOLUTION

Construct Specific Equations

Find W_w

$$\left[(W_c+W_p)h + \frac{1}{2g}(W_w+W_c+2W_p)v^2\right] - \left[h(W_w+W_p)\right] = 0$$

$$W_c h + W_p h + \frac{W_w v^2}{2g} + \frac{W_c v^2}{2g} + \frac{W_p v^2}{g} - W_w h - W_p h = 0$$

$$W_w\left(\frac{v^2}{2g} - h\right) = -W_c h - \frac{W_c v^2}{2g} - \frac{W_p v^2}{g}$$

$$W_w = \frac{W_c\left(h + \frac{v^2}{2g}\right) + \frac{W_p v^2}{g}}{h - \frac{v^2}{2g}}$$

Check Units

$$W_w = \frac{lbs\left(m + \frac{(m/s)^2}{(m/s^2)}\right) + lbs.\frac{(m/s)^2}{(m/s^2)}}{m - \frac{(m/s)^2}{m/s^2}}$$

$$= \frac{lbs \cdot m}{m} = lbs,\ ok ✓$$

EXECUTE the PLAN

Calculate Target Quantity(ies)

$$W_w = \frac{(60 lbs)\left(3m + \frac{(0.6 m/s)^2}{2(9.8 m/s^2)}\right) + (5 lbs)\frac{(0.6 m/s)^2}{9.8 m/s^2}}{3m - \frac{(0.6 m/s)^2}{2(9.8 m/s^2)}}$$

$$= 60.8 lbs.$$

EVALUATE the ANSWER

Is Answer Properly Stated?

Yes, W_w has units of force (lbs.)

Is Answer Unreasonable?

No. Weight should be > 60 lbs, but not too much greater. Also, as $v \uparrow$, $W_w \uparrow$, heavier counterweight = faster speed.

Is Answer Complete?

The counterweight should not be larger than 60.8 lbs. (it should be between 60 and 60.8 lbs.)

(extra space if needed)

Quiz 4 12/2/11

Instructions:
1. Put your name and student ID# on the scantron sheet.
2. Answer the 5 multiple choice questions on the scantron sheet.
3. Put your problem solutions on the paper provided.
4. Start each problem on a new sheet.
5. Fill out the information at the top of every sheet that you use.

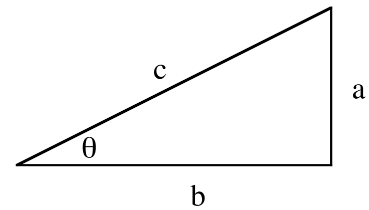
Ground rules:
1. This is a closed book, closed notes quiz. Calculators are permitted.
2. Calculations **MUST** begin with one of the mathematical relationships on the back of this sheet.
3. Mathematics must begin with one of the fundamental relationships on the back of this page.
4. Don't plug in numbers until the very end.

Grading rubric:
Your solution should include:
- A clear re-description of the problem that helps you to solve it and communicates what you are doing.
- An explicit statement of the physics principles and assumptions used in solving the problem.
- The solution should follow a logical progression and be easily followed by someone else.
- The answer should be explicitly evaluated for obvious signs of incorrectness in at least 2 ways.

Common conversion factors and constants

1 mile = 5280 ft. $g = 9.8\ m/s^2 = 32\ ft/s^2$ 1 m = 3.28 ft. 9.8 N = 2.2 lbs.

Useful Mathematical Relationships



For a right triangle: $\sin\theta = \frac{a}{c},\quad \cos\theta = \frac{b}{c},\quad \tan\theta = \frac{a}{b}$

$$a^2 + b^2 = c^2,\quad \sin^2\theta + \cos^2\theta = 1$$

For a circle: $C = 2\pi R,\quad A = \pi R^2$

For a sphere: $A = 4\pi R^2,\quad V = \frac{4}{3}\pi R^3$

sin(A+B) = sin A cos B + cos A sin B

cos(A+B) = cos A cos B – sin A sin B

If $Ax^2 + Bx + C = 0$, then $x = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$$\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + C \qquad \int \left(f[x] + g[x]\right)dx = \int f[x]dx + \int g[x]dx$$

Fundamental definitions

$$\vec{v} = \frac{d\vec{r}}{dt}\quad \left(v_x = \frac{dx}{dt}\right), \qquad \vec{a} = \frac{d\vec{v}}{dt}\quad \left(a_x = \frac{dv_x}{dt}\right)$$

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}\quad \left(v_{avg,x} = \frac{\Delta x}{\Delta t}\right), \qquad \vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}\quad \left(a_{avg,x} = \frac{\Delta v_x}{\Delta t}\right)$$

$$speed_{avg} = \frac{\text{distance traveled}}{\Delta t}$$

$$\vec{F}_{net} = \Sigma F = m\vec{a} \qquad E_f - E_i = E_{in} - E_{out} \qquad \vec{p}_f - \vec{p}_i = \vec{p}_{in} - \vec{p}_{out}$$

$$\vec{L}_f - \vec{L}_i = \vec{L}_{in} - \vec{L}_{out}$$

$$E_{in}, E_{out} = \left|\int \vec{F} \bullet d\vec{l}\right| \qquad \vec{p}_{in}, \vec{p}_{out} = \int \vec{F}\,dt$$

$$\vec{L}_{in}, \vec{L}_{out} = \int \vec{\tau}\,dt = \int \left(\vec{r} \times \vec{F}\right)dt$$

$$\vec{p} = m\vec{v} \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$$

Under certain conditions:

$$\Delta x = \frac{1}{2} a_x (\Delta t)^2 + v_{ix} \Delta t \qquad \Delta v_x = a_x \Delta t \qquad a = \frac{v^2}{R}$$

$$F = \mu_k F_N \qquad F \le \mu_s F_N$$

$$W = mg \qquad F = k\,\Delta x$$

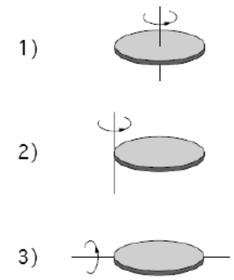
$$E = \frac{1}{2} m v^2 \qquad PE = mgh$$

$$PE = \frac{1}{2} k \Delta x^2 \qquad I = mR^2$$

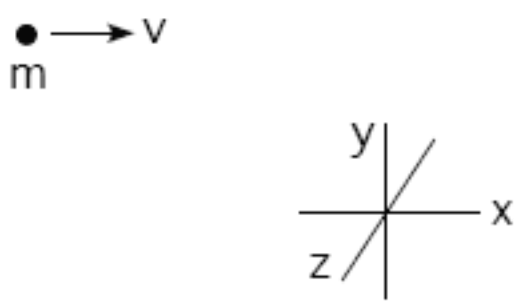
$$s = R\theta \qquad v = R\omega$$

$$a = R\alpha \qquad RKE = \frac{1}{2} I \omega^2$$

Multiple choice questions (5 points each)



1. For which rotational axis of the three shown does the disc have the smallest moment of inertia?
   A. 1
   B. 2
   C. 3
   D. 1 and 3
   E. All three have the same I



2. In the above situation, a particle is moving in the x-y plane with a constant velocity. The magnitude of its angular momentum L relative to the origin
   A. increases then decreases
   B. decreases then increases
   C. remains constant
   D. is zero because this is not circular motion
   E. none of the above

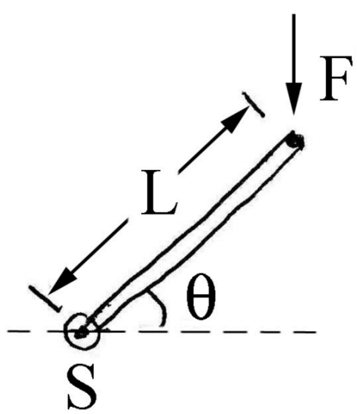
3. A person sits at rest on a stool whose seat can rotate with very little friction. This person is then handed a spinning bicycle wheel and holds it by the axle such that when viewed from above, the wheel appears to spin clockwise in a horizontal plane. The person then flips the wheel over so that when viewed from the top, the wheel appears to spin counter-clockwise in a horizontal plane. As a result
   A. the person on the stool begins to rotate clockwise as viewed from above
   B. the person on the stool begins to rotate counter-clockwise as viewed from above
   C. the direction of rotation of the person depends on in which direction the wheel was flipped over
   D. the person remains at rest
   E. this choice is incorrect, so don't choose it.

4. A figure skater stands on one spot on the ice (with negligible friction) and spins around with her arms held in. She then extends her arms, changing her rotational speed. During this process, her rotational kinetic energy

A. increases
B. decreases
C. remains the same
D. the answer depends on how far she extends her arms
E. none of the above

5. A force of magnitude F is applied to one end of a lever of length L as shown at right. What is the magnitude of the torque about the point S?

A. F L sin θ
B. F L cos θ
C. F L tan θ
D. F L
E. None of the above



Problems (25 points each)

1. You are helping to design a display at a toy store and decide to build a suspended track on which a toy train will run. The track will be a horizontal circle hung from three thin wires attached to a pivot on the ceiling so that it can rotate freely. The train is started from rest and accelerates without slipping to a final speed of 1.0 m/s relative to the track. Because the store owner is worried about the possibility of an accident where the train jumps off the track and falls on a customer, you are asked to find its final speed relative to the floor. The mass of the train is 400 g and the mass of the track is 2 kg. The radius of the circular track is 1.5 m.

2. You are helping to design ladders for a hardware company and have been asked to make sure that the rubber feet of the ladder are sufficiently skid-proof to make the ladder safe. One particular model has a length of 20 feet when extended and weighs 40 lb. If the coefficient of friction between the rubber feet of the ladder and a typical floor is 0.6, what is the maximum angle from the vertical at which the ladder can be leaned against the wall and still be safe for a 250 lb. person to climb to a maximum height of 17 ft up the ladder. For simplicity, you decide to ignore friction between the ladder and the wall.

Answers to Multiple Choice Questions

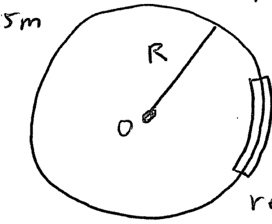
1. **C**
2. **C**
3. **A**
4. **B**
5. **B**

Solutions to Long Problems:

FOCUS the PROBLEM Picture and Given Information

Before train starts (top view)

$R = 1.5\text{ m}$



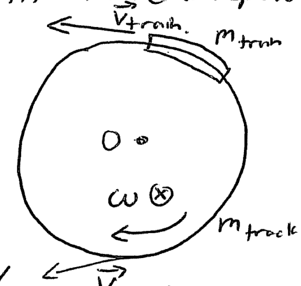
$m_{train} = 400\text{ g}$

$m_{track} = 2\text{ kg}$

Since train travels 1.0 m/s relative to track,

$v_{train} + v_{track} = v = 1.0\text{ m/s}$

After train @ full speed



Moving train makes suspended track rotate in opposite direction. Actual speed of train relative to floor should be less than 1.0 m/s.

Question(s)

How fast is train going relative to floor?

Approach

Cons. of Ang. Momentum. System: Track + train. Origin at center of track

$L_i = 0$

$L_f = L_{train} + L_{track}$

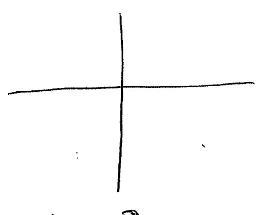
$L_{in}, L_{out} = 0$

Assumptions

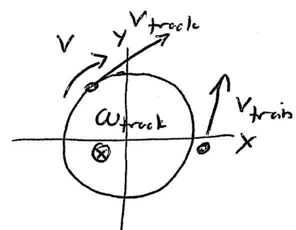
- Ignore all friction and dissipative forces.
- Treat track as thin hoop.

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities



no xfer



$I_{track} = m_{track} R^2$ (all mass at rim)

$\omega_{track} = \frac{v_{track}}{R}$ ← tangential v of a point on track.

$L_i = 0$

$$L_f = L_{track} + L_{train}$$
$$= -I_{track}\omega + m_{train} v_{train} R$$
$$= -m_{track} R^2 \left(\frac{v_{track}}{R}\right) + m_{train} v_{train} R$$

Target Quantity(ies)

v_{train} (speed of train relative to ground)

Quantitative Relationships

$$L_f - L_i = L_{in} - L_{out}$$

$$-m_{track} R^2 \left(\frac{v_{track}}{R}\right) + m_{train} v_{train} R - 0 = 0 - 0$$

$$-m_{track} v_{track} + m_{train} v_{train} = 0$$

2 - 40

also $v_{track} + v_{train} = v = 1.0\text{ m/s}$ since they move in opposite directions

PLAN the SOLUTION
Construct Specific Equations

Find v_{train}

$$-m_{track} v_{track} + m_{train} v_{train} = 0$$

Find v_{track}

$$v_{track} + v_{train} = v$$

$$v_{track} = v - v_{train}$$

$$\rightarrow -m_{track}(v - v_{train}) + m_{train} v_{train} = 0$$

$$-m_{track} v + m_{track} v_{train} + m_{train} v_{train} = 0$$

$$v_{train} = \frac{m_{track}\, v}{m_{train} + m_{track}}$$

Check Units

$$v_{train} = \frac{[kg]\,[m/s]}{[kg]} = m/s$$

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$v_{train} = \frac{(2.0\ kg)(1.0\ m/s)}{(2.0\ kg + 0.4\ kg)}$$

$$v_{train} = 0.83\ m/s$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes, v_{train} has units of speed.

Is Answer Unreasonable?

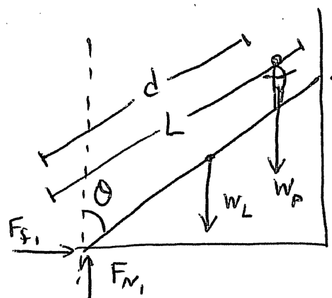
No. It is smaller than 1 m/s. Also, $m_{track} \uparrow$, $v_{train} \uparrow$, as track gets more massive, it moves less and v_{train} is closer to 1 m/s.

Is Answer Complete?

Yes, we found v of train relative to ground.

(extra space if needed)

FOCUS the PROBLEM Picture and Given Information



$L = 20$ ft.
$W_L = 40$ lbs.
$W_p = 250$ lbs.
$d = 17$ ft.
$\mu = 0.6$

Assumptions:

- Person's weight is applied 17ft up the ladder
- No bending of ladder
- Person is just standing
- Friction with wall is negligible.

Question(s)

What is largest angle θ for which the ladder is still in equilibrium?

Approach

Dynamics

System: Ladder + person

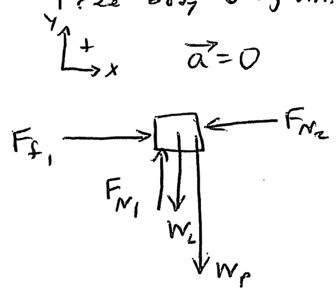
t: Person is standing 17ft up the ladder

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

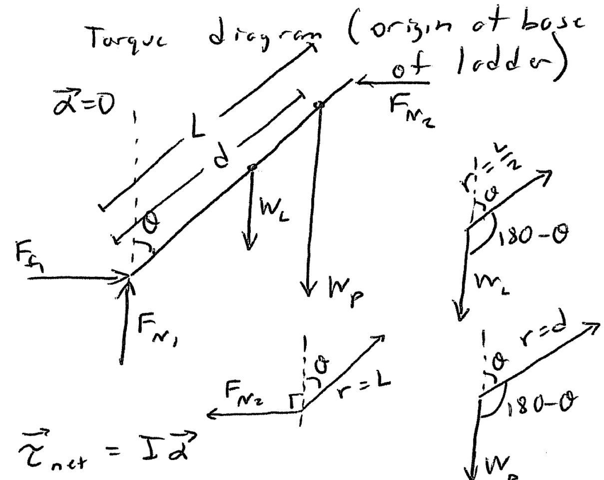
Free-body diagram.

$\vec{a} = 0$



Torque diagram (origin at base of ladder)

$\vec{\alpha} = 0$



Target Quantity(ies)

θ

Quantitative Relationships

$\vec{F}_{net} = m\vec{a}$

$F_{f_1} - F_{N_2} = 0$

$F_{N_1} - W_L - W_p = 0$

$F_{f_1} = \mu F_{N_1}$

$\vec{\tau}_{net} = I\vec{\alpha}$

$-W_L \frac{L}{2} \sin(180 - \theta) - W_p d \sin(180 - \theta) + F_{N_2} L \sin(90 + \theta) = 0$

$-W_L \frac{L}{2} \sin\theta - W_p d \sin\theta + F_{N_2} L \cos\theta = 0$

PLAN the SOLUTION
Construct Specific Equations

Find θ

$$F_{N_2} L\cos\theta - W_L \frac{L}{2}\sin\theta - W_p d\sin\theta = 0$$

Find F_{N_2}

$$\mu F_{N_1} - F_{N_2} = 0$$

Find F_{N_1}

$$F_{N_1} - W_L - W_p = 0$$

$$F_{N_1} = W_L + W_p$$

$$\mu(W_L + W_p) = F_{N_2}$$

$$\mu(W_L + W_p) L\cos\theta - W_L \frac{L}{2}\sin\theta - W_p d\sin\theta = 0$$

$$\mu(W_L + W_p) L\cos\theta = \sin\theta\left(W_L \frac{L}{2} + W_p d\right)$$

$$\tan\theta = \frac{\mu(W_L + W_p)L}{W_L \frac{L}{2} + W_p d}$$

$$\theta = \tan^{-1}\left[\frac{\mu(W_L + W_p)L}{W_L \frac{L}{2} + W_p d}\right]$$

Check Units

θ = radians OK

argument of $\tan^{-1}$ should be dimensionless. $\frac{[N][m]}{[N][m]} = 1$ ✓

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$\theta = \tan^{-1}\left[\frac{(0.6)(290\text{ lbs.})(20\text{ ft})}{(40\text{ lbs.})(10\text{ ft}) + (250\text{ lbs.})(17\text{ ft})}\right]$$

$$\theta = \tan^{-1}(0.748)$$

$$\theta = 36.8^{\circ}$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes, θ is an angle.

Is Answer Unreasonable?

The max angle is about 37° from the vertical. This seems reasonable for a ladder.

Is Answer Complete?

Yes. This is the max angle for a person standing 17ft. up the ladder. If the person

(extra space if needed)

is moving, his/her effective weight will be larger and the answer may change.

Other checks

$\mu \uparrow$, $\theta \uparrow$ more friction, can put the ladder more horizontal

Final Exam 12/21

Instructions:

1. Put your name and student ID# on the scantron sheet.
2. Answer the 12 multiple choice questions on the scantron sheet.
3. Put your problem solutions on the paper provided.
4. Start each problem on a new sheet.
5. Fill out the information at the top of every sheet that you use.

Ground rules:

1. This is a closed book, closed notes quiz. Calculators are permitted.
2. Calculations **MUST** begin with one of the mathematical relationships on the back of this sheet.
3. Mathematics must begin with one of the fundamental relationships on the back of this page.
4. Don't plug in numbers until the very end.

Grading rubric:

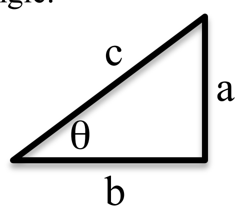
Your solution should include:

- A clear re-description of the problem that helps you to solve it and communicates what you are doing.
- An explicit statement of the physics principles and assumptions used in solving the problem.
- The solution should follow a logical progression and be easily followed by someone else.
- The answer should be explicitly evaluated for obvious signs of incorrectness in at least 2 ways.

Common conversion factors and constants

1 mile = 5280 ft. $g = 9.8\ m/s^2 = 32\ ft/s^2$ 1 m = 3.28 ft.
9.8 N = 2.2 lbs.

Useful Mathematical Relationships
For a right triangle:



$\sin\theta = \frac{a}{c},\ \cos\theta = \frac{b}{c},\ \tan\theta = \frac{a}{b},\quad a^2 + b^2 = c^2,\ \ \sin^2\theta + \cos^2\theta = 1$

For a circle: $C = 2\pi R,\quad A = \pi R^2$ For a sphere:
$A = 4\pi R^2,\quad V = \frac{4}{3}\pi R^3$

sin(A+B) = sin A cos B + cos A sin B cos(A+B) = cos A cos B – sin A sin
B

If $Ax^2 + Bx + C = 0$, then $x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$\vec{A} \bullet \vec{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$ $\vec{A} \times \vec{B} = AB\sin\theta\hat{n}$

Fundamental definitions

$\vec{v} = \frac{d\vec{r}}{dt}\ \left(v_x = \frac{dx}{dt}\right),\qquad \vec{a} = \frac{d\vec{v}}{dt}\ \left(a_x = \frac{dv_x}{dt}\right)$

$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}\ \left(v_{avg,x} = \frac{\Delta x}{\Delta t}\right),\qquad \vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}\ \left(a_{avg,x} = \frac{\Delta v_x}{\Delta t}\right)$

$speed_{avg} = \frac{\text{distance traveled}}{\Delta t}$

$\vec{F}_{net} = \Sigma F = m\vec{a}$ $E_f - E_i = E_{in} - E_{out}$ $\vec{p}_f - \vec{p}_i = \vec{p}_{in} - \vec{p}_{out}$
$\vec{L}_f - \vec{L}_i = \vec{L}_{in} - \vec{L}_{out}$

$E_{in}, E_{out} = \left|\int \vec{F} \bullet d\vec{l}\right|$ $\vec{p}_{in}, \vec{p}_{out} = \int \vec{F}\,dt$
$\vec{L}_{in}, \vec{L}_{out} = \int \vec{\tau}\,dt = \int \left(\vec{r} \times \vec{F}\right) dt$

$\vec{p} = m\vec{v}$

$\vec{\tau} = \vec{r} \times \vec{F}$

$\vec{L} = \vec{r} \times \vec{p} = I\vec{\omega}$

Under certain conditions:

$\Delta x = \frac{1}{2} a_x (\Delta t)^2 + v_{ix} \Delta t$

$\Delta v_x = a_x \Delta t$

$a = \frac{v^2}{R}$

$x_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$

$F = \mu_k F_N$

$F \le \mu_s F_N$

$W = mg$

$F = k\,\Delta x$

$KE = \frac{1}{2} mv^2$

$PE = mgh$

$PE = \frac{1}{2} k \Delta x^2$

$I = mR^2$

$s = R\theta$

$v = R\omega$

$a = R\alpha$

$RKE = \frac{1}{2} I\omega^2$

$\omega = \sqrt{\frac{k}{m}}$

$\omega = \sqrt{\frac{g}{L}}$

$\omega = 2\pi f = \frac{2\pi}{T}$

$x[t] = A\sin(Bt + C),\ A\cos(Bt + C),\ A\sin(Bt) + C\cos(Bt)$

Multiple choice questions (2 points each)

1. The position of a particle, in meters, is given by $y = 3.0t^2 + 2.0t + 5.0$. What is the acceleration of this particle at t = 3.00 s?

(a) 0
(b) 2.0 m/s^2
(c) 6.0 m/s^2
(d) 20.0 m/s^2
(e) 38.0 m/s^2

2. Which of the following statements is true, neglecting wind resistance and the curvature of the Earth?
I. If a ball is thrown upward at 2.00 m/s, when it returns to the height at which it is thrown, it will have a velocity greater than 2.00 m/s.
II. A projectile dropped vertically from the edge of a cliff will hit the ocean below the cliff before a similar projectile fired horizontally from the same point hits the ocean.
III. A projectile fired with an initial velocity of v_0 at an angle of 30° above the horizontal has the same range as a projectile fired with the same initial velocity at an angle of 60° above the horizontal.

(a) Statement I only
(b) Statement II only
(c) Statement III only
(d) Statements I and II
(e) Statements II and III

3. You take a karate class and learn how to shatter a concrete block with your fist. Assume that your fist has a mass of 0.70 kg, that it is moving at 5.0 m/s just before impacting the block and that it stops within 6.0 mm of the point of contact. What is the average force exerted by the block on your fist?

(a) 970 N
(b) 1110 N
(c) 1240 N
(d) 1315 N
(e) 1460 N

4. In a skid test, a recent model BMW 530xi was able to travel in a circle of radius 45.7 m in 15.2 s without skidding. Assuming the car's speed was constant, what is the minimum value of the coefficient of static friction between the tires of the car and the road?

(a) 0.561
(b) 0.603
(c) 0.692

(d) 0.717
(e) 0.797

5. In the Twister ride at an amusement park, the cars travel through a full loop of radius *r* and at one point the riders are completely upside down. What is the minimum velocity required for the car in order to keep an unbelted upside down rider from falling out?

(a) gr
(b) gr^2
(c) $(gr)^{1/2}$
(d) g^2r
(e) $gr^{1/2}$

6. Suppose that a light block and a heavy block are sliding towards you on a frictionless table. Both blocks have the same momentum and you exert the same force to stop each block. Which of the following statements is true?

(a) It takes a longer distance and a longer time to stop the light block.
(b) It takes a longer distance to stop the light block, but the time to stop both blocks is the same.
(c) It takes a longer distance and a longer time to stop the heavy block.
(d) It takes a longer distance to stop the heavy block, but the time to stop both blocks is the same.
(e) It takes a longer distance and a shorter time to stop the heavy block.

7. A particle of mass 2.00 kg experiences a force (in Newtons) $F(x) = 2x + 1$. The work done by this force moving this particle from $x = 1.00$ m to $x = 2.00$ m is

(a) 1.00 J.
(b) 2.00 J.
(c) 3.00 J.
(d) 4.00 J.
(e) 5.00 J.

8. A constant force is exerted on a cart that is initially at rest on an air track. Friction between the cart and the track is negligible. The force acts for a short time interval and gives the cart a certain final speed. To reach the same speed with a force that is only half as big, that force must be exerted on the cart for a time interval that is

(a) four times as long.
(b) twice as long.
(c) equal.
(d) half as long.
(e) a quarter as long.

9. Rank the following torques from largest to smallest.

I: A 20 N force applied at 90° to the end of a 0.2 m long wrench
II. A 20 N force applied at 90° to the end of an 0.15 m long wrench
III. A 20 N force applied at 60° to the end of a 0.2 m long wrench
IV. A 40 N force applied at 90° to the end of a 0.1 m long wrench

(a) IV > I > III > II
(b) IV > II > I > III
(c) I > IV > II > III
(d) I = IV > II > III
(e) I = IV > III > II

10. A merry-go-round of radius 2.0 m and moment of inertia 500 kg-m^2 is rotating about a frictionless pivot, making one revolution every 5.0 s. A child of mass 25 kg originally standing in the center of the merry-go-round walks out to the rim. The length of time now required for the merry-go-round to make a complete revolution is

(a) 4 s.
(b) 5 s.
(c) 6 s.
(d) 7 s.
(e) 8 s.

11. A mass with a weight of 8.00 N is suspended asymmetrically by two wires attached to a flat ceiling. One wire makes an angle of 30.0° with the ceiling. The other wire makes an angle of 60.0° to the ceiling. What is the force exerted by the mass on the wire with the larger tension?

(a) 2.67 N
(b) 4.00 N
(c) 5.45 N
(d) 6.93 N
(e) 7.47 N

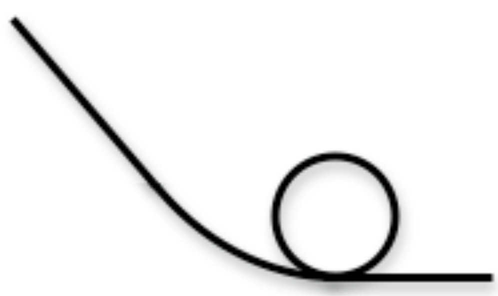
12. A 3.0 kg mass attached to a spring oscillates with an amplitude of 4.0 cm and a period of 2.0 s. What is the maximum speed of the object?

(a) 0.02 m/s
(b) 0.13 m/s
(c) 0.39 m/s
(d) 2.0 m/s
(e) 12.6 m/s

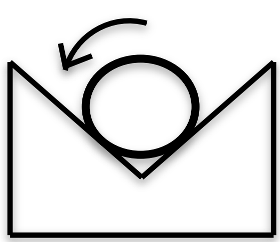
Problems (25 points each)

1. You are designing a projectile for a fireworks display. After being launched from the ground, you wish the projectile to explode into a shower of sparklers at the maximum height of its trajectory, 3.5 seconds after launch, directly above a point on the ground that is 125 feet away from its launch point. Find the necessary speed and the angle above the horizontal at which the projectile must be launched. Assume that air resistance can be neglected.

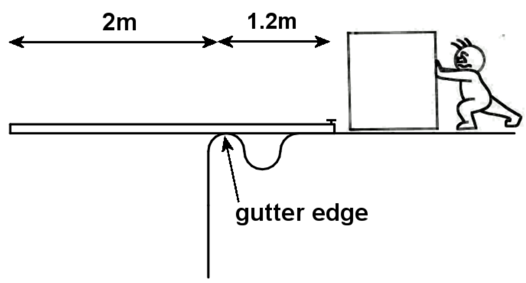
2. You are designing a track for an ice skating show. Starting from rest, an ice skater will slide down an ice-covered ramp and then through the inside of a vertical loop as shown in the picture. The vertical circular loop will have a diameter of 30 feet. Because you are worried about the structural integrity of the loop, you would like to determine the conditions required for the skater to make it through the loop safely with the loop remaining intact. Specifically, what is the maximum vertical height from the floor to the top of the starting ramp so that the skater exerts a force on the top of the loop that is no larger than the skater's weight? You may neglect friction.



4. A 200 lb. shaft consisting of a long solid cylinder 1.2 ft. in diameter rests in a V-shaped groove with both of its walls angled at 45° from the horizontal as shown. How much torque must be supplied by a motor to keep the shaft turning at a constant angular velocity? The contact surface between the shaft and the groove is well-lubricated so the coefficient of kinetic friction between the two surfaces is only 0.20. The moment of inertia of the cylinder is ½ of what it would be if all its mass were concentrated at its rim.



4. A stiff, massless plank is placed across a gutter on a roof and sticks out 2.0 meters to the left from the edge of the gutter. The right end of the plank is permanently attached to the roof 1.2 meter from the gutter edge by a nail. A 75 kg person pushes a 100 kg box out onto the left end of the plank, nails it to the plank, and sits on it.

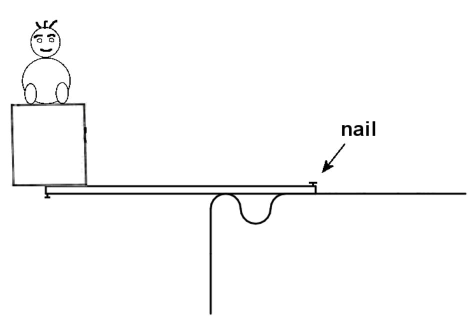


(a) What is the force (magnitude and direction) **on the edge of the gutter** due to the plank and its cargo.

(b) What is the force (magnitude and direction) **on the nail** holding down the right end of the plank due to the plank and its cargo?

Our person notices that the left end of the plank deflected downward 2.0 cm as he moved the box and himself out to the end. He then jumps off and notices the box oscillating up and down.

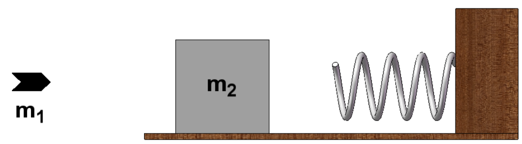
(c) What is the natural frequency (cycles per second) of the motion of the box?



5. A wooden block with mass m_2 rests on a frictionless surface to the left of a spring. A bullet of mass m_1 is fired with velocity v into the block where it sticks inside. The block then travels to the right and hits a massless spring with spring constant k, and sticks to the end of the spring.

(a) What is the period of oscillation for the block stuck on the end of the spring? Express your answer in terms of m_1, m_2, v, k, and g.

(b) What is the amplitude of the oscillation? Express your answer in terms of m_1, m_2, v, k, and g.

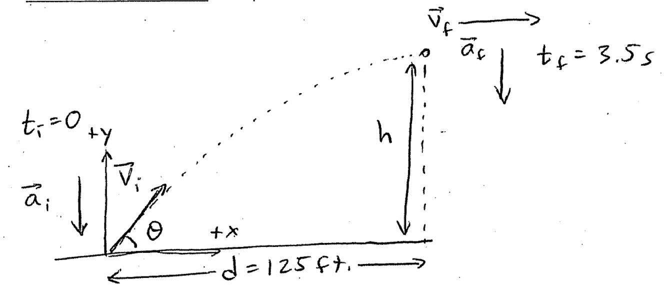


Answers to Multiple Choice Questions

1. **C**
2. **C**
3. **E**
4. **E**
5. **C**
6. **B**
7. **D**
8. **B**
9. **E**
10. **C**
11. **D**
12. **B**

Solutions to Long Problems:

FOCUS the PROBLEM Picture and Given Information



Assumptions:

- Neglect air resistance
- Accel. is constant

Question(s)

What is the initial velocity (magnitude + direction) of the projectile?

Approach

Kinematics

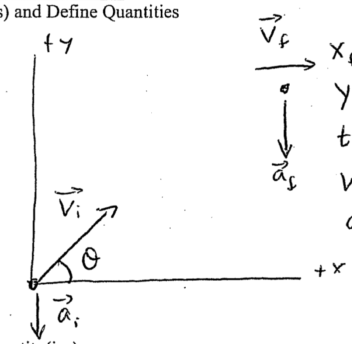
System: projectile.

t_i = just after launch

t_f = top of trajectory

DESCRIBE the PHYSICS
Diagram(s) and Define Quantities

$x_i = 0$
$y_i = 0$
$t_i = 0$
$v_i = ?$
$a_i = g$



$x_f = d$
$y_f = h$
$t_f = 3.5s$
$v_f = ? = v_{ix}$
$a_f = g$

$v_{ix} = v_i \cos\theta$
$v_{iy} = v_i \sin\theta$

Target Quantity(ies)

v_i, θ

Quantitative Relationships

$a = \frac{dv}{dt} \Rightarrow v_f - v_i = a(t_f - t_i)$ x: $v_{fx} = v_i \cos\theta$

y: $0 - v_i \sin\theta = (-g)\, t_f$

$v = \frac{dr}{dt} \Rightarrow x_f = \frac{1}{2}a(t_f - t_i)^2 + v_{ix}(t_f - t_i) + x_i$ x: $d = v_i \cos\theta\, t_f$

y: $h = \frac{1}{2}(-g)t_f^2 + v_i \sin\theta\, t_f$

$125 ft = 38.1 m$

PLAN the SOLUTION
Construct Specific Equations

Unknowns v_{fx}, v_i, θ, h

Find v_i

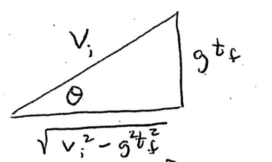
$d = v_i \cos\theta \, t_f$

Find θ

$-v_i \sin\theta = -g t_f$

$\theta = \sin^{-1}\left(\frac{g t_f}{v_i}\right)$

$d = t_f v_i \cos\left[\sin^{-1}\left(\frac{g t_f}{v_i}\right)\right]$



$d = t_f v_i \left[\frac{\sqrt{v_i^2 - g^2 t_f^2}}{v_i}\right] = t_f$

$d^2 = t_f^2 (v_i^2 - g^2 t_f^2)$

$t_f^2 v_i^2 - g^2 t_f^4 - d^2 = 0$

Check Units

$v_i = \frac{\pm\sqrt{4(t_f^2)(g^2 t_f^4 + d^2)}}{2 t_f^2}$

Use + root for a physically meaningful answer.

