The Robustness of Multilevel Multiple Imputation for Handling Missing Data in Hierarchical Linear Models

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Dedication

I would like to dedicate this dissertation to my mother Lemlem Kidane. She did a wonderful job of raising three children on her own despite tremendous adversity. It is because of her that I will never give up on my goals. I would also like to dedicate this dissertation to my sister Senayet who helped raise both my younger sister and me despite the fact that she was a child herself. She grew up early so that we could enjoy being children. I also dedicate this dissertation to my sister Mehret who has made it a joy to be an older brother. I also dedicate this dissertation to my beautiful future wife Kristen Neigebauer. Thank you for always being understanding even when pursuing my PhD meant less time to spend together and for always being a source of calm and support.

Abstract

Missing data often present problems for credible statistical analyses. Luckily there are valid methods for dealing with missing data but the context in which the data are missing can impact the performance of these methods. Relatively little is known about the proper way to handle missing data in multilevel data structures. This study used a Monte Carlo simulation to compare the performance of three missing data methods on multilevel data (multilevel multiple imputation, multiple imputation ignoring the multilevel structure, and listwise deletion). The comparison of these methods was made under conditions known or believed to influence both the performance of missing data methods and multilevel modeling. The results suggest that listwise deletion performs well compared to multilevel multiple imputation but multiple imputation ignoring the multilevel structure performed poorly. The implications of these results for educational research are discussed.

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Chapter 1

Introduction

1.1 Statement of the problem

Educational research can present unique challenges. The fact that educational data are often captured on students that exist within hierarchical structures (e.g., classrooms, schools, school districts) can pose a challenge to researchers. Because students in a particular classroom share common experiences (e.g., are instructed by a common teacher, receive the same instruction, read from the same material, have access to similar resources), they tend to perform similarly. That is, by virtue of being in a similar environment, students within the same class are more likely to have test data that are more similar to one another relative to students in different classes. One of the implications of this dependence is that traditional statistical analyses which assume outcomes are independent of one another (such as multiple regression) can produce biased hypothesis tests because of biased standard errors.

One of the statistical methods that takes into account the dependence that can exist in hierarchical/nested data is known as hierarchical linear modeling (HLM). HLM partitions the variance in the outcome between different levels of the units of analysis (e.g., variance due to students vs. variance due to classrooms)(S. W. Raudenbush & Bryk, 2002). This approach allows researchers to obtain accurate standard errors so that corresponding hypothesis tests are unbiased.

Another common challenge present in educational research is missing data. Data can be missing for a variety of reasons. For example, a student may be sick one day and therefore not have test data. Alternatively, a student may skip a question in a survey. The existence of missing data can have a negative impact on statistical analyses. When missing data do exist, the researcher must decide how to treat them. One way of treating missing data is to remove observations from statistical analyses if any variables of interest have missing values (Graham, 2009). This approach is known as listwise deletion (LD). LD is the default in many statistical software programs.

The use of LD can be problematic but the extent to which it is a problem depends on why the data are missing. In the best case scenario, the data can be missing completely at random (MCAR). When the data are MCAR the probability of missingness is unrelated to either observed or unobserved variables and therefore the use of LD would lead to unbiased results. However, when the data are not MCAR the subsequent statistical analyses can be biased.

There are alternatives to the use of LD for treating missing data but some of these alternatives have adverse effects on subsequent analyses. For example, single imputation is the process of replacing missing values with plausible values using the available data but it does not take into account the uncertainty associated with the missing data (Acock, 2005). Newer treatments for missing data such as multiple imputation (MI) and maximum likelihood estimation (ML) are an improvement over the more traditional methods such as LD.

Despite the prevalence of both missing data and nested data in educational research, the treatment of missing data in the context of a nested data structure has received little attention. Multilevel multiple imputation (MLMI) is a missing data technique that performs multiple imputation in the context of nested data and it appears to be a viable solution. A small number of programs exist that will perform MLMI. Additionally, few studies have examined competing approaches to handling missing values in a nested data structure.

The purpose of this study is to compare three missing data techniques; LD,MI, and MLMI when applied to nested data structures that contain missing data. Furthermore, this study compares these techniques under factors known or believed to impact the performance of both missing data techniques (percentage of missing data, the reason for missingness) and HLM (level two sample size, intraclass correlation, and distribution of random effects).

Chapter 2 reviews the literature on HLM, missing data, and multilevel missing data. It is within chapter 2 that the reasons for multilevel modeling and missing data treatments are outlined. Chapter 3 outlines the research question as well as the methods used to answer the research question. Chapter 4 presents the results from this study and synthesizes the results to arrive at answers to the research question. Finally, chapter 5 discusses the results and implications of the findings for applied and methodological researchers.

1.2 Significance of the study

MI has been established as a viable modern missing data treatment (Schafer & Graham, 2002; Schafer, 1997) and HLM is a statistical technique which has been growing in popularity. Despite the common occurrence of missing data in educational research many researchers fail to report the impact of missing data on their statistical analyses and inferences. Because the use of multiple imputation in a multilevel context is complicated it is important to understand the conditions under which MLMI contributes to valid inferences and when imputing missing data provides an advantage over simpler alternatives such as MI and LD.

Some studies that have examined the performance of missing data treatments in a nested data structure have applied missing data treatments in a way that is inconsistent with the nested data structure. These studies shed little light on the performance of missing data treatments in a nested data structure because the performance of these methods is confounded with ignoring the structure of the data. This study hopes to examine the performance of a properly specified MLMI procedure. Additionally, this study hopes to examine the robustness of this missing data treatment by examining its performance under various factors that can impact both missing data treatments and HLM.

Chapter 2

Literature Review

2.1 Missing data and implications for research findings

Research involving human participants can present unique challenges. Among these challenges are things like adequately representing and measuring constructs of interest (e.g. mathematical knowledge), and obtaining representative samples of the target population. Another common problem is missing data. Even after obtaining a sample of participants as well as all of the required consents, participants may drop out of the study or not participate fully (e.g. not answer all questions). When this occurs, the result is a data matrix with missing values. Missing data can produce biased estimates of sample statistics and model parameters when the missing data are treated inappropriately (Peng, Harwell, Liou, & Ehman, 2006; Rubin, 1987; Schafer, 1997).

2.2 Missing data mechanisms

Rubin (1976) classified three different mechanisms of missing data. These three mechanisms have come to be called MCAR (missing complete at random), MAR (missing at random), and MNAR (missing not at random) and can be defined in terms of the probability of a given variable being unobserved (i.e. having a missing value). Let Y be a matrix of variables with n rows and p columns. Let R be a matrix of response indicators with n rows and p columns. R can take on the value of 1 if the corresponding variable value is missing and 0 if the corresponding value is observed (i.e. if $R_{12} = 1$, then Y_{12} is missing, if $R_{12} = 0$, then Y_{12} is observed). The probability that a given variable is missing can be written as $P(R|Y)$. Because the data matrix Y can have both observed and missing values (Y_{obs}, Y_{mis}), the probability that a given variable is missing can be written as $P(R|Y_{obs}, Y_{mis})$. When the missing data mechanism does not depend on either observed or missing values, the condition of MCAR is satisfied and $P(R|Y_{obs}, Y_{mis}) = P(R)$. That is, the probability that a variable is missing is unrelated to either observed or unobserved variables and is therefore a constant across individuals. One example of when missing data may be MCAR include when a respondent accidentally skips an item on a survey.

When the missing data mechanism depends on observed data but not on unobserved data, MAR is satisfied and $P(R|Y_{obs}, Y_{mis}) = P(R|Y_{obs})$. For example, consider a study in which one asks respondents for their income. If the likelihood of a respondent answering the income question was related to their education which was also collected, the missing values for the income variable are MAR. Finally, when the probability of missingness depends on the missing values themselves, the missing data mechanism is said to be MNAR and $P(R|Y_{obs}, Y_{mis}) = P(R|Y_{mis})$. In this case, the probability that a given variable is observed is directly a function of the values that would have been observed for that variable. Again, consider the example of a study in which one asks for the income of respondents. If those with higher incomes are less likely to respond to this question, the missing values for the income variable are MNAR.

2.3 Ignorable and non-ignorable missingness

These three missing data mechanisms (MCAR, MAR, and MNAR) can be classified into the categories of ignorable or nonignorable missingness. To introduce these definitions, consider two additional terms θ and ϕ. θ represents the set of unknown parameters governing the data model, and ϕ represents the set of unknown parameters governing the missing data mechanism. Ignorable missingness reflects the case when the parameter governing the distribution of the data is distinct from the parameter governing the missingness mechanism conditional on the data (Raessler et al., 2008; Schafer, 1997). Nonignorable missingness reflects the case when the mechanism is not ignorable.

MCAR is an ignorable missing data mechanism because the probability of the observed data can be written under this missing data mechanism as $P(R, Y_{obs}|\theta, \phi) = P(R|\phi)P(Y_{obs}|\theta)$. In the case of MCAR, the missing data can be thought of as a simple random sample of the full data matrix and sample statistics based on the observed data are unbiased. Aside from a loss of statistical power (ability to detect an effect when one exists), the deletion of cases with missing data under MCAR is an appropriate technique for dealing with such a dataset. This is because under MCAR the observed data do not differ in any way from the complete data (Acock, 2005; Rubin, 1987; Peugh & Enders, 2004; Schafer & Graham, 2002; Horton & Kleinman, 2007). More formally, under MCAR the probability of a missing value for a given variable is unrelated to either observed or unobserved variables rendering the missing data mechanism independent of the distribution governing the observed data. In some cases researchers can use this characteristic of MCAR to their advantage as when one imposes planned missingness (Graham, Hofer, & MacKinnon, 1996; Graham, 2009). Consider a situation in which a researcher is developing an item pool for an academic achievement test. The researcher may be considering hundreds of items for the item pool. However, giving each respondent all the items is not feasible as the respondents may experience fatigue. In such a case, the researcher can administer a subset of items to each respondent and assign the subset of items to respondents randomly. Because the missing data are MCAR, summary statistics such as correlations, means, and variances are unbiased.

Despite the advantages of MCAR many researchers suggest that it may not always be a tenable assumption (Acock, 2005; Puma, Olsen, Bell, & Price, 2009; Schafer & Graham, 2002). A less restrictive assumption of the missing data mechanism is that the data are missing at random (MAR). This assumption, in essence, states that conditional on observed covariates, the data are MCAR and can be written as $P(R, Y_{obs}|\theta, \phi) = P(R|Y_{obs}, \phi)P(Y_{obs}|\theta)$. This assumption is less restrictive than the MCAR assumption in that it allows for the missingness to depend on observed variables while MCAR requires that the missingness is unrelated to either observed or unobserved data. However, for this assumption to be upheld, the researcher must collect data on variables that explain why the data are missing. When the missing data are MAR, and the parameters θ and ϕ are distinct (i.e. ignorable missing data mechanism), the use of likelihood or Bayesian-based inferences allows the analyst to ignore the missing data mechanism and work with the observed data likelihood (Rubin, 1976, 1987; Schafer, 1997). Specifically, when one uses maximum likelihood estimation (ML) under MAR the data analyst does not need to model the missing data mechanism, but rather can simply include the covariates allowing for the MAR assumption to be tenable (Schafer, 1997). This point is an important one and will be discussed within the context of Hierarchical Linear Modeling later in this paper.

MNAR represents a non-ignorable missing data mechanism (Schafer & Graham, 2002). In the case of MNAR, it is generally required that the data analyst incorporate a model for the missing data mechanism and include it in subsequent analyses for inferences to be unbiased (Puma et al., 2009). Ways of dealing with MNAR are being explored (Schafer & Graham, 2002).

2.4 Traditional missing data treatments

What does one do when they have missing data? Traditional methods of dealing with missing data include listwise deletion, pairwise deletion, mean imputation, and regression-based imputation.

2.4.1 Listwise deletion

Listwise deletion (LD) involves the removal of cases with missing values on any of the variables used in the statistical model. LD is the default missing data treatment in many statistical software packages. While the application of LD is convenient, its ability to produce unbiased estimates is determined by the missing data mechanism (Sinharay, Stern, & Russell, 2001). Furthermore, the use of LD results in a loss of statistical power. In the case of MCAR, LD leads to unbiased estimates because the missing data can be thought of as a simple random sample of the complete data matrix (Horton & Kleinman, 2007). As mentioned above, under MCAR the probability of a given variable having a missing value is independent of either observed or unobserved variables. This means each case in the data matrix has an equal probability of having a missing value for a given variable therefore cases with at least one missing value on a variable in the statistical model, which would be excluded in LD, can be thought of as a random sample of the sampled cases. As a result, sample statistics obtained from the complete data are unbiased estimates under MCAR.

2.4.2 Pairwise deletion

An alternative approach to LD is pairwise deletion (PD). PD makes use of all available data by only excluding cases from calculations if they have missing values. For example, consider three continuous variables (X_1, X_2, X_3) each of which contains a certain proportion of missing data. Under pairwise deletion the correlation between X_1 and X_2 would be based on those cases that had values for both variables X_1 and X_2 regardless of whether those cases had missing values on X_3 . Likewise, the correlation between X_1 and X_3 would be based on cases with values for both variables X_1 and X_3 regardless of whether those cases had missing values for X_2. PD can result in different sample sizes for the separate correlations as they are not based on a single sample. When the data are MCAR, sample statistics based on PD are unbiased. Additionally, unlike LD, PD makes use of all available data. However, when the data are MAR or MNAR PD can result in biased parameter estimates because the probability of missingness is related to either observed or unobserved variables. Another disadvantage of PD is that it is unclear to which population the inferences generalize because statistics are based on different subsamples. Additionally, if the missing data mechanism is not MCAR PD can present computational problems such as covariance matrices that are not of full rank (Schafer & Graham, 2002; Acock, 2005; Graham, 2009).

Treatment of missing data through LD and PD have been referred to as ad hoc missing data treatments as they are often applied by default and without consideration to the nature of the missing data (Peng et al., 2006). If the data are not MCAR and one uses LD or PD as the missing data treatment the inferences made from the remaining data are likely to be biased and may not represent the original sample. Furthermore, these ad hoc missing data treatments are model dependent in that sample size differs as a function of the models specified (i.e. the inclusion or exclusion of a covariate with missing values will result in a different sample when either LD or PD is used). It is important to note that the assumption of the missing data mechanism is not an assumption that can be checked (Schafer & Graham, 2002; Stuart, 2010). It is for this reason that a naive assumption of MCAR and subsequent use of LD or PD is considered to be inappropriate for statistical analyses.

2.4.3 Single imputation

Another method of dealing with missing data is to impute the missing values. Imputation is the process of substituting plausible values for the missing values in a data set (Schafer, 1999). One approach, referred to as mean imputation, involves replacing the missing values of a variable with the mean of the observed values for that variable. This process is not recommended as it results in a decrease of the variance of variables for which mean imputation is applied. It will therefore decrease the covariance between the mean imputed variable and other variables in the dataset (Acock, 2005; Peugh & Enders, 2004; Sinharay et al., 2001).

Another single imputation method involves the use of statistical models (model-based approach) based on the observed data to predict missing values on variables. Multiple regression (in the case of a continuous variable with missing values) is a method that can be used to impute missing values based on observed data. However, the use of regression-based imputation can inflate the relationship between variables in a dataset because imputed values are completely a function of other variables in the dataset. This inflation can be mitigated by introducing random error in the estimates for the missing values (Buck, 1960; Peugh & Enders, 2004; Schafer & Graham, 2002). While the inclusion of random error in a regression-based single imputation can help to mitigate the inflation of the relationship between variables, subsequent analyses do not take into account the uncertainty of the missing values that were imputed (Gelman & Hill, 2006; Schafer, 1999; Stuart, 2010). This is true of all single imputation procedures. The actual missing values are unknown and single-imputation procedures do not take this fact into account.

Despite the weakness of traditional missing data treatments, evidence suggests that their use is still prevalent. Peng et al. (2006) reviewed 11 education journals from 1988 to 2004 and found that, out of the studies that reported dealing with missing data, the majority used either LD or PD. Peugh and Enders (2004) reviewed 23 journal volumes published in 2003 and found that the majority of studies used LD. Additionally, Black (2008) reviewed papers published in the American Educational Research Journal, Journal of Educational Research, and the Journal of Educational Psychology in 2006 and 2007 and found that the majority of studies used LD.

The deficiencies of traditional missing data treatments led to the development of more appropriate methods which are often referred to as modern missing data treatments.

2.5 Modern missing data treatments

Modern missing data treatments include multiple imputation, likelihood-based procedures, and Bayesian procedures among others (Schafer, 1997; Acock, 2005). The first two modern missing data treatments (multiple imputation and likelihood-based procedures), which are the focus of this paper, are described below.

2.5.1 Multiple imputation

An alternative to single-imputation procedures is the use of multiple imputation (MI). MI was originally developed by Rubin (1987) as a method for creating public-use data files based on complex survey data. It was also developed for the condition where there may be several reasons for why data are missing.

MI consists of two steps. First, the missing portions of the data set are imputed a number of times (m). As in the case with single imputation methods, the goal is to replace missing values with plausible values that maintain the relationships among the data. When the missingness is related to observed data it is logical to incorporate the variables that help explain missingness in the imputation. MI accomplishes this by using special procedures such as a Monte Carlo Markov Chain (MCMC) procedure which has as its goal the generation of random numbers from a probability distribution (Schafer, 1999, 1997). In the case of continuous variables, it is often convenient to assume a multivariate normal distribution for the variables (Cai, 2008; Horton & Kleinman, 2007; Jacobusse, Buuren, & Groothuis-Oudshoorn, 2005; Reiter, Raghunathan, & Kinney, 2006; Stuart, 2010). Let Y represent a data matrix consisting of the variables $Y_1, Y_2, ..., Y_J$. We assume that the density function of the multivariate distribution is multivariate normal $f(Y) \sim N(\mu, \sigma^2)$. Assuming multivariate normality, and assuming that missingness is related to observed variables (MAR) one can obtain random draws from the joint distribution of the data using the Gibbs sampler (Schafer, 1997). Initially, a variable in the data matrix is sampled from the marginal distribution of that variable given all other variables in the data set $Y_1^{(t+1)} \sim P(Y_1|Y_2^{(t)}, Y_3^{(t)}, ..., Y_J^{(t)})$ assuming an initial value for each of the variables $Y_2^{(t)}, Y_3^{(t)}, ..., Y_J^{(t)}$. Then, the next variable in the data set is sampled from the marginal distribution of that variable, given all the variables from the previous step $Y_2^{(t+1)} \sim P(Y_2|Y_1^{(t+1)}, Y_3^{(t)}, ..., Y_J^{(t)})$. This process continues for all variables in the dataset resulting in a set of point estimates for all variables in $P(Y_{mis}|Y_{obs})$. As $t \to \infty$ the distribution of Y becomes the target distribution from where imputed values are drawn. In this way MCMC is used to generate random draws of the missing values from $P(Y_{mis}|Y_{obs})$. MI is a missing data treatment that precedes the inferential analysis of the data.

Multiple imputation is sometimes criticized. Researchers that study missing data techniques have addressed many if not all of these criticisms (e.g. Rubin, 1996; Schafer, 1999). This paper will not rehash all of the criticisms and the responses to those criticisms, however the author will discuss two criticisms in particular.

Given the nature of MI and its reliance on simulation, some may argue that MI is simply making up data. Schafer (1999) argues that this is not an accurate representation of multiple imputation. This criticism would be true of single imputation which treats the missing values as if they are known. In contrast, MI simply attempts to take into account the uncertainty of the statistical model due to having missing values, and not knowing what those missing values would have been. Alternative approaches such as LD and single imputation assume there is no uncertainty associated with missing data.

Another criticism is that when using MI, one is assuming that there is more data, or more information, than is actually present. When the inferential analysis is performed on each complete dataset, the parameter estimates can be combined using Rubins rules which calculate the variance of the parameter estimate as a combination of the within imputation variance and between imputation variance. Therefore, as the proportion of missing information increases, and the uncertainty due to missingness increases, the uncertainty in the parameter estimate also increases (Rubin, 1996). In this way, more uncertainty due to missing data results in a penalization in the certainty of parameter values.

An alternative missing data treatment used in conjunction with the inferential analysis is maximum likelihood estimation (ML) (Black, 2008; Graham, 2009; Sinharay et al., 2001).

2.5.2 Maximum likelihood estimation

Maximum likelihood estimation (ML) is an estimation technique that has as its goal the maximization of the likelihood of the observed data, conditional on a set of parameters of interest $L(Y|\theta)$. In the case where the data matrix Y consists of both observed and missing values, the likelihood function can be written as $L(Y_{obs}, Y_{mis}|\theta, \phi)$ or $L(Y_{obs}, R|\theta, \phi)$. Rubin (1976) and Rubin (1987) showed that under ignorability, and when using likelihood-based or Bayesian approaches to estimate the parameters of interest, one does not need to incorporate a model for the missing data mechanism. Because the parameters describing the missing data mechanism and the parameters of interest are distinct under the ignorability assumption, this allows for the factorization of the likelihood function pertaining to the observed data and the missing data mechanism into separate components. This factorization leaves the likelihood function pertaining to the observed data (observed data likelihood) proportional to the likelihood function of the parameters given the observed data (Schafer, 1997). Therefore under MAR, inferences about the parameters governing the distribution of the dependent variable can be made based on the observed-data likelihood.

Schafer (1997) illustrates the observed-data likelihood function for the example of multivariate normal data with arbitrary patterns of missingness. This example is presented below.

$$\prod_{i=1}^{S} \prod_{i \in I(s)} |\Sigma_s^*| exp(-\tfrac{1}{2}(y_i^* - \mu_s^*)^T \Sigma_s^{*-1}(y_i^* - \mu_s^*)$$

Where s is a distinct missing data pattern among the variables, i is the number of cases within a distinct missing data pattern (s), y_i^* is the observed data within a case, μ_s^* is the vector of means for the observed variables in a given missing data pattern (s), and Σ_s^* is the variance/covariance matrix for the observed variables in a given missing data pattern(s).

Schafer (1997) points out that such a likelihood function is complex and difficult to estimate and therefore requires iterative approaches such as the EM algorithm (Beale & Little, 1975; Arthur, Nan, & Donald, 1977; Rubin, 1976). The EM algorithm consists of two steps, the expectation step and the maximization step. In the expectation step the algorithm computes the expected value of the complete-data sufficient statistics in the presence of missing data, given initial values for the parameters of interest. In the maximization step the expected values in the previous step (the output of the expectation step) are used to maximize the parameters of interest.

Schafer (1997) illustrates the EM algorithm by stating that in any data problem where missing data are present the distribution of the complete data (Y) can be written as $P(Y|\theta) = P(Y_{obs}|\theta)P(Y_{mis}|Y_{obs})$. This can be written as a likelihood function in terms of θ as $l(\theta|Y) = l(\theta|Y_{obs}) + log(P(Y_{mis}|Y_{obs}, \theta)) + c$, where c is an arbitrary constant. Because Y_{mis} is unobserved, its predictive distribution $P(Y_{mis}|Y_{obs}, \theta)$ cannot be calculated. Instead $l(\theta|Y)$ is averaged over $log(P(Y_{mis}|Y_{obs}, \theta))$ given a preliminary estimate of $\theta(\theta^{(t)})$. This averaging is performed in the E step of the EM algorithm and yields $\int l(\theta|Y)P(Y_{mis}|Y_{obs}, \theta^{(t)})dY_{mis}$.

The value of θ that maximizes the function above ($\theta^{(t+1)}$) is computed in the M step of the EM algorithm. This new estimate is then used in the next E step and the process is repeated until the observed data likelihood converges, yielding maximum likelihood estimates of the parameters of interest.

2.6 Nested data

In order to study missing data strategies in multilevel models it is necessary to first define nested data. Nested data refers to the presence of a hierarchical structure of data where units of observation exist within other units of observation (S. W. Raudenbush, 1988). For example, students are nested within classrooms. In this example, students are considered to be at level one (the lower level unit of analysis) while classrooms are at level two (the higher level unit of analysis).

While this paper focuses on nested data structures consisting of two levels, it is possible to have nested data that consists of more than two levels (e.g. students nested within classrooms that are nested within schools). Furthermore, in the current study, the focus is on cross-sectional nested data. With cross-sectional data, units of observation are measured at one point in time on variables of interest. For example, students who are nested in classrooms may be measured on their performance in a state mandated eighth grade mathematics test and hours spent studying prior to the test. The researcher may also collect data on classroom characteristics, such as each teachers years of mathematics education to examine its impact on student test performance. This example differs from a situation in which a researcher were to collect data on the mathematics performance of these students from eighth grade through twelfth grade each year. The latter would be considered repeated measures/longitudinal data. While valuable inferences can be gleaned from such data, it is important to note that this paper focuses on cross-sectional nested data and that the missing data treatments discussed do not directly apply to missing longitudinal data.

2.7 Implications of nested data

Traditional statistical techniques like multiple regression assume the responses from units of observation (i.e. students in the previous example) are independent of one another (S. W. Raudenbush, 1988; S. W. Raudenbush & Bryk, 2002). This assumption may be violated in nested data structures. For example, students within the same classroom may be more similar to one another than to students in a different classroom. The mechanisms underlying the violation of this assumption are intuitive in the example presented here. Students within a classroom share many characteristics that are likely to impact their performance in a state mandated mathematics test. This may include having the same teacher, textbooks, and assignments. Violation of the assumption of independence renders the use of traditional statistical procedures such as multiple regression problematic. In a hierarchical/nested data set, the violation of the independence assumption corresponds to a correlation among the error terms within classrooms. As such, the estimation of standard errors in a traditional linear regression is likely to be biased downward resulting in higher type I errors in the tests of parameter estimates (Maas & Hox, 2005). As the dependence within level two units increases, the bias in standard errors produced by traditional methods such as single-level regression increases (Julian, 2001; Walsh, 1947).

2.8 Hierarchical linear modeling

The failure of traditional analyses to accommodate nested data well spurred the development of a new class of statistical procedures for analyzing these data (Draper, 1995;

S. W. Raudenbush, 1988; S. Raudenbush & Bryk, 1986). One of these statistical modeling procedures is known as hierarchical linear modeling (HLM). HLM accommodates nested data by allowing the researcher to partition the variance in the outcome variable among different levels of the model (S. W. Raudenbush & Bryk, 2002). To illustrate HLM this chapter will use an example of a two-level model in which students (level one units) are nested within classrooms (level two units). An unconditional HLM, one with no predictors at either level of the model, has the form:

$$Y_{ij} = \beta_{0j} + e_{ij} \tag{2.1}$$

$$\beta_{0j} = \gamma_{00} + u_{0j} \tag{2.2}$$

$$e_{ij} \sim N(0, \sigma^2) \tag{2.3}$$

$$u_{0j} \sim N(0, \tau_{00} \tag{2.4}$$

In the model above, there are no predictors at either level one or level two. Level two includes a random effect (u_{0j}) which allows for the intercept to vary across the level two units (j). A fully unconditional model corresponds to a one-way random effects analysis of variance (ANOVA) (S. W. Raudenbush & Bryk, 2002). This fully unconditional random intercepts model allows the researcher to partition the variance in the outcome (Y_{ij}) into between classroom variance and within classroom variance (τ_{00}, σ^2, respectively) (S. W. Raudenbush & Bryk, 2002). The ratio of the between classroom variance to the total variance ($\rho = \frac{\tau_{00}}{\tau_{00}+\sigma^2}$) represents the proportion of the variance in the outcome that can potentially be explained by classroom level predictors and is referred to as the intraclass correlation (ICC) (S. W. Raudenbush & Bryk, 2002). This value also represents the within level two dependence. While there is no consensus on how much of the variance in the outcome should be between level two units to warrant the use of an HLM, as this value increases the use of a traditional multiple regression becomes more and more problematic. One can calculate the design effect which estimates the expected downward bias in the standard errors in a complex sample compared to a random sample by using the formula $D = (1+(\hat{n}_j-1)\times\rho)$ where $\hat{n}_j$ is the average within group sample size (Kish, 1965). The way in which HLM partitions the variance in the outcome to between and within variance is what allows for dependence within level two units.

Including predictors in the first level of the model attempts to explain the within classroom variance and including predictors at the second level of the model attempts to explain the between classroom variance. Consider a model with one level one predictor whose effect is allowed to vary across level two units:

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{1ij} + e_{ij} \tag{2.5}$$

$$\beta_{0j} = \gamma_{00} + u_{0j} \tag{2.6}$$

$$\beta_{1j} = \gamma_{10} + u_{0j} \tag{2.7}$$

To illustrate the utility of such a model, consider a study in which a researcher is interested in decreasing the impact of socioeconomic status (SES) on students performance on a standardized test. The student/level one model (2.5) predicts an outcome (Y_{ij}) using a student level variable, X_{1ij} (e.g. SES). Therefore, β_{1j} represents the relationship between SES and performance on the standardized test within a classroom (j). This value, β_{1j}, is allowed to vary across classrooms, which conceptually would mean that the relationship between SES and test performance can vary from classroom to classroom. By including predictors at level two for β_{1j}, (2.7) one attempts to predict the relationship between SES and performance with classroom level variables (e.g. the impact of number of hours a teacher receives in instructional training on the relationship between SES and test performance).

In an example where there are several variables predicting the outcome at level one, Y_{ij}, the relationship between these variables and the outcome for each classroom can be described using a vector of parameter estimates $\beta_{pj} = [\beta_{0j}, \beta_{1j}, \beta_{2j}, ..., \beta_{pj}]$, where p represents the level one predictor and j represents a particular classroom. Again, modeling these parameter estimates at level two enables the researcher to examine the variation in the relationship between covariates at level one and the outcome variable across level two units. This is another powerful feature of HLM. If a classroom level predictor is a significant predictor of a slope at level one (e.g., 2.7), it suggests that the relationship between a covariate at level one and the outcome is a function of a classroom level variable.

2.9 Estimation of parameters in HLM

With regard to HLMs, there are three types of parameters that need to be estimated; fixed effects (γ), the random level one coefficients (β_{qj}), and the variance/covariance components (τ, σ^2). This section describes how each type of parameter is estimated in the statistical software package HLM 6 (S. W. Raudenbush, 2004) which is used by many researchers when analyzing multilevel data.

2.9.1 Fixed effects

Estimation of fixed effects is accomplished through weighted least squares in which the precision of the estimate for each group is taken into account. Using matrix notation, the equation for the fixed effects can be expressed as $\hat{\gamma} = (\sum W_j^T \Delta_j^{-1} W_j^{-1}) \sum W_j^T \Delta_j^{-1} \hat{\beta}_j$, where W_j is a $(Q+1)$ by F matrix of predictors, Δ_j^{-1} is the inverse of the variance/covariance matrix and is referred to as the precision matrix (S. W. Raudenbush & Bryk, 2002). This precision matrix is made up of parameter dispersion and error dispersion terms $\Delta_j^{-1} = T + V_j = Var(u_j + e_j)$. In the event that each level two unit has the same number of level one units, each level two unit has the same set of level one covariates, and the same set of level two predictors for each level one slope, the expression above for the fixed effects would reduce to the OLS regression estimator $\hat{\gamma} = (\sum W_j^T W_j)^{-1} \sum W_j^T \hat{\beta}_j$. Therefore, it can be seen that the estimation of fixed effects is accomplished by weighting the contribution of a level two unit's OLS estimate by the precision matrix for that level two unit and then combining them into a single estimate.

2.9.2 Random level one coefficients

The estimation of random level one coefficients (β_{qj}) is accomplished through empirical Bayes estimation in which the estimates are again weighted by the precision $\beta_j^* = \Lambda_j \hat{\beta}_j + (I - \Lambda_j) W_j \hat{\gamma}$, where $\hat{\beta}_j = (X_j^T X_j)^{-1} X_j^T Y_j$ and is the OLS regression estimator for group j. The term $\Lambda_j = T(T + V_j)^{-1}$ reflects the reliability of the random level one coefficient. The more precise or reliable the estimate for a group, the more weight is given to the OLS regression estimator for that group in estimating the random level one coefficient. The less precise the estimate for a group, the more weight is given to the fixed effect for that coefficient when estimating the random level one coefficient.

2.9.3 Variance/covariance components

The variance/covariance components are typically estimated through maximum likelihood or Bayes estimation. With regard to maximum likelihood, one can use either full (FML) or restricted maximum likelihood (RML) estimation. The distinction between these two approaches when estimating variance/covariance components is that RML adjusts the variance/covariance components for uncertainty about the fixed effects (S. W. Raudenbush, 1988; S. W. Raudenbush & Bryk, 2002) whereas FML does not. The level one error variance will often be similar between the two techniques whereas differences may arise in the variance/covariance component associated with the random effects. However, when there is a large number of level two units, both methods will produce similar results with regard to the variance/covariance components.

2.9.4 Expectation-maximization algorithm

The estimation of the coefficients in HLM depends on the variance components and the estimation of variance components depends on the coefficients. This mutual dependence between the coefficients and the variance components in HLM lends itself to the use of an iterative procedure such as the expectation-maximization (EM) algorithm. S. W. Raudenbush and Bryk (2002) describe the use of the EM algorithm in the context of HLM by stating that while the values for the dependent variable for individuals within level two units (Y_{ij}) are observed, the random effects (u_j) are not observed and can therefore be considered missing data. The authors show that if both Y_{ij} and u_j were observed, each of the parameters could be estimated. However, given that the random effects are not observed, the sufficient statistics necessary to estimate the parameters can be obtained by using Y_{ij} and initial guesses for the parameters γ, τ, σ^2.

In the first E-step of the EM algorithm, the initial guesses for the parameters are used to estimate the sufficient statistics. Then, in the M-step the sufficient statistics estimated from the previous E-step are substituted into the complete data ML estimates of the parameters. This process represents a single iteration of the EM algorithm. The process is repeated until the algorithm converges, such that the change in consecutive iterations of the algorithm produces little change in the likelihood function (S. W. Raudenbush & Bryk, 2002; Schafer, 1997).

2.10 Statistical assumptions of HLM

HLM, like other statistical analysis procedures, requires that a set of assumptions be met for the inferences to be valid. Violations of assumptions for HLM may render the estimates and/or statistical tests of estimates problematic. For a two-level HLM, the residuals at level one are assumed to be independent of the covariates at level one ($e_{ij} \perp x_{ij}$). The random effects at level two are assumed to be independent of the covariates at level two ($u_j \perp W_j$). The random effects associated with the coefficients at level one are assumed to be independent of one another ($u_j \perp u'_j$). Additionally, the covariates at either level are assumed to be independent of the covariates at the other level ($x_{ij} \perp W_j$). There are also distributional assumptions associated with both level one residuals as well as the level two random effects. Specifically, the level one error is assumed to be normally distributed with a mean of 0 and variance of σ^2. The level two random effects are assumed to have a multivariate normal distribution with mean 0 and variance τ_{qq}, where q is the random effect of interest.

2.10.1 Sample size

The statistical power of HLM is highly dependent on the number of units at the highest level. As mentioned above, HLMs can be fit using ML. However, ML methods used in multilevel models require large sample sizes (Maas & Hox, 2004a). Failing to obtain an appropriate sample size at the highest level can have an adverse impact on the estimates obtained in HLMs. For two-level HLM analyses, Gibson and Olejnik (2003) suggested a minimum sample size at level two of 30. However, Maas and Hox (2005) found downward bias of regression coefficients when the number of level two units was less than 50. While small sample sizes within level two units is typically not a problem for the estimation of fixed effects, they do impact the testing of random slope variances because the sample size within level two units influences the reliability of the estimates of random slopes (Snijders, 2005). Maas and Hox (2005) simulated data for a two-level model with one predictor at each level under varying sample sizes at both levels (30, 50, and 100 at level two; 5,30, and 50 within level two units) and varying ICCs (.1, .2, .3). In general they found that the fixed effects and the corresponding standard errors were estimated accurately under their conditions. However, the authors also found that the level two variance components were too small when the number of level two units was considerably lower than 100.

Sample size at both level one and two can also have mitigating effects for the impact of assumption violations. For example, Maeda (2007) examined the effect of the omission of a key level one covariate which manifests itself as a violation of the assumption of independence between the level one error term and the level two covariates. Maeda found that the accuracy of fixed effects estimates improved as both level one and level two sample size increased.

Sample size at level two also has implications for the estimation of standard errors in HLM. Smaller sample sizes at level two tend to result in downwardly biased standard errors for both the fixed effects and variance components at level two, with the latter more susceptible to this bias (Maas & Hox, 2004b). When the sample size at level two is small, RML estimation can help to alleviate some of the problems with bias in standard errors (Brownel & Draper, 2000). In addition to sample size, the ICC has implications for the estimation of standard errors. Hox and Maas (2001) analyzed simulated multilevel data using a Structural Equation Model (SEM) and found that bias in the standard errors at level two is exacerbated by a low ICC. They also examined the impact of unbalanced designs where the number of level one units within level two units differs. The authors suggested that researchers not use multilevel SEM when the number of level two units is less than 100 and the ICC is low, which they defined as being below .25. An ICC of .25, while considered to be the cutoff for a low ICC by (Hox & Maas, 2001), may be higher than the ICC found in many educational research settings. For example, Hedges and Hedberg (2007) compiled ICC values for mathematics and reading achievement data for grades K-12 in the United States and found that the average unconditional ICC across all grades for mathematics achievement was .22. This would imply that educational researchers need to have relatively large sample sizes for level two units to appropriately use HLM. The statistical power of multilevel models can be estimated (in simple cases) using available software such as Optimal Design (S. W. Raudenbush, Spybrook, Congdon, Liu, & Martinez, 2011). Snijders (2005) provides formulas which illustrate the effect of sample sizes on standard errors and statistical power in multilevel models.

2.10.2 Distribution of random effects

Another key assumption of HLM is that the random effects follow a multivariate normal distribution with mean 0 and variance τ (Maas & Hox, 2004a). When this assumption is violated, even in the presence of a large number of level two units, the standard errors are inaccurate (Maas & Hox, 2004a). Maas and Hox (2004a) conducted a simulation study in which one of the factors of interest was the distribution of the level two random effects. They found that when random effects followed either a uniform or Laplace distribution, significance tests of the variance component associated with level two were inaccurate. This was not the case when the random effects followed a Chi-square distribution (df=1). While violations of the normality assumption of the random effects produce bias in tests of variance components, they have very little effect on fixed effects estimates Maas and Hox (2004a, 2004b). The bias in the standard errors associated with the variance components was partially mitigated by the use of the Huber/White corrected standard errors (sandwich estimator).

The simulation studies mentioned above looked at the impact of sample size and the distribution of random effects on the performance of HLM. However, these simulation studies did not impose missingness on the data. That is, these simulation studies consisted of complete data for the simulated variables. While these simulation studies provide needed information on the performance of HLM under violations of assumptions, researchers are likely to have missing data in many educational research studies containing nested data.

2.11 Multilevel missing data

In the illustrative example discussed throughout this paper, students nested within classrooms, there are two possible sources of missing data. First, data may be missing for level one units (e.g. students). Second, data may be missing for level two units (e.g. classrooms). Reasons why data may be missing at level one (students) include factors such as the student being sick and therefore absent the day of the test or the student skipped class that day. Reasons for missing data at level two (classrooms) may include things like; some teachers are unfamiliar with computers or online survey tools used for data collection.

Classrooms may differ in terms of the reasons why data are missing. For example, if high school students are at level one and high schools are at level two, the likelihood of data being missing because students decided to skip class may differ if some high schools have closed campus policies whereas others do not. If the reason for missingness differs across level two units, it will have implications for the more tenable assumption of MAR.

2.12 Multiple imputation of multilevel missing data

Because MI has many desirable properties above and beyond single imputation, it should be considered as a missing data treatment to be used with nested data. However, the process of MI should be undertaken thoughtfully. The imputation procedure should take into account the goals of future analyses. Again, imagine a situation in which a researcher is interested in quantifying the impact of the years of mathematics education a teacher received on how students perform on a standardized mathematics test. As discussed earlier, students' mathematics scores are likely to be more similar to one another within a classroom than they are between classrooms. For ease of illustration of the impact of ignoring the nested structure of data during imputation, assume the imputer chooses to use mean imputation of missing standardized mathematics scores. Furthermore, because the imputer is ignoring the nested data structure, he/she uses the mean of the observed values on the mathematics score across all students to replace the missing values on this outcome variable. Ignoring the bias incurred from using mean imputation in general, it is easy to see that the use of an imputation procedure ignoring the nested data structure can further obscure the true dependence of the outcomes in the population. A more principled approach would be for the imputer to use the mean of the observed values within level two units (e.g. classrooms) for the missing values on the outcome variable within the same level two units. This example illustrates how failing to take into account the nested data structure in the imputation phase can result in, among other things, a decrease in the ICC, the proportion of variance in the outcome that is between classrooms, which violates the goal of preserving relationships among variables.

2.13 Uncongeniality

Uncongeniality refers to the situation where there is inconsistency between the imputation phase and analysis phase (Meng, 1994). When the imputation process and data analysis process are uncongenial, the results of the data analyses can be biased because the imputation phase failed to take into account the relationship among variables. In the case of uncongeniality between the imputation and analysis process of nested data, the relationship between variables at higher levels with those at lower levels of the data structure are not taken into account.

2.13.1 Uncongenial multilevel multiple imputation

The consequences of failing to take into account a nested data structure when imputing missing values has been studied. Andridge (2011) examined the effect of multiple imputation using fixed effects for group membership when cluster randomized trials were analyzed using HLM. The results suggested MI of missing data at level one using fixed effects can produce severe overestimation of variance of group means. This overestimation was more severe for small cluster sizes and small ICCs. Ignoring the nested data structure during MI resulted in an increase in type I error, where the effect of interest is a treatment effect at level two. This approach, which corresponds to multiple imputation under a single level framework, consistently performed poorly. The strength of the relationship between a covariate used in the imputation model and the missing variable did not improve the performance of this MI approach.

Taljaard, Donner, and Klar (2008) studied imputation strategies for cluster randomized trials that were subsequently analyzed using a modified t-test (a t-test adjusted for the ICC). As expected, MI ignoring the nested structure of the data consistently resulted in an underestimated ICC. Having acknowledged the need to take into account the nested data structure in multilevel missing data, Puma et al. (2009) published a technical report with guidelines on how to treat missing data in cluster randomized trials. In the report, the authors recommended the use of regression-based multiple imputation as well as ML using the EM algorithm for imputation of missing values.

2.14 Recommendations for multilevel multiple imputation

The Puma et al. (2009) report as well as the studies by Taljaard et al. (2008) and Andridge (2011) dealt with the situation where the inference of interest was on a treatment variable at level two and where level two units (clusters) were randomly assigned to either a treatment or control group. While recommendations from these studies are useful to the understanding of MLMI, it is important to note that cluster randomized trials (CRTs) differ from observational nested data in ways that have implications for the adoption of an appropriate multilevel imputation strategy.

For example, the Puma et al. (2009) report suggested that while the use of a missing data treatment referred to as dummy variable adjustment has been scrutinized in the academic literature, it may be appropriate for CRTs because the treatment is unrelated to the covariates with missing data. CRTs randomly assign clusters into the treatment conditions of interest. This helps to make the treatment variable independent of the covariates at the cluster level allowing for unbiased estimates of the treatment effect. When the inference of interest is on the treatment variable at level two (cluster), the analyst can use an HLM with a random intercept only that contains a predictor at level two indicating treatment exposure without additional predictors for the intercept. The restriction of the inference to the intercept only model under CRTs allows one to use an imputation approach, for level one missing data, that incorporates dummy variables for the level two units (clusters) in the imputation model (Graham, 2009).

When inferences about random slopes are also of interest, Graham (2009) recommends the imputation be carried out separately within clusters. Gelman and Hill (2006) generally recommend imputing data separately by level while perhaps using data from one level to impute missing data at a different level.

As illustrated by the recommendations by Gelman and Hill (2006), Graham (2009) and the Puma et al. (2009) report, the choice of the imputation approach for nested data should take into account the inferences of interest as well as the design of the study. In general, authors of research literature on MI have suggested that MI strategies should be at least as general as the analyses planned for the complete data. Additionally, MI should incorporate as much information that can aid in the prediction of missing values as possible (Meng, 1994; Peugh & Enders, 2004; Reiter et al., 2006; Schafer, 1994, 1999).

A MI approach consistent with this recommendation within the context of multilevel missing data involves the use of both fixed and random effects in an imputation.

2.15 Multilevel multiple imputation using PAN

Schafer (2001) describes a procedure for multilevel multiple imputation (MLMI) implemented in an S-PLUS package called PAN which is reviewed in this section. In his paper, Schafer described the implementation of PAN within the context of a longitudinal data structure. However, PAN can be implemented in a cross-sectional nested data structure as well.

PAN uses a multivariate extension of the linear mixed-effects model which allows the imputer to specify several outcomes that correspond to variables with missing data and predictors represent covariates that are used to impute missing values. Schafer (2001) states that the purpose of PAN is to impute missing responses so as to preserve the relationships among variables, not to construct a theoretically meaningful model. Therefore, it is not problematic if one uses the exact same set of covariates to impute missing values on outcomes that are completely different from one another. It is important however, to include covariates in the imputation model that will be used subsequently in the analysis phase. It is also important to include auxiliary variables in the imputation model (variables that help to explain missingness but may not be of substantive interest in the subsequent analyses).

2.15.1 How PAN works

Like the general MI procedure described above, the PAN package makes use of the Gibbs sampler. However, rather than sampling from the conditional distribution of individual variables given all other variables, the algorithm in PAN does the following:

The algorithm begins by (a) first drawing random values for the random effects using some possible values for 1) the missing data, 2) the fixed effects, 3) the error variance and 4) the variance/covariance matrix of the random effects. It then (b) draws new random values for the fixed effects, error variance, and variance-covariance matrix of the random effects. Finally, (c) it draws random values for the missing data given the random effects used in (a) and the parameters obtained in (b). This process represents a single cycle (k=1) and is repeated until the algorithm converges. Upon convergence, the k^{th} cycle is statistically independent of the values in the initial cycle.

For MI, this process is repeated m number of times with the k^{th} cycle producing an imputed data set. After producing m imputed data sets, the data analyst can then use HLM to estimate the parameters of interest. These parameter estimates are then be averaged over the m imputed data sets using Rubins rules (Rubin, 1987). An important caveat to PAN is that it does not impute missing values at level two by default. However, if one imposes a distribution on the covariates such as multivariate normality, missing values at level two can be imputed.

2.16 Maximum likelihood vs. multiple imputation

ML is common in the analysis of nested data and is implemented in available software for HLM such as HLM 6 and a procedure within SAS 9.2 called SAS PROC MIXED (S. W. Raudenbush & Bryk, 2002; Littell, 2006, respectively). The use of ML procedures provides unbiased estimates under the assumption of MAR (Rubin, 1976; Schafer, 1997).

As described previously, the goal of ML estimation is to maximize the likelihood function for a set of unknown parameters (θ) given the observed data or $L(\theta|Y_{obs}) = \int L(\theta|Y_{obs}, Y_{mis})dY_{mis})$, where $L(\theta|Y_{obs}, Y_{mis})$ denotes the likelihood function that one would use if no data were missing. Schafer (2001) describes the integration of the likelihood function presented above as being equivalent to the averaging over $P(Y_{mis}|Y_{obs})$ that takes place when one uses MI. However, there are some key advantages to dealing with missing data using MI rather than within the context of fitting an HLM through ML. One of these advantages is that the imputation process can easily make use of variables which explain missingness in the data and are not of substantive interest in subsequent analyses (Graham, Olchowski, & Gilreath, 2007; Schafer, 2001). The separation of the imputation phase from the analysis phase also allows for special attention to be paid to the assumption of MAR. Once the multiply imputed datasets have been created there is no longer a need to incorporate those variables that are thought to explain missingness but are unrelated to the outcome of interest in the statistical model, thereby leading to a more parsimonious model.

Another advantage of MI over ML within HLM is that ML does not allow for data missing at level two. Level two units with missing values for covariates in HLMs are eliminated from the analysis along with all of the observations at level one within those level two units. This can be particularly problematic when the number of level two units is already relatively small (e.g. 30 or so) (Gibson & Olejnik, 2003). One reason this is particularly problematic is because statistical power in HLMs depends heavily on the number of level two units (Scherbaum & Ferreter, 2009), especially as the ICC increases (Maas & Hox, 2005). To further explain, as the ICC increases, the dependency of units at level one within a level two unit increases resulting in less unique information (Scherbaum & Ferreter, 2009) provided by a member of a level two unit (e.g. student in a particular classroom). Through the use of MI of level one and level two variables, the loss of statistical power due to missing data is eliminated.

Another advantage of MI over ML is that uncertainty due to the missing data is separated from the uncertainty due to sampling variability (Schafer, 1997, 2001). HLM through ML does not have this characteristic in that sampling variability and uncertainty due to the missing data are combined. While MI has some advantages, it can also be less efficient than ML for dealing with missing data as it requires simulation (Schafer, 2001). However, because of the access to computing power available to many researchers this should not be seen as a significant barrier.

2.17 Previous literature on MI vs. ML for cross-sectional HLM

The effect of violations of the assumptions of HLM have been assessed through various simulation studies (Maas & Hox, 2004a, 2004b, 2005; Hox & Maas, 2001). Studies of the impact of missing data mechanisms on the performance of HLM have also appeared in the literature (Cai, 2008; Jacobusse et al., 2005; Petrin, 2006; Shin, Davison, & Long, 2009; Yucel, 2008). However, there is very little literature on the impact of both missing data and violations of assumptions within the context of HLM. Shin et al. (2009) did investigate the impact of missing data but did not consider MI and focused on repeated measures data.

The lack of research taking into account these issues simultaneously is concerning given the results of previous studies which have concluded that tests of parameters in HLM are dependent on various factors such as the ICC, missing data, presence or absence of a balanced design, sample size, and distribution of random effects. The cross-section of missing data, relatively small sample sizes at level two (e.g. less than 50) and violations of distributional assumptions is where many studies in educational research may find themselves. The possibility of excluding level two units increases as the number of covariates in the HLM at level two increases. Missing data can impact the inferences being made from hierarchical data. It is unclear how missing data impact inferences being made when studies do not explicitly discuss the missing data or the plausibility of MAR. Even when MAR is plausible, it is only a tenable assumption when covariates that explain missingness are included in the models, in either the imputation phase or data analysis phase. It is for these reasons that the impact of missing data on the performance of HLM and the possibility of mitigating some of these adverse effects of missing data through MI should be explored.

Gibson and Olejnik (2003) point out that research on methods for treating missing data within a nested data structure have been largely focused on longitudinal data instead of cross-sectional data. Researchers that have focused on cross-sectional HLM tend to focus on data missing at level one. Gibson and Olejnik (2003) considered the question of the impact of missing data at level two and assessed the performance of MI. While their simulation study included MI as a factor, their imputation procedure did not simultaneously impute data for levels one and two. Instead, the data were imputed separately for level two units and merged back with the level one file. This may have resulted in uncongeniality between the imputation and data analysis phases. Taking this under consideration, it is not surprising that the authors found MI performed poorly in estimating parameters in the HLM. While Gibson and Olejnik (2003) considered missing data treatments such as MI, the distribution of random effects was not a factor in their study.

Cai (2008) examined the performance of different missing data treatments including LD, restrictive MI (non-comprehensive MI model), and inclusive MI (comprehensive MI model) for missing data under MAR. Cai found that MI did not produce accurate or precise estimates of either fixed or random effects and concluded that LD produced the most accurate parameter estimates for all the HLM parameters. It should be noted however that the author did not use a multilevel MI strategy, meaning that the imputation and analysis phases were uncongenial.

Petrin (2006) examined the effect of three missing data treatments: MI ignoring the nested structure, multistage MI, and a hybrid multistage MI. In the multistage MI, imputation is performed in two steps with the first step performed on data missing at level two and the second step performed by incorporating the complete data at level two in conjunction with level one covariates and a categorical variable indicating group membership to impute missing data at level one. In the hybrid multistage MI level two data are imputed separately followed by the use of an HLM-based multiple imputation strategy developed by Goldstein (1995). Petrin (2006) made use of secondary data with no missing values which he then used to generate HLM parameter estimates that served as the "true" values in the simulation. He then imposed different types of missingness (MCAR, MAR, MNAR) and found that, in general, the MI procedures performed better than LD. Because a real dataset was used however, other factors outside of the missing data mechanism and imputation procedure were not manipulated (e.g. ICC, distribution of random effects, etc.).

Black (2008) examined the effects of four missing data treatments (ML, MI under the normal model, MI under HLM, and LD) on parameter estimates with three rates of missing data on the outcome variable (10%, 30%, 50%) when the missing data were MAR. She found that in general fixed effects were estimated accurately under all missing data treatments except for the fixed effect which reflected a cross-level interaction. The cross-level interaction term was more accurately estimated under the MI under HLM. In general, Black found that ML and MI under HLM were superior to other missing data treatments and performed similarly on the estimation of fixed effects and variance components. However, she noted that the sample statistics based on the incomplete data were similar to that of the complete data suggesting that the MAR condition imposed on the data was trivial (i.e. similar to MCAR). She speculated that the trivial MAR condition may have led to a better than expected performance of LD. Additionally, the author examined missing data on the outcome only and did not consider the effect of the distribution of random effects.

Zhang (2005) examined the performance of HLM and multilevel SEM in terms of their statistical power and accuracy of parameter estimates in the presence of nonnormal incomplete data. In doing so, MI was used to impute missing values. However, the multiple imputation procedure was uncongenial with subsequent multilevel analyses because it was carried out ignoring the nested data structure. Furthermore, the concern in this study was on the effect of nonnormality of the observed data. Despite the uncongeniality of the imputation and analysis phases, the author found that neither HLM nor multilevel SEM appeared to be sensitive to the violation of the assumption of multivariate normality.

2.18 Contributions of this study

Multiple imputation has been established as a viable modern missing data treatment (Schafer & Graham, 2002; Schafer, 1997) and HLM is a statistical technique which has been growing in popularity. Despite the common occurrence of missing data in educational research many researchers fail to report the impact of missing data on their statistical analyses and inferences.

A preliminary step in determining whether HLM is necessary is to consider the unconditional ICC, amount of variance in the outcome due to the level two grouping. This is estimated through an unconditional random intercept model. Because the model does not include covariates at either level, the assumption of MAR is not a plausible assumption. Additionally, subsequent conditional models will make use of covariates that may have different proportions of missing data thereby changing the sample being analyzed. There are at least two possible negative consequences of proceeding in this fashion. 1) The unconditional ICC may be misleading when the outcome has missing values. 2) The assessment of the proportion of variance explained at level two may be inaccurate (i.e. $\frac{\tau'_{qq}}{\tau_{qq}}$, where τ_{qq} represents the unconditional variance component at level two associated with the random effect q, τ'_{qq} is the conditional variance associated with the random effect q) leading the researcher to attribute an incorrect amount of predictability to their set of covariates.

MI in a multilevel context is more complicated than within a single level context. Because of this added complexity, it is important to understand the conditions under which MLMI contributes to valid inferences within an HLM framework. It is also important to understand when imputing missing data provides an advantage over ML using only the observed data.

Chapter 3

Methods

While the importance of addressing missing data has received increased attention over time, there is little evidence practitioners make use of more principled approaches when dealing with multilevel missing data. This study examines how competing approaches to handling missing data perform for various conditions.

Research Question

Which missing data method is most robust in the presence of missing data on the lower level units of analysis in a multilevel dataset and under what conditions does each of the missing data methods (listwise deletion; LD, multiple imputation ignoring the nested data structure; MI, and multilevel multiple imputation; MLMI) perform optimally?

This research question is designed to provide evidence about the best missing data method (i.e. across conditions) to apply in multilevel settings. It is also designed to tease out the effects associated with the performance of the missing data methods by examining their performance under various conditions.

It is hypothesized that MLMI will result in the best coverage rate of the fixed effects (proportion of times the parameter is contained within a 95% confidence interval), and will produce the least biased estimates of the fixed effects and variance components. The reason for this hypothesis is that MLMI results in a complete dataset where the multilevel nature of the data structure as well as the relationship between variables is preserved. Therefore, fixed effects associated with cross-level interactions which would otherwise be underestimated (and closer to 0) under MI and LD should be less biased under MLMI. It is also hypothesized that the inadequate performance of MI and LD will be exacerbated by a higher percentage of missing data at level one, smaller sample size at level two, and higher ICCs. Accordingly, MI and LD should perform best under smaller ICCs, smaller percentages of missing data at level one, and larger sample size at level two.

The choice of missing data methods to be compared in this study is guided by the missing data literature and the ease of implementation for practitioners. Ease of implementation is considered because while a more sophisticated approach for dealing with multilevel missing data may outperform all others its complexity may deter practitioners.

What follows in this chapter is the description of the conditions under which the study was conducted. The design of the simulation and the conditions are described first, followed by a description of the source of the parameter values used for generating the simulated data. A description of the process used to generate the data is given and then an explanation of how the missing data methods were evaluated.

3.1 Multilevel missing data conditions and practices

Several conditions of the simulation study described in this section were informed by a review of current conditions and practices in published empirical studies in educational research that contained multilevel missing data. The studies considered for review were restricted to those that used cross-sectional multilevel models, as this was the focus of the current study. Additionally, the review was restricted to journal articles that appeared in five research journals: American Educational Research Journal (AERJ), Educational Evaluation and Policy Analysis (EEPA), Journal of Experimental Education (JEE), Journal of Educational Psychology (JEP), and Sociology of Education (SOE).

The review was further restricted to articles published in 2010 and 2011 to obtain information about current conditions and practices with regard to the treatment of multilevel missing data. The aforementioned sources were selected because they are well known, peer-reviewed, educational research journals and are thought to contain studies representative of conditions and practices in educational research. Articles within these journals were retrieved by conditioning the online search within the journals on the following keywords: HLM, hierarchical, multilevel, and mixed effects.

This review processes resulted in the retrieval of 40 articles from AERJ, 22 articles from EEPA, 15 articles from JEE, 22 articles from JEP, and 19 articles from SOE for a total of 118 articles. The articles were then reviewed for consistency with the goals of the present study, in order to eliminate irrelevant studies such as those that used single level analyses or longitudinal models. Many of the retrieved articles made reference to multilevel models but did not fit such models. Additionally, many of the retrieved articles used longitudinal analyses which are not a focus of this study. The final number of articles retained was 45.

Of the 45 articles, 23 did not reference the number or percentage of cases with missing data at either level of the multilevel model. A few articles stated that they dealt with the missing data using ML (four studies). It is important to note however, that the treatment of missing data through ML is only relevant for the variable being modeled (i.e. the dependent variable). If covariates in the model contain missing data and those covariates themselves are not modeled explicitly, the use of ML does not guarantee unbiased parameter estimates (even if the missingness on the covariates are MAR given other covariates in the model).

It was somewhat common for the authors in the studies to deal with missing data by excluding cases when determining the appropriate sample to be used for analysis (five studies). For example, if the goal was to examine the impact of an intervention on test performance among African American students, students with missing data on race were excluded when defining the sample of interest. In four studies, the authors stated that they used LD for cases at level one. In nine studies, the authors reported using MI at level one. In only one study did the authors report using MLMI for missing level one data. In two studies, the authors used group mean imputation.

Assuming that the articles in this sample are representative of educational research studies using multilevel models, it appears that many studies do not address missing data. Among those that do the most frequent approach is MI (9/22 or 41%). While only 4 or 18% of these studies reported using LD, this number is likely an underestimate. There are two reasons why this percentage is likely an underestimate. First, in the presence of missing data on covariates at either level, the default among software such as HLM, which was used by many of the studies in the sample is to apply LD.

Second, Peng et al. (2006) reviewed 11 educational research journals and found that approximately 97% of the studies that reported dealing with missing data used either listwise or pairwise deletion. Given the prevalence of both LD and MI, both will be included in the simulation study.

3.2 Simulation Study

This study made use of a Monte Carlo simulation. A simulation was chosen because it allows one to examine the performance of statistical methods when analytic derivations are not possible. Additionally, simulations allow one to examine the performance of statistical methods when assumptions are violated whereas analytic derivations sometimes require that the assumptions be met. The details of the simulation used in this study are presented in this section. A graphical representation of the steps used in simulating the data within each cell of the factorial design is also provided in appendix I

3.2.1 MNMAP

Some aspects of the data simulated in this study are based on parameter estimates obtained from a multilevel model fit to data from the Minnesota Mathematics Achievement Project (MNMAP). This was done to increase the generalizability of the results from this simulation study to educational research studies with missing data. MNMAP was a multisite research project based at the University of Minnesota. MNMAP researchers studied the relationship between high school mathematics curricula and college mathematics performance (MNMAP, 2013). MNMAP researchers collected data on students who graduated from a Minnesota High School in either Spring of 2001 or Spring of 2002 and subsequently enrolled in a post-secondary institution. Data pertaining to more than 25,000 students from 52 post-secondary institutions were collected. MNMAP researchers assessed the impact of high school mathematics curricula on college mathematics performance by examining various outcomes in college. Among these outcomes was the grade a student received in their first college mathematics course. More information on findings from MNMAP can be found in the following published papers: Harwell et al. (2009), Post et al. (2010), and Harwell, Dupuis, Post, Medhanie, and LeBeau (in press).

