Essays in Relational Contract Theory

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Dedication

To my parents, who always remind me that I am able to handle my PhD study.

Abstract

This dissertation consists of 3 essays. The first essay studies a dynamic principal-agent problem where the agent's outside option is endogenously determined by the stock of effort. The compensation comes from 2 channels: the principal's explicit wage payment and the implicit outside option growth. On the one hand, the agent gains inherent work incentives under an increasing outside option. On the other hand, outside option growth makes the agent's participation constraint more stringent. I show that an agent is paid for his work in short-term contracts and for staying on the position in long-term contracts, rather than for work per se. The principal won't allow for infinite periods of work since it's costly to maintain an agent with outside options sufficiently high. The optimal contract is consistent with some academic empirical evidence on economics professors in the States.

The second essay contributes to the understanding of the fast-track effect, the phenomenon that one is likely to be promoted faster in the future, given that he is promoted faster previously. The model is based on the assumption that both the principal and the agent are risk-neutral. It is found that the date of the first success has impacts throughout the entire career as well as the duration of the relationship. The ones who achieve their first success at an earlier date would experience faster growth on outside options throughout the entire career. Therefore, the selection of fast-tracks is not based on individual differences but the stochastic output realizations. In addition, career

advancement also differs in the rate of outside option growth. Academic empirical evidence with a sample of top economists is found to support the fast-track results.

In the third essay, I vary the risk-neutral assumption and look at the contract with a risk-averse agent. With a risk-averse agent, the fast-track effect no longer exists. In this case, higher performance pay not only induces incentives but also imposes higher risk which in turn generates a risk premium to be paid by the principal. Given that the agent has already delivered some high output, realizing high output would cost a higher risk premium and lead to less effort to be implemented in the next period.

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Wage Dynamics with Outside Option Growth

1.1 Introduction

In relational contracts, it is possible for the principal to base future terms of trade on the success of the present trade. Current models such as Levin(2003) assume that the agent's outside option is constant and the principal faces a stationary problem in every period.

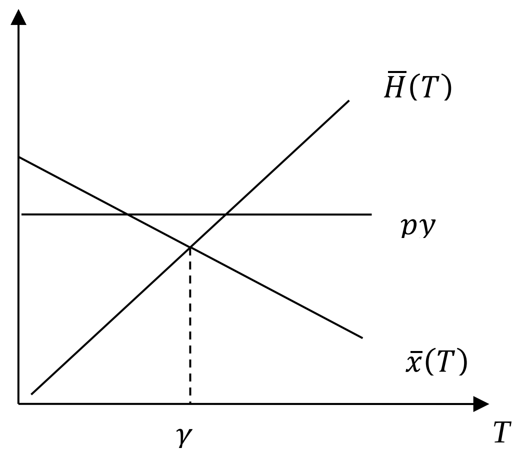
However, a constant outside option is not always a realistic assumption. It's often seen that a worker gains skills and public reputation through work, which adds to his value in the job market, i.e. growth of his outside option. If an agent's outside option is endogenously increasing in the stock of effort, the principal no longer faces the same profit maximizing problem in every period. How does the increasing outside option interact with the agent's incentive and participation constraints? What is the principal's optimal contract? Does inefficiency occur so that the principal sometimes purposely keeps a shirking agent? How does the optimal contract vary with parameters such as productivity, the growth rate of outside option and the time discount factor?

I study an infinite-horizon principal-agent problem where an agent's outside option is increasing in the stock of effort. Compensations consist of 2 parts: (i) wage payment from the principal that is based on the production history; (ii) outside option growth as one builds up his stock of effort.

An outside option that is increasing in the stock of effort affects both the incentive and participation constraints. A growing outside option provides work incentives and saves the principal's wage payment, since it serves as part of the agent's compensation. Meanwhile, it discourages the principal from implementing effort, since it is costly to retain an agent with high outside options.

In view of career concerns, an agent has an inherent incentive to work, when his outside option grows through work (rather than unchanged). The principal takes advantage of the growing outside option, and exploits the agent by paying a lower wage. I show that the principal's average per period wage payment is decreasing in the stock of effort for convex outside option growth and is increasing for the concave case.

On the other hand, the participation constraint becomes more stringent as an agent builds up the stock of effort. The agent's participation constraint becomes binding before the stock of effort goes infinity. When the principal implements work for all periods, Figure 1-1 average per period wage payment in contract duration below illustrates how the average per period wage payment changes with the contract duration.



When the outside option growth exhibits convexity and the principal implements work throughout a T- period relationship,

(i) $\bar{H}(T)$ is the average per period wage payment so that the participation constraint binds;

(ii) $\bar{x}(T)$ is the average per period wage payment so that the incentive constraint binds.

For $T < \gamma$, the incentive constraint is binding and participation constraint slack.

For $T \geq \gamma$, the participation constraint becomes binding and the agent has inherent incentives to work.

FIGURE 1-1 AVERAGE PER PERIOD WAGE PAYMENT IN CONTRACT DURATION

I assume that productivity doesn't change with the stock of effort. A reasonable explanation for this would be that productivity is sometimes determined by working conditions such as the capital-labor ratio. Therefore, it may not grow with the stock of effort. Under the assumption of constant productivity, the principal will terminate the relationship, when it becomes too costly to maintain an agent.

This paper studies 2 situations of outside option growth: convexity and concavity. The differences in growth rates might come from the public's perceptions of an agent's ability. When an agent's ability doesn't manifest itself and is gradually known through work, a senior worker is better publicly recognized. Rosen (1982) describes this as "the magnification effect" as the returns to ability are convex and senior workers' impact is magnified. On the other hand, if the public perceives an agent's ability as soon as he starts work, the agent's outside option may grow fast at the early stages of his career.

For the convex case, I show that in order to satisfy the incentive constraint, the average per period wage payment is decreasing in the contract length. However, the average wage payment to satisfy the participation constraint increases with the contract length. Therefore, an agent is paid according to incentive constraint in short-term contracts. For contracts lasting long enough, it is the participation constraint for working in every period that binds. Thus the wage payment is determined by the participation constraint. This suggests that the agent obtains work incentives from the principal's explicit wage payment in short-term relationships. For long-term relationships, his outside option growth accelerates as he stocks up effort. Thus an agent becomes self-motivated and is paid according to the participation constraint.

This is consistent with Gibbons and Murphy (1992), which shows that explicit wage payment and implicit career concern are substitutes in compensation. Eugene Fama (1980) and Holmstrom (1982) also show that incentives can be provided for career concerns, even without any explicit wage payment related to performance.

The findings are also consistent with some empirical evidence in academic fields. Ehrenberg, Pieper, & Willis (1998) finds the trade-off between the probability that an assistant professor obtains tenure and salaries, using the data on new assistant professors' salaries and probabilities of receiving tenures at economics departments in the States, from1974-75 to 1980-81. Evidence is found that assistant professors are willing to accept lower salaries with higher tenure probabilities. In addition, evidence shows that "departments that offered low tenure probabilities to assistant professors also paid higher salaries to their tenured faculty". The authors attribute this to the highly productive tenured faculty, who exerted lots of effort at the beginning of their careers. Although I don't consider any productivity growth by stock of effort, it is also found in my model that higher salaries are paid to senior workers, in order to retain those with higher outside options by satisfying their participation constraints.

For the concave case, an agent's outside option eventually grows very slowly and he lacks incentives to work, as the stock of effort grows. Similar results are found in Gibbons and Murphy (1992) that "explicit incentives from the optimal compensation contract should be strongest for workers close to retirement because career concerns are weakest for those workers". However my model differs in the sense that workers gain implicit incentives due to outside option growth, rather than perceptions of talent.

Depending on parameters such as work productivity, the growth rate of outside options and the time discount factor, the principal determines the effort to implement, as well as the retention decision. This paper finds that the principal's optimal contract is either to implement work followed by infinite shirks, or more periods of work before leaving the agent with his outside option.

Both the convex and concave cases deliver similar results on comparative statics. Higher work productivity, faster outside option growth and a smaller discount factor all lead to the principal's implementing work followed by terminating the relationship. My results show that the principal will stop implementing work, before the stock of effort reaches infinity. Similar results are also found in Holmstrom (1982) that the principal sometimes doesn't want the agent to over work.

Levin (2003) studies a principal-agent problem on an infinite horizon with fixed outside options and unobservable efforts, where the principal solves a stationary profit-maximizing problem in each period. Hopenhayn and Werning (2008) considers the equilibrium default model in which the outside option is the agent's private information and the principal observes human capital. My model differs in the sense that it involves the agent's moral hazard where both his effort and outside option are private information. The principal only observes output and rewards are given based on the production history. Output is observable by both parties, but the principal can't infer an agent's effort from stochastic realizations of output. Therefore, the settings are also different from MacLeod (2003) and Fuchs (2007), where output is evaluated privately by the principal. The analysis in this paper is related to career concerns. I show that compensation comes from the principal's explicit wage payment as well as the implicit outside option growth. Similar results are found in Eugene Fama (1980) and Holmstrom (1982) that a growing outside option provides the agent with work incentive, even in the absence of explicit performance-related wage payment. Rosen (1982) illustrates the effect of convex returns on ability so that senior workers have higher returns. This is similar to my results that under the convex growth of outside option, a senior worker's outside option growth accelerates as he builds up stock of effort. For the concave case, the outside option eventually grows very slowly and an agent lacks incentives to work, as his stock of effort grows. This is related to Gibbons and Murphy (1992) that workers exert effort for their career concerns. However in their paper, an agent's career concerns raise from perceptions of talent, rather than the endogenous outside option growth as in my model. Empirical evidence is found in academic careers of economics professors in the States, by Ehrenberg, Pieper, & Willis (1998).

The paper is organized as follows.

The model is introduced in section 1.2. Section 1.3 analyzes the optimal contract in an infinite horizon, where an agent's outside option grows as a convex function in the stock of effort. Section 1.4 studies the concave case and section 1.5 concludes.

1.2 Model

There are one principal and one agent, who are risk neutral and live for infinitely many periods. The 2 parties interact in each period, dated as $t = (0,1,2,\ldots,\infty)$. People are not perfectly patient and the common time discount factor is $\delta \in (0,1)$.

At date t, outside option $s(k_t)$ is endogenously determined as an increasing function in the stock of effort k_t. If the agent chooses to leave the relationship with his outside option, the relationship ends permanently.

The wage payment is contingent on production history. For any given wage offer, the agent decides whether to participate in the relationship or to pick up his outside option.

Conditional on participating, the agent either works or shirks. If he works, effort is exerted at the cost of c and $e_t = 1$. If he shirks, we have $e_t = 0$ without any effort cost. The stock of effort evolves according to

$$k_{t+1} = k_t + e_t\,,$$

where e_t is the agent's effort at date t. The agent starts with initial stock of effort $k_0 = 0$, which is known by the both parties.

Output possibly takes on 2 values $y_t = \{0, y\}$ with probability distributions

$$prob(y_t|e_t = 1) = p \;\&\; prob(y_t|e_t = 0) = q,\, 0 < q < p < 1, \forall t.$$

An underlying assumption of the model is that at each date, the principal has the same level of effort to implement and the same firing decision, independent of production histories. [1] Therefore, the principal would not punish the agent for delivering low output by implementing a lower level of effort in the next period.

For each date t, a relational contract specifies (i) a wage scheme contingent on output realizations offered by the principal; (ii) an agent's decision on whether to participate or not; (iii) conditional on participating, an agent's choice of effort for production.

For a given wage scheme w_t, the principal's expected profit is given by

[1] An example is the artist agency agreement, which may specify the contract duration and the number of albums to make for the artist.

$$E\left\{\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left[I_\tau(y_\tau - w_\tau)\right]\right\},$$

and the agent's expected utility is given by

$$E\{\sum_{\tau=t}^{\infty}\delta^{\tau-t}\left[I_\tau(w_\tau - ce_\tau) + (1-I_\tau)(1-\delta)s(k_\tau)\right]\},$$

where $I_t = 1$ if the agent participates and $I_t = 0$ otherwise.

I consider 2 situations of outside option growth: (i) as a convex function in the stock of effort (section 3); (ii) as a concave function in the stock of effort (section 4).

2 basic assumptions are introduced before continuing on to the next section,

Assumption 1-1. *For all integers $k \geq 0$, we have*

(1)

$$c - \delta s(k+1) + s(k) > 0.$$

This is equivalent to

$$-c + \delta s(k+1) < s(k),$$

which will imply that an agent will not work for free merely for the sake of working experiences.

Assumption 1-2. *$\forall$ integer $k \geq 0$, we have*

(2)

$$s(k+1) - s(k) > \delta[s(k+2) - s(k+1)].$$

If

(*2*) is not true, we have

$$s(k+1) - s(k) \leq \delta[s(k+2) - s(k+1)], \text{ for some } k.$$

In terms of present discount values, the agent expects a higher growth on his outside option, by continuing working in the next period. Therefore the agent keeps chasing higher outside option growth in the next period, and the principal can take advantage of the outside option growth and retain the agent by implementing the same effort in the next period.

Assumption 1-3. *The agent has limited liability and wage payment is non-negative.*

1.3 The Optimal Contract under Convex Outside Option Growth

This section studies the principal's profit-maximizing contract, when outside option growth exhibits convexity:

$$s(1) - s(0) < s(2) - s(1) < s(3) - s(2) < \cdots.$$

In section 1.3.1, I look into the participation constraint and the result shows that the principal will terminate the relationship before the stock of effort grows infinity. Therefore, I study the incentive constraint for implementing work in a finite T-period relationship in section 1.3.2. Section 1.3.3 summarizes the principal's optimal contract. The analysis for the concave outside option growth in section 1.3.4 is similar to the convex case.

1.3.1 Participation Constraint

When implementing work on an agent with the stock of effort $(k+1)$, the binding condition for the participation constraint is given by

$$-c+\delta s(k+1)+h(k)=s(k), k\geq 1,$$

where $h(k)$ is the minimum wage payment such that the participation constraint for taking effort binds.

$h(k)$ can be written as:

$$h(k)=s(k)+c-\delta s(k+1), k\geq 1. \quad (3)$$

Note that the minimum wage payment to satisfy the participation constraint for the kth unit of effort is endogenously determined by the stock of effort.

Claim. *$h(k)=s(k-1)+c-\delta s(k)$ is increasing in k.*

Proof:

For any integer $k\geq 1$, applying equation (3), we have

$$h(k+1)-h(k)=[s(k)+c-\delta s(k+1)]-[s(k-1)+c-\delta s(k)]$$
$$=[s(k)-s(k-1)]-\delta[s(k+1)-s(k)]>0.$$

Figure 1-2 illustrates the minimum wage payment $h(k)$ to satisfy the participation constraint of taking the kth unit of effort and the output py from taking effort.

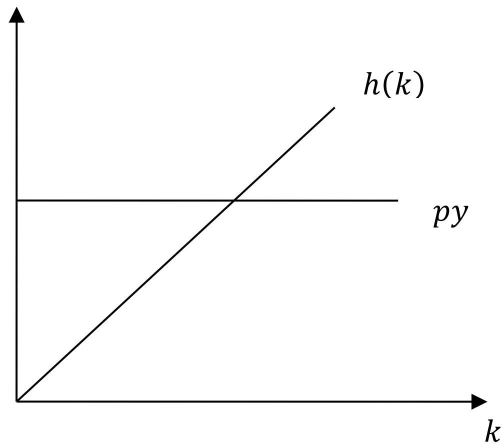


Figure 1-2: The Wage Payment When the Participation Constraint for an Agent with Stock of Effort $\boldsymbol{k}$ to Work Binds

The principal won't allow the stock of effort to grow infinitely, since the expected wage payment to implement work will exceed the expected output, for the agent whose stock of effort is sufficiently high.

Notation. *Suppose the principal implements work in every period of a finite T-period relationship. The date 0 discounted wage payment so that the participation constraint binds is given by*

$$H(T) = \sum_{k=1}^{T} \delta^{k-1} h(k).$$

Proposition 1-1. *For a relationship that may last indefinitely, if the principal finds it profit-maximizing to implement shirk at date* t $(t \geq 0)$, *he will also implement shirk at date* $(t + 1)$.

Proof:

Suppose it's not true and the principal finds it profit-maximizing to implement work at date $(t + 1)$. Since the agent shirks at date t, his stock of effort doesn't grow and $s(k_t) = s(k_{t+1})$, where k_t and k_{t+1} are the agent's stock of effort at date t and $(t + 1)$.