$v_i = \sqrt{g^2 t_f^2 + \frac{d^2}{t_f^2}}$

Check units. $v_i = \sqrt{\left[\frac{m^2}{s^4}\right]\left[s^2\right] + \frac{[m^2]}{[s^2]}} = \frac{m}{s}$ ✓

EXECUTE the PLAN
Calculate Target Quantity(ies)

$v_i = \sqrt{g^2 t_f^2 + \frac{d^2}{t_f^2}} = \sqrt{(9.8 m/s^2)^2 (3.5 s)^2 + \frac{(38.1 m)^2}{(3.5 s)^2}}$

$= 36 m/s$

$\theta = \sin^{-1}\left(\frac{(9.8 m/s^2)(3.5 s)}{(36 m/s)}\right) = 72.4°$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes. v_i has units of speed and θ is in degrees.

Is Answer Unreasonable?

No. θ is between 0 and 90° and 36 m/s seems reasonable for a projectile.

Is Answer Complete?

Yes, all parts have been answered.

(extra space if needed)

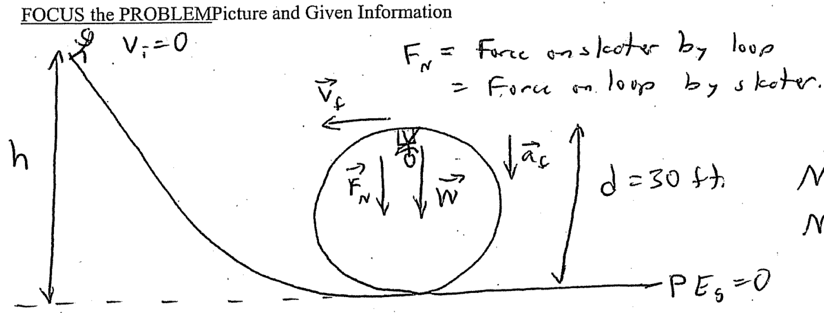
Other checks on v_i

as $d \uparrow, v_i \uparrow$ (need more initial speed to go further)

as $g \uparrow, v_i \uparrow$ (more grav. pull, need more initial speed)

2 - 41

FOCUS the PROBLEM Picture and Given Information



Assumptions:
Neglect air drag
Neglect friction

Question(s)

From what max height can skater start so that max force on loop is $\leq W$?

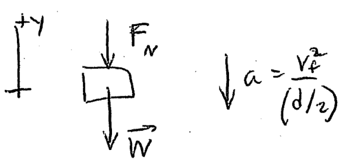
Approach

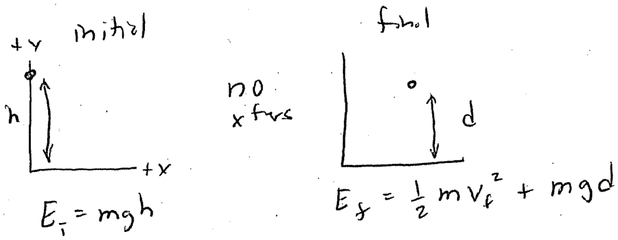
Dynamics
System: Skater
t: At top of loop.

Conservation of Energy
System: Skater + Earth
t_i = top of ramp
t_f = top of loop.

E_i = grav. pot. E
E_f = grav. pot E + KE
$E_{in} = E_{out} = 0$

DESCRIBE the PHYSICS
Diagram(s) and Define Quantities





Target Quantity(ies) h

Quantitative Relationships

Dynamics: $-F_N - W = -m\left(\frac{v_f^2}{(d/2)}\right)$ want $F_N \leq mg$

$$-mg - mg = -m\left(\frac{2v_f^2}{d}\right) \Longrightarrow g = \frac{v_f^2}{d}$$

Energy: $\left(\frac{1}{2} m v_f^2 + mgd\right) - mgh = 0 \Longrightarrow \frac{1}{2} v_f^2 + gd - gh = 0$

PLAN the SOLUTION
Construct Specific Equations

Find h

$\frac{1}{2} v_f^2 + g d - g h = 0$

Find v_f

$g = \frac{v_f^2}{d}$

$v_f = \sqrt{g d}$

$\frac{1}{2}(g d) + g d - g h = 0$

$\frac{3}{2} d = h$

h and d have same units, so units OK.

Check Units

EXECUTE the PLAN
Calculate Target Quantity(ies)

$h = \frac{3}{2} d = \frac{3}{2}(30 \text{ ft.}) = 45 \text{ ft.}$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes, h has units of ft.

Is Answer Unreasonable?

h is somewhat taller than 30 ft., but not excessively. This seems OK. Also, as $d\uparrow$, $h\uparrow$.

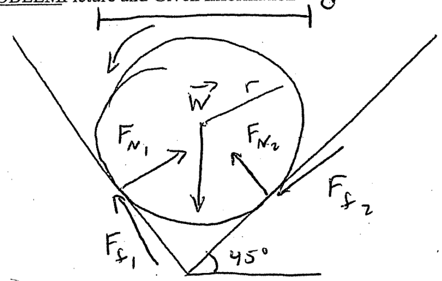
Is Answer Complete?

Yes, question is answered

(extra space if needed)

FOCUS the PROBLEM Picture and Given Information

$d = 1.2\ ft. = 2r$



$W = 200\ lbs.$

$\mu = 0.2$

Question(s)

What torque is necessary to keep cyl. turning @ constant ω?

Approach

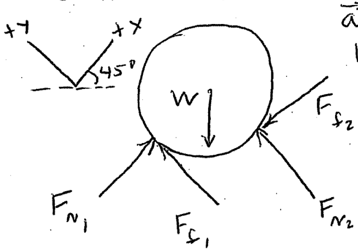
Dynamics

System: Shaft

t: While turning.

DESCRIBE the PHYSICS

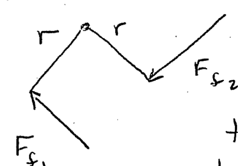
Diagram(s) and Define Quantities



$\vec{a} = 0$

$W = 200\ lbs.$

Torques - center of shaft is origin



applied torque must be equal to torques due to friction

Target Quantity(ies)

τ

Quantitative Relationships

$$F_{N_1} - F_{f_2} - W\sin\theta = 0$$

$$F_{f_1} + F_{N_2} - W\cos\theta = 0$$

$$F_{f_1} = \mu F_{N_1}$$

$$F_{f_2} = \mu F_{N_2}$$

$$\tau - rF_{f_1} - rF_{f_2} = 0$$

PLAN the SOLUTION
Construct Specific Equations

Find τ

$$\tau - \left(\frac{d}{2}\right)\mu F_{N_1} - \left(\frac{d}{2}\right)\mu F_{N_2} = 0$$

Find F_{N_1}

$$F_{N_1} - \mu F_{N_2} - W\sin\theta = 0$$

Find F_{N_2}

$$\mu F_{N_1} + F_{N_2} - W\cos\theta = 0$$

$$F_{N_2} = W\cos\theta - \mu F_{N_1}$$

$$F_{N_1} - \mu\left[W\cos\theta - \mu F_{N_1}\right] - W\sin\theta = 0$$

$$F_{N_1} + \mu^2 F_{N_1} = W\sin\theta + \mu W\cos\theta$$

$$F_{N_1} = \frac{W(\sin\theta + \mu\cos\theta)}{1+\mu^2}$$

$$F_{N_2} = \frac{W(\cos\theta - \mu\sin\theta)}{1+\mu^2}$$

$$\tau = \frac{d}{2}\mu\left[F_{N_1} + F_{N_2}\right]$$

Check Units

$$= \frac{d\mu}{2}\left[\frac{W\sin\theta + \mu W\cos\theta + W\cos\theta - \mu W\sin\theta}{1+\mu^2}\right]$$

$$\tau = \frac{Wd\mu}{2}\left[\frac{\sin\theta + \mu\cos\theta + \cos\theta - \mu\sin\theta}{1+\mu^2}\right]$$

for $\theta = 45°$, $\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$

$$\tau = \frac{Wd\mu}{\sqrt{2}(1+\mu^2)}$$

units $\tau = [N][m]$

units ok.

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$\tau = \frac{(200\text{ lbs})(1.2\text{ ft.})(0.2)}{\sqrt{2}\,(1 + (0.2)^2)}$$

$$= 32.6\text{ ft}\cdot\text{lb.}$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes, torque has units of force - length.

Is Answer Unreasonable?

Answer does not seem unreasonably large or small. If $W\uparrow$, $\tau\uparrow$ and if $\mu\uparrow$, $\tau\uparrow$ which seem correct.

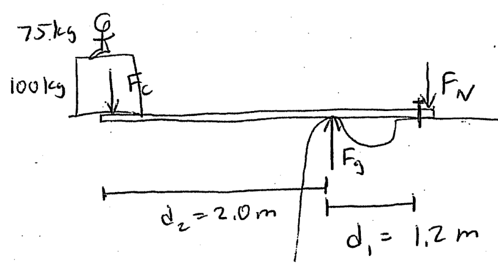
Is Answer Complete?

Yes, we've calculated the necessary torque.

(extra space if needed)

2-41

FOCUS the PROBLEM Picture and Given Information



F_c - force on plank by cargo

F_N - force on plank by nail

F_g - force on plank by gutter

Assumptions

- Plank is massless.

Question(s) (a) What is force on gutter due to plank? (Same magnitude as F_g) opposite direction

(b) What is force on nail due to plank? (Same magnitude as F_N) opposite direction

(c) What is frequency of the motion of the box?

Treat box like mass on spring.

$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m_{box}}}$ We know m, use Dynamics to find equivalent k.

Approach

Dynamics

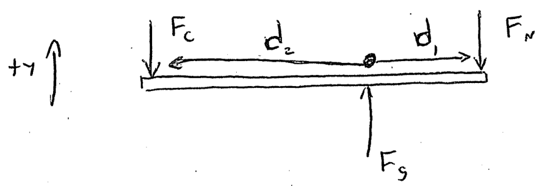
System: Plank.

t: Person sitting on box already

DESCRIBE the PHYSICS

Diagram(s) and Define Quantities

$a = 0$



Since no motion or accel,

$F_c = W_{box + person}$

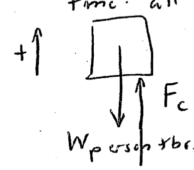
Take gutter edge as origin for rotations.

(c) Dynamics.

System: Person + box.

time: all at rest, plank deflected down $\Delta x = 2\,cm$.

Treat plank as spring.



$F_c - W_p = 0$

$k\Delta x - W_p = 0$

$k\Delta x = mg$ — person + box

Target Quantity(ies)

F_g, F_N

Quantitative Relationships

$F_g - F_c - F_N = 0$

$d_2 F_c - d_1 F_N = 0$

PLAN the SOLUTION
Construct Specific Equations

Find F_g

$$F_g - F_c - F_N = 0$$

Find F_N

$$d_2 F_c - d_1 F_N = 0$$

$$F_N = \frac{d_2}{d_1} F_c$$

units: $F_N = \frac{[m]}{[m]}[N] = [N]$ ok✓

$$F_g = F_c + F_N = F_c + \frac{d_2}{d_1} F_c$$

$$F_g = F_c\left(1 + \frac{d_2}{d_1}\right)$$

$F_g = [N] * \left[\frac{[m]}{[m]}\right] = [N]$ ✓ ok.

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$F_N = \frac{d_2}{d_1} F_c = \frac{(2.0\,m)}{(1.2\,m)}(175\,kg)(9.8\,m/s^2)$$

$$\boxed{F_N = 2\,858\ N\ \text{up}}$$

$$F_g = (175\,kg)(9.8\,m/s^2)\left(1 + \frac{(2.0\,m)}{(1.2\,m)}\right)$$

$$\boxed{F_g = 4573\ N\ \text{down}}$$

~~EVALUATE the ANSWER~~
Is Answer Properly Stated?

Yes, Forces have units of N, and directions are specified.

Is Answer Unreasonable?

Answers are of same order of magnitude as weight of person + box. Seems not unreasonable. Also, as $d_2 \uparrow$, forces also $\uparrow$ which makes sense.

Is Answer Complete?

Yes, magnitude + direction are given. Using Newton's third law, we found forces on the nail and on the gutter from the forces they exert on the plank.

(extra space if needed)

(c) Find f

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_{box}}}$$

Find k

$$k\Delta x = mg \quad \text{(person + box)}$$

$$k = \frac{mg}{\Delta x}$$

Check Units

$$f = \frac{1}{2\pi}\sqrt{\frac{mg}{\Delta x\, m_{box}}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{m_{person+box}\, g}{\Delta x\, m_{box}}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{(175\,kg)(9.8\,m/s^2)}{(0.02\,m)(100\,kg)}}$$

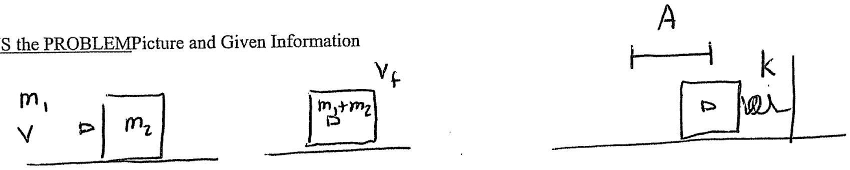
units $\rightarrow \sqrt{\frac{kg\ m/s^2}{m\ kg}} = \sqrt{\frac{1}{s^2}} = \frac{1}{s}$

$$\boxed{f = 4.66\ \text{cycles}/s}$$

answer has correct units, answers the question. 5 vibrations/s seems not unreasonable.

2-41

FOCUS the PROBLEM Picture and Given Information



Assumptions:
- Surface is frictionless
- Spring is massless.

Question(s) (a) What is period of the oscillation?
(b) What is amplitude of the oscillation?

Approach

(a) $\omega = \sqrt{\frac{k}{m}}$

$\rightarrow T = 2\pi\sqrt{\frac{m}{k}}$

(b). Conservation of Momentum

System: bullet + block
i: just before impact
f: just after impact

$p_i = p_{bullet}$ $\quad p_{in}, p_{out} = 0$

$p_f = p_{block}$

Conservation of Energy

System: bullet/block + spring
i: just before block hits spring
f: spring at max compression

$E_i = KE_{block/bullet}$ $\quad E_{in}, E_{out} = 0$

$E_f = PE_{spring}$

DESCRIBE the PHYSICS
Diagram(s) and Define Quantities

$p_i = m_1 v$ $\qquad p_f = (m_1+m_2)v_f$

$E_i = \frac{1}{2}(m_1+m_2)v_f^2$ $\qquad E_f = \frac{1}{2}kA^2$

max compression of spring = amplitude of oscillation.

Target Quantity(ies) A, T

Quantitative Relationships

momentum: $(m_1+m_2)v_f - m_1 v = 0$

energy: $\frac{1}{2}kA^2 - \frac{1}{2}(m_1+m_2)v_f^2 = 0$

result for mass on spring oscillations.

$T = 2\pi\sqrt{\frac{m}{k}}$

(from $T = \frac{1}{f} = \frac{2\pi}{\omega}$ and $\omega = \sqrt{\frac{k}{m}}$)

PLAN the SOLUTION
Construct Specific Equations

(a) Find T

$$T = 2\pi\sqrt{\frac{m}{k}} \qquad m = m_1 + m_2$$

(b) Find A

$$\frac{1}{2}kA^2 - \frac{1}{2}(m_1+m_2)v_f^2 = 0$$

Find v_f

$$(m_1+m_2)v_f - m_1 v = 0$$

$$v_f = \frac{m_1}{m_1+m_2}v$$

$$kA^2 - (m_1+m_2)\left[\frac{m_1}{(m_1+m_2)}v\right]^2 = 0$$

$$kA^2 = \frac{m_1^2 v^2}{m_1+m_2}$$

$$A^2 = \frac{m_1^2 v^2}{k(m_1+m_2)}$$

$$A = \frac{m_1 v}{\sqrt{k(m_1+m_2)}}$$

Check Units

$$T = \sqrt{\frac{[kg]}{[N/m]}} = \sqrt{[s]^2} = [s] \qquad N = \frac{kg\cdot m}{s^2}$$

ok ✓

$$A = \frac{[kg][m/s]}{\sqrt{\frac{kg\cdot m}{s^2}\frac{1}{m}kg}} = \frac{kg\cdot m/s}{\sqrt{\frac{kg^2}{s^2}}} = m$$

ok ✓

EXECUTE the PLAN
Calculate Target Quantity(ies)

$$T = 2\pi\sqrt{\frac{m_1+m_2}{k}}$$

$$A = \frac{m_1 v}{\sqrt{k(m_1+m_2)}}$$

EVALUATE the ANSWER
Is Answer Properly Stated?

Yes, answers are given in terms of specified quantities.

Is Answer Unreasonable?

T: m↑, T↑. more mass, longer period. k↑, T↓ stiffer spring, faster osc.

A: v↑, A↑ faster bullet more osc. k↑, A↓ stiffer spring, smaller osc.

Is Answer Complete?

Yes, all parts completed.

(extra space if needed)

Appendix 5. Fall 2011 Surveys

Survey for Computer Coaches

Survey for students attempting 10 or more computer coaches on the palweb site.
(https://palweb.spa.umn.edu/student/)
1.Describe what you liked **best** about the computer coaches.

2. Describe what you liked **least** about the computer coaches.

3. Approximately how many of the computer coaches did you attempt during this semester (there were 35 total) ?
A. 10-14
B. 15-19
C. 20-24
D. 25-29
E. 30-35

4. Which of the following statements **best describes** how you completed the homework in this course?
A. I tried to solve the problems on my own and used WebAssign to check the answer. I did not use the computer coaches regularly.
B. I tried to solve the problems on my own and used WebAssign to check the answer. I only used the computer coaches when I ran out of WebAssign submissions.
C. I tried to solve the problems on my own and used WebAssign to check the answer. I used the computer coaches to see another way to solve the problem.
D. I tried to solve the problems on my own and used the computer coaches to check my solution method.
E. I tried to solve the problems on my own and used the computer coaches for help if I got stuck.
F. I worked through the computer coaches before trying to solve the problems on my own.
G. I typically did not do the homework.

5. Which type of computer coach did you find the **most useful at the beginning of the course**?
A. Type 1 (The computer guides you through the decisions to make in the process of solving a problem and checks your work.)
B. Type 2 (You guide the computer through the decisions to make in the process of solving a problem and check the computer's work.)
C. Type 3 (The computer asks you to solve a problem on your own first, then offers help if necessary.)

6. Which type of computer coach did you find the **least useful at the beginning of the course**?
A. Type 1 (The computer guides you through the decisions to make in the process of solving a problem and checks your work.)
B. Type 2 (You guide the computer through the decisions to make in the process of

solving a problem and check the computer's work.)
C. Type 3 (The computer asks you to solve a problem on your own first, then offers help if necessary.)

7. I used the computer coaches
A. Mostly at the beginning of the semester and very little near the end of the semester.
B. More or less evenly throughout the semester.
C. Mostly towards the end of the semester.
D. I mostly did not use the computer coaches.
E. I never used the computer coaches.

8. I worked on the computer coaches
A. when they first became available.
B. spaced out about evenly every few days.
C. in a bunch shortly before they were due.

9. Which of the following **best describes** how you usually worked on the computer coaches?
A. By myself.
B. With other people.

10. Which one of the following **best describes** how you usually worked through a single computer coach?
A. I usually gave it my full attention to complete it in one uninterrupted session.
B. Although I gave it my full attention, I often took one or more breaks of a few minutes.
C. I often interrupted it for a long period of time to do something else before coming back to it and completing it.
D. I was often multitasking to conduct other activities at the same time.

11. Which type of computer coach did you find the **most useful at the end of the course**?
A. Type 1 (The computer guides you through the decisions to make in the process of solving a problem and checks your work.)
B. Type 2 (You guide the computer through the decisions to make in the process of solving a problem and check the computer's work.)
C. Type 3 (The computer asks you to solve a problem on your own first, then offers help if necessary.)

12. Which type of computer coach did you find the **least useful at the end of the course**?
A. Type 1 (The computer guides you through the decisions to make in the process of solving a problem and checks your work.)
B. Type 2 (You guide the computer through the decisions to make in the process of solving a problem and check the computer's work.)
C. Type 3 (The computer asks you to solve a problem on your own first, then offers help if necessary.)

Rank questions 13 – 26 on a scale of A to E with A being strongly agree, B agree, C neither agree nor disagree, D disagree, and E strongly disagree.

13. When using the computer coaches, it was usually clear how to proceed.

14. The **type 1 computer coaches** (computer guides you through the decisions to make in the process of solving a problem and checks your work) were too repetitive.

15. The computer coaches did not help improve my problem-solving.

16. The **type 2 computer coaches** (you guide the computer through the decisions to make in the process of solving a problem and check the computer's work) were useful to me.

17. The computer coaches helped my conceptual knowledge of physics.

18. The **type 3 computer coaches** (computer asks you to solve a problem on your own first, then offers help if necessary.) were too repetitive.

19. The computer coaches helped me identify what I needed to get help with from other sources.

20. The feedback offered by the computer coaches was not helpful.

21. The **type 3 computer coaches** (computer asks you to solve a problem on your own first, then offers help if necessary) were not useful to me.

22. The computer coaches helped my problem solving for classes other than physics.

23. The **type 2 computer coaches** (you guide the computer through the decisions to make in the process of solving a problem and check the computer's work) were too repetitive.

24. The **type 1 computer coaches** (computer guides you through the decisions to make in the process of solving a problem and checks your work) were useful to me.

25. Using the computer coaches for homework made the homework take too long.

26. I intend to work through some of the computer coaches again to help me study for the final exam.

27**. Rank the components of the physics class in order from most useful (1) to least useful (18) to your learning. If you did not use a particular component, then omit it from your ranking.**
Lectures (Monday through Wednesday class sessions)

Discussion Section (Thursday morning problem-solving sessions)
Lab Section
Quizzes (taken Thursday/Friday mornings in class)
Textbook reading
Textbook problems
In-class clicker questions
Writing lab reports
Computer coaches (palweb)
TA help in the Tutor Room (Tate 137)
The Competent Problem Solver supplementary text
Practice quizzes (posted on the physics website)
WebAssign
Posted lecture notes
Your own lecture notes
Studying physics with other students outside of class
Discussions with the professor outside of class
Discussions with your TA outside of class

28. What suggestions do you have for changes that should be made to the computer coaches to make them more useful?

29. What aspect(s) of the computer coaches should **not** be changed no matter what other students say?

Survey for WebAssign

Survey for students attempting fewer than 10 computer coaches on the palweb site.

1.Describe what you liked **best** about WebAssign.

2. Describe what you liked **least** about WebAssign.

3. Approximately how many of the computer coaches did you attempt during this semester (there were 35 total) ?
A. 0-4
B. 5-9

4. Which of the following statements **best describes** how you completed the homework in this course?
A. I tried to solve the problems on my own and used WebAssign to check the answer. I did not use the computer coaches regularly.
B. I tried to solve the problems on my own and used WebAssign to check the answer. I only used the computer coaches when I ran out of WebAssign submissions.
C. I tried to solve the problems on my own and used WebAssign to check the answer. I used the computer coaches to see another way to solve the problem.
D. I tried to solve the problems on my own and used the computer coaches to check my solution method.
E. I tried to solve the problems on my own and used the computer coaches for help if I got stuck.
F. I worked through the computer coaches before trying to solve the problems on my own.
G. I typically did not do the homework.

5. Which aspect of WebAssign did you find the **most useful at the beginning of the course**?
A. The multiple submissions allowed for each homework problem
B. The instant feedback about the correctness of my answer
C. The web forum for posting questions about the homework if WebAssign said my answer was wrong

6. Which aspect of WebAssign did you find the **least useful at the beginning of the course**?
A. The multiple submissions allowed for each homework problem
B. The instant feedback about the correctness of my answer
C. The web forum for posting questions about the homework if WebAssign said my answer was wrong

7. I used WebAssign
A. Mostly at the beginning of the semester and very little near the end of the semester.
B. More or less evenly throughout the semester.
C. Mostly towards the end of the semester.
D. I mostly did not use WebAssign.
E. I never used WebAssign.

8. I worked on the WebAssign homework problems
A. when they first became available.
B. spaced out about evenly every few days.
C. in a bunch shortly before they were due.

9. Which of the following **best describes** how you usually worked on the WebAssign homework?
A. By myself.
B. With other people.

10. Which one of the following **best describes** how you usually worked through a single WebAssign homework problem?
A. I usually gave it my full attention to complete it in one uninterrupted session.
B. Although I gave it my full attention, I often took one or more breaks of a few minutes.
C. I often interrupted it for a long period of time to do something else before coming back to it and completing it.
D. I was often multitasking to conduct other activities at the same time.

11. Which aspect of WebAssign did you find the **most useful at the <u>end</u> of the course**?
A. The multiple submissions allowed for each homework problem
B. The instant feedback about the correctness of my answer
C. The web forum for posting questions about the homework if WebAssign said my answer was wrong

12. Which aspect of WebAssign did you find the **least useful at the <u>end</u> of the course**?
A. The multiple submissions allowed for each homework problem
B. The instant feedback about the correctness of my answer
C. The web forum for posting questions about the homework if WebAssign said my answer was wrong

Rank questions 13 – 26 on a scale of A to E with A being strongly agree, B agree, C neither agree nor disagree, D disagree, and E strongly disagree.

13. When doing the WebAssign homework, it was usually clear how to proceed.

14. The WebAssign homework problems were too repetitive.

15. The WebAssign homework did not help improve my problem-solving.

16. The WebAssign homework was useful to me.

17. The WebAssign homework helped my conceptual knowledge of physics.

18. The feedback from WebAssign was not detailed enough.

19. The WebAssign homework helped me identify what I needed to get help with from other sources.

20. The feedback offered by the WebAssign homework was not helpful.

21. The multiple submissions allowed for each WebAssign homework question were not useful to me.

22. The WebAssign homework helped my problem solving for classes other than physics.

23. Entering equations in WebAssign was too difficult.

24. The web forum for posting questions about the homework if WebAssign said my answer was wrong was useful to me.

25. Using WebAssign for homework made the homework take too long.

26. I intend to work through some of the WebAssign homework problems again to help me study for the final exam.

27**. Rank the components of the physics class in order from most useful (1) to least useful (18) to your learning. If you did not use a particular component, then omit it from your ranking.**
Lectures (Monday through Wednesday class sessions)
Discussion Section (Thursday morning problem-solving sessions)
Lab Section
Quizzes (taken Thursday/Friday mornings in class)
Textbook reading
Textbook problems
In-class clicker questions
Writing lab reports
Computer coaches (palweb)
TA help in the Tutor Room (Tate 137)
The Competent Problem Solver supplementary text
Practice quizzes (posted on the physics website)
WebAssign
Posted lecture notes
Your own lecture notes
Studying physics with other students outside of class
Discussions with the professor outside of class
Discussions with your TA outside of class

28. What suggestions do you have for changes that should be made to the WebAssign homework to make it more useful?

29. What aspect(s) of the WebAssign homework should **not** be changed no matter what

other students say?

Final Exam

This is a closed book, closed notes quiz. Calculators are permitted. The only formulas that may be used are those given below. ***No other equation sheet is allowed.*** If any other modifications to the equations below were made or needed, make sure they are justified physically and explained as part of the solution.

MAKE SURE YOUR NAME, ID #, and TAs NAME ARE ON EACH PAGE TO BE GRADED!

Define all symbols and justify all mathematical expressions used. Please state all of the assumptions used to solve a problem. Credit will be given only for a logical and complete solution that is clearly communicated with correct units. Partial credit will be given for a well communicated problem solving strategy based on correct physics.

Each problem is worth 25 points. For each problem it is expected that a picture or equivalent translation of the problem, a procedure (what is the plan and physics involved), the solution (the solution path and final answer) and a proper evaluation (does it make sense, why or why not) is included. Each of these parts is required for full credit.

Each of the multiple choice questions is worth 5 points each.

Relevant Equations

Kinematics:

$$\Delta\vec{r} = \int_{t_i}^{t} \vec{v}\,(t')dt'$$

$$\Delta\vec{v} = \int_{t_i}^{t} \vec{a}\,(t')dt'$$

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$v = |\vec{v}|$$

$$\vec{a} = \frac{d\vec{v}}{dt}$$

$$x = x_0 + v_{0x}(\Delta t) + \tfrac{1}{2}a_x(\Delta t)^2$$

$$v_x = v_{0x} + a_x\Delta t$$

Dynamics:

$$\sum \vec{F}_{ext} = m\vec{a}$$

$$\vec{F}_{A\to B} = -\vec{F}_{B\to A}$$

$$F_x = -k\Delta x$$

$$f_s \le \mu_s N$$

$$f_{r,k} = \mu_{r,k}N$$

Systems:

$$Mx_{cm} = \sum_i m_i x_i$$

$$M\vec{r}_{cm} = \int \vec{r}\,dm$$

Energy:

$$\Delta E_{\text{system}} = \Delta E_{\text{transfer}}$$

$$\Delta K + \Delta U = W$$

$$P = \frac{dW}{dt}$$

$$K = \frac{1}{2}mv^2$$

$$K_r = \frac{1}{2}I\omega^2$$

$$W = \int_a^b \vec{F}_{\text{net}}\cdot d\vec{l}$$

$$F_x = -\frac{dU}{dx}$$

$$U_s = \frac{1}{2}kx^2$$

$$U_g = mgy + U_0$$

Momentum:

$$\Delta\vec{p}_{\text{system}} = \Delta\vec{p}_{\text{transfer}}$$

$$\sum \vec{F}_{ext} = \frac{d\vec{p}_{sys}}{dt}$$

$$\vec{p} = m\vec{v}$$

$$\vec{I} = \int \vec{F}dt \approx \vec{F}_{\text{avg}}\Delta t$$

Rotations:

$$\Delta\theta = \omega_i(\Delta t) + \tfrac{1}{2}\alpha(\Delta t)^2$$

$$\Delta\omega = \alpha\Delta t$$

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$$I = \int r^2 dm$$

$$I \approx \sum_i m_i r_i^2$$

$$I = I_{CM} + mh^2$$

$$s = r(\Delta\theta)$$

$$v_T = \omega r$$

$$a_T = \alpha r$$

$$a_c = {v^2}/{R}$$

$$\sum \vec{\tau}_{ext} = I\vec{\alpha}$$

$$\vec{\tau} = \vec{r}\times\vec{F}$$

Angular Momentum:

$$\Delta\vec{L}_{\text{system}} = \Delta\vec{L}_{\text{transfer}}$$

$$\vec{L} = \vec{r}\times\vec{p}$$

$$\vec{L} = I\vec{\omega}$$

$$\sum \vec{\tau}_{ext} = \frac{d\vec{L}}{dt}$$

$$\Delta\vec{L}_{\text{transfer}} = \int \vec{\tau}dt$$

Oscillations:

$$\ddot{x} = -\omega^2 x$$

$$x(t) = A\cos(\omega t + \delta)$$

$$f = \frac{1}{T}$$

$$\omega = 2\pi f$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{g}{L}}$$

Constants:

1 mi = 5280 ft

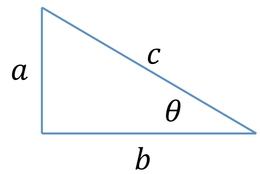
1 m = 3.28 ft

1 yd = 3 ft

1 lb = 4.45 N

$g = 9.81\,\frac{\text{m}}{\text{s}^2} = 32.2\,\frac{\text{ft}}{s^2}$

Math:



For a right triangle:

$\sin\theta = \frac{a}{c}$, $\cos\theta = \frac{b}{c}$,

$\tan\theta = \frac{a}{b}$

$a^2 + b^2 = c^2$,

$\sin^2\theta + \cos^2\theta = 1$

Dot (scalar) product:

$\vec{A}\cdot\vec{B}$

$= A_iB_i + A_jB_j + A_kB_k$

$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta_{AB}$

For a circle:

$C = 2\pi R,\ A = \pi R^2$

For a sphere:

$A_s = 4\pi R^2,\ V = \frac{4}{3}\pi R^3$

For $Ax^2 + Bx + C = 0$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Cross (vector) Product:

$|\vec{A}\times\vec{B}| = |A|B_\perp = A_\perp|B|$

$|\vec{A}\times\vec{B}| = |A||B|\sin\theta_{AB}$

Problem #1 (25 pts)

As the safety engineer for a new roller coaster ride, you are concerned with the safety harness keeping the riders in their seats over a particularly hilly section of the ride. The top of one hill can be approximated as a circular path with a radius of 10 m. When approaching the hill, the roller coaster travels through a flat section going at a speed of 20 m/s. What force (direction and magnitude) must the safety harness supply to the rider when the rider is at the top of the hill? The roller coaster has a weight of 15000 N and the riders have a maximum allowed weight of 1750 N. It is safe to assume that the roller coaster rolls without friction through this section of the ride.

Problem #2 (25 pts)

While hiking, you notice a dead tree trunk 4 meters long resting on level ground. You are curious as to how heavy it is so you try to lift it. You find that discover that a force of 900 N is just enough to lift one end of the tree. Walking to the other end of the fallen tree, you find that a force of 500 N is just enough to lift this other end. Remembering your physics, you realize that you now know enough to calculate the mass of the tree trunk.

Problem #3 (25 pts)

You are in charge of setting up cannons on top of a wall at a fort for a reenactment of a famous battle. In order to make sure the cannons will be safe to fire, you need to determine if the wall is wide enough to accommodate the recoil of the cannon. The cannons, which fire 10 kg cannonballs horizontally with a speed of 200 m/s, are permanently mounted on a wagon. Because the wheels of the wagon are locked to minimize movement, the 3500 kg cannon slides during the firing with a coefficient of kinetic friction between the wheels and the wall a value of 0.6.

Problem #4 (25 pts)

A fireworks company has hired you as a ballistic engineer to help them design a new class of fireworks. Your partner accidently lights a test firework which launches at a speed of $45\,\frac{\text{m}}{\text{s}}$ at the unsafe angle of 30° above horizontal. However, it doesn't fully explode, only breaking into two pieces, both with velocities parallel to the ground, when it reaches the highest point of its trajectory, which happens to be directly above your head. You find one 2 kg piece on the ground 15 m from your position, between you and the launch point. If the total mass of the firework is 3.54 kg, where should you look for the other piece? Because you were worried about this exact situation, your handy stopwatch estimates the time of descent for the piece to be around 2.3 seconds.