MNMAP data were obtained through archival data sets at each of the participating post-secondary institutions. Consequently, it was not possible for MNMAP researchers to use a randomized field trial when examining the impact of the primary independent variable of interest (high school mathematics curriculum). For this reason it was necessary to control for both pre-existing differences and additional potential confounders. The simulation in the current study was based on one covariate at level two and two covariates at level one so that the number of fixed effects and random effects in the model is more manageable for synthesizing results.

3.2.2 HLM model used for MNMAP data

The data simulation performed in this study made use of parameter estimates from a two-level cross-sectional HLM fit to the MNMAP data in which grade in first college mathematics course was the dependent variable, high school mathematics GPA (4.0 scale) and ACT mathematics score were predictors at level one, and the average high school mathematics GPA was the predictor at level two. The corresponding model fit to the MNMAP data took the form:

$$Y_{ij} = \beta_{0j} + \beta_{1j}X_{1ij} + \beta_{2j}X_{2ij} + e_{ij} \tag{3.1}$$

$$\beta_{0j} = \gamma_{00} + \gamma_{01}W_{1j} + u_{0j} \tag{3.2}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11}W_{1j} + u_{1j} \tag{3.3}$$

$$\beta_{2j} = \gamma_{20} + \gamma_{21}W_{1j} + u_{2j} \tag{3.4}$$

Where X_{1ij} represents the high school mathematics GPA of student i in post-secondary institution j, X_{2ij} represents the ACT mathematics score of student i in post-secondary institution j, W_{1j} represents the average high school mathematics GPA in school j. The parameter estimates for the fixed effects $(\gamma_{00}, \gamma_{01}, \gamma_{10}, \gamma_{11}, \gamma_{20}, \gamma_{21})$ from the above model fit to the MNMAP data were used as parameters in the simulation. The estimates of the fixed effects and the variance components obtained from fitting the HLM model above to the MNMAP data are provided in table 3.1.

It is important to note that the parameter estimates presented in table 3.1 reflect estimates obtained from an HLM using covariates (high school math GPA and ACT math score) and an outcome (grade) often found in educational research. Using the

Table 3.1: Parameter values obtained from MNMAP HLM

Fixed effect	Estimate
Intercept	2.65
High school Math GPA	0.57
ACT Math Score	0.02
Average High school Math GPA	0.41
ACT Math Score×Average High school Math GPA	0.04
High school Math GPA×Average High school Math GPA	0.02
Variance of Intercept	0.052
Variance of High school Math GPA Slope	0.024
Variance of ACT Math Score Slope	0.001
Variance of level-1 residual	0.95

parameter estimates from MNMAP data helps the results of this simulation study to generalize to similar research studies (e.g. studies that use similar variables in a multilevel modeling framework). The presence of the somewhat small variance component for the high school math GPA slope will also allow these results to generalize to multilevel models with somewhat small variance components for slopes (i.e. when the variance is close to zero). Finally, while the covariates at level one in the MNMAP data had a correlation of .43, this correlation was constrained to zero in the simulation study as was the correlation among the random effects. Therefore, only the values for the fixed effects estimates and variance of the random effects presented in table 3.1 were used in the simulation study to generate the complete data (see equation 3.5 for the exact model used to produce complete data).

3.3 Missingness

This study simulated missing data under the condition of MAR and MCAR. MNAR is not used because, while it is possible that studies encounter data which are MNAR it is difficult to handle such cases and can require complex models that are study-specific.

Following recommended practice (Hoaglin & Andrews, 1975) the simulation study followed a $4 \times 4 \times 4 \times 3 \times 3 \times 2$ factorial design in which the independent variables defining conditions of the simulation served as factors. The conditions and their levels considered in this simulation study are presented below.

3.4 Missing data method

Three missing data methods were examined: 1) listwise deletion (LD), 2) multiple imputation (MI), 3) multilevel multiple imputation (MLMI). The first level of this condition (LD) was chosen because it is the default in multilevel statistical software packages such as HLM. The second level (MI) was used because the literature review suggested that this is the most common missing data method among the studies reviewed. The final level (MLMI) was chosen because it is considered a congenial missing data method for multilevel data (Schafer & Yucel, 2002) and as such, parameter estimates in a multilevel model should be unbiased after the missing data are subjected to MLMI.

3.5 Percentage of missing data at level one

The percentage of missing data was also a factor manipulated in this study. The review of journal articles revealed that, of those studies that reported the percentage of cases at level one with missing data, the 25th, 50th and 75th percentiles were 4.7, 10, and 23.3. These values were rounded to use as levels of the factor so that the percentages were 5%, 10%, and 25%. A final level of 50% missing data at level one was added to this condition to reflect a high percentage of missing data. This value is close to the highest percentage of missing data (47%) observed in the articles reviewed as well as the highest level used for the study conducted by Black (2008). Therefore, the percentage of cases with missing data at level one consisted of four levels (5%, 10%, 25%, and 50%)[1] .

3.6 Random effects

The distributional condition of the random effects (u_{0j}, u_{1j}, u_{2j}) took on four levels; (a) multivariate normal, (b)χ^2_1, (c) Laplace with location parameter zero and scale parameter one, and (d) approximate Cauchy distribution with location parameter zero

[1] These percentages of missingness were imposed separately for the level one predictor X_{1ij} and the outcome Y_{ij}. Because they were imposed separately, but using the same missing data mechanism (missingness depended on X_{2ij}), the actual percentage of cases with missing data can be slightly higher than the target percentages of 5%, 10%, 25% and 50%. The way in which missingness was imposed is discussed later in section 3.12

and scale parameter one[2] .

These distributional conditions, with the exception of the approximate Cauchy, are modeled after those considered in Maas and Hox (2004a). They were chosen to examine the impact of distributions with wider (approximate Cauchy) and thinner tails (Laplace) compared to a normal distribution and the impact of asymmetric distributions (χ^2).

Originally Maas and Hox (2004a) chose the uniform distribution as a distribution reflecting heavier tales (compared to the normal distribution). The uniform distribution was replaced with a approximate Cauchy distribution because it is thought that a uniform distribution is less plausible with real data. That is, at least with regard to an unconditional random intercepts model, it is unlikely that one would have randomly sampled level two units that are uniformly distributed in terms of average performance on the outcome of interest. Such a distribution would suggest that one is equally likely to observe an average performance score (e.g. average ACT math score) independent of where the score is along the scale. Instead, what is more likely than having a uniform distribution is that through random sampling, or perhaps through purposive sampling, one obtains more higher achieving (with respect to the outcome) level two units and more lower achieving level two units than one would expect if achievement were normally distributed, with the bulk of level two units centered around the mean. In the case of purposive sampling one may want to ensure that they have adequate representation among higher and lower performing level two units (e.g. high schools, universities).

3.7 Sample size at level two

Sample size at level two (i.e. the number of clusters) consisted of three levels: (1) J=30, (2) J=100, (3) J=200. The choice of these levels was influenced by a review of the literature. First, Gibson and Olejnik (2003) suggested a minimum sample size of 30 for level two units. Second, Maas and Hox (2004a, 2005) state that in order to obtain accurate standard errors for variance components sample sizes close to 100 should be obtained. This value was doubled ($100 \times 2 = 200$) to create a condition under which accurate estimates of all aspects of the model would be ensured (when all the other

[2] The ICC (a factor also manipulated in this study) could not be controlled when simulating data where the random effects followed the approximate Cauchy distribution because the Cauchy distribution has undefined variance.

assumptions are met). Accurate estimates of fixed effects and the associated standard errors are achievable with a smaller number of level two units.

Because sample size at level two is of primary interest and because within group sample size has little effect on statistical power[3] , the within group sample size was held at 50 prior to generating missing data. Given the above three levels for level two sample size and a fixed within level two sample size of 50, the total sample sizes for the complete data were 2,500; 5,000; and 10,000.

3.8 Intraclass correlations

The ICC was another factor manipulated in the simulation study. The values chosen for the ICC were .05, .1, .2, and .5. The value of .05 was chosen to reflect a small ICC and is consistent with the unconditional ICC observed in the MNMAP data. The values of .1 and .2 are chosen to reflect increasing within group dependence with .2 approximating the average unadjusted ICC observed in a review conducted by Hedges and Hedberg (2007) for educational data. The highest value for ICC of .5 was chosen to reflect the case with a large amount of within group dependence. This may prove to be very problematic for missing data treatments that do not account for the nested data structure (the three highest unconditional ICCs in the review conducted for this study were between .45 and .58).

It is important to note that the ICCs created in the simulation study are conditional ICCs. That is, the ICCs are based on the conditional models ($\frac{\tau_{00,conditional}}{\tau_{00,conditional}+\sigma^2_{00,conditional}}$) because the outcome variable is generated through the specified variance components. This is consistent with previous simulation studies that have controlled for the ICC (Maeda, 2007).

[3] The small impact of within level two sample size can be observed through the use of the optimal design software in which power to detect a level two effect is plotted as a function of within group sample size for a fixed number of level two units. Increases in within group sample size are associated with only a slight increase in statistical power whereas the impact of level two units on sample size (for a fixed within group sample size) is dramatic. The relative impact of these two factors are presented in Appendix A

3.9 Other features of the simulation study

The simulation study was conducted under the condition of no missing data at level two. While it is possible to have missing data at level two in practice, it was difficult to ascertain the percentage of cases with missing data at level two among the articles reviewed earlier. Most articles made no reference to missing data at level two (30 out of 45 studies). Additionally, six studies reported having no missing data at level two, either because the level two variables were aggregates of level one variables or because the sampling of level two units was conditional on complete data at level two. When studies did discuss missing data at level two they often made references to percentages of missing data for certain variables (e.g. the percentage of missing data for teachers education level was 10%). This made it impossible to determine the percentage of cases with missing data on any of the level two variables, which would be the exclusion criterion in statistical software packages using LD at level two.

3.10 Generating complete data

The complete datasets were simulated using the parameter estimates obtained from MNMAP (presented in Table 3.1 above) in the following way:

1. Random effects: Recall that there are four levels of the distribution of the random effects (multivariate normal, χ^2_1, Laplace, and approximate Cauchy). The condition of multivariate normality (MVN) for the random effects was imposed on the data by generating multivariate normal variables using the mtvnorm package in the statistical software program R (Team et al., 2008). The variance covariance matrix for the random effects were simulated under the variances presented in Table 3.1. For the three non-normal distribution conditions, the random effects were generated under the corresponding marginal univariate distributions (χ^2_1, Laplace with location parameter 0 and scale parameter 1, and a approximate Cauchy distribution with location parameter 0 and scale parameter 1). The χ^2_1 distribution was rescaled to have a mean of 0 as this was the only non-symmetric distribution considered for the random effects.

2. Covariates: The covariates at level one were created by generating univariate

standard normal random variables. The level two covariate was created in the same way. Both level one and level two covariates were uncorrelated within and across levels (i.e. $X_1 \perp X_2, X \perp W$).

3. Error term: The error term at level one was created by generating a univariate normal random variable with mean 0 and variance such that the target conditional ICCs were maintained. That is, the level one error term was generated as having a mean of 0 and variance of $\sigma^2_{00,conditional}$ where $(\frac{\tau_{00,conditional}}{\tau_{00,conditional}+\sigma^2_{00,conditional}})$ is equal to either .05, .10, .20, or .50. For example, when the random effects of the intercept are distributed as χ^2_1 the expected variance is 2×df (2 times the degrees of freedom) or 2 and therefore the corresponding level one conditional error variance for the conditional ICC value of .50 is 2 $(\frac{\tau_{00,conditional}}{\tau_{00,conditional}+\sigma^2_{00,conditional}} = \frac{2}{2+\sigma^2_{00,conditional}} = 0.5)$. Therefore, through the use of four different level one conditional error variances for each distribution of random effects, the four levels of the conditional ICCs are maintained. It is also worth noting that the level one error term was simulated independent of the random effects at level two ($e_{ij} \perp u_j$) and always followed a univariate normal distribution regardless of the distribution of the random effects.

After steps 1 through 3 above were complete the values for the dependent variable were generated by combining the random effects, covariates, and level one error terms using the following (mixed effects) expression:

$$Y_{ij} = (\gamma_{00}+\gamma_{01}W_{1j}+u_{0j})+(\gamma_{10}+\gamma_{11}W_{1j}+u_{1j})X_{1ij}+(\gamma_{20}+\gamma_{21}W_{1j}+u_{2j})X_{2ij}+e_{ij} \quad (3.5)$$

While the factorial design of the simulation study involved 1,152 cells (4 × 4 × 4 × 3 × 3 × 2), three of the six factors do not involve the creation of the complete data set. Specifically, the conditions of percentage of missing data at level one (5%, 10%, 25%, 50%), missing data mechanism (MAR, and MCAR) and the subsequent treatment of the missing data (LD, MI, or MLMI) are conditions that do not impact the simulation of the raw data themselves. Therefore, there were 48 (4 × 4 × 3) unique combinations of conditions that resulted in complete datasets (three levels of sample size at level two, four levels of distributional conditions of random effects, and four ICCs). Within each of these 48 cells, 500 replications (i.e. complete data sets) were generated. The conditions under which the complete data were simulated are presented below in Table 3.2. Each replication resulted in a single dataset containing complete data under each of the 48 unique combinations of simulation conditions. All 500 datasets were saved for later use in the simulation (i.e. to generate missing data, apply the missing data treatment, and estimate parameters in the multilevel model).

Table 3.2: Number of replications per condition for simulation of complete data

ICC	Level two Sample size	MVN	χ_1^2	Laplace	approximate Cauchy	Total
	30	500	500	500	500	2,000
.05	100	500	500	500	500	2,000
	200	500	500	500	500	2,000
	30	500	500	500	500	2,000
.10	100	500	500	500	500	2,000
	200	500	500	500	500	2,000
	30	500	500	500	500	2,000
.20	100	500	500	500	500	2,000
	200	500	500	500	500	2,000
	30	500	500	500	500	2,000
.50	100	500	500	500	500	2,000
	200	500	500	500	500	2,000
	Total	6000	6000	6000	6000	24000

Note: MVN=Multivariate normal distribution

3.11 Evaluating the complete data

The generation of the complete data was evaluated for the condition of 200 level two units and MVN random effects by 1) calculating the percent relative estimate[4] using $100 \times \frac{\hat{\theta}}{\theta}$ (Maas & Hox, 2004a, 2004b, 2005) where θ is the value of the parameter for a particular fixed effect or variance component (appearing in table 3.1 above), and $\hat{\theta}$ is the estimate of the corresponding fixed effect or variance component, and 2) examining the coverage probability of the fixed effects by creating an indicator variable reflecting whether the 95% confidence interval for the fixed effect estimates covered the corresponding parameter values. For both criteria, the simulated complete data should

[4] Previous authors have referred to this measure as ”percent relative bias”

show an average percent relative estimate of 100% and coverage of 0.95[5] when the random effects follow a multivariate normal distribution and there are 200 level two units (reflecting the case when the assumptions of HLM are best met).

3.12 Generating missing data

For the condition of MAR, missing data were generated by first creating a probability of missingness $P(R)$ for both the outcome Y_{ij} variable and the level one predictor variable X_{1ij}. These probabilities were generated for each of the variables using the following formula $P(R_{ij}) = \frac{e^{\beta_0+\beta_1 X_{2ij}}}{1+e^{\beta_0+\beta_1 X_{2ij}}}$. This expression was used because it allows for the probability of missingness to depend on the covariate X_{2ij} and keeps the probability between 0 and 1. The missing data indicator variables $R_{Y_{ij}}$ and $R_{X_{1ij}}$ were then generated by sampling from a binomial distribution with probability equal to $P(R_{ij})$ for each observation ($N = 1$). If the indicator variable for a case is 1, the variable value is deleted. For example, if $R_{Y11} = 1$ then the value for the dependent variable Y for student 1 in school 1 was deleted. The values used for β_0 and β_1 in the equation for the probability of missingness were -5.77 and -3 for $P(R) = .05$, -4.48 and -3 for $P(R) = .10$, -2.35 and -3 for $P(R) = .25$, and 0 and -3 for $P(R) = .5$ These values were chosen because they resulted in an average probability of missingness of .05, .10, .25, and .50 respectively. Given that the missingness was specified separately for the outcome and the level one covariate, the percentage of level one cases with missing data was not held fixed. Because the probability of missingness was dependent on X_{2ij} the missing data mechanism for both the outcome and the level one predictor X_{1ij} is MAR (i.e. missingness for both variables is related to X_{2ij}) (Petrin, 2006). For the condition of MCAR, the missing data were generated in the same way as described above for MAR except that the slope β_1 in the formula $\frac{e^{\beta_0+\beta_1 X_{2ij}}}{1+e^{\beta_0+\beta_1 X_{2ij}}}$ was constrained to equal zero.

[5] The value of 100% for percent relative estimate indicates that on average the estimate is equal to the parameter value. Values higher than 100% for percent relative estimate indicate that on average the estimate is higher than the parameter value whereas values lower than 100% indicate that on average the estimate is lower than the parameter value. The percent relative estimate can also be negative in the event the estimate (numerator) is negative and the parameter value (denominator) is positive. With regard to coverage probability, values equal to 0.95 indicate that coverage is equal to the nominal rate. Values higher than 0.95 indicate that coverage is greater than the nominal coverage rate whereas values lower than 0.95 indicate that coverage is lower than the nominal rate

3.13 Treatment of missing data

The missing data methods examined in this study were LD, MI, and MLMI. LD was applied to the dataset before estimating model parameters. MI was applied by passing the incomplete data through the R package NORM, and using all variables in the imputation, prior to estimating the parameters in the model. MLMI was applied by passing the incomplete data through the R package PAN prior to estimating the parameters in the model (the imputation model is described in chapter 4).

3.14 Estimating parameters

The fixed effects and corresponding standard errors as well as the variance components were estimated by passing the appropriate data (either incomplete data or multiply imputed data) through a statistical software package in R. All of the estimates for the variance components, fixed effects and their standard errors were then saved into an external data file.

3.15 Evaluating the missing data methods

The performance of the missing data methods was evaluated by 1) calculating the percent relative estimate in the fixed effects and variance components using the formula given above, and 2) examining the coverage of the fixed effects by creating an indicator variable reflecting whether the 95% confidence interval for the fixed effects estimates covered the corresponding parameter values.

While the full factorial design consisted of 1,152 cells ($4 \times 4 \times 4 \times 3 \times 3 \times 2$), the main effects of interest are those associated with the missing data treatment, ICC, percentage of missing data, and level two sample size. Therefore, while the distribution conditions of the random effects are simulated, the evaluation of the missing data treatments is conducted separately for each distributional condition of the random effects. This resulted in an analytic factorial design with 144 cells ($4 \times 4 \times 3 \times 3$), within which there were 500 replications in each cell. It is within each of these 144 cells that mean percent relative estimate and the proportion of replications in which the 95% confidence intervals contained the parameter value, were computed. These summaries of the variance components, fixed effects, and their standard errors were used to answer the research questions regarding the appropriate treatment of multilevel missing data.

Chapter 4

Results

4.1 Additional simulation details

The simulation study was carried out using a custom built windows based desktop with a six-core intel i7 processor operating at 3.8 Ghz with 16 gigabytes of ram.

Recall that each replication of the 500 replications consisted of a single dataset. Each of these datasets had complete data for all 48 unique combinations of simulation conditions (three levels of sample size at level two, four levels of distributional conditions of random effects, and four ICCs). Because of the demanding computational nature of this simulation study, the 500 replications were separated into 10 groups (50 replications each)[1] . Each replication was first subjected to the creation of missing data, then each missing data treatment (LD, MI, MLMI), and finally fitting a multilevel model to the subsequent data files (one for LD, five for MI, and five for MLMI). The multilevel model results from the multiply imputed files under MI and MLMI were combined using Rubin's rules (Rubin, 1987) so that after the same dataset was subjected to each of the three missing data treatments, there were three separate rows of results (one for LD, one for MI, and one for MLMI). In this way each missing data treatment was imposed on the same dataset. The entire simulation completed in approximately four days and twelve hours.

Even though it was the author's intent to use the HLM 6 software to estimate

[1] These 10 groups represented ten separate initiations of the statistical software package R 2.15.2 (Team et al., 2008)

the parameters within the simulation study, this proved to be a challenge and was abandoned. The problem was that the HLM 6 software would only estimate the models one at a time which would have required several months for the simulation to complete. Instead the author used the open source statistical software package R for all aspects of the simulation study. The program R was chosen because multiple instances of R can run at the same time using multiple processors. Specifically, this simulation was completed by initiating 10 R sessions simultaneously on the same computer allowing for each session to focus on 50 replications. In this way, each of the 10 R sessions were simultaneously producing parameter estimates under the various simulation conditions and pooling these results. This splitting up of the task allowed the author to cut down the simulation time from approximately 45 days to 4.5 days.

The multilevel models were estimated using the lme4 package (Bates, 2007) within R[2] . To ensure that the estimation of the parameters within R would not differ substantially from the estimation of the parameters in HLM 6, one hundred of the five hundred replications were executed using both HLM 6 and lme4. The similarity between the two programs was evaluated by calculating the correlation between the parameter estimates as well as the mean difference. These results are presented in tables 4.1 through 4.6.

Table 4.1: Correlation between HLM 6 and lmer fixed effects estimates

	γ_{00}	γ_{10}	γ_{20}	γ_{01}	γ_{11}	γ_{21}
γ_{00}	≈ 1					
γ_{10}		$\approx$1				
γ_{20}			$\approx$1			
γ_{01}				$\approx$1		
γ_{11}					$\approx$1	
γ_{21}						$\approx$1

Note: lmer estimates are along the columns and HLM 6 estimates are along the rows. The smallest correlation was .9999989.

The results suggest that the parameter estimates obtained under HLM 6 and lmer are very similar. The correlation of nearly one for the fixed effects, their standard errors, and the variance components suggests that the simulation conditions (across which the correlations are calculated) influence the parameter estimates in the same way whether

[2] lmer does not estimate standard errors for the variance components, nor does it produce significance tests for the fixed effects. It is for these reasons that neither the coverage for the random effects nor the Type I error rate for the fixed effects are presented in this study.

Table 4.2: Correlation between HLM 6 and lmer standard errors of fixed effects

	γ_{00}	γ_{10}	γ_{20}	γ_{01}	γ_{11}	γ_{21}
γ_{00}	.986					
γ_{10}		.988				
γ_{20}			.978			
γ_{01}				.984		
γ_{11}					.988	
γ_{21}						.979

Note: lmer estimates are along the columns and HLM 6 estimates are along the rows.

Table 4.3: Correlation between HLM 6 and lmer variance estimates

	τ_{00}	τ_{10}	τ_{20}	τ_{11}	τ_{21}	τ_{22}
τ_{00}	≈ 1					
τ_{10}		≈ 1				
τ_{20}			≈ 1			
τ_{11}				≈ 1		
τ_{21}					≈ 1	
τ_{22}						≈ 1

Note: lmer estimates are along the columns and HLM 6 estimates are along the rows. The smallest correlation was .9998856.

the estimates are obtained through HLM 6 or lmer. This means that whatever the influence of the simulation conditions, one can expect the impact of those factors to manifest themselves in the same way on the parameter estimates independent of which statistical software package is used to estimate the parameters.

The mean difference was calculated to determine if either program (HLM 6 or lmer) produces a consistently different (higher or lower) estimate for the parameters of interest in this simulation study. The results above show that for the fixed effects estimates lmer and HLM 6 produced, on average, nearly identical estimates (to at least four decimal places). The results for the standard errors of the fixed effects show that lmer produces standard errors that are slightly larger than those obtained from the HLM 6 program. This means that, all things being equal, one can expect fewer significant tests of fixed effects using lmer compared to HLM 6.

The mean differences in the estimates of the variance components shows that, for the parameters of interest in this study ($\tau_{00}, \tau_{11}, \tau_{22}$), the results produced by HLM 6 and lmer are very similar. The average differences for the first two variance components

Table 4.4: Mean difference between lmer and HLM 6 fixed effects estimates

Parameter	Mean
γ_{00}	≈ 0
γ_{10}	≈ 0
γ_{20}	≈ 0
γ_{01}	≈ 0
γ_{11}	≈ 0
γ_{21}	≈ 0

Note: HLM 6 estimates were subtracted from corresponding lmer estimates. The largest absolute value was zero to four decimal places.

Table 4.5: Mean difference between lmer and HLM 6 standard errors of fixed effects

Parameter	Mean
γ_{00}	0.02
γ_{10}	0.11
γ_{20}	0.08
γ_{01}	0.01
γ_{11}	0.12
γ_{21}	0.08

Note: HLM 6 estimates were subtracted from corresponding lmer estimates.

of interest (τ_{00}, τ_{11}) represent approximately 3.6% and 8.7% of the parameter value suggesting that, on average while the HLM 6 estimates are higher, it is by a small magnitude. The largest average difference is observed for the parameter estimate of τ_{22} which was 0.01577 which is approximately sixteen times larger than the parameter value of 0.001. However, this is to be expected when the parameter value is so close to zero because even a slight deviation from zero can be much larger than the parameter value.

Given the results above, it was determined that the conclusions drawn from the simulation study using R would not differ substantially from the conclusions drawn had the author used the HLM 6 software to estimate the multilevel models.

4.1.1 Specifying the multiple imputation models

The multiple imputation model ignoring the nested structure of the data (MI) assumed the data come from a multivariate normal distribution and was implemented using the package "norm" in R (Novo & Schafer, 2010). This imputation technique will be

Table 4.6: Mean difference between lmer and HLM 6 variance estimates

Parameter	Mean
τ_{00}	$\approx$0
τ_{10}	$\approx$0
τ_{20}	0.02
τ_{11}	$\approx$0
τ_{21}	$\approx$0
τ_{22}	0.02

Note: HLM 6 estimates were subtracted from corresponding lmer estimates. Values with ≈0 were zero to at least two decimal places.

referred to as multiple imputation under the normal model throughout the remainder of this paper. MI is considered an uncongenial missing data treatment for multilevel data because it fails to take into account the multilevel data structure (i.e. it assumes the ICC is zero, no variability in slopes etc.). This differs from the multilevel multiple imputation (MLMI) approach also used in this study.

The MLMI procedure was implemented in the R package PAN. Because the data had missing values for both the dependent variable Y_{ij} and the independent variable X_{1ij} the author specified a multivariate MLMI model within PAN so that missing values for both variables would be imputed.

The multivariate MLMI model implemented in PAN had the following structure:

$$Y_{ij} = (\gamma_{00} + \gamma_{01}W_{1j} + u_{0j}) + (\gamma_{10} + \gamma_{11}W_{1j} + u_{1j})X_{2ij} + e_{Yij}$$
$$X_{1ij} = (\gamma_{00} + \gamma_{01}W_{1j}) + (\gamma_{10} + \gamma_{11}W_{1j})X_{2ij} + e_{X1ij}$$
$$e \sim (0, \begin{matrix} \sigma_Y^2 & \sigma_{X_1,Y} \\ \sigma_{Y,X_1} & \sigma_{X_1}^2 \end{matrix})$$

Whereas the variance/covariance matrix for e was estimated in its entirety, only the variances for u were estimated. The covariances for u were constrained to be equal to zero. This model is an appropriate imputation model because the response variables are allowed to correlate through the variance/covariance matrix for e. Additionally, the fact that the true model contains both a random intercept for Y_{ij} and a random slope for X_{2ij} is also taken into account via the diagonal elements of the variance/covariance matrix for u. Constraining the off-diagonal elements of the variance/covariance matrix for u is also consistent with the true model given that the random effects for the intercept and X_{2ij} slope were not allowed to covary with one another.

4.1.2 Number of iterations for multiple imputation

Five imputations were used for each of the imputation-based missing data treatments, MI and MLMI. The author chose to use five imputations because, as Schafer (1999) shows with 50% missing data, an estimate based on five imputations has a standard deviation that is only about 5 % wider than an estimate based on an infinite number of imputations[3] .

In order to obtain imputations from the algorithm implemented in PAN, it was necessary to assess the convergence of the algorithm under the condition with the highest percentage of missing data (50%). Time-series plots of the parameter estimates of the MLMI model were produced and are presented in figures B.2 through B.6 in Appendix B. Both the time-series plots and the plots of the autocorrelations for the parameters in the MLMI model suggested that the algorithm converged in less than about 100 iterations. The time-series plots showed a rectangular shaped pattern to the estimates over iterations beyond 100. Additionally, the plot of the autocorrelations showed that the autocorrelations dropped rapidly within 100 iterations. To guard against an insufficient number of iterations in the simulation, the number of iterations was set to 2,000 with 2,000 iterations between draws from the posterior distribution (i.e. missing data were imputed after every 2,000th cycle of the Markov Chain). The multiple imputation package in R used to impute missing data under the normal model "norm" had the same specifications.

4.2 Validating the generation of complete data

In order to validate the simulation of complete multilevel data, the complete data were simulated under the condition of level two sample size equal to 200, ICC equal to .05, and a multivariate normal distribution of random effects. The data were then analyzed by fitting the multilevel model used to generate the data. This process was repeated 500 times. If the multilevel data were generated correctly, the parameter estimates obtained

[3] This is based on the formula given by Rubin (1987) $\sqrt{1+\frac{\lambda}{m}}$ where λ is the rate of missing information.

from the multilevel data should be unbiased and have a 95% confidence interval coverage probability close to the nominal rate of .95. The percent relative estimate was calculated for both fixed effects and variance components. Additionally, the coverage probability around the fixed effects estimates was calculated. The results are summarized in table 4.7.

Table 4.7: Percent relative estimate and coverage validating complete data

Parameter	%relative estimate	coverage of 95% CI
γ_{00}	99.97	.95
γ_{10}	100.03	.94
γ_{20}	96.89	.95
γ_{01}	100.18	.96
γ_{11}	102.52	.95
γ_{21}	100.31	.96
τ_{00}	99.97	
τ_{11}	99.68	
τ_{22}	149.66	

The results above suggest the method used to generate the complete data generally performed as intended. This inference is supported by how well the parameter estimates were recovered by the appropriate statistical model (i.e. % relative estimate close to 100, and coverage probability of 95% CI close to .95).

The percent relative estimate for the fixed effects and variance components was nearly 100% for all of the parameter estimates. The most problematic percent relative estimate was found for τ_{22} which had a value of 149.66. However, it should be noted that the parameter value is 0.001. Given how close the parameter value is to zero, it is not surprising that the percent relative estimate is so large as a slight deviation from zero can have a large percent relative estimate. This would suggest that obtaining an unbiased estimate of this variance component τ_{22} is difficult even under otherwise good conditions (i.e. multivariate normal distribution of random effects, and 200 level two units).

4.3 Results from evaluation of the performance of missing data methods

As mentioned earlier the performance of the missing data treatments was assessed using the percent relative estimate for the fixed effects estimates and the variance components, as well as a coverage probability for the fixed effects. The performance of these metrics was examined visually through the use of plots. The coverage probabilities of the 95% confidence intervals are presented first followed by the percent relative estimate.

4.4 Coverage of 95% confidence interval

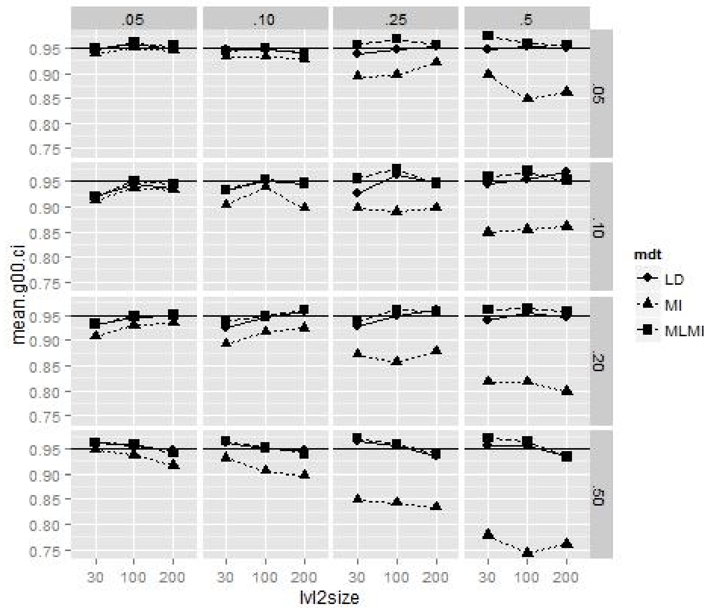
The performance of the missing data treatments was informed by examining the coverage of the 95% confidence intervals around the fixed effects estimates. Figures 4.1 through 4.6 depict the proportion of replications where the 95% confidence interval around the fixed effect contained the corresponding parameter value when the data were MAR and the random effects followed a multivariate normal distribution. The results for the coverage of the 95% CI under the other distributional conditions, as well as when the data were MCAR are presented in Appendix C and Appendix D respectively.

Figure 4.1 presents the proportion of simulations where the 95% confidence interval around the estimate of γ_{00} contained the parameter value (2.65).

Multivariate normal random effects

Figure 4.1 shows the coverage probability of the parameter γ_{00} when the random effects followed a multivariate normal distribution. In general, listwise deletion (LD) and multilevel multiple imputation (MLMI) resulted in proper coverage under most simulation conditions. Multiple imputation under the normal model (MI) consistently resulted in very poor coverage which was exacerbated by higher proportions of missing data. This suggests that with respect to the fixed effect for the intercept in a two level model, when the random effects follow a multivariate normal distribution, using either LD or MLMI will produce a coverage probability close to the nominal probability (.95). However, under the same conditions, using MI will result in a conservative coverage probability (lower than the nominal coverage probability)that becomes more conservative as the proportion of missing data increases. MI produced coverage probabilities

Figure 4.1: 95% CI coverage of estimate of γ_{00} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

close to the nominal rate when the percentage of missing data was smallest (5%) and the number of clusters was large (100 or 200). Under all simulation conditions, the coverage probability for this parameter remained between roughly .75 and .98.

Chi-square distribution of random effects

When the random effects followed a Chi-square distribution coverage ranged between approximately .82 and .96. This range is narrower than when the random effects followed a multivariate normal distribution. With regard to both LD and MLMI, the coverage probability began as being too conservative when the number of clusters was lowest (30) and approached the nominal rate as the number of clusters increased. Under many of the simulation conditions, the coverage probability under LD was closest to the nominal coverage probability (.95). MI generally produced a conservative coverage probability which decreased further as the percentage of missing data increased.

Laplace distribution of random effects

When the random effects followed a Laplace distribution the coverage probability ranged between approximately .84 and .96. Similar to what was observed when the random effects followed a Chi-square distribution, both LD and MLMI produced coverage probabilities that were close to the nominal coverage rate. This was particularly true of LD and MLMI as the number of clusters increased. When LD and MLMI performed differently with respect to coverage it was LD that tended to have coverage closest to the nominal rate (.95). Again, as with the previous distributions of the random effects, MI produced conservative coverage probabilities under almost all simulation conditions and this was exacerbated as either the percentage of missing or ICC increased.

Approximate Cauchy distribution of random effects

When the random effects followed an approximate Cauchy distribution the coverage probability ranged between roughly .82 and 1. Both LD and MLMI tended to have coverage that was too high (i.e. above 95%) across all the simulation conditions. The tendency to have a coverage probability that was too high was observed for MI as well except for when the percentage of missing data was high (25% or 50%) in which case MI tended to have a coverage probability that was too conservative.

Figure 4.2 presents the proportion of simulations where the 95% confidence interval around the estimate of γ_{01} contained the parameter value (0.41).

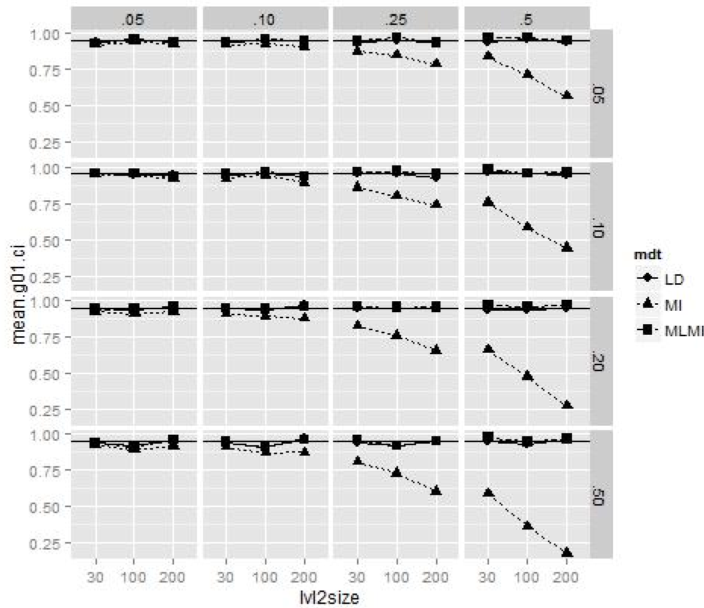
Multivariate normal random effects

Figure 4.2 shows the coverage probability of the parameter γ_{01} when the random effects followed a multivariate normal distribution. The coverage probability ranged between approximately .2 and .96 across the simulation conditions. Both MLMI and LD tended to have a coverage probability that was very close to the nominal coverage probability (.95). However, when MI was used, the coverage probability tended to be conservative, especially as either the percentage of missing data or ICC increased. This suggests that if one is interested in the fixed effect for a level two predictor of the intercept in a two level model and the random effects follow a multivariate normal distribution, using either LD or MLMI will produce a coverage probability close to the nominal rate (.95). However, when MI is used, the coverage rate will be too conservative, particularly if there is a large amount of missing data or dependence is large. MI is less problematic when the percentage of missing data is small (5%).

Chi-square distribution of random effects

When the random effects followed a Chi-square distribution the coverage probability

Figure 4.2: 95% CI coverage of estimate of γ_{01} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

ranged between roughly .74 and .98. The relative coverage of the missing data treatments was similar to when the random effects followed a multivariate normal distribution. Again, MI tended to produce conservative coverage probabilities under many simulation conditions, particularly as the percentage of missing data or the ICC increased. However, when the random effects followed a Chi-square distribution the range of coverage probabilities (across simulation conditions) was narrower than when the random effects followed a multivariate normal distribution.

Laplace distribution of random effects

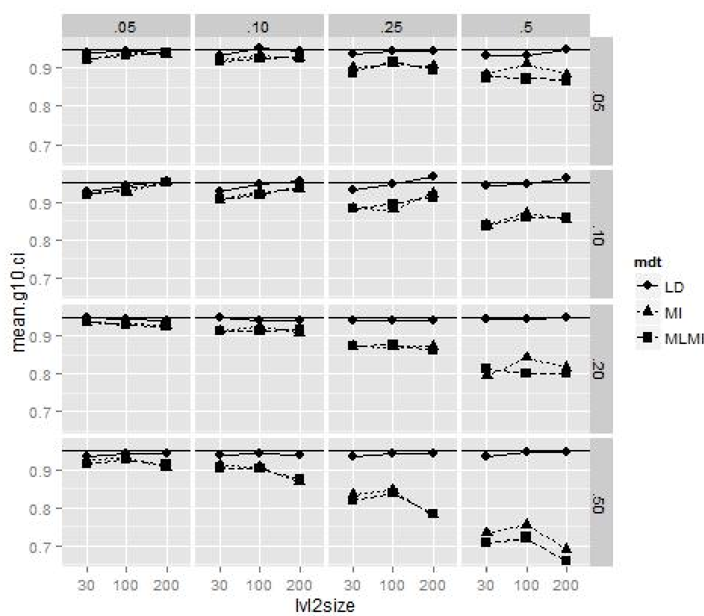
When the random effects followed a Laplace distribution the coverage probabilities ranged between roughly .75 and .96. Generally MLMI and LD produced similar coverage probabilities across simulation conditions and these coverage probabilities were often very close to the nominal coverage probability (.95). On the other hand, MI produced conservative coverage probabilities as either the percentage of missing data or ICC increased. MI produced coverage probabilities closes to the nominal probability when the percentage of missing data was smallest (5%).

Approximate Cauchy distribution of random effects

When the random effects followed an approximate Cauchy distribution the coverage probability ranged between roughly .75 and .96. Similar to the results observed when the random effects followed the previous distributions (multivariate normal, chi-square, Laplace), MLMI and LD produced coverage probabilities very similar to one another under almost all simulation conditions. Additionally, this coverage was closest to the nominal coverage rate for both MLMI and LD when the number of clusters was largest (200) or when the ICC was either at its smallest or largest value (.05 or .5). MI produced conservative coverage probabilities as the percentage of missing data increased. MI produced coverage probabilities that were closes to the nominal coverage probability when the percentage of missing data was smallest (5%).

Figure 4.3 presents the proportion of simulations where the 95% confidence interval around the estimate of γ_{10} contained the parameter value (0.57).

Figure 4.3: 95% CI coverage of estimate of γ_{10} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

Multivariate normal random effects

Figure 4.3 shows the coverage probabilities for the parameter γ_{10} when the random effects followed a multivariate normal distribution. The coverage probabilities ranged between about .65 and .95. The coverage probability was closest to the nominal probability for LD under almost all simulation conditions. As the percentage of missing data increased both MLMI and MI had conservative coverage probabilities. Both MLMI and MI produced coverage probabilities close to the nominal rate only when the percentage of missing data was smallest (5%). This suggests that when one is interested in the coverage of the fixed effect associated with a level one covariate in a two level model, generally the treatment of missing data with LD would result in adequate coverage whereas the treatment of missing data with either MI or MLMI would result in poor coverage.

Chi-square distribution of random effects

When the random effects followed a chi-square distribution, the missing data treatments performed similar to when the random effects followed a multivariate normal distribution. Specifically, LD had coverage probabilities closest to the nominal rate compared to either MI or MLMI. Additionally, LD had coverage probabilities closest to the nominal rate as the number of clusters increased. Both MLMI and MI had conservative coverage probabilities which became more conservative as either the percentage of missing data or ICC increased. Under all simulation conditions the coverage probabilities stayed between roughly .68 and .96.

Laplace distribution of random effects

When the random effects followed a Laplace distribution LD produced coverage probabilities nearly identical to the nominal rate under almost all simulation conditions. MI and MLMI produced coverage probabilities closest to the nominal rate only when the percentage of missing data was smallest (5%). Additionally, as either the percentage of missing data or the ICC increased, MLMI and MI produced conservative coverage probabilities. Under all simulation conditions the coverage probabilities remained between roughly .65 and .95.

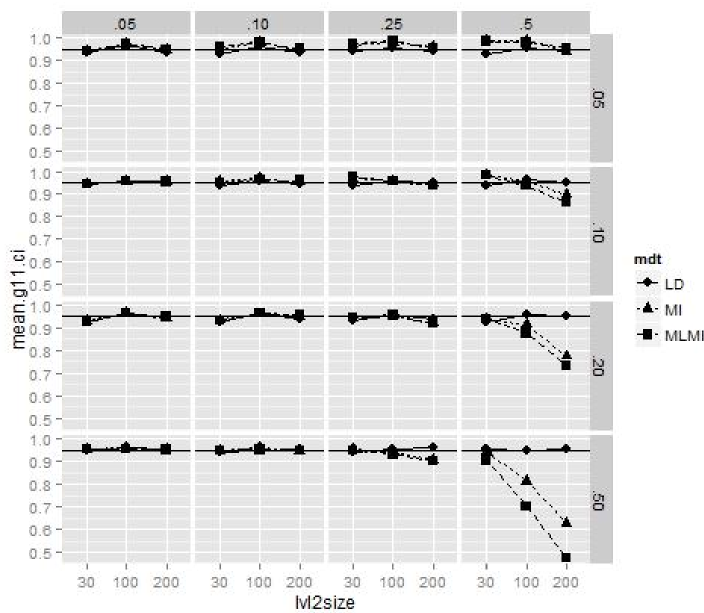
Approximate Cauchy distribution of random effects

When the random effects followed an approximate Cauchy distribution, the coverage probability for LD tended to be too high (i.e. more than .95) across all conditions.

MLMI and MI also had coverage probabilities that were too high when the percentage of missing data was smallest (5%). Additionally, MLMI and MI produced coverage probabilities that were too conservative as the percentage of missing data increased. Under all simulation conditions the coverage probabilities remained between roughly .55 and 1.

Figure 4.4 presents the proportion of simulations where the 95% confidence interval around the estimate of γ_{11} contained the parameter value (0.02).

Figure 4.4: 95% CI coverage of estimate of γ_{11} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

Multivariate normal random effects

Figure 4.4 shows the coverage probability for the parameter γ_{11} when the random effects followed a multivariate normal distribution. The coverage probabilities ranged between about .48 and 1. The three missing data treatments tended to have similar coverage over most conditions in the simulation and in general they tended to produce coverage probabilities very close to the nominal coverage rate. However, when the percentage of missing data was highest (50%), MLMI and MI showed conservative coverage probabilities as the within group dependence increased. This suggests that if one is interested in the coverage of the fixed effect associated with the cross-level interaction between a level one covariate and a level two predictor all missing data treatments produce appropriate coverage with the exception of MLMI and MI when the percentage of missing data is highest and the dependence is large (.20 or higher).

Chi-square distribution of random effects

When the random effects followed a chi-square distribution the coverage probabilities remained between approximately .92 and 1. LD tended to have coverage probabilities closest to the nominal rate of .95 compared to the other missing data treatments. In general, as the percentage of missing data increased, MLMI and MI tended to have coverage probabilities that exceeded the nominal rate. This finding was in contrast to LD which produced coverage probabilities that were too conservative when the percentage of missing data was highest and dependence increased.

Laplace distribution of random effects

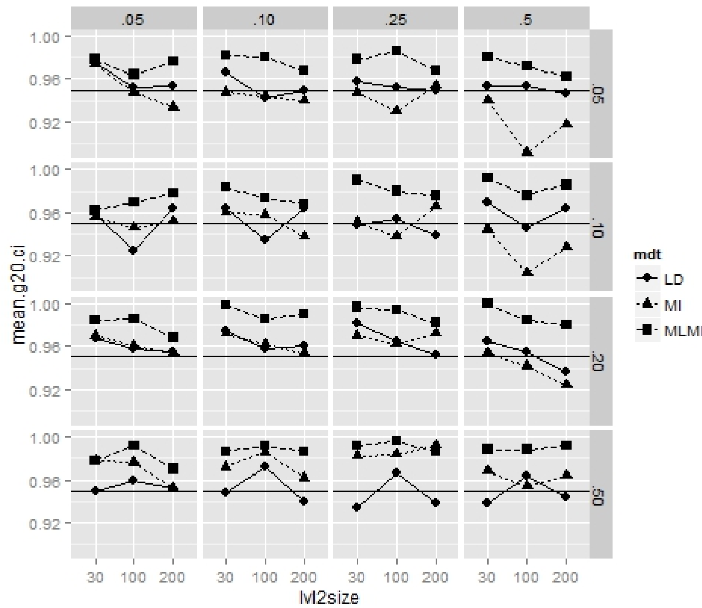
When the random effects followed a Laplace distribution, the coverage probabilities ranged between roughly .8 and .95. The pattern of coverage for the missing data treatments when the random effects followed the Laplace distribution was similar to the pattern when the random effects followed a Chi-square distribution. Specifically, both MLMI and MI produced coverage probabilities that exceeded the nominal rate as the percentage of missing data increased. In general LD produced coverage probabilities closest to the nominal rate compared to the other missing data treatments.

Approximate Cauchy distribution of random effects

When the random effects followed an approximate Cauchy distribution, the coverage probabilities of the parameter γ_{11} ranged between about .91 and .99. MLMI and MI tended to produce coverage probabilities that were larger than the nominal rate as the percentage of missing data increased. This pattern was different for LD in that LD produced coverage probabilities close to the nominal rate as sample size increased regardless of simulation conditions. MLMI and MI produced coverage probabilities closest to the nominal rate when the percentage of missing data was lowest (5%) and the dependence was highest (.5).

Figure 4.5 presents the proportion of simulations where the 95% confidence interval around the estimate of γ_{20} contained the parameter value (0.02).

Figure 4.5: 95% CI coverage of estimate of γ_{20} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

Multivariate normal random effects

Figure 4.5 shows the coverage probability for the parameter γ_{20} when the random effects followed a multivariate normal distribution. When the random effects followed a multivariate normal distribution the coverage probabilities ranged between approximately .90 and 1. In general MLMI produced coverage probabilities that were higher than the nominal rate (.95). LD produced coverage probabilities closest to the nominal rate (.95) compared to both MI and MLMI. MI produced coverage probabilities closest to the nominal rate when the percentage of missing data was smallest (5%) and often produced coverage probabilities closer to the nominal rate relative to MLMI. These results suggest that if one is interested in the fixed effect associated with a level one covariate in a two level model MLMI will tend to produce coverage probabilities that exceed the nominal rate whereas the coverage produced by MI will depend on the proportion of missing data. In general, when the percentage of missing data is small (5%) both MI and LD produce coverage probabilities close to the nominal rate.

Chi-square distribution of random effects

When the random effects followed a chi-square distribution the coverage probabilities ranged between about .78 and .96. LD and MLMI had similar coverage probabilities. For both LD and MLMI coverage probabilities approached the nominal rate when dependence was lowest (.05) and they produced conservative coverage probabilities as the dependence increased. MI consistently produced coverage probabilities that were too conservative, particularly as either the proportion of missing data or dependence increased.

Laplace distribution of random effects

When the random effects followed a Laplace distribution the coverage probabilities for the parameter γ_{20} ranged between roughly .80 and .97. Both LD and MLMI had similar coverage probabilities and their coverage tended to be close to the nominal rate (.95) under all simulation conditions, particularly when the number of clusters was highest (200). As with the other distributions of the random effects, MI tended to have coverage probabilities that were too conservative, especially as the percentage of missing data increased.

Approximate Cauchy distribution

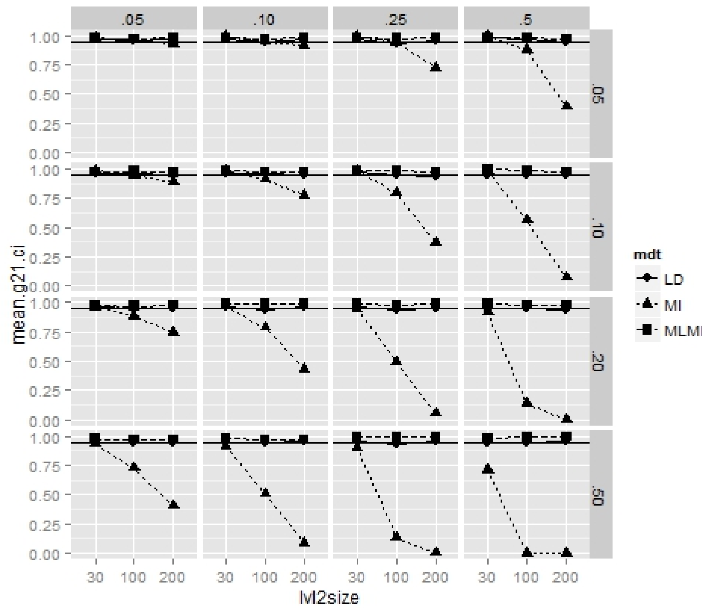
When the random effects followed an approximate Cauchy distribution the coverage probabilities ranged between about .84 and .98. LD and MLMI had similar coverage probabilities and both tended to have coverage probabilities that exceeded the nominal rate under all simulation conditions. MI produced coverage probabilities that were closest to the nominal rate when the percentage of missing data was smallest (5%) and produced coverage probabilities that were too conservative as the percentage of missing data increased.

Figure 4.6 presents the proportion of simulations where the 95% confidence interval around the estimate of γ_{21} contained the parameter value (0.04).

Multivariate normal random effects

Figure 4.6 shows the coverage probability for the parameter γ_{21} when the random effects followed a multivariate normal distribution. When the random effects followed a multivariate normal distribution coverage probabilities ranged between about 0 and 1. Both LD and MLMI consistently had coverage probabilities close to the nominal rate under all simulation conditions. However, MI produced coverage probabilities close

Figure 4.6: 95% CI coverage of estimate of γ_{21} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

to the nominal rate only when the percentage of missing data was smallest (5%) and dependence was smallest. Additionally, as the percentage of missing data increased, MI produced conservative coverage probabilities. These coverage probabilities approached 0 as the percentage of missing data approached 50%. This tendency did not appear to be mitigated by a larger number of clusters. This would suggest that if one is interested in the fixed effect associated with a cross-level interaction between a level two predictor and a level one predictor in a two-level model the use of either LD or MLMI would produce the appropriate coverage whereas MI would be problematic except under very restrictive conditions (i.e. 5% missing data and ICC less than .05).

Chi-square distribution of random effects

When the random effects followed a Chi-square distribution coverage probabilities ranged between about .92 and 1. MI consistently had coverage that exceeded the nominal rate (.95). The coverage probabilities produced by LD and MLMI were similar and tended to be close to the nominal rate. However, LD tended to produce coverage probabilities closest to the nominal rate.

Laplace distribution of random effects

When the random effects followed a Laplace distribution the coverage probabilities ranged between roughly .90 and 1. Both LD and MLMI produced coverage probabilities similar to one another. These coverage probabilities were also closer to the nominal rate as the number of clusters increased. MI produced coverage probabilities that exceeded the nominal rate as the percentage of missing data increased. All three missing data treatments produced coverage probabilities closest to the nominal rate when the percentage of missing data was lowest (5%) and the ICC was .20.

Approximate Cauchy distribution of random effects

When the random effects followed an approximate Cauchy distribution coverage probabilities ranged between roughly .93 and 1. LD tended to have coverage that was closest to the nominal rate (.95). However, both LD and MLMI tended to perform similarly with regard to the coverage probabilities. MI produced coverage probabilities that exceeded .95, particularly as the percentage of missing data increased.

For the sake of parsimony, while the figures of the coverage probabilities (proportion of simulations where the 95% confidence interval around the fixed effects estimates) were produced for each of the levels of the distribution conditions in this study, they were only referenced in the discussion of coverage but not presented in the body of this paper. Instead, the figures depicting the coverage of the fixed effects under the remainder of the distribution conditions and MCAR can be found in Appendix C and Appendix D respectively.

4.5 Summary of confidence interval results

In order to summarize the coverage of the parameters, the coverage probability of the 95% confidence interval was calculated after collapsing all the distributional conditions for the random effects (multivariate normal,χ^2,Laplace,approximate Cauchy). This was done to represent what may be expected on average across distributional conditions. The results for the coverage of the three missing data treatments are summarized in tables 4.8 through 4.13. These results are summarized by parameter.

With regard to the coverage of the fixed effects, LD was most robust to the simulation

Table 4.8: Table summarizing coverage of γ_{00} by missing data treatment

	95% CI Coverage Probability		
Missing Data Treatment	$< .95$	$\approx .95$	$> .95$
LD	N/A	All Conditions	N/A
MLMI	N/A	All Conditions	N/A
MI	$\uparrow$ Mis	ICC=.05, Mis=5%	N/A

Note: $\uparrow$*=increasing,*$\downarrow$*=decreasing,Mis= %Missing, N/A=Not Applicable.*

Table 4.9: Table summarizing coverage of γ_{01} by missing data treatment

	95% CI Coverage Probability		
Missing Data Treatment	$< .95$	$\approx .95$	$> .95$
LD	N/A	All Conditions	N/A
MLMI	N/A	All Conditions	N/A
MI	$\uparrow$ Mis,$\uparrow$ ICC	N/A	N/A

Note: $\uparrow$*=increasing,*$\downarrow$*=decreasing,Mis= %Missing, N/A=Not Applicable.*

conditions in terms of proper coverage (i.e. .95). That is, LD tended to have proper coverage of all fixed effects under all conditions except for the fixed effect γ_{11}. In the case of γ_{11} LD showed proper coverage when the number of clusters (level two units) was greater than or equal to 100.

MLMI performed similar to LD in that it showed proper coverage of most of the fixed effects (γ_{00},γ_{01},γ_{20},γ_{21}). On the other hand, the adequate coverage of two fixed effects (γ_{10} & γ_{11}) depended on other factors. Specifically, MLMI resulted in adequate coverage of γ_{10} when the percentage of missing data was at its lowest (5%) and tended to produce conservative coverage probabilities as the percentage of missing data increased. MLMI resulted in adequate coverage of γ_{11} when the ICC was at least .2 and the percentage of missing data was less than 25%. However, as the ICC remained at less than or equal to .1 and the percentage of missing data increased, MLMI tended to produce coverage probabilities that were too high.

Finally, MI performed the worst relative to the other two missing data treatments (LD,MLMI). MI tended to produce coverage probabilities that were too conservative as the percentage of missing data increased and as the ICC increased. MI tended to show proper coverage only when the percentage of missing data was smallest (5%).

Table 4.10: Table summarizing coverage of γ_{10} by missing data treatment

	95% CI Coverage Probability		
Missing Data Treatment	$< .95$	$\approx .95$	$> .95$
LD	N/A	All Conditions	N/A
MLMI	↑ICC,↑ Mis	Mis=5%	N/A
MI	↑ICC,↑Mis	Mis=5%	N/A

Note: ↑=increasing,↓=decreasing,Mis= %Missing, N/A=Not Applicable.

Table 4.11: Table summarizing coverage of γ_{11} by missing data treatment

	95% CI Coverage Probability		
Missing Data Treatment	$< .95$	$\approx .95$	$> .95$
LD	size=30	size≥100	N/A
MLMI	Mis=50%,ICC=.5	ICC≥.2 & Mis ≤ 25%	ICC≤.1 & ↑Mis
MI	Mis=50% & ICC=.5	ICC≥.2 & Mis≤25%	ICC≤.1 & ↑Mis

Note: ↑=increasing,↓=decreasing,Mis= %Missing, N/A=Not Applicable.

4.6 Percent relative estimate

The results for the percent relative estimate of the fixed effects and variance components are presented in figures 4.7 through 4.15. These figures portray the impact of the intraclass correlation, percentage of missing data, level two sample size, and missing data treatment when the data are MAR and when the random effects follow a multivariate normal distribution. These graphs are restricted to the condition where the random effects follow a multivariate normal distribution to examine the impact of the other factors when the distributional assumption of the random effects is met. The performance of the missing data treatments across the other levels of the distribution of random effects (Chi-square, Laplace, and approximate Cauchy) is presented in the figures in Appendix C. Additionally, the performance of the missing data treatments when the data are MCAR is presented in the figures in Appendix D.

Figure 4.7 presents the percent relative estimate for the estimate of γ_{00} which has a parameter value of 2.65. This estimate is the estimate of the average conditional intercept in the multilevel model.

Multivariate normal random effects

Figure 4.7 shows the average percent relative estimate for the estimate of γ_{00} when the random effects followed a multivariate normal distribution. When the random effects

Table 4.12: Table summarizing coverage of γ_{20} by missing data treatment

	95% CI Coverage Probability		
Missing Data Treatment	$< .95$	$\approx .95$	$> .95$
LD	N/A	All Conditions	N/A
MLMI	N/A	All Conditions	N/A
MI	↑Mis,ICC$\geq$.1	N/A	N/A

Note: ↑=increasing,↓=decreasing,Mis= %Missing, N/A=Not Applicable.

Table 4.13: Table summarizing coverage of γ_{21} by missing data treatment

	95% CI Coverage Probability		
Missing Data Treatment	$< .95$	$\approx .95$	$> .95$
LD	N/A	All Conditions	N/A
MLMI	N/A	All Conditions	N/A
MI	↑Mis & ICC$\geq$.1	Mis=5% & size=30	size=30 & ICC $\leq$.1

Note: ↑=increasing,↓=decreasing,Mis= %Missing, N/A=Not Applicable.

follow a multivariate normal distribution and the data are MAR, the estimates for the fixed effect of the conditional intercept is relatively unbiased. This is determined by the fact that in all of the plots in figure 4.7, the average percent relative estimate ranges between approximately 100.3 and 99.9. This suggests that if one is interested in obtaining an unbiased estimate of the fixed effect associated with conditional intercept in a two level model, all three missing data treatments will perform well when the random effects follow a multivariate normal distribution.