Since we only consider the participation constraint for this section, the expected wage payment to implement work at both date t and $(t + 1)$ are the same. If it is optimal to implement work at date $(t + 1)$, so is it at date t. This contradicts with our assumption that it's profit-maximizing to implement shirk at date t. Q.E.D.

Corollary. *The principal's optimal contract would take one of the following 2 possibilities:*

(i) *Implementing work before leaving the agent with his outside option;*

(ii) *implementing work followed by infinite shirks.*

Note that implementing shirk for all periods is considered in (ii): when the principal implements shirk from the very beginning of the relationship.

The above corollary is derived from the result of Proposition 1-1 .

1.3.2 Incentive Constraint in a Finite Relationship

Since the principal would only implement work for finitely many periods, I focus on the incentive constraint for implementing work throughout a finite T- period relationship.

For this section, I only consider the incentive constraint and assume the participation constraint is slack.

The principal wants to implement work in every period of a finite T- period relationship ($T < \infty$). At date 0, the principal announces a wage scheme $W(Y_{T-1})$ that pays the agent at date T, based on the entire production history $(y_o, y_1, \dots, y_{T-1})$. Proposition 1-2 describes the principal's optimal wage scheme of implementing work in a finite relationship. Proposition 1-3 illustrates the principal's expected wage payment as a function of the contract length.

Notation. *In a finite T-period relationship, denote Y_t as the production history up to date t: $Y_t = (y_0, \dots, y_t)$.*

At date t $(0 \leq t \leq T-1)$, the agent either stays in the relationship or picks up his outside option. A participating agent is choosing a profile of actions $[\, e_t^*(Y_{t-1}), e_{t+1}^*(Y_t), \dots, e_{T-1}^*(Y_{T-2})]$ that is contingent on the production history. [2]

Definition 1-1. *At date t, the agent's utility maximizing profile of actions $[e_t^*(Y_{t-1}), e_{t+1}^*(Y_t), \dots, e_{T-1}^*(Y_{T-2})]$ is a profile of actions that is contingent on the*

[2] Since we have assumed for this section that the agent's participation constraint is slack.

production history and maximizes his expected utility under a given wage scheme $W(Y_{T-1})$. Formally:

(4)

$$[e_t^*(Y_{t-1}), e_{t+1}^*(Y_t), \dots, e_{T-1}^*(Y_{T-2})]$$

$$\in \underset{(e_t,\dots,e_{T-1})}{argmax} \sum_{Y_{T-1}|Y_{t-1}} prob[Y_{T-1}|(e_t, \dots e_{T-1})] \left\{ \delta^{T-t} W(Y_{T-1}) + \delta^{T-t} s(k_T) - \sum_{i=t}^{T-1} \delta^{i-t} c e_i \right\},$$

where for the given profile of actions and the production history Y_{t-1}, $prob[Y_{T-1}|(e_t, \dots e_{T-1})]$ denotes the probability of realizing Y_{T-1}, $s(k_T)$ gives the agent's outside option at date T and $\sum_{i=t}^{T-1} \delta^{i-t} c e_i$ is the date t discounted future effort cost.

Definition 1-2. $W^*(Y_{T-1})$ *is the principal's cost-minimizing wage scheme to satisfy the incentive for working throughout a T-period relationship, if*

$$W^*(Y_{T-1}) \in Argmin \textstyle\sum_{Y_{T-1}} prob[Y_{T-1}|(e_0^*, \dots, e_{T-1}^*)] W(Y_{T-1}),$$

where the agent's utility maximizing action profile $(e_0^*, \dots, e_{T-1}^*)$ is to work in every period.

Notation. $x(T)$ *is the date 0 discounted wage payment, under the cost-minimizing wage scheme that satisfies the incentive constraint to work for all periods of a finite T- period relationship:*

$$x(T) = \sum_{Y_{T-1}} \delta^T prob[Y_{T-1} | (e_0^*, \dots, e_{T-1}^*)] W^*(Y_{T-1}).$$

Proposition 1-2. *Suppose the principal implements work for all periods of a finite T-period relationship and the agent's outside option grows as a convex function in the stock of effort. It is cost-minimizing for the principal's to offer a wage scheme $W^*(Y_{T-1})$ such that a deviant agent maximizes utility by shirking till the end, once being off the equilibrium path.*

Proof:

I will prove by induction.

At date $(T-1)$, it is trivially true that a deviant agent will shirk till the end of the relationship. Suppose it is true for all $r \geq (t+1)$ such that a deviant agent who shirks at date r will shirk for all the succeeding periods. It's sufficient to prove that a deviant agent who shirks at date t will shirk for all the succeeding periods.

Notations.

- $w(y_t, \dots, y_{T-1} | k_t = t)$ is the wage paid at date T based on the production history from date t to $(T-1)$, if the agent enters date t with the stock of effort t.

- $w^*(y_t, \dots, y_{T-1}|k_t = t)$ is the wage scheme that maximizes the principal's expected profit at date t, if the agent enters at date t with the stock of effort t.

An agent enters date t with the stock of effort $k_t = t$ and the principal's is solving the following cost-minimizing problem

$$\min_{w} \sum_{Y_{T-1}|Y_{t-1}} prob(y_t, \dots, y_{T-1}) w(y_t, \dots, y_{T-1}|k_t = t)$$

such that an obedient agent who works at date t will maximize his utility by working in all the succeeding periods according to the wage scheme w_t:

$$(e_t = 1, \dots, e_{T-1} = 1) \in \underset{(e_t, \dots, e_{T-1})}{argmax}\, E_t[(-c + w + s)|Y_{t-1}, k_t = t]$$

Recall that an agent is choosing a profile of actions according to the wage scheme. Lemma 1-1 (see below) points out a critical point that specifies how a deviant agent at date t will choose his effort.

Lemma 1-1. *Suppose that the cost-minimizing wage scheme* $w^*(y_{t+1}, \dots, y_{T-1}|k_{t+1} = t + 1)$ *is such that an agent is incentive compatible to work from date* $(t + 1)$ *on and a deviant agent who shirks at date* $(t + 1)$ *will shirk for all the succeeding periods. If the wage scheme* $w(y_t, \dots, y_{T-1}|k_t = t)$ *satisfies the following conditions:*

(i) $w(y_t, \dots, y_{T-1}|k_t = t) \geq w^*(y_{t+1}, \dots, y_{T-1}|k_{t+1} = t + 1), \forall(y_{t+1}, \dots, y_{T-1})$,

(ii) $w(y_t = y, \ldots, y_{T-2} = y, y_{T-1} = y | k_t = t) - w(y_t = y, \ldots, y_{T-2} = y, y_{T-1} = 0 | k_t = t) \leq \frac{c - \delta s(t+1) + \delta s(t)}{\delta(p-q)}$,

(iii) $w(y_t, \ldots, y_{T-1} | k_t = t) = w^*(y_{t+1}, \ldots, y_{T-1} | k_{t+1} = t+1)$, $\forall (\mathrm{y_t}, \ldots, \mathrm{y_{T-1}}) \neq (\mathrm{y}, \ldots, \mathrm{y}, \mathrm{y})$& $(\mathrm{y}, \ldots, \mathrm{y}, 0)$,

a deviant agent who shirks at date t will maximize his utility by shirking till the end of the relationship.

Proof:

By assumptions, we know that $w^*(y_{t+1}, \ldots, y_{T-1} | k_{t+1} = t+1)$ is such that a deviant agent who shirks at date $(t+1)$ will shirk till the end of the relationship. As long as a deviant agent yields any low output before date $(T-1)$, he will expect a wage payment that discourages any deviant agent from returning to work. Therefore, it is sufficient to prove that a deviant agent who shirks at date t yet yields high output up to date $(T-2)$ is not willing to work at date $(T-1)$:

$$-c + \delta p w(y_t = y, \ldots, y_{T-2} = y, y_{T-1} = y | k_t = t) + \delta(1-p) w(y_t = y, \ldots, y_{T-2} = y, y_{T-1} = 0 | k_t = t) + \delta s(t+1)$$

$$\leq \delta q w(y_t = y, \ldots, y_{T-2} = y, y_{T-1} = y | k_t = t) + \delta(1-q) w(y_t = y, \ldots, y_{T-2} = y, y_{T-1} = 0 | k_t = t) + \delta s(t),$$

which is equivalent to

$$w(y_t = y, \dots, y_{T-2} = y, y_{T-1} = y | k_t = t) - w(y_t = y, \dots, y_{T-2} = y, y_{T-1} = 0 | k_t = t)$$
$$\leq \frac{c - \delta s(t+1) + \delta s(t)}{\delta(p-q)}.$$

This completes the proof of Lemma 1-1.   Q.E.D.

Lemma 1-1 is important in that it illustrates how a deviant agent at date t will maximize utility according in the succeeding periods. Figure 1-3 below illustrates the utility maximizing actions of a deviant agent.

$$\begin{array}{l} w_t(y_t = y, \dots, y_{T-2} = y, y_{T-1} = \mathrm{y}) \\ -w_t(y_t = y, \dots, y_{T-2} = y, y_{T-1} = 0) \end{array} \quad \underbrace{\qquad\qquad\qquad\qquad}_{\text{A deviant agent will shirk till the end}} \overset{\frac{c - \delta s(t+1) + \delta s(t)}{\delta(p-q)}}{|} \underbrace{\qquad\qquad\qquad\qquad}_{\text{A deviant agent will return to work for at least 1 period period}}$$

Figure 1-3: The critical point for a deviant agent to return to work

When the wage scheme $w(y_t, \dots, y_{T-1} | k_t = t)$ is such that

$$w(y_t, \dots, y_{T-1} | k_t = t) \ = w^*(y_{t+1}, \dots, y_{T-1} | k_{t+1} = t+1) \ \forall (y_t, y_{t+1}, \dots, y_{T-1}),$$

$w(y_t, \dots, y_{T-1} | k_t = t)$ will not provide enough incentive for the agent to work at date t. By Lemma 1-1, we know that a deviant agent at date t will maximize utility by shirking till the end of the relationship. Therefore, an agent strictly prefers to shirk from date t on.

(5)

$$E_t[(-c+w+s)|(e_t=1,\dots,e_{T-1}=1)] - E_t[(-c+w+s)|(e_t=0,\dots,e_{T-1}=0)] < 0.$$

Equation (5) gives the difference in expected utility if an agent works rather than shirks at date t, under the wage scheme such that $w(y_t,\dots,y_{T-1}|k_t=t) = w^*(y_{t+1},\dots,y_{T-1}|k_{t+1}=t+1)$.

Since the agent is not incentive compatible to work at date t, the principal needs to update the wage scheme so that the incentive constraint binds with the left hand side of equation (5) = 0.

Definition 1-4. *For a given production history* $(y_t,\dots,y_{T-1})$, *the likelihood ratio* $L(y_t,\dots,y_{T-1}|w)$ *is the ratio in probabilities of realizing the production history* $(y_t,\dots,y_{T-1})$, *when an agent chooses to work versus to shirk at date* t, *given that he is maximizing utility in all the succeeding periods according to the wage scheme* $w(y_t,\dots,y_{T-1})$. *Formally, we can write it as*

(6) $$L(y_t,\dots,y_{T-1}|w) = \frac{prob(y_t,\dots,y_{T-1}|w,e_t=1)}{prob(y_t,\dots,y_{T-1}|w,e_t=0)}$$

$with\ prob(y_t,\dots,y_{T-1}|w,e_t=0) \neq 0.$

Note that the wage scheme affects the likelihood ratio in the sense that an agent maximizes utility according to it. Therefore, a change on the wage scheme possibly alters the agent's profit-maximizing profile of actions from date t on, as well as the likelihood ratio.

The likelihood ratio $L(\cup\,|w)$ for a set $\cup$ of production histories is defined on the joint probability of realizing each of the production history:

$$L(\cup\,|w) = \frac{\sum_{(y_t,\dots,y_{T-1})\in\cup} prob(y_t,\dots,y_{T-1}|w,e_t=1)}{\sum_{(y_t,\dots,y_{T-1})\in\cup} prob(y_t,\dots,y_{T-1}|w,e_t=0)}$$

Lemma 1-2. *Among all wage schemes w such that an agent is incentive compatible to work from date* t *on, the cost-minimizing wage scheme* $w^*(y_t,\dots,y_{T-1}|k_t=t)$ *is such that*

(i) *There exists a set* $\cup^*$ *of production histories with* $L(\cup^*\,|w^*) \geq L(y_t,\dots,y_{T-1}|w^*)$, $\forall(y_t,\dots,y_{T-1})$.

(ii) $\forall\ (y_t,\dots,y_{T-1}) \in \cup^*:\ w^*(\,y_t,\dots,y_{T-1}|k_t=t) > w^*(\,y_{t+1},\dots,y_{T-1}|k_{t+1}=t+1)$,

(iii) $\forall\ \ (y_t,\dots,y_{T-1}) \notin \cup^*, w^*(\,y_t,\dots,y_{T-1}|k_t=t) = w^*(\,y_{t+1},\dots,y_{T-1}|k_{t+1}=t+1)$.

Proof :

Let's update the wage scheme to

$$w(y_t,\dots,y_{T-1}|k_t=t) = w^*(y_{t+1},\dots,y_{T-1}|k_{t+1}=t+1)\ +\Delta\,(y_t,\dots,y_{T-1})],$$

$$\forall(y_t,\dots,y_{T-1}).$$

For $\triangle\,(y_t, \dots, y_{T-1})$ sufficiently small: a deviant agent at date t will still maximize his utility by shirking till the end of the relationship. Therefore the expected effort cost and date T outside option don't change under the updated wage scheme, for both the obedient and deviant agent. A small wage increment $\triangle\,(y_t, \dots, y_{T-1})$ alters the utility difference in equation (5) by

$$\delta^{T-t} \sum_{(y_t,\dots,y_{T-1})} [prob(y_t, \dots, y_{T-1} | e_t = 1, \dots, e_{T-1} = 1) - prob(y_t, \dots, y_{T-1} | e_t = 0, \dots, e_{T-1} = 0)] \triangle\,(y_t, \dots, y_{T-1}).$$

Let's consider the following problem for the principal:

$$min_{\triangle(y_t,\dots,y_{T-1})\geq 0} \sum_{(y_t,\dots,y_{T-1})} [\triangle\,(y_t, \dots, y_{T-1}) prob(y_t, \dots, y_{T-1} | e_t = 1, \dots, e_{T-1} = 1)],$$

Subject to:

$$\delta^{T-t} \sum_{(y_t,\dots,y_{T-1})} [prob(y_t, \dots, y_{T-1} | e_t = 1, \dots, e_{T-1} = 1) - prob(y_t, \dots, y_{T-1} | e_t = 0)] \triangle\,(y_t, \dots, y_{T-1}) = \epsilon,$$

where ϵ>0 is small enough such that a deviate agent at date t will shirk till the end of the relationship.

At date t, the principal is looking for the cost-minimizing way to increase the difference in utility between the obedient and deviant agents by ϵ.

The Dual problem is given as follows.

$$\max_{\lambda} \lambda\epsilon$$

such that

$$\lambda\delta^{T-t}[prob(y_t,\dots,y_{T-1}|e_t=1,\dots,e_{T-1}=1)-prob(y_t,\dots,y_{T-1}|e_t=0,\dots,e_{T-1}=0)]\geq[prob(y_t,\dots,y_{T-1}|e_t=1,\dots,e_{T-1}=1),\forall(y_t,\dots,y_{T-1}).$$

The dual problem is solved as follows.

For $[prob(y_t,\dots,y_{T-1}|e_t=1,\dots,e_{T-1}=1)-prob(y_t,\dots,y_{T-1}|e_t=0)]\leq 0$: any $\lambda\geq 0$ satisfies the constraint.