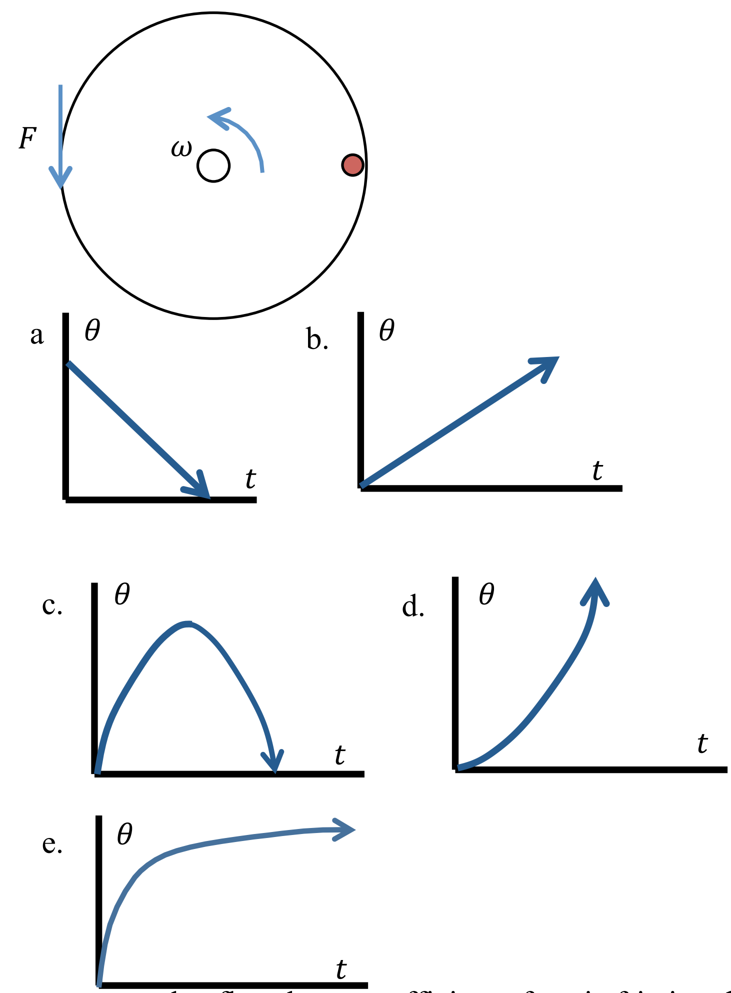
Problem #5 (25 pts)

You are in a research group investigating the mechanisms by which a virus attaches itself to a healthy cell and injects its genetic material into the cell. In the virus you are studying, the head of the virus, shaped roughly like a sphere, is attached to one end of a long, thin tubule. When the virus collides with a cell, the free end of the tubule attaches to the cell and the tubule acts as a spring. Just after the collision, the head of the cell oscillates along the direction of the tubule. You need to determine the maximum speed of the head as it oscillates because you think that this helps the virus inject its genetic material into the cell. From a micro-video of the process, you know the maximum distance of the oscillation from the stable position and the period of the oscillation, but you do not know anything else about the virus.

Multiple Choice

(Please record answers on the provided bubble sheet.)

1. Consider a turntable rotating counterclockwise as seen from above with a continuous force applied as shown in the picture. Which plot best represents the angular position of the filled dot as a function of time?



2. A piano on a wooden floor has a coefficient of static friction that is larger than its coefficient of kinetic friction. A force is applied that is large enough to start the piano sliding. If this same force continues to be applied, the piano will

    a. eventually stop moving.

b. move at constant velocity.
c. speed up at a constant rate.
d. move in a series of jerks.
e. none of the above.

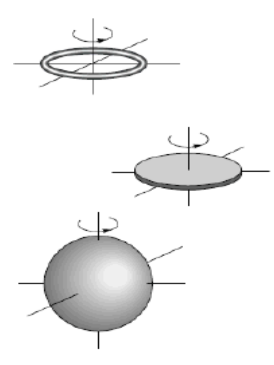
3. Which force(s) perform non-zero work between any two different points during the motion of a simple pendulum, i.e., a mass hanging from a string that swings back and forth?
   a. Tension
   b. Gravitational force
   c. The centripetal force
   d. Both a. and b.
   e. a. b. and c.

4. A ping pong ball and a bowling ball, both with the same momentum, roll toward you. Suppose you
   exert the same amount of force on each ball to stop it. Which ball takes less time to stop?

   a. The ping pong ball
   b. The bowling ball
   c. They both take the same amount of time to stop.
   d. Not enough information to answer the question.

5. The three solid shapes shown have the same mass and radius. Which has/have the smallest moment of inertia about the rotational axis shown?

   a. Thin ring
   b. Disk
   c. Solid sphere
   d. The ring and disk
   e. All three have the same I



6. While standing at the edge of a roof, two players decide to throw two identical baseballs at the same time. Ball A is thrown straight up while Ball B is thrown horizontally with the same initial speed. Ignoring air resistance, which of the following statements are true after the baseballs have left the players' hands?

    a. The acceleration of ball A initially points up.
    b. The acceleration of Ball B is initially horizontal then becomes vertical.
    c. The speed of each baseball just before hitting the ground is identical.
    d. The velocity of each baseball points in the direction as its acceleration.
    e. Both baseballs hit the ground at the same time.

7. Cart A and Cart B collide elastically on a frictionless track. Cart A is initially at rest. If the direction of cart B's final velocity is opposite to its initial velocity, which of the following statements is true?

    a. Cart A added energy to the system of the two carts.
    b. Cart A is less massive than Cart B.
    c. Cart A is more massive than Cart B.
    d. Cart A and Cart B have the same mass.
    e. It depends on the speed of Cart B before the collision.

8. An oscillator consisting of a mass attached to a spring has a maximum velocity of v. If you double both the amplitude and the period, the new maximum velocity is

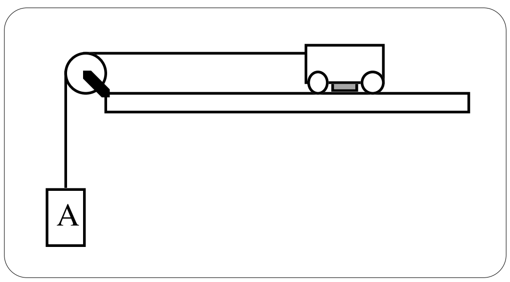
    a. ½ v
    b. v
    c. 2v
    d. 4v
    e. None of the above

9. A skydiver with an open parachute has a relatively safe terminal velocity. When the skydiver is at terminal velocity, which of the following statements is true?

    a. Gravity no longer does work on the skydiver.
    b. The energy output by the drag force is larger than the energy input by gravity.
    c. The energy output by the drag force is equal to the energy input by gravity.
    d. The energy output by the drag force is less than the energy input by gravity.
    e. Cannot determine the answer with the provided information.

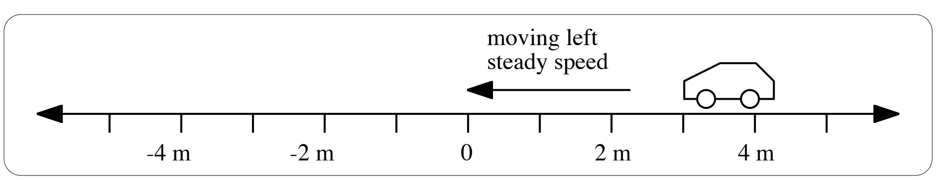
10. The picture at right shows a cart with a friction pad on a track attached to a string. When a mass of 0.20 kg is hung from the string at the location marked A, the cart is pulled along at a constant speed of 1.5 m/s. If a mass of 0.40 kg were hung from the string instead, the cart would be pulled along:

a. at a constant speed of 3 m/s.
b. at a constant speed greater than 1.5 m/s, but not necessarily 3 m/s.
c. for a while at a constant speed greater than 1.5 m/s, then with an increasing speed.
d. for a while with an increasing speed, then with a constant speed thereafter.
with a continuously increasing speed.

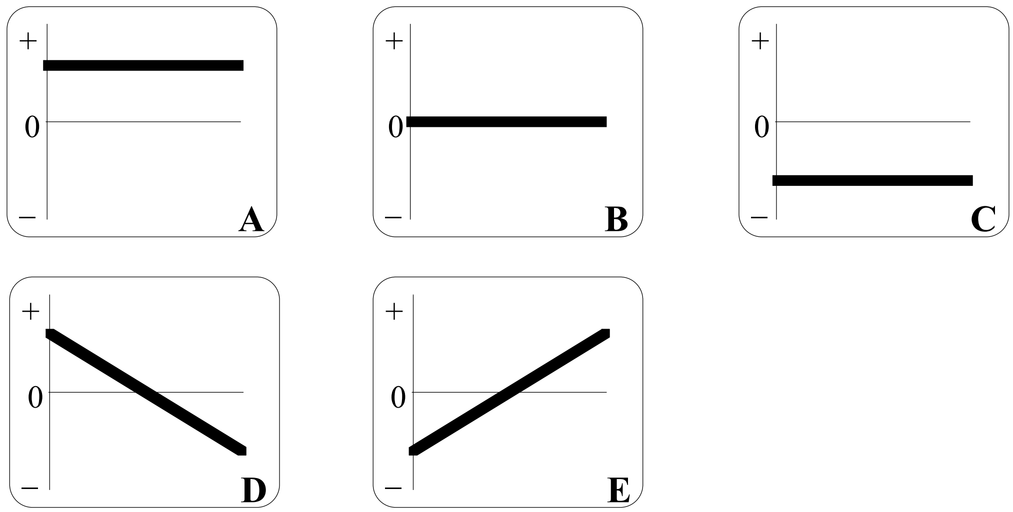


11. A truck is pulling a trailer behind it and both the truck and trailer are speeding up. Which of the following is true?

a. The force of the truck on the trailer is larger than the force of the trailer on the truck.
b. The force of the truck on the trailer is as large as the force of the trailer on the truck.
c. The force of the truck on the trailer is less than the force of the trailer on the truck.
d. There are no frictional forces on either the truck or the trailer.
e. The answer depends on the masses of the truck and trailer.



12. A toy car moves to the left at a constant speed. Using the coordinate system shown above, which of the graphs below (A-E) best represents the <u>velocity v. time</u> of the car?


A
B
C
D
E

Calculus Version

(New Questions F2005)

Please:

- Do not write anything on this questionnaire -- scratch paper should be provided.
- Mark your answers on the answer sheet provided.

---

1. Given $\mathbf{t^2 + 4t - 12 = 0}$, which of the following is correct?

   (a) $(t + 2)(t - 6) = 0$ (b) $(t + 4)(t - 3) = 0$ (c) $(t - 1)(t + 12) = 0$

   (d) $(t + 3)(t - 4) = 0$ (e) $(t + 6)(t - 2) = 0$

2. If **z = asin(bt)**, where **a** and **b** are constants, then $\frac{d^2z}{dt^2}$ = ?

   (a) acos(b) (b) abcos(t) (c) absin(bt)

   (d) acos(bt) (e) $-ab^2\sin(bt)$

3. If you know **at = b** and **cx + dt = f** and the values of **a, b, c, d** and **f**, but you don't know the value of **t**, solve for the value of **x**.

   (a) $\frac{f + dt}{c}$ (b) $\frac{b + f}{c(a + d)}$ (c) $\frac{f}{c} - \frac{db}{ac}$

   (d) $\frac{f}{c} - \frac{db}{a}$ (e) $\frac{b}{a}$

4. If $\frac{dz}{dx} = x - a$, where **a** is not a function of **x**, then **z** could be:

   (a) 1 (b) x^2 (c) $\frac{x^2}{2} - ax$

   (d) $\frac{ax^2}{2}$ (e) $\frac{x^2}{2} - a$

5. $\dfrac{4\times10^{-3}}{10^{-4}} = ?$

(a) 4×10^{-7} (b) $4\times10^{-3/4}$ (c) 4

(d) 40 (e) 4×10^{7}

6. If $\mathbf{z = ax^3 + bx + c}$, then $\dfrac{\mathbf{dz}}{\mathbf{dx}} = ?$

(a) $ax^2 + b$ (b) $a + b + c$ (c) $3ax^2 + 2b$

(d) $3ax^2 + b + c$ (e) $3ax^2 + b$

7. Solve for **x** in the equation $\mathbf{d + px^2 + kx = 0}$

(a) $\dfrac{k}{2p}\left(-1 \pm \sqrt{1-\dfrac{4pd}{k^2}}\right)$ (b) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (c) $\pm\sqrt{\dfrac{-d+k}{p}}$

(d) $\pm\sqrt{\dfrac{-k\left(\dfrac{d}{k}+x\right)}{p}}$ (e) $\dfrac{-d-px^2}{k}$

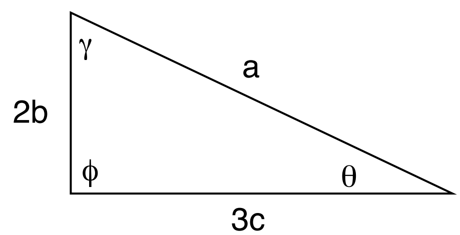
8. If $\dfrac{\mathbf{dz}}{\mathbf{dt}} = \mathbf{-ab^2 \sin(b^2 t)}$, where **a** and **b** are constants, then **z** = ?

(a) $2ab\cos(t) + k$ (b) $-2ab\sin(b^2 t) + k$ (c) $-2ab\sin(bt) + k$

(d) $a\cos(b^2 t) + k$ (e) $-2ab\cos(bt) + k$

9. For this right triangle, $\cos\theta = ?$

(a) 2b/3c (b) a/3c (c) 2b/a
(d) 3c/a (e) a/2b



10. For this right triangle, find c in terms of a and b.

(a) $c = \dfrac{a^2 - (2b)^2}{3}$ (b) $c = \dfrac{a^2 + 2b^2}{3}$ (c) $c = \dfrac{a - 2b}{3}$

(d) $c = \sqrt{\dfrac{a^2 - 4b^2}{3}}$ (e) $c = \dfrac{\sqrt{a^2 - 4b^2}}{3}$

11. If $\dfrac{\mathbf{dz}}{\mathbf{dx}} = \mathbf{ax^3 + bx^2 + cx + f}$, where **a**, **b**, **c**, and **f** are constants, then **z** could be:

(a) $3ax^2 + 2bx + c$ (b) $ax^4 + bx^3 + cx^2 + fx$ (c) $\dfrac{a}{4}x^4 + \dfrac{b}{3}x^3 + \dfrac{c}{2}x^2 + fx$

(d) $a + b + c + f$ (e) $\dfrac{ax^4}{3} + \dfrac{bx^3}{2} + cx^2 + fx$

12. Solve for **c** in the equation $\dfrac{\mathbf{ax + b}}{\left(\dfrac{\mathbf{c}}{\mathbf{y}}\right)} = \mathbf{d}$

(a) $\dfrac{axy + by}{c}$ (b) $\dfrac{ax + b - d}{y}$ (c) $\dfrac{dy}{ax + b}$

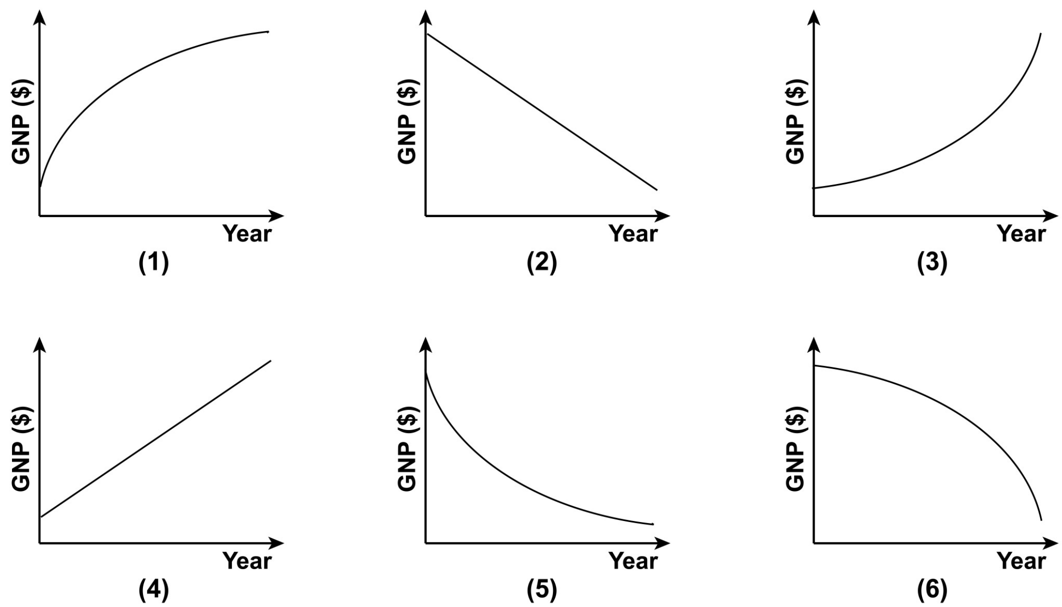
(d) $\dfrac{ax + b}{dy}$ (e) $\dfrac{axy + by}{d}$

13. If $\mathbf{z = ae^{bt}}$, where **a** and **b** are not functions of **t**, then $\frac{\mathbf{dz}}{\mathbf{dt}} = ?$

(a) bz (b) ae^b (c) az

(d) abe^t (e) abe^b

For the next three questions, refer to the charts below. Each chart shows a country's Gross National Product (GNP) for a period of several years.



14. Which chart(s) above depict a GNP that continually *decreases* during the years shown?

(a) 3 and 6 only (b) 2, 5, and 6 only (c) 2 only
(d) 5 and 6 only (e) 2 and 4 only

15. Which chart(s) above depict a GNP with a *rate of increase* that continually *increases* during the years shown?

(a) 3 and 6 only (b) 1, 3, and 4 only (c) 4 only
(d) 3 only (e) 1 and 5 only

16. Which chart(s) above depict a GNP with a *rate of decrease* that continually *increases* during the years shown?

(a) 5 only (b) 2, 5, and 6 only (c) 6 only
(d) 1, 3, and 4 only (e) 1 and 5 only

17. If you know $\mathbf{\frac{b}{2}y^2 - cd^2 = 0}$, $\mathbf{ax + y = d}$ and the values of **a**, **b**, **c** and **d** but you don't know the value of **y**, solve for the value of **x**.

(a) $\frac{y-d}{a}$ (b) $\frac{d}{a}\left(1 \pm \sqrt{\frac{2c}{b}}\right)$ (c) $\frac{d}{a} \pm \frac{1}{a}\sqrt{\frac{2cd}{b}}$

(d) $\frac{b}{2}(d-ax)^2 - cd^2$ (e) $\frac{d}{a} - \frac{2cd^2}{ab}$

18. If $\mathbf{\frac{dx}{dt}} = \mathbf{5at^3 + b}$, where **a** and **b** are constants, then **x** =?

(a) $15at^2$ (b) $\frac{5}{4}at^4 + bt + c$ (c) $\frac{5}{4}at^4 + b$

(d) $5at^2$ (e) $\frac{5}{4}at^4$

19. Solve for **x** in the equation $\mathbf{ax + bcx^2 + a^2 = 0}$

(a) $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ (b) $\frac{a}{2bc}(-1 \pm \sqrt{1-4bc})$ (c) $\pm\sqrt{\frac{-a^2 + a}{bc}}$

(d) $\pm\sqrt{\frac{-a(a+x)}{bc}}$ (e) $\frac{-a^2 - bcx^2}{a}$

20. If $\mathbf{f = b\sin(ax)}$, where **a** and **b** are constants, then $\frac{\mathbf{d^2f}}{\mathbf{dx^2}} = ?$

(a) bcos(a) (b) $-a^2f$ (c) bf

(d) bcos(ax) (e) -absin(a)

21. If you know $\mathbf{z} = \frac{\mathbf{b-a}}{\mathbf{t}}$ and $\mathbf{c} = \frac{\mathbf{f-d}}{\mathbf{t}}$ and $\mathbf{z} = \frac{\mathbf{d+f}}{\mathbf{2}}$ and the values of **a, c, d** and **f** but you don't know the values of **t** or **z**, solve for the value of **b**.

(a) $b = \frac{d+f}{2}t + a$ (b) $b = \frac{f^2 - d^2}{2c} + a$ (c) $b = \frac{(f-d)^2}{2}c + a$

(d) $b = \frac{f-d}{c}z + a$ (e) $b = a - 2f$

22. If $\mathbf{z = ax^2 + b}$, **a** and **b** are constants, and **x** is a function of **t**, then $\frac{\mathbf{dz}}{\mathbf{dt}} = ?$

(a) $2ax\frac{dx}{dt}$ (b) 2ax (c) $(ax^2 + b)\frac{dx}{dt}$

(d) $2ax + b\frac{dx}{dt}$ (e) 2a

23. Solve for **y** in the equation $\frac{\mathbf{ax+b}}{\mathbf{cy+d}} = \mathbf{f}$

(a) $\frac{ax + b - df}{cf} = y$ (b) $\frac{ax+b}{f+d}$ (c) $\frac{ax+b}{d}\left(\frac{1}{cf}\right)$

(d) $\frac{ax+b}{cf+d}$ (e) $\frac{1}{c}\left(\frac{f}{ax+b} - d\right)$

24. If $\mathbf{y = ax^2 + bx + c}$, for what value of **x** does **y** have its minimum value? Assume that **a**, **b**, and **c** are each positive and constant.

(a) 0 (b) $-\infty$ (c) $-\frac{a+b}{c}$

(d) $-\frac{b}{2a}$ (e) $-\frac{a}{2}$

25. $(5 \times 10^{-3})(3 \times 10^{2}) = ?$

(a) 1.5×10^{-2} (b) 15×10^{5} (c) 1.5

(d) 1.5×10^{-5} (e) 15×10^{-5}

26. If $\mathbf{y = ke^{bx}}$, where **b** and **k** are not functions of **x**, then $\frac{\mathbf{dy}}{\mathbf{dx}} = ?$

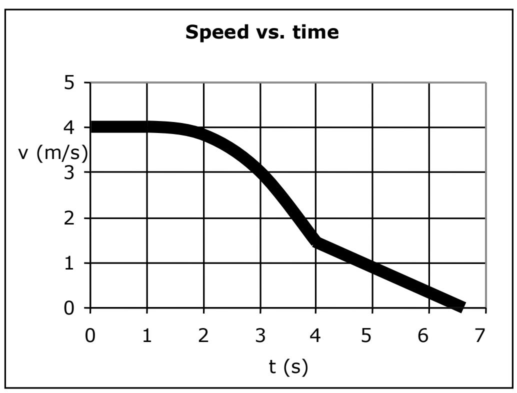
(a) bke^{b} (b) ke^{b} (c) $k^{2}e^{bx}$

(d) ke^{x} (e) bke^{bx}

---

27. Indicate the best estimate for the area under the curve between t = 1s and t = 3s.

(a) 7.5 m (b) 11.5 m (c) 15 m
(d) 15.5 m (e) 31 m



Appendix 8. Background Survey Questions (given at the beginning of the semester, attached after pre FCI)

1301 Student Background Information

Please take a moment to complete this questionnaire. The information you provide will help the Physics Department evaluate the usefulness of the laboratory. Your name will be used to match this evaluation with the other questionnaires you have completed this quarter. Your answers and comments will be kept confidential. Completing this questionnaire is voluntary and will not affect your grade in this or any other course. Your cooperation is appreciated.

YOUR BACKGROUND:

What is your intended major? Answer only once for questions 31-32.

31.

A. Biological Science
B. Chemistry
C. Computer Science
D. Engineering
E. Mathematics

32.

A. Pre-medical
B. Physics/Astrophysics
C. Social Science
D. Other
E. Undecided

Your physics and math background

Questions 33 through 41 are concerned with your math and science background.

33. How well **prepared** do you feel to deal with the subject matter of physics?
 a Totally unprepared
 b Unprepared
 c Somewhat prepared
 d Prepared
 e Very well prepared

34. Have you taken a physics course before? (select only one)
 a No

b Yes, regular high school only
c Yes, advanced placement high school only
d Yes, college only
e Yes, both college and high school

35. Are you repeating this course?
    a No.
    b Yes. I took this course before at the University of Minnesota.
    c Yes. I took a similar course at another college or university.

36. What was the last high school math class you completed?
    a Algebra
    b Geometry or Trigonometry
    c Pre-calculus, Functions, or Analysis
    d Calculus
    e Other, more advanced math in high school

37. What was the last college math class you completed **prior** to taking this course?
    a I have not taken a college math class
    b Algebra, Geometry, Trigonometry
    c Pre-calculus, Functions, or Analysis
    d Calculus
    e Other, more advanced math

38. When did you take your most recently completed math course?
    a Last term
    b Two terms ago
    c Last year
    d 2-3 years ago
    e More than three years ago

39. Are you enrolled in a math course this semester?
    a No
    b Yes

40. How many science classes, other than this course, have you taken in college?
    a This is my first college science course
    b I'm taking another first, college science course concurrently with this class
    c 1-2
    d 3-4
    e 5 or more

41. How computer literate do you consider yourself?
    a Uncomfortable with computers
    b Marginally computer literate
    c Fairly computer literate
    d Very computer literate
    e Extremely computer literate

Your academic workload background
Questions 44 through 50 are concerned with your academic background.

42. What is your approximate college GPA on a 4.0 system?
    a I do not have a GPA at the University of Minnesota

b 3.4-4.0
c 2.8-3.3
d 1.8-2.7
e Below 1.8

43. What grade do you expect to receive in this course?
a A
b B
c C
d D
e F

44. Approximately how much time per week do you anticipate spending on this course in addition to regular class sessions?
a Less than two hours per week
b 2-5 hours per week
c 6-10 hours per week
d 10-15 hours per week
e More than 15 hours per week

45. How many total course credits are you taking this semester?
a 0-4
b 5-8
c 9-12
d 13-16
e More than 16

46. How many hours per week are you employed?
a None
b 1-10 hours per week
c 11-20 hours per week
d 21-30 hours per week
e More than 31 hours per week

47. What is your age?
a 17 or younger
b 18-20
c 21-23
d 24-29
e 30 or older

48. What type of residence do you live in (choose only one)?
a Dormitory on-campus
b Living near campus
c Living off-campus
d With parents/family
e Other

Appendix 9. Spring 2013 Surveys

Mid-Semester Survey

1) On average, how many hours per week do you spend on this class (not including time spent in class)?

2) In an average week, on what part of this course do you spend the most time outside of class time?

3) How many different computer coaches have you attempted so far?
Zero 1-2 3-6 7-10 More than 10

4) Think about the topics addressed by the coaches. Order the following from the one with which the coaches helped you the most (1) to the one which the coaches helped you the least (10). Please do not use any ties.
Getting started solving a problem.
Using the physics concepts.
Deciding on what physics approaches to use.
Doing better on the quizzes.
Gaining confidence solving problems.
Determining that you need outside help.
Doing the math.
Understanding the lectures.
Deciding on which equations to use.
Deciding what the problem is about.

5) Rank the following components of the physics class in order from most useful (1) to least useful (11) to your learning. Do NOT use any ties. If you did not use a particular component, then omit it from your ranking.
Lectures
Thursday Discussion sections
Labs
Computer coaches
Textbook
Tutor room (137 Tate)
Clicker questions
Learning Assistants
The Competent Problem Solver (pdf on website)
WebAssign
Professor's office hours

Rank these questions on a scale of A to E with A being strongly agree, B agree, C neither agree nor disagree, D disagree, and E strongly disagree.

6) When using the computer coaches, it was usually clear how to proceed.

A B C D E

7) Using the coaches improved my confidence solving problems.
A B C D E

8) Using the computer coaches for homework made the homework take too long.
A B C D E

9) The computer coaches were useful to me.
A B C D E

10)
The computer coaches did not help improve my problem solving in this class.
A B C D E

11) The feedback offered by the computer coaches was helpful.
A B C D E

12) Using the coaches helped my course grade.
A B C D E

13) The computer coaches did not help me identify what I was confused about.
A B C D E

14) The coaches were too repetitive.
A B C D E

15) The computer coaches helped improve my conceptual knowledge of physics.
A B C D E

16) Using the computer coaches was unpleasant.
A B C D E

Short answer.

17) Describe what you like best about the computer coaches.

18) Describe what you liked least about the computer coaches.

19) Rank the three types of coaches from the one you find most useful to your learning (1) to the ones that you find least useful to your learning (3). If you did not use that type of coach, do not give it a rank.
Type 1
(The computer guides you through the problem solving decisions.)
Type 2
(You guide the computer through the problem solving decisions.)

Type 3
(You choose help you want.)

20) Why do you use the coaches? (Short answer)

End-Semester Survey

1. Consider all of the coaches you have attempted in the semester. On average, how much time did you spend on an individual coach?

2. How many different computer coaches have you attempted so far?
Zero 0-8 9-17 18-26 More than 27

3. Do not use any ties. Think about the topics addressed by the coaches. Order the following from the one with which the coaches helped you the most (1) to the one which the coaches helped you the least (10). DO NOT USE ANY TIES.
Interpreting the problem text.
Determining that you need outside help.
Applying the physics concepts to a specific problem.
Deciding on what physics to use. (eg. Kinematics, Conservation of Energy, etc.)
Doing the math.
Gaining confidence solving problems.
Understanding the lectures.
Applying the appropriate equations to a particular problem.
Doing better on the quizzes.
Getting started solving a problem.

4. Do NOT use any ties. Rank the following components of the physics class in order from most useful (1) to least useful (11) to your learning. If you did not use a particular component, then omit it from your ranking. DO NOT USE ANY TIES
Lectures
Learning Assistants (LAs) in lecture
Thursday Discussion sections
Labs
Computer coaches
Textbook
Tutor room (137 Tate)
Doing the homework
Clicker questions
The Competent Problem Solver (pdf on website)
Feedback from WebAssign

5. Rank this question on a scale of A to E with A being strongly agree, B agree, C neither agree nor disagree, D disagree, and E strongly disagree.

The computer coaches did not help improve my problem solving in this class.
A B C D E

6. Rank this question on a scale of A to E with A being strongly agree, B agree, C neither agree nor disagree, D disagree, and E strongly disagree.

Using the coaches improved my confidence in solving non-coached problems.
A B C D E

7. Rank this question on a scale of A to E with A being strongly agree, B agree, C neither agree nor disagree, D disagree, and E strongly disagree.

The feedback offered by the computer coaches was helpful in learning the physics.
A B C D E

8. Rank this question on a scale of A to E with A being strongly agree, B agree, C neither agree nor disagree, D disagree, and E strongly disagree.

Using the coaches helped me do better on the quizzes.
A B C D E

9. Rank this question on a scale of A to E with A being strongly agree, B agree, C neither agree nor disagree, D disagree, and E strongly disagree.

Using the coaches improved my confidence when starting new, unknown problems.(eg. Quiz problems)
A B C D E

10. Rank this question on a scale of A to E with A being strongly agree, B agree, C neither agree nor disagree, D disagree, and E strongly disagree.

The computer coaches did not help me identify what I was confused about.
A B C D E

11. Rank this question on a scale of A to E with A being strongly agree, B agree, C neither agree nor disagree, D disagree, and E strongly disagree.

The computer coaches helped improve my conceptual knowledge of physics.
A B C D E

12. Rank this question on a scale of A to E with A being strongly agree, B agree, C neither agree nor disagree, D disagree, and E strongly disagree.

The feedback offered by the computer coaches was helpful in learning how to solve problems.
A B C D E

13. Rank this question on a scale of A to E with A being strongly agree, B agree, C neither agree nor disagree, D disagree, and E strongly disagree.

Using the computer coaches was not unpleasant.

A B C D E

14. Which of the following statements best describes how you used the coaches in this course?
A. I tried to solve the problems on my own and used WebAssign to check the answer. I did not use the computer coaches regularly. B. I tried to solve the problems on my own and used WebAssign to check the answer. I only used the computer coaches when I was running out of WebAssign submissions. C. I tried to solve the problems on my own and used WebAssign to check the answer. I used the computer coaches to see another way to solve the problem after I submitted the correct answer. D. I tried to solve the problems on my own and used the computer coaches to check my solution method. E. I tried to solve the problems on my own and used the computer coaches for help if I got stuck. F. I worked through the computer coaches before trying to solve the problems on my own. G. I typically did not do the homework or the coaches. H. I only used the coaches to study for quizzes. I. Other

If you selected other, please describe:

15. Consider all of the coaches that were available for you to use throughout the semester. Why did you choose to use particular coaches?

Please answer honestly.

16. Consider all of the coaches that were available for you to use throughout the semester. Why did you choose to not use particular coaches?

Please answer honestly.

17. Rank the three types of coaches from the one you found most useful to your learning (1) to the ones that you found the least useful to your learning (3) at the beginning of the semester. If you did not use that type of coach at the beginning of the semester, do not give it a rank. Do not use any ties
Type 1
(The computer guides you through the problem solving decisions.)
Type 2
(You guide the computer through the problem solving decisions.)
Type 3
(You choose help you want.)

18. Rank the three types of coaches from the one you found most useful to your learning (1) to the ones that you found the least useful to your learning (3) at the end of the semester. If you did not use that type of coach during the semester, do not give it a rank. Do not use any ties.
Type 1
(The computer guides you through the problem solving decisions.)

Type 2
(You guide the computer through the problem solving decisions.)
Type 3
(You choose help you want.)

19. Describe one thing that was the most useful in the coaches.

Please answer honestly.

20. Describe the one thing that could be improved the most in the coaches.

Please answer honestly.

Appendix 10. Rubric Training Material

The purpose of problem-solving rubric training is to introduce to people who have never used the rubric before what it is and how it's supposed to be used. For example, it can be given to Physics TAs to show that the students in a physics course should be given a grade that actually correspond to their problem solving ability and in what aspects those problem solving ability consists.

The problem solving training material consists of two parts. In part 1, people are first given an introductory physics problem and an instructor's sample solution. Then they are given several real students solutions to be scored from 0-25. In part 1, people are just doing the regular grading like how they would do for an exam and the rubric is not introduced to them. In part 2, we give them the rubric and an explanation of the meaning of each category. Some sample student solutions scored by rubric are also provided as well as the reason of scoring. After that, they are asked to score the same students solutions they did in part 1 using the rubric. They can compare their grading to the rubric scores to see the correlation between grading and problem solving abilities.

The training materials are attached below. It was modified based on the training material that had been used in previous research by Jennifer Docktor (Docktor,2009).

Introduction:

In this task you will be asked to assess the quality of student solutions to a physics exam problem using a prescribed scoring technique. Your scores and comments are meant to help you reflect on your own teaching practices.

Instructions for the scoring task:

There are two parts to this task. The first part will be a take-home exercise in preparation for the second part.

PART I (To be done **BEFORE** the problem-solving session): This is the preparation for your evaluating student problem solutions in tomorrow's class.

1. Read "What is Problem Solving?" by M. A. Martinez in Section 3 of your selected reading packet.
2. Write down your solution to the provided physics problem. This is the problem the students solved.
3. After you have a written problem solution, compare it to the instructor's solution (other side). Note that there are two possible solutions, and the problem requires a unit conversion.
4. You have 6 student solutions to this problem labeled F – K. Give each of them a grade of 0 and 25 with 25 being a perfect solution. Just use your judgment to determine the grade. You will report these grades in class. For reference here is a mapping of numerical grades to letter grades used by some of the classes:
   A : 25-21 B: 20-17 C: 16-14 D : 13-11 F : 10-0

   The University grading policy gives the meaning of these grades as:
   A - Represents achievement that is outstanding relative to the level necessary to meet course requirements
   B - Represents achievement that is significantly above the level necessary to meet course requirements
   C - Represents achievement that meets the course requirements in every respect
   D - Represents achievement that is worthy of credit even though it fails to meet fully the course requirements

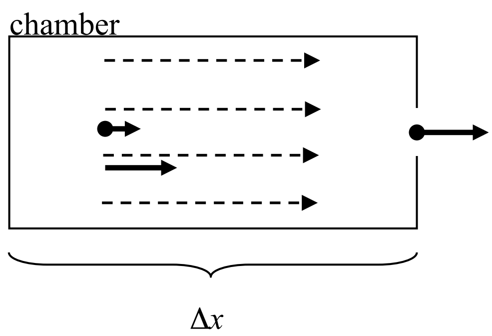
5. Read the scoring document (rubric) and category descriptions printed after these instructions. If there is anything you find unclear in the wording, write down your comments on page 2 of the scoring sheet (last page of the packet). Write down any features of a good problem solution that are not represented by these categories.