Chi-square distribution of random effects

When the random effects followed a chi-square distribution, the percent relative estimate ranged between roughly 98 and 101 across the simulation conditions. This range is close to the ideal value of 100%.

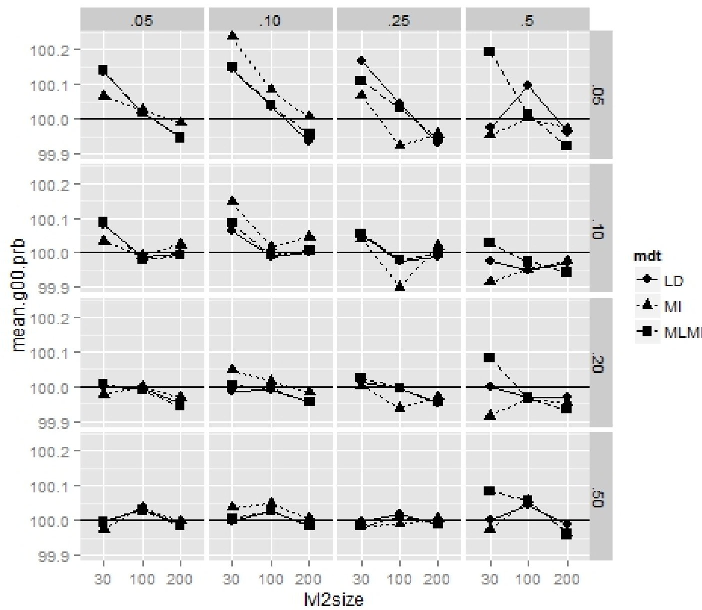
Laplace distribution of random effects

As with both the multivariate normal and chi-square distribution of the random effects, when the random effects followed a Laplace distribution, the percent relative estimate remained very close to 100 under all simulation conditions (i.e. the range was between about 100 and 102).

Approximate Cauchy distribution of random effects

When the random effects followed an approximate Cauchy distribution, the percent

Figure 4.7: Percent relative estimate of γ_{00} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

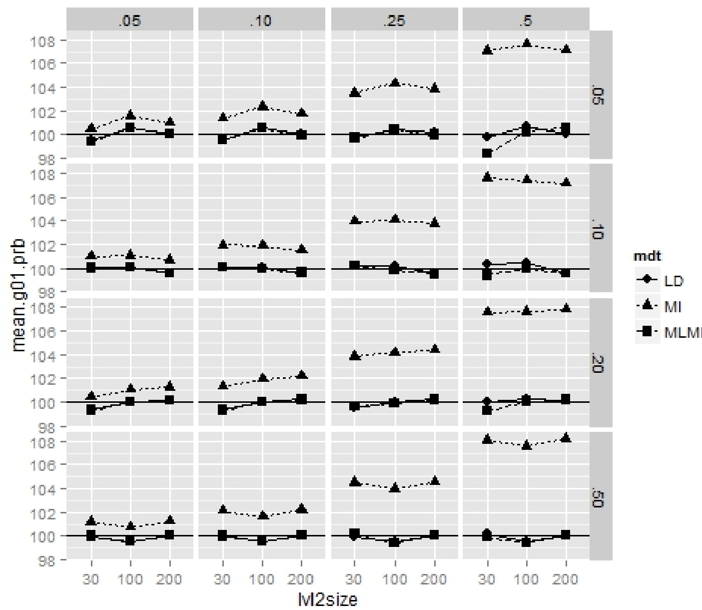
relative estimate ranged between about -10 and 220. Additionally, the percent relative estimate was somewhat erratic across simulation conditions. The effects of the number of clusters was not consistent. For example, under the approximate Cauchy distribution, for an ICC of 0.5 and a level two sample size of 30, the parameter estimate was first underestimated to a small degree, as the number of clusters increased to 100, it was over estimated to a large degree, then when the number of clusters was highest (200), it was again underestimated to a large degree. It does not appear as if the bias follows a particular pattern when the random effects follow an approximate Cauchy distribution.

Figure 4.8 illustrates the average percent relative estimate for the estimate of γ_{01} which has a parameter value of 0.41. This estimate is the estimate of the impact of the level two predictor W_{1j} on the intercept.

Multivariate normal random effects

Figure 4.8 shows the average percent relative estimate for the estimate of γ_{01} when the random effects followed a multivariate normal distribution. When the random effects follow a multivariate normal distribution the percent relative estimate ranged between

Figure 4.8: Percent relative estimate of γ_{01} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

approximately 98 and 108. MI tended to over estimate this parameter, particularly as the percentage of missing data increased. LD and MLMI performed similarly with regard to bias. Both LD and MLMI consistently produced an unbiased estimate of this parameter under almost all simulation conditions (i.e. percent relative estimate was nearly 100). This suggests that if one is interested in the estimate of the fixed effect associated with a level two predictor of the intercept in a two level model, generally either LD or MLMI will produce an unbiased estimate.

Chi-square distribution of random effects

When the random effects followed a Chi-square distribution, the percent relative estimate ranged between approximately 92 and 112. Relative to the multivariate normal distribution of random effects, this parameter estimate showed a somewhat larger degree of bias when the random effects followed a Chi-square distribution. Under the Chi-square distribution, when the number of level two units was small (i.e. 30) and the within group dependence was relatively large (i.e. 0.20), this parameter was under estimated after using both LD and MLMI. In contrast, when the within group dependence was largest (i.e. 0.5), the estimate of this parameter was relatively unbiased for both LD and MLMI, whereas MI overestimated this parameter as the percentage of missing data increased. All three missing data treatments performed similarly and well (i.e. produced an unbiased estimate) when the percentage of missing data was lowest (5%) and the ICC was lowest (.05).

Laplace distribution of random effects

When the random effects followed a Laplace distribution the percent relative estimate ranged between about 95 and 113. Both LD and MLMI performed similarly. LD and MLMI produced a relatively unbiased estimate when the percentage of missing data was under 25% and the ICC was less than or equal to .20. MI produced an overestimate of this parameter as the percentage of missing data increased. In general all three missing data treatments performed similarly and well when the percentage of missing data was smallest (5%).

Approximate Cauchy distribution of random effects

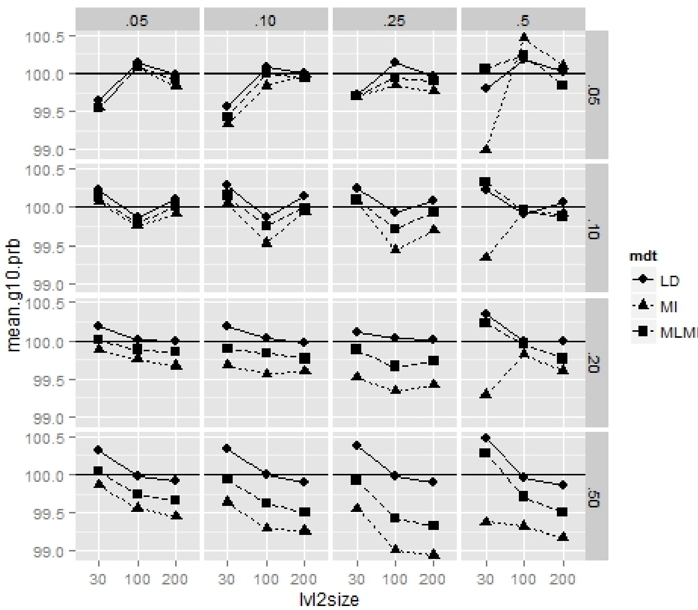
When the random effects followed an approximate Cauchy distribution the bias was more dramatic in that it ranged between about -3400 and 1200. The large values for the range were due primarily to the performance of MI under a few circumstances. For example, when the ICC was equal to .20 and the number of clusters was 30, MI dramatically underestimated this parameter, particularly as the percentage of missing data increased. However, it should be noted that MLMI also performed particularly poorly under certain conditions (e.g. percentage of missing data = 50% and number of clusters = 200). In general, as the within group dependence increased, so did the bias. Additionally, sample size did not appear to mitigate the impact of dependence on the bias.

Figure 4.9 illustrates the average percent relative estimate for the estimate of γ_{10} which has a parameter value of 0.57. This estimate is the conditional estimate of the average impact of the level one predictor X_{1ij} on Y_{ij} in the multilevel model.

Multivariate normal random effects

Figure 4.9 shows the average percent relative estimate for the estimate of γ_{10} when the random effects followed a multivariate normal distribution. When the random effects followed a multivariate normal distribution the percent relative estimate of the estimate of the parameter γ_{10} ranged between roughly 99 and 100.5 indicating that the estimate

Figure 4.9: Percent relative estimate of γ_{10} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

of this parameter was relatively unbiased under almost all conditions and missing data treatments.

Chi-square distribution of random effects

When the random effects followed a chi-square distribution the percent relative estimate ranged between about 90 and 105. LD and MLMI tended to perform similarly with regard to bias and the estimate of this parameter after using either LD or MLMI tended to be unbiased as the number of clusters increased. When the percentage of missing data was highest (50%) and the ICC was .10 or greater, MI tended to underestimated this parameter.

Laplace distribution of random effects

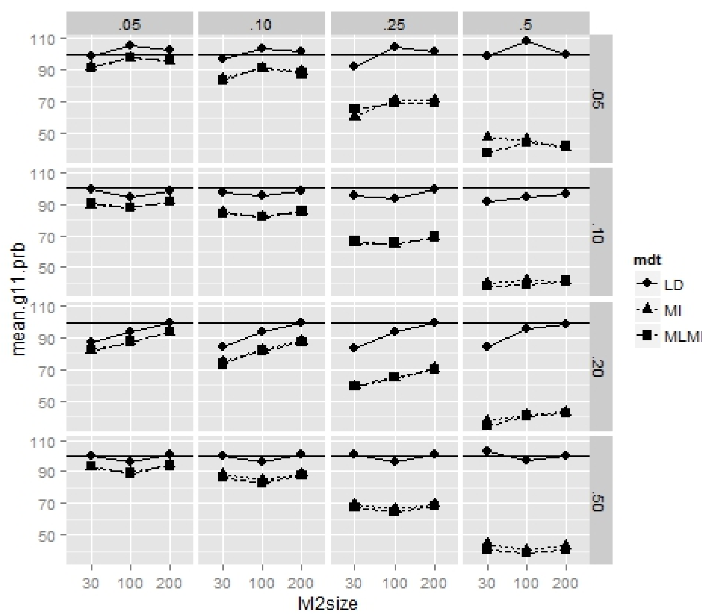
When the random effects followed a Laplace distribution the percent relative estimate ranged between about 90 and 105. In general the different missing data treatments performed similarly. One exception is for MI when the percentage of missing data was highest (50%) and the ICC was highest (0.5) in which case MI underestimated this parameter with percent relative estimate between about 90 and 95. When dependence was highest (.50) all three missing data treatments tended to underestimate this parameter.

Approximate Cauchy distribution of random effects

When the random effects followed an approximate Cauchy distribution the bias in this estimate was dramatic. It was least biased under all conditions when the ICC was either 0.05, 0.20, or 0.50. However, even under these conditions the bias was still large (e.g. 200%, -42%). The coverage was extremely poor when the ICC was 0.10. Additionally, the three missing data treatments performed similarly with respect to the bias under all simulation conditions.

Figure 4.10 illustrates the average percent relative estimate for the estimate of γ_{11} which has a parameter value of 0.02. This estimate is the estimate of the impact of the level two predictor W_j on the slope β_{1j} in the multilevel model and therefore reflects a cross-level interaction.

Figure 4.10: Percent relative estimate of γ_{11} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

Multivariate normal random effects

Figure 4.10 shows the average percent relative estimate for the estimate of γ_{11} when the random effects followed a multivariate normal distribution. When the random effects followed a multivariate normal distribution the percent relative estimate of the estimate of γ_{11} ranged between about 40 and 110. This is the estimate of the fixed effect reflecting the cross-level interaction between the level two predictor and the slope of X_{1ij} on Y_{ij}. The estimate was normally either relatively unbiased or was underestimated. In particular, whereas under MI and MLMI the estimate for this fixed effect tended to be underestimated as the percentage of missing data increased, under LD estimates tended to be unbiased even as the percentage of missing data increased (except when the ICC was equal to .20 and the number of level two units was equal to 30 which resulted in underestimation). This suggests that if one is interested in the fixed effect associated with a cross-level interaction between a level two predictor and a level one covariate, LD will tend to produce an unbiased estimate of this effect whereas MI and MLMI will underestimate this effect.

Chi-square distribution of random effects

When the random effects followed a chi-square distribution the percent relative estimate ranged between roughly 0 and 300. In most cases the three missing data treatments performed similarly but in some cases both MI and MLMI outperformed LD. For all missing data treatments, the estimate of this parameter tended to become less biased as sample size increased. The estimate of this parameter was most biased when dependence was smallest (ICC=.05) and the number of clusters was smallest (30).

Laplace distribution of random effects

When the random effects followed a Laplace distribution the percent relative estimate ranged between about 20 and 250. In general this parameter was rarely unbiased. It was either overestimated or underestimated to a relatively large degree. The three missing data treatments performed similarly with respect to percent relative estimate in about half of the conditions. When the missing data treatments performed differently and level two sample size was either 100 or 200, LD outperformed the other two missing data treatments.

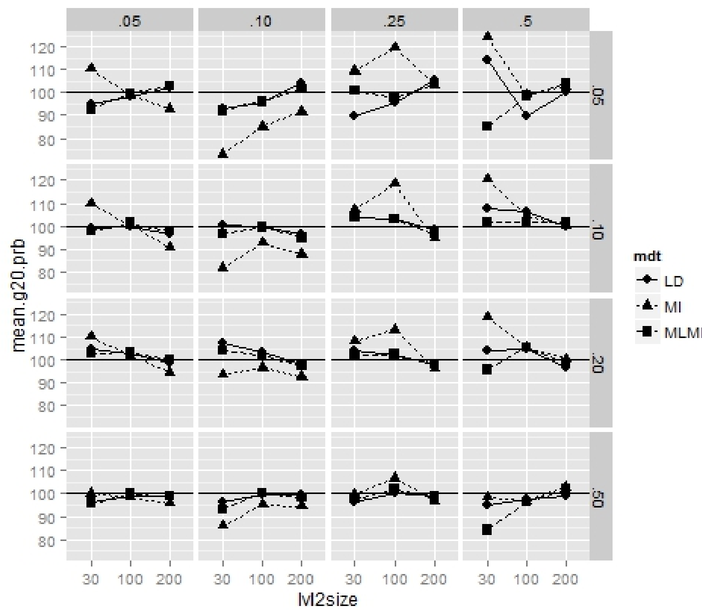
Approximate Cauchy distribution of random effects

Under the approximate Cauchy distribution this parameter estimate was biased to a large degree with all three missing data treatments performing poorly and similarly. The bias under all simulation conditions was extremely large.

Figure 4.11 illustrates the average percent relative estimate for the estimate of γ_{20}

which has a parameter value of 0.02. This estimate is the conditional estimate of the average impact of the level one predictor X_{2ij} on Y_{ij} in the multilevel model.

Figure 4.11: Percent relative estimate of γ_{20} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

Multivariate normal random effects

Figure 4.11 shows the average percent relative estimate for the estimate of γ_{20} when the random effects followed a multivariate normal distribution. When the random effects followed a multivariate normal distribution the percent relative estimate of the estimate of γ_{20} ranged between roughly 72 and 128. This estimate is of the fixed effect associated with the slope of X_{2ij} on Y_{ij}. The results appear to be somewhat mixed. In general, larger sample sizes were associated with less bias. Additionally, as the amount of within group dependence increased, the estimate tended to be less biased for all missing data treatments. While there are situations under which MLMI appeared to produce a less biased estimate of this fixed effect relative to LD (e.g. ICC=.10, 50% missing data, sample size=30), under many situations LD and MLMI produced similar results. Both LD and MLMI performed similarly and well when the percentage of missing data was smallest (5%). This suggests that if one is interested in the fixed effect associated with a level one covariate in a two level model and the random effects follow a multivariate normal distribution generally either LD or MLMI will produce relatively unbiased estimates.

Chi-square distribution of random effects

When the random effects followed a chi-square distribution the percent relative estimate ranged between approximately -25 and 325. LD and MLMI tended to perform similarly. When the number of clusters was smallest (30) MI produced biased estimates with the degree of bias (underestimation or overestimation) varying relative to the amount of within group dependence. All three missing data treatments produced relatively biased estimates under most simulation conditions with the least amount of bias observed when the number of clusters was highest (200) and as the ICC increased.

Laplace distribution of random effects

When the random effects followed a Laplace distribution the percent relative estimate ranged between approximately -100 and 325. Both LD and MLMI performed similarly. MI tended to produce estimates that were more biased than the other two missing data treatments under many of the simulation conditions. However, all three missing data treatments produced biased estimates under many of the simulation conditions. The estimates were least biased when the number of clusters was highest (200) and the ICC was highest.

Approximate Cauchy distribution of random effects

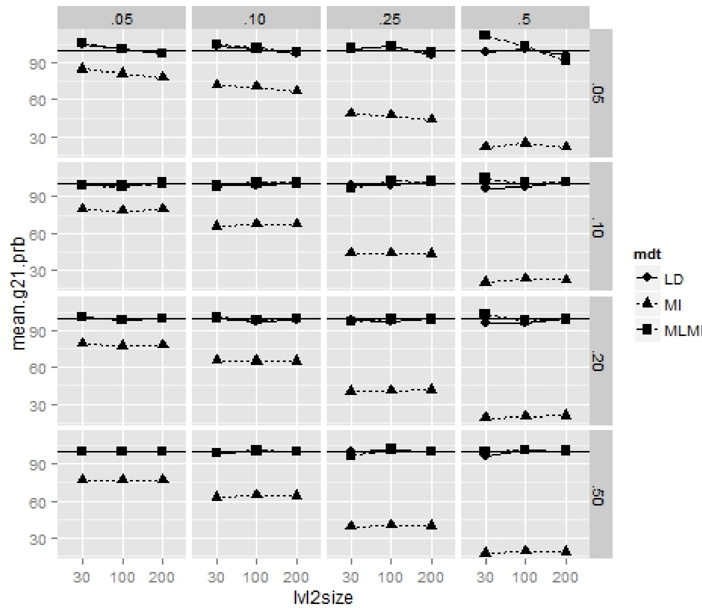
When the random effects followed an approximate Cauchy distribution the estimate of this parameter was extremely biased, particularly when the ICC was .05.

Figure 4.12 illustrates the average percent relative estimate for the estimate of γ_{21} which has a parameter value of 0.04. This estimate is the estimate of the impact of the level two predictor W_j on the slope β_{2j} in the multilevel model and therefore reflects a cross-level interaction.

Multivariate normal random effects

Figure 4.12 shows the average percent relative estimate for the estimate of γ_{21} when the random effects followed a multivariate normal distribution. When the random effects followed a multivariate normal distribution the percent relative estimate ranged between about 15 and 100. This estimate is the estimate of the fixed effect associated with the cross-level interaction between the level two covariate and the slope for X_{2ij} on Y_{ij}.

Figure 4.12: Percent relative estimate of γ_{21} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

This estimate was underestimated when using MI and the degree of underestimation increased as the percentage of missing data increased. LD and MLMI appear to be similar with regard to percent relative estimate under the various conditions in that they both produce relatively unbiased estimates across simulation conditions. This suggests if one is interested in the estimate of the fixed effect associated with a cross-level interaction between a level two covariate and a level one covariate when the random effects follow a multivariate normal distribution, both LD and MLMI will produce an unbiased estimate whereas MI will tend to underestimate this effect.

Chi-square distribution of random effects

When the random effects followed a chi-square distribution the percent relative estimate ranged between roughly 0 and 175. Both MLMI and LD produced similar estimates of this parameter under most of the conditions of the simulation study. However, all missing data treatments produced somewhat biased estimates under most simulation conditions. LD and MLMI differed most when the percentage of missing data was largest (50%). MI consistently underestimated this parameter, particularly as the percentage of missing data increased.

Laplace distribution of random effects

When the random effects followed a Laplace distribution the percent relative estimate ranged between about 0 and 200. MI consistently underestimated this parameter under most simulation conditions and this bias was more severe as the percentage of missing data increased. MLMI and LD tended to produce unbiased estimates as the number of level two units increased.

Approximate Cauchy distribution of random effects

When the random effects followed an approximate Cauchy distribution, the bias in the parameter estimate was dramatic and somewhat erratic. For example, when the ICC was .20, this parameter was dramatically underestimated under a small number of level two units (30) whereas this parameter was overestimated under the same number of level two units when the ICC was .50.

The next set of plots is for the average percent relative estimate of the variance components of interest ($\tau_{00}, \tau_{11}, \tau_{22}$). These estimates correspond to the variance of the conditional intercept, the variance of the slope associated with $X_{1ij}(\beta_{1j})$, and the variance of the slope associated with $X_{2ij}(\beta_{2j})$, respectively. The covariances between the random effects are not presented here because the complete data were generated under zero covariance among the random effects.

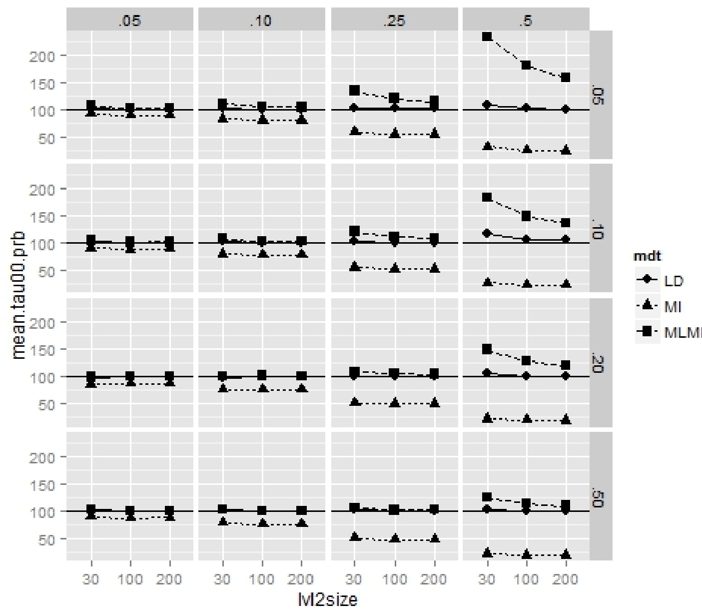
The percent relative estimate was not calculated for any of the variance components under the approximate Cauchy distribution as there was no parameter value to use for the calculation of bias (because the approximate Cauchy distribution has an undefined variance).

Figure 4.13 depicts the percent relative estimate for the variance component τ_{00} (which has a parameter value of 0.052) across the simulation conditions (various levels of the ICC, percent of missing data, sample size, and missing data treatment under the condition of MAR when the random effects follow a multivariate normal distribution).

Multivariate normal random effects

Figure 4.13 shows the average percent relative estimate for the estimate of τ_{00} when the random effects followed a multivariate normal distribution. When the random effects followed a multivariate normal distribution the percent relative estimate ranged

Figure 4.13: Percent relative estimate of τ_{00} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

between about 0 and 275. MI underestimated the variance associated with the intercept, particularly as the percentage of missing data increased. Also, while in general the estimate obtained after using LD and MLMI tended to be similar and unbiased, under the highest percentage of missing data (50%), MLMI tended to overestimate the parameter compared to LD. This suggests that if one is interested in the estimate of the variance component associated with an intercept in a two level model and the random effects follow a multivariate normal distribution LD will provide an unbiased estimate as will MLMI but under more restricted conditions. MI should be avoided.

Chi-square distribution of random effects

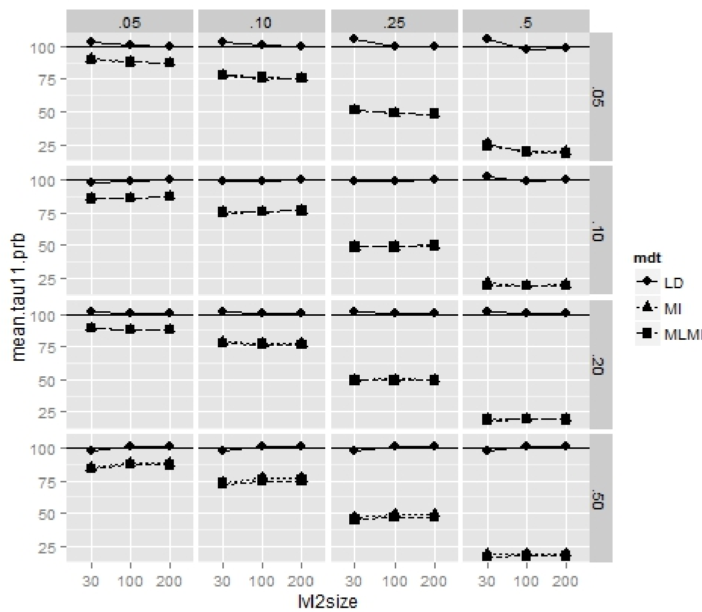
When the random effects followed a chi-square distribution the percent relative estimate ranged between about 35 and 110. This parameter was estimated with slightly more bias than when the random effects followed a multivariate normal distribution. Again, both LD and MLMI produced similar results, whereas MI tended to underestimate this parameter, especially as the percentage of missing data increased.

Laplace distribution of random effects

When the random effects followed a Laplace distribution the percent relative estimate ranged between about 35 and 118. These results were similar to the results observed under the condition of multivariate normality. Both LD and MLMI performed similarly and well (i.e. percent relative estimate close to 100) whereas MI consistently underestimated this parameter.

Figure 4.14 depicts the percent relative estimate for the variance component τ_{11} (which has a parameter value of 0.024).

Figure 4.14: Percent relative estimate of τ_{11} under multivariate normal random effects



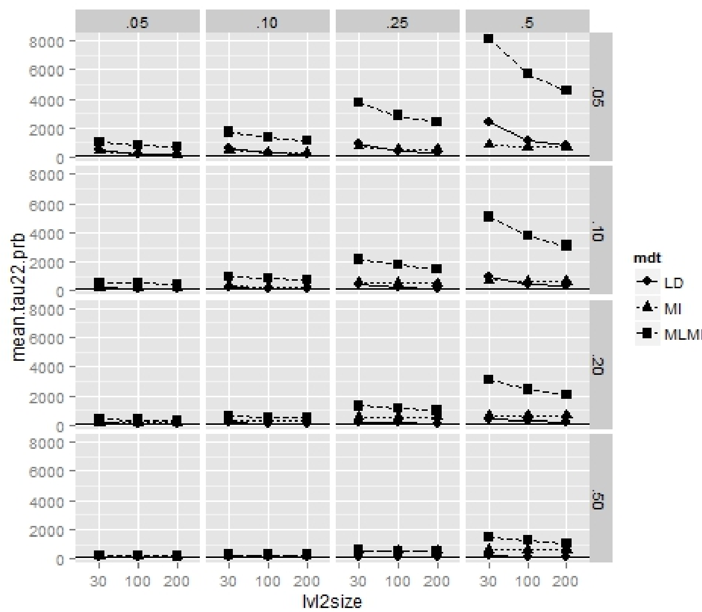
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

Multivariate normal random effects

Figure 4.14 shows the average percent relative estimate for the estimate of τ_{11} when the random effects followed a multivariate normal distribution. When the random effects followed a multivariate normal distribution the percent relative estimate for the estimate of τ_{11} ranged between about 12 and 113. MI and MLMI tended to underestimate this parameter as the percentage of missing data increased. In contrast, LD tended to be unbiased across all simulation conditions. This pattern was nearly identical when the random effects followed a chi-square and Laplace distribution as well. Therefore the results of the percent relative estimate of this parameter are not discussed separately for either the chi-square or Laplace distribution of the random effects. The performance of the missing data treatments suggests that when one is interested in the estimate of the variance component for a slope in a two level model LD is preferable to either MI or MLMI.

Figure 4.15 depicts the average percent relative estimate for the variance component τ_{22} (which has a parameter value of 0.001).

Figure 4.15: Percent relative estimate of τ_{22} under multivariate normal random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows.

Multivariate normal random effects

Figure 4.15 shows the average percent relative estimate for the estimate of τ_{22} when the random effects followed a multivariate normal distribution. When the random effects followed a multivariate normal distribution the percent relative estimate for the estimate of τ_{22} ranged between about 0 and 8000. This parameter tended to be overestimated and this overestimation was more prevalent with higher rates of missing data. Additionally, under higher percentages of missing data and smaller ICCs, LD tended to overestimate this parameter relative to MLMI. MI also tended to produce an overestimate of this parameter with higher rates of missing data (25 and 50% missing data). The three missing data treatments produced the least amount of bias when the percentage of missing data was smallest (5%) and dependence was highest (0.5).

Chi-square distribution of random effects

When the random effects followed a chi-square distribution the percent relative estimate ranged between about 35 and 125 which is a much smaller range than when the random effects followed a multivariate normal distribution. MI tended to underestimate this parameter and this underestimation increased with larger percentages of missing data. LD and MLMI performed similarly and well under all conditions. This also differs from the results observed when the random effects followed a multivariate normal distribution.

Laplace distribution of random effects

When the random effects followed a Laplace distribution the bias in this parameter estimate was similar to that observed under the chi-square distribution. Specifically, the range in percent relative estimate was roughly between 35 and 125. Both LD and MLMI performed similarly and well whereas MI underestimated the parameter as sample size increased.

Again, for the sake of parsimony, the figures of the percent relative estimate for the fixed effects and variance components under the other levels of the distributional conditions for the random effects are presented in Appendix C.

4.7 Summary of percent relative estimate results

In order to summarize the percent relative estimate of the parameters, the percent relative estimate for each parameter was calculated after collapsing three of the four distributional conditions for the random effects (multivariate normal,χ^2,Laplace). The approximate Cauchy distribution was removed as this produced such large values for the average percent relative estimate that none of the missing data treatments showed adequate percent relative estimate (i.e. 100%) when the approximate Cauchy distribution was included. The results were collapsed across distributions to get a sense of the average performance of the missing data treatments. The results of the percent relative estimate of the three missing data treatments are summarized in tables 4.14 through

4.19. These results are summarized by parameter.

Table 4.14: Table summarizing percent relative estimate of γ_{00} by missing data treatment

	Percent Relative Estimate		
Missing Data Treatment	< 100	≈ 100	> 100
LD	N/A	All Conditions	N/A
MLMI	N/A	All Conditions	N/A
MI	N/A	All Conditions	N/A

Note: ↑=*increasing*,↓=*decreasing*,*Mis*= *%Missing, N/A=Not Applicable.*

Table 4.15: Table summarizing percent relative estimate of γ_{01} by missing data treatment

	Percent Relative Estimate		
Missing Data Treatment	< 100	≈ 100	> 100
LD	N/A	All Conditions	N/A
MLMI	N/A	All Conditions	N/A
MI	N/A	Mis=5%	↑Mis

Note: ↑=*increasing*,↓=*decreasing*,*Mis*= *%Missing, N/A=Not Applicable.*

Table 4.16: Table summarizing percent relative estimate of γ_{10} by missing data treatment

	Percent Relative Estimate		
Missing Data Treatment	< 100	≈ 100	> 100
LD	N/A	All Conditions	N/A
MLMI	N/A	All Conditions	N/A
MI	↑ICC,↑Mis	N/A	N/A

Note: ↑=*increasing*,↓=*decreasing*,*Mis*= *%Missing, N/A=Not Applicable.*

With regard to the percent relative estimate of the fixed effects, listwise deletion (LD) was most robust to the simulation conditions. Specifically, LD showed very little bias under all simulation conditions for the fixed effects γ_{00},γ_{01}, and γ_{10}. For the remaining fixed effects (γ_{11},γ_{20}, and γ_{21}), LD produced unbiased estimates as the number of clusters increased and it tended to produce biased estimates when the number of clusters was smallest.

Multilevel multiple imputation (MLMI) performed nearly identical to LD for all fixed effects estimates except for γ_{11}. For γ_{11}, MLMI produced unbiased estimates under a

Table 4.17: Table summarizing percent relative estimate of γ_{11} by missing data treatment

	Percent Relative Estimate		
Missing Data Treatment	< 100	≈ 100	> 100
LD	N/A	size$\geq$100	size=30
MLMI	$\uparrow$Mis & size$\geq$100	Mis=5% & size$\geq$100	size=30 & ICC=.05 OR .5 & Mis$\leq$10%
MI	$\uparrow$Mis & size $\geq$100	Mis=5% & size$\geq$100	size=30 & ICC=.05 OR .5 & Mis$\leq$10%

Note: $\uparrow$=increasing,$\downarrow$=decreasing,Mis= %Missing, N/A=Not Applicable.

Table 4.18: Table summarizing percent relative estimate of γ_{20} by missing data treatment

	Percent Relative Estimate		
Missing Data Treatment	< 100	≈ 100	> 100
LD	size=30 & ICC=.5	size=200	size=30 & Mis=5% & ICC $\leq$.2
MLMI	size=30 & ICC=.5	size=200	size=30 & Mis=5% & ICC $\leq$.2
MI	size=30 & ICC=.5, size=30 &Mis=10%	Mis=25% & size=100	size=30 & Mis=5% & ICC $\leq$.2

Note: $\uparrow$=increasing,$\downarrow$=decreasing,Mis= %Missing, N/A=Not Applicable.

more restrictive set of conditions relative to LD. Specifically, MLMI produced unbiased estimates for γ_{11} when the percentage of missing data was at its lowest (5%) and the number of clusters was at least 100.

Multiple imputation under the normal model (MI) performed the worst relative to the other two missing data treatments (LD,MLMI). As the percentage of missing data increased, MI resulted in downwardly biased estimates of the fixed effects γ_{10},γ_{11}, and γ_{21}. MI tended to only produce unbiased estimates when the percentage of missing data was smallest (5%).

With regard to the variance components for the random effects, LD produced unbiased estimates under all conditions for both τ_{00} and τ_{11}. LD produced unbiased

Table 4.19: Table summarizing percent relative estimate of γ_{21} by missing data treatment

	Percent Relative Estimate		
Missing Data Treatment	< 100	≈ 100	> 100
LD	N/A	↑size	size=30 & ICC=.1
MLMI	N/A	↑size	size=30 & ICC=.1
MI	↑Mis	N/A	N/A

Note: ↑=increasing,↓=decreasing,Mis= %Missing, N/A=Not Applicable.

Table 4.20: Table summarizing percent relative estimate of τ_{00} by missing data treatment

	Percent Relative Estimate		
Missing Data Treatment	< 100	≈ 100	> 100
LD	N/A	All Conditions	N/A
MLMI	N/A	Mis≤10%	Mis=50% & ↓ICC
MI	↑Mis	N/A	N/A

Note: ↑=increasing,↓=decreasing,Mis= %Missing, N/A=Not Applicable.

estimates of $\tau_{22}$when the ICC was .50 and tended to overestimate τ_{22} as the ICC decreased.

MLMI produced unbiased estimates for τ_{00} when the percentage of missing data was less than 10%. However, MLMI underestimated τ_{11} as the percentage of missing data increased and overestimated τ_{22} under all conditions.

MI underestimated both τ_{00} and τ_{11} as the percentage of missing data increased and overestimated τ_{22} under all conditions.

4.8 Overall summary of simulation results

In general LD and MLMI performed similarly with respect to both the coverage of the fixed effects and the percent relative estimate of the fixed effects. When the results differed for the fixed effects, they tended to be in favor of LD. These results do not support the hypothesis that MLMI would outperform LD both in terms of the coverage and bias of the fixed effects. However, the hypothesis with regard to MI was supported. Specifically, MI did tend to perform poorly relative to the other two missing data treatments in terms of both coverage and bias and its performance was much more dependent on

Table 4.21: Table summarizing percent relative estimate of τ_{11} by missing data treatment

Missing Data Treatment	Percent Relative Estimate < 100	≈ 100	> 100
LD	N/A	All Conditions	N/A
MLMI	↑Mis	N/A	N/A
MI	↑Mis	N/A	N/A

Note: ↑=increasing,↓=decreasing,Mis= %Missing, N/A=Not Applicable.

Table 4.22: Table summarizing percent relative estimate of τ_{22} by missing data treatment

Missing Data Treatment	Percent Relative Estimate < 100	≈ 100	> 100
LD	N/A	ICC=.50	↓ICC
MLMI	N/A	N/A	All Conditions
MI	N/A	N/A	All Conditions

Note: ↑=increasing,↓=decreasing,Mis= %Missing, N/A=Not Applicable.

the percentage of missing data and the ICC (compared to LD and MLMI).

Whereas LD and MLMI performed similarly for the fixed effects, LD showed an advantage in terms of the variance components for the random effects. Specifically, LD produced unbiased estimates of the parameters τ_{00} and τ_{11} under all conditions and produced unbiased estimates of τ_{22} when the ICC was .50. This performance of LD is in contrast to MLMI which only produced unbiased estimates of τ_{00} when the percentage of missing data was less than or equal to 10%. MLMI also consistently underestimated τ_{11} (especially as the percentage of missing data increased) and overestimated τ_{22} under all conditions. In contrast to both LD and MLMI, MI consistently produced biased estimates of all three variance components (τ_{00},τ_{11},τ_{22}).

The superior performance of LD relative to MLMI in terms of the variance of the random effects does not support the hypothesis that MLMI would outperform LD. However, the hypothesis of the poor performance of MI relative to both LD and MLMI was supported. The implications of these results are discussed in the next chapter.

Chapter 5

Conclusion and Discussion

5.1 Conclusion

The purpose of this study was to examine the performance of three missing data treatments for multilevel missing data; multilevel multiple imputation (MLMI), listwise deletion (LD), and multiple imputation under the normal model (MI). This study used a simulation to compare the three missing data treatments under conditions known to influence both missing data treatments and hierarchical linear modeling. This study simulated multilevel data under a variety of conditions (number of level two units, distribution of random effects, percentage of missing data, intraclass correlation) and imposed missingness on both the outcome and a level one covariate such that the missingness was either missing at random (MAR) or missing completely at random (MCAR). These data were then subjected to all three missing data treatments for comparison in their ability to both retrieve parameter estimates close to their true values and obtain unbiased estimates.

Research literature on the effect of missing data suggested that MLMI would outperform LD and MI. The author believed this to be the case because literature on missing data shows that when LD is applied to data that are not MCAR results can be biased (Peng et al., 2006; Rubin, 1987; Schafer, 1997). This bias associated with LD was thought to apply to multilevel missing data as well. Furthermore,it was hypothesized that MI would perform poorly because of literature on the implications of uncongenial (i.e. fail to take into account subsequent analyses) missing data treatments (Meng, 1994;

Puma et al., 2009; Taljaard et al., 2008). Contrary to the hypotheses, LD performed best with respect to bias of estimates as well as the coverage of the fixed effects. While MLMI and LD performed similarly under most conditions, MLMI performed poorly with regard to the estimation of the variance components of the random effects. This is noteworthy because the estimation of variance components may be of interest in studies utilizing a multilevel model.

This study also found that while the distribution of random effects influences the bias and coverage of parameter estimates, the relative performance of the missing data treatments tends to be unaffected by the distribution of the random effects. That is, LD tended to outperform the other two missing data treatments regardless of the distribution of random effects.

MI proved to be a poorly performing missing data treatment, particularly as the percentage of missing data increased and as the within group dependence increased.

Factors known to influence the performance of missing data treatments in single level models proved to be influential in multilevel models as well. When the percentage of missing data was lowest (5%) all three missing data treatments performed similarly. Additionally, as the number of level two units (i.e. number of clusters) increased, the negative effects of missing data tended to decrease.

5.2 Discussion

The remainder of this chapter consists of four sections. The first section gives possible explanations for the unexpected results observed in this study. The second section gives recommendations for future research in the area of missing data treatments for multilevel data. The third section gives recommendations to both applied researchers fitting cross-sectional multilevel models to datasets that contain missing data and methodological researchers. Finally, the fourth section describes the limitations of this study.

5.2.1 Making sense of the results

Surprisingly LD was superior under almost all conditions. The answer to how LD performed so well in this study may be found in Allison (2002). Allison states that when covariates in a regression have missing values, there is a special case in which

LD produces the least biased results (relative to other missing data treatments). This ideal situation for LD occurs when the probability of missingness for the independent variables does not depend on the values of the dependent variable. Allison shows this result with the following expression:

$$f(Y|X, A = 1) = \frac{P(A=1|Y,X)f(Y|X)f(X)}{P(A=1|X)f(X)} = f(Y|X, A = 1) = f(Y|X)$$

Where $A = 1$ if all the variables are complete and $A = 0$ when at least one variable has missing values. The key is that when the probability of missingness for the independent variables X is unrelated to the dependent variable, then $P(A = 1|Y, X) = P(A = 1|X)$ and therefore $f(Y|X, A = 1) = f(Y|X)$. As a result, when the probability of missingness on the independent variable is unrelated to the values of the dependent variable, regression based on only complete cases provides unbiased results (i.e. results are similar to what would have been obtained if there were no missing data).

In this simulation study missingness on the outcome, Y_{ij}, and the level one covariate, X_{1ij}, depended on a level one covariate, $P(R_{Yij}|X_{2ij})$, and $(P(R_{X_{1ij}}|X_{2ij}))$ respectively. Therefore, using the notation of Allison (2002), the probability of a case having at least one missing value ($A = 0$), or conversely, having complete data ($A = 1$) depended on just the covariate X_{2ij}, $P(A = 1|X_{2ij})$. In this way missingness on the level one covariate did not depend on the outcome, $P(A = 1|Y_{ij}, X_{2ij}) = P(A = 1|X_{2ij})$. It may be the case that the optimal performance of LD described above holds true for multilevel models as well.

It is important to keep in mind that this study imposed missingness on the outcome Y_{ij} as well. However, missingness on the outcome was MAR conditional on X_{2ij} which was in the model. As noted in chapter 2, when the outcome is MAR and the model is fit using ML, the inclusion of the covariate that explains missingness on the outcome results in unbiased estimates. The combination of using ML for an outcome which was MAR and the independence of the probability of missingness on the covariate and the values of the dependent variable may account for the stellar performance of LD in this simulation study.

The result of the poor performance of MI is consistent with the findings of Black (2008). Black (2008) found that MI resulted in more biased estimates of both the fixed effects and variance components for the random effects relative to LD and MLMI. In her study missingness was generated for the dependent variable alone conditional on both a level one predictor and a level two predictor (so that missingness on the outcome was MAR). Using MI to impute missing values for multilevel data ignores the multilevel nature of the data. Because MI does not take into account the multilevel nature of the data, imputation is carried out as though there is no dependence among the observations ($Y_{i.} \perp Y_{i.}$) and therefore the ICC is assumed to be equal to zero during imputation. Additionally, under MI relationships between variables are thought to be homogeneous across level two units (i.e. $cor(X_{ij}, Y_{ij}) = cor(X_{i.}, Y_{i.})$) and therefore variances of slopes are assumed to be equal to zero. These assumptions of the imputation model can bias the subsequent multilevel analyses, particularly as these assumptions are less and less likely (e.g. because of a large ICC, or heterogeneity in slopes) and the percentage of missing data increases (which means more of the imputed values depart from the observed relationship among the variables).

When the random effects followed an approximate Cauchy distribution the bias of the estimates was much more dramatic and erratic compared to the results obtained under the other distributional conditions. The approximate Cauchy distribution of the random effects did not have this effect on the coverage probabilities. This is because the standard errors under the approximate Cauchy distribution were much larger than the standard errors under the other distributions resulting in adequate coverage despite the fact that bias was rather dramatic. It is important to note that the approximate Cauchy distribution does not have a defined variance. This may have contributed to the erratic behavior of all the missing data treatments as it is a violation of the assumption that the random effects have variance τ_{qq}.

5.2.2 Future work

As mentioned above the exact reason for the superior performance of LD in this study is unknown. The proof provided by Allison (2002) shows how LD will perform optimally within a single level modeling framework (e.g. multiple regression) if missingness does not depend on Y_i. If this proof holds for the multilevel framework, and LD performs optimally when missingness in a multilevel dataset is not dependent on Y_{1ij}, that may explain why LD performed best in this study. To examine if the reason for the optimal performance of LD was due to missingness being independent of Y_{1ij}, a small follow up simulation was conducted. In the follow up simulation missingness on the covariate X_{1ij} depended on values of the dependent variable Y_{ij}. This follow up simulation was restricted to the situation where the percentage of missing data was 50%, ICC was equal to .20 and the random effects followed a multivariate normal distribution. Furthermore, this simulation consisted of 100 replications (rather than 500 as in the previous simulation). Within the context of this small follow up simulation MLMI did in fact outperform LD. This result suggests that the superior performance of LD in the main simulation study was in fact because missingness on the covariate was independent of the dependent variable. However, given the limited nature of this follow up simulation a more thorough investigation is warranted. Such an investigation would help further determine the exact conditions under which LD is a viable option to the researcher dealing with multilevel data.

It should be noted that it was difficult to specify the conditions under which PAN produced multiple imputations for this study. Specifically, PAN requires the user to specify the prior distributions for both the variance/covariance matrix of the variables to be imputed (σ) as well as the variance/covariance matrix of the random effects for the multivariate MLMI model (Ψ). Any deviation from a diagonal matrix for the variance/-covariance matrix of the random effects resulted in PAN not imputing values. While a diagonal matrix for the variance/covariance matrix of the random effects was appropriate for the multiple imputation model used in this study, there may be situations where a researcher would need to deviate from a diagonal matrix for an appropriate specification of the imputation model. Currently the number of programs that perform MLMI are limited and somewhat difficult to use making MLMI less accessible to researchers. Given the limited availability of such software, it would be useful to know how the available programs that perform MLMI perform relative to one another (e.g. PAN, REALCOM-IMPUTE, MLwin). Such information would help to guide practitioners in their choice of programs among those that have a graphic user interface (GUI) such as MLwin compared to a syntax-driven program such as PAN.

MLMI remains a complex missing data treatment and therefore more accessible and viable alternatives should be considered. It is for this reason that the performance of multiple imputation separately for each level two unit as recommended by Gelman and Hill (2006) such be studied. This approach involves the separate imputation of missing data within each level two unit before the level two units are then combined to form the complete dataset. By performing the imputations separately within level two units, one could possibly preserve the multilevel data structure. Additionally, such an approach would presumably be easier for non-data analysts to carry out relative to MLMI (regardless of the program used for MLMI).

5.2.3 Recommendations for applied researchers

Prior to collecting the data

The results of this study highlight the importance of minimizing missing data during the data collection phase of a research study. Even after using a sophisticated missing data treatment designed to preserve the complex relationships present in a multilevel dataset, key parameter values were estimated poorly as the percentage of missing data increased. The most successful approach to decreasing the adverse effects of missing data is to make sure that missingness is minimized. Prior to data collection, the researcher should spend time thinking through the different reasons why study participants may not provide data and attempt to decrease these occurrences. For example, research utilizing surveys may decrease the impact of missing data by paying close attention to the wording of questions that make participants feel uncomfortable and increase the probability of non-response. Furthermore, having clear instructions for respondents can decrease the frequency with which respondents may not respond because they are unsure or confused about what the researcher is asking.

In addition to minimizing the extent to which questions may yield non-response, researchers should collect data on variables thought to be related to non-response so that these auxiliary variables[1] can act as candidates for inclusion in a missing data treatment. Researchers should also ask respondents if they are willing to be followed up with in case there are missing data.

If the researcher is collecting archival data, he/she should become familiar with the data capturing system and understand how and under what circumstances data are captured. This understanding can help the researcher determine if auxiliary variables that can be used in a missing data treatment should be included when retrieving data

[1] auxiliary variables are variables that help to explain why data are missing

from the data capturing system.

After collecting the data

After collecting data, the researcher(s) should examine the missing data for patterns. The review of the data for missingness should take place as soon as possible so that respondents can be contacted for follow-up so that missing values can be replaced with the correct responses. The replacement of missing values through follow-up eliminates the uncertainty associated with missingness.

Once the data have been collected, the data should undergo a cleaning process in which implausible values are examined, values are appropriately coded, relevant codebooks are created etc. During this process researchers should distinguish between values that are missing and should be included as candidates for imputation and those that should not be imputed (e.g. because the response to a question is "I don't know") (Acock, 2005). Variables for which missing data should not be imputed should be coded a particular value that will indicate exclusion from imputation (e.g. -8888) which would differ in value from a numeric code indicating missing values that are candidates for imputation (e.g. -9999).

Based on the results in this study, in general LD may be less problematic when missingness is very small (i.e. 5% or less). However, as a general rule of thumb, if the amount of missing data in a multilevel dataset is larger, researchers should attempt to examine the patterns of missing data so that they can provide evidence as to the missing data mechanism. Despite the poor performance of MLMI in this study, MLMI using other software packages may prove fruitful. The appropriate use of imputation techniques such as MLMI can eliminate the need for data analysts to consider the impact of missing data on outcomes over the duration of a research project or life of a dataset. In contrast, with significant amounts of missing data ad hoc approaches such as LD would require the analyst to revisit the question of the impact of missing data with each analysis.

As with all imputation techniques, if MLMI is used, the MLMI model should incorporate all the available information that may help to explain missingness (including variables that may be considered as dependent variables in subsequent analyses). Additionally, the MLMI model should be as general as possible (e.g. including all relevant random effects). The data analyst should examine diagnostic plots of the imputation model parameters to determine the appropriate number of cycles in the chain from which imputations will be drawn. Poor performance of the model can sometimes be diagnosed through such plots (Schafer, 1997).

Under no circumstance should MI be used with a multilevel dataset. Using MI imposes a "flattening" of the relationships thereby undermining the multilevel structure of the data.

Conducting the analysis

Even if the proof showing the optimal performance of LD presented by Allison (2002) holds for multilevel models, in a typical research study the researcher will not know the parameter values or the missing data mechanism. In the absence of this knowledge it would be unwise to assume that LD would perform as well as it did in this study. However if the proof offered by Allison (2002) does hold for multilevel models and a researcher finds that missingness is only a problem for the covariate(s) in the model, this author recommends the following. The researcher should create an indicator variable reflecting whether a case has a missing value for each covariate with missing data. Then, the researcher should regress the indicator variable(s) on the dependent variable through a multilevel logistic regression. In the event the dependent variable in this model is not a significant predictor of missingness on the covariate(s), this would suggest that the assumption that missingness on the covariate is independent of the values of the dependent variable is supported and as such LD for the multilevel model of interest may yield unbiased results.

Researchers should report the prevalence of missing data in their study as well as the exact sample sizes used in the analyses so that readers can consider the potential impact of missing data (Peng et al., 2006). Even if a researcher uses LD, the reporting of the exact sample sizes used in each of the analyses and prevalence of missing data will help readers juxtapose the results against the prevalence of missing data and would help the reader have a better understanding of the population to which the results do generalize.

5.2.4 Recommendations for methodological researchers

Methodological researchers interested in examining the performance of missing data treatments through simulation should pay very close attention to how missingness is generated in their simulation studies. Simulation studies that simulate MAR often do so by using different proportions of missingness on a variable conditional on intervals of another variable (Black, 2008; Puma et al., 2009). For example, a study may simulate missingness on Y such that

$$P(R_{Yij}) = \begin{cases} .2 & : -\infty < X_{1ij} < 0 \\ .4 & : 0 \leq X_{1ij} < \infty \end{cases}$$

The specification of such a mechanism imposes what may be an unrealistic mechanism of missing data. It is for this reason that methodological researchers specifying a missing data mechanism should use a logistic function which allows the researcher to impose more control and a more realistic missing data mechanism (e.g. $P(R_{Yij}) = \frac{e^{\beta_0+\beta_1 X_{1ij}}}{1+e^{\beta_0+\beta_1 X_{1ij}}}$). Through the use of a logistic function the researcher is given more flexibility in modifying the strength of the relationship between a covariate and the probability of missingness and this approach also allows the researcher to create a more sophisticated missing data mechanism (one that may depend on more than one variable) more easily.

Methodological researchers focusing on missing data through simulation should also examine the implications of using the precise missing data mechanism specified in the simulation study. In the current study missingness did not depend on the values of the dependent variable. This aspect of the simulation may have accounted for the superior performance of LD. Failing to acknowledge this possibility may lead one to erroneously conclude that LD is usually the best missing data treatment for multilevel data (i.e. over generalize the results). Because a simulation study requires the researcher to create the entire reality on which the study is based (e.g. sample size, relationship among variables, the missing data mechanism etc.), inferences are limited to the conditions in the simulation study. Authors of simulation studies should clearly define their conditions and the scope of their recommendations given the conditions.

5.2.5 Limitations

The multilevel models fit in this study were fit using the lmer function in the lme4 package in the statistical software program R. The function lmer does not give significance tests for the fixed effects or variance components. Therefore the type I error rate was not considered as a criterion in this study and so the results of this study do not speak to this aspect of the impact of the missing data treatments. While the question of type I error rate was not assessed in this study, the criterion of whether the 95 % confidence interval covered the true value was a criterion in this study.

This simulation study used the R package "PAN" to perform MLMI. However, performing MLMI through PAN would not be accessible to some practitioners. PAN requires the user to interface solely through syntax and specify aspects of the MLMI model through matrices. For this reason, the results of this simulation study may not generalize to programs that would be used by most practitioners. For practitioners that do not have experience using syntax-driven programs, the learning curve associated with utilizing such a program appropriately may outweigh the benefit of performing MLMI, particularly if the rate of missing data is relatively small.

Specification of the variance/covariance matrix of the random effects in the multivariate MLMI model used by PAN was restricted to a diagonal matrix. The implication of this restriction is that random effects in the imputation model are not allowed to covary. If such a specification is not general enough to preserve the relationships among the variables in the multilevel dataset, a data analyst would need to use some other program to perform MLMI. Because of the restriction of the variance/covariance matrix in this study the results of this simulation do not speak to more complicated multilevel data structures (i.e. ones that involve covariances between random effects)

Finally, The simulation in this study forced lmer to report parameter estimates at all times. Even when lmer resulted in a false convergence, the simulation extracted results from the last iteration. In this way, this study did not track convergence problems under the various conditions in the simulation study. Had they been tracked, convergence problems would likely have arisen more frequently under the conditions when the percentage of missing data was highest (Black, 2008) and LD was used. Therefore, while LD appeared to perform well in this study, the results may have been different if convergence was a criterion by which the missing data treatments were evaluated.

