BUILDING A THEORETICAL MODEL OF METACOGNITIVE PROCESSES IN COMPLEX MODELING ACTIVITIES: A WINDOW INTO THE DEVELOPMENT OF STUDENTS' METACOGNITIVE ABILITIES

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continue my education despite many hardships,
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Abstract

A theoretical model of metacognition in complex modeling activities has been developed based on existing frameworks, by synthesizing the re-conceptualization of metacognition at multiple levels by looking at the three sources that trigger metacognition. Using the theoretical model as a framework, this study was designed to explore how students' thinking becomes metacognitive while collaboratively solving a complex mathematical modeling task.

This study used a series of Model-Eliciting Activities (MEAs), which are a type of problem-solving activity in which participants are required to verbalize their thoughts while working within a group, as an authentic method for analyzing verbal metacognitive actions, addressing several criticisms of self-report methods. Multiple cycles of data analysis, including a finer-grained analysis of conversational statements and a cross-case analysis, were conducted.

Results from the data analysis provided empirical evidence supporting the soundness and appropriateness of the theoretical model of metacognition on multiple levels in identifying and interpreting students' metacognitive activities in complex mathematical modeling tasks. This study identified several patterns and tendencies of students' spontaneous metacognitive activities. This study provided empirical evidence supporting the potential similarity of students' developing metacognitive abilities to their developing cognitive abilities with respect to the dimensions of development. In addition, this study identified the circumstances facilitating or interfering with students' spontaneous metacognitive activities.

This study furthers our understanding about how one develops metacognitive abilities within problem-solving processes, and ultimately informs how to effectively encourage students' metacognition and improve their problem-solving achievement.

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Chapter 1

Introduction

In learning environments, some students rely upon teachers for constant support in learning, while others have active control over the processes of thinking engaged in learning. One way to characterize differences between students is to consider their metacognitive abilities. Possessing self-reflective thinking abilities, which were defined by Flavell (1976) using the term “metacognition,” enables students to be effective learners and to obtain higher achievement (e.g., Schraw & Dennison, 1994). Students with strong metacognitive skills are aware of what they need in order to learn effectively by efficiently organizing, monitoring and regulating what they know.

Metacognition is frequently defined as “thinking about one’s own thinking.” The need and rationale for studying metacognition have been widely advocated based on the view that metacognition is a major aspect of successful learning (e.g., Bransford, Brown, & Cocking, 2000; Campione, Brown, & Connell, 1989). In particular, success in mathematical problem solving and reasoning, which centers on current visions for the effective teaching and learning of mathematics (NCTM, 2000), has been linked to the critical role of metacognition (Goos, 1994; Muir, Beswick, & Williamson, 2008; Schoenfeld, 1987, 1992; Schraw, 1998).

One criticism of the research on metacognition is that the term is too poorly defined to be the object of scientific inquiry (e.g., Campione et al., 1989; Garofalo & Lester, 1985). However, several research studies have successfully described metacognitive behaviors, based on empirical evidence, during problem solving (Goos, 2002; Goos, Galbraith, & Renshaw, 2002; Iiskala, Vauras, Lehtinen, & Salonen, 2011; Magiera, 2008; Magiera & Zawojewski, 2011; Wilson & Clarke, 2002, 2004).

Still, most studies fail to go beyond the description of these metacognitive behaviors to directly link them with students' improved future outcomes. Lesh and Zawojewski (2007) argued that this limitation is due to the view of metacognition and cognition as hierarchically separate and the lack of the regard for the social, contextual and situated nature of metacognition. They suggested the need for the application of developmental perspectives to metacognition, similar to studies on children's conceptual development. Such an application can provide a window into the development of students' metacognitive abilities–a mechanism making it possible to organize, monitor, and regulate what one knows in order to successfully attain a goal (Lesh & Zawojewski, 2007; Schoenfeld, 1992). Thus, it can ultimately inform school practice. The current study addresses the need for application, with the expectation to inform school practice.

A Theoretical Model of Metacognition on Multiple Levels

In a pilot study, the researcher identified how one student's interactions with a given problem (environmental level) and with others (social level) became a catalyst for thinking metacognitively (Kim, Park, Moore, & Varma, 2013). The pilot study, which provided the foundation for the current study, led to the development of a theoretical model of metacognition in order to address several issues and challenges for research on metacognition, as briefly described above.

A theoretical model of metacognition in complex modeling activities has been developed based on the models and modeling perspectives (MMP) and Goos' (2002) framework for analyzing verbal protocols in a collaborative problem-solving setting, by drawing on a re-conceptualization of metacognition at multiple levels, which entails looking at the sources that trigger metacognition at the individual, social, and environmental levels (Kim et al., 2013).

First, the theoretical model of metacognition adopted the models and modeling perspectives (MMP) as a theoretical base in order to investigate metacognition by treating cognition and metacognition as integrated; i.e., they develop in parallel and interactively. The MMP also values the social, contextual and situated nature of metacognition (Lesh, Lester, & Hjalmarson, 2003; Lesh & Zawojewski, 2007). The adoption of the MMP on metacognition allows the application of a developmental perspective to metacognition, with the expectation to inform school practice. As a result, it allows researchers to conduct developmental investigation in order to explore what mechanisms facilitate students' metacognition, and how these mechanisms manifest within problem-solving processes and through experiences.

Second, the theoretical model focuses on the functions of metacognition itself, i.e. the monitoring function (involving the assessment of one's own thinking) and the regulatory function (based on the assessment processes), rather than its components; therefore, this model allows for agreement on the definition of metacognition (Efklides, 2006; Goos, 2002; Wilson & Clarke, 2002). In particular, Goos' (2002) framework for analyzing verbal protocols in a collaborative problem-solving setting, which adopts a linear progression viewpoint of metacognition, was adopted for the theoretical model with a rearrangement to represent the MMP on problem solving–a multiple dimensional progression viewpoint of metacognition during problem solving (Kim et al., 2013).

Finally, the researcher re-conceptualized metacognition at multiple levels by looking at three of the sources that trigger metacognition: (a) oneself (the individual level); (b) one's interactions with others (the social level); or (c) one's interactions with learning environments, such as textbooks and computers (the environmental level) (Kim et al., 2013). This theoretical model helps make clear the distinction of cognitive (i.e., "thinking with" cognitive components) and metacognitive (i.e., "thinking about" cognitive components) behaviors (Lesh et al., 2003).

As a result, several issues and challenges for research in metacognition are addressed by using the theoretical model of metacognition on multiple levels. Through a single case study, the researcher illustrated how the theoretical model was used to study students' metacognition (Kim et al., 2013). Multiple case studies involving a series of problems and several student groups are required to examine the appropriateness and soundness of the theoretical model for research in metacognition. Furthermore, the need to apply developmental perspectives to metacognition research (Lesh & Zawojewski, 2007), as mentioned above, and the need for in-depth investigation of how an individual's thinking becomes metacognitive in context by using robust theories (Schoenfeld, 1999) lead to the justification for the current study. By employing the theoretical model of metacognition on multiple levels as a framework, the current study conducted a developmental investigation to explore how students' metacognition develops in social contexts. This study intends to further our understanding about how one develops metacognitive abilities within collaborative problem-solving processes, and ultimately, to inform how to effectively encourage students' metacognition and improve their problem-solving achievement.

The Purpose of the Study

First, the current study is designed to examine the soundness and appropriateness of the theoretical model of metacognition on multiple levels in identifying and interpreting students' metacognitive activities. The appropriateness and soundness of the theoretical model will be examined with respect to how the model helps make clear the distinction between cognition and metacognition, and the distinction among metacognitive functions, and with respect to how it helps interpret students' metacognitive activities within and across problem-solving sessions.

Second, the current study is designed to explore how students' thinking becomes metacognitive while collaboratively solving a complex mathematical modeling task. In particular, based on students' metacognitive activities identified by using the theoretical model of metacognition on multiple levels as an analytical framework, this study (a) explores the patterns and tendencies of the students' metacognitive activities; (b) investigates the nature of students' metacognitive abilities; and (c) identifies the circumstances facilitating or interfering with students' spontaneous metacognitive activities.

Research Questions

The primary research questions and sub-questions that guide this study are:

1) How appropriate is the theoretical model for identifying and interpreting metacognitive activities in complex modeling activities?

2) How does students' thinking become metacognitive during complex modeling activities?
    a) How do metacognitive abilities manifest during complex modeling activities?
    b) What is the nature of students' metacognitive abilities manifested during complex modeling activities?
    c) What are the circumstances in which metacognitive activity is abandoned?

To address these questions, a multiple case study was employed to study students' metacognitive activities during problem solving. This study used a series of model-eliciting activities (MEAs)–a type of problem-solving activity in which students generate mathematical models to solve them–to generate qualitative data from multiple case studies of 23 students in seven groups of three to four seventh-grade students, collaboratively working on three different problem-solving sessions for the 2011-2012 school year. MEAs require participants to verbalize their thoughts while working within a group; thus, MEAs work as an authentic method for analyzing verbal metacognitive actions, addressing several criticisms of self-report methods (Ericsson & Simon, 1984). The primary sources of data for this study are audio transcripts of the student groups, solutions to the MEAs, and researcher field notes. Multiple cycles of data analysis, including a finer-grained analysis of conversational statements from each transcript of the problem-solving sessions and a cross-case analysis, were conducted.

Results from the multiple case study illustrate how the theoretical model helps appropriately distinguish metacognition from cognition and make the distinction among metacognitive functions, and how it helps interpret students' metacognitive activities. Findings from the multiple cycles of data analysis also allowed me to explore the patterns and tendencies of the students' metacognitive activities within and across the three problem-solving sessions, potential dimensions along which students' metacognitive abilities develop, and critical events that facilitate or interfere with students' metacognition.

Organization of the Study

This study consists of five chapters. Chapter I is the introduction, as presented above. Chapter II reviews a subset of the expansive literature on metacognition, focusing on the issues and challenges for research on metacognition and pertaining to the theoretical framework for this study. Chapter III describes the research methodology and the usefulness of model-eliciting activities (MEAs) for this study. Chapter IV presents an analysis of the data and findings pertaining to the research questions. Chapter V presents a summary of the discussion and limitations, discusses the implications and recommendations for future research, and presents a conclusion with final remarks.

Chapter 2

Review of the Literature

For this chapter, a subset of the prolific literature on metacognition is reviewed, focusing on the issues and challenges for research on metacognition. A review of the literature on metacognition related to the theoretical framework for this study is also presented. The chapter concludes with how the current study has been influenced by previous research.

Research on Metacognition

In spite of advocating for the critical role of metacognition in successful learning and the need of ongoing research, metacognition has been considered a challenging topic to research due to a number of reasons, three of which are detailed here.

First, there is lack of an agreed-upon definition of metacognition (Garofalo & Lester, 1985; Wilson & Clarke, 2002). Metacognition is commonly defined as "thinking about one's own thinking." However, the definition is not instrumental enough to examine students' metacognitive activities when solving mathematical problems and when learning mathematics (Schoenfeld, 1987; Wilson & Clarke, 2002). Over the last few decades, various definitions of metacognition have emerged in the educational literature. They involve different views on the components of metacognition, how to categorize the components, and how to establish the relationship among them (e.g., Efklides, 2006; Flavell, 1976; Garofalo & Lester, 1985; Norman, 1981; Schoenfeld, 1987; Wilson & Clarke, 2002). For example, some researchers partition "beliefs" as a separate classification from "knowledge about one's own thinking" (Schoenfeld, 1987), while others regard beliefs as a kind of subjective knowledge (Efklides, 2006). These disparities have led to confusion with respect to the term "metacognition," and furthermore, there are many claims considering metacognition as an ill-defined object that is inappropriate for scientific inquiry (Campione et al., 1989; Garofalo & Lester, 1985).

Second, it is not always easy to distinguish metacognition from cognition (i.e., thinking about the concept at hand), even with an agreed-upon definition (Garofalo & Lester, 1985; Wilson & Clarke, 2002). There fails to exist a coherent model with respect to research on metacognition, effectively identifying and interpreting students' metacognitive activities. Third, there is also lack of an authentic methodological tool to provide a window into students' mental processes, in particular metacognition (Garofalo & Lester, 1985; Wilson & Clarke, 2002). Most empirical studies have drawn on self-report methods to investigate metacognition, despite several criticisms–for example, when student participants look at a video of themselves working, what they say about their own thinking is what they are currently thinking, rather than what they were thinking at the time (Ericsson & Simon, 1980; Goos & Galbraith, 1996).

Despite these reported difficulties related to studying metacognition, several research studies have successfully provided empirical evidence for metacognitive behaviors during problem solving (e.g., Goos, 2002; Goos et al., 2002; Iiskala et al., 2011; Magiera, 2008; Magiera & Zawojewski, 2011; Whitebread, Bingham, Grau, Pino Pasternak, & Sangster, 2007; Wilson & Clarke, 2002, 2004).

For example, Goos et al. (2002) investigated students' metacognitive activities while pairs of students solved mathematical problems. Based on their three-year study of senior secondary school students (ages 11–12), Goos and her colleagues found that students' critical engagement with each other's thinking led to productive metacognitive decisions for successful problem solving, while their lack of critical engagement led to poor metacognitive decisions, and in turn, unsuccessful problem solving. Furthermore, Goos (2002) developed the "red flags" model, including "lack of progress," "error detection," and "anomalous result"–a warning system to identify situations that lead to students' poor metacognitive decisions, and thus, metacognitive failures.

By studying eight high-achieving students (age 10) solving mathematical problems of different difficulty levels, Iiskala et al. (2011) also identified both students' poor and productive metacognitive activities while collaboratively working in pairs. Based on their observational study investigating nonverbal behaviors of young children (ages 3-5) while individually or collaboratively solving problems, Whitebread et al. (2007) provided empirical evidence of young children's spontaneous metacognitive activities, in particular, metacognitive activities focused on evaluation. They also found a predominance of young children's spontaneous metacognitive activities during collaborative, as opposed to individual problem solving.

Wilson and Clarke (2002, 2004) used self-report methods and a card-sorting activity, in which sixth-grade students were asked to arrange metacognitive action cards in order–to construct and reconstruct their thought processes after problem solving and while watching a video recording of their problem-solving activity, respectively. From these methods, the researchers investigated the nature of students' metacognitive activity during individual problem solving. Based on an operational definition of metacognition involving the three types of metacognitive activities (awareness, regulation, and evaluation), they found that metacognitive evaluation was the most frequent of the metacognitive activities during individual problem solving, followed by metacognitive regulation, and finally metacognitive awareness.

More recently, Magiera and Zawojewski (2011), whose research originated from Magiera's (2008) doctoral dissertation, replicated these findings from Wilson and Clarke (2002, 2004), adopting their framework and methodology. Furthermore, they extended the framework and methodology for studying students' metacognitive activity during individual problem solving in order to study students' metacognitive activity during collaborative problem solving, namely model-eliciting activities (MEAs). Based on this extension, they also found a predominance of students' metacognitive activities, which were "social-based," compared to "self-based" ones.

However, research on metacognition has still made little progress and has had little impact on school practice (Lesh & Zawojewski, 2007), not only because of the difficulties related to studying metacognition as mentioned above, but also because of its lack of a theoretical base. In addition, the lack of impact on school practice is due to the view of metacognition and cognition as hierarchically separate (Lesh & Zawojewski, 2007).

Most researchers have assumed that metacognition is a higher-order entity that is separate from lower-order cognitive processes, such as the development of an understanding of mathematical concepts. They have also assumed that specific metacognitive functions are always productive, for example, regardless of the stages of problem solving, specific content and contexts. These assumptions result in few studies based on developmental perspectives, which investigate a direct link between instruction in metacognition to the teaching of mathematics and improved problem-solving performance. They also result in a disregard of the social, contextual and situated nature of metacognition (Lesh et al., 2003; Lesh & Zawojewski, 2007). For example, many educators and researchers' priority may be the development of students' cognitive abilities. They may consider the development of students' metacognitive abilities as a later step because of this hierarchical view of cognition and metacognition. Therefore, metacognitive aspects may be mainly thought of as pertaining to a few students who have higher achievement, but not for all students in natural classroom settings.

Models and Modeling Perspectives on Metacognition

As discussed above, most researchers have assumed that metacognition and cognition cannot develop at the same time. As a result, a few studies have adopted developmental perspectives to investigate metacognition, which is unlike research on the nature of students' developing conceptual understanding in mathematics education (Lesh et al., 2003; Lesh & Zawojewski, 2007). To address this issue, the researcher has developed a theoretical model of metacognition by adopting the models and modeling perspectives (MMP) on metacognition (Kim et al., 2013). The adoption of the MMP on metacognition allows a developmental investigation to explore (a) the mechanisms of metacognition; (b) the means by which to successfully motivate students' metacognition within the domain of problem solving; and (c) how the development of students' metacognitive abilities can be promoted, with the expectation to inform school practice.

In contrast to the traditional perspectives on metacognition as described before, the MMP provides new insights into the existing paradigm of research on metacognition, as follows (Lesh et al., 2003; Lesh & Zawojewski, 2007). First, the MMP emphasizes the critical role of holistic conceptual systems that individuals possess in successful problem solving and learning. A conceptual system that a student possesses includes both cognitive (e.g., understanding, skills) and metacognitive (e.g., beliefs, awareness) components. They interactively and bi-directionally influence each other (Lesh et al., 2003). For example, as students increase their understanding, their metacognition is effectively triggered. Likewise, students develop better understanding as their metacognition is actively encouraged.

The MMP operates under the assumption that thinking becomes metacognitive when people change from “thinking WITH” cognitive components to “thinking ABOUT” them, via monitoring and regulating them (Lesh et al., 2003). As a result, the MMP assumes that metacognition and cognition develop in parallel and interactively at the same time, rather than hierarchically. Thus, instruction in metacognition can be embedded in the teaching of mathematics and problem-solving activities (Lesh et al., 2003; Lesh & Zawojewski, 2007).

Second, an alternative focus of MMP research can be found in the situated, contextual, and social nature of metacognition. The MMP operates under the assumption that metacognition is closely associated with specific content and contexts because it makes use of individuals’ interpretations of these content and contexts rooted in their conceptual systems. In valuing the situated and contextual nature of metacognition, the MMP operates under the assumption that the effectiveness of metacognitive functions frequently differs across problems, and even across the stages of problem solving. This can be explained by the focus of the (sub) tasks varying across problems and changing during the problem-solving processes. For instance, it may be more efficient to brainstorm at the early stages of problem-solving processes as opposed to later stages, such as assessment (Lesh et al., 2003).

The social nature of metacognition is another focal point of the MMP. Research from the MMP frequently examines problem solving, while students are involved in collaborative teams, and this research compares teams with individuals. The MMP implies that investigations centering on a team are often a fruitful way to ascertain how one person's mind works in context. Specifically, such investigations help determine how one person's thinking becomes metacognitive (Lesh et al., 2003; Magiera & Zawojewski, 2011), given that thinking becomes metacognitive when one person begins to function as if s/he were a team of several agents, such as planner, monitor, assessor, and so on, who work together within the person (Lesh et al., 2003).

Multiple Levels of Metacognition

Based on the models and modeling perspectives (MMP), the researcher has also re-conceptualized metacognition on multiple levels in the theoretical model, focusing on an individual as a unique agent who has access to the sources that trigger metacognition at the individual, social, and environmental levels (Kim et al., 2013). Although investigations focusing on a team as one unit of problem solver–an entire agent of “metacognition at the social level”–often provide a useful window into how one individual’s thinking becomes metacognitive (e.g., Iiskala, Vauras, & Lehtinen, 2004; Iiskala et al., 2011; Vauras, Iiskala, Kajamies, Kinnunen, & Lehtinen, 2003; Volet, Summers, & Thurman, 2009; Volet, Vauras, & Salonen, 2009), the active and unique agent of metacognition is an individual, regardless of whether the individual engages in collaborative teams working together or isolated individuals working independently, because metacognition, in and of itself, is a mental process within an individual (Kim et al., 2013). The sources of metacognition, to which an individual has access for eliciting metacognition (e.g., others’ conceptual systems), are not active agents, but rather a kind of starting place. In the theoretical model, the term “trigger” is used to emphasize this starting place that causes individuals to monitor and regulate their cognitive components.

As a result, the theoretical model of metacognition, which was used for this study as a theoretical framework, focuses on an individual as a unique agent and multiple sources that trigger the individual’s spontaneous metacognitive activities (Kim et al., 2013). This re-conceptualization of metacognition reflects the views on metacognition through the lens of individual cognitive theories, as well as through the lens of social cognitive theories, and the situated, contextual, and social nature of metacognition (Kim et al., 2013). Schoenfeld (1999) pointed out a lack of theories informing how an individual’s mind works in social contexts. He highlighted not only the need for their development, but also their application to practice.

By employing the theoretical model, the current study addresses the need for an in-depth investigation of how an individual’s thinking becomes metacognitive in social contexts, with the expectation for potential application to school practice. The following sections describe the distinctions among metacognition on multiple levels–the individual, social, and environmental levels–in terms of which level of sources triggers metacognition.

Metacognition triggered at the individual level. The sources that trigger metacognition at the individual level are individuals' conceptual systems. The internal sources of metacognition are cumulated by the individuals' prior knowledge and experiences, and thus differ in quantity, as well as in quality (Kim et al., 2013). Based on differing conceptual systems, which include both cognitive and metacognitive components, the individuals interpret contextual variables, such as particular contents and situations, and their thinking becomes metacognitive (Lesh et al., 2003). Consequently, individuals themselves "trigger" metacognition at the individual level, by beginning the work of monitoring or regulating their cognitive components.

Metacognition triggered at the social level. On the other hand, individuals may go beyond the limits of internal sources of metacognition by having access to external sources of metacognition. One of the external sources is obtained by drawing on the conceptual systems of other individuals, such as peers or teachers. Social interactions comprise the main sources that trigger metacognition at the social level (Kim et al., 2013). Through interactions with peers and teachers, students are often encouraged to retest their current thinking, to monitor their current level of knowledge and understanding, and to detect and correct their misconceptions (e.g., Carr & Biddlecomb, 1998; Goos, 1994, 2002; Goos & Galbraith, 1996; Goos et al., 2002; Kramarski &

Mevarech, 2003; Lesh et al., 2003; Lesh & Zawojewski, 2007; Schraw, 1998). The social processes through interactions with others are the main sources of metacognition triggered at the social level (Kim et al., 2013). Consequently, social interactions “trigger” metacognition at the social level, by making individuals begin the work of monitoring or regulating their cognitive components.

Metacognition triggered at the environmental level. Another external source that triggers metacognition originates from the interactions between a person (or persons) and the learning environment. Interactions with the learning environment are also the main sources that trigger metacognition, in particular, at both the individual and social levels (Kim et al., 2013; Volet, Vauras, et al., 2009). They are potential sources that maximize opportunities for students to retest their current thinking, to monitor their current level of knowledge and understanding, and to unpack and correct their misconceptions, by stimulating spontaneous metacognitive activities triggered at both the individual and social levels. Consequently, learning environments “trigger” metacognition at the environmental level, by making individuals, either themselves or via others, begin the work of monitoring or regulating their cognitive components. For example, classroom activities, in and of themselves, the types of problem-solving activities (e.g., Lesh et al., 2003; Lesh & Zawojewski, 2007; Magiera & Zawojewski, 2011), and task complexity (e.g., Helms-Lorenz & Jacobse, 2008; Iiskala et al., 2004, 2011; Prins, Veenman, & Elshout, 2006; Stahl, Pieschl, & Bromme, 2006; Vauras et al., 2003) are sources of metacognition triggered at the environmental level.

In particular, Magiera and Zawojewski (2011) identified problem-solving situations that stimulate spontaneous students' metacognitive activities triggered at both the individual and social levels, respectively as follows: (a) three self-based contexts, characterized as "seeking personal satisfaction," "making experience-based quantitative judgments," and "making personal projections" into problem situations; and (b) three social-based contexts, characterized as "interpreting diverse perspectives," "engaging in explanations," and "seeking mathematical consensus." In addition, social contexts (e.g., team-oriented problem-solving activities) and problem situations that require students to make their own definitions of qualitative constructs and negotiate their individual and social meanings are identified as good environmental sources to stimulate students' metacognitive activities, triggered at both the individual and social levels (Kim et al., 2013). Figure 2.1 below summarizes how metacognition (or cognition) is triggered on multiple levels.

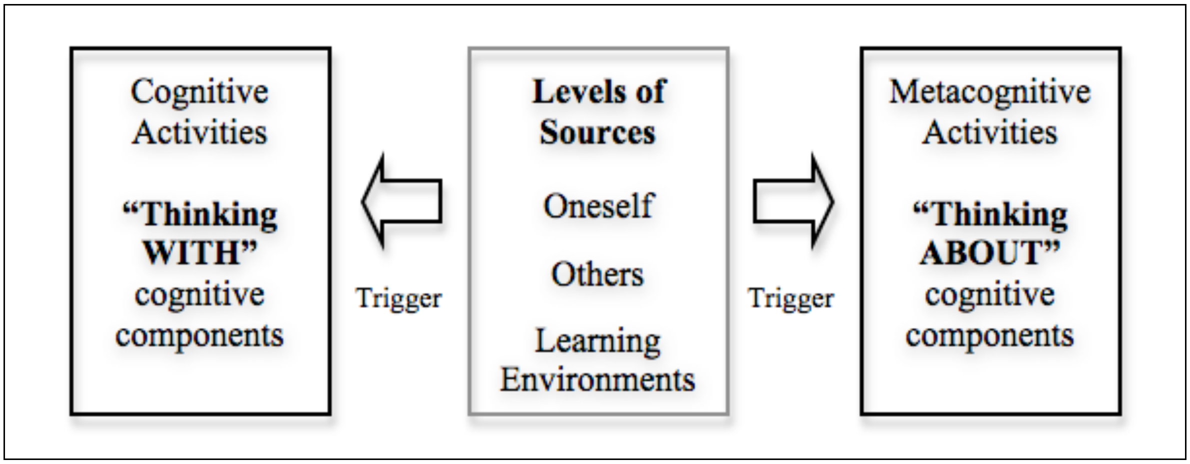


Figure 2.1. Multiple levels of metacognition and cognition.

Construct of Metacognition on Multiple Levels

As described above, the theoretical model of metacognition on multiple levels, which was employed as the framework in the current study, adopted the MMP view of metacognition and a re-conceptualization involving metacognitive triggers. Figure 2.2 below illustrates the framework employed in this research to study students' metacognitive activities (Kim et al., 2013). It involves the operational definition for the construct of metacognition on multiple levels adopted in the current study.

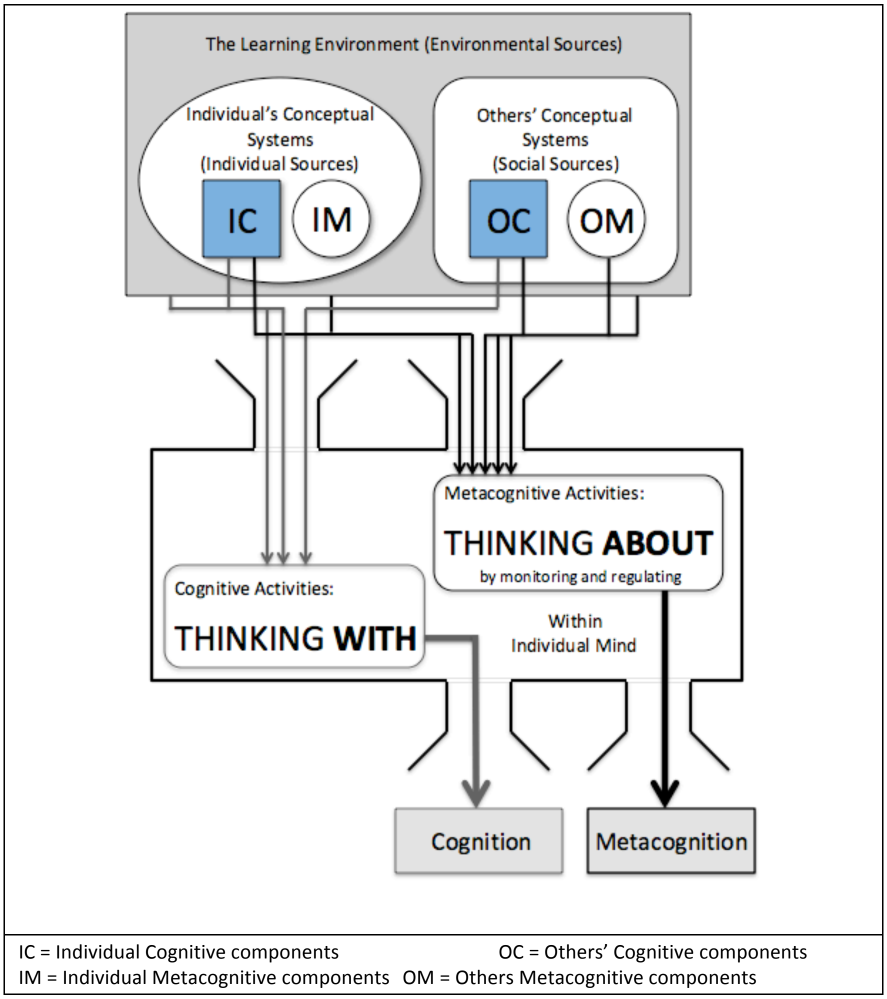


Figure 2.2. Diagram illustrating how the various internal and external triggers impact cognition and metacognition (Kim, Park, Moore, & Varma, 2013).

For instance, when a student working within a group uses the mathematical concept of “ratio” to work out a problem in the absence of any evaluation, he/she thinks with the cognitive component “knowledge of ratio” from herself/himself (the individual level) or from group members (the social level). On the contrary, when someone judges whether the concept of “ratio” is appropriate for the problem situation involving some cues of evaluation, he/she thinks about the cognitive component, and her/his thinking shifts to being metacognitive (the individual level or the social level, respectively).

The diagram on the top half of Figure 2.2 shows the extended sources that an individual can access to activate metacognition. In particular, usage of the Venn diagram points out that the origins of metacognition are not active agents, but rather initiating points for metacognitive activities (Kim et al., 2013). As shown, the origins of metacognition are classified as internal and external sources. First, the internal sources consist of individuals’ conceptual systems, which include both cognitive and metacognitive components. Second, the external sources involve (a) others’ conceptual systems within collaborative activities; and (b) environmental origins (the learning environment).

The function machine at the bottom half of Figure 2.2 indicates how individuals’ thinking becomes metacognitive, as well as how metacognition is distinguished from cognition. In particular, using the function machine representation puts an emphasis on the focus regarding the functions of metacognition that an individual, as the unique agent of metacognition, activates (Kim et al., 2013). The focus of the theoretical model regarding the metacognitive functions themselves, i.e., the monitoring function and the regulatory function–rather than its components, how to categorize the components, and how to establish the relationship among them–also allows for agreement on the definition of metacognition (Efklides, 2006; Goos, 2002; Wilson & Clarke, 2002).

By focusing on the metacognitive functions themselves, there is consensus as to the operational definition of metacognition, i.e., metacognition as an outward appearance of the monitoring and regulatory functions. For instance, Wilson and Clarke (2002) put special emphasis on the two non-regulatory functions of metacognition: "awareness individuals have of their own thinking" (*metacognitive awareness*) and "their evaluation of that thinking" (*metacognitive evaluation*) and one regulatory function of metacognition: "their regulation of that thinking" (*metacognitive regulation*), in order to define the construct of metacognition. Similarly, Efklides (2006) showed three aspects of metacognition, differentiated by their outward appearances as a function of monitoring and control. The two monitoring functions consist of (a) *metacognitive knowledge*: knowledge regarding one's own cognition; and (b) *metacognitive experiences*: metacognitive judgment and assessment rooted in monitoring the attributes or results of the given task. The control function of metacognition is referred to as *metacognitive skills*: knowledge of the procedures required to regulate cognition (Efklides, 2006).

In particular, the theoretical model of metacognition on multiple levels employed in this study has adopted a portion of Goos' (2002) metacognitive constructs framework, which was developed on the basis of the episode-based frameworks of Schoenfeld (1985) and Artzt and Armour-Thomas (1992), to identity students' spontaneous metacognitive activities while collaboratively working on a complex mathematical task (Kim et al., 2013). Goos' (2002) framework for analyzing verbal protocols within a collaborative problem-solving context focused on the dual role of metacognition, the monitoring and regulatory functions that would be most applicable and expected at each stage of the problem-solving processes. In particular, assessment of one's own thinking is considered central to the monitoring function. The monitoring function involves the *assessment of knowledge*, *assessment of understanding*, *assessment of strategy appropriateness*, *assessment of progress toward goal*, *assessment of strategy execution*, and *assessment of accuracy or sense of result*. These assessing processes lead to manifestations of the regulatory function. They involve identifying new (alternative) information (strategy)–*new idea*, reinterpreting problem–*reinterpretation*, *changing strategy*, and *correcting errors* (Goos, 2002).

While Goos (2002) put forth this framework as linear, indicating a linear progression viewpoint of problem solving, the MMP, adopted as a base for the theoretical model of metacognition on multiple levels, has a multiple dimensional progression viewpoint of problem solving (Kim et al., 2013). As a result, a rearrangement of Goos' (2002) framework has been made for the theoretical model to represent the MMP on problem solving, as well as a multiple dimensional progression viewpoint of metacognition during problem solving (Kim et al., 2013). Figure 2.3 below shows how students' spontaneous metacognitive activities, four regulating activities and six monitoring activities, occur during problem solving in terms of the MMP viewpoint.

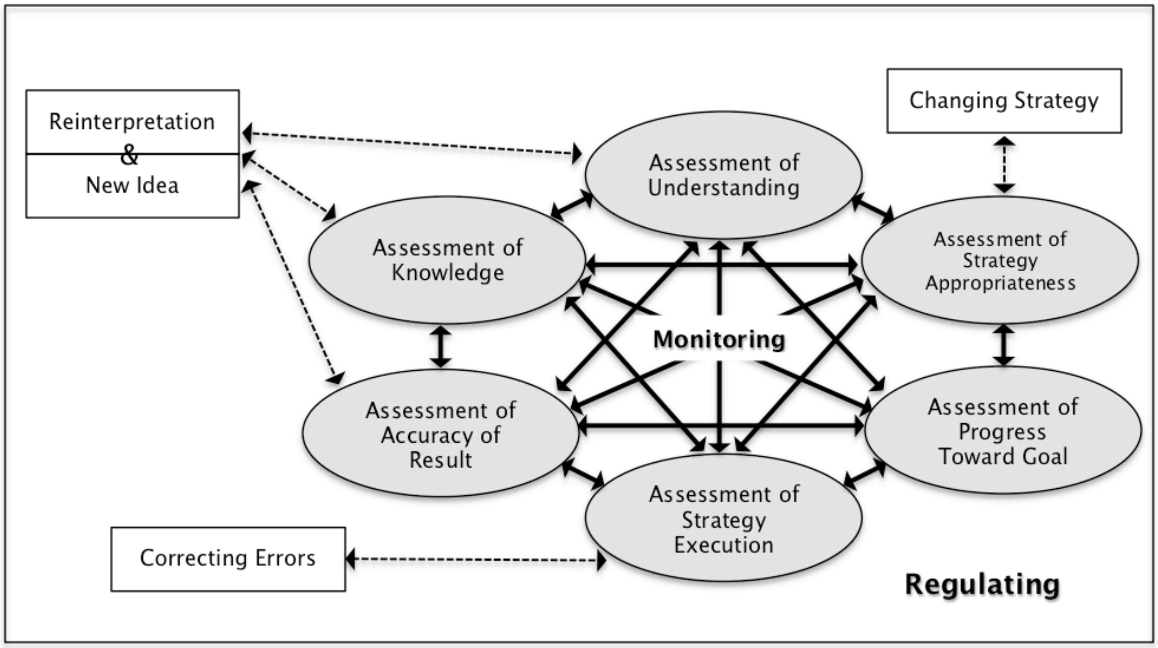


Figure 2.3. An MMP view of metacognitive activities during problem solving (Kim, Park, Moore, & Varma, 2013).

Consequently, a theoretical model of metacognitive processes in complex modeling activities has been developed based on the models and modeling perspectives (MMP), by synthesizing the re-conceptualization of metacognition on multiple levels and Goos' (2002) classification of metacognitive activities as manifestations of the monitoring and regulatory functions, to identify students' metacognitive activities while collaboratively solving a complex mathematical task (Kim et al., 2013). How this has been done is summarized in Figures 2.2 and 2.3 above. A single case study illustrated how the model helps make clear the distinction between cognition and metacognition, and the distinction among metacognitive functions (Kim et al., 2013). The single case study also illustrated how the model helps interpret students' metacognitive activities within a problem-solving session.

Furthermore, the current study employs a multiple case study to examine the appropriateness and soundness of the theoretical model for research in metacognition. In addition, the current study employs the theoretical model of metacognition on multiple levels as an analytical framework to explore how students' thinking becomes metacognitive, while collaboratively solving complex mathematical tasks. Based on the students' metacognitive activities identified by using the theoretical model, the study investigates (a) how students' metacognitive abilities manifest themselves during complex modeling tasks; (b) the nature of students' metacognitive abilities manifested during complex modeling tasks; and (c) critical events that facilitate or interfere with students' metacognition. The following chapter presents the research methodology and the usefulness of model-eliciting activities (MEAs) for this study.

Chapter 3

Research Methodology

As previously highlighted, the first purpose of this study was to examine the soundness and appropriateness of the theoretical model of metacognition on multiple levels in identifying and interpreting students' metacognitive activities, with respect to (a) how the model helps make clear the distinction between cognition and metacognition, and the distinction among metacognitive functions; and (b) how it helps interpret students' metacognitive activities within and across problem-solving sessions. The second purpose was to explore how students' thinking becomes metacognitive while collaboratively solving a complex mathematical modeling task. In particular, this study (a) explored the patterns and tendencies of the students' metacognitive activities; (b) investigated the nature of students' metacognitive abilities; and (c) identified the circumstances facilitating or interfering with students' spontaneous metacognitive activities.

A qualitative study was conducted in order to illustrate how the theoretical model provides a window into the development of metacognitive abilities and in order to explore potential mechanisms for the development of metacognitive abilities. In addition, this qualitative study illustrates how research on metacognition may benefit from studies that investigate students' spontaneous metacognitive activities by engaging students in model-eliciting activities (MEAs).

Research Design

Through a multiple case study, the current study examined the appropriateness and soundness of the theoretical model for research in metacognition. Based on students' metacognitive activities identified by using the theoretical model of metacognition on multiple levels, the current study also investigated students' spontaneous metacognitive activities during small-group problem solving and the nature of students' metacognitive abilities via the following research questions:

1) How appropriate is the theoretical model for identifying and interpreting metacognitive activities in complex modeling activities?
2) How does students' thinking become metacognitive during complex modeling activities?
    a) How do metacognitive abilities manifest during complex modeling activities?
    b) What is the nature of students' metacognitive abilities manifested during complex modeling activities?
    c) What are the circumstances in which metacognitive activity is abandoned?

A multiple case study was conducted in order to address these questions. The study generated qualitative data from multiple case studies of 23 students in seven groups of three to four seventh-grade students, collaboratively working on three different problem-solving sessions. The focus of the study was to investigate students' spontaneous metacognitive activities emerging when working together in natural classroom settings and to build an understanding of how students' thinking becomes metacognitive within and across problem-solving processes. As a result, these descriptive, naturalistic, and inductive characteristics of the study met the needs of qualitative research methods (Bogdan & Biklen, 2003; Creswell, 2006).

Research Site

The study took place in a conveniently selected public suburban middle school in the Midwest and was conducted in block-scheduled mathematics classes (Two consecutive mathematics classes meet for approximately 80 minutes total, four days a week). The school serves approximately 750 students in the sixth through eighth grades. The demographics of the school are 79% white, 8% Hispanic, 6% black, 6% Asian and less than 1% American Indian. Thirty percent of the school's student population receives free or reduced-price lunch. The school did not make Adequate Yearly Progress (AYP) in Mathematics for 2011. As one goal to improve mathematics performance, the school adopted block scheduling in mathematics classrooms. Even though the mathematics courses involve a STEM (Science, Technology, Engineering, and Mathematics) integration curriculum, the school could be described as traditional in terms of both the mathematics curriculum and the overall learning environment.

Participants

The participants in this study were seventh-grade students in a higher-level mathematics course entitled "Algebra with Statistics." This course was intended for students who demonstrated high mathematics achievement on district-wide assessments, the MAP Tests. Twenty-six participants worked in eight groups of three to four during three different problem-solving sessions for the 2011-2012 school year. The teacher allowed the students to decide upon their own groups before the first problem-solving session, emphasizing that no member change was allowed over the three problem-solving sessions. The group members worked consistently and coherently during each problem solving session, allowing for sufficient identification of the metacognitive processes in which they engaged. No member change also allowed me to explore the overall patterns of each student's metacognitive activities within and across the three problem-solving sessions.

For each problem-solving session, the students were presented with one model-eliciting activity (MEA) described in the next section. Most participants had prior experience with the type of problems selected for the study. In the previous school year, they had participated in an MEA entitled Bigfoot as part of a STEM integration curriculum. One of the eight groups had a student who did not submit the student and parental consent forms necessary to participate; consequently, the student group was removed from this study. Thus, the study involved 23 students in seven groups of three to four, regardless of the group members' positive attitudes during their problem-solving sessions and the degree to which they verbalized their conceptualization of problems over the course of problem solving. In reporting from this study, pseudonyms were used to protect the students' identities.

Data Collection

This study used a series of three model-eliciting activities (MEAs) to collect data (see Table 3.1 below). MEAs require participants to verbalize their thoughts spontaneously while collaboratively working in a group in natural classroom settings;

thus, MEAs work as an authentic method for analyzing verbal metacognitive actions (Kim et al., 2013). The primary sources of data for this study were audio transcripts of student groups, student group solutions to the MEAs (i.e., group letters to imaginary clients), and researcher field notes during the three problem-solving sessions.

Table 3.1

Description of the Three MEAs Used in the Study

Title	Problem and Student Solution Product	Main Strands
Summer Jobs	Key question: *How do you develop a productive management scheme in order to decide who to rehire for summer job positions when reviewing employees' records from last year?* Students are given information about the previous year park vendors' performance (i.e., numbers of hours they worked, amount of money they collected, and the overall park profile). They are asked to analyze the performance of nine vendors with a goal of developing a system for selecting three for a full-time and three for a half-time summer job position.	Number and Operations, Algebra, Data Analysis, Statistics, Problem Solving, and Communication
Volleyball	Key question: *How do you develop a system that will make fair teams in order to have more competition in the volleyball summer camp's tournament based on the information about some of the players from tryouts and from the coaches?* Students are given numeric and non-numeric information about each player. The data include height, vertical leap, speed, serving percentage, spike results, and the coach's comments. The students are asked to develop a procedure for creating equal teams of volleyball players.	Number and Operations, Algebra, Data Analysis, Statistics, Problem Solving, and Communication
Paper Airplane	Key question: *How do you create a fair judging scheme for a paper airplane contest when looking for the most accurate paper airplane and the best floater?* Students are given a set of numeric and visual data recorded during a paper airplane contest. The data include information about time in the air, distance traveled, and accuracy landing. The students are asked to develop a procedure for selecting the winners of the paper airplane contest in two categories, Most Accurate and Best Floater. The problem solvers need to define for themselves each of these categories before they can select data relevant to each category and develop a model for analyzing the relevant performance.	Number and Operations, Algebra, Data Analysis and Probability Problem Solving, and Communication

For the block-scheduled mathematics class periods of approximately 80 minutes, each problem-solving session took two days, for a total of about 160 minutes. The three problem-solving sessions were conducted in a natural classroom setting. Each of the three problem-solving sessions captured collaborative interactions of the student group members as they worked on challenging problems. Each group of three to four students was audio and video recorded to explore student conversations throughout each MEA group activity period. The group conversations of the three to four students in this study were transcribed by using both the audio- and video-recorded data, which allowed me to ensure the students' identities.

The multiple case study, which involved 23 students in seven groups working on three different problem-solving sessions, generated enough qualitative data to examine the soundness and appropriateness of the theoretical model of metacognition on multiple levels in identifying and interpreting students' metacognitive activities (the first research question). The multiple cases of 23 students in seven groups provided rich examples to illustrate (a) how the model helps make clear the distinction between cognition and metacognition, and the distinction among metacognitive functions; and (b) how it helps interpret students' metacognitive activities within and across problem-solving sessions.