Problem:

You are designing part of a machine to detect carbon monoxide (CO) molecules (28 g/mol) in a sample of air. In this part, ultraviolet light is used to produce singly charged ions (molecules with just one missing electron) from air molecules at one side of a chamber. A uniform electric field then accelerates these ions from rest through a distance of 0.8 m through a hole in the other side of the chamber. Your job is to calculate the direction and magnitude of the electric field needed so that CO^+ ions created at rest at one end will have a speed of 8×10^4 m/s when they exit the other side.

Example Instructor Solution:

Description

	v_i = 0; initial velocity of the CO^+ molecule v_f = $8x10^4$m/s; final velocity of the CO^+ molecule E: uniform electric field in the chamber Δx = 0.8m; distance to hole in chamber q = $1.602x10^{-19}$C; charge of a CO^+ molecule m = mass of a CO^+ molecule a_x= acceleration of the CO^+ molecule F_E = force on the CO^+ molecule in the uniform electric field

Target: calculate the electric field, *E*

Solution Approach 1: Use Newton's Second Law to relate the force on the molecule to its acceleration; use kinematics to write an expression for acceleration in terms of velocity and distance. Assume gravity is negligible. Convert the mass of CO into kilograms per molecule.

$$\sum F_X = ma_X : \quad qE = ma_X \quad solve\ for\ the\ electric\ field : E = \frac{ma_X}{q}$$

$$v_f^2 = v_i^2 + 2a_X \Delta x \quad solve\ for\ acceleration : a_X = \frac{v_f^2 - v_i^2}{2\Delta x}$$

$$m = \frac{28g}{mol} = \frac{0.028kg}{mol} \bullet \frac{1 mol}{6.022 \times 10^{23} molecules} \approx 4.65 \times 10^{-26} kg / molecule\, CO^+$$

$$E = \frac{m(v_f^2 - v_i^2)}{2q\Delta x} = \frac{4.65 \times 10^{-26} kg \left((8 \times 10^4 m/s)^2 - 0\right)}{2(1.602 \times 10^{-19} C)(0.8m)} = \boxed{1160\, N/C}\ direction\ is\ same\ as\ v\ (to\ the\ right.)$$

Solution Approach 2: Use conservation of energy to relate the electric potential energy transferred to the molecule and its final kinetic energy. Assume gravity is negligible. Convert the mass of CO into kilograms per molecule.

$$E_{final} - E_{initial} = E_{in} - E_{out}: \qquad \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2} = q\Delta V - 0 \;\; OR \;\; \frac{1}{2}mv^2 - \frac{1}{2}mv_i^{\,2} = \int \vec{F}\cdot d\vec{s} - 0$$

$$for\ uniform\ electric\ field: \; \Delta V = \int \vec{E}\cdot d\vec{s} = E\Delta x \quad and \quad \int \vec{F}\cdot d\vec{s} = F_E\Delta x = qE\Delta x$$

$$\frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_i^{\,2} = qE\Delta x \;\; solve\ for\ electric\ field$$

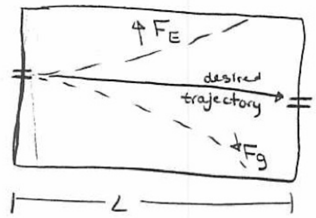
$$m = \frac{28g}{mol} = \frac{0.028kg}{mol} \bullet \frac{1mol}{6.022\times 10^{23} molecules} \approx 4.65\times 10^{-26} kg/molecule\ CO^+$$

$$E = \frac{m\left(v_f^2 - v_i^2\right)}{2q\Delta x} = \frac{4.65\times 10^{-26} kg\left(\left(8\times 10^4 m/s\right)^2 - 0\right)}{2\left(1.602\times 10^{-19} C\right)\left(0.8m\right)} = \boxed{1160 N/C} \; direction\ is\ same\ as\ v\,(to\ the\ right.)$$

Check: The units are correct for electric field. We expect that for a particle with larger mass or higher final velocity the electric field would need to be stronger, which is consistent with the equation obtained.

F

picture:



$q = e$ $(=+1.602 \times 10^{-19} C)$
$v_i = 0\ m/s$
$v_f = 8 \times 10^4\ m/s$
$m = 28\ g/mol$
$L = 0.8\ m$

Question: What magnitude and direction of an electric field should be used for charged particles to reach a velocity of 8×10^4 m/s and experience no net force to make it through the hole on the other side?

Approach: Use Newton's Laws to find the value of $\vec{F}_E$ and then use that information to solve for $\vec{E}$.

$F_{total} = F_E + F_g = 0 \rightarrow F_g = |-F_E| \longrightarrow mg = qE$

and $\vec{E} = \frac{\vec{F}_E}{q_0}$ $\qquad \vec{F}_E = q\vec{E}$

$\boxed{E = \frac{mg}{q}}$

$F_g = mg \rightarrow m: \frac{28\ g}{mol\ CO} \cdot \frac{1\ mol}{6.02 \times 10^{23}\ molecule} \cdot \frac{1\ kg}{1000\ g} = 4.65 \times 10^{-26}\ kg/molecule$

$F_g = (4.65 \times 10^{-26}\ kg)(9.8\ m/s^2) = 4.56 \times 10^{-25}\ N$

$4.56 \times 10^{-25}\ N = qE \rightarrow E = \frac{4.56 \times 10^{-25}\ N}{q} = \frac{4.56 \times 10^{-25}\ N}{1.602 \times 10^{-19}\ C}$

11

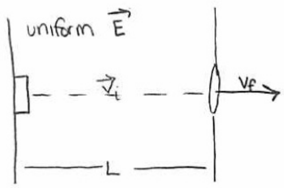
$\boxed{E = 2.85 \times 10^{-6}\ \frac{N}{C} \text{ straight upward}}$

check units: $\frac{N}{C}$ is correct for

G

Focus the Problem

Diagram:



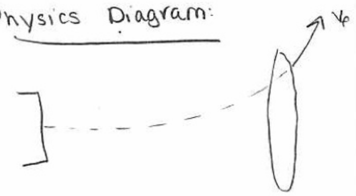
$v_f = 8 \times 10^4$ m/s
$L = 0.8$ m

Question:
What is the direction & magnitude of the electric field such that the final velocity of the ion is 8×10^4 m/s?

Approach:
Use $\vec{E} = \frac{\vec{F}}{q_0}$, dynamics, Newton's Laws ($F = ma$), & Kinematics to determine $\vec{E}$ field; find magnitude 1st then decide direction

Planning the Solution:

Physics Diagram:



$\vec{E} = \frac{\vec{F}}{q_0}$ $\quad F = ma \quad a = \frac{v^2}{R}$

$E_f - E_{in} = 0$
$\Rightarrow E_f = E_i$

Solving the Problem:

① $a = \frac{v^2}{R}$ } know all but a

② plug ① into $F = ma$
$\Rightarrow F = \frac{mv^2}{R}$

③ $E = \frac{F}{q}$
$\Rightarrow E = \frac{mv^2}{qR}$

]

Evaluating the Solution

$E = \frac{mv^2}{qR}$ } don't know m

⟹ use CO 28 g/mol

we will arbitrarily use 1 mol of substance:

$$\frac{28\,g}{mol} \cdot \frac{1\,mol}{1} \cdot \frac{1\,kg}{1000\,g} = 0.028\,kg$$

$$\Rightarrow E = \frac{mv^2}{qR}$$

$m = 0.028\,kg$
$v = 8 \times 10^4\,m/s$
$q = -1.602 \times 10^{-19}\,C$
$R = 0.8\,m$

$$E = \frac{(0.028\,kg)(8 \times 10^4\,\frac{m}{s})^2}{(-1.602 \times 10^{-19}\,C)(0.8\,m)} = -1.398 \times 10^{27}\,\frac{N}{C}$$

implies $\vec{E}$ field is inverted

⟹ $\vec{E}$ direction must be downward

$\vec{E}$ direction: downward

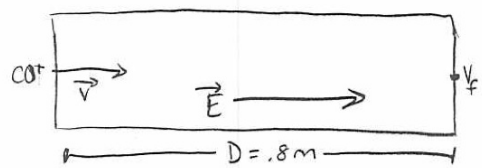
E magnitude: $1.398 \times 10^{27}\,\frac{N}{C}$

Checking the Answer

units:

$$E = \frac{N}{C} \Rightarrow E = \frac{kg \cdot \frac{m^2}{s^2}}{C \cdot m} = \frac{N \cdot \cancel{m}}{C \cdot \cancel{m}} = \frac{N}{C}\ ✓$$

H



$KE = \frac{1}{2}mv^2$

$v_i = 0$

$v_f = 8E4\ m/s$

$\Sigma F = F_E = ma$ $\quad F_E = Eq$

Q: Calculate direction & magnitude of Electric field.

Approach/Solve:

Because CO^+ is a positive charge, the electric field will be moving in the direction of CO^+.

To find magnitude of E:

$\Sigma F = F_e = ma$ $\qquad m_{CO^+} = 28g = .028\ kg$

$v_f^2 = \cancel{v_i^2}^{\,0} + 2a\Delta x$

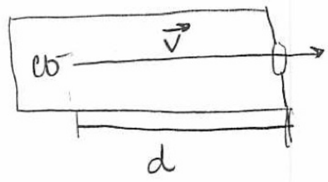
$v_f^2 = 2aD$

$a = \frac{v_f^2}{2D} = \frac{(8E4\ \frac{m}{s})^2}{2(.8m)} = 50{,}000\ \frac{m}{s^2}$

$F_e = ma = .028\ kg \times 50{,}000\ \frac{m}{s^2} = 1400 = Eq$

$E = \frac{1400}{q}$

I



Question: Find the direction and magnitude of the electric field needed to move the CO molecules through the hole at 8×10^4 m/s

$d = .8$ m
$\vec{v} = 8 \times 10^4$ m/s

Approach: use conservation of energy
assume gravity to be negligible in comparison to electric force

System: CO^+, box, Earth
t_i = when CO^+ molecule is at rest
t_f = when CO^+ molecule is leaving the box
$E_i = -\int \vec{F} \cdot d\vec{s}$
$E_f = \frac{1}{2}mv^2$
$E_{input} = q_0\vec{E}$
$E_{output} = 0$

Quantitative Relationships

$\vec{E} = \frac{\vec{F}}{q_0}$ $\quad PE = -\int \vec{F} \cdot d\vec{s}$ $\quad E_f - E_i = E_{in} - E_{out}$

$KE = \frac{1}{2}mv^2$

$\sum F = 0$

15

We want the velocity and field parallel to make this motion.

$$-\int \vec{F} \cdot d\vec{s} = -q\vec{E}d\cos 0^\circ = -qEd$$

Now all that's left is

$$\frac{1}{2}mv^2 + q\vec{E}d = q_0\vec{E}$$

$$\frac{1}{2}mv^2 = q_0\vec{E}(1-d)$$

$$\vec{E} = \frac{\frac{1}{2}mv^2}{q(1-d)}$$

$$m_{CO^+} = 28\frac{g}{mol} \cdot \frac{mol}{6.022 \times 10^{23} ions}$$

$$m_{CO^+} = 4.65 \times 10^{-23}\ g/ion = 4.65 \times 10^{-} kg$$

$q_0 = 1.602 \times 10^{-19} C$ because it has a +1 charge

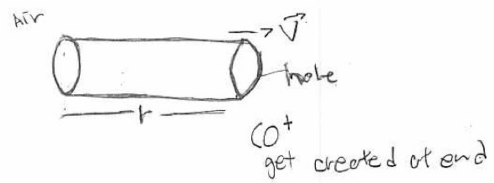
$$\vec{E} = \frac{\frac{1}{2}(4.65 \times 10^{-20} kg)(8 \times 10^4 m/s)^2}{1.602 \times 10^{-19} C\ (1-.8m)}$$

units $\frac{N}{C}$ ✓

$$\vec{E} = 4.64 \times 10^9\ N/C\ !$$

That is ridiculously high for the field but then again making a particle move from rest to 8,000 m/s in less than a meter is ridiculous too.

J



$V = 8 \times 10^4$ m/s
$r = 0.8$ m
$CO = 28$ g/mol.

Question.

- Calculate the direction and magnitude of the electric field needed so that ions created at rest at one end will have a speed of 8×10^4 m/s when the exit the other side.

Approach

Use coulom's Law. to find out the magnitube of the electric field

Direction should be. right side

Solution.

$$F = K_e \frac{q_1 q_2}{r^2} \qquad K_e = 9.00 \times 10^9 \frac{N \cdot m^2}{C^2}$$

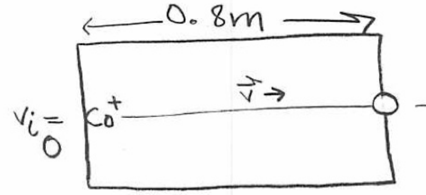
$$\vec{F} = q_1 \vec{V} \times \vec{B}$$

F=

$$B = \frac{F}{q_1 V} = \frac{K_e q_1 q_2}{r^2} \cdot \frac{1}{q_1 V} = \frac{K_e q_2}{r^2 \cdot V} = \frac{9.00 \times 10^9 \frac{N \cdot m^2}{C^2} \times 28 \text{ g/mol}}{0.64 \text{ m}^2 \cdot 8 \times 10^4 \text{ m/s}} = 4.92 \times 10^6 \frac{N \cdot g}{C^2 \cdot m}$$

Unit ✓ kind of weird unit
answer ✓
reasonable?. ✓ yes

K



$s = 0.8$ m
$v_f = 8 \times 10^4$ m/s
$m_{CO} = 28$ g/mol

Question: Calculate the direction and magnitude of the electric field needed so that CO^+ ions created at rest at one end will have a speed of 8×10^4 m/s when they exit the other side

Approach: Use conservation of energy

system: Co^+ particle

intial time: right as CO^+ enters the box

final time: right as CO^+ leaves the box

$E_i = 0 \quad v_i = 0$

$E_f = \frac{1}{2}mv^2$ $\quad$ E_{in} = electric potential energy

$E_{out} = 0$

Electric potential energy $= \Delta V q = -q\int \vec{E} \cdot ds$

because the electric field is constant

$PE_e = -qE\int ds$

$PE_e = -qES$ ← just want magnitude so can leave negative sign off

$E_f - E_i = E_{in} - E_{out}$

$\frac{1}{2}mv^2 = qEs$

$E = \dfrac{\frac{1}{2}mv^2}{qs}$

<u>check units</u>

$\frac{1}{2}mv^2 \rightarrow$ energy units

$\frac{E \cdot s}{V} \cdot \frac{q}{C}$ = energy units

units ok ✓

$$|E| = \frac{\frac{1}{2}mv^2}{qS} = \qquad J = n \cdot m$$

$CO - e^- = CO^+$

$.028\,Kg - 9.11 \times 10^{-31}\,Kg = 0.028\,Kg$

$$|E| = \frac{\frac{1}{2}(0.028\,Kg)(8 \times 10^4\,m/s)^2}{(1.602 \times 10^{-19}\,C)(0.8\,m)}$$

$$\boxed{|E| = 6.99 \times 10^{26}\ N/C}$$

units ✓

$$\frac{Kg \cdot m^2/s^2}{C \cdot m} = N/C$$

evaluate

✓ units OK

✓ seems like a rather large magnitude for an electric field but it would make sense because it takes a lot to accelerate a Co^+ particle from rest to 8×10^4 m/s in such a short distance

The Electric field should be pointing right because it is the only force on the Co^+ particle and since the particle is being accelerated to the right, the force should be to the right.

PART II (To be done during the class):

1. Read scored example solution A and discuss the score of each category. These scores represent an evaluation of the student's strength in that area and should not be confused with grading. Discuss the basis on which the evaluator might have justified each score. Discuss within your group whether or not the scores give you a picture of the strengths of the student in solving this problem. Discuss how the scores could give you an indication of where the student needs coaching. Repeat step 5 for example solutions B-E as time allows.

Note: Each of the scored example solutions **A-E** include rubric scores at the top and score comments in boxes distributed throughout the solutions. Some features of the scored example solutions **A-E** are:

A. Logical progression is good (the solution process is clear) but the application of physics is incorrect

B. Physics approach and math calculations are unnecessary for this solver (NA – Solver)

C. The solution is unfocused and does not progress to an answer

D. Example of a score "1" in physics approach

E. A description is unnecessary for this solver (NA-Solver)

2. Look at student solution **F**. Individually use the rubric to assign a separate score of **0, 1, 2, 3, 4, 5, NA(Solver), or NA(Problem)** for each of the five categories. On the scoring sheet, record the scores for student solution **F** and any relevant notes. Do not get any help from other members of your group.
   On the scoring sheet, record your scores for student solution F and any relevant notes. Refer back to the example scores **A-E** as necessary. Remember, these scores do not represent a grade and would not be added together to arrive at one. However, use this experience to revise your own grade for this student's solution (at the top of the scoring grid) if you think it is appropriate.

3. Answer the questions on the scoring sheet. Record comments and scoring difficulties on page 2 of the scoring sheet.

4. Repeat steps 7 and 8 for student solutions **G-K** as time allows.

The session will finish with a discussion of the extent to which an awareness of the features of problem solving such as those in the rubric can help you make both teaching and grading decisions.

	5	4	3	2	1	0	NA(Problem)	NA(Solver)
USEFUL DESCRIPTION	The description is useful, appropriate, and complete.	The description is useful but contains minor omissions or errors.	Parts of the description are not useful, missing, and/or contain errors.	Most of the description is not useful, missing, and/or contains errors.	The entire description is not useful and/or contains errors.	The solution does not include a description and it is necessary for this problem /solver.	A description is not necessary for this problem. (i.e., it is given in the problem statement)	A description is not necessary for this solver.
PHYSICS APPROACH	The physics approach is appropriate and complete.	The physics approach contains minor omissions or errors.	Some concepts and principles of the physics approach are missing and/or inappropriate.	Most of the physics approach is missing and/or inappropriate.	All of the chosen concepts and principles are inappropriate.	The solution does not indicate an approach, and it is necessary for this problem/ solver.	An explicit physics approach is not necessary for this problem. (i.e., it is given in the problem)	An explicit physics approach is not necessary for this solver.
SPECIFIC APPLICATION OF PHYSICS	The specific application of physics is appropriate and complete.	The specific application of physics contains minor omissions or errors.	Parts of the specific application of physics are missing and/or contain errors.	Most of the specific application of physics is missing and/or contains errors.	The entire specific application is inappropriate and/or contains errors.	The solution does not indicate an application of physics and it is necessary.	Specific application of physics is not necessary for this problem.	Specific application of physics is not necessary for this solver.
MATHE-MATICAL PROCEDURES	The mathematical procedures are appropriate and complete.	Appropriate mathematical procedures are used with minor omissions or errors.	Parts of the mathematical procedures are missing and/or contain errors.	Most of the mathematical procedures are missing and/or contain errors.	All mathematical procedures are inappropriate and/or contain errors.	There is no evidence of mathematical procedures, and they are necessary.	Mathematical procedures are not necessary for this problem or are very simple.	Mathematical procedures are not necessary for this solver.
LOGICAL PROGRESSION	The entire problem solution is clear, focused, and logically connected.	The solution is clear and focused with minor inconsistencies	Parts of the solution are unclear, unfocused, and/or inconsistent.	Most of the solution parts are unclear, unfocused, and/or inconsistent.	The entire solution is unclear, unfocused, and/or inconsistent.	There is no evidence of logical progression, and it is necessary.	Logical progression is not necessary for this problem. (i.e., one-step)	Logical progression is not necessary for this solver.

Category Descriptions:

Useful Description assesses a solver's skill at organizing information from the problem statement into an appropriate and useful representation that summarizes essential information symbolically and visually. The description is considered "useful" if it guides further steps in the solution process. A *problem description* could include restating known and unknown information, assigning appropriate symbols for quantities, stating a goal or target quantity, a visualization (sketch or picture), stating qualitative expectations, an abstracted physics diagram (force, energy, motion, momentum, ray, etc.), drawing a graph, stating a coordinate system, and choosing a system.

Physics Approach assesses a solver's skill at selecting appropriate physics concepts and principle(s) to use in solving the problem. Here the term *concept* is defined to be a general physics idea, such as the basic concept of "vector" or specific concepts of "momentum" and "average velocity". The term *principle* is defined to be a fundamental physics rule or law used to describe objects and their interactions, such as the law of conservation of energy, Newton's second law, or Ohm's law.

Specific Application of Physics assesses a solver's skill at applying the physics concepts and principles from their selected approach to the specific conditions in the problem. If necessary, the solver has set up specific equations for the problem that are consistent with the chosen approach. A *specific application of physics* could include a statement of definitions, relationships between the defined quantities, initial conditions, and assumptions or constraints in the problem (i.e., friction negligible, massless spring, massless pulley, inextensible string, etc.)

Mathematical Procedures assesses a solver's skill at following appropriate and correct mathematical rules and procedures during the solution execution. The term *mathematical procedures* refers to techniques that are employed to solve for target quantities from specific equations of physics, such as isolate and reduce strategies from algebra, substitution, use of the quadratic formula, or matrix operations. The term *mathematical rules* refers to conventions from mathematics, such as appropriate use of parentheses, square roots, and trigonometric identities. If the course instructor or researcher using the rubric expects a symbolic answer prior to numerical calculations, this could be considered an appropriate mathematical procedure.

Logical Progression assesses the solver's skills at communicating reasoning, staying focused toward a goal, and evaluating the solution for consistency (implicitly or explicitly). It checks whether the entire problem solution is clear, focused, and organized logically. The term *logical* means that the solution is coherent (the solution order and solver's reasoning can be understood from what is written), internally consistent (parts do not contradict), and externally consistent (agrees with physics expectations).

Student F	Initial Grade:		Revised Grade:	
Rubric Part	**Score**	**Notes**		
Useful Description				
Physics Approach				
Specific App. of Physics				
Mathematical Procedures				
Logical Progression				

Student G	Initial Grade:		Revised Grade:	
Rubric Part	**Score**	**Notes**		
Useful Description				
Physics Approach				
Specific App. of Physics				
Mathematical Procedures				
Logical Progression				

Student H	Initial Grade:		Revised Grade:	
Rubric Part	**Score**	**Notes**		
Useful Description				
Physics Approach				

Specific App. of Physics		
Mathematical Procedures		
Logical Progression		

Please write your name here:____

Student I	**Initial Grade:**		**Revised Grade:**	
Rubric Part	**Score**	**Notes**		
Useful Description				
Physics Approach				
Specific App. of Physics				
Mathematical Procedures				
Logical Progression				

Student J	**Initial Grade:**		**Revised Grade:**	
Rubric Part	**Score**	**Notes**		
Useful Description				
Physics Approach				
Specific App. of Physics				
Mathematical Procedures				
Logical Progression				

Student K	Initial Grade:		Revised Grade:	
Rubric Part	**Score**	**Notes**		
Useful Description				
Physics Approach				
Specific App. of Physics				
Mathematical Procedures				
Logical Progression				

Please write your name here:____

Please write your name here:____

Questions:

1. What features do you usually look for when grading a student exam paper? How does that compare with the rubric categories?

Comments about the rubric scoring activity:

2. What difficulties did you encounter during this activity?

a. Difficulties understanding the scoring task

b. Difficulties using the scoring rubric

Please write your name
here:____

3. Additional comments:

Appendix 11. Results to All Survey Questions

Fall 2011	# of A	# of B	# of C	# of D	# of E	# of F	# of G	# of respons es
Q3	7	13	22	37	53	0	0	132
Q4	1	10	5	11	71	34	0	132
Q5	83	31	18	0	0	0	0	132
Q6	7	31	93	0	0	0	0	131
Q7	4	113	13	2	0	0	0	132
Q8	39	42	51	0	0	0	0	132
Q9	118	14	0	0	0	0	0	132
Q10	98	28	2	4	0	0	0	132
Q11	38	44	50	0	0	0	0	132
Q12	30	38	64	0	0	0	0	132
Q13	21	98	10	3	0	0	0	132
Q14	16	56	31	25	4	0	0	132
Q15	2	8	17	73	32	0	0	132
Q16	23	68	23	17	1	0	0	132
Q17	24	83	18	7	0	0	0	132
Q18	13	28	31	40	20	0	0	132
Q19	14	84	28	6	0	0	0	132
Q20	4	12	38	64	14	0	0	132
Q21	13	32	22	51	14	0	0	132
Q22	3	15	68	44	2	0	0	132
Q23	4	38	34	54	2	0	0	132
Q24	39	78	11	4	0	0	0	132
Q25	4	28	31	58	11	0	0	132
Q26	36	58	20	16	2	0	0	132

Table 18. Responses to all survey questions

Appendix 12. Rubric Training Process

Rubric Training and Reliability Check

Two assessors, a PER graduate student and a faculty member with a PhD in PER each scored half of the students' solutions using the rubric. The agreement of the rubric training process (calibration) is in Figure 6. When scoring the actual solution set, to ensure inter-rater reliability, they always checked their reliability before they start a new problem. First they scored the same 10 student solutions, comparing and discussing their ratings. The agreement for the reliability check process is in Figure 7.

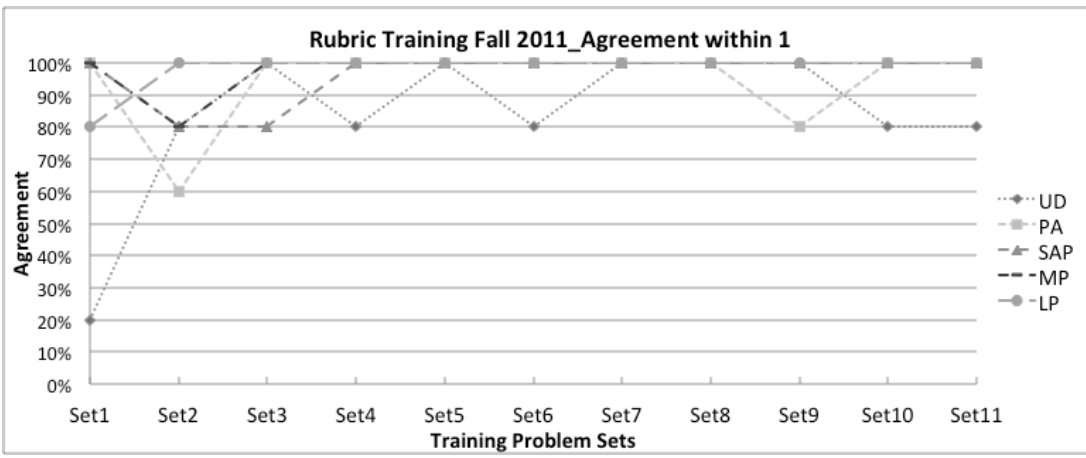


Figure 4. Agreement (within 1) between two raters on each of the five rubric categories during the training (calibration) process in Fall 2011 (using 11 sets of 8 problems from Spring 2011).

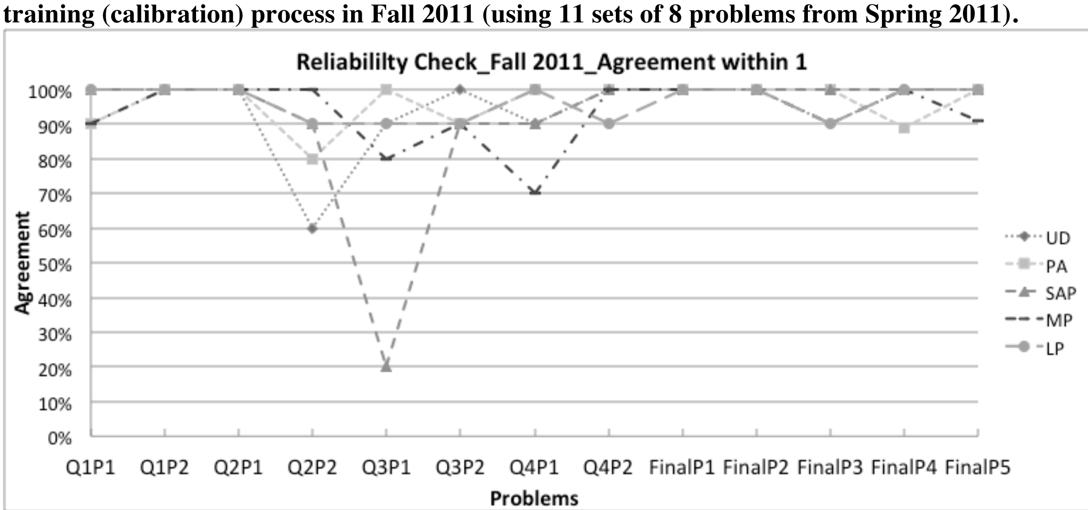


Figure 5. Agreement (within 1) between two raters on each of the five rubric categories during the reliability check process (scoring all 13 problems from Fall 2011.

Training Set (1-8) (Figure 4): Set 1

C31

P1 (25 pts). Just as a passenger train that you boarded a few moments before is beginning to pull out of the station, it is passed by a freight train traveling at 45. mi/h along a parallel track in the same direction that your train is headed. If your train undergoes constant acceleration, how far will you have traveled before your train passes the freight train, assuming that the freight train's speed remains constant? All that you know about your train's acceleration is that it takes 3.5 miles for the train to reach a speed of 60. mi/h, starting from rest.

my train   freight train

$V_{freight} = 45 mi/h$

$V_f^2 - V_i^2 = 2 \cdot a \cdot (x_f - x_i)$

$a_{train} = \frac{(60 mi/h)^2 - 0}{3.5 miles \times 2} = 514.286 mi/h^2$

$P_{freight} = V \cdot t = 45 mi/h \times t$

$P_{train} = x_0 + v_0 t + \frac{1}{2}at^2 = \frac{1}{2} \times 514.286 \times t^2$

When my train passes the freight train, $P_{my\ train} = P_{freight\ train}$

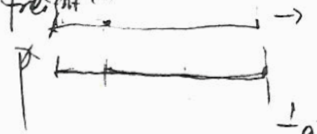
$\frac{1}{2} \times 514.286 \times t^2 = 45t \Rightarrow t = 0.175h$

$P_{train} = \frac{1}{2}at^2 = \frac{1}{2} \times 514.286 \times (0.175)^2$

$= 7.875 miles.$

P1 (25 pts). Just as a passenger train that you boarded a few moments before is beginning to pull out of the station, it is passed by a freight train traveling at 45. mi/h along a parallel track in the same direction that your train is headed. If your train undergoes constant acceleration, how far will you have traveled before your train passes the freight train, assuming that the freight train's speed remains constant? All that you know about your train's acceleration is that it takes 3.5 miles for the train to reach a speed of 60. mi/h, starting from rest.

freight train 45. mi/h



$\frac{1}{2}at^2 = \Delta x$.

knows

$V_{freight\ train} = 45$ mi/h.

$a = ?$

Target · $a_{passenger}$, t, x.

$V_f^2 = 2ax$

$a = \frac{V_p^2}{2x}$ $\frac{360}{7}$

Passenger train $x = \frac{1}{2}at^2$

travel distance is $x = \frac{1}{2}at^2$

freight train travels distance is $x_f = V_f t$

set the distance is equal

$\frac{1}{2}at^2 = V_f t$

$t = V_f \cdot 2x$

$\frac{1}{2}\frac{V_p^2}{2x}t = V_f$

$t = \frac{V_f \cdot 2x \cdot 2}{V_p^2}$

Dimensional Analysis

$L = \frac{(\frac{L}{T})^2 \times L}{(\frac{L}{T})^2}$

$T = \frac{\frac{L}{T} \times L}{(\frac{L}{T})^2}$

$x_p = \frac{1}{2}at^2$

$= \frac{1}{2}\frac{V_p^2}{2x}\left(\frac{V_f \cdot 2x \cdot 2}{V_p^2}\right)^2$

$= \frac{1}{2}\frac{V_f^2 \times 2x \cdot 4}{V_p^2}$

substituting in numbers

$X =$ ~~4.72~~ 5 mile;

C33

P1 (25 pts). Just as a passenger train that you boarded a few moments before is beginning to pull out of the station, it is passed by a freight train traveling at 45. mi/h along a parallel track in the same direction that your train is headed. If your train undergoes constant acceleration, how far will you have traveled before your train passes the freight train, assuming that the freight train's speed remains constant? All that you know about your train's acceleration is that it takes 3.5 miles for the train to reach a speed of 60. mi/h, starting from rest.

$$v_f^2 - v_i^2 = 2a(x_f - x_i) \qquad \sout{v = at + v_0}$$

$$2a = \frac{v_f^2 - v_i^2}{x_f - x_i}$$

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{60^2 \text{ mi}^2/\text{hr}^2}{2 \cdot 3.5^2 \text{ mi}^2} = \frac{3{,}600 \text{ mi}}{24.5 \text{ hr}^2} = \underline{147 \text{ mi/hr}^2}$$

$$v = at + v_0$$

$$at = v$$

$$t = \frac{v}{a} = \frac{60 \text{ mi/hr}}{147 \text{ mi/hr}^2} = 0.41 \text{ hr}$$

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

$$x = \frac{1}{2} a t^2$$

$$= \frac{1}{2} 147 \text{ mi/hr}^2 \cdot 0.41^2 \text{ hr}^2$$

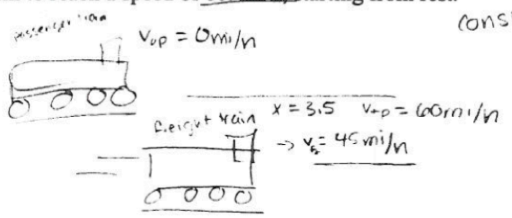
$$\boxed{= 12 \text{ mi}}$$

C34

NAME ______

P1 (25 pts). Just as a passenger train that you boarded a few moments before is beginning to pull out of the station, it is passed by a freight train traveling at 45. mi/h along a parallel track in the same direction that your train is headed. If your train undergoes constant acceleration, how far will you have traveled before your train passes the freight train, assuming that the freight train's speed remains constant? All that you know about your train's acceleration is that it takes 3.5 miles for the train to reach a speed of 60. mi/h, starting from rest.

constant acceleration



• If your train undergoes constant acceleration, how far will you have traveled before your train passes the freight train, assuming that the freight train's speed remains constant?