For $[prob(y_t,\dots,y_{T-1}|e_t=1,\dots,e_{T-1}=1)-prob(y_t,\dots,y_{T-1}|e_t=0)]>0$:

$$\lambda\leq\left(\frac{1}{\delta^{T-t}}\right)\frac{[prob(y_t,\dots,y_{T-1}|e_t=1,\dots,e_{T-1}=1)}{[prob(y_t,\dots,y_{T-1}|e_t=1,\dots,e_{T-1}=1)-prob(y_t,\dots,y_{T-1}|e_t=0)}\ \forall(y_t,\dots,y_{T-1}).$$

Therefore, the solution is given by

$$\lambda=min_{(y_t,\dots,y_{T-1})}\left(\frac{1}{\delta^{T-t}}\right)\frac{prob(y_t,\dots,y_{T-1}|e_t=1,\dots,e_{T-1}=1)}{[prob(y_t,\dots,y_{T-1}|e_t=1,\dots,e_{T-1}=1)-prob(y_t,\dots,y_{T-1}|e_t=0,\dots,e_{T-1}=0)]}.$$

Equivalently, the principal is looking for the maximum of $\frac{prob(y_t,\dots,y_{T-1}|e_t=1,\dots,e_{T-1}=1)}{prob(y_t,\dots,y_{T-1}|e_t=0,\dots,e_{T-1}=0)}$.

The maximum $\frac{p^{T-t}}{q^{T-t}}$ is given by the production history $(y_t=y,\dots,y_{T-1}=y)$, where a deviant agent will shirk till the end of the relationship.

$$L(y_t=y,\dots,y_{T-1}=y|w+\triangle)=\frac{p^{T-t}}{q^{T-t}}\,.$$

According to Proposition 1-2, we know that a deviant agent at date t will shirk till the end of the relationship, as long as $\vartriangle$ is such that

$$\vartriangle(y, \dots, y, y) + w^*(y, \dots, y, y | k_{t+1} = t+1) - w^*(y, \dots, y, 0 | k_{t+1} = t+1) \leq \frac{c - \delta[s(t+1) - s(t)]}{\delta(p-q)}.$$

The critical point where a deviant agent at date t is just indifferent between shirking till the last period and working for only one period is given by

$$\vartriangle^*(y, \dots, y, y) = \frac{c - \delta[s(t+1) - s(t)]}{\delta(p-q)} - w^*(y, \dots, y, y | k_{t+1} = t+1) + w^*(y, \dots, y, 0 | k_{t+1} = t+1).$$

Case 1: The agent's incentive constraint to work from date t on becomes binding before $\vartriangle$ reaches the critical the critical level $\vartriangle^*(y, \dots, y)$. The cost-minimizing wage scheme (to provide work incentive) is such that a deviant agent at date t will shirk for all the succeeding periods and is characterized by

(i) $w^*(y_t, \dots, y_{T-1} | k_t = t) \geq w^*(y_{t+1}, \dots, y_{T-1} | k_{t+1} = t+1), \forall (y_{t+1}, \dots, y_{T-1})$;

(ii) $w^*(y, \dots, y, y | k_t = t) - w^*(y, \dots, y, 0 | k_{t+1} = t+1) \leq \frac{c - \delta[s(t+1) - s(t)]}{\delta(p-q)}$;

(iii) $w^*(y_t, \dots, y_{T-1} | k_t = t) = w^*_{t+1}(y_{t+1}, \dots, y_{T-1} | k_{t+1} = t+1)$, $\forall (y_t, \dots, y_{T-1}) \neq (y, \dots, y, y)$.

Case 2: The agent's incentive constraint to work from date t on doesn't bind at the wage scheme given in case 1 above. Following the same logic, we increase the wage payment on realizing the production history that gives the maximum likelihood ratio.

- If raising $w(y_t = y, \dots, y_{T-1} = y | k_t = t)$ so that a deviant agent will return to work for only one period: the likelihood ratio $\leq \left(\frac{p}{q}\right)^{T-t-1}$;
- If raising $w(y_t = y, \dots, y_{T-1} = y | k_t = t)$ and $w_t\ (y_t = y, \dots, y_{T-1} = y | k_t = 0)$ simultaneously so that a deviant agent won't return to work:

the likelihood ratio $= \left(\frac{p}{q}\right)^{T-t-1}$.

Note that although the above 2 possibilities are equally optimal (cost-minimizing) for the principal, under the first possibility, the number of periods that an agent returns to work after shirking at date t is dependent on the parameters. Therefore, a general solution would be:

(i) $w^*(\,y, \dots, y, y | k_t = t) - w^*(\,y, \dots, y, 0 | k_t = t) \leq \frac{c - \delta[s(t+1) - s(t)]}{\delta(p-q)}$,

(ii) *for* $(y_t, \dots, y_{T-1}) = (y, \dots, y, y)$ *or* $(y, \dots, y, 0)$ *we have* $w^*(y_t, \dots, y_{T-1} | k_t = t) >\ w^*(\,y_{t+1}, \dots, y_{T-1} | k_{t+1} = t+1)$,

(iii) $w_t^*(\,y_t, \dots, y_{T-1}) = w_{t+1}^*(\,y_{t+1}, \dots, y_{T-1})$

$\forall (y_t, \dots, y_{T-1}) \neq (y, \dots, y, y)$ *or* $(y, \dots, y, 0)$.

Intuitively, under the convex outside option growth, a deviant agent with a lower stock of effort has his outside option growing not as fast (than the obedient agent). Therefore, he lacks work incentives. The principal's profit-maximizing wage scheme that makes it incentive compatible for an obedient agent to work won't provide sufficient work incentives for a deviant agent.

Definition 1-3. *Suppose the principal implements work in every period of a finite* $\boldsymbol{T}$*-period relationship and* $x(T)$ *is the date 0 discounted wage payment so that the incentive constraint binds. The average wage payment per period is given by*

$$\bar{x}(T) = \left(\frac{1}{\sum_{k=1}^{T} \delta^{k-1}}\right) x(T).$$

$\bar{x}(T)$ is the weighted average of $x(T)$. In terms of the date 0 discounted value, the wage payment $x(T)$ is equivalent to a payment stream of $\bar{x}(T)$ per period.

Similarly, the average output per period from working for all periods of a finite T-period relationship is given by

$$\left(\frac{1}{\sum_{k=1}^{T} \delta^{k-1}}\right)(py + \delta py + \cdots + \delta^{T-1} py) = py$$

Proposition 1-3. *Suppose the principal implements work for all periods of a finite relationship and the outside option grows as a convex function in the stock of effort. The average wage payment per period so that the incentive constraint binds is decreasing in the contract length.*

Proof :

Let $w^*(\, y_{t+1}, \ldots, y_{T-1} | k_{t+1} = t+1)$ be the cost minimizing wage scheme to implement work from date $(t+1)$ on, where an agent enters date $(t+1)$ with the stock of effort $k_{t+1} = t+1$. Denote χ_t as the corresponding expected wage obtained by an obedient agent and χ_t' is the expected wage obtained by a deviant agent at date t. According to Proposition 1-2, we know that an agent who shirks at date t will shirk for all the succeeding periods.

Since $w^*(\, y_{t+1}, \ldots, y_{T-1} | k_{t+1} = t+1)$ is the cost-minimizing wage scheme of implementing work from date $(t+1)$ on, the corresponding incentive constraint is binding and an agent is indifferent between working and shirking at date $(t+1)$.

$$
\begin{aligned}
u_{t+1} &= -c\frac{1-\delta^{T-(t+1)}}{1-\delta} + \delta^{T-(t+1)} s(T) + \delta^{T-(t+1)} \chi_{t+1} \\
&= \delta^{T-(t+1)} s(t+1) + \delta^{T-(t+1)} \chi'_{t+1}
\end{aligned}
$$

At date t, if the wage scheme is such that

$$
w(y_t, y_{t+1}, \ldots, y_{T-1} | k_t = t) \;\; = w^*_{t+1}(\, y_{t+1}, \ldots, y_{T-1} | k_{t+1} = t+1), \forall (y_{t+1}, \ldots, y_{T-1}),
$$

an obedient agent's expected utility is given by

$$
(7) \qquad -c + \delta u_{t+1},
$$

and a deviant agent's expected utility is given by

(8) $$\delta\chi'_{t+1} + \delta^{T-t}s(t)$$

$$= \delta[u_{t+1} - \delta^{T-(t+1)}s(t+1)] + \delta^{T-t}s(t).$$

The difference in utility between an obedient and a deviant agent at date t is given by (7) −(8):

(9)

$$-c + \delta u_{t+1} - \{\delta[u_{t+1} - \delta^{T-(t+1)}s(t+1)] + \delta^{T-t}s(t)\}$$

$$= -c + \delta^{T-t}[s(t+1) - s(t)].$$

When the contract extends to $(T+1)$ periods, the wage payment for implementing work from date 0 to date $(T-1)$ will decrease. Since:

- The difference in utility between an obedient and a deviant agent decreases:

$$-c + \delta^{T-t}[s(t+1) - s(t)] < -c + \delta^{(T+1)-(t+1)}[s(t+2) - s(t+1)], \forall 0 \leq t \leq T-1.$$

- The likelihood ratio for realizing $(y_t = y, \dots, y_{T-1} = y) \& (y_t = y, \dots, y_{T-2} = y, y_{T-1} = 0)$ is non-decreasing: it is tougher for a deviant agent to imitate the obedient ones.

In addition, the wage payment to implement work in the last period is lower, as the contract extends:

(10) $$c - \delta[s(T+1) - s(T)] < c - \delta[s(T) - s(T-1)].$$

Therefore, the average per period wage payment is decreasing in contract length.

Q.E.D.

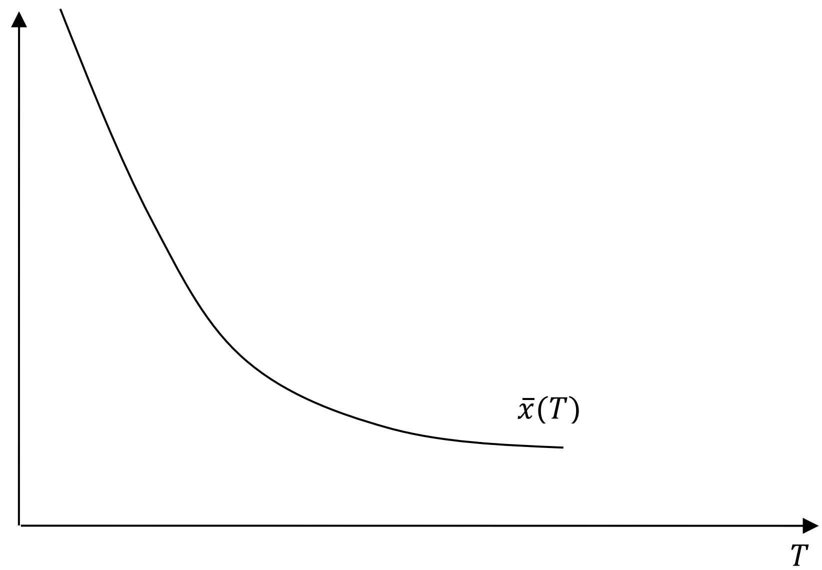


Figure 1-4 The average per period wage payment to satisfy the incentive constraint is decreasing in contract length

The intuition for Proposition 1-3 is that under the convex outside option growth, an agent gets more work incentives from the outside option growth. In addition, according to Proposition 1-1, a deviant agent won't work, once being off the equilibrium path. Therefore, it's tougher for a deviant agent to imitate the obedient ones over a longer time period.

Recall that the date 0 discounted wage payment so that the participation constraint binds for working throughout a finite T- period relationship is given by

$$H(T) = \sum_{k=1}^{T} \delta^{k-1} h(k).$$

The average wage payment per period is correspondingly given by

$$\bar{H}(T) = \frac{1}{\left(\sum_{k=1}^{T} \delta^{k-1}\right)} H(T).$$

Claim. $\bar{H}(T)$ *is increasing in the contract length.*

Proof: $\bar{H}(T) = \frac{1}{\left(\sum_{\tau=1}^{T-1} \delta^{\tau-1} + \delta^{T-1}\right)} \{H(T-1) + \delta^{T-1} h(T)\}$

$$= \frac{1}{\left(\sum_{\tau=1}^{T-1} \delta^{\tau-1} + \delta^{T-1}\right)} \left\{\bar{H}(T-1)\left(\sum_{\tau=1}^{T-1} \delta^{\tau-1}\right) + \delta^{T-1} h(T)\right\}$$

$$= \frac{\bar{H}(T-1)\left(\sum_{\tau=1}^{T-1} \delta^{\tau-1}\right)}{\left(\sum_{\tau=1}^{T-1} \delta^{\tau-1} + \delta^{T-1}\right)} + \frac{\delta^{T-1} h(T)}{\left(\sum_{\tau=1}^{T-1} \delta^{\tau-1} + \delta^{T-1}\right)}$$

$$= \bar{H}(T-1) + \frac{\delta^{T-1}}{\left(\sum_{\tau=1}^{T-1} \delta^{\tau-1} + \delta^{T-1}\right)} [h(T) - \bar{H}(T-1)]$$

$$> \bar{H}(T-1)$$

The inequality is obtained from the fact that $h(.)$ is increasing in the stock of effort.

Q.E.D.

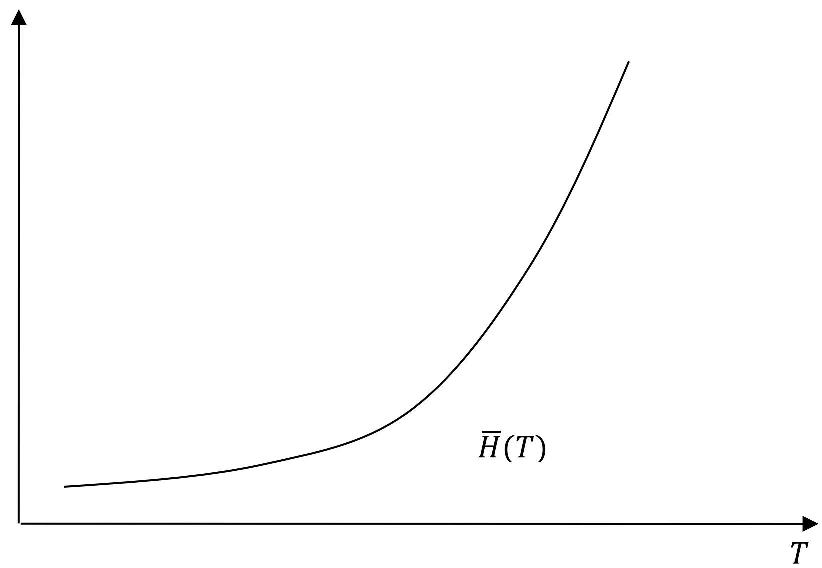


Figure 1-5The average per period wage payment to satisfy the participation constraint is increasing in contract length

Proposition 1-4. *Suppose the principal implements work for all periods of a finite relationship. There exists a γ such that:*

(i) For $T < \gamma, \bar{x}(\gamma) > \bar{H}(\gamma)$ and average wage payment per period is decreasing in the contract length. The agent's incentive constraint is binding and participation constraint slack.

(ii) For $T \geq \gamma, \bar{x}(\gamma) \leq \bar{H}(\gamma)$ and average wage payment per period is increasing in the contract length. The agent's participation constraint is binding and incentive constraint slack.

Corollary. *It's not profit-maximizing to implement work for less than γ periods, before terminating the relationship.*

Proof:

If the relationship lasts for less than γ periods, the wage payment is determined by the incentive constraint and $\bar{x}(T)$ is decreasing for $T < \gamma$. Therefore, the principal can increase his effort by implementing work till the stock of effort reaches γ. Q.E.D.

1.3.3 Optimal Contract

We know from Proposition 1-4 that the principal won't allow the stock of effort to grow infinity. The optimal contract takes on 2 possibilities and I will analyze each situation individually.

(i) Implementing work till the stock of effort reaches k_1 followed by terminating the relationship.

The principal will implement work till the stock of effort reaches the level k_1 where the additional wage paid to satisfy the participation constraint of taking one more unit of effort equals its expected output:

$$h(k_1) = py \tag{11}$$

The wage scheme is such that at date 0, the principal will offer a wage scheme that pays at date k_1 according to the production history up to date k_1. Any wage scheme that preserves the work incentives yet satisfies the agent's participation constraint would be a solution. A general solution would be

$$W\left(Y_{k_1-1}\right) = W^*\left(Y_{k_1-1}\right) + [H(k_1) - x(k_1)],$$

where $W^*\left(Y_{k_1-1}\right)$ is the cost-minimizing wage scheme so that the agent is incentive compatible to work till the stock of effort reaches k_1, $H(k_1)/x(k_1)$ are the wage payment so that the agent's participation/incentive constraints bind for working till the stock of effort reaches k_1.

By paying $W^*\left(Y_{k_1-1}\right)$, the agent is incentive compatible to work. In order to get his participation constraint binds, we have to lump-sum add $[H(k_1) - x(k_1)]$ towards the wage paid on all output realizations.