The qualitative data from the multiple case study also allowed the researcher to probe beneath the surface of students' metacognition and to explore how students' thinking becomes metacognitive, while collaboratively solving problems (the second research question). In particular, the data from the multiple case study spanning three problem-solving sessions without group member changes allowed the researcher to (a) explore the patterns and tendencies of the students' metacognitive activities; and (b) investigate the nature of students' metacognitive abilities (the first two sub-questions of the second research question). The data from the multiple case study also provided a rich context to identify the circumstances facilitating or interfering with students' spontaneous metacognitive activities (the last sub-question of the second research question).

The following section discusses two main reasons for why model-eliciting activities (MEAs) were used for this study: (a) as an environmental source supporting students' spontaneous metacognitive activities, and (b) as a methodological tool for verbal protocol analysis to investigate students' spontaneous metacognitive activities in collaborative problem-solving settings.

Model-Eliciting Activities (MEAs)

During each problem-solving session, the participants worked on one model-eliciting activity (MEA). MEAs are the most frequent kind of problem-solving activity in the area of the MMP (Lesh & Zawojewski, 2007). They are team-oriented and realistic problem-solving tasks that require the participants to develop their own mathematical models as conceptual systems to interpret the presented problem situations (Chamberlin & Moon, 2005; Diefes-Dux, Moore, Zawojewski, Imbrie, & Follman, 2004; Lesh & Doerr, 2003; Lesh, Hoover, Hold, Kelly, & Post, 2000; Moore & Diefes-Dux, 2004; Moore, Diefes-Dux, & Imbrie, 2006; Moore & Hjalmarson, 2010).

Each MEA contains an individual activity to allow all students to have relevant background information about a real-life context in which the actual MEA is set up (see examples of MEA problem statements in the Appendices). The students worked individually for approximately 15 minutes on the reading and question activity. This pre-activity helped them organize their understanding of the problem situation, and motivated the need for creating mathematical models to describe the presented situation (English, 2008; Hamilton, Lesh, Lester, & Brilleslyper, 2008; Lesh & Doerr, 2003; Lesh & Harel, 2003). Then students were required to work in groups for approximately 105 minutes, including time for a break and for preparing a five-minute group presentation. Finally, the student groups presented their preliminary solutions to the whole class (40 minutes). Each of the three MEAs used in this study is briefly summarized below in order, in which the problems were presented to the study participants (see Table 3.1). Their full versions are found in the Appendices.

The following sections provide additional details about MEAs in terms of two main reasons as to why they were chosen for this research to study metacognition, in particular, how they are expected to address the need of an authentic methodological tool for studying metacognition.

Model-eliciting activities (MEAs) as an environmental source. One of the environmental considerations making MEAs excellent research sites in which to study metacognition is that MEAs can work as an effective environmental source in order to stimulate students' spontaneous metacognitive activities triggered at the individual and social levels.

MEAs are complex, open-ended problems, in which students are required to make their own definitions of qualitative constructs (definition building), then to make the qualitative constructs measurable (operationalizing definitions), such as by quantifying, converting, and sampling (Kim et al., 2013). For example, in the Paper Airplane MEA (Table 3.1 or Appendix C), students need to define what is necessary to be accurate in a particular context, and then they need to quantify their definition of “accurate,” such as by sampling and choosing variables.

The two problem-solving processes, “definition building” and “operationalizing definitions,” encourage students to think about thinking, both in terms of their own and others’, and to monitor or regulate likely alternative processes. Specifically, such problem-solving processes engage students in self-assessing their own knowledge and understanding (individual level), as well as in assessing others’ (social level). Based on their existing knowledge and prior experience, students need to make judgments regarding their own ways of thinking. These problem-solving processes also engage students in negotiating their individual and social meanings of problem situations or problem-solving processes (Kim et al., 2013).

MEAs are team-oriented problem-solving tasks that require students to verbalize their thoughts spontaneously (according to their own needs or motivations) while collaboratively working in a group in natural classroom settings (English, 2008; Diefes-Dux et al., 2004; Hamilton et al., 2008; Lesh & Doerr, 2003; Lesh et al., 2000). Students need to share their multiple perspectives, and test and revise their assumptions on which they build their diverse interpretations of problem situations or problem-solving processes (Hamilton et al., 2008; Lesh & Doerr, 2003; Lesh et al., 2003). Thus, engaging in communication and making team agreements are necessary for productive problem solving. MEAs are also client-driven, realistic problem-solving tasks that require students to develop their own mathematical, scientific, or engineering models to meet the needs of an imaginary client in a real-life context involving problem-solving situations (e.g., Hamilton et al., 2008). Thus, students often project themselves into the imaginary problem situations or problem-solving processes, and work as mathematicians, scientists, or engineers do in the real world.

These characteristics may enable MEAs to work as an effective environmental source that stimulates students' spontaneous metacognitive activities, triggered at both the individual and social levels. This assumption is supported by the findings of Magiera and Zawojewski (2011). By using five MEAs and self-report methods, they identified and characterized six contexts that stimulated spontaneous students' metacognitive activities triggered at both the individual and social levels, as presented before in chapter 2. The six contexts: three self-based contexts, characterized as (a) "seeking personal satisfaction," (b) "making experience-based quantitative judgments," and (c) "making personal projections"; and three social-based contexts, characterized as (d) "interpreting diverse perspectives," (e) "engaging in explanations," and (f) "seeking mathematical consensus" are consistent with the characteristics of MEAs, as described above.

In addition, several research studies have provided a rationale for studying metacognition in the context of complex tasks, such as MEAs, by documenting that task difficulty, in terms of task complexity, tends to stimulate the need for metacognitive thinking (e.g., Efklides, 2006; Helms-Lorenz & Jacobse, 2008; Iiskala et al., 2004, 2011; Prins et al., 2006; Vauras et al., 2003).

As a result, MEAs have the potential as an effective environmental source to stimulate students' spontaneous metacognitive activities in a natural classroom setting. The following section discusses how MEAs can address the need of an authentic methodological tool for studying metacognition.

Model-eliciting activities (MEAs) as a methodological tool for verbal protocol analysis. Another reason why MEAs were chosen for this research to study metacognition is that MEAs can work as an authentic methodological tool for studying metacognition on multiple levels. While working on an MEA, students are engaged in a group work in natural classroom settings without any interactions with researchers. The role of teachers as a facilitator and observer avoids any influence upon students' solutions. The students are required to verbalize their thoughts only by their own needs (e.g., Lesh et al., 2000; Lesh & Lamon, 1992). The verbal protocols of this problem-solving session can be considered as consistent with an individuals' typical behavior during problem solving (Ericsson & Simon, 1984) and can provide an accurate assessment of metacognitive activity (e.g., Presley, 2000; Veenman, 2005). Thus, MEAs work well as an authentic method for verbal protocol analysis. For that reason, during each problem-solving session in the study, the students should not be given any specific directions besides being asked to actively and collaboratively work on the problem. In particular, the researcher should only be an observer to make field notes without any interactions with the students.

In addition, MEAs can address several criticisms regarding self-report methods that have commonly been used in research on metacognition (Kim et al., 2013), including *accessibility*, *retrieval issues*, *veridicality*, *reactivity* as a result of a contrived setting, and *completeness* (e.g., Ericsson & Simon, 1980; Garofalo & Lester, 1985; Goos & Galbraith, 1996; Schoenfeld, 1985; Wilson & Clarke, 2002).

MEAs require students to provide both product-oriented (models) and process-oriented (modeling) documentation of a problem solution. Thus, the need to share decision processes through speaking and writing makes MEAs naturally thought revealing (thereby allowing *accessibility* to students' mental processes). Students share, test, and revise their multiple perspectives throughout their participation in MEAs (thereby addressing *retrieval issues*: What students are saying reflects what they *are* thinking, as opposed to what they *were* thinking). Students spontaneously verbalize their thoughts to achieve a common goal according to their own needs, rather than by request (thereby addressing the issue of *veridicality*). MEAs are fundamentally social, team-oriented tasks in natural classroom settings (thereby reducing the problem of *reactivity*). This collaborative nature of MEAs guarantees much more verbalization of students' thinking, which in turn, can address the issue of *completeness* (Schoenfeld, 1985). The collaborative facet of MEAs naturally provokes considerable metacognitive functions, monitoring and regulating of their own and one another's thinking (e.g., Goos & Galbraith, 1996).

Consequently, MEAs can work as a methodological tool for verbal protocol analysis to investigate students' spontaneous metacognitive activities in collaborative problem-solving settings. The following section discusses the data analysis and presents the step-by-step process for the data analysis.

Data Analysis

The data analysis was conducted by multiple cycles of viewing and reviewing the audio transcripts and video records of the problem-solving sessions, the student group solutions, and the field notes. The coding scheme that was used for this study to analyze the data is consistent with the theoretical model in Figures 2.2 and 2.3, involving the preset categories (codes): NI (*new idea*), RI (*reinterpretation*), CS (*changing strategy*), CE (*correcting errors*), A-K (*assessment of knowledge*), A-U (*Assessment of understanding*), A-S (*assessment of strategy appropriateness*), A-P (*assessment of progress toward goal*), A-E (*assessment of strategy execution*), and A-R (*assessment of accuracy or sense of result*).

The types of regulating activity, *changing strategy* and *correcting errors*, inherently involve the types of monitoring activity, *assessment of strategy appropriateness* and *assessment of strategy execution*, respectively; thus, students' comments identified as *changing strategy* and *correcting errors* would be coded as CS & A-S and CE & A-E, respectively. In addition, cross-indexing was allowed, if needed.

A finer-grained analysis of the conversational statements was conducted to identify each student's problem-solving behaviors, the monitoring and regulating activities, and to decide on the levels of sources triggering metacognition. Based on the coding results, students' conversational comments were annotated to indicate metacognitive functions, four regulating activities and six monitoring activities, and the levels of sources triggering them.

By focusing on an individual as a unique agent of metacognition, the unit for analyzing verbal protocols was each comment made by an individual. However, the coding decision for each comment was made on the basis of the overall scenario of the students' dialogues within the group, rather than on the basis of each individual statement made. For example, when a student led a problem-solving process by suggesting a new idea with the comment, "We need to do the money they made over the hours they worked." This comment was coded at the individual level not because of the subject "we," but due to the overall scenario.

Also, non-verbal and verbal cues were considered as important factors in making coding decisions. For example, if students' comments involved some cues of evaluation, such as "Well," "Oh," "I (don't) think," "So, (no) wait," "…don't we?" "Wait, you have to," and so on, the comments could be coded as metacognitive activities because they indicate evaluations occurring. On the other hand, the comment, "So we'd each take a pilot. There are only 3 pilots and there are 3 of us. And then you'll average each of these," could be not coded for metacognition because there was no cue of any evaluation based on the overall scenario.

Each vignette made by students was labeled with symbols (R, X, Y, Z) representing (a) the student (R); (b) the group to which the student belongs (X); (c) the MEA (Y)–Summer Jobs (SJ), Volleyball (VB), and Paper Airplane (PA); and (d) the respective transcript segment (Z). For example, the comment illustrated above, "We need to do the money they made over the hours they worked," was labeled with the symbol (H3, G1, SJ, 67). This indicates that the comment made by student H3 within Group 1 during the Summer Jobs MEA was in the transcript on line 67. In addition, the letter “T” was used to represent the teacher. Table 3.2 below summarizes the step-by-step process for the data analysis.

Table 3.2

Step-by-Step Process for Data Analysis

Step	Description	Example	Coding decision
Step 1	Deciding whether each student’s comment was a cognitive activity or a metacognitive activity, based on the overall scenario: Cognitive activity, *if there is no cue of any evaluation or regulation*	(S2, G5, PA, 72) “So we’d each take a pilot. There are only 3 pilots and there are 3 of us. And then you’ll average each of these.” (No cue of any evaluation or regulation based on the overall scenario)	Cognitive activity
	Metacognitive activity, *if there is a cue of evaluation or regulation*	(S2, G5, PA, 66) “So, I have an idea, Brittany [S3]. I don’t know if it’ll work, but it’s an idea. I was thinking that what we could do is we could average each of these for all of the planes.” (A cue of regulation: “So, I have an idea,”)	Metacognitive activity
Step 2	Deciding which level (oneself, others, or learning environments) triggered the metacognitive activity, after identifying a metacognitive activity, based on the overall scenario		Individual level
Step 3	Deciding which metacognitive function it was among the four regulating activities by using the codes: NI, RI, CS, and CE or among the six monitoring activities by using the codes: A-K, A-U, A-S, A-P, A-E, and A-R, based on the overall scenario		New idea

Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Table 3.3 below describes the coding scheme used in this study for distinguishing metacognitive activities (thinking ABOUT) from cognitive activities (thinking WITH), and for deciding on the distinction among the multiple levels.

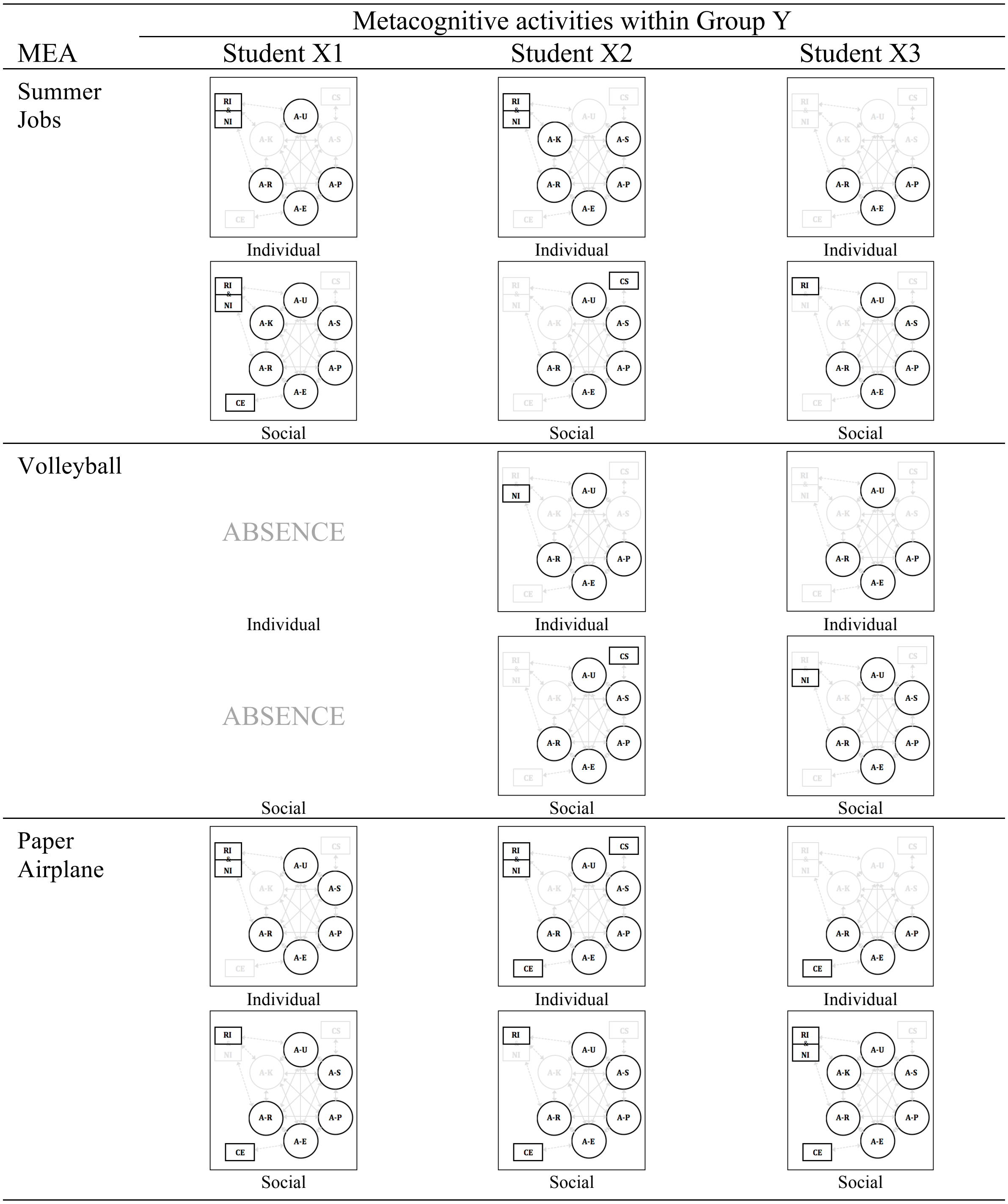
Table 3.3

Coding Scheme for Deciding on the Distinction Between Metacognition and Cognition, and the Distinction Among the Multiple Levels (Kim, Park, Moore, & Varma, 2013)

Problem-Solving Behaviors		Sources to Which an Individual Has Access		
Distinction	Description	Individual Level	Social Level	Environmental Level
Cognitive Activities: Think WITH	Without any evaluation and regulation, an individual thinks WITH ***a cognitive component:*** e.g., Given a set of numeric data, student X thinks with "knowledge of the average" to solve a problem.	Thinking WITH the cognitive component due to ***oneself*** e.g., Student X's thinking with her own "knowledge of the average"	Thinking WITH the cognitive component due to ***others*** e.g., Student X's thinking with "knowledge of the average" is triggered by another student's idea to use the average.	Thinking WITH the cognitive component due to ***a learning environment*** e.g., Student X's thinking with her own "knowledge of the average" due to something in the problem that indicates that concept, such as, "What is the average of the numbers?"
Metacognitive Activities: Think ABOUT	With evaluation or regulation, an individual thinks ABOUT ***a cognitive or metacognitive component***: e.g., Given a set of numeric data, student X thinks about "knowledge of the average" as to whether it is proper to solve a problem.	Thinking ABOUT the cognitive or metacognitive component due to ***oneself*** e.g., Student X's thinking about her own "knowledge of the average" is triggered by her own realization of a mistake.	Thinking ABOUT the cognitive or metacognitive component due to ***others*** e.g., Student X's thinking about her own "knowledge of the average" is triggered by another student pointing out a mistake.	Thinking ABOUT the cognitive or metacognitive component due to ***a learning environment*** e.g., Student X's thinking about her own "knowledge of the average" is triggered by something in the problem that makes her original solution not viable, such as competing variables in the problem.

After completely coding the data, several frequency tables of the identified students' metacognitive activities were developed. The type of table, including frequency counts, reveals general patterns in the data, even though it is not appropriate for statistical analysis (Miles & Huberman, 1994). The frequencies of the identified metacognitive activities were explored and compared across the seven groups, across the three MEAs, and across the group members to figure out general patterns of students' spontaneous metacognitive activities, triggered on multiple levels.

Finally, Figure 3.1 below was used as an analytical tool, which is consistent with the theoretical model in Figures 2.2 and 2.3. This figure was used to explore the patterns of each student's metacognitive activities, triggered at the individual and social levels, within each MEA. By using this figure, a cross-case analysis for the three MEAs was also conducted to explore the overall patterns of each students' metacognitive activities across the problem-solving sessions, and to investigate the nature of the students' metacognitive abilities.



NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result

Figure 3.1. An example of a figure used for analyzing each student's metacognitive activities, triggered at the individual and social levels, within and across the three MEAs.

As shown above, a figure used for analyzing each student's metacognitive activities originated from the theoretical model in Figure 2.3. For each student (in columns) within each group, the types of identified metacognitive activities, which were respectively triggered at the individual and social levels, are highlighted in each figure across the three MEAs–Summer Jobs, Volleyball, and Paper Airplane MEAs (in rows). For example, during the Paper Airplane MEA (in the last row of Figure 3.1), all of the metacognitive activities, except for *changing strategy* (CS), were identified at the social level from student X3 (in the last column of Figure 3.1); thus, they were highlighted in the figure. On the other hand, one regulating activity–*correcting errors* (CE) and three monitoring activities–*assessment of progress toward goal* (A-P), *assessment of strategy execution* (A-E), and *assessment of accuracy or sense of result* (A-R)–were identified at the individual level from student X3; thus, they too were highlighted in the figure.

During multiple cycles of data analysis, open coding via annotations, which allowed new categories to emerge from the data (Corbin & Strauss, 2008; Miles & Huberman, 1994), was also conducted to identify the circumstances facilitating or interfering with students' metacognitive activities.

Consequently, multiple cycles of data analysis, including a finer-grained analysis of the conversational statements by using two coding strategies–preset and open coding, and a cross-case analysis for the three problem-solving sessions, were conducted to (a) examine the soundness and appropriateness of the theoretical model of metacognition on multiple levels in identifying and interpreting students' metacognitive activities; (b) explore how students' metacognitive abilities manifest while collaboratively solving complex mathematical modeling tasks; (c) investigate the nature of students' metacognitive abilities manifested during complex mathematical modeling tasks; and (d) identify the circumstances facilitating or interfering with students' spontaneous metacognitive activities. The results and findings from these multiple cycles of data analysis are presented in the following chapter in detail.

Chapter 4

Data Analysis and Results

Through a multiple case study, the current research examined how the theoretical model of metacognition on multiple levels helps appropriately distinguish metacognition from cognition, and how it helps make the distinction and interpretation among students' metacognitive activities. By using the theoretical model of metacognition on multiple levels as a framework, this study was also conducted to explore how students' thinking becomes metacognitive, while collaboratively solving complex mathematical modeling tasks. Based on students' metacognitive activities, identified by using the theoretical model, the study explored how students' metacognitive abilities manifest themselves during complex modeling tasks. In addition, the study investigated the nature of students' metacognitive abilities manifested during complex modeling tasks, and critical events that facilitate or interfere with students' metacognition. A series of Model-Eliciting Activities (MEAs) was used as a method for analyzing verbal metacognitive actions to gain external access to students' metacognition, which is largely internal. Data collection and coding procedures used for this study are explained in Chapter 3. In this chapter, the analysis of the coded data employs qualitative and quantitative methods.

This chapter consists of two main sections. The first section presents the analysis and findings pertaining to the first research question, which examined the potential of the theoretical model of metacognition on multiple levels as a coherent model in identifying and interpreting students' metacognitive activities in complex modeling activities. The identified metacognitive activities are discussed and accompanied by students' excerpts from the collected data. The findings described in this section are organized around the theoretical model of metacognition on multiple levels, which is consistent with the coding scheme used in this study; that is, four regulating activities (*new idea*, *changing strategy*, *correcting errors*, and *reinterpretation*) and six monitoring activities (*assessment of knowledge*, *assessment of understanding*, a*ssessment of strategy appropriateness*, *assessment of progress toward goal*, *assessment of strategy execution*, and *assessment of accuracy or sense of result*), triggered at the individual, social, and environmental levels (Figure 2.3).

The second section of this chapter presents the analysis and findings pertaining to the second research question, which explored how students' thinking becomes metacognitive during complex mathematical modeling tasks. This section is organized around the three sub-questions of the second research question. First, the analysis and findings pertaining to the first two of the three sub-questions, which sought to explore patterns and tendencies of students' metacognitive activities within and across the three problem-solving sessions, and the nature of students' metacognitive abilities manifested during the problem-solving sessions, respectively, are presented. Patterns of students' metacognitive activities are reported on, and discussed and accompanied by illustrations drawn from the collected data. Second, the analysis and findings pertaining to the third part of the second research question, which sought to explore what circumstances interfered with students' metacognitive activities, are presented. The identified events that might interfere with students' metacognition are reported and illustrated with students' excerpts from the collected data. Finally, the results pertaining to the second research question are summarized, based on those to the three sub-questions.

The transcript excerpts use the same notation in this chapter as the one used in Chapter 3. For instance, for the transcript of student Group 5, the letter “T” stands for the teacher, and the letters “S1,” “S2,” and “S3” represent the group members. Each scenario is also classified with symbols (R, X, Y, Z) that represent the student (R), the group to which the student belongs (X), the MEA (Y)–Summer Jobs (SJ), Volleyball (VB), and Paper Airplane (PA), and the respective transcript segment (Z). With respect to the coding results, students’ conversational comments are highlighted to convey students’ problem-solving behaviors, monitoring and regulatory activities, and the levels of sources triggering them.

How Appropriate is the Theoretical Model for Identifying and Interpreting Metacognitive Activities in Complex Modeling Activities?

Model-Eliciting Activities (MEAs) were used as a method for verbal protocol analysis, where individual students’ spontaneous verbalization of their thoughts was required while collaboratively working in a group in the natural classroom setting–this allows MEAs to address several criticisms of self-report methods, described previously. By using the theoretical model of metacognition on multiple levels as an analytical framework, students’ spontaneous verbal actions during MEAs were mined to identify metacognitive activities triggered at the individual, social, and environmental levels. As a result, the students’ metacognitive activities triggered at all of the multiple levels (i.e., the individual, social, and environmental levels) were identified, and all of the metacognitive activities–four regulating activities (*new idea*, *changing strategy*, *correcting errors*, and *reinterpretation*) and six monitoring activities (*assessment of knowledge*, *assessment of understanding*, a*ssessment of strategy appropriateness*, *assessment of progress toward goal*, *assessment of strategy execution*, and *assessment of accuracy or sense of result*)– were also identified at both the individual and social levels.

First, this section will present rich excerpts of students to illustrate how the theoretical model of metacognition helps appropriately distinguish metacognition from cognition. In particular, these examples show how the theoretical model helped more or less clearly distinguish metacognitive activities (thinking ABOUT) from cognitive activities (thinking WITH), according to the existence of any evaluation or regulation cues: (a) metacognitive activity, if a student's spontaneous verbal action involved a cue of evaluation or regulation; and (b) cognitive activity, if a student's spontaneous verbal action did not involve any cue of evaluation or regulation.

Second, the students' excerpts presented in this section illustrate how the theoretical model helped make the distinction among metacognitive activities, based on an agreed-upon definition of metacognition, focusing on the monitoring function and the regulatory function. In particular, they show how the theoretical model helped classify the students' verbal metacognitive actions into four regulating activities (*new idea*, *changing strategy*, *correcting errors*, and *reinterpretation*) and six monitoring activities (*assessment of knowledge*, *assessment of understanding*, *assessment of strategy appropriateness*, *assessment of progress toward goal*, *assessment of strategy execution*, and *assessment of accuracy or sense of result*).

Finally, this section is made up of sub-sections presenting students' spontaneous metacognitive activities triggered at all of the multiple levels (i.e., the individual, social, and environmental levels). The students' excerpts presented in each sub-section illustrate how the theoretical model identified the particular level triggering a student's metacognitive activity, by looking at the sources activating the students' verbal metacognitive actions, whether they were triggered: (a) at the individual level by oneself; (b) at the social level by others; or (c) at the environmental level by something in the learning environment. Specifically, these examples show how the re-conceptualization of metacognition on multiple levels in the theoretical model helped interpret how students' thinking became metacognitive in context.

In these ways, this section demonstrates that the theoretical model of metacognition on multiple levels is appropriate and useful for identifying and interpreting metacognitive activities in complex modeling activities. This section starts this demonstration with a sub-section presenting overall examples of students' spontaneous metacognitive activities triggered at the individual, social, and environmental levels, as follows.

Metacognitive activities triggered at the individual, social, and environmental levels. A large number of metacognitive activities triggered at all of the multiple levels (i.e., the individual, social, and environmental levels) were extensively identified across the three MEAs–Summer Jobs, Volleyball, and Paper Airplane MEAs, across the seven groups, and across the group members. The following sections present some illustrations of the identified metacognitive activities triggered at all of the multiple levels.

Metacognitive activities triggered at the individual and social levels. The following excerpt illustrates how students' metacognitive activities during the Paper Airplane MEA were manifested in the transcripts at the individual and social levels. The excerpt below was drawn from the transcript of student Group 5, where three girls (S1, S2, and S3) worked on the Paper Airplane MEA.

(*Group 5, Paper Airplane MEA, Transcript Lines 66-74*)

[66] *S2*: So, I have an idea, Brittany [S3]. I don't know if it'll work, but it's an idea. I was thinking that what we could do is we could average each of these for all of the planes. (*New idea, Individual Level*)

[71] *S3*: Hold on, so what? (*Assessment – understanding: Thinking ABOUT S2's way of thinking, Social Level*)

[72] *S2*: So we'd each take a pilot. There are only 3 pilots and there are 3 of us. And then you'll average each of these. (*Not coded for Metacognition*)

[74] *S3*: But they're all different. How would that—? (*Assessment – strategy appropriateness: Thinking ABOUT S2's way of thinking, Social Level*)

In the excerpt from the beginning of the problem solving, student S2 herself triggered her new idea, "I don't know if it'll work, but it's an idea. I was thinking that what we could do is we could average each of these for all of the planes" (S2, G5, PA, 66), using the concept of the average at the individual level. Then, student S2's new idea triggered student S3's assessment of her understanding of student S2's way of thinking at the social level: "Hold on, so what?" (S3, G5, PA, 71). Afterward, student S2's additional explanation about her way of thinking, "So we'd each take a pilot. There are only 3 pilots and there are 3 of us. And then you'll average each of these" (S2, G5, PA, 72), triggered student S3's assessment of strategy appropriateness, suggested by student S2 at the social level: "But they're all different. How would that—?" (S3, G5, PA, 74).

Metacognitive activities triggered at the environmental level. Illustrated in Table 4.1 below are various ways in which the students' metacognitive activities were triggered at the environmental level by something in the learning environment, such as calculators, computers, types of problems, constraints of the problem, and so on. For example, the existence of a calculator triggered student S3's assessment of strategy appropriateness: "Well, we kind of have to work together because we don't all have a calculator," (S3, G5, PA, 82) and, "Go get a calculator" (S3, G5, SJ, 221). Student S2's assessment of strategy appropriateness was also triggered by the existence of a calculator, as illustrated by the statement, "What'd we do if we didn't use calculators. Then you'd have to figure out another way to do it on the calculators" (S2, G5, PA, 158). The existence of a protractor also triggered student M2's assessment of strategy appropriateness, as illustrated in the statement, "It seems like everybody used a protractor. I feel like we did this wrong even though it is right" (M2, G3, PA, 212).

Table 4.1

Examples of Students' Metacognitive Activities Triggered at the Environmental Level

Transcript Segment	Description of Metacognitive Activities Triggered at the Environmental Level
(S3, G5, PA, 82)	Well, we kind of have to work together because we don't all have a calculator. *(Assessment – strategy appropriateness, Environmental Level)*
(S2, G5, PA, 158)	What'd we do if we didn't use calculators. Then you'd have to figure out another way to do it on the calculators. *(Assessment – strategy*

	appropriateness, Environmental Level)
(M2, G3, PA, 212)	It seems like everybody used a protractor. I feel like we did this wrong even though it is right. *(Assessment – strategy appropriateness, Environmental Level)*
(S3, G5, SJ, 221)	Go get a calculator. *(Assessment – strategy appropriateness: Thinking ABOUT S1's way of thinking, Environmental Level)* [Recognizing the allowance of using a calculator–"You're allowed to use a calculator." (S1, G5, SJ, 218), when S1 shared her thinking–"Like, show how many hours they worked versus how much money they get?" (S1, G5, SJ, 215)]
(S2, G5, PA, 351)	You know, we were stupid. We were supposed to do both. *(Assessment – accuracy or sense of result, Environmental Level)* [Going back to the problem, and figuring out four competing variables for two subtasks, the most accurate problem and the best floater problem]
(P4, G6, SJ, 158)	We're not writing it yet. We still have tomorrow. *(Assessment – progress toward goal, Environmental Level)*
(P2, G6, VB, 609)	And the letter isn't even due until tomorrow. We aren't supposed to do this until tomorrow. *(Assessment – progress toward goal, Environmental Level)*
(W2, G7, SJ, 144)	We have to make sure we pick all of the best people. We have tomorrow too. (*Assessment – progress toward goal, Environmental Level)*
(H4, G1, PA, 366)	You just got to remember we don't get homework when we do this, so it's all worth it. *(Assessment – strategy appropriateness, Environmental Level)*

Another example of the students' metacognitive activities triggered at the environmental level, which were triggered by the type of problem involving four competing variables for two subtasks, is illustrated when student S2 assessed the accuracy or sense of result her group developed: "You know, we were stupid. We were supposed to do both" (S2, G5, PA, 351). The student went back to the problem and figured out four competing variables for two subtasks, the most accurate problem and the best floater problem. The constraint of the problem activated her assessment of accuracy or sense of result.

Finally, the students evaluated their own progress toward the goal, considering various factors in the learning environment. For example, having enough periods of time given for problem solving made not only a positive impact on students' assessment of progress toward their goal for productive problem solving, as illustrated by student W2's way of thinking, "We have to make sure we pick all of the best people. We have tomorrow too" (W2, G7, SJ, 144), but also a negative impact on students' assessment of progress toward their goal for productive problem solving, as illustrated in student P2's statement, "And the letter isn't even due until tomorrow. We aren't supposed to do this until tomorrow" (P2, G6, VB, 609). In addition, student H4's statement, "You just got to remember we don't get homework when we do this, so it's all worth it" (H4, G1, PA, 366), exemplified a negative impact of the homework burden on her assessment of strategy appropriateness for productive problem solving.

All of the metacognitive activities triggered at both the individual and social levels. While collaboratively working in a group of three to four during three different problem-solving sessions, the students revealed their spontaneous metacognitive activities by their own needs at the individual level or in response to one another's way of thinking at the social level. As illustrated with the data excerpts in this section, all of the metacognitive activities were identified at both the individual and social levels: four regulating activities (*new idea*, *changing strategy*, *correcting errors*, and *reinterpretation*) and six monitoring activities (*assessment of knowledge*, *assessment of understanding*, a*ssessment of strategy appropriateness*, *assessment of progress toward goal*, *assessment of strategy execution*, and *assessment of accuracy or sense of result*) at both the individual and social levels. This section will present detailed illustrations of how the theoretical model of metacognition on multiple levels helps appropriately distinguish metacognition from cognition, and how it helps make the distinction and interpretation among students' metacognitive activities while they collaboratively solve complex mathematical modeling tasks.

Regulating activities: new idea. Across the three MEAs–Summer Jobs, Volleyball, and Paper Airplane MEAs, the students demonstrated their use of the first type of regulating activity, *new idea*, at both the individual and social levels, as illustrated with the data excerpts in this section. This is a metacognitive activity because it involves a regulating process in which students identify new (alternative) information (strategy), based on an assessment of their own thinking. *New idea* occurred when potentially useful information sprouted from the students, mainly in the beginning of problem solving, or when an alternative approach came to light during problem solving.

New idea triggered at the individual level. First, Table 4.2 illustrates examples of students' regulating activities identified as a *new idea* triggered at the individual level. In each excerpt, the student herself or himself triggered her or his new idea. In the beginning of problem solving, the students shared potentially useful information, which they identified through assessing their own thinking of the problems, with other group members to lead problem solving, such as, "I have an idea. We are going to determine a system in which everything they do is worth points" (M2, G3, VB, 44); "I think we should first see who is fast on their feet" (K2, G2, VB, 6); "And then whichever one has the highest average is the winner" (P2, G6, PA, 56); and so on. While student S2's *new idea,* "Okay, the first thing we need to do is we need to compare how many hours they worked and how much money they earned when they worked" (S2, G5, SJ, 150), was not adopted immediately, student H3's *new idea,* "We need to do the money they made over the hours they worked" (H3, G1, SJ, 67), led the group to a strategic change.

Table 4.2

Examples of Students' Regulating Activities Identified as New Idea Triggered at the Individual Level

Transcript Segment	Description of Regulating Activities Identified as *New Idea* Triggered at the Individual Level
(H3, G1, SJ, 67)	We need to do the money they made over the hours they worked. *(New idea: Thinking ABOUT a way of thinking, Individual Level)*
(M1, G3, SJ, 238)	Hours to money ratio, guys. Divide the hours by the money. *(New idea: Thinking ABOUT a way of thinking, Individual Level)*
(S2, G5, SJ, 150)	Okay, the first thing we need to do is we need to compare how many hours they worked and how much money they earned when they worked. *(New idea: Thinking ABOUT a way of thinking, Individual Level)*
(P2, G6, SJ, 22)	I think we should go through the busy first. The slow is going to definitely make the biggest difference because, if you can sell when it's slow, it means that you're probably the best. *(New idea: Thinking ABOUT a way of thinking, Individual Level, Individual Level)*
(H4, G1, VB, 61)	So my plan is that we put the people of the same…like there is five ten, five ten, five eight, or five nine. We put each player…and we make sure they can jump the same height. *(New idea: Thinking ABOUT a way of thinking, Individual Level, Individual Level)*
(M2, G3, VB, 44)	I have an idea. We are going to determine a system in which everything they do is worth points. *(New idea: Thinking ABOUT a way of thinking, Individual Level, Individual Level)*
(S2, G5, VB, 38)	And then I'll take the top person and the worst person, second best, second worst, third best, third worst. Kind of do it like that. *(New idea: Thinking ABOUT a way of thinking, Individual Level)*
(K2, G2, VB, 6)	I think we should first see who is fast on their feet. *(New idea: Thinking ABOUT a way of thinking, Individual Level)*
(P2, G6, PA, 56)	And then whichever one has the highest average is the winner. *(New idea:*

	Thinking ABOUT a way of thinking, Individual Level)
(K2, G2, PA, 71)	I want to find the average so that I can find the best one. *(New idea: Thinking ABOUT a way of thinking, Individual Level)*
(S2, G5, PA, 353)	Well, time in the air is the best floater, but it's still C for me. And the distance and the time go together and the distance and angle from target go together. *(New idea: Thinking ABOUT a way of thinking, Individual Level)*
(M2, G3, PA, 319)	We need to find a way to incorporate diameter [of the circles of the plane landings] and distance [from the target]. *(New idea: Thinking ABOUT a way of thinking, Individual Level, Individual Level)*

New idea also occurred when the students confronted some problems with their prior strategies, and they needed an alternative approach to solve the problems. For example, student M2 shared an alternative approach: "We need to find a way to incorporate diameter [of the circles of the plane landings] and distance [from the target]" (M2, G3, PA, 319), to correct a flaw in their prior strategy. When recognizing a flaw in their final solution, student S2 shared her *new idea,* "Well, time in the air is the best floater, but it's still C for me. And the distance and the time go together and the distance and angle from target go together" (S2, G5, PA, 353).

New idea triggered at the social level. As shown below, Table 4.3 illustrates examples of the identified students' regulating activities, a *new idea*, triggered at the social level, which was triggered through one's interactions with others. First, *new idea* came to light from the results of students' monitoring activities, such as a*ssessment of strategy appropriateness*, which was prompted by others' way of thinking. For example, assessing student M2's way of thinking, student M3 shared his *new idea,* "I mean. You need to divide slow hours by something to make them the same as busy hours" (M3, G3, SJ, 242). When student H3 shared her way of thinking, "…but it doesn't matter about slow or busy…" (H3, G1, SJ, 66), student H1 disagreed with her and suggested *new idea,* "Well, we need to know the average of the busy hours divided by the average of the slow. And then we can combine them" (H1, G1, SJ, 69).

Table 4.3

Examples of Students' Regulating Activities Identified as New Idea Triggered at the Social Level

Transcript Segment	Description of Regulating Activities Identified as *New Idea* Triggered at the Social Level
(M3, G3, SJ, 242)	I mean. You need to divide slow hours by something to make them the same as busy hours. *(New Idea: Thinking ABOUT M2's way of thinking, Social Level)* [Triggered by H3's way of thinking–"Don't you mean that busy hours are more than slow hours because slow hours, you are just sitting there" (M2, G3, SJ, 240)]
(H4, G1, SJ, 44)	So do we need to find a common denominator [of hours worked]? (*New Idea: Thinking ABOUT H2 and H3's way of thinking, Social Level*) [Prompted by assessment of accuracy or sense of result from which H2 and H3 made though looking over the data]
(H1, G1, SJ, 69)	Well, we need to know the average of the busy hours divided by the average of the slow. And then we can combine them. *(New Idea: Thinking ABOUT H3's way of thinking, Social Level)* [Triggered by H3's way of thinking– "…but it doesn't matter about slow or busy…" (H3, G1, SJ, 66)]
(P1, G6, SJ, 30)	Wait, let's figure out what we would suggest is a good day of hours and what would be a bad day. Fifteen plus hours in a month? *(New idea: Thinking ABOUT P4's way of thinking, Social Level)* [Triggered by P4's way of thinking–"So, we'll go through and circle the highest hours and the highest paid or what you think is good" (P4, G6, SJ, 25)]
(S3, G5, VB, 74)	Oh! You can make a key. Yeah. Make a key [Star = good; A smiley face = closer look at; A line = middle; & A sad face = not the best]. *(New idea: Thinking ABOUT S2's way of thinking, Social Level)* [Triggered by S2's way of thinking–"I'm going to put a smiley face next to her" (S2, G5, VB, 73)]
(P4, G6, VB, 236)	1 and 6, 2 and 3, and 4 and 5. *(New idea: Thinking ABOUT P2's way of thinking, Social Level)* [Triggered by P2's way of thinking–"So we can have 2 of these people on each team. So, we could have number 1 and—" (P2, G6, VB, 234)]

(M2, G3, VB, 11)	We have to make three equal teams. Why are we putting them in order tallest to shortest? What if they are just fast? *(New idea: Thinking ABOUT M3's way of thinking, Social Level)* [Triggered by M3's way of thinking–"How about we sort them tallest to shortest?" (M3, G3, VB, 10)]
(R3, G4, VB, 16)	Not always though. I think we should use the leaps… *(New idea: Thinking ABOUT R1's way of thinking, Social Level)* [Triggered by R1's way of thinking–"But the shorter people can jump higher" (R1, G4, VB, 15)]
(S3, G5, PA, 226)	Then, put— [Writing down 1st, 1st, 3rd, and 4th] *(New idea: Thinking ABOUT S2's way of thinking, Social Level)* [Triggered by S2's way of thinking–"What if it's a tie?" (S2, G5, PA, 225)]
(H3, G1, PA, 139)	Only the ones that are relatively close. *(New Idea: Thinking ABOUT H1's way of thinking, Social Level)* [Triggered by H1's way of thinking–"Wait, you have to angle every single throw?" (H1, G1, PA, 138)]
(K1, G2, PA, 130)	Tie breaker! *(New idea: Thinking ABOUT K3's way of thinking, Social Level)* [Triggered by K3's way of thinking–"There is a problem. We have two winners" (K3, G2, PA, 128)]
(M3, G3, PA, 316)	Then, what if we just measured the distance of all of them? *(New idea: Thinking ABOUT M3's way of thinking, Social Level)* [Triggered by M3's way of thinking–"That our theory would be proven wrong? But, it's okay" (M3, G3, PA, 315)]

The students also shared their *new idea*, as a response to others, to complement others' ways of thinking, such as, "Wait, let's figure out what we would suggest is a good day of hours and what would be a bad day. Fifteen plus hours in a month?" (P1, G6, SJ, 30) and, "Oh! You can make a key. Yeah. Make a key [Star = good; A smiley face = closer look at; A line = middle; & A sad face = not the best]" (S3, G5, VB, 74). For example, when student R1 shared his way of thinking, "But the shorter people can jump higher" (R1, G4, VB, 15), student R3 partially agreed with him and suggested *new idea,* "Not always though. I think we should use the leaps…" (R3, G4, VB, 16).

New idea also occurred at the social level when the students mentioned an alternative idea to others' way of thinking. For example, when student M3 shared his way of thinking, "How about we sort them tallest to shortest?" (M3, G3, VB, 10), student M2 suggested an alternative idea: “We have to make three equal teams. Why are we putting them in order tallest to shortest? What if they are just fast?” (M2, G3, VB, 11). Finally, the students revealed a *new idea* triggered at the social level as a solution to some problems, which were recognized and issued by others. For example, when student S2 and student K3 issued the similar problem, “What if it’s a tie?” (S2, G5, PA, 225), and, “There is a problem. We have two winners” (K3, G2, PA, 128), while they worked within each group on the Paper Airplane MEA, student S3 and student K1 shared their *new idea* to solve the problem: “Then, put— [Writing down 1st, 1st, 3rd, and 4th]” (S3, G5, PA, 226), and, “Tie breaker!” (K1, G2, PA, 130) within their groups, respectively.

Regulating activities: changing strategy. The second type of regulating activity, *changing strategy*, is a metacognitive activity because it involves a regulating action in which students change their strategy, based on an assessment of the adequacy of their own thinking. *Changing strategy* occurred when an alternative approach was adopted by the students who originally suggested it at the individual level, or by others at the social level. This type of regulating activity inherently involves a type of monitoring activity, a*ssessment of strategy appropriateness*, as described in Chapter 3; thus, cross-annotating was used, as illustrated with the data excerpts below.

