Essays on Liquidity Management and Aggregate Fluctuations

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Dedication

To my parents, for their faith. To Elisa, for her love and support. To my family, past, present and future, for being an inexhaustible source of strength. To God for everything.

Abstract

This dissertation consists of two essays that focus on the role that frictions to intertemporal exchange have in shaping the response of aggregate activity to exogenous changes in credit conditions.

The first essay approaches this issue from a general equilibrium perspective. I study credit crises by considering the fact that they do not always affect all agents in the same manner. I show that during a household credit crisis, the more the productive sector saves, the more disruptive the contraction in household debt is. That is, the response of the economy is closely connected to the financial asset position of the productive sector, a statistic that can be calculated directly from aggregate data. An important feature of the model is that it is household debt, as opposed to household savings, which has a productive role. The low interest rates that ensue the contraction in household debt not only make investment cheaper, but also alter its composition. Consequently, capital ends being used in a less efficient manner, and the overall effect in economic activity is ultimately a quantitative issue. I solve the model numerically and perform a numerical evaluation of this novel channel.

The second essay, co-authored with Maria Elisa Belfiori, provides a decision theoretical model of loan commitments, with the purpose of exploiting aggregate data on loan commitments to identify credit crunches. The usual difficulty for identification is that interest rate movements can be misleading due to the nature of credit markets. For instance, during a credit crunch, interest rates could go down just because funds *flight to quality*, or because lending contracts are indexed to policy interest rates. We sidestep this problem by exploiting data on used of unused balances of loan commitments contracts. Commercial and Industrial Loans in the U.S. are predominantly implemented using these contracts. We study the evolution of the used and unused portions during the last recession to shed light on the origin of this episode. We find that the type of movements observed in aggregate data regarding aggregate quantities can indeed be consistent with real shocks.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

It resonates as an undeniable truth that financial markets are among the most important factors influencing aggregate economic activity. To the extent that they function properly, financial markets should allow economic agents to smooth their consumption and investment decisions. That is, they facilitate the inter-temporal allocation of resources by making funds to flow from savers to borrowers. The presence of frictions impairs this smoothing process. Furthermore, when these frictions vary over time, fluctuations at the aggregate level can occur. This dissertation study this relationship between frictions and aggregate activity from two different angles. It is structured as follows.

- Chapter 2 presents the essay "Credit Crises and Private Liquidity: The Role of Household Debt". In this essay I take into account that variations in credit conditions do not necessarily affect all agents in the same manner. By focusing on the household sector, I uncover the essential role played by household debt in facilitating the efficient use of productive resources.

- Chapter 3 presents the essay "Understanding Credit Crunches through a Model of Loan Commitments". This essay provides a model to interpret aggregate series regarding used and unused balances of loan commitment contracts. By doing so, it offers a new perspective to identify credit crunches from aggregate data without relying on movements in the interest rate, which are known to be misleading.

In both essays, the analysis is conveyed using models of heterogeneous agents that face idiosyncratic uncertainty within an incomplete market structure. Crises are modeled in a stark way as exogenous changes in credit conditions of economic agents. In the first essay, these represent exogenous changes in borrowing limits. In the second, these are changes in the terms of the loan commitment contract. Throughout, I remain agnostic regarding what causes these changes and focus merely on the response of the economic system. The supplemental material of both essays is deferred to the appendix.

Chapter 2

Credit Crises and Private Liquidity: The Role of Household Debt

2.1 Introduction

In this paper I study the consequences of credit crises by considering the fact they do not always affect all agents in the same manner. I adopt a stark definition of credit crises as exogenous contractions in the borrowing capacity of economic agents. In a large class of economic models featuring market incompleteness, these contractions trigger a de-leveraging process due to the strengthening of the precautionary motive to save. I study the consequences of this process in order to address the following questions: how does the reduction in the stock of private debt affect economic activity? Does it matter which agents, consumers or producers, are affected by the change in financial conditions? What type of intervention is needed, if any, to alleviate the effects of these changes?

In answering these questions, I emphasize the role that household liabilities play in promoting the efficient use of productive resources. In the model developed here, the aggregate stock of debt is a source of *liquidity*, in the sense that it allows agents to save and concentrate their spending in those periods in which they need it the most.

Therefore, credit crises can be seen as episodes of *liquidity shortages*. This paper shows that from producers' viewpoint, the larger their asset position the more important liquidity is. In particular, when producers are net lenders to the rest of the economy, household debt (or any other sort of debt for that matter) becomes effectively an input for production and a household credit crisis can trigger output downturns.

To start building intuition towards this result, observe that credit crises involve a reduction in agents' ability to pledge the value of their physical assets as collateral. The effect of this apparent destruction of *collateralizable* wealth depends on which agents are mostly affected by it. As the economy strives to save more, the interest rate decreases and the collateral in hands of those who were unaffected becomes relatively more valuable. Hence, the credit constraints of these agents loosen, inducing them to take on more debt and soak up these extra savings. As agents adjust their asset position, a reallocation of resources ensues and the economy transits to a new equilibrium.

Consider then the case of a household credit crisis. As households de-leverage, producers reduce their asset position to restore the equilibrium in the asset market. Due to the presence of credit constraints, the depletion of assets makes more difficult for producers to correlate their investment decisions with their productivity level, reducing the efficiency in the use of productive resources. Starting from a situation in which producers are net lenders, the consequences of this reallocation of resources will show up in the aggregate as a reduction in measured TFP.

I develop the argument in an extended version of the heterogeneous agents model used in most quantitative research. Households and firms display a motive to save using a risk free asset which is in zero net supply. Negative asset positions require to pledge collateral, which households own in the form of houses and firms in the form of capital. The saving motive in the firm side stems from idiosyncratic shocks that affect the return on capital. When this return is low, firms seek to postpone investment and they accomplish this goal by saving.

In this environment, the stock of private debt corresponds to the sum of negative asset positions across all agents. As it was mentioned above, private debt provides liquidity in the sense that it facilitates the intertemporal allocation of resources by creating stores of value. In the productive side, the availability of stores of value with high return encourages the disposal of unattractive investment opportunities. This allows capital to find its way to its most efficient use, increasing aggregate output. During a credit crisis, deleveraging makes stores of value scarce and drives its price upwards (interest rate downwards). Although this benefits the lucky firms that incur in leveraged investment, it also hurts the unlucky ones that sit in their wealth waiting for sunny days. The overall effect in economic activity is ultimately a quantitative issue and this paper makes progress in that direction.

I deliver my results by considering financial conditions varying independently for households and firms. This separation turns out to be crucial to understand the effects of liquidity in aggregate dynamics. I study first comparative statics across steady states. It is not surprising that a credit tightening in the firm side decreases aggregate output, consumption and investment. But this outcome mixes the effect of the reduction of the value of capital as collateral with that of the increase in firms' demand for liquidity resulting from tighter financial conditions. By considering a credit tightening in the household side I am able to isolate the effect of a pure liquidity shortage on economic activity. In this case, the mechanism considered in this paper drives output down only if the productive sector is a net saver. Put differently, it is only when the productive sector cannot generate enough liquidity by itself that the scarcity of stores of value hinders aggregate productivity.

To dig further in the effects of liquidity shortages, I examine the transition of the market equilibrium following permanent changes in financial conditions of private agents. This exercise is challenging because the path of prices becomes a state variable for each agent's problem along the transition, and this requires to keep track of the evolution of the wealth distribution in both sectors. Remarkably, a household credit tightening looks very much like a housing bubble burst, featuring low interest rates and a severe drop in the price of houses. However, in the short run the economy experiences a boom in production since the effect of interest rate on the investment decisions of lucky firms dominates.

It is interesting to frame this finding in the context of the last recession. The main indicators of economic difficulty that preceded this episode were the substantial increase in mortgage delinquency rates and the large decline in house prices. Clearly, both events point towards a reduction in households' borrowing capacity. However, it is less clear whether these tighter financial conditions ultimately spread over to the productive sector by putting banks under distress. This suggests the need to make precise the mechanism capable of translating the original impulse in the household sector into a drop in output, without necessarily assuming that all private agents faced tighter financial constraints. This paper makes progress in that direction by studying the circumstances under which household deleveraging drives down economic activity due to a shortage of stores of value. The reason why this can happen is because these instruments play effectively a productive role, but they require private agents to take leveraged asset positions. Hence, an increment in private savings can be harmful for the economy, but only because it disrupts the reallocation of productive resource. In particular, this mechanism is unrelated to any sort of goods market frictions that seem to be embedded in the logic of the keynesian paradox of thrift.

In the model presented here, productive efficiency requires from producers the ability to store value and as a result, firms have an incentive to hold financial assets in their balance sheets. An important question in this context is whether a shortage of stores of value would grant any particular form of intervention. According to the analysis developed here, the answer is not so straightforward. On one hand, a trivial solution to the problem of limited pledgeability would involve the intervention of a government endowed with the ability to issue debt fully backed up by its tax collection powers[1] . On the other hand, output contractions triggered by credit crises are not necessarily associated with liquidity shortages. In other words, the reason behind aggregate activity contracting so sharply on impact might be unrelated to the aggregate supply of saving instruments and consequently would not require any type of intervention in this respect.

The rest of this section places this paper in the literature. The next section provides an empirical motivation using aggregate data. Section 3 presents the model economy and Section 4 does a partial characterization of the solution. In Section 5, I perform comparative statics, focusing on the effect of changes in the tightness of the financial constraints and changes in the degree of idiosyncratic uncertainty. Section 6 presents the transition of the market equilibrium following permanent changes in financial conditions. Section 7 concludes.

[1] For a more detailed discussion regarding this alternative, see [1] and [2]

Related literature. The study of the aggregate effect of changes in the stock of debt has important precedents in the economic literature, which can be be explained in the context of this paper. First, debt issued by households acts as a good substitute for debt issued by firms, limiting their possibilities of leveraging up their investment. This competition effect resembles that studied by [3] in his celebrated analysis of national debt. Second, private debt provides unlucky firms with an instrument to fulfill their desire to delay investment, acting as a complement to it, a point first stressed by [4].

This paper contributes to the literature on financial frictions that stems from the work by [5] and [6]. These seminal contributions were more concerned about the role of frictions as an amplification mechanism of TFP shocks. In contrast, recent contributions to this literature explore the idea of financial shocks as independent drivers of output fluctuations. In general, these shocks are conceived as perturbations to the borrowing capacity of economic agents. Regarding the recent crisis, a line of research presumes that the initial impulse in the household sector was transmitted to the productive sector by pushing firms' potential lenders into financial distress. A recession, the argument goes, is the result of the difficulties in accessing external funds to finance production (see for instance [7]). Other authors dispense with that presumption and focus on the effects of a household credit tightening over economic activity (see [8] or [9]). The view in these papers is that recessions are driven by households pulling back private consumption. Instead, this paper focus on the effect of the deleveraging process on economic activity through the scarcity of stores of value.

This paper is also related to [10] (hereafter KM) who study the role of *liquidity shocks* in driving output fluctuations. It should be stressed however that the concept of liquidity in KM's is different from the one used here. In their paper, the term is used as a property of assets that refers to how fast they can be deployed in order to engage in intratemporal exchange[2] . A salient feature of KM's model and many of its relatives is to assume that households act as hand to mouth agents. This modeling choice abstracts from the role that households' intertermporal decisions can play in the dynamics of the economy, rendering their model not well equipped to address the issues posed in this

[2] This distinction is not as sharp as one would wish. In an economy with *illiquid assets*, the ability to store value across time is limited, e.g. it is an *illiquid environment*. In this paper I adopt the view that claims backed by collateral are the only source of liquidity. Consequently, for the purpose of storing value across time, all other assets are equally illiquid.

paper.

This paper is also related to [11] who studies the link between aggregate savings and economic activity in an economy in which agents face uninsured idiosyncratic investment risk. Unlike in the seminal work of [12], that paper shows that more precautionary savings do not necessarily imply a larger capital stock relative to a complete market economy. This is because households are risk averse and they allocate a share of their savings to the risk free asset (which is in zero net supply), the share being increasing in the amount of idiosyncratic risk agents face. In this paper, households do not save directly in the risky asset so that the logic of a *flight to quality* does not apply. Moreover, more savings push the interest rate downwards and reduce firms' opportunity cost of holding capital. However, when firms are net savers, this does imply less aggregate output since capital ends up being poorly allocated. This is because household debt is effectively playing a productive role in the economy[3] .

In modeling the household side, this paper abstracts from imperfect housing markets. This is in order to focus on the role that collateralized debt plays in the reallocation of capital in the presence of financial frictions. This approach is also taken in [13]. These authors note that houses, like money, help to ameliorate credit market frictions because they are storable goods that can serve the role of collateral. Therefore housing prices are likely to carry a *liquidity premium*, which might vary over time for reasons that are unrelated to their fundamental value. This observation leads them to study the dynamics of housing prices, in order to understand the emergence of bubbles in an economy with financing constraints.

In fact, the mechanism studied here is reminiscent of the ideas pioneered in the literature of rational bubbles, which also refers to the availability of instruments to move resources intertemporally as *liquidity*. In general, models with bubbles admit equilibria in which liquidity expands the production possibilities of the economy by improving the intratemporal allocation of resources (see [1] or [2] for a clear exposition of this idea). Hence, to some extent bubbles in this literature serve the same purpose that household

[3] In neoclassical theory, the natural stabilizing mechanism to a drop in investment is an increase in the real rate of return that attracts more savings and therefore increase investment. [11] shows that this logic fails when these savings are not channeled to productive use. In this paper more savings (less debt) also hurts but for different reasons related to the provision of *liquidity* and the role it plays for the efficient allocation of resources.

debt serves in this paper. From the perspective of the firm, there is a distinction between *inside liquidity*, used to denote claims originated within the productive sector, and *outside liquidity*, used to denote claims originated elsewhere. Bubbles are a form of outside liquidity. A key difference with respect to this literature is that, in my model, the aggregate amount of outside liquidity is linked to the optimal behavior of households.

This paper is also related to [14], which shows that modeling explicitly heterogeneity in production in the presence of financial frictions makes transparent how the misallocation of productive resources can translate into productivity losses. As in their paper, the asset market structure is exogenously incomplete. [15] has pointed out that with complete insurance markets, the presence of collateral constraints can only weakly affect aggregate output when shocks originate in the productive sector. In other words, the possibility of hedging against idiosyncratic shocks severely downplays the credit channel. Although I do not relax the assumption regarding the asset market structure, the study of household credit crises conducted in this paper resembles [15] point that the destruction of collateral outside the productive sector can ultimately affect aggregate activity. An advantage of the model in this paper is that it is more suited for quantitative research.

Finally, the role of outside sources of liquidity as a support of investment is also the focus of [16]. The modeling device that generates a demand for liquidity in their model is quite different. There, liquidity is necessary to bring current investment to completion whereas here it is necessary to postpone it and take advantage of future opportunities. Furthermore, firms' motive to save is completely absent in their model. In a related work, [17] develop an overlapping generations model of entrepreneurs that have no option other than saving in their first period of their life. Their analysis focuses on the role of bubbles as sources of liquidity, whereas here the focus is on the role of the private debt issued by the household sector. Besides this difference, many of their insights regarding the role that outside liquidity plays in the economy also apply here.

2.2 Empirical Motivation

I organize this section in three parts to gain some insight about the following issues

1. Household indebtedness

2. Precautionary savings in the productive sector

3. Net Financial Asset Position by economic sector

I also focus on the period 1980Q1 to date, which comprises five NBER recessions and an era of substantial deregulation of in the mortgage market[4] .

2.2.1 Household indebtedness

This subsection presents two statistics reflecting household indebtedness constructed from aggregate data. Most analysts are used to keep track of the debt to income ratio as a measure of household indebtedness. However, in some sense this indicator says little about how close to their constraints households feel. In this respect, [19] report that 75% of household debt is actually collateralized by housing structures and when one considers durable goods, the percentage of collateralized debt goes up to 90%. This indicates that household access to borrowing depends much of the market value of its non financial assets. The important thing to consider is that fluctuations in the price of collateral can lead them to feel more constrained. This is something that cannot be fully captured by the debt to income ratio. Hence in addition to this statistic, I also report two versions of what I call the debt to (collateralizable) wealth ratio. The first version is the ratio of all household debt relative to the market value of non financial assets. The second considers only mortgage debt divided by the value of housing structures.