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Appendix A

Statistical power and sample size in HLM

Figure A.1: Power as a function of within level two unit sample size

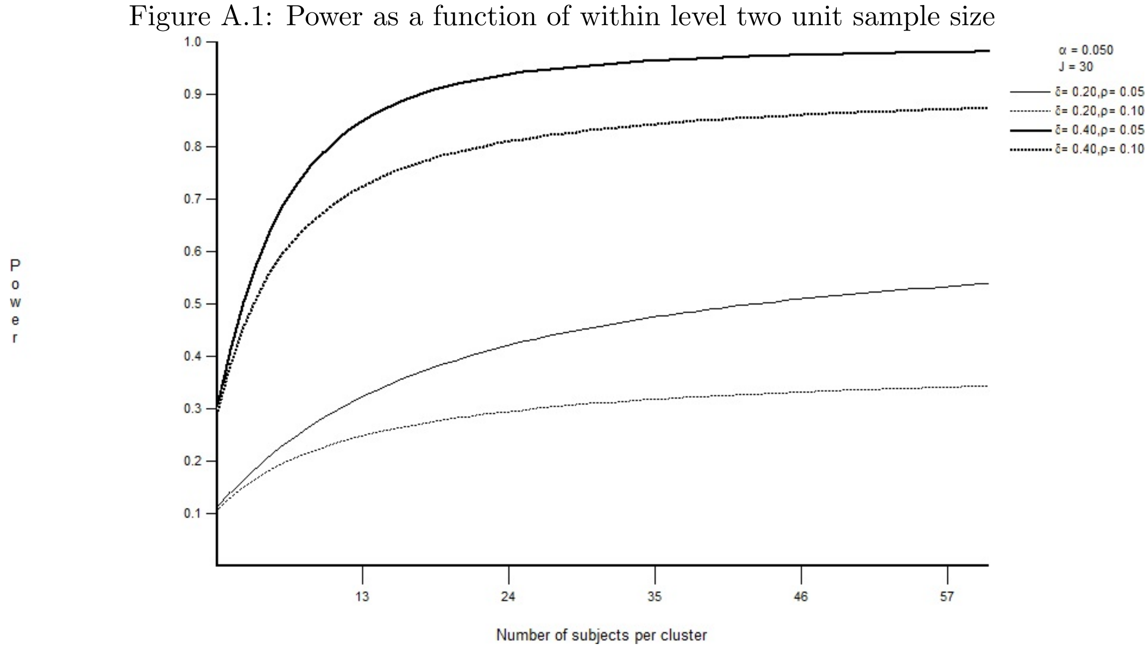
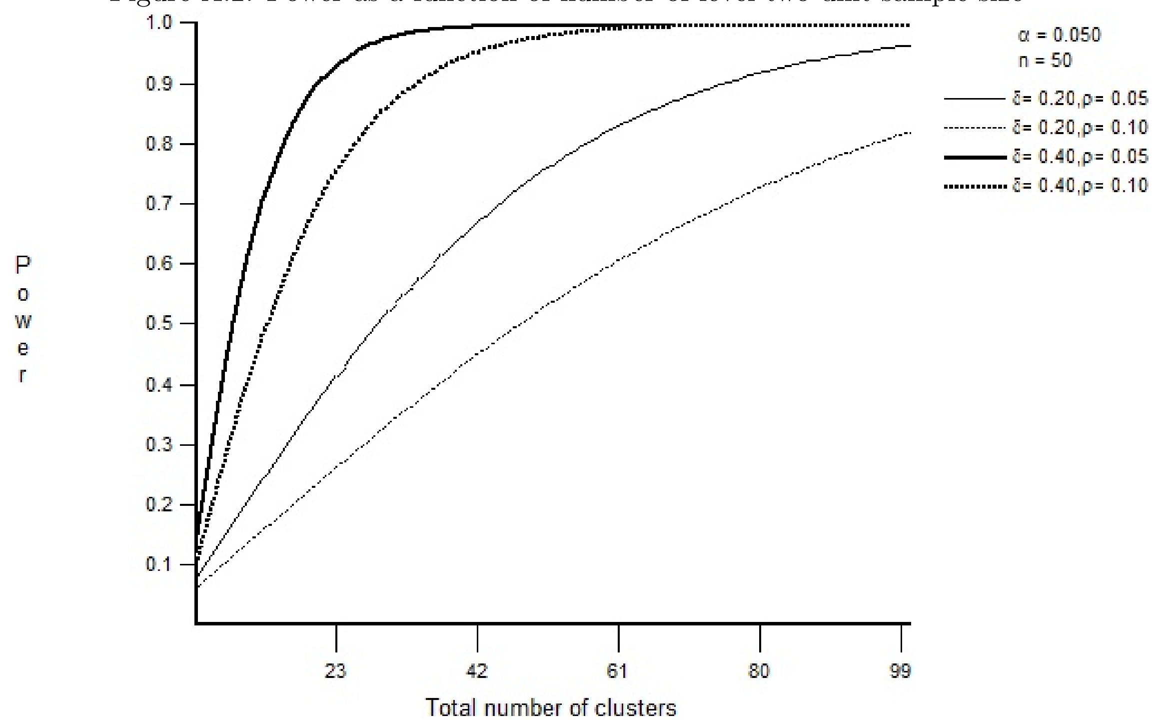


Figure A.2: Power as a function of number of level two unit sample size



Appendix B

Autocorrelations and plots of MLMI model parameters

Figure B.1: Plot of autocorrelation of variance components for multilevel multiple imputation model

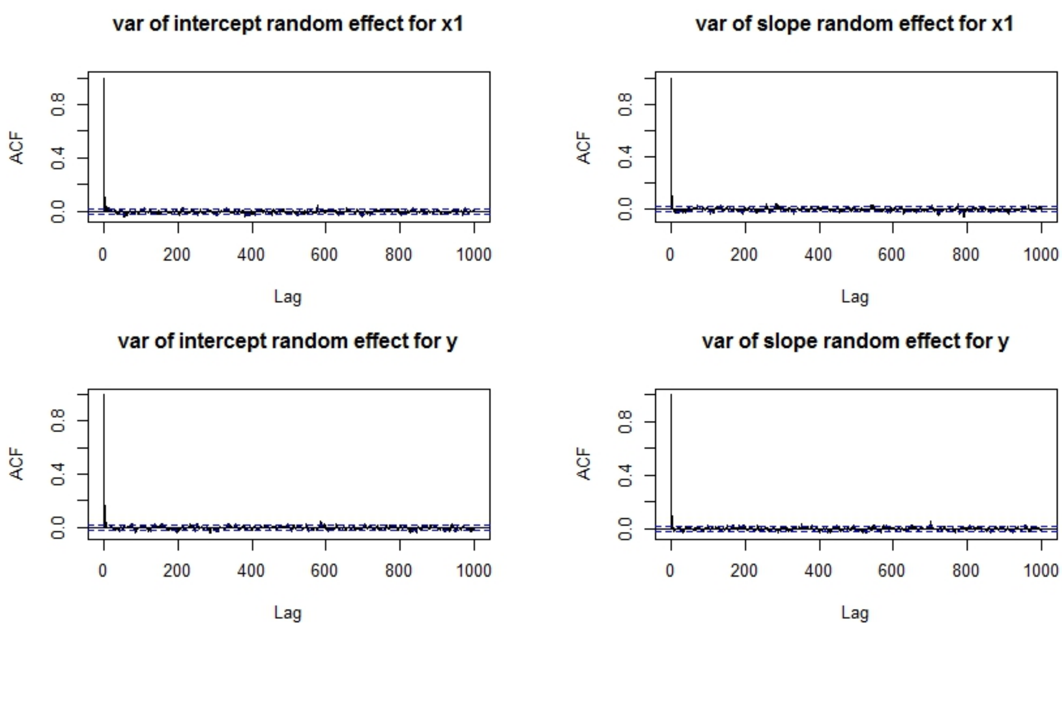


Figure B.2: Plot of values of random effects from multilevel multiple imputation model

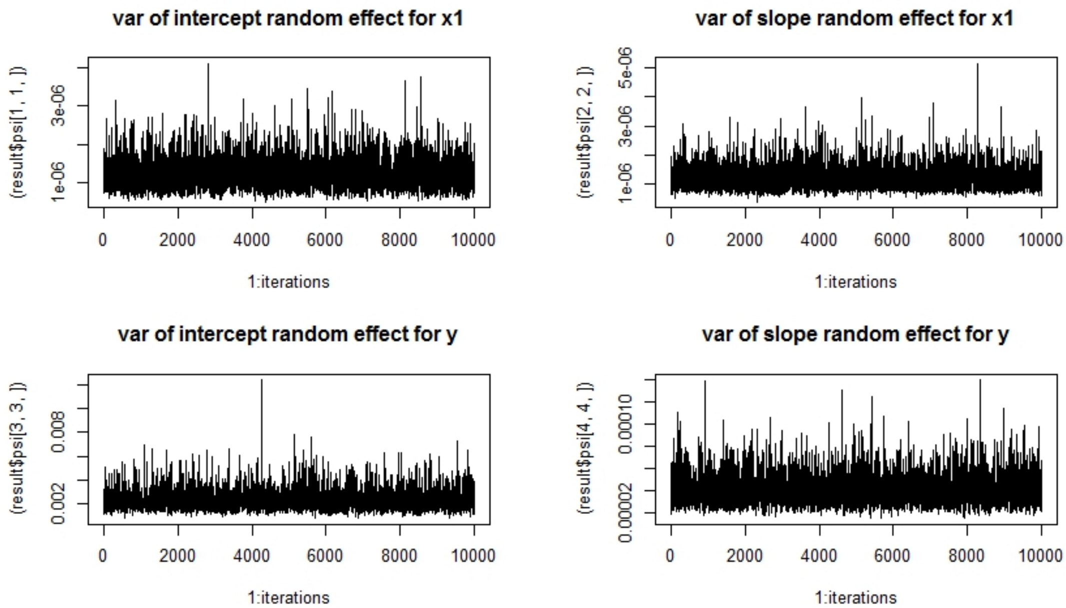


Figure B.3: Plot of autocorrelations of error variances for level one outcomes in multilevel multiple imputation model

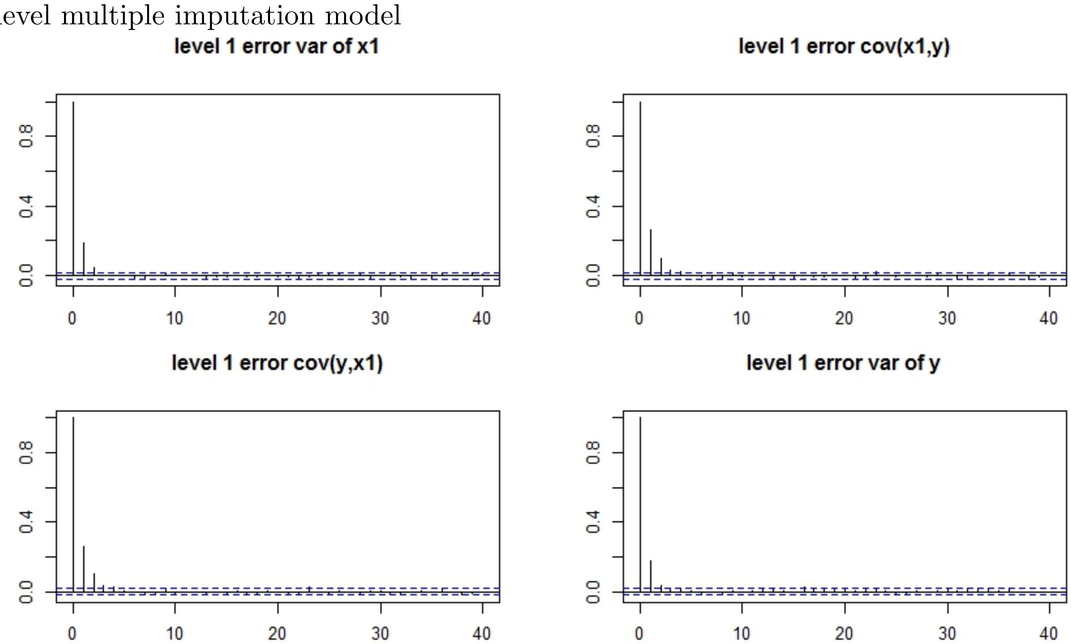


Figure B.4: Plot of values of error variances for level one outcomes in multilevel multiple imputation model

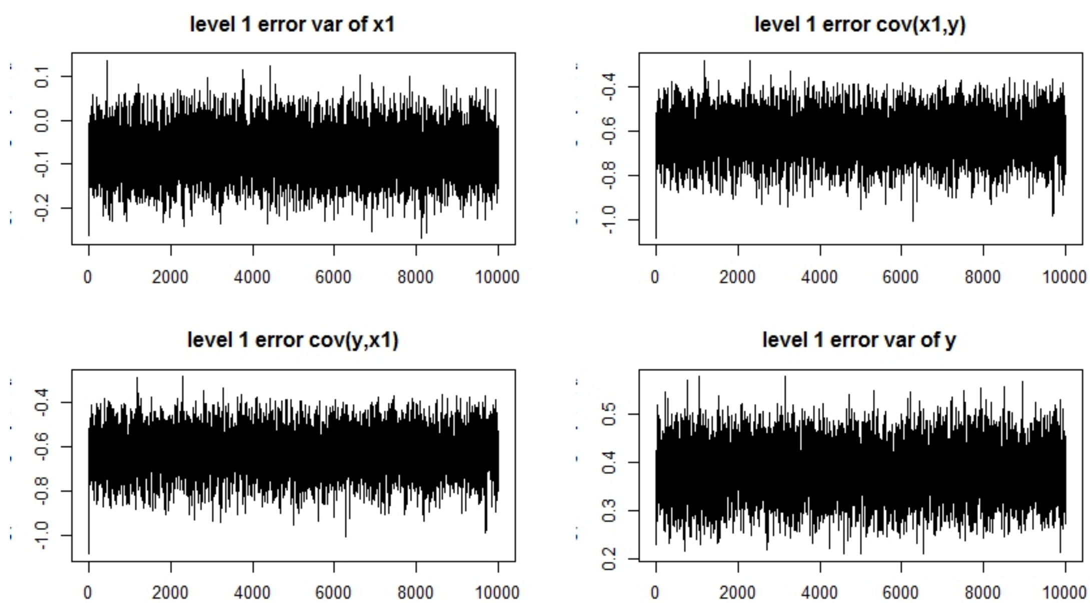


Figure B.5: Plot of autocorrelations of fixed effects estimates from multilevel multiple imputation model

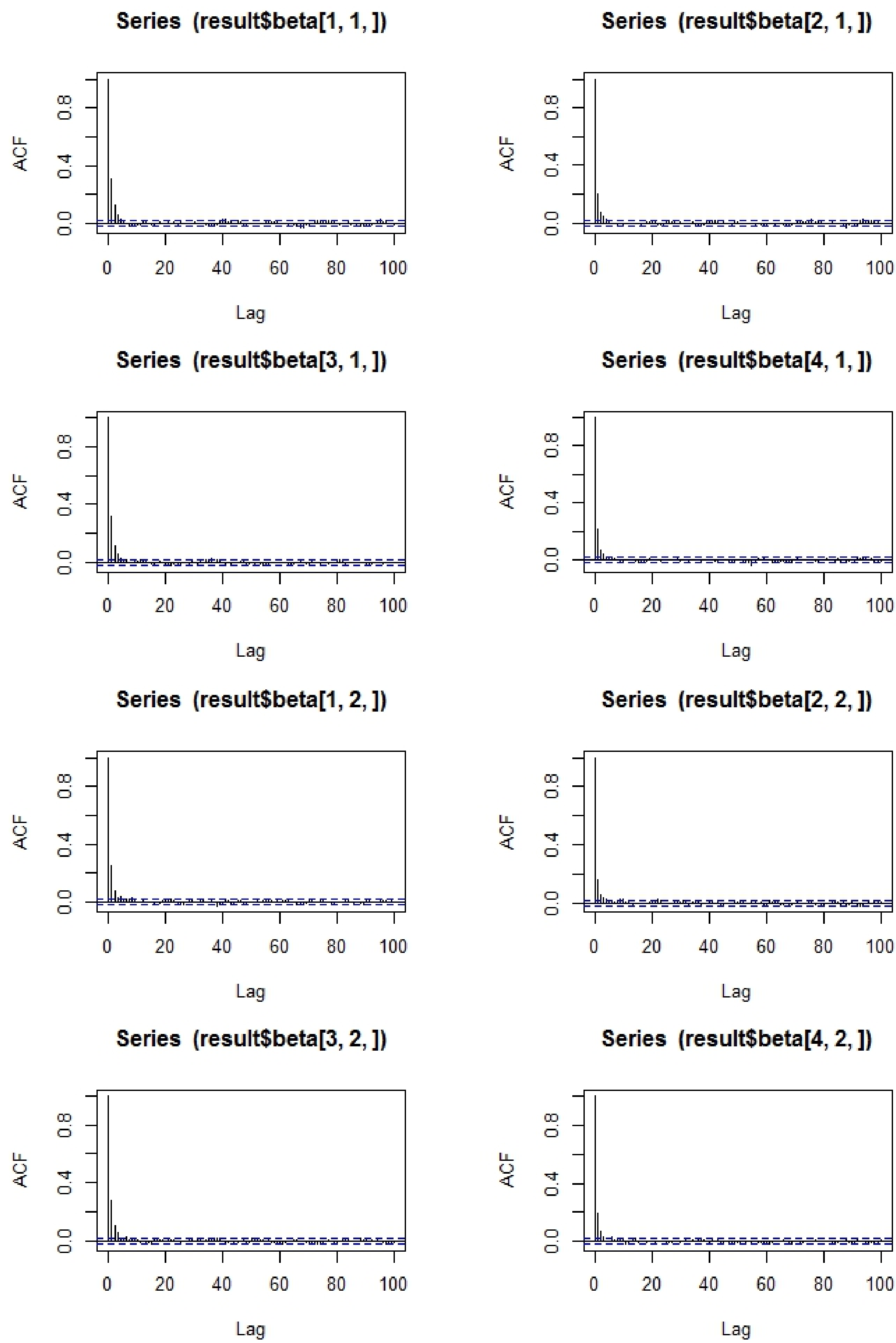
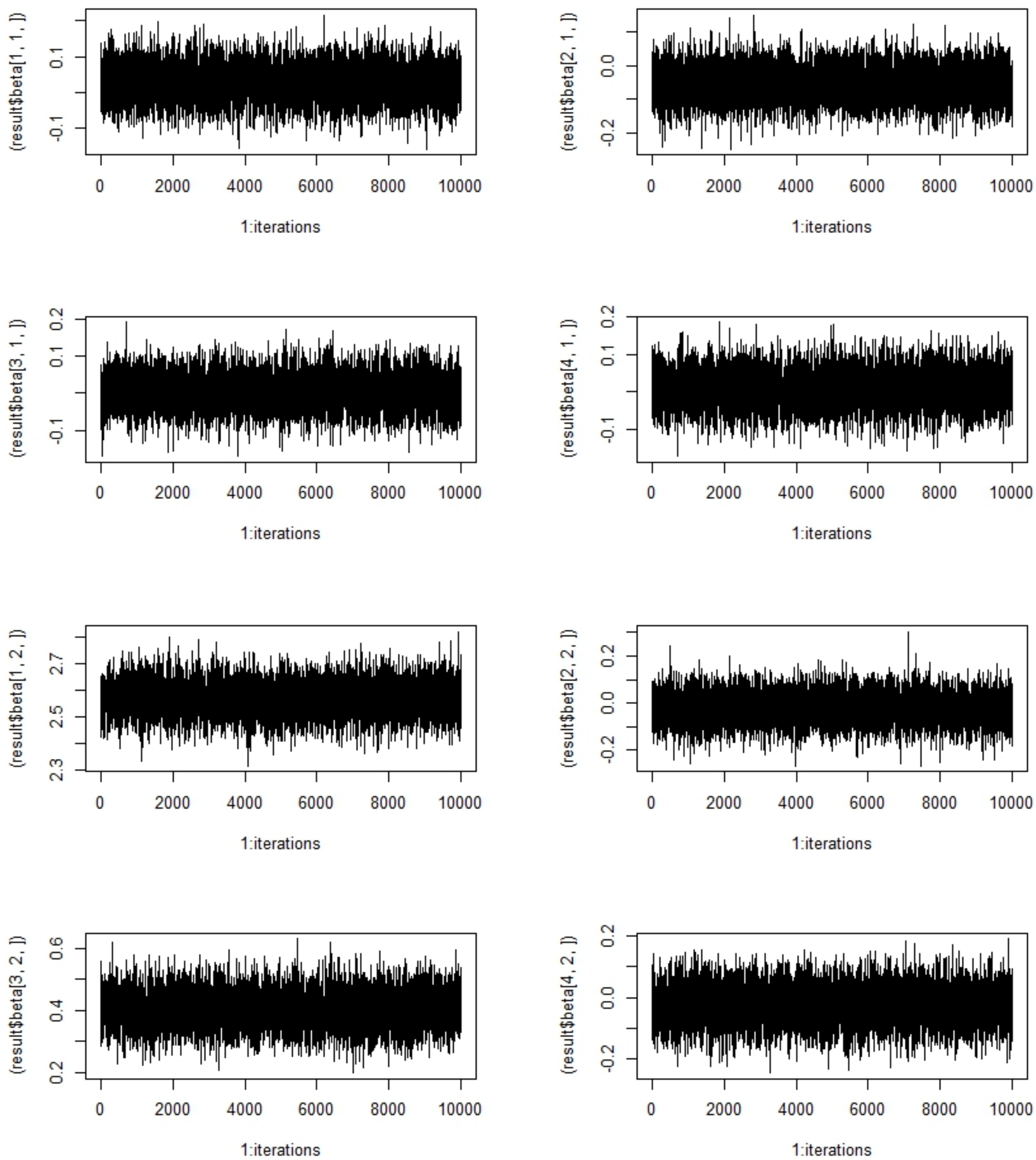


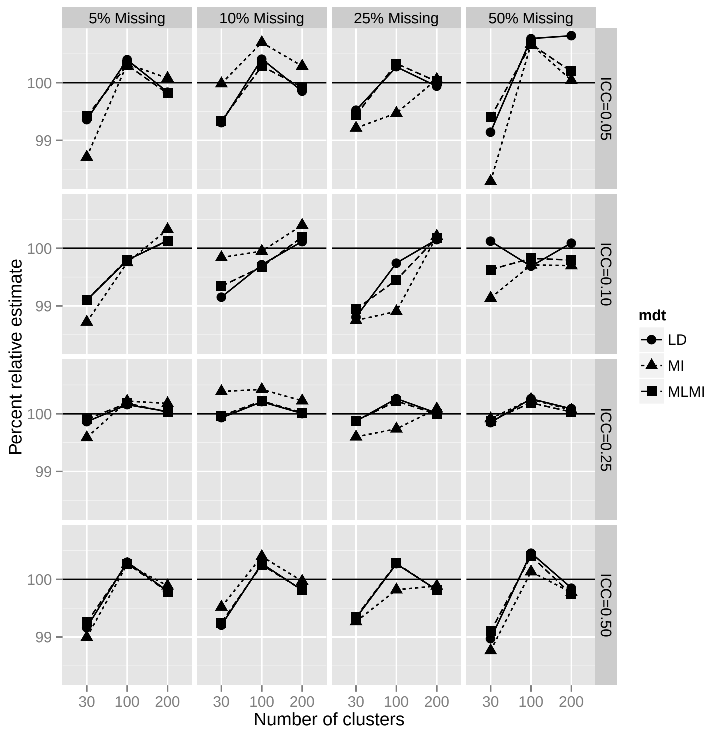
Figure B.6: Plot of values of fixed effects estimates from multilevel multiple imputation model



Appendix C

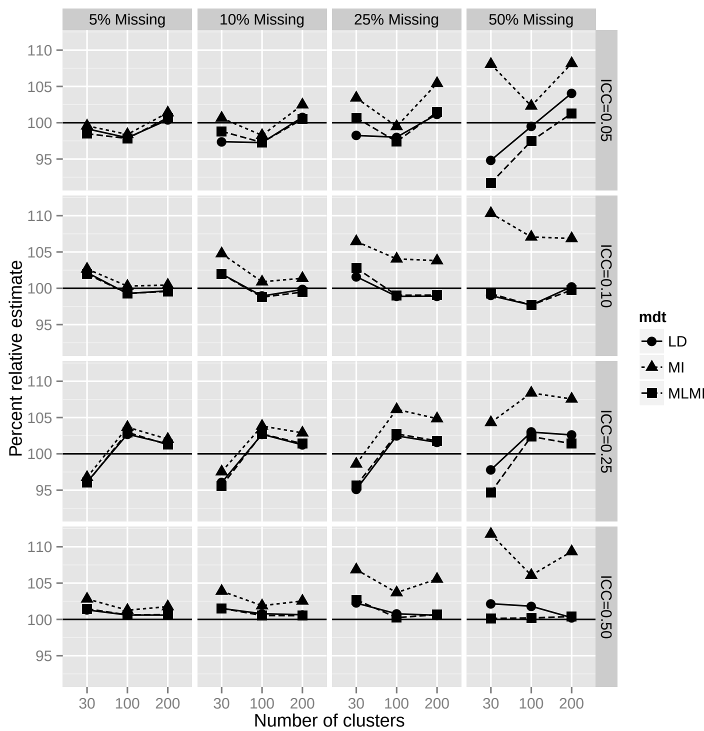
Percent relative estimate and coverage under MAR

Figure C.1: Percent relative estimate of estimate of γ_{00} under chi-square distribution of random effects



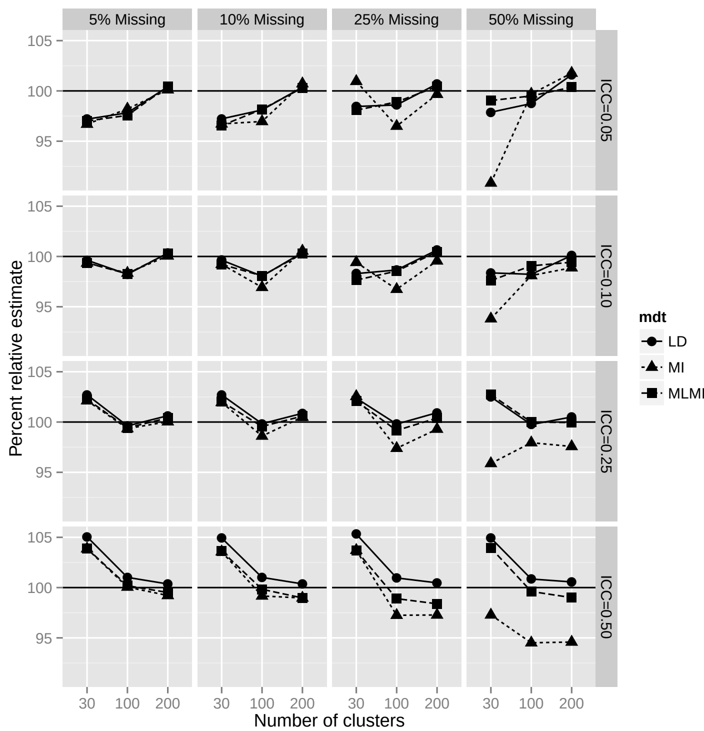
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.2: Percent relative estimate of estimate of γ_{01} under chi-square distribution of random effects



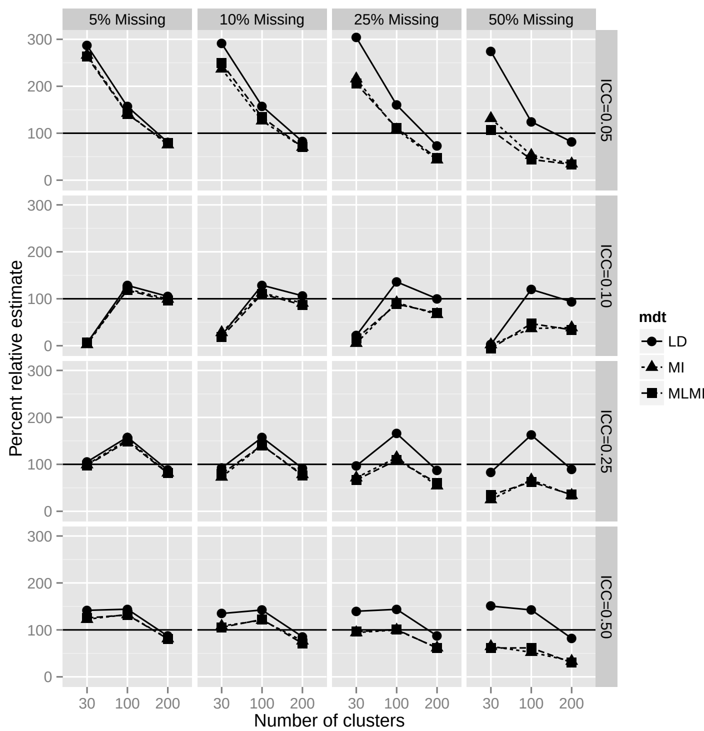
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.3: Percent relative estimate of estimate of γ_{10} under chi-square distribution of random effects



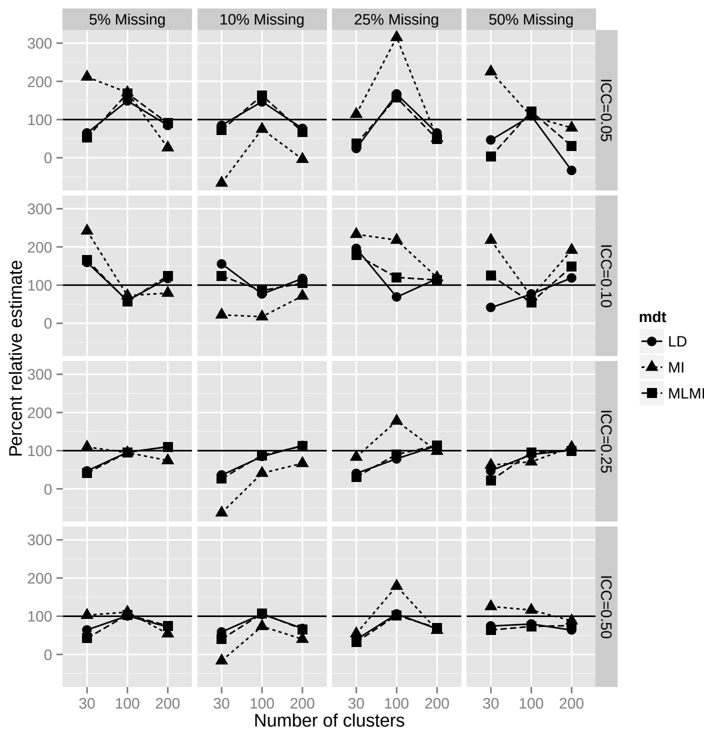
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.4: Percent relative estimate of estimate of γ_{11} under chi-square distribution of random effects



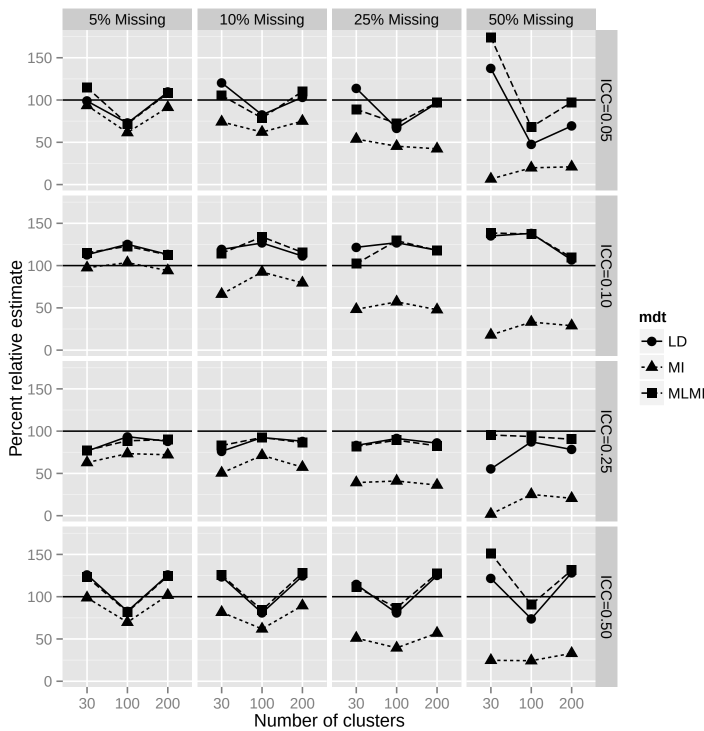
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.5: Percent relative estimate of estimate of γ_{20} under chi-square distribution of random effects



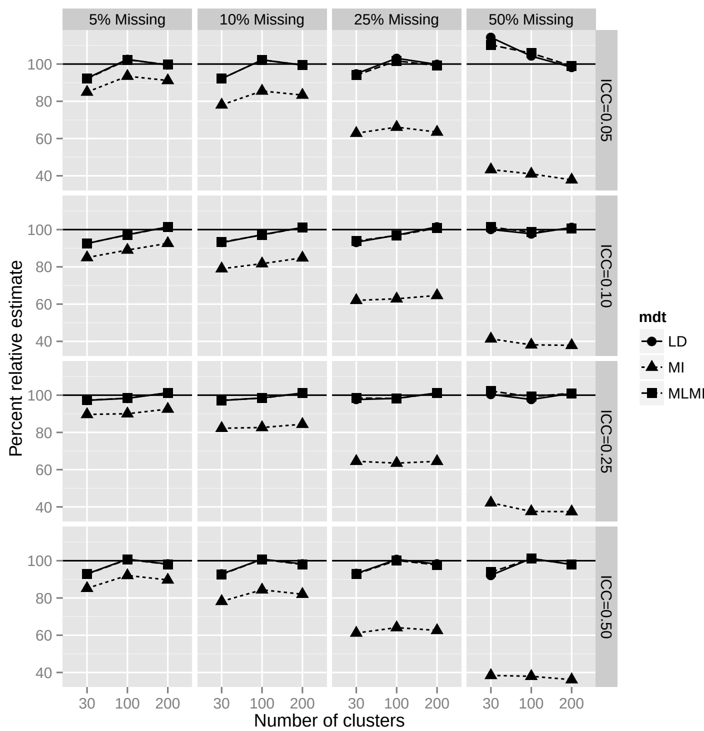
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.6: Percent relative estimate of estimate of γ_{21} under chi-square distribution of random effects



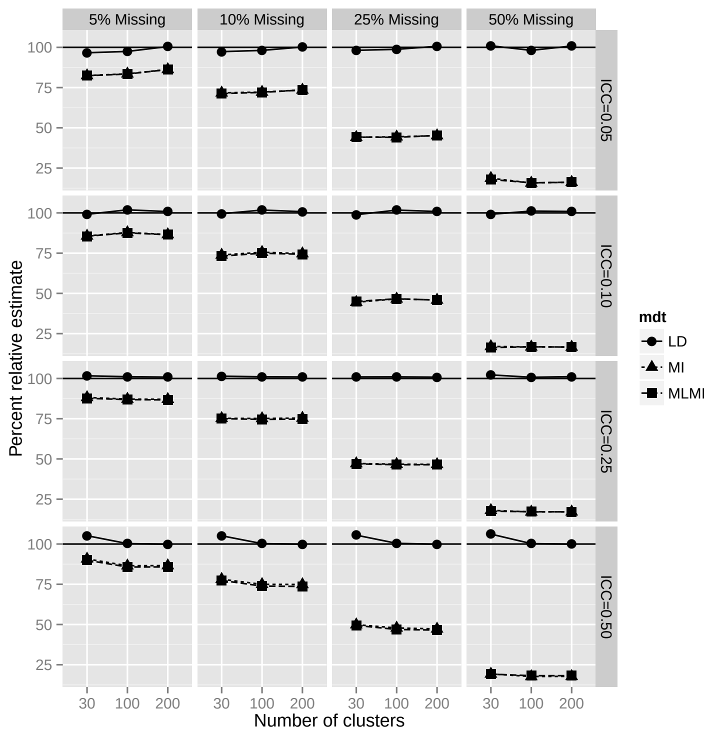
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.7: Percent relative estimate of estimate of τ_{00} under chi-square distribution of random effects



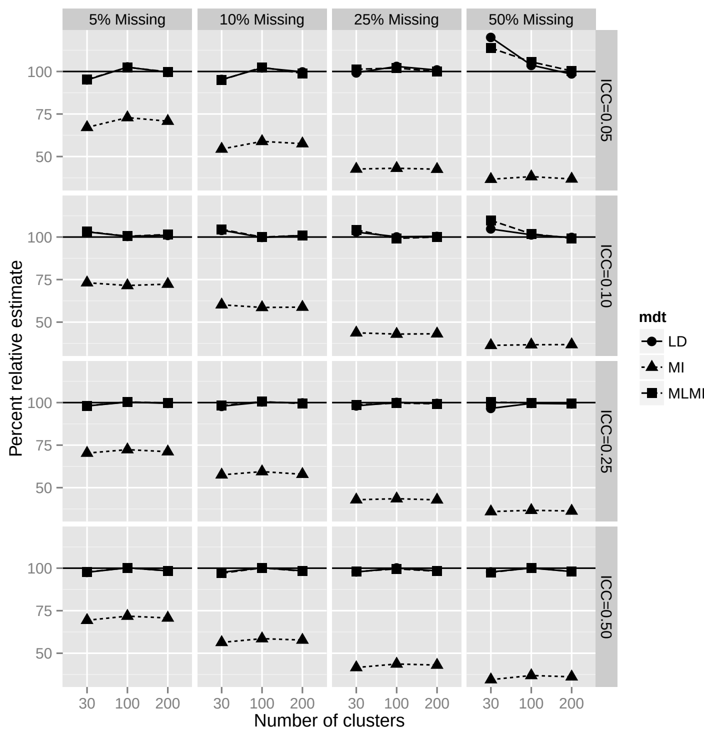
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.8: Percent relative estimate of estimate of τ_{11} under chi-square distribution of random effects



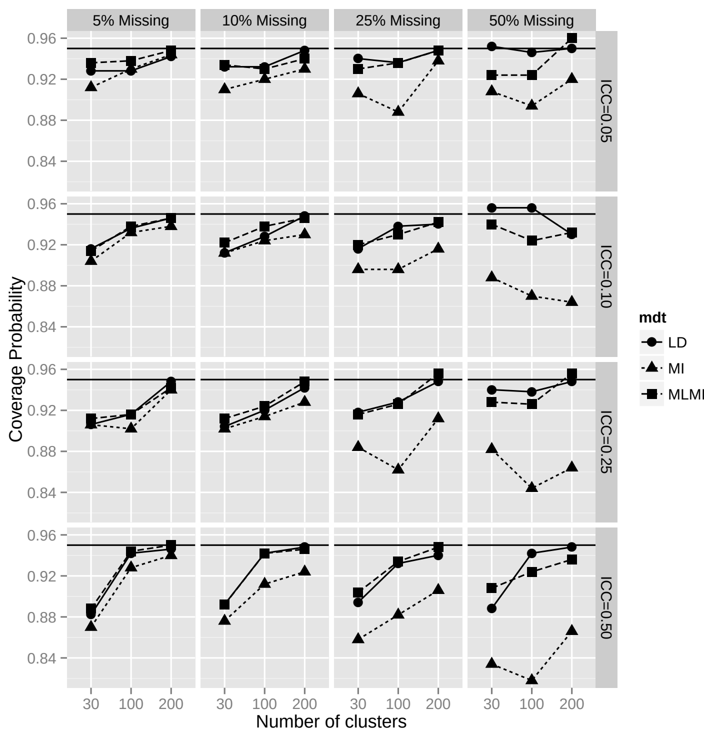
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.9: Percent relative estimate of estimate of τ_{22} under chi-square distribution of random effects



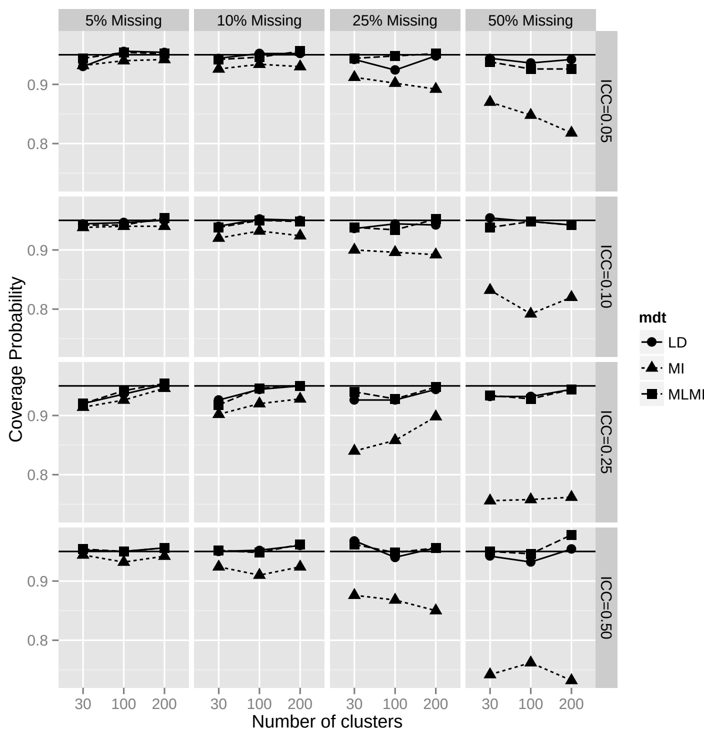
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.10: 95% CI coverage of estimate of γ_{00} under chi-square distribution of random effects



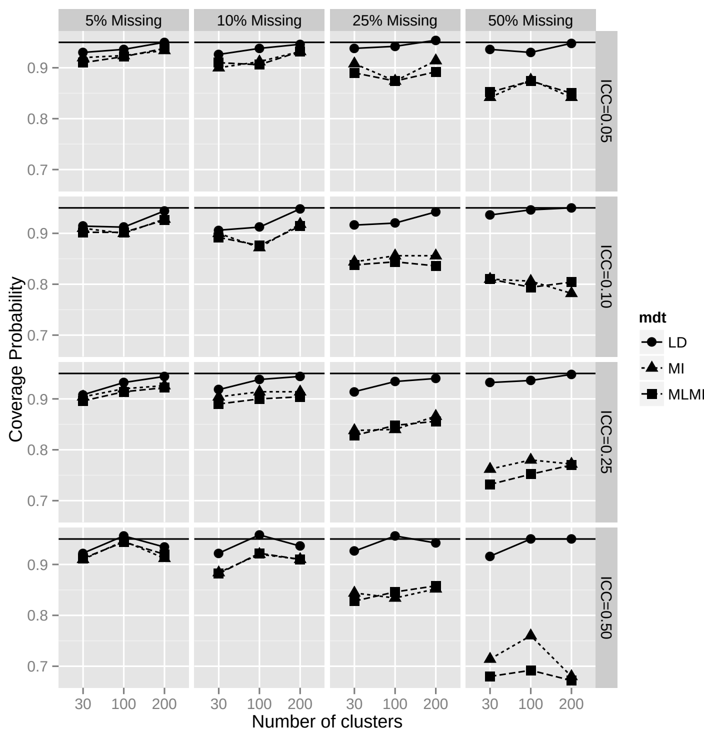
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.11: 95% CI coverage of estimate of γ_{01} under chi-square distribution of random effects



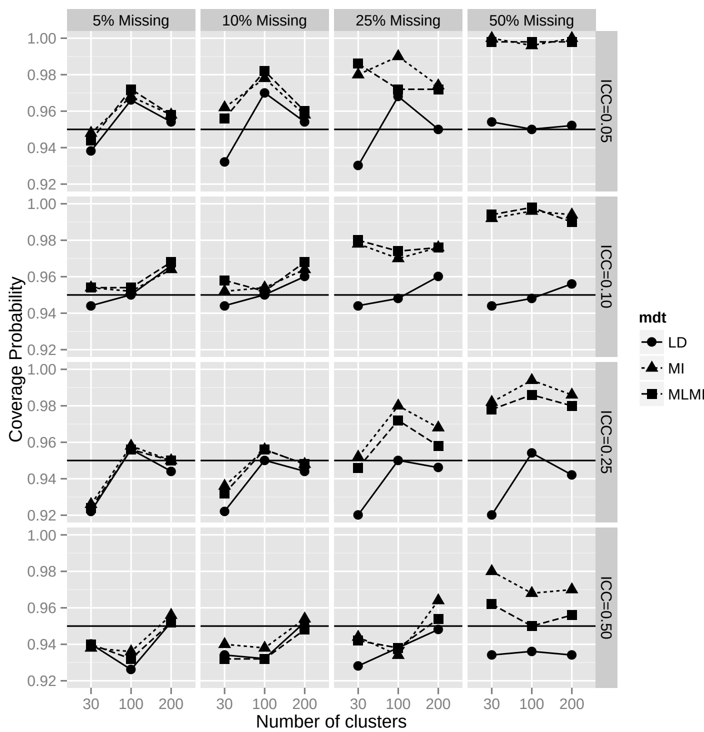
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.12: 95% CI coverage of estimate of γ_{10} under chi-square distribution of random effects



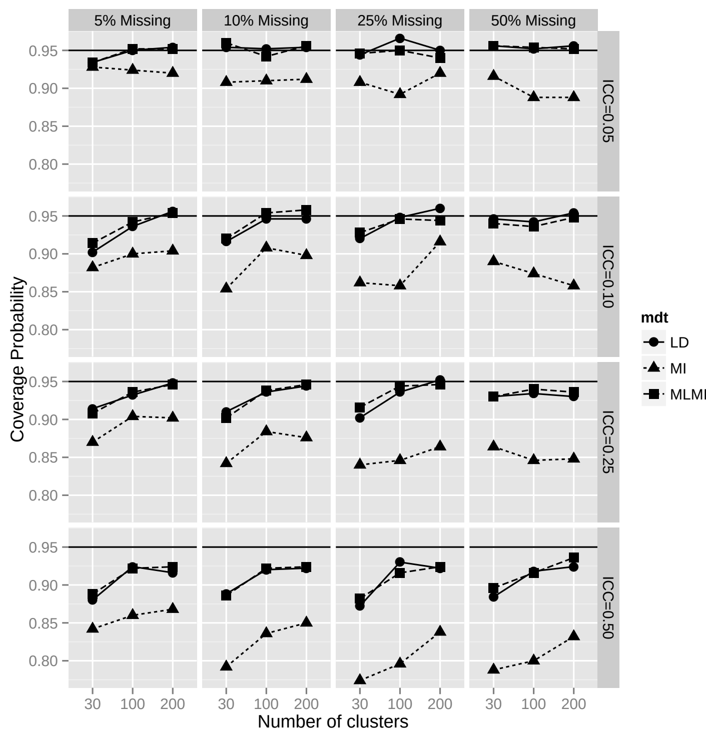
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.13: 95% CI coverage of estimate of γ_{11} under chi-square distribution of random effects



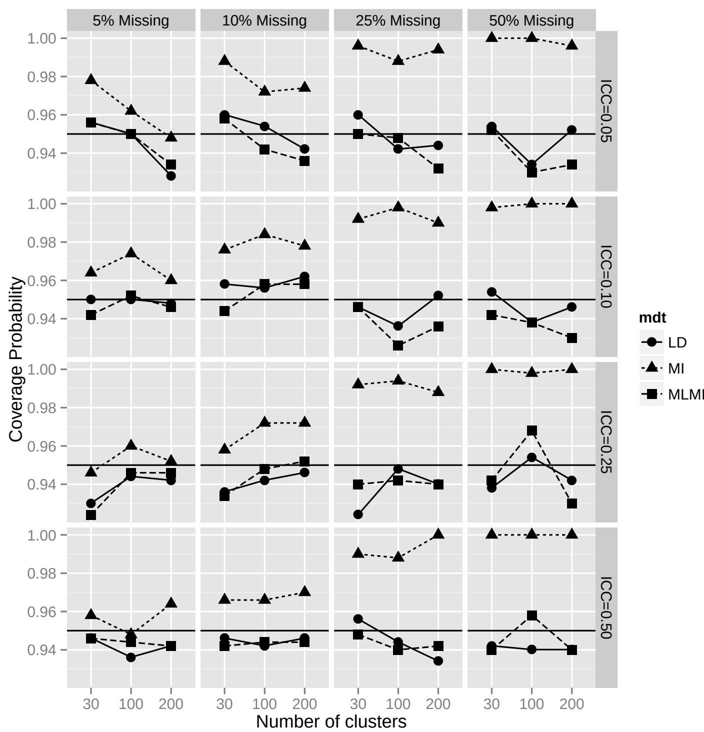
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.14: 95% CI coverage of estimate of γ_{20} under chi-square distribution of random effects



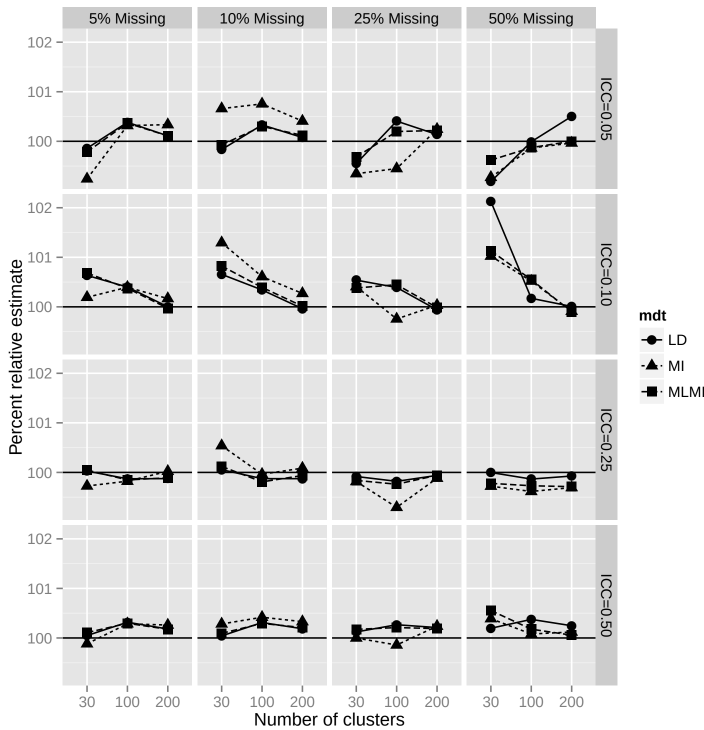
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.15: 95% CI coverage of estimate of γ_{21} under chi-square distribution of random effects



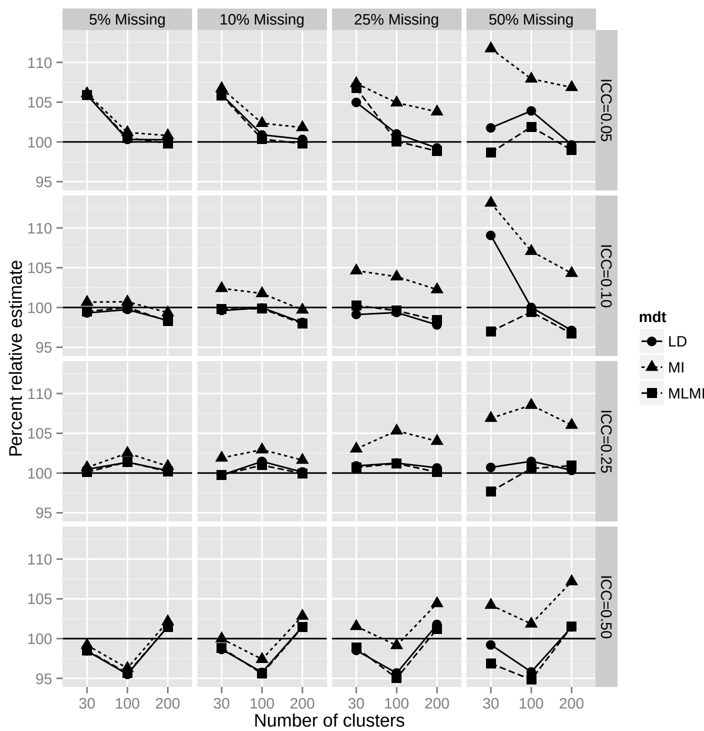
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.16: Percent relative estimate of estimate of γ_{00} under Laplace distribution of random effects



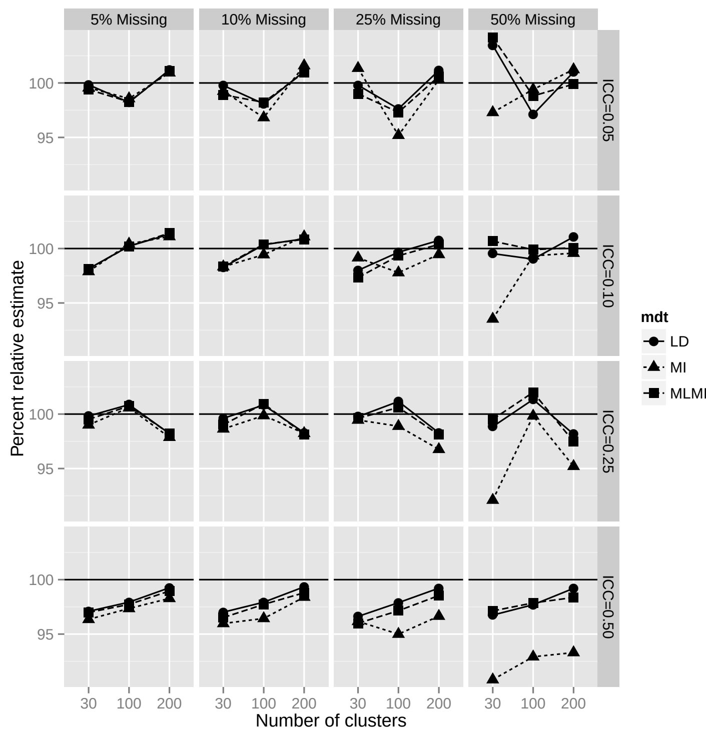
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.17: Percent relative estimate of estimate of γ_{01} under Laplace distribution of random effects



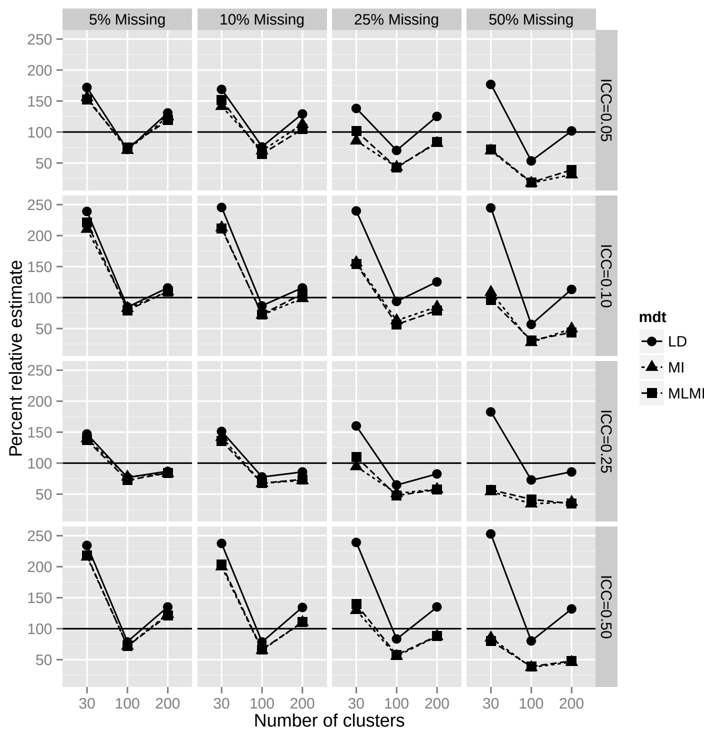
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.18: Percent relative estimate of estimate of γ_{10} under Laplace distribution of random effects



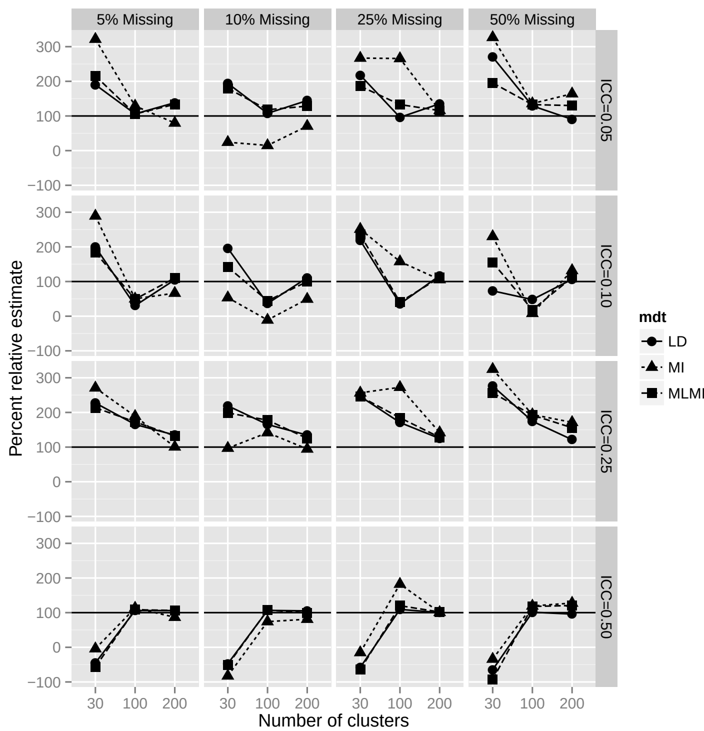
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.19: Percent relative estimate of estimate of γ_{11} under Laplace distribution of random effects



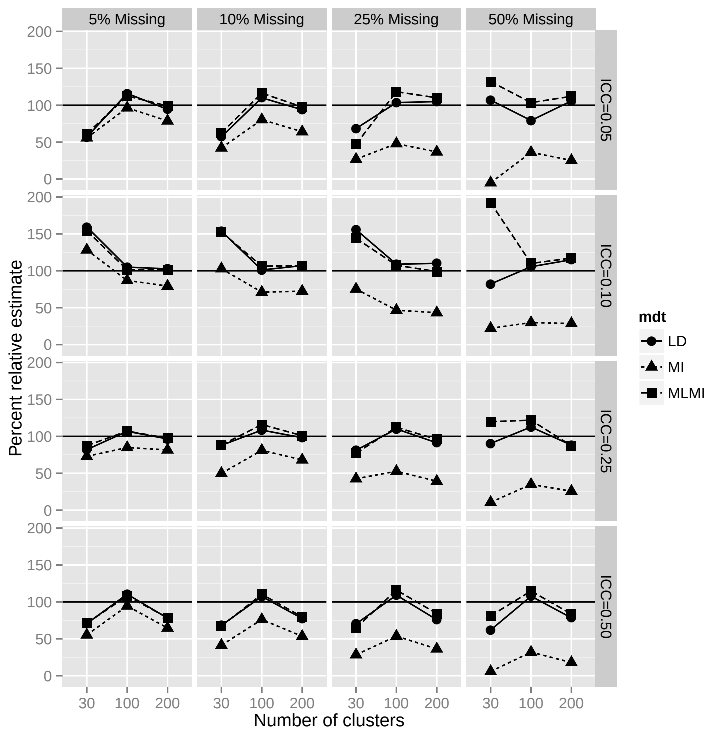
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.20: Percent relative estimate of estimate of γ_{20} under Laplace distribution of random effects



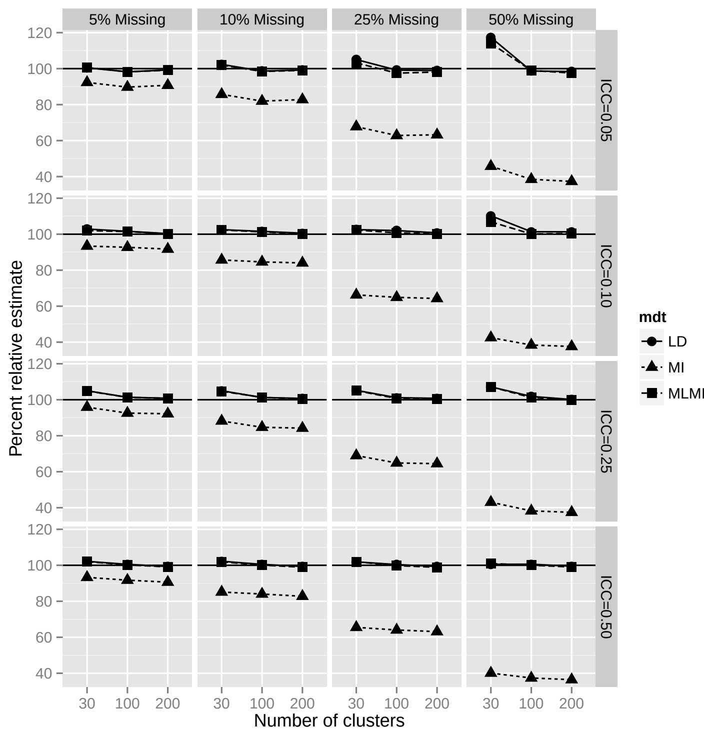
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.21: Percent relative estimate of estimate of γ_{21} under Laplace distribution of random effects



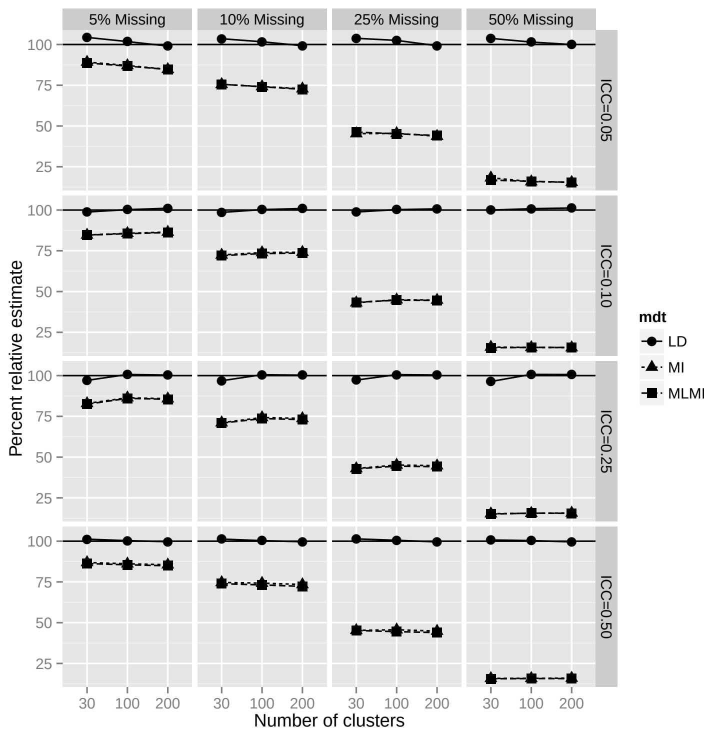
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.22: Percent relative estimate of estimate of τ_{00} under Laplace distribution of random effects



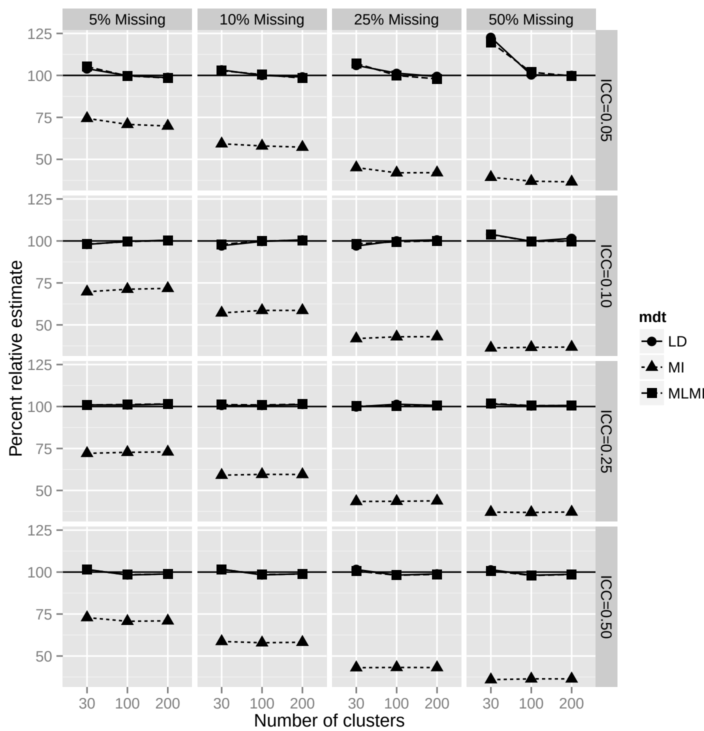
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.23: Percent relative estimate of estimate of τ_{11} under Laplace distribution of random effects



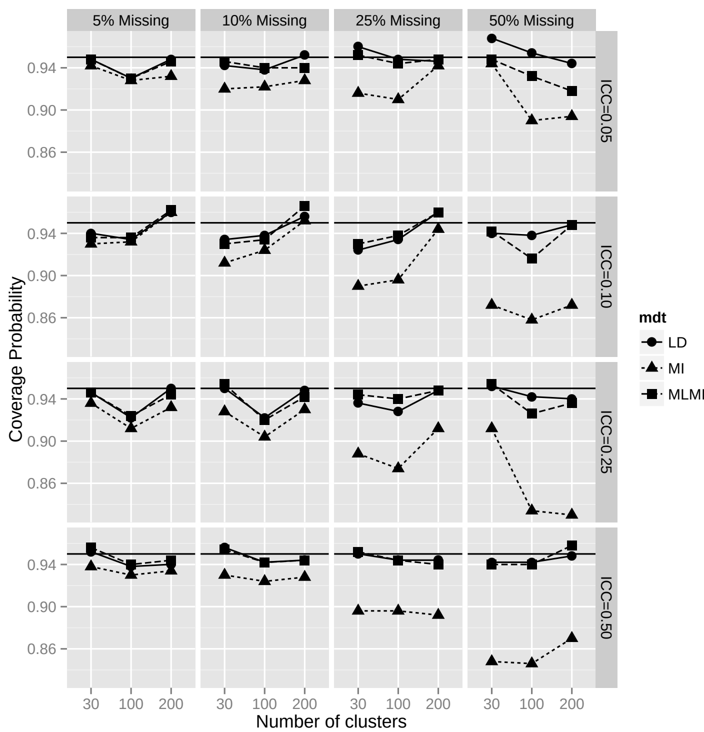
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.24: Percent relative estimate of estimate of τ_{22} under Laplace distribution of random effects