Changing strategy triggered at the individual level. First, the student herself or himself immediately changed previously implemented strategies based on the results from *assessment of strategy appropriateness* triggered at the individual level (Table 4.4). For example, followed by other group members’ acceptance of the alternative approach through assessing the appropriateness, student W1 and student H4 immediately changed prior strategies based on their assessment of their adopting an alternative approach, respectively: “Okay, this doesn’t really matter. We are averaging how much they made throughout the whole year. Then we average it all together and divide by 2 [after adding the average of hours to the average of money] and we will see the top 3 and the bottom 3” (W1, G7, SJ, 102), and, “But their skills would even it out. We should label if they are good or bad next to them” (H4, G1, VB, 187).

Table 4.4

Examples of Students’ Regulating Activities Identified as Changing Strategy Triggered at the Individual Level

Transcript Segment	Description of Regulating Activities Identified as *Changing Strategy* Triggered at the Individual Level
(W1, G7, SJ, 102)	Okay, this doesn’t really matter. We are averaging how much they made throughout the whole year. Then we average it all together and divide by 2 [after adding the average of hours to the average of money] and we will see the top 3 and the bottom 3. *(Assessment – strategy appropriateness & Changing Strategy: Thinking ABOUT a way of thinking, Individual Level)*
(H4, G1, VB, 187)	But their skills would even it out. We should label if they are good or bad next to them. *(Assessment – strategy appropriateness & Changing Strategy: Thinking ABOUT a way of thinking, Individual Level)*
(R3, G4, VB, 24)	We are going to use these three [the vertical leap, the forty-meter dash, and serve results]. David [R2], I want who has the best and who has the worst. Best is number 1. *(Assessment – strategy appropriateness & Changing Strategy: Thinking ABOUT a way of thinking, Individual Level)*
(M2, G3, PA, 120)	We should do it by which one had a tighter grouping around X. C’s grouping is like that. *(Assessment – strategy appropriateness & Changing Strategy: Thinking ABOUT a way of thinking, Individual Level)*
(M2, G3, PA, 131)	So we do not have a circle maker of any kind? If we had a circle maker, we could make the circle. *(Assessment – strategy appropriateness & Changing Strategy: Thinking ABOUT a way of thinking, Individual Level)*
(S2, G5, PA, 356)	These two go together and these two go together. This is the best out of these two, you find the best letter out of these two; you find the best accuracy. *(Assessment – strategy appropriateness & Changing Strategy: Thinking ABOUT a way of thinking, Individual Level)*

With a tacit agreement from the group members, the students also immediately adopted alternative approaches, as illustrated by the statements "We are going to use these three [the vertical leap, the forty-meter dash, and serve results]. David [R2], I want who has the best and who has the worst. Best is number 1" (R3, G4, VB, 24), and, "These two go together and these two go together. This is the best out of these two, you find the best letter out of these two; you find the best accuracy" (S2, G5, PA, 356). Without additional issues from other members, the groups used the alternative approach, respectively.

Changing strategy triggered at the social level. Illustrated in Table 4.5 are examples of how the students' regulating activity*, Changing Strategy*, was triggered at the social level. *Changing strategy* occurred at the social level when the students adopted an alternative approach, which was not immediately implemented by the students who originally suggested it, after assessing the appropriateness of the alternative approach. For example, when *new idea,* "Hours to money ratio, guys. Divide the hours by the money" (M1, G3, SJ, 238), came to light, the alternative approach from student M1 was evaluated, and afterward implemented by other group members. This was illustrated by the statement, "Let's just do total hours over total money earned. Find out how much money they earned" (M2, G3, SJ, 285). Similarly, by adopting student S1's alternative approach, "Like, show how many hours they worked versus how much money they get?" (S1, G5, SJ, 215), student S2 changed their prior strategy, as illustrated by the statement, "Okay. Now we should kind of compare. We could—" (S2, G5, SJ, 226).

Table 4.5

Examples of Students' Regulating Activities Identified as Changing Strategy Triggered at the Social Level

Transcript Segment	Description of Regulating Activities Identified as *Changing Strategy* Triggered at the Social Level
(M2, G3, SJ, 285)	Let's just do total hours over total money earned. Find out how much money they earned. *(Assessment – strategy appropriateness & Changing Strategy: Thinking ABOUT M1's way of thinking, Social Level)* [Triggered by M1's way of thinking–"Hours to money ratio, guys. Divide the hours by the money" (M1, G3, SJ, 238)]
(S2, G5, SJ, 226)	Okay. Now we should kind of compare. We could— *(Assessment – strategy appropriateness & Changing Strategy: Thinking ABOUT S1's way of thinking, Social Level)* [Triggered by S1's way of thinking–"Like, show how many hours they worked versus how much money they get?" (S1, G5, SJ, 215)]
(H1, G1, SJ, 72)	You add up those hours and then you add up the money collected and then for June you do steady. If we all do it, we have more chances it will be right. *(Assessment – strategy appropriateness & Changing Strategy: Thinking ABOUT H3's way of thinking, Social Level)* [Triggered by H3's way of thinking–"We need to do the money they made over the hours they worked" (H3, G1, SJ, 67)]
(S2, G5, VB, 75)	Star equals good players. *(Assessment – strategy appropriateness & Changing Strategy: Thinking ABOUT S3's way of thinking, Social Level)* [Triggered by S3's way of thinking–"Oh! You can make a key. Yeah. Make a key" (S3, G5, VB, 74)]
(M3, G3, PA, 283)	Yes. We found the two points farthest away from each other and then drew a circle on the line so this was the diameter. The circle that had the smallest diameter and the center being closest to X was the most accurate and the one that had the longest distance and the circle that had the biggest diameter was the best floater. *(Assessment – strategy appropriateness & Changing Strategy: Thinking ABOUT M2's way of thinking, Social Level)* [Triggered by M2's way of thinking–"Because my idea was just how close every circle was to the X determines accuracy" (M2, G3, PA, 271)]
(M3, G3, PA, 320)	This one would win most accurate because all five of the points are closer to X than these ones, but combined… It's fine, Paul [M2]. *(Assessment – strategy appropriateness & Changing Strategy: Thinking ABOUT M2's way of thinking, Social Level)* [Triggered by M2's way of thinking–"We need to find a way to incorporate diameter and distance" (M2, G3, PA, 319)]

(H1, G1, PA, 316)	Then…the closest it lands to the target…let's find the average…(Mumbled)… where's that sheet? *(Assessment – strategy appropriateness & Changing Strategy: Thinking ABOUT H3's way of thinking, Social Level)* [Triggered by H3's way of thinking–"Just draw a little circle in the middle surrounding the closest ones" (H1, G1, PA, 312)]

In addition, *changing strategy* immediately occurred at the social level as a judgment to an alternative approach, which was suggested by others, reflecting immediate assessment of strategy appropriateness at the social level. Once student S3 suggested, "Oh! You can make a key. Yeah. Make a key" (S3, G5, VB, 74), student S2 adopted and implemented her alternative approach, "Star equals good players" (S2, G5, VB, 75). Also, student M2's way of thinking, "Just draw a little circle in the middle surrounding the closest ones" (H1, G1, PA, 312), led student M3 to immediately adopt the strategic change, as illustrated by the statement, "Then…the closest it lands to the target…let's find the average…(Mumbled)… where's that sheet?" (H1, G1, PA, 316).

Regulating activities: correcting errors. While collaboratively working in a group, the students also demonstrated their use of the third type of regulating activity, *correcting errors*, triggered at both the individual and social levels. This is a metacognitive activity because it involves a regulating action in which students correct their errors during collaborative problem solving, based on an assessment of their own thinking. As described in Chapter 3, this type of regulating activity inherently involves the type of monitoring activity, *assessment of strategy execution*; thus, cross-annotating was used, as illustrated with the data excerpts below. At the individual level, *correcting errors* occurred when the students corrected their own errors or others' through their own *assessment of strategy execution*–without requesting anything from others. At the social level, *correcting errors* occurred when the students recognized and corrected errors based on others' *assessment of strategy execution*.

Correcting errors triggered at the individual level. First, as illustrated in Table 4.6 below, the students' conversational statements conveys thinking focused on self-correcting based on the results from their own monitoring activity, *assessment of strategy execution*. For example, the students evaluated their own strategy execution, and recognized and corrected errors, as illustrated with the statements, "Oh, I forgot to add the coach's comments! That means Gertrude is a negative one" (M3, G3, VB, 155), "I should have divided it by 3 because there are three teams" (M1, G3, VB, 237), and, "I did the wrong two points farthest from each other. So the correct diameter of that circle is actually 4.2, which means that C was actually not that accurate as far as average goes" (M2, G3, PA, 220).

Table 4.6

Examples of Students' Regulating Activities Identified as Correcting Errors Triggered at the Individual Level

Transcript Segment	Description of Regulating Activities Identified as *Correcting Errors* Triggered at the Individual Level
(K1, G2, SJ, 27)	You divide it by what? *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT a way of thinking, Individual Level)*
(H4, G1, SJ, 80)	You don't divide. Oh you do to get the average. And do the same thing for money. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT a way of thinking, Individual Level)*
(P4, G6, SJ, 38)	No, just do 15 or higher. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT a way of thinking, Individual Level)*
(M3, G3, VB, 155)	Oh, I forgot to add the coach's comments! That means Gertrude is a negative one. *(Assessment – strategy execution & Correcting Errors: Thinking*

	ABOUT a way of thinking, Individual Level)
(M1, G3, VB, 237)	I should have divided it by 3 because there are three teams. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT a way of thinking, Individual Level)*
(M1, G3, VB, 164)	For the speed you have to do above that time is minus one. Because the slower the time the faster you are. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT a way of thinking, Individual Level)*
(H1, G1, VB, 221)	Not good. Christina I'm putting as bad. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT a way of thinking, Individual Level)*
(K1, G2, PA, 136)	That is not 2. It is 1.988888889. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT a way of thinking, Individual Level)*
(S2, G5, PA, 399)	So, no wait. We put distance from start and time in the air to decide the best floater. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT a way of thinking, Individual Level)* [Looking at the formula written down the letter]
(M2, G3, PA, 220)	I did the wrong two points farthest from each other. So the correct diameter of that circle is actually 4.2, which means that C was actually not that accurate as far as average goes. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT a way of thinking, Individual Level)*
(M3, G3, PA, 121)	No, C's grouping is like this. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT a way of thinking, Individual Level)*

Correcting errors also occurred at the individual level when the students evaluated others' strategy execution–despite no request from others, and they found and corrected errors, as illustrated with the statements, "You divide it by what?" (K1, G2, SJ, 27), " No, just do 15 or higher" (P4, G6, SJ, 38), "So, no wait. We put distance from start and time in the air to decide the best floater" (S2, G5, PA, 399), and, "No, C's grouping is like this" (M3, G3, PA, 121).

Correcting errors triggered at the social level. As revealed in Table 4.7 below, the students also corrected errors depending on others' monitoring activities at the social level. Others' *assessment of strategy execution* led the students to detect and correct errors. For example, student H3's *assessment of strategy execution,* "You divided those by three, right?" (H3, G1, SJ, 149) allowed student H1 to detect and correct errors: "Wait, I didn't divide Jose's by 3" (H1, G1, SJ, 151). Student S2's *assessment of strategy execution,* "We divide the money earned by the time worked to—" (S2, G3, SJ, 378), also led student S1 to correct errors: "Oh, I get it now" (S1, G5, SJ, 379). Triggered by others' monitoring activities, the students also evaluated their strategy execution and corrected errors, as illustrated by the statements, "Well then put her as average" (H3, G1, VB, 304); "Oh yeah! That's on the edge of the circle, wow" (M3, G3, PA, 412); and, "Oh wait. I'd be 1st, 1st, 4th, 4th, right? No. 1st, 1st, 3rd, 3rd. You are right" (S2, G5, PA, 236).

Table 4.7

Examples of Students' Regulating Activities Identified as Correcting Errors Triggered at the Social Level

Transcript Segment	Description of Regulating Activities Identified as *Correcting Errors* Triggered at the Social Level
(H1, G1, SJ, 151)	Wait, I didn't divide Jose's by 3. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT H3's way of thinking, Social Level)* [Triggered by H3's way of thinking–"You divided those by three, right?" (H3, G1, SJ, 149)]
(S1, G5, SJ, 233)	No, 699 divided by 10. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT S3's way of thinking, Social Level)* [Triggered by S3's way of thinking–"…divided by 699, right?" (S2, G3, SJ, 232)]
(S1, G5, SJ, 379)	Oh, I get it now. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT S2's way of thinking, Social Level)* [Triggered by M2's way of thinking–"We divide the money earned by the time worked to—" (S2, G3, SJ, 378)]
(H1, G1, VB, 265)	I'm writing 'bad.' Tina needs to be bad. Actually, I'm writing average. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT H3's way of thinking, Social Level)* [Triggered by H3's way of thinking–"I think she's average" (H3, G1, VB, 264)]

(H3, G1, VB, 304)	Well then put her as average. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT H1 and H2's way of thinking, Social Level)* [Considering H1's negative evaluation and H2's positive evaluation about the given data]
(M3, G3, PA, 412)	Oh yeah! That's on the edge of the circle, wow. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT M2's way of thinking, Social Level)* [Triggered by M2's way of thinking–"Okay. No, that's not the circle. You did it wrong" (M2, G3, PA, 411)]
(S2, G5, PA, 190)	I have 22—no 23.4. No, wait. 22. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT S3's way of thinking, Social Level)* [Triggered by S3's way of thinking–"You might want to divide it" (S3, G5, PA, 189)]
(S2, G5, PA, 236)	Oh wait. I'd be 1st, 1st, 4th, 4th, right? No. 1st, 1st, 3rd, 3rd. You're right. *(Assessment – strategy execution & Correcting Errors: Thinking ABOUT S3's way of thinking, Social Level)*

Regulating activities: reinterpretation. As illustrated with the data excerpts in this section, the students demonstrated their use of the final type of regulating activity, *reinterpretation*, triggered at both the individual and social levels. This is a metacognitive activity because it involves a regulating process in which students reinterpret problems or problem situations, based on an assessment of their own thinking. This type of regulating activity occurred when the students reinterpreted problems, previously implemented strategies, or alternative approaches in order to support their own or others' understanding of them, and ultimately to be able to support their monitoring activities.

Reinterpretation triggered at the individual level. The students reinterpreted their problem-solving process according to their own needs (Table 4.8 below). During problem solving, the students reinterpreted what they were supposed to do, as illustrated by the statements, "No, he wants to know how to do it" (H3, G1, SJ, 278); "All you've got to do is make sure that why we want these people…" (S1, G5, SJ, 212); and, "William [P2], we're supposed to find a formula" (P4, G6, PA, 381). To share and support their ways of thinking, student H3 and student S2 reinterpreted problem situations, respectively, as follows: “Now we should figure out… Sure, she works more hours, but how much money is she making? The less hours she works and the more money she brings in, it will be better. If it takes her 50 hours to get $1,000 and it takes her 10 hours to get $2,000 then who is doing better? She’s doing better” (H3, G1, SJ, 183), and, “But he worked here and he didn’t make that much. But it was slow. So, we need to find out how much money they make every hour” (S2, G5, SJ, 155).

Table 4.8

Examples of Students’ Regulating Activities Identified as Reinterpretation Triggered at the Individual Level

Transcript Segment	Description of Regulating Activities Identified as *Reinterpretation* Triggered at the Individual Level
(H3, G1, SJ, 183)	Now we should figure out… Sure, she works more hours, but how much money is she making? The less hours she works and the more money she brings in, it will be better. If it takes her 50 hours to get $1,000 and it takes her 10 hours to get $2,000 then who is doing better? She’s doing better. *(Reinterpretation: Thinking ABOUT a way of thinking, Individual Level)*
(H3, G1, SJ, 278)	No, he wants to know how to do it. *(Reinterpretation: Thinking ABOUT a way of thinking, Individual Level)*
(S1, G5, SJ, 212)	All you’ve got to do is make sure that why we want these people… *(Reinterpretation: Thinking ABOUT a way of thinking, Individual Level)*
(S2, G5, SJ, 155)	But he worked here and he didn’t make that much. But it was slow. So, we need to find out how much money they make every hour. *(Reinterpretation: Thinking ABOUT a way of thinking, Individual Level)*
(M3, G3, VB, 286)	How to get equal teams… would this procedure work for 200 people? More than how long have we been doing this? An hour? *(Reinterpretation: Thinking ABOUT a way of thinking, Individual Level)*
(P4, G6, PA, 381)	William [P2], we’re supposed to find a formula. *(Reinterpretation: Thinking ABOUT a way of thinking, Individual Level)*
(S2, G5, PA, 330)	But we don’t need to know which one was the best. What we need to do is

	figure out a formula. And our formula would be to average everything and then— *(Reinterpretation: Thinking ABOUT a way of thinking, Individual Level)*
(M2, G3, PA, 184)	So just figure out the diameter of all the circles of the plane landings. How do we find out the floater? So that does not work because some goes up and then floats to the ground like that at a ridiculous distance from the target. *(Reinterpretation: Thinking ABOUT a way of thinking, Individual Level)*
(M2, G3, PA, 308)	But it also had to have a good time because otherwise, it was not a very good floater. If it goes fast and then just hits the ground, then it was not a very good floater. *(Reinterpretation: Thinking ABOUT a way of thinking, individual Level)*

On the other hand, student M2 reinterpreted problem situations to issue flaws, which the previously implemented strategy involved, as illustrated by the statements, “So just figure out the diameter of all the circles of the plane landings. How do we find out the floater? So that does not work because some goes up and then floats to the ground like that at a ridiculous distance from the target” (M2, G3, PA, 184), and, “But it also had to have a good time because otherwise it was not a very good floater. If it goes fast and then just hits the ground, then it was not a very good floater” (M2, G3, PA, 308).

Reinterpretation triggered at the social level. The students reinterpreted problem-solving processes in response to others’ way thinking, in other words, according to others’ needs (Table 4.9 below). For example, when student S3 asked, “I can do more information?” (S3, G5, SJ, 214), student S1 provided a reinterpretation to support student S3’s understanding of the alternative approach, which was suggested by her: “Like, show how many hours they worked versus how much money they get? So, add up and divide this by that number for each of them and see which—” (S1, G5, SJ, 215). Student H3 reinterpreted the alternative approach, which was suggested by student H4: “So we write down all the good people, all the medium people and all the bad people, and then mix them together" (H3, G1, VB, 65), to support her own and peers' understanding of the alternative approach.

Table 4.9

Examples of Students' Regulating Activities Identified as Reinterpretation Triggered at the Social Level

Transcript Segment	Description of Regulating Activities Identified as *Reinterpretation* Triggered at the Social Level
(M3, G3, SJ, 239)	No, but slow hours are worth more than busy hours. (*Reinterpretation: Thinking ABOUT M1's way of thinking, Social Level)* [Triggered by M1's way of thinking–"Hours to money ratio, guys. Divide the hours by the money" (M1, G3, SJ, 238)]
(M3, G3, SJ, 245)	If you ask me if you add the slow hours and the steady hours together, you get at least some busy hours. *(Reinterpretation: Thinking ABOUT M3's way of thinking, Social Level)* [Triggered by M3's way of thinking–"I mean. You need to divide slow hours by something to make them the same as busy hours" (M3, G3, SJ, 242)]
(S1, G5, SJ, 215)	Like, show how many hours they worked versus how much money they get? So, add up and divide this by that number for each of them and see which— *(Reinterpretation: Thinking ABOUT S3's way of thinking, Social Level)* [Triggered by S3's way of thinking–"I can do more information?" (S3, G5, SJ, 214)]
(H3, G1, VB, 65)	So we write down all the good people, all the medium people and all the bad people, and then mix them together. *(Reinterpretation: Thinking ABOUT H4 and H2's way of thinking, Social Level)* [Based on H3 and H2's assessment of appropriateness of strategy suggested by H4]
(R2, G4, VB, 14)	Actually, it does because if you are taller, you can reach over the net easier. *(Reinterpretation: Thinking ABOUT R1's way of thinking, Social Level)* [Triggered by R1's way of thinking–"Yes, it [height] does not really matter" (R1, G4, VB, 13)]
(S3, G5, PA, 344)	Okay, well, we still need a formula that fits the whole thing. *(Reinterpretation: Thinking ABOUT S2's way of thinking, Social Level)*
(M3, G3, PA, 305)	These ones landed in a tighter area, and these ones are more spread out. These ones flew the farthest and we were basing this one…where we got the largest diameter was the best floater because it flew the farthest. *(Reinterpretation: Thinking ABOUT the teacher's way of thinking, Social Level)* [Triggered by the teacher's way of thinking–"Not necessarily, did you guys not say something about how small or big the circle was?" (T, G3, PA, 297)]

The students also reinterpreted problem situations at the social level to refute others' ways of thinking. When student M1 and student R1 shared their ways of thinking, respectively, "Hours to money ratio, guys. Divide the hours by the money" (M1, G3, SJ, 238), and, "Yes, it [height] does not really matter" (R1, G4, VB, 13), student M3 and student R2 reinterpreted problem situations to disprove their ways of thinking, respectively: "No, but slow hours are worth more than busy hours" (M3, G3, SJ, 239), and, "Actually, it does because if you are taller, you can reach over the net easier" (R2, G4, VB, 14).

Finally, when the teacher asked, "Not necessarily, did you guys not say something about how small or big the circle was?" (T, G3, PA, 297), student M3 reinterpreted problem situations to defend their implemented strategy, as follows: "These ones landed in a tighter area, and these ones are more spread out. These ones flew the farthest and we were basing this one…where we got the largest diameter was the best floater because it flew the farthest" (M3, G3, PA, 305).

Monitoring activities: assessment of knowledge. This section will illustrate students' excerpts, which were identified as the first type of monitoring activity, *assessment of knowledge,* triggered at both the individual and social levels. This type of monitoring activity is a metacognitive activity because it involves an evaluating process in which students reflect on their own thinking of their knowledge. *Assessment of knowledge* was triggered when the students assessed their ways of thinking, focusing on their knowledge, such as relevant mathematical knowledge and task-specific knowledge at the individual level, or in response to others' thinking, which focused on the other students' knowledge at the social level.

Assessment of knowledge triggered at the individual level. First, the students, themselves, triggered an assessment of their knowledge at the individual level, as revealed in Table 4.10 below. The students revealed their own ways of thinking about task-specific knowledge, as illustrated by the statements, "I think I know how to do this" (M3, G3, SJ, 22); "It's like the flight-delay that we had last year" (M1, G3, SJ, 25); and, "I know how to solve for who's the best floater" (P2, G6, PA, 51). Student K3 assessed her own way of thinking, focusing on her knowledge about the concept of rate: "Well, I know that the rate…find out the rate of money she has made per hour" (K3, G2, SJ, 10). Student W2, herself, also triggered an assessment of her knowledge about the concepts of the average, as illustrated with the statement, "Divided by what [to do the average]?" (W2, G7, PA, 151). The students also evaluated their own ways of thinking, focusing on their knowledge about how to use a protractor, as follows: "We don't get it at all [because we don't know how to use the protractor]" (H2, G1, PA, 109), and, "I don't know how to use a protractor! How do you use it, William [P2]?" (P1, G6, PA, 155).

Table 4.10

Examples of Students' Monitoring Activities Identified as Assessment of Knowledge Triggered at the Individual Level

Transcript Segment	Description of Monitoring Activities Identified as *Assessment of Knowledge* Triggered at the Individual Level
(S2, G5, SJ, 556)	I think we should change "succeeds" to "exceeds" in sales. Because "succeeds" sounds just okay, but "exceeds"—what do you guys think? *(Assessment – knowledge: Thinking ABOUT a way of thinking, Individual Level)*

(M3, G3, SJ, 22)	I think I know how to do this. *(Assessment – knowledge: Thinking ABOUT a way of thinking, Individual Level)*
(M1, G3, SJ, 25)	It's like the flight-delay that we had last year. *(Assessment – knowledge: Thinking ABOUT a way of thinking, Individual Level)*
(M3, G3, SJ, 26)	I'm pretty sure I know how to do this. *(Assessment – knowledge: Thinking ABOUT a way of thinking, Individual Level)*
(K3, G2, SJ, 10)	Well, I know that the rate…find out the rate of money she has made per hour. *(Assessment of Knowledge: Thinking ABOUT a way of thinking, Individual Level)*
(H2, G1, PA, 109)	We don't get it at all [because we don't know how to use the protractor]. *(Assessment – knowledge: Thinking ABOUT a way of thinking, Individual Level)*
(W2, G7, PA, 151)	Divided by what [to do the average]? *(Assessment – knowledge: Thinking ABOUT a way of thinking, Individual Level)*
(P2, G6, PA, 51)	I know how to solve for who's the best floater. *(Assessment – knowledge: Thinking ABOUT a way of thinking, Individual Level)*
(P1, G6, PA, 155)	I don't know how to use a protractor! How do you use it, William [P2]? *(Assessment – knowledge: Thinking ABOUT a way of thinking, Individual Level)*

Assessment of knowledge triggered at the social level. Second, as shown below, Table 4.11 illustrates examples of how the students' monitoring activity*, assessment of knowledge*, was triggered at the social level. For example, the students' *assessment of knowledge* occurred at the social level when the students made judgments in response to others' ways of thinking, which focused on the other students' knowledge.

Table 4.11

Examples of Students' Monitoring Activities Identified as Assessment of Knowledge Triggered at the Social Level

Transcript Segment	Description of Monitoring Activities Identified as *Assessment of Knowledge* Triggered at the Social Level

(M3, G3, SJ, 28)	Yes. At least for the money part I know how. *(Assessment – knowledge: Thinking ABOUT M1's way of thinking, Social Level)* [Triggered by M1's way of thinking–"It's like the flight-delay that we had last year" (M1, G3, SJ, 25)]
(S1, G5, SJ, 558)	Let's try it. *(Assessment – knowledge: Thinking ABOUT S2's way of thinking, Social Level)* [Triggered by S2's way of thinking–"I think we should change 'succeeds' to 'exceeds' in sales. Because 'succeeds' sounds just okay, but 'exceeds'—what do you guys think?" (S2, G5, SJ, 556)]
(W1, G7, PA, 142)	Yeah. *(Assessment – knowledge: Thinking ABOUT W2's way of thinking, Social Level)* [Triggered by W2's way of thinking–"Then divide by 18 [to do average]?" (W2, G7, PA, 141)]
(P4, G6, PA, 159)	Then, William [P2], after I add all these up, what do I divide by? Do I divide by 3, 4? Do I divide by 12? *(Assessment – knowledge: Thinking ABOUT P2's way of thinking, Social Level)*
(S3, G5, PA, 242)	So mine goes—wait. So, what does mine do then? The negative confuses me. *(Assessment – knowledge: Thinking ABOUT S1 and S2's way of thinking, Social Level)* [Prompted by the conversation with S1 and S2 about how to treat negative numbers]
(S3, G5, PA, 244)	But the negative will still count. That doesn't even make sense. *(Assessment – knowledge: Thinking ABOUT S2's way of thinking, Social Level)* [Triggered by S2's way of thinking–"Just pretend that the negative's not there" (S2, G5, PA, 243)]
(R3, G4, PA, 43)	Yes, so I do not know how to do it so I want to skip it. *(Assessment – knowledge: Thinking ABOUT R2's way of thinking, Social Level)* [Triggered by R2's way of thinking–"So we do not do the last column because it has negatives? Oh, it is angle" (R2, G4, PA, 42)]

When student M1 shared, "It's like the flight-delay that we had last year" (M1, G3, SJ, 25), student M3 assessed student M1's way of thinking about task-specific knowledge, as illustrated by the statement, "Yes. At least for the money part I know how" (M3, G3, SJ, 28). By mentioning, "Let's try it" (S1, G5, SJ, 558), student S1 revealed her agreement with student S2's *assessment of knowledge* about the words *succeed* and *exceed*: "I think we should change 'succeeds' to 'exceeds' in sales. Because 'succeeds' sounds just okay, but 'exceeds'—what do you guys think?" (S2, G5, SJ, 556).

At the social level, *assessment of knowledge* also occurred through the students' interactions with others. Prompted by others, the students assessed their own ways of thinking, which focused on their knowledge about the concept of average or negative numbers, as illustrated by the statements, "Then, William [P2], after I add all these up, what do I divide by? Do I divide by 3, 4? Do I divide by 12?" (P4, G6, PA, 159); "But the negative will still count. That doesn't even make sense" (S3, G5, PA, 244); and, "Yes, so I do not know how to do it so I want to skip it" (R3, G4, PA, 43).

Monitoring activities: assessment of understanding. The second type of monitoring activity, *assessment of understanding,* was extensively identified at both the individual and social levels across the three MEAs, across the seven groups, and across the group members. This section will illustrate examples of the identified students' *assessment of understanding,* triggered at the individual level and social level, respectively. This type of monitoring activity is a metacognitive activity because it involves an evaluating process in which students reflect on their own thinking of their understanding. *Assessment of understanding* occurred when the students assessed their own understanding of problems or problem situations at the individual level. On the other hand, *assessment of understanding* occurred at the social level when the students assessed their own understanding of others' ways of thinking, in and of itself, or when the students assessed others' ways of thinking, which focused on the other students' understanding of problems or problem situations.

Assessment of understanding triggered at the individual level. As illustrated in Table 4.12 below, the students revealed their own assessment of understanding of what needed to be done as follows: “What does she need us to do? She needs us to see who she needs to hire?” (R1, G4, SJ, 20); “Cam if you read, it says they have to be of equal skill” (M2, G3, VB, 8); and, “Do we have to write her a letter?” (M2, G3, SJ, 21). The statements from students conveyed their thinking, which focused on their understanding, indicating that they did not understand something about the problems: “I am adding them up. What is worse, in the net, or out of bounds?” (K3, G2, VB, 78); “I do not know what this [all the letters on the graphs] means” (R3, G4, PA, 46); and, “What would be accuracy? I can’t figure it out” (W2, G7, PA, 114). Student M3 also assessed his own understanding of problem situations, as illustrated by the statement, “We need to figure out… It’s a problem that the pilots aren’t flying consistent. They’re all like randomly thrusting them forward at different waves” (M3, G3, PA, 34).

Table 4.12

Examples of Students’ Monitoring Activities Identified as Assessment of Understanding Triggered at the Individual Level

Transcript Segment	Description of Monitoring Activities Identified as *Assessment of Understanding* Triggered at the Individual Level
(H3, G1, SJ, 31)	So we just have to figure out who she should hire, right? *(Assessment – understanding: Thinking ABOUT a way of thinking, Individual Level)*
(R1, G4, SJ, 20)	What does she need us to do? She needs us to see who she needs to hire? *(Assessment – understanding: Thinking ABOUT a way of thinking, Individual Level)*
(M2, G3, SJ, 21)	Do we have to write her a letter? *(Assessment – understanding: Thinking ABOUT a way of thinking, Individual Level)*
(P4, G6, SJ, 16)	Okay, so we have to evaluate how well the different vendors did last year for the business and decide which ones she should rehire. *(Assessment – understanding: Thinking ABOUT a way of thinking, Individual Level)*

(K3, G2, VB, 78)	I am adding them up. What is worse, in the net, or out of bounds? *(Assessment – understanding: Thinking ABOUT a way of thinking, Individual Level)*
(S3, G5, VB, 410)	Wait. Who are we writing to? Camp counselors? Or should we do camp organizers? *(Assessment – understanding: Thinking ABOUT a way of thinking, Individual Level)*
(M2, G3, VB, 8)	Cam if you read it says they have to be of equal skill. *(Assessment – understanding: Thinking ABOUT a way of thinking, Individual Level)*
(W2, G7, VB, 12)	So we need to do 18 [players] divided by 3 [teams]. 18 divided by 3 is 6. *(Assessment – understanding: Thinking ABOUT a way of thinking, Individual Level)*
(M3, G3, PA, 34)	We need to figure out… It's a problem that the pilots aren't flying consistent. They're all like randomly thrusting them forward at different waves. *(Assessment – understanding: Thinking ABOUT a way of thinking, Individual Level)*
(R3, G4, PA, 46)	I do not know what this [all the letters on the graphs] means. *(Assessment – understanding: Thinking ABOUT a way of thinking, Individual Level)*
(S2, G5, PA, 33)	Figure 1. So, there are 4 different planes. *(Assessment – understanding: Thinking ABOUT a way of thinking, Individual Level)*
(W2, G7, PA, 114)	What would be accuracy? I can't figure it out. *(Assessment – understanding: Thinking ABOUT a way of thinking, Individual Level)*

Assessment of understanding triggered at the social level. Illustrated in Table 4.13 below are examples of how the students' monitoring activity*, assessment of understanding*, was triggered by others at the social level. The students revealed their assessment of understanding of others' ways of thinking, in and of itself, as follows: "What are you talking about?" (S3, G5, VB, 414); "What does that mean?" (P2, G6, PA, 130); and, "I have no clue what you're talking about" (H2, G1, PA, 330). The students also shared their ways of thinking to complement others' ways of thinking, which focused on the other students' understanding of problems or problem situations, as illustrated by the statements, "So we have to fire the bad people" (R1, G4, SJ, 7); "No, we'll have to get rid of 12 people" (P2, G6, SJ, 178); and, "Yes, because there are eight teams. Oh, there are eighteen teams and if we divide them up into six, then that will leave us with three teams" (K2, G2, VB, 3).

Table 4.13

Examples of Students' Monitoring Activities Identified as Assessment of Understanding Triggered at the Social Level

Transcript Segment	Description of Monitoring Activities Identified as *Assessment of Understanding* Triggered at the Social Level
(H4, G1, SJ, 93)	What? *(Assessment – understanding: Thinking ABOUT H2's way of thinking, Social Level)* [Triggered by H2's way of thinking–"Yeah you add the busy and divide by how many people there are" (H2, G1, SJ, 92)]
(R1, G4, SJ, 7)	So we have to fire the bad people. *(Assessment – understanding: Thinking ABOUT R2's way of thinking, Social Level)* [Triggered by R2's way of thinking–"There was a vendor last year. Last year they hired 9 people and they can only hire 6 people this year and we have to figure it out" (R2, G4, SJ, 5)]
(S3, G5, SJ, 35)	We want to find someone who works a lot during busy and not a lot during— *(Assessment – understanding: Thinking ABOUT S1's way of thinking, Social Level)* [Triggered by S1's way of thinking–"So, basically, we're trying to find 3—" (S1, G5, SJ, 34)]
(P2, G6, SJ, 178)	No, we'll have to get rid of 12 people. *(Assessment – understanding: Thinking ABOUT P4's way of thinking, Social Level)* [Triggered by P4's way of thinking–"So, 3. We'll have to get rid of 3 people" (P4, G6, SJ, 177)]
(K2, G2, VB, 3)	Yes, because there are eight teams. Oh, there are eighteen teams and if we divide them up into six, then that will leave us with three teams. *(Assessment – understanding: Thinking ABOUT K3's way of thinking, Social Level)* [Triggered by S2's way of thinking–"We need to pick six players for each team" (K3, G2, VB, 2)]
(R3, G4, VB, 89)	I am trying to figure out these. *(Assessment – understanding: Thinking ABOUT R1's way of thinking, Social Level)* [Triggered by R1's way of thinking–"Two good players, two bad players, and two medium players. I think that is what we should do" (R1, G4, VB, 88)]
(S3, G5, VB, 414)	What are you talking about? *(Assessment – understanding: Thinking ABOUT S2's way of thinking, Social Level)* [Triggered by S2's way of thinking–"It's

	funny. We ended up with the same top 3 people here. Wait a minute! No we didn't" (S2, G5, VB, 413)]
(P4, G6, VB, 26)	Is it teams of 3? *(Assessment – understanding: Thinking ABOUT P1's way of thinking, Social Level)* [Triggered by P1's way of thinking–"Okay. So we need to find the 3 best players and put them on 3 different teams" (P1, G6, VB, 25)]
(P2, G6, PA, 130)	What does that mean? *(Assessment – understanding: Thinking ABOUT P4's way of thinking, Social Level)* [Triggered by P4's way of thinking–"I've got the time in flight" (P4, G6, PA, 129)]
(H2, G1, PA, 330)	I have no clue what you're talking about. *(Assessment – understanding: Thinking ABOUT H1's way of thinking, Social Level)* [Triggered by H1's way of thinking–"Instead of going this way it went that way" (H1, G1, PA, 329)]
(M3, G3, PA, 172)	Why? *(Assessment – understanding: Thinking ABOUT M2's way of thinking, Social Level)* [Triggered by M2's way of thinking–"We should figure out which pilots have which letter of plane" (M2, G3, PA, 171)]
(R1, G4, PA, 47)	With all the letters on it? Those are where the planes landed. *(Assessment – understanding: Thinking ABOUT R3's way of thinking, Social Level)* [Triggered by R3's way of thinking–"I do not know what this means" (R3, G4, PA, 46)]

Monitoring activities: assessment of strategy appropriateness. As illustrated with the students' excerpts in this section, the type of monitoring activity, *assessment of strategy appropriateness,* triggered at both the individual and social levels, was identified. This type of monitoring activity is a metacognitive activity because it involves an evaluating process in which students reflect on their own thinking of their strategy. *Assessment of strategy appropriateness*, which mainly followed the type of regulating activity–*new idea*, occurred when potentially useful information or alternative approaches were evaluated by the students who originally suggested them at the individual level, or by others at the social level. Also, *assessment of strategy appropriateness* occurred when the students themselves triggered an assessment of prior strategies, which were being implemented with explicit acceptance or tacit agreement from the group members, at the individual level, or when the students responded to others' ways of thinking, which focused on potentially useful information, alternative approaches, or previously implemented strategies at the social level.

Assessment of strategy appropriateness triggered at the individual level. Illustrated in Table 4.14 below are examples of how the students' monitoring activity, *assessment of strategy appropriateness*, was triggered at the individual level. The students sometimes strongly advocated the appropriateness of strategy suggested by them, as illustrated by the statements, "We should say slow plus steady equals busy. It should just be total hours worked anyway" (M1, G3, SJ, 282), and, "So yeah, I think we should look at this graph with this information" (S2, G5, PA, 48). Sometimes the students more or less passively supported their assessment of strategy appropriateness, which they originally shared, as follows: "Are we going to do the ratios?" (M3, G3, SJ, 266); "I'm just thinking it should be based off the average" (P2, G6, SJ, 283); "Do you think our plan is good? I think it works" (M2, G3, PA, 179); and, "I thought we had to come up with a new rule" (H3, G1, PA, 219). During problem solving, student M3 positively appraised the group's implemented strategy: "I don't think anyone else is doing our point system" (M3, G3, VB, 249). On the other hand, student P3 himself triggered a negative assessment of a previously implemented strategy: "I'm trying to think what I want to do. There's not a much more complicated approach than this" (P3, G6, PA, 277).

Table 4.14

Examples of Students' Monitoring Activities Identified as Assessment of Strategy Appropriateness Triggered at the Individual Level

Transcript Segment	Description of Monitoring Activities Identified as *Assessment of Strategy Appropriateness* Triggered at the Individual Level
(M1, G3, SJ, 282)	We should say slow plus steady equals busy. It should just be total hours worked anyway. *(Assessment – strategy appropriateness: Thinking ABOUT a way of thinking, Individual Level)*
(M3, G3, SJ, 266)	Are we going to do the ratios? *(Assessment – strategy appropriateness: Thinking ABOUT a way of thinking, Individual Level)*
(S2, G5, SJ, 157)	True. *(Assessment – strategy appropriateness: Thinking ABOUT a way of thinking, Individual Level)* [Once she shared her own way of thinking–"So, we need to find out how much money they make every hour" (S2, G5, SJ, 155)]
(P2, G6, SJ, 283)	I'm just thinking it should be based off the average. *(Assessment – strategy appropriateness: Thinking ABOUT a way of thinking, Individual Level)*
(K3, G2, VB, 126)	It will probably be the worst way and probably not that accurate, but I am going to try it. *(Assessment – strategy appropriateness: Thinking ABOUT a way of thinking, Individual Level)*
(M3, G3, VB, 249)	I don't think anyone else is doing our point system. *(Assessment – strategy appropriateness: Thinking ABOUT a way of thinking, Individual Level)*
(M2, G3, PA, 33)	Nah, a point system won't work for this. *(Assessment – strategy appropriateness: Thinking ABOUT a way of thinking, Individual Level)*
(M2, G3, PA, 179)	Do you think our plan is good? I think it works. *(Assessment – strategy appropriateness: Thinking ABOUT a way of thinking, Individual Level)*
(S2, G5, PA, 48)	So yeah, I think we should look at this graph with this information. *(Assessment – strategy appropriateness: Thinking ABOUT a way of thinking, Individual Level)*
(H1, G1, PA, 148)	I'll start making—wait, Barbara [H2] let me see this. Wait, did we know that they already had this [Angle from Target] all written down for us? *(Assessment – strategy appropriateness: Thinking ABOUT a way of thinking, Individual Level)* [While measuring the angles on the graphs to determine the most accurate plane]
(H3, G1, PA, 219)	I thought we had to come up with a new rule. *(Assessment – strategy appropriateness: Thinking ABOUT a way of thinking, Individual Level)*
(P3, G6, PA, 277)	I'm trying to think what I want to do. There's not a much more complicated approach than this. *(Assessment – strategy appropriateness: Thinking ABOUT a way of thinking, Individual Level)*

Assessment of strategy appropriateness triggered at the social level. As shown below in Table 4.15, at the social level, the students' monitoring activities*, assessment of strategy appropriateness*, occurred when the students evaluated potentially useful information or alternative approaches, which were suggested by others. *Assessment of strategy appropriateness* was also triggered at the social level in response to others' ways of thinking, which focused on potentially useful information, alternative approaches, or previously implemented strategies. For example, when student K2 shared, "I want to find the average so that I can find the best one" (K2, G2, PA, 71), student K1 positively evaluated her way of thinking: "That is a good idea," (K1, G2, PA, 72). Similarly, when student M1 suggested, "I think what we should do is go by hours work first. Whoever works the least" (M1, G3, SJ, 63), student M2 actively accepted his suggestion: "Total of their busy, steady, and compare them. We should drop them down to a number that they both worked" (M2, G3, SJ, 66). The students also agreed with others' assessment of strategy appropriateness, as illustrated by the statements, "I think we should do that. That would make it much simpler" (K1, G2, SJ, 14), and, "That's what we've been doing" (S1, G5, SJ, 152).