[4] The deregulation in the mortgage market made easier for consumers to use housing as collateral for loans. The Alternative Mortgage Transaction Parity Act (AMTPA), passed in 1982, allowed banks to innovate in mortgage products. Before it, they were forbidden to offer households anything but conventional fixed-rate amortizing mortgages. Later on, the Depository Institutions Deregulation and Monetary Control Act abolished interest rate usury caps. It is often argued that such deregulation made the price mechanism (i.e. the interest rate) to bear the full burden of discriminating among borrowers instead of implementing screening practices to identify good borrowers. See [18] and [19] for a full discussion

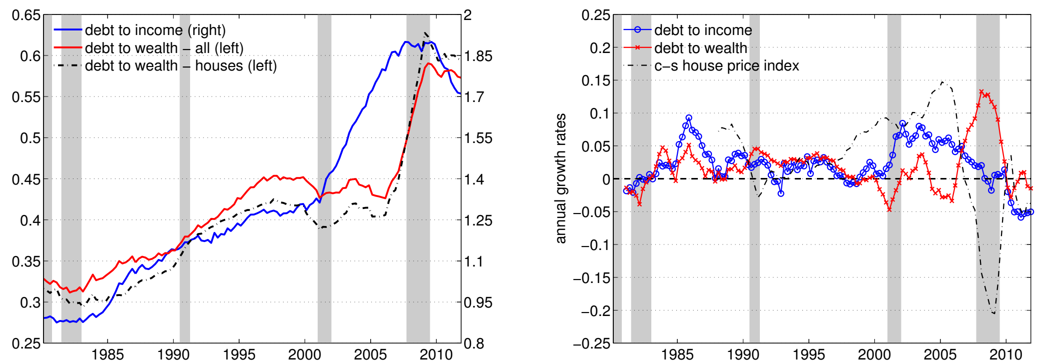

Figure 2.1: Measures of Household Indebtedness: Debt to Income Ratio and Debt to Collateralizable Wealth. Source: Tables B100 and F7 from the Flow of Funds. Debt to Income = B100.31/F7.3, Debt to Wealth - All = B100.31/B100.2, Debt to Wealth - Houses = B100.33/B100.4

Both statistics are reported in Figure 2.1. The debt to income ratio ramped up since the beginning of the century, but it flattens out right before the last recession was called. Some analysts refer to the increase of this ratio as an over borrowing episode. Yet, as the debt to wealth ratio suggests, alongside with this increase we observe that the debt to wealth ratio remained flat. This suggest that household borrowing behavior was particularly prudent. They maintained this ration constant around 40%, even when the price of their collateral was increasing. It is well known that alongside this increase in the price of collateral, the ability to pledge houses as collateral was also on the rise. The model in the next section relates these fluctuations in the price of collateral to exogenous fluctuations in the ability to pledge collateral. The view I take here is that the latter are associated to the bad performance of mortgage loans, but I make no attempt to explain what triggered an increase in delinquency rates.

2.2.2 Precautionary savings in the productive sector

In this subsection, I present the financing gaps for the productive sector in the U.S. This statistic is calculated by subtracting the amount of internal funds from capital expenditures. In Figure 2.2, I scale this difference by the amount of capital expenditures, to present it as a ratio.

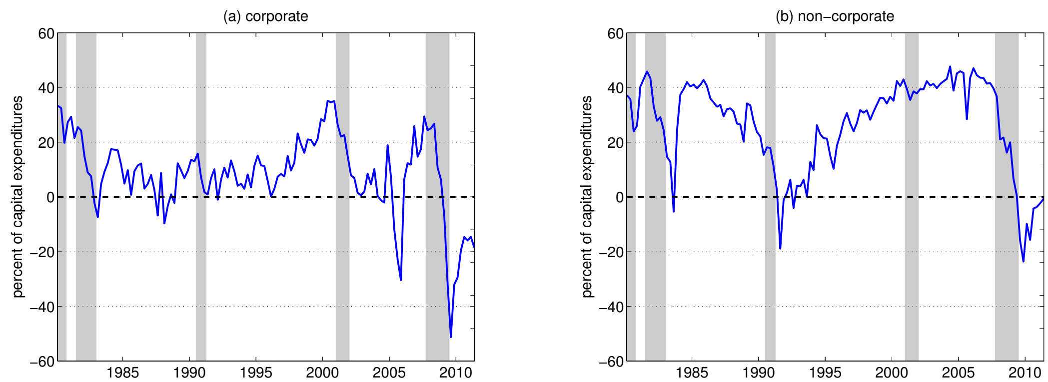

Figure 2.2: Financing gaps for Non-Financial Businesses. Source: Tables F102 and F103 from the Flow of Funds. Corporate sector = F102.54/F102.11, Non-Corporate sector = F103.7/F103.4.

Two things are worth to notice. First, the ratio can go negative if producers use their profits mostly to accumulate financial assets in their balance sheet. Second, the fluctuations of this ratio disclose little information about how difficult is for non-financial businesses to access external financing. Higher values could be explained by producers either borrowing funds heavily or reducing capital expenditures drastically. In other words, it is not possible to identify whether fluctuations in financing gaps are supply or demand driven. However, Figure 2.2 does indicate that *rainy days* are indeed associated with this gap going smaller and even negative. This turns out to be true for both corporate and non-corporate businesses and especially so during the last recession. I interpret this evidence as producers displaying a precautionary saving behavior and when times are bad, they actually *save for sunny days*.

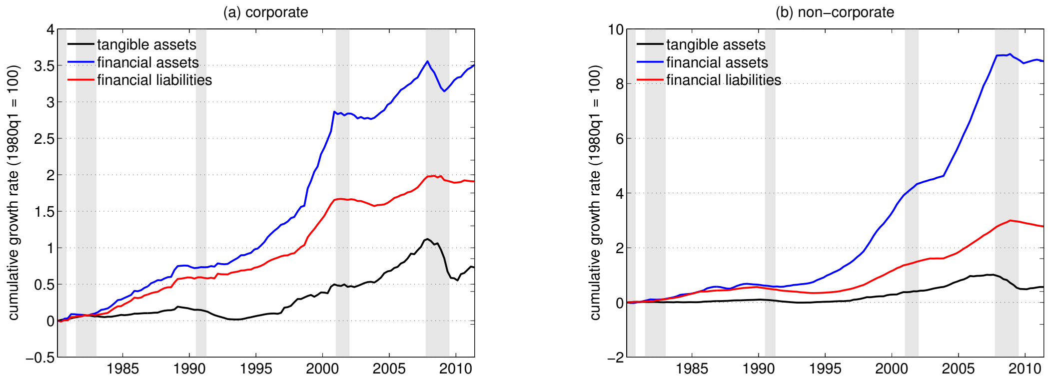

Figure 2.3: Evolution of Balance Sheet Structure for Non-Financial Businesses. Source: Tables B102 and B103 from the Flow of Funds. Corporate sector: B102.2, B102.6 and B102.21; Non-Corporate sector: B103.2, B103.10 and B103.24.

Another piece of evidence that points towards the substantial increase of savings in the productive sector is the balance sheet structure. Its evolution is displayed in of Figure (2.3), which reveals how substantial the accumulation of financial assets in the productive sector has been. It is at least suggestive that this large accumulation has occurred just about the same time in which household debt ramped up. The asset position of the two sides of the economy were actually going in opposite directions. This suggest that both the deregulation and innovation of the financial market served economic agents in different ways.

2.2.3 Net financial asset position by economic sector

To end this section, I document here the co-movement of the asset positions of the household sector and the corporate sector in the U.S. This feature is of special interest for this paper, since the theory laid out in the following section has stark predictions regarding this co-movement.

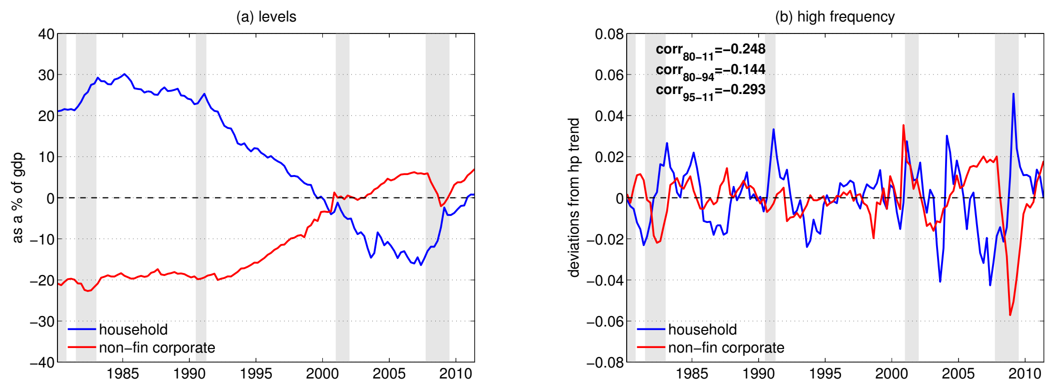

Figure 2.4: Net Financial Asset Position by Economic Sectors. Source: Tables B100 and B102 from the Flow of Funds. Households = B100.9 + B100.14 - B100.31, Corporate = B102.6 - B102.21

The left panel of Figure 2.4 shows that over the last twenty years, households have build up a negative asset position (net of equity holdings). This is due mainly to the dramatic increase in the stock of household debt through mortgage related liabilities. In the same period, the productive sector has experienced a somewhat opposite phenomenon. The aggregate stock of financial assets in the balance sheet of corporations has increased substantially, leading to the corporate sector to become a net lender to the rest of the economy[5] . Another way of saying describing this phenomenon is that households borrow from other households through the productive sector. The right panel of Figure 2.4 shows that this co-movement is also present at the high frequency and that it has become somewhat stronger in the las decade.

2.3 Model

This section lays out a model with heterogeneous households and producers. I will restrict attention to stationary environments and this will be reflected in the notation.

[5] OECD Data shows that among developed countries, the productive sector being a net lender is a phenomenon more common than what one would have expected. There are few papers that address this issue. An exception that tries to build around this fact for the case of the U.S. is [20]

The analysis of non stationary equilibria is delayed until Section 2.6, where I study transitional dynamics.

2.3.1 Environment

Time is discrete and infinite. There is a single non storable consumption good which requires the use of capital and labor to be produced. There are two physical assets: houses owned by households and capital owned by producers. The household side incorporates a housing decision into an otherwise standard problem of idiosyncratic income fluctuations. In the productive side, the assumption of decreasing returns to scale and time varying productivity shocks delivers a motive to save which will be key in the analysis. Finally, there is a reduced form of financial intermediation that allows agents to save and borrow funds by pledging the physical assets they own as collateral. I describe in turn the producers' problem, the households' problem, the market structure and the notion of equilibrium.

2.3.2 Producers

There is a unit mass of producers. Each producer has access to a production technology that transforms k units of capital installed the period before and l units of labor hired in the current period into y units of the consumption good according to the following production function:

$$y = F(z, k, l) = (zk)^{\alpha_1} l^{\alpha_2} \tag{2.1}$$

The production technology displays decreasing returns to scale, e.g. $\alpha_1 + \alpha_2 < 1$. As it is customary, I will refer to $\alpha_0 \equiv 1 - \alpha_1 - \alpha_2$ as the span of control, which is uniform across producers. The random variable z_j represents an idiosyncratic productivity shock with the structure $\ln z_j = x_j + \epsilon_j$.

Assumption 1. *The random variables x and ϵ are uncorrelated, i.i.d both over time and across producers and normally distributed with $x \sim N(-\frac{\sigma_x^2}{2}, \sigma_x^2)$ and $\epsilon \sim N(-\frac{\alpha_1 \sigma_\epsilon^2}{2}, \sigma_\epsilon^2)$*

The parameters of the normal distribution are chosen to ensure that $\mathbb{E}[e^x] = 1$ and $\mathbb{E}[e^{\alpha_1 \epsilon}] = 1$. The assumption of independence over time is not trivial. As it has been pointed out by [21], the persistence of the idiosyncratic shock strengthens the motive to save and this affects the dynamics of the transition. I explore and discuss the implications of relaxing this assumption in the quantitative section[6] .

Each individual producer learns the realization of x before capital is installed. Hence, the shock x will be often referred to as an *investment opportunity shock*, because it fully reveals the (expected) productivity of capital. In contrast, producers learn the value of ϵ_j only after they decide their production plans. Therefore, this shock can be interpreted as pure idiosyncratic investment risk[7] .

To set things up recursively, I adopt a cash in hand formulation. The timing of events goes as follows. Each producer starts the period with total resources worth ω_F and observes the realization of the random variable x. Then, he decides how to distribute these funds among dividends d, physical capital k' and savings m' in order to satisfy the flow of funds constraint given by[8]

$$d + k' + m' \leq \omega_F \tag{2.2}$$

As it was mentioned earlier, financial intermediation is introduced in a reduced form way. More precisely, each producer has the possibility to leverage up their physical investment by borrowing external funds. This borrowing capacity is constrained by the stock of installed capital, which is the only resource they can pledge. The collateral constraint for the firm has the form:

$$\mathbf{R}(m')m' + \phi_F(1-\delta)k' \geq 0 \tag{2.3}$$

The parameter $\phi_F \in (0,1)$ measures the pledgeability of the undepreciated capital stock and it is assumed to be uniform across firms. Notice that the interest rate $\mathbf{R}$ is allowed to depend on m' to consider the possibility of a spread between the borrowing and saving interest rates, which would reflect costly financial intermediation.

At the end of the period, each producer chooses the amount of labor input to utilize in production. This is an unconstrained static decision that solves the following problem

[6] In the quantitative section I will also allow some autocorrelation in the *investment opportunity shock* by positing a stochastic process of the form $x = x^P + x^T$, where x^P is an AR(1) process and x^T is white noise.

[7] For instance, the work of [11] focuses on ϵ-shocks, whereas the work of [21] or [14] focus on x-shocks.

[8] The model formulated here assumes that markets for insurance are absent, e.g. agents cannot trade claims that are contingent on their individual state. [15] shows that this assumption might not be trivial since allowing for hedging mutes the amplification of technology shocks created by credit constraints. See also [22] and [23]

$$\overline{\Pi}(k,x) = \max_{l\geq 0} \int (e^{x+\epsilon}k)^{\alpha_1} l^{\alpha_2} dG(\epsilon) - \mathbf{w}l \tag{2.4}$$

where G denotes the cumulative distribution of ϵ. After this decision is made, the idiosyncratic productivity shock ϵ realizes. Observe that producers face no constraint while financing the wage bill. The implicit assumption is that the pledgeability of cashflows is high enough so that the asynchronicity between receipts and outlays does not impose a problem regarding short term financing. The value of the static problem $\overline{\Pi}$ denotes the expected profit function. In turn, let Π denote the realized profit function. It is straightforward to obtain these functions in closed form, as the next result states.

Lemma 1. *The expected and realized profit functions are*

$$\begin{aligned} \overline{\Pi}(k,x) &= (1-\alpha_2)(\frac{\alpha_2}{\mathbf{w}})^{\frac{\alpha_2}{1-\alpha_2}}(e^x k_1^{\alpha})^{\frac{1}{1-\alpha_2}} \\ \Pi(k,x,\epsilon) &= (\frac{e^{\alpha_1\epsilon}-\alpha_2}{1-\alpha_2})\overline{\Pi}(k,x) \end{aligned}$$

Proof. From first order conditions, the labor demand is $l(k,x) = (\frac{\alpha_2}{\mathbf{w}})^{\frac{1}{1-\alpha_2}}(e^x k)^{\frac{\alpha_1}{1-\alpha_2}}$. Plugging this expression into the profit functions delivers the closed form expressions. □

We can use these expressions to write the law of motion for producers' total resources as follows

$$\omega_F'(k',m',x,\epsilon') = \Pi(k',x,\epsilon') + (1-\delta)k' + \mathbf{R}(m')m' \tag{2.5}$$

The individual state of an individual producer consists of the pair (ω_F, x). A production plan is a vector (d, k', m'). An optimal production plan is the solution to the right hand side of the following functional equation

$$\begin{aligned} J(\omega_F,x) = \max_{d,k',m'} \quad & d + \beta_F \mathbb{E}\left[J(\omega_F'(k',m',x,\epsilon'),x')\right] \\ s.t. \quad & (2.2)\ \ and\ (2.3) \end{aligned} \tag{2.6}$$

The presence of x as a state variable might seem confusing at first, given Assumption 1, but it is due the *cash in hand* formulation of the producer's problem. In order to calculate future resources, it does not suffice to know only current resources, but also how productive capital investment will be, e.g. the value of x. It is also important to note that producers are assumed to discount cash flows at the constant rate $\beta_F^{-1} - 1$. I discuss the appropriateness of this assumption at the end of this section.

2.3.3 Households

There is a unit mass of households. Each household is infinitely lived, supplies labor inelastically, and derives utility from consumption and housing services according to the instantaneous utility function $u(c, h)$. It is useful to think of the household problem as a standard income fluctuation problem that incorporates a housing decision. Each period, households are hit by an idiosyncratic labor income shock which is denoted by θ.

Assumption 2. *The random variable θ is i.i.d across households and follows an AR(1) process with persistence ρ and variance of the innovation $(1-\rho)\sigma_\theta^2$*

I will assume that housing is a continuous choice variable. For the purposes of this paper, this is a convenient formulation and no additional insight can be gained from assuming discrete choice or housing sizes[9] .