$V_{op} = 0mi/h$

$X_{pt} = 3.5$ at $V_{tp} = 60mi/h$

$V_{ft} = 45mi/h$

$X_{ptf} = ?$ (How far you go before passes)

$V_f^2 - V_i^2 = 2a(x_f - x_i)$

$\frac{V_f^2 - V_i^2}{2a} = x_f - x_i$

$\frac{(V_f^2 - V_i^2)}{2a} + x_i = x_f$

$\frac{(V_{tp}^2 - V_{ft}^2)}{2 \cdot a} + x_{pt} = x_{ptf}$

$\frac{(60mi/h)^2 - (45mi/h)^2}{2 \cdot 9.81 m/s^2} + 3.5mi$

21.95 mi/h

$9.81m/s^2 = 32.2ft/s^2$

$\frac{32.2ft}{s^2} \cdot \frac{1mi}{5780ft} \cdot \frac{3600s}{1h}$

21.95 mi/h

$\frac{3600 - 2025}{43.9} + 3.5mi = x_{ptf}$

$35.88 + 3.5 = x_{ptf}$

$39.4 = x_{ptf}$ (sig fig = 2)

39 miles

$1mi + mi = mi$

C35

P1 (25 pts). Just as a passenger train that you boarded a few moments before is beginning to pull out of the station, it is passed by a freight train traveling at 45. mi/h along a parallel track in the same direction that your train is headed. If your train undergoes constant acceleration, how far will you have traveled before your train passes the freight train, assuming that the freight train's speed remains constant? All that you know about your train's acceleration is that it takes 3.5 miles for the train to reach a speed of 60. mi/h, starting from rest.

Useful Equation

$X = X_0 + V_0 t + \frac{1}{2} a t^2$

$V_f^2 - V_i^2 = 2a(X_F - X_i)$

Train 1 — 1 mile per min speed

acceleration is

$\frac{\left(\frac{V_f^2}{X_f - X_i}\right)}{2} = a \quad \frac{\frac{1}{3.5}}{2} = .143 \text{ mile/min}^2$ acceleration

$3.5 = \frac{1}{2} .143\, t^2$

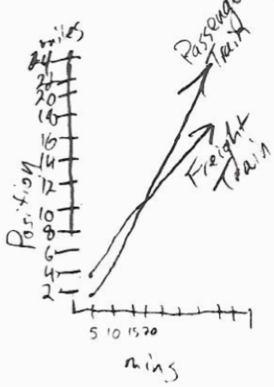
7mins = 3.5 mile
10mins = 6.5 miles
15mins = 11.5
14mins = 10.5 miles ✓

Train 2 — .75m/min speed

5mins = 3.75
10mins = 7.5
15 = 11.25
20mins = 15miles
14mins = 10.5 miles ✓

Graph converges at 14minutes & 10.5 miles

Graph



Answer

You will have traveled 14 mins and 10.5 miles before you pass the freight train.

Units check

position = miles

L = L ✓

Training Set (1-8) (Figure 4): Set 2

C31

P2 (25 pts). A ball is thrown straight upward and returns to the thrower's hand after 3.00 s in the air. A second ball is thrown at an angle of 30.0° relative to the horizontal. At what speed must the second ball be thrown so that it reaches the same height as the one thrown vertically?

The First Ball

$t = 3s$

$g = 9.81 m/s^2$

Single Ride

$t = \frac{t_{total}}{2} = \frac{3}{2} = 1.5s$

$g = 9.81 m/s^2$

$X = X_0 + V_0 t + \frac{1}{2} g t^2$

$= 0 + V + \frac{1}{2} \times 9.81 \times 1.5^2$

$= 11.036m$ meters

So, the heigh the firsh reaches is 11.036 meters.

The Second Ball

30°

Angle = 30°

$g = 9.81 m/s^2$

$H = 11.036m$

$V = ?$

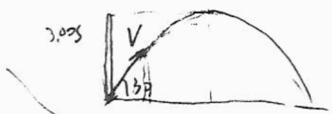
$V_{vertical} = V \times \sin 30° = \frac{1}{2} V$

$V_f^2 - V_i^2 = 2a(x_f - x_i)$

$(\frac{1}{2}V)^2 - 0 = 2 \times 9.81 \times 11.036 \Rightarrow V = 29.71 m/s$

C32

P2 (25 pts). A ball is thrown straight upward and returns to the thrower's hand after 3.00 s in the air. A second ball is thrown at an angle of 30.0° relative to the horizontal. At what speed must the second ball be thrown so that it reaches the same height as the one thrown vertically?



Target V_0

equation: $\frac{V}{a} = t$

Because the motion is symmetrical time that a ball is thrown straight upward is equal to that returns to the throwers

$$\frac{0 - V_0 \sin\theta}{-g} = \frac{t}{2}$$

$$V_0 = \frac{gt}{2\sin\theta}.$$

Dimention Analysis

$$\frac{L}{T^2} \times T = \frac{L}{T}$$

substituting in numbers

$$V_0 = \frac{9.8\,m/s^2 \times 3s}{2 \times \frac{1}{2}} = 29.4\ m/s$$

C33

P2 (25 pts). A ball is thrown straight upward and returns to the thrower's hand after 3.00 s in the air. A second ball is thrown at an angle of 30.0^{O} relative to the horizontal. At what speed must the second ball be thrown so that it reaches the same height as the one thrown vertically?

Ball 1

$v = at + v_0$

$v_0 = v - at$

$= 0 - -9.81\ m/s^2 \cdot 3.00 s$

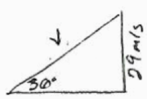
$= 29.43\ m/s$

~~$y = y_0 + v_0 t + \frac{1}{2} a t^2$~~

~~$= 29.43\ m/s \cdot 3.00 s + \frac{1}{2} \cdot -9.81\ m/s^2 \cdot 3.00^2 s^2$~~

~~$= 73.575\ m$~~

unnecessary

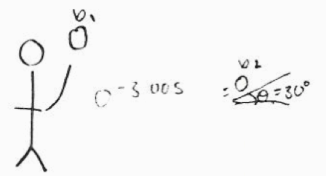


$\sin 30° = \frac{29\ m/s}{v}$

$v \sin 30° = 29\ m/s$

$v = \frac{29\ m/s}{\sin 30°} = \boxed{59.9\ m/s}$

P2 (25 pts). A ball is thrown straight upward and returns to the thrower's hand after 3.00 s in the air. A second ball is thrown at an angle of 30.0° relative to the horizontal. At what speed must the second ball be thrown so that it reaches the same height as the one thrown vertically?



• At what speed must the second ball be thrown so that it reaches the same height as the one thrown vertically?

$t_1 = 3.00s$

$\theta = 30.0°$

$V_{b_2} = ?$

$y_{b_1} = ?$

$y_{b_2} = ?$

$V = at + V_0$ $\quad V_{0b_1} = 0$

$V_{b_1} = 9.81 \, m/s^2 \cdot 3.00s + 0$

$V_{b_1} = 29.43 \, m/s$

$y \sin\theta$

$29.43 \sin 30$

$= 14.72 \, m/s$

$y = y_0 + v_0 t + \frac{1}{2} a t^2$

$= 0 + 0$

$y = \frac{1}{2} a t^2$

$y = \frac{1}{2} \cdot 9.81 \cdot 3^2$

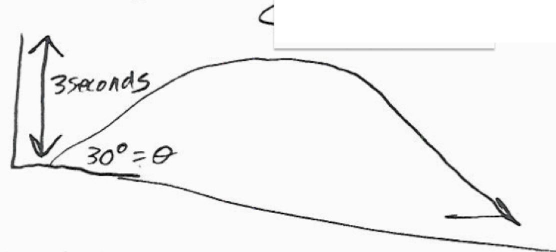
$y = 44.15$

C35

P2 (25 pts). A ball is thrown straight upward and returns to the thrower's hand after 3.00 s in the air. A second ball is thrown at an angle of 30.0° relative to the horizontal. At what speed must the second ball be thrown so that it reaches the same height as the one thrown vertically?

Useful Equations

$x = x_0 + V_0 t + \frac{1}{2} a t^2$



Ball 1

$y = \frac{1}{2} a t^2$

$y = \frac{1}{2} 9.81 \cdot 3^2$

$y = 44.145$ meter

↑ 44.145 meters

Ball 2

$y = V_0 t + \frac{1}{2} a t^2$

$\frac{y - \frac{1}{2} g \sin\theta \, t^2}{t} = V_0$

$\frac{44.145 - \frac{1}{2}(9.81 \sin 30) \cdot 3^2}{3} =$ 7.3575 m/s (answer)

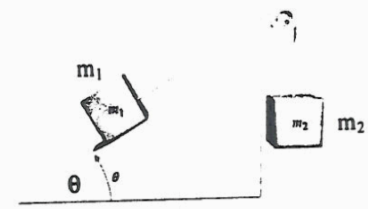
Units do checkout

$\frac{L - \frac{L}{t^2} \cdot \frac{t^2}{1}}{t} = \frac{L}{t}$

Training Set (1-8) (Figure 4): Set 3

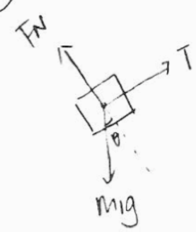
C31

P1 (25 pts). A massless string, as shown in the figure below, connects two objects. The incline and the massless pulley are frictionless. Find the acceleration of the objects and the tension in the string in terms of θ, m_1 and m_2.

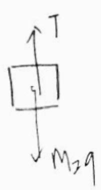


These 2 objects are connected together, So they have the same acceleration and tension acting on them are the same. we have m_1, m_2, θ now we want to find T and a

(m_1)



(m_2)



① if the acceleration of m_2 is downward.

then $T - m_1 g\sin\theta = m_1 a$

$m_2 g - T = m_2 a$

So. $a = \dfrac{g(m_2 - m_1\sin\theta)}{m_1 + m_2}$

$T = m_2 g - \dfrac{m_2 g(m_2 - m_1\sin\theta)}{m_1 + m_2}$

② if the acceleration of m_2 is upward

$m_1 g\sin\theta - T = m_1 a$

$T - m_2 g = m_2 a$

So $a = \dfrac{g(m_1\sin\theta - m_2)}{m_1 + m_2}$

$T = m_2 g + \dfrac{m_2 g(m_1\sin\theta -}{m_1 + m_2}$

③ If $a = 0$

then $T = m_2 g$ or

$T = m_1 g\cos\theta$

$a = 0$

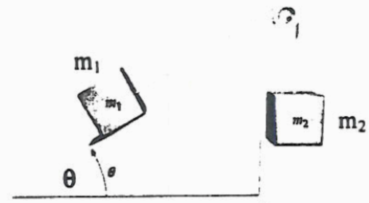
$[M]/[T]^2 = [M]/[T]^2 \times \dfrac{[M]}{[M]} = [M],$

$[F] = [M] + [M]/[T]$

$= [F]$

They are reasonable

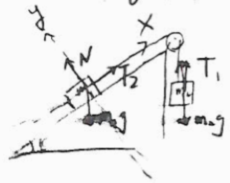
P1 (25 pts). A massless string, as shown in the figure below, connects two objects. The incline and the massless pulley are frictionless. Find the acceleration of the objects and the tension in the string in terms of θ, m_1 and m_2.



physics concept Newton 2nd Law
Newton 3rd Law

useful equation $F = ma$

known θ m_1, m_2 target a, T



$\Sigma F_y = 0$ $N = m_1 g\cos\theta$

$\Sigma F_x = ma$ $T_2 - m_1 g\sin\theta = m_1 a_1$ ①

$m_2 g - T = m_2 a_2$ ②

∵ $a_1 = a_2$ in magnitude because they are connected by one string

$T_1 = T_2$ in magnitude according to Newton 3rd Law

$m_1 g\sin\theta + m_1 a_1 = m_2 g - m_2 a_2$

$m_2 g - m_1 g\sin\theta = (m_1 + m_2)a$

$a = \dfrac{m_2 g - m_1 g\sin\theta}{m_1 + m_2}$

Dimension analysis $\dfrac{L}{T^2} = \dfrac{M\times\frac{L}{T^2} - M\times\frac{L}{T^2}}{M} = \dfrac{L}{T^2}$ OK

reasonable analysis: if m_2 is larger a will be larg[e]. So m_2 is in the numerator is reasonable

if $m_1 + m_2$ is larger, the acceleration will decre[ase]. So it is reasonable $m_1 + m_2$ in the denominator

acceleration decrease

if θ increase ↑ $m_1 g\sin\theta$ also increase, $-m_1 g\sin\theta$ decrease. So acceleration decrease. so θ in the numinator is reasonable

I think state neat & properly anser is completely because I solve target.

$T = m_2 g - m_2 a$

$= m_2 g - m_2\dfrac{(m_2 g - m_1 g\sin\theta)}{m_1 + m_2}$

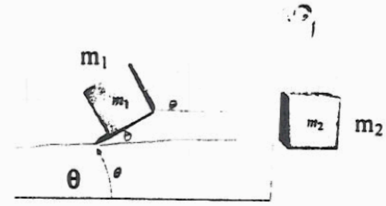
$F = M\frac{L}{T^2} - M\left(\dfrac{M\frac{L}{T^2} - M\times\frac{L}{T^2}}{M}\right)$

because a is reasonable so T is also reasonable

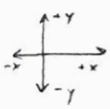
if $m_1 = m_2 = 1kg$ $\theta = 30°$ $g = 10 m/s^2$

$T = 7.5N$ it is reasonable

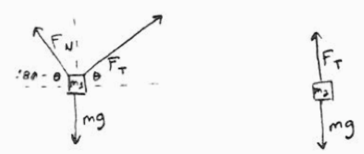
P1 (25 pts). A massless string, as shown in the figure below, connects two objects. The incline and the massless pulley are frictionless. Find the acceleration of the objects and the tension in the string in terms of θ, m_1 and m_2.



Question: What is the acceleration (a) of the objects?
What is the tension (T) of the string?



Physics: Newton's 2nd Law ($F=ma$)
Newton's 3rd Law ($F_{12}=-F_{21}$)



m_2
$F_x = 0$
$F_y = T - m_2 g$

$a_{m_2} = \frac{T - m_2 g}{m_2}$

m_1
$F_x = T\cos\theta - m_1 g\cos(180° - \theta)$
$F_y = m_1 g\cos(\theta - 90°) + T\cos(90° - \theta) - m_1 g$
$F = T\cos\theta - m_1 g\cos(180° - \theta) + m_1 g\cos(\theta - 90°) + T\cos(90° - \theta) - m_1 g$
$= T\cos\theta + T\cos\theta_2 + m_1 g\cos\theta - m_1 g + m_1 g\cos\theta_2$
$= T(\cos\theta + \cos\theta_2) + m_1 g(\cos\theta - 1 + \cos\theta_2)$

$a_{m_1} = \frac{\cos\theta(T + m_1 g) + \cos\theta_2(T + m_1 g) - m_1 g}{m_1} = \frac{T - m_2 g}{m_2} = a_{m_2}$

T =

next page

$\underline{m_1}$

$$F_x = m_2 g \cos\theta + m_1 g \cos\theta$$

$$F_y = m_1 g \cos\theta_2 + m_2 g \cos\theta_2 - m_1 g$$

$$F = m_1 g(\cos\theta + \cos\theta_2 - 1) + m_2 g(\cos\theta + \cos\theta_2)$$

m_2

$$F_y = m_1 g - m_2 g$$

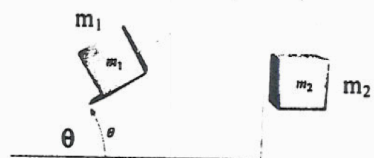
$$a = \frac{F}{m} = \frac{m_1 g(\cos\theta + \cos\theta_2 - 1) + m_2 g(\cos\theta + \cos\theta_2)}{m_1} = \frac{m_1 g - m_2 g}{m_2}$$

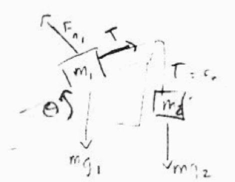
$$m_1 m_2 g(\cos\theta + \cos\theta_2 - 1) + m_2^2 g(\cos\theta + \cos\theta_2$$

C34

P1 (25 pts). A massless string, as shown in the figure below, connects two objects. The incline and the massless pulley are frictionless. Find the acceleration of the objects and the tension in the string in terms of θ, m_1 and m_2.

* find acceleration + tension
θ, m_1, m_2





$F = mg$

$F = ma$

$F/m = a$

$F_{b1} = m_1 g \sin\theta + T\cos\theta - m_1$ $\quad F_{b2} = m_2 g$

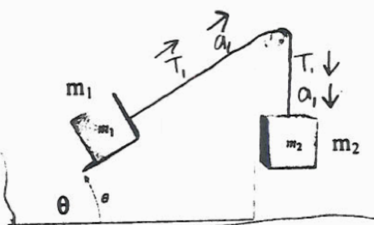
$a = \frac{m_2 g}{m}$

$F_{b1} = m_1 g \sin\theta + T\cos\theta - m_1 g \quad F_{b2} = m_2 g$

$\ldots \theta + T\cos\theta - m_1 g = m_2 g$

C35

P1 (25 pts). A massless string, as shown in the figure below, connects two objects. The incline and the massless pulley are frictionless. Find the acceleration of the objects and the tension in the string in terms of θ, m_1 and m_2.



Physics Concepts

- Newton's 2^{nd} law

Analyze the Problem

Knowns	Unknown
m_1	T_1
m_2	a_1
θ	

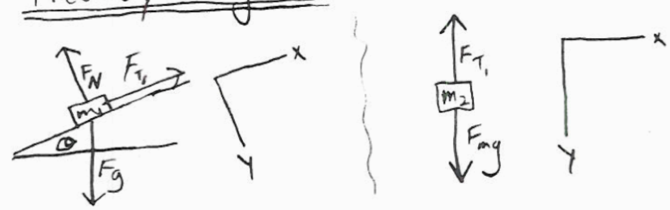
Useful Equations

$F = ma$

$F_y net$ = sum of y forces

$F_x net$ = sum of x forces

Freebody Diagrams



Constructing the solution

M_2

$\Sigma F_y net = m_2 g - T_1$

m_1

$\Sigma F_x net = T_1 - mg\cos\theta$

$T_1 - m_1 g\cos\theta = m_2 g - F_{T_1}$

$\frac{2T_1}{g} = \frac{m_1 g\cos\theta + m_2 g}{g}$

Solution

$\frac{2T_1}{g} = m_1\cos\theta + m_2$

Evaluate Answer

The answer makes sense because the units check

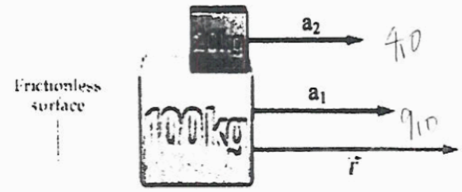
Units Check

$\left[\frac{m(L/T^2)}{(L/T^2)} = m\right] = [m = m]$

Training Set (1-8) (Figure 4): Set 4

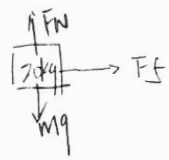
C31

P2 (25 pts). A ~~100~~ kg mass is pulled along a frictionless surface by a horizontal force $\vec{F}$ such that its acceleration is 9.0 m/s^2. A 20 kg mass slides along the top of the 100 kg mass and has an acceleration of 4.0 m/s^2. (It thus slides backward relative to the 100 kg mass.)



What is the force $\vec{F}$ *and* what is the coefficient of kinetic friction between the two masses?

The force causes the 20 kg mass's acceleration is the frictional force between these objects and the force causes 100 kg mass's acceleration is the net force of $\vec{F}$ and Ff



$F_N = mg = 20kg \times 9.81 m/s^2 = 196.2 N$

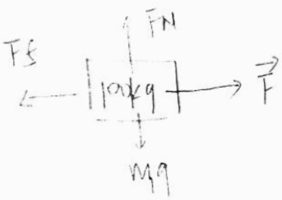
$F_f = F_N \cdot U_K = 196.2 \times U_K$

$F_f = m_2 a_2 = 20kg \times 9.0 m/s^2 = 180 N$

$180N = 196.2 N \cdot U_K$

$U_K = \frac{180N}{196.2} = 0.92$

cofficient of kinetic friction should betwen 0~1, though 0.92 is a little large, it's still reasonable



$\vec{F} - F_f = m_1 a_1$

$\vec{F} - 180N = 100kg \times 9 m/s^2$

$\vec{F} = 100kg \times 9 m/s^2 + 180 N$

$= 1080 N$

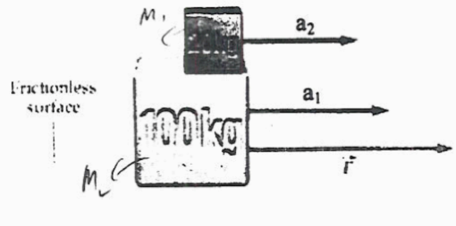
$[F] = [M \times L/T^2] + [F]$

$= [F] + [F]$

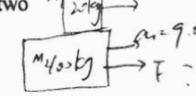
$= [F]$

so it's reasonable.

P2 (25 pts). A 100 kg mass is pulled along a frictionless surface by a horizontal force $\bar{F}$ such that its acceleration is 9.0 m/s^2. A 20 kg mass slides along the top of the 100 kg mass and has an acceleration of 4.0 m/s^2. (It thus slides backward relative to the 100 kg mass.)



physics concept: Newton 2nd law, Newton 3rd law, force of friction, kinematic coefficient friction

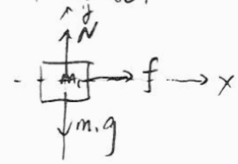


What is the force $\bar{F}$ *and* what is the coefficient of kinetic friction between the two masses?

known $m_1 = 20kg$ $m_2 = 100kg$ $a_1 = 9.0 m/s^2$ $a_2 = 4.0 m/s^2$

Target $\vec{F}$ and μ.

useful Equation $\Sigma F = ma$, $f = \mu_k N$



$\Sigma F_Y = 0$ $m_1 g - N = 0$

$\Sigma F_x = ma$ $f_1 = m_1 a_2$

$f_1 = \mu N$

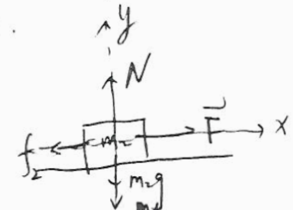
$\therefore$ $m_1 a_2 = \mu m_1 g$

$\mu = \frac{a_2}{g}$

Subsetitute in number $\mu = \frac{4 m/s^2}{9.81 m/s^2} = 0.41$

Dimension analysis $1 = \frac{L/T^2}{L/T^2}$ OK

I think statement is proper answer is completely because F μ is solved.



$\Sigma F_Y = 0$ $m_2 g + N_1 = N$ $N_1 = m_1 g$ according to New 3rd l

$\Sigma F_x = ma$ $F - f = m_2 a_1$ $f = f_1 = m_1 a_2$

$\therefore F = m_2 a_1 + m_1 a_2$

Dimension analysis $F = M \frac{L}{T^2} + M \frac{L}{T^2}$ OK. plug in number

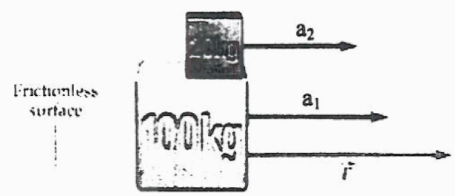
$F = 100kg \times 9 m/s^2 + 20kg \times 4.0 m/s^2$

$= 980N$

the value of F is reasonable because the acceleration of $m_1 m_2$ is large.

reasonable analysis Although μ is larger than the value we consider in life it is still acceptable.

P2 (25 pts). A 100 kg mass is pulled along a frictionless surface by a horizontal force $\vec{F}$ such that its acceleration is 9.0 m/s^2. A 20 kg mass slides along the top of the 100 kg mass and has an acceleration of 4.0 m/s^2. (It thus slides backward relative to the 100 kg mass.)



What is the force $\vec{F}$ *and* what is the coefficient of kinetic friction between the two masses?

$F = ma$

$F = 120\,kg \cdot 9.0\,m/s^2 = 1{,}080\ N = \vec{F}$

$f = \mu n = \mu\, 20\,kg \cdot 9.81\,m/s^2 = \mu\, 196.2\,N$

$F = ma = 20\,kg \cdot 4.0\,m/s^2 = 80\,N$

$\mu 196.2\,N$

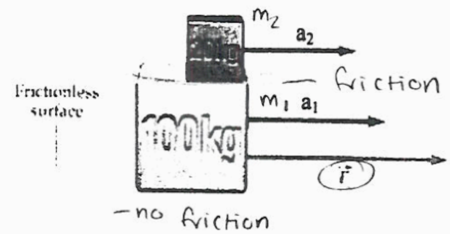
$1080\,N - \mu \cdot 196.2\,N = 80\,N$

$\mu \cdot 196.2\,N = 1000\,N$

$\mu = 5.1$

C34

P2 (25 pts). A 100 kg mass is pulled along a frictionless surface by a horizontal force $\bar{F}$ such that its acceleration is 9.0 m/s². A 20 kg mass slides along the top of the 100 kg mass and has an acceleration of 4.0 m/s². (It thus slides backward relative to the 100 kg mass.)

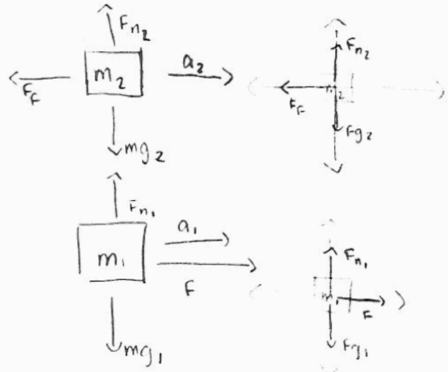


$F_n = mg$

What is the force $\bar{F}$ *and* what is the coefficient of kinetic friction between the two masses?

$\vec{F}$? $\mu_k = ?$

$F = ma$ ->
$F = \mu_k n$
$a_1 = 9.0 m/s^2$
$a_2 = 4.0 m/s^2$
$m_1 = 100 kg$
$m_2 = 20 kg$



$F - F_{b1} = m_1 \cdot a_1$

$F_{b2} = m_2 \cdot a_2$

$F - F_{b1} = m_1 \cdot a_1$

$F_{b1} = 100 kg \cdot 9.0 m/s^2$

900 N

$F_{b2} = m_2 \cdot a_2$

$F_{b2} = 20 kg \cdot 4.0 m/s^2$

80 N

$F_f = \mu_k n$

$\frac{F_f}{n} = \mu_k$

$\frac{80}{1176} = \boxed{0.07}$

$\vec{F} = m_1 a_1 + m_2 a_2$

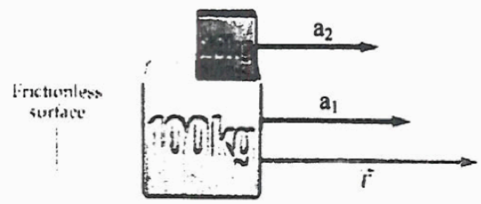
$\vec{F} = 100 kg \cdot 9.0 m/s^2 + 20 kg \cdot 4.0 m/s^2$

$\vec{F} = 900 N + 80 N$

$\boxed{\vec{F} = 980 N}$

C35

P2 (25 pts). A 100 kg mass is pulled along a frictionless surface by a horizontal force $\vec{F}$ such that its acceleration is 9.0 m/s^2. A 20 kg mass slides along the top of the 100 kg mass and has an acceleration of 4.0 m/s^2. (It thus slides backward relative to the 100 kg mass.)



What is the force $\vec{F}$ *and* what is the coefficient of kinetic friction between the two masses?

Physics Concepts

- 1D Kinematics
- Friction
- Newtons 1st + 2nd law

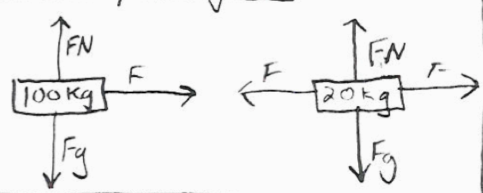
Useful Equations

$F = \mu_k n$ $\quad F_{net}$ = Sum of forces

$F = ma$

$a = \frac{V}{t}$

Free body Diagram



Constructing a solution

$F = [(m_1)(a_1)] + [(m_2)(a_2)]$

$F = [(100 \cdot 9)] + (20 \cdot 4)$

answer For F = $F = 980$ Newtons

$\frac{F}{n} = \frac{\mu_k n}{n}$

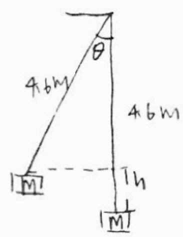
answer for μ_k $\quad \frac{80}{196.2} = .41 = \mu_k$

Training Set (1-8) (Figure 4): Set 5

C31

P1 (25 pts). Walking by a pond, you find a rope attached to a tree limb that is 5.2 m above ground level. You decide to use the rope to swing out over the pond. The rope is a bit frayed, but supports your 70 kg mass. You estimate that the rope might break if the tension is 80 N greater than your weight. You grab the rope at a point 4.6 m from the limb and move back to swing out over the pond. (Model yourself as a point particle attached to the rope 4.6 m from the limb.)
(a) What is the maximum safe initial angle between the rope and the vertical at which it will not break during the swing?
(b) If you begin at this maximum angle, and the surface of the pond is 1.2 m below the level of the ground, with what speed will you enter the water if you let go of the rope when the rope is vertical?

(a)



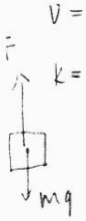
suppose the maximum safe initial angle is θ.

So at the start point, I have no kinetic energy, but I have all potential energy. And as i move downward, my potential energy is decreasing, kinetic energy is increasing when kinetic energy reaches the maximum, the force acting on the string is the largest

$h = 4.6 - 4.6\cos\theta = 4.6(1-\cos\theta)$ ①

$V = mgh = 70 \times 9.81 \times 4.6(1-\cos\theta) = 3158.82(1-\cos\theta)$ ②

$k = \frac{1}{2}mv^2 = \frac{1}{2} \times 70v^2 = 35v^2$ ③



$F_{Net} = F - mg = \frac{v^2}{R}m$ ④

$F = mg + 80N$ ⑤

From all 5 equations above, we get

$40 \times 4.6 = 3158.82(1-\cos\theta)$

$\therefore \theta = 19.65°$

(b)

At point 1, the mass has all potential energy, and at point 3, the mass has all kinetic energy

So. $V = mg(h + h_1 + h_2)$

$= 70 \times 9.81 (0.268 + 0.6 + 1.2)$

$= 1420.02J$

$E = \frac{1}{2}mv^2 = \frac{1}{2} \times 70\, v^2 = 35v^2$

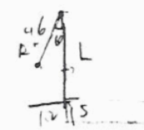
$V = E$

$\therefore 1420.02J = 35v^2$

$\therefore v = 6.37 m/s$

C32

P1 (25 pts). **Walking by a pond, you find a rope attached to a tree limb that is 5.2 m above ground level. You decide to use the rope to swing out over the pond. The rope is a bit frayed, but supports your 70 kg mass. You estimate that the rope might break if the tension is 80 N greater than your weight. You grab the rope at a point 4.6 m from the limb and move back to swing out over the pond. (Model yourself as a point particle attached to the rope 4.6 m from the limb.)**
(*a*) What is the maximum safe initial angle between the rope and the vertical at which it will not break during the swing?
(*b*) If you begin at this maximum angle, and the surface of the pond is 1.2 m below the level of the ground, with what speed will you enter the water if you let go of the rope when the rope is vertical?



(a)

$T - mg = m\frac{v^2}{R}$

$mgR(1-\cos\theta) = \frac{1}{2}mv^2$

$\cos\theta = \frac{3mg-T}{2mg}$

Physical concept: work kinetic energy theorem
Newton second Law

$R = 4.6\,m$
$L = 5.2\,m$
$m = 70\,kg$
$S = 1.2\,m$

$T - mg = 80N$
$T = 80N + mg$

$\theta = \arccos\frac{3mg-T}{2mg}$ [illegible]

$\theta = \arccos = \frac{3\times 70kg\times 9.81m/s^2 - (80N + 70kg\times 9.8m/s^2)}{2\times 70kg\times 9.81m/s^2}$
$\approx 20°$

dimensial analysis
$\frac{M\frac{L}{T^2} - M\frac{L}{T^2}}{M\frac{L}{T^2}} = 1$ OK

W = ΔK

b). $mgR(1-\cos\theta) + mg(L-R+S) = \frac{1}{2}mv^2$

Dimensional analysis

$v = \sqrt{2gR(1-\cos\theta) + 2g(L-R+S)} = 6.4\ m/s$

$= \sqrt{2\times 9.8m/s^2\times 4.6(1-\cos 20) + 2\times 9.8m/s(5.2m - 4.6m + 1.2m)} = 6.4m/s$

$\frac{L}{T} = \sqrt{\frac{L}{T^2}\times L + \frac{L}{T^2}\times L} = \frac{L}{T}$ ✓

Evaluation: the dimension is correct and the speed and angle is reasonable

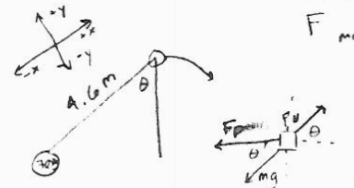
P1 (25 pts). Walking by a pond, you find a rope attached to a tree limb that is 5.2 m above ground level. You decide to use the rope to swing out over the pond. The rope is a bit frayed, but supports your 70 kg mass. You estimate that the rope might break if the tension is 80 N greater than your weight. You grab the rope at a point 4.6 m from the limb and move back to swing out over the pond. (Model yourself as a point particle attached to the rope 4.6 m from the limb.)

(a) What is the maximum safe initial angle between the rope and the vertical at which it will not break during the swing?

(b) If you begin at this maximum angle, and the surface of the pond is 1.2 m below the level of the ground, with what speed will you enter the water if you let go of the rope when the rope is vertical?

$70\ kg \cdot 9.81\ m/s^2 = 686.7\ N$

$F_{max} = 686.7\ N + 80\ N = 766.7\ N$

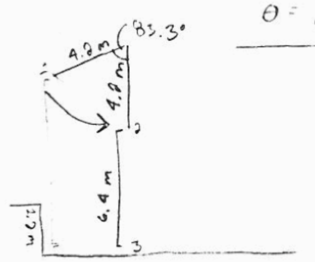


$F_T = mg\cos\theta + mg = F_{max}$

$mg\cos\theta = F_{max} - mg$

$\theta = \arccos \frac{F_{max} - mg}{mg} = \arccos\left(\frac{80}{686.7\ N}\right) = \boxed{83.3°}$ a)

$U = mgy = 766.7\ N \cdot$



$V_{12} = \sqrt{9.81\ m/s^2 \cdot 4.2\ m} = 6.419\ m/s$ (c)

$U_{23} = 686.7\ N \cdot 6.4\ m \cdot \cos 83.3° = 366136.9952\ J = K_{23}$

$K = \frac{1}{2}mv^2$

$v^2 = \frac{2K}{m}$

$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \cdot 366136.9952\ J}{70\ kg}} = 605\ m/s$

$F = ma$
$1 = kg \cdot m/s^2$

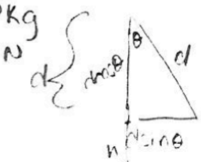
P1 (25 pts). Walking by a pond, you find a rope attached to a tree limb that is 5.2 m above ground level. You decide to use the rope to swing out over the pond. The rope is a bit frayed, but supports your 70 kg mass. You estimate that the rope might break if the tension is 80 N greater than your weight. You grab the rope at a point 4.6 m from the limb and move back to swing out over the pond. (Model yourself as a point particle attached to the rope 4.6 m from the limb.)
(*a*) What is the maximum safe initial angle between the rope and the vertical at which it will not break during the swing?
(*b*) If you begin at this maximum angle, and the surface of the pond is 1.2 m below the level of the ground, with what speed will you enter the water if you let go of the rope when the rope is vertical?