Table 4.15

Examples of Students' Monitoring Activities Identified as Assessment of Strategy Appropriateness Triggered at the Social Level

Transcript Segment	Description of Monitoring Activities Identified as *Assessment of Strategy Appropriateness* Triggered at the Social Level
(K1, G2, SJ, 14)	I think we should do that. That would make it much simpler. *(Assessment – strategy appropriateness: Thinking ABOUT K3's way of thinking, Social Level)* [Triggered by K3's way of thinking–"For each person. Or should we

	just add it all up and then divide?” (K3, G2, SJ, 13)]
(H3, G1, SJ, 175)	It wouldn’t matter. It doesn’t matter how they do as a whole if they hire an individual person. *(Assessment – strategy appropriateness: Thinking ABOUT H4’s way of thinking, Social Level)* [Triggered by H4’s way of thinking–“So we found the averages for everything?” (H4, G1, SJ, 173)]
(M2, G3, SJ, 66)	Total of their busy, steady, and compare them. We should drop them down to a number that they both worked. *(Assessment – strategy appropriateness: Thinking ABOUT M1’s way of thinking, Social Level)* [Triggered by M1’s way of thinking–“I think what we should do is go by hours work first. Whoever works the least” (M1, G3, SJ, 63)]
(S1, G5, SJ, 152)	That’s what we’ve been doing. *(Assessment – strategy appropriateness: Thinking ABOUT S2’s way of thinking, Social Level)* [Triggered by S2’s way of thinking–“Okay, the first thing we need to do is we need to compare how many hours they worked and how much money they earned when they worked” (S2, G5, SJ, 150)]
(H4, G1, VB, 64)	No, we split teams. *(Assessment – strategy appropriateness: Thinking ABOUT H2’s way of thinking, Social Level)* [Triggered by H2’s way of thinking–“You cannot have all the good people on the same team” (H2, G1, VB, 63)]
(S3, G5, VB, 26)	Well, why don’t we read this first and then you can say your idea. *(Assessment – strategy appropriateness: Thinking ABOUT S2’s way of thinking, Social Level)* [Triggered by S2’s way of thinking–“I’ve got an idea. What we could do is we could list it out and then—” (S2, G5, VB, 25)]
(M3, G3, VB, 48)	I wrote down tallest to shortest… *(Assessment – strategy appropriateness: Thinking ABOUT M2’s way of thinking, Social Level)* [Triggered by M2’s way of thinking–“And we are going to come up with a system for every single…” (M2, G3, VB, 46)]
(P2, G6, VB, 197)	No, actually I think this is easier. *(Assessment – strategy appropriateness: Thinking ABOUT P4’s way of thinking, Social Level)* [Triggered by P4’s way of thinking–“The thing is, we’re not cutting people. This would be much easier if we were cutting people” (P4, G6, VB, 195)]
(S1, G5, PA, 397)	That’s a formula? What’s the formula for averaging? Result 1 + result 2 + result 3 divided by 3? *(Assessment – strategy appropriateness: Thinking ABOUT S2’s way of thinking, Social Level)* [Triggered by S2’s way of thinking–“So, you average it and then you separate them” (S2, G5, PA, 396)]
(W3, G7, PA, 117)	Well distance to target is how far away it lands from the target, so if it is far away it would not be very accurate. *(Assessment – strategy appropriateness: Thinking ABOUT W2’s way of thinking, Social Level)* [Triggered by W2’s way of thinking–“Seriously, how would distance to target help the flight? Oops, I mean the accuracy” (W2, G7, PA, 116)]

(P4, G6, PA, 314)	We don’t need to know the straightest! *(Assessment – strategy appropriateness: Thinking ABOUT P2’s way of thinking, Social Level)* [Triggered by P2’s way of thinking–“For the straightest, I want to say that it goes to C” (P2, G6, PA, 313)]
(K1, G2, PA, 72)	That is a good idea. *(Assessment – strategy appropriateness: Thinking ABOUT K2’s way of thinking, Social Level)* [Triggered by K2’s way of thinking–“I want to find the average so that I can find the best one” (K2, G2, PA, 71)]

On the other hand, when student P2 suggested an alternative approach, “For the straightest, I want to say that it goes to C” (P2, G6, PA, 313), student P4 negatively evaluated his way of thinking: “We don’t need to know the straightest!” (P4, G6, PA, 314). When an alternative approach came to light from student S2 using the average of the average, “So, you average it and then you separate them” (S2, G5, PA, 396), student S1 doubted the appropriateness of her way of thinking: “That’s a formula? What’s the formula for averaging? Result 1 + result 2 + result 3 divided by 3?” (S1, G5, PA, 397). The students also disagreed with others’ ways of thinking, which focused on alternative approaches, as illustrated by the statements, “It wouldn’t matter. It doesn’t matter how they do as a whole if they hire an individual person” (H3, G1, SJ, 175); “No, we split teams” (H4, G1, VB, 64); and, “No, actually I think this is easier” (P2, G6, VB, 197).

Monitoring activities: assessment of progress toward goal. The type of monitoring activity, *assessment of progress toward goal,* was extensively identified at both the individual and social levels across the three MEAs, across the seven groups, and across the group members, as shown below. This type of monitoring activity is a metacognitive activity because it involves an evaluating process in which students reflect on their own thinking of progress toward their goal. At the individual level, *assessment of progress toward goal* occurred when the students made judgments regarding their own ways of thinking, which focused on progress toward their goal. At the social level, *assessment of progress toward goal* occurred when the students evaluated others' ways of thinking, which focused on progress toward their goal.

Assessment of progress toward goal triggered at the individual level. As shown below in Table 4.16, the students made judgments regarding what needed to be done, what had been done, or where they were in their problem-solving process, as illustrated by the statements, "Okay let's go through and see who is good or bad" (W3, G7, VB, 30); "So we are going to decide each of the coaches' comments if it is minus one, plus one, or zero" (M2, G3, VB, 49); "Now we have to do it for busy and slow, unless you want to do the average for steady" (K3, G2, SJ, 20); "You guys are going to have to do these either way, so why not start with those" (K1, G2, PA, 31); "Okay, here we go guys" (H2, G1, PA, 180); and, "We are going to total them" (M2, G3, SJ, 133).

Table 4.16

Examples of Students' Monitoring Activities Identified as Assessment of Progress Toward Goal Triggered at the Individual Level

Transcript Segment	Description of Monitoring Activities Identified as *Assessment of Progress Toward Goal* Triggered at the Individual Level
(S3, G5, SJ, 147)	Okay. You look at that, I look at that, and you look at that. Okay. So, wait. *(Assessment – progress toward goal: Thinking ABOUT a way of thinking, Individual Level)*
(K3, G2, SJ, 20)	Now we have to do it for busy and slow, unless you want to do the average for steady. *(Assessment – progress toward goal: Thinking ABOUT a way of thinking, Individual Level)*
(H2, G1, SJ, 89)	And then, guys, we need to do all of them down by how many people there are. *(Assessment – progress toward goal: Thinking ABOUT a way of thinking, Individual Level)*

(M2, G3, SJ, 133)	We are going to total them. *(Assessment – progress toward goal: Thinking ABOUT a way of thinking, Individual Level)*
(S2, G5, VB, 321)	Okay, I'm going to start going over them now. Because I can't really go down much. *(Assessment – progress towards goal: Thinking ABOUT a way of thinking, Individual Level)*
(W3, G7, VB, 30)	Okay let's go through and see who is good or bad. *(Assessment – progress toward goal: Thinking ABOUT a way of thinking, Individual Level)*
(K3, G2, VB, 31)	Maybe we should look at the other ones. *(Assessment – progress toward goal: Thinking ABOUT a way of thinking, Individual Level)*
(M2, G3, VB, 49)	So we are going to decide each of the coaches' comments if it is minus one, plus one, or zero. *(Assessment – progress toward goal: Thinking ABOUT a way of thinking, Individual Level)*
(S3, G5, PA, 304)	Okay, so, how are we going to figure it out? *(Assessment – progress toward goal: Thinking ABOUT a way of thinking, Individual Level)*
(K1, G2, PA, 31)	You guys are going to have to do these either way, so why not start with those. *(Assessment – progress towards goal: Thinking ABOUT a way of thinking, Individual Level)*
(H2, G1, PA, 180)	Okay, here we go guys. *(Assessment – progress towards goal: Thinking ABOUT a way of thinking, Individual Level)*
(M2, G3, PA, 102)	I'm going to look at the data and try to figure out what we should do to measure this. *(Assessment – progress towards goal: Thinking ABOUT a way of thinking, Individual Level)*

Assessment of progress toward goal triggered at the social level. Illustrated in Table 4.17 are examples of how the students' monitoring activity*, assessment of progress toward goal*, was triggered at the social level. The students evaluated others' ways of thinking, which focused on progress toward their goal. For example, when student M2 asked, "All right, so are we going to total hours or the busy hours?" (M2, G3, SJ, 81), student M1 made a judgment to move toward their goal: "Let's total all the busy, steady, and slow hours for each person" (M1, G3, SJ, 83). The students agreed with others' ways of thinking, which focused on progress toward their goal, as illustrated by the statements, "I am just going to add his money up as we can" (K1, G2, SJ, 22), and, "We are going to get back to that one. We are doing this first" (K3, G2, VB, 15).

Table 4.17

Examples of Students' Monitoring Activities Identified as Assessment of Progress Toward Goal Triggered at the Social Level

Transcript Segment	Description of Monitoring Activities Identified as *Assessment of Progress Toward Goal* Triggered at the Social Level
(M1, G3, SJ, 83)	Let's total all the busy, steady, and slow hours for each person. *(Assessment – progress toward goal: Thinking ABOUT M2's way of thinking, Social Level)* [Triggered by M2's way of thinking–"All right, so are we going to total hours or the busy hours?" (M2, G3, SJ, 81)]
(H3, G1, SJ, 177)	But, it's not everyone together. It doesn't have to do with anything. *(Assessment – progress towards goal: Thinking ABOUT H2's way of thinking, Social Level)* [Triggered by H2's way of thinking–"We already have the average of everybody" (H2, G1, SJ, 176)]
(W3, G7, SJ, 139)	Anna [W1], wait. We are going to rank them with money and then we will rank them on here. This one is for hours. *(Assessment – progress towards goal: Thinking ABOUT W1's way of thinking, Social Level)* [Triggered by W1's way of thinking–"Let's rank them on the bottom part and make little numbers by their names on the sheet" (W1, G7, SJ, 137)]
(K1, G2, SJ, 22)	I am just going to add his money up as we can. *(Assessment – progress towards goal: Thinking ABOUT K3's way of thinking, Social Level)* [Triggered by K3's way of thinking–"Now we have to do it for busy and slow, unless you want to do the average for steady" (K3, G2, SJ, 20)]
(M2, G3, VB, 97)	No, we're going to find the average for each one and if you're over… *(Assessment – progress towards goal: Thinking ABOUT M1's way of thinking, Social Level)* [Triggered by M1's way of thinking–"Add up the total?" (M1, G3, VB, 96)]
(W3, G7, VB, 206)	No, I am doing the top half. Okay what did you do? *(Assessment – progress towards goal: Thinking ABOUT W2's way of thinking, Social Level)* [Triggered by W2's way of thinking–"Your doing the bottom half right?" (W2, G7, VB, 205)]
(S3, G5, VB, 262)	I don't think we need this much more, do we? Maybe we keep it in case we want to look at it. *(Assessment – progress towards goal: Thinking ABOUT S2's way of thinking, Social Level)* [Triggered by S2's way of thinking–"So, I'll go on to here now from this face" (S2, G5, VB, 261)]

(K3, G2, VB, 15)	We are going to get back to that one. We are doing this first. *(Assessment – progress towards goal: Thinking ABOUT K1's way of thinking, Social Level)* [Triggered by K1's way of thinking – "You have to pay attention to this too" (K1, G2, VB, 14)]
(S1, G5, PA, 305)	Well, we need to look at all of these. *(Assessment – progress toward goal: Thinking ABOUT S3's way of thinking, Social Level)* [Triggered by S3's way of thinking–"Okay, so, how are we going to figure it out?" (S3, G5, PA, 304)]
(K1, G2, PA, 29)	You did not do either of them, so you have to do that one. *(Assessment – progress towards goal: Thinking ABOUT K2's way of thinking, Social Level)* [Triggered by K2's way of thinking–"Now do that one" (K2, G2, PA, 28)]
(W2, G7, PA, 94)	No we have to do this one first! *(Assessment – progress towards goal: Thinking ABOUT W1's way of thinking, Social Level)* [Triggered by W1's way of thinking–"We should do this one first" (W1, G7, PA, 93)]
(R3, G4, PA, 29)	No, you do time in flight. *(Assessment – progress towards goal: Thinking ABOUT R2's way of thinking, Social Level)* [Triggered by R2's way of thinking–"Do I do distance of flight from the next column?" (R2, G4, PA, 28)]

On the other hand, the students disagreed with others' judgments regarding what needed to be done, what had been done, or where they were in their problem-solving process, as illustrated by the statements, "Anna [W1], wait. We are going to rank them with money and then we will rank them on here. This one is for hours" (W3, G7, SJ, 139); "You did not do either of them, so you have to do that one" (K1, G2, PA, 29); "But, it's not everyone together. It doesn't have to do with anything" (H3, G1, SJ, 177); "No, we're going to find the average for each one and if you're over…" (M2, G3, VB, 97); and, "No we have to do this one first!" (W2, G7, PA, 94).

Monitoring activities: assessment of strategy execution. The type of monitoring activity, *assessment of strategy execution,* was also extensively identified at both the individual and social levels across the three MEAs, across the seven groups, and across the group members, as illustrated below. This type of monitoring activity is a metacognitive activity because it involves an evaluating process in which students reflect on their own thinking of strategy execution. *Assessment of strategy execution* occurred when the students assessed their own strategy execution or others'–despite no request from others–at the individual level, or when the students assessed their strategy execution in response to others' ways of thinking, which focused on strategy execution at the social level.

Assessment of strategy execution triggered at the individual level. Table 4.18 presents examples of how the students' monitoring activity*, assessment of strategy execution*, was triggered at the individual level. The students evaluated whether they, themselves, were properly executing their strategy, as illustrated by the statements, "So, Maria worked 10 hours in busy, divided by 699, right?" (S3, G5, SJ, 232); "I don't think we can have exactly equal numbers, but we can get close" (M3, G3, VB, 223); and, "Oh, I went across 33.5" (R1, G4, PA, 20). Also, the students assessed whether others were correctly executing their strategy while collaboratively working on the MEAs–despite no request from others, as follows: "Did you go like this and then add these three and divide it by three? Just copy" (H4, G1, SJ, 102); "How'd you get 5.9 for your time in flight? You didn't divide" (S2, G5, PA, 171); and, "Hold on. Let me think. No add these ones, right here. Distance from start and time of flight for A. Add all of these together" (W1, G7, PA, 123).

Table 4.18

Examples of Students' Monitoring Activities Identified as Assessment of Strategy Execution Triggered at the Individual Level

Transcript Segment	Description of Monitoring Activities Identified as *Assessment of Strategy Execution* Triggered at the Individual Level
(S3, G5, SJ, 232)	So, Maria worked 10 hours in busy, divided by 699, right? *(Assessment – strategy execution: Thinking ABOUT a way of thinking, Individual Level)*
(H4, G1, SJ, 102)	Did you go like this and then add these three and divide it by three? Just copy. *(Assessment – strategy execution: Thinking ABOUT a way of thinking, Individual Level)*
(R3, G4, SJ, 81)	Divide them. What do you get? Maria. *(Assessment – strategy execution: Thinking ABOUT a way of thinking, Individual Level)*
(W1, G7, SJ, 62)	When it's average, I get 17.86 repeating. *(Assessment – strategy execution: Thinking ABOUT a way of thinking, Individual Level)*
(M3, G3, VB, 223)	I don't think we can have exactly equal numbers, but we can get close. *(Assessment – strategy execution: Thinking ABOUT a way of thinking, Individual Level)*
(S2, G5, VB, 63)	So, she might not be the best server, but she sounds to be pretty good at jumping, running, and she's pretty tall. So, I'm going to put a little star next to her on mine just to say that she's someone who we might want to look at. *(Assessment – strategy execution: Thinking ABOUT a way of thinking, Individual Level)*
(W3, G7, VB, 101)	So 12 times 5, which is 60. Plus 9 equal 69 plus 18 equals 87. So that is Rebecca's vertical height. So do Ruth. So do 5 times 12 plus 3 plus 26. That equals 89 for Ruth's vertical height and jump. *(Assessment – strategy execution: Thinking ABOUT a way of thinking, Individual Level)*
(R3, G4, VB, 73)	Okay. I am getting the teams. Even though Anna is not the fastest person, she is number one on the other three, so I think she should be on number one. *(Assessment – strategy execution: Thinking ABOUT a way of thinking, Individual Level)*
(R1, G4, PA, 20)	Oh, I went across 33.5 *(Assessment – strategy execution: Thinking ABOUT a way of thinking, Individual Level)*
(S2, G5, PA, 171)	How'd you get 5.9 for your time in flight? You didn't divide. *(Assessment – strategy execution: Thinking ABOUT a way of thinking, Individual Level)*
(H1, G1, PA, 138)	Wait, you have to angle every single throw? *(Assessment – strategy execution: Thinking ABOUT a way of thinking, Individual Level)*
(W1, G7, PA, 123)	Hold on. Let me think. No add these ones, right here. Distance from start and time of flight for A. Add all of these together. *(Assessment – strategy execution: Thinking ABOUT a way of thinking, Individual Level)*

Assessment of strategy execution triggered at the social level. The students' monitoring activity*, assessment of strategy execution*, was triggered at the social level when the students evaluated others' ways of thinking, which focused on strategy execution (Table 4.19). For example, student H2 assessed their strategy execution: "No, we're doing the hours. We're doing it by person" (H2, G1, SJ, 83), in response to student H1's way of thinking, which focused on strategy execution: "You're going across?" (H1, G1, SJ, 82). When student W2 shared, "Okay I am confused what do I do now?" (W2, G7, PA, 162), student W1 shared her way of thinking, which focused on strategy execution: "Add these two and these two" (W1, G7, PA, 163). The students also supported one another to properly execute their strategy, as illustrated by the statements, "Here are the numbers I got for that one" (M1, G3, SJ, 257); "No, we should do stars, see, star and sad face, star and sad face" (S2, G5, VB, 272); "Yes. It is all the same thing. They returned it. They just hit it later" (R1, G4, VB, 65); and, "You might want to divide it" (S3, G5, PA, 189).

Table 4.19

Examples of Students' Monitoring Activities Identified as Assessment of Strategy Execution Triggered at the Social Level

Transcript Segment	Description of Monitoring Activities Identified as *Assessment of Strategy Execution* Triggered at the Social Level
(S1, G5, SJ, 395)	But they didn't earn any money at all. Like, these didn't work as many hours, but they still earned money. *(Assessment – strategy execution: Thinking ABOUT S3's way of thinking, Social Level)* [Triggered by S3's way of thinking–"But the no-times worked the least hours" (S3, G5, SJ, 394)]
(H2, G1, SJ, 83)	No, we're doing the hours. We're doing it by person. *(Assessment – strategy execution: Thinking ABOUT H1's way of thinking, Social Level)* [Triggered

	by H1's way of thinking–"You're going across?" (H1, G1, SJ, 82)]
(W2, G7, SJ, 99)	No, they average all of them together. So if we divided it by three it would be more than the actual average. *(Assessment – strategy execution: Thinking ABOUT W1's way of thinking, Social Level)* [Triggered by W1's way of thinking–"And the Willy and then Maria. Shouldn't we divide it by 3?" (W1, G7, SJ, 98)]
(M1, G3, SJ, 257)	Here are the numbers I got for that one. *(Assessment – strategy execution: Thinking ABOUT M2's way of thinking, Social Level)* [Triggered by M2's way of thinking–"There's no way I can fit those numbers there" (M2, G3, SJ, 256)]
(S2, G5, VB, 272)	No, we should do stars, see, star and sad face, star and sad face. *(Assessment – strategy execution: Thinking ABOUT S3's way of thinking, Social Level)* [Triggered by S3's way of thinking–"Probably. Now let's maybe do the double smiley-faces. Do we have any of those?" (S3, G5, VB, 271)]
(M3, G3, VB, 65)	They returned a dink, which is good, because she dinked it, but it returned, so that's a zero. *(Assessment – strategy execution: Thinking ABOUT M2's way of thinking, Social Level)* [Triggered by M2's way of thinking–"Out of bounds, that's minus one. There's not many of those" (M2, G3, VB, 63)]
(W2, G7, VB, 159)	I think Gertrude should be first. *(Assessment – strategy execution: Thinking ABOUT W3's way of thinking, Social Level)* [Triggered by W3's way of thinking–"So would she be before or after Gertrude then?" (W3, G7, VB, 158)]
(R1, G4, VB, 65)	Yes. It is all the same thing. They returned it. They just hit it later. *(Assessment – strategy execution: Thinking ABOUT R3's way of thinking, Social Level)* [Triggered by R3's way of thinking–"Well, how did you get three when there are only two returned? Are you counting dink returned too?" (R3, G4, VB, 64)]
(S3, G5, PA, 189)	You might want to divide it. *(Assessment – strategy execution: Thinking ABOUT S2's way of thinking, Social Level)* [Triggered by S2's way of thinking–"I have -70. Oh, wait. I don't have -70…" (S2, G5, PA, 188)]
(W1, G7, PA, 163)	Add these two and these two. *(Assessment – strategy execution: Thinking ABOUT W2's way of thinking, Social Level)* [Triggered by W2's way of thinking–"Okay I am confused what do I do now?" (W2, G7, PA, 162)]
(H1, G1, PA, 361)	Zero degrees so it's perfect. *(Assessment – strategy execution: Thinking ABOUT H2's way of thinking, Social Level)* [Triggered by H2's way of thinking–"So if you have the most accurate plane then you have zero?" (H2, G1, PA, 359)]
(R3, G4, PA, 25)	The highest on this one because it is distance from the start. *(Assessment – strategy execution: Thinking ABOUT R2's way of thinking, Social Level)* [Triggered by R2's way of thinking–"The highest or the lowest?" (R2, G4, PA, 24)]

Monitoring activities: assessment of accuracy or sense of result. This section will illustrate the students' excerpts identified as the type of monitoring activity, *assessment of accuracy or sense of result,* triggered at the individual level and social level, respectively. This type of monitoring activity is a metacognitive activity because it involves an evaluating process in which students reflect on their own thinking of problem-solving results. *Assessment of accuracy or sense of result* occurred when the students made judgments regarding their own ways of thinking, which focused on accuracy or a sense of result at the individual level. Otherwise, *assessment of accuracy or sense of result* occurred at the social level when the students made judgments regarding their own ways of thinking in response to others' assessment of the results.

Assessment of accuracy or sense of result triggered at the individual level. Illustrated in Table 4.20 are examples of how the students' monitoring activity, *assessment of accuracy or sense of result*, was triggered at the individual level. The students shared their own assessment about their sense of the results as follows: "I think Jose should have a part time job" (H1, G1, SJ, 209); "We need six people to keep, so we are keeping Terry, Maria, and Kim for full time" (M2, G3, SJ, 348); and, "So, I think they should be together. So I think Robin and Beth…" (P1, G6, VB, 355). Also, the students shared their own ways of thinking, which focused on assessment about the accuracy of the results, as illustrated by the statements, "There is a problem. We have two winners" (K2, G2, PA, 128); "I still say C is the best," (M3, G3, PA, 176); and, "So, yeah. C is the best for me in everything" (S2, G5, PA, 210).

Table 4.20

Examples of Students' Monitoring Activities Identified as Assessment of Accuracy or Sense of Result Triggered at the Individual Level

Transcript Segment	Description of Monitoring Activities Identified as *Assessment of Accuracy or Sense of Result* Triggered at the Individual Level
(K3, G2, SJ, 42)	Terry is out, Chad is out, Thomas … I have no idea if Cheri should be out. *(Assessment – accuracy or sense of result: Thinking ABOUT a way of thinking, Individual Level)*
(H1, G1, SJ, 209)	I think Jose should have a part time job. *(Assessment – accuracy or sense of result: Thinking ABOUT a way of thinking, Individual Level)*
(M2, G3, SJ, 348)	We need six people to keep, so we are keeping Terry, Maria, and Kim for full time. *(Assessment – accuracy or sense of result: Thinking ABOUT a way of thinking, Individual Level)*
(S1, G5, SJ, 204)	I still think Jose would be better. *(Assessment – accuracy or sense of result: Thinking ABOUT a way of thinking, Individual Level)*
(P1, G6, VB, 355)	So, I think they should be together. So I think Robin and Beth… *(Assessment – accuracy or sense of result: Thinking ABOUT a way of thinking, Individual Level)*
(W1, G7, VB, 104)	So this team is going to be a little bit better because they don't have the really bad player. So Gertrude and Christina will be on either of these teams. *(Assessment – accuracy or sense of result: Thinking ABOUT a way of thinking, Individual Level)*
(K3, G2, VB, 115)	I have concluded who are the top three. *(Assessment – accuracy or sense of result: Thinking ABOUT a way of thinking, Individual Level)*
(S3, G5, VB, 289)	I think we should do Christina, Rebecca, Ruth, Nikki. Wait, no! Nikki. *(Assessment – accuracy or sense of result: Thinking ABOUT a way of thinking, Individual Level)*
(S2, G5, PA, 210)	So, yeah. C is the best for me in everything. *(Assessment – accuracy or sense of result: Thinking ABOUT a way of thinking, Individual Level)*
(K2, G2, PA, 128)	There is a problem. We have two winners. *(Assessment – accuracy or sense of result: Thinking ABOUT a way of thinking, Individual Level)*
(M3, G3, PA, 176)	I still say C is the best. *(Assessment – accuracy or sense of result: Thinking ABOUT a way of thinking, Individual Level)*
(P2, G6, PA, 323)	The most accurate, I'm going to say is C. Because look at this. H is all over

	the place, but look! When he threw plane C, he was all on target. *(Assessment – accuracy or sense of result: Thinking ABOUT a way of thinking, Individual Level)*

Assessment of accuracy or sense of result triggered at the social level. As shown below, Table 4.21 presents the students' excerpts identified as *assessment of accuracy or sense of result,* triggered at the social level. Prompted by others, the students made judgments regarding their own ways of thinking, which focused on accuracy or a sense of the results in response to others'. For example, when student W1 shared, "So Gertrude and Christina will be on either of these teams" (W1, G7, VB, 105), student W2 assessed her way of thinking, which focused on a sense of the results: "Why don't you give this team another medium player?" (W2, G7, VB, 106). Student W2's assessment of accuracy of the results, "No, it would be D" (W2, G7, PA, 174), occurred in response to student W3's assessment of the accuracy of the results: "All right, B is the best floater" (W3, G7, PA, 173). The students also shared their assessment to complement others' ways of thinking regarding the accuracy or a sense of the results, as illustrated by the statements, "We will take the top 6 because we need part time too. Tony is first, Robin is two, Kim is 3, Jose is 4, Terry is 5, Chad is 6" (W1, G7, SJ, 94); "So, I think that we should switch out Cheri with Maria" (S3, G5, SJ, 344); and, "Yeah, we need to add some good players. I think we should add Beth to their team! So, we should switch Beth with [Kate]…" (H1, G1, VB, 250).

Table 4.21

Examples of Students' Monitoring Activities Identified as Assessment of Accuracy or Sense of Result Triggered at the Social Level

Transcript Segment	Description of Monitoring Activities Identified as *Assessment of Accuracy or Sense of Result* Triggered at the Social Level
(W1, G7, SJ, 94)	We will take the top 6 because we need part time too. Tony is first, Robin is two, Kim is 3, Jose is 4, Terry is 5, Chad is 6. *(Assessment – accuracy or sense of result: Thinking ABOUT W2's way of thinking, Social Level)* [Triggered by W2's way of thinking–"We need a 6th one, that's 5 [Terry]" (W2, G7, SJ, 93)]
(H4, G1, SJ, 182)	I think the people we should hire back for the first three is Robin, Tony and Kim. *(Assessment – accuracy or sense of result: Thinking ABOUT H3's way of thinking, Social Level)* [Triggered by H3's way of thinking–"Robin has the highest total. Robin, Robin, Tony for hours. Busy steady, slow. Busy money is Kim, Tony, Tony" (H3, G1, SJ, 180)]
(S3, G5, SJ, 344)	So, I think that we should switch out Cheri with Maria. *(Assessment – accuracy or sense of result: Thinking ABOUT S2's way of thinking, Social Level)* [Triggered by S2's way of thinking–"Maybe we should get rid of Maria" (S2, G5, SJ, 342)]
(P2, G6, SJ, 327)	All right. These are mine. The three that are going home with all the work that I did, the three that are going home are Terry, Tony, and Jose. *(Assessment – accuracy or sense of result: Thinking ABOUT P1's way of thinking, Social Level)* [Triggered by P1's way of thinking–"The three that are going home on mine are Willy, Chad, and Terry" (P1, G6, SJ, 326)]
(S2, G5, VB, 328)	Wait. Let's look at her scores again. *(Assessment – accuracy or sense of result: Thinking ABOUT S3's way of thinking, Social Level)* [Triggered by S3's way of thinking–"Probably Tina, and then Kate or Rhonda" (S3, G5, VB, 327)]
(H1, G1, VB, 250)	Yeah, we need to add some good players. I think we should add Beth to their team! So, we should switch Beth with [Kate]… *(Assessment – accuracy or sense of result: Thinking ABOUT H4's way of thinking, Social Level)* [Triggered by H4's way of thinking–"Team 2 is an average team" (H4, G1, VB, 249)]
(W2, G7, VB, 106)	Why don't you give this team another medium player? *(Assessment – accuracy or sense of result: Thinking ABOUT W1's way of thinking, Social Level)* [Triggered by W1's way of thinking–"So Gertrude and Christina will be on either of these teams" (W1, G7, VB, 105)]
(P4, G6, VB, 299)	I think it's better because Kate is very good at getting to the ball after serves. And, Nikki is a good server, so it would just be a good combination. *(Assessment – accuracy or sense of result: Thinking ABOUT P2's way of thinking, Social Level)* [Triggered by P2's way of thinking–"Kate does? Oh boy did I screw that up. Oh, Kate and Nikki for mine are on the same team. Whoops" (P2, G6, VB, 297)]

(S2, G5, PA, 359)	No, it's a tie between B and D because this one is—so this one wins B. C wins for the other one. *(Assessment – accuracy or sense of result: Thinking ABOUT S1's way of thinking, Social Level)* [Triggered by S1's way of thinking–"Well, C got best floater and most accurate" (S1, G5, PA, 358)]
(W2, G7, PA, 174)	No, it would be D. *(Assessment – accuracy or sense of result: Thinking ABOUT W3's way of thinking, Social Level)* [Triggered by W3's way of thinking–"All right, B is the best floater" (W3, G7, PA, 173)]
(P3, G6, PA, 328)	No! We're not even done yet. (*Assessment – accuracy or sense of result: Thinking ABOUT P2's way of thinking, Social Level)* [Triggered by P2's way of thinking–"C is the most accurate! C is the most accurate!" (P2, G6, PA, 327)]
(K3, G2, PA, 108)	It is not D. I want to add all of these up and then divide it by nine. *(Assessment – accuracy or sense of result: Thinking ABOUT K2's way of thinking, Social Level)* [Triggered by K2's way of thinking–"I think it [the best floater] is D" (K2, G2, PA, 107)]

Coherent model of metacognition in appropriately identifying and interpreting students' metacognitive activities. Using the theoretical model of metacognition on multiple levels, a large number of students' spontaneous metacognitive activities, which were identified at all of the multiple levels–the individual, social, and environmental levels, and which were identified as all types of metacognitive activities–four regulating activities and six monitoring activities, were also extensively identified across the seven groups, across the group members, and across the three MEAs–Summer Jobs, Volleyball, and Paper Airplane MEAs.

Students' metacognitive activities were triggered at the environmental level by something in the learning environment, such as the existence of a calculator or a protractor, the type of problem, the periods of time given for problem solving, and the homework burden, which made either positive or negative impacts on students' spontaneous metacognitive activities for productive problem solving.

At the individual level, the students triggered their *new idea,* according to their own needs, in order to lead problem solving, mainly in the beginning of their problem solving, or in order to revise previously implemented strategies during problem solving. At the social level, *new idea* came to light from the results of students' interactions with one another. In particular, the results of students' monitoring activities, which were prompted by others' ways of thinking, triggered their *new idea*. The students also shared their *new idea*, as a response to others, suggesting alternative approaches and complementing others' ways of thinking. With explicit acceptance or tacit agreement from the group members, *changing strategy* occurred when an alternative approach was adopted and implemented, either by the students who originally suggested it at the individual level, or by others at the social level.

Correcting errors occurred at the individual level when the students detected and corrected not only their own errors through their own *assessment of strategy execution*, but also others' errors through their voluntary *assessment of strategy execution*. *Correcting errors* also occurred at the social level when others' *assessment of strategy execution* led the students to correct the errors. At the individual level, *reinterpretation* occurred when the students reinterpreted problem situations in order to support their own ways of thinking or in order to address some problems that they recognized during problem solving. At the social level, *reinterpretation* occurred when the students reinterpreted problem situations in order to support others' understanding of them or in order to refute others' ways of thinking.

The type of monitoring activity, *Assessment of knowledge*, was triggered at the individual level when the students assessed their own ways of thinking, which focused on their knowledge, such as relevant mathematical knowledge and task-specific knowledge. It was also triggered at the social level when the students made judgments regarding their own ways of thinking in response to others' ways of thinking, which focused on the other students' knowledge. *Assessment of understanding* occurred at the individual level when the students assessed their understanding of problems or problem situations. It was also activated at the social level when the students assessed their own understanding, either in response to others' ways of thinking, which focused on the other students' understanding of a problem or problem situations, or others' ways of thinking, in and of itself, in order to complement their own or others' understanding of the problem/problem situations.

Assessment of strategy appropriateness was triggered at the individual level when the students made judgments regarding their own ways of thinking, which focused on a new idea that they originally suggested, or strategies previously implemented during problem solving. This type of monitoring activity was more frequently triggered at the social level. The students made judgments regarding their own ways of thinking, either focusing on a new idea that was originally suggested by others, or in response to others' ways of thinking, which focused on a new idea or strategies previously implemented. *Assessment of progress toward goal* occurred at the individual level when the students assessed their own ways of thinking, which focused on progress toward their goal, such as what needed to be done, what had been done, where they were in their problem-solving process, and so on. *Assessment of progress toward goal* also occurred at the social level when the students assessed their ways of thinking in response to others' ways of thinking, which focused on progress toward their goal, in order to suggest, confirm, or revise their directions toward their goal.

At the individual level, *assessment of strategy execution* was triggered when the students assessed their own ways of thinking, which focused on strategy execution, or they voluntarily assessed others' strategy execution–even when others did not solicit feedback from them. At the social level, it was triggered when the students made judgments regarding their own ways of thinking in response to others' ways of thinking, which focused on strategy execution, in order to support one another to correctly execute their strategy. The type of monitoring activity, *assessment of accuracy or sense of result*, occurred at the individual level when the students made judgments regarding their own ways of thinking, which focused on accuracy or sense of result. It also occurred at the social level when the students assessed their own ways of thinking in response to others' assessment of accuracy or sense of result.

As summarized above, this multiple case study provided empirical evidence to show that the theoretical model of metacognition on multiple levels is appropriate in identifying and interpreting metacognitive activities in complex modeling activities; thus, this model proved to be very efficient for research on metacognition. First, the theoretical model of metacognition helped more or less clearly distinguish metacognitive activities (thinking ABOUT) from cognitive activities (thinking WITH), according to the existence of any evaluation or regulation cues. Second, the theoretical model helped make the distinction among metacognitive activities, focusing on the monitoring function and the regulatory function. Finally, the re-conceptualization of metacognition on multiple levels in the theoretical model helped interpret how students' thinking became metacognitive in context.

These results from the multiple case study together demonstrate that the theoretical model of metacognition on multiple levels employed in this study is a coherent model of metacognition that helps make clear the distinction between cognition and metacognition, and the distinction among metacognitive functions, and thus, ultimately indicates its usefulness in identifying and interpreting metacognitive activities in complex modeling activities.

In addition, the results illustrate how MEAs work as a research method for verbal protocol analysis by providing a window into individual students' spontaneous verbal actions, and thus, providing access to their metacognitive activities, and how MEAs work as an environmental source to stimulate students' spontaneous metacognitive activities at the individual and social levels.

How Does Students' Thinking Become Metacognitive During Complex Modeling Activities?

By using the theoretical model of metacognition on multiple levels, a large enough sample of metacognitive activities was identified to begin the work of exploring general patterns. Several frequency tables of the identified students' metacognitive activities were developed to explore how the students' thinking became metacognitive within each MEA, and across the three MEAs–Summer Jobs, Volleyball, and Paper Airplane MEAs.

As mentioned already in Chapter 3, these tables, including frequency counts, were developed to explore general patterns from the data analysis, not to do a statistical analysis. Through comparing the frequencies of the identified metacognitive activities across the seven groups, across the three MEAs, and across the group members, several meaningful patterns were found. In the following sections, the identified patterns of the students' metacognitive activities are described. The following sections are organized around the three sub-questions of the second research question, and finally, a summary section is provided for the second research question overall.

a) How do metacognitive abilities manifest during complex modeling activities? This section will present the analysis and findings pertaining to the first part of the second research question, which sought to explore patterns and tendencies of students' metacognitive activities, within and across the three problem-solving sessions. Table 4.22 below reports the overall summary of the identified metacognitive activities with respect to the regulating and monitoring activities. The total number of transcript segments identified as metacognitive activities across the three MEAs for all of the seven groups is 4,069, which is the sum of 177 regulating activities and 3,892 monitoring activities. The quantity of transcript segments found to be regulating and monitoring activities are analyzed with respect to the entire set of transcript segments and are listed as a percentage of the total number of transcript segments.

Table 4.22

Metacognitive Activities Identified Overall

		Metacognitive activities	
Group	Total number of metacognitive activities (%)	Total number of regulating activities (%)	Total number of monitoring activities (%)
Total	4069 (100%)	177 (4.3%)	3892 (95.7%)

A predominance of monitoring activities compared to regulating activities. As shown above in Table 4.22, there was a clear predominance of monitoring activities and a relative infrequency of regulating activities, overall. The overwhelming majority of metacognitive activities was identified as monitoring activities compared to regulating activities (95.7% vs. 4.3% respectively), based on a ratio of the numbers of transcript segments identified as monitoring activities and regulating activities to the total number of identified metacognitive activities, respectively.

A predominance of metacognitive activities overall triggered at the social level. Table 4.23 below summarizes the frequencies with which the students' metacognitive activities were identified at both the individual and social levels across all of the three MEAs. The numbers of transcript segments, which were identified as metacognitive activities triggered at the individual and social levels, are first indicated, respectively. The numbers of transcript segments identified for regulating or monitoring activities triggered at the individual and social levels are secondly indicated, respectively. The numbers are followed by the percentage of identified transcript segments in parentheses, respectively.

Table 4.23

Metacognitive Activities Identified Overall at the Individual and Social Levels

			Metacognitive activities			
Group	Total number of metacognitive activities (%)		Total number of regulating activities (%)		Total number of monitoring activities (%)	
	Individual	Social	Individual	Social	Individual	Social
Total	1352 (33.2%)	2717 (66.8%)	129 (3.2%)	48 (1.2%)	1223 (30.1%)	2669 (65.6%)

Overall, metacognitive activities triggered at the social level were more frequent than those at the individual level. As illustrated in Table 4.23, metacognitive activities triggered at the social level were identified more than twice as often as those at the individual level (66.8% vs. 33.2%). This predominance of metacognitive activities triggered at the social level might indicate that through interactions with others, which provide access to others' conceptual systems, students can have more external, social sources that elicit their metacognitive activities. This predominance of students' metacognitive activities triggered at the social level, versus those at the individual level, might also indicate that students' metacognitive activities are likely to be frequently triggered on the basis of external, social sources.

A frequency of monitoring activities triggered at the social level. In particular, the prevalence of metacognitive activities triggered at the social level was slightly more evident during monitoring activities. While the ratio of the number of transcript segments identified as metacognitive activities triggered at the social level to the number of transcript segments identified as metacognitive activities triggered at the individual level is 2.0 (2717 transcript segments vs. 1352 transcript segments), the ratio of the number of transcript segments identified as monitoring activities triggered at the social level to the number of transcript segments identified as monitoring activities triggered at the individual level is 2.2 (2669 vs. 1223). Thus, the prevalence ratio of metacognitive activities triggered at the social level increased slightly from 2.0 to 2.2.

An infrequency of regulating activities triggered at the social level. On the other hand, regulating activities triggered at the social level were relatively more infrequent than those at the individual level. As shown above in Table 4.23, regulating activities triggered at the individual level were identified almost threefold as often as those at the social level (3.2% vs. 1.2%).

This infrequency of regulating activities triggered at the social level compared to those at the individual level might indicate that regulating activities tends to be dependent upon more individual sources, and thus, internal rather than external. In other words, the type of metacognitive activity, regulating activities – *new idea*, *changing strategy*, *correcting errors*, and *reinterpretation* – tend to be triggered by oneself, rather than through one's interaction with others (3.2% vs. 1.2%). However, this infrequency of regulating activities triggered at the social level seems to be mainly due to the prevalence of a *new idea* triggered at the individual level, as shown below in Table 4.24. A *new idea* triggered at the individual level was identified almost seven times as often as that at the social level (2.1% vs. 0.3%), while other regulating activities seemed to be relatively balanced on both the individual and social levels.

Table 4.24

Overall Four Regulating Activities Identified at the Individual and Social Levels

Regulating Activities	Individual	Social	Total
Total number of *NI (%)	84 (2.1%)	13 (0.3%)	97 (2.4%)
Total number of *RI (%)	23 (0.6%)	15 (0.4%)	38 (0.9%)
Total number of *CS (%)	6 (0.1%)	7 (0.2%)	13 (0.3%)
Total number of *CE (%)	16 (0.4%)	13 (0.3%)	29 (0.7%)
Total	129 (3.2%)	48 (1.2%)	177 (4.3%)

*NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; and CE = Correcting Errors.

This tendency of students' regulating activities, in particular, *new idea*, depending on more individual sources, and their overwhelming infrequency versus monitoring activities, which was previously identified, might support the prior finding: Students' metacognitive activities tend to be triggered on the basis of external, social sources.

Assessment of progress toward goal, strategy execution, and accuracy or sense of result vs. assessment of knowledge, understanding, and strategy appropriateness. More interesting patterns are shown in Table 4.25 below when reorganizing the monitoring activities into two categories with respect to different foci of assessment, cognition and procedure: (a) *assessment of cognition,* which is made up of the monitoring activities: assessment of knowledge, assessment of understanding, and assessment of strategy appropriateness; and (b) *assessment of procedure,* which includes the monitoring activities: assessment of progress toward goal, assessment of strategy execution, and assessment of accuracy or sense of result. *Assessment of procedure* seems to occur based on more tangible evaluation results through more explicit actions, while *assessment of cognition* seems not to be directly associated with explicit actions, but rather largely mentally running.

Table 4.25

Monitoring Activities Identified Overall at the Individual and Social Levels

Group	Total number of monitoring activities (%)		Monitoring activities			
			Total number of *A-K, A-U, & A-S (%)		Total number of *A-P, A-E, & A-R (%)	
	Individual	Social	Individual	Social	Individual	Social
Total	1223 (30.1%)	2669 (65.6%)	93 (2.3%)	481 (11.8%)	1130 (27.8%)	2188 (53.8%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

For example, depending on their own notes or numeric outcomes during problem solving, students make an assessment of strategy execution. However, they need to make an assessment of strategy appropriateness, depending only on their mental runs without aid from tangible evaluation results.

A relative infrequency of monitoring activities overall in assessment of cognition. As shown above in Table 4.25, there was a relative infrequency of monitoring activities overall in *assessment of cognition* than in *assessment of procedure* (14.1% vs. 81.6%). This infrequency may be, in part, due to characteristics of monitoring activities in *assessment of cognition*, which may be mainly dependent upon mental runs with limited cues through less explicit actions that are produced during problem solving, such as students' notes and numeric outcomes. On the other hand, monitoring activities in *assessment of procedure* may be directly associated with these by-products of problem-solving processes, which are more tangible evaluation results through more explicit actions. Depending on these tangible sources, the students might have more chances to trigger their spontaneous metacognitive activities.

Consequently, this infrequency of monitoring activities in *assessment of cognition* might indicate that students' metacognitive activities tend to be triggered on the basis of external, tangible (concrete) sources. However, this infrequency of monitoring activities in *assessment of cognition* could be due to their characteristics, in and of themselves: They are largely mentally running, and thus, may not be externalized in terms of students' spontaneous verbal actions and are thus, unidentified.

A relatively more evident prevalence of monitoring activities triggered at the social level in assessment of cognition. As shown above in Table 4.25, a relative prevalence of metacognitive activities triggered at the social level was more evident in *assessment of cognition* than that in *assessment of procedure*. Monitoring activities triggered at the social level were identified more than quintuple as often as those at the individual level (11.8% vs. 2.3%) in *assessment of cognition*, while monitoring activities triggered at the social level were identified about double as often as those at the individual level (53.8% vs. 27.8%) in *assessment of procedure*.

The results indicate that perhaps as the students had access to peers' conceptual systems through interaction with them, their monitoring activities in *assessment of cognition* were more actively triggered, depending on social sources, which might have led to more explicit actions, such as feedback from peers. In particular, *assessment of understanding* and *assessment of strategy appropriateness* were identified at the social level dramatically often, compared to those at the individual level (5.0% vs. 0.9% and 6.6% vs. 1.1% respectively), as shown below in Table 4.26. The steep increase in the frequencies of students' assessment of understanding and of the strategy appropriateness triggered at the social level may indicate that students' metacognitive activities tend to be triggered on the basis of external, social sources, which lead to more explicit actions and more concrete and tangible evaluation results.