Each household starts the period with total wealth given by ω_H and decides consumption, asset holdings a' and housing status h', so as to satisfy the budget constraint

$$c + a' + \mathbf{p}h' \leq \omega_H \tag{2.7}$$

This constraint indicates that there are two ways in which households can save in this environment: by holding debt instruments issued by other agents, or by purchasing houses. In addition, they can borrow by issuing collateralized debt, where the only asset that can be pledged as collateral are houses[10] . The corresponding collateral constraint can be expressed as follows

$$\mathbf{R}\left(a'\right) a' + \phi_H \mathbf{p} h' \geq 0 \tag{2.8}$$

As it was the case with firms, the interest rate $\mathbf{R}$ is allowed to depend on a' to consider the possibility of costly financial intermediation. The parameter ϕ_H measures the

[9] This can be also justified by underscoring that the focus of this paper is on the role of limited ability to pledge assets as collateral, rather than on the trading frictions that arise from indivisibilities.

[10] The largest component of household debt corresponds to collateralized borrowing in the form of mortgage credit. According to the Flow of Funds of the Federal Reserve Board, this type of credit accounts for more than two thirds of aggregate household debt. [18] report that according to the Survey of Consumers Finance, the share of collateralized debt increases to around 80% if one considers other type of durable goods.

pledgeability of the housing stock[11] and it is assumed to be uniform across households. Finally, the law of motion for household's wealth is given by

$$\omega_H'\left(a',h',\theta'\right) = \theta'\mathbf{w} + \mathbf{R}\left(a'\right)a' + \mathbf{p}h' + \mathbf{d} \tag{2.9}$$

where θ represents the idiosyncratic labor income shock. The bold weighted variables $\mathbf{w}$, $\mathbf{p}$ and $\mathbf{d}$ correspond to the wage, the price of houses and the aggregate dividend payout respectively. An important assumption, as it was mentioned before, is that the housing market displays no frictions whatsoever. Dispensing with this assumption could bring other interesting issues to the table, but since the focus of the paper is on credit market frictions and the equilibrium in the market for loanable funds, it will be maintained throughout.

The individual state for the household consists of the triple (ω_H, θ). A consumption plan corresponds to a triple (c, a', h'). An optimal consumption plan solves the right hand side of the following functional equation

$$\begin{aligned} V(\omega_H,\theta) \quad &= \quad \max_{c,a',h'} u(c,h') + \beta\mathbb{E}[V(\omega_H'(a',h',\theta'),\theta')] \\ s.t. \quad & \quad (2.7)\,,\ \ (2.8)\,,\ and\ \ (2.9) \end{aligned} \tag{2.10}$$

The assumption that the housing decision affects current instantaneous utility is innocuous and convenient regarding the numerical implementation.

2.3.4 Markets

Let ψ_H and ψ_F represent the stationary measures of households and firms over their respective individual state. Since, both of them participate in the asset market, the corresponding market clearing condition can be written as follows

$$\int a'(\omega_H,\theta)d\psi_H(\omega_H,\theta) + \int m'(\omega_F,x)d\psi_F(\omega_F,x) = \overline{B} \tag{2.11}$$

where $\bar{B}$ is an exogenously supply of bonds (i.e. government bonds). The idea is that the economy might have sources of liquidity other than households' and producers' liabilities.

[11] Alternatively, the required downpayment to buy a house equals $1-\phi_H$ percent of its market value.

Recall that the labor and housing markets have an inelastic supply, so their market clearing conditions are

$$\int l(\omega_F, x) d\psi_F(\omega_F, x) = 1 \quad (2.12)$$

$$\int h'(\omega_H, \theta) d\psi_H(\omega_H, \theta) = 1 \quad (2.13)$$

By Walras' Law, market clearing for the final good is satisfied whenever these conditions hold.

2.3.5 Stationary equilibrium

This formulation suggests a straightforward definition of equilibrium. To this end, let $\mathcal{C}_H$ denote the set of continuous functions mapping $\mathbb{R}_+ \times \mathbb{R}$ into $\mathbb{R}$ and $\mathcal{C}_F$ denote the set of continuous functions mapping $\mathbb{R}_+ \times \mathbb{R}$ into $\mathbb{R}_+$. Let $\mathcal{S}_H$ and $\mathcal{S}_F$ denote the Borel sigma algebra defined respectively over $\mathcal{C}_H$ and $\mathcal{C}_F$. A generic element of $\mathcal{S}_H$ (resp. $\mathcal{S}_F$) is a set $S \equiv (S_\omega, S_\theta)$ (resp. $S \equiv (S_\omega, S_x)$). Finally, let Δ_H and Δ_F be the set of all probability measures defined over $\mathcal{S}_H$ and $\mathcal{S}_F$ and define the updating operator over $\mathcal{S}_H$ and $\mathcal{S}_F$ as follows

$$\mathbf{T}\widetilde{\psi}_H(S) = \Pr(\theta' \in S_\theta) \int \mathbf{I}\left(\omega'_H(\omega_H, \theta, \theta') \in S_\omega\right) d\widetilde{\psi}_H(\omega_H, \theta) \quad (2.14)$$

$$\mathbf{T}\widetilde{\psi}_F(S) = \Pr(x' \in S_x) \int \mathbf{I}\left(\omega'_F(\omega_F, x, \epsilon') \in S_\omega\right) \widetilde{\psi}_F(\omega_F, x) \quad (2.15)$$

for any set $\widetilde{\psi}_H$ and $\widetilde{\psi}_H$ belonging alternatively to Δ_H or Δ_B. We are now in position to introduce our notion of equilibrium.

Definition 1. *A stationary recursive competitive equilibrium consists of value functions* $V \in \mathcal{C}_H$ *and* $J \in \mathcal{C}_F$*, decision rules for the firm* $\{d, k', m'\} \in \mathcal{C}_F$ *and for the household* $(c, a', h') \in \mathcal{C}_H$*, aggregate prices* $(\mathbf{w}, \mathbf{p}, \mathbf{R})$ *and probability measures* $\psi^*_F \in \Delta_F$ *and* $\psi^*_H \in \Delta_H$*, satisfying*

1. *Firm Optimality:* J *and* (d, k', m') *solve* (2.6)

2. *Household Optimality:* V *and* (c, a', h') *solve* (2.10)

3. *Market Clearing: conditions* (2.11)*,* (2.12)*, and* (2.13) *are satisfied*

4. Stationarity: (ψ_F, ψ_H) *are fixed points of* (2.14) *and* (2.15) *respectively*

The choice of the rate producers use to discount future dividends deserves some discussion. The fact that dividends are rebated to households (as reflected in (2.7)) implicitly assumes that they own the firm, although they are not able to trade shares. From this perspective, the appropriate discount factor should incorporate marginal utilities so that the objective of the productive sector is consistent with households' interests. Alternatively, one could impose an objective assuming a fixed discount rate (as it is done here), and let producers be net present value maximizers. In this case, if shares were traded in the market, for any given β_F, households would price these shares accordingly just as they would price a Lucas tree. In this way we could implicitly calculate the value of a firm. Furthermore, from a corporate finance perspective, the appropriate rate of discount should be the opportunity cost of capital - the expected return that could be earned by investing instead in a financial asset with a similar risk profile. Although there is no such alternative asset in this economy, if there were it would carry a risk premium which would drive the discount rate above the market interest rate. All that is needed in the model is that the opportunity cost of capital does not move together with the equilibrium interest rate. Setting $\beta_F = \beta$ is too simplistic, but it certainly accomplish the goal. Alternatively (and perhaps preferably), the economy could be just interpreted as having two type of agents, households and producers, with different preferences and access to backyard technologies. In such a case, dividends should be interpreted as consumption of the producers[12] .

2.4 Equilibrium Properties

This section characterizes some properties of stationary equilibria, emphasizing the productive side of the economy. The main purpose is to understand the relationship between aggregate output and the interest rate. At the end of the section, I propose an heuristic representation of the equilibrium in the spirit of [24].

[12] Another alternative is to assume that the firm is in hands of a fictitious representative consumer. In that case, discount factors would coincide in a stationary equilibrium.

2.4.1 Gains from reallocation

As it will be seen, in the numerical section I will assume that the economy starts from a situation in which the productive sector is a net lender to the rest of the economy. To understand better the need for this assumption, this section focuses on the relationship between aggregate output and interest rate[13] . Since deleveraging makes liquidity scarce and drives its price upwards (interest rate downwards), in order for this process to decrease total output, the productive sector must be demanding more liquidity than what it is able to create by itself[14] .

It is useful to start by defining the *weighted capital stock* as follows

$$\mathbf{K_e} \equiv \int e^x k(\omega_F, x) d\psi_F(\omega_F, x) \tag{2.16}$$

This can be thought of as the aggregate amount of capital measured in efficiency units. Alternatively, this measure of the capital stock can be written as follows

$$\mathbf{K_e} \equiv \left(1 + \mathbf{cov}\left(e^x, \kappa(\omega_F, x)\right)\right) \mathbf{K} \tag{2.17}$$

where $\kappa(\omega_F, x) \equiv k(\omega_F, x) / \int k(\omega_F, x) d\psi_F(\omega_F, x)$ is the share of capital in hands of the mass of producers with individual state (ω_F, x). This expression indicates that weighted capital is larger than the aggregate stock of capital, the larger the covariance term between the investment opportunity shock and the individual capital stock[15] . The next result obtains an expression for the aggregate production function in closed form.

Proposition 1. *In a stationary competitive equilibrium, aggregate output is given by*

$$\mathbf{Y} \approx \left[1 - \varphi \mathbf{cv}(e^x k(\omega_x, x))^2\right]^{1-\alpha_2} \left[1 + \mathbf{cov}\left(e^x, \kappa(\omega_F, x)\right)\right]^{\alpha_1} \mathbf{K}^{\alpha_1} \tag{2.18}$$

where **cv** *denotes the coefficient of variation.*

[13] [17] obtain a similar result in a model with bubbles. In their environment, the relation between output and interest rate becomes positive when the economy features abundant outside liquidity, as measured by the availability of stores of value originated outside the productive sector.

[14] This condition prevails in several developed countries, included the U.S. In addition, as it was documented in Section 2.2, in the U.S. it is also true that in the same period in which the producers became net lenders, households reduced their net holdings of riskless assets to the point of becoming net borrowers.

[15] It is straightforward to show that for any given **K**, productive efficiency requires more capital to be allocated to those producers with larger realizations of x. Since both random variables are normalized, this covariance is a direct and unit-free measure of productive efficiency in this environment.

The proof is relegated to the appendix. This result is interesting for two reasons. First, it indicates that the aggregate production function also displays decreasing returns to scale with respect to the aggregate capital stock $\mathbf{K}$. Second, it associates the TFP term with two unit-free statistics of the equilibrium allocation of capital

1. Dispersion across producers
2. Covariance with the x-shock.

The comparative statics are straightforward. More dispersion reduces TFP, whereas positive covariance increases it. Hence, this result offers a way to understand the relationship between output and the real interest rate. To start with, suppose we were in an environment with a representative producer that faces no aggregate risk. In such a case, the only thing that would matter is the effect of the interest rate on the aggregate capital stock. If the representative producer must borrow to take a leveraged position, a larger interest rate will reduce his borrowing capacity and output would decline.

In contrast, in an environment with heterogeneous producers, one needs to consider also what happens to the dispersion and the correlation terms, which will affect aggregate measured TFP. Two types of dispersion need to be considered. It is useful to refer to these as *uncorrelated* dispersion and *correlated* dispersion, paraphrasing [25]. The former has to do with the fact that producers with the same x, could end up with different levels of capital. The latter has to do with the fact that producers with larger x, could end up with more capital. Both types of dispersion reduce the first term of (2.18), but *correlated dispersion* increases the second term, or equivalently, it increases the *weighted capital stock*[16] .

[16] In the presence of decreasing returns to scale, uncorrelated dispersion always carry losses. When returns to scale are constant, $\varphi = 0$ and dispersion has no effect. See the appendix.

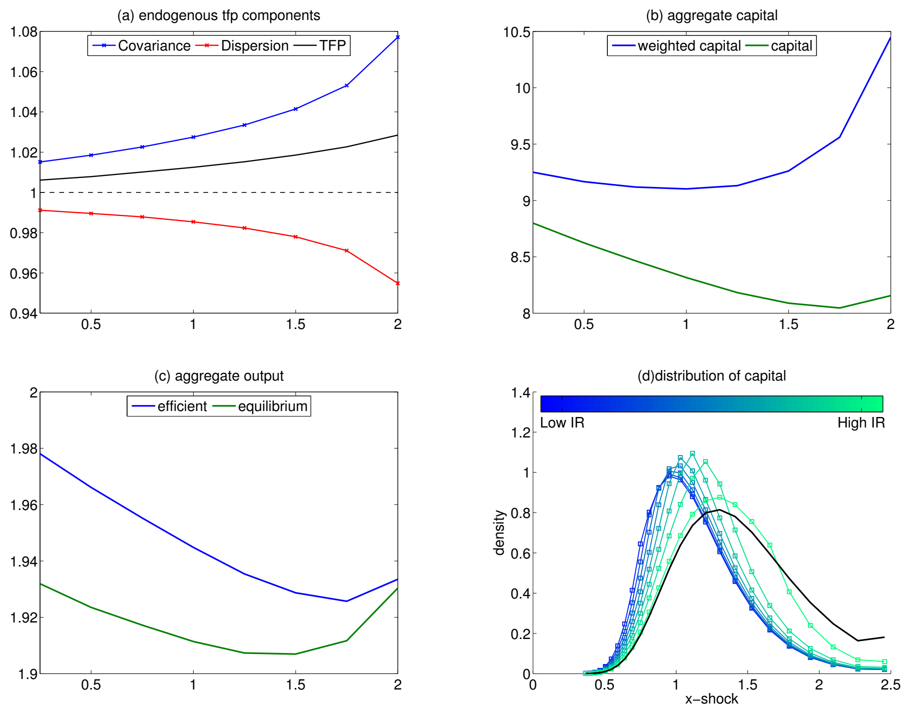

Figure 2.5: Productive Sector Aggregates: Endogenous TFP components and Measures of Capital Stock.

Both components of TFP and the measures of capital stock are displayed in the top of Figure (2.5)[17] . The conclusion that can be drawn from these figures is that larger interest rates in general reduce the aggregate capital stock, but they also unambiguously augment measured TFP. In fact, as the same figure portrays, the measures of the capital stock are not monotone as a function of the interest rate. Both ramp up eventually as the interest rate increases, and the weighted capital stock does it earlier due to the covariance term which is indeed monotone increasing in the interest rate.

High interest rates boost wealth accumulation and allow producers to take advantage of good investment opportunities. Going back to the expression for aggregate output, when interest rates are low, their effect on the dispersion term and the aggregate stock of capital (measured in standard units) dominate. In contrast, when interest rates are

[17] The figures in this section already take into account labor market clearing condition. Since the price of houses does not affect directly producers' problem, it is easy to solve numerically for productive sector aggregates as a function of the interest rate.

high, the covariance term kicks in and output increases due to a better allocation of resources. Producers save to correlate their choice of capital with their realization of x. Hence, across steady states, the larger the return on savings, the easier is to achieve such a goal. These effects are portrayed at the bottom of Figure 2.5[18] .

This observation stresses the role of private debt in the reallocation of capital. By affecting the equilibrium interest rate, deleveraging sets in motion two effects on productive decisions that go in opposite directions. First there is a *leverage effect*, which affects lucky producers that incur in leveraged investment. This captures the spirit of [3] analysis of the effect of public debt on capital accumulation. Second, there is a *liquidity effect*, which affects unlucky producers that sit in their wealth waiting for better investment opportunities. This second effect was first stressed by [4]. In the aggregate, the response of total output on the relative strength of these effects, considering that larger interest rates favor those producers who accumulate financial assets.

2.4.2 The price of houses

In the previous subsection, there was no need to make reference to the price of houses since it does not enter into producers' decision problem. Yet, when I study transitions in Section 2.6, the evolution of the price of houses is key to understand the equilibrium path of prices and quantities. Following the same logic of Proposition 1 we obtain the following result.

Proposition 2. *In a stationary competitive equilibrium, the price of houses satisfies*

$$\mathbf{p} \approx \left\{ \frac{\mathbf{R}}{(1-\beta)\mathbf{R} - (1-\beta\mathbf{R})\phi_H} \right\} \left\{ \frac{2+\nu(\nu+1)\mathbf{cv}(h)^2}{2+\sigma(\sigma+1)\mathbf{cv}(c)^2} \right\} \gamma \mathbf{C}^{\sigma} \tag{2.19}$$

where $\mathbf{cv}$ *denotes the coefficient of variation.*

Hence, up to a second order approximation, the price of houses increases with aggregate consumption and decreases with both the interest rate and the tightness of borrowing constraints. Intuitively, loose credit (high ϕ_H) makes households to bid up for houses, increasing their price.