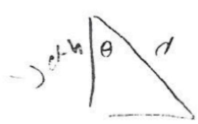
$F = ma$
$F = 70 \cdot 9.81$
$F = 686.7\,N$
$F = 687\,N$

$mg = 687\,N$
$d = R$

Known
$h = 5.2\,m$
$m_y = 70\,kg$
$T = 80\,N$
$\theta = ?$

$h = \frac{T \cdot R}{2Mg}$





$\Sigma F = F_c = T - mg + mg$

$F_c = m\frac{v^2}{R} = T$

$m\frac{v^2}{R} = T$

$v^2 = \frac{T \cdot R}{m}$

$K_i + K_f + U_i - U_f = 0$
$\frac{1}{2}mv_f^2 - mgh = 0$
$\frac{1}{2}mv_f^2 = mgh$
$v_f^2 = 2gh$

$\frac{2gh}{2g} = \frac{T \cdot R}{m}$

$h = \frac{T \cdot R}{\ldots}$

$\theta = \cos^{-1}\left(1 - \frac{\frac{T \cdot R}{2mg}}{d}\right)$

$\theta = \cos^{-1}\left(1 - \frac{T}{2mg}\right)$

$\theta = \cos^{-1}\left(1 - \frac{80\,N}{2 \cdot 687\,N}\right)$

$\theta = \cos^{-1}\left(1 - \frac{80}{1374}\right)$

$\theta = \cos^{-1}(1 - 0.06)$

$\theta = \cos^{-1}(0.94)$

$\theta = 19.9°$

$d\cos\theta = d - h$
$\cos\theta = \frac{d-h}{d}$
$\theta = \cos^{-1}\left(1 - \frac{h}{d}\right)$

b) $v^2 = \frac{T \cdot R}{m}$

$v = \sqrt{\frac{T \cdot R}{m}}$

$v = \sqrt{\frac{80\,N \cdot 5.2}{687\,N}}$

$v = \sqrt{0.61}$

$v = 0.78$

94
0.07
0.61

C35

P1 (25 pts). Walking by a pond, you find a rope attached to a tree limb that is 5.2 m above ground level. You decide to use the rope to swing out over the pond. The rope is a bit frayed, but supports your 70 kg mass. You estimate that the rope might break if the tension is 80 N greater than your weight. You grab the rope at a point 4.6 m from the limb and move back to swing out over the pond. (Model yourself as a point particle attached to the rope 4.6 m from the limb.)
(*a*) What is the maximum safe initial angle between the rope and the vertical at which it will not break during the swing?
(*b*) If you begin at this maximum angle, and the surface of the pond is 1.2 m below the level of the ground, with what speed will you enter the water if you let go of the rope when the rope is vertical?

Picture | Target Quantities | Equations

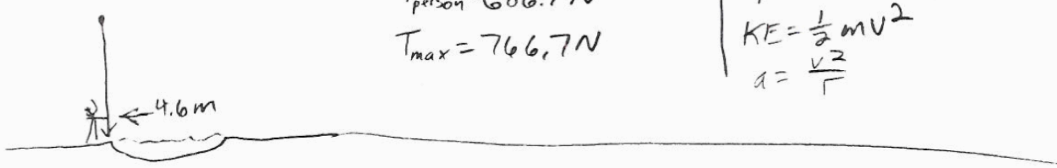
$F_{person} = 686.7\ N$

$T_{max} = 766.7\ N$

$F = ma$

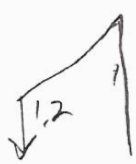
$KE = \frac{1}{2} m v^2$

$a = \frac{v^2}{r}$



$766.7 = mg \sin\theta$

$\theta = \frac{686.7}{766.7} \sin^{-1}$ $\boxed{\theta_{max} = 63.59°}$



$= 1.2 = \frac{1}{2} 9.81 t^2$

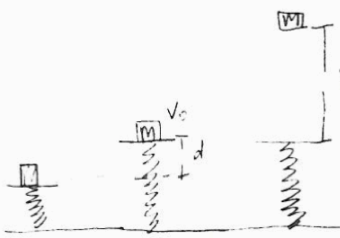
$t = .5$

$v = 9.81 \cdot .5$

$\boxed{V = 4.9\ m/s}$

C31

P2 (25 pts). A spring-loaded gun is cocked by compressing a short, strong spring by a distance *d*. It fires a signal flare of mass *m* directly upward. The flare has speed v_0 as it leaves the spring and is observed to rise to a maximum height *h* above the point where it leaves the spring. After it leaves the spring, effects of drag force by the air on the flare are significant. (Express answers in terms of *m*, v_0, *d*, *h*, and *g*.) (*a*) How much work is done on the spring during the compression? (*b*) What is the value of the force constant *k*? (*c*) Between the time of firing and the time at which maximum elevation is reached, how much mechanical energy is dissipated by the drag force?



(a) The work done on the spring is converted to kinetic energy and potential energy of the mass

So $W_s = K_{mi} + U_{mi} = \frac{1}{2}mv_0^2 + mgd$

(b) The work done on the spring also equal to the potential energy of the spring

$\therefore U_s = \frac{1}{2}kd^2 = \frac{1}{2}mv_0^2 + mgd$

$\therefore k = \frac{mv_0^2 + 2mgd}{d^2}$

(c) total energy when firing $E_i = \frac{1}{2}mv_0^2 + mgd$

total energy when reached maximum elevation

$E_f = mg(d+h)$

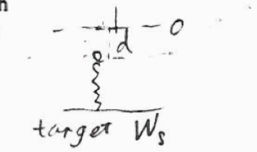
$\therefore$ Energy dissipated $= E_i - E_f$

$= \frac{1}{2}mv_0^2 + mgd - mg(d+h)$

$= \frac{1}{2}mv_0^2 - mgh$

C32

P2 (25 pts). A spring-loaded gun is cocked by compressing a short, strong spring by a distance d. It fires a signal flare of mass m directly upward. The flare has speed v_0 as it leaves the spring and is observed to rise to a maximum height h above the point where it leaves the spring. After it leaves the spring, effects of drag force by the air on the flare are significant. (Express answers in terms of m, v_0, d, h, and g.) (*a*) How much work is done on the spring during the compression? (*b*) What is the value of the force constant k? (*c*) Between the time of firing and the time at which maximum elevation is reached, how much mechanical energy is dissipated by the drag force? $W = \Delta k$



(a) $W_S - mgd = \frac{1}{2}mV_0^2 - 0$

$W_S = \frac{1}{2}mV_0^2 + mgd$

Physical concept

work - kinetic energy theorem

Dimensional analysis

$M(\frac{L}{T})^2 + M\frac{L}{T^2}L = M(\frac{L}{T})^2$

ok.

work energy theorem

$W_{ex} = \Delta K + \Delta U_g + \Delta U_s$

$0 = \frac{1}{2}mV_0^2 - 0 + mgd + (0 - \frac{1}{2}kd^2)$

(b) $U(s) = \frac{1}{2}kx^2 = \frac{1}{2}kd^2$

$\frac{1}{2}kd^2 = \frac{1}{2}mV_0^2 + mgd$

$k = \frac{mV_0^2 + 2mgd}{d^2}$

Dimensional analysis $= \frac{M(\frac{L}{T})^2 + M\frac{L}{T^2}L}{L^2}$

$k = \frac{M}{T^2}$

$= \frac{M}{T^2}$ ✓

(c)

$E_i = E_f$

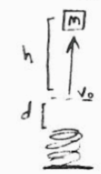
$E_{mechanic_i} = E_{mechanic 2} + E_{dissipa..}$

$\frac{1}{2}mV_0^2 + mgd = mgh + E_{dissipate}$

$E_{dissipate} = \frac{1}{2}mV_0^2 + mgd - mgh$

* My answer is reasonable because dimension is correct.

P2 (25 pts). A spring-loaded gun is cocked by compressing a short, strong spring by a distance d. It fires a signal flare of mass m directly upward. The flare has speed v_0 as it leaves the spring and is observed to rise to a maximum height h above the point where it leaves the spring. After it leaves the spring, effects of drag force by the air on the flare are significant. (Express answers in terms of m, v_0, d, h, and g.) (*a*) How much work is done on the spring during the compression? (*b*) What is the value of the force constant k? (*c*) Between the time of firing and the time at which maximum elevation is reached, how much mechanical energy is dissipated by the drag force?



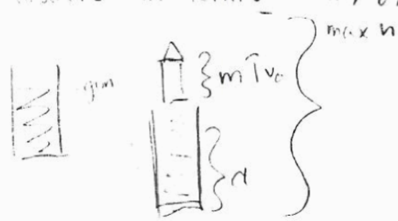
a) $W = Fd = mgd^2$

b) d^2

c) $\dfrac{\frac{1}{2} m v_f^2}{\frac{1}{2} m v_0^2} = \dfrac{v_f^2}{v_0^2}$

P2 (25 pts). A spring-loaded gun is cocked by compressing a short, strong spring by a distance *d*. It fires a signal flare of mass *m* directly upward. The flare has speed v_0 as it leaves the spring and is observed to rise to a maximum height *h* above the point where it leaves the spring. After it leaves the spring, effects of drag force by the air on the flare are significant. (Express answers in terms of *m*, v_0, *d*, *h*, and *g*.) (*a*) How much work is done on the spring during the compression? (*b*) What is the value of the force constant *k*? (*c*) Between the time of firing and the time at which maximum elevation is reached, how much mechanical energy is dissipated by the drag force?

answers in terms of $m, v_0, d, h + g$



$E_{mech} = K + U$

E_{lost} by the drag force

Work done on spring?

$W = \Delta K$

$W = F \cdot d$

$F_{spring} = -kx$

$W_{total} = \Delta K + \Delta U$

$W_{total} = K_i + K_f + U_i - U_f$

$\frac{1}{2}mv^2 - mgh$

$= \frac{1}{2}kx^2 - mgh$

$= \frac{1}{2}kx^2 + mgh$

$W_{total} = \frac{1}{2}kx^2 + mgh$

$W_{total} = \frac{1}{2}kv^2 + mgh$

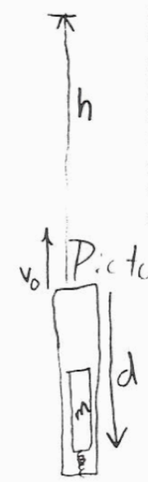
k?

$W_{total} = \frac{1}{2}kx^2 + mgh$

$W_{total} - mgh = \frac{1}{2}kx^2$

$\frac{2(W_{total} - mgh)}{x^2} = k$

P2 (25 pts). A spring-loaded gun is cocked by compressing a short, strong spring by a distance ***d***. It fires a signal flare of mass ***m*** directly upward. The flare has speed v_0 as it leaves the spring and is observed to rise to a maximum height ***h*** above the point where it leaves the spring. After it leaves the spring, effects of drag force by the air on the flare are significant. (Express answers in terms of ***m***, v_0, ***d***, ***h***, and ***g***.) (*a*) How much work is done on the spring during the compression? (*b*) What is the value of the force constant ***k***? (*c*) Between the time of firing and the time at which maximum elevation is reached, how much mechanical energy is dissipated by the drag force?



Picture

Quantities
d = distance of spring compression
m = mass of flare
v_0 = speed of flare as it leaves the spring
h = height flare reaches after leaving spring
g = gravity

Part A)
Target - Work done on spring
Equations - $W = F \cdot d$
$F = -kx$
$-k = \frac{F}{x} = \frac{mg}{x}$
$W = \left[\left(\frac{mg}{x}\right) x\right] \cdot d$
$\boxed{W = mgd}$

Part B
Target = k constant
$-k = \frac{mg}{d}$
$\boxed{k = \left(-\frac{mg}{d}\right)}$

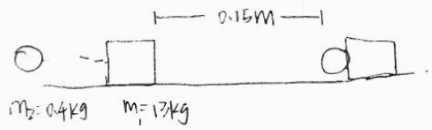
Part C
$E_{mech} = K + U$
$U = mgy$
$KE = \frac{1}{2}mv^2$

$E_{mech\,initial} - E_{mech\,Final} = E_{mech\,Dissipated}$
$\left(\frac{1}{2}mv_0^2 + 0\right) - \left(0 + mgh\right)$
$= \boxed{\frac{1}{2}mv_0^2 - mgh = E_{mech\,Dissipated}}$

Training Set (1-8) (Figure 4): Set 7

C31

P1 (25 pts). A 13-kg cubic block, 30 cm on a side, is at rest on a level floor. A 400-g glob of putty is thrown at the block perpendicular to one face of the block so that the putty travels horizontally, hits the block in the center of the face, and sticks to it. The block and putty slide 15 cm along the floor. If the coefficient of kinetic friction is 0.40, what is the initial speed of the putty?



$m_1 = 13kg$

$m_2 = 0.4kg$

$\mu_k = 0.4$

v_1 putty = ?

$F_f = \mu_k F_N = \mu_k \times (m_1 + m_2) g$

$a = \frac{F_f}{m} = \frac{\mu_k (m_1 + m_2) g}{m_1 + m_2} = \mu_k g = 0.40 \times 9.81 m/s^2 = 3.924 m/s^2$

$v_f^2 - v_i^2 = 2 \cdot a \cdot x$

$\therefore 0^2 - v_1^2 = 2 \cdot (-3.924) \times 0.15$

$\therefore v_1^2 = 1.1772 m/s^2$

$\therefore v_1 = 1.08 m/s$

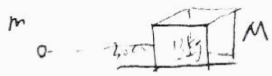
conservation of linear momentum

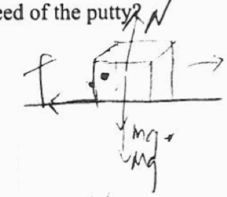
$m_p v_p = m_t v_t$

$\therefore 0.4 \times v_p = (13 + 0.4) \times 1.08$

$v_p = 36.18 m/s$

P1 (25 pts). A 13-kg, cubic block, 30 cm on a side, is at rest on a level floor. A 400-g glob of putty is thrown at the block perpendicular to one face of the block so that the putty travels horizontally, hits the block in the center of the face, and sticks to it. The block and putty slide 15 cm along the floor. If the coefficient of kinetic friction is 0.40, what is the initial speed of the putty?





Physics concept: Newton Second Law

conservation of momentum

Work - kinetic energy theorem

target V_i

$I_i = I_f$

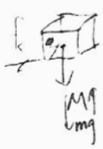
$mV_i = (M+m)V_f$

$W = \Delta K$

$-fs = 0 - \frac{1}{2}(m+M)V_f^2$ $\quad V_f = \frac{mV_i}{M+m}$

$fs = \frac{(mV_i)^2}{2(M+m)}$

$V_i = \sqrt{\frac{2(M+m)\cdot fs}{m^2}}$ $\quad$ dimensional analysis $\frac{L}{T} = \sqrt{\frac{M \cdot \frac{L}{T^2} L}{M^2}} = \sqrt{\left(\frac{L}{T}\right)^2} = \frac{L}{T}$ ✓



$f = \mu N$

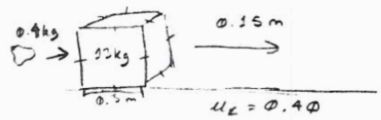
$f = \mu(M+m)g$ $\quad V_i = \sqrt{\frac{2(M+m)^2 \times \mu g s}{m^2}} = \sqrt{\frac{2\times(13kg+0.4kg)^2 \times 0.4 \times 9.81 m/s^2 \times 0.15m}{(0.4kg)^2}}$

$V_i = 36.3\ m/s$

I think my answer is reasonable. if the distance of block and putty sliding together is larger, the initial velocity will be larger.

C33

P1 (25 pts). A 13-kg, cubic block, 30 cm on a side, is at rest on a level floor. A 400-g glob of putty is thrown at the block perpendicular to one face of the block so that the putty travels horizontally, hits the block in the center of the face, and sticks to it. The block and putty slide 15 cm along the floor. If the coefficient of kinetic friction is 0.40, what is the initial speed of the putty?



Question: How fast must the putty be thrown for the block and the putty to move 0.15 m?

Physics Concepts: Force (F=ma), acceleration, friction

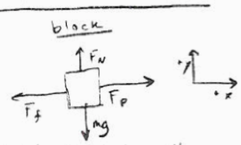
Approximations/Constraints: perfectly inelastic collision, no resistance other than friction

Analyze the Problem

Target quantity: v_i (of putty)

Useful equations: $F=ma$, $F=mg$, $v_f^2 - v_i^2 = 2a(x_f - x_i)$, $F = \mu_k mg$,
$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$

Construct a Solution



velocity of block + putty

$v_f^2 - v_i^2 = 2a(x_f - x_i)$

$v_i = \sqrt{2x\mu_k g}$

$ma = \mu_k mg$
$a = \mu_k g$

putty, block

$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v$

$m_1 v_1 = (m_1 + m_2) \cdot \sqrt{2x\mu_k g}$

$$v_p = \frac{(m_p + m_b) \cdot \sqrt{2x\mu_k g}}{m_p}$$

Numerical Answer

$$v_p = \frac{(0.4\,kg + 13\,kg) \cdot \sqrt{2 \cdot 0.15\,m \cdot 0.40 \cdot 9.81\,m/s^2}}{0.4\,kg} = \boxed{36.3\,m/s}$$

Evaluation: This answer is reasonable because a little piece of putty would need to move fast.

reasonable because a little piece of putty would need to move fast.

C34

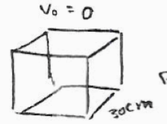
P1 (25 pts). A 13-kg, cubic block, 30 cm on a side, is at rest on a level floor. A 400-g glob of putty is thrown at the block perpendicular to one face of the block so that the putty travels horizontally, hits the block in the center of the face, and sticks to it. The block and putty slide 15 cm along the floor. If the coefficient of kinetic friction is 0.40, what is the initial speed of the putty?

$400g = \frac{kg}{1000g}$

$0.4 kg$

→ inelastic collision

before



known

$m_b = 13 kg$

side, = 30cm

$m_p = 400g = 0.4 kg$

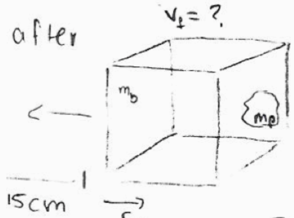
$d = 15cm$

$\mu_k = 0.40$

unknown

$v_{ip} = ?$

after



$F_f = \mu_k F_n \quad F_f = \mu_k \cdot mg$

$K_i = \frac{1}{2} m_p v_i^2 + \frac{1}{2} m_b v_i^2 \rightarrow v_{bi} = 0$

$K_i = \frac{1}{2} m_p v_i^2$

$K_f = \frac{1}{2} m_b v_f^2 + \frac{1}{2} m_p v_f^2 + \mu_k \cdot mg$

$K_f = \frac{1}{2}(m_b + m_p) v_f^2 + \mu_k \cdot mg$

$\sqrt{\frac{2\mu_k mg}{(m_b + m_p)}} = v_f$

$\frac{1}{2} m_p v_i^2 = \frac{1}{2}(m_b + m_p)\left(\frac{\sqrt{2\mu_k mg}}{(m_b + m_p)}\right) + \mu_k \cdot mg$

v_i

7.848

13.4

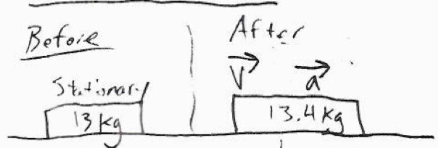
0.603497

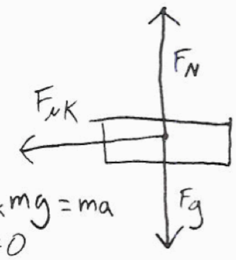
0.776896

9.1297

P1 (25 pts). A 13-kg, cubic block, 30 cm on a side, is at rest on a level floor. A 400-g glob of putty is thrown at the block perpendicular to one face of the block so that the putty travels horizontally, hits the block in the center of the face, and sticks to it. The block and putty slide 15 cm along the floor. If the coefficient of kinetic friction is 0.40, what is the initial speed of the putty?

Elastic Collision:





$F = Ma$

$F = \mu_K n$

~~$m_1 v_1 = V_2(m_1 + m_2)$~~

$v = v_0 + at$

~~$x = x_0 + vt + \frac{1}{2}at^2$~~

$F_x = \mu_K n = \mu_K mg = ma$

$F_y = F_N - F_g = 0$

$$W = F \cdot D = \frac{1}{2}mv^2 \quad (\Delta KE)$$

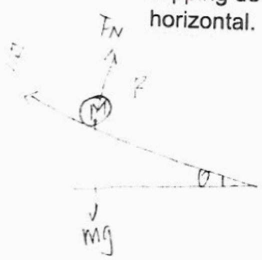
$$\sqrt{\frac{2F \cdot D}{m}} = V_0$$

$$\sqrt{\frac{2(.4 \cdot 13.4 \cdot 9.81) \cdot .15}{13.4}} = \boxed{1.18\ m/s = V_{initial}}$$

Training Set (1-8) (Figure 4): Set 8

C31

P2 (25 pts). A solid sphere with mass M and radius R rolls without slipping down a plane inclined at an angle θ with respect to horizontal. What is its acceleration?



mass = M
Radius = R
Angle = θ
acceleration = ?

mg and F_N are all both acting on the center of mass. So F_f is the only force acting as torque.

According to Newton's second law

$mg\sin\theta - F_f = ma$

Also,

$\tau = F_f \cdot R = I\alpha = I\frac{a}{R}$

So $F_f \cdot R = I\frac{a}{R}$

$(mg\sin\theta - ma)\cdot R = I\frac{a}{R}$

$\therefore a = \frac{mg\sin\theta}{(I/R + mR)}$

We know I of a solid sphere is $\frac{2}{5}mR^2$

So $a = \frac{mg\sin\theta}{(\frac{2}{5}\frac{mR^2}{R} + mR)} = \frac{5\, g\sin\theta}{7R}$

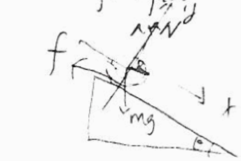
C32

P2 (25 pts). A solid sphere with mass M and radius R rolls without slipping down a plane inclined at an angle θ with respect to horizontal. What is its acceleration?

target a

physical concept
Newton Second Law
Newton Second Law for rotation

Free body diagram



$\sum F_{net} = ma$

$\sum F_x \quad mg\sin\theta - f = ma$

$\sum F_y = mg\cos\theta - N = 0$

$\tau = I\alpha$

Newton Second law for rotation $\quad fR = \frac{2}{5}mR^2 \cdot \alpha$

$\alpha = \frac{a}{R}$

$fR = \frac{2}{5}mR^2\frac{a}{R}$

$f = \frac{2}{5}ma$

$mg\sin\theta - \frac{2}{5}ma = ma$

$a = \frac{5}{7}g\sin\theta$ $\Rightarrow$ dimensional analysis $\frac{L}{T^2} = \frac{L}{T^2}$ ✓

I think the answer is reasonable, the acceleration of solid sphere is larger than the acceleration of hoop which is $\frac{1}{2}g\sin\theta$ in the same mass and radius because moment of inertia of solid sphere is smaller than the hoop's

P2 (25 pts). A solid sphere with mass *M* and radius *R* rolls without slipping down a plane inclined at an angle θ with respect to horizontal. What is its acceleration?

Understand the Problem

What is the linear acceleration of the sphere?
(Inertia, angular acceleration)

Analyze the Problem

Target quantity: a (acceleration)

Inertia of solid sphere: $I = \frac{2}{3} M R^2$

$\omega = r \times F$

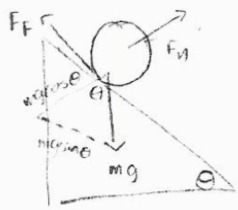
Construct a Solution

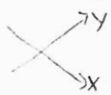
$\omega = r \times F = R \cdot F \sin\theta$

$\omega = \alpha t$

P2 (25 pts). A solid sphere with mass M and radius R rolls without slipping down a plane inclined at an angle θ with respect to horizontal. What is its acceleration?

*without slipping = F_f





Unknown
$a = ?$

Known
M = mass
R = radius
$\theta = \theta$

$a = \alpha R$

$F_f = \mu_k F_n$

$\alpha = a/R$

$\Sigma F_x = mg\sin\theta - F_f = ma$ $\quad F_f = \mu_k F_n \quad F_n = mg \quad \Sigma \tau = F \times R$

$\Sigma F_y = F_n - mg\cos\theta = 0$ $\quad F_f = \mu_k mg$

$mg\sin\theta - F_f = ma$

$-1(-F_f = ma - mg\sin\theta)$

$F_f = -ma + mg\sin\theta$

Solid sphere $= I = 2/5 mR^2$

~~$K_r = 1/2 I\omega^2$~~

~~$K = 1/2 (2/5 mR^2)\omega^2$~~

~~$K = 1/2 (2/5 mR^2)\left(\frac{a}{R}\right)^2$~~

~~$K = 1/2 (2/5 mR^2)\left(\frac{a^2}{R^2}\right)$~~

~~$= 1/5 ma$~~

$F = 2/5 ma$

$-ma + mg\sin\theta = 2/5 ma$

$mg\sin\theta = 7/5 ma + ma$

$g\sin\theta = 7/5 a + a$

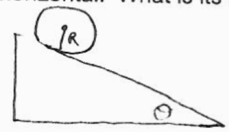
$g\sin\theta = 7/5 a$

$\boxed{a = 7/5 g\sin\theta}$

$m/s^2 = m/s^2$

C35

P2 (25 pts). A solid sphere with mass *M* and radius *R* rolls without slipping down a plane inclined at an angle θ with respect to horizontal. What is its acceleration?



Freebody of sphere

Equations

$I = \frac{1}{2}MR^2$

~~$\tau = FR$~~

$F_x = Mg\cos\theta$

$L = I\alpha$

$\alpha = \frac{a}{r}$

Knowns

θ = angle of incline

R = Radius of sphere

F_x = Force of gravity

g = gravity

M = Mass of sphere

Newtons 3rd law pairings

$F = MA = I\alpha$

$Mg\cos\theta = \frac{1}{2}MR^2 \cdot \frac{a}{R}$

$2Mg\cos\theta = \frac{MR^2 a}{R}$

$\frac{2Mg\cos\theta}{MR} = a$

Answer

Training Set (1-8) (Figure 4): Set 9

C26 C25

P1 (25 pts). Just as a passenger train that you boarded a few moments before is beginning to pull out of the station, it is passed by a freight train traveling at 45. mi/h along a parallel track in the same direction that your train is headed. If your train undergoes constant acceleration, how far will you have traveled before your train passes the freight train, assuming that the freight train's speed remains constant? All that you know about your train's acceleration is that it takes 3.5 miles for the train to reach a speed of 60. mi/h, starting from rest.

$x = x_0 + v_0 t + \frac{1}{2} a t^2$ $v = at + v_0$ $v_f^2 - v_i^2 = 2a(x_f - x_i)$

45 m/h

3.5 mi, 60 m/h, x

$\frac{at}{v}$ $a = \frac{v - v_0}{t}$ $v_f^2 - v_i^2 = \left(\frac{2v - 2v_0}{t}\right)(x_f - x_i)$

$0 = 0 +$ $x = x_0 + v_0 t + \frac{1}{2}\left(\frac{v - v_0}{t}\right)t^2$ P = when 45 gets passed

$x =$ $v_0 = v - at$ $v - v_0 = at$

115 mph

$3.5 = 0 + 60_{mi/h} t + \frac{1}{2} a t^2$ $x = 45t + \frac{1}{2} 0 t^2$ $60 = 514.28t \cdot 0$

$x = 3.5 + 60t + \frac{1}{2} t^2$

$45t = 3.5 \cdot 60t$

$= 15t$

$t =$ $x < 6.125$ miles

$x = 0 + \frac{at}{v} t + \frac{1}{2} a t^2$

$v_1 = 45$ mph

$v_2 = 60$ mph

5.3 miles

C27

P1 (25 pts). Just as a passenger train that you boarded a few moments before is beginning to pull out of the station, it is passed by a freight train traveling at 45. mi/h along a parallel track in the same direction that your train is headed. If your train undergoes constant acceleration, how far will you have traveled before your train passes the freight train, assuming that the freight train's speed remains constant? All that you know about your train's acceleration is that it takes 3.5 miles for the train to reach a speed of 60. mi/h, starting from rest.

45mi/h

Freight train

your train

speed of freight train = 45 mi/h

your train

$v_0 = 0$

your train

speed: $\frac{\text{dist.}}{t}$

$60 = \frac{3.5}{t}$

$t = .0583\,s$

$V_{ave} = \frac{\Delta x}{\Delta t}$

$= \frac{3.5}{.0583}$

$V = 60$

$a_{ave} = \frac{\Delta V}{\Delta t}$

$a = \frac{60}{.0583}$

$a = 1028.6\,m/s^2 \rightarrow \frac{1820}{3600} = 520.01\,mi/hr$

$x = x_0 + V_0 t + \frac{1}{2}a(t)^2$

$x = 0 + 0 + \frac{1}{2}at^2$

$x = \frac{1}{2}(520.0)(0.0583)$

$x = 15.16\,mi$

$T \le 5$ **C28**

P1 (25 pts). Just as a passenger train that you boarded a few moments before is beginning to pull out of the station, it is passed by a freight train traveling at 45. mi/h along a parallel track in the same direction that your train is headed. If your train undergoes constant acceleration, how far will you have traveled before your train passes the freight train, assuming that the freight train's speed remains constant? All that you know about your train's acceleration is that it takes 3.5 miles for the train to reach a speed of 60. mi/h, starting from rest.

velocity of freight train $= V_{FT} = 45\ mi/h$

acceleration of your train $= a_{yt} =$

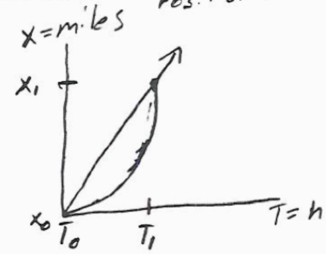
(1)* time for youre train to reach 60 mi/h

$3.5 mi = 60\ mi/h$

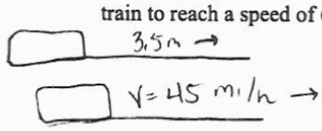
$h = \frac{60 mi}{3.5 mi}$

$h = \frac{60}{3.5}$

Position function



P1 (25 pts). Just as a passenger train that you boarded a few moments before is beginning to pull out of the station, it is passed by a freight train traveling at 45. mi/h along a parallel track in the same direction that your train is headed. If your train undergoes constant acceleration, how far will you have traveled before your train passes the freight train, assuming that the freight train's speed remains constant? All that you know about your train's acceleration is that it takes 3.5 miles for the train to reach a speed of 60. mi/h, starting from rest.



$v_f^2 - v_i^2 = 2a(x_f - x_i)$

$v_{pt} = (60\,mi/h)\left(\frac{5280\,ft}{1\,mi}\right)\left(\frac{1\,h}{3600\,s}\right) = 88\,ft/s$

$x_f = (3.5\,mi)\left(\frac{5280\,ft}{1\,mi}\right) = 18480$

$v_{ft} = (45\,mi/h)\left(\frac{5280\,ft}{1\,mi}\right)\left(\frac{1\,h}{3600\,s}\right) = 66\,ft/s$

$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)}$

$a = \frac{(88\,ft/s)^2 - 0^2}{2(18480\,ft - 0)}$

$a = 0.21\,ft/s^2$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$\sqrt{\frac{2x}{a}} = t$

$\sqrt{\frac{2(18480\,ft)}{0.21\,ft/s^2}} = t$

$t = 419.52\,s$

$x_{ft} = v_0 t = 66\,ft/s\,(419.52\,s) = 27688.32\,ft$

$x_{pt} = \frac{1}{2}at^2 = \frac{1}{2}\,0.21\,ft/s^2 \cdot (419.52\,s)^2 = 18479.69\,ft$

$88\,ft/s - 66\,ft/s = 22\,ft/s$

27688.32 ft
− 18479.69 ft
= 9208.63 ft
(between the trains)

$d = rt$

$t = \frac{d}{r}$

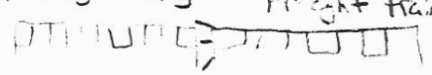
$\frac{9208.63\,ft}{22\,ft/s}$

418.57 s
+ 419.52 s
= 838.09 s

838.09 s
or 13.97 min
or 0.23 h

P1 (25 pts). Just as a passenger train that you boarded a few moments before is beginning to pull out of the station, it is passed by a freight train traveling at 45. mi/h along a parallel track in the same direction that your train is headed. If your train undergoes constant acceleration, how far will you have traveled before your train passes the freight train, assuming that the freight train's speed remains constant? All that you know about your train's acceleration is that it takes 3.5 miles for the train to reach a speed of 60. mi/h, starting from rest.

Passenger trains Freight train



3.5 miles to reach 60 mi/h

45 mi/h

$V_F = 45$ mi/h $a_F = 0$ $X_{0F} = 0$

$V_{0P} = 0$ m/h

$a_P =$ constant $X_{0P} = 0$

$t = ?$

Time to reach 60 mi/h is equal to $= \frac{3.5 \text{ mi}}{\frac{60 \text{ mi}}{h}} = 3.5 \text{ mi} \cdot \frac{h}{60 \text{ mi}} = .058$ hours to reach 60 mi/h

$V_{fP} = a_P t + V_{0P}$ $V_{fP} = 60$ mi/h $t = .058$ hours $V_{0P} = 0$

$60 = .058a$ $a_P = 1034 \text{ mi}/h^2$

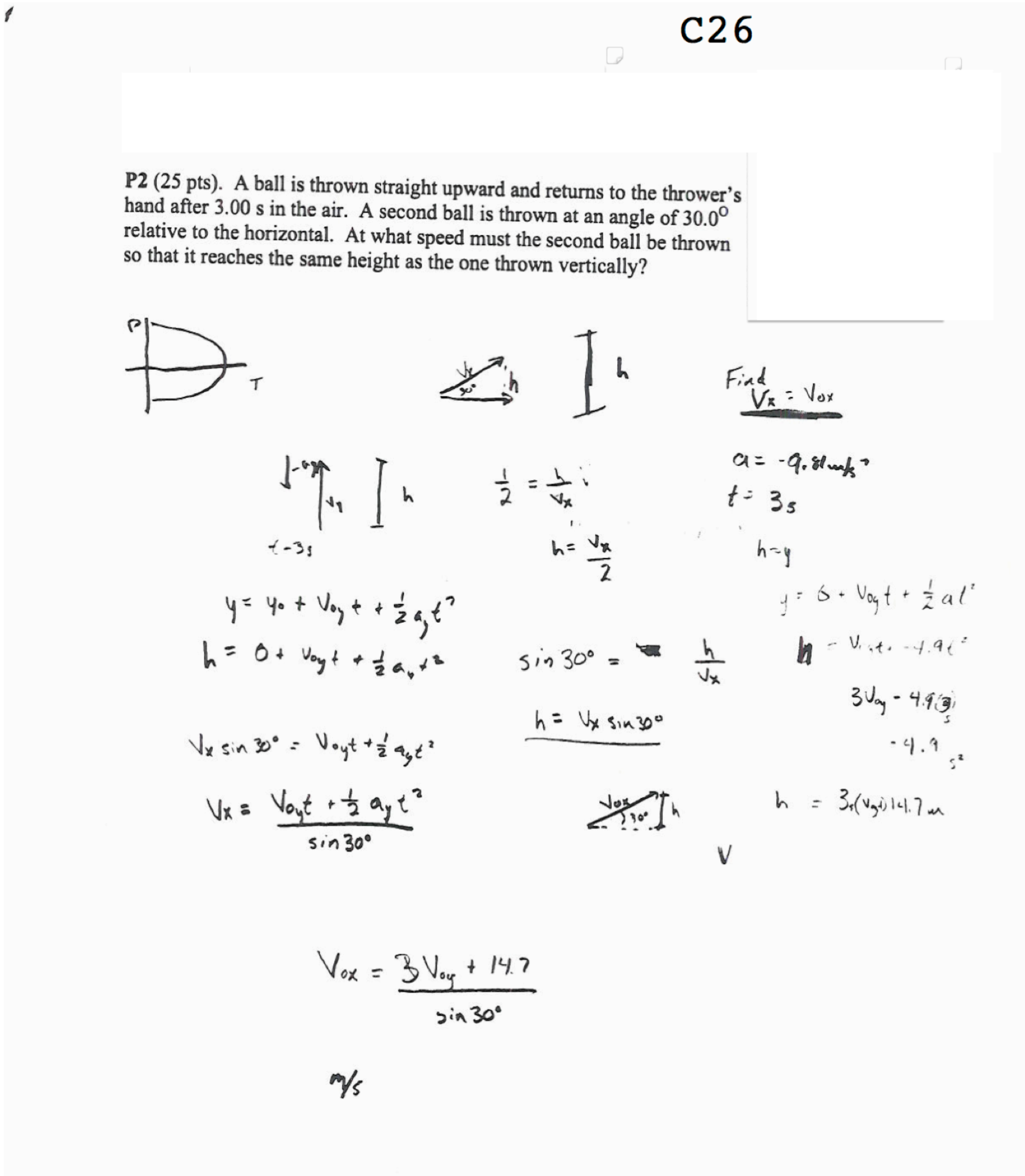
Then position of passenger train and frieght train must equal each other.