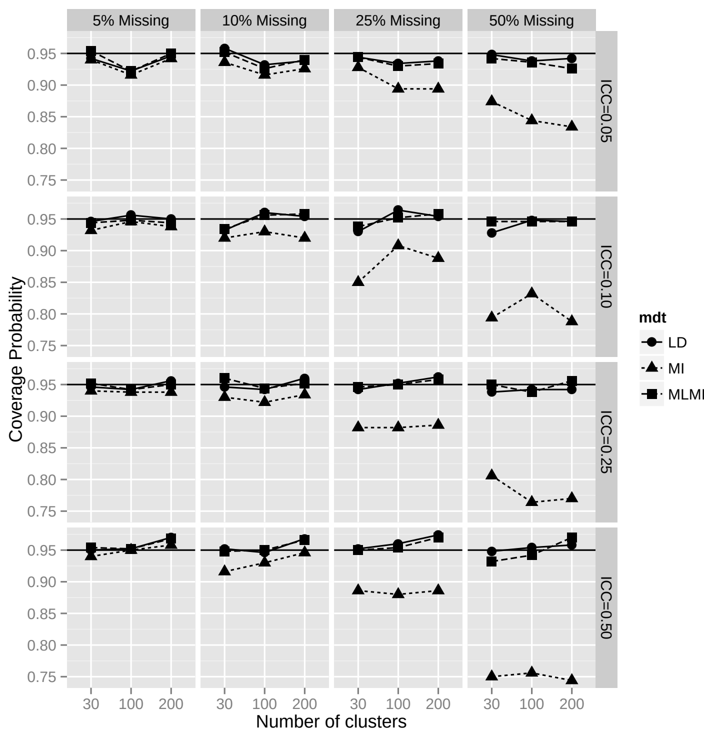
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.25: 95% CI coverage of estimate of γ_{00} under Laplace distribution of random effects



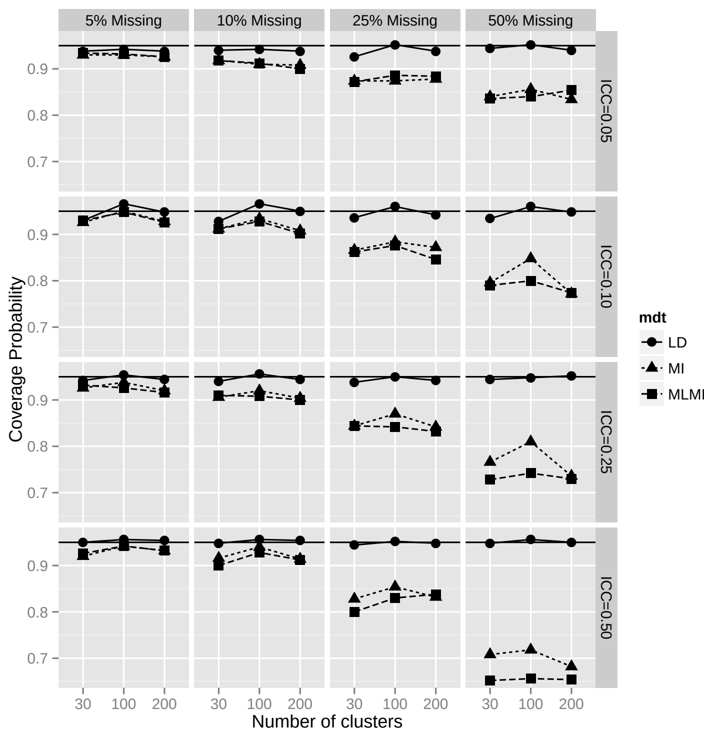
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.26: 95% CI coverage of estimate of γ_{01} under Laplace distribution of random effects



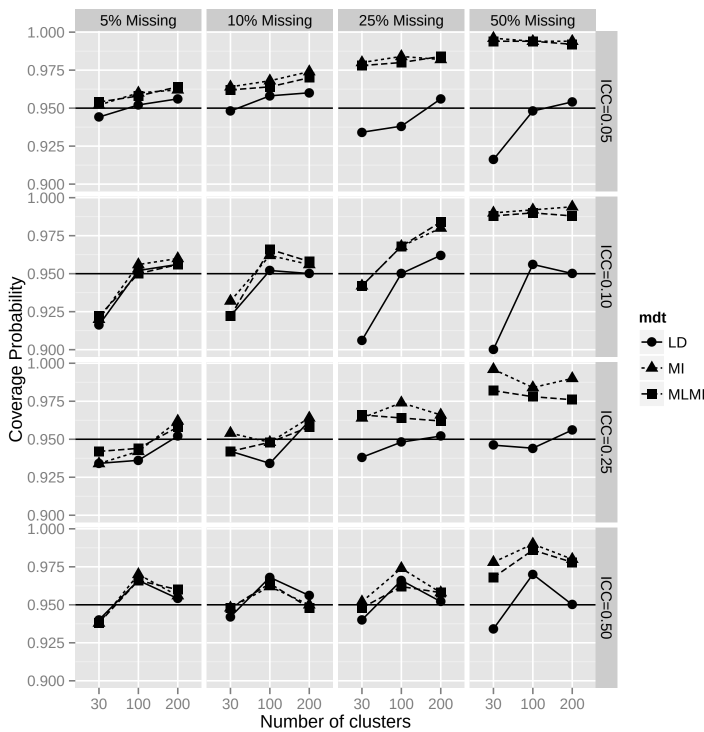
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.27: 95% CI coverage of estimate of γ_{10} under Laplace distribution of random effects



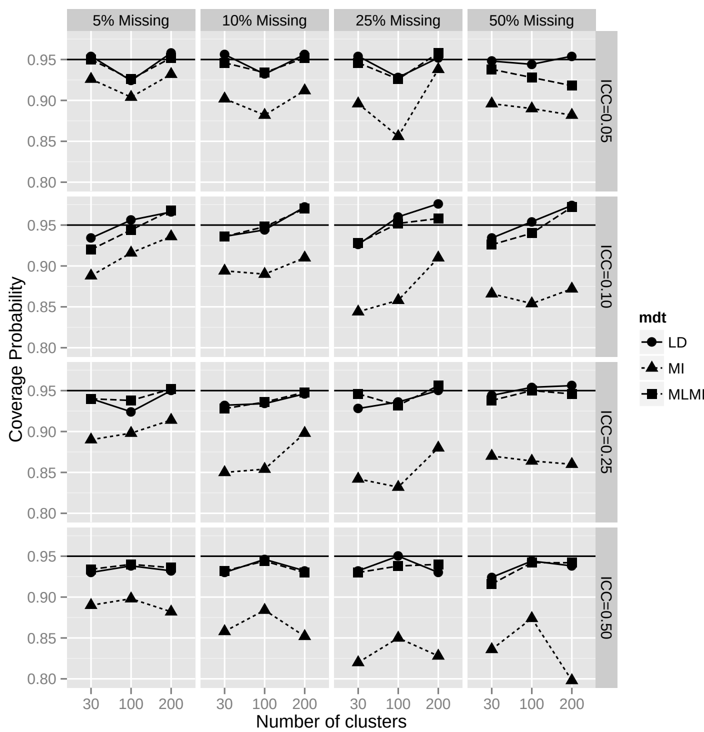
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.28: 95% CI coverage of estimate of γ_{11} under Laplace distribution of random effects



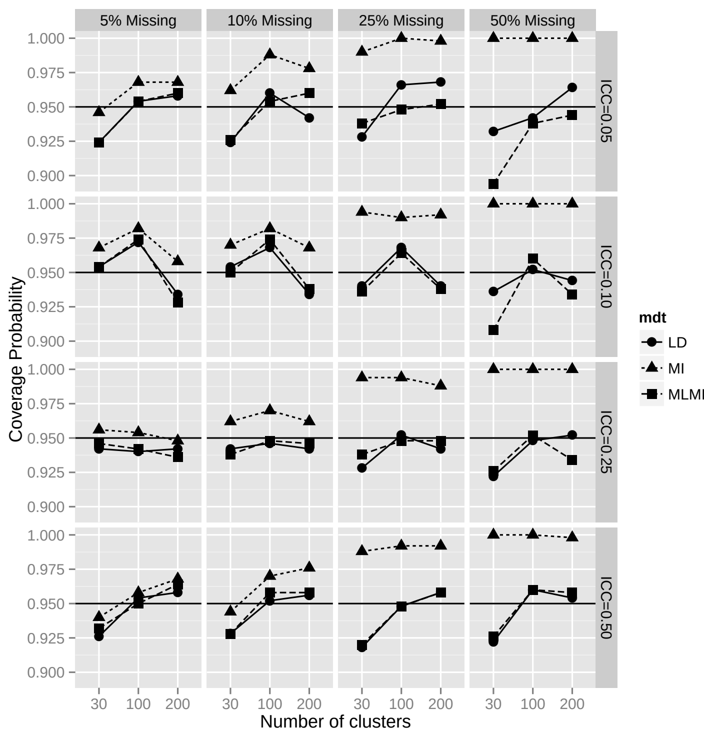
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.29: 95% CI coverage of estimate of γ_{20} under Laplace distribution of random effects



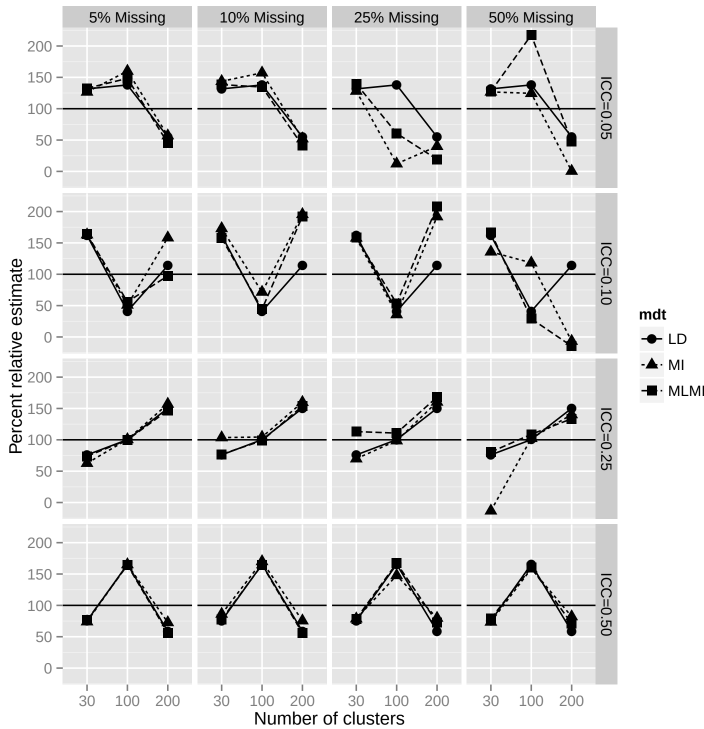
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.30: 95% CI coverage of estimate of γ_{21} under Laplace distribution of random effects



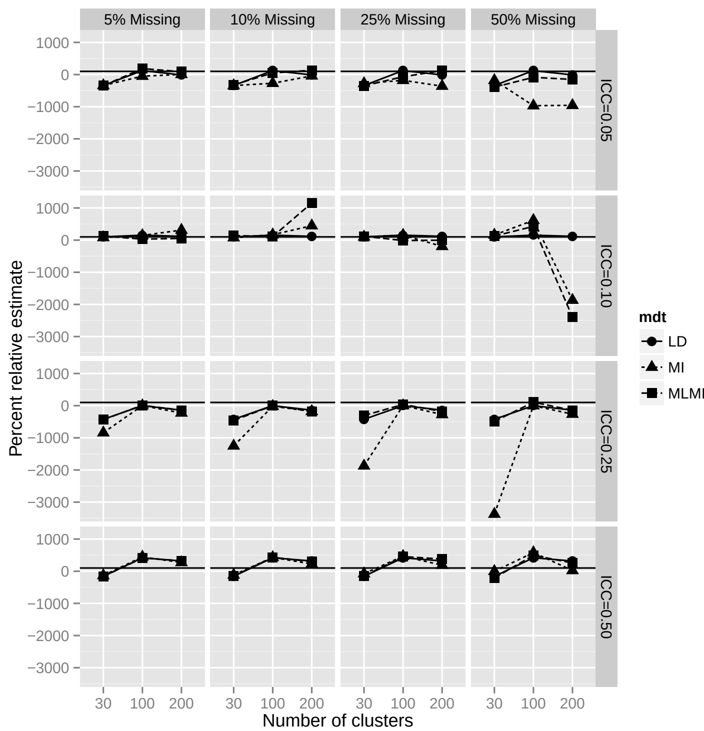
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.31: Percent relative estimate of estimate of γ_{00} under Cauchy distribution of random effects



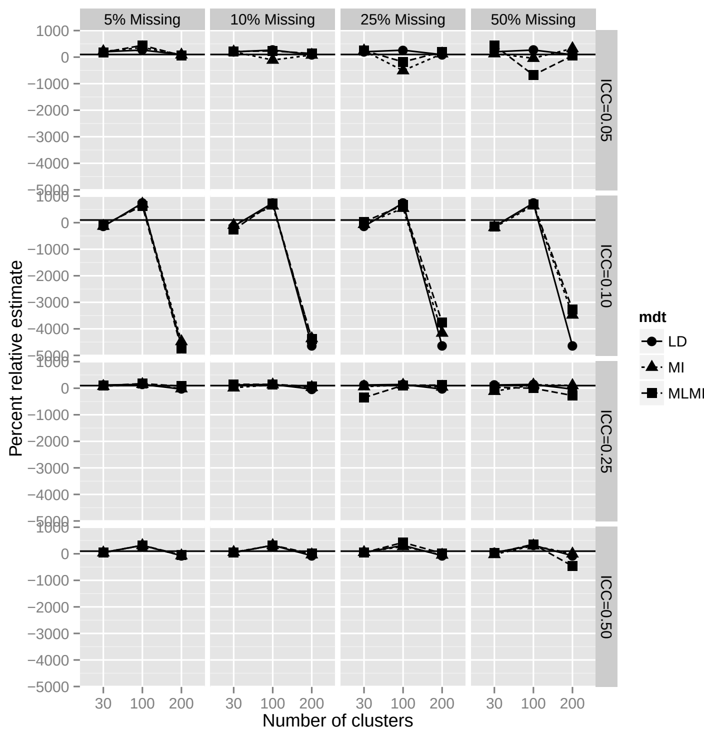
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.32: Percent relative estimate of estimate of γ_{01} under Cauchy distribution of random effects



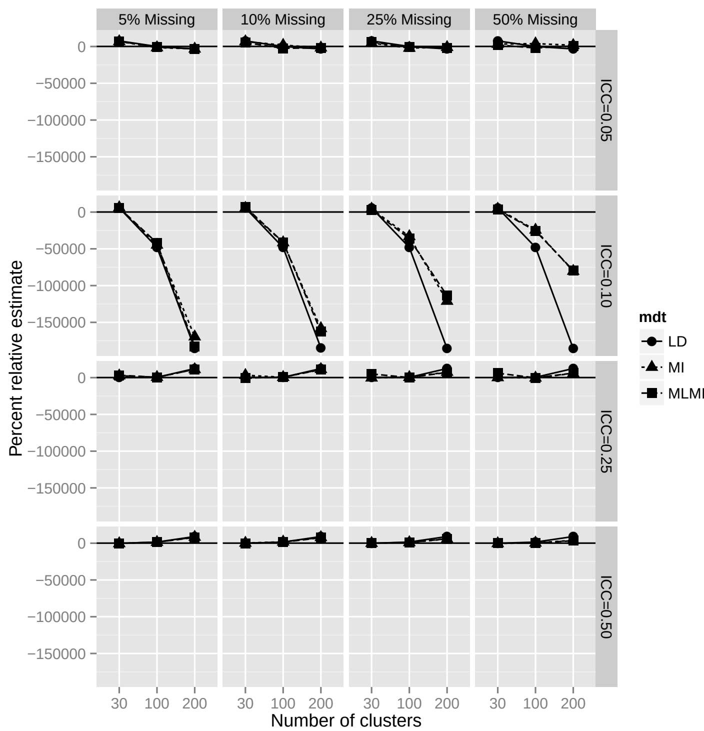
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.33: Percent relative estimate of estimate of γ_{10} under Cauchy distribution of random effects



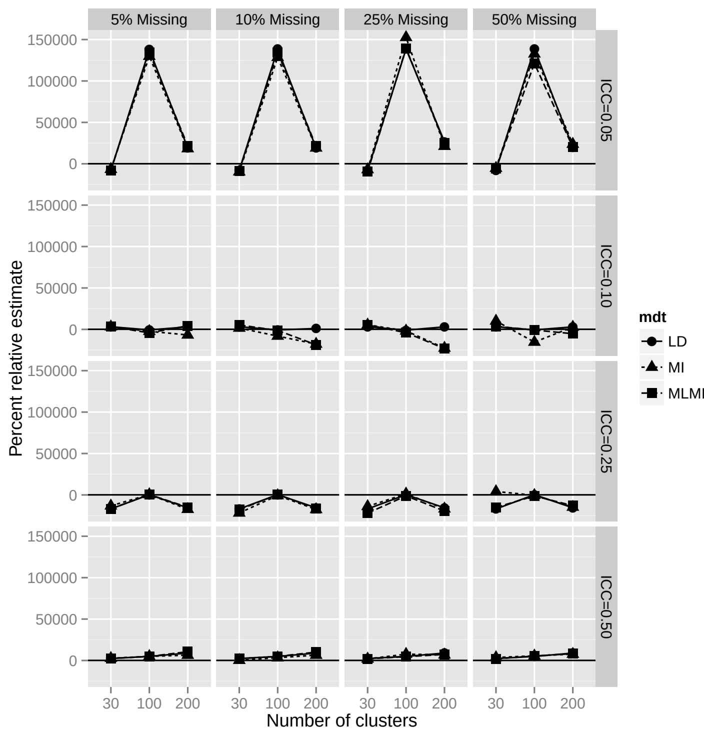
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.34: Percent relative estimate of estimate of γ_{11} under Cauchy distribution of random effects



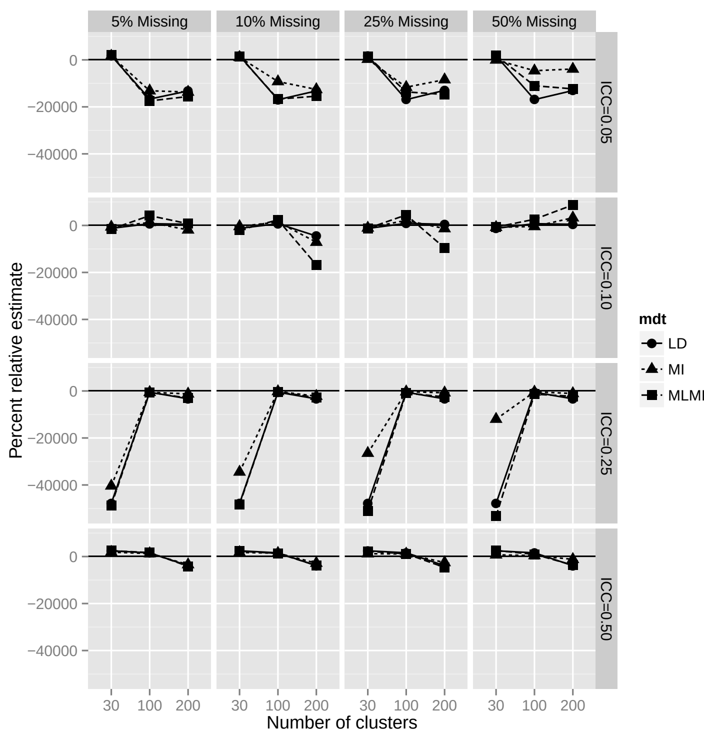
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.35: Percent relative estimate of estimate of γ_{20} under Cauchy distribution of random effects



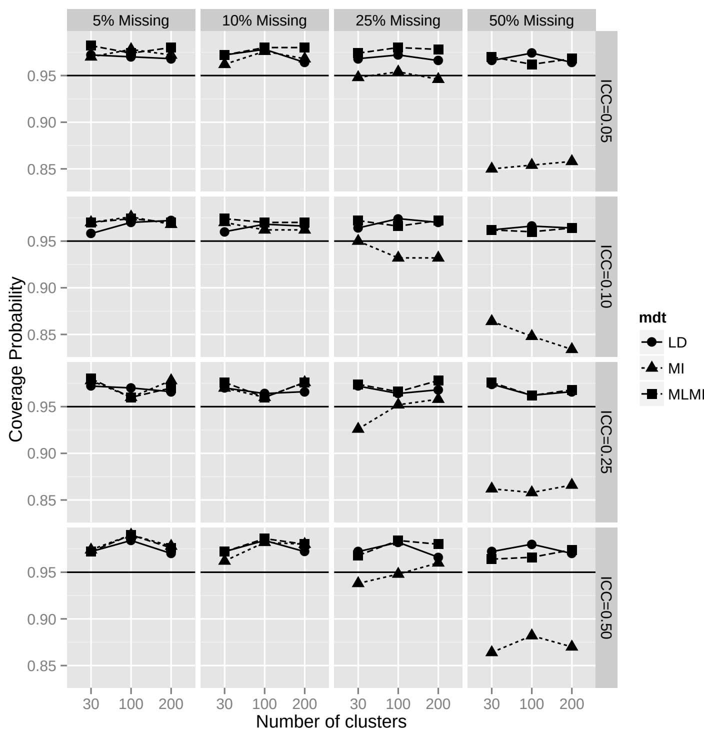
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.36: Percent relative estimate of estimate of γ_{21} under Cauchy distribution of random effects



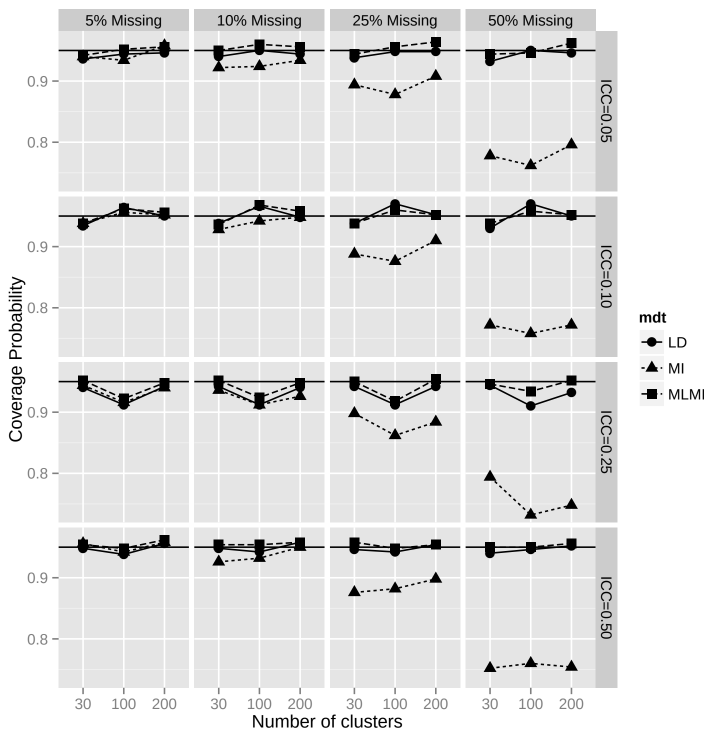
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.37: 95% CI coverage of estimate of γ_{00} under Cauchy distribution of random effects



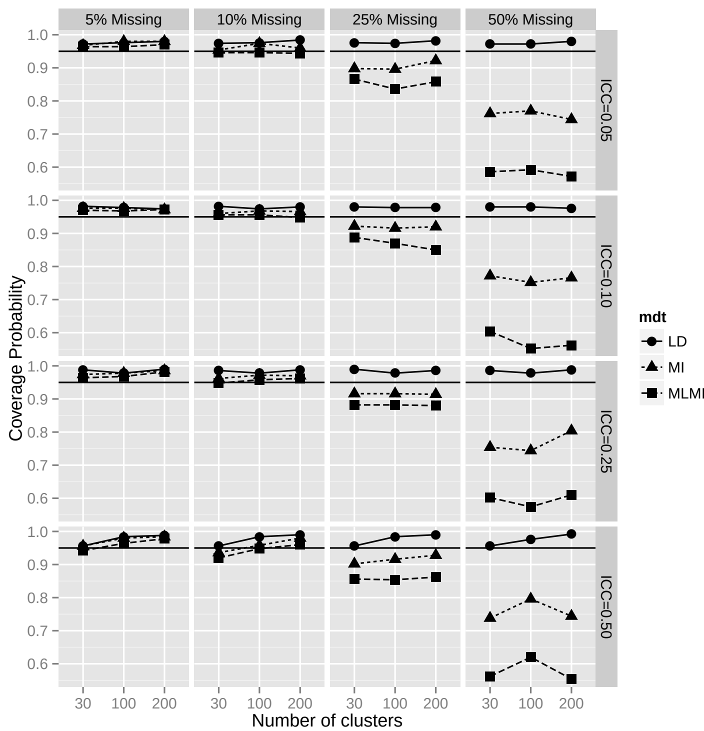
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.38: 95% CI coverage of estimate of γ_{01} under Cauchy distribution of random effects



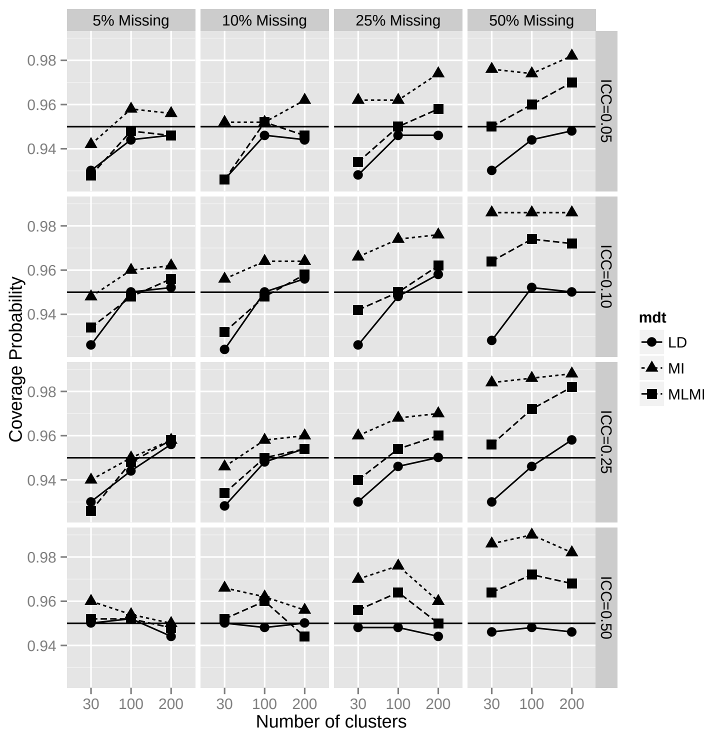
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.39: Percent relative estimate of estimate of γ_{10} under Cauchy distribution of random effects



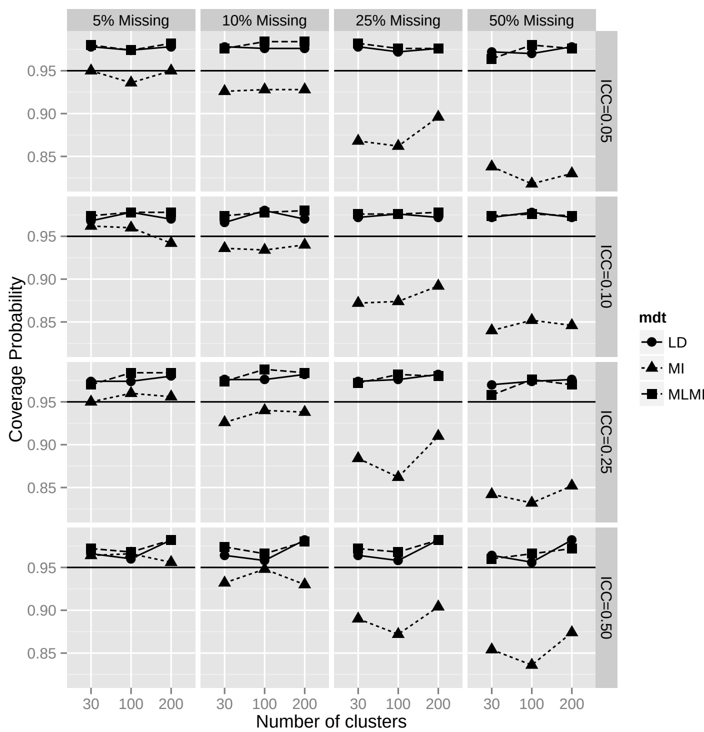
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.40: 95% CI coverage of estimate of γ_{11} under Cauchy distribution of random effects



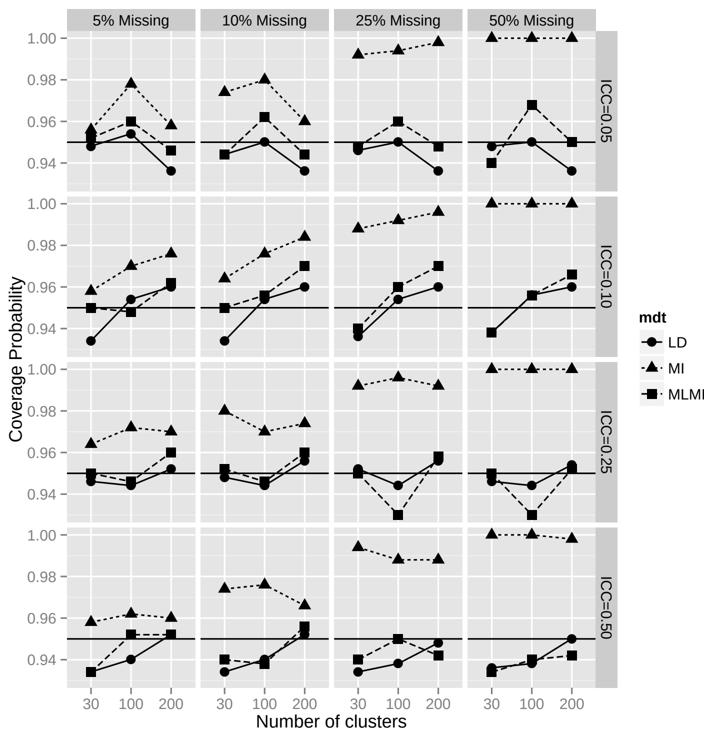
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.41: 95% CI coverage of estimate of γ_{20} under Cauchy distribution of random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure C.42: 95% CI coverage of estimate of γ_{21} under Cauchy distribution of random effects

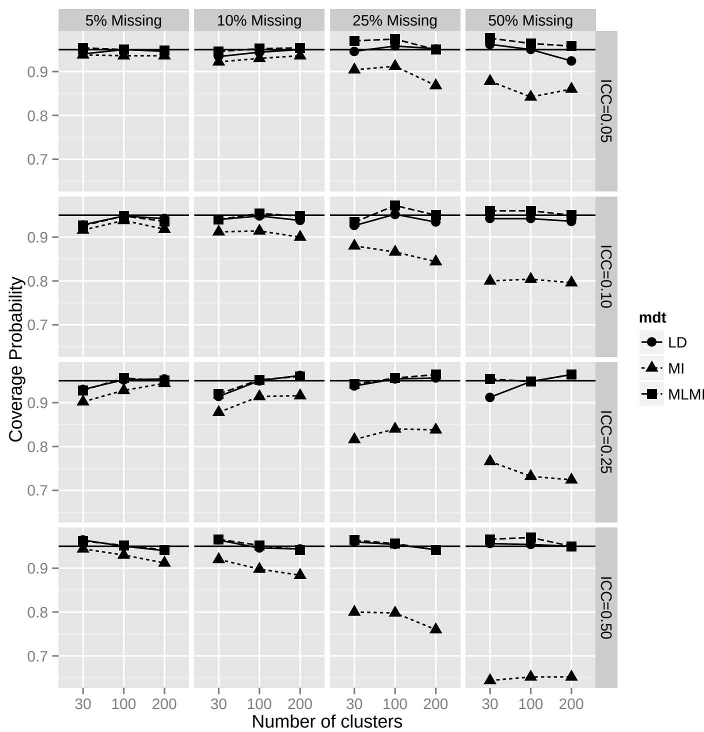


Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Appendix D

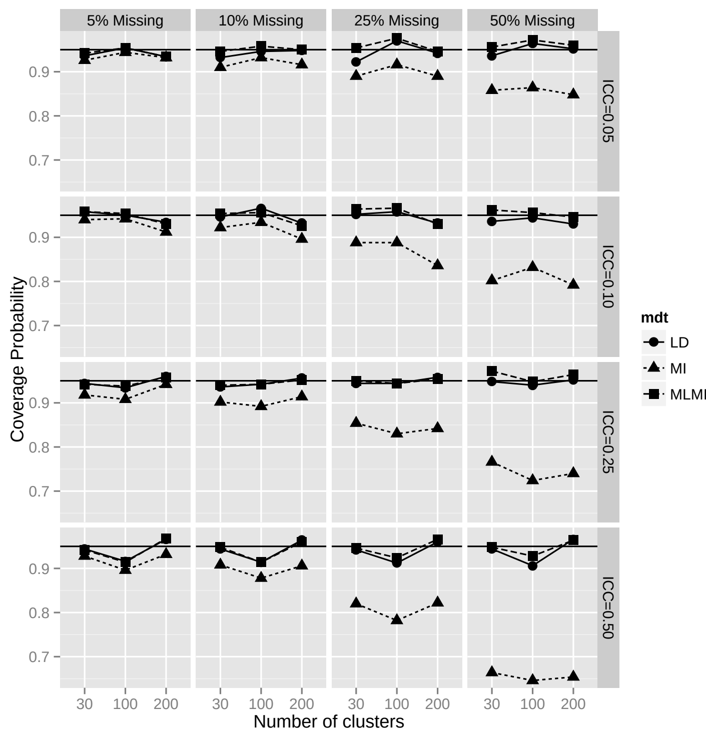
Percent relative estimate and coverage under MCAR

Figure D.1: 95% CI coverage of γ_{00} under multivariate normal distribution of random effects



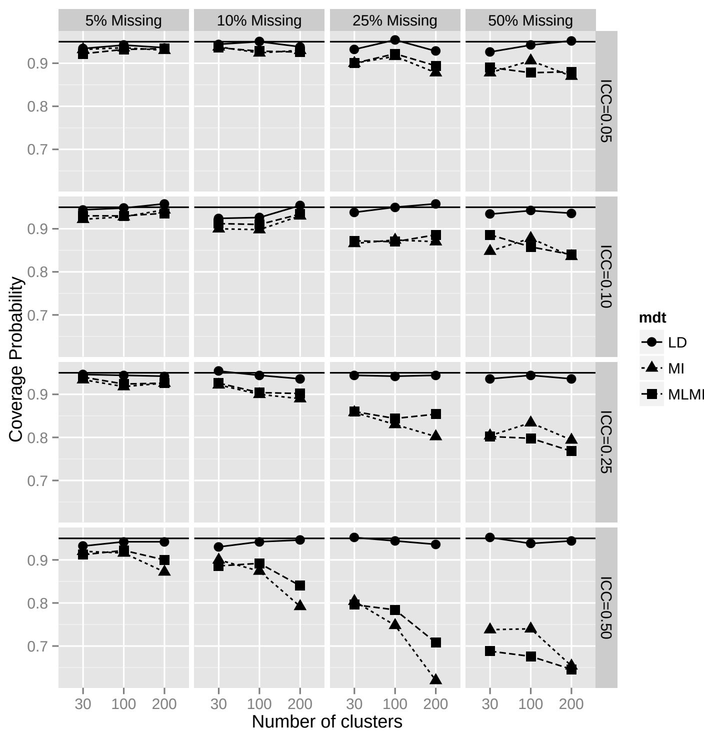
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.2: 95% CI coverage of estimate of γ_{01} under multivariate normal distribution of random effects



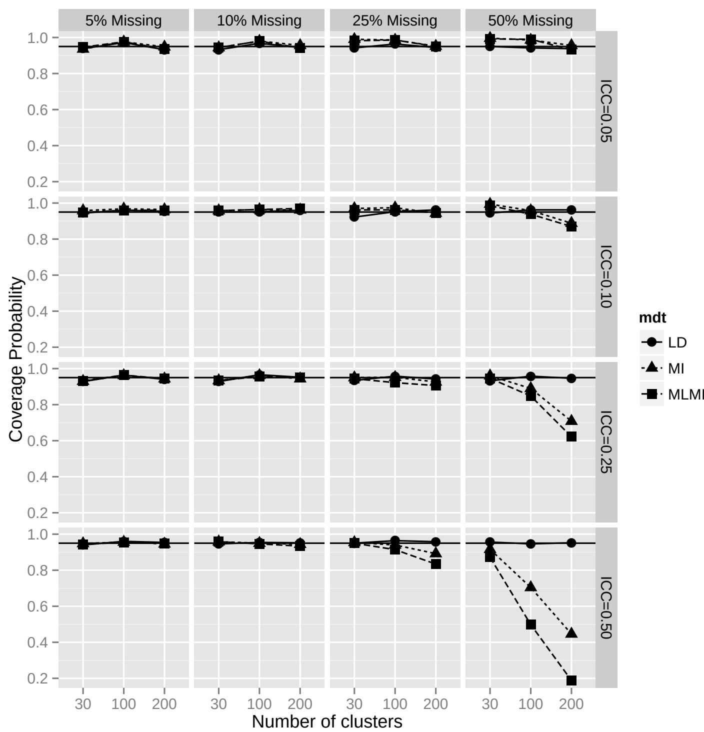
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.3: 95% CI coverage of estimate of γ_{10} under multivariate normal distribution of random effects



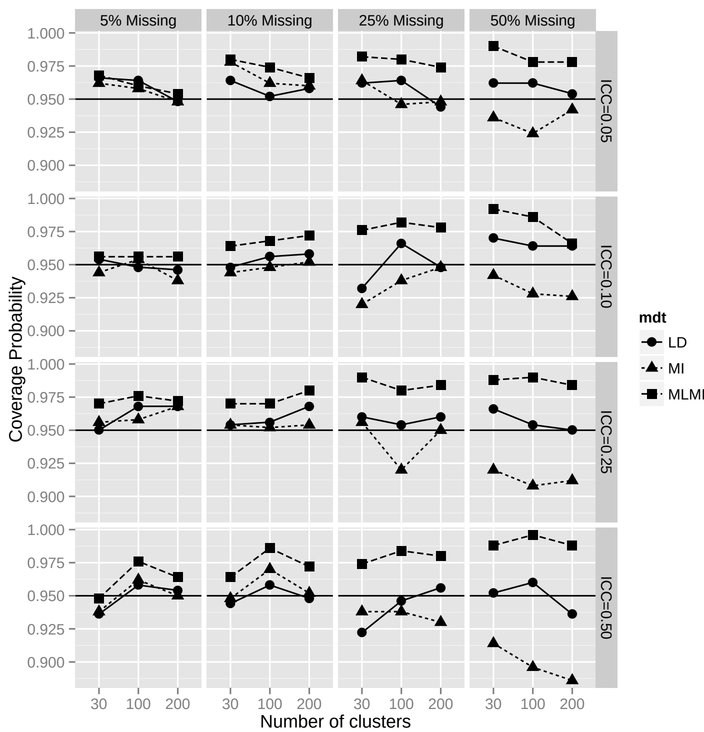
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.4: 95% CI coverage of estimate of γ_{11} under multivariate normal distribution of random effects



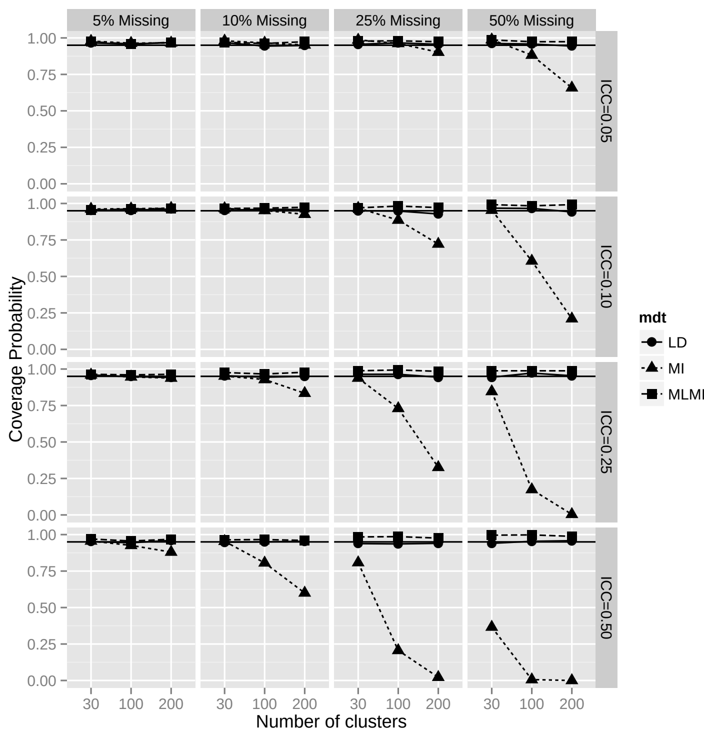
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.5: 95% CI coverage of estimate of γ_{20} under multivariate normal distribution of random effects



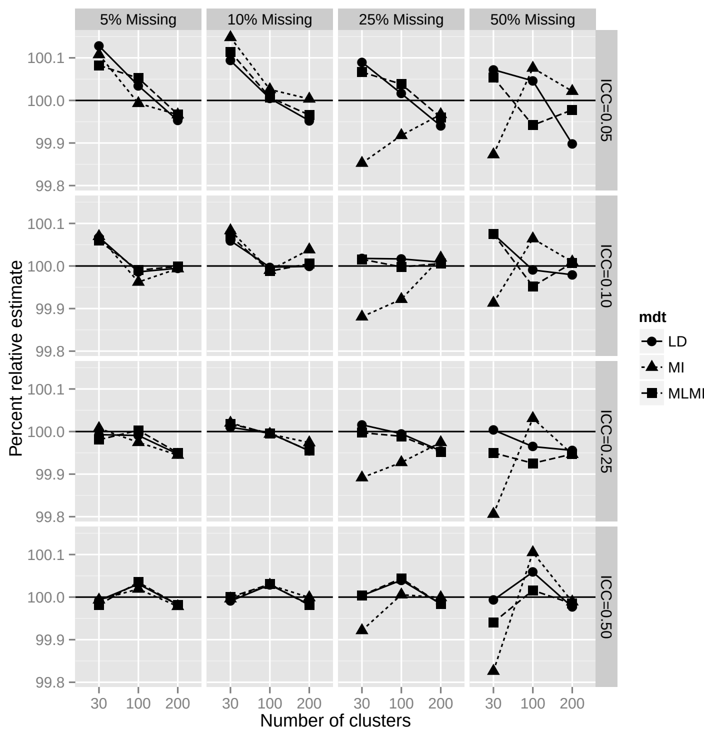
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.6: 95% CI coverage of estimate of γ_{21} under multivariate normal distribution of random effects



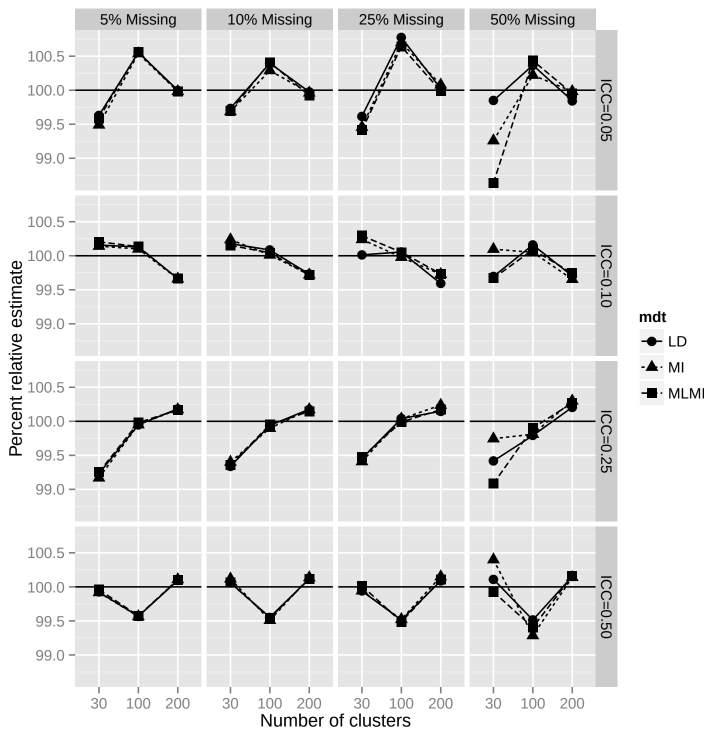
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.7: Percent relative estimate of estimate of γ_{00} under multivariate normal distribution of random effects



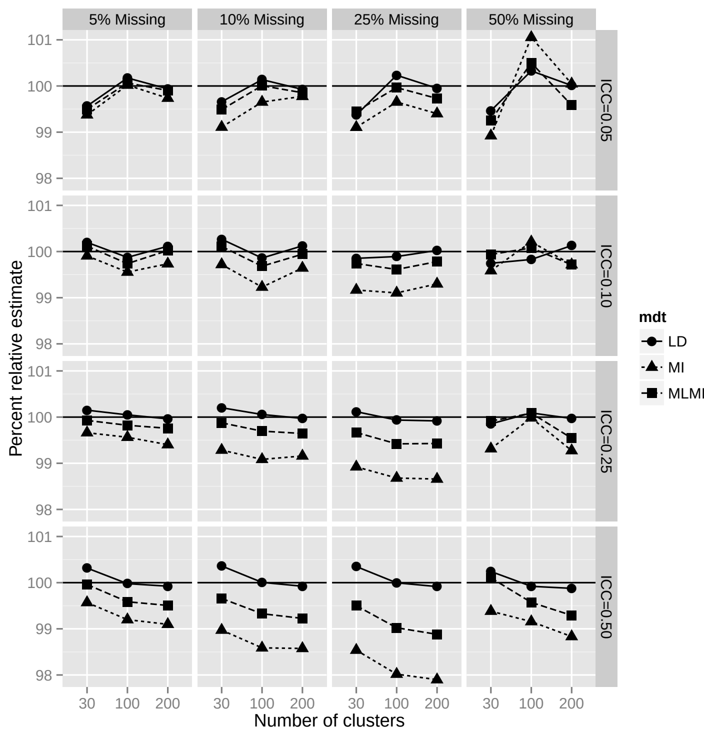
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.8: Percent relative estimate of estimate of γ_{01} under multivariate normal distribution of random effects



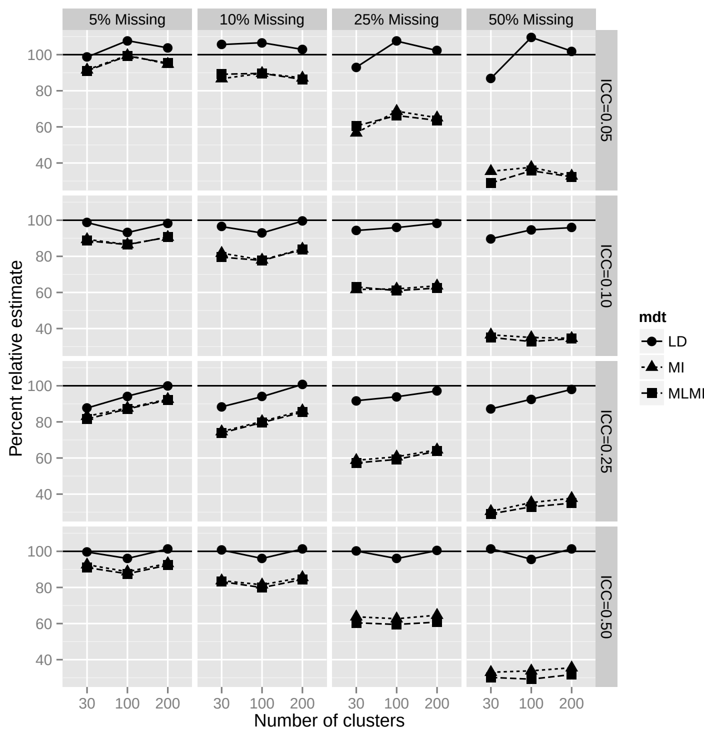
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.9: Percent relative estimate of estimate of γ_{10} under multivariate normal distribution of random effects



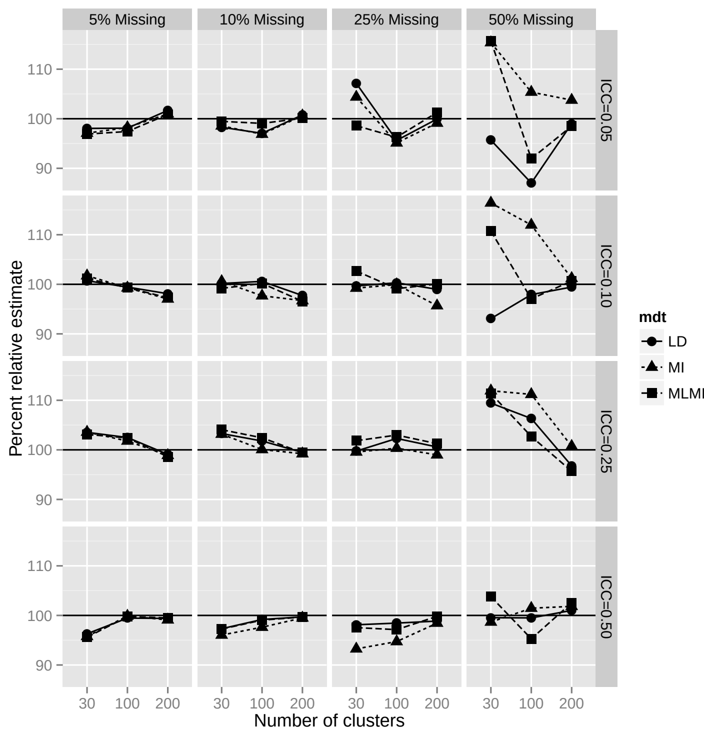
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.10: Percent relative estimate of estimate of γ_{11} under multivariate normal distribution of random effects



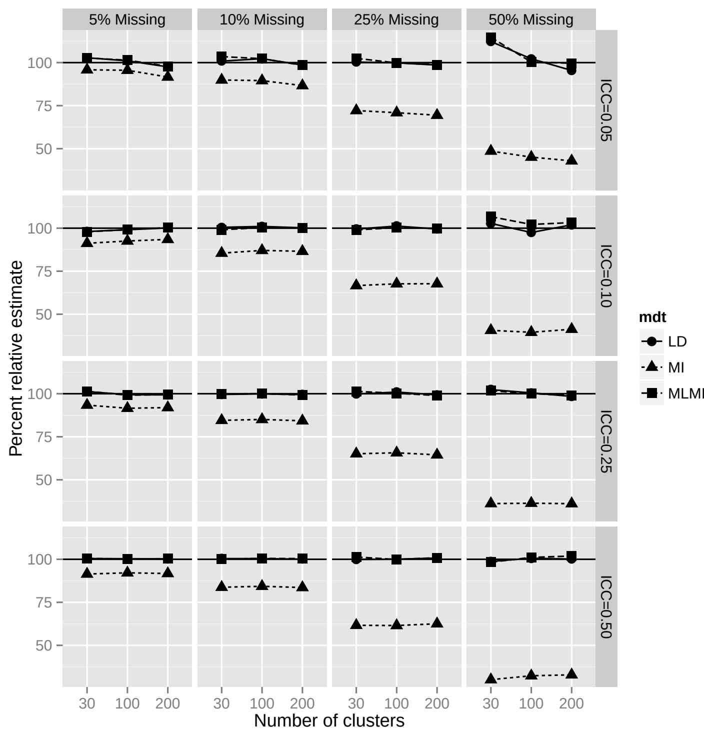
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.11: Percent relative estimate of estimate of γ_{20} under multivariate normal distribution of random effects



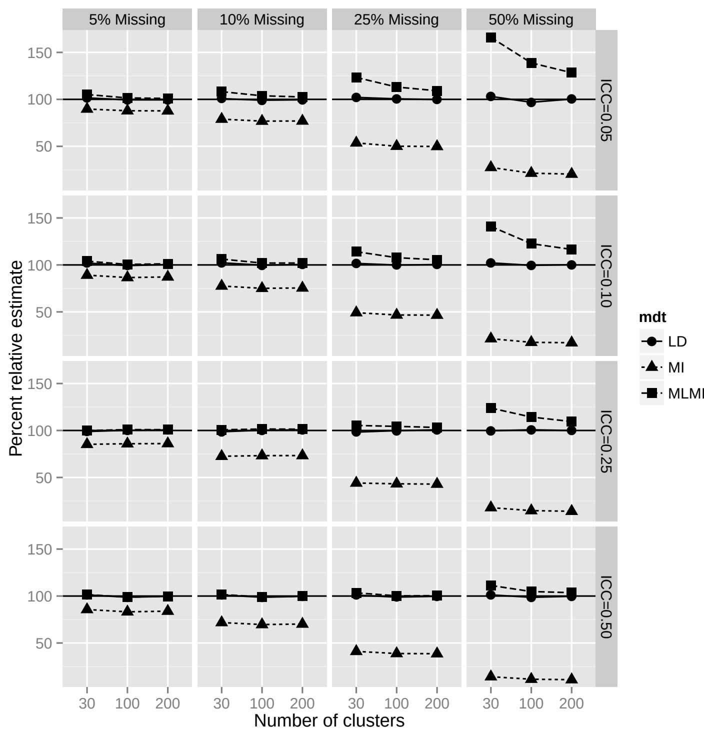
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.12: Percent relative estimate of estimate of γ_{21} under multivariate normal distribution of random effects



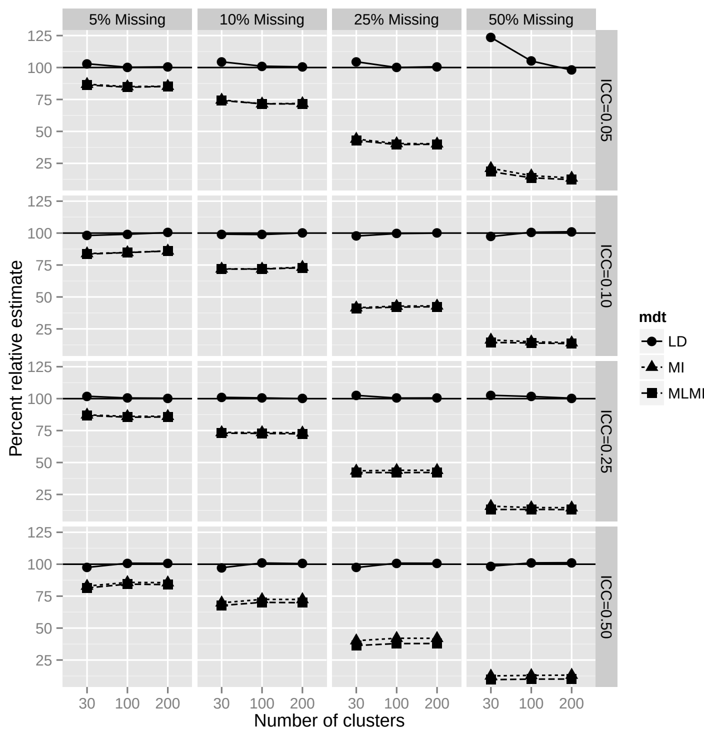
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.13: Percent relative estimate of estimate of τ_{00} under multivariate normal distribution of random effects



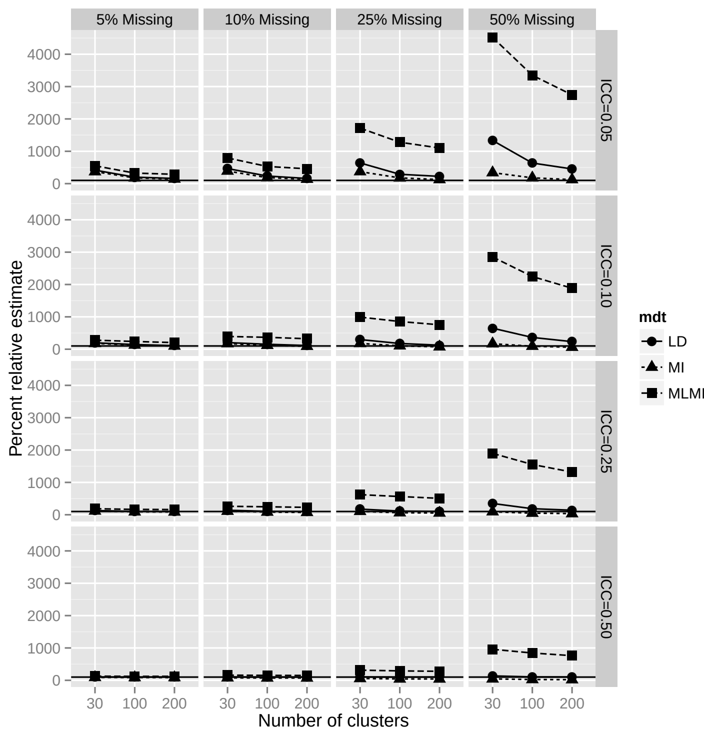
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.14: Percent relative estimate of estimate of τ_{11} under multivariate normal distribution of random effects



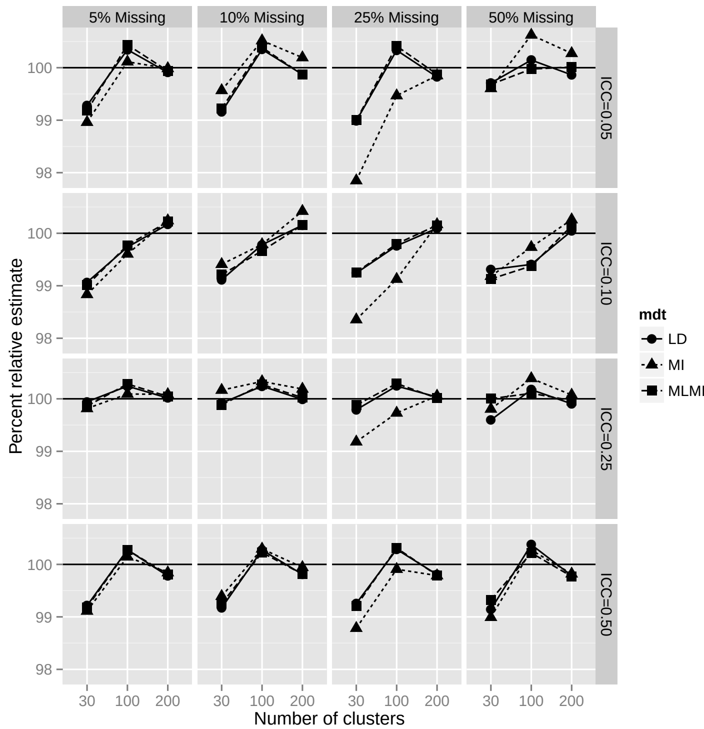
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.15: Percent relative estimate of estimate of τ_{22} under multivariate normal distribution of random effects



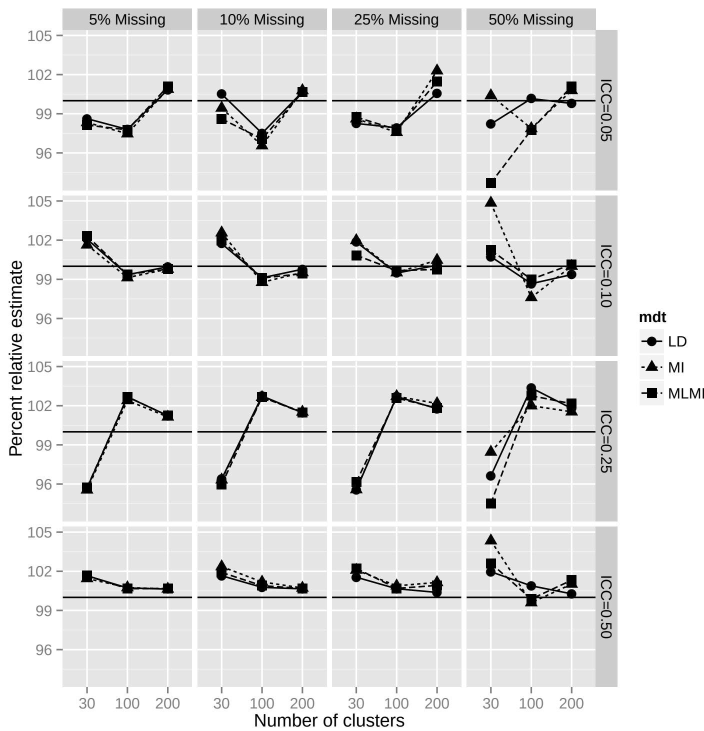
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.16: Percent relative estimate of estimate of γ_{00} under chi-square distribution of random effects