[18] The efficient level of output is calculated by aggregating $k^{\star}(x;\mathbf{K})$ across producers, which is defined as the solution to $\max_{k(x)} \left\{ \int (e^x k(x))^{\alpha} dG(x) \mid \int k(x) dG(x) \leq \mathbf{K} \right\}$, where $\mathbf{K}$ is the aggregate capital stock. This solution is used to depict the efficient distribution capital in the bottom right panel of Figure 2.5.

2.4.3 Liquidity premium

As it is standard in this class of models, a necessary condition for existence of a stationary competitive equilibrium is $\min\{\beta, \beta_F\}\mathbf{R} < 1$. The next proposition states that when this condition holds, the value function J inherits the features of the production technology. The proof of this result uses standard dynamic programming techniques and it is therefore omitted.

Lemma 2. *If $\beta\mathbf{R} < 1$ the value function J is continuous and strictly increasing in ω_F and x and strictly concave in ω_F.*

Using this Lemma, we can obtain the next result, much in the spirit of [11]. The proof is relegated to the appendix.

Proposition 3. *An unconstrained producer assigns a premium to the return on capital only if $\sigma_\epsilon > 0$.*

In this model economy, the return on physical investment will be larger than the return on the risk free bond This differential is due to the presence of a risk premium associated with σ_ϵ and to the fact there is a positive mass of producers borrowing heavily and failing to equalize their marginal return on capital with the marginal cost of borrowing (e.g. facing binding borrowing constraints). This makes difficult to label this wedge as a *liquidity premium* since in the absence of volatility in ϵ, producers holding the financial asset would not pay for it more than its opportunity cost. Therefore, it is of interest that even in the absence of a liquidity premium per se, the concept of liquidity remains relevant for the analysis of the aggregate consequences of credit tightening. This will indeed become clearer in the quantitative section[19] .

[19] Models in which the price of the financial asset commands a premium rely either on the role these assets play in facilitating intra temporal exchange, as in [26]), or on the role they serve as collateral when markets are incomplete, as in [14]. Although the model here is more associated with the second view, notice that assets are not held because they are pledgeable (only physical assets are) but rather because they allow agents to store value. This modeling choice makes clear that the premium on the financial asset is associated with agents bidding up for the financial asset due to their precautionary motive to save. Hence, this modeling choice emphasizes that the fundamental feature of the equilibrium is the limited supply of stores of value, which in turn is tightly linked to the limited pledgeability of the physical assets.

2.4.4 Optimal Policies

Producers' problem can also be formulated in terms of ratios indicating how much to retain from total funds and how much to leverage up physical investment, as the following functional equation suggests

$$J(\omega_F, x) = \max_{\sigma,\lambda} \quad (1-\sigma)\,\omega_F + \beta_F \mathbb{E}\left[J\left(\omega_F'\left(\sigma,\lambda,x,\epsilon'\right),x'\right)\right]$$
$$s.t. \quad 0 \leq \sigma \leq 1 \text{ and } 0 \leq \lambda \leq \frac{1}{1-\phi_F(1-\delta)\,\mathbf{R}(\lambda)^{-1}}$$

where $\omega_F'(\sigma,\lambda,x,\epsilon') = \Pi(\lambda\sigma\omega_F, x, \epsilon') + ((1-\delta)\lambda + (1-\lambda)\mathbf{R})\,\sigma\omega_F$. In this formulation, σ denotes the fraction of firm's resources that are retained and $\sigma\lambda$ denotes the fraction of those resources that are invested in physical capital. Therefore, leveraged investment occurs whenever $\lambda > 1$, otherwise the firm saves. The assumption of decreasing returns to scale implies there will always be positive physical investment.

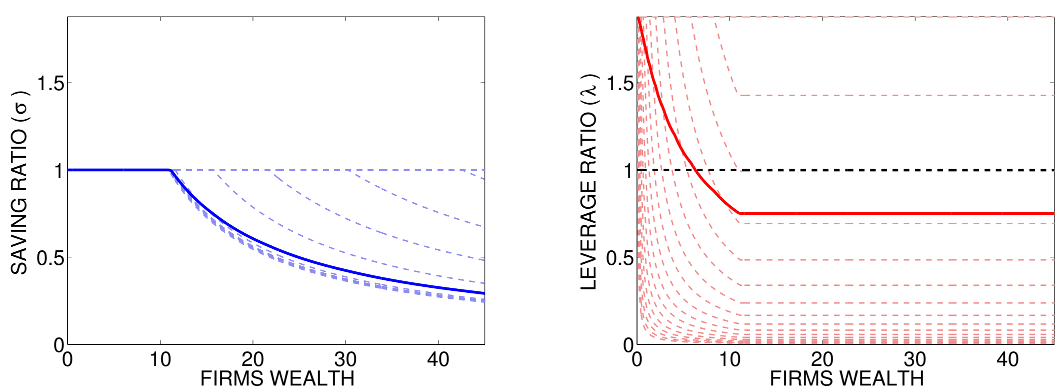

Figure 2.6: Producers' Decision Rules. Each dashed line corresponds to a value of x. The solid line corresponds to the average of the dashed lines using the marginal distribution of x

The optimal decision rules of this alternative formulation are illustrated in Figure 2.6. Observe that producers delay distribution of dividends until they reach a certain level of wealth. Since the level of ω_F is associated with firm size, as measured by the level of installed capital, this means small producers are in general more leveraged and distribute dividends less often. It is worth to notice that both borrowing and saving occur along the entire support of the distribution for ω_F. This implies that producers decide to hold the financial asset even if they have enough funds so as to finance the optimal level of physical capital. This is the sense in which producers display a precautionary motive to save.

It is possible to cast households' problem in a similar way. To avoid excessive notation, here we also use σ to denote the saving rate and λ to denote the share of the saved resources allocated to purchasing houses. The problem of the household becomes

$$\begin{aligned} V(\omega_H, \theta) = \max_{\sigma,\lambda} \quad & u\left((1-\sigma)\,\omega_H, \frac{\lambda\sigma}{\mathbf{p}}\omega_H\right) + \beta\mathbb{E}\left[V\left(\omega_H'(\sigma,\lambda,\theta), \theta'\right)\right] \\ s.t. \quad & 0 \leq \sigma \leq 1 \; and \; 0 \leq \lambda \leq \frac{1}{1-\phi_H \mathbf{p}\mathbf{R}(\lambda)^{-1}} \end{aligned}$$

where now $\omega_H'(\sigma,\lambda,x) = \mathbf{w}\theta' + \mathbf{d} + (\lambda + (1-\lambda)\,\mathbf{R})\,\sigma\omega_H$. Instead of depicting the optimal policies, which one can anticipate that look standard, I choose here to illustrate the distribution of asset holdings and housing in steady state.

2.4.5 Equilibrium Representation

In this subsection I present a heuristic representation of the equilibrium in the market of loanable funds to understand how output is determined. Since both households and producers display a precautionary motive to save, depending on the intensity of this motive in each sector, the equilibrium could deliver producers being net lenders or net borrowers. These two possibilities are represented on Figure 2.7. One should think of the red line as the aggregate supply of savings and the blue line as the aggregate demand for savings. Since the financial asset is assumed to be in zero net supply, this line must correspond to the aggregate marginal productivity of capital.

The aggregate stock of capital is determined by the point in which these two lines cross each other. Total output would correspond to the area below the blue curve up to the value of the aggregate capital stock. The figure also displays the net supply of funds by the productive sector, which is represented by the dotted green line. This line corresponds to total assets in producers' balance sheet, which in turn are composed of financial and physical assets. The asset position of the productive sector can be deduced from the location of the green line with respect to the red line at the equilibrium point.

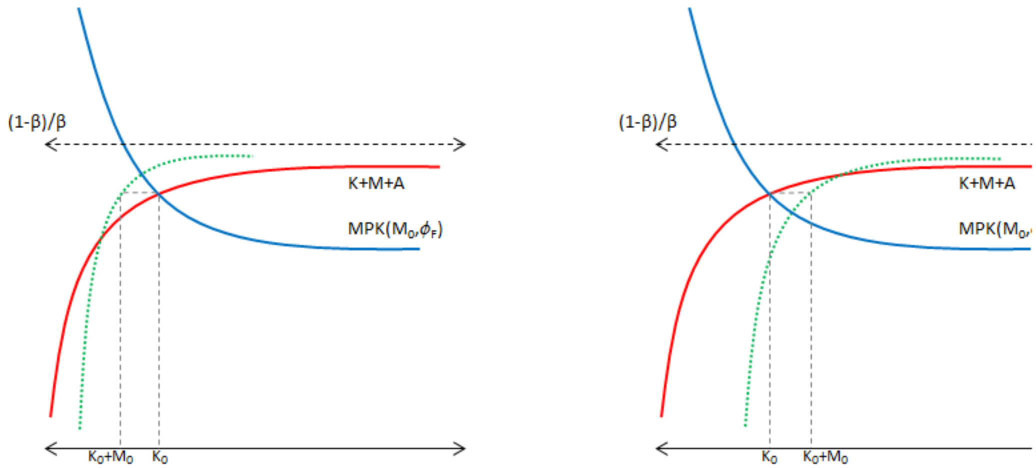

Figure 2.7: Equilibrium Representation. The location of the dashed green line indicates the asset position of the productive sector. Producers as net borrowers on the left, producers as net lenders on the right.

Suppose the precautionary motive is strengthened due to a credit crisis that tightens the collateral constraints of private agents. In this case, the red line in Figure 2.7 would shift to the right, as it would indeed do in a standard heterogeneous agent model. But in this model, that is not the end of the story. As I will argue momentarily, in response to a fall in the interest rate the marginal product (blue line) will also shift downwards. Moreover, this shift will occur even if it is the household sector the one affected by the credit tightening and its magnitude will depend on the asset position of the productive sector at the moment the tightening occurs.

The previous discussion suggests that if the downward shift is large enough, the economy produces less output although it holds more capital. This is because capital is being poorly allocated among productive units. Figure 2.8 illustrates two possible scenarios assuming it is households' precautionary motive which increases from the initial situation depicted in Figure 2.7. This implies that the dashed green line remains unchanged. Notice that since the asset market must be in zero net supply, the net asset position of the productive sector decreases in both cases. This alters the marginal product of capital and the figure assumes the downward shift is larger when the productive sector is a net lender (later, it will be shown numerically that this is indeed the case). Consequently, the area below the blue curve up to the value of the new capital stock is more likely to be lower than before.

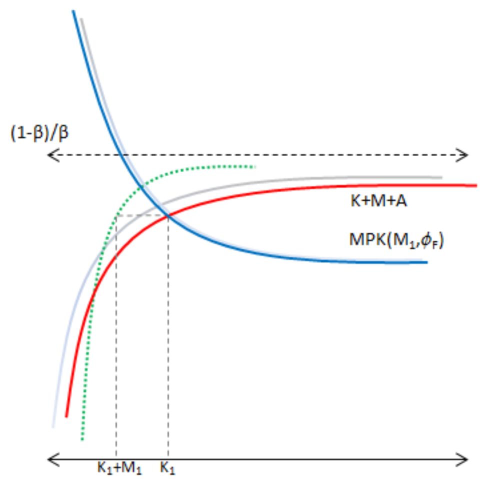

Figure 2.8: Representation of the Effect of a Household Credit Tightening. Producers as net borrowers on the left, producers as net lenders on the right.

In summary, to achieve productive efficiency, low productivity producers must be induced to postpone their investment decisions. Since in equilibrium, this is possible only if other agents are willing to increase current consumption / investment through collateralized borrowing decisions, credit crises affect productive activity by strengthening the precautionary motive to save of private agents. Moreover, this logic applies even if the crisis does not hit the productive sector directly.

2.5 Comparative Statics

Since deleveraging pushes interest downwards, the response of aggregate output to the type of perturbation is, both in the short run and in the long run, ultimately a quantitative issue. Notice that when collateral constraints are tighter, the extent in which the market can make resources to be allocated to their most efficient use is more limited. The idea of this section is to perform a quantitative exploration of the aggregate consequences of a tightening of collateral constraints that induce a strengthening of the precautionary motive to save and examine whether they have different empirical implications. Depending on where these shocks hit, e.g. either households or producers, we have two cases of study which we discuss in the following paragraphs.

2.5.1 Baseline Parameterization

The model is parameterized so that its steady state matches some key statistics for the pre crisis period[20] . The calibration is summarized in Table 2.1. Since the model is still very stylized, most of the choices rely on previous studies. The reader should keep in mind though that most estimates are not directly applicable and further sensitivity analysis are a pending task.

The pledgeability of houses should be interpreted as the loan to value ratio of home equity loans. Alternatively, one could think of $1 - \phi_H$ as the downpayment required to purchase a house. In both cases, the choice corresponds roughly to the financial conditions before the recent crisis. All preference parameters are chosen exogenously. A candidate target to calibrate some of these parameters is the pre-crisis debt to income ratio of the household sector. Regarding the producer side of the economy, the factor shares imply a span of control parameter of 0.1. This is an important parameter and in the literature it usually ranges from .1 (see [28]) to .25 (see [29]). The pledgeability of capital is chosen to match the capital to income ratio in the United States. Finally, a crucial parameter is the extent of idiosyncratic risk that firms face. The larger this risk, the more intense the motive to save in the firm side. For instance, [11] considers a standard deviation for this shock of 20% and 40%, although he notes that it is difficult

[20] An alternative strategy is to calibrate the model to a period of relative stability and then feed in a leverage cycle, e.g. a period of loose credit easing followed by a credit crisis. This is the strategy pursued in a recent paper by [27]

to come with precise estimates of the level of idiosyncratic investment risk. In the results presented I go with the upper bound of that range, but it should be pointed that this amount of risk overshoots the ratio of the financial asset position to tangible capital of the productive sector. An easy way out is to consider the presence of fixed (and highly illiquid, in the sense they can't be traded) factors of production. At least conceptually, it is easy to extend the model in that dimension, but it carries the cost of complicating the numerical computation[21] .

Table 2.1: Parameters for Baseline Economy

Exogenous	**Symbol**	**Value**	**Source**
Pledgeability h	ϕ_H	.95	Zillow.com
Span of control	$1-\alpha_1-\alpha_2$	.1	Khan and Thomas (2011)
Home ownership	$\overline{H}$	.66	U.S. Census Bureau
Persistence θ	ρ_θ	.8	Guvenen (2009)
St Dev ϵ_θ	σ_{ϵ_θ}	.1	-
Depreciation rate	δ	.05	-
Risk Aversion	σ	2	-
Discount factor	β	.98	-
Calibrated			**Target**
Preference shifter	γ	.15	Debt to Income Ratio
Pledgeability k	ϕ_F	.5	Capital to Income Ratio
St Dev of ϵ_z	σ_{ϵ_z}	.4	Net Financial Assets / Tangible Capital

2.5.2 Financial shocks to firms

Table 2.2 displays the effect of a permanent credit tightening in the firms side over the main aggregate variables. The baseline economy considers that 65% of the undepreciated capital stock that an individual firm will own at the beginning of the next period can be pledged as collateral. The next two columns consider the steady states to which the economy will transit if the pledgeability of capital decreases to 50% and 35% respectively.

[21] The results presented here are based in the model presented in the body of the paper, but in the appendix, I present an alternative formulation of the producer's problem that incorporates that extension.

The pledgeability of housing wealth is fixed at 95%.

Table 2.2: Comparative Statics following a Firm Credit Tightening

Aggregate	**Baseline**	**Crisis 1**	**Crisis 2**
Output	2.0662	2.0397	2.0142
Capital	8.6866	8.5038	8.3164
Weighted Capital	13.9799	12.8778	11.9465
Household NFA	-1.8412	-2.2164	-2.5516
Housing Wealth	4.4423	4.5595	4.7307
Interest Rate	1.6630	1.5319	1.4009

Certain things are worth to mention in the household side of the economy. Housing wealth increases across steady states. Since the supply of houses is fixed, this reveals an increase in the price of houses. This can be interpreted as firms demanding more liquidity and the economy producing it by making households more levered. In the productive side, it is not surprising that tighter financial conditions drive the economy to a lower output and stock of capital. But it cannot be deduced whether this is due to the destruction of (collateralizable) wealth or to the fact that stores of value are relatively scarce. Moreover, it is worth to notice that since the anticipation of tighter financial conditions intensifies firms savings (just as it does with households in standard heterogeneous agents models), this scarcity is indeed demand driven.