Table 4.26

Overall Six Monitoring Activities Identified at the Individual and Social Levels

Monitoring Activities		Individual	Social	Total
Assessment of cognition	Total number of *A-K (%)	10 (0.2%)	9 (0.2%)	19 (0.5%)
	Total number of *A-U (%)	37 (0.9%)	205 (5.0%)	242 (5.9%)
	Total number of *A-S (%)	46 (1.1%)	267 (6.6%)	313 (7.7%)
Assessment of procedure	Total number of *A-P (%)	400 (9.8%)	598 (14.7%)	998 (24.5%)
	Total number of *A-E (%)	428 (10.5%)	996 (24.5%)	1424 (35.0%)
	Total number of *A-R (%)	302 (7.4%)	594 (14.6%)	896 (22.0%)
Total		1223 (30.1%)	2669 (65.6%)	3892 (95.7%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

However, *assessment of knowledge* was rarely identified at both the individual and social levels. *Assessment of knowledge* may especially occur in individual heads, and thus, is not easy to identify, or it may be a sensitive assessment for individuals to unpack.

A predominance of monitoring activities triggered at the individual level in assessment of procedure. On the other hand, monitoring activities in *assessment of procedure–assessment of progress toward goal*, *assessment of strategy execution*, and *assessment of accuracy or sense of result*–were more often identified at the individual level (9.8%, 10.5%, and 7.4%, respectively), compared to other metacognitive activities triggered at the individual level (Table 4.24 and Table 4.26).

These results support the assumption mentioned above when reorganizing six monitoring activities into *assessment of cognition* and *assessment of procedure*: Monitoring activities in *assessment of procedure* seem to occur based on more tangible evaluation results through more explicit actions. Thus, an individual herself or himself may have more opportunities to trigger her or his metacognitive activities during problem solving. Consequently, this predominance of monitoring activities triggered at the individual level in *assessment of procedure* shows that students' metacognitive activities tend to be triggered on the basis of more concrete and tangible results of assessment via more explicit actions.

As shown above in Table 4.26, the frequencies of monitoring activities triggered at the social level in *assessment of procedure* also mildly increased by 14.7% (598 transcript segments out of 4069), 24.5% (996 out of 4069), and 14.6% (594 out of 4069), respectively. Table 4.27 below summarizes all of the patterns identified so far.

Table 4.27

Identified Patterns of Students' Metacognitive Activities

All of the patterns identified so far for the full set of data
1. A predominance of monitoring activities compared to regulating activities
2. A predominance of metacognitive activities overall triggered at the social level, compared to those at the individual level
1) A relatively more evident prevalence of monitoring activities triggered at the social level, compared to that at the individual level
2) An infrequency of regulating activities, in particular, *new idea*, triggered at the social level, compared to those at the individual level
3. A relative infrequency of monitoring activities overall in *assessment of cognition* compared to those in *assessment of procedure*
4. A relatively more evident prevalence of monitoring activities triggered at the social level in *assessment of cognition*, compared to that in *assessment of procedure*
5. A predominance of monitoring activities triggered at the individual level in *assessment of procedure*, compared to other metacognitive activities triggered at the individual level

One example of metacognition triggered at the environmental level: the three MEAs. Shown below in Table 4.28 are the data separated according to each of the three MEAs–Summer Jobs, Volleyball, and Paper Airplane MEAs. The number of transcript segments, which were identified as (a) overall metacognitive activities; (b) monitoring and regulating activities; and (c) *assessment of cognition* and *assessment of procedure* at the individual and social levels, respectively, for each MEA is presented. The percentage of the segments also follows in parentheses, respectively.

Table 4.28

Metacognitive Activities Identified Across MEAs

MEA	Total number of metacognitive activities (%)		Metacognitive activities					
			Total number of regulating activities (%)		Total number of monitoring activities (%)			
					*A-K, A-U, & A-S		*A-P, A-E, & A-R	
	Individual	Social	Individual	Social	Individual	Social	Individual	Social
Summer Jobs	452 (31.0%)	1007 (69.0%)	36 (2.5%)	18 (1.2%)	26 (1.8%)	126 (8.6%)	390 (26.7%)	863 (59.2%)
Volleyball	540 (33.1%)	1091 (66.9%)	23 (1.4%)	10 (0.6%)	16 (1.0%)	114 (7.0%)	501 (30.7%)	967 (59.3%)
Paper Airplane	360 (36.8%)	619 (63.2%)	70 (7.2%)	20 (2.0%)	51 (5.2%)	241 (24.6%)	239 (24.4%)	358 (36.6%)
Total	1352 (33.2%)	2717 (66.8%)	129 (3.2%)	48 (1.2%)	93 (2.3%)	481 (11.8%)	1130 (27.8%)	2188 (53.8%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

The results show that even when separating the data in terms of each MEA, the same overall tendencies of students' metacognitive activities, which have been revealed so far, are once again revealed (Table 4.27 above). However, there are also certain differences not only in the quantity of the identified metacognitive activities, as shown below in Table 4.28, but also perhaps, in the quality of those outcomes. These differences in the students' metacognitive activities identified across the three MEAs–Summer Jobs, Volleyball, and Paper Airplane MEAs–might be able to illustrate metacognitive activities triggered at the environmental level, which are triggered by problem-solving tasks with different characteristics, such as different levels in the conceptual and cognitive demands of problem-solving processes.

For example, during the Paper Airplane MEA, the students' regulating activities triggered at the individual level were identified more than quintuple as often as those at the individual level during the Volleyball MEA (7.2% vs. 1.4%), and almost triple as often as those at the individual level during the Summer Jobs MEA (7.2% vs. 2.5%). Also, similar patterns hold in *assessment of cognition* triggered at the individual level during the Paper Airplane MEA, which were identified more than quintuple and almost threefold as often as those at the individual level during the Volleyball MEA (5.2% vs. 1.0%) and during the Summer Jobs MEA (5.2% vs. 1.8%), respectively.

These differences might be explained by different characteristics of the Paper Airplane MEA. Unlike the two MEAs, the Paper Airplane MEA (Appendix C) involves two subtasks–the *best floater* problem and the *most accuracy* problem–requiring two problem-solving processes, which are required in MEAs: (a) definition building–the process to define a qualitative construct; and (b) operationalizing definitions–the process to make a qualitative construct measurable, such as quantifying, converting, and sampling. Thus, the Paper Airplane MEA requires two sets of definition building and operationalizing definitions, while the two MEAs involve only one set of the two problem-solving processes. As a result, the three MEAs illustrate how they differently served as an environmental source to stimulate students' spontaneous metacognitive activities triggered at the individual and social levels.

Patterns of students' metacognitive activities across groups. The frequencies with which each student group revealed metacognitive activities triggered at both the individual and social levels across all of the three MEAs were summarized in Table 4.29

below. The quantity of transcript segments, indicated as (a) the entire range of metacognitive activities; (b) all monitoring and regulating activities; and (c) *assessment of cognition* and *assessment of procedure*, from the individual and social perspectives for each group of students, is shown. The segment percentages follow in parentheses.

Table 4.29

Metacognitive Activities Identified Across Groups

Group	Total number of metacognitive activities (%)		Metacognitive activities					
			Total number of regulating activities (%)		Total number of monitoring activities (%)			
					*A-K, A-U, & A-S		*A-P, A-E, & A-R	
	Individual	Social	Individual	Social	Individual	Social	Individual	Social
Group 1	199 (33.0%)	404 (67.0%)	24 (4.0%)	11 (1.8%)	15 (2.5%)	73 (12.1%)	160 (26.5%)	320 (53.1%)
Group 2	99 (38.7%)	157 (61.3%)	8 (3.1%)	1 (0.4%)	5 (2.0%)	36 (14.1%)	86 (33.6%)	120 (46.9%)
Group 3	258 (34.4%)	491 (65.6%)	27 (3.6%)	13 (1.7%)	27 (3.6%)	100 (13.4%)	204 (27.2%)	378 (50.5%)
Group 4	66 (30.6%)	150 (69.4%)	5 (2.3%)	2 (0.9%)	5 (2.3%)	23 (10.6%)	56 (25.9%)	125 (57.9%)
Group 5	268 (31.8%)	576 (68.2%)	32 (3.8%)	18 (2.1%)	18 (2.1%)	117 (13.9%)	218 (25.8%)	441 (52.3%)
Group 6	329 (31.1%)	730 (68.9%)	18 (1.7%)	2 (0.2%)	13 (1.2%)	100 (9.4%)	298 (28.1%)	628 (59.3%)
Group 7	133 (38.9%)	209 (61.1%)	15 (4.4%)	1 (0.3%)	10 (2.9%)	32 (9.4%)	108 (31.6%)	176 (51.5%)
Total	1352 (33.2%)	2717 (66.8%)	129 (3.2%)	48 (1.2%)	93 (2.3%)	481 (11.8%)	1130 (27.8%)	2188 (53.8%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

As shown above, the patterns identified for the entire dataset (Table 4.27 above, p. 111) are evident across the seven student groups, even though there are clear quantitative differences in the students' metacognitive activities identified within each group. The quantitative differences in the students' metacognitive activities identified across the seven groups, which were directly due to the different quantities of audio transcripts, indicate different degrees to which group members were engaged in group work during the three MEAs. For example, the relatively small numbers of transcript segments, which were identified as metacognitive activities from Group 2 and Group 4 across the three MEAs (total of 256 transcript segments and 216 respectively) compared to other groups, indicate that the group members passively engaged in collaboration to solve problems. They even infrequently verbalized their thoughts, and thus, their verbal metacognitive actions were seldom identified across the three MEAs.

Patterns of each student's metacognitive activities. Shown in Table 4.30 – 4.36 below are the total number of transcript segments, identified as metacognitive activities triggered at the individual and social levels, respectively–with respect to overall metacognitive activities, monitoring and regulating activities, and *assessment of cognition* and *assessment of procedure* over the three MEAs–for each student within each group. The percentage of the segments is also shown in parentheses, respectively.

Table 4.30

Metacognitive Activities Identified Across Students within Group 1 Over the Three MEAs

Student	Total number of metacognitive activities (%)		Metacognitive activities					
			Total number of regulating activities (%)		Total number of monitoring activities (%)			
					*A-K, A-U, & A-S		*A-P, A-E, & A-R	
	Individual	Social	Individual	Social	Individual	Social	Individual	Social
Student H1	63 (36.4%)	110 (63.6%)	7 (4.0%)	6 (3.5%)	5 (2.9%)	20 (11.6%)	51 (29.5%)	84 (48.6%)
Student H2	36 (25.5%)	105 (74.5%)	1 (0.7%)	0 (0.0%)	2 (1.4%)	29 (20.6%)	33 (23.4%)	76 (53.9%)
Student H3	50 (35.2%)	92 (64.8%)	9 (6.3%)	3 (2.1%)	4 (2.8%)	12 (8.5%)	37 (26.1%)	77 (54.2%)
Student H4	50 (34.0%)	97 (66.0%)	7 (4.8%)	2 (1.4%)	4 (2.7%)	12 (8.2%)	39 (26.5%)	83 (56.5%)
Total	199 (33.0%)	404 (67.0%)	24 (4.0%)	11 (1.8%)	15 (2.5%)	73 (12.1%)	160 (26.5%)	320 (53.1%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Table 4.31

Metacognitive Activities Identified Across Students within Group 2 Over the Three MEAs

Student	Total number of metacognitive activities (%)		Metacognitive activities					
			Total number of regulating activities (%)		Total number of monitoring activities (%)			
					*A-K, A-U, & A-S		*A-P, A-E, & A-R	
	Individual	Social	Individual	Social	Individual	Social	Individual	Social
Student K1	37 (40.2%)	55 (59.8%)	4 (4.3%)	1 (1.1%)	0 (0.0%)	20 (21.7%)	33 (35.9%)	34 (37.0%)
Student K2	22 (32.8%)	45 (67.2%)	2 (3.0%)	0 (0.0%)	1 (1.5%)	9 (13.4%)	19 (28.4%)	36 (53.7%)
Student K3	40 (41.2%)	57 (58.8%)	2 (2.1%)	0 (0.0%)	4 (4.1%)	7 (7.2%)	34 (35.1%)	50 (51.5%)
Total	99 (38.7%)	157 (61.3%)	8 (3.1%)	1 (0.4%)	5 (2.0%)	36 (14.1%)	86 (33.6%)	120 (46.9%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Table 4.32

Metacognitive Activities Identified Across Students within Group 3 Over the Three MEAs

Student	Total number of metacognitive activities (%)		Metacognitive activities					
			Total number of regulating activities (%)		Total number of monitoring activities (%)			
					*A-K, A-U, & A-S		*A-P, A-E, & A-R	
	Individual	Social	Individual	Social	Individual	Social	Individual	Social
Student M1	48 (33.3%)	96 (66.7%)	5 (3.5%)	1 (0.7%)	2 (1.4%)	13 (9.0%)	41 (28.5%)	82 (56.9%)
Student M2	101 (33.0%)	205 (67.0%)	13 (4.2%)	4 (1.3%)	13 (4.2%)	42 (13.7%)	75 (24.5%)	159 (52.0%)
Student M3	109 (36.5%)	190 (63.5%)	9 (3.0%)	8 (2.7%)	12 (4.0%)	45 (15.1%)	88 (29.4%)	137 (45.8%)
Total	258 (34.4%)	491 (65.6%)	27 (3.6%)	13 (1.7%)	27 (3.6%)	100 (13.4%)	204 (27.2%)	378 (50.5%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Table 4.33

Metacognitive Activities Identified Across Students within Group 4 Over the Three MEAs

Student	Total number of metacognitive activities (%)		Metacognitive activities					
			Total number of regulating activities (%)		Total number of monitoring activities (%)			
					*A-K, A-U, & A-S		*A-P, A-E, & A-R	
	Individual	Social	Individual	Social	Individual	Social	Individual	Social
Student R1	23 (31.9%)	49 (68.1%)	2 (2.8%)	0 (0.0%)	1 (1.4%)	8 (11.1%)	20 (27.8%)	41 (56.9%)
Student R2	10 (20.0%)	40 (80.0%)	0 (0.0%)	1 (2.0%)	2 (4.0%)	8 (16.0%)	8 (16.0%)	31 (62.0%)
Student R3	33 (35.1%)	61 (64.9%)	3 (3.2%)	1 (1.1%)	2 (2.1%)	7 (7.4%)	28 (29.8%)	53 (56.4%)
Total	66 (30.6%)	150 (69.4%)	5 (2.3%)	2 (0.9%)	5 (2.3%)	23 (10.6%)	56 (25.9%)	125 (57.9%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Table 4.34

Metacognitive Activities Identified Across Students within Group 5 Over the Three MEAs

Student	Total number of metacognitive activities (%)		Metacognitive activities					
			Total number of regulating activities (%)		Total number of monitoring activities (%)			
					*A-K, A-U, & A-S		*A-P, A-E, & A-R	
	Individual	Social	Individual	Social	Individual	Social	Individual	Social
Student S1	35 (27.3%)	93 (72.7%)	8 (6.3%)	7 (5.5%)	5 (3.9%)	23 (18.0%)	22 (17.2%)	63 (49.2%)
Student S2	143 (38.6%)	227 (61.4%)	23 (6.2%)	5 (1.4%)	12 (3.2%)	36 (9.7%)	108 (29.2%)	186 (50.3%)
Student S3	90 (26.0%)	256 (74.0%)	1 (0.3%)	6 (1.7%)	1 (0.3%)	58 (16.8%)	88 (25.4%)	192 (55.5%)
Total	268 (31.8%)	576 (68.2%)	32 (3.8%)	18 (2.1%)	18 (2.1%)	117 (13.9%)	218 (25.8%)	441 (52.3%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Table 4.35

Metacognitive Activities Identified Across Students within Group 6 Over the Three MEAs

Student	Total number of metacognitive activities (%)		Metacognitive activities					
			Total number of regulating activities (%)		Total number of monitoring activities (%)			
					*A-K, A-U, & A-S		*A-P, A-E, & A-R	
	Individual	Social	Individual	Social	Individual	Social	Individual	Social
Student P1	68 (35.4%)	124 (64.6%)	1 (0.5%)	1 (0.5%)	3 (1.6%)	12 (6.3%)	64 (33.3%)	111 (57.8%)
Student P2	104 (29.7%)	246 (70.3%)	7 (2.0%)	0 (0.0%)	5 (1.4%)	29 (8.3%)	92 (26.3%)	217 (62.0%)
Student P3	52 (29.7%)	123 (70.3%)	2 (1.1%)	0 (0.0%)	2 (1.1%)	20 (11.4%)	48 (27.4%)	103 (58.9%)
Student P4	105 (30.7%)	237 (69.3%)	8 (2.3%)	1 (0.3%)	3 (0.9%)	39 (11.4%)	94 (27.5%)	197 (57.6%)
Total	329 (31.1%)	730 (68.9%)	18 (1.7%)	2 (0.2%)	13 (1.2%)	100 (9.4%)	298 (28.1%)	628 (59.3%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Table 4.36

Metacognitive Activities Identified Across Students within Group 7 Over the Three MEAs

Student	Total number of metacognitive activities (%)		Metacognitive activities					
			Total number of regulating activities (%)		Total number of monitoring activities (%)			
					*A-K, A-U, & A-S		*A-P, A-E, & A-R	
	Individual	Social	Individual	Social	Individual	Social	Individual	Social
Student W1	42 (42.0%)	58 (58.0%)	7 (7.0%)	0 (0.0%)	1 (1.0%)	9 (9.0%)	34 (34.0%)	49 (49.0%)
Student W2	45 (35.4%)	82 (64.6%)	5 (3.9%)	0 (0.0%)	7 (5.5%)	12 (9.4%)	33 (26.0%)	70 (55.1%)
Student W3	46 (40.0%)	69 (60.0%)	3 (2.6%)	1 (0.9%)	2 (1.7%)	11 (9.6%)	41 (35.7%)	57 (49.6%)
Total	133 (38.9%)	209 (61.1%)	15 (4.4%)	1 (0.3%)	10 (2.9%)	32 (9.4%)	108 (31.6%)	176 (51.5%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Even when the data are analyzed from the perspective of individual students within each student group, all of the patterns identified so far are evident across each group member (Table 4.27, p. 111), such as the prevalence of metacognitive activities triggered at the social level, and the overall predominance of monitoring activities compared to regulating activities.

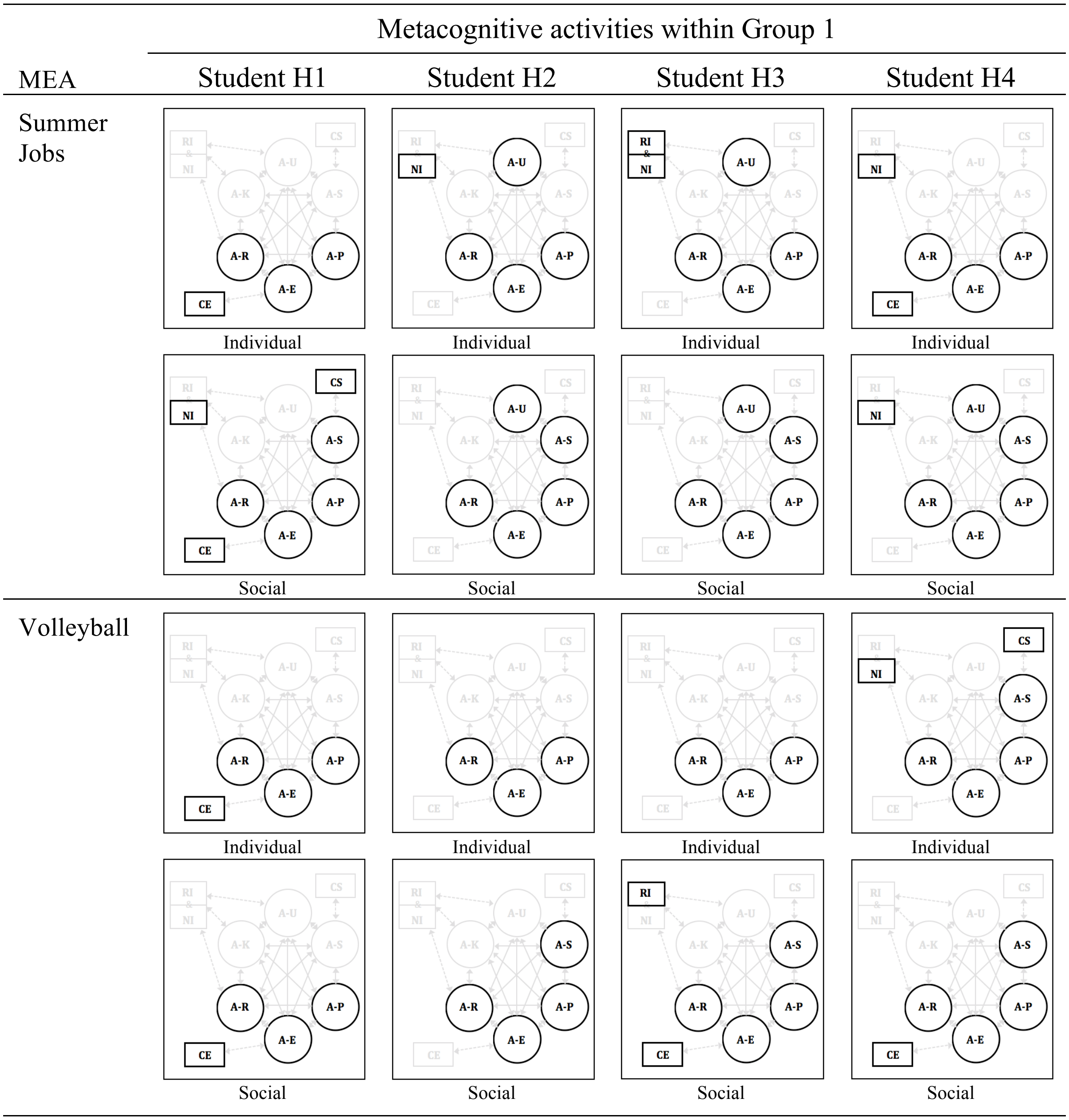
Summary of patterns of the students' metacognitive activities. Through exploring a large enough sample of students' spontaneous metacognitive activities, which were identified at both the individual and social levels across the three MEAs–Summer Jobs, Volleyball, and Paper Airplane MEAs, several patterns of students' metacognitive activities were identified. The first pattern was that monitoring activities were again and again reported as the overwhelming majority of identified metacognitive activities, versus regulating activities across groups, students, and the three MEAs. The second pattern was that overall, metacognitive activities were more frequently revealed at the social level versus those at the individual level across groups, students, and the three MEAs. In particular, the frequency of metacognitive activities triggered at the social level was slightly more evident in monitoring activities across groups, students, and the three MEAs. However, regulating activities, in particular, *new idea*, were more likely to be frequently activated at the individual level across groups, students, and the three MEAs.

The third pattern was that overall, monitoring activities in *assessment of cognition* (*assessment of knowledge*, *assessment of understanding*, and *assessment of strategy appropriateness*) were relatively less frequent than those in *assessment of procedure* (*assessment of progress toward goal*, *assessment of strategy execution*, and *assessment of accuracy or sense of result*) across groups, students, and the three MEAs. The fourth pattern was that a relative predominance of monitoring activities triggered at the social level was more evident in *assessment of cognition* versus in *assessment of procedure* across groups, students, and the three MEAs. Finally, a predominance of monitoring activities triggered at the individual level in *assessment of procedure* was revealed across groups, students, and the three MEAs, versus other metacognitive activities triggered at the individual level.

These patterns of students' metacognitive activities indicate that they tend to be triggered on the basis of external, social, and concrete sources. As students have access to others' conceptual systems through interaction with them, their metacognitive activities become more actively triggered, depending on more concrete and tangible evaluation results via more explicit actions. Consequently, the tendencies of students' metacognitive activities indicate that as students have access to more external sources, such as others' conceptual systems, they have more opportunities to develop their abilities efficiently to organize, monitor, and regulate what they know to reach a goal successfully.

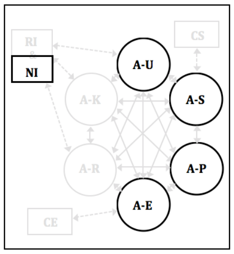
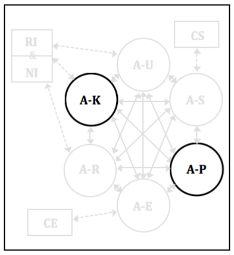
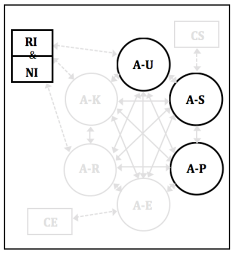
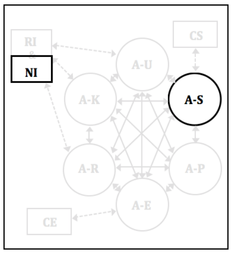
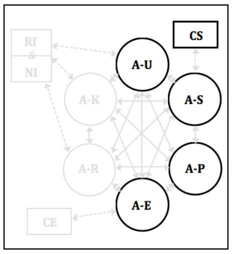
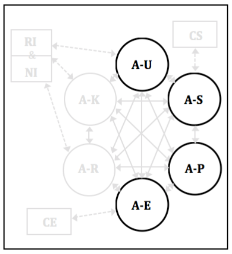
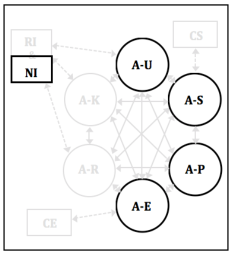
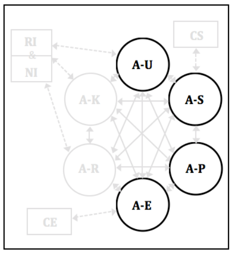
b) What is the nature of students' metacognitive abilities manifested during complex modeling activities? This section will present the analysis and findings pertaining to the second part of the second research question, which sought to explore the nature of students' metacognitive abilities. A cross-case analysis for the three MEAs was conducted to address this sub-question.

Shown in Figures 4.1 – 4.7 below are metacognitive activities identified at the individual and social levels from each student within each group across the three MEAs– Summer Jobs, Volleyball, and Paper Airplane MEAs. As mentioned in Chapter 3, this figure, which is consistent with the theoretical model in Figures 2.2 and 2.3, was used as an analytical tool to explore the overall patterns of each student's metacognitive activities within and across the three MEAs, and the nature of students' metacognitive abilities manifested during the problem-solving sessions. During each MEA (in rows), the types of identified metacognitive activities from each student (in columns) within each group were highlighted in each figure (at the individual and social levels, respectively).



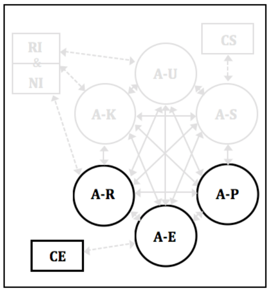
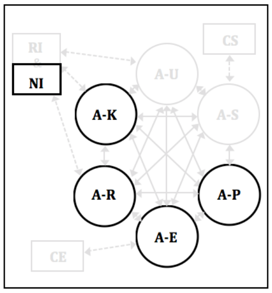
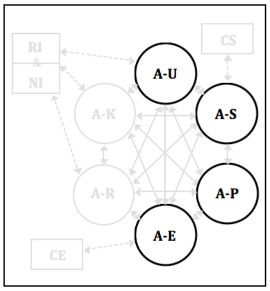
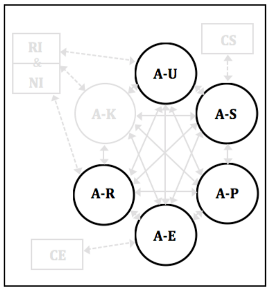
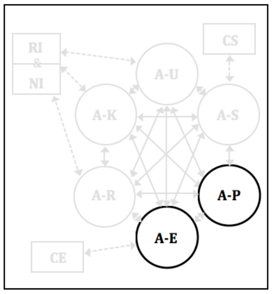
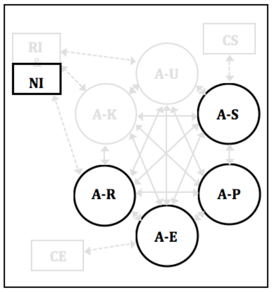
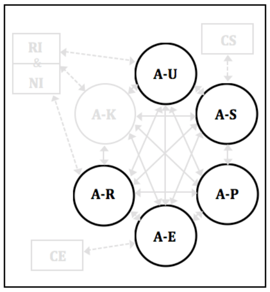
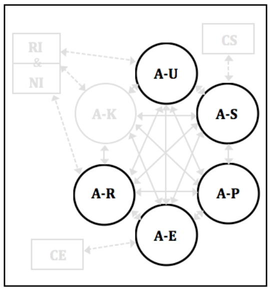
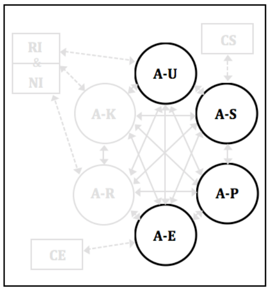
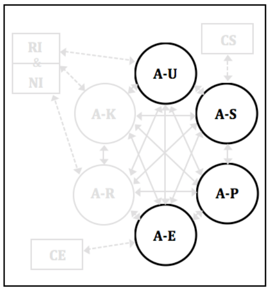
Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.1. Each student's metacognitive activities within Group 1 across MEAs.

	Metacognitive activities within Group 1			
MEA	Student H1	Student H2	Student H3	Student H4
Paper Airplane	 Individual	 Individual	 Individual	 Individual
	 Social	 Social	 Social	 Social

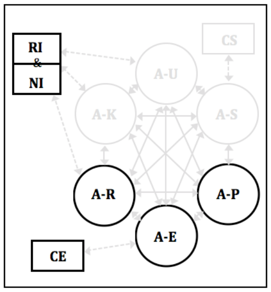
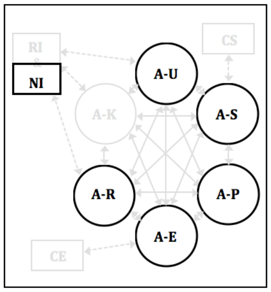
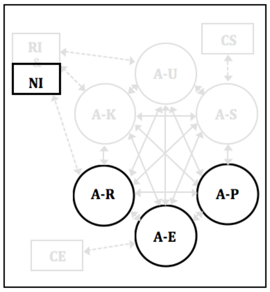
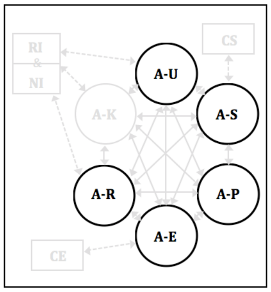
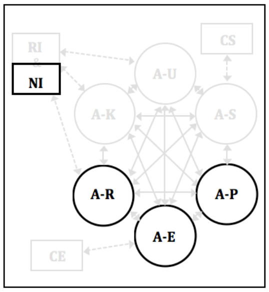
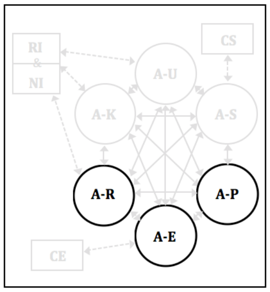
Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.1. continued.

	Metacognitive activities within Group 2		
MEA	Student K1	Student K2	Student K3
Summer Jobs	 Individual	ABSENCE Individual	 Individual
	 Social	ABSENCE Social	 Social
Volleyball	 Individual	 Individual	 Individual
	 Social	 Social	 Social

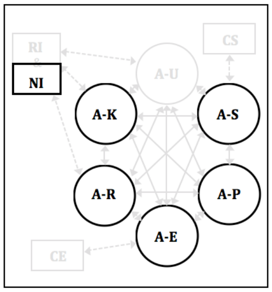
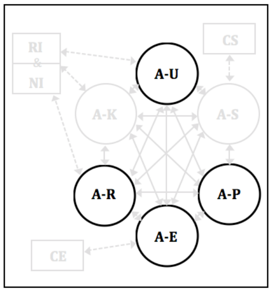
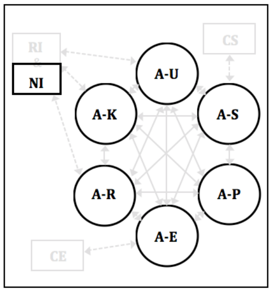
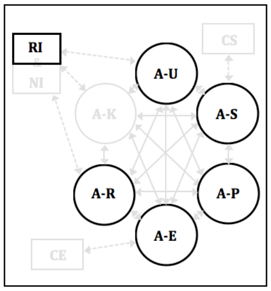
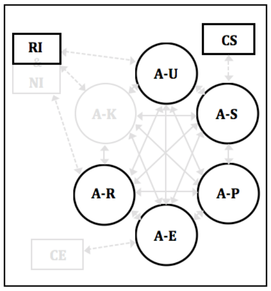
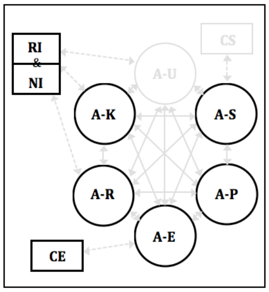
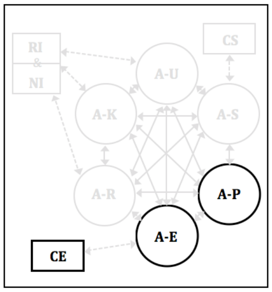
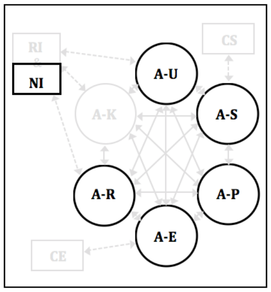
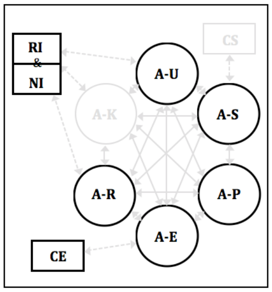
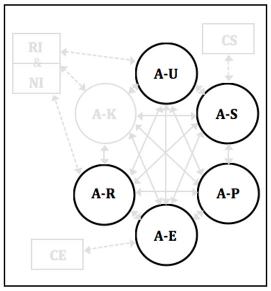
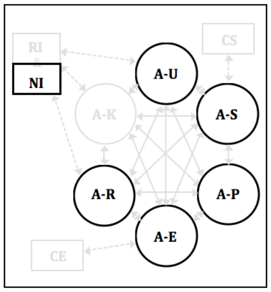
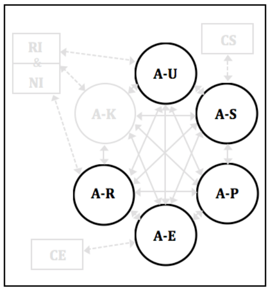
Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.2. Each student's metacognitive activities within Group 2 across MEAs.

	Metacognitive activities within Group 2		
MEA	Student K1	Student K2	Student K3
Paper Airplane	 Individual  Social	 Individual  Social	 Individual  Social

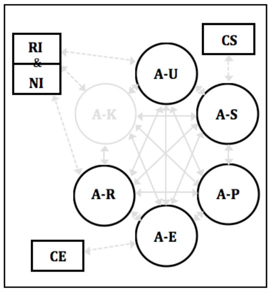
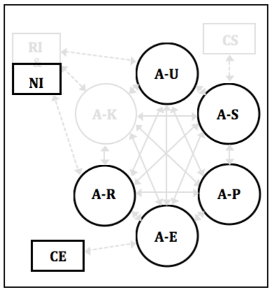
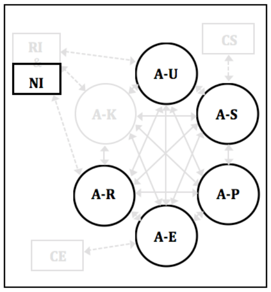
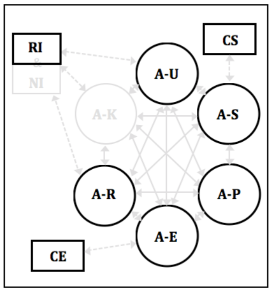
Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.2. continued.

	Metacognitive activities within Group 3		
MEA	Student M1	Student M2	Student M3
Summer Jobs	 Individual	 Individual	 Individual
	 Social	 Social	 Social
Volleyball	 Individual	 Individual	 Individual
	 Social	 Social	 Social

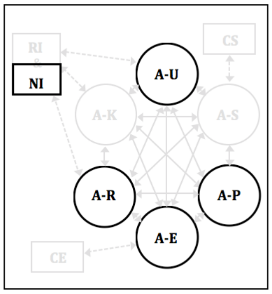
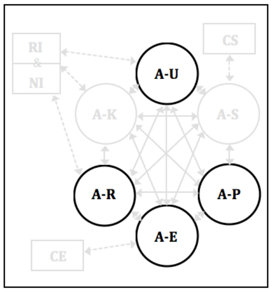
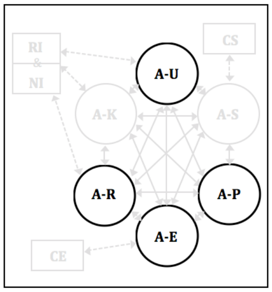
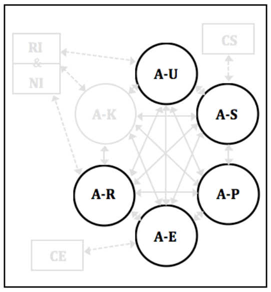
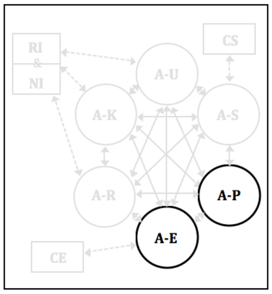
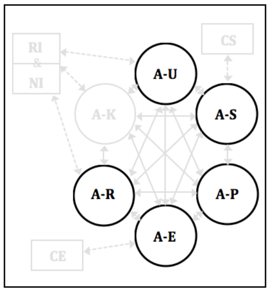
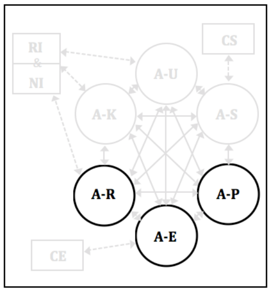
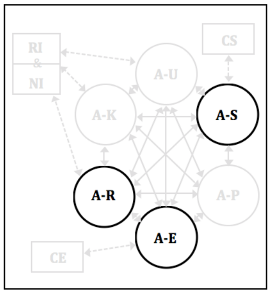
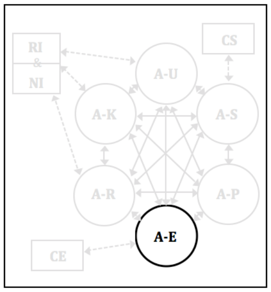
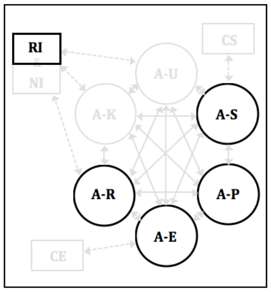
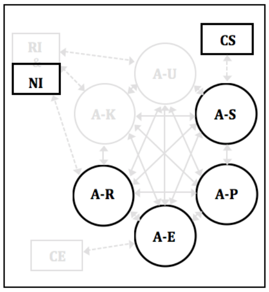
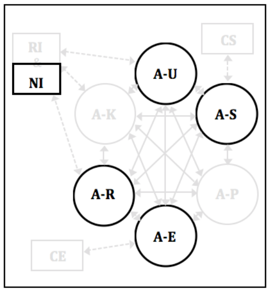
Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.3. Each student's metacognitive activities within Group 3 across MEAs.

	Metacognitive activities within Group 3		
MEA	Student M1	Student M2	Student M3
Paper Airplane	ABSENCE Individual	 Individual	 Individual
	ABSENCE Social	 Social	 Social

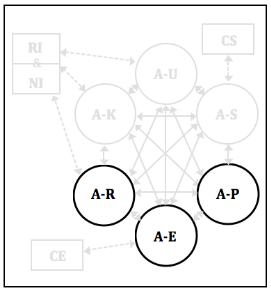
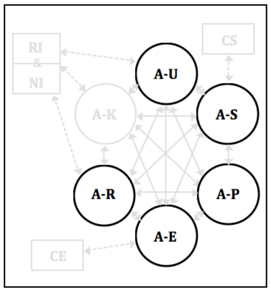
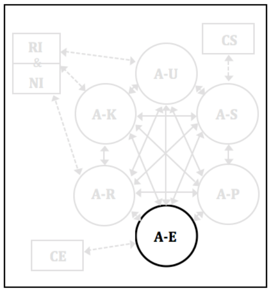
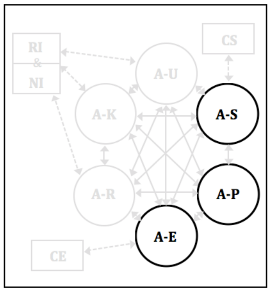
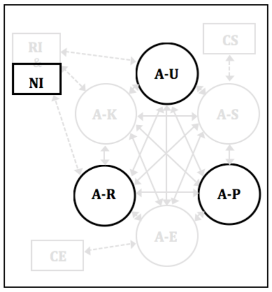
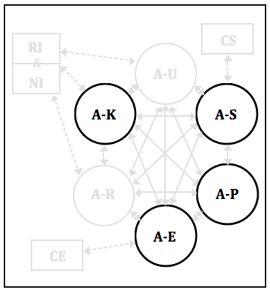
Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.3. continued.

	Metacognitive activities within Group 4		
MEA	Student R1	Student R2	Student R3
Summer Jobs	 Individual  Social	 Individual  Social	 Individual  Social
Volleyball	 Individual  Social	 Individual  Social	 Individual  Social

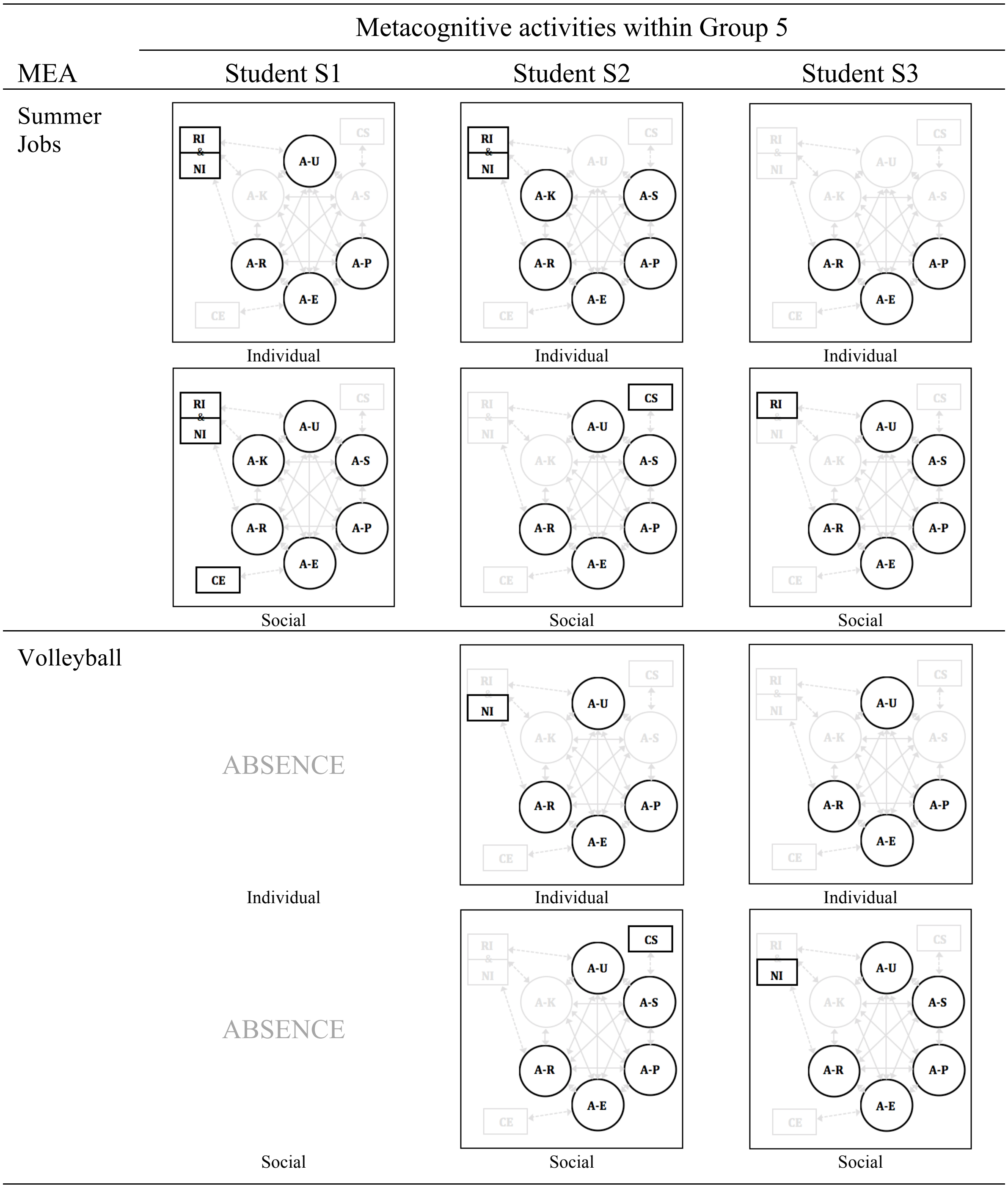
Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.4. Each student's metacognitive activities within Group 4 across MEAs.