$X_F = V_F t$ $X_P = \frac{1}{2}at^2$ $V_F t = \frac{1}{2}at^2$

$0 = \frac{1}{2}at^2 - V_F t$ $0 = t(\frac{1}{2}at - V_F)$ $0 = t(517t - 4$

$0 = 517t - 45$ $\boxed{t = .087 \text{ hours}}$

Training Set (1-8) (Figure 4): Set 10



C26

P2 (25 pts). A ball is thrown straight upward and returns to the thrower's hand after 3.00 s in the air. A second ball is thrown at an angle of 30.0° relative to the horizontal. At what speed must the second ball be thrown so that it reaches the same height as the one thrown vertically?

Find $V_x = V_{ox}$

$a = -9.81 m/s^2$

$t = 3 s$

$h = y$

$\frac{1}{2} = \frac{1}{V_x}$

$h = \frac{V_x}{2}$

$y = y_0 + V_{oy}t + \frac{1}{2}a_y t^2$

$h = 0 + V_{oy}t + \frac{1}{2}a_y t^2$

$\sin 30° = \frac{h}{V_x}$

$h = V_x \sin 30°$

$V_x \sin 30° = V_{oy}t + \frac{1}{2}a_y t^2$

$V_x = \frac{V_{oy}t + \frac{1}{2}a_y t^2}{\sin 30°}$

$y = 0 + V_{oy}t + \frac{1}{2}at^2$

$h = V_{oy}t - 4.9t^2$

$3V_{oy} - 4.9(3)$

-4.9

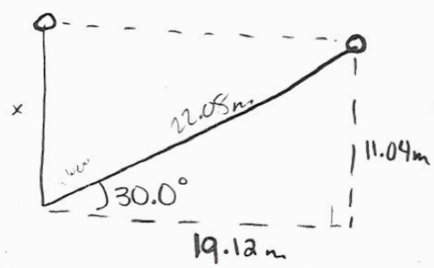
$h = 3(V_{oy}) 14.7 m$

$V_{ox} = \frac{3V_{oy} + 14.7}{\sin 30°}$

m/s

C27

P2 (25 pts). A ball is thrown straight upward and returns to the thrower's hand after 3.00 s in the air. A second ball is thrown at an angle of 30.0° relative to the horizontal. At what speed must the second ball be thrown so that it reaches the same height as the one thrown vertically?



$t = 3.00 s$

$a = 9.81 m/s^2$

$x = x_0 + v_0 t + \frac{1}{2} a t^2$

$x = 0 + 0 + \frac{1}{2} a t^2$

$V = V_0 \cos\theta$

$x = \frac{1}{2}(9.81)(1.5)^2$ 1.5 = half the first ball's journey

$x = 11.04m$

$V_f^2 - V_i^2 = 2a(x_f - x_i)$

$V_f = 0$ $a = -9.81$

$x_f = 22.08$ $x_i = 0$

$-V_i^2 = 2(-9.81)(22.08)$

$-V_i^2 = -433.2096$

$V_i^2 = 433.2096$

$V_i = 20.81 m/s$

C28

V_{SB} = Velocity of second Ball

$T_1 = 3.00$s **P2** (25 pts). A ball is thrown straight upward and returns to the thrower's hand after 3.00 s in the air. A second ball is thrown at an angle of 30.0° relative to the horizontal. At what speed must the second ball be thrown so that it reaches the same height as the one thrown vertically?

X_{su} = meters Straight up

(1)* $X_{su} = \frac{1}{2}9.8T^2 + V_0T + \cancel{x_0}$

(2)*

(3)* $y_2 = X_{su} = \frac{1}{2}9.8T^2 + V_0$

(4)*

(5) $V = (aT + V_0)$

(6) $V_0 = (V) - aT$ (5) int (6)

(7) $V_0 = aT + (aT + V_0)$

unknowns

$V_0 =$

$X_{su} =$

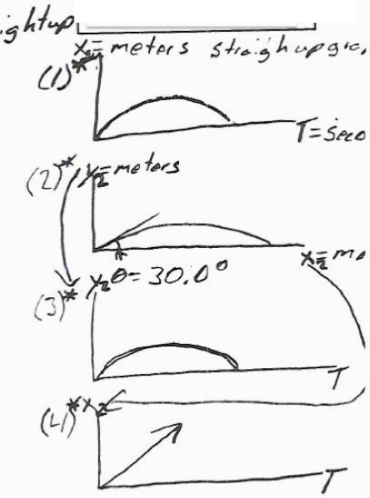
$V_{SB} =$

y_2

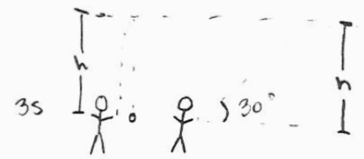
Velocity initial = V_0

Velocity = V

=



P2 (25 pts). A ball is thrown straight upward and returns to the thrower's hand after 3.00 s in the air. A second ball is thrown at an angle of 30.0° relative to the horizontal. At what speed must the second ball be thrown so that it reaches the same height as the one thrown vertically?



V_{0} = ?
h = ?

$X = X_0 + V_0 t + \frac{1}{2} a t^2$

$V = at + V_0$

$V_0 = V - at$

(top h) $V_0 = 0 - (-9.8 m/s^2) \cdot (1.5 s)$

$V_{0_1} = 14.7 m/s$

$V_{0y} = \sin\theta \, V_0$

$h = V_{0y} t + \frac{1}{2} a t^2$

$V_f^2 - V_i^2 = 2a(x_f - x_i)$

$X = V_0 t + \frac{1}{2} a t^2$

$0 - V_0^2 = 2(-9.8 m/s^2)(11.025 m)$

$= 14.7 m/s \cdot (1.5 s) + \frac{1}{2}(-9.8 m/s^2)(1.5)$

$h = 11.025 m$

$V_{0y} = \sqrt{216.09 m^2/s^2}$

$= 14.7 m/s$

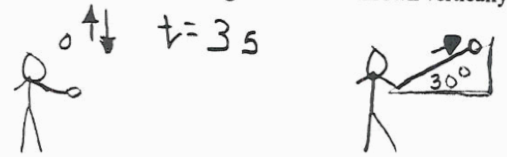
$V_0 = \frac{V_{0y}}{\sin\theta}$

$= \frac{14.7 m/s}{\sin 30°}$

$V_0 = 29.4 m/s$

The dimensions match, the V_0 is reasonable because it will take a higher V_0 to reach h at an angle

P2 (25 pts). A ball is thrown straight upward and returns to the thrower's hand after 3.00 s in the air. A second ball is thrown at an angle of 30.0° relative to the horizontal. At what speed must the second ball be thrown so that it reaches the same height as the one thrown vertically?



h = same as first ball   t = 3 s

$V_2 = ?$

Ball must be in air for same amount of time as first ball.

$$Y(t) = Y_0 + V_{0y} t + \tfrac{1}{2} a t^2$$

$a = -9.8$

$V_{0y} = V_0 \cdot \sin 30°$

$$Y(t) = V_0 \sin 30° t - 4.91 t^2$$

height has to equal first ball

hight of ball 1 – $y_1(t) = y_0 + V_{0y} t + \tfrac{1}{2} a t^2$ $t = 3\,s$ $y_1(t) = y_0 = 0$

$0 = V_{0y} t - 4.91 t^2$ $44.19 = 3 V_{0y}$

$V_{0y} = 14.73\ m/s$

$$V_{0y} t + \tfrac{1}{2} a t^2 = V_0 \sin 30° t - 4.91 t^2$$

– Set heights of first ball and second ball equal to each other then solved for V_0 of second ball

$$V_{0y} t = V_0 \sin 30° t$$

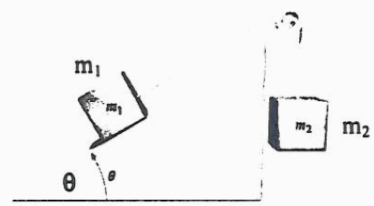
$$V_{0y} = V_0 \sin 30°$$

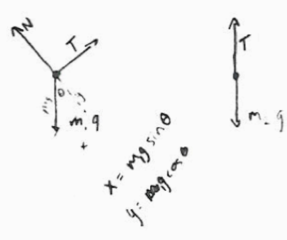
$$\boxed{V_0 = 29.5\ m/s \text{ for second ball}}$$

Training Set (1-8) (Figure 4): Set 11

C26

P1 (25 pts). A massless string, as shown in the figure below, connects two objects. The incline and the massless pulley are frictionless. Find the acceleration of the objects and the tension in the string in terms of θ, m_1 and m_2.





$x = m_1 g \sin\theta$

$y = m_1 g \cos\theta$

$M_2 g = m_1 g \sin\theta$

$\sum F = ma$

$T =$

$g = \frac{T}{M_2}$

$m_1 g \sin\theta - T = (m_1 + m_2) a$

$m_2 g - T = (m_1 + m_2) a \quad T = m_2 g - (m_1 + m_2) a$

unknown-
acceleration = ?
T = ?

know
θ m_1 m_2

$M_2 g - m_1 g \sin\theta = (m_1 + m_2) a$

$$a = \frac{m_2 g - m_1 g \sin\theta}{(m_1 + m_2)}$$

$$\theta = \arcsin\left(\frac{m_2}{m_1}\right)$$

$$T = m_2 g - \frac{m_2 g - m_1 g \sin\theta}{(m_1 + m_2)} (m_1 + m_2)$$

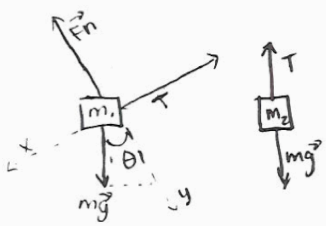
$T = m_2 g - m_2 g - m_1 g \sin\theta$

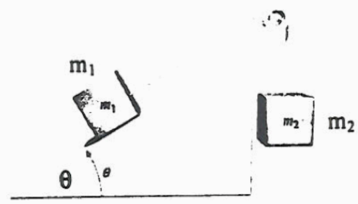
$$T = 2(m_2 g) - m_1 g \sin\theta$$

C27

P1 (25 pts). A massless string, as shown in the figure below, connects two objects. The incline and the massless pulley are frictionless. Find the acceleration of the objects and the tension in the string in terms of θ, m_1 and m_2.

Free body diagrams





m_1

$\Sigma F_x = T_1 + m_1 g_x \tan\theta = m_1 a$

$\Sigma F_y = F_N + m_1 g \cos\theta = 0$

$a_1 = \frac{T}{m_1} + g \tan\theta$

$T_1 = m_1 a_1 - m_1 g_x \tan\theta$

m_2

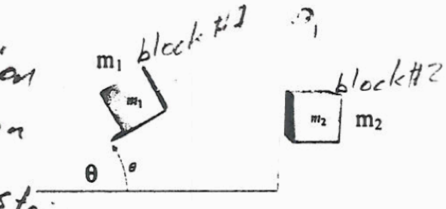
$\Sigma F_x = m_2 a = 0$

$\Sigma F_y = T_2 + m_2 g = m_2 a_2$

$a_2 = \frac{T}{m_2} + g \qquad T_2 = m_2 a_2 - m_2 g$

P1 (25 pts). A massless string, as shown in the figure below, connects two objects. The incline and the massless pulley are frictionless. Find the acceleration of the objects and the tension in the string in terms of θ, m_1 and m_2.

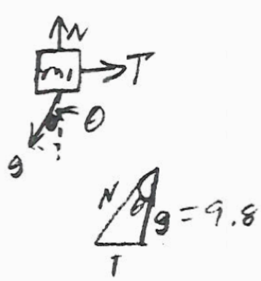
a_2 = acceleration block #2
a_1 = acceleration block #1
T = Tension of string



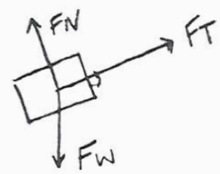
Target
$a_1 =$
$a_2 =$
$T = \frac{g\sin\theta}{\cos\theta}$

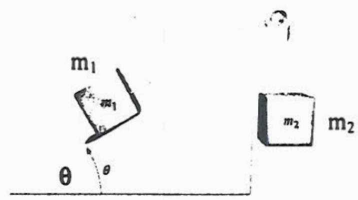
$T = N\sin\theta = \frac{g\sin\theta}{\cos\theta}$

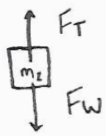
$N = \frac{g}{\cos\theta}$



P1 (25 pts). A massless string, as shown in the figure below, connects two objects. The incline and the massless pulley are frictionless. Find the acceleration of the objects and the tension in the string in terms of θ, m_1 and m_2.







$ma = F_T - F_W$
$F_W = mg$
$F_{T_1} = \sin\theta\, mg$
$F_N = \cos\theta\, mg$

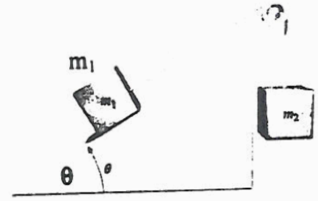
$F_{T_1} = \sin\theta\, ma$
$F_{T_2} = ma + F_W$

$F_T = \sin\theta\, mg + ma + m_2 g$

$F_{T_1} = \sin\theta\, m_1 a$
$\sin\theta\, m_1 a = F_T - m_1 g$
$\sin\theta\, m_1 a - ma = -m_1 g$
$a(\sin\theta\, m_1 - m) = -m_1 g$
$a = \dfrac{-m_1 g}{\sin\theta\, m_1 - m}$

C30

P1 (25 pts). A massless string, as shown in the figure below, connects two objects. The incline and the massless pulley are frictionless. Find the acceleration of the objects and the tension in the string in terms of θ, m_1 and m_2.



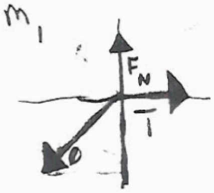
$T = ?$

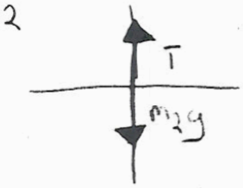
$a = ?$

$T - m_2 g = m_2 a$

$m_1 g \cos\theta - T = m_1 a$

$m g \sin\theta - F_N = 0$





$T = m_2(a+g)$, $T = m_1(g\cos\theta - a)$

$m_2 a + m_1 a = m_1 g\cos\theta - m_2 g$

$a(m_2+m_1) = g(m_1\cos\theta - m_2)$

$$a = \frac{g(m_1\cos\theta - m_2)}{m_2+m_1}$$

$$T = m_2 g\left(\frac{(m_1\cos\theta - m_2)}{m_2+m_1} + 1\right)$$

Acceleration for both blocks woulde be the same.

Evaluate:

$a = \frac{d}{t^2} \cdot d \cdot \frac{1}{d} = d/t^2$ correct un

$T = m \cdot \frac{m}{t^2}\left(\frac{m}{m} + 1\right) = m/t^2$ Correct unit

makes sense because as the mass of m_2 increases so does Tension, & as the total mass increases acceleration decreases.

Appendix 13. FC and LC Group

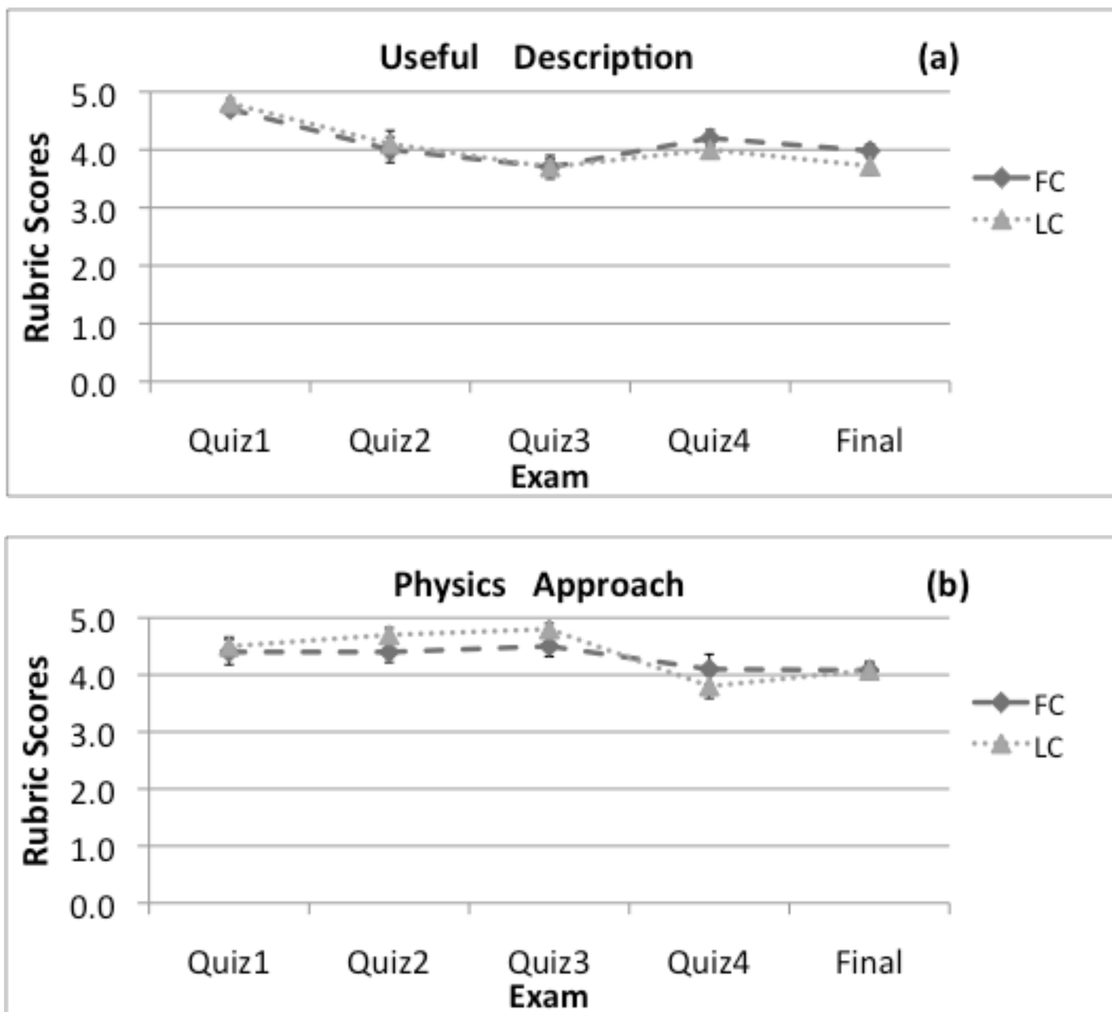
Diagnostic Exam Scores (Pre & Post) and Quiz Scores

Matched groups	Top 24 FC (frequent completers)	Bottom 24 LC (less frequent completers)
Gender balance	5 F, 19 M (20.8% F)	3 F, 21 M (12.5% F)
FCI pre	59.9% ± 4.0%	59.9% ± 4.1%
FCI post	81.0% ± 3.2%	81.3% ± 2.9%
Math pre	66.2% ± 4.1%	66.2% ± 3.7%
Math post	74.5% ± 3.1%	76.2% ± 3.1%
CLASS pre	64.6% ± 3.4%	64.6% ± 3.2%
CLASS post	61.5% ± 3.6%	66.0% ± 3.4%
avg. # coaches completed	32.79 (30-35)	11.42 (5-15)
avg. # coaches attempted	33.88 (30-35)	21.21 (7-32)
fraction completed	0.97	0.60
Q1 P1	19.7 ± 0.9	21.0 ± 0.9
Q1 P2	11.8 ± 1.2	12.8 ± 1.1
Q2 P1	19.0 ± 1.2	19.5 ± 1.3
Q2 P2	14.3 ± 1.4	15.3 ± 1.3
Q3 P1	15.8 ± 0.9	14.6 ± 1.1
Q3 P2	15.3 ± 1.0	14.6 ± 0.8
Q4 P1	12.9 ± 0.8	14.3 ± 1.0
Q4 P2	20.1 ± 1.2	17.8 ± 1.2
Final P1	19.3 ± 1.1	18.5 ± 1.3

Final P2	17.8 ± 1.0	18.0 ± 0.9
Final P3	11.4 ± 0.9	10.4 ± 0.9
Final P4	14.3 ± 1.2	14.3 ± 1.1
Final P5	15.9 ± 1.8	17.0 ± 1.5

Table 19. Matched FC and LC groups performance on: diagnostic exams, quizzes and final exam.

Rubric Scores



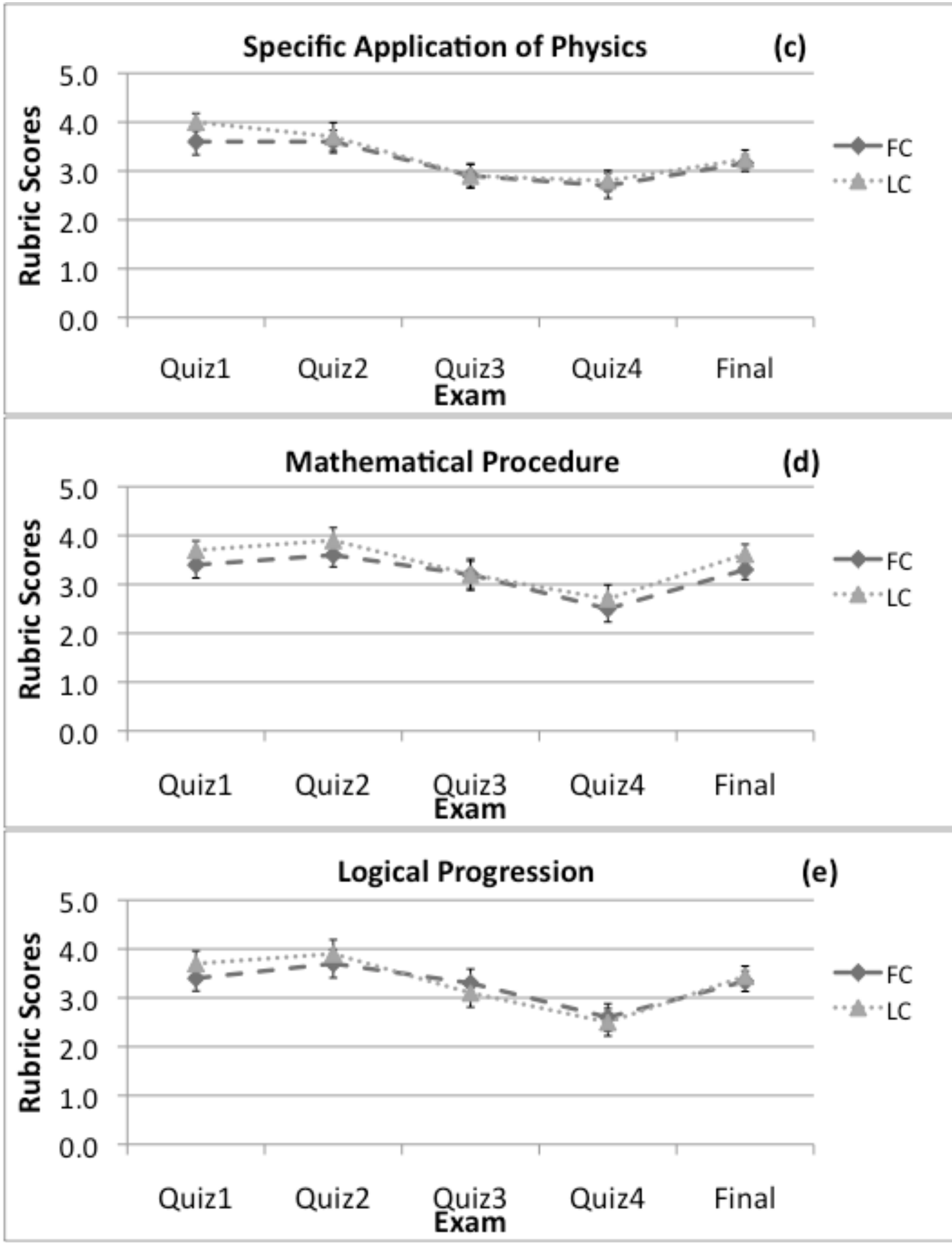
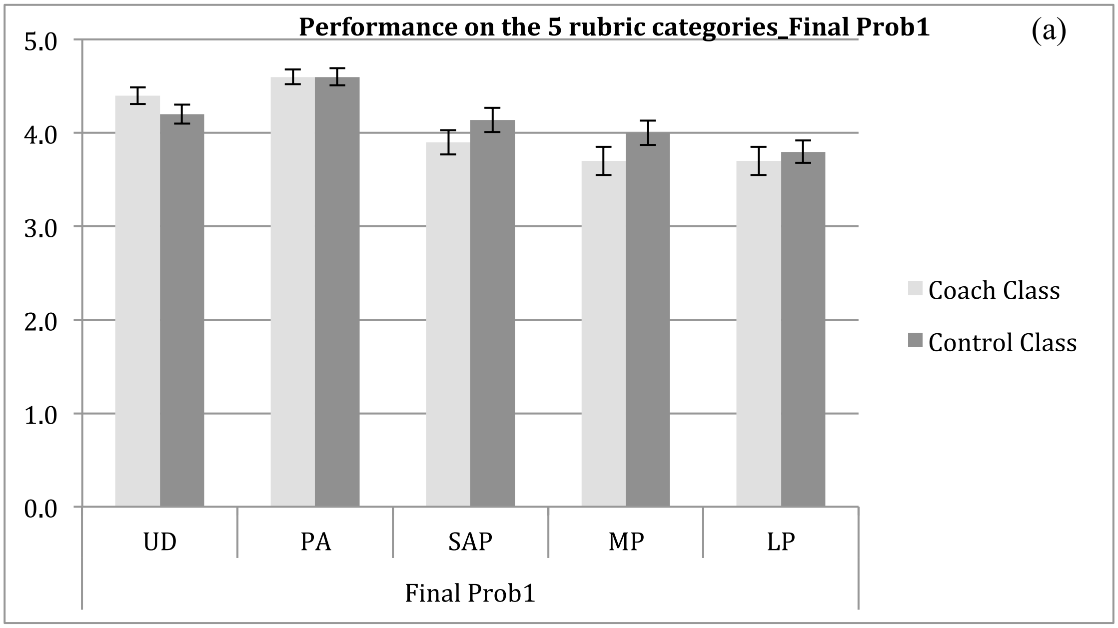
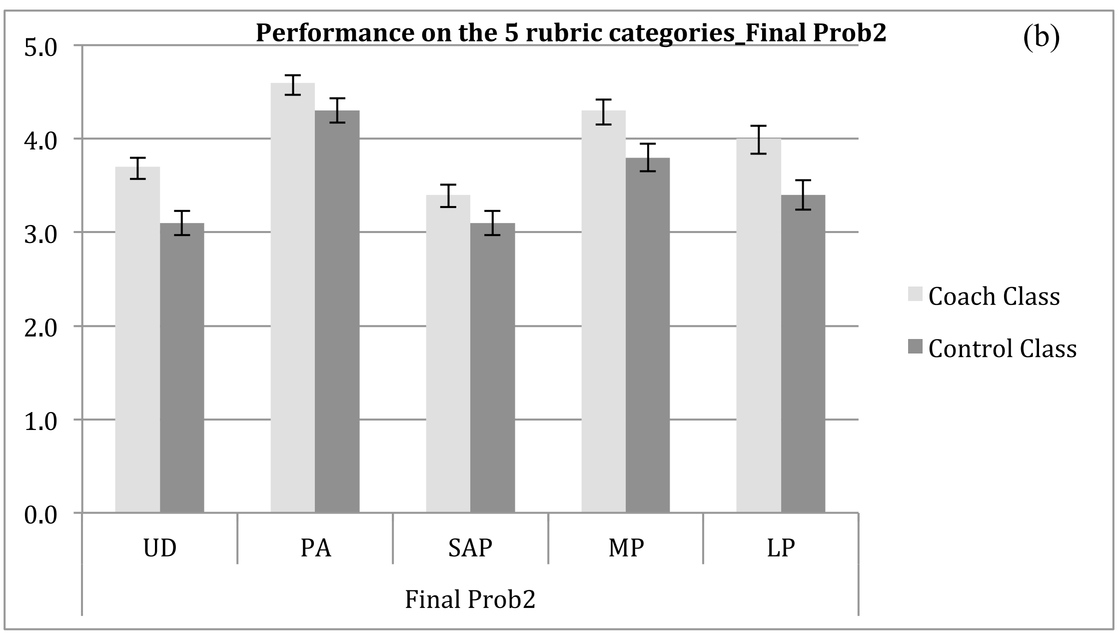


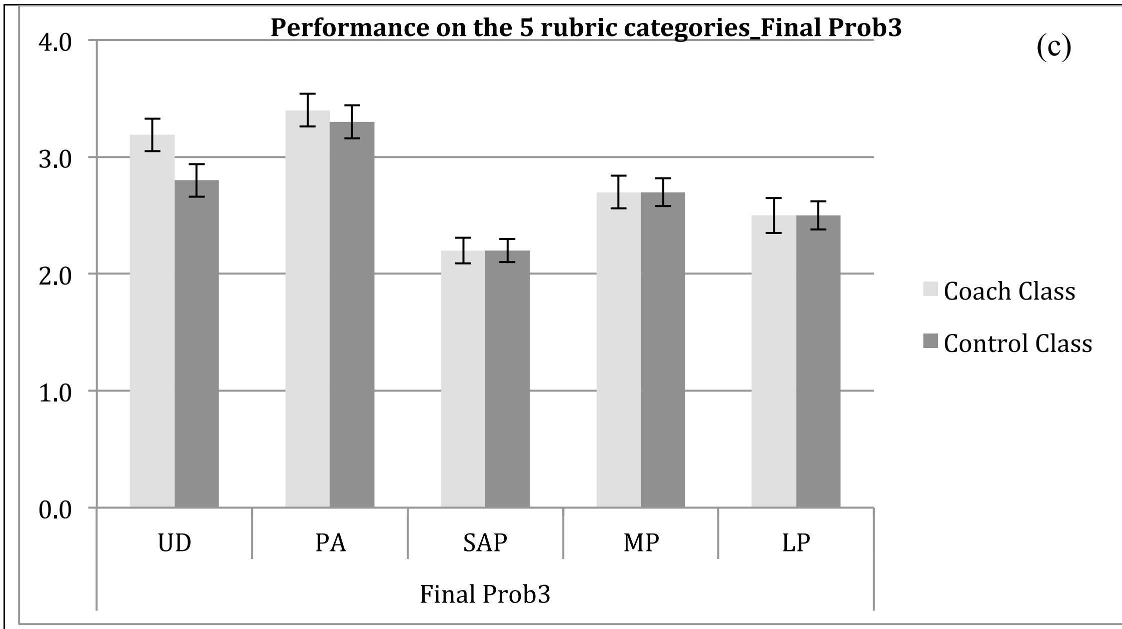
Figure 6 (a-e). Average scores of the matched FC and LC groups on each of the five rubric categories for 4 quizzes and final exam (averaged together). Lines are included only to guide the eye.