(ii) Implementing work till the stock of effort reaches k_2 followed by infinite shirks

The expected profit from (ii) is given by

$$\frac{1}{(\sum_{\tau=1}^{k_2} \delta^{\tau-1})}[py - \overline{H}(k_2)] + \delta^{k_2}[\frac{qy}{1-\delta} - s(k_2)],$$

where the principal implements work until the stock of effort reaches k_2 , followed by infinite shirks.

The wage scheme is such that at date 0, the principal will offer a wage scheme that pays at date k_2 according to the production history. The wage scheme is as described in (i). After that, the principal pays the shirking agent for his outside option depreciation, with an expected wage of $(1-\delta)s(k_2)$ for every period.

Claim. $k_1 \geq k_2$.

Proof:

Note that in the second situation, the profit from implementing infinite shirks after the stock of effort reaches k_2 must be non-negative:

$$\frac{qy}{1-\delta} - s(k_2) \geq 0.$$

Suppose $k_1 < k_2$: it is not optimal to let the agent leave till the stock of effort reaches k_1, since the principal can make positive profits by implementing infinite shirks, which is better than terminating the relationship.

$$\frac{qy}{1-\delta} - s(k_1) > \frac{qy}{1-\delta} - s(k_2) \geq 0$$

This completes the proof that $k_1 \geq k_2$. Q.E.D

Yet we do not know which situation delivers a higher profit. Comparative statics is given to compare the profits under the 2 situations.

Proposition 1-5. *The following 3 types of shocks make it more profitable for the principal to implement work before terminating the relationship:*

- *Raises in productivity from work, such as an increase in y, p or a decrease in q;*
- *A higher growth rate on outside options, such as an increase on $s(k), \forall k \geq 1$;*
- *People become less patient, such as a lower δ.*

Proof:

Recall that the optimal contract takes 1 of the following possibilities:

(i) Implementing work before leaving the agent with his outside option;

(ii) Implementing work followed by infinite shirks.

Let's vary the parameters $[p, q, y, \delta, s(k)]$ and see the corresponding changes in profits and work length under the 2 situations. Note that the difference in profit is given by:

(12)

$$\pi_1 - \pi_2 = \sum_{i=k_2+1}^{k_1} \delta^{i-1}\,[py - h(i)] - \delta^{k_2}\left[\frac{qy}{1-\delta} - s(k_2)\right]$$

By equation (11), a raise in y or p implies a higher k_1, which says that it is profitable to allow for a higher stock of effort.

If k_2 increases as much as k_1, the profit increase in (ii) won't be as much as in (i). Under (ii), the profit is partly offset by the increase in wage payment to maintain a shirking agent. Therefore, k_2 doesn't increase as much as k_1 and $(k_1 - k_2)$ increases.

According to equation (12) we know that $(\pi_1 - \pi_2)$ also increases.

For large enough shocks, the principal would find it profitable to implement more periods of work before ending the relationship.

Figure 1-6 illustrates the shock of an increase in y or p.

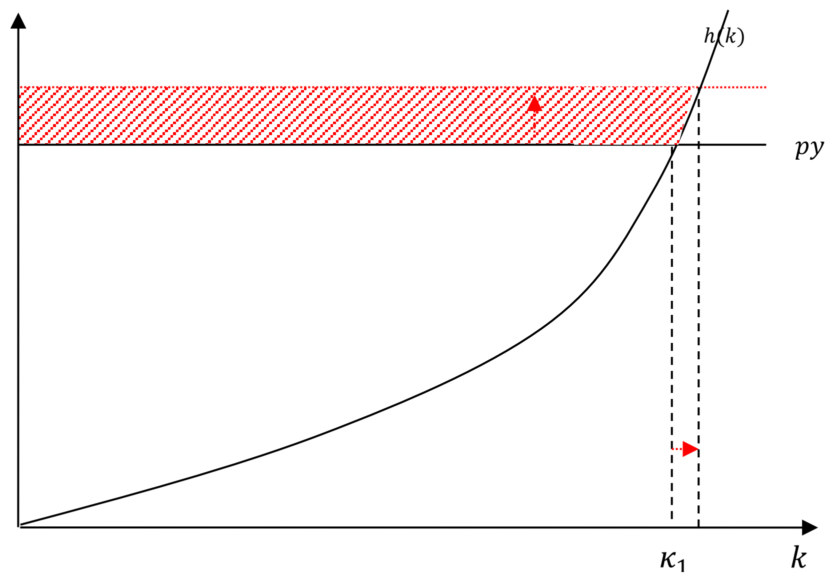


Figure 1-6: An Increase in p or y

An increase in q would raise a shirking agent's productivity. This doesn't affect the profit and work length in (i). For q sufficiently large, implementing shirk becomes profitable and the work length in (ii) will decrease.

When $s(k)$ is higher for all integer $k \geq 1$, it becomes cheaper to implement k_1 units of effort in (i) as $H(k_1)$ decreases. Therefore, it's profitable to implement more periods of work. However, it is not possible for k_2 to increase as much as k_1. Since the profit is partly offset by the wage paid to maintain a shirking agent with a higher outside option. Therefore, the optimal contract is such that the principal will implement more periods of work followed by leaving the agent with his outside option.

For δ sufficiently large, people are patient enough towards future profits and keeping the agent shirking from the very beginning of the relationship dominates:

$$\frac{qy}{1-\delta} \to \infty, as\ \delta \to 1.$$

1.4 Optimal contract under concave outside option growth

In this section, I study the optimal contract under the concave outside option growth. A concave $s(k)$ can be written as:

$$s(1) - s(0) > s(2) - s(1) > s(3) - s(2) > \cdots$$

The optimal contract is derived through similar methods as for the convex case.

Note that Proposition 1-3 no longer holds for the concave case. Under the concave outside option growth, an agent with a lower stock of effort has stronger work incentives. Any contract that provides work incentive for the high type agent would also satisfy the incentive constraint of the low type. Therefore, the wage scheme can't prevent a deviant agent from returning to work, once being off the equilibrium path.

Corollary. *Suppose the principal implements work for all periods of a finite relationship and the outside option grows as a concave function in the stock of effort. The average wage payment per period so that the incentive constraint binds is increasing in the stock of effort.*

Proof: Refer to the proof of Proposition 1-3.

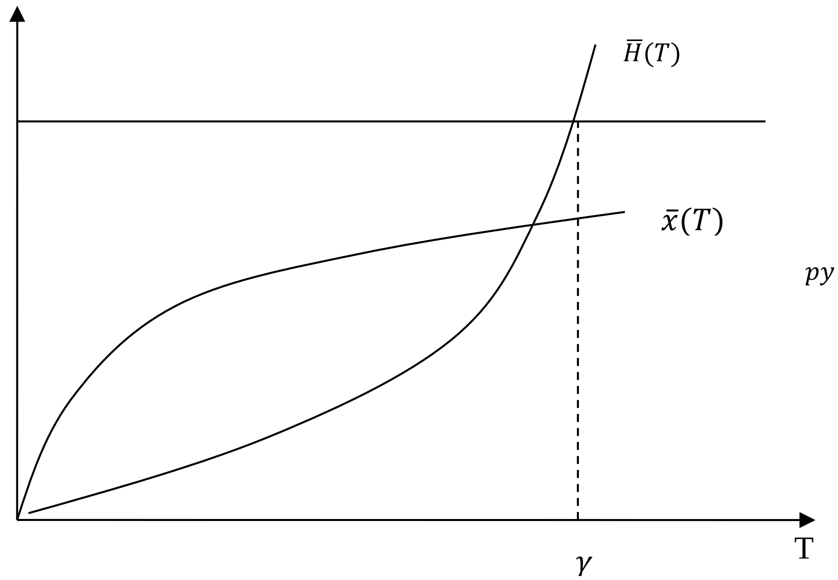


FIGURE 1-7: THE PER PERIOD AVERAGE WAGE PAYMENT TO IMPLEMENT WORK FOR ALL PERIODS OF A FINITE T-PERIOD RELATIONSHIP, UNDER CONCAVE OUTSIDE OPTION GROWTH

Figure 1-7 above depicts the average wage payment and output per period, where $\bar{H}(T)$& $\bar{x}(T)$ are as previously defined. An agent's participation constraint becomes binding, when his stock of effort reaches γ.

Results for comparative statics are similar as in the convex case.

It is noteworthy that in the concave case, it is possible that the principal maximizes profit by offering a wage scheme such that an agent is paid according to the incentive constraint, when productivity from work is not sufficiently high, i.e. py not large enough. See Figure 1-8.

Therefore, unlike the convex case, the agent who is not as productive may never get paid according to his participation constraint independent of the contract length, when his outside option growth exhibits concavity.

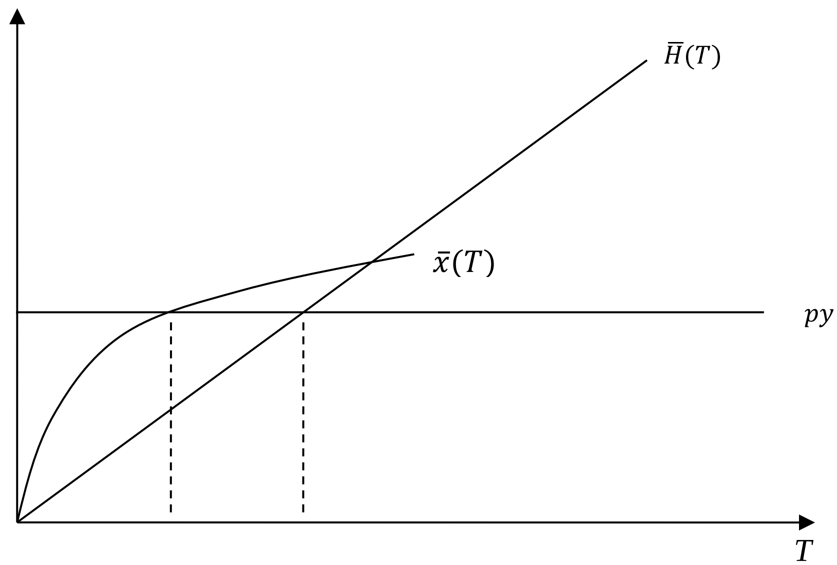


Figure 1-8:Optimal contact for the Concave case

1.5 Conclusion

I study the wage dynamics under the assumption of increasing outside options. Results show that when outside option growth exhibits convexity, a worker is paid for work incentives under short-term contracts, whereas he is paid for staying on the position rather than work performances, when the relationship lasts for sufficiently long. For the concave case, outside options eventually grow very slowly and an agent lacks incentives, unless being well paid so that his incentive constraint binds. Comparative statics shows that higher productivity from work, faster growth of outside options and a smaller time discount factor all lead to implementing more periods of work followed by terminating the relationship. Therefore, the agent builds up a higher stock of effort throughout the relationship, if being highly productive. On the other hand, the principal is unlikely to retain an agent with outside options that grow too quickly. Results for the convex case are consistent with some empirical evidence found in academia, where assistant professors' salaries, the probability of becoming tenured, and the salaries for senior faculties are correlated. See Ehrenberg, Pieper, & Willis (1998).

Potential studies in the future may consider the continuous choices of effort. In this paper, an agent can only choose 2 levels of effort: either to work or shirk. It would be interesting to see how the agent's effort changes as he builds up his stock of effort, under a set of continuously distributed effort choices, rather than only two.

Chapter 2

Outside Option Growth and the Fast-Track Effect

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2.1 Introduction

Moral hazard may arise when the principal offers a wage payment that is independent of production histories. A solution for the moral hazard problem would be a wage payment contingent on output realizations that provides the agent with work incentives. Levin (2003) studies a principal-agent problem on an infinite horizon with fixed outside options, where effort is the agent's private information and the principal faces a stationary profit-maximizing problem in each period.

This paper develops a principal-agent problem in a finite relationship under the assumptions that the agent has a continuous set of effort choices and the principal's wage payment is contingent on production histories. The results are related to the fast- track effect, the phenomenon that time to promotion is serially correlated. It is found that the date of the first success has impacts on outside option growth throughout the entire career. In addition, career advancement also differs in the rate of outside option growth.

When his outside option is endogenously determined as an increasing function in the stock of effort, an agent receives compensation from 2 channels: the wage paid by the principal that is contingent on output realizations, as well as the outside option growth that saves the principal's wage payment in order to generate incentives. However, outside option growth endows an agent with bargaining power over the current employer and therefore it becomes more expensive to maintain the agent.

I construct a principal-agent problem in a finite relationship with the following features. First, the principal announces a wage scheme before production takes place and wage payments are contingent on production histories. Second, the duration of the contract is fixed and independent of production histories. Third, the agent has a continuous set of effort choices and the principal can't observe effort but only output. Therefore, the agent chooses effort according to the production history and wage scheme. Fourth, the agent's outside option grows as he builds up his stock of effort. This chapter answers questions such as how an agent's effort changes throughout his career path, what are the impacts that output realizations would have on an agent's career advancement, whether it is profit-maximizing for the principal to implement effort contingent on production histories.

The paper finds that the date of realizing the first high output plays an important role in one's career development. An agent's stock of effort will grow at a faster rate, if his first high output is delivered at an earlier date. Intuitively, it generates incentives to reward the high output with better prospects into the future. Similar results are found in Meyer (1992) that the principal tend to favor the early winner in order to generate incentives.

Fast-track effect has been seen in the labor market. According to my model, the fast-track arises as a reward for delivering high output and it saves the principal's wage payment to generate incentives. Because the principal only sees the stochastic realizations of output, the selection of fast-track agents is not driven by individual differences, for instance, the stock of effort. Ariga, Ohkusa and Brunello (1999) study the promotion policy in large Japanese manufacturing firms that "individuals promoted faster earlier are more likely to be promoted faster later on". It is found in their paper that the fast-track effect exists, even if controlling for other factors such as the innate ability.

Empirical evidence is also found in academia. Smeets (2004) studies a sample of top economists to see if the time spent as an assistant professor has impacts on the time spent as an associate professor. Serial correlations are found that "the individuals promoted quickly at the beginning were also the ones who experienced the fastest and most successful careers".

In the previous chapter, I studied a relational contract where an agent has only 2 levels of effort to choose from: work ($e = 1$) or shirk ($e = 0$), and the principal's firing decision is not contingent on output realizations. According to the results in chapter 1, the principal implements work till the stock of effort reaches a certain level and the agent's effort changes in jumps, i.e. from full effort ($e = 1$) to zero ($e = 0$), without taking on any intermediate values.

In this chapter, I relax the assumption that the principal implements effort independent of production histories. The profit-maximizing principal punishes an agent for delivering low output by implementing lower effort in the next period. This saves the wage payment since an agent has stronger incentives, for the fear of getting less work opportunities in the future. However, this punishment disappears after the delivery of the first high output. Once an agent has delivered some high output, the effort implemented by the principal no longer depends on the random realizations of output. It is noteworthy that although an agent's incentives are reduced after realizing the first high output, this effect is offset by the increase of incentives in earlier periods.

I study 2 situations of outside option growth: convexity and concavity. The differences in growth rates might come from the public's perceptions of an agent's ability. When an agent's ability doesn't manifest itself and is gradually known through work, a senior worker is more publicly recognized. Rosen (1982) describes this as "the magnification effect" as the returns to ability are convex and senior workers' impact is magnified. On the other hand, if the public perceives an agent's ability as soon as he starts work, the agent's outside option may grow fast at the early stages of his career.

This paper gives a comparison between the type of job with the convex outside option growth and the one with the concave growth. The key difference is that the principal implements effort more conservatively, when outside option growth exhibits concavity. The intuition is that building up the stock of effort would accelerate outside option growth and save wage payment in the convex case. Whereas for the concave case, implementing effort has a negative impact towards the rate of outside option growth, which also reduces the agent's inherent work incentives.

I don't consider the productivity growth in the stock of effort. One possible explanation is that productivity is determined by working conditions too as well, such as the capital-labor ratio. Evidence has been found that workers tend to be more productive in larger firms, see TL Idson (1999).

Related Literature Levin (2003) studies a principal-agent problem on an infinite horizon with fixed outside options, where the principal can't observe an agent's effort and faces a stationary profit-maximizing problem in each period. Hopenhayn and Werning (2008) considers the equilibrium default model in which the outside option is the agent's private information and the principal observes human capital. MacLeod (2003) and Fuchs (2007) study the relational contract where output is evaluated privately by the principal. My model changes the assumption of fixed outside options and looks at the optimal contract with growing outside options. Output is observable by both parties. Rewards are given based on the production history and effort is the agent's private information.