2.5.3 Financial shocks to households

Consider instead the long run effects from a tightening of household financial conditions. The results of such experiment are displayed in Table 2.3. The baseline economy assumes that the pledgeability of housing wealth equals 95%. The following two columns correspond to the case in which this parameters is reduced to 80% and 65% respectively. The pledgeability of undepreciated capital is fixed at 50%.

The results in terms of output and capital are qualitatively similar to those of the previous exercise, but much more modest in magnitude. Although this is suggestive of how small the effects of a liquidity shortage can be, these numbers should be taken with caution mostly because the parameters for a realistic calibration in the context of this model are not well known. I interpret this result as indicating that liquidity shortages induced by private deleveraging can indeed drive down aggregate output due to reduced total factor productivity. This is yet another channel through which a shock in the household sector can deliver a contraction of economic activity.

Table 2.3: Comparative Statics following a Household Credit Tightening

Aggregate	**Baseline**	**Crisis 1**	**Crisis 2**
Output	2.0417	2.0397	2.0352
Capital	8.4984	8.5038	8.5181
Weighted Capital	13.0170	12.8778	12.6412
Household NFA	-2.7466	-2.2164	-1.4130
Housing Wealth	4.6947	4.5595	4.4028
Interest Rate	1.5855	1.5319	1.4388

2.6 Transitional Dynamics

The question that arises naturally from the previous analysis is whether or not these effects are stronger on impact. The study of these issues is challenging because at the moment financial conditions change, the path of prices becomes a state variable for each individual problem along the transition to the new steady state.

I perform two exercises. First, I investigate the effects of a borrowing constraint shock to the productive side. Second, I explore the effects of a borrowing constraint shock in the household side. This can be interpreted as a decrease in the loan to value ratio of home equity loans, or as an increase in the downpayment requirement to purchase houses. One could also assume that at the moment the collateral shock hits, intermediation costs increase, driving a wedge between the borrowing and lending interest rate. In the appendix, I perform these exercises and show that the conclusions of this section remain unchanged.

2.6.1 Financial shock to firms

Figure 2.9 displays the effect of a collateral constraint shock to firms on the main aggregates. There is a sense in which the analysis of this case and the bubble burst studied in [2] are similar: they both produce a destruction of firms wealth and a relative scarcity of stores of value. In fact, the evolution of output resembles the one reported by that paper in his analysis of the consequences of bubble bursts: output drops more drastically in the short run. But notice that the dynamics also display an increase in the price of houses. This feature makes difficult to relate the initial impulse considered in this subsection with the narrative of a housing bubble burst. An important difference between both exercises is that here the scarcity is entirely demand driven (recall that the productive sector was assumed to be a net saver), while the idea of a bubble burst suggests a drop in the supply of liquid assets.

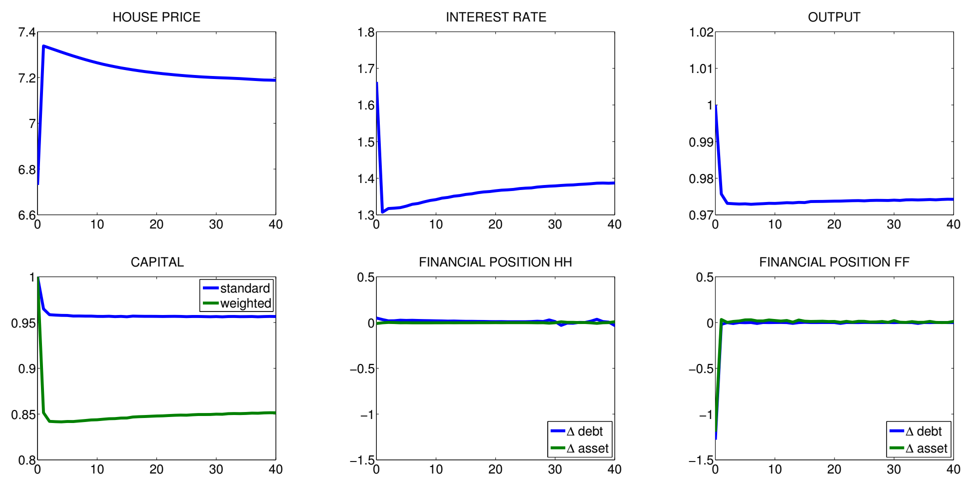

Figure 2.9: Transitional Dynamics following a Firm Credit Tightening

To better understand these dynamics, the bottom of the figure displays how the deleveraging process is absorbed by the economy. It can be seen that half of it is absorbed by renters, which see the drop in interest rates as an opportunity to become owners, but they need to build up their savings accounts to do so. Since the housing market is in fixed supply, this imposes an upward pressure in the price of houses, which converges to a higher level. In addition, notice that some households are indeed increasing their leveraged position. Hence, these dynamics are consistent with the household sector having a larger gross debt position. In the productive side, not only the accumulation of physical capital is affected, but also the allocation of it to its most efficient use is hindered due to fall in the return of the saving instrument.

2.6.2 Financial shock to households

Figure 2.10 shows the transitional dynamics of the main aggregates following a permanent contraction in the pledgeability of the value of houses ϕ_H. Remarkably, a household credit tightening looks very much like a housing bubble burst, featuring low interest rates and a severe drop in the price of houses. However, in the short run the economy experiences a boom in production. This response can be explained by the relative strength of the leverage and liquidity effect explained in the previous sections. In the short run, the effect of the interest rate on the investment decisions of lucky firms dominates.

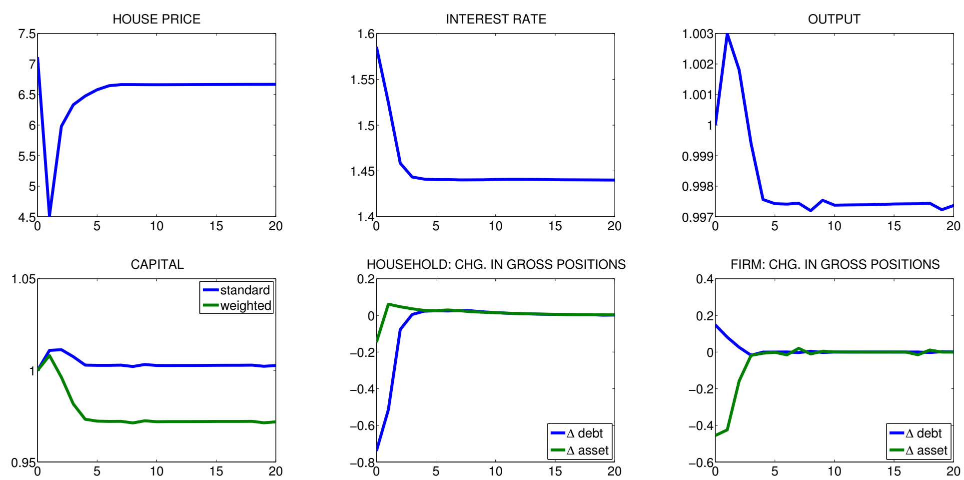

Figure 2.10: Transitional Dynamics following a Household Credit Tightening

It is interesting to compare this result with that in [8]. In their model, the response of the economy to a household credit tightening features an interest rate overshooting and a recession in the short run[22]. Since the financial asset is in zero net supply only across households, the short run response is a consequence of the the effect induced by the interest rate along the asset distribution. In other words, overshooting must induce

[22] The production technology in their model uses only labor, which is endogenously chosen by the household. The dynamics of output is shaped by the composition of the labor supply.

agents at the top to deplete assets as rapidly as those at the bottom deleverage, which they do almost without paying attention to the interest rate.

The reasoning here is similar, but it is mostly the productive sector which soaks up household debt instead of wealthy households. As households deleverage, firms need to be discouraged save in order for the asset market to clear. The results show that they do so very rapidly and there is no need for an overshooting. It is true though that the decline of household debt effectively affects economic activity in the long run by contracting the production possibilities of the economy.

2.6.3 Extensions

Capital adjustment costs It would be interesting to inquire into what adjustment cost would do the the mechanism explored here. In the analysis of the household credit tightening, the leverage effect dominated in part because capital is being allocated to high productivity firms (the covariance term is large). Adjustment costs would dampen this effect precisely by making reallocation costly. It is not clear what would it take to have a productive sector as a net saver or whether liquidity helps in any way when unproductive capital is stuck at individual firms. This is certainly an interesting issue to explore, although computationally it complicates the analysis by adding a state variable to the firms' problem.

Liquidity for short term financing In addition to the role that financial assets play in firms' balance sheet studied here, the literature also recognizes the role of liquid assets in financing working capital. The analysis has deliberately abstracted from this motive to hold financial assets in order to maintain a symmetry in the treatment of households' and firms' financial conditions.

Land in hands of firms In this model, firms do not participate in the housing market. This assumption was useful to isolate the effect of a liquidity shortage, but one might wonder what would happen if they do. The interpretation would be to assume that both households and firms value land services provided by the physical asset called houses. Then, following a credit tightening in the household side, firms' wealth would be destroyed by the drop in the value of houses. The point is that the outcome of such exercise would again mix the effect of variations in wealth with that of scarcity of stores of value.

Other legal forms of organization It could also be interesting to reinsert the figure of the entrepreneur: a risk averse agent with access to a backyard technology. If these agents are in general more leveraged that firms, as it seems to be the case, they would be among those benefited by the fall in the interest rate. However, insofar they own houses and use it as collateral to finance their activities, they would also suffer a wealth loss even if they experienced no change in their financial conditions.

2.7 Conclusions

This paper presented a general equilibrium model in which household debt provides firms with the liquidity necessary to organize production intertemporally. I introduced a motive to save in the productive side, which stems from combining decreasing returns to scale, time varying productivity shocks and collateral constraints. I considered the case in which firms are net lenders to the rest of the economy and studied the consequences of a deleveraging process triggered by a tightening of the financial conditions of private agents. In this context, I have shown that a credit crisis originated in the household side depresses economic activity by producing a liquidity shortage. More savings in the household side hurt the economy by pushing the interest rate downwards and making liquid assets more expensive. I have also contrasted the implications of a credit tightening in the household sector and the productive sector and argued that the case of a pure liquidity shortage corresponds to the former, since the latter mixes the effect of a relative scarcity with the destruction of wealth. Moreover, a household credit tightening resembled a housing bubble burst in terms of the behavior of the interest rate and the price of houses, but it produces a boom in production in the short run.

The findings in this paper have also implications for policy. If the tightening of private agents' financial conditions had triggered an unambiguous output drop in the short run, the government could attenuate this effect by injecting debt that pays a high interest rate ([2]). However, this paper shows that it matters whose wealth is destroyed. When the credit tightening affects the household sector, the short run effects grant no intervention. Of course, these implications are reversed if one considers the effect of a credit tightening in the firm side. However, it is not clear whether tighter financial conditions affect all agents at the same time nor whether those affecting one sector ultimately spread over to the rest of the economy. In the context of the last recession, many presumed that the poor performance of mortgage loans put banks under distress and triggered a credit tightening in the firm side. Although this logic is persuasive, it is still subject of some dispute ([30]).

Overall, models that study financial shocks as independent drivers of business fluctuations are far from generating consensus among researchers. They are appealing because financial conditions happen to vary a great deal with the cycle, but more research is needed to understand how these conditions are determined in equilibrium, and how private choices of financial contracts are affected by changes in fundamentals. These are the issues that I plan to address in my future research.

Chapter 3

Understanding Credit Crunches Through a Model of Loan Commitments

3.1 Introduction

The last financial crisis has frequently been associated with a credit crunch, a situation in which banks suddenly slow lending activity. However, bank lending data provides inconclusive evidence about this event since the stock of loans did not fall at all when the crisis was at its peak. A substantial decline in bank lending was indeed observed later, when the economy was already going through a deep recession. Moreover, since the real activity slowed down first, it was not clear whether the decline in lending was due to banks being reluctant to lend or due to firms not willing to borrow in the absence of good projects to be financed. That is, it was not clear if it was supply or demand driven.

Faced with this evidence, some researchers suggested that the evolution of loans is not necessarily informative about the occurrence of a credit crunch. Since a salient feature of this type of lending is that it is often implemented through committed lines of credit, they argued that one should take a closer look to the evolution of the unused balances of loan commitments[1] . This suggestion embeds a simple logic: a credit crunch implies a decline of newly extended loans but not necessarily of committed ones. In particular, in the presence of committed loans, a lag in the fall of loans is consistent with a credit crunch as long as the unused balances decline simultaneously[2] .

In principle, a plain examination of aggregate bank lending to firms does not help to give a verdict. This is not only due to its aggregate nature, but also to the well known fact that movements in the interest rate can be misleading due to the presence of a flight to quality effect in lending practices. That is, interest rates can decline during a credit crunch just because the share of credit flowing to borrowers with high agency costs decreases.

We start by documenting that bank lending to firms is mainly implemented through loan commitments or revolving credit lines. We also show that in the last two recessions, the evolution of the aggregate used and unused balances of commercial and industrial loans are qualitatively different. Finally, we document that according the Senior Loan Officers, reluctance to lend and weak incentives to borrow seem to coincide during bad times, making hard to infer whether one causes the other or if they are simultaneously sparkled by another single phenomenon.

We then investigate if the behavior of aggregate balances can be informative about the source of the perturbation hitting the economy. To this end we develop a model in which loan commitment contracts play an essential role in production which is to overcome revenue shocks. We allow agents to choose optimally the contracts that better fit their financial needs and analyze the transition of an economy following an exogenous deterioration of terms of lending (financial shock) and a exogenous deterioration of business prospects (real shock). We then contrast the predictions with the data. We see this approach as useful because it provides a framework to interpret aggregate trends. Of course, the virtues of disaggregated data are unquestionable, but the correct interpretation of the aggregates grants policymakers the ability to make opportune interventions when needed. This paper contributes to this goal.

In terms of modeling choice, our model borrows from the analytical framework used in much quantitative macroeconomics, e.g. agents facing idiosyncratic shock within an

[1] Throughout the paper, the terms Loan commitments and Credit lines will be used indistinctly. The main features of this type of contracts are detailed in section 2.

[2] For a discussion on both sides of this debate see [31]and [32]

incomplete market structure. We add two important ingredients into this environment. First, we do no restrict borrowing to one period debt contracts, which gives agents more flexibility to manage their debt balances. Second, we allow borrowing limits to be determined in equilibrium. In this way, we stand against the idea that these limits are set exogenously (or even unilaterally) and examine how these arrangements are shaped by the optimal choices of economic agents.

In the real world, the importance of bank lending relative to other sources of funds is a debatable issue. Conventional wisdom suggests that bank loans are important, at least for small and medium sized firms which have a strong dependence for trade credit (see papers on credit chains) which in practice impose the necessity of transforming maturities since in general revenues do not arrive when expenditures are due. As for large firms, the broader access to external funds seems to imply that bank lending can be easily substituted. We believe that some of the properties of bank lending made to firms are essential for their operation. For instance, just as an individual makes use of their credit card for day to day purchases instead of liquidating money market funds, it is plausible to imagine that firms prefer to draw funds from loan commitments rather than issue equity or bonds.

We do not take a stand on what constraints indebtedness. We allow firms to optimally choose their limits recognizing that this will imply to keep them fixed for a while. In the real world, many contracts have fixed duration which could push the firm to try more expensive sources of financing when they are pushed against their debt limits and they cannot change it.

Related Literature The interest on loan commitments starts as early as the 80's. The theoretical literature focused on why commitments exist. The view is that it is adequate to provide contingency which has been studied by the finance literature (see [33], or [34]). Others see them as a solution to a moral hazard problem (see [35], [36]) or adverse selection (see [37] or[38]). Other stream of the theoretical literature has focused on the credit risk exposure through balance sheet effects that loan commitments might have in banks (see [39]).

In the economic literature, [16] model of financial intermediation sees the existence of lines of credit as a private solution to the problem of avoiding inefficient discontinuation of projects in the presence of lack of commitment.Through lines of credit, the intermediary can redistribute excess liquidity from lucky firms to unlucky ones, something that the market would not do due to ex post differences in liquidity needs. Intermediaries play the essential role of acting as liquidity pools that avoid the waste of liquid funds.

Finally, [40] studies the link between loan commitments and capital structure and argues that rather than being a substitute for, loan commitments can actually complement with other sources of funds and lower the cost of borrowing and therefore affects the debt structure of firms.

We start our analysis by documenting the evolution of bank lending to firms during recessions. In Section 3 we setup the model. In Section 4 we discuss the properties of the optimal decision rules and perform steady state comparisons Section 5 is devoted to the analysis of transitional dynamics and the effects of aggregate uncertainty. We discuss our results and the alleys for future research in Section 6.

3.2 Data Description

The purpose of this section is twofold. First, we want to stress the importance of loan commitment contracts in bank lending to firms in the United States. Second, we want to document the evolution of both the used and unused balances of these loans during the recent financial crisis. The particular behavior of these two variables is the main empirical observation that motivates this paper.