	Metacognitive activities within Group 4		
MEA	Student R1	Student R2	Student R3
Paper Airplane	 Individual  Social	 Individual  Social	 Individual  Social

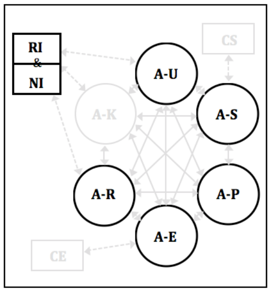
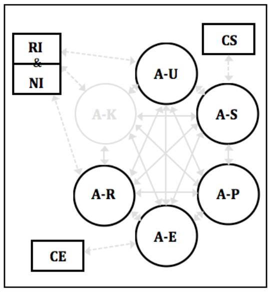
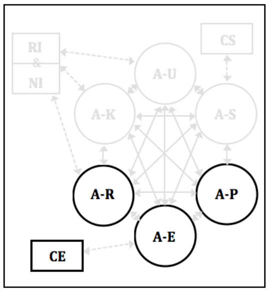
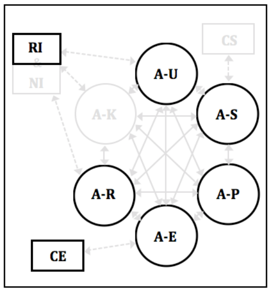
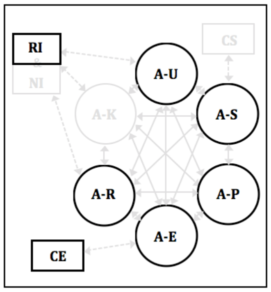
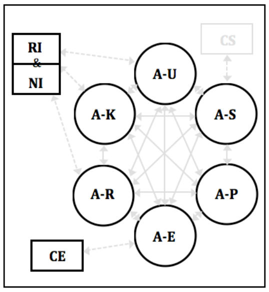
Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.4. continued.



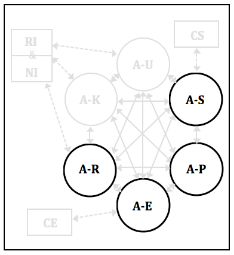
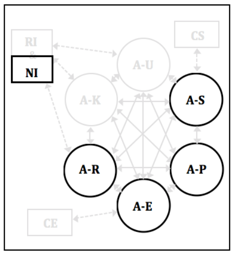
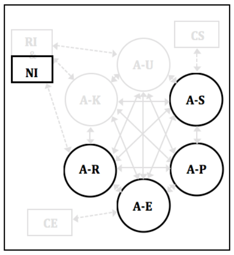
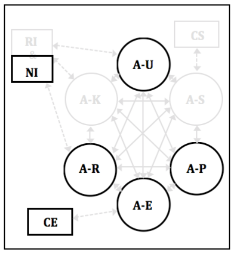
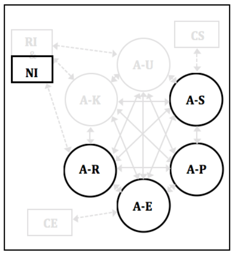
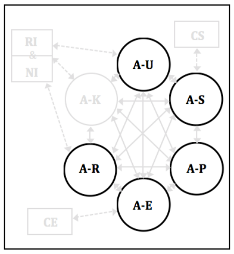
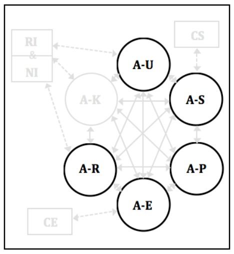
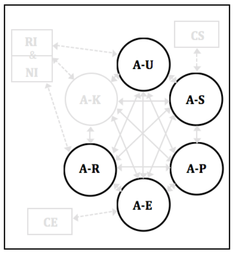
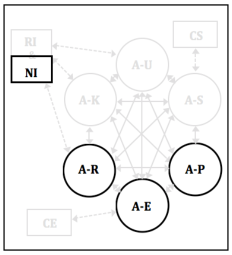
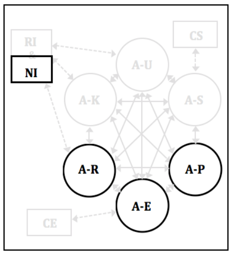
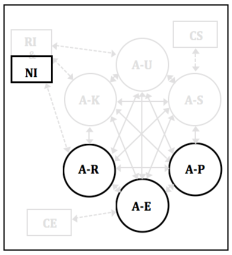
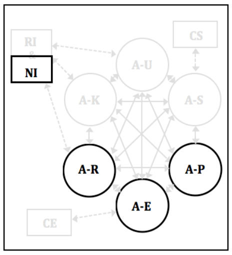
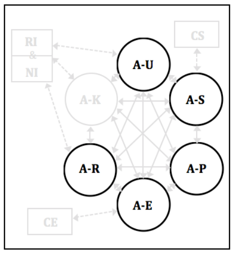
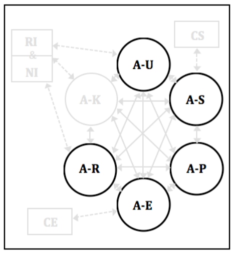
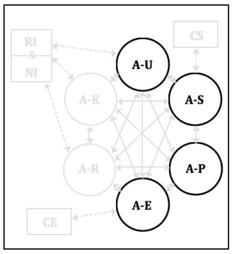
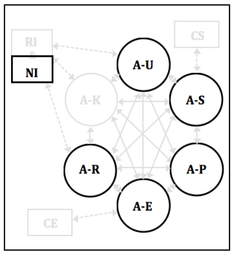
Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.5. Each student's metacognitive activities within Group 5 across MEAs.

	Metacognitive activities within Group 5		
MEA	Student S1	Student S2	Student S3
Paper Airplane	 Individual	 Individual	 Individual
	 Social	 Social	 Social

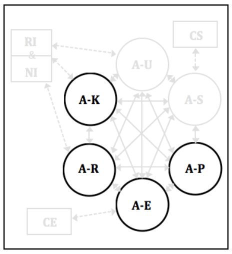
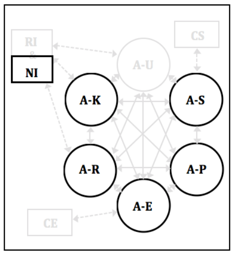
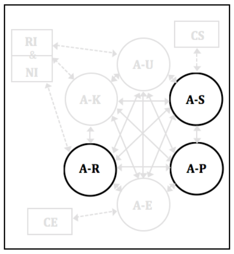
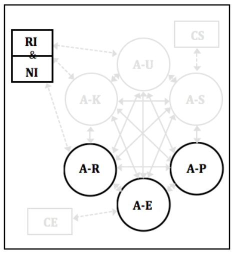
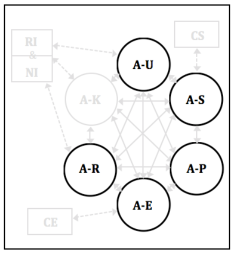
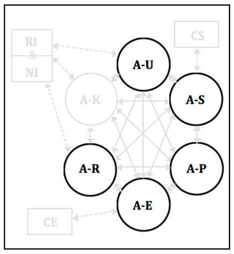
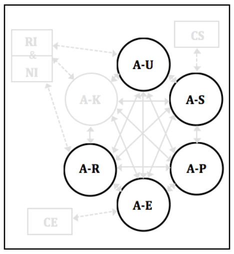
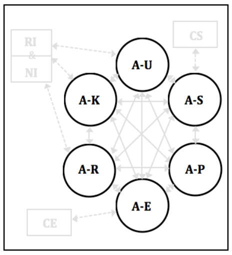
Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.5. continued.

	Metacognitive activities within Group 6			
MEA	Student P1	Student P2	Student P3	Student P4
Summer Jobs	 Individual	 Individual	 Individual	 Individual
	 Social	 Social	 Social	 Social
Volleyball	 Individual	 Individual	 Individual	 Individual
	 Social	 Social	 Social	 Social

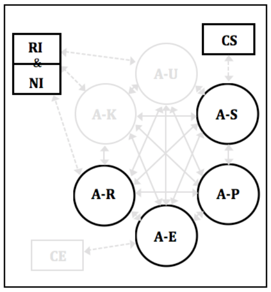
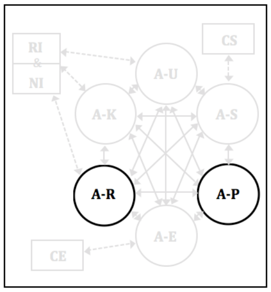
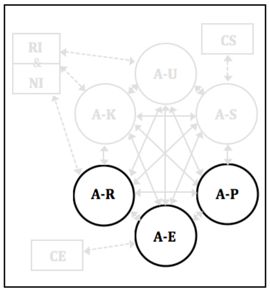
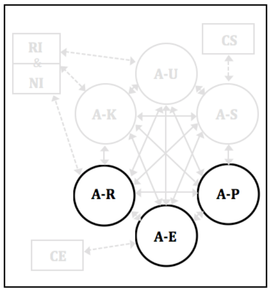
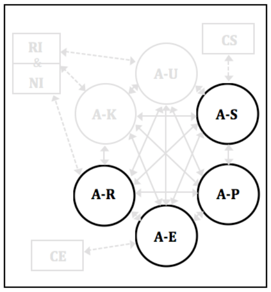
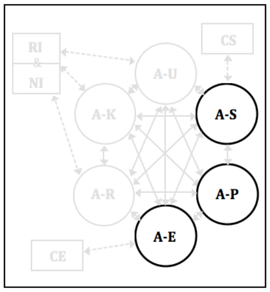
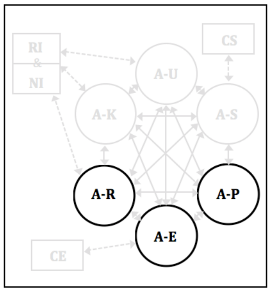
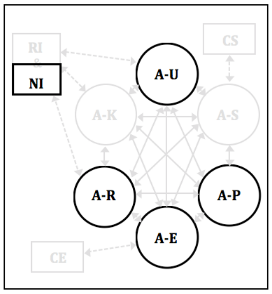
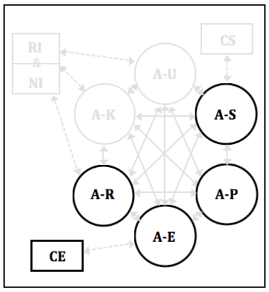
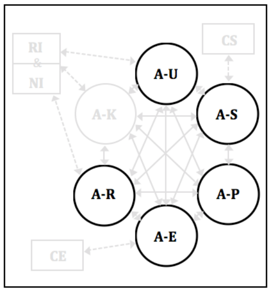
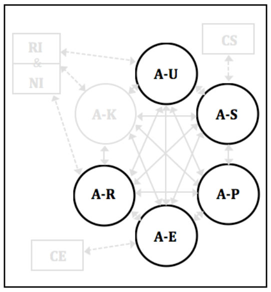
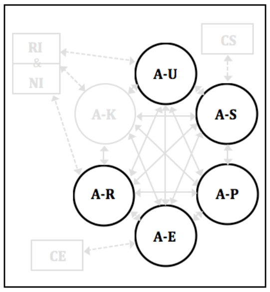
Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.6. Each student's metacognitive activities within Group 6 across MEAs.

	Metacognitive activities within Group 6			
MEA	Student P1	Student P2	Student P3	Student P4
Paper Airplane	 Individual	 Individual	 Individual	 Individual
	 Social	 Social	 Social	 Social

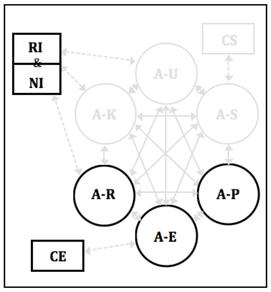
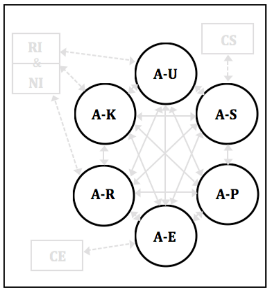
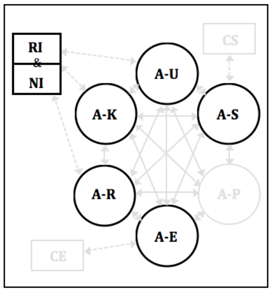
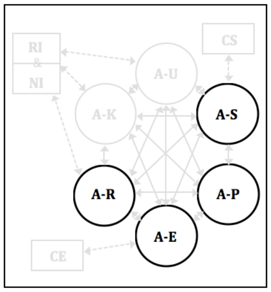
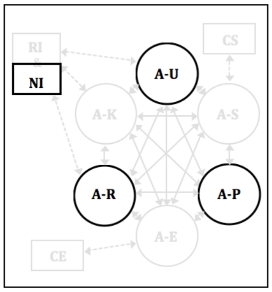
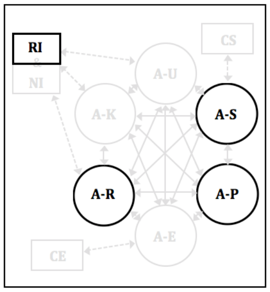
Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.6. continued.

	Metacognitive activities within Group 7		
MEA	Student W1	Student W2	Student W3
Summer Jobs	 Individual	 Individual	 Individual
	 Social	 Social	 Social
Volleyball	 Individual	 Individual	 Individual
	 Social	 Social	 Social

Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.7. Each student's metacognitive activities within Group 7 across MEAs.

	Metacognitive activities within Group 7		
MEA	Student W1	Student W2	Student W3
Paper Airplane	 Individual  Social	 Individual  Social	 Individual  Social

Note. NI = New Idea; RI = Reinterpretation; CS = Changing Strategy; CE = Correcting Errors; A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Figure 4.7. continued.

Also, as shown below, Tables 4.37 – 4.43 report the number of transcript segments, identified as (a) overall metacognitive activities; (b) monitoring and regulating activities; and (c) *assessment of cognition* and *assessment of procedure* triggered at the individual and social levels, respectively, for each student within each group across the three MEAs. The percentage of the segments also follows in parentheses, respectively.

Table 4.37

Metacognitive Activities Identified Across Students within Group 1

				Metacognitive activities					
		Total number of metacognitive activities (%)		Total number of regulating activities (%)		Total number of monitoring activities (%)			
						*A-K, A-U, & A-S		*A-P, A-E, & A-R	
		Individual	Social	Individual	Social	Individual	Social	Individual	Social
Summer Jobs	Student H1	17 (33.3%)	34 (66.7%)	1 (2.0%)	4 (7.8%)	0 (0.0%)	3 (5.9%)	16 (31.4%)	27 (52.9%)
	Student H2	6 (16.7%)	30 (83.3%)	1 (2.8%)	0 (0.0%)	1 (2.8%)	3 (8.3%)	4 (11.1%)	27 (75.0%)
	Student H3	27 (40.9%)	39 (59.1%)	4 (6.1%)	0 (0.0%)	1 (1.5%)	5 (7.6%)	22 (33.3%)	34 (51.5%)
	Student H4	11 (28.2%)	28 (71.8%)	3 (7.7%)	1 (2.6%)	0 (0.0%)	3 (7.7%)	8 (20.5%)	24 (61.5%)
	Total	61 (31.8%)	131 (68.2%)	9 (4.7%)	5 (2.6%)	2 (1.0%)	14 (7.3%)	50 (26.0%)	112 (58.3%)
Volley-ball	Student H1	29 (37.7%)	48 (62.3%)	2 (2.6%)	1 (1.3%)	0 (0.0%)	0 (0.0%)	27 (35.1%)	47 (61.0%)
	Student H2	22 (32.8%)	45 (67.2%)	0 (0.0%)	0 (0.0%)	0 (0.0%)	2 (3.0%)	22 (32.8%)	43 (64.2%)
	Student H3	13 (23.2%)	43 (76.8%)	0 (0.0%)	2 (3.6%)	0 (0.0%)	1 (1.8%)	13 (23.2%)	40 (71.4%)
	Student H4	35 (39.8%)	53 (60.2%)	3 (3.4%)	1 (1.1%)	1 (1.1%)	1 (1.1%)	31 (35.2%)	51 (58.0%)
	Total	99 (34.4%)	189 (65.6%)	5 (1.7%)	4 (1.4%)	1 (0.3%)	4 (1.4%)	93 (32.3%)	181 (62.8%)
Paper Airplane	Student H1	17 (37.8%)	28 (62.2%)	4 (8.9%)	1 (2.2%)	5 (11.1%)	17 (37.8%)	8 (17.8%)	10 (22.2%)
	Student H2	8 (21.1%)	30 (78.9%)	0 (0.0%)	0 (0.0%)	1 (2.6%)	24 (63.2%)	7 (18.4%)	6 (15.8%)
	Student H3	10 (50.0%)	10 (50.0%)	5 (25.0%)	1 (5.0%)	3 (15.0%)	6 (30.0%)	2 (10.0%)	3 (15.0%)
	Student H4	4 (20.0%)	16 (80.0%)	1 (5.0%)	0 (0.0%)	3 (15.0%)	8 (40.0%)	0 (0.0%)	8 (40.0%)
	Total	39 (31.7%)	84 (68.3%)	10 (8.1%)	2 (1.6%)	12 (9.8%)	55 (44.7%)	17 (13.8%)	27 (22.0%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Table 4.38

Metacognitive Activities Identified Across Students within Group 2

				Metacognitive activities					
		Total number of metacognitive activities (%)		Total number of regulating activities (%)		Total number of monitoring activities (%)			
						*A-K, A-U, & A-S		*A-P, A-E, & A-R	
		Individual	Social	Individual	Social	Individual	Social	Individual	Social
Summer Jobs	Student K1	6 (37.5%)	10 (62.5%)	1 (6.3%)	0 (0.0%)	0 (0.0%)	4 (25.0%)	5 (31.3%)	6 (37.5%)
	Student K2	Absence	Absence	Absence	Absence	Absence	Absence	Absence	Absence
	Student K3	10 (41.7%)	14 (58.3%)	1 (4.2%)	0 (0.0%)	1 (4.2%)	3 (12.5%)	8 (33.3%)	11 (45.8%)
	Total	16 (40.0%)	24 (60.0%)	2 (5.0%)	0 (0.0%)	1 (2.5%)	7 (17.5%)	13 (32.5%)	17 (42.5%)
Volley-ball	Student K1	7 (25.0%)	21 (75.0%)	0 (0.0%)	0 (0.0%)	0 (0.0%)	7 (25.0%)	7 (25.0%)	14 (50.0%)
	Student K2	11 (34.4%)	21 (65.6%)	1 (3.1%)	0 (0.0%)	1 (3.1%)	4 (12.5%)	9 (28.1%)	17 (53.1%)
	Student K3	18 (38.3%)	29 (61.7%)	0 (0.0%)	0 (0.0%)	3 (6.4%)	4 (8.5%)	15 (31.9%)	25 (53.2%)
	Total	36 (33.6%)	71 (66.4%)	1 (0.9%)	0 (0.0%)	4 (3.7%)	15 (14.0%)	31 (29.0%)	56 (52.3%)
Paper Airplane	Student K1	24 (50.0%)	24 (50.0%)	3 (6.3%)	1 (2.1%)	0 (0.0%)	9 (18.8%)	21 (43.8%)	14 (29.2%)
	Student K2	11 (31.4%)	24 (68.6%)	1 (2.9%)	0 (0.0%)	0 (0.0%)	5 (14.3%)	10 (28.6%)	19 (54.3%)
	Student K3	12 (46.2%)	14 (53.8%)	1 (3.8%)	0 (0.0%)	0 (0.0%)	0 (0.0%)	11 (42.3%)	14 (53.8%)
	Total	47 (43.1%)	62 (56.9%)	5 (4.6%)	1 (0.9%)	0 (0.0%)	14 (12.8%)	42 (38.5%)	47 (43.1%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Table 4.39

Metacognitive Activities Identified Across Students within Group 3

				Metacognitive activities					
		Total number of metacognitive activities (%)		Total number of regulating activities (%)		Total number of monitoring activities (%)			
						*A-K, A-U, & A-S		*A-P, A-E, & A-R	
		Individual	Social	Individual	Social	Individual	Social	Individual	Social
Summer Jobs	Student M1	32 (36.8%)	55 (63.2%)	3 (3.4%)	1 (1.1%)	2 (2.3%)	10 (11.5%)	27 (31.0%)	44 (50.6%)
	Student M2	22 (25.3%)	65 (74.7%)	0 (0.0%)	2 (2.3%)	1 (1.1%)	16 (18.4%)	21 (24.1%)	47 (54.0%)
	Student M3	22 (29.7%)	52 (70.3%)	2 (2.7%)	3 (4.1%)	4 (5.4%)	14 (18.9%)	16 (21.6%)	35 (47.3%)
	Total	76 (30.6%)	172 (69.4%)	5 (2.0%)	6 (2.4%)	7 (2.8%)	40 (16.1%)	64 (25.8%)	126 (50.8%)
Volley-ball	Student M1	16 (28.1%)	41 (71.9%)	2 (3.5%)	0 (0.0%)	0 (0.0%)	3 (5.3%)	14 (24.6%)	38 (66.7%)
	Student M2	26 (22.8%)	88 (77.2%)	1 (0.9%)	1 (0.9%)	2 (1.8%)	7 (6.1%)	23 (20.2%)	80 (70.2%)
	Student M3	41 (34.7%)	77 (65.3%)	3 (2.5%)	0 (0.0%)	2 (1.7%)	3 (2.5%)	36 (30.5%)	74 (62.7%)
	Total	83 (28.7%)	206 (71.3%)	6 (2.1%)	1 (0.3%)	4 (1.4%)	13 (4.5%)	73 (25.3%)	192 (66.4%)
Paper Airplane	Student M1	Absence	Absence	Absence	Absence	Absence	Absence	Absence	Absence
	Student M2	53 (50.5%)	52 (49.5%)	12 (11.4%)	1 (1.0%)	10 (9.5%)	19 (18.1%)	31 (29.5%)	32 (30.5%)
	Student M3	46 (43.0%)	61 (57.0%)	4 (3.7%)	5 (4.7%)	6 (5.6%)	28 (26.2%)	36 (33.6%)	28 (26.2%)
	Total	99 (46.7%)	113 (53.3%)	16 (7.5%)	6 (2.8%)	16 (7.5%)	47 (22.2%)	67 (31.6%)	60 (28.3%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Table 4.40

Metacognitive Activities Identified Across Students within Group 4

				Metacognitive activities					
		Total number of metacognitive activities (%)		Total number of regulating activities (%)		Total number of monitoring activities (%)			
						*A-K, A-U, & A-S		*A-P, A-E, & A-R	
		Individual	Social	Individual	Social	Individual	Social	Individual	Social
Summer Jobs	Student R1	15 (32.6%)	31 (67.4%)	2 (4.3%)	0 (0.0%)	1 (2.2%)	1 (2.2%)	12 (26.1%)	30 (65.2%)
	Student R2	7 (23.3%)	23 (76.7%)	0 (0.0%)	0 (0.0%)	2 (6.7%)	4 (13.3%)	5 (16.7%)	19 (63.3%)
	Student R3	16 (34.0%)	31 (66.0%)	0 (0.0%)	0 (0.0%)	0 (0.0%)	2 (4.3%)	16 (34.0%)	29 (61.7%)
	Total	38 (30.9%)	85 (69.1%)	2 (1.6%)	0 (0.0%)	3 (2.4%)	7 (5.7%)	33 (26.8%)	78 (63.4%)
Volley-ball	Student R1	5 (29.4%)	12 (70.6%)	0 (0.0%)	0 (0.0%)	0 (0.0%)	5 (29.4%)	5 (29.4%)	7 (41.2%)
	Student R2	1 (9.1%)	10 (90.9%)	0 (0.0%)	1 (9.1%)	0 (0.0%)	3 (27.3%)	1 (9.1%)	6 (54.5%)
	Student R3	11 (33.3%)	22 (66.7%)	2 (6.1%)	1 (3.0%)	1 (3.0%)	3 (9.1%)	8 (24.2%)	18 (54.5%)
	Total	17 (27.9%)	44 (72.1%)	2 (3.3%)	2 (3.3%)	1 (1.6%)	11 (18.0%)	14 (23.0%)	31 (50.8%)
Paper Airplane	Student R1	3 (33.3%)	6 (66.7%)	0 (0.0%)	0 (0.0%)	0 (0.0%)	2 (22.2%)	3 (33.3%)	4 (44.4%)
	Student R2	2 (22.2%)	7 (77.8%)	0 (0.0%)	0 (0.0%)	0 (0.0%)	1 (11.1%)	2 (22.2%)	6 (66.7%)
	Student R3	6 (42.9%)	8 (57.1%)	1 (7.1%)	0 (0.0%)	1 (7.1%)	2 (14.3%)	4 (28.6%)	6 (42.9%)
	Total	11 (34.4%)	21 (65.6%)	1 (3.1%)	0 (0.0%)	1 (3.1%)	5 (15.6%)	9 (28.1%)	16 (50.0%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Table 4.41

Metacognitive Activities Identified Across Students within Group 5

		Metacognitive activities							
		Total number of metacognitive activities (%)		Total number of regulating activities (%)		Total number of monitoring activities (%)			
						*A-K, A-U, & A-S		*A-P, A-E, & A-R	
		Individual	Social	Individual	Social	Individual	Social	Individual	Social
Summer Jobs	Student S1	18 (25.4%)	53 (74.6%)	4 (5.6%)	4 (5.6%)	1 (1.4%)	7 (9.9%)	13 (18.3%)	42 (59.2%)
	Student S2	33 (34.4%)	63 (65.6%)	4 (4.2%)	1 (1.0%)	2 (2.1%)	6 (6.3%)	27 (28.1%)	56 (58.3%)
	Student S3	29 (30.9%)	65 (69.1%)	0 (0.0%)	1 (1.1%)	0 (0.0%)	8 (8.5%)	29 (30.9%)	56 (59.6%)
	Total	80 (30.7%)	181 (69.3%)	8 (3.1%)	6 (2.3%)	3 (1.1%)	21 (8.0%)	69 (26.4%)	154 (59.0%)
Volley-ball	Student S1	Absence	Absence	Absence	Absence	Absence	Absence	Absence	Absence
	Student S2	72 (38.7%)	114 (61.3%)	3 (1.6%)	1 (0.5%)	2 (1.1%)	12 (6.5%)	67 (36.0%)	101 (54.3%)
	Student S3	46 (26.9%)	125 (73.1%)	0 (0.0%)	1 (0.6%)	1 (0.6%)	14 (8.2%)	45 (26.3%)	110 (64.3%)
	Total	118 (33.1%)	239 (66.9%)	3 (0.8%)	2 (0.6%)	3 (0.8%)	26 (7.3%)	112 (31.4%)	211 (59.1%)
Paper Airplane	Student S1	17 (29.8%)	40 (70.2%)	4 (7.0%)	3 (5.3%)	4 (7.0%)	16 (28.1%)	9 (15.8%)	21 (36.8%)
	Student S2	38 (43.2%)	50 (56.8%)	16 (18.2%)	3 (3.4%)	8 (9.1%)	18 (20.5%)	14 (15.9%)	29 (33.0%)
	Student S3	15 (18.5%)	66 (81.5%)	1 (1.2%)	4 (4.9%)	0 (0.0%)	36 (44.4%)	14 (17.3%)	26 (32.1%)
	Total	70 (31.0%)	156 (69.0%)	21 (9.3%)	10 (4.4%)	12 (5.3%)	70 (31.0%)	37 (16.4%)	76 (33.6%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Table 4.42

Metacognitive Activities Identified Across Students within Group 6

		Total number of metacognitive activities (%)		Metacognitive activities					
				Total number of regulating activities (%)		Total number of monitoring activities (%)			
						*A-K, A-U, & A-S		*A-P, A-E, & A-R	
		Individual	Social	Individual	Social	Individual	Social	Individual	Social
Summer Jobs	Student P1	26 (27.1%)	70 (72.9%)	0 (0.0%)	1 (1.0%)	2 (2.1%)	2 (2.1%)	24 (25.0%)	67 (69.8%)
	Student P2	53 (31.0%)	118 (69.0%)	3 (1.8%)	0 (0.0%)	3 (1.8%)	11 (6.4%)	47 (27.5%)	107 (62.6%)
	Student P3	28 (25.2%)	83 (74.8%)	1 (0.9%)	0 (0.0%)	1 (0.9%)	9 (8.1%)	26 (23.4%)	74 (66.7%)
	Student P4	44 (31.9%)	94 (68.1%)	2 (1.4%)	0 (0.0%)	3 (2.2%)	10 (7.2%)	39 (28.3%)	84 (60.9%)
	Total	151 (29.3%)	365 (70.7%)	6 (1.2%)	1 (0.2%)	9 (1.7%)	32 (6.2%)	136 (26.4%)	332 (64.3%)
Volley-ball	Student P1	32 (47.8%)	35 (52.2%)	1 (1.5%)	0 (0.0%)	0 (0.0%)	5 (7.5%)	31 (46.3%)	30 (44.8%)
	Student P2	30 (24.8%)	91 (75.2%)	1 (0.8%)	0 (0.0%)	0 (0.0%)	11 (9.1%)	29 (24.0%)	80 (66.1%)
	Student P3	21 (42.9%)	28 (57.1%)	1 (2.0%)	0 (0.0%)	0 (0.0%)	6 (12.2%)	20 (40.8%)	22 (44.9%)
	Student P4	40 (30.1%)	93 (69.9%)	1 (0.8%)	1 (0.8%)	0 (0.0%)	11 (8.3%)	39 (29.3%)	81 (60.9%)
	Total	123 (33.2%)	247 (66.8%)	4 (1.1%)	1 (0.3%)	0 (0.0%)	33 (8.9%)	119 (32.2%)	213 (57.6%)
Paper Airplane	Student P1	10 (34.5%)	19 (65.5%)	0 (0.0%)	0 (0.0%)	1 (3.4%)	5 (17.2%)	9 (31.0%)	14 (48.3%)
	Student P2	21 (36.2%)	37 (63.8%)	3 (5.2%)	0 (0.0%)	2 (3.4%)	7 (12.1%)	16 (27.6%)	30 (51.7%)
	Student P3	3 (20.0%)	12 (80.0%)	0 (0.0%)	0 (0.0%)	1 (6.7%)	5 (33.3%)	2 (13.3%)	7 (46.7%)
	Student P4	21 (29.6%)	50 (70.4%)	5 (7.0%)	0 (0.0%)	0 (0.0%)	18 (25.4%)	16 (22.5%)	32 (45.1%)
	Total	55 (31.8%)	118 (68.2%)	8 (4.6%)	0 (0.0%)	4 (2.3%)	35 (20.2%)	43 (24.9%)	83 (48.0%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Table 4.43

Metacognitive Activities Identified Across Students within Group 7

		Metacognitive activities							
		Total number of metacognitive activities (%)		Total number of regulating activities (%)		Total number of monitoring activities (%)			
						*A-K, A-U, & A-S		*A-P, A-E, & A-R	
		Individual	Social	Individual	Social	Individual	Social	Individual	Social
Summer Jobs	Student W1	16 (47.1%)	18 (52.9%)	4 (11.8%)	0 (0.0%)	1 (2.9%)	0 (0.0%)	11 (32.4%)	18 (52.9%)
	Student W2	6 (27.3%)	16 (72.7%)	0 (0.0%)	0 (0.0%)	0 (0.0%)	2 (9.1%)	6 (27.3%)	14 (63.6%)
	Student W3	8 (34.8%)	15 (65.2%)	0 (0.0%)	0 (0.0%)	0 (0.0%)	3 (13.0%)	8 (34.8%)	12 (52.2%)
	Total	30 (38.0%)	49 (62.0%)	4 (5.1%)	0 (0.0%)	1 (1.3%)	5 (6.3%)	25 (31.6%)	44 (55.7%)
Volley-ball	Student W1	12 (42.9%)	16 (57.1%)	0 (0.0%)	0 (0.0%)	0 (0.0%)	3 (10.7%)	12 (42.9%)	13 (46.4%)
	Student W2	24 (35.8%)	43 (64.2%)	1 (1.5%)	0 (0.0%)	2 (3.0%)	5 (7.5%)	21 (31.3%)	38 (56.7%)
	Student W3	28 (43.8%)	36 (56.3%)	1 (1.6%)	0 (0.0%)	1 (1.6%)	4 (6.3%)	26 (40.6%)	32 (50.0%)
	Total	64 (40.3%)	95 (59.7%)	2 (1.3%)	0 (0.0%)	3 (1.9%)	12 (7.5%)	59 (37.1%)	83 (52.2%)
Paper Airplane	Student W1	14 (36.8%)	24 (63.2%)	3 (7.9%)	0 (0.0%)	0 (0.0%)	6 (15.8%)	11 (28.9%)	18 (47.4%)
	Student W2	15 (39.5%)	23 (60.5%)	4 (10.5%)	0 (0.0%)	5 (13.2%)	5 (13.2%)	6 (15.8%)	18 (47.4%)
	Student W3	10 (35.7%)	18 (64.3%)	2 (7.1%)	1 (3.6%)	1 (3.6%)	4 (14.3%)	7 (25.0%)	13 (46.4%)
	Total	39 (37.5%)	65 (62.5%)	9 (8.7%)	1 (1.0%)	6 (5.8%)	15 (14.4%)	24 (23.1%)	49 (47.1%)

*A-K = Assessment of Knowledge; A-U = Assessment of Understanding; A-S = Assessment of Strategy Appropriateness; A-P = Assessment of Progress Toward Goal; A-E = Assessment of Strategy Execution; and A-R = Assessment of Accuracy or Sense of Result.

Continuously, through exploring each student's metacognitive activities triggered at the individual and social levels, some meaningful patterns, which will be described in the following sections, were additionally identified. Also, additional evidence was identified in order to support patterns of students' metacognitive activities, which were identified so far.

A catalyst for the development of metacognition within a group. Figures 4.1 – 4.7 above indicate that during an MEA, there was a student who worked as a catalyst for the development of metacognition within each group. The student's metacognitive activities were relatively triggered in an active manner at both the individual and social levels when considering both the frequencies and types of metacognitive activities–four regulating activities and six monitoring activities, in particular, focusing on regulating activities triggered at the individual level, and monitoring activities triggered at the individual level in *assessment of cognition* (*assessment of knowledge*, *assessment of understanding*, and *assessment of strategy appropriateness*). On the other hand, others' metacognitive activities were relatively biased on the social level.

For example, during the Summer Jobs MEA, student H3 within Group 1 (Figure 4.1), student M3 within Group 3 (Figure 4.3), student S2 within Group 5 (Figure 4.5), both student P2 and student P4 within Group 6 (Figure 4.6), and student W1 within Group 7 (Figure 4.7) were likely to serve as a catalyst for the development of metacognition within their respective groups. Their metacognitive activities were relatively triggered in an active manner at both the individual and social levels.

For example, as shown above in Figure 4.1, the patterns of student H2 and student H3's metacognitive activities identified during the Summer Jobs MEA are very similar, but Table 4.37 above shows that student H3's metacognitive activities were more actively triggered than students H2's (a total of 66 transcript segments vs. 36). Also, student H3's metacognitive activities triggered at the individual level were identified more often than student H2's at the individual level (40.9% vs. 16.7%), in particular, regulating activities triggered at the individual level (6.1% vs. 2.8%). Thus, student H3 revealed that her metacognitive activities were relatively triggered in an active manner, at both the individual and social levels versus student H2's. She seemed to serve as a catalyst for the development of metacognition within Group 1 during the MEA.

Similarly, Figure 4.6 above shows that the patterns of student P2 and student P3's metacognitive activities identified during the Summer Jobs MEA are the same with respect to the types of identified metacognitive activities, but as shown above in Table 4.42, student P2's metacognitive activities were more actively triggered than students P3's (a total of 171 transcript segments vs. 111). Student P2's metacognitive activities triggered at the individual level were also identified more often than student P3's at the individual level (31.0% vs. 25.2%), in particular, regulating activities triggered at the individual level (1.8% vs. 0.9%) and monitoring activities triggered at the individual level in *assessment of cognition* (1.8% vs. 0.9%). The results indicate that student P2 worked as a catalyst for the development of metacognition within Group 6 during the Summer Jobs MEA.

The role of student P2 as a catalyst for the development of metacognition within Group 6 was more evident during the Paper Airplane MEA. He revealed his metacognitive activities, which were relatively triggered at both the individual and social levels in an active manner when considering both the frequencies and types of identified metacognitive activities. This tendency of student P2 as a catalyst for the development of metacognition within Group 6 may also be well illustrated by his statement during the Paper Airplane MEA: "Stay positive. We aren't going to fail. We're just going [to need to work together] to be last minute" (P2, G6, PA, 311).

Social toward individual (external toward internal). On the other hand, during the Summer Jobs MEA, metacognitive activities identified from more students within each group, such as student H1 and student H2 within Group 1 (Figure 4.1); student M2 within Group 3 (Figure 4.3); student S1 and student S3 within Group 5 (Figure 4.5); student P3 within Group 6 (Figure 4.6); and student W2 and student W3 within Group 7 (Figure 4.7), were relatively biased on the social level, compared to the student who worked as a catalyst for the development of metacognition within each group. Their metacognitive activities seemed to be relatively dependent upon social sources. As shown above in the figures, the students revealed few types of metacognitive activities triggered at the individual level, compared to those at the social level. Otherwise, the number of transcript segments that were identified as their metacognitive activities triggered at the social level versus those at the individual level, is relatively large.

For example, student H1 revealed few types of metacognitive activities triggered at the individual level, compared to her metacognitive activities triggered at the social level during the Summer Jobs MEA (Figure 4.1). On the other hand, although Figure 4.1 above shows that the patterns of student H2 and student H3's metacognitive activities identified during the Summer Jobs MEA are very similar with respect to the types of identified metacognitive activities, student H2's metacognitive activities were more often identified at the social level, versus those at the individual level (83.3% vs. 16.7%) (Table 4.37). The result indicates that student H2's metacognitive activities were relatively biased on the social level, depending on external, social sources.

As discussed above, additional evidence was identified through continuously exploring each student's metacognitive activities triggered at the individual and social levels to support the several patterns identified beforehand through comparing the frequencies of the identified metacognitive activities across the seven groups, across the group members, and across the three MEAs. The patterns supported here by the empirical evidence are as follows: (a) a predominance of students' metacognitive activities triggered at the social level, versus those at the individual level; (b) overall an overwhelming infrequency of regulating activities–being dependent upon more individual sources–versus monitoring activities; and (c) a relatively steep increase in the frequencies of students' assessment of understanding and of the strategy appropriateness triggered at the social level. All of the findings indicate that students' metacognitive activities tend to be triggered on the basis of external, social sources.

Consequently, within each group, more students' metacognitive activities were dependent upon external, social sources, while there was a student who served as a catalyst for the development of metacognition. In other words, more students' metacognitive activities seem to be frequently triggered on the basis of external, social sources, rather than by internal, individual sources. These patterns were identified and hold across the three MEAs–Summer Jobs, Volleyball, and Paper Airplane MEAs, as shown above in Figures 4.1 – 4.7. Considering the student differences in the identified patterns, and with the expectation for developmental potential, these findings may indicate social toward individual or external toward internal dimensions to the development of students' metacognitive abilities.

Concrete toward abstract (simple toward complex). In addition, Figures 4.1 – 4.7 show that types of monitoring activities triggered at the individual level, which were identified from most students across the three MEAs, were relatively biased in *assessment of procedure*, compared to those at the social level. Within each MEA, only a few students' types of monitoring activities triggered at the individual level were relatively balanced in both *assessment of cognition* and *assessment of procedure*, compared to other group members'. In particular, they were mainly the identical students who served as a catalyst for the development of metacognition within each group. Also, the biased patterns on types of monitoring activities triggered at the individual level in *assessment of procedure* were slightly more evident in monitoring activities identified from the students who revealed relatively few metacognitive activities triggered at the individual level, versus those at the social level. As identified before in the previous section, the students' metacognitive activities tend to be triggered on the basis of external, social sources, rather than by internal, individual sources.

For example, the students who were already illustrated before in the previous section: student H1 and student H2 within Group 1 (Figure 4.1); student M2 within Group 3 (Figure 4.3); student S1 and student S3 within Group 5 (Figure 4.5); student P3 within Group 6 (Figure 4.6); and student W2 and student W3 within Group 7 (Figure 4.7) revealed relatively few types of monitoring activities triggered at the individual level in *assessment of cognition*, compared to those at the social level, across the three MEAs.

These patterns are especially well illustrated in student S3's and student P3's metacognitive activities identified across the three MEAs. When looking more closely at student S3's and student P3's metacognitive activities, identified at the individual and social levels across the three MEAs, they revealed relatively few metacognitive activities triggered at the individual level versus those at the social level, compared to other group members within Group 5 and Group 6, respectively (Table 4.41 and Table 4.42, respectively). Across the three MEAs, they also revealed few types of monitoring activities triggered at the individual level in *assessment of cognition*, compared to those at the social level (Figure 4.5 and Figure 4.6, respectively). These patterns are supported by their infrequencies of monitoring activities triggered at the individual level in *assessment of cognition* versus that in *assessment of procedure*. For example, student P3's monitoring activities triggered at the individual level were relatively biased in *assessment of procedure* versus those in *assessment of cognition*: 23.4% vs. 0.9% during the Summer Jobs; 40.8% vs. 0.0% during the Volleyball MEA; and 13.3% vs. 6.7% during the Paper Airplane MEA (Table 4.42).

The narrow types of monitoring activities triggered at the individual level, which were biased in *assessment of procedure–assessment of progress toward goal*, *assessment of strategy execution*, and *assessment of accuracy or sense of result*, might indicate that student S3's and student P3's evaluation about their ways of thinking were mainly triggered on the basis of more concrete and tangible by-products of problem-solving processes, such as numeric outcomes. These types of monitoring activities also tend to be relatively simple, rather than complex–because they are mainly made based on concrete and tangible outcomes, and thus involve less complexity in assessments–compared to other monitoring activities, such as *assessment of strategy appropriateness*, which requires individuals to activate mental runs to make assessments involving many possible ways.

These patterns of most students' monitoring activities triggered at the individual level across the three MEAs, which were relatively biased in *assessment of procedure* compared to those at the social level, are compatible with the patterns identified beforehand, through comparing the frequencies of the identified metacognitive activities across the seven groups, across the group members, and across the three MEAs. The patterns identified beforehand are as follows: (a) a relatively steep increase in the frequencies of students' assessment of understanding and of the strategy appropriateness triggered at the social level; and (b) a predominance of monitoring activities triggered at the individual level in *assessment of procedure*, versus those in *assessment of cognition*. All of the findings indicate that students' metacognitive activities tend to be triggered on the basis of more concrete and tangible results of assessment via more explicit actions.

Consequently, most students' monitoring activities across the seven groups were dependent upon concrete and tangible outcomes in problem-solving processes, and thus less complex because of fewer mental demands of assessment. In other words, most students' metacognitive activities seem to be frequently triggered on the basis of concrete and tangible sources, rather than by abstract sources, and they also seem to be simple, rather than complex, preferring fewer mental demands of assessment. Considering the student differences in the above-mentioned patterns, and with the expectation for developmental potential, these findings may indicate concrete toward abstract or simple toward complex dimensions to the development of students' metacognitive abilities.

Situated toward decontextualized (specific toward general). A cross-case analysis for the three MEAs–Summer Jobs, Volleyball, and Paper Airplane MEAs was conducted for each group to explore the overall patterns of each student's metacognitive behaviors across the problem-solving sessions, as shown above in Figures 4.1 – 4.7. The results from the cross-case analysis provide additional evidence that the development of students' metacognitive activities might have a potential similarity of dimensions to the development of students' cognitive abilities, that is, situated toward decontextualized or specific toward general dimensions to the development of students' metacognitive abilities.