The results of this chapter are related to the fast-track effect in labor market, the phenomenon that times to promotions are serially correlated. Empirical evidence is found in Smeets (2004) with a sample of top economists that "the individuals promoted quickly at the beginning were also the ones who experienced the fastest and most successful careers". Meyer (1992) provides theoretical support for the observation that principals favors the early winner in order to generate incentives. My paper finds that the time of realizing the first high output plays an important role in one's career development. Specifically, those who deliver their first high output earlier will build up the stock of effort faster.

Ariga, Ohkusa and Brunello (1999) study the promotion policy in large Japanese manufacturing firms and it is found that workers being promoted faster previously are more likely to be promoted faster in the future. In addition, this fast-track effect exists, even if controlling for other factors such as the innate ability. The casual effect of promotion in compensation is presented by Belzil and Bognanno (2006), through examining the panel data of American executives. According to my paper, the selection of fast-track workers is due to the stochastic output realizations, rather than individual differences in the stock of effort.

The paper is organized as follows.

The model is introduced in section 2.2. Section 2.3 studies principal's profit-maximizing contract. Section 2.4 concludes.

2.2 Model

There are one principal and one agent, who are risk neutral. The 2 parties interact in each period, dated as $t = 0,1,2 \ldots, T$. People are not perfectly patient and the common time discount factor is $\delta \in (0,1)$.

At date t, the agent's outside option $s(k_t)$ is endogenously determined as an increasing function in the stock of effort k_t, where $s(0) = 0, s'(.) > 0$.

The wage payment w_t is contingent on the production history. With a given wage scheme, the agent at date t decides whether to participate in the relationship or pick up his outside option.

A participating agent at date t chooses his effort $e_t \in [0,1]$ at the cost of $c(e_t)$, where $c(0) = 0, c'(.) > 0, c''(.) > 0, c'(1) = \infty$. Output realizations are stochastic: high $(y_t = y)$ or low $(y_t = 0)$. The stock of effort evolves according to

$$k_{t+1} = k_t + e_t.$$

The probability distribution of realizing high output is given by

$$prob(y_t = y) = e_t.$$

Hence, the expected output is linearly increasing in effort.

The agent starts with the initial stock of effort $k_0 = 0$, which is known by both parties.

At each date t with production history $(y_0, y_1, \dots, y_{t-1})$, a relational contract specifies (i) a wage scheme contingent on output realizations offered by the principal; (ii) an agent's decision on whether to participate or not; (iii) conditional on participating, an agent's choice of effort for production.

Given the wage scheme, the principal's expected profit at date t is given by

$$E_t\{\sum_{\tau=t}^{T} \delta^{\tau-t}[I_\tau(y_\tau - w_\tau)]\}$$

and the agent's expected utility is given by

$$E_t\{\sum_{\tau=t}^{T}\delta^{\tau-t}I_\tau[w_\tau - c(e_\tau)] + \frac{(1-I_\tau)(1-\delta)}{[1-\delta^{(\sum_{\tau=t}^{\tau=T} I_\tau)}]}s(k_\tau)\},$$

where $I_\tau = 1$ if the agent participates and $I_\tau = 0$ otherwise.

I consider 2 situations of outside option growth: (i) as a convex function in the stock of effort; (ii) as a concave function in the stock of effort.

Before starting the next section, I will introduce some basic assumptions.

Assumption 2-1. *The effort cost $c(e)$ is continuous on $e_t \in [0,1]$ and twice differentiable on $e_t \in (0,1)$.*

Assumption 2-2. *The outside option $s(k)$ is increasing in k and twice differentiable on $k \in (0,\infty)$.*

Assumption 2-3. *$s''(k)$ is decreasing in k, $\forall k \in (0,\infty)$.*

Assumption 2-4. *For any $e \in [0,1]$, $s(k) - \delta s(k+e)$ is increasing in k.*

Suppose we don't have Assumption 2-4. For $k' > k$, we would have

$$s(k') - \delta s(k'+e) \leq s(k) - \delta s(k+e),$$

or $c(e) + s(k') - \delta s(k'+e) \leq c(e) + s(k) - \delta s(k+e)$.

The participation constraint doesn't become more stringent as the stock of effort grows. Therefore, it is easier for the principal to retain an agent with a higher outside option, which is not a proper assumption.

Assumption 2-5. *For any* $e \in [0,1]$ *and* $k \geq 0$, $c'(e) > \delta s'(k+e)$.

Assumption 2-5 states that a worker can't work for free without any payment. Since the marginal effort cost would exceed marginal increase in outside options.

Assumption 2-6. *The agent has limited liability and wage payment is non-negative.*

2.3 Finite relationship

Since the principal and agent are both risk neutral, it is weakly dominant for the principal to pay the agent at the end of the relationship.

2.3.1 Contract

At date 0, the principal announces a wage scheme $w(y_0, \dots, y_{T-1})$ that pays at the end of the relationship (at date T), according to the entire production history $(y_0, \dots, y_{T-1})$. See Figure 2-1.

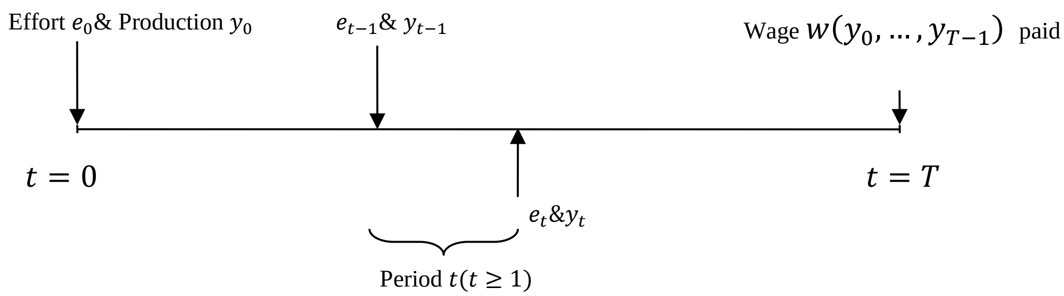

Figure 2-1 The production of wage payment in a finite $\boldsymbol{T}$-period relationship

Denote Y_t as the production history up to date t: $Y_t = (y_0, \dots, y_t)$.

At date t, with a given wage scheme $w(Y_{T-1})$, an agent chooses $e_t^*(Y_{t-1}, k_t)$ according to his stock of effort k_t and the production history Y_{t-1}. Since the date 0 stock of effort $k_0 = 0$, for any given history Y_{t-1}, the agent's stock of effort k_t can be inferred from his optimal level of effort at all the previous dates: $k_t(Y_{t-1})$. Therefore, I simplify the notation $e_t^*(Y_{t-1}, k_t)$ as $e_t^*(Y_{t-1})$ for the agent's optimal effort level at date t, following production history Y_{t-1}.

2.3.2 The agent's problem

By assumption 2-5, An agent won't work for free without any wage payment. Since wage payment won't be made till the end of the relationship, it is not optimal for the agent to quit the relationship before the end of the relationship.

At date $t \in \{0, \dots, T-1\}$, with a given wage scheme and stock of effort k_t, the agent is choosing a profile of actions $[e_t^*(Y_{t-1}), e_{t+1}^*(Y_t), \dots, e_{T-1}^*(Y_{T-2})]$[3] contingent on production histories, such that

(i) $[e_t^*(Y_{t-1}), e_{t+1}^*(Y_t), \dots, e_{T-1}^*(Y_{T-2})]$ maximizes the agent's expected utility at date t:

$$[e_t^*(Y_{t-1}), e_{t+1}^*(Y_t), \dots, e_{T-1}^*(Y_{T-2})] \in Argmax \sum_{(y_t, \dots, y_{T-1})} prob(y_t, \dots, y_{T-1}) \left[\delta^{T-t} w(Y_{T-1}) - \sum_{\tau=t}^{T-1} \delta^{\tau-t} c(e_\tau) + \delta^{T-t} s(k_T)\right];$$

(ii) the agent's participation constraint is satisfied so that he will stay in the relationship:

$$\sum_{(y_t, \dots, y_{T-1})} prob(y_t, \dots, y_{T-1}) \left[\delta^{T-t} w(Y_{T-1}) - \sum_{\tau=t}^{T-1} \delta^{i-t} c(e_\tau^*) + \delta^{T-t} s(k_T)\right] \geq s(k_t), \forall t \in [0, T-1].$$

[3] When $t = 0$, the profile of actions is $[e_0^*, e_1^*(Y_0), \dots, e_{T-1}^*(Y_{T-2})]^3$

2.3.3 The principal's problem

Since it is not optimal for the agent to quit early, a participating agent is expected to stay in the entire relationship. The principal chooses a wage scheme that maximizes his expected profit at date 0:

$$\max_{w(Y_{T-1})} \left\{ \sum_{Y_{T-1}} prob(Y_{T-1}) \left[\sum_{\tau=0}^{T-1} \delta^{\tau} e_{\tau}^{*} y - \delta^{T} w(Y_{T-1}) \right] \right\},$$

such that the agent solves his utility maximizing problem by choosing $[e_0^*, e_1^*(Y_0), \ldots, e_{T-1}^*(Y_{T-2})]$.

Alternatively, we can interpret the principal's profit maximization problem as choosing a profile of actions to implement, so that his date 0 expected profit is maximized. Formally:

$$\max_{[e_0, e_1(Y_0), \ldots, e_{T-1}(Y_{T-2})]} \left\{ \sum_{Y_{T-1}} prob(Y_{T-1}) \left[\sum_{\tau=0}^{T-1} \delta^{\tau} e_{\tau} y - \delta^{T} w^{*}(Y_{T-1}) \right] \right\},$$

where $w^*(Y_{T-1})$ is the wage scheme that satisfies the agent's incentive and participation constraints for taking the profile of actions $[e_0, e_1(Y_0), \ldots, e_{T-1}(Y_{T-2})]$.

I look at the incentive and participation constraints individually in sections 2.3.5, 2.3.6 and 2.3.7. Section 2.3.8 summarizes the principal's optimal contract that maximizes his expected profit at date 0.

2.3.4 Notations

Suppose the agent chooses a profile of actions $[e_t(Y_{t-1}), \dots, e_{T-1}(Y_{T-2})]$ from date t on, for a given wage scheme $w(Y_{T-1})$.

(i) The agent's date t expected utility is given by

$$E_t(u) = \sum_{(y_t,\dots,y_{T-1})} prob(y_t, \dots, y_{T-1}) \left[\delta^{T-t} w(Y_{T-1}) - \sum_{\tau=t}^{T-1} \delta^{\tau-t} c(e_i) + \delta^{T-t} s(k_T) \right].$$

(ii) The date t expected wage payment is given by

$$E_t(w) = \sum_{(y_t,\dots,y_{T-1})} prob(y_t, \dots, y_{T-1})\, \delta^{T-t} w(Y_{T-1}).$$

(iii) The date t expected future effort cost is given by

$$E_t(c) = \sum_{(y_t,\dots,y_{T-1})} prob(y_t, \dots, y_{T-1}) \left[\sum_{\tau=t}^{T-1} \delta^{\tau-t} c(e_\tau) \right].$$

(iv) The date t expected outside option at the end of the relationship is given by

$$E_t(s) = \delta^{T-t} \left[\sum_{(y_t,\dots,y_{T-1})} prob(y_t, \dots, y_{T-1})\, s(k_T) \right].$$

Furthermore, denote $E_t(s') = \delta^{T-t}\left[\sum_{(y_t,\dots,y_{T-1})} prob(y_t, \dots, y_{T-1})\, s'(k_T)\right]$.

Assumption 2-7. *For all profile of effort* $\{e_0, e_1(y_0), \dots, e_{T-1}(y_0, \dots, y_{T-1})\}$, we have:

$$\frac{\partial[c(e_t) - E_t(s)]}{\partial e_t}$$

increasing in e_t, $\forall t \in [0, T-1]$, $\forall (y_0, \dots, y_{t-1})$.

According to Assumption 2-5, an agent will not work without receiving any wage payment. Assumption 2-7 says that if an agent works without any wage payment, the change in effort cost has to be significant enough so that the marginal disutility is increasing in effort.

2.3.5 Incentive Constraint

In this section, I study the principal's wage payment to implement a profile of actions $[e_0, e_1(Y_0), \dots, e_{T-1}(Y_{T-2})]$ so that the agent is incentive compatible. To focus on the incentive constraint, I assume the participation constraint is slack for section 2.3.5 only.

According to Figure 2-2, for a given production history Y_{t-1}, the agent's expected utility at date t can be written as

(13)

$$E_t(u) = -c(e_t) + \delta[prob(y_t = y)E_{t+1}(u|y_t = y) + prob(y_t = 0)E_{t+1}(u|y_t = 0)]$$

$$= -c(e_t) + \delta[e_t E_{t+1}(u|y_t = y) + (1 - e_t)E_{t+1}(u|y_t = 0)].$$

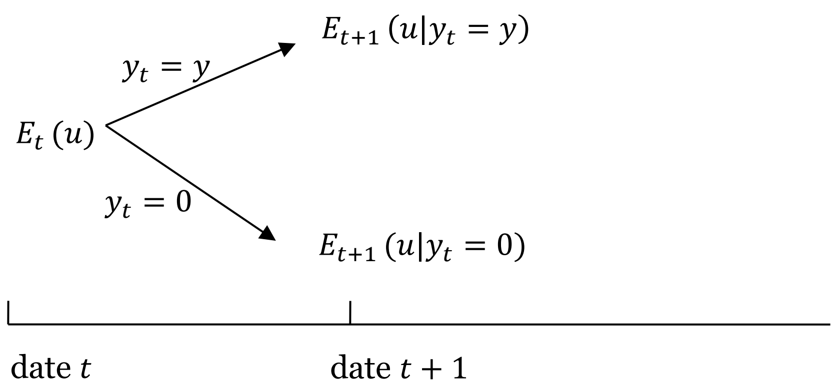

FIGURE 2-2 THE AGENT'S EXPECTED UTILITY AT DATE $\boldsymbol{t}$

It can be shown that the agent's date t utility is concave in e_t. Therefore, the first order condition is sufficient for the agent's utility maximization.

CLAIM: *The date t expected utility in equation (13) is concave in e_t.*

Proof:

Taking the first order derivative of $E_t(u)$ as in equation (13) with respect to e_t:

(14)

$$-c'(e_t) + \delta[E_{t+1}(u|y_t = y) - E_{t+1}(u|y_t = 0)] + \delta\left[e_t \frac{\partial E_{t+1}(u|y_t = y)}{\partial e_t} + (1 - e_t)\frac{\partial E_{t+1}(u|y_t = 0)}{\partial e_t}\right]$$

$$= -c'(e_t) + \delta[E_{t+1}(u|y_t = y) - E_{t+1}(u|y_t = 0)] + \delta[e_t E_{t+1}(s'|y_t = y) + (1 - e_t)E_{t+1}(s'|y_t = 0)]$$

By Assumptions 2-5, we know that (14) is decreasing in e_t, since

$$\frac{\partial(\mathbf{14})}{\partial e_t} = -c''(e_t) + \frac{\partial \delta[E_{t+1}(u|y_t = y) - E_{t+1}(u|y_t = 0)]}{\partial e_t} + \frac{\partial \delta[e_t E_{t+1}(s'|y_t = y) + (1 - e_t)E_{t+1}(s'|y_t = 0)]}{\partial e_t}$$

$$= -c''(e_t) + \frac{\partial \delta[E_{t+1}(s|y_t = y) - E_{t+1}(s|y_t = 0)]}{\partial e_t} + \frac{\partial \delta[e_t E_{t+1}(s'|y_t = y) + (1 - e_t)E_{t+1}(s'|y_t = 0)]}{\partial e_t}$$

$$= \frac{\partial^2[\mathrm{c}(\mathrm{e_t}) - E_t(s)]}{\partial e_t^2} < 0$$

The last step is obtained from Assumption 2-5. Since $\frac{\partial[c(e_t) - E_t(s)]}{\partial e_t}$ is decreasing in e_t, therefore we have

$$\frac{\partial^2[c(e_t) - E_t(s)]}{\partial e_t^2} < 0.$$

The agent's date t expected utility is concave in e_t. Q.E.D.