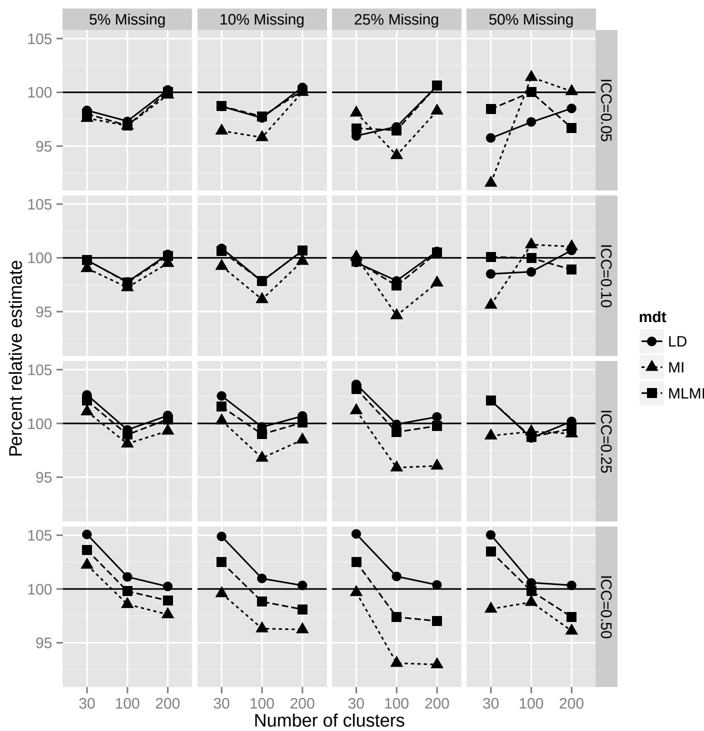
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.17: Percent relative estimate of estimate of γ_{01} under chi-square distribution of random effects



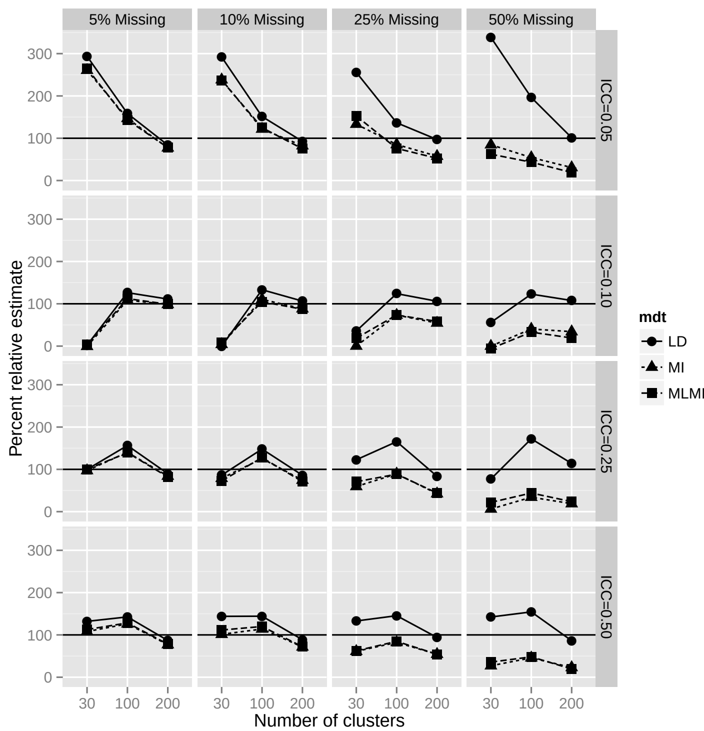
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.18: Percent relative estimate of estimate of γ_{10} under chi-square distribution of random effects



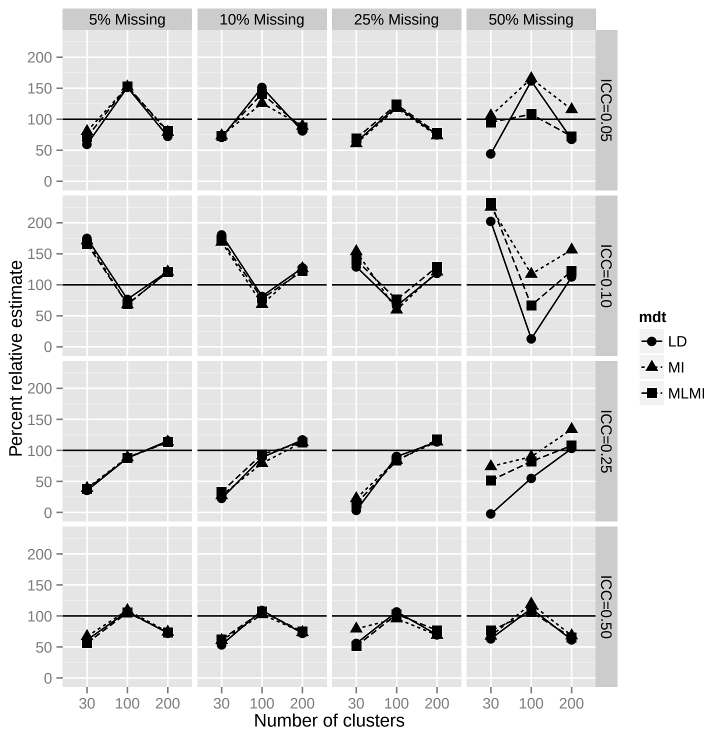
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.19: Percent relative estimate of estimate of γ_{11} under chi-square distribution of random effects



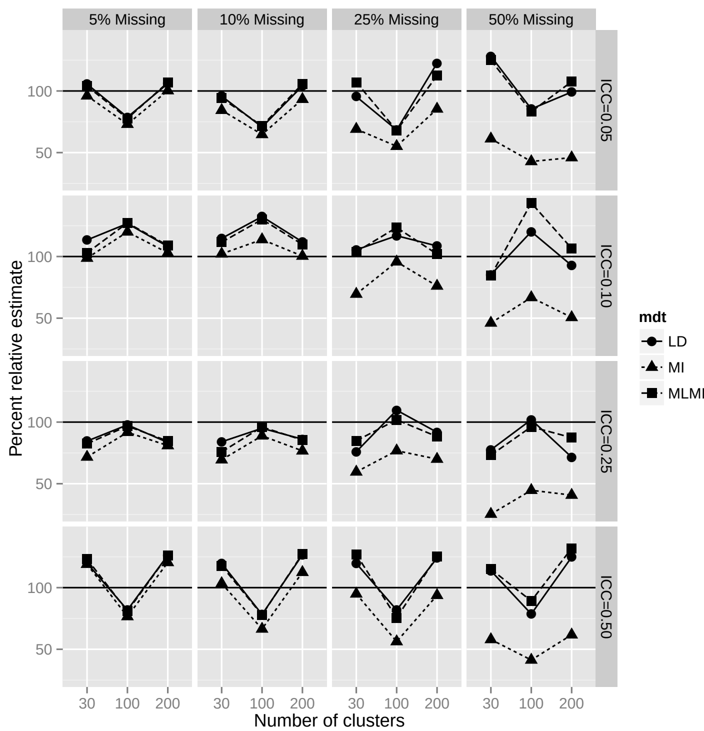
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.20: Percent relative estimate of estimate of γ_{20} under chi-square distribution of random effects



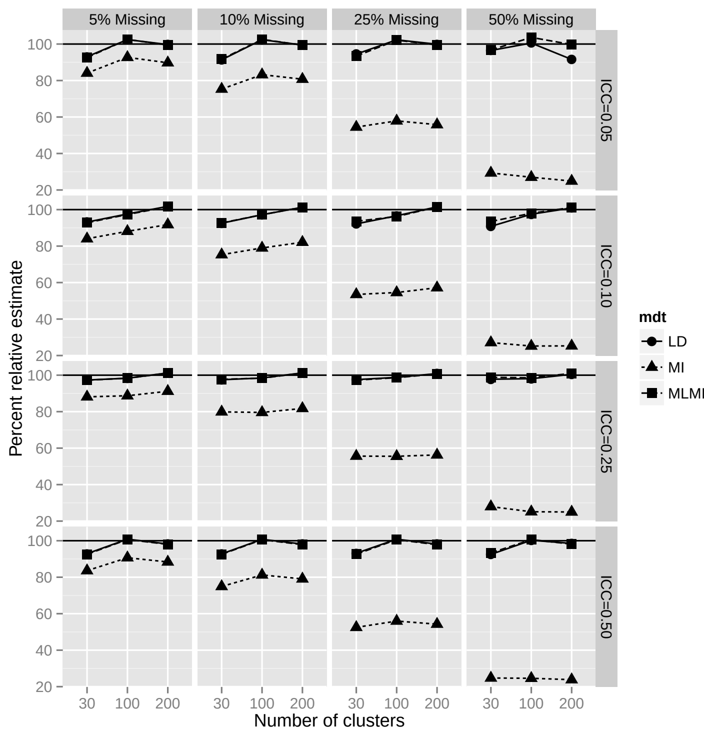
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.21: Percent relative estimate of estimate of γ_{21} under chi-square distribution of random effects



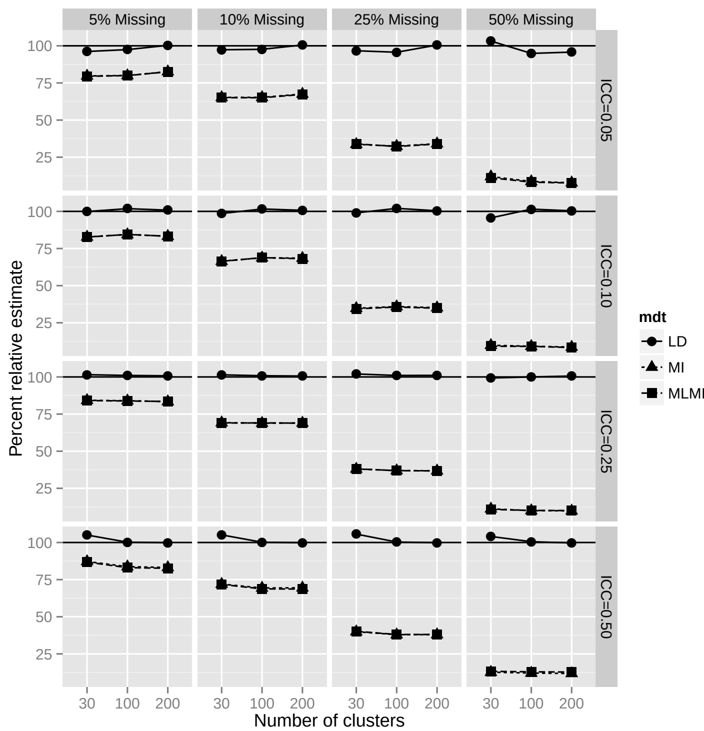
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.22: Percent relative estimate of estimate of τ_{00} under chi-square distribution of random effects



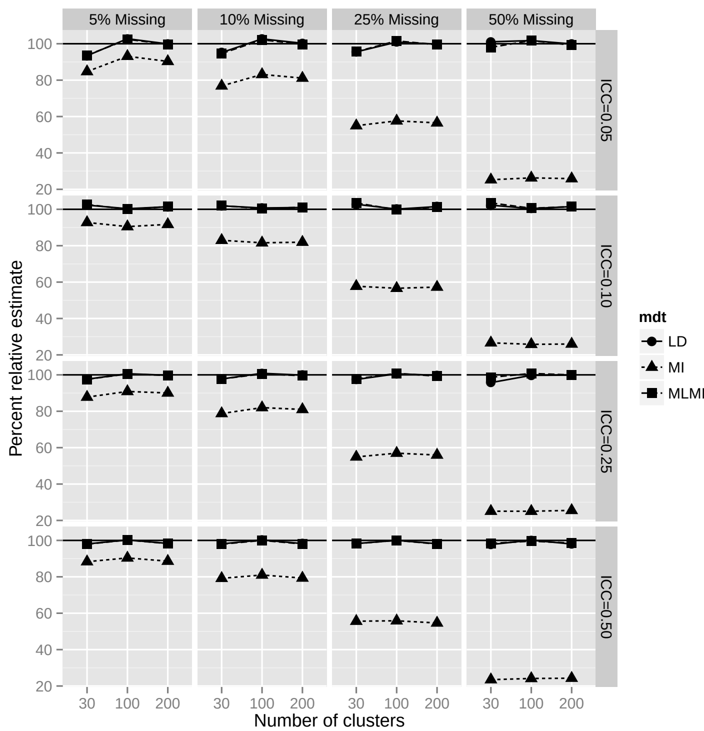
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.23: Percent relative estimate of estimate of τ_{11} under chi-square distribution of random effects



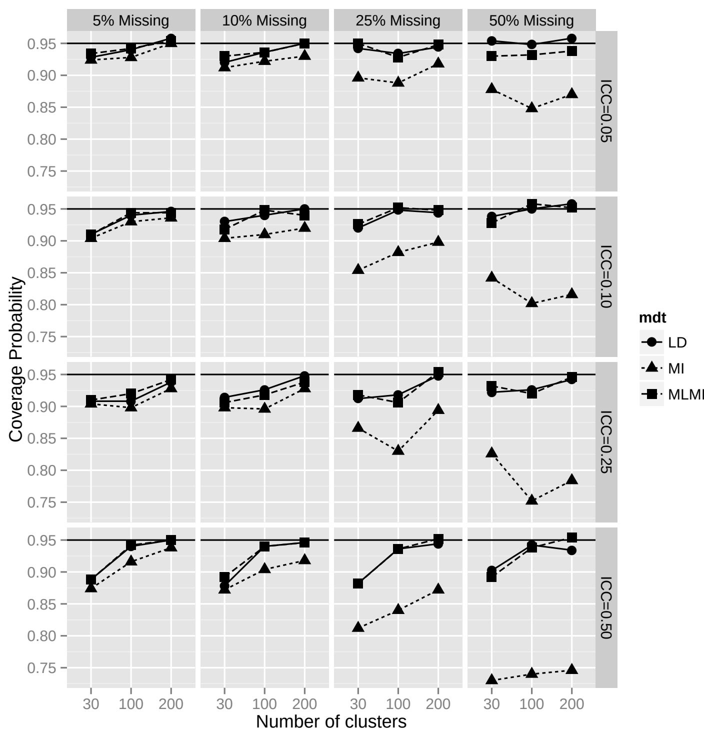
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.24: Percent relative estimate of estimate of τ_{22} under chi-square distribution of random effects



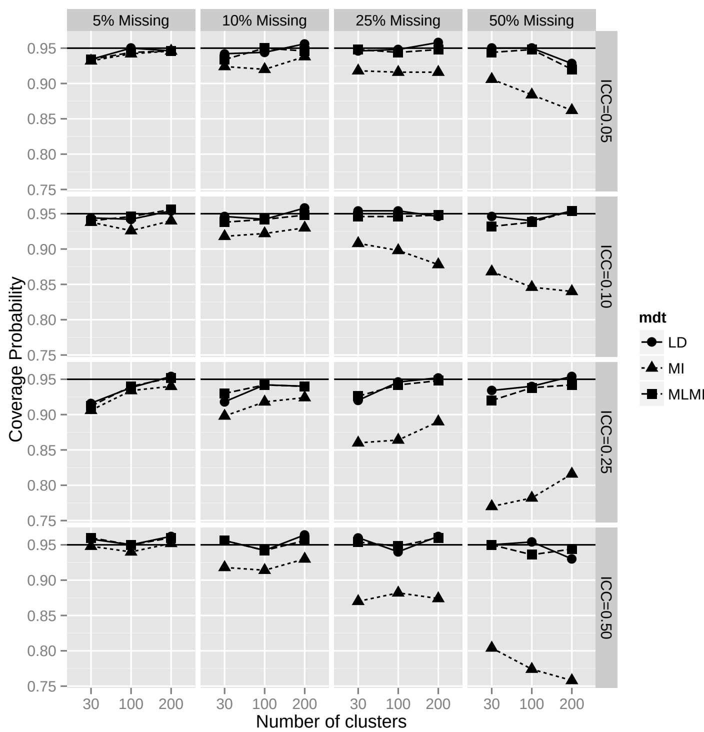
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.25: 95% CI coverage of estimate of γ_{00} under chi-square distribution of random effects



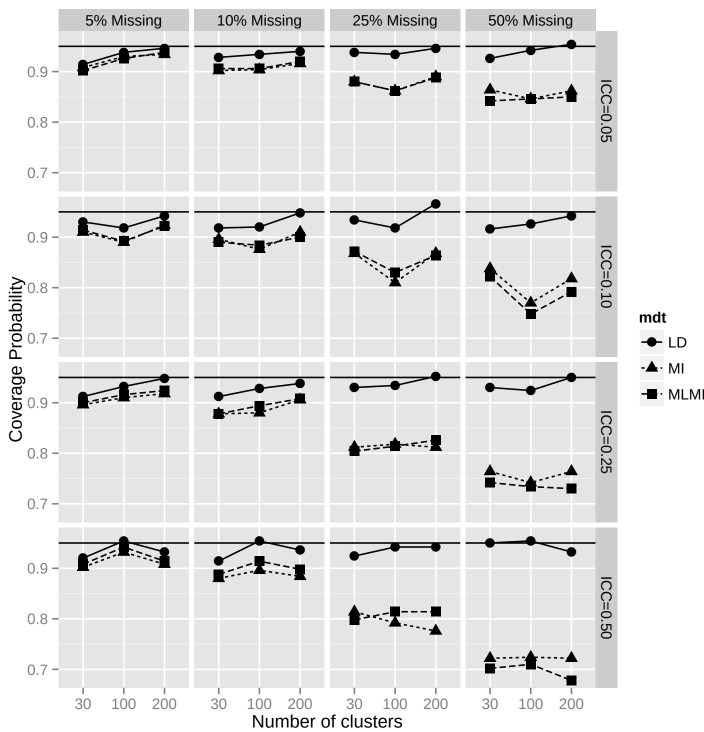
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.26: 95% CI coverage of estimate of γ_{01} under chi-square distribution of random effects



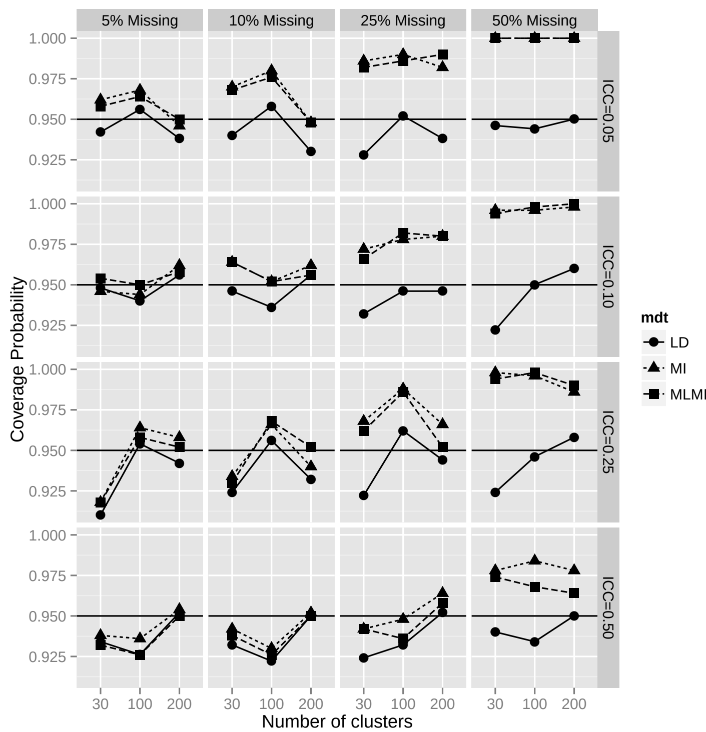
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.27: 95% CI coverage of estimate of γ_{10} under chi-square distribution of random effects



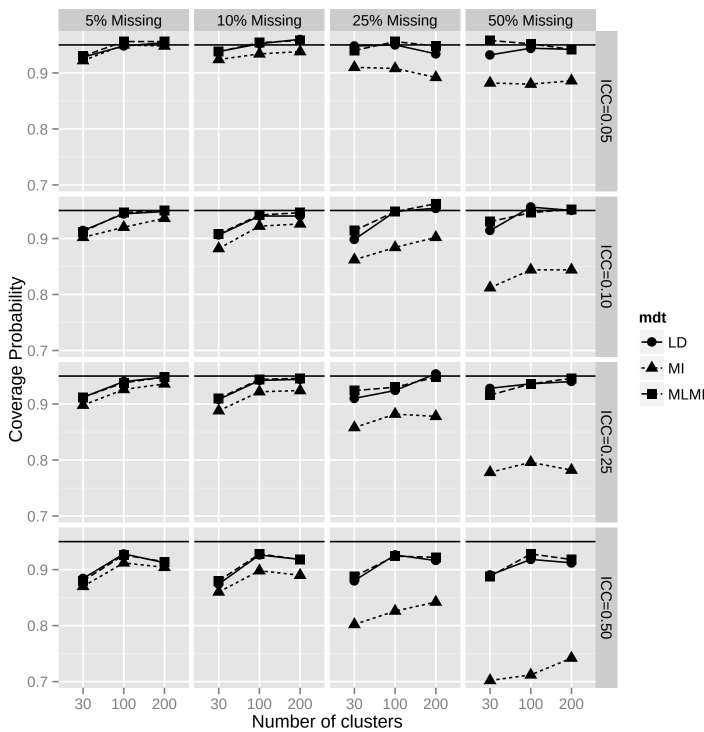
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.28: 95% CI coverage of estimate of γ_{11} under chi-square distribution of random effects



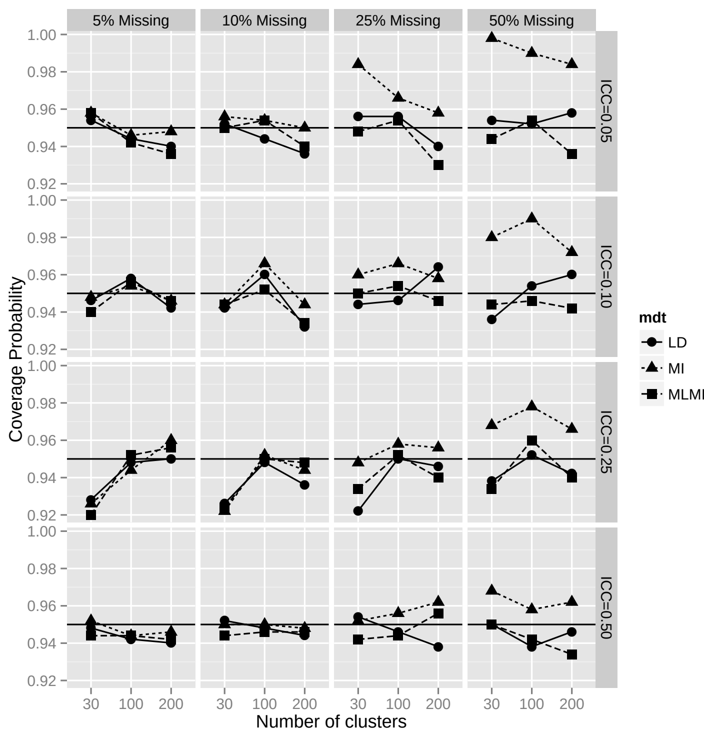
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.29: 95% CI coverage of estimate of γ_{20} under chi-square distribution of random effects



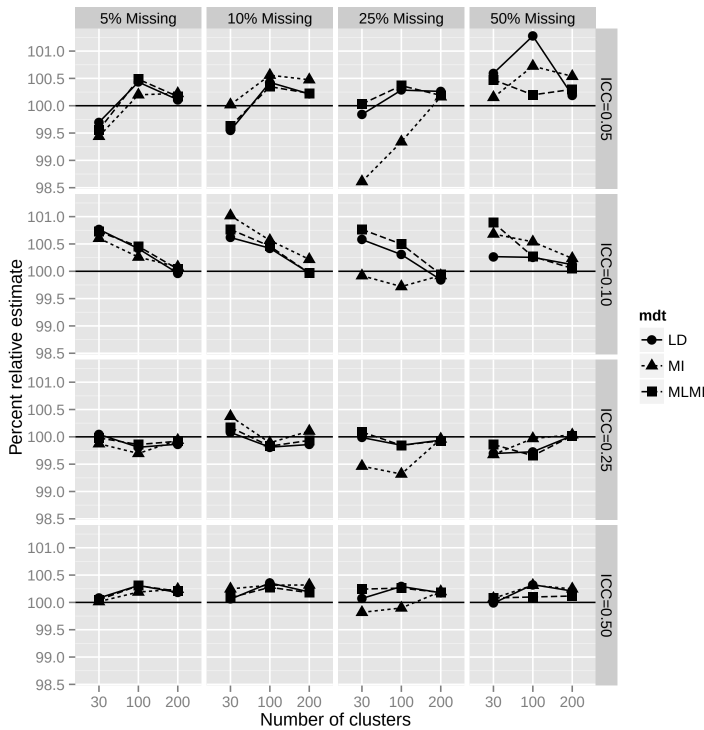
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.30: 95% CI coverage of estimate of γ_{21} under chi-square distribution of random effects



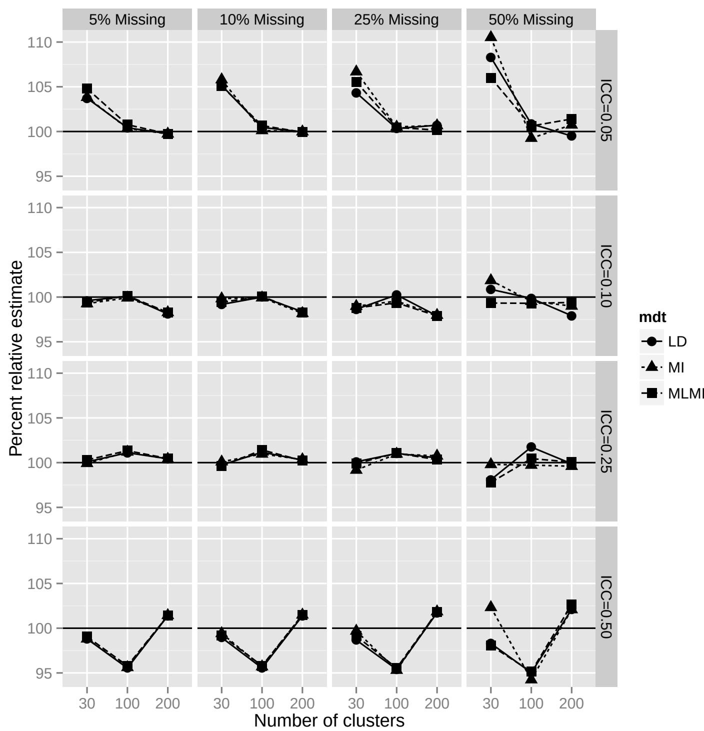
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.31: Percent relative estimate of estimate of γ_{00} under Laplace distribution of random effects



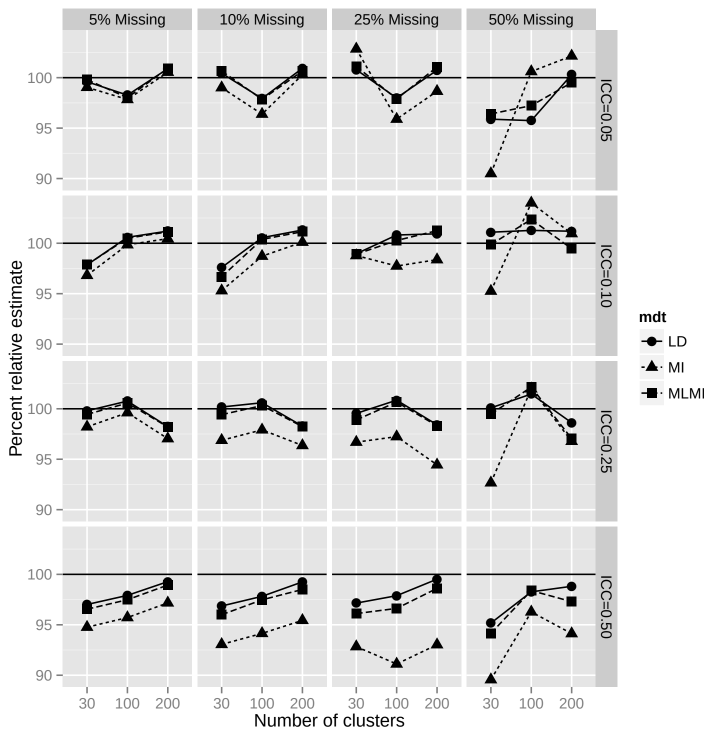
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.32: Percent relative estimate of estimate of γ_{01} under Laplace distribution of random effects



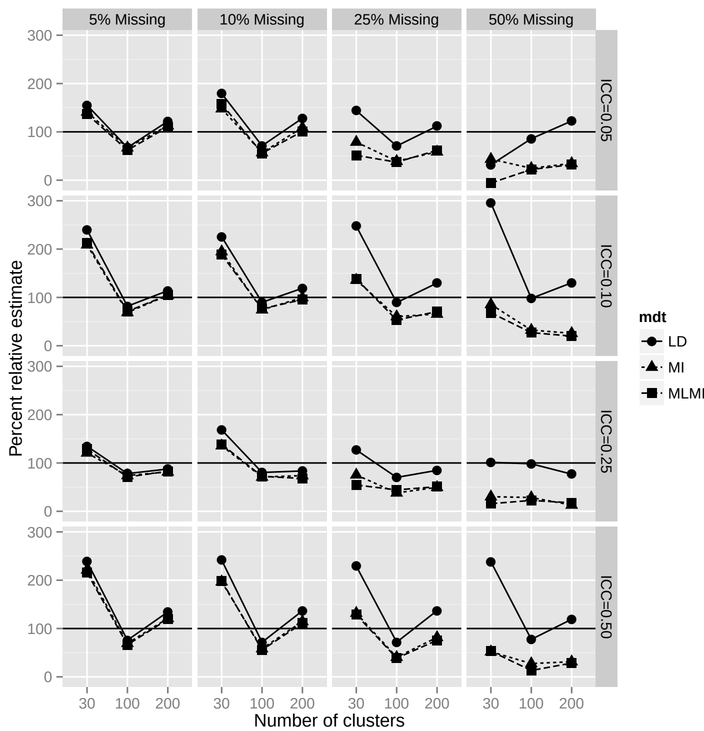
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.33: Percent relative estimate of estimate of γ_{10} under Laplace distribution of random effects



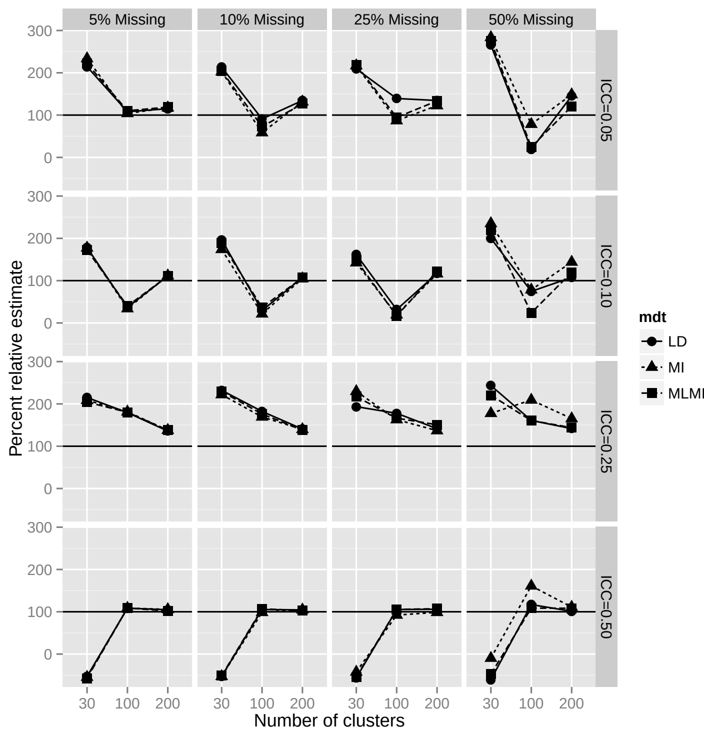
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.34: Percent relative estimate of estimate of γ_{11} under Laplace distribution of random effects



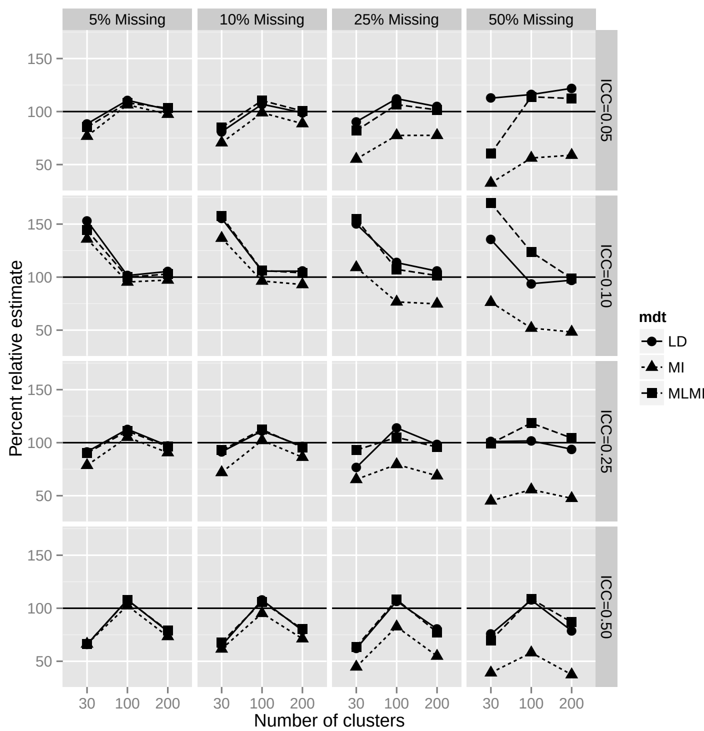
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.35: Percent relative estimate of estimate of γ_{20} under Laplace distribution of random effects



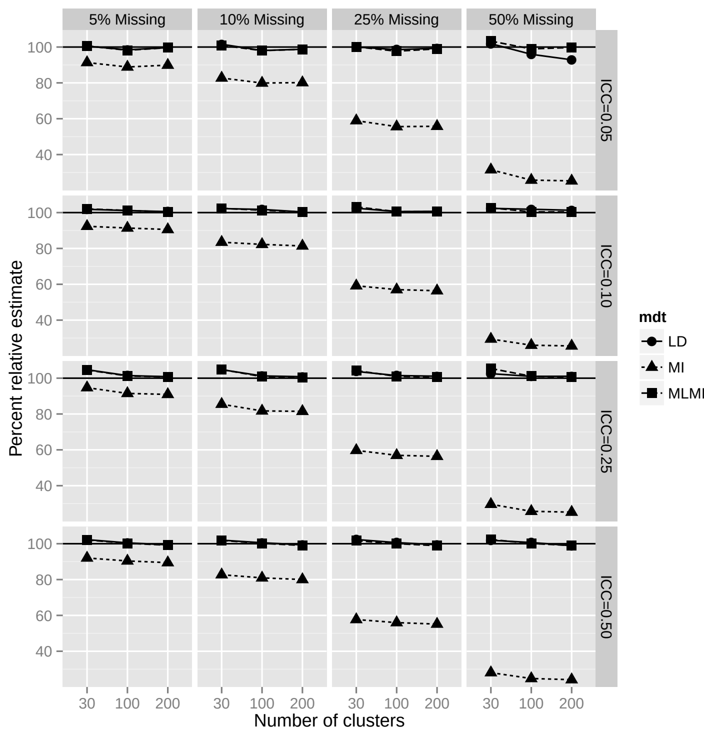
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.36: Percent relative estimate of estimate of γ_{21} under Laplace distribution of random effects



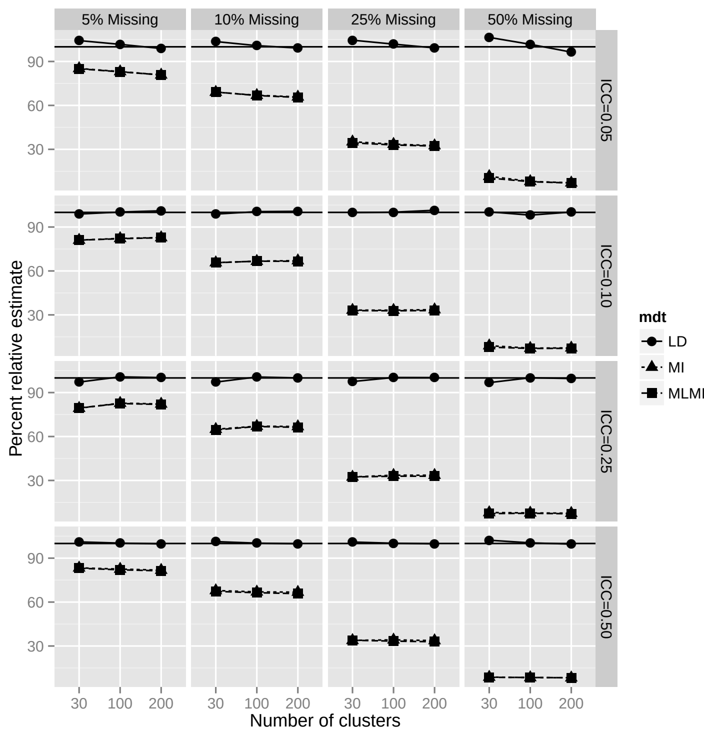
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.37: Percent relative estimate of estimate of τ_{00} under Laplace distribution of random effects



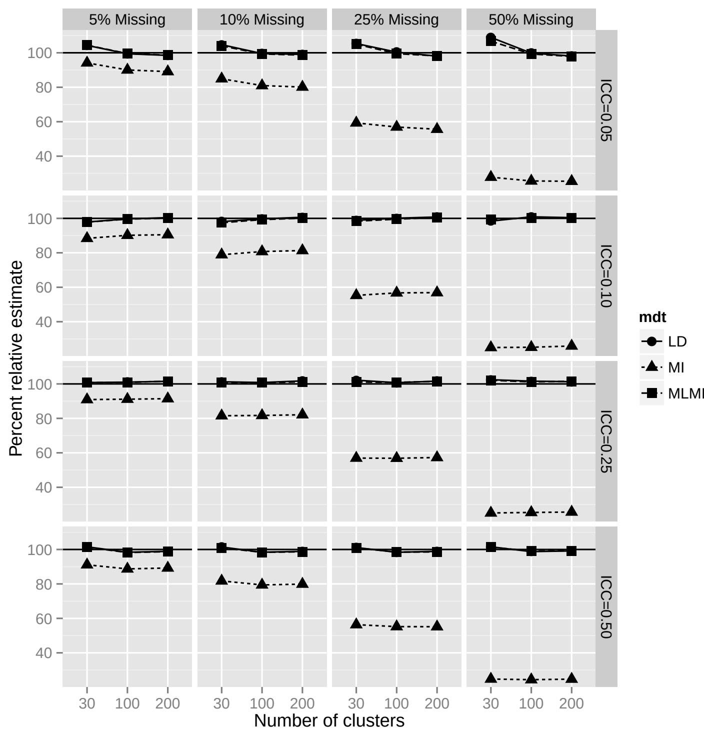
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.38: Percent relative estimate of estimate of τ_{11} under Laplace distribution of random effects



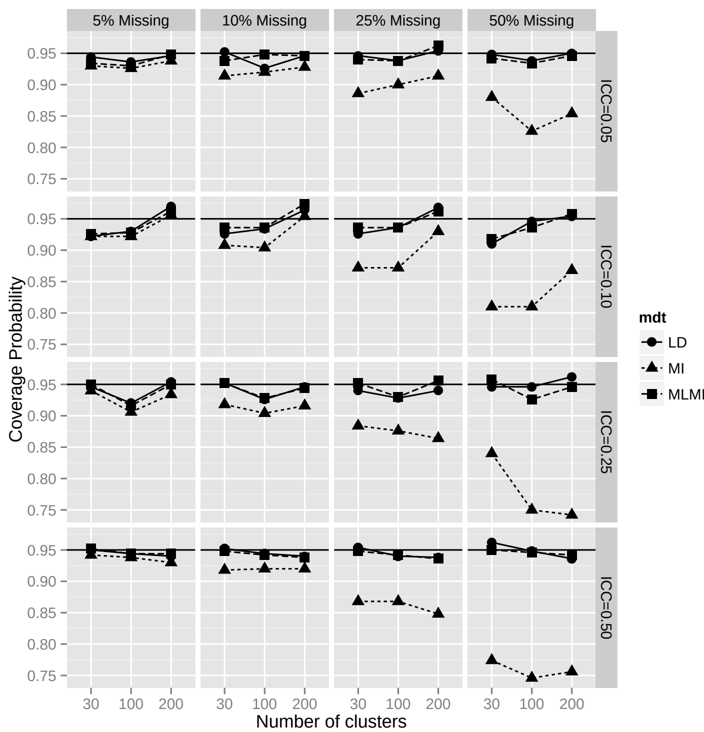
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.39: Percent relative estimate of estimate of τ_{22} under Laplace distribution of random effects



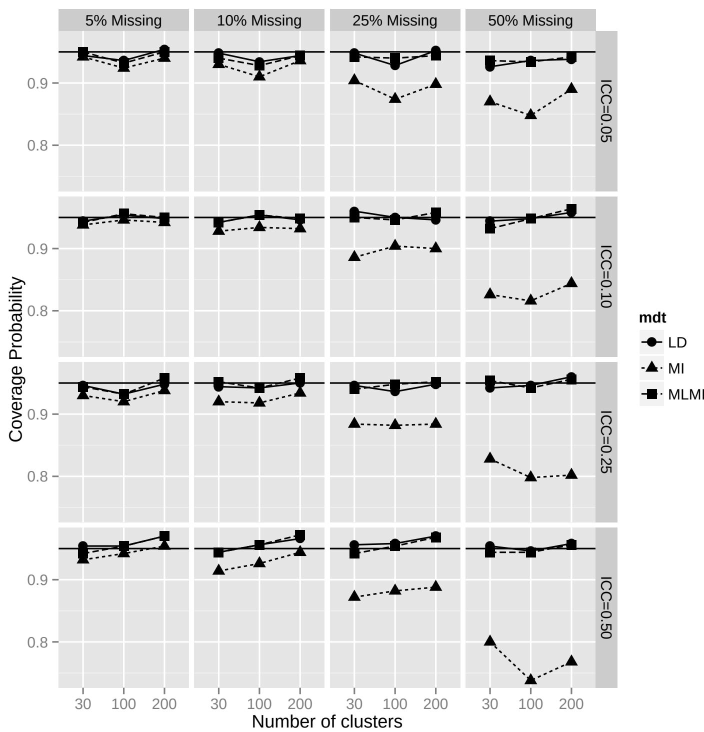
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.40: 95% CI coverage of estimate of γ_{00} under Laplace distribution of random effects



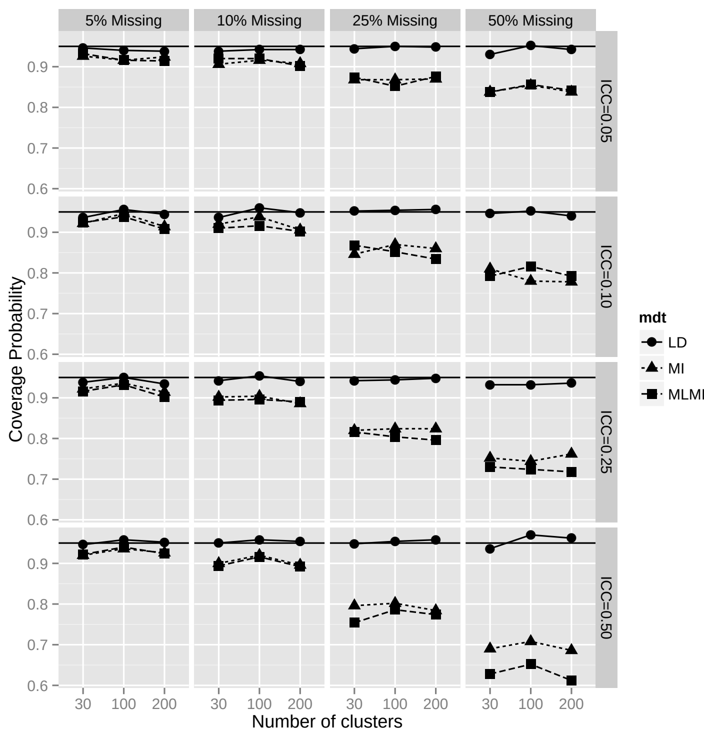
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.41: 95% CI coverage of estimate of γ_{01} under Laplace distribution of random effects



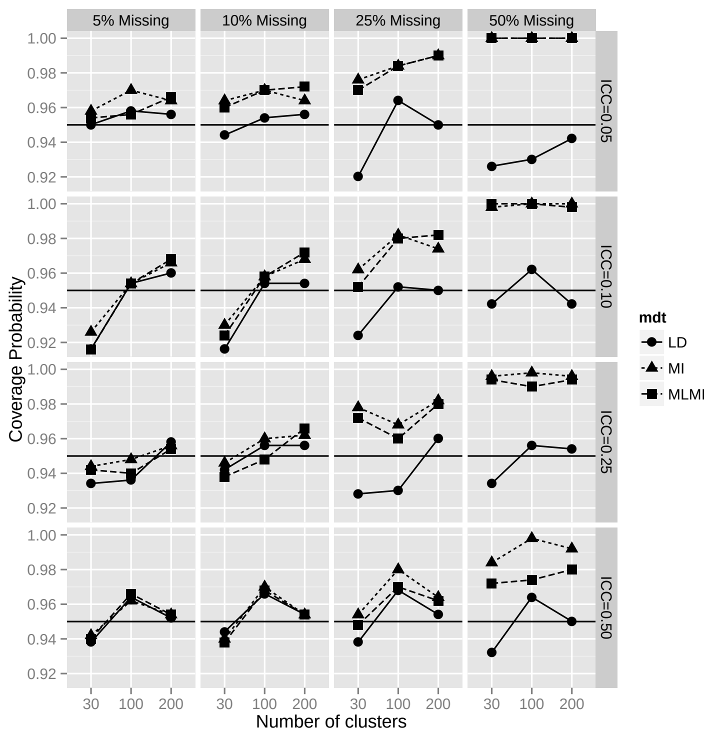
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.42: 95% CI coverage of estimate of γ_{10} under Laplace distribution of random effects



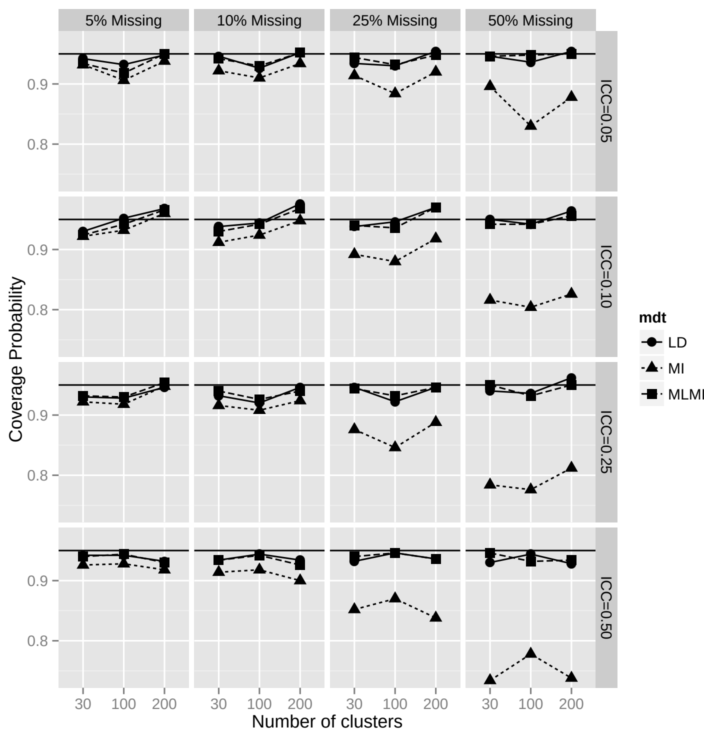
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.43: 95% CI coverage of estimate of γ_{11} under Laplace distribution of random effects



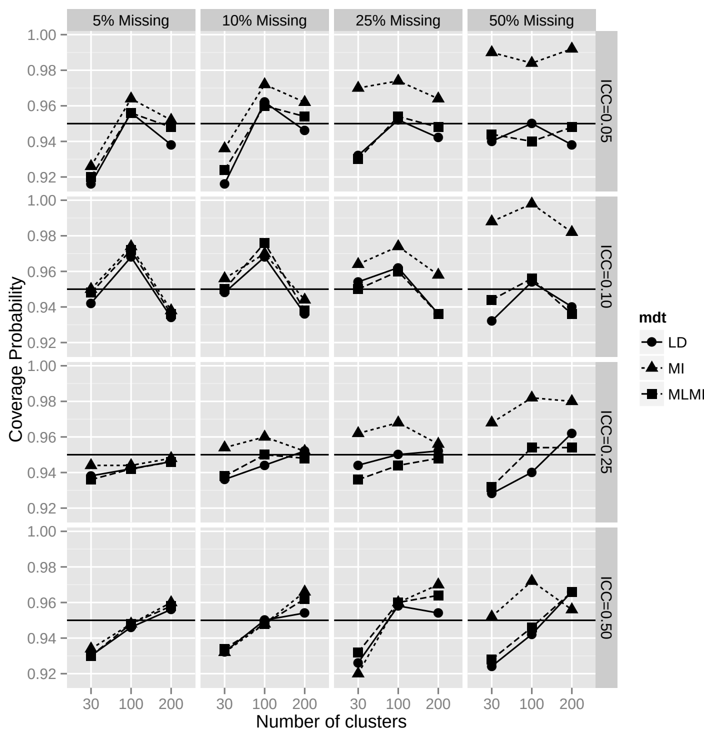
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.44: 95% CI coverage of estimate of γ_{20} under Laplace distribution of random effects



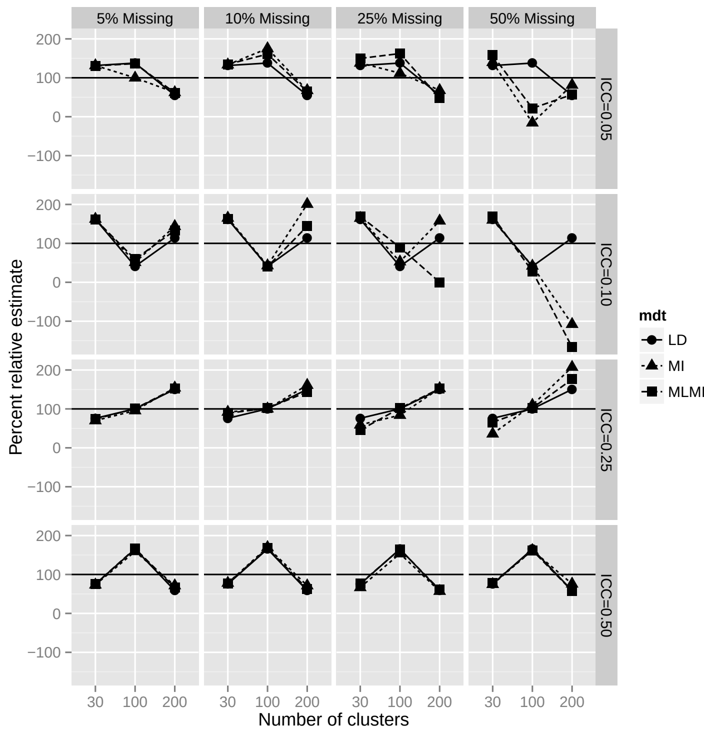
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.45: 95% CI coverage of estimate of γ_{21} under Laplace distribution of random effects



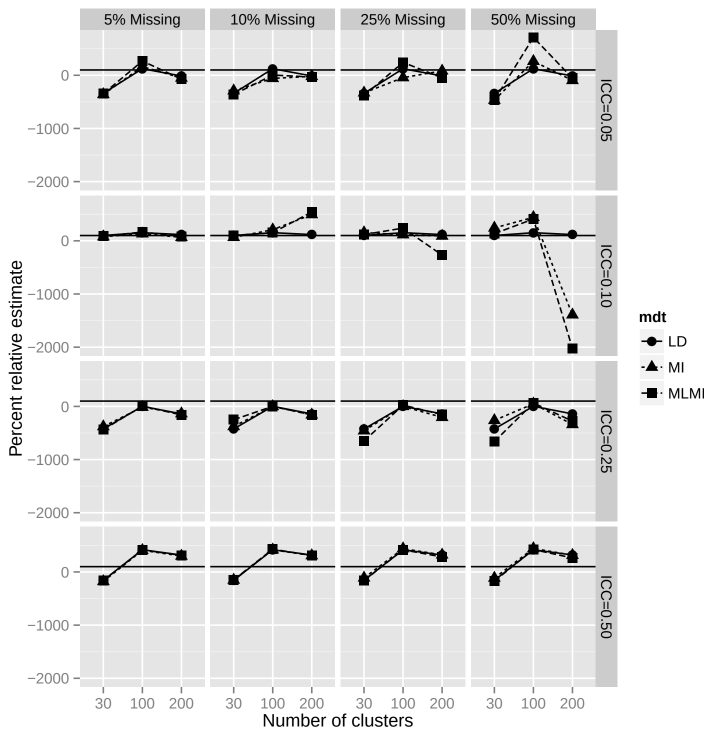
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.46: Percent relative estimate of estimate of γ_{00} under Cauchy distribution of random effects



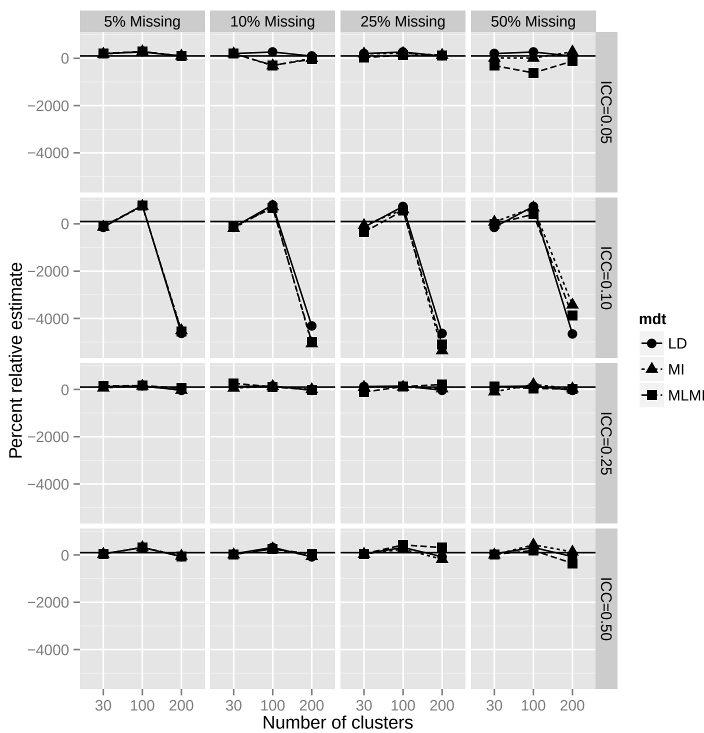
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.47: Percent relative estimate of estimate of γ_{01} under Cauchy distribution of random effects



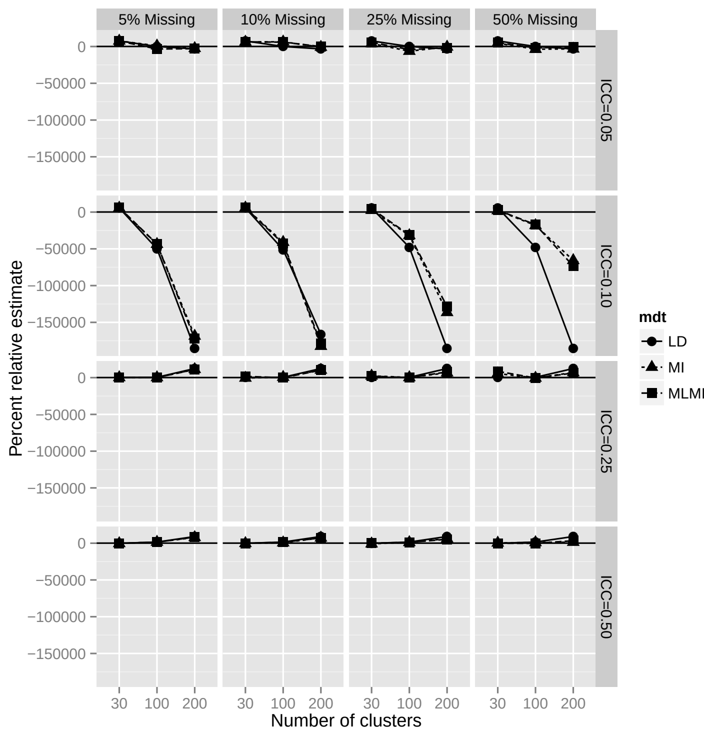
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.48: Percent relative estimate of estimate of γ_{10} under Cauchy distribution of random effects



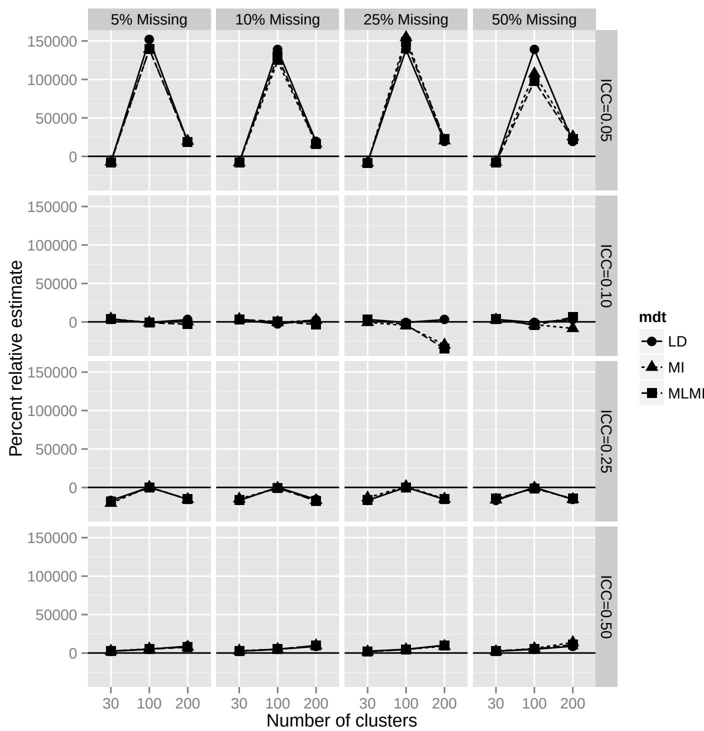
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.49: Percent relative estimate of estimate of γ_{11} under Cauchy distribution of random effects