3.2.1 Features of Bank Lending to firms

A distinctive feature of bank lending to firms is that it is generally implemented through loan commitment contracts (also known as revolving credit facilities or lines of credit). These contracts grant firms access to funds for a predetermined period and up to a credit limit (the committed amount). In exchange for the option to access funds at will, firms are charged an interest rate on the funds borrowed and a commitment fee. In most cases, the interest rate is expressed as a spread over the bank's prime rate or the LIBOR rate and the fee is proportional to the committed amount.

Several papers have documented that loan commitments are widely used by firms in the U.S. For instance, [41] reports that about 75% of public firms have a line of credit and that funds borrowed from these lines represent more than a quarter of their outstanding debt[3] . In addition, [42] report that about the same fraction of private firms possesses a line of credit[4] . Furthermore, they find that 92% of large firms (those with total sales above $1 billion) have a credit line and 75% of small firms do. Conditional on having borrowed from those lines, there is a higher fraction of small firms that have positive debt balance in their credit line and small firms draw down more funds, as a fraction of the committed amount they have available. Overall, the evidence suggests that lines of credit are an important instrument of bank lending to firms in the US and, despite the fact that small firms may rely on them more than large firms, the use of these instruments is widely spread among all firms.

At the aggregate level, the Survey of Terms Business Lending (henceforth STBL) released by the Federal Reserve Board, reveals that a large share of Commercial and Industrial Loans made by all commercial banks is actually made under commitment. Using the statistical releases since 1997, in Figure 3.1 we plot the share of total loans that are actually made under commitment, classified by loan size. On average (blue line in the figure), this share is around 75% of all C&I Loans and it reaches almost 90% for loans of medium size. Figure 3.2 verifies that there is a great deal of comovement between the interest rate charged in these contracts and the policy interest rate, which is likely to reflect the indexed nature of this rate. This points out that the pricing mechanism might be of little help in determining what drives the evolution of quantities. One needs to exploit other sources of data.

[3] While standard firm level databases do not include information on credit lines, [41] combines data on Compustat from 1996 to 2003 and information from the annual 10-K SEC filings to derive summary statistics on lines of credit usage in the US. For a detailed description of the data, see Section 1 of [41].

[4] [42] use data from a survey realized to 794 CFOs from 31 countries in North America, Europe and Asia. The statistics reported above correspond only to the U.S. sample.

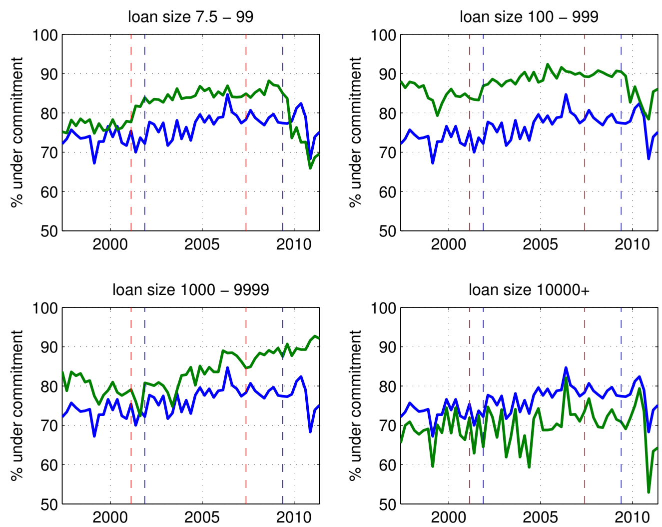

Figure 3.1: Percentage made under Commitment by Loan Size (STBL)

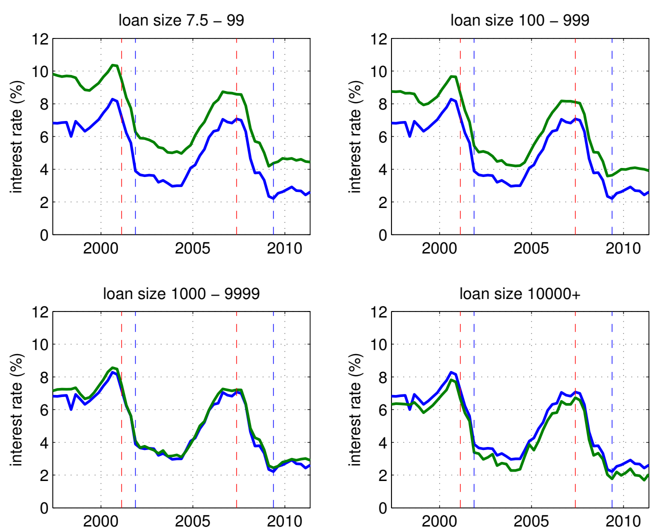

Figure 3.2: Weighted Average Loan Rate by Loan Size (STBL)

Finally, in Figure 3.3 one can see that although recessions are times in which the terms of these contracts are reviewed more frequently, the average durations is quite large, averaging ten months in bad times.

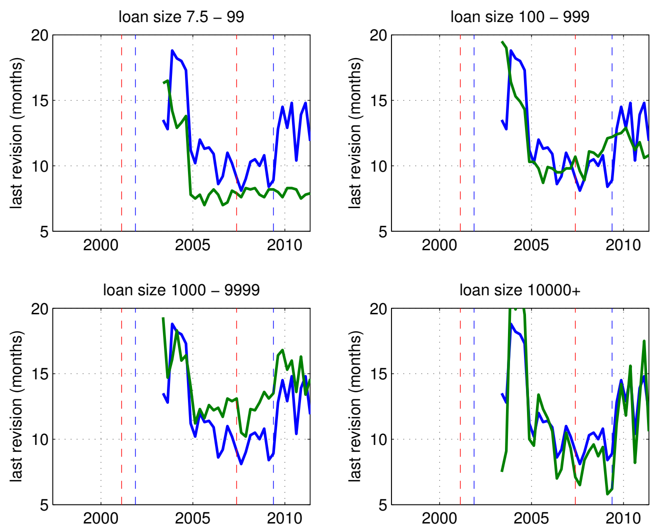

Figure 3.3: Moths since Last Revision by Loan Size (STBL)

3.2.2 Bank Lending During Recessions

Lending activity is known to be highly pro cyclical. Since interest rates are misleading and quantities alone do not allow to give a accurate diagnosis regarding the drivers of lending activity, one would think that banks should know better. Figure 3.4 organizes data from the Senior Loan Officer Opinion Survey (SLOOS), also released by the Federal Reserve Board. It is apparent that in the last two recessions there has been a drastic tightening of lending terms and standards. However it is also true that demand became weaker in periods of low economic activity. Two stories fit this pattern of responses[5] . First, the response of standards and the terms of lending is a consequence of the new pool of borrowers banks face due to a weaker demand. Second, the weak demand is a consequence of tighter lending conditions. The survey does not help to elucidate which

[5] Net percentage is calculated as the weighted average of respondants.

of these two stories is more adequate.

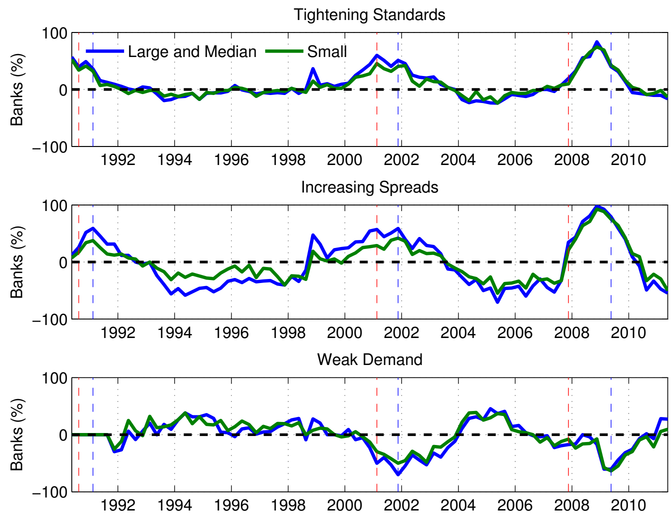

Figure 3.4: Evolution of Lending Practices (SLOOS)

We now turn to data from the Report of Condition and Income (Call Report) reported in Figures 3.5. The first column displays al the period for which data is available whereas the other two zoom in the last two recessions. In the top, we plot C&I Loans and Unused Commitments of C&I Loans divided by GDP. In the bottom we plot the Implicit Return (Interest Income on C&I Loans over the stock of C&I Loans) and the realized charge-off rates on these loans[6] [7] .

One way to describe the data on the top of the figure is by focusing on the size of the gap between them. In the last recession, one observes an immediate widening of this gap which lasts for several quarters. Towards the end of the recession, this gap starts

[6] Before the first quarter of 2010 banks were not required to report separately the unused part of commercial and industrial loans. The data we use corresponds to the item "Other unused commitments" which includes the kind of securities offered to firms (e.g. overdraft facilities, commercial lines of credit, retail check credit and related plans) and excludes those offered to households (e.g. home equity lines of credit). According to the new information released, about 45% of these "Other unused commitments" correspond to C&I loans so we plot 45% of the original series as a proxy to measure the unused balances of C&I loans. In addition, in line with the information from the STBL mentioned above, we consider only 65% of the total outstanding balance of C&I loans as a proxy of the used balances of credit lines.

[7] The dashed red line indicates the start of the recession and the dashed blue line the end of it.

to narrow. This evolution contrasts with that corresponding to the previous crisis, in which the gap starts to narrow simultaneously with the arrival of the recession[8] . In addition, it is interesting to note that the increment in delinquency rates for this type of loans kicks in several quarters after the recession is called. The evolution of the implicit return seems to indicate however that this delay is illusory, a mere artifact of accounting procedures. The implicit return decreases as loans perform badly, and then they become non-accrual. This would fit a credit crunch logic. But again, since most of these contracts are strongly linked to policy interest rates, it is not possible to know for sure which is the correct interpretation of these series.

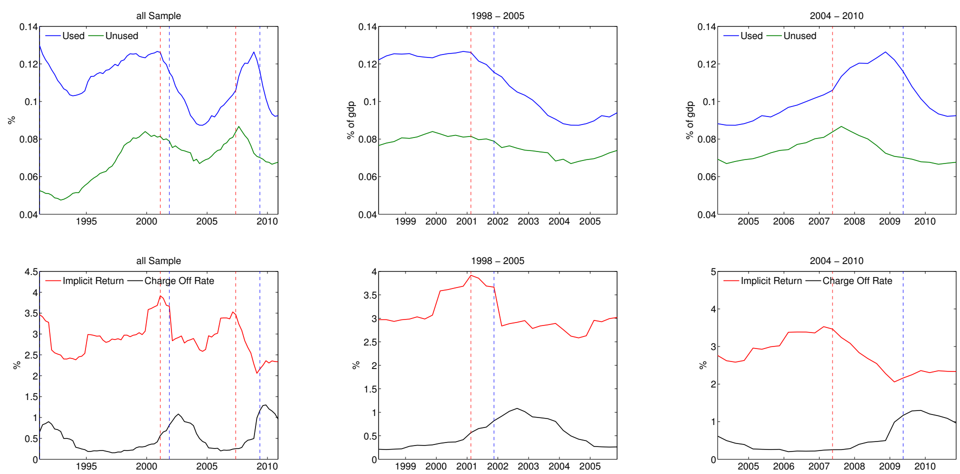

Figure 3.5: Commercial and Industrial Loans(Call Reports)

To sum up, one could explain the evolution of these series in the last recession using a credit crunch logic. Except for an initial period in which firms withdrew from their committed lines of credit, loans decreased perhaps because banks made them more expensive. At the same time, unused balances kept falling because credit limits were drastically reduced. However, an alternative explanation as likely to fit this pattern involves a deterioration of firms' prospects. The lack of good projects not only induces a reduction in indebtedness but also makes larger limits less attractive. As long as these limits are determined based on the value firms assign to them, data could display

[8] Although the data about the unused balances only goes as back as 1991Q1, it is interesting to notice that this narrowing seems to have occurred in that recession as well.

a decrease in both the used and the unused balances of loans as a result of a decrease in this value. In our opinion, the idea of borrowing limits exogenously given or unilaterally set by banks favors the credit crunch hypothesis and precludes other candidate explanations. We argue instead that the evolution of used and unused balances are equilibrium outcomes which can be better understood by considering the optimal choices of economic agents. In order to address this issue, in the next section we develop a model of loan commitments which we use later to study the behavior of the aggregate statistics when the economy is hit by aggregate shocks.

3.3 A Model of Loan Commitments

We consider the problem of a firm that faces uncertain revenues and finances the purchases of its inputs through a line of credit. We start with an example of how the possibility of changing limits affects available funds to the firm. We then describe the model recursively. When we populate the environment with a continuum of firms, the model admits a stationary distribution which can be used to calculate aggregate statistics. Steady state comparisons are reserved for the next section.

3.3.1 An Example

Consider a firm that operates a decreasing returns to scale technology using a single input factor h, which we call labor. Their revenues per period are given by the following function

$$R(z,h) = zAh^{\alpha} - wh \tag{3.1}$$

where $w > 0$ is the wage per unit of labor. Revenues are uncertain to theextent that z is a random variable that realizes in every period after the labor decision has been made. Firms rely on their ability to borrow funds up to a limit L in order to pay for the wage bill. Once revenues are realized, cash is used to pay debt and dividends.

Suppose the borrowing limits were not fixed but vary over time. Let $\{L_t\}$ be an arbitrary sequence of credit limits defining the constraint on the outstanding level of debt that the firm can carry over across periods. In this environment, a firm maximizing the expected discounted value of dividends would solve

$$\begin{aligned}\Omega(\{L_t\}) = \max_{\{h_t d_t, b_{t+1}\}} \quad & \mathbb{E}_0\left[\left\{\sum \delta^t g(d_t)\right\}\right] \\ s.t. \quad & d_t + (1+r)\, b_t \leq R(z_t, h_t) + b_{t+1} \\ & 0 \leq wh_t \leq L_t - b_t \\ & 0 \leq b_{t+1} \leq L_{t+1}\end{aligned}$$

Notice we allow dividends to be valued according to the function g, which in principle can be linear. We provide a specific functional form later in the paper. As in [24], let $a_t = L_t - b_t$ denote the unused balances in period t. The constraint set can be rewritten as follows

$$\begin{aligned}& d_t + a_{t+1} \leq R(z_t, h_t) + (1+r)\, a_t - rL_t + (L_{t+1} - L_t) \\ & 0 \leq wh_t \leq a_t \\ & 0 \leq a_{t+1} \leq L_{t+1}\end{aligned}$$

In this modified constraint set, a firm manages debt by choosing directly the unused balances in its line of credit. Observe that when dividends are paid, cash in hand includes the expression $(L_{t+1} - L_t)$. Not surprisingly, limits varying across time affects the resources available to the firm and as a consequence, its dividends payout decision. In the next section we lay down a model in which firms directly choose these limits, for which they have to pay a proportional fee every time they do it.

3.3.2 The Model

Time is discrete and the time horizon is infinite. There is a unit measure of firms which hire labor h as an input to obtain revenues $R(z, h)$, where $z \in Z \subset \mathbb{R}$ is an idiosyncratic revenue shock. For simplicity we assume these shocks are i.i.d. over time[9] . Since labor must be paid in advance, firms rely on loan commitment contracts to finance their production. A contract θ is a triple consisting of a credit limit $L(\theta)$, an interest rate $r(\theta)$, and a commitment fee $q(\theta)$. These contracts are not perpetual, which implies that the borrowing capacity of firms varies over time. We model this feature by introducing

[9] The analysis in sections 4 and 5 considers Markov shocks so that z is added as an individual state variable.

a random variable $\lambda \in \{0,1\}$ with $\Pr[\lambda = 1] = \gamma$, which indicates whether or not the firm is allowed to switch to a new contract θ' within a period[10] . In this version of the model, we consider the case in which r and q are taken as given, so that instead of choosing a contract, the firm will choose directly the credit limit.

Timing A firm enters the period with an unused balance a and a credit limit L. Labor services are decided and paid for at the beginning of the period. Subsequently, all uncertainty is revealed and firms learn their idiosyncratic shocks (z, λ). Since bad shocks translate into low revenues, contracts with larger limits are preferred insofar they allow to overcome more contingencies. The timing of events within a period is summarized in Figure 2.