Lemma 2-1. *For a given wage scheme* $w(Y_{T-1})$*, if it is incentive compatible for the agent to take effort* e_t*, the wage scheme must satisfy the following condition:*

(15)

$$c'(e_t) - E_t(s') = \delta[E_{t+1}(-c + s + w|y_t = y) - E_{t+1}(-c + s + w|y_t = 0)]$$

$$= \delta[E_{t+1}(u|y_t = y) - E_{t+1}(u|y_t = 0)].$$

Equation (15) says that in order to make it incentive compatible for the agent to take effort e_t, the change in effort cost net outside option growth must be compensated by the increase in expected utility upon realizing high output.

Proof:

Since the agent's date t utility is concave in e_t, the first order derivatives of the expected utility $E_t(u)$ in equation (13) respect to e_t gives:

$$-c'(e_t) + \delta[E_{t+1}(u|y_t = y) - E_{t+1}(u|y_t = 0)] + \delta e_t \frac{\partial E_{t+1}(u|y_t = y)}{\partial e_t}$$

$$+ (1 - e_t) \frac{\partial E_{t+1}(u|y_t = 0)}{\partial e_t} = 0,$$

which is equivalent to

$$-c'(e_t) + \delta[E_{t+1}(u|y_t = y) - E_{t+1}(u|y_t = 0)] + \delta e_t \frac{\partial E_{t+1}(s|y_t = y)}{\partial e_t}$$

$$+ (1 - e_t) \frac{\partial E_{t+1}(s|y_t = 0)}{\partial e_t} = 0.$$

Since

$$\frac{\partial E_{t+1}(s)}{\partial e_t} = \frac{\partial \delta^{T-(t+1)}\left[\sum_{(y_{t+1},\dots,y_{T-1})} prob(y_{t+1},\dots,y_{T-1}) s'(k_T)\right]}{\partial e_t} = E_{t+1}(s'),$$

we have

$-c'(e_t) + \delta[E_{t+1}(u|y_t = y) - E_{t+1}(u|y_t = 0)] + \delta e_t E_{t+1}(s') + (1 - e_t)E_{t+1}(s') = 0.$

Re-arranging：

$\delta[E_{t+1}(w|y_t = y) - E_{t+1}(w|y_t = 0)] = c'(e_t) - E_t(s') + \delta\{E_{t+1}(c - s|y_t = y) - E_{t+1}[(c - s)|y_t = 0]\}$ Q.E.D.

Lemma 2-2. *The principal's date* 0 *expected wage payment so that it is incentive compatible for the agent to take the profile of actions* $[e_0, e_1(Y_0), \ldots, e_{T-1}(Y_{T-2})]$ *is given by*

(16)

$$f(e_0, \ldots, e_{T-1}) = E_0(c - s) + e_0[c'(e_0) - E_0(s')] - c(e_0) + \sum_{\tau=1}^{T-1} \delta^\tau \{e_\tau[c'(e_\tau) - E_\tau(s')] - c(e_\tau)|Y_{\tau-1} = (0, \ldots, 0)\} + \delta^T E_T[s|Y_{T-1} = (0, \ldots, 0)].$$

Equation (16) is obtained from equations (14) & (15).

Proof:

By Lemma 2-1, we know that

$$\delta e_t[E_{t+1}(w|y_t = y) - E_{t+1}(w|y_t = 0)] = e_t[c'(e_t) - E_t(s')] + \delta e_t[E_{t+1}(c - s|y_t = y) - E_{t+1}(c - s|y_t = 0)].$$

Therefore, we have

(17)

$$f(e_0, \ldots, e_{T-1}) = \delta e_0 E_1(w|y_0 = y) + \delta(1 - e_0)E_1(w|y_0 = 0)$$

$$= \delta e_0[E_1(w|y_0 = y) - E_1(w|y_0 = 0)] + \delta E_1(w|y_0 = 0)$$

$$= e_0[c'(e_0) - E_0(s')] + \delta e_0\{E_1[(c - s)|y_0 = y] - E_1[(c - s)|y_0 = 0]\}$$
$$+ \delta E_1(w|y_0 = 0).$$

Continuing substituting $E_1(w|y_0 = 0)$, we obtain by induction:

$$f(e_0, \ldots, e_{T-1}) = E_0(c - s) + e_0[c'(e_0) - E_0(s')] - c(e_0)$$
$$+ \sum_{\tau=1}^{T-1} \delta^\tau \{e_\tau[c'(e_\tau) - E_\tau(s')] - c(e_\tau)|Y_{\tau-1} = (0, \ldots, 0)\}$$
$$+ \delta^T E_T[s|Y_{T-1} = (0, \ldots, 0)].$$

Suppose the principal implements a profile of actions $[e_0, e_1(Y_0), \ldots, e_{T-1}(Y_{T-2})]$. For a given production history Y_{t-1}, the marginal wage payment so that an agent is incentive compatible to take e_t is given by

(18) $$\frac{\partial f(e_0, \ldots, e_t)}{\partial e_t},$$

where $f(e_0, \dots, e_t)$ is the wage payment so that it is incentive compatible for the agent to take the profile of actions.

Claim: *$f(e_0, \dots, e_t)$ is convex in e_t.*

Proof:

Take the derivative of $f(e_0, \dots, e_t)$ with respect to e_t yields the following results.

(i) For given $Y_{t-1} \neq (0, \dots, 0)$ such that the first high output occurs at date $i < (t-1)$:

(19)

$$\frac{\partial f(e_0, \dots, e_t)}{\partial e_t} = \frac{\partial E_0(c-s)}{\partial e_t} - \sum_{\tau=1}^{i} \delta^\tau e_\tau \frac{\partial E_\tau[s'|Y_{\tau-1} = (0, \dots, 0)]}{\partial e_t}$$

(ii) For given Y_{t-1} such that $y_i = 0 \ \forall i \leq (t-1)$:

(20)

$$\frac{\partial f(e_0, \dots, e_t)}{\partial e_t} = \frac{\partial E_0(c-s)}{\partial e_t} - e_0 \frac{\partial E_0(s')}{\partial e_0} - \sum_{\tau=1}^{t-1} \delta^\tau e_\tau \frac{\partial E_\tau[s'|Y_{\tau-1} = (0, \dots, 0)]}{\partial e_t}$$

$$+ \delta^t \frac{\partial \{e_t[c'(e_t) - E_t(s')] - c(e_t)\}}{\partial e_t} + \delta^T \frac{\partial E_T[s|Y_{T-1} = (0, \dots, 0)]}{\partial e_t}$$

$$= \frac{\partial E_0(c-s)}{\partial e_t} - e_0 \frac{\partial E_0(s')}{\partial e_0} - \sum_{\tau=1}^{t-1} \delta^\tau e_\tau \frac{\partial E_\tau[s'|Y_{\tau-1} = (0, \dots, 0)]}{\partial e_t}$$

$$+\delta^t e_t\left[\frac{\partial c'(e_t)-E_t(s')}{\partial e_t}\right]-\{\delta^t E_t(s')-\delta^T E_T[s'|Y_{T-1}=(0,\dots,0)]\}.$$

It can be shown that the above first order derivatives in equations (19) and (20) are increasing in e_t. See details below.

- By Assumption 2-7, $\frac{\partial E_0(c-s)}{\partial e_t}$ is increasing in e_t.
- By Assumption 2-3,

$$-e_0\frac{\partial E_0(s')}{\partial e_0}$$

$$-\sum_{\tau=1}^{t-1}\delta^\tau e_\tau\frac{\partial E_\tau[s'|Y_{\tau-1}=(0,\dots,0)]}{\partial e_t}-\{\delta^t E_t(s')$$

$$-\delta^T E_T[s'|Y_{T-1}=(0,\dots,0)]\},$$

is increasing in e_t.

- By Assumptions 2-1 and 2-3,

$$\delta^t e_t\left[\frac{\partial c'(e_t)-E_t(s')}{\partial e_t}\right]$$

is increasing in e_t.

Therefore, $f(e_0,\dots,e_t)$ is convex in e_t.

Knowing that the expected output is linear and the expected wage payment $f(e_0,\dots,e_t)$ is convex in e_t , the first order condition is sufficient for the principal's profit maximizing problem. Q.E.D.

Lemma 2-3. *Suppose that the participation constraint is slack and consider only the incentive constraint. For* $t \geq 1$ *and* $Y_{t-1} \neq (0, \dots, 0)$, *the profit-maximizing principal will implement* $e_{t+1}(y_t = y) = e_{t+1}(e_t = 0)$.

According to Lemma 2-3, the marginal wage payment subject to incentive compatibility is independent of the output realization ever since the realization of the first high output.

Proof:

Given $Y_{t-1} \neq (0, \dots, 0)$ such that the first high output occurs at date $i < (t-1)$, the marginal wage payment

$$\frac{\partial f(e_0, \dots, e_t)}{\partial e_t} = \frac{\partial E_0(c-s)}{\partial e_t} - \sum_{\tau=1}^{i} \delta^{\tau} e_{\tau} \frac{\partial E_{\tau}[s' | Y_{\tau-1} = (0, \dots, 0)]}{\partial e_t}$$

is independent of production histories for all e_t.

Therefore, the effort implemented by the principal is independent of the production history, once the agent has already delivered some high output. Q.E.D.

The intuition is given in the discussion below.

2.3.6 Discussion for Lemma 2-3

Consider the following contract illustrated in Figure 2-3.

Suppose the principal increases $e_{t+2}(y_t = y, y_{t+1} = 0)$ by 1 unit while implementing the same effort for all other production histories and dates. According to lemma 2-3, the decrease in work incentives at date $(t+2)$ is offset by the increase in work incentives at date t.

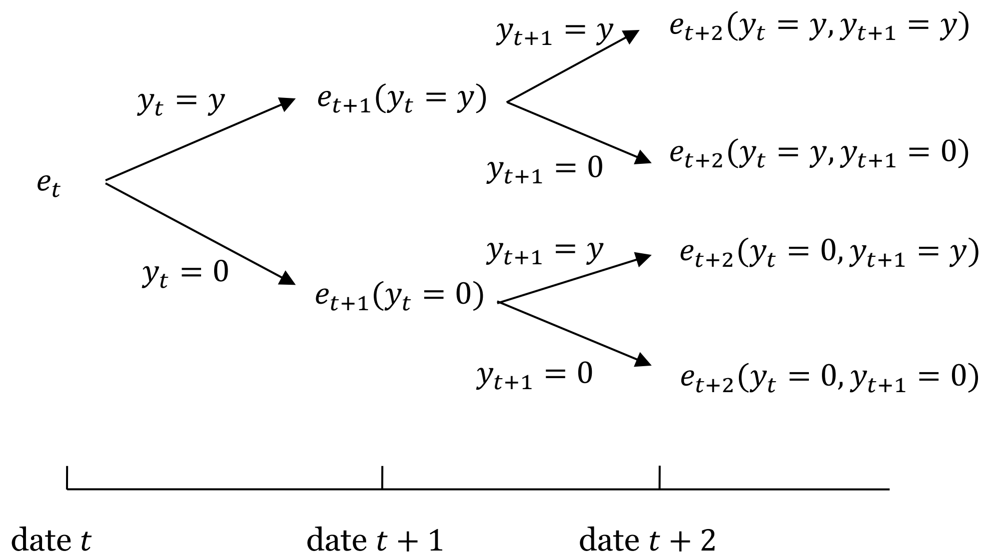

Figure 2-3 The effort to be implemented from date t on

(i) Given that $y_t = y$ and $y_{t+1} = 0$, 1 unit of increase in e_{t+2} will raise the expected utility at date $(t+1)$ by $\frac{\partial E_{t+1}(u|y_t=y)}{\partial e_{t+2}}$. The increase in the expected wage at date $(t+1)$ so that the agent is incentive compatible is given by

(21) $$\frac{\partial E_{t+1}(w)}{\partial e_{t+2}} = \frac{\partial E_{t+1}(u|y_t=y)}{\partial e_{t+2}} + \frac{\partial E_{t+1}(c-s)}{\partial e_{t+2}}.$$[4]

Decomposing equation (21):

- $\frac{\partial E_{t+1}(c-s)}{\partial e_{t+2}}$ is compensation for the effort cost net outside option changes;
- $\frac{\partial E_{t+1}(u|y_t=y)}{\partial e_{t+2}}$ is the increase in wage payment due to the decrease in incentives at date $(t+1)$.

(ii) The condition for incentive compatibility of taking e_t is given by

(22)

$$c'(e_t) - E_t(s') = \delta[E_{t+1}(u|y_t = y) - E_{t+1}(u|y_t = 0)],$$

Given that $y_t = y$ and $y_{t+1} = 0$, 1 unit of increase in e_{t+2} will save the expected wage payment at date t by

(23)

$$\frac{\partial \delta[E_{t+1}(u|y_t=y) - E_{t+1}(u|y_t=0)] + E_t(s')}{\partial e_{t+2}} = \frac{\partial \delta E_{t+1}(u|y_t=y)}{\partial e_{t+2}} + \frac{\partial E_t(s')}{\partial e_{t+2}}:$$

- $\frac{\partial E_t(s')}{\partial e_{t+2}}$ is the savings in the wage payment due to the change of the marginal increase on outside options;

[4] Since $E_{t+1}(u|y_t = y) = E_{t+1}(-c + s + w|y_t = y)$.

- $\frac{\partial \delta E_{t+1}(u|y_t=y)}{\partial e_{t+2}}$ is the savings in the wage payment due to the increase of incentives at date t.

The total effect of the decrease of incentives at date $(t+1)$ and increase of incentives at date t is given by

$$-\delta \frac{\partial E_{t+1}(u|y_t=y)}{\partial e_{t+2}} + \frac{\partial E\delta_{t+1}(u|y_t=y)}{\partial e_{t+2}} = 0.$$

2.3.7 Participation Constraint

This section studies the agent's participation constraint.

Denote $h(k,e)$ as the minimum wage payment to satisfy the participation constraint, if the principal wants to implement effort e on an agent with the stock of effort k. The binding condition for the participation constraint is

(24)

$$s(k) = -c(e) + h(k,e) + \delta s(k+e).$$

Equivalently, we have

(25)

$$h(k,e) = s(k) + c(e) - \delta s(k+e).$$

CLAIM: *$h(k,e)$ is increasing in k & e.*

Proof:

(i) For a given e, suppose $k_1 > k_2 \geq 0$.

$$h(k_1, e) - h(k_2, e)$$

$$= [s(k_1) + c(e) - \delta s(k_{t+1} + e)] - [s(k_2) + c(e) - \delta s(k_2 + e)]$$

$$= [s(k_1) - s(k_2)] - \delta[s(k_1 + e) - s(k_2 + e)] > 0,$$

according to Assumption 2-5. Q.E.D.

At date t, for a given production history Y_{t-1}, the expected wage paid to implement $[e_t, e_{t+1}(Y_t), \dots, e_{T-1}(Y_{T-2})]$ so that the participation constraint binds is given by

(26)

$$\sum_{(y_t, \dots, y_{T-1})} \sum_{\tau=t}^{T-1} prob(y_t, \dots, y_{T-1}) \delta^{\tau-t} h(k_\tau, e_\tau)$$

$$= E_t(c - s | Y_{t-1}) + s(k_t) .$$

In particular, the principal's expected wage payment date 0to implement $[e_0, e_1(Y_0), \dots, e_{T-1}(Y_{T-2})]$ so that the participation constraint binds is given by:

(27)

$$E_0(c) + s(k_0) - E_0(s) = E_0(c - s).$$

By Assumption 2-4, we know that the wage payment in equation (27) is convex in effort.

Suppose the principal implements a profile of actions $[e_0, e_1(Y_0), \dots, e_{T-1}(Y_{T-2})]$. At date t, with a given production history Y_{t-1}, the marginal wage payment to satisfy the participation constraint is given by:

(28)

$$\frac{\partial E_0(c-s)}{\partial e_t},$$

where $E_0(c-s)$ is the principal's expected wage payment to implement the profile of actions so that the participation constraint binds. See equation (28).

Bringing together the incentive and participation constraints, the expected wage payment for implementing e_t^* is given by

$$\int_0^{e_t^*} max\left[\frac{\partial E_0(c-s)}{\partial e_t}, \frac{\partial f(e_0, \dots, e_{T-1})}{\partial e_t}\right] de_t,$$

where $E_0(c-s)$ and $f(e_0, \dots, e_{T-1})$ are the wage payment to satisfy the participation and incentive constraints respectively.