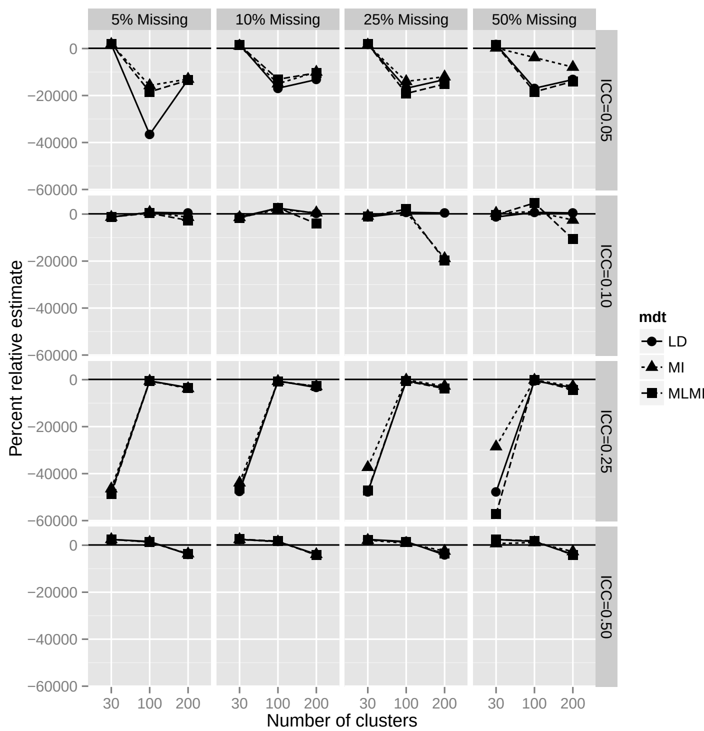
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.50: Percent relative estimate of estimate of γ_{20} under Cauchy distribution of random effects



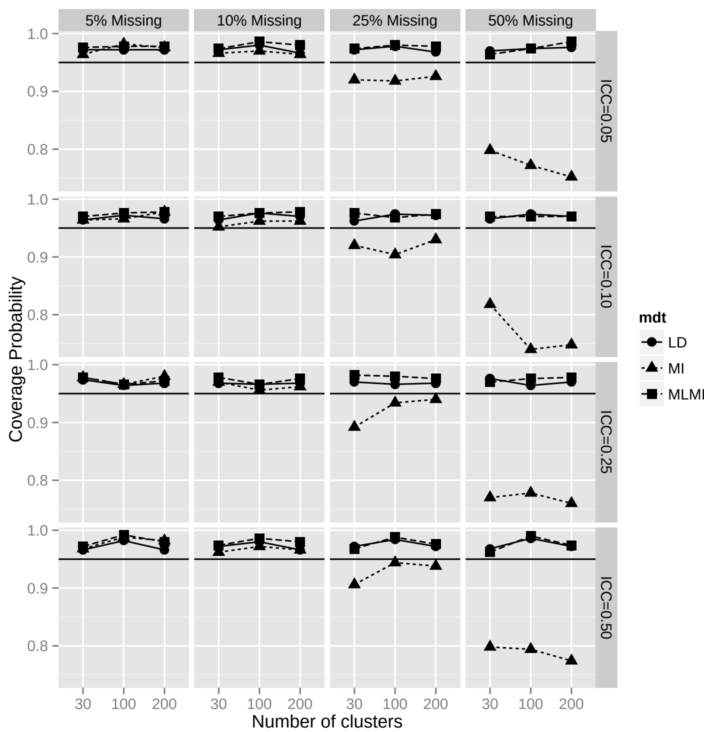
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.51: Percent relative estimate of estimate of γ_{21} under Cauchy distribution of random effects



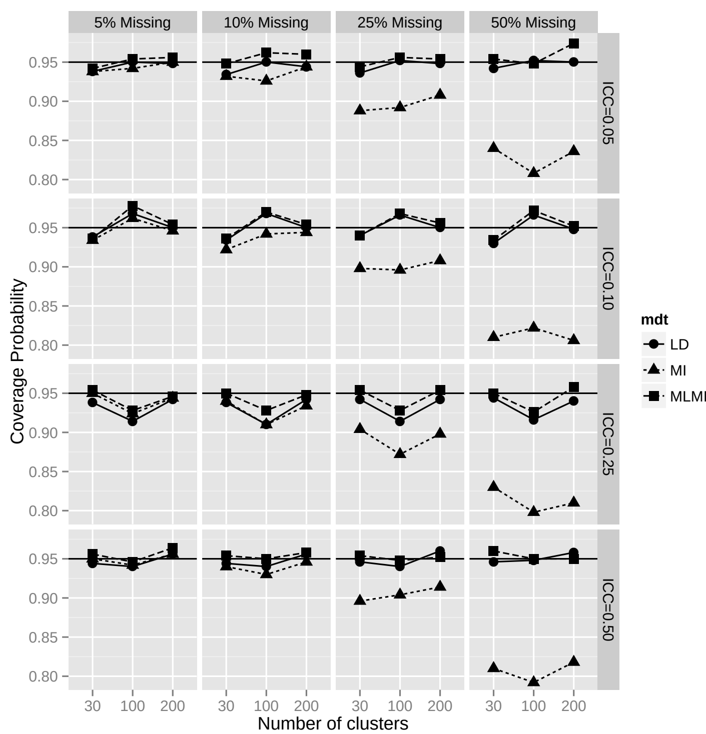
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.52: 95% CI coverage of estimate of γ_{00} under Cauchy distribution of random effects



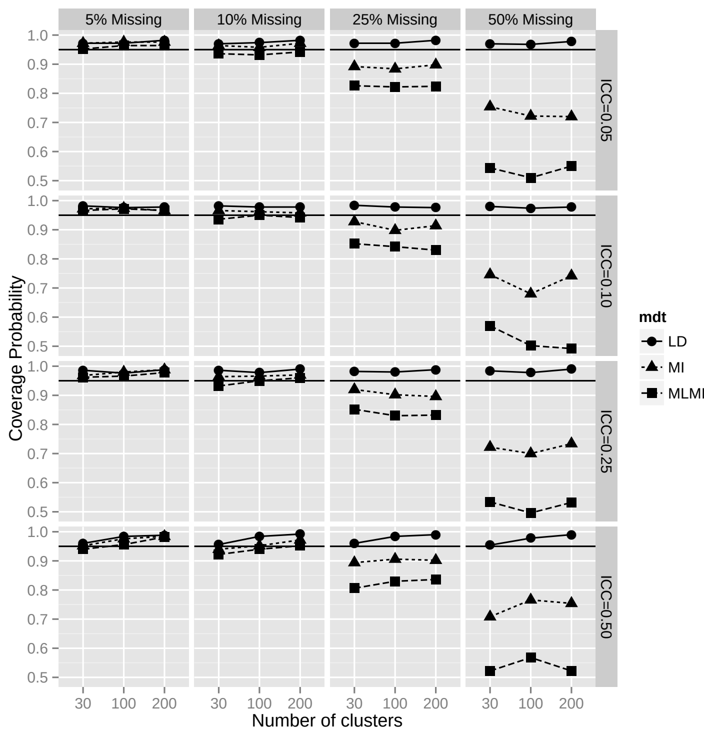
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.53: 95% CI coverage of estimate of γ_{01} under Cauchy distribution of random effects



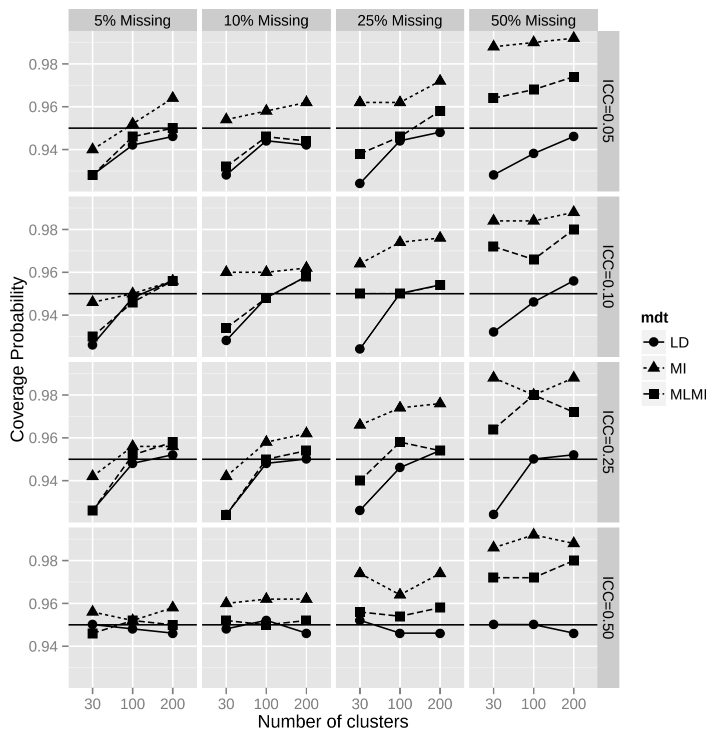
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.54: Percent relative estimate of estimate of γ_{10} under Cauchy distribution of random effects



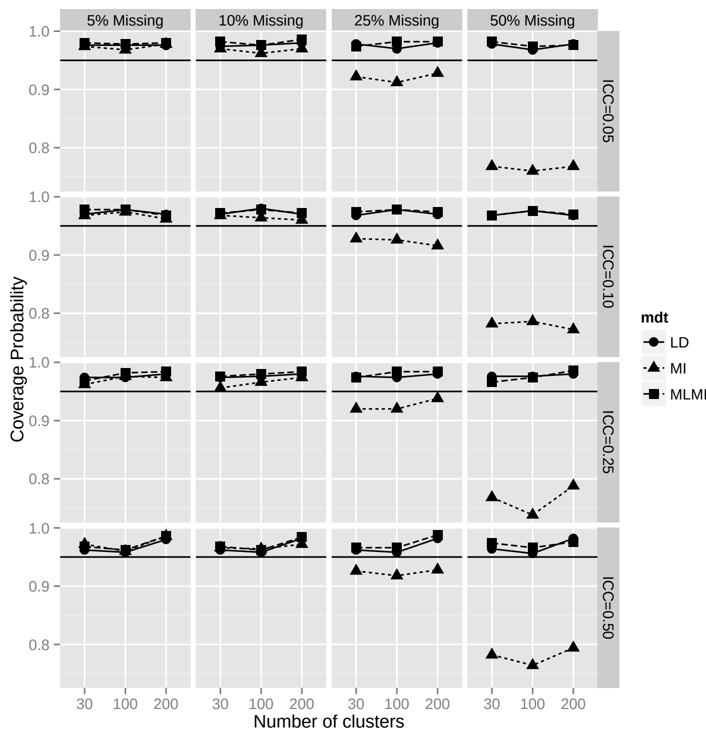
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.55: 95% CI coverage of estimate of γ_{11} under Cauchy distribution of random effects



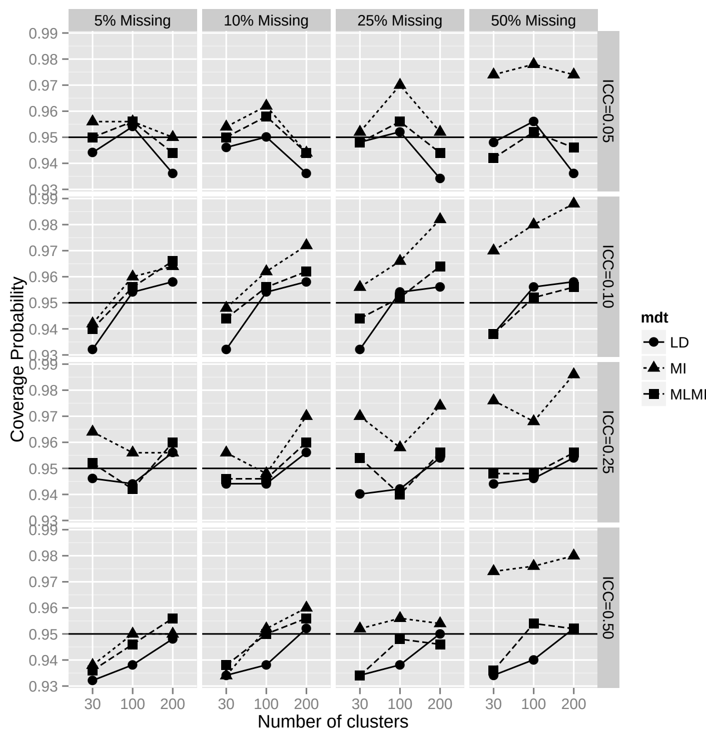
Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.56: 95% CI coverage of estimate of γ_{20} under Cauchy distribution of random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Figure D.57: 95% CI coverage of estimate of γ_{21} under Cauchy distribution of random effects



Note: Levels of missing data are represented across the columns while levels of ICC are represented across the rows

Appendix E

Tables of percent relative estimate under MAR

Table E.1: Percent relative estimate of γ_{00} under MAR

			MDT											
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	132	99	100	100	127	99	99	100	133	99	100	100
		100	138	100	100	100	160	100	100	100	148	100	100	100
		200	55	100	100	100	57	100	100	100	45	100	100	100
	0.1	30	132	99	100	100	144	100	101	100	138	99	100	100
		100	138	100	100	100	157	101	101	100	135	100	100	100
		200	55	100	100	100	52	100	100	100	41	100	100	100
	0.25	30	132	100	100	100	128	99	99	100	140	99	100	100
		100	138	100	100	100	12	99	99	100	61	100	100	100
		200	55	100	100	100	40	100	100	100	19	100	100	100
	0.5	30	132	99	99	100	127	98	99	100	128	99	100	100
		100	138	101	100	100	125	101	100	100	218	101	100	100
		200	55	101	101	100	0	100	100	100	48	100	100	100
0.1	0.05	30	162	99	101	100	163	99	100	100	164	99	101	100
		100	41	100	100	100	51	100	100	100	56	100	100	100
		200	114	100	100	100	158	100	100	100	97	100	100	100
	0.1	30	162	99	101	100	173	100	101	100	158	99	101	100
		100	41	100	100	100	72	100	101	100	44	100	100	100
		200	114	100	100	100	196	100	100	100	192	100	100	100
	0.25	30	162	99	101	100	158	99	100	100	158	99	100	100
		100	41	100	100	100	36	99	100	100	53	99	100	100
		200	114	100	100	100	192	100	100	100	208	100	100	100
	0.5	30	162	100	102	100	136	99	101	100	166	100	101	100
		100	41	100	100	100	118	100	101	100	29	100	101	100
		200	114	100	100	100	− 7	100	100	100	− 15	100	100	100
0.2	0.05	30	76	100	100	100	63	100	100	100	74	100	100	100
		100	100	100	100	100	101	100	100	100	100	100	100	100
		200	150	100	100	100	157	100	100	100	147	100	100	100
	0.1	30	76	100	100	100	104	100	101	100	77	100	100	100
		100	100	100	100	100	104	100	100	100	99	100	100	100
		200	150	100	100	100	160	100	100	100	154	100	100	100
	0.25	30	76	100	100	100	70	100	100	100	113	100	100	100
		100	100	100	100	100	99	100	99	100	111	100	100	100
		200	150	100	100	100	160	100	100	100	168	100	100	100
	0.5	30	76	100	100	100	− 13	100	100	100	81	100	100	100
		100	100	100	100	100	102	100	100	100	109	100	100	100
		200	150	100	100	100	140	100	100	100	134	100	100	100
0.5	0.05	30	75	99	100	100	74	99	100	100	76	99	100	100
		100	165	100	100	100	165	100	100	100	164	100	100	100
		200	59	100	100	100	73	100	100	100	56	100	100	100
	0.1	30	75	99	100	100	86	100	100	100	78	99	100	100
		100	165	100	100	100	170	100	100	100	165	100	100	100
		200	59	100	100	100	76	100	100	100	56	100	100	100
	0.25	30	75	99	100	100	79	99	100	100	78	99	100	100
		100	165	100	100	100	148	100	100	100	167	100	100	100
		200	59	100	100	100	80	100	100	100	73	100	100	100
	0.5	30	75	99	100	100	74	99	100	100	79	99	101	100
		100	165	100	100	100	159	100	100	100	160	100	100	100
		200	59	100	100	100	82	100	100	100	71	100	100	100

Table E.2: Percent relative estimate of γ_{10} under MAR

								MDT						
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	200	97	100	100	215	97	100	100	179	97	99	100
		100	267	98	98	100	361	98	99	100	445	98	98	100
		200	92	100	101	100	87	100	101	100	66	100	101	100
	0.1	30	200	97	100	100	202	97	99	99	216	97	99	99
		100	266	98	98	100	−110	97	97	100	231	98	98	100
		200	92	100	101	100	80	101	102	100	137	100	101	100
	0.25	30	200	98	100	100	242	101	101	100	248	98	99	100
		100	262	99	98	100	−509	96	95	100	−181	99	97	100
		200	93	101	101	100	127	100	100	100	199	100	101	100
	0.5	30	200	98	103	100	130	91	97	99	444	99	104	100
		100	267	99	97	100	−42	100	99	100	−678	100	99	100
		200	92	102	101	100	326	102	101	100	55	100	100	100
0.1	0.05	30	−142	100	98	100	−127	99	98	100	−86	99	98	100
		100	737	98	100	100	708	98	100	100	635	98	100	100
		200	−4639	100	101	100	−4475	100	101	100	−4743	100	101	100
	0.1	30	−138	100	98	100	−95	99	98	100	−261	99	98	100
		100	742	98	100	100	634	97	99	100	726	98	100	100
		200	−4634	100	101	100	−4365	101	101	100	−4391	100	101	100
	0.25	30	−142	98	98	100	−61	99	99	100	32	98	97	100
		100	743	99	100	100	545	97	98	99	658	99	99	100
		200	−4639	101	101	100	−4157	100	99	100	−3756	100	100	100
	0.5	30	−142	98	100	100	−178	94	94	99	−144	98	101	100
		100	739	98	99	100	643	98	99	100	693	99	100	100
		200	−4639	100	101	100	−3476	99	100	100	−3264	99	100	100
0.2	0.05	30	122	103	100	100	74	102	99	100	95	102	100	100
		100	143	100	101	100	148	99	101	100	174	99	101	100
		200	−33	101	98	100	−15	100	98	100	75	100	98	100
	0.1	30	122	103	100	100	17	102	99	100	147	102	99	100
		100	143	100	101	100	134	99	100	100	147	100	101	100
		200	−35	101	98	100	35	100	98	100	64	101	98	100
	0.25	30	122	102	100	100	61	103	99	100	−356	102	100	100
		100	143	100	101	100	124	97	99	99	111	99	101	100
		200	−34	101	98	100	49	99	97	99	119	100	98	100
	0.5	30	122	102	99	100	−114	96	92	99	33	103	100	100
		100	143	100	101	100	131	98	100	100	0	100	102	100
		200	−33	101	98	100	107	98	95	100	−274	100	97	100
0.5	0.05	30	47	105	97	100	41	104	96	100	46	104	97	100
		100	320	101	98	100	317	100	97	100	321	100	98	100
		200	−72	100	99	100	−68	99	98	99	−46	100	99	100
	0.1	30	47	105	97	100	61	104	96	100	40	104	97	100
		100	320	101	98	100	317	99	96	99	322	100	98	100
		200	−73	100	99	100	−32	99	98	99	3	99	99	100
	0.25	30	47	105	97	100	57	104	96	100	43	104	96	100
		100	320	101	98	100	278	97	95	99	428	99	97	99
		200	−72	100	99	100	−44	97	97	99	18	98	99	99
	0.5	30	47	105	97	100	−23	97	91	99	22	104	97	100
		100	319	101	98	100	305	95	93	99	359	100	98	100
		200	−72	101	99	100	0	95	93	99	−462	99	98	100

Table E.3: Percent relative estimate of γ_{20} under MAR

			LD Distribution				MDT MI Distribution				MLMI Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	−7854	65	190	95	−6577	212	322	110	−7863	53	216	93
		100	137859	149	106	98	129866	171	129	99	134565	169	106	99
		200	19159	85	138	102	18357	26	80	93	21772	90	133	103
	0.1	30	−7862	85	194	93	−9552	−66	24	73	−8416	73	180	92
		100	138451	147	108	95	128548	75	15	85	134535	163	118	96
		200	19185	77	145	104	19176	−4	71	91	21259	68	128	101
	0.25	30	−7852	24	218	90	−6916	114	267	109	−9424	37	186	101
		100	138564	167	96	95	152586	315	266	119	139351	157	133	97
		200	26642	65	135	106	21193	51	116	103	25010	49	116	104
	0.5	30	−7848	46	270	114	−5286	225	327	124	−5624	4	195	85
		100	138796	110	129	90	132706	108	136	99	121057	121	133	99
		200	19772	−33	90	100	23779	78	164	103	20049	31	130	104
0.1	0.05	30	3198	160	200	99	3132	242	289	110	3250	166	184	98
		100	−656	59	30	100	−2319	73	50	100	−4198	58	49	101
		200	3023	118	105	97	−6731	79	66	91	4308	124	110	98
	0.1	30	3217	156	196	100	1577	22	53	82	5323	123	142	96
		100	−634	77	37	100	−8097	17	−11	93	−1304	86	44	100
		200	1073	117	111	96	−17718	72	49	88	−18562	105	100	95
	0.25	30	3167	196	218	104	5495	233	251	107	5363	179	235	104
		100	−939	69	35	103	−1797	218	158	118	−3977	120	41	103
		200	3024	117	117	98	−22554	120	106	95	−23153	113	111	97
	0.5	30	3195	41	73	108	10154	218	230	121	3664	125	155	102
		100	−635	76	48	107	−15365	70	8	105	−766	55	18	102
		200	3021	119	106	100	2815	192	132	100	−5237	149	111	101
0.2	0.05	30	−16884	47	227	105	−13159	110	271	110	−16826	42	213	103
		100	333	96	165	102	509	95	189	102	419	96	173	103
		200	−15546	110	135	99	−17349	74	101	95	−14863	110	132	100
	0.1	30	−16878	36	219	107	−21755	−63	97	94	−17842	27	199	104
		100	331	84	165	103	−687	41	142	97	574	88	178	102
		200	−15524	114	135	98	−17538	67	95	93	−16580	112	125	97
	0.25	30	−16888	41	246	104	−13814	83	257	108	−21947	31	246	102
		100	332	79	171	102	1323	178	273	113	−1157	90	185	102
		200	−15528	113	125	98	−16741	99	142	96	−19417	114	128	98
	0.5	30	−16894	48	276	104	4175	62	325	119	−15133	21	256	96
		100	332	90	175	105	−413	71	194	106	−1205	95	192	105
		200	−15570	102	121	96	−14680	109	171	100	−13006	100	156	99
0.5	0.05	30	2413	64	−45	97	2685	103	−4	100	2372	42	−57	96
		100	4932	101	106	99	5018	111	113	98	4894	104	108	100
		200	8983	72	106	99	6390	54	87	96	10587	75	106	99
	0.1	30	2411	59	−47	96	285	−16	−83	86	2202	40	−51	93
		100	4884	105	106	100	3389	74	74	95	4941	107	107	100
		200	8972	68	105	99	6433	40	81	95	10152	66	99	98
	0.25	30	2415	39	−58	96	1895	54	−15	99	2074	32	−64	98
		100	4902	106	109	100	7674	179	182	107	4636	103	120	102
		200	8981	67	100	99	6840	63	101	97	7331	70	102	99
	0.5	30	2424	74	−65	95	3618	126	−34	99	2093	64	−93	84
		100	4881	80	101	98	5641	116	119	97	5362	73	118	97
		200	8976	64	96	99	7665	88	128	103	8343	75	121	102

Table E.4: Percent relative estimate of γ_{11} under MAR

								MDT						
			LD					MI				MLMI		
			Distribution					Distribution				Distribution		
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	7639	287	172	99	6893	267	156	92	7103	263	153	91
		100	40	157	72	105	−1413	143	71	98	−831	141	75	98
		200	−3243	81	131	103	−3210	76	125	96	−3184	79	119	96
	0.1	30	7638	291	169	97	6770	237	142	84	5378	249	152	83
		100	30	158	76	104	1910	127	70	91	−2412	134	65	91
		200	−3248	82	129	102	−1686	70	113	89	−1772	70	105	88
	0.25	30	7640	304	138	92	4828	216	86	60	5868	206	102	66
		100	0	160	70	105	−2075	107	44	71	−707	111	42	69
		200	−3248	73	125	101	−1050	43	82	71	−1797	47	84	69
	0.5	30	7617	274	177	99	3278	132	70	47	2062	107	72	38
		100	39	124	53	109	3941	53	18	46	−1973	44	19	44
		200	−3249	82	102	100	1719	35	31	41	833	34	39	42
0.1	0.05	30	5657	6	239	99	6179	3	210	89	5628	6	221	91
		100	−47983	128	85	95	−44719	121	83	88	−42079	119	79	88
		200	−185659	105	116	98	−169545	100	110	92	−182887	96	110	92
	0.1	30	5584	22	246	97	5379	28	213	85	6803	19	211	84
		100	−48266	129	87	95	−41450	113	72	82	−41103	110	73	82
		200	−184925	106	116	98	−158199	91	99	85	−162362	87	106	85
	0.25	30	5653	22	240	95	4281	5	157	65	3097	15	154	66
		100	−48052	136	94	94	−33102	92	63	64	−36010	89	57	65
		200	−185614	100	125	99	−121086	67	85	69	−113816	70	79	69
	0.5	30	5650	3	245	92	4220	2	109	39	3332	−6	96	38
		100	−47924	120	57	95	−24336	37	28	42	−25586	47	31	39
		200	−185659	94	113	97	−80343	39	50	41	−79418	34	44	41
0.2	0.05	30	272	105	147	87	2646	99	140	82	3231	98	137	82
		100	438	158	77	94	595	153	78	88	505	149	72	88
		200	12377	88	87	100	11186	82	84	94	11316	81	84	94
	0.1	30	269	92	151	84	3260	73	142	75	−620	80	135	73
		100	442	157	78	94	492	140	67	82	809	141	68	82
		200	12368	92	86	99	11342	79	72	88	10774	77	74	87
	0.25	30	264	97	160	83	−276	71	95	59	5010	67	110	60
		100	443	166	65	94	496	115	51	65	110	109	48	65
		200	12405	87	83	100	7589	54	58	70	7188	60	57	70
	0.5	30	276	83	183	84	159	25	54	37	6020	35	57	35
		100	452	162	73	95	−360	67	34	41	−262	62	42	40
		200	12401	89	86	99	5840	34	37	43	6169	35	34	42
0.5	0.05	30	−192	142	235	100	−407	123	216	93	−214	125	218	93
		100	1525	144	79	96	1367	133	72	90	1415	132	72	89
		200	9077	87	135	101	8317	81	124	94	7829	81	121	94
	0.1	30	−191	135	237	100	248	108	200	88	−91	105	204	87
		100	1541	142	79	96	1810	121	65	84	1388	122	66	83
		200	9060	85	135	101	7878	77	110	89	7454	71	110	87
	0.25	30	−192	140	240	101	0	94	130	69	−55	96	140	68
		100	1539	144	83	96	1018	100	56	66	1096	101	58	65
		200	9071	87	135	101	5700	62	88	69	6023	61	88	69
	0.5	30	−198	151	253	103	−354	65	85	45	−35	61	80	41
		100	1514	143	80	97	391	52	38	40	172	62	39	39
		200	9062	82	132	100	3907	33	46	43	3423	30	48	41

Table E.5: Percent relative estimate of γ_{01} under MAR

							MDT							
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	−340	99	106	99	−340	100	106	100	−347	98	106	99
		100	124	98	100	101	−51	98	101	102	186	98	101	101
		200	−13	100	100	100	4	101	101	101	88	101	100	100
	0.1	30	−340	97	106	100	−342	101	107	101	−321	99	106	100
		100	124	97	101	101	−271	98	102	102	50	97	100	101
		200	−13	101	100	100	−42	102	102	102	133	100	100	100
	0.25	30	−340	98	105	100	−276	103	107	103	−350	101	107	100
		100	124	98	101	100	−180	99	105	104	−68	97	100	100
		200	−13	101	99	100	−360	105	104	104	120	101	99	100
	0.5	30	−340	95	102	100	−181	108	112	107	−392	92	99	98
		100	124	99	104	101	−969	102	108	108	−87	97	102	100
		200	−13	104	100	100	−958	108	107	107	−152	101	99	101
0.1	0.05	30	101	102	99	100	80	103	101	101	130	102	100	100
		100	154	99	100	100	145	100	101	101	29	99	100	100
		200	118	100	98	100	312	100	99	101	53	100	98	100
	0.1	30	101	102	100	100	75	105	102	102	141	102	100	100
		100	154	99	100	100	167	101	102	102	110	99	100	100
		200	118	100	98	100	448	101	100	102	1155	99	98	100
	0.25	30	102	102	99	100	99	106	105	104	119	103	100	100
		100	154	99	99	100	155	104	104	104	−13	99	100	100
		200	118	99	98	100	−195	104	102	104	−2	99	98	100
	0.5	30	101	99	109	100	159	110	113	108	126	99	97	99
		100	154	98	100	100	616	107	107	107	427	98	99	100
		200	118	100	97	100	−1868	107	104	107	−2394	100	97	100
0.2	0.05	30	−425	96	100	99	−843	97	101	100	−434	96	100	99
		100	2	103	101	100	2	104	103	101	12	103	101	100
		200	−141	101	100	100	−227	102	101	101	−145	101	100	100
	0.1	30	−425	96	100	99	−1252	98	102	101	−458	96	100	99
		100	2	103	101	100	−18	104	103	102	−2	103	101	100
		200	−141	101	100	100	−178	103	102	102	−175	101	100	100
	0.25	30	−424	95	101	100	−1874	99	103	104	−306	96	101	100
		100	1	102	101	100	4	106	105	104	39	103	101	100
		200	−141	102	101	100	−276	105	104	104	−181	102	100	100
	0.5	30	−424	98	101	100	−3373	104	107	108	−490	95	98	99
		100	2	103	101	100	−2	108	109	108	107	102	101	100
		200	−141	103	100	100	−263	108	106	108	−143	101	101	100
0.5	0.05	30	−158	101	98	100	−133	103	99	101	−162	101	99	100
		100	418	101	96	100	445	101	96	101	409	101	96	100
		200	319	101	102	100	272	102	102	101	325	101	101	100
	0.1	30	−158	102	99	100	−116	104	100	102	−149	102	99	100
		100	418	101	96	100	430	102	97	102	433	101	96	100
		200	319	101	101	100	226	103	103	102	298	101	101	100
	0.25	30	−159	102	99	100	−75	107	102	104	−147	103	99	100
		100	418	101	96	99	466	104	99	104	455	100	95	99
		200	319	101	102	100	188	106	104	105	379	101	101	100
	0.5	30	−159	102	99	100	−2	112	104	108	−210	100	97	100
		100	418	102	96	100	580	106	102	108	494	100	95	99
		200	318	100	101	100	22	109	107	108	256	100	101	100

Table E.6: Percent relative estimate of γ_{21} under MAR

							MDT							
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	1752	99	57	105	1622	94	56	85	2039	115	61	106
		100	−16674	73	116	101	−13152	61	96	81	−17474	72	113	101
		200	−13233	109	94	98	−13692	91	79	78	−15603	108	99	97
	0.1	30	1751	120	57	104	1090	74	42	72	1473	105	62	105
		100	−16963	82	110	101	− 9224	62	80	70	−16680	79	116	102
		200	−13215	103	94	98	−12581	75	64	67	−15467	110	98	99
	0.25	30	1759	114	68	101	280	54	27	48	1464	89	47	102
		100	−16921	67	104	103	−11629	45	48	48	−13634	72	118	103
		200	−12972	97	105	96	− 8419	42	37	43	−14719	97	110	98
	0.5	30	1761	137	107	99	− 39	7	− 5	21	1817	174	132	112
		100	−16907	48	79	101	− 4619	20	36	25	−11016	68	104	103
		200	−13056	69	106	96	− 3894	21	25	21	−12340	97	112	92
0.1	0.05	30	− 1242	113	159	99	− 623	97	128	80	− 1557	115	154	99
		100	747	125	105	99	1255	103	87	79	4169	123	101	98
		200	486	113	103	100	− 1909	94	79	80	729	113	102	101
	0.1	30	− 1243	119	154	99	− 430	66	103	66	− 1967	115	152	98
		100	733	127	101	100	1456	92	71	68	2355	134	106	101
		200	− 4509	111	107	101	− 7137	80	72	67	−16778	116	107	101
	0.25	30	− 1276	122	156	99	− 1081	48	75	44	− 1270	102	144	97
		100	876	127	109	99	2150	57	47	44	4482	129	108	103
		200	490	118	110	102	− 1341	48	43	43	− 9602	118	99	102
	0.5	30	− 1238	135	82	97	− 800	18	22	20	− 705	139	192	105
		100	743	138	105	97	− 450	33	30	23	2612	137	110	101
		200	487	107	115	102	3257	29	29	22	8708	109	117	101
0.2	0.05	30	−47855	76	82	101	−40402	63	73	79	−48711	77	87	101
		100	− 521	93	107	98	− 567	73	85	77	− 593	89	107	99
		200	− 3320	88	97	100	− 1230	72	82	78	− 3012	90	97	100
	0.1	30	−47855	76	88	100	−34474	50	50	65	−48312	83	88	100
		100	− 521	92	108	98	− 210	71	81	65	− 397	93	116	98
		200	− 3359	88	98	99	− 2207	57	68	65	− 2803	86	101	99
	0.25	30	−47863	83	81	98	−26476	39	43	40	−51119	82	77	97
		100	− 519	91	110	98	− 318	41	53	41	− 919	89	112	99
		200	− 3356	86	91	99	− 756	36	39	41	− 2506	83	96	99
	0.5	30	−47867	55	90	95	−11992	2	11	18	−53254	95	120	103
		100	− 524	87	113	96	− 407	25	35	20	− 1357	94	122	98
		200	− 3344	78	88	99	− 1047	21	26	21	− 2274	90	87	99
0.5	0.05	30	2449	126	71	100	1867	99	55	77	2481	123	71	100
		100	1559	83	110	101	1263	70	94	77	1690	82	108	100
		200	− 3885	126	78	100	− 3425	102	65	77	− 4148	124	78	100
	0.1	30	2451	124	68	99	1770	82	41	63	2265	125	67	99
		100	1578	81	107	101	1419	62	76	65	1363	84	110	101
		200	− 3888	124	78	100	− 2808	89	53	64	− 3794	128	80	100
	0.25	30	2453	114	70	100	1202	51	28	39	2245	111	65	97
		100	1592	81	109	101	1110	39	54	40	1155	87	116	102
		200	− 3887	125	76	100	− 2546	57	36	39	− 4614	127	84	100
	0.5	30	2453	122	62	97	720	25	6	17	2567	151	81	100
		100	1576	74	108	101	420	24	32	19	945	91	115	101
		200	− 3880	128	78	100	− 1187	33	18	18	− 3643	131	84	101

Table E.7: Percent relative estimate of τ_{00} under MAR

			MDT											
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	NA	92	100	102	NA	85	92	91	NA	92	101	105
		100	NA	103	98	100	NA	93	90	89	NA	102	98	102
		200	NA	99	99	99	NA	91	91	89	NA	100	99	101
	0.1	30	NA	92	102	101	NA	78	86	81	NA	92	102	110
		100	NA	102	99	100	NA	86	82	79	NA	102	98	105
		200	NA	99	99	100	NA	83	83	79	NA	99	99	103
	0.25	30	NA	95	105	103	NA	63	68	57	NA	94	103	133
		100	NA	103	99	101	NA	66	63	54	NA	102	97	119
		200	NA	100	99	101	NA	63	63	53	NA	99	98	114
	0.5	30	NA	114	117	108	NA	43	46	31	NA	110	114	232
		100	NA	104	99	100	NA	41	38	25	NA	106	99	179
		200	NA	98	98	98	NA	38	37	24	NA	99	98	157
0.1	0.05	30	NA	93	103	102	NA	85	93	91	NA	93	102	104
		100	NA	97	102	99	NA	89	93	88	NA	97	101	101
		200	NA	101	100	100	NA	93	92	89	NA	101	100	101
	0.1	30	NA	93	103	102	NA	79	86	80	NA	93	102	106
		100	NA	97	102	100	NA	82	85	78	NA	97	101	102
		200	NA	101	101	100	NA	85	84	78	NA	101	100	102
	0.25	30	NA	93	103	103	NA	62	66	54	NA	94	102	120
		100	NA	97	102	100	NA	63	65	52	NA	97	101	111
		200	NA	101	101	101	NA	65	64	52	NA	101	100	109
	0.5	30	NA	100	110	115	NA	41	43	26	NA	101	107	183
		100	NA	98	101	105	NA	38	38	23	NA	99	100	149
		200	NA	101	101	106	NA	38	38	22	NA	101	100	137
0.2	0.05	30	NA	97	105	99	NA	90	96	87	NA	97	105	100
		100	NA	98	101	100	NA	90	93	88	NA	98	101	101
		200	NA	101	101	101	NA	93	92	88	NA	101	101	101
	0.1	30	NA	97	105	99	NA	82	88	77	NA	97	105	102
		100	NA	99	101	100	NA	83	85	77	NA	98	101	102
		200	NA	101	101	101	NA	84	84	77	NA	101	100	102
	0.25	30	NA	98	105	100	NA	65	69	51	NA	98	105	109
		100	NA	98	101	100	NA	64	65	50	NA	98	101	106
		200	NA	101	101	101	NA	64	64	50	NA	101	100	105
	0.5	30	NA	100	107	105	NA	42	43	23	NA	102	107	149
		100	NA	98	102	102	NA	38	38	21	NA	99	101	129
		200	NA	101	100	102	NA	37	37	20	NA	101	100	120
0.5	0.05	30	NA	93	102	101	NA	85	93	88	NA	93	102	101
		100	NA	101	101	99	NA	92	92	86	NA	101	100	99
		200	NA	98	99	100	NA	90	91	87	NA	98	99	100
	0.1	30	NA	93	102	101	NA	78	85	77	NA	93	102	102
		100	NA	101	101	99	NA	84	84	75	NA	101	100	99
		200	NA	98	99	100	NA	82	83	75	NA	98	99	100
	0.25	30	NA	93	102	101	NA	61	66	50	NA	93	102	105
		100	NA	101	100	99	NA	64	64	47	NA	100	100	101
		200	NA	98	99	100	NA	63	63	48	NA	98	99	101
	0.5	30	NA	92	101	102	NA	38	40	21	NA	94	101	124
		100	NA	101	101	99	NA	38	37	18	NA	101	100	112
		200	NA	98	99	100	NA	36	36	18	NA	98	99	109

Table E.8: Percent relative estimate of τ_{11} under MAR

			MDT											
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	NA	97	104	103	NA	82	89	90	NA	82	89	89
		100	NA	97	102	101	NA	84	87	88	NA	83	87	88
		200	NA	101	99	100	NA	86	85	87	NA	86	85	87
	0.1	30	NA	97	104	103	NA	72	76	78	NA	71	76	78
		100	NA	98	102	101	NA	72	74	76	NA	72	74	76
		200	NA	100	99	100	NA	74	73	76	NA	74	73	76
	0.25	30	NA	98	104	106	NA	44	45	51	NA	44	46	51
		100	NA	99	103	100	NA	44	45	49	NA	44	45	49
		200	NA	101	99	100	NA	45	44	48	NA	45	44	48
	0.5	30	NA	101	104	106	NA	19	18	26	NA	18	17	24
		100	NA	98	101	98	NA	16	16	19	NA	16	16	19
		200	NA	101	100	99	NA	16	15	20	NA	16	15	18
0.1	0.05	30	NA	99	99	98	NA	86	85	86	NA	85	85	86
		100	NA	102	100	99	NA	88	86	86	NA	88	86	86
		200	NA	101	101	100	NA	87	86	88	NA	86	86	88
	0.1	30	NA	99	99	99	NA	74	72	75	NA	73	72	75
		100	NA	102	100	99	NA	76	74	76	NA	75	73	76
		200	NA	101	101	100	NA	75	74	77	NA	74	74	77
	0.25	30	NA	99	99	99	NA	45	43	49	NA	45	43	49
		100	NA	102	100	99	NA	47	45	49	NA	47	45	49
		200	NA	101	101	101	NA	46	45	50	NA	46	45	50
	0.5	30	NA	99	100	103	NA	17	16	21	NA	16	16	19
		100	NA	101	101	99	NA	17	16	20	NA	17	16	19
		200	NA	101	101	101	NA	17	16	20	NA	17	16	19
0.2	0.05	30	NA	102	97	102	NA	88	83	89	NA	88	82	89
		100	NA	101	101	101	NA	87	86	88	NA	87	86	88
		200	NA	101	100	100	NA	87	86	88	NA	87	85	88
	0.1	30	NA	101	97	102	NA	75	71	78	NA	75	71	78
		100	NA	101	101	101	NA	75	74	77	NA	75	74	77
		200	NA	101	100	100	NA	75	74	77	NA	75	73	77
	0.25	30	NA	101	97	102	NA	47	43	50	NA	47	43	49
		100	NA	101	101	100	NA	47	45	50	NA	46	44	49
		200	NA	101	100	100	NA	47	45	50	NA	46	44	49
	0.5	30	NA	102	96	102	NA	18	15	20	NA	18	15	19
		100	NA	101	101	100	NA	17	16	20	NA	17	16	19
		200	NA	101	101	100	NA	17	16	20	NA	17	16	19
0.5	0.05	30	NA	105	101	98	NA	91	87	85	NA	90	86	84
		100	NA	100	100	101	NA	87	86	88	NA	86	85	87
		200	NA	100	100	101	NA	87	86	88	NA	86	85	87
	0.1	30	NA	105	101	98	NA	78	75	74	NA	77	74	73
		100	NA	100	100	101	NA	75	74	76	NA	74	73	75
		200	NA	100	100	101	NA	75	73	77	NA	74	72	75
	0.25	30	NA	106	101	98	NA	50	45	47	NA	49	45	45
		100	NA	100	101	101	NA	48	46	49	NA	47	45	47
		200	NA	100	100	101	NA	47	45	49	NA	47	44	47
	0.5	30	NA	106	101	98	NA	19	16	18	NA	19	16	16
		100	NA	100	100	101	NA	18	16	19	NA	18	16	17
		200	NA	100	100	101	NA	18	16	19	NA	18	16	17

Table E.9: Percent relative estimate of τ_{22} under MAR

			MDT											
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	NA	95	104	490	NA	67	74	425	NA	95	105	1067
		100	NA	103	100	243	NA	73	71	233	NA	102	100	817
		200	NA	100	99	182	NA	71	70	188	NA	100	98	724
	0.1	30	NA	95	103	577	NA	54	59	495	NA	95	103	1718
		100	NA	102	100	298	NA	59	58	308	NA	102	101	1348
		200	NA	100	99	193	NA	58	57	272	NA	99	98	1150
	0.25	30	NA	99	106	920	NA	43	45	749	NA	101	107	3720
		100	NA	103	101	456	NA	43	42	546	NA	102	100	2843
		200	NA	101	99	296	NA	43	42	528	NA	100	98	2364
	0.5	30	NA	120	123	2425	NA	37	39	838	NA	114	120	8121
		100	NA	104	100	1175	NA	38	37	705	NA	106	102	5694
		200	NA	98	100	826	NA	37	37	680	NA	100	100	4591
0.1	0.05	30	NA	103	98	229	NA	73	70	237	NA	103	98	582
		100	NA	100	100	156	NA	72	71	183	NA	100	100	533
		200	NA	101	100	116	NA	72	72	154	NA	101	100	467
	0.1	30	NA	104	97	278	NA	60	57	324	NA	105	98	968
		100	NA	100	100	183	NA	59	59	268	NA	100	100	854
		200	NA	101	101	124	NA	59	59	244	NA	101	100	742
	0.25	30	NA	103	97	436	NA	44	42	586	NA	104	98	2175
		100	NA	100	100	251	NA	43	43	526	NA	99	100	1792
		200	NA	100	101	167	NA	43	43	508	NA	100	100	1525
	0.5	30	NA	105	104	1007	NA	36	36	715	NA	110	104	5095
		100	NA	101	100	483	NA	37	37	658	NA	102	100	3755
		200	NA	100	102	323	NA	37	37	652	NA	99	100	3093
0.2	0.05	30	NA	98	101	143	NA	70	72	178	NA	98	101	365
		100	NA	100	101	106	NA	72	73	157	NA	100	101	335
		200	NA	100	101	102	NA	71	73	158	NA	100	102	316
	0.1	30	NA	98	101	160	NA	58	59	261	NA	98	101	559
		100	NA	100	101	110	NA	59	60	251	NA	101	101	516
		200	NA	100	101	104	NA	58	60	252	NA	99	101	479
	0.25	30	NA	98	100	228	NA	43	43	529	NA	99	100	1277
		100	NA	100	101	133	NA	44	44	531	NA	100	100	1098
		200	NA	100	101	115	NA	43	44	530	NA	99	101	973
	0.5	30	NA	97	101	432	NA	36	37	623	NA	100	102	3127
		100	NA	100	100	241	NA	37	37	645	NA	100	101	2420
		200	NA	99	101	175	NA	36	37	640	NA	100	100	2032
0.5	0.05	30	NA	98	102	101	NA	69	73	162	NA	98	101	194
		100	NA	100	98	101	NA	72	71	165	NA	100	98	190
		200	NA	98	99	101	NA	71	71	166	NA	98	99	185
	0.1	30	NA	98	102	103	NA	56	59	267	NA	97	102	270
		100	NA	100	98	102	NA	59	58	269	NA	100	99	264
		200	NA	98	99	100	NA	58	58	268	NA	98	99	253
	0.25	30	NA	98	102	114	NA	42	43	554	NA	98	101	568
		100	NA	100	98	101	NA	44	43	552	NA	100	98	515
		200	NA	98	99	101	NA	43	43	556	NA	98	99	484
	0.5	30	NA	98	101	173	NA	34	36	624	NA	98	101	1488
		100	NA	100	98	112	NA	37	36	622	NA	100	98	1239
		200	NA	98	99	100	NA	36	36	627	NA	98	99	1082

Appendix F

Tables of coverage of 95% CI under MAR

Table F.1: Coverage of 95% CI around γ_{00} under MAR

			MDT											
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	0.97	0.93	0.95	0.95	0.97	0.91	0.94	0.94	0.98	0.94	0.95	0.95
		100	0.97	0.93	0.93	0.96	0.98	0.93	0.93	0.95	0.97	0.94	0.93	0.96
		200	0.97	0.94	0.95	0.95	0.97	0.94	0.93	0.94	0.98	0.95	0.95	0.95
	0.1	30	0.97	0.93	0.94	0.95	0.96	0.91	0.92	0.93	0.97	0.93	0.95	0.94
		100	0.98	0.93	0.94	0.95	0.98	0.92	0.92	0.93	0.98	0.93	0.94	0.95
		200	0.96	0.95	0.95	0.94	0.97	0.93	0.93	0.93	0.98	0.94	0.94	0.94
	0.25	30	0.97	0.94	0.96	0.94	0.95	0.91	0.92	0.89	0.97	0.93	0.95	0.96
		100	0.97	0.94	0.95	0.95	0.95	0.89	0.91	0.90	0.98	0.94	0.94	0.97
		200	0.97	0.95	0.95	0.95	0.95	0.94	0.94	0.92	0.98	0.95	0.95	0.96
	0.5	30	0.97	0.95	0.97	0.95	0.85	0.91	0.94	0.90	0.97	0.92	0.95	0.97
		100	0.97	0.95	0.95	0.95	0.85	0.89	0.89	0.85	0.96	0.92	0.93	0.96
		200	0.96	0.95	0.94	0.95	0.86	0.92	0.89	0.86	0.97	0.96	0.92	0.96
0.1	0.05	30	0.96	0.92	0.94	0.92	0.97	0.90	0.93	0.91	0.97	0.91	0.94	0.92
		100	0.97	0.94	0.93	0.94	0.98	0.93	0.93	0.94	0.97	0.94	0.94	0.95
		200	0.97	0.95	0.96	0.94	0.97	0.94	0.96	0.93	0.97	0.95	0.96	0.94
	0.1	30	0.96	0.91	0.93	0.93	0.97	0.91	0.91	0.90	0.97	0.92	0.93	0.93
		100	0.97	0.93	0.94	0.95	0.96	0.92	0.92	0.94	0.97	0.94	0.93	0.95
		200	0.97	0.95	0.96	0.95	0.96	0.93	0.95	0.90	0.97	0.95	0.97	0.95
	0.25	30	0.96	0.92	0.92	0.93	0.95	0.90	0.89	0.90	0.97	0.92	0.93	0.96
		100	0.97	0.94	0.93	0.96	0.93	0.90	0.90	0.89	0.97	0.93	0.94	0.97
		200	0.97	0.94	0.96	0.95	0.93	0.92	0.94	0.90	0.97	0.94	0.96	0.95
	0.5	30	0.96	0.96	0.94	0.95	0.86	0.89	0.87	0.85	0.96	0.94	0.94	0.96
		100	0.97	0.96	0.94	0.95	0.85	0.87	0.86	0.85	0.96	0.92	0.92	0.97
		200	0.96	0.93	0.95	0.97	0.83	0.86	0.87	0.86	0.96	0.93	0.95	0.95
0.2	0.05	30	0.97	0.91	0.95	0.93	0.98	0.91	0.94	0.91	0.98	0.91	0.95	0.93
		100	0.97	0.92	0.92	0.95	0.96	0.90	0.91	0.93	0.96	0.92	0.92	0.95
		200	0.97	0.95	0.95	0.95	0.98	0.94	0.93	0.93	0.97	0.94	0.94	0.95
	0.1	30	0.97	0.90	0.95	0.92	0.97	0.90	0.93	0.89	0.98	0.91	0.95	0.94
		100	0.96	0.92	0.92	0.95	0.96	0.91	0.90	0.92	0.96	0.92	0.92	0.95
		200	0.97	0.94	0.95	0.96	0.98	0.93	0.93	0.92	0.98	0.95	0.94	0.96
	0.25	30	0.97	0.92	0.94	0.93	0.93	0.88	0.89	0.87	0.97	0.92	0.94	0.94
		100	0.96	0.93	0.93	0.95	0.95	0.86	0.87	0.86	0.97	0.93	0.94	0.96
		200	0.97	0.95	0.95	0.96	0.96	0.91	0.91	0.88	0.98	0.96	0.95	0.96
	0.5	30	0.97	0.94	0.95	0.94	0.86	0.88	0.91	0.82	0.98	0.93	0.95	0.96
		100	0.96	0.94	0.94	0.96	0.86	0.84	0.83	0.82	0.96	0.93	0.93	0.96
		200	0.97	0.95	0.94	0.95	0.87	0.86	0.83	0.80	0.97	0.96	0.94	0.96
0.5	0.05	30	0.97	0.88	0.95	0.96	0.97	0.87	0.94	0.95	0.97	0.89	0.96	0.96
		100	0.98	0.94	0.94	0.96	0.99	0.93	0.93	0.94	0.99	0.94	0.94	0.96
		200	0.97	0.95	0.94	0.95	0.98	0.94	0.93	0.92	0.98	0.95	0.94	0.94
	0.1	30	0.97	0.89	0.96	0.96	0.96	0.88	0.93	0.93	0.97	0.89	0.95	0.97
		100	0.98	0.94	0.94	0.95	0.98	0.91	0.92	0.91	0.99	0.94	0.94	0.95
		200	0.97	0.95	0.94	0.95	0.98	0.92	0.93	0.90	0.98	0.95	0.94	0.94
	0.25	30	0.97	0.89	0.95	0.96	0.94	0.86	0.90	0.85	0.97	0.90	0.95	0.97
		100	0.98	0.93	0.94	0.96	0.95	0.88	0.90	0.84	0.98	0.93	0.94	0.96
		200	0.97	0.94	0.94	0.93	0.96	0.91	0.89	0.83	0.98	0.95	0.94	0.94
	0.5	30	0.97	0.89	0.94	0.96	0.86	0.83	0.85	0.78	0.96	0.91	0.94	0.97
		100	0.98	0.94	0.94	0.96	0.88	0.82	0.85	0.74	0.97	0.92	0.94	0.96
		200	0.97	0.95	0.95	0.94	0.87	0.87	0.87	0.76	0.97	0.94	0.96	0.93

Table F.2: Coverage of 95% CI around γ_{10} under MAR

								MDT						
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	0.97	0.93	0.94	0.94	0.97	0.92	0.93	0.92	0.96	0.91	0.93	0.92
		100	0.98	0.94	0.94	0.95	0.98	0.92	0.93	0.94	0.96	0.92	0.93	0.93
		200	0.98	0.95	0.94	0.94	0.98	0.93	0.93	0.94	0.97	0.94	0.93	0.94
	0.1	30	0.97	0.93	0.94	0.93	0.95	0.90	0.92	0.92	0.95	0.91	0.92	0.92
		100	0.98	0.94	0.94	0.95	0.97	0.91	0.91	0.93	0.95	0.91	0.91	0.93
		200	0.98	0.95	0.94	0.94	0.96	0.93	0.91	0.93	0.94	0.93	0.90	0.93
	0.25	30	0.98	0.94	0.93	0.94	0.90	0.91	0.87	0.90	0.87	0.89	0.87	0.89
		100	0.97	0.94	0.95	0.94	0.90	0.87	0.87	0.91	0.84	0.87	0.89	0.92
		200	0.98	0.95	0.94	0.94	0.92	0.91	0.88	0.91	0.86	0.89	0.88	0.90
	0.5	30	0.97	0.94	0.94	0.93	0.76	0.84	0.84	0.88	0.59	0.85	0.84	0.88
		100	0.97	0.93	0.95	0.93	0.77	0.88	0.86	0.91	0.59	0.87	0.84	0.87
		200	0.98	0.95	0.94	0.95	0.74	0.84	0.83	0.88	0.57	0.85	0.85	0.87
0.1	0.05	30	0.98	0.91	0.93	0.93	0.98	0.91	0.93	0.92	0.97	0.90	0.93	0.92
		100	0.98	0.91	0.97	0.94	0.98	0.90	0.95	0.93	0.97	0.90	0.95	0.93
		200	0.97	0.94	0.95	0.95	0.97	0.93	0.93	0.95	0.97	0.93	0.93	0.95
	0.1	30	0.98	0.91	0.93	0.93	0.96	0.90	0.91	0.91	0.96	0.89	0.91	0.91
		100	0.97	0.91	0.97	0.95	0.97	0.87	0.93	0.93	0.96	0.88	0.93	0.92
		200	0.98	0.95	0.95	0.96	0.97	0.92	0.91	0.93	0.95	0.91	0.90	0.94
	0.25	30	0.98	0.92	0.94	0.93	0.92	0.84	0.87	0.88	0.89	0.84	0.86	0.88
		100	0.98	0.92	0.96	0.95	0.92	0.86	0.88	0.88	0.87	0.84	0.88	0.89
		200	0.98	0.94	0.94	0.97	0.92	0.86	0.87	0.92	0.85	0.84	0.85	0.91
	0.5	30	0.98	0.94	0.93	0.94	0.77	0.81	0.80	0.84	0.60	0.81	0.79	0.84
		100	0.98	0.95	0.96	0.95	0.75	0.81	0.85	0.87	0.55	0.79	0.80	0.86
		200	0.98	0.95	0.95	0.96	0.77	0.78	0.77	0.85	0.56	0.80	0.77	0.86
0.2	0.05	30	0.99	0.91	0.94	0.95	0.97	0.90	0.93	0.94	0.96	0.90	0.93	0.94
		100	0.98	0.93	0.95	0.94	0.98	0.92	0.94	0.93	0.97	0.91	0.93	0.93
		200	0.99	0.94	0.94	0.94	0.99	0.93	0.92	0.93	0.98	0.92	0.92	0.92
	0.1	30	0.99	0.92	0.94	0.95	0.96	0.90	0.91	0.91	0.95	0.89	0.91	0.91
		100	0.98	0.94	0.96	0.94	0.97	0.91	0.92	0.92	0.96	0.90	0.91	0.91
		200	0.99	0.94	0.94	0.94	0.97	0.91	0.90	0.91	0.96	0.90	0.90	0.92
	0.25	30	0.99	0.91	0.94	0.94	0.92	0.84	0.84	0.87	0.88	0.83	0.84	0.87
		100	0.98	0.93	0.95	0.94	0.92	0.84	0.87	0.87	0.88	0.85	0.84	0.88
		200	0.99	0.94	0.94	0.94	0.91	0.87	0.84	0.87	0.88	0.86	0.83	0.86
	0.5	30	0.99	0.93	0.94	0.94	0.75	0.76	0.77	0.79	0.60	0.73	0.73	0.81
		100	0.98	0.94	0.95	0.94	0.74	0.78	0.81	0.84	0.57	0.75	0.74	0.80
		200	0.99	0.95	0.95	0.95	0.80	0.77	0.74	0.82	0.61	0.77	0.73	0.80
0.5	0.05	30	0.96	0.92	0.95	0.93	0.96	0.91	0.92	0.93	0.94	0.91	0.93	0.91
		100	0.98	0.96	0.96	0.94	0.98	0.95	0.94	0.93	0.96	0.94	0.94	0.93
		200	0.99	0.93	0.95	0.94	0.98	0.91	0.93	0.91	0.98	0.92	0.93	0.91
	0.1	30	0.96	0.92	0.95	0.94	0.94	0.88	0.92	0.91	0.92	0.88	0.90	0.90
		100	0.98	0.96	0.96	0.94	0.96	0.92	0.94	0.91	0.95	0.92	0.93	0.90
		200	0.99	0.94	0.95	0.94	0.98	0.91	0.91	0.87	0.96	0.91	0.91	0.87
	0.25	30	0.96	0.93	0.94	0.94	0.90	0.84	0.83	0.84	0.86	0.83	0.80	0.82
		100	0.98	0.96	0.95	0.94	0.92	0.83	0.85	0.85	0.85	0.85	0.83	0.84
		200	0.99	0.94	0.95	0.94	0.93	0.85	0.83	0.78	0.86	0.86	0.84	0.78
	0.5	30	0.96	0.92	0.95	0.93	0.74	0.71	0.71	0.73	0.56	0.68	0.65	0.71
		100	0.98	0.95	0.96	0.95	0.80	0.76	0.72	0.75	0.62	0.69	0.66	0.72
		200	0.99	0.95	0.95	0.95	0.74	0.68	0.68	0.69	0.55	0.67	0.65	0.66