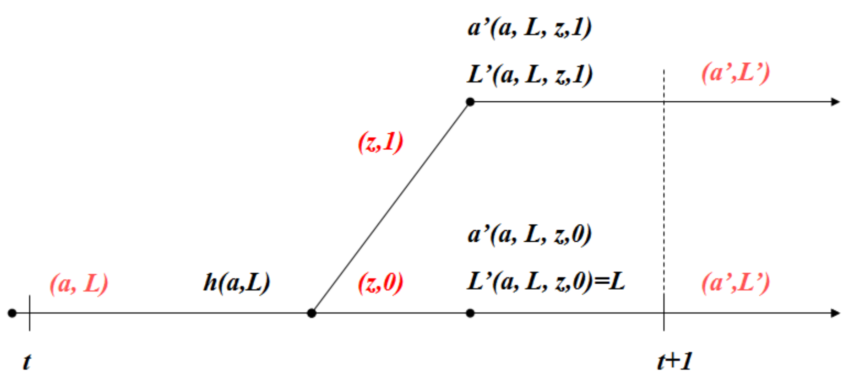

Figure 3.6: Timing within a period

Recursive formulation A *production plan* is a collection of functions h, $d(z,\lambda)$, $a'(z,\lambda)$, and $L'(z,\lambda)$ defined over $\mathbb{R}_+ \times \mathcal{L}$[11] . The first three functions map into the real line, whereas the last maps into $\mathcal{L}$. Firm's choices must satisfy four conditions. First, labor services can only paid by drawing funds from the unused part of their credit

[10] This modeling choice allows to consider different scenarios for the average duration of a contract. More explicitly, in the model there is a one to one mapping between the probability of switching contracts $(\lambda = 1)$ and the average duration of a contract.

[11] The set $\mathcal{L}$ over which the choice of the credit limit takes place is bounded above by $\overline{L}$. The choice of $\overline{L}$ affects the aggregate ratio of used balances to credit limit, which can be interpreted as a leverage ratio.

line:

$$0 \leq wh \leq a \tag{3.2}$$

When firms are not allowed to switch contracts, $L'(z,0) = L$ and the only remaining decisions are $d(z,0)$ and $a'(z,0)$. These must satisfy the following constraints

$$\begin{aligned} 0 &\leq a'(z,0) \leq L && (3.3) \\ d(z,0) + a'(z,0) &\leq R(z,h) + (1+r)a - rL && (3.4) \end{aligned}$$

The first constraint states that the unused balances cannot exceed the outstanding credit limit. The second is just a budget constraint. When switchingcontracts is allowed, $d(z,1)$, $a'(z,1)$ and $L'(z,1)$ are chosen so that they satisfy

$$\begin{aligned} 0 &\leq a'(z,1) \leq L && (3.5) \\ d(z,1) + a'(z,1) &\leq R(z,h) + (1+r)a - rL - (1-q)L'(z,1) - L && (3.6) \end{aligned}$$

Observe that the budget constraint incorporates the payment of the commitment fee for the newly chosen limit. Let $\Omega(a,L)$ be the value of a firm with unused balances a and credit limit L. This value must solve the following functional equation

$$\Omega(a,L) = \max \mathbb{E}\left[g(d(z,\lambda)) + \delta\Omega\left(a'(z,\lambda), L'(z,\lambda)\right)\right] \tag{3.7}$$

where the maximization on the right hand side is made over *production plans* and subject to (3.2) - (3.6). Observe that we do not assume limited liability, which implies unused balances can be replenished by running negative dividends. This introduces in a simple way the possibility of relaxing (3.2). We consider the following functional form for g

$$g(d) = \begin{cases} d & if \quad d \geq \overline{d} \\ d - \chi\left(\overline{d} - d\right)^2 & if \quad d < \overline{d} \end{cases}$$

where we allow $\chi > 0$ to consider the case in which paying out low dividends is costly for the firm. We can interpret this modeling choice in several ways. It could be that firms replenish unused balances by using other sources of funds which are costly when used in a relatively smaller scale. Alternatively, one might think that firms anticipating liquidity shortages respond by hoarding cash, which is costly since those funds could have been used in other profitable investment projects.

3.4 Steady State Analysis

In this version of our model, r and q are taken as parameters. We are still considering many ways of refining how these price schedules could be determined. It seems that the most obvious way of doing it is by introducing a competitive banking sector. For instance, since the model precludes the possibility of default, we could assume that the interest rate carries no premium, so that r equals the cost of external funds to the banks. It is less clear what the commitment fee should compensate the banks for. We have been working in a version in which q is set so that it compensates for the opportunity cost of the funds that the bank receives back by the end of the period, which can only be stored using an inferior technology with return $\mu < r$. By restricting attention to linear schedules, q could be summarized by a scalar that must satisfy a zero profit condition in terms of aggregate statistics. This extension could potentially enrich the dynamics of the model. We aim to explore it in future versions of this project.

Nevertheless, our environment still admits a stationary distribution over individual states. We consider a Markov structure for the productivity shock z. The stationary distribution is a fixed point of the operator T that maps the space of probability measures defined over the space $\mathcal{S} \equiv (\mathbb{R}_+ \times \mathcal{L} \times Z)$ into itself, which is defined using the optimal decisions of the firm. We denote such a distribution by Ψ^*. Our steady state analysis proceeds as follows. We first discuss the properties of the optimal policies, which have been solved for numerically. We then use the distribution Ψ^* to define the aggregate statistics we are interested in and perform comparative statics. The details of the parametrization of the model can be found in the appendix . We delay the study of aggregate uncertainty for the next section.

3.4.1 Optimal Policies

In this section we report and discuss the optimal policies that solve the right hand side of (3.7), which are depicted in the four panels of Figure 3. The optimal choices of the credit limits and unused balances are of particular interest. Since firms are heterogeneous, they will optimally choose different limits when they are allowed to do so.

The optimal choice of labor is depicted in Panel (A) of Figure 3. We can observe that there are three regions of interest. When the unused balances are low, firms will hire as much labor as possible so that the constraint (3.2) is effectively binding. On the other hand, when the line of credit remains highly unused, the labor choice is optimal in the sense that it maximizes expected revenues $\mathbb{E}\left[R\left(z,h\right) \mid z_{-}\right]$. There is a middle range of values for the unused balances for which the labor choice is interior. This is a consequence of the presence of borrowing limits and the choice of the functional form of g. As we motivated above, low revenues might require the firm to adjust its scale of production in order to avoid decreasing dividends. Finally, for a given credit limit, the optimal choice of h is increasing in the unused balances a.

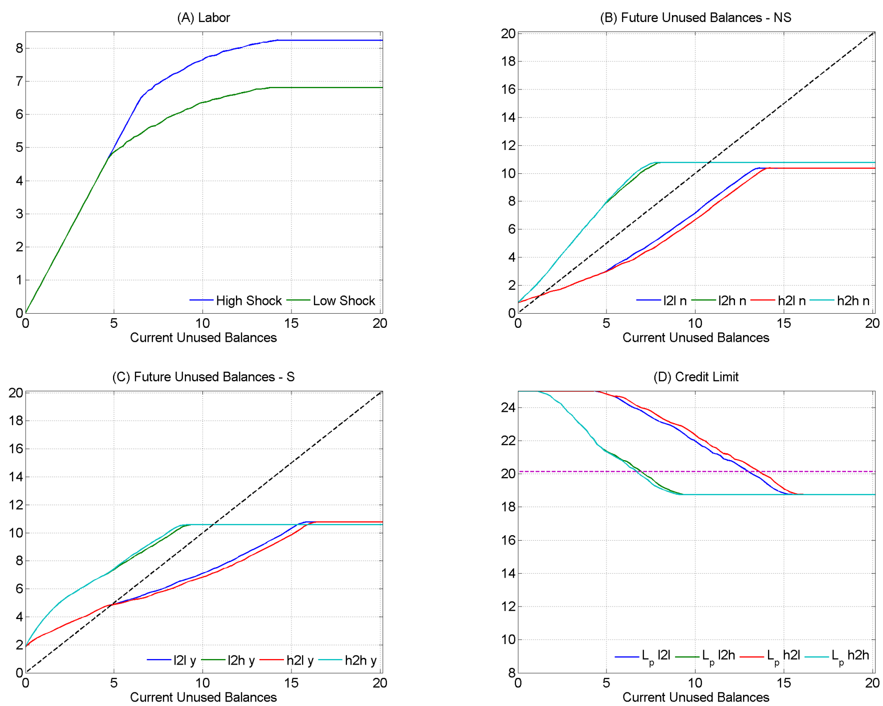

Figure 3.7: Optimal Policies

Panel (B) and (C) display the optimal choices of future unused balances. These balances are perceived as an asset to the firm insofar they are essential to undertake production. As a consequence, the optimal decision rules are standard and resemble those that arise in a standard income fluctuation problem with incomplete markets. In the context of our model, firms build up their unused balances in good times and deplete them in bad times, irrespective of whether or not they are allowed to switch contracts.

Finally, Panel (D) displays the optimal choice of the credit limit when firms are allowed to switch contracts. As we mentioned earlier, larger limits are valuable as they allow firms to overcome more contingencies. At the same time, firms understand that bad luck cannot last forever and hence, the larger the limit, the lower the marginal value of increasing it. Since firms are charged a proportional fee every time they choose a new contract, they will not choose arbitrarily large limits.

3.4.2 Comparative Statics

It is straightforward to calculate aggregate statistics in the steady state version of our model. We are ultimately interested in how these aggregates respond to aggregate shocks. The aggregate level of credit limits, used and unused balances are calculated using the stationary distribution Ψ^* as follows:

$$\mathbb{L} = \int L\Psi^* (da, dL, dz) \tag{3.8a}$$

$$\mathbb{B} = \int (L - a)\, \Psi^* (da, dL, dz) \tag{3.8b}$$

$$\mathbb{UB} = \int a\Psi^* (da, dL, dz) \tag{3.8c}$$

In order to get some insight about what the model can deliver we perform the following exercise. We calculate the steady state in four different scenarios. The first two differ only on the interest rate charged over the used balances. By studying how the aggregates vary when we move from a low to a high interest rate, we gain some insight about the consequences of an exogenous increase in the cost of funds. We associate this exercise with the effects of a *financial shock* since the perturbation is external to the firm[12] . The remaining two scenarios differ only on the level of aggregate productivity as given by the value of A in (3.1). In this case, by comparing the aggregates, we associate the results with the effects of a *real shock*, since the perturbation is technological and internal to the firm.

[12] In a version of this model with a competitive banking sector, a financial shock would represent an increase in banks' cost of raising external funds, which translates into the real sector through the interest rate.

We summarize the results of this exercise in Table 1. As it can be seen, steady state responses differ both in magnitude and in the direction depending on the source of the bad news. When these news come from a financial shock, the aggregate credit limit and the outstanding debt decrease, whereas the aggregate unused balance goes up. It should be stressed that the movements in the aggregates are not the consequence of an accounting identity. That is, it is not necessarily true that when the used balances decrease, the unused balances must increase. In fact, as the last two columns of Table 1 show, when the bad news come from a real shock, both the used and the unused balances fall. This is so because the aggregate credit limit falls more than the used balances.

Table 3.1: Steady State Effects of Bad News

	Interest Rate r		Productivity A	
1. Aggregates	Low	High	High	Low
Agg Credit Limit (L)	22.27	19.43	20.91	19.93
Used Balances (B)	12.90	9.59	11.41	11.24
Unused Balances (UB)	9.37	9.84	9.50	8.69
2. Ratios				
B/L	.58	.49	.55	.56
L/Y	1.87	1.60	1.83	2.16

To understand these results, notice that a higher interest rate makes debt more expensive so firms reduce the used balances of their credit lines. This reduction makes less likely that the firm will hit the credit limit and as a result larger credit limits become less valuable. On the other hand, when aggregate productivity goes down firms find optimal to operate at a smaller scale. As a consequence, the shadow value of larger limits and unused balances go down.

The results so far indicate that the origin of perturbations have qualitatively different implications regarding the evolution of the aggregates. To complete the analysis, we shall evaluate how these aggregates evolve over time when these shocks hit the economy.

3.5 Transitional Dynamics

In this section, we further study the effects of aggregate shocks by computing the transitional dynamics of the economy when moving from one steady state to another. To ease the exposition, we refer to the scenarios 1 and 3 computed in the previous section as the good states and to scenarios 2 and 4 as the bad states. Let $s = \{g, b\}$ denote the aggregate state. We focus on the transition from the good state to the bad state. We perform two sets of experiments which differ on how firms form expectations about future aggregate shocks. The details are explained below.

Experiment 1. In this experiment, we assume firms ignore that the economy can be hit by aggregate shocks. In particular, firms take as given the current aggregate productivity level and the interest rate they pay on their loans and make their decisions as if these values were never going to change. The steady states of these deterministic economies are the ones computed in Table 1. When firms are surprised by a switch of the aggregate state, they instantaneously adjust their optimal decisions to the new scenario. The economy goes through a transition where firms ' used and unused balances and credit limits gradually adjust until they are consistent with the bad steady state balances. We compute the aggregate levels of used and unused loans and the credit limits as the distribution of firms transits from the good steady state stationary distribution to bad steady state one. Specifically, suppose the economy reaches the new steady state in T periods and let Ψ_s^* be the stationary distribution for $s = \{g, b\}$. We calculate the sequence $\left\{\Psi^{g*}, \Psi_1^b, \Psi_2^b, ..., \Psi_{T-1}^b, \Psi^{b*}\right\}$ with the transition matrix implied by the optimal behavior of firms in the bad state and use it to compute the corresponding aggregates as defined in (3.8).

Figure 4 shows the evolution of credit limits, used and unused balances during the transition. With respect to average levels, the real shock corresponds to a decrease from $+3.3\%$ to of -3.3% in the aggregate productivity while the interest rate remains at the average level of 5%. Analogously, the financial shock corresponds to a decrease from $+1\%$ to -1% in the interest rate while the aggregate productivity remains at the average value of 3. The values are normalized to one at the moment of the state switch. The picture shows that both financial and real shocks can affect the economy. In particular, a higher interest rate makes debt more expensive so firms reduce the used balances of their credit lines. This reduction implies that it is less likely that the firm will hit the credit limit. Hence, larger limits become less valuable and firms gradually adjust their limits to a lower level as well. On the other hand, a real shock reduces the holdings of unused balances and credit limits. Although the effects of the financial shock are larger, the model in general delivers little action in terms of its responses to shocks.

One possible reason for this lack of response is the fact that we have assumed that prices are set exogenously and, in particular, that they do not depend on the aggregate shocks. Another reason is that, in this experiment, we have assumed that firms do not take into account the aggregate shock when forming their expectations. As a consequence there is little room for the aggregate shocks to affect the economy.

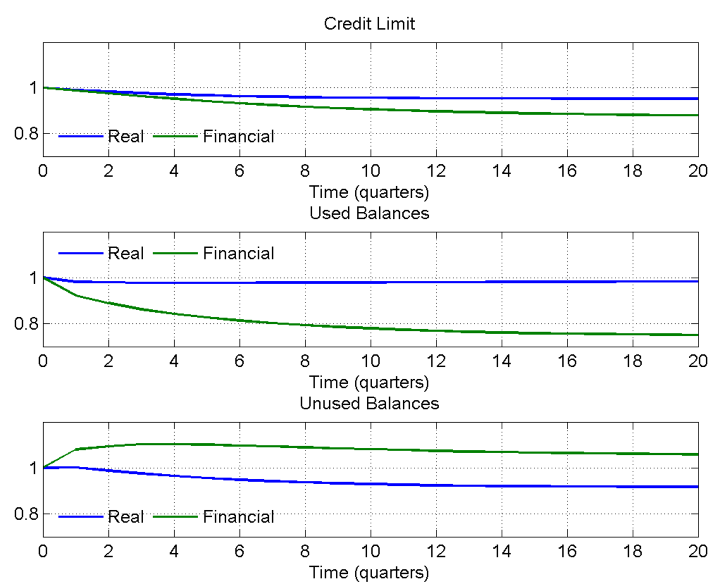

Figure 3.8: Transitional Dynamics after Bad News

Experiment 2. The second experiment introduces aggregate uncertainty. In this economy, firms make decisions knowing that they live in a stochastic world where the aggregate state of the economy changes with positive probability. Since we assumed that prices are exogenous in our model, the introduction of aggregate uncertainty is relatively simple and only costly in terms of the computational burden. In particular, we need to extend the model to allow firms to condition their optimal decisions on the aggregate state of the economy. We assume all uncertainty occurs simultaneously. Consequently, the decision rules are calculated in the same way as before, but a cross sectional stationary distribution no longer exists. The computation of the aggregates is analogous to the previous experiment with the caveat that, in this case, the optimal decisions and hence, the transition matrix we use to update the distribution, take into account the stochastic nature of the economy.