2.3.8 Principal's Optimal Contract

This section studies the principal's optimal contract that maximizes his expected profit at date 0, by bringing together the incentive and participation constraints.

The principal is solving the following profit maximization problem:

(29)

$$\max_{[e_0^*, e_1^*(Y_0), \ldots, e_{T-1}^*(Y_{T-2})]} \sum_{\tau=0}^{T-1} \sum_{Y_{T-1}} prob(Y_{T-1})\{\delta^\tau e_\tau^* y$$

$$- \int_0^{e_\tau^*} max\left[\frac{\partial E_0(c-s)}{\partial e_\tau}, \frac{\partial f(e_0, \ldots, e_{T-1})}{\partial e_\tau}\right] de_\tau\}.$$

Since $\frac{\partial E_0(c-s)}{\partial e_\tau}$ and $\frac{\partial f(e_0,\ldots,e_{T-1})}{\partial e_\tau}$ are both increasing in e_τ, the maximum function $max\left[\frac{\partial E_0(c-s)}{\partial e_t}, \frac{\partial f(e_0,\ldots,e_{T-1})}{\partial e_t}\right]$ is also increasing in e_τ. Therefore, the principal's date 0 expected profit in equation (29) is concave in effort and the first order condition is sufficient for profit-maximization. The profit-maximizing condition is given by

(30)

$$\frac{\partial[\sum_{\tau=0}^{T-1} \sum_{Y_{T-1}} prob(Y_{T-1})(\delta^i e_\tau^* y)]}{\partial e_t^*} = max\left[\frac{\partial E_0(c-s)}{\partial e_t^*}, \frac{\partial f(e_0, \ldots, e_{T-1})}{\partial e_t^*}\right], \forall t \in [0, T-1].$$

Proposition 2-1. *Suppose the production history* $Y_{t-1} \neq (0, \ldots, 0)$ *and the principal implements effort* e_t *at date* t.

- *For convex* $s(k)$, *the marginal wage payment to satisfy the participation constraint for taking* e_t *is higher than incentive constraint.*
- *For concave* $s(k)$, *the marginal wage payment to satisfy the incentive constraint for taking* e_t *is higher than participation constraint.*

Proof:

Comparing equations (29) & (30) :

$\sum_{u=0}^{i-1} \delta_u e_u \frac{\partial E_u(s')}{\partial e_t} > 0$ for $s(k)$ convex, we have

$$\frac{\partial f(e_0, \dots, e_t)}{\partial e_t} = \frac{\partial E_0(c-s)}{\partial e_t} - \sum_{\tau=1}^{i} \delta^\tau e_\tau \frac{\partial E_\tau[s'|Y_{\tau-1} = (0, \dots, 0)]}{\partial e_t} < \frac{\partial E_0(c-s)}{\partial e_t},$$

$$\text{therefore } max\left[\frac{\partial E_0(c-s)}{\partial e_t}, \frac{\partial f(e_0,\dots,e_{T-1})}{\partial e_t}\right] = \frac{\partial E_0(c-s)}{\partial e_t}.$$

Similarly, we know that $\sum_{u=0}^{i-1} \delta_u e_u \frac{\partial E_u(s')}{\partial e_t} < 0$ for $s(k)$ concave. Therefore we obtain

$$max\left[\frac{\partial E_0(c-s)}{\partial e_t}, \frac{\partial f(e_0,\dots,e_{T-1})}{\partial e_t}\right] = \frac{\partial E_0(w)}{\partial e_t}. \text{ Q.E.D.}$$

The intuition for Proposition 2-1is that as one builds up his stock of effort, for $s(k)$ convex, the accelerating outside option growth would give additional work incentives and save the principal's wage payment. Therefore, the marginal wage payment to satisfy the incentive constraint is lower than the participation constraint. The intuition for the concave case is analogous.

According to Proposition 2-1, given that high output has been delivered before date t, the marginal wage payment of implementing e_t is determined by the participation constraint for $s(k)$ convex and the incentive constraint for $s(k)$ concave.

Proposition 2-2. *At date t of a finite T-period relationship, given that* $Y_{t-1} \neq (0, \ldots, 0)$*, the principle maximizes his expected profit by implementing* $e^*_{t+1}\,(y_t = y) = e^*_{t+1}\,(y_t = 0)$.

Proof:

I will prove the convex case and the concave case is similar.

At date $(T-1)$, with any given Y_{T-1}, the profit maximizing condition gives

$$[c'(e^*_{T-1}) - \delta s'(k_T)] = y.$$

Therefore we have $e^*_{T-1}(y_{T-2} = y) = e^*_{T-1}(y_{T-2} = 0)$.

By backward induction, for any given $Y_{\tau-1} \neq (0, \ldots, 0)$, the first order condition gives

$$e^*_{\tau+1}(y_\tau = y) = e^*_{\tau+1}(y_\tau = 0),\ \forall \tau \in [t, T-1].$$

As long as the agent has delivered high output, his effort in the future is no longer dependent on the production history. Q.E.D.

Proposition 2-2 states that the principal would implement the same level of effort at date $(t+1)$, independent of the date t output realization, given that the agent has already delivered some high output before date t.

For the convex case, given that the agent has already delivered some high output, the marginal wage payment for implementing effort is determined by the participation constraint and thus has no impacts on incentives.

For the concave case, according to Proposition 2-1, the marginal wage payment is determined by the incentive constraint. Given that high output has already been delivered at date $i < t$, implementing $e_{t+1}(y_t = y) = e_{t+1}(y_t = 0)$ instead of $e_{t+1}(y_t = y) > e_{t+1}(y_t = 0)$ would decrease the incentives at date t, but this effect is offset by the increase of incentives at date i, due to the better prospects into the future success at date i. the principal is to reward the agent with some wage paid at the end of the relationship. If seeing low output at date t, the principal punishes the agent by deducting money from the wage payment for the previous high output. Therefore, implementing the same effort in the next period would not affect incentives, independent of the date t output realizations.

Corollary. *At date t of a finite T-period relationship, for a given production history $Y_{t-1} = (0, \dots, 0)$, we have $e^*_{t+1}(y_t = y) \geq e^*_{t+1}(y_t = 0)$.*

Proof:

I will prove the convex case and the concave case is similar.

The profit-maximizing condition for $e^*_{t+1}(y_t = y)$ yields

$$y = c'[e^*_{t+1}(y_t = y)] - E_{t+1}(s').$$

For $y_t = 0$, we do not know exactly whether the marginal wage payment to satisfy the incentive or participation constraint is higher.

- If the marginal wage payment to satisfy the incentive constraint is higher:

$$e_{t+1}^*(y_t = y) > e_{t+1}^*(y_t = 0).$$

- If the marginal wage payment to satisfy the participation constraint is higher:

$$e_{t+1}^*(y_t = y) = e_{t+1}^*(y_t = 0).$$

Therefore, we have $e_{t+1}^*(y_t = y) \geq e_{t+1}^*(y_t = 0)$. Q.E.D.

Corollary. *In a finite T-period relationship, for T sufficiently large:*

$$e_{T-1}^*[Y_{T-2} = (0, \dots, 0)] \to 0.$$

Proof:

For T sufficiently large, the marginal wage payment to satisfy the incentive constraint is given by

(31)

$$\frac{\partial E_{T-1}(w)}{\partial e_{T-1}^*} = \sum_{i=0}^{T-1} -\frac{1}{prob(Y_{T-1})} \delta^{T-i} \left\{ \frac{\partial e_i^* E_i(s')}{\partial e_{T-1}^*} | Y_{i-1} = (0, \dots, 0) \right\}$$

$$+ \frac{1}{prob(Y_{T-1})} \{ \frac{\partial e_{T-1}^*[c'(e_{T-1}^*) - E_{T-1}(s')]}{\partial e_{T-1}^*} | Y_{T-1} = (0, \dots, 0) \}$$

$$+ \frac{\partial E_{T-1}(c - s)}{\partial e_{T-1}^*} + \delta \left[\frac{E_T(s')}{prob(Y_{T-1})} \middle| Y_{T-1} = (0, \dots, 0) \right].$$

Suppose e^*_{T-1} doesn't converge to zero: the stock of effort won't converge.

The marginal wage payment in equation (31) grows infinity and e^*_{T-1} goes to zero. This contradicts with the assumption that e^*_{T-1}does not converge to zero. Q.E.D.

Proposition 2-3. *In a finite T- period relationship, for the given wage scheme and production history $Y_{t-1} \neq (0, \ldots, 0)$, we have* $e^*_\tau < e^*_{\tau+1}, \forall \tau \geq t$.

Proof:

I will prove for the convex case and the concave is similar.

For $s(k)$ convex, the profit-maximizing condition yields

$$c'(e^*_\tau) - \delta^{T-\tau} E_T(s') = c'(e^*_{\tau+1}) - \delta^{T-\tau-1} E_T(s)$$

which implies $e^*_\tau < e^*_{\tau+1}$. Q.E.D.

Proposition 2-3 states that the agent's effort will increase over time once he has delivered some high output. The intuition is that as the agent gets closer to the termination date, he expects to exercise his outside option sooner and thus has more incentives.

Proposition 2-4. *For $s(k)$ convex, given 2 histories of production Y_{T-1}&Y'_{T-1} in a finite T- period relationship such that*

$$Y_{T-1} = (y_0, \ldots, y_{T-1}) \text{ with } y_i = y \& y_\tau = 0, \forall \tau < i$$

$\& \;\; Y'_{T-1} = (y'_0, \dots, y'_{T-1})$ with $y'_{i+1} = y \& y'_\tau = 0, \forall \tau < i + 1,$

we have $\forall t \geq i$*:* $s(k_t) \geq s(k'_t)$ *&* $e^*_t \leq {e'_t}^*$*, where* k_t *&* e^*_t (or k'_t *&* ${e'_t}^*$) *are the outside option and effort (implemented by the principal) at date* t*, if the production history follows* Y_{T-1} (or Y'_{T-1}).

Proof:

Since $y_i = y \& y'_i = 0$, according to the Corollary of 2-2, we know that

$$e^*_i \geq {e'_i}^* \;\&\; s(k_{i+1}) \geq s(k'_{i+1}).$$

For $t > i$: the profit maximizing condition gives

$$c'(e^*_t) - \delta^{T-t} s'(k_T) = c'({e'_t}^*) - \delta^{T-t} s'(k'_T) = y.$$

Since $s(k_i) \geq s(k'_i)$, we have $e_t \geq e'^*_t$, $\forall\; t \geq i$, if $s(k)$ convex. Q.E.D.

Proposition 2-4 says that under convex outside option growth, if an agent succeeds at an earlier date, he will gain more job opportunities with a higher level of effort stock for the rest of his career. It is the random output realizations that determine one's career advancement. According to the model, the selection of fast-tracks is not due to individual differences such as the stock of effort but the stochastic realizations of output. The intuition is that for convex outside option growth, the fast-track individuals with a higher stock of effort has higher returns from effort. The principal would then take advantage of their inherent work incentives and implement more effort. Therefore, the one with an early success expects more job opportunities (higher effort) and higher stock of effort throughout the entire career path.

Corollary. *For $s(k)$ concave: given 2 histories of production Y_{T-1}& Y'_{T-1} in a finite T-period relationship such that*

$$Y_{T-1} = (y_0, \dots, y_{T-1}) \text{ with } y_i = y \& y_\tau = 0, \forall \tau < i$$

$$\& \;\; Y'_{T-1} = (y'_0, \dots, y'_{T-1}) \text{ with } y'_{i+1} = y \& y'_\tau = 0, \forall \tau < i+1,$$

we have $s(k_T) > s(k'_T)$ *, where* $s(k_T)$ or $s(k'_T)$ *is the outside option at date* T*, if the production history follows* Y_{T-1}or Y'_{T-1}.

Proof:

For Y_{T-1} , the marginal wage payment to implement e_t^* is given by

$$\frac{\partial E_0(c-s)}{\partial e_t^*} - e_0 \frac{\partial E_0(s')}{\partial e_t^*} - \sum_{\tau=1}^{i} \delta^\tau e_\tau \frac{\partial E_\tau[s'|Y_{\tau-1} = (0, \dots, 0)]}{\partial e_t^*}.$$

For Y'_{T-1} , the marginal wage payment to implement $e_t'^*$ is given by

$$\frac{\partial E_0(c-s)}{\partial e_t'^*} - e_0 \frac{\partial E_0(s')}{\partial e_t'^*} - \sum_{\tau=1}^{i+1} \delta^\tau e_\tau \frac{\partial E_\tau[s'|Y_{\tau-1} = (0, \dots, 0)]}{\partial e_t'^*}.$$

The profit-maximizing condition requires that

$$c'(e_t^*) - \delta^{T-t} s'(k_T) > c'(e_t'^*) - \delta^{T-t} s'(k'_T).$$

Suppose that $s(k_T) \leq s(k'_T)$, we have $s'(k'_T) \geq s'(k'_T)$ and $c'(e^*_t) \geq c'(e'^*_t)\ \forall t > i$. However, it is not possible to have $e^*_t \geq e'^*_t\ \forall t > i$, yet $k_T \leq k'_T$. So we conclude that $s(k_T) \geq s(k'_T)$.   Q.E.D.

If high output starts at a late date, the agent's future expected stock of effort would be low. The intuition is that an agent with a lower stock of effort expects a higher return from effort, which saves the principal's wage payment to generate incentives at date t. Therefore, an agent who failed to deliver high output for a long time may get a temporary rapid growth on the stock of effort. However, he builds up his stock of effort slower through the entire relationship.

2.3.9 Empirical Result

Empirical evidence has been found in Smeets (2004) that a handicapping policy does exist so that the ones promoted quickly in the past are not likely to get a quick promotion again. However, this paper illustrates that the handicapping policy is relative and it does not survive during the entire career so that the fast-track individuals are overall experiencing a more successful career. Under the assumption of the concave outside option growth, I interpret the handicapping policy as the possibility that the principal may implement a lower level of effort on those who realizes their first successes at an earlier date. My model also finds that the handicapping policy doesn't exist throughout the entire relationship and earlier success would bring a higher stock of effort throughout the entire relationship.

According to my model, the timing of realizing the first success has impacts throughout one's entire career path. Those who are fortunate to accomplish the first success at an earlier date would go onto the fast track that gives them more opportunities with higher stocks of effort at all future dates. Therefore, the selection for fast-tracks is solely based on random output realizations, instead of individual differences such as capability.

Academic evidence is found in Smeets (2004), who studies a sample of top economists to see if the time spent as an assistant professor has impacts on the time spent as an associate professor. It's found that the fast-track individuals who receive tenure in less than 7 years are generally more productive throughout the entire career. According his paper, fast-track effects are found that "the individuals promoted quickly at the beginning were also the ones who experienced the fastest and most successful careers".

2.3.10 Comparison between fast-track and slower-track

It has been previously shown that

$$h(k,e) = c(e) + s(k) - \delta s(k+e)$$

is (i) increasing & convex in e (ii) increasing in k.

If the agent realizes his first high output at a later date, he is thus on a slower track with a lower stock of effort at date $(T-1)$. The principal's marginal profit by implementing effort at date $(T-1)$ is higher for the slower track individual.

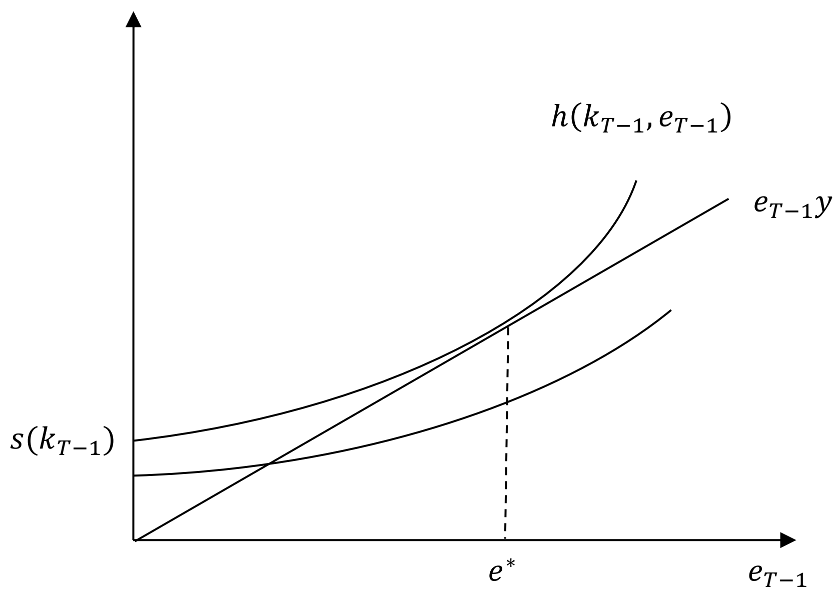

Figure 2-4 For $s(k)$ convex: The principal's marginal profit at date $(T-1)$ is higher for the slower track individual

2.4 Conclusions

This paper develops a principal-agent problem under the assumptions that the agent has a continuous set of effort choices and the duration of the relationship is contingent on output realizations. The results are related to the fast- track effect, the phenomenon that time to promotion is serially correlated. It's found that the date of realizing the first high output plays an important role in one's career development. An agent's stock of effort will grow at a faster rate, if his first high output is realized at an earlier date.