Figures 4.1 – 4.7 show the overall different patterns of each student's metacognitive activities triggered at the individual and social levels across the three MEAs. These differences may also be supported by differences in the quantity of metacognitive activities identified from each student within each group across MEAs, as reported above in Tables 4.37 – 4.43. For example, all group members within Group 1 (Figure 4.1) revealed remarkably different patterns of metacognitive activities triggered at the individual and social levels across the three MEAs, with respect to the types of identified metacognitive activities. These patterns indicate that the students' metacognitive activities were situated and specific in each context.

On the other hand, student S2 and student P2, who served as a catalyst for the development of metacognition during the Summer Jobs MEA within Group 5 and Group 6, respectively, revealed relatively stable patterns of metacognitive activities across the three MEAs, with respect to the types of identified metacognitive activities (Figure 4.5

and Figure 4.6, respectively). These patterns might indicate that their metacognitive abilities had developed toward being more context-free and general, compared to other group members'. Within and across the three MEA, the highest frequencies of metacognitive activities identified from the students within their respective groups (Table 4.41 and Table 4.42, respectively) might also indicate that their metacognitive abilities had relatively developed in a stable manner, compared to other group members'.

Consequently, the overall patterns of each student's metacognitive activities were different across the problem-solving sessions. Only a few students' metacognitive activities were relatively stable across the three MEAs. In other words, students' metacognitive activities likely tend to be situated, rather than decontextualized, and they also tend to be specific in a given context, rather than context-free and general. Considering the student differences in the identified patterns, and with the expectation for developmental possibility, these findings may indicate a potential similarity to the development of students' metacognitive abilities from that of students' cognitive abilities: situated toward decontextualized or specific toward general dimensions to their development.

Development of metacognitive abilities vs. development of cognitive abilities. Through exploring each student's metacognitive activities triggered at the individual and social levels within and across the three MEAs, this study identified several patterns of students' metacognitive activities, as reported above. The patterns identified based on empirical evidence describe the nature of students' metacognitive abilities. The findings may indicate that the development of students' metacognitive abilities has potentially similar dimensions to that of students' cognitive abilities, which are mentioned as follows: (a) social toward individual; (b) external toward internal; (c) concrete toward abstract; (d) simple toward complex; (e) situated toward decontextualized; and (f) specific toward general.

The following section addresses the last part of the second research question by presenting the identified interesting events that might interfere with students' spontaneous metacognitive activities.

c) What are the circumstances in which metacognitive activity is abandoned?

This section will present the analysis and findings pertaining to the third part of the second research question, which sought to explore the circumstances that interfered with students' metacognitive activities.

After going through iterative cycles of rereading the audio transcripts of student groups, each transcript segment was carefully examined to ensure the fidelity of making coding decisions. During multiple cycles of data analysis, annotations were also made to identify the circumstances facilitating or interfering with students' metacognitive activities. Two categories emerged from the annotations describing the circumstances in which metacognitive activity is abandoned. The identified circumstances might interfere with students' spontaneous metacognitive activities themselves. Thus, they might deprive students' chances of achieving more productive outcomes. The following sections describe the two categories and illustrations for them, drawn from the collected data.

Learning environments in which interruptions to spontaneous metacognitive activities occurred. In addition to their internal psychological resources, students have access to external sources to trigger their metacognitive activities. One of the external sources comes from interacting with something in the learning environment. However, the environmental triggers may sometimes interfere with students' spontaneous metacognitive activities, and thus, may cause less productive outcomes, as illustrated below.

Distorted function of mathematics supplies. One interesting event that might have interfered with students' metacognition was identified in unexpected circumstances. The existence of mathematics supplies, which were distributed by the teacher, made not only a positive impact, but also a negative impact on students' spontaneous metacognitive activities. For example, the existence of a protractor could have made a negative impact on the students' metacognitive activities within Group 3 through abandoning their way of thinking, as illustrated by student M2's assessment of strategy appropriateness: "It seems like everybody used a protractor. I feel like we did this wrong, even though it is right" (M2, G3, PA, 212).

There was an example indicating a positive impact of the existence of a calculator on the students' spontaneous metacognitive activities: The accessibility to calculators allowed student S3 to make a positive evaluation on S1's strategy: "Go get a calculator" (S3, G5, SJ, 221). However, the existence of a calculator was also a possible circumstance that might have had a negative impact on the students' spontaneous metacognitive activities, as indicated with several illustrations. For example, student S3's assessment of strategy appropriateness–"Well, we kind of have to work together because we don't all have a calculator" (S3, G5, PA, 82)–might have deprived other group members chances to try different ways of thinking. Another example indicates the possibility of distorted evaluation for strategy appropriateness, and chance interruption for better ideas, as illustrated by student S2's assessment of strategy appropriateness: "What'd we do if we didn't use calculators. Then you'd have to figure out another way to do it on the calculators" (S2, G5, PA, 158).

Other factors within classroom management systems. Another interesting event that have might interfered with students' metacognition was associated with the classroom management system. Several circumstances that might have had a negative impact on the students' spontaneous metacognitive activities were identified in this category. For example, as illustrated at the beginning of this chapter, periods of time given for problem solving could have had not only a positive impact, but also a negative impact on students' assessment of progress toward their goal for productive problem solving. Here are additional illustrations, which are related to a negative impact of periods of time given, showing the possibility of chance interruption for students to activate spontaneous metacognitive activities: "No, letter [is for] tomorrow. Everything else should be done today" (P2, G6, PA, 332); and, "We're not writing it yet. We still have tomorrow" (P4, G6, SJ, 158).

Another circumstance, which might have had a negative impact on the students' spontaneous metacognitive activities, and thus, might have been able to cause less productive outcomes, was associated with the student assessment. Although someone may argue about the negative effects of grading, the following illustration may indicate that students might not have been motivated to trigger their spontaneous metacognitive activities in order to get better outcomes if their efforts were not rewarded, based on a sufficient distinction between excellent and poor performance.

(*Group 1, Paper Airplane MEA, Transcript Lines 302-307*)

[302] *H1*: We just have to measure the accuracy and best floater. *(Assessment – understanding: Thinking ABOUT a way of thinking, Individual Level)*

[303] *H3*: Watch our way be totally off compared to everyone else's. *(Assessment – strategy appropriateness: Thinking ABOUT a way of thinking, Individual Level)*

[304] *H2*: There is no right answer. *(Assessment – strategy appropriateness: Thinking ABOUT H3's way of thinking, Social Level)*

[305] *H4:* Do we get graded on this?

[306] *H1*: No.

[307] *H3*: No.

As the problem solving was approaching its final phrase, student H3 made an assessment of strategy appropriateness: "Watch our way be totally off compared to everyone else's" (H3, G1, PA, 303). Then, student H4 tried to check whether their work on the MEA would be graded or not: "Do we get graded on this?" (H4, G1, PA, 305). After the conversation, they kept going with a prior strategy without considering any other alternative ways. The group members did not seem to care about the quality of their solution.

The final example of circumstances, which might have interfered with students' spontaneous metacognitive activities, was due to students' homework burden. As already illustrated at the beginning of this chapter, the homework burden seemed to have a negative impact when the students made an assessment of strategy appropriateness, by leading them to abandon a more careful assessment for productive problem solving, as follows:

(*Group 1, Paper Airplane MEA, Transcript Lines 363-371*)

[363] *H3*: How about how long it was in the air past a certain height, does it give us the heights? Nope. *(New idea: Thinking ABOUT a way of thinking, individual Level)*

[364] *H1*: How long it was in the air one foot above the ground? *(Assessment – understanding: Thinking ABOUT H3's way of thinking, Social Level)*

[365] *H3*: Yeah. *(Assessment – understanding: Thinking ABOUT H1's way of thinking, Social Level)*

[366] *H4*: You just got to remember we don't get homework when we do this, so it's all worth it. *(Assessment – strategy appropriateness: Thinking ABOUT H3's way of thinking, Social Level)*

[367] *H2*: Exactly. *(Assessment – strategy appropriateness: Thinking ABOUT H4's way of thinking, Social Level)*

[Irrelevant conversation]

[370] *H1*: We're pretty much done. *(Assessment – progress towards goal: Thinking ABOUT a way of thinking, Individual Level)*

[371] *H3*: We're done? That was easier. I think we were definitely over thinking it. *(Assessment – progress towards goal: Thinking ABOUT H1's way*

of thinking, Social Level)

When the students worked on operationalizing the definition for the best floater, in which they needed to quantify what "best floating" means for themselves, student H3 herself triggered her new idea, "How about how long it was in the air past a certain height, does it give us the heights? Nope" (H3, G1, PA, 363). Then, the new idea triggered student H1's assessment of her understanding of student H3's way of thinking at the social level: "How long it was in the air one foot above the ground? (H1, G1, PA, 364). Afterward, the homework burden made a negative impact on the students' spontaneous metacognitive activities by causing student H4 and student H2's unsuitable assessment of strategy appropriateness, "You just got to remember we don't get homework when we do this, so it's all worth it" (H4, G1, PA, 366) and, "Exactly" (H2, G1, PA, 367). Without any additional assessment, the students tried to conclude the problem solving.

Social interactions from which interruptions to spontaneous metacognitive activities occurred. Another external source, which provides students with rich resources to trigger their metacognitive activities, originates from other people interacting with them. However, as environmental triggers do, social triggers also may sometimes interfere with students' spontaneous metacognitive activities, and thus, may interrupt students from achieving more productive outcomes. The following sections illustrate the circumstances that were identified in the study, interfering with students' spontaneous metacognitive activities.

Teacher's inappropriate intervention. Undoubtedly, the role of teachers is critical in eliciting or confounding students' metacognitive activities. One circumstance, which might have interfered with students' spontaneous metacognitive activities, came from the teacher's inappropriate intervention. It is well illustrated by the following examples.

At the beginning of the problem-solving session of the Paper Airplane MEA, when the teacher led a whole class discussion for sharing students' answers to the readiness questions (Appendix C), the teacher mentioned, "I have protractors that I will hand out, which will help you to measure the angles on the graphs if you want to do that. So, you can draw straight lines and find the actual angles for each of these points plotted. That will better help you determine how accurate the throw actually was" (T, G1, PA, 96).

This guided direction from the teacher reflecting his own way of thinking seemed to unintentionally interfere with students' spontaneous metacognitive activities by reducing chances for the students to work on definition building, in which they needed to define what "accurate" means for themselves. For example, student Group 2 depended on the teacher's way of thinking without any evaluation, by using protractors. Many groups also spent a substantial amount of time measuring the angles on the graphs, even though it was unnecessary because they had already been given the numeric data.

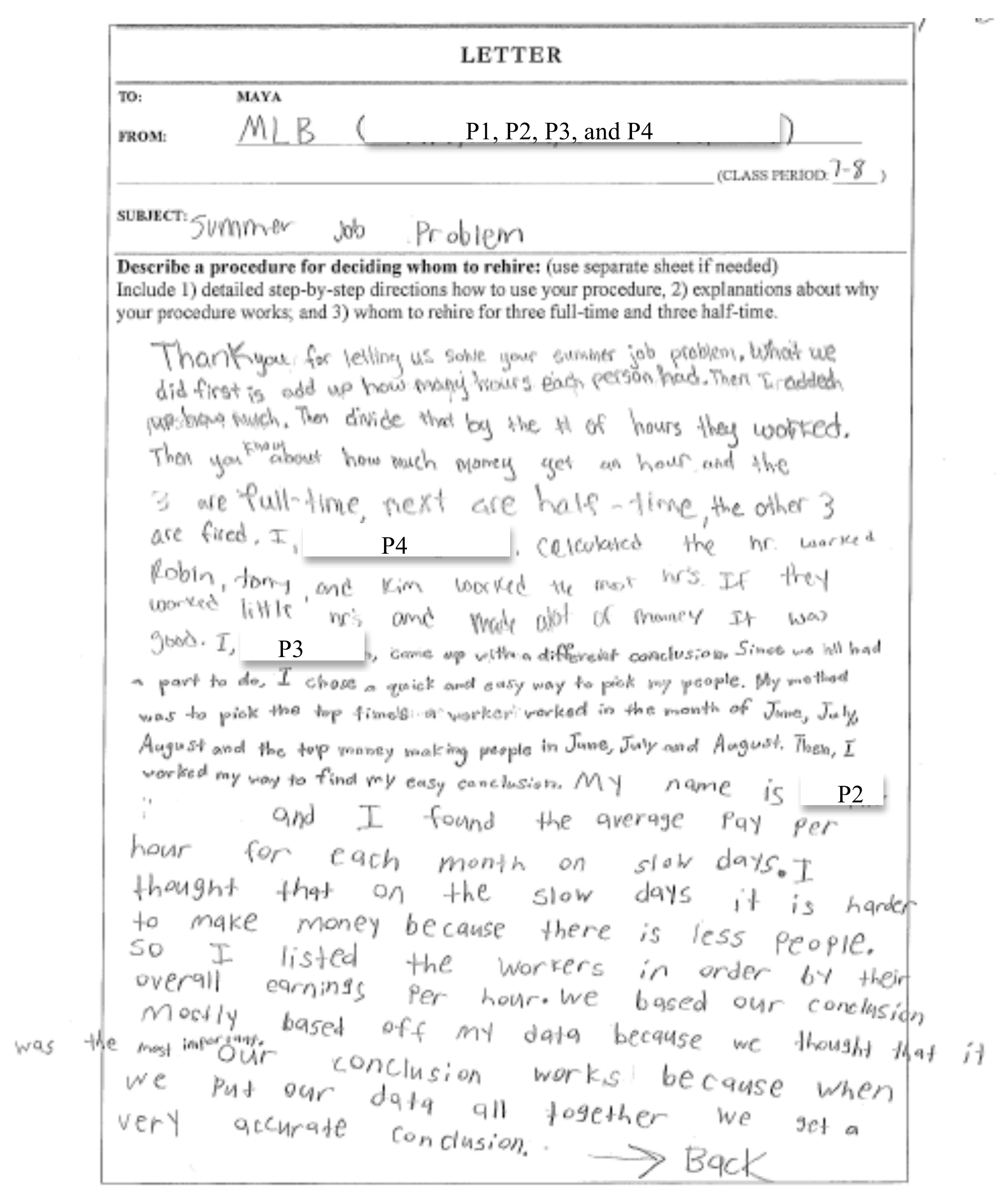
Even though some students indicated this point, such as, "Okay he told me what to do. This is positive, and this is negative. Okay, so … (mumbling)… so like this already has all the information why are we even doing this? Okay this is positive and this is negative" (W1, G7, PA, 57), the teacher adhered to his erroneous way of thinking and seemed to enforce it to the students, as illustrated by the statements, “Some people were asking why they needed to measure the angles with a protractor if they already gave us the degrees. Well, here’s why” (T, G3, PA, 157); and, “Which one of those is sixteen degrees? Can you tell by looking at it? Just by looking at it without measuring it? No, that is why I gave you the protractors to measure. Now, you might think that’s really tedious and will take me forever. Well, I have extras in case your group wants more than one. Do you feel better about protractors and how to find the angles?” (T, G3, PA, 160).

A negative impact of mixed-gender groups. When collaboratively working in a group, problem-solving performance may be affected by many factors, such as group size and group composition. These factors may influence the quality of one’s interactions with others, which also elicit or confound metacognitive activities. In addition to the teacher’s inappropriate intervention, another circumstance identified in the study, which might have interfered with students’ spontaneous metacognitive activities, is associated with the group composition in terms of gender, that is, mixed-gender grouping.

This study involves two mixed-gender groups among seven, which had been formed by the students themselves. As mentioned in Chapter 3, the teacher allowed students to decide upon their own groups before the first problem-solving session, emphasizing that no member change was allowed over the three problem-solving sessions. The mixed-gender groups are Group 2, where two girls (K2 and K3) and one boy (K1) worked together and Group 4, where one girl (R3) and two boys (R1 and R2) worked together. The results from the data analysis show the relatively small quantities of transcript segments, which were identified as metacognitive activities from Group 2 (a total of 256 transcript segments) and Group 4 (a total of 216 transcript segments) compared to those from Group 1, Group 3, Group 5, Group 6, and Group 7 (a total of 603 transcript segments, 749, 844, 1059, and 342, respectively) (Table 4.29). As mentioned before, the differences in the quantity of metacognitive activities identified from each group were directly due to the different quantities of audio transcripts–how often they verbalized their thoughts, and thus, revealed their verbal metacognitive actions. Also, the differences in the quantity of the audio transcripts indicate different degrees to which group members were engaged in collaboration to solve the problems.

Consequently, the relatively small quantities of transcript segments, which were identified as metacognitive activities from the two mixed-gender groups, may indicate a negative impact of mixed-gender grouping on students' spontaneous metacognitive activities by causing lower levels of interaction and cooperation.

A lack of collaborative working skills. The final example of circumstances, which might have interfered with the students' spontaneous metacognitive activities for productive problem solving, involved a lack of collaborative working skills in negotiating individual and social meanings about problems, making a common goal, and deciding a strategy to successfully attain that goal. The problem-solving processes of Group 6 working on the Summer Jobs MEA well illustrate this circumstance. Figure 4.8 below shows the final letter from Group 6 to an imaginary client in the MEA. The group solution to the MEA directly illustrates how a lack of collaborative working skills could have caused disconnections on group interaction, which might have interrupted the students' spontaneous metacognitive activities for productive problem solving.



LETTER

TO: MAYA

FROM: MLB (P1, P2, P3, and P4)

(CLASS PERIOD: 7-8)

SUBJECT: Summer Job Problem

Describe a procedure for deciding whom to rehire: (use separate sheet if needed)
Include 1) detailed step-by-step directions how to use your procedure, 2) explanations about why your procedure works; and 3) whom to rehire for three full-time and three half-time.

Thankyou for letting us solve your summer job problem. What we did first is add up how many hours each person had. Then I added up how much. Then divide that by the # of hours they worked. Then you know about how much money get an hour and the 3 are full-time, next are half-time, the other 3 are fired. I, P4, calculated the hr. worked Robin, tony, and Kim worked the most hrs. If they worked little hrs and made alot of money It was good. I, P3, came up with a different conclusion. Since we all had a part to do, I chose a quick and easy way to pick my people. My method was to pick the top time's a worker worked in the month of June, July, August and the top money making people in June, July and August. Then, I worked my way to find my easy conclusion. My name is P2 and I found the average pay per hour for each month on slow days. I thought that on the slow days it is harder to make money because there is less people. So I listed the workers in order by their overall earnings per hour. We based our conclusion mostly based off my data because we thought that it was the most important. Our conclusion works because when we put our data all together we get a very accurate conclusion. → Back

Figure 4.8. Letter from student Group 6 (solution to the Summer Jobs MEA).

As shown in the letter, each student within Group 6 reported individual solutions by using different strategies in the letter, rather than a group solution, such as "I, P3, comes up with a different conclusion…My method was to pick the top times a worker worked in the month of June, July, August…" and, 'My name is P2 and I found the average pay per hour for each month on slow days."

Throughout the problem solving, the students' metacognitive activities were often interrupted due to a lack of agreement for a strategy as a whole group in order to attain a common goal. The students' assessment of progress toward goal, strategy execution, and accuracy or sense of result was often triggered separately, based on different criteria for different goals. Sometimes the students worked in pairs, but infrequently in a whole group, indicating a lack of collaboration, which might have interfered with students' spontaneous metacognitive activities for productive problem solving.

The circumstances that interfere with students' metacognition. As presented above, a tiny subset of circumstances, in which metacognitive activity is abandoned, was identified in this study. The first category includes a distorted function of mathematics supplies, periods of time given for problem solving, no grade points, and the homework burden. The second category involves teacher's inappropriate intervention, mixed-gender groups, and a lack of collaborative working skills. The findings may need to be verified in additional studies. However, it would be useful and valuable enough to consider the circumstances in order to create metacognitive learning environments that support students' spontaneous development of metacognitive abilities.

Potential mechanisms for the development of metacognitive abilities during complex modeling activities. This section summarizes the findings pertaining to the second research question, which explored how students' thinking becomes metacognitive during complex modeling activities. The findings from the multiple cycles of data analysis pertaining to the three sub-questions of the second research question have together informed our understanding of the potential mechanisms for the development of metacognitive abilities during complex mathematical modeling tasks.

First, students' spontaneous metacognitive activities tend to be triggered on the basis of external, social, and tangible (concrete) sources. Through interaction with others, and through interaction with something in the learning environment, students are likely to have access to more external sources spontaneously to trigger their metacognitive activities. This increasing access to metacognitive triggers may provide students with more opportunities to develop their abilities efficiently to organize, monitor, and regulate what they know to reach a goal successfully. Students' spontaneous metacognitive activities also tend to be relatively biased in the focus of assessment, procedure, rather than cognition. Students are likely to prefer fewer mental demands involving less complexity (simple) in assessments, depending on concrete and tangible outcomes in problem-solving processes.

Second, all of these tendencies, which are summarized above, hold in most students' metacognitive activities identified across the three MEAs. In addition, the overall patterns of most students' metacognitive activities were inconsistent across the three MEAs with respect to the types of identified metacognitive activities. Most students' metacognitive activities tend to be situated and specific in a given context, rather than decontextualized and context-free and general. Only a few students, who served as a catalyst for the development of metacognition within each group, revealed relatively balanced metacognitive activities. First, these few students' metacognitive activities were relatively balanced, compared to those of most students, in terms of the level of sources triggering metacognition–individual and social sources. Second, these few students' metacognitive activities were relatively balanced in terms of the foci of assessment–cognition and procedure, while most students' metacognitive activities tended to be biased toward the focus of procedure. Finally, these few students' metacognitive activities were relatively balanced across the problem-solving sessions with respect to the types of identified metacognitive activities, compared to those of most students. As a result, these findings may indicate a potential similarity to the development of students' metacognitive abilities from that of students' cognitive abilities: (a) social toward individual; (b) external toward internal; (c) concrete toward abstract; (d) simple toward complex; (e) situated toward decontextualized; and (f) specific toward general dimensions to their development.

Finally, social and environmental triggers make not only a positive impact, but also a negative impact on students' spontaneous metacognitive activities. Several circumstances that might interfere with students' spontaneous metacognitive activities themselves originate from social interactions, such as a teacher's inappropriate intervention, mixed-gender groups, and a lack of collaborative working skills. In addition, students' spontaneous metacognitive activities may sometimes be interrupted by environmental triggers, such as the distorted function of math supplies (e.g., calculators and protractors), the duration of time given to solve problems, no credit given, and an aversion to homework.

As summarized above, the findings pertaining to the three sub-questions of the second research question together further our understanding about how students' thinking becomes metacognitive during complex modeling activities. They also inform how to effectively encourage students' metacognition, with the expectation to improve their problem-solving achievement.

The following chapter presents the discussion involving potentially important contributions of the current study, limitations, implications for both research and education, and recommendations for future research.

Chapter 5

Discussion, Limitations, Implications, and Future Directions

This dissertation aims to contribute to the still underdeveloped progress in research on metacognition, which lacks a theoretical base and currently has a weak impact on school practice. This chapter presents how this study has addressed these areas so as to provide metacognition research with a rich theoretical foundation and to maximize its impact on school practice. This chapter consists of five sections. The first section presents a summary discussion, in which the results reported in Chapter 4 are discussed in light of the existing research on metacognition. This section includes eight potential contributions of this study to research in metacognition. The second section of this chapter reports the limitations of the study. The third section discusses the implications of the current study for research on metacognition, and for teacher education and professional development programs. The fourth section describes how the current study can serve as a springboard for future research directions. In particular, several recommendations for further research questions are proposed in terms of one smaller-scale qualitative study, and furthermore, larger-scale research. Finally, the fifth section presents a conclusion containing brief closing remarks.

Discussion

Several difficulties and issues related to studying metacognition have been reported as follows: (a) the lack of a theoretical base, with an agreed-upon definition of metacognition, which clearly enables the distinction between metacognition and cognition; (b) the lack of an authentic method to observe and analyze metacognition; and

(c) the weak impact of research in metacognition on school practice, due to the view of metacognition and cognition as hierarchically separate, and disregard for the social, contextual and situated nature of metacognition (e.g., Garofalo & Lester, 1985; Lesh et al., 2003; Lesh & Zawojewski, 2007; Schoenfeld, 1987; Wilson & Clarke, 2002). The current study has continued the work to address these issues and challenges for research on metacognition.

The theoretical model of metacognition on multiple levels. As the first step to address these issues and challenges, the researcher developed a theoretical model of metacognition in complex modeling activities. The theoretical model was developed based on existing frameworks for analyzing problem-solving behaviors. It involved the re-conceptualization of metacognition at multiple levels by looking at the three sources that trigger metacognition.

First, the theoretical model adopted the models and modeling perspectives (MMP). The MMP assumes that individuals' conceptual systems consist of both cognitive (e.g., understanding, skills) and metacognitive (e.g., beliefs, awareness) components, and that thinking becomes metacognitive when individuals shift from "thinking WITH" cognitive components to "thinking ABOUT" them, by monitoring, which involves evaluation and regulation of them (Lesh et al., 2003). This assumption indicates that cognition and metacognition are essentially integrated; thus, they develop in parallel and interactively. Also, the MMP values the situated, contextual, and social nature of metacognition (Lesh et al., 2003). Consequently, the adoption of the MMP view as a theoretical base to develop the theoretical model of metacognition has allowed the application of a developmental perspective to metacognition, with the expectation to inform school practice.

Second, metacognition has been re-conceptualized by focusing on an individual as a unique agent, and by looking at the sources that trigger a student's metacognition at the individual, social, and environmental levels. With the MMP on metacognition, this re-conceptualization of metacognitive triggers has allowed for in-depth investigations of how one individual's thinking becomes metacognitive in context (Kim et al., 2013).

Finally, Goos' (2002) classification of metacognitive activities has been adopted for developing the theoretical model of metacognition in this study. Her framework focuses on the functions of metacognition itself–the monitoring function, which involves the assessment of cognitive components, and the regulatory function rather than its components. Focusing on the functions of metacognition itself (e.g., Efklides, 2006; Wilson & Clarke, 2002), the theoretical model of metacognition employed in this study has enabled consensus on the operational definition of metacognition.

The current study adopted the theoretical model, extrapolating from a single case study to a multiple case study to examine the appropriateness and soundness of the theoretical model for research in metacognition. Using the theoretical model as a framework, this study also explored how students' thinking becomes metacognitive, while collaboratively solving a complex mathematical modeling task. In this study, the theoretical model of metacognition on multiple levels has been proved appropriate in identifying and interpreting metacognitive activities in complex modeling activities; in addition, it has shown to be potentially efficient for research on metacognition, as supported by the following sections. This study also successfully identified (a) several patterns of students' spontaneous metacognitive activities, within and across the problem-solving sessions; (b) the nature of students' metacognitive abilities; and (c) critical events that might interfere with students' spontaneous metacognitive activities.

The following sections present eight potential contributions of this study to research in metacognition.

The theoretical model of metacognition on multiple levels as a coherent model of metacognition. The first major contribution of this study is in response to the need for a coherent model with respect to research on metacognition, with an agreed-upon definition of metacognition, appropriately identifying and interpreting students' metacognitive activities. As illustrated in Chapter 4 and summarized below, the theoretical model of metacognition on multiple levels employed in this study has the potential as a coherent model of metacognition to help more or less clearly distinguish metacognition from cognition and to make the distinction among metacognitive functions. The results of the study ultimately prove the appropriateness and usefulness of the theoretical model of metacognition on multiple levels in identifying and interpreting metacognitive activities in complex modeling activities. The results also show the model's efficiency for research on metacognition by providing a window into the development of students' metacognitive abilities.

By employing the theoretical model of metacognition on multiple levels, a great number of students' spontaneous metacognitive activities were extensively identified, in terms of all of the multiple levels–the individual, social, and environmental levels and all types of metacognitive activities–four regulating activities and six monitoring activities, across the seven student groups, across the team members, and across the three MEAs–Summer Jobs, Volleyball, and Paper Airplane MEAs.

Results showed that students' metacognitive activities were triggered at the environmental level by stimuli in the learning environment. Some examples included the existence of mathematics supplies, the type of problem, the duration of time given to solve problems, and aversion to homework, all of which may have had positive or negative influences on students' spontaneous metacognitive activities for effective problem solving.

In working together as teams of three or four, and throughout three problem-solving meetings, students also spontaneously showed all types of metacognitive activities triggered via their own needs at the individual level, or by evaluating other people's thinking at the social level: four regulating activities (*new idea*, *changing strategy*, *correcting errors*, and *reinterpretation*) and six monitoring activities (*assessment of knowledge*, *assessment of understanding*, *assessment of strategy appropriateness*, *assessment of progress toward goal*, *assessment of strategy execution*, and *assessment of accuracy or sense of result*) at both the individual and social levels.

Looking at the individual level, students themselves triggered their *new idea*, which is the first type of regulating activity, based on their own needs so as to lead problem solving, primarily at the beginning of their problem solving, or in order to change prior implemented strategies during problem solving. Then, at the social level, students gained a *new idea* from the results of students' monitoring activities, which were prompted by others' ways of thinking. The students also shared their *new idea*, in response to others, offering alternative approaches or supporting others' manners of thinking. By direct acceptance or subtle agreement from group members, the second type of regulating activity, *changing strategy*, happened when prior strategies were changed, based on students' assessment of their adopting an alternative approach, by the student who originally suggested the new idea at the individual level, or by others at the social level. Third, *correcting errors* occurred when students detected and corrected others' errors–despite no request from others–as well as their own errors through their own *assessment of strategy execution* at the individual level, or when students corrected their errors detected through others' *assessment of strategy execution* at the social level. Finally, students triggered their *reinterpretation* about problems, previously implemented strategies, or alternative approaches in order to support their own ways of thinking, and ultimately, to be able to support their monitoring activities at the individual level, or in order to support others' ways of thinking and others' monitoring activities at the social level.

The first type of monitoring activity, *assessment of knowledge*, was triggered when students evaluated their own manner of thinking, focusing on their knowledge, such as relevant mathematical and task-specific knowledge, at the individual level, or in response to others' thinking, which focused on the other students' knowledge, at the social level. Second, *assessment of understanding* was triggered when students assessed their own ways of thinking, which focused on their comprehension of problems and problem contexts at the individual level, or when students assessed their own comprehension of others' ways of thinking, in and of itself, or others' ways of thinking, which focused on others' understanding of problems and problem contexts, in order to complement their own or others' understanding of problems at the social level. The third type of monitoring activity, *assessment of strategy appropriateness,* occurred when students assessed a new idea, which they originally suggested, or previously implemented strategies during problem solving at the individual level, or when students assessed their own ways of thinking, either focusing on a new idea originally suggested by others, or in response to others' ways of thinking, which focused on a new idea or previously implemented strategies at the social level. The fourth type of monitoring activity, *assessment of progress toward goal*, took place when students made judgments either regarding their own ways of thinking, which focused on progress toward their goal (e.g., what needed to be done, what had been done, where they were in their problem-solving process, and so on) at the individual level, or in response to others' ways of thinking, which focused on the progress toward their goal, in order to suggest, confirm, or revise their directions toward their goal at the social level. Fifth, *assessment of strategy execution* occurred when students assessed their own strategy execution or others'– despite no request from others–at the individual level, or when students assessed their strategy execution in response to others' ways of thinking, which focused on strategy execution, in order to support one another to properly execute their strategy at the social level. Finally, the sixth type of monitoring activity, *assessment of accuracy or sense of result*, occurred when students made judgments either regarding their own ways of thinking, which focused on accuracy or a sense of result at the individual level, or in response to others' assessment of the results at the social level.

As summarized above, the results from the multiple case study illustrated (a) how the theoretical model of metacognition helped more or less clearly distinguish metacognition from cognition, according to the existence of any cue of evaluation or regulation; (b) how it helped make the distinction among metacognitive activities, focusing on the monitoring function and the regulatory function; and (c) how it helped interpret students' metacognitive activities in context by looking at the sources that triggered them. These results together demonstrate that the theoretical model of metacognition on multiple levels employed in this study is a coherent model of metacognition, which is appropriate in identifying and interpreting metacognitive activities in complex modeling activities. The soundness and appropriateness of the theoretical model of metacognition on multiple levels are also supported by the following section, in which the results of this study are discussed in light of existing research on metacognition.

The identification of patterns and tendencies of students' spontaneous metacognitive activities. The second major contribution of this study is the identification of several patterns and tendencies of students' metacognitive activities, involving a replication of findings from prior research. Even though the current study adopted a different theoretical framework and different metrology, some of the findings, which are summarized below, are similar to or consistent with findings of prior research. This replication may also indirectly support the soundness and appropriateness of the theoretical model of metacognition on multiple levels.

Through exploring a large enough sample of students' spontaneous metacognitive activities, which were identified by using the theoretical model of metacognition on multiple levels, several meaningful patterns of students' metacognitive activities were found and hold across groups, across students, and across the three MEAs – Summer Jobs, Volleyball, and Paper Airplane MEAs. Table 5.1 below summarizes all of the patterns and tendencies of students' spontaneous metacognitive activities identified from the current study. Each pattern (in the first column) indicates the tendencies of students' spontaneous metacognitive activities (in the second column). In particular, all of the tendencies seen below are related to the tendencies of cognitive development that are generally accepted. Researchers argue that cognitive development moves from (a) social toward individual; (b) external toward internal; (c) concrete toward abstract; (d) simple toward complex; (e) situated toward decontextualized; and (f) specific toward general (e.g., Lesh et al., 2003; Zawojewski & Lesh, 2003).

Table 5.1

Summary of the Identified Patterns and Tendencies of Students' Spontaneous Metacognitive Activities

Identified patterns of students' spontaneous metacognitive activities	Tendencies of students' spontaneous metacognitive activities
1. A predominance of monitoring activities compared to regulating activities	External
2. A predominance of metacognitive activities overall triggered at the social level, compared to those at the individual level	External Social
1) A relatively more evident prevalence of monitoring	External

activities triggered at the social level, compared that at the individual level	Social
2) An infrequency of regulating activities, in particular, *new idea*, triggered at the social level, compared to those at the individual level	External Social
3. A relative infrequency of monitoring activities overall in *assessment of cognition*, compared to those in *assessment of procedure*	External Concrete Simple
4. A relatively more evident prevalence of monitoring activities triggered at the social level in *assessment of cognition*, compared to that in *assessment of procedure*	External Social Concrete
5. A predominance of monitoring activities triggered at the individual level in *assessment of procedure*, compared to other metacognitive activities triggered at the individual level	External Concrete Simple

First, monitoring activities were consistently reported as the overwhelming majority of identified metacognitive activities, compared to regulating activities across groups, across students, and across the three MEAs. This finding is similar to the findings of prior research, even though this study used a different framework and methodology (Magiera, 2008; Magiera & Zawojewski, 2011; Whitebread et al., 2007; Wilson & Clarke, 2002, 2004). For example, using a theoretical model of metacognition adopted from Wilson and Clarke (2002, 2004) involving the three types of metacognitive activities–metacognitive awareness, regulation, and evaluation, and using self-report methods and five MEAs, Magiera and Zawojewski (2011) have recently identified the overall prevalence of metacognitive evaluation, followed by metacognitive regulation, and then metacognitive awareness.

As previously mentioned in Chapter 2, the theoretical model of metacognition on multiple levels for this study focuses on the functions of metacognition, in and of itself, i.e., metacognition as manifestations of the monitoring and regulating functions. Considering roughly metacognitive awareness and evaluation as the two non-regulatory activities that are monitoring activities, the overall predominance of monitoring activities and a relative infrequency of regulating activities are consistent with the findings of Magiera and Zawojewski (2011) and Wilson and Clarke (2002, 2004).

Second, the overall metacognitive activities were more frequently identified at the social level, compared those at the individual level across groups, students, and the three MEAs. This finding is also similar to the findings of prior research, which used a different framework and methodology (Magiera, 2008; Magiera & Zawojewski, 2011; Whitebread et al., 2007). Magiera and Zawojewski (2011) argued that this overall pattern could be partially due to the study designs involving self-report methods and small-group problem-solving sessions, which inherently might provide the participants with more opportunities to identify and explain their metacognitive activities triggered at the social level–because of explicit actions through interaction with peers, thus being visible in the video recording–during self-reports, compared to their metacognitive activities triggered at the individual level.

By using MEAs as a method for analyzing verbal metacognitive actions, which addresses several criticisms of self-report methods, and by using the theoretical model of metacognition on multiple levels, this study additionally builds a possible explanation about the predominance of metacognitive activities triggered at the social level: Perhaps as students have access to others' conceptual systems, they can have more external resources through their interactions with others that elicit their metacognitive activities. The predominance of metacognitive activities triggered at the social level is not due to self-report methods. Consequently, this predominance of students' metacognitive activities triggered at the social level, versus those at the individual level, shows that students' spontaneous metacognitive activities tend to be triggered by external, social sources.

In particular, the frequency of metacognitive activities triggered at the social level was slightly more evident in monitoring activities across groups, students, and the three MEAs. Again, roughly considering metacognitive awareness and evaluation as monitoring activities, this relative prevalence of monitoring activities triggered at the social level is consistent with the findings of Magiera (2008).

On the other hand, regulating activities, in particular, *new idea*, were more likely to be frequently triggered at the individual level in the overall study. This finding is inconsistent with the findings of Magiera (2008). Her finding showed a predominance of students' regulating activities triggered at the social level, compared to those at the individual level. Unlike the model of metacognition, which was used in her doctoral dissertation, the theoretical model of metacognition for the current study classifies metacognitive activities focused on *planning* into the type of monitoring activity– *assessment of progress toward goal*, rather than the regulating activity. This difference in the theoretical framework could be a possible reason for the inconsistency in her findings.

Another possible explanation about the infrequency of regulating activities triggered at the social level compared to those at the individual level is that the type of metacognitive activity, regulating activities–*new idea*, *changing strategy*, *correcting errors*, and *reinterpretation*, tends to be dependent upon potentially more internal, individual sources, rather than external, social sources. Considering the overall overwhelming infrequency of regulating activities versus monitoring activities, which was previously identified, this tendency of regulating activities, in particular, *new idea*, which are dependent upon more individual sources supports the finding mentioned above: Students' spontaneous metacognitive activities tend to be triggered by external, social sources.

Third, overall monitoring activities in *assessment of cognition* (*assessment of knowledge*, *assessment of understanding*, and a*ssessment of strategy appropriateness*) were relatively more infrequent than those in *assessment of procedure* (*assessment of progress toward goal*, *assessment of strategy execution*, and *assessment of accuracy or sense of result*) across groups, students, and the three MEAs. Considering the characteristics of monitoring activities in *assessment of procedure*, which may be directly associated with more tangible evaluation results through more explicit actions, this infrequency of monitoring activities in *assessment of cognition*, which may be mainly dependent upon mental runs with limited cues through less explicit actions, may indicate that students' spontaneous metacognitive activities tend to be triggered on the basis of external, tangible (concrete) sources; thus, they tend to be simple with respect to mental demands.

Fourth, the relative predominance of monitoring activities triggered at the social level was more evident in *assessment of cognition* than that in *assessment of procedure* across groups, students, and the three MEAs. In particular, the relatively steep increase in the frequencies of students' assessment of understanding and of the strategy appropriateness triggered at the social level may indicate that students' spontaneous metacognitive activities tend to triggered on the basis of external, concrete, social sources that lead to more explicit actions. On the other hand, the infrequency of students' assessment of knowledge, triggered at both the individual and social levels, may be due to its characteristics, in and of itself: It is largely a mental run, and thus, may not be externalized. As a result, it is not easy to identify, or it may be a sensitive assessment for individuals to unpack.

Finally, a predominance of monitoring activities triggered at the individual level in *assessment of procedure* (*assessment of progress toward goal*, *assessment of strategy execution*, and *assessment of accuracy or sense of result*) was identified across groups, students, and the three MEAs, compared to other metacognitive activities triggered at the individual level. This predominance of monitoring activities triggered at the individual level in *assessment of procedure* supports the prior finding: Students' spontaneous metacognitive activities tend to be triggered on the basis of more tangible (concrete) evaluation results through more explicit actions, such as social interactions, and with students preferring fewer mental demands (simple).

The current study reorganized Goos' (2002) classification of six monitoring activities into two categories, *assessment of cognition* and *assessment of procedure*, with respect to the different foci of assessment–cognition and procedure. As summarized above, in addition to replicating findings from prior research, this categorization of monitoring activities allowed the researcher to identify several new patterns and tendencies of students' metacognitive activities.

The identification of the nature of students' metacognitive abilities. The third major contribution of this study lies in the empirical identification of the nature of students' metacognitive abilities. This study may provide empirical evidence supporting an argument from prior research: Students' metacognitive abilities naturally develop along with dimensions similar to their cognitive development (e.g., Lesh et al., 2003; Zawojewski & Lesh, 2003). Figure 5.1 below summarizes the identified nature of students' metacognitive abilities, based on the empirical evidence. This figure indicates how students' metacognition could develop in parallel and interactively with cognition as they are triggered spontaneously, while collaboratively working in a group during problem solving in natural classroom settings.

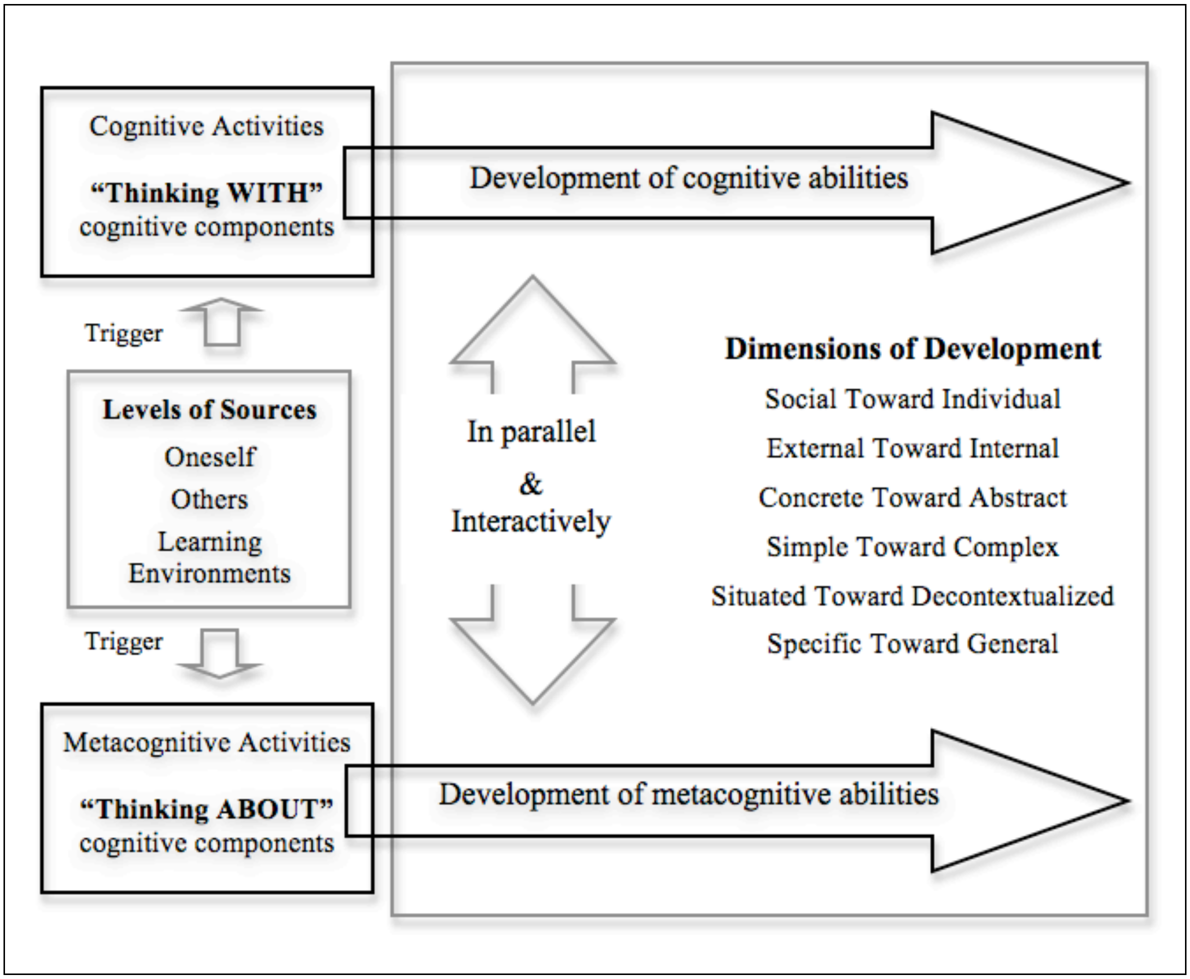


Figure 5.1. The nature of students' metacognitive abilities.