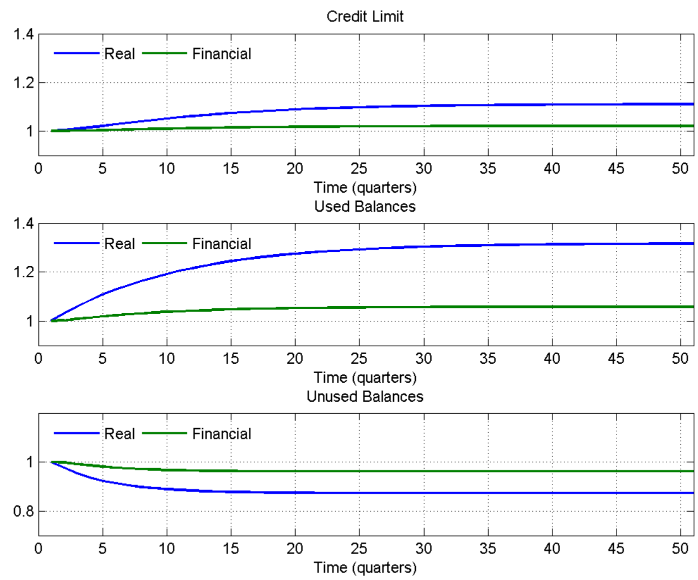

Figure 3.9: Transitional Dynamics after Bad News with Aggregate Uncertainty

We analyze the transitional dynamics of the economy when it switches to the bad state after a long period of stability in the good state[13] . We chose the Markov transition matrix for the aggregate state so that the average duration of and $r(b) = 5.5\%$. It is important to highlight the fact that the aggregate financial shock does not affect the

[13] The experiment consists on simulating a 50 year realization of the good state after which a permanent switch to the bad state takes place. The approach of the exercise is close to that used by [43] to study the real effects of different monetary policies.

transition matrix of the idiosyncratic productivity process. We modeled the real shock as an increase of 3.3% in aggregate productivity during good times and a decrease of 3.3% in bad times with respect to the average level. In both cases, the value of the other parameter (A in the first case, r in the second) is set to its expected value with respect to the invariant distribution of the proposed transition matrix.

Figure 5 shows the transitional dynamics after the switch in the aggregate state. Once we allow firms to internalize the fact that they are living in a stochastic world, the effects of real and financial shocks change substantially. First of all, the aggregates move in the same direction regardless of the nature of the shock. Second, the magnitude of these movements are larger after the real shock. To understand this result, observe that our choice of the transition matrix for the aggregate shock implies that a bad realization is perceived as transitory by the firms. In the case of the financial shock, this explains the increase of the used balances. Firms do not adjust their debt levels since they expect the interest rate to go down in the short run. As for the real shock, since the fall in revenues is only transitory firms maintain their scale of production and let the unused balances decrease. By doing so, they avoid decreasing dividends and getting closer to the threshold $\overline{d}$.

We emphasize that the size and the direction of the effects of both real and financial shocks depend crucially on the parametrization of the model we have chosen. In order to provide a quantitative interpretation of the result, a cautious calibration must be done. The exercises presented in this paper are meant to provide a flavor of how we can use the model to understand the recent evolution of bank lending data.

3.6 Concluding Remarks

This paper proposes a model of loan commitments to elucidate whether the evolution of bank lending to firms during the last crisis is unquestionably associated with a credit crunch. We find that the credit crunch argument relies at least partially on the presumption that banks set credit limits unilaterally. When we drop this assumption, the type of movements that we observe in the data can be consistent with real shocks, i.e. changes in the internal conditions of the firm.

We think our model provides a simple framework in which the effect of aggregate shocks on bank lending can be analyzed. Yet, our analysis needs to be extended in several directions. First, prices are not endogenously determined in the model. This constraints the possibility of studying other interesting issues. For instance, one could be interested in analyzing the case in which interest rate adjustments are induced by changes in the condition of the borrowers, rather than being determined exogenously. Another direction in which the model can be extended is by deriving a supply of loanable funds by introducing a household sector. Overall, these extensions would allow us to perform a more careful calibration of the model. We are currently working on these issues.

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Appendix A

Supplemental Material to Credit Crises and Private Liquidity: The Role of Household Debt

A.1 Proofs

A.1.1 Proof of Proposition 1

From first order condition of problem 2.4, individual labor demand is

$$l(\omega_F, x) = (\frac{\alpha_2}{\mathbf{w}})^{\frac{1}{1-\alpha_2}}(e^x k(\omega_F, x))^{\frac{\alpha_1}{1-\alpha_2}}$$

Hence, individual output can be written as

$$y(\omega_F, x) = (\frac{\alpha_2}{\mathbf{w}})^{\frac{\alpha_2}{1-\alpha_2}}(e^x k(\omega_F, x))^{\frac{\alpha_1}{1-\alpha_2}}$$

Aggregate output corresponds to the integral with respect to the producers' distribution

$$\mathbf{Y} = (\frac{\alpha_2}{\mathbf{w}})^{\frac{\alpha_2}{1-\alpha_2}} \int (e^x k(\omega_F, x))^{\frac{\alpha_1}{1-\alpha_2}} d\psi_F(\omega_F, x)$$

Labor market clearing implies

$$\frac{\mathbf{w}}{\alpha_2} = \left[\int (e^x k(\omega_F, x))^{\frac{\alpha_1}{1-\alpha_2}} d\psi_F(\omega_F, x)\right]^{1-\alpha_2}$$

Plugging this into the expression for aggregate output yields

$$\mathbf{Y} = \left[\int (e^x k(\omega_F, x))^{\frac{\alpha_1}{1-\alpha_2}} d\psi_F(\omega_F, x)\right]^{1-\alpha_2}$$

Using the definition of *weighted capital stock*, we can write aggregate output as follows

$$\mathbf{Y} = \left[\int \left(\frac{e^x k(\omega_F, x)}{\int e^x k(\omega_F, x) d\psi_F(\omega_F, x)}\right)^{\frac{\alpha_1}{1-\alpha_2}} d\psi_F(\omega_F, x)\right]^{1-\alpha_2} \mathbf{K_e}^{\alpha_1}$$

or since ψ_F is a probability measure

$$\mathbf{Y} = \left[\frac{\mathbb{E}\left[(e^x k(\omega_F, x))^{\frac{\alpha_1}{1-\alpha_2}}\right]}{\mathbb{E}\left[e^x k(\omega_F, x)\right]^{\frac{\alpha_1}{1-\alpha_2}}}\right]^{1-\alpha_2} \mathbf{K_e}^{\alpha_1}$$

Now, since e^x and $k(\omega_x, x)$ are random variables, a second order Taylor expansion of the argument in the numerator delivers[1]

$$\mathbf{Y} \approx \left[1 - \frac{\alpha_1}{1-\alpha_2}\left(1 - \frac{\alpha_1}{1-\alpha_2}\right)\frac{\mathbf{cv}(e^x k(\omega_x, x))^2}{2}\right]^{1-\alpha_2} \mathbf{K_e}^{\alpha_1}$$

where $\mathbf{cv}(e^x k(\omega_x, x))$ is the coefficient of variation of the random variable $e^x k(\omega_x, x)$. Letting $\varphi = \frac{1}{2}\frac{\alpha_1}{1-\alpha_2}\left(1 - \frac{\alpha_1}{1-\alpha_2}\right)$ and using again the definition of *weighted capital stock* delivers the expression in the main text.

A.1.2 Proof of Proposition 2

From first order conditions of (2.10), we can obtain

$$\begin{aligned} u_c(c) &= \beta \mathbf{R}\left[\mathbb{E}[u_c(c')] + \mu_{CC}\right] \\ v_h(h') &= \mathbf{p}\left\{\beta\left[\mathbb{E}[u_c(c')] + \phi_H \mu_{CC}\right] - u_c(c)\right\} \end{aligned}$$

where μ_{CC} denotes the Lagrange multiplier of (2.8). These two can be combined to obtain

$$v_h(h') = \mathbf{p}\left\{(1-\phi_H)\beta\mathbb{E}[u_c(c')] - \frac{\mathbf{R} - \phi_H}{\mathbf{R}} u_c(c)\right\}$$

[1] A second order Taylor expansion of $\mathbf{E}[f(x)]$ delivers

$$\mathbf{E}[f(\mu_x + (x-\mu_x))] \approx \mathbf{E}[f(\mu_x) + f'(\mu_x)(x-\mu_x) + \frac{1}{2}f''(\mu_x)(x-\mu_x)^2] = f(\mu_x) + \frac{1}{2}f''(\mu_x)\sigma_x^2$$

where $\mu_x = \mathbb{E}[x]$ and $\sigma_x^2 = \mathbf{Var}[x]$

Integrating both sides with respect to the stationary household measure delivers

$$\int v_h = \mathbf{p}\left\{\frac{(1-\phi_H)\beta\mathbf{R}-(\mathbf{R}-\phi_H)}{\mathbf{R}}\right\}\int u_c$$

Finally, we perform again a second order Taylor expansion to both sides of this expression and use the housing market clearing condition (2.13) obtain

$$\mathbf{p} \approx \left\{\frac{\mathbf{R}}{(1-\beta\mathbf{R})\phi_H-(1-\beta)\mathbf{R}}\right\}\left\{\frac{2+\sigma(\sigma+1)\mathbf{cv}(c)^2}{2+\gamma\nu(\nu+1)\mathbf{cv}(h)^2}\right\}\mathbf{C}^{\sigma}$$

where the closed forms follows from assuming $U(c,h)$ is separable and of the power utility form (and as it is usual, $\sigma = 1$ or $\nu = 1$ imply log preferences). As it is standard in models with partial insurance, equilibrium implies $\beta\mathbf{R} < 1$ and hence the first order effect of an exogenous contraction of ϕ_H is a reduction in the prices of houses.

A.1.3 Proof of Proposition 3

First of all, from an individual perspective, first order conditions require the following condition to hold for each producer with state (ω_F, x)

$$\mathbb{E}\left[\frac{\partial J(\omega_F'(k,x,\epsilon'),x')}{\partial\omega_F'(k,x,\epsilon')}\frac{\partial\omega_F'(k,x,\epsilon')}{\partial k}\right] \geq \mathbb{E}\left[\frac{\partial J(\omega_F'(k,x,\epsilon'),x')}{\partial\omega_F'(k,x,\epsilon')}\right]\mathbf{R} \qquad \text{(A.1)}$$

with strict inequality when the collateral constraint (2.3) is binding. Consider the situation in which a producer is temporarily unconstrained. Equation $(A.1)$ can be written as follows:

$$\mathbb{E}\left[\frac{\partial\omega_F'}{\partial k}\right] = \mathbf{R} - \mathbf{cov}\left[\frac{\partial J\left(\omega_F',x'\right)}{\partial\omega_F'}, \frac{\partial\omega_F'\left(k,x,\epsilon'\right)}{\partial k}\right]\Big/\mathbb{E}\left[\frac{\partial J\left(\omega_F',x'\right)}{\partial\omega_F'}\right] \qquad \text{(A.2)}$$

Since $\dfrac{\partial\omega_F'(k,x,\epsilon')}{\partial k} = \left(\dfrac{e^{\alpha_1\epsilon}-\alpha_2}{1-\alpha_2}\right)\dfrac{\partial\overline{\Pi}\left(k,x\right)}{\partial k} + (1-\delta)$, the concavity of J implies that the covariance term must be negative as long as ϵ is a random variable. Furthermore, notice that it does not matter how the uncertainty of ϵ affects ω_F' since no matter the way it does, the first derivative of the value function would move in the opposite direction. This indicates that an expenditure shocks formulation of ϵ would deliver the same sign for the covariance term.

A.2 The problem of the firm with illiquid capital

The idea now is that capital the capital stock has a liquid and an illiquid component. Investment in the latter is irreversible. To be more precise, it is assumed that within each producer

$$k = \Phi\left(k_L, k_I\right) \tag{A.3}$$

and Φ is assumed to be a CES production function, e.g. $\Phi\left(k_L, k_I\right) = (ak_L^\sigma + (1-a)k_I^\sigma)^{\frac{1}{\sigma}}$. We will guess and later verify that the individual state for a producer consists of the triple (ω_F, k_I, x). The control variables are (d, k', k'_I, m'). The constraint set is

$$d + k' + m' \leq \omega_F \tag{A.4}$$

$$\mathbf{R}m' + \phi_F(1-\delta)k' \geq 0 \tag{A.5}$$

$$k' \geq (1-\delta)k_I \tag{A.6}$$

$$k'_I \geq (1-\delta)k_I \tag{A.7}$$

$$k'_L \leq k' - k'_I \tag{A.8}$$

and we should also consider the law of motion for ω'

$$\omega'_F(k'_L, k'_I, m', x, \epsilon') = \Pi(\Phi(k'_L, k'_I), x, \epsilon') + (1-\delta)k' + \mathbf{R}(m')m' \tag{A.9}$$

I reformulate this problem by allowing the producer to choose directly (σ, λ, ν) and denoting $\varphi = \dfrac{k_I}{\omega_F}$. The constraint set now becomes

$$0 \leq \sigma \leq 1 \tag{A.10}$$

$$0 \leq \lambda \leq \frac{1}{1 - \frac{\phi_F(1-\delta)}{\mathbf{R}}} \tag{A.11}$$

$$0 \leq \nu \leq 1 \tag{A.12}$$

$$\sigma\lambda \geq \varphi \tag{A.13}$$

$$\sigma\lambda\nu \geq \varphi \tag{A.14}$$

and the law of motions for the endogenous states

$$\omega'_F(\sigma, \lambda, \nu, x, \epsilon') = \Pi(\Phi((1-\mu)\sigma\lambda\omega_F, \mu\sigma\lambda\omega_F, x, \epsilon') + (1-\delta)\sigma\lambda\omega_F + \mathbf{R}(1-\lambda)\sigma\omega_F \tag{A.15}$$

$$\varphi'(\sigma, \lambda, \nu, x, \epsilon') = \frac{\sigma\lambda\nu\omega_F}{\omega'_F(\sigma, \lambda, \nu, x, \epsilon')} \tag{A.16}$$

In order to write down the maximization problem as a nested maximization problem, it is useful to write to constraint set as follows

$$\frac{\varphi}{1-\frac{\phi_F(1-\delta)}{\mathbf{R}}} \leq \sigma \leq 1 \tag{A.17}$$

$$\frac{\varphi}{\sigma} \leq \lambda \leq \frac{1}{1-\frac{\phi_F(1-\delta)}{\mathbf{R}}} \tag{A.18}$$

$$\frac{\varphi}{\sigma\lambda} \leq \nu \leq 1 \tag{A.19}$$

and the law of motions for the endogenous states remain the same.

My conjecture is that this model with illiquid capital is isomorphic to a mode with dividend and/or profit tax. This result would be remarkable since it allows me to deal with a much simpler producers' problem and introduce a tax on dividends that potentially helps to improve the quantitative performance of the model.

Appendix B

Supplemental Material to Understanding Credit Crunches Through a Model of Loan Commitments

B.1 Numerical Procedure

We include in this appendix the details of the numbers we have used to report the results included in main text. We classify the parameters we need to specify into two groups depending on whether they affect the real side or the financial side of the firm. We call them real and financial parameters. The model is not entirely quantitative since it is not clear which would be the data counterpart of some of the model features. Despite the fact that we have not undertaken a serious calibration, we have tried to choose plausible parameter values. Table 2 reports all the parameters and the values we have chosen. A discussion of our choices for each group follows. Since the financial parameters are less standard, we discuss them first.

Table B.1: Parameters for the Numerical Exercises

Real		**Financial**	
Discount Factor (δ)	.92	Upper Credit Limit $(\overline{L})$	25
Dividend Threshold ($\overline{d}$)	2	Lower Credit Limit	0
Labor Share (α)	.67	Prob os Switching Contract (γ)	.2
Wage (w)	1	Commitment Fee (q)	.06
Aggregate Productivity (A)	3	Interest Rate (r)	.045 - .055
Idiosyncratic Shocks (Z)	$\{.57; 1.18\}$		
Transision Matrix for z	$[.7, .3; .6, .4]$		

Financial Parameters. We specified the grid of credit limits available to the firm so that with the maximum credit limit, an unconstrained firm can finance 3 times its wage bill. The lower limit is set to zero even though, in steady state, firms do not demand credit limits lower than the unconstrained value of the wage bill, which is positive. The grid for the unused balances is endogenous since it is bounded above by the current credit limit. The probability of having the option to change the credit line contract is set so that the average duration of a credit line is 15 months. This number is consistent with the information reported in the STBL regarding the average number of months passed since the terms of new C&I loans were set. The fee was set to 6% of the credit limit and the interest rates vary from 4.5% to 5.5% depending on the state.

Real Parameters. The discount factor was chosen so that firms are impatient enough to be willing to borrow. In steady state, the parametrization implies a rate of utilization of the credit line $\left(\frac{B}{L}\right)$ of about 60%. The value for the labor share is standard and the wage was set to one. The idiosyncratic productivity shock was chosen so that, under the bad shock z_l, operating at the unconstrained level of labor delivers negative revenues to the firm. The choice of the productivity shock implies a variance of 8% and a mean of 1 while the aggregate level of productivity was set to 3.