According to the model, the fast-track effect arises so that the profit-maximizing principal can save the wage payment for generating incentives. Since the principal only sees the stochastic realizations of output, the selection of workers into the pool of fast-track is not driven by individual differences in the stock of effort.

My model finds that an earlier delivery of the first high output would bring the agent more work opportunities in the future and his stock of effort grows more quickly. Therefore, an agent with early success stocks up effort in a faster way.

In this chapter, I relax the assumption that effort implemented by the principal is independent of the production history (as in chapter 1). Therefore, the profit-maximizing principal punishes an agent for the delivery of low output by implementing lower effort in the next period. This saves the wage payment since an agent has stronger incentives to exert effort, for the fear of getting less working opportunities in the future. However, this punishment disappears since the delivery of the first high output. Once an agent has delivered some high output, the effort implemented by the principal no longer depends on the random realizations of output.

Chapter 3 Incentives and Risk-Aversion

3.1 Introduction

The principal-agent problems in the previous two chapters are built under the assumption that both parties are risk-neutral. However, people are sometimes risk-averse over the wage paid according to stochastic output realizations. In this chapter, I vary the assumption and look at the contract with a risk-averse agent, with the restriction that the wage has to be paid at the last date.

According to the results in the previous chapter, the date of the first success plays an important role in one's career advancement. The one who delivers his first success earlier would enter the fast-track and build up his stock of effort faster. The principal maximizes his profit by implementing the same effort that is independent of production history, after the delivery of the first high output. Under the assumption of the risk-averse agent, does an early success necessarily induce fast tracks?

The fast-track effect no longer exists with a risk-averse agent. When the agent is risk-averse, higher performance pay not only induces incentives but also imposes higher risk which in turn generates a risk premium to be paid by the principal. Given that high output is already delivered, realizing high output would cost a higher risk premium and lead to less effort in the next period. Similar results are also found in Allen and Lueck (1992), Lafontaine (1992) and Aggarwal and Samwick (1999) with negative trade-offs between uncertainty and pay-for-performance sensitivity.

The optimal finite T-period contract is analyzed in section 3-2.

Besides Assumptions 2-1 through 2-4, we have 2 more assumptions for this chapter.

Assumption 3-1. *The risk-averse agent's utility is additively separable in the effort cost, wage payment and outside option:*

$$E_t(u) = E_t[-c + s + \mu(w)],$$

where $\mu(.)$ *is the utility over wage payment such that*

$$\mu(.) \geq 0, \mu' > 0, \mu'' < 0 \textit{ on } (0, \infty).$$

It can be shown that the date t expected utility $E_t(u)$ is increasing and concave in in effort. The agent's date t expected utility is given by

(32)

$$E_t\{\sum_{\tau=t}^{\infty} \delta^{\tau-t} I_\tau[\mu(w_\tau) - c(e_\tau)] + \frac{(1 - I_\tau)(1 - \delta)}{[1 - \delta^{(\sum_{\tau=t}^{\tau=T} I_\tau)}]} s(k_\tau\},$$

where $I_\tau = 1$ if the agent participates and $I_\tau = 0$ otherwise. (32) states that the date t expected utility comes from the wage payment and outside option at the termination date of the relationship, net of the disutility of effort.

Assumption 3-2. *The principal is risk-neutral.*

3.2 Finite *T*- period relationship

This section studies the profit-maximizing contract over a finite T- period relationship, with the restriction that the wage has to be paid at the end.

At date 0, the principal announces a wage scheme $w(y_0, \dots, y_{T-1})$ that pays at the end of the relationship (at date T), according to the entire production history $(y_0, \dots, y_{T-1})$.

3.2.1 The agent's problem

Similar with chapter 2, it is not optimal for a participating agent to quit early.

At date $t \in [0, T-1]$, with a given wage scheme, the agent is choosing a profile of actions $[e_t^*, e_{t+1}^*(Y_t), \dots, e_{T-1}^*(Y_{T-2})]$ contingent on production histories, such that

(i) $[e_t^*, e_{t+1}^*(Y_t), \dots, e_{T-1}^*(Y_{T-2})]$ maximizes the agent's expected utility at date t:

$$[e_t^*, e_{t+1}^*(Y_t), \dots, e_{T-1}^*(Y_{T-2})]$$

$$\in Arg \max_{w(Y_{T-1})} \sum_{(y_t, \dots, y_{T-1})} prob(y_t, \dots, y_{T-1}) \{\delta^{T-t}\mu[w(Y_{T-1})]$$

$$- \sum_{i=t}^{T-1} \delta^{i-t} c(e_i) + \delta^{T-t} s(k_T)\};$$

(ii) the agent's participation constraint is satisfied so that he will stay in the relationship:

$$\sum_{(y_t,\dots,y_{T-1})} prob(y_t,\dots,y_{T-1})\,\{\delta^{T-t}\mu[w(Y_{T-1})]-\sum_{i=t}^{T-1}\delta^{i-t}c(e_i^*)+\delta^{T-t}s(k_T)\}$$

$\geq s(k_t), \forall t \in [0, T-1]$.

3.2.2 The principal's problem

Since risk-neutral, the principal's problem is to maximize the expected output less wage payment at date 0:

$$\max_{w(Y_{T-1})}\left\{\sum_{Y_{T-1}} prob(Y_{T-1})\left[\sum_{\tau=0}^{T-1}\delta^{\tau}e_{\tau}y-\delta^{T}w(Y_{T-1})\right]\right\},$$

such that the agent solves his utility maximizing problem by choosing $[e_0^*, e_1^*(Y_0), \dots, e_{T-1}^*(Y_{T-2})]$.

. *For a given wage scheme $w(Y_{T-1})$, if the agent is incentive compatible to take effort e_t, the wage scheme must satisfy the following condition:*

(33)

$$\delta\{E_{t+1}[\mu(w)|y_t=y]-E_{t+1}[\mu(w)|y_t=0]\}$$
$$=c'(e_t)-E_t(s')+\delta[E_{t+1}(c-s|y_t=y)-E_{t+1}(c-s|y_t=0)].$$ [5]

(33) characterizes the incentive created by wage payment. With a risk-averse agent, the incentive not only depends on the wage difference, but also the particular wage level for delivering high and low output. The proof of Lemma 3-1 follows that of Lemma 2-1.

To see that, let's fix the wage difference for delivering high and low output. Suppose the principal increases the wage payment for realizing all output histories by the same constant $a > 0$. Due to the concavity of $\mu(.)$, the incentive decreases:

$$\delta\{E_{t+1}[\mu(w+a)|y_t = y] - E_{t+1}[\mu(w+a)|y_t = 0]\}$$
$$< \delta\,\{E_{t+1}[\mu(w)|y_t = y] - E_{t+1}[\mu(w)|y_t = 0]\}.$$

Lemma 3-1. *The agent's date 0 expected utility from wage payment when he is incentive compatible to take the profile of actions* $[e_0, e_1(Y_0), \dots, e_{T-1}(Y_{T-2})]$ *is given by*

(34)

$$E_0[\mu(w)] = E_0(c - s) + e_0[c'(e_0) - E_0(s')] - c(e_0)$$
$$+ \sum_{\tau=1}^{T-1} \delta^\tau\{e_\tau[c'(e_\tau) - E_\tau(s')] - c(e_\tau)|Y_{\tau-1} = (0, \dots, 0)\}$$
$$+ \delta^T E_T[s|Y_{T-1} = (0, \dots, 0)].$$

---

[5] This is equivalent to: $c'(e_t) - E_t(s') = \delta[E_{t+1}(u|y_t = y) - E_{t+1}(u|y_t = 0)]$

The proof of Lemma 3-1 follows the same logic as Lemma 2-2.

Suppose the principal implements a profile of actions $[e_0, e_1(Y_0), \dots, e_{T-1}(Y_{T-2})]$. The marginal utility over the wage payment so that an agent is incentive compatible to take $\mathbf{e_t}$ is given by

$$\frac{\partial E_0[\mu(w)]}{\partial e_t},$$

where $E_0[\mu(w)]$ is the expected utility over wage payment, if he is incentive compatible to take the profile of actions $[e_0, e_1(Y_0), \dots, e_{T-1}(Y_{T-2})]$.

When participation constraint for taking e_t binds, we have

$$s(k_t) = -E_t(c) + E_t[\mu(w)] + \delta E_t[s(k_{T-1})].$$

Specifically, the agent's date 0 expected utility from wage payment for exerting $[e_0, e_1(Y_0), \dots, e_{T-1}(Y_{T-2})]$ so that the participation constraint binds is given by:

(35)

$$E_0(c) + s(k_0) - E_0(s) = E_0(c - s).$$

Suppose the principal implements a profile *of actions* $[e_0, e_1(Y_0), \dots, e_{T-1}(Y_{T-2})]$. *At date* t, *the marginal utility over wage payment to satisfy the participation constraint is given by*

$$\frac{\partial E_0(c-s)}{\partial e_t},$$

where $E_0(c - s)$ is the agent's expected utility at date 0, when his participation constraint binds for taking the profile of actions $[e_0, e_1(Y_0), \dots, e_{T-1}(Y_{T-2})]$.

Lemma 3-2. *The expected wage payment to implement the profile of actions* $[e_0, e_1(Y_0), \ldots, e_{T-1}(Y_{T-2})]$ *is increasing and convex in* e_t.

Proof:

It is easy to show that the marginal utility from wage payment to satisfy both the incentive and participation constraints are increasing and convex in e_t. I will prove that the marginal wage to satisfy the incentive constraint of taking e_t is increasing.

To implement additional effort at date t, the principal has to increase the wage payment for all production histories with $y_t = y$ by the same constant.

Clearly, we have $\frac{\partial E_0(w)}{\partial e_t} > 0$.

Since $u(.)$ concave, an increasing $\frac{\partial E_0[u(w)]}{\partial e_t}$ implies an increasing $\frac{\partial E_0(w)}{\partial e_t}$.

Therefore, the marginal wage payment for implementing e_t is increasing and convex.

Q.E.D

Proposition 3-1. *At date* t *of a finite* T*-period relationship, given that* $Y_{t-1} \neq (0, \ldots, 0)$, *the principle maximizes his expected profit by implementing* $e^*_{t+1}(y_t = y) < e^*_{t+1}(y_t = 0)$.

Proof:

I will prove Proposition 3-1by looking at the marginal wage payment to satisfy the incentive and participation constraints.

(i) Suppose the participation constraint is slack and only consider the incentive constraint.

Holding the effort e_τ unchanged for all $\tau \geq t+2$, if the principal implements additional effort at date $(t+1)$, he has to raise the wage payment $w(y_0, \dots, y_{T-1})$ for all outcomes with $y_{t+1} = y$. Otherwise it will change the agent's incentives after date $(t+1)$.

Suppose $e^*_{t+1}(y_t = y) = e^*_{t+1}(y_t = 0)$. According to the incentive constraint, we have

(36)

$$\delta\{E_{t+2}[u(w)|y_t = y, y_{t+1} = y] - E_{t+2}[u(w)|y_t = y, y_{t+1} = y]\} =$$
$$\delta\{E_{t+2}[u(w)|y_t = 0, y_{t+1} = y] - E_{t+2}[u(w)|y_t = 0, y_{t+1} = y]\}.$$

(ii) Suppose the incentive constraint is slack and only consider the participation constraint.

It is easy to see that

$$\frac{\partial E_0[c-s]}{\partial e^*_{t+1}(y_t = y)} = \frac{\partial E_0[c-s]}{\partial e^*_{t+1}(y_t = 0)}.$$

Holding the effort e_τ unchanged for all $\tau \geq t+2$, if the principal implements additional effort at date $(t+1)$, he has to raise the wage payment $w(y_0, \dots, y_{T-1})$ for all outcomes with $y_t = y$. Otherwise it will change the agent's incentives after date $(t+1)$.

Suppose $e_{t+1}^{*}\,(y_t = y) = e_{t+1}^{*}\,(y_t = 0)$. According to the incentive constraint, we have

(37)

$$\delta\{E_{t+2}[u(w)|y_t = y, y_{t+1} = y] - E_{t+2}[u(w)|y_t = y, y_{t+1} = y]\} =$$
$$\delta\{E_{t+2}[u(w)|y_t = 0, y_{t+1} = y] - E_{t+2}[u(w)|y_t = 0, y_{t+1} = y]\}.$$

For any given $(y_{t+1}, y_{t+2}, \dots, y_{T-1})$, it is clear that

(38)

$$w(Y_{T-1}|y_t = y) > w(Y_{T-1}|y_t = 0),$$

otherwise the agent won't get incentives working at date t.

Due to the concavity of the utility function $u(.)$, marginal utility decreases with wage.

Combining equations (37) and (38), we know that

$$\delta[E_{t+2}(w|y_t = y, y_{t+1} = y) - E_{t+2}(w|y_t = y, y_{t+1} = 0)]$$
$$> \delta[E_{t+2}(w|y_0 = y, y_{t+1} = y) - E_{t+2}(w|y_0 = y, y_{t+1} = 0)].$$

Since the marginal wage payment is higher when $y_{t+1} = y$, it is not profit maximizing to implement $e_{t+1}^{*}\,(y_t = y) = e_{t+1}^{*}\,(y_t = 0)$. Instead, the principal maximizes profit by implementing $e_{t+1}^{*}\,(y_t = y) < e_{t+1}^{*}\,(y_t = 0)$. Q.E.D.

The intuition is that the agent who delivers high output with a higher expectation on wage payment needs a higher risk premium, since a risk averse agent has decreasing marginal utility from wage.

Recall the results from Chapter 2 under the risk-neutral agent: After his delivery of the first high output, the principal would implement the same effort on the agent, independent of production realizations.

With the introduction of the risk-averse agent, this result no longer holds. Given that the agent has already produced some high output, the principal would implement a lower level of effort in the next period, upon the delivery of high output. The effort implemented by the principal still depends on the output realizations after the first high output. Therefore, realizing the first high output doesn't necessarily imply the fast-track.

Proposition 3-2. *At date T of a finite T-period relationship, given production histories $(y_0, \dots, y_{T-1})$ and $(y'_0, \dots, y'_{T-1})$ such that*

$$(y_0, \dots, y_{i-1}) = (y'_0, \dots, y'_{i-1}) \neq (0, \dots 0), (y_{i+1}, \dots, y_{T-1}) = (y'_{i+1}, \dots, y'_{T-1})$$

$$y_i = y, y'_i = 0 \textit{ for some } i \geq 1,$$

we have $s(k_T|y_0, \dots, y_{T-1}) < s(k_T|y'_0, \dots, y'_{T-1})$.

Proof:

It is clear that $e_\tau(y_0, \dots, y_{\tau-1}) = e_\tau(y'_0, \dots, y'_{\tau-1})$ for all $\tau < i$.

According to Proposition 3-2, we know that $e_\tau(y_0, \dots, y_{\tau-1}) > e_\tau(y'_0, \dots, y'_{\tau-1})$, for all $\tau \geq i$.

Therefore, one has a higher stock of effort at date T, following the production history $(y'_0, \dots, y'_{T-1})$.

3.3 Conclusions

As an extension of Chapter 2, this chapter develops a principal-agent problem with a risk-averse agent and a risk neutral principal. Under the new assumptions, the fast-track effect described in Chapter 2 is mitigated: implementing higher effort following the realization of the first high output would not only provide work incentives but also generate higher risk-premium to be paid by the principal. Therefore, delivering the first high output doesn't necessarily imply fast tracks. Given that the first success is realized, the delivery of high output would rather delay the career advancement, due to the disincentives from wage increases.

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