First, while there was a student who revealed metacognitive activities that were relatively triggered in an active manner, at both the individual and social levels, more students' metacognitive activities within each group were relatively biased on the social level, depending on external, social sources, rather than internal, individual sources. These findings may indicate social toward individual or external toward internal dimensions to the development of students' metacognitive abilities, which are mentioned as dimensions to that of students' cognitive abilities by prior research (e.g., Lesh et al., 2003; Zawojewski & Lesh, 2003).

Second, within and across the three MEA, most students across the seven groups revealed relatively biased types of monitoring activities triggered at the individual level in *assessment of procedure*, compared to those at the social level, depending on concrete and tangible sources, rather than abstract sources, and thus less complex mental processes of assessment. These findings may indicate concrete toward abstract or simple toward complex dimensions to the development of students' metacognitive abilities. These dimensions are also consistent with dimensions of the development of students' cognitive abilities, which are mentioned by prior research (e.g., Lesh et al., 2003; Zawojewski & Lesh, 2003).

Finally, the results from the cross-case analysis for the three MEAs show differences in the overall patterns of each student's metacognitive activities across the three MEAs. These findings may indicate situated toward decontextualized or specific toward general dimensions to the development of students' metacognitive abilities, which are also mentioned as dimensions to that of students' cognitive abilities (e.g., Lesh et al., 2003; Zawojewski & Lesh, 2003).

Consequently, these empirical findings of the current study may support the argument that students' metacognitive abilities could develop along with dimensions similar to their cognitive development as follows: (a) social toward individual; (b) external toward internal; (c) concrete toward abstract; (d) simple toward complex; (e) situated toward decontextualized; and (f) specific toward general (e.g., Lesh et al., 2003; Zawojewski & Lesh, 2003).

In particular, this empirical evidence for the potential similarity to the development of students' metacognitive abilities from that of students' cognitive abilities may support the theoretical assumptions made by the models and modeling perspectives (MMP): (a) treating metacognition and cognition as integrated, not hierarchically separate–thus, metacognition develops naturally in parallel and interactively with cognition, as it is triggered spontaneously during small-group problem-solving activities; and (b) valuing the situated, environmental, and social nature of metacognition–spontaneous metacognitive activities are closely related to particular content, situations, and contexts, as well as social interactions (Lesh et al., 2003; Lesh & Zawojewski, 2007; Zawojewski & Lesh, 2003).

The identification of circumstances in which metacognitive activity is abandoned. The fourth major contribution of this study is the identification of circumstances in which metacognitive activity is abandoned. While Goos (2002) identified the circumstances related to metacognitive failures, in which less successful outcomes are produced–described as metacognitive blindness, metacognitive vandalism, and metacognitive mirage, this study has identified the circumstances that might interfere with students' spontaneous metacognitive activities themselves; thus, they might deprive students' chances of achieving more productive outcomes.

The first category of circumstances that might interfere with students' spontaneous metacognitive activities originates from learning environments. It includes a distorted function of mathematics supplies (e.g., calculators and protractors), duration of time given to solve problems, no grade points, and aversion to homework. In particular, a negative impact of the existence of calculators on students' spontaneous metacognitive activities has also been identified in my prior research with Korean eighth-grade students working on the Paper Airplane MEA (Kim et al., 2013). The current study has also identified that an implementation of problem-solving activities with no grade points might be able to interfere with students' spontaneous metacognitive activities. This empirical finding is inconsistent with prior research arguing several negative effects of grading (e.g., Kohn, 2011).

The second category of circumstances that might interfere with students' spontaneous metacognitive activities comes from social interactions. It includes teacher's inappropriate intervention, mixed-gender groups, and a lack of collaborative working skills. Research has identified many factors influencing problem-solving performance in social contexts, such as group size, group composition, seating arrangement, and role assignments (e.g., Heller & Hollabaugh, 1992). These factors may also have an impact on the quality of social interactions, and thus have an impact on students' spontaneous metacognitive activities.

In particular, one interesting event that might interfere with students' spontaneous metacognitive activities is due to the group composition in terms of gender. The current study has identified a negative impact of mixed-gender grouping on students' spontaneous metacognitive activities by causing lower levels of interaction and cooperation. This finding is similar to the findings of prior research indicating a negative impact of majority-girls or boys-mixed-gender groups on group interaction (e.g., Wilkinson & Fung, 2001). Another interesting event identified in the study, which might interfere with students' spontaneous metacognitive activities, originated from students' lack of collaborative working skills. In particular, students revealed a lack of collaboration working as a whole group in the two problem-solving processes, "definition building" and "operationalizing definitions," in which different levels of complexity also influence students' spontaneous metacognitive activities (Kim et al., 2013).

Social interactions as rich resources that trigger students' spontaneous metacognitive activities. The fifth contribution of this study is the confirmation that as students have access to social sources through interactions with others, which provide access to others' conceptual systems, they can go beyond their knowledge or regulation of cognition in a natural way, and have increasing opportunities spontaneously to develop their metacognitive abilities. This confirmation supports the findings of prior research emphasizing the importance of social interactions that stimulate students' metacognitive activities (e.g., Goos, 1994, 2002; Goos et al., 2002; Iiskala et al., 2004, 2011; Kim et al., 2013; Magiera & Zawojewski, 2011).

Model-Eliciting Activities (MEAs) as an environmental source triggering metacognition. The sixth major contribution of this study is the confirmation that Model-Eliciting Activities (MEAs), which are a specific type of problem-solving activity, serve as an effective environmental source that stimulates students' spontaneous metacognitive activities triggered at both the individual and social levels. The current study identified a large number of students' spontaneous metacognitive activities, which were identified at the individual and social levels, during the three MEAs–Summer Jobs, Volleyball, and Paper Airplane MEAs. The characteristics of MEAs, which are described in detail in Chapter 3, may establish effective contexts that stimulate students' spontaneous metacognitive activities, triggered at both the individual and social levels.

In particular, this confirmation supports the findings of Magiera and Zawojewski (2011), who identified and characterized problem-solving situations that stimulate spontaneous students' metacognitive activities, which may be provided by MEAs. They identified three social-based contexts that stimulate spontaneous students' metacognitive activities triggered at the social level, characterized as "interpreting diverse perspectives," "engaging in explanations," and "seeking mathematical consensus." They also identified three self-based contexts that stimulate spontaneous students' metacognitive activities triggered at the individual level, characterized as "seeking personal satisfaction," "making experience-based quantitative judgments," and "making personal projections" (Magiera & Zawojewski, 2011). As discussed in Chapter 3 and illustrated in Chapter 4, MEAs can establish these six contexts effectively, and thus they can work as an effective environmental source that stimulates students' spontaneous metacognitive activities, triggered at both the individual and social levels.

Model-Eliciting Activities (MEAs) for research on metacognition. The seventh major contribution of this study is the confirmation that MEAs could substitute for self-report methods, which have been used in most research on metacognition, with using the theoretical model of metacognition on multiple levels. Through providing detailed accounts of data analysis procedures and a large enough sample of the identified metacognitive activities, the current study has illustrated how MEAs could work as a method for verbal protocol analysis, by providing access to students' metal processes, in particular, their metacognitive activities, addressing several criticisms of self-report methods, described previously.

In-depth investigation of how an individual's thinking becomes metacognitive in context.

Finally, the eighth contribution of this study is in response to the need for an in-depth investigation of how an individual's mind works in social contexts, with the expectation for potential application to school practice (Schoenfeld, 1999). The theoretical model employed in the current study has re-conceptualized metacognition on multiple levels by focusing on an individual as a unique agent, and by looking at the sources that trigger a student's metacognition at the individual, social, and environmental levels. This re-conceptualization of metacognition on multiple levels provides a window into the development of students' metacognitive abilities by allowing an in-depth investigation of how an individual's thinking becomes metacognitive in context.

Limitations

Although this research involves multiple case studies of 23 students in seven groups of three to four, collaboratively working on three different problem-solving sessions during an academic year, reasonable caution must be exercised in that the generalizability of the study's results is limited. Some aspects of the results may not be the same with different students, different group compositions, different types of problems (beyond modeling activities), and in different settings.

However, the consistency of the patterns and the tendencies of the identified students' metacognitive activities, which were triggered at the individual, social, and environmental levels, held across students, across groups, and across the three problem sessions. This consistency might indicate the potential for generalizability of the study's results. In addition, the results of the current study were similar to or consistent with those of prior research, which involved different frameworks and different mythologies, as well as different subjects, tasks, or settings (e.g., Magiera, 2008; Magiera & Zawojewski, 2011; Whitebread et al., 2007; Wilson & Clarke, 2002, 2004).

Another limitation is that the identified circumstances interfering with students' spontaneous metacognitive activities are undoubtedly a tiny subset that may need to be verified in additional studies. However, it may be helpful and useful enough for us to design metacognitive learning environments, as illustrated below in the following section of implications for education.

The current study may also involve several potential limitations, which are due to the study's design. First, a limitation of the study's results is that the relative infrequency of monitoring activities in *assessment of cognition–assessment of knowledge*, *assessment of understanding*, and *assessment of strategy appropriateness*–compared to those in *assessment of procedure* may be an underestimate of the actual frequency. A possible explanation for this potential underestimate of the actual frequency is similar to explanations from Magiera and Zawojewski (2011) and Wilson and Clarke (2004) for the relative infrequency of metacognitive awareness, which is one of the components in their model of metacognition. They pointed out that the infrequency of metacognitive awareness might be due to the design of the study. Students' metacognitive awareness might be largely subconscious, and thus, may not be externalized or visible in the video recording during the self-reports; likewise, it may not be observable to the researchers.

Similar to these arguments, monitoring activities in *assessment of cognition* may be largely running mentally. As a result, they may not be externalized in terms of students' spontaneous verbal actions, and are thus, unidentified.

Another limitation of this study might be the narrow range of metacognitive activities identified at the environmental level, which might originate from the narrow range of problem-solving tasks used. This possible limitation provides opportunities for future research by suggesting studies to investigate what types of problems (beyond model-eliciting activities) and learning environments elicit various levels of metacognition. In addition, the current study has not examined the different levels of complexity in the problem-solving tasks as environmental sources for triggering metacognition, which could also be an area for further research. The narrow range of problem-solving tasks used, in and of itself, might also be a limitation of the study's results. However, as Magiera and Zawojewski (2011)–who used five MEAs in their research on metacognition–argued, the findings of the current study that were similar to or consistent with prior research using different types of problems (Whitebread et al., 2007; Wilson & Clarke, 2002, 2004) might make this possible limitation less problematic.

Finally, a limitation of this study is that the changes in some groups, which were caused by the absence of group members, might have influenced both the quantity of the identified metacognitive activities across group members within respective groups, as well as the quality of those outcomes. The following section presents implications of this study with respect to education and research.

Implications

In addition to addressing the two main research questions concerning (a) the appropriateness of the theoretical model of metacognition on multiple levels in identifying and interpreting students' metacognitive activities; and (b) the nature of students' metacognitive abilities during problem solving, the current study has several meaningful implications for education and research. The following points need to be considered with respect to implications.

Implications for education. The findings of the study enrich our understanding of how to design metacognitive learning environments. The current study has the potential to guide teachers, teacher educators, and curriculum developers to create circumstances that support students' spontaneous development of metacognitive abilities. It also has the potential to guide the development of effective instructional methods to integrate these circumstances into existing curricula.

First, in order to create circumstances that support students' spontaneous development of metacognitive abilities, it is necessary to consider the patterns and tendencies of students' metacognitive abilities. The current study has suggested empirical evidence for the theoretical assumptions that students' metacognitive abilities naturally develop along with dimensions similar to their cognitive development during small-group problem solving in natural classroom settings (e.g., Lesh et al., 2003; Magiera & Zawojewski, 2011; Zawojewski & Lesh, 2003). As a result, the following circumstances are recommended in order to support students' spontaneous development of their metacognitive abilities: (a) involving social contexts (e.g., team-oriented problem-solving activities) to ensure rich social sources; (b) providing multiple representations, in particular, visualizations that provide more concrete and tangible sources; and (c) involving similar toward different mathematical structures to provide more stable sources.

Second, when designing metacognitive learning environments, it is necessary to consider the minimization of circumstances that interfere with students' spontaneous metacognitive activities. For example, teachers can minimize the distorted function of mathematics supplies and manipulatives, which was identified as one of the circumstances interfering with students' spontaneous metacognitive activities in the current study, by equipping an arsenal of mathematics supplies and manipulatives in a corner of the classroom where students can stop by, based on their own needs, rather than handing them out. In addition, teachers need to avoid comments or questions that steer their students toward a particular solution reflecting the teachers' own way of thinking, which was identified as another circumstance interfering with students' spontaneous metacognitive activities in the current study. Instead, teachers always need to maintain their role as a facilitator in order to encourage student teams to concentrate on their own issues when implementing small-group problem-solving activities, such as a modeling activity. Finally, students need to improve a lack of collaborative working skills, which was identified as a circumstance interfering with students' spontaneous metacognitive activities in the current study. It would be a possible way to provide students with many opportunities to engage in problem situations requiring them to make their own definitions of qualitative constructs and negotiate their individual and social meanings as a whole group. In particular, this frequent engagement in definition building would also be a good way to stimulate students' metacognitive activities, in particular, their *assessment of knowledge* and *assessment of understanding*, which can lead to tracking and correcting their misunderstanding.

Third, in order to engage students in active, higher-order learning, it is necessary to develop effective instructional methods to integrate circumstances supporting students' spontaneous development of metacognitive abilities into existing curricula. The current study has illustrated how MEAs work as an environmental source to stimulate students' spontaneous metacognitive activities in a natural classroom setting. MEAs effectively establish both individual-based and social-based contexts that stimulate students' spontaneous metacognitive activities triggered at the individual and social levels, respectively. They involve a set of data via multiple representations, and are inherently mathematical structures-oriented. In particular, MEAs require students to define a qualitative construct (definition building) and to make a qualitative construct measurable in order to formalize the goal of a problem (operationalizing definitions).

Finally, writing a letter to an imaginary client, which is the final product required in MEAs, has been identified as a rich source that stimulates students' spontaneous metacognitive activities. While students summarized the entire problem-solving process in writing a letter, their metacognitive activities were re-triggered. In addition to these implications for education, the current study has several implications for research on metacognition, as discussed in the following section.

Implications for research. As illustrated by an abundant sample of the identified students' spontaneous metacognitive activities, the theoretical model of metacognition on multiple levels employed in this study has the potential as a coherent model with respect to research on metacognition appropriately identifying and interpreting students' metacognitive activities. As a result, the theoretical model of metacognition on multiple levels can provide a window into research on metacognition by facilitating an in-depth investigation of how students' metacognitive abilities develop in contexts, and how to promote the development of students' metacognitive abilities. In particular, re-conceptualizing metacognition on multiple levels–the individual level, the social level, and the environmental level–makes an in-depth investigation of the development of students' metacognitive abilities possible.

The current study has also illustrated how MEAs work as a research method for verbal protocol analysis, addressing several criticisms of self-report methods. MEAs require students to verbalize their thoughts spontaneously while collaboratively working in a group in natural classroom settings. Individual students' spontaneous verbal actions, which are expressed according to their own needs during MEAs, provides access to their mental processes, in particular, metacognitive activities during small-group problem solving.

Consequently, the theoretical model of metacognition on multiple levels employed in this study provides a theoretical base for research on metacognition, addressing several issues and challenges, which were reported earlier. Model-Eliciting Activities (MEAs) as a research tool also provide an authentic method to observe and analyze students' metacognitive activities in natural classroom settings. Based on these implications for research, the current study serves as a springboard for future research directions as recommended in the following section.

Recommendations for Future Research

By using the theoretical model of metacognition on multiple levels, several areas of future research can build on this study. First, as suggested in the limitations section above, future research is needed to explore what different types of problems (beyond model-eliciting activities) and learning environments elicit various levels of metacognition, and how different levels of complexity in problem-solving tasks function differently as environmental sources for triggering metacognition. Further research is also needed to understand what types of environments effectively encourage metacognition on the individual and social levels. Several possible questions for a smaller scale qualitative research are: (a) *What factors within classroom management systems effectively elicit various levels of metacognition?*; (b) *What factors in social contexts (e.g., group size and group composition) effectively elicit various levels of metacognition?*; and (c) *How do different environmental sources (textbooks, curricula, and technology) encourage students' metacognition on multiple levels?* Second, future research is needed to understand what types of social sources effectively encourage metacognition. A possible question for a smaller scale qualitative research is, *What is the role of the teacher in eliciting or confounding metacognition?* Another is, *How do different social interactions influence the productivity of different types of metacognitive activity?* Finally, the empirical identification of two categories for circumstances in which metacognitive activity is abandoned provides a springboard for continued investigation. Longitudinal study is also needed to explore the relationship between the development of students' metacognitive abilities and their improvement in problem-solving achievement.

Conclusion

The empirical evidence in this study has supported the soundness and appropriateness of the theoretical model of metacognition on multiple levels in identifying and interpreting students' metacognitive activities in complex modeling activities. By using the theoretical model of metacognition on multiple levels, this study has identified several patterns and tendencies of students' spontaneous metacognitive activities in complex modeling activities. This study has also provided empirical evidence for the potential similarity of students' developing metacognitive abilities to their developing cognitive abilities with respect to the dimensions of development. The study's results may indicate that students' metacognition develops in parallel and interactively with cognition as they are triggered spontaneously while collaboratively working in a group during problem solving in natural classroom settings. The current study is expected to further our understanding about how students develop metacognitive abilities within problem-solving processes, and to inform how to effectively encourage students' metacognition and improve their problem-solving achievement. Finally, the theoretical model of metacognition on multiple levels can provide a window into the development of students' metacognitive abilities.

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APPENDICES

Appendix A: Summer Jobs MEA

Newspaper Article: Heat Is on for Summertime Jobs.

Saturday is the Minneapolis Middle School's annual job fair, organized by the Youth Action Club. The fair gives students an early start to plan how they will earn money during the summer.

It also honors innovative work efforts. This year's Earnest Earner award went to seventh-grader Tyrone Wakes. His Kids on the Run service provided mail pickup, errand running, and pet walking for residents in his apartment complex.

"Summer jobs are big business," claims Randy Tye, President of Youth Action. "Our fair gives ideas about the kinds of jobs kids can get and the money they can earn. We will hand out information on pay rates for different jobs and advice on 'talking money' with employers."

The fair will show how to track down hard-to-get jobs. Student Greg Maby reports, "The basic idea is to go after the jobs. Don't wait for them to come to you. It's up to kids to show what they can do. We'll give money-making tips, like how to win baby-sitting jobs by getting to know young children in the neighborhood."

Students Jack Elliot drums up yard work by mowing his own lawn early on Saturday mornings. "Wake up and smell the grass clippings! Is my motto. When people see the job I do on my parents' lawn, they often ask me to do theirs." Showing off your skills is a good way to attract business.

Visitors to the fair can take a lesson on washing cars for fun and profit. Prices for commercial car washes are high enough hat young people can compete for the business. Tye says, "Most adults don't care who washes their car. If you provide services as good as others but at a better price, you'll get the job."

A profit-making venture often overlooked is services for the elderly. As life expectancy increases, people live longer, and they can use different kinds of help. Senior citizens often need help shopping, cleaning house, or doing yard work. Households with infirm elderly may need someone to visit with then, so that family members can take a break.

Jenny Crisp says, "Believe it or not, my baby-sitting job is with my next-door neighbor's grandmother. She has Alzheimer's disease. We read stories together. Sometimes I play my flute. We have a good time and I also get paid. I'm going into medical research when I grow up. I'd like to find a cure for Alzheimer's."

Today's students use brochures to get summer jobs. Teacher Helen Soleski says, "Our computer lab has software for designing flyers that advertise job skills and experience. Some students design resumes giving potential employers information about their training and previous work experience."

Eighth-grader Justin Shar says, "My resume tells about the baby-sitting I've done in the past, my class in first aid, and my advanced swimmer's certificate. I listed references with names and phone numbers of people I worked for. They gave me such good recommendations that I got five job offers."

Sixth-grader Marsha Turi advises, "Don't give up. Last year I made flyers, went door-to-door, and advertised on bulletin boards. I must have asked a hundred people for work. I got ninety-nine no's and one very good job that lasted all summer. By September, I had put $350 in the bank. I did such a good job that I expect to get the job again next summer."

Typical Pay for Student Summer Jobs

Washing Cars

Own Parents		Others
$2/car	MEDIAN	$4/car
$2 to $5	LOW/HIGH	$3 to $6

Lawn Mowing

Own Parents		Others
$5/lawn	MEDIAN	$10/car
$3 to $10	LOW/HIGH	$5 to $15

Paper Routes

MEDIAN	$12.50/week
LOW/HIGH	$7 to $25

READINESS QUESTIONS

1. About how much money might a student earn mowing his or her parent's lawn once a week for a month? Be sure to show your calculations below.

2. About how much might a student earn mowing 10 average-size lawns once a week for a month? Show your calculations below.

3. About how much might a student earn in a month mowing 5 small-size lawns once a week and 5 average-size lawns once every two weeks? Show your calculations below.

4. Next summer Jack Elliot wants to do more than just yard work. He plans to mow 10 yards, get a newspaper route, and wash cars on Saturdays. Jack wants to know about how much money he can realistically expect to make during the summer, but this depends on many things, such as how often he mows a lawn or how big the lawn is.

   Estimate what his earnings might be and list the assumptions you make. Give at least 5 of your assumptions.

Problem Statement

Last summer Maya started a concession business at Wild Days Amusement Park. Her vendors carry popcorn and drinks around the park, selling wherever they can find customers. Maya needs your help deciding which workers to rehire next summer.

Last year Maya had nine vendors. This summer, she can have only six – three full-time and three half-time. She wants to rehire the vendors who will make the most money for her. She doesn't know how to compare them because they worked different numbers of hours. Also, when they worked makes a big difference. After all, it is easier to sell more on a crowded Friday night than a rainy afternoon.

Maya reviewed her records from last year. For each vendor, she totaled the number of hours worked and the money collected – when business in the park was busy (high attendance), steady (medium attendance), and slow (low attendance). (See the tables.) Please evaluate how well the different vendors did last year for the business and decide which three she should rehire full-time and which three she should rehire half-time.

Write a letter to Maya giving your results. In your letter describe how you evaluated the vendors. Give details so Maya can check your work, and give a clear explanation so she can decide whether your method is a good one for her to use.

	HOURS WORKED LAST SUMMER								
	June			**July**			**August**		
	Busy	*Steady*	*Slow*	*Busy*	*Steady*	*Slow*	*Busy*	*Steady*	*Slow*
MARIA	12.5	15	9	10	14	17.5	12.5	33.5	35
KIM	5.5	22	15.5	53.5	40	15.5	50	14	23.5
TERRY	12	17	14.5	20	25	21.5	19.5	20.5	24.5
JOSE	19.5	30.5	34	20	31	14	22	19.5	36
CHAD	19.5	26	0	36	15.5	27	30	24	4.5
CHERI	13	4.5	12	33.5	37.5	6.5	16	24	16.5
ROBIN	26.5	43.5	27	67	26	3	41.5	58	5.5
TONY	7.5	16	25	16	45.5	51	7.5	42	84
WILLY	0	3	4.5	38	17.5	39	37	22	12

	MONEY COLLECTED LAST SUMMER								
	June			**July**			**August**		
	Busy	*Steady*	*Slow*	*Busy*	*Steady*	*Slow*	*Busy*	*Steady*	*Slow*
MARIA	690	780	452	699	758	835	788	1732	1462
KIM	474	874	406	4612	2032	477	4500	834	712
TERRY	1047	667	284	1389	804	450	1062	806	491
JOSE	1263	1188	765	1584	1668	449	1822	1276	1358
CHAD	1264	1172	0	2477	681	548	1923	1130	89
CHERI	1115	278	574	2972	2399	231	1322	1594	577
ROBIN	2253	1702	610	4470	993	75	2754	2327	87
TONY	550	903	928	1296	2360	2610	615	2184	2518
WILLY	0	125	64	3073	767	768	3005	1253	253

Figures are given for times when park attendance was high (busy), medium (steady), and low (slow).

Appendix B: Volleyball MEA

Newspaper Article: Volleyball Champions

St. Paul, Minnesota - For years, volleyball has been the focal point of the fall semester at several middle schools. Roseville and Battle Creek Middle School in Minnesota are perennially among the best in the nation. Their success has not come by accident. Great leadership and excellent coaching have helped to put these two schools on the map for their success. Coach Smith of Battle Creek Middle School attributes his team's success to their work ethic.

Each morning the athletes come in to lift weights. Once a week, athletes and coaches at each school study films to observe techniques and to watch traits of their competitors. In addition to lifting weights and watching films, the teams practice six days a week.

Not only do the athletes work hard, they also have strong attitudes about keeping alive their winning streaks. Upon being interviewed, athletes at each school point to the fact that they feel an obligation to keep the winning tradition alive. "We feel like we owe it to the girls that preceded us," states Mariana Sailors of Roseville. "They were so successful and we just don't want to disappoint them."

Another thing that athletes and coaches agree has helped each team continue in their winning ways is a summer volleyball camp. This camp is held each summer from the second to the fourth week of June in St. Paul, Minnesota. University of Minnesota hosts the camp, and professional players are brought in from the summer circuit when time permits.

At the end of the camp, a volleyball tournament is held. Middle school girls from across the Midwest, ages 13-15, are paired with one another for teams. The organizers of the camp are coaches from the Big Ten and the MAC (Mid-American Conference). They make an attempt to split girls from the same school so that no advantage exists for one team.

In the past few years some teams have drastically defeated the other teams. Mariana Sailors recalls coming to the camp last year and losing one match 15-3, 15-2, 15-6. "It was a humbling experience," states Sailors. "I came to the camp for two reasons. First, I wanted to improve as a player. Second, I wanted to compete against other teams and individuals." With the current system in place, it is not uncommon to get destroyed by the competition.

Because of the lack of competition in the tournament, interest in the camp is decreasing. The camp does offer players the chance to learn and improve their volleyball skills, but the tournament isn't as fun as it could be. Even the players that win regularly in the tournament are getting bored. They too would rather face stiff competition than win their games easily.

VOLLEYBALL SPIKES! In volleyball, spikes are often classified as follows:
Kill: The other team was unable to return the ball.
Out of Bounds: The hitter spiked the ball out of bounds so the other team gets the serve.
Returned: The other team returned the spike.
Dink-unreturned: The hitter faked the spike and only tipped the ball over the net. The other team failed to return the drink.
Dink-returned: The hitter faked the spike and only tipped the ball over the net. The other team returned the dink.
In the Net: The hitter failed to hit the ball over the net.

Volleyball Readiness Questions

After reviewing the article and the data in the table on the following page, please answer the following questions.

1) What problem is the camp having?

2) What type of a spike would be classified as a Dink-Unreturned with the proposed tryout system?

3) What type of a spike would be classified as a Kill with the proposed tryout system?

4) Who is the tallest player among the players listed in the table?

5) Which player can jump the highest? Is this the same person as the player that can reach the highest point? Why or why not?

Data From Tryouts

Name	**Height of player**	**Vertical Leap in inches**	**40 Meter Dash in seconds**	**Serve Results (Number of serves successfully completed out of 10)**	**Spike Results (Out of 5 attempts)**
Gertrude	6'1"	20	6.21	8	Dink-Returned Dink-Unreturned Kill In the Net Returned
Beth	5'2"	25	5.98	7	Kill Returned Out of Bounds Dink-Returned Kill
Jill	5'10"	24	6.44	8	Out of Bounds Returned Returned Kill In the Net
Amy	5'10"	27	6.01	9	Kill Kill Dink-Unreturned Kill Returned
Ana	5'6"	25	6.95	10	Out of Bounds In the Net Returned Returned Dink-Returned
Kate	5'8"	17	7.12	6	Kill Dink-Unreturned Kill Returned Kill
Rhonda	5'3"	21	6.34	5	Out of Bounds Kill In the Net In the Net Dink-Returned
Christina	5'5"	23	7.34	8	In the Net Kill Kill Kill Dink-Unreturned
Andrea	5'5"	24	6.32	9	In the Net Out of Bounds In the Net Out of Bounds Returned
Nikki	5'7"	19	8.18	10	Dink-Unreturned Kill Kill Out of Bounds Returned
Kim	5'9"	23	6.75	7	Dink-Returned Kill Returned Out of Bounds Kill
Robin	5'8"	15	5.87	8	Kill Kill Kill Dink-Unreturned In the Net
Ermalinda	5'4"	21	6.72	8	Kill Returned Out of Bounds In the Net Dink-Returned
Lori	5'7"	19	6.88	9	Out of Bounds In the Net In the Net Kill Returned
Tina	5'1"	24	6.27	6	Dink-Unreturned Dink-Returned Dink-Returned Kill Out of Bounds
Angie	5'10"	23	6.54	8	Out of Bounds Kill Out of Bounds Out of Bounds Dink-Returned
Ruth	5'3"	26	7.01	9	Dink-Unreturned In the Net Kill Kill Kill
Rebecca	5'9"	18	6.78	10	In the Net Out of Bounds Kill Dink-Returned Kill

Key for Spike Results

Kill: The other team was unable to return the ball.

Out of Bounds: The hitter spiked the ball out of bounds so the other team gets the serve.

Returned: The other team returned the spike

Dink-unreturned: The hitter faked the spike and only tipped the ball over the net. The other team failed to return the dink.

Dink-returned: The hitter faked the spike and only tipped the ball over the net. The other team returned the dink.

In the Net: The hitter failed to hit the ball over the net.

COACH'S COMMENTS

Gertrude: Gertrude is slow getting to the ball.
Beth: She is very agile on her feet.
Jill: Jill's height could prove to be an asset for any team.
Amy: She is an awesome leaper, but she needs to know when to use it.
Ana: She comes from teams that have not been successful.
Kate: Kate has great quickness to get to the ball after serves.
Rhonda: Rhonda plays best when the team is playing well.
Christina: Her family life has negatively impacted her ability to play well.
Andrea: She is exceptionally strong for her age.
Nikki: She does many things well. In particular she serves well.
Kim: Kim is a great blocker.
Robin: Robin is the hardest worker we've ever had at the high school.
Ermalinda: Ermalinda is a girl that others want to be with because whatever event she's in, she seems to always find a way to win.
Lori: Lori does not always get her serve over the net.
Tina: She is one of the most intense players we have ever seen.
Angie: Her father coaches at a local school.
Ruth: Ruth's sister is a very good volleyball player at University of Alabama.
Rebecca: Rebecca is very coachable.

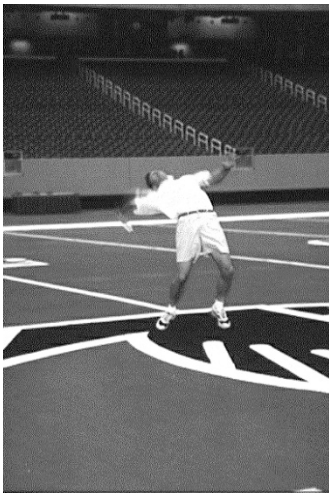
The Volleyball Problem

Information: The organizers of the volleyball summer camp want to have more competition in the camp's tournament. Thus, they need a way to fairly divide the campers into teams. They have compiled information about some of the players from tryouts and from the coaches. This information should be used to put together three teams of equal abilities to play volleyball.

The Problem: The camp organizers need you to split the players into **three equal teams**. In addition to forming these three teams, they need you to **write a letter** to them describing how you created your three equal teams. They will use your process for the next camp when they need to split a LARGE number of players into equal teams. Thus, you need to make sure that your process for creating teams will also work for a very large number of players.

Appendix C: Paper Airplane MEA

Students to fly away with paper airplane contest in the Twin Cities



St. Paul, MN –If you stop by Amy Frank's eighth grade science classes this week you are likely to find a very busy group of kids. Ms. Frank's students will follow in the footsteps of the Wright Brothers, engineers, and pilots as they design and fly paper airplanes in the Twin Cities Annual Paper Airplane Rodeo held in the Metrodome.

Frank's students will be designing, creating, and flying paper airplanes throughout the week. The students are learning how engineers work as they plan, create, test, and redesign their paper airplanes. They won't be using aluminum parts or jet engines for these planes. All they will need are pieces of paper – and a whole lot of imagination.

Students will need to design planes that are able to fly long distances as well as stay in the air for a long period of time. Each contestant will design a plane to try to win prizes in one of two categories: Best Floater and Most Accurate.

Said Frank, "The contest is designed to require the students to be very thoughtful about making their planes, so students who want to enter the paper airplane contest must follow a few rules." The rules are as follows: each plane must be made using a single sheet of 8.5"x11" paper. No cuts can be made in the paper; and, no tape, staples, glue, or paper clips can be used to hold the plane together or to change the plane's weight or balance. Also, each entry must qualify as being able to fly. For example, last year, a spitball and a dart were disqualified because they didn't really fly - even though it was possible to throw them so that they stayed in the air for a long time. Parachutes and helicopters also were disqualified because they don't go anywhere. For each throw, the judges will measure the time spent in the air, the distance the plane lands from the starting point, the distance the plane lands from the target, and the angle the plane lands from the target.

Because all paper airplanes are minutely different, it is difficult to make decisions about which plane is best. In order to make the competition as fair as possible, the judges are implementing two new processes for the contest. First, to minimize thrower advantages, the contest will have three neutral pilots to throw all planes in the contest. Second, the judges are designing a new scoring system to fairly judge the two winners.

"Some students are really getting into this contest – I've heard a couple who said they're bringing in-flight refreshments, crash helmets, and parachutes," said Frank. "It will be lots of fun and very interesting."

Questions to Get You Started

1. What are the categories for which the airplanes will be judged?

2. What types of measurements do you believe should be taken for each throw to fairly judge the contest?

3. How would you decide which airplane is the best floater?

4. How would you decide which airplane is the most accurate?

5. What are the judges doing differently this year than in years past? Why are they doing it?

Problem

In past competitions, the judges have had problems deciding how to select a winner for each award (Most Accurate and Best Floater). They don't know what to consider from each path to determine who wins each award. Some sample data from a practice competition and a description of how measurements were made have been included. To make decisions about things like being the best floater, the judges want to be as objective as possible. This is because there usually are only small differences among the best paper airplanes - and it seems unfair if different judges use different information or different formulas to calculate scores. So, this year, when the planes are flown, the judges want to use the same rules to calculate each score.

Write a brief 1- or 2-page letter to the judges of the paper airplane contest. Give them a rule or a formula which will allow them to use the kind of measurements that are given in Table 1 to decide which airplane is: (a) the most accurate flier, and (b) the best floater. … Table 1 shows a sample of data that were collected for four planes last year. Three different pilots threw each of the four planes. This is because paper airplanes often fly differently when different pilots throw them. So, the judges want to "factor out" the effects due to pilots. They want the awards to be given to the best airplanes – regardless who flies them.

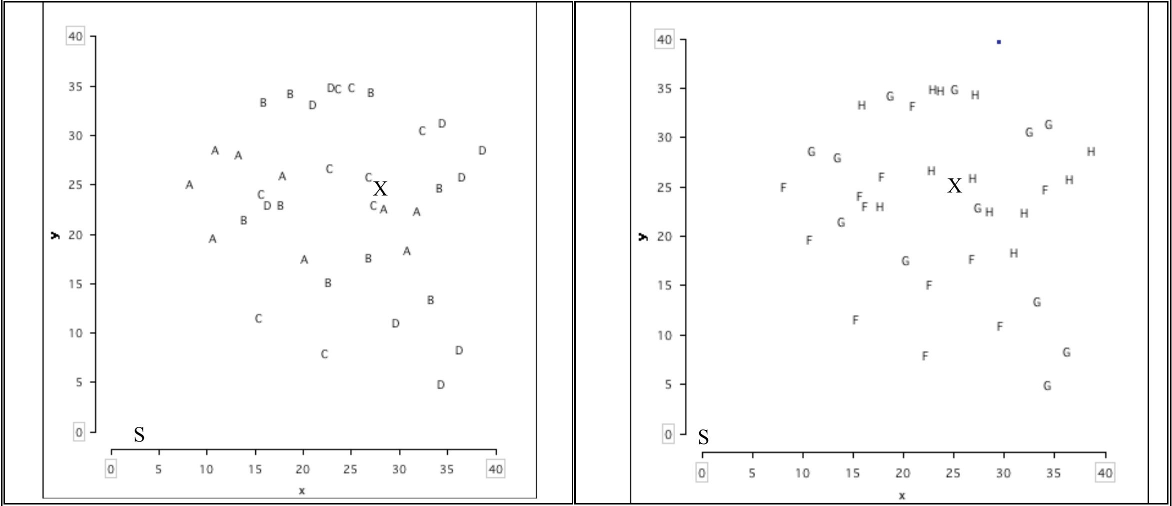
Use the data in Table 1 to show exactly how your rule or formula works – because the judges need to use your recommendation for planes that will be flown during the actual competition this year.

Note: The paper airplanes were thrown in a large 40-foot by 40-foot area in the arena. Each paper plane was thrown by a pilot who was standing at the point that is marked with by the letter S in the lower left hand side of each graph in Figure 1. So, this starting point is located at the point (0,0) on the graph. Similarly, the target is near the center of each graph, and it is marked with the letter X. So, the target is located at the point (25,25) on the graph.
In Table 1, the angles are measured in degrees. Positive angles are measured in a counter clockwise direction – starting from a line drawn from the lower left hand corner of the graphs to the upper right hand corner of the graphs (or starting from the point S and passing through the point X). Negative angles are measured in a clockwise direction starting from this same line.

Table 1: Information about Four Paper Airplanes Flown by Three Different Pilots

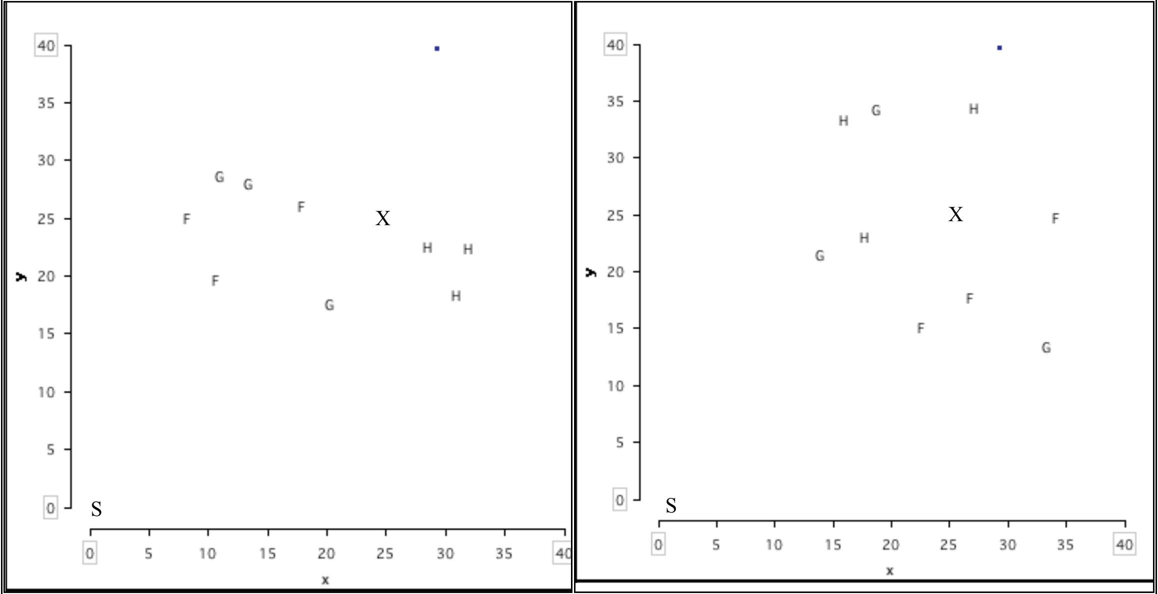
		Pilot F				Pilot G				Pilot H			
Plane	Flight	Distance from Start	Time in Flight	Distance To Target	Angle from Target	Distance from Start	Time in Flight	Distance To Target	Angle from Target	Distance from Start	Time in Flight	Distance To Target	Angle from Target
A	1	22.4	1.7	15.2	16	30.6	1.6	14.5	23	39	1.8	7.5	-10
	2	26.3	1.7	16.7	26	31.1	1.6	11.9	19	36.3	1.7	4.3	-6
	3	31.6	1.7	7.1	10	26.7	2.2	8.9	-4	35.9	2.2	9	-14
B	1	32.1	1.9	7.6	-11	35.9	1.9	14.3	-23	43.7	2.0	9.5	6
	2	42.2	2.0	9.2	-9	39	2.1	11.1	16	29	2.0	7.6	7
	3	27.2	2.1	10.2	-11	25.6	2.0	11.7	12	36.9	1.9	12.4	19
C	1	19.2	1.8	16.6	-8	42.9	2.0	9.8	9	35.1	1.6	2.8	4
	2	28.7	1.9	9.3	11	44.6	2.0	9.3	-1	37.2	2.2	2	-1
	3	23.6	2.1	17.3	-25	35.7	2.2	3.2	-5	42	2.1	9.8	10
D	1	28.1	1.5	8.9	9	37.2	2.1	20.2	-32	41.7	2.2	10.1	11
	2	31.6	1.6	14.8	-24	46.6	2.0	11.4	-2	48	1.9	14.1	-8
	3	39.3	2.3	9.1	12	34.7	1.8	22.2	-36	44.7	1.7	11.5	-9

Figure 1: Landing Points for Four Paper Airplanes Thrown by Three Pilots



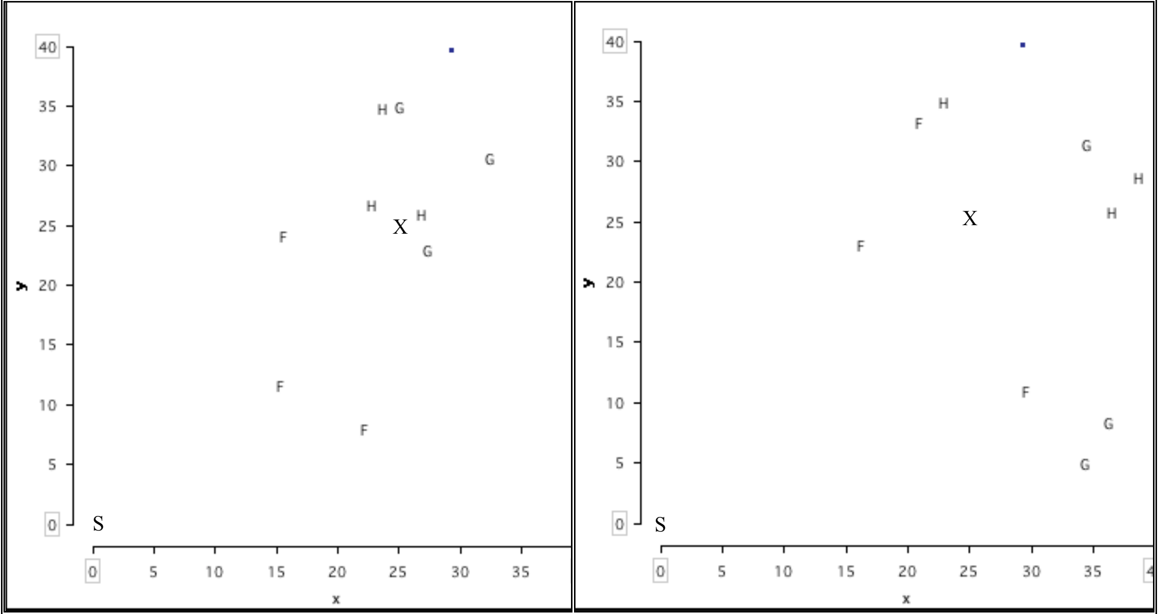
Landing Points for the Four Planes | **Landing Points for the 3 Pilots**

Figure 2: Separate Graph for Four Paper Airplanes



Plane A's Landing Points for Three Pilots

Plane B's Landing Points for Three Pilots



Plane C's Landing Points for Three Pilots

Plane D's Landing Points for Three Pilots