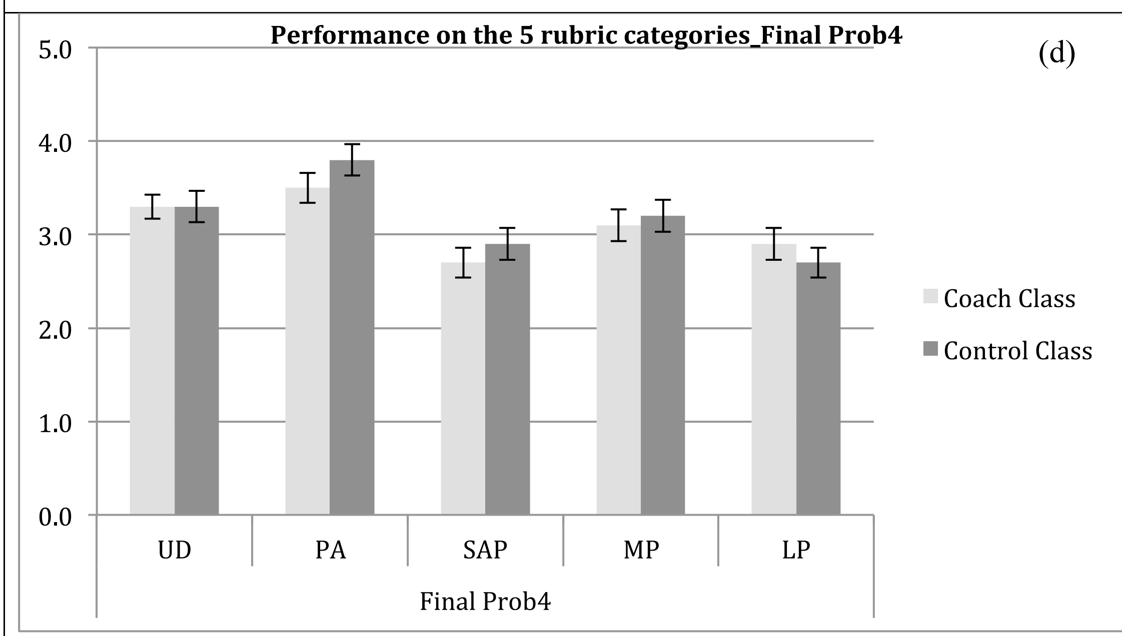
Appendix 14. Performances Between Coach and Control Class

Rubric Average on Final Exam






Performance on the 5 rubric categories_Final Prob3
(c)
4.0
3.0
2.0
1.0
0.0
UD
PA
SAP
MP
LP
Final Prob3
Coach Class
Control Class


Performance on the 5 rubric categories_Final Prob4
(d)
5.0
4.0
3.0
2.0
1.0
0.0
UD
PA
SAP
MP
LP
Final Prob4
Coach Class
Control Class

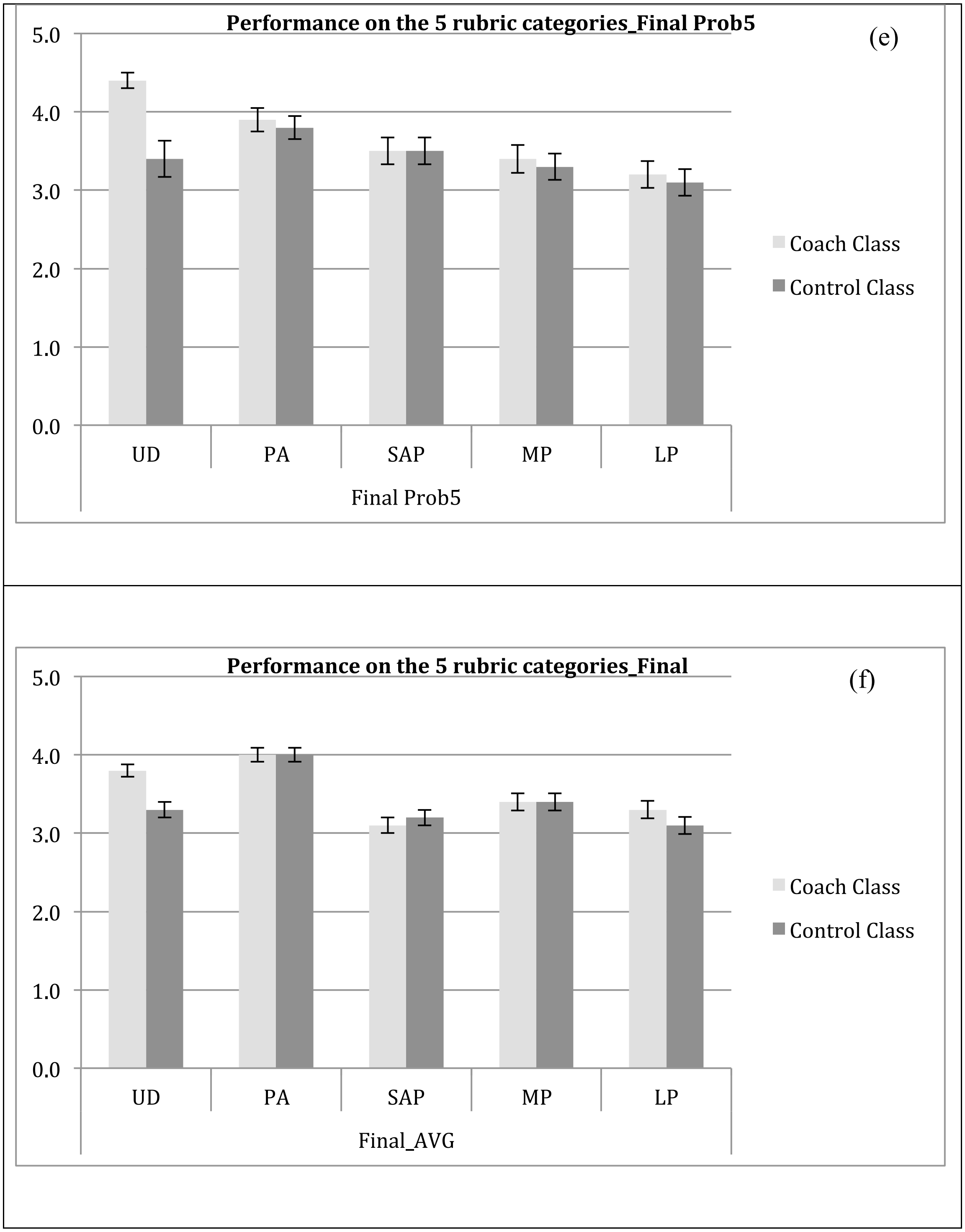


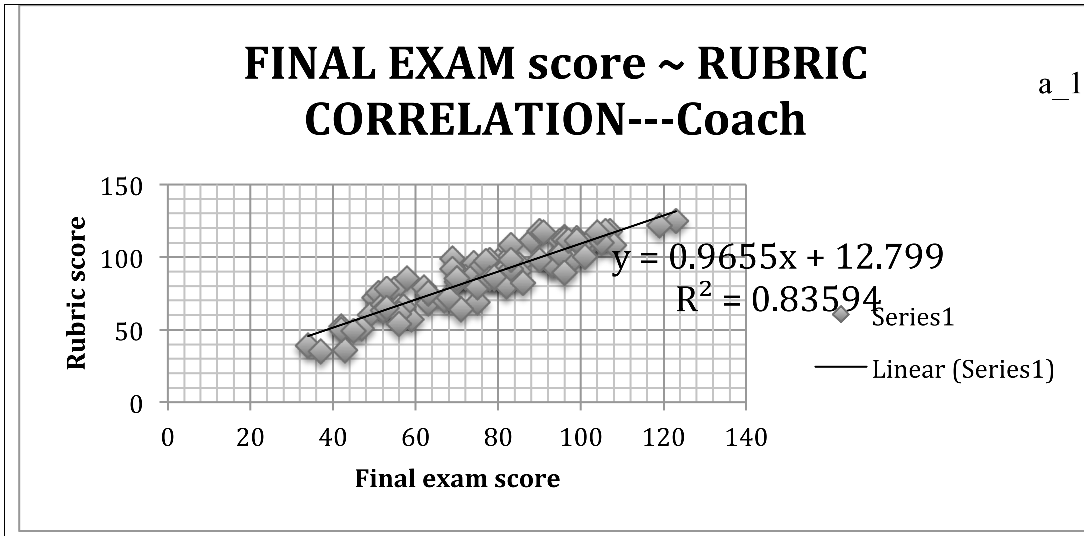
Figure 7 (a-f). Average scores of the matched coach and control class on each of the five rubric categories for each of the five final exam problems and the overall averaged

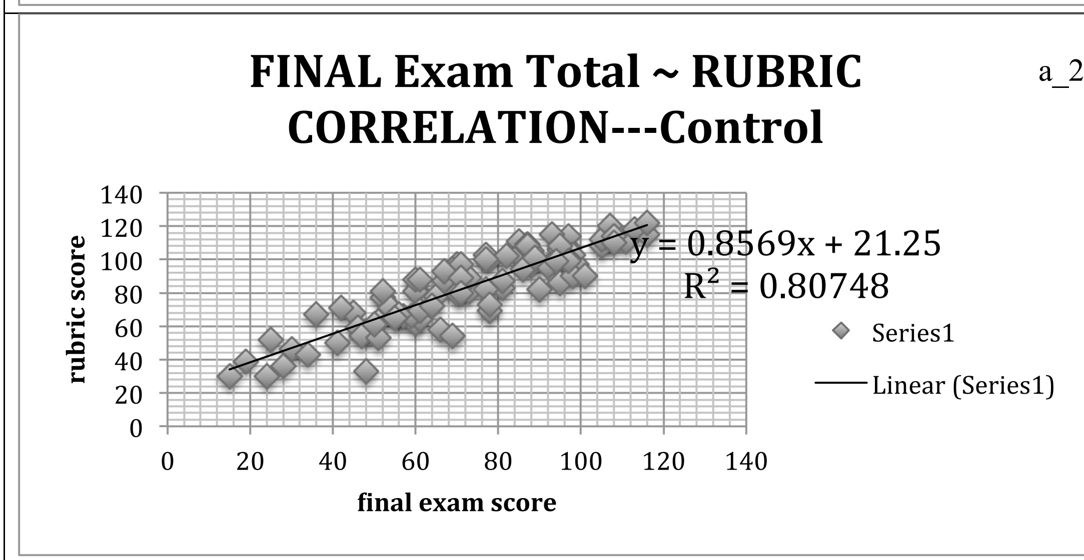
Statistics (t-test)

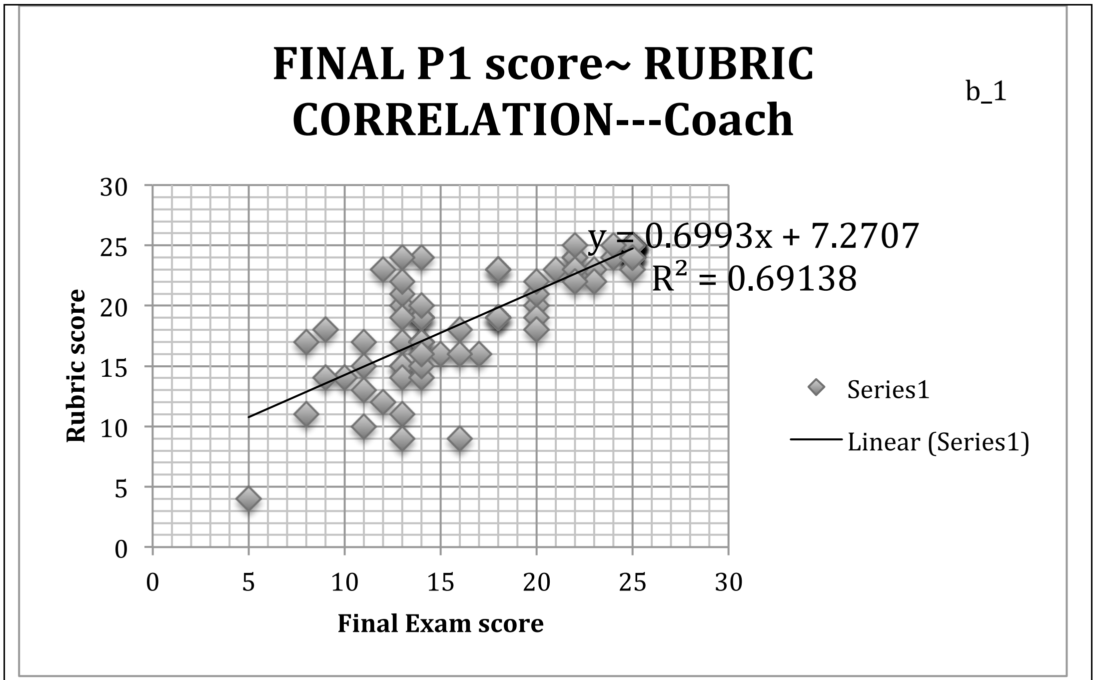
Two Sample t test	UD	PA	SAP	MP	LP
Final P1	p = 0.18	p = 0.73	p = 0.08	p = 0.07	p = 0.65
Final P2	p <0.01	p < 0.05	p = 0.06	p < 0.05	p <0.01
Final P3	p = 0.05	p = 0.48	p = 1.00	p = 0.86	p = 0.86
Final P4	p = 0.79	p = 0.16	p = 0.38	p = 0.81	p = 0.45
Final P5	p <0.01	p = 0.78	p = 0.86	p = 0.49	p = 0.58
Final Average	p <0.01	p = 0.70	p = 0.83	p = 0.83	p = 0.23

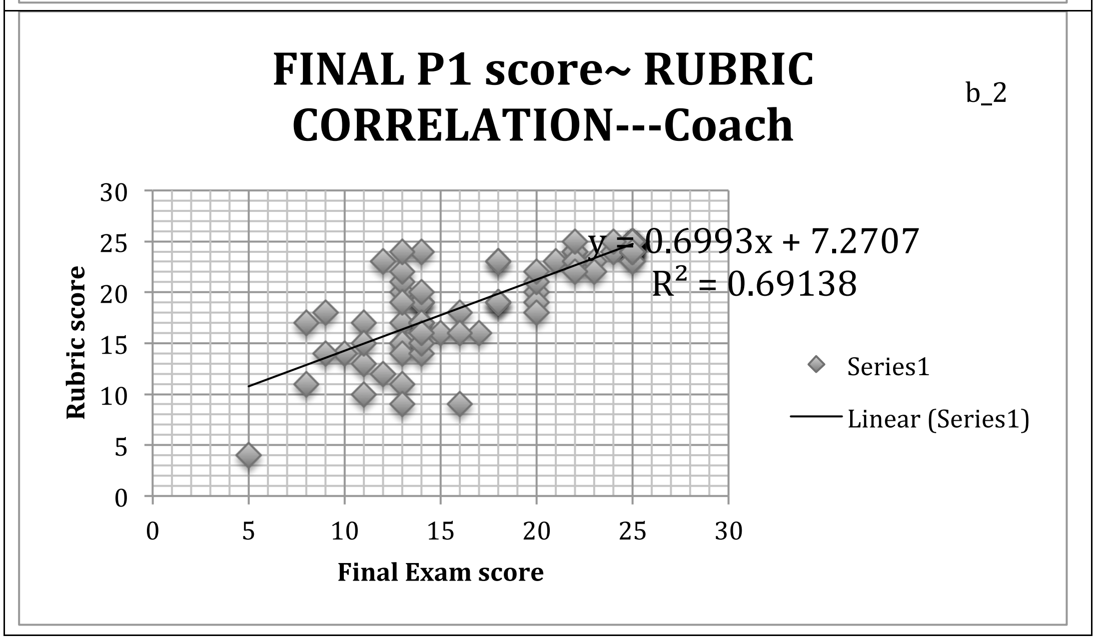
Table 20. P-value for two sample t-test between the matched coach and control class: average on each of the five rubric categories for each of the five final exam problems and the overall averaged

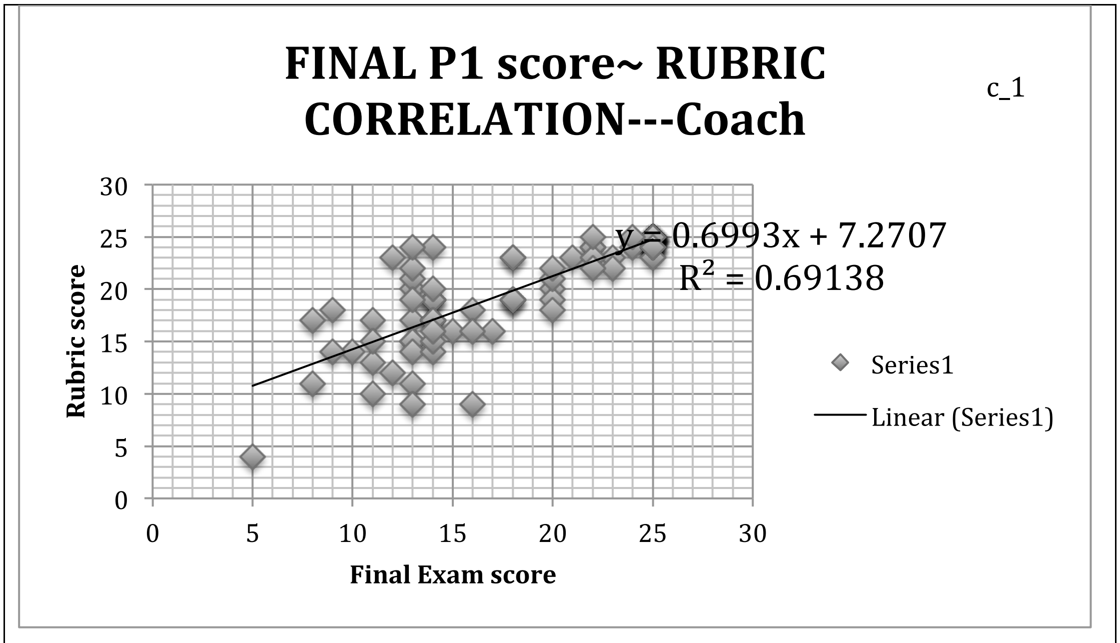
Exam Score and Rubric Score Correlation

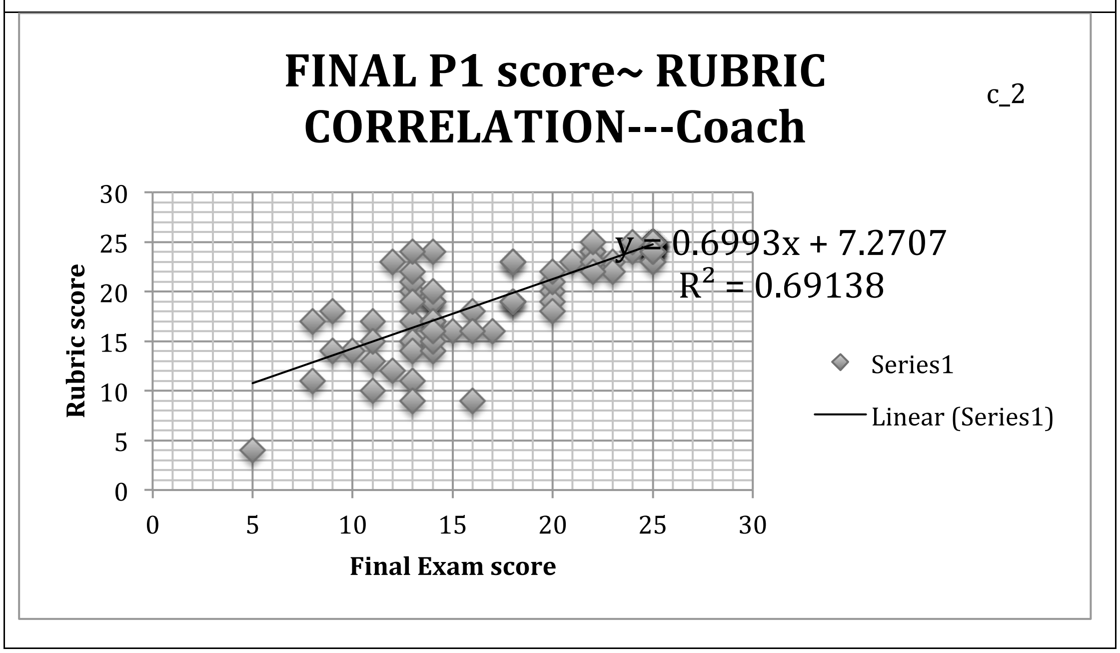


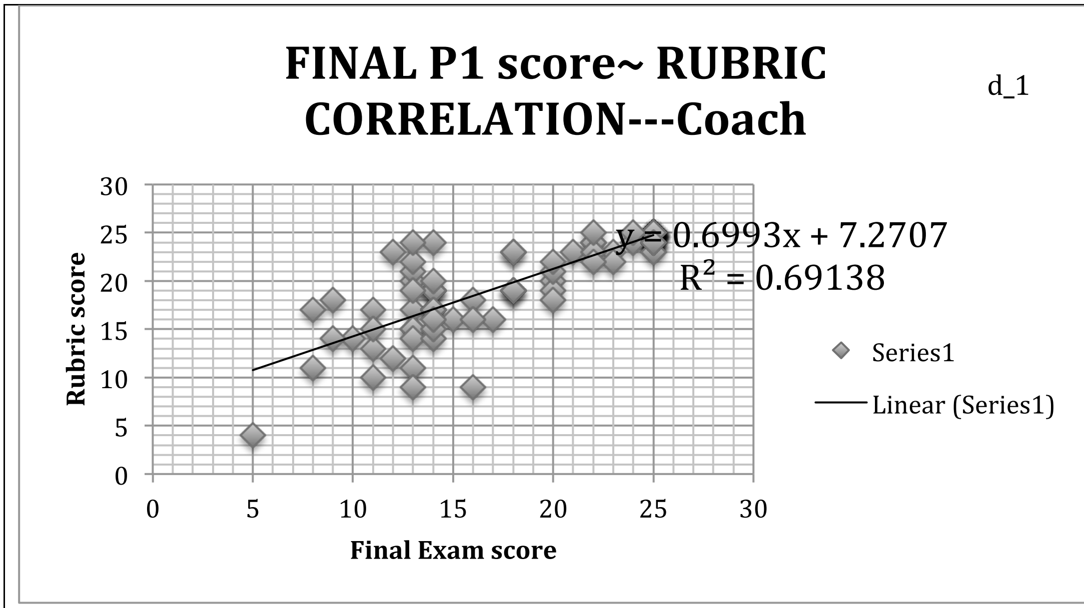


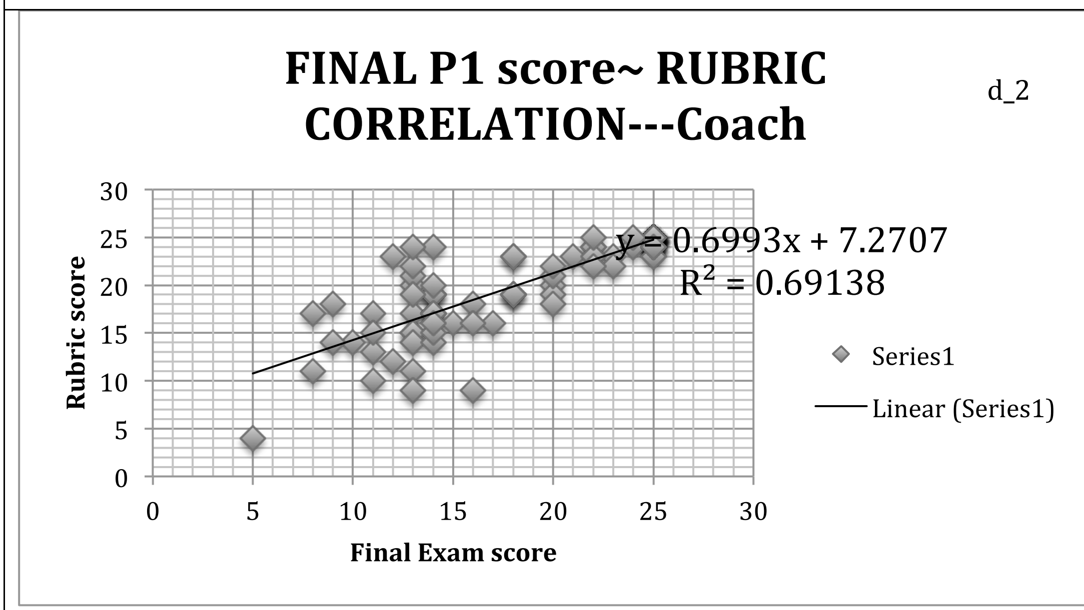

FINAL P1 score~ RUBRIC CORRELATION---Coach
b_1
y = 0.6993x + 7.2707
R^2 = 0.69138
Rubric score
Final Exam score
Series1
Linear (Series1)

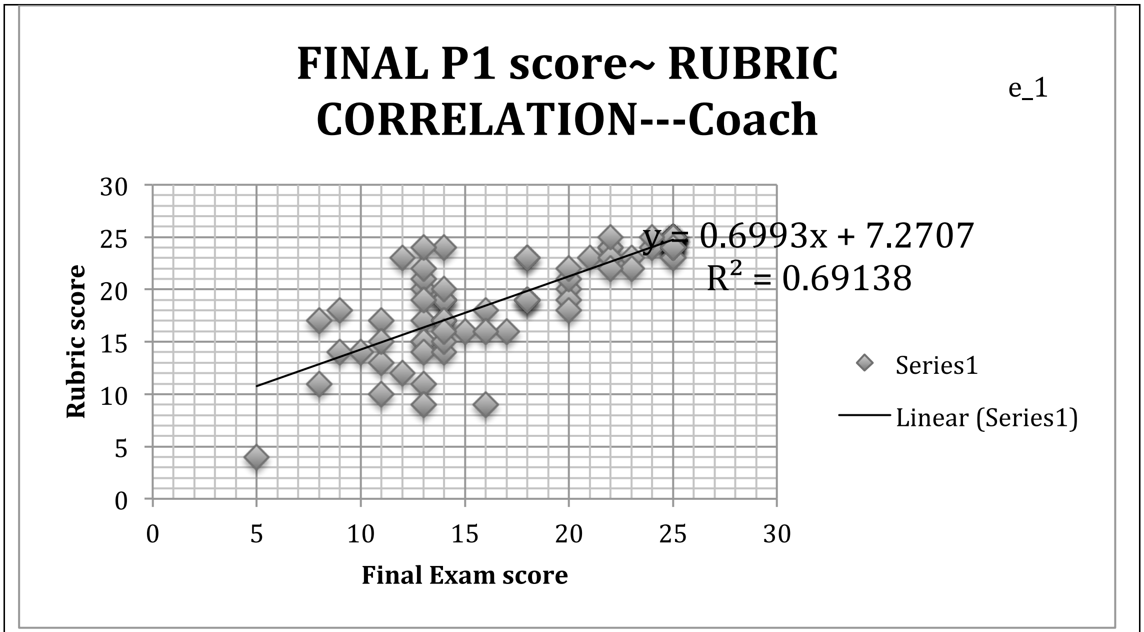

FINAL P1 score~ RUBRIC CORRELATION---Coach
b_2
y = 0.6993x + 7.2707
R^2 = 0.69138
Rubric score
Final Exam score
Series1
Linear (Series1)


FINAL P1 score~ RUBRIC CORRELATION---Coach
c_1
y = 0.6993x + 7.2707
R^2 = 0.69138
Rubric score
0
5
10
15
20
25
30
Final Exam score
Series1
Linear (Series1)


FINAL P1 score~ RUBRIC CORRELATION---Coach
c_2
y = 0.6993x + 7.2707
R^2 = 0.69138
Rubric score
0
5
10
15
20
25
30
Final Exam score
Series1
Linear (Series1)


FINAL P1 score~ RUBRIC CORRELATION---Coach
d_1
y = 0.6993x + 7.2707
R² = 0.69138
Rubric score
Final Exam score
Series1
Linear (Series1)
0
5
10
15
20
25
30


FINAL P1 score~ RUBRIC CORRELATION---Coach
d_2
y = 0.6993x + 7.2707
R² = 0.69138
Rubric score
Final Exam score
Series1
Linear (Series1)
0
5
10
15
20
25
30



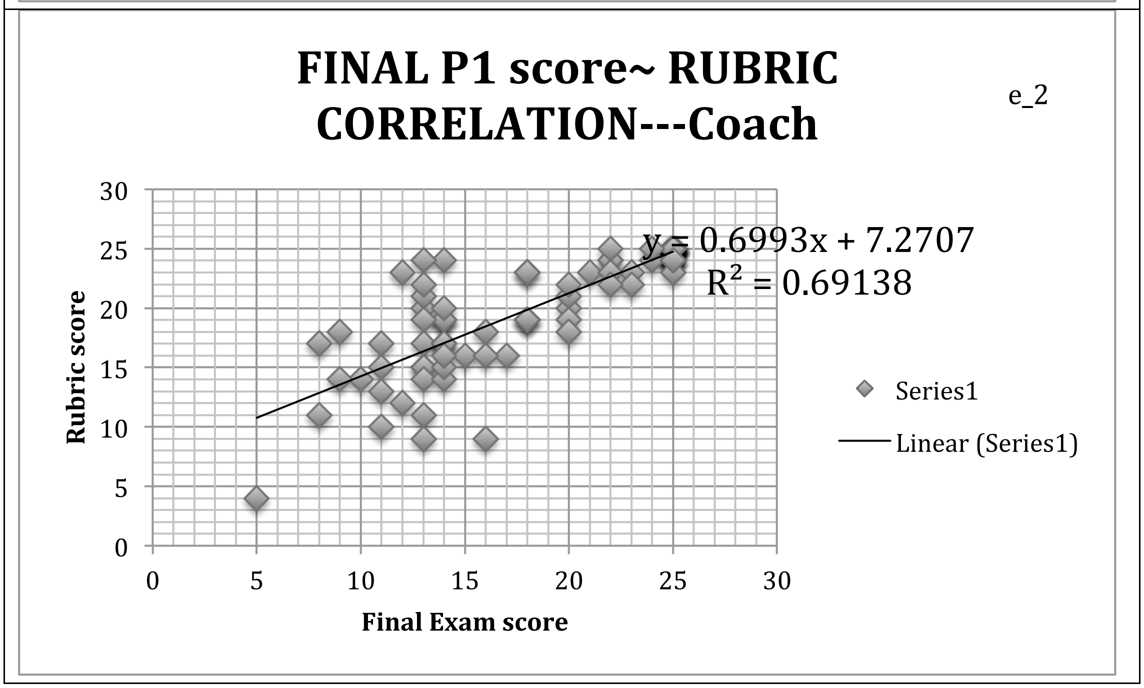


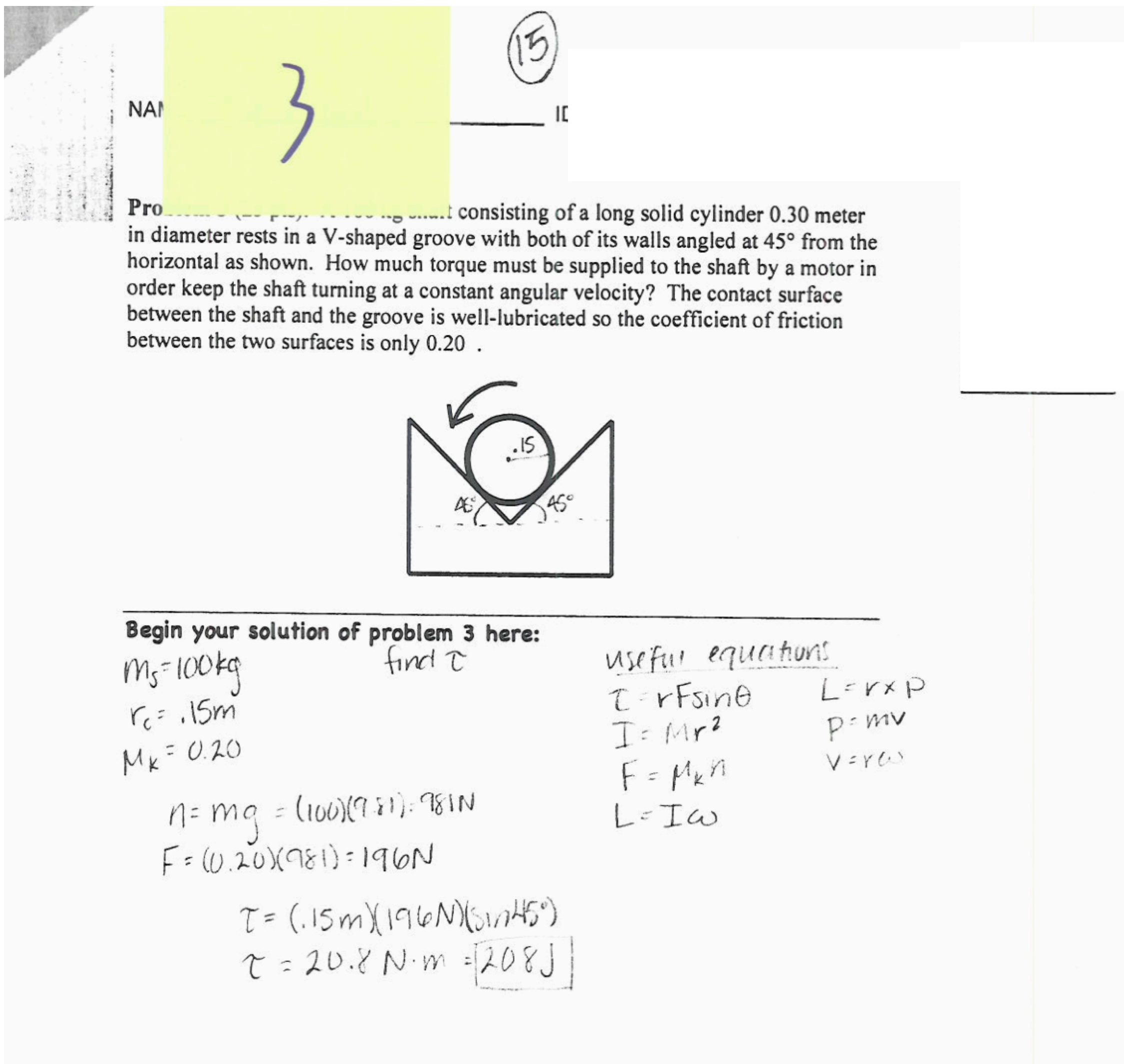
Figure 8 (a1-e2). Correlation graphs between rubric score (sum of all 5 categories, total 25 pts) and score assigned by a TA (total 25 pts) for the 5 problems respectively and all problems together.

Appendix 16. Examples of Inappropriate Grading

Sample Solutions

Student 3, TA grade is 15, Rubric score is: UD 1 PA 1 SAP 1 MP 1 LP 1 (total 5)

Student 41, TA grade is 10, Rubric score is: UD 4 PA 1 SAP 1 MP 2 LP 1 (total 9)



41

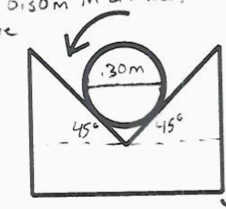
(10)

NAM ______________ ID_

Prol... consisting of a long solid cylinder 0.30 meter in diameter rests in a V-shaped groove with both of its walls angled at 45° from the horizontal as shown. How much torque must be supplied to the shaft by a motor in order keep the shaft turning at a constant angular velocity? The contact surface between the shaft and the groove is well-lubricated so the coefficient of friction between the two surfaces is only 0.20 .

Known: • 100kg cylinder 0.30m in diameter
• V-shaped groove (45°)
• $\mu = 0.20$

unknown: τ supplied to keep shaft turning with constant angular velocity



I of solid cylinder = $\frac{1}{2} MR^2$

Begin your solution of problem 3 here:

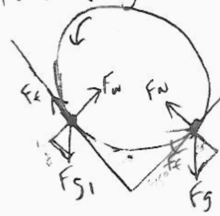
$\vec{\tau} = \vec{r} \times \vec{F}$

$\omega = \frac{v}{r}$

$\omega = \omega_0 + \alpha t$

$\alpha = 0$

Force Diagram



$F_g = mg = 100\,kg\,(9.8\,\frac{m}{s^2})$
$= 980$

Over 2 surfaces, so divide by 2

$\frac{980\,N}{2} = 490\,N$

$F_f = 490\,N$

Forces opposing motion $= 2F_f + F_{g2} - F_{g1}$

$F = 980\,N$

$F_N = \frac{980\,N}{(0.20)} = 4900$

$\tau = r \times F$?

$\tau = (0.15\,m)(4900)$

$= 735\,N \cdot m$

Appendix 17. Gender Ratio and Pre-FCI differences between the Frequent Completer vs. Less-frequent Completers in Fall 2011

Pre-test	Frequent completers			Less-frequent completers		
	Overall	Male	Female	Overall	Male	Female
N	47	24	23	47	41	6
FCI	47%±2.9%	58%±4.0%	36% ±2.7%	67% ±2.9%	70% ±2.7%	41% ±8.0%
Math	61%±2.7%	65%±4.0%	57% ±3.5%	68% ±2.7%	69% ±2.8%	60% ±8.7%
CLASS	61%±2.2%	62%±3.4%	59% ±2.8%	67% ±2.5%	68% ±2.7%	61% ±5.0%

Table 21. Gender ratio and pre diagnostic scores for the frequent completers and less-frequent completers