Table F.3: Coverage of 95% CI around γ_{20} under MAR

			MDT											
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	0.98	0.93	0.95	0.97	0.95	0.93	0.93	0.97	0.98	0.93	0.95	0.98
		100	0.97	0.95	0.92	0.95	0.94	0.92	0.90	0.95	0.97	0.95	0.93	0.96
		200	0.98	0.95	0.96	0.95	0.95	0.92	0.93	0.93	0.98	0.95	0.95	0.98
	0.1	30	0.98	0.95	0.96	0.97	0.93	0.91	0.90	0.95	0.98	0.96	0.95	0.98
		100	0.98	0.95	0.93	0.94	0.93	0.91	0.88	0.94	0.98	0.94	0.93	0.98
		200	0.98	0.95	0.96	0.95	0.93	0.91	0.91	0.94	0.98	0.96	0.95	0.97
	0.25	30	0.98	0.94	0.95	0.96	0.87	0.91	0.90	0.95	0.98	0.95	0.95	0.98
		100	0.97	0.97	0.93	0.95	0.86	0.89	0.86	0.93	0.98	0.95	0.93	0.99
		200	0.98	0.95	0.95	0.95	0.90	0.92	0.94	0.95	0.98	0.94	0.96	0.97
	0.5	30	0.97	0.96	0.95	0.95	0.84	0.92	0.90	0.94	0.96	0.96	0.94	0.98
		100	0.97	0.95	0.94	0.95	0.82	0.89	0.89	0.89	0.98	0.95	0.93	0.97
		200	0.98	0.96	0.95	0.95	0.83	0.89	0.88	0.92	0.98	0.95	0.92	0.96
0.1	0.05	30	0.97	0.90	0.93	0.96	0.96	0.88	0.89	0.96	0.97	0.91	0.92	0.96
		100	0.98	0.94	0.96	0.92	0.96	0.90	0.92	0.95	0.98	0.94	0.94	0.97
		200	0.97	0.96	0.97	0.96	0.94	0.90	0.94	0.95	0.98	0.95	0.97	0.98
	0.1	30	0.97	0.92	0.94	0.96	0.94	0.85	0.89	0.96	0.97	0.92	0.94	0.98
		100	0.98	0.95	0.94	0.93	0.93	0.91	0.89	0.96	0.98	0.95	0.95	0.97
		200	0.97	0.95	0.97	0.96	0.94	0.90	0.91	0.94	0.98	0.96	0.97	0.97
	0.25	30	0.97	0.92	0.93	0.95	0.87	0.86	0.84	0.95	0.98	0.93	0.93	0.99
		100	0.98	0.95	0.96	0.95	0.87	0.86	0.86	0.94	0.98	0.95	0.95	0.98
		200	0.97	0.96	0.98	0.94	0.89	0.92	0.91	0.97	0.98	0.94	0.96	0.98
	0.5	30	0.97	0.95	0.93	0.97	0.84	0.89	0.87	0.94	0.97	0.94	0.93	0.99
		100	0.98	0.94	0.95	0.95	0.85	0.87	0.85	0.90	0.98	0.94	0.94	0.98
		200	0.97	0.95	0.97	0.96	0.85	0.86	0.87	0.93	0.97	0.95	0.97	0.99
0.2	0.05	30	0.97	0.91	0.94	0.97	0.95	0.87	0.89	0.97	0.97	0.91	0.94	0.98
		100	0.97	0.93	0.92	0.96	0.96	0.90	0.90	0.96	0.98	0.94	0.94	0.99
		200	0.98	0.95	0.95	0.95	0.96	0.90	0.91	0.95	0.98	0.95	0.95	0.97
	0.1	30	0.98	0.91	0.93	0.97	0.93	0.84	0.85	0.97	0.97	0.90	0.93	1.00
		100	0.98	0.94	0.93	0.96	0.94	0.88	0.85	0.96	0.99	0.94	0.94	0.99
		200	0.98	0.94	0.95	0.96	0.94	0.88	0.90	0.95	0.98	0.95	0.95	0.99
	0.25	30	0.97	0.90	0.93	0.98	0.88	0.84	0.84	0.97	0.97	0.92	0.95	1.00
		100	0.98	0.94	0.94	0.96	0.86	0.85	0.83	0.96	0.98	0.94	0.93	0.99
		200	0.98	0.95	0.95	0.95	0.91	0.86	0.88	0.97	0.98	0.95	0.96	0.98
	0.5	30	0.97	0.93	0.94	0.96	0.84	0.86	0.87	0.95	0.96	0.93	0.94	1.00
		100	0.97	0.93	0.95	0.95	0.83	0.85	0.86	0.94	0.98	0.94	0.95	0.98
		200	0.98	0.93	0.96	0.94	0.85	0.85	0.86	0.92	0.97	0.94	0.95	0.98
0.5	0.05	30	0.97	0.88	0.93	0.95	0.96	0.84	0.89	0.98	0.97	0.89	0.93	0.98
		100	0.96	0.92	0.94	0.96	0.97	0.86	0.90	0.98	0.97	0.92	0.94	0.99
		200	0.98	0.92	0.93	0.95	0.96	0.87	0.88	0.95	0.98	0.92	0.94	0.97
	0.1	30	0.96	0.89	0.93	0.95	0.93	0.79	0.86	0.97	0.97	0.89	0.93	0.99
		100	0.96	0.92	0.95	0.97	0.95	0.84	0.88	0.99	0.97	0.92	0.94	0.99
		200	0.98	0.92	0.93	0.94	0.93	0.85	0.85	0.96	0.98	0.92	0.93	0.99
	0.25	30	0.96	0.87	0.93	0.93	0.89	0.77	0.82	0.98	0.97	0.88	0.93	0.99
		100	0.96	0.93	0.95	0.97	0.87	0.80	0.85	0.98	0.97	0.92	0.94	1.00
		200	0.98	0.92	0.93	0.94	0.90	0.84	0.83	0.99	0.98	0.92	0.94	0.99
	0.5	30	0.96	0.88	0.92	0.94	0.85	0.79	0.84	0.97	0.96	0.90	0.92	0.99
		100	0.96	0.92	0.94	0.96	0.84	0.80	0.87	0.95	0.97	0.92	0.94	0.99
		200	0.98	0.92	0.94	0.94	0.87	0.83	0.80	0.96	0.97	0.94	0.94	0.99

Table F.4: Coverage of 95% CI around γ_{11} under MAR

								MDT						
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	0.93	0.94	0.94	0.93	0.94	0.95	0.95	0.94	0.93	0.94	0.95	0.94
		100	0.94	0.97	0.95	0.97	0.96	0.97	0.96	0.97	0.95	0.97	0.96	0.97
		200	0.95	0.95	0.96	0.94	0.96	0.96	0.96	0.95	0.95	0.96	0.96	0.95
	0.1	30	0.93	0.93	0.95	0.93	0.95	0.96	0.96	0.95	0.93	0.96	0.96	0.96
		100	0.95	0.97	0.96	0.96	0.95	0.98	0.97	0.98	0.95	0.98	0.96	0.98
		200	0.94	0.95	0.96	0.94	0.96	0.96	0.97	0.95	0.95	0.96	0.97	0.95
	0.25	30	0.93	0.93	0.93	0.94	0.96	0.98	0.98	0.97	0.93	0.99	0.98	0.97
		100	0.95	0.97	0.94	0.95	0.96	0.99	0.98	0.98	0.95	0.97	0.98	0.99
		200	0.95	0.95	0.96	0.94	0.97	0.97	0.98	0.96	0.96	0.97	0.98	0.95
	0.5	30	0.93	0.95	0.92	0.93	0.98	1.00	1.00	0.99	0.95	1.00	0.99	0.98
		100	0.94	0.95	0.95	0.95	0.97	1.00	0.99	0.99	0.96	1.00	0.99	0.98
		200	0.95	0.95	0.95	0.94	0.98	1.00	0.99	0.94	0.97	1.00	0.99	0.95
0.1	0.05	30	0.93	0.94	0.92	0.94	0.95	0.95	0.92	0.95	0.93	0.95	0.92	0.94
		100	0.95	0.95	0.95	0.96	0.96	0.95	0.96	0.96	0.95	0.95	0.95	0.96
		200	0.95	0.97	0.96	0.95	0.96	0.96	0.96	0.96	0.96	0.97	0.96	0.95
	0.1	30	0.92	0.94	0.92	0.94	0.96	0.95	0.93	0.95	0.93	0.96	0.92	0.95
		100	0.95	0.95	0.95	0.96	0.96	0.95	0.96	0.97	0.95	0.95	0.97	0.96
		200	0.96	0.96	0.95	0.95	0.96	0.96	0.96	0.95	0.96	0.97	0.96	0.96
	0.25	30	0.93	0.94	0.91	0.94	0.97	0.98	0.94	0.97	0.94	0.98	0.94	0.98
		100	0.95	0.95	0.95	0.96	0.97	0.97	0.97	0.96	0.95	0.97	0.97	0.96
		200	0.96	0.96	0.96	0.95	0.98	0.98	0.98	0.95	0.96	0.98	0.98	0.94
	0.5	30	0.93	0.94	0.90	0.94	0.99	0.99	0.99	0.99	0.96	0.99	0.99	0.98
		100	0.95	0.95	0.96	0.96	0.99	1.00	0.99	0.96	0.97	1.00	0.99	0.94
		200	0.95	0.96	0.95	0.95	0.99	0.99	0.99	0.90	0.97	0.99	0.99	0.86
0.2	0.05	30	0.93	0.92	0.93	0.93	0.94	0.93	0.93	0.93	0.93	0.92	0.94	0.93
		100	0.94	0.96	0.94	0.97	0.95	0.96	0.94	0.97	0.95	0.96	0.94	0.96
		200	0.96	0.94	0.95	0.95	0.96	0.95	0.96	0.94	0.96	0.95	0.96	0.95
	0.1	30	0.93	0.92	0.94	0.93	0.95	0.94	0.95	0.94	0.93	0.93	0.94	0.93
		100	0.95	0.95	0.93	0.96	0.96	0.96	0.95	0.96	0.95	0.96	0.95	0.97
		200	0.95	0.94	0.96	0.94	0.96	0.95	0.96	0.95	0.95	0.95	0.96	0.96
	0.25	30	0.93	0.92	0.94	0.93	0.96	0.95	0.96	0.94	0.94	0.95	0.97	0.94
		100	0.95	0.95	0.95	0.96	0.97	0.98	0.97	0.95	0.95	0.97	0.96	0.96
		200	0.95	0.95	0.95	0.94	0.97	0.97	0.97	0.92	0.96	0.96	0.96	0.92
	0.5	30	0.93	0.92	0.95	0.92	0.98	0.98	1.00	0.94	0.96	0.98	0.98	0.94
		100	0.95	0.95	0.94	0.96	0.99	0.99	0.98	0.91	0.97	0.99	0.98	0.88
		200	0.96	0.94	0.96	0.95	0.99	0.99	0.99	0.77	0.98	0.98	0.98	0.73
0.5	0.05	30	0.95	0.94	0.94	0.95	0.96	0.94	0.94	0.95	0.95	0.94	0.94	0.95
		100	0.95	0.93	0.97	0.96	0.95	0.94	0.97	0.96	0.95	0.93	0.97	0.96
		200	0.94	0.95	0.95	0.95	0.95	0.96	0.96	0.96	0.95	0.95	0.96	0.95
	0.1	30	0.95	0.93	0.94	0.94	0.97	0.94	0.95	0.95	0.95	0.93	0.95	0.95
		100	0.95	0.93	0.97	0.95	0.96	0.94	0.96	0.96	0.96	0.93	0.96	0.95
		200	0.95	0.95	0.96	0.95	0.96	0.95	0.95	0.95	0.94	0.95	0.95	0.95
	0.25	30	0.95	0.93	0.94	0.94	0.97	0.94	0.95	0.96	0.96	0.94	0.95	0.95
		100	0.95	0.94	0.97	0.95	0.98	0.93	0.97	0.94	0.96	0.94	0.96	0.93
		200	0.94	0.95	0.95	0.96	0.96	0.96	0.96	0.91	0.95	0.95	0.96	0.90
	0.5	30	0.95	0.93	0.93	0.95	0.99	0.98	0.98	0.94	0.96	0.96	0.97	0.91
		100	0.95	0.94	0.97	0.95	0.99	0.97	0.99	0.81	0.97	0.95	0.99	0.70
		200	0.95	0.93	0.95	0.95	0.98	0.97	0.98	0.62	0.97	0.96	0.98	0.47

Table F.5: Coverage of 95% CI around γ_{01} under MAR

						MDT								
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	0.94	0.93	0.94	0.94	0.94	0.93	0.94	0.92	0.94	0.94	0.95	0.93
		100	0.94	0.96	0.92	0.96	0.93	0.94	0.92	0.94	0.95	0.95	0.92	0.96
		200	0.95	0.95	0.95	0.94	0.96	0.94	0.94	0.93	0.96	0.95	0.95	0.94
	0.1	30	0.94	0.94	0.96	0.94	0.92	0.93	0.94	0.92	0.95	0.94	0.95	0.94
		100	0.95	0.95	0.93	0.95	0.92	0.93	0.92	0.93	0.96	0.95	0.93	0.96
		200	0.94	0.95	0.94	0.95	0.93	0.93	0.93	0.90	0.96	0.96	0.94	0.95
	0.25	30	0.94	0.94	0.94	0.94	0.89	0.91	0.93	0.87	0.94	0.94	0.94	0.95
		100	0.95	0.92	0.93	0.95	0.88	0.90	0.89	0.85	0.96	0.95	0.93	0.97
		200	0.95	0.95	0.94	0.94	0.91	0.89	0.89	0.79	0.96	0.95	0.93	0.94
	0.5	30	0.93	0.94	0.95	0.94	0.78	0.87	0.87	0.84	0.94	0.94	0.94	0.97
		100	0.95	0.94	0.94	0.96	0.76	0.85	0.84	0.72	0.95	0.93	0.94	0.97
		200	0.95	0.94	0.94	0.94	0.80	0.82	0.83	0.57	0.96	0.93	0.93	0.95
0.1	0.05	30	0.93	0.94	0.95	0.95	0.94	0.94	0.93	0.95	0.94	0.94	0.94	0.95
		100	0.96	0.95	0.96	0.95	0.96	0.94	0.95	0.95	0.96	0.94	0.95	0.95
		200	0.95	0.95	0.95	0.94	0.95	0.94	0.94	0.91	0.96	0.95	0.94	0.93
	0.1	30	0.94	0.94	0.93	0.95	0.93	0.92	0.92	0.92	0.94	0.94	0.93	0.95
		100	0.97	0.95	0.96	0.96	0.94	0.93	0.93	0.94	0.97	0.95	0.96	0.96
		200	0.95	0.95	0.95	0.93	0.95	0.92	0.92	0.89	0.96	0.95	0.96	0.94
	0.25	30	0.94	0.94	0.93	0.95	0.89	0.90	0.85	0.86	0.94	0.94	0.94	0.97
		100	0.97	0.94	0.96	0.95	0.88	0.90	0.91	0.80	0.96	0.93	0.95	0.97
		200	0.95	0.94	0.95	0.92	0.91	0.89	0.89	0.74	0.95	0.95	0.96	0.95
	0.5	30	0.93	0.95	0.93	0.96	0.77	0.83	0.79	0.76	0.94	0.94	0.95	0.98
		100	0.97	0.95	0.95	0.95	0.76	0.79	0.83	0.58	0.96	0.95	0.95	0.95
		200	0.95	0.94	0.95	0.94	0.77	0.82	0.79	0.44	0.95	0.94	0.95	0.96
0.2	0.05	30	0.94	0.92	0.95	0.94	0.94	0.91	0.94	0.92	0.95	0.92	0.95	0.94
		100	0.91	0.94	0.94	0.93	0.92	0.93	0.94	0.91	0.92	0.94	0.94	0.94
		200	0.94	0.95	0.96	0.96	0.94	0.95	0.94	0.92	0.95	0.95	0.95	0.96
	0.1	30	0.94	0.93	0.95	0.95	0.94	0.90	0.93	0.91	0.95	0.92	0.96	0.95
		100	0.91	0.94	0.94	0.94	0.91	0.92	0.92	0.89	0.92	0.95	0.94	0.94
		200	0.94	0.95	0.96	0.96	0.93	0.93	0.93	0.88	0.95	0.95	0.95	0.96
	0.25	30	0.94	0.93	0.94	0.95	0.90	0.84	0.88	0.82	0.95	0.94	0.95	0.96
		100	0.91	0.93	0.95	0.95	0.86	0.86	0.88	0.76	0.92	0.93	0.95	0.95
		200	0.94	0.94	0.96	0.95	0.88	0.90	0.89	0.65	0.95	0.95	0.96	0.95
	0.5	30	0.94	0.93	0.94	0.94	0.79	0.76	0.81	0.66	0.95	0.93	0.95	0.96
		100	0.91	0.93	0.94	0.93	0.73	0.76	0.76	0.48	0.93	0.93	0.94	0.96
		200	0.93	0.94	0.94	0.95	0.75	0.76	0.77	0.28	0.95	0.94	0.96	0.97
0.5	0.05	30	0.95	0.95	0.95	0.94	0.96	0.94	0.94	0.93	0.95	0.95	0.95	0.94
		100	0.94	0.95	0.95	0.92	0.94	0.93	0.95	0.89	0.95	0.95	0.95	0.91
		200	0.96	0.96	0.97	0.97	0.96	0.94	0.96	0.92	0.96	0.96	0.97	0.96
	0.1	30	0.95	0.95	0.95	0.94	0.93	0.92	0.92	0.91	0.95	0.95	0.95	0.95
		100	0.94	0.95	0.95	0.91	0.93	0.91	0.93	0.87	0.95	0.95	0.95	0.91
		200	0.96	0.96	0.97	0.97	0.95	0.92	0.95	0.87	0.96	0.96	0.97	0.96
	0.25	30	0.95	0.97	0.95	0.95	0.88	0.88	0.89	0.81	0.96	0.96	0.95	0.96
		100	0.94	0.94	0.96	0.92	0.88	0.87	0.88	0.73	0.95	0.95	0.95	0.92
		200	0.95	0.96	0.97	0.95	0.90	0.85	0.89	0.60	0.95	0.96	0.97	0.95
	0.5	30	0.94	0.94	0.95	0.95	0.75	0.74	0.75	0.59	0.95	0.95	0.93	0.98
		100	0.95	0.93	0.95	0.93	0.76	0.76	0.76	0.36	0.95	0.95	0.94	0.95
		200	0.95	0.95	0.96	0.96	0.75	0.73	0.74	0.18	0.96	0.98	0.97	0.97

Table F.6: Coverage of 95% CI around γ_{21} under MAR

			LD Distribution				MDT MI Distribution				MLMI Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	0.95	0.96	0.92	0.97	0.96	0.98	0.95	0.98	0.95	0.96	0.92	0.98
		100	0.95	0.95	0.95	0.95	0.98	0.96	0.97	0.96	0.96	0.95	0.95	0.97
		200	0.94	0.93	0.96	0.97	0.96	0.95	0.97	0.93	0.95	0.93	0.96	0.98
	0.1	30	0.94	0.96	0.92	0.97	0.97	0.99	0.96	0.99	0.94	0.96	0.93	0.98
		100	0.95	0.95	0.96	0.95	0.98	0.97	0.99	0.95	0.96	0.94	0.95	0.97
		200	0.94	0.94	0.94	0.97	0.96	0.97	0.98	0.92	0.94	0.94	0.96	0.98
	0.25	30	0.95	0.96	0.93	0.97	0.99	1.00	0.99	0.99	0.95	0.95	0.94	0.99
		100	0.95	0.94	0.97	0.93	0.99	0.99	1.00	0.94	0.96	0.95	0.95	0.97
		200	0.94	0.94	0.97	0.96	1.00	0.99	1.00	0.72	0.95	0.93	0.95	0.98
	0.5	30	0.95	0.95	0.93	0.98	1.00	1.00	1.00	1.00	0.94	0.95	0.89	0.98
		100	0.95	0.93	0.94	0.97	1.00	1.00	1.00	0.88	0.97	0.93	0.94	0.98
		200	0.94	0.95	0.96	0.95	1.00	1.00	1.00	0.39	0.95	0.93	0.94	0.97
0.1	0.05	30	0.93	0.95	0.95	0.96	0.96	0.96	0.97	0.98	0.95	0.94	0.95	0.98
		100	0.95	0.95	0.97	0.96	0.97	0.97	0.98	0.94	0.95	0.95	0.97	0.98
		200	0.96	0.95	0.93	0.94	0.98	0.96	0.96	0.88	0.96	0.95	0.93	0.97
	0.1	30	0.93	0.96	0.95	0.96	0.96	0.98	0.97	0.99	0.95	0.94	0.95	0.99
		100	0.95	0.96	0.97	0.96	0.98	0.98	0.98	0.91	0.96	0.96	0.97	0.98
		200	0.96	0.96	0.93	0.95	0.98	0.98	0.97	0.78	0.97	0.96	0.94	0.98
	0.25	30	0.94	0.95	0.94	0.97	0.99	0.99	0.99	0.99	0.94	0.95	0.94	0.98
		100	0.95	0.94	0.97	0.95	0.99	1.00	0.99	0.80	0.96	0.93	0.96	0.98
		200	0.96	0.95	0.94	0.94	1.00	0.99	0.99	0.37	0.97	0.94	0.94	0.98
	0.5	30	0.94	0.95	0.94	0.95	1.00	1.00	1.00	0.98	0.94	0.94	0.91	0.99
		100	0.96	0.94	0.95	0.95	1.00	1.00	1.00	0.56	0.96	0.94	0.96	0.99
		200	0.96	0.95	0.94	0.95	1.00	1.00	1.00	0.07	0.97	0.93	0.93	0.97
0.2	0.05	30	0.95	0.93	0.94	0.96	0.96	0.95	0.96	0.96	0.95	0.92	0.95	0.97
		100	0.94	0.94	0.94	0.95	0.97	0.96	0.95	0.89	0.95	0.95	0.94	0.97
		200	0.95	0.94	0.94	0.95	0.97	0.95	0.95	0.74	0.96	0.95	0.94	0.98
	0.1	30	0.95	0.94	0.94	0.96	0.98	0.96	0.96	0.97	0.95	0.93	0.94	0.99
		100	0.94	0.94	0.95	0.93	0.97	0.97	0.97	0.79	0.95	0.95	0.95	0.99
		200	0.96	0.95	0.94	0.96	0.97	0.97	0.96	0.43	0.96	0.95	0.95	0.99
	0.25	30	0.95	0.92	0.93	0.97	0.99	0.99	0.99	0.95	0.95	0.94	0.94	1.00
		100	0.94	0.95	0.95	0.94	1.00	0.99	0.99	0.49	0.93	0.94	0.95	0.98
		200	0.96	0.94	0.94	0.95	0.99	0.99	0.99	0.05	0.96	0.94	0.95	0.99
	0.5	30	0.95	0.94	0.92	0.96	1.00	1.00	1.00	0.92	0.95	0.94	0.93	0.99
		100	0.94	0.95	0.95	0.96	1.00	1.00	1.00	0.14	0.93	0.97	0.95	0.98
		200	0.95	0.94	0.95	0.95	1.00	1.00	1.00	0.00	0.95	0.93	0.93	0.98
0.5	0.05	30	0.93	0.95	0.93	0.95	0.96	0.96	0.94	0.93	0.93	0.95	0.93	0.98
		100	0.94	0.94	0.95	0.95	0.96	0.95	0.96	0.73	0.95	0.94	0.95	0.97
		200	0.95	0.94	0.96	0.94	0.96	0.96	0.97	0.40	0.95	0.94	0.96	0.97
	0.1	30	0.93	0.95	0.93	0.95	0.97	0.97	0.94	0.91	0.94	0.94	0.93	0.98
		100	0.94	0.94	0.95	0.94	0.98	0.97	0.97	0.51	0.94	0.94	0.96	0.97
		200	0.95	0.95	0.96	0.95	0.97	0.97	0.98	0.09	0.96	0.94	0.96	0.97
	0.25	30	0.93	0.96	0.92	0.95	0.99	0.99	0.99	0.91	0.94	0.95	0.92	0.99
		100	0.94	0.94	0.95	0.93	0.99	0.99	0.99	0.13	0.95	0.94	0.95	0.99
		200	0.95	0.93	0.96	0.95	0.99	1.00	0.99	0.00	0.94	0.94	0.96	0.99
	0.5	30	0.94	0.94	0.92	0.94	1.00	1.00	1.00	0.71	0.93	0.94	0.93	0.98
		100	0.94	0.94	0.96	0.94	1.00	1.00	1.00	0.00	0.94	0.96	0.96	0.99
		200	0.95	0.94	0.95	0.96	1.00	1.00	1.00	0.00	0.94	0.94	0.96	0.99

Appendix G

Tables of percent relative estimate under MCAR

Table G.1: Percent relative estimate of γ_{00} under MCAR

							MDT							
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	132	99	100	100	132	99	99	100	131	99	100	100
		100	138	100	100	100	100	100	100	100	136	100	100	100
		200	55	100	100	100	62	100	100	100	61	100	100	100
	0.1	30	132	99	100	100	134	100	100	100	135	99	100	100
		100	138	100	100	100	175	101	101	100	161	100	100	100
		200	55	100	100	100	68	100	100	100	65	100	100	100
	0.25	30	132	99	100	100	138	98	99	100	150	99	100	100
		100	138	100	100	100	112	99	99	100	162	100	100	100
		200	55	100	100	100	68	100	100	100	48	100	100	100
	0.5	30	131	100	101	100	140	100	100	100	158	100	100	100
		100	138	100	101	100	− 16	101	101	100	22	100	100	100
		200	55	100	100	100	82	100	101	100	57	100	100	100
0.1	0.05	30	162	99	101	100	162	99	101	100	161	99	101	100
		100	41	100	100	100	51	100	100	100	59	100	100	100
		200	114	100	100	100	144	100	100	100	133	100	100	100
	0.1	30	162	99	101	100	165	99	101	100	163	99	101	100
		100	41	100	100	100	43	100	101	100	40	100	100	100
		200	114	100	100	100	201	100	100	100	144	100	100	100
	0.25	30	162	99	101	100	165	98	100	100	169	99	101	100
		100	41	100	100	100	53	99	100	100	90	100	100	100
		200	114	100	100	100	158	100	100	100	0	100	100	100
	0.5	30	162	99	100	100	160	99	101	100	169	99	101	100
		100	41	99	100	100	42	100	101	100	28	99	100	100
		200	114	100	100	100	−108	100	100	100	−167	100	100	100
0.2	0.05	30	76	100	100	100	70	100	100	100	74	100	100	100
		100	100	100	100	100	96	100	100	100	101	100	100	100
		200	150	100	100	100	154	100	100	100	153	100	100	100
	0.1	30	76	100	100	100	92	100	100	100	89	100	100	100
		100	100	100	100	100	100	100	100	100	102	100	100	100
		200	150	100	100	100	161	100	100	100	143	100	100	100
	0.25	30	76	100	100	100	59	99	99	100	46	100	100	100
		100	100	100	100	100	84	100	99	100	102	100	100	100
		200	150	100	100	100	153	100	100	100	152	100	100	100
	0.5	30	76	100	100	100	36	100	100	100	66	100	100	100
		100	100	100	100	100	110	100	100	100	102	100	100	100
		200	150	100	100	100	207	100	100	100	176	100	100	100
0.5	0.05	30	75	99	100	100	73	99	100	100	76	99	100	100
		100	165	100	100	100	159	100	100	100	166	100	100	100
		200	59	100	100	100	71	100	100	100	66	100	100	100
	0.1	30	75	99	100	100	78	99	100	100	77	99	100	100
		100	165	100	100	100	170	100	100	100	167	100	100	100
		200	59	100	100	100	71	100	100	100	63	100	100	100
	0.25	30	75	99	100	100	67	99	100	100	77	99	100	100
		100	165	100	100	100	154	100	100	100	165	100	100	100
		200	59	100	100	100	57	100	100	100	61	100	100	100
	0.5	30	75	99	100	100	75	99	100	100	78	99	100	100
		100	165	100	100	100	162	100	100	100	162	100	100	100
		200	59	100	100	100	76	100	100	100	58	100	100	100

Table G.2: Percent relative estimate of γ_{10} under MCAR

			LD Distribution				MDT MI Distribution				MLMI Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	200	98	100	100	181	98	99	99	207	98	100	99
		100	267	97	98	100	285	97	98	100	287	97	98	100
		200	92	100	101	100	104	100	101	100	88	100	101	100
	0.1	30	200	99	100	100	198	96	99	99	202	99	101	99
		100	267	98	98	100	−309	96	96	100	−296	98	98	100
		200	92	100	101	100	−2	100	100	100	−40	100	101	100
	0.25	30	200	96	101	99	190	98	103	99	27	97	101	99
		100	267	97	98	100	213	94	96	100	137	96	98	100
		200	92	101	101	100	135	98	99	99	112	101	101	100
	0.5	30	199	96	96	99	14	92	90	99	−308	98	96	99
		100	267	97	96	100	11	101	101	101	−617	100	97	100
		200	92	99	100	100	276	100	102	100	−120	97	100	100
0.1	0.05	30	−144	100	98	100	−129	99	97	100	−99	100	98	100
		100	784	98	101	100	752	97	100	100	769	98	100	100
		200	−4639	100	101	100	−4511	100	100	100	−4560	100	101	100
	0.1	30	−143	101	98	100	−183	99	95	100	−132	101	97	100
		100	800	98	101	100	730	96	99	99	679	98	100	100
		200	−4316	101	101	100	−5064	100	100	100	−4999	101	101	100
	0.25	30	−142	100	99	100	−62	100	99	99	−350	100	99	100
		100	739	98	101	100	604	95	98	99	561	97	100	100
		200	−4639	101	101	100	−5359	98	98	99	−5106	100	101	100
	0.5	30	−142	99	101	100	81	96	95	100	−4	100	100	100
		100	739	99	101	100	686	101	104	100	407	100	102	100
		200	−4659	101	101	100	−3427	101	101	100	−3879	99	99	100
0.2	0.05	30	122	103	100	100	65	101	98	100	150	102	99	100
		100	143	99	101	100	147	98	100	100	162	99	101	100
		200	−33	101	98	100	−30	99	97	99	54	100	98	100
	0.1	30	122	103	100	100	59	100	97	99	260	102	99	100
		100	143	100	101	100	129	97	98	99	107	99	100	100
		200	−34	101	98	100	−2	98	96	99	−24	100	98	100
	0.25	30	122	104	100	100	94	101	97	99	−111	103	99	100
		100	142	100	101	100	101	96	97	99	119	99	101	99
		200	−34	101	98	100	34	96	94	99	207	100	98	99
	0.5	30	121	102	100	100	−97	99	93	99	121	102	99	100
		100	143	99	101	100	210	99	102	100	42	99	102	100
		200	−34	100	99	100	33	99	97	99	10	100	97	100
0.5	0.05	30	47	105	97	100	38	102	95	100	42	104	97	100
		100	320	101	98	100	319	99	96	99	317	100	97	100
		200	−72	100	99	100	−40	98	97	99	−60	99	99	100
	0.1	30	47	105	97	100	16	100	93	99	25	102	96	100
		100	320	101	98	100	265	96	94	99	256	99	97	99
		200	−80	100	99	100	−58	96	95	99	38	98	99	99
	0.25	30	47	105	97	100	39	100	93	99	49	103	96	100
		100	320	101	98	100	243	93	91	98	426	97	97	99
		200	−75	100	100	100	−178	93	93	98	313	97	99	99
	0.5	30	46	105	95	100	10	98	90	99	22	103	94	100
		100	319	101	98	100	429	99	96	99	190	100	98	100
		200	−71	100	99	100	130	96	94	99	−353	97	97	99

Table G.3: Percent relative estimate of γ_{20} under MCAR

			LD				MDT MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	− 7864	60	214	98	− 7175	81	234	97	− 7772	71	222	97
		100	151710	151	108	98	139440	152	104	98	139282	153	110	97
		200	19140	72	115	102	19909	79	117	101	19005	81	119	101
	0.1	30	− 7863	71	213	98	− 7941	74	203	99	− 7855	73	204	99
		100	138764	151	91	97	123761	126	59	97	129889	141	73	99
		200	19183	81	134	101	16508	89	131	101	16393	86	127	100
	0.25	30	− 7860	63	209	107	− 8386	61	217	104	− 8536	69	219	99
		100	138715	121	139	96	154613	118	87	95	148882	124	95	96
		200	19183	75	135	100	20436	74	122	99	22879	78	133	101
	0.5	30	− 7867	44	265	96	− 7205	106	284	115	− 8116	95	276	116
		100	138715	162	19	87	107492	166	78	105	97872	108	25	92
		200	19178	68	146	99	25701	116	148	104	22466	72	121	99
0.1	0.05	30	3195	175	178	101	4087	171	177	102	3936	166	172	101
		100	− 591	77	37	99	− 860	68	34	99	− 1012	69	40	99
		200	3027	122	111	98	473	121	112	97	− 2921	121	111	97
	0.1	30	3198	181	197	100	3576	169	174	101	3083	172	188	99
		100	− 2688	82	29	101	− 500	69	22	98	662	78	37	100
		200	2885	127	108	98	2376	126	105	97	− 3286	122	107	97
	0.25	30	3200	129	162	100	− 532	154	142	99	2884	140	148	103
		100	− 644	66	32	100	− 4648	60	19	100	− 3579	76	17	99
		200	3027	119	117	99	− 29741	121	116	96	− 34581	129	122	100
	0.5	30	3224	202	200	93	2840	225	235	116	3907	232	220	111
		100	− 643	13	75	98	− 3459	117	78	112	− 4277	67	24	97
		200	3033	113	108	99	− 8370	157	144	101	6215	122	119	101
0.2	0.05	30	− 16872	35	215	104	− 20302	39	209	104	− 18476	37	204	103
		100	337	88	179	102	456	89	181	102	166	88	180	102
		200	− 15439	115	136	99	− 15249	114	138	99	− 15058	113	138	99
	0.1	30	− 17002	22	232	103	− 14733	27	222	103	− 16266	33	229	104
		100	329	89	182	102	− 701	79	169	100	− 539	94	176	102
		200	− 15544	117	140	99	− 16664	113	140	99	− 17525	113	138	99
	0.25	30	− 16890	3	193	100	− 13261	23	230	100	− 16152	14	217	102
		100	320	90	178	102	1440	84	163	100	254	83	166	103
		200	− 15512	114	142	101	− 14782	114	137	99	− 15080	117	151	101
	0.5	30	− 16873	− 3	243	109	− 15924	74	178	112	− 14400	52	220	111
		100	303	55	161	106	− 445	90	209	111	− 1244	82	161	103
		200	− 15506	103	142	97	− 15152	134	165	101	− 14444	108	144	96
0.5	0.05	30	2408	61	− 53	96	2591	67	− 55	96	2452	56	− 57	96
		100	4915	107	108	99	5465	109	109	100	5114	105	109	100
		200	8988	72	105	99	8039	74	106	99	7593	73	102	99
	0.1	30	2414	53	− 53	97	3028	61	− 53	96	2386	62	− 51	97
		100	4897	109	105	99	4928	102	99	98	5048	107	107	99
		200	8623	72	104	100	9804	74	105	99	9838	75	103	100
	0.25	30	2404	56	− 56	98	1864	79	− 42	93	1960	51	− 54	98
		100	4892	107	105	98	4675	96	92	95	4594	103	106	97
		200	10225	70	106	99	8990	69	99	98	9953	77	107	100
	0.5	30	2408	63	− 61	100	2140	68	− 10	99	2976	76	− 47	104
		100	4880	112	117	99	6040	119	161	101	5206	106	109	95
		200	8973	61	101	101	13910	68	111	102	11012	65	108	103

Table G.4: Percent relative estimate of γ_{11} under MCAR

			LD Distribution				MDT MI Distribution				MLMI Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	7637	294	155	99	7627	261	141	92	7396	265	137	91
		100	43	158	67	108	991	147	67	100	−3321	143	62	99
		200	−3245	84	122	104	−2461	77	113	95	−2991	78	110	95
	0.1	30	7616	292	179	106	6163	238	148	87	6347	237	158	89
		100	36	151	71	107	6439	122	57	90	5947	125	56	90
		200	−3246	93	128	103	−829	82	109	87	−62	75	101	86
	0.25	30	7644	256	144	93	4098	133	79	57	5636	153	51	61
		100	38	137	71	108	−6311	85	39	69	−4084	76	37	66
		200	−3247	97	112	102	−521	58	59	65	−1852	53	62	64
	0.5	30	7640	338	32	87	4514	85	44	35	5191	62	−6	29
		100	43	196	86	110	−3594	54	25	38	−1862	43	22	36
		200	−3241	101	123	102	−3230	31	35	33	−672	19	32	32
0.1	0.05	30	5631	2	240	99	5648	−1	209	89	6215	3	212	89
		100	−49912	126	82	93	−43736	109	69	87	−43541	112	71	86
		200	−185655	111	114	98	−168621	99	104	91	−172248	99	105	91
	0.1	30	5647	0	226	96	5798	4	195	82	5903	9	189	80
		100	−51165	133	90	93	−40948	110	74	78	−42940	105	77	78
		200	−166287	106	119	100	−182474	88	100	84	−179027	87	95	84
	0.25	30	5666	36	248	94	2649	0	136	62	4507	19	138	63
		100	−47969	124	89	96	−31739	73	60	62	−31063	73	53	61
		200	−185656	106	130	98	−136454	54	65	64	−128427	58	71	62
	0.5	30	5678	56	295	90	2119	0	85	37	2596	−6	68	35
		100	−47939	123	98	95	−18127	40	32	35	−17044	33	27	33
		200	−185187	108	130	96	−65661	34	26	35	−73445	19	20	34
0.2	0.05	30	274	100	135	88	211	97	121	83	304	100	128	82
		100	445	156	78	94	186	140	74	88	159	140	71	87
		200	12405	90	88	100	11282	84	82	93	11064	82	83	92
	0.1	30	273	86	168	88	−517	79	136	75	1679	73	139	74
		100	444	148	80	94	686	126	71	80	−69	128	72	80
		200	12405	86	83	101	10347	74	74	86	10162	71	68	85
	0.25	30	287	123	127	92	2379	59	75	59	1986	71	54	57
		100	434	165	70	94	225	89	38	61	43	89	44	59
		200	12400	84	85	97	6641	42	50	65	7752	44	51	64
	0.5	30	248	77	101	87	5585	6	30	30	8546	22	16	29
		100	451	172	98	93	−521	34	29	35	−507	44	22	33
		200	12405	114	77	98	7027	19	13	38	6252	24	18	35
0.5	0.05	30	−191	132	238	100	−341	108	221	93	−87	113	216	91
		100	1546	143	76	96	1532	126	69	89	1337	128	66	88
		200	9069	87	134	101	7846	77	121	93	8353	78	119	92
	0.1	30	−198	144	242	101	−182	101	197	84	−153	112	199	83
		100	1541	144	72	96	661	114	58	81	1310	120	56	80
		200	9097	89	137	101	7250	72	115	86	7177	72	111	84
	0.25	30	−187	133	229	100	−766	61	132	64	−47	62	129	60
		100	1538	145	72	96	1082	82	41	63	1222	85	39	60
		200	9108	94	137	100	6129	54	82	65	5354	54	76	61
	0.5	30	−184	142	238	101	355	27	52	33	−99	36	53	30
		100	1556	154	78	96	994	46	28	34	−245	48	13	29
		200	9080	86	119	101	1373	23	31	36	3366	19	29	32

Table G.5: Percent relative estimate of γ_{01} under MCAR

								MDT						
				LD				MI				MLMI		
				Distribution				Distribution				Distribution		
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	−340	99	104	100	−360	98	104	99	−346	98	105	100
		100	124	98	100	101	163	97	100	101	268	98	101	101
		200	−13	101	100	100	−50	101	100	100	−72	101	100	100
	0.1	30	−340	101	105	100	−290	99	106	100	−361	99	105	100
		100	124	98	100	100	−59	97	100	100	4	97	101	100
		200	−13	101	100	100	−21	101	100	100	−31	101	100	100
	0.25	30	−340	98	104	100	−330	99	107	99	−381	99	106	99
		100	123	98	100	101	−44	98	101	101	243	98	100	101
		200	−13	101	101	100	82	102	101	100	−49	101	100	100
	0.5	30	−340	98	108	100	−467	100	111	99	−467	94	106	99
		100	123	100	101	100	264	98	99	100	712	98	101	100
		200	−13	100	99	100	−97	101	101	100	−56	101	101	100
0.1	0.05	30	102	102	100	100	78	102	99	100	86	102	99	100
		100	153	99	100	100	133	99	100	100	166	99	100	100
		200	118	100	98	100	66	100	98	100	83	100	98	100
	0.1	30	101	102	99	100	66	103	100	100	99	102	99	100
		100	154	99	100	100	209	99	100	100	153	99	100	100
		200	119	100	98	100	495	100	98	100	542	99	98	100
	0.25	30	102	102	99	100	155	102	99	100	118	101	99	100
		100	153	99	100	100	116	100	100	100	239	100	99	100
		200	118	100	98	100	90	100	98	100	−269	100	98	100
	0.5	30	101	101	101	100	243	105	102	100	135	101	99	100
		100	154	99	100	100	442	98	100	100	411	99	99	100
		200	118	99	98	100	−1391	100	99	100	−2026	100	99	100
0.2	0.05	30	−425	96	100	99	−379	96	100	99	−437	96	100	99
		100	2	103	101	100	−14	102	101	100	7	103	101	100
		200	−141	101	100	100	−137	101	100	100	−161	101	100	100
	0.1	30	−425	96	100	99	−377	96	100	99	−251	96	100	99
		100	2	103	101	100	−1	103	101	100	2	103	101	100
		200	−141	101	100	100	−159	102	100	100	−159	101	100	100
	0.25	30	−424	96	100	99	−458	96	99	99	−650	96	100	99
		100	2	103	101	100	0	103	101	100	24	103	101	100
		200	−141	102	101	100	−206	102	101	100	−155	102	100	100
	0.5	30	−424	97	98	99	−263	98	100	100	−661	95	98	99
		100	2	103	102	100	49	102	100	100	64	103	100	100
		200	−141	102	100	100	−340	102	100	100	−278	102	100	100
0.5	0.05	30	−158	102	99	100	−178	101	99	100	−159	102	99	100
		100	418	101	96	100	407	101	96	100	416	101	96	100
		200	319	101	101	100	300	101	101	100	311	101	101	100
	0.1	30	−158	102	99	100	−151	102	99	100	−148	102	99	100
		100	418	101	96	100	417	101	96	100	429	101	96	100
		200	319	101	101	100	308	101	101	100	309	101	101	100
	0.25	30	−158	102	99	100	−112	102	100	100	−163	102	99	100
		100	418	101	95	100	439	101	95	100	417	101	96	99
		200	319	100	102	100	321	101	102	100	283	101	102	100
	0.5	30	−159	102	98	100	−114	104	102	100	−173	103	98	100
		100	417	101	95	100	442	100	94	99	421	100	95	99
		200	320	100	102	100	315	101	102	100	264	101	103	100

Table G.6: Percent relative estimate of γ_{21} under MCAR

			MDT											
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	1753	106	88	103	1832	96	77	96	1997	104	85	103
		100	−36581	79	111	101	−15722	73	106	96	−18452	78	108	101
		200	−13207	106	102	98	−12989	100	98	92	−13475	107	103	98
	0.1	30	1756	96	81	101	1113	84	71	90	1564	94	85	103
		100	−16899	71	107	102	−14969	65	99	90	−13094	72	110	102
		200	−13216	104	99	99	−10000	93	89	87	−10372	106	101	98
	0.25	30	1751	95	90	100	1560	69	55	72	1905	107	82	102
		100	−16919	68	112	100	−13988	55	77	71	−19152	68	107	100
		200	−13222	123	105	99	−12002	85	78	69	−15124	112	102	99
	0.5	30	1751	128	113	112	259	61	33	49	1478	125	60	115
		100	−16915	85	116	102	−3831	43	56	45	−18445	83	114	100
		200	−13215	99	122	95	−7901	46	59	43	−14005	108	112	100
0.1	0.05	30	−1244	113	153	98	−1303	99	136	91	−1227	103	144	98
		100	728	127	102	99	805	120	95	93	392	127	100	99
		200	490	108	105	100	−1338	103	97	93	−2682	109	103	100
	0.1	30	−1242	115	155	100	−1403	102	137	85	−1744	112	158	99
		100	2563	132	106	101	1893	114	96	87	2693	130	106	100
		200	341	112	106	100	604	100	93	87	−4077	110	104	100
	0.25	30	−1244	105	150	99	−808	70	109	67	−949	104	155	99
		100	752	117	114	101	786	96	77	68	2074	124	107	100
		200	488	109	106	100	−18852	76	75	68	−19657	102	101	100
	0.5	30	−1248	85	135	103	437	46	76	41	−328	85	170	107
		100	741	120	94	97	1051	67	52	40	4741	143	124	102
		200	492	93	97	102	−2529	51	48	41	−10604	107	99	103
0.2	0.05	30	−47856	85	91	101	−46411	72	79	93	−48626	83	90	101
		100	−524	98	113	99	−574	92	105	92	−605	97	111	99
		200	−3360	83	97	99	−3741	81	91	92	−3459	85	96	100
	0.1	30	−47607	84	91	100	−43890	69	72	85	−46428	76	93	100
		100	−524	95	111	100	−669	89	102	85	−828	96	112	100
		200	−3356	86	97	99	−2853	77	86	84	−2760	85	96	99
	0.25	30	−47855	76	77	100	−37307	60	65	65	−47290	85	93	101
		100	−524	110	114	101	−104	77	79	66	−640	102	105	100
		200	−3361	92	98	99	−2695	70	69	64	−3806	88	96	99
	0.5	30	−47844	78	101	102	−28520	25	45	36	−57241	73	99	102
		100	−521	102	102	100	−156	45	56	36	−46	96	119	100
		200	−3360	72	94	98	−2782	41	48	36	−4301	87	104	99
0.5	0.05	30	2450	120	66	100	2475	119	66	91	2455	123	66	100
		100	1583	82	108	100	1298	77	102	92	1426	81	108	100
		200	−3887	126	78	100	−3592	120	73	92	−3790	126	79	100
	0.1	30	2449	120	65	100	2470	103	62	84	2546	118	68	100
		100	1577	78	108	100	1462	66	95	84	1737	78	106	100
		200	−3862	127	79	100	−3884	112	71	84	−4247	127	80	100
	0.25	30	2452	120	62	100	2155	95	45	62	2318	127	64	101
		100	1577	82	107	100	845	56	83	62	1334	75	108	100
		200	−4146	124	81	101	−2483	94	55	63	−3603	125	77	101
	0.5	30	2439	114	76	99	677	58	39	30	2431	115	70	98
		100	1582	79	108	101	1153	41	58	32	1859	89	109	101
		200	−3890	125	79	100	−2692	62	37	33	−4135	132	87	102

Table G.7: Percent relative estimate of τ_{00} under MCAR

								MDT						
				LD				MI				MLMI		
				Distribution				Distribution				Distribution		
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	NA	93	100	102	NA	84	91	90	NA	93	101	105
		100	NA	102	98	99	NA	93	89	88	NA	103	98	102
		200	NA	100	100	99	NA	90	90	88	NA	99	100	101
	0.1	30	NA	91	102	101	NA	75	83	79	NA	92	101	108
		100	NA	102	98	99	NA	83	80	77	NA	103	98	104
		200	NA	99	99	100	NA	81	80	77	NA	99	99	103
	0.25	30	NA	94	100	102	NA	54	59	54	NA	93	100	123
		100	NA	102	99	101	NA	58	56	50	NA	102	98	113
		200	NA	100	99	100	NA	56	56	50	NA	100	99	109
	0.5	30	NA	96	102	103	NA	29	32	28	NA	97	103	166
		100	NA	101	96	97	NA	27	26	21	NA	104	99	139
		200	NA	92	93	100	NA	25	25	20	NA	100	100	128
0.1	0.05	30	NA	93	102	102	NA	84	92	89	NA	93	102	104
		100	NA	98	101	99	NA	88	91	87	NA	97	101	101
		200	NA	102	101	100	NA	92	91	87	NA	102	100	101
	0.1	30	NA	93	102	102	NA	75	83	78	NA	93	102	106
		100	NA	97	102	100	NA	79	82	75	NA	97	101	102
		200	NA	101	100	100	NA	82	81	75	NA	101	100	102
	0.25	30	NA	92	102	102	NA	53	59	49	NA	94	103	114
		100	NA	97	101	100	NA	55	57	47	NA	96	101	108
		200	NA	101	101	100	NA	57	56	47	NA	101	101	105
	0.5	30	NA	91	102	102	NA	27	29	21	NA	93	103	141
		100	NA	97	102	100	NA	25	26	18	NA	98	100	123
		200	NA	101	101	100	NA	25	26	17	NA	101	100	117
0.2	0.05	30	NA	97	105	99	NA	88	95	85	NA	97	105	100
		100	NA	98	101	100	NA	89	91	86	NA	98	101	101
		200	NA	101	101	101	NA	91	91	86	NA	101	101	101
	0.1	30	NA	97	105	99	NA	80	85	73	NA	98	105	101
		100	NA	98	101	100	NA	80	82	73	NA	98	101	102
		200	NA	101	101	101	NA	82	81	73	NA	101	100	101
	0.25	30	NA	98	104	98	NA	56	60	44	NA	97	104	105
		100	NA	99	101	100	NA	56	57	43	NA	99	101	104
		200	NA	101	101	101	NA	56	56	43	NA	101	101	103
	0.5	30	NA	98	103	100	NA	28	30	18	NA	99	105	124
		100	NA	98	101	101	NA	25	26	15	NA	99	101	114
		200	NA	100	101	100	NA	25	25	14	NA	101	101	109
0.5	0.05	30	NA	93	102	101	NA	84	92	86	NA	93	102	101
		100	NA	101	100	99	NA	91	90	83	NA	101	100	99
		200	NA	98	99	100	NA	88	89	84	NA	98	99	100
	0.1	30	NA	93	102	101	NA	75	83	72	NA	93	102	102
		100	NA	101	101	99	NA	81	81	70	NA	101	100	99
		200	NA	98	99	100	NA	79	80	70	NA	98	99	100
	0.25	30	NA	93	102	101	NA	52	58	41	NA	93	102	103
		100	NA	101	101	99	NA	56	56	39	NA	101	100	100
		200	NA	98	99	100	NA	54	55	39	NA	98	99	100
	0.5	30	NA	92	102	101	NA	25	28	14	NA	93	102	111
		100	NA	100	101	99	NA	25	25	11	NA	101	100	105
		200	NA	99	99	100	NA	24	24	11	NA	98	99	104

Table G.8: Percent relative estimate of τ_{11} under MCAR

								MDT						
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	NA	96	104	103	NA	80	85	87	NA	79	85	86
		100	NA	97	102	100	NA	80	83	85	NA	80	83	85
		200	NA	100	99	101	NA	83	81	85	NA	82	81	85
	0.1	30	NA	97	104	104	NA	65	69	75	NA	65	69	74
		100	NA	98	101	101	NA	65	67	72	NA	65	67	72
		200	NA	101	99	101	NA	68	66	72	NA	67	65	71
	0.25	30	NA	97	105	105	NA	34	35	44	NA	34	34	43
		100	NA	96	102	100	NA	32	34	41	NA	32	33	40
		200	NA	101	99	101	NA	34	33	40	NA	34	32	40
	0.5	30	NA	103	106	124	NA	12	12	21	NA	11	10	18
		100	NA	95	102	105	NA	9	8	15	NA	8	8	14
		200	NA	96	96	98	NA	8	7	14	NA	8	7	12
0.1	0.05	30	NA	100	99	98	NA	83	81	84	NA	83	81	84
		100	NA	102	100	99	NA	84	82	85	NA	84	82	85
		200	NA	101	101	100	NA	83	83	86	NA	83	83	86
	0.1	30	NA	99	99	99	NA	66	66	72	NA	66	66	72
		100	NA	102	101	99	NA	69	67	72	NA	69	67	72
		200	NA	101	101	100	NA	68	67	73	NA	68	67	73
	0.25	30	NA	99	100	98	NA	35	33	42	NA	34	33	41
		100	NA	102	100	100	NA	36	33	43	NA	36	33	42
		200	NA	100	101	100	NA	35	34	43	NA	35	33	42
	0.5	30	NA	96	100	97	NA	10	9	16	NA	9	8	14
		100	NA	102	98	101	NA	9	7	15	NA	9	7	14
		200	NA	100	100	101	NA	9	7	14	NA	8	7	13
0.2	0.05	30	NA	102	97	102	NA	84	79	87	NA	84	80	87
		100	NA	101	101	101	NA	84	83	86	NA	84	83	85
		200	NA	101	100	100	NA	83	82	86	NA	83	82	86
	0.1	30	NA	101	97	101	NA	69	65	74	NA	69	65	73
		100	NA	101	101	101	NA	69	67	74	NA	69	67	73
		200	NA	101	100	100	NA	69	67	73	NA	69	66	72
	0.25	30	NA	102	98	103	NA	38	32	43	NA	38	32	42
		100	NA	101	100	100	NA	37	34	44	NA	37	33	42
		200	NA	101	100	101	NA	37	34	44	NA	37	33	42
	0.5	30	NA	99	97	103	NA	11	8	16	NA	11	7	13
		100	NA	100	100	102	NA	10	8	15	NA	10	8	13
		200	NA	101	100	100	NA	10	8	15	NA	10	7	13
0.5	0.05	30	NA	105	101	97	NA	87	83	83	NA	87	83	81
		100	NA	100	100	101	NA	84	83	86	NA	83	82	84
		200	NA	100	100	101	NA	83	82	86	NA	83	81	84
	0.1	30	NA	105	101	97	NA	72	68	70	NA	72	67	68
		100	NA	100	100	101	NA	69	67	72	NA	69	66	70
		200	NA	100	100	100	NA	69	67	72	NA	69	66	70
	0.25	30	NA	106	101	97	NA	40	34	40	NA	40	34	36
		100	NA	100	100	101	NA	38	34	42	NA	38	33	38
		200	NA	100	100	101	NA	38	34	42	NA	38	33	38
	0.5	30	NA	104	102	98	NA	13	9	13	NA	13	9	10
		100	NA	101	100	101	NA	12	8	13	NA	13	9	10
		200	NA	100	100	101	NA	12	8	13	NA	13	8	10

Table G.9: Percent relative estimate of τ_{22} under MCAR

								MDT						
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	NA	93	104	417	NA	85	94	370	NA	94	104	551
		100	NA	103	99	200	NA	93	90	178	NA	102	100	326
		200	NA	100	99	160	NA	90	89	143	NA	100	99	289
	0.1	30	NA	95	105	466	NA	77	85	389	NA	95	104	794
		100	NA	103	100	235	NA	83	81	186	NA	102	99	531
		200	NA	100	99	160	NA	81	80	137	NA	100	99	459
	0.25	30	NA	96	105	640	NA	55	59	371	NA	96	105	1716
		100	NA	101	100	289	NA	58	57	177	NA	102	100	1280
		200	NA	100	98	225	NA	57	56	126	NA	99	98	1102
	0.5	30	NA	101	109	1334	NA	25	28	339	NA	98	107	4520
		100	NA	102	100	637	NA	26	26	179	NA	102	99	3349
		200	NA	100	98	454	NA	26	25	124	NA	99	98	2744
0.1	0.05	30	NA	102	98	198	NA	93	88	179	NA	103	98	278
		100	NA	100	100	144	NA	90	90	128	NA	100	100	239
		200	NA	101	100	110	NA	92	91	99	NA	101	100	204
	0.1	30	NA	102	98	204	NA	83	79	174	NA	102	97	393
		100	NA	101	100	152	NA	82	81	120	NA	100	99	368
		200	NA	101	101	114	NA	82	81	91	NA	101	100	323
	0.25	30	NA	103	99	298	NA	58	55	178	NA	103	98	992
		100	NA	100	100	177	NA	57	57	105	NA	100	100	855
		200	NA	101	101	124	NA	57	57	73	NA	101	101	755
	0.5	30	NA	102	98	649	NA	27	25	169	NA	103	99	2850
		100	NA	100	101	364	NA	26	25	92	NA	101	100	2246
		200	NA	101	100	234	NA	26	26	61	NA	101	100	1885
0.2	0.05	30	NA	98	101	129	NA	88	91	116	NA	98	101	186
		100	NA	101	101	104	NA	91	91	90	NA	100	101	166
		200	NA	100	102	100	NA	90	91	86	NA	100	102	161
	0.1	30	NA	98	101	139	NA	79	82	110	NA	98	101	262
		100	NA	101	101	107	NA	82	82	84	NA	101	101	246
		200	NA	100	102	99	NA	81	82	74	NA	100	101	229
	0.25	30	NA	97	102	174	NA	55	57	105	NA	98	101	621
		100	NA	101	101	113	NA	57	57	63	NA	101	101	563
		200	NA	100	102	105	NA	56	57	54	NA	99	101	507
	0.5	30	NA	96	102	349	NA	25	25	94	NA	99	102	1897
		100	NA	100	102	187	NA	25	25	49	NA	101	101	1554
		200	NA	100	101	141	NA	26	26	35	NA	100	102	1319
0.5	0.05	30	NA	98	102	104	NA	88	91	90	NA	98	101	128
		100	NA	100	98	100	NA	90	89	86	NA	100	98	123
		200	NA	98	99	102	NA	89	89	86	NA	98	99	124
	0.1	30	NA	98	101	105	NA	79	82	77	NA	98	101	159
		100	NA	100	98	101	NA	81	79	72	NA	100	98	152
		200	NA	98	99	101	NA	79	80	72	NA	98	99	150
	0.25	30	NA	98	101	107	NA	56	56	56	NA	98	101	314
		100	NA	100	98	100	NA	56	55	42	NA	100	98	291
		200	NA	98	99	103	NA	55	55	41	NA	98	99	279
	0.5	30	NA	98	102	137	NA	23	25	44	NA	98	101	959
		100	NA	100	99	106	NA	24	24	24	NA	100	99	844
		200	NA	98	99	105	NA	24	25	20	NA	99	99	762

Appendix H

Tables of coverage of 95% CI under MCAR

Table H.1: Coverage of 95% CI around γ_{00} under MCAR

			MDT											
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	0.97	0.93	0.94	0.94	0.96	0.92	0.93	0.94	0.98	0.93	0.93	0.95
		100	0.97	0.94	0.94	0.95	0.98	0.93	0.93	0.94	0.98	0.94	0.93	0.95
		200	0.97	0.96	0.95	0.95	0.98	0.95	0.94	0.94	0.98	0.95	0.95	0.95
	0.1	30	0.97	0.92	0.95	0.93	0.97	0.91	0.91	0.92	0.97	0.93	0.94	0.95
		100	0.98	0.94	0.93	0.94	0.97	0.92	0.92	0.93	0.99	0.94	0.95	0.95
		200	0.97	0.95	0.95	0.95	0.96	0.93	0.93	0.94	0.98	0.95	0.95	0.95
	0.25	30	0.97	0.94	0.95	0.95	0.92	0.90	0.89	0.90	0.97	0.95	0.94	0.97
		100	0.98	0.93	0.94	0.96	0.92	0.89	0.90	0.91	0.98	0.93	0.94	0.97
		200	0.97	0.94	0.95	0.95	0.93	0.92	0.91	0.87	0.98	0.95	0.96	0.95
	0.5	30	0.97	0.95	0.95	0.96	0.80	0.88	0.88	0.88	0.96	0.93	0.94	0.98
		100	0.97	0.95	0.94	0.95	0.77	0.85	0.83	0.84	0.97	0.93	0.93	0.96
		200	0.98	0.96	0.95	0.92	0.75	0.87	0.85	0.86	0.99	0.94	0.95	0.96
0.1	0.05	30	0.96	0.91	0.92	0.93	0.96	0.90	0.92	0.92	0.97	0.91	0.93	0.93
		100	0.97	0.94	0.93	0.95	0.97	0.93	0.92	0.94	0.98	0.94	0.93	0.95
		200	0.97	0.95	0.97	0.94	0.98	0.94	0.96	0.92	0.98	0.94	0.96	0.94
	0.1	30	0.96	0.93	0.93	0.94	0.95	0.90	0.91	0.91	0.97	0.92	0.94	0.94
		100	0.98	0.94	0.93	0.95	0.96	0.91	0.90	0.91	0.98	0.95	0.94	0.95
		200	0.97	0.95	0.96	0.94	0.96	0.92	0.95	0.90	0.98	0.94	0.97	0.95
	0.25	30	0.96	0.92	0.93	0.93	0.92	0.85	0.87	0.88	0.98	0.93	0.94	0.93
		100	0.97	0.95	0.94	0.95	0.90	0.88	0.87	0.87	0.97	0.95	0.94	0.97
		200	0.97	0.94	0.97	0.93	0.93	0.90	0.93	0.84	0.97	0.95	0.96	0.95
	0.5	30	0.97	0.94	0.91	0.94	0.82	0.84	0.81	0.80	0.97	0.93	0.92	0.96
		100	0.97	0.95	0.95	0.94	0.74	0.80	0.81	0.80	0.97	0.96	0.94	0.96
		200	0.97	0.96	0.95	0.94	0.75	0.82	0.87	0.80	0.97	0.95	0.96	0.95
0.2	0.05	30	0.97	0.91	0.95	0.93	0.98	0.90	0.94	0.90	0.98	0.91	0.95	0.93
		100	0.96	0.91	0.92	0.95	0.97	0.90	0.91	0.93	0.97	0.92	0.92	0.96
		200	0.97	0.94	0.95	0.95	0.98	0.93	0.93	0.94	0.97	0.94	0.95	0.95
	0.1	30	0.97	0.91	0.95	0.91	0.97	0.90	0.92	0.88	0.98	0.91	0.95	0.92
		100	0.97	0.93	0.93	0.95	0.96	0.90	0.90	0.91	0.97	0.92	0.93	0.95
		200	0.97	0.95	0.95	0.96	0.96	0.93	0.92	0.92	0.98	0.94	0.94	0.96
	0.25	30	0.97	0.91	0.94	0.94	0.89	0.87	0.88	0.82	0.98	0.92	0.95	0.94
		100	0.97	0.92	0.93	0.95	0.93	0.83	0.88	0.84	0.98	0.91	0.93	0.96
		200	0.97	0.95	0.94	0.96	0.94	0.89	0.86	0.84	0.98	0.95	0.96	0.96
	0.5	30	0.98	0.92	0.95	0.91	0.77	0.83	0.84	0.77	0.97	0.93	0.96	0.95
		100	0.96	0.93	0.95	0.95	0.78	0.75	0.75	0.73	0.98	0.92	0.93	0.95
		200	0.97	0.94	0.96	0.96	0.76	0.78	0.74	0.72	0.98	0.95	0.95	0.96
0.5	0.05	30	0.97	0.89	0.95	0.96	0.97	0.87	0.94	0.94	0.97	0.89	0.95	0.96
		100	0.98	0.94	0.94	0.95	0.99	0.92	0.94	0.93	0.99	0.94	0.94	0.95
		200	0.97	0.95	0.94	0.94	0.98	0.94	0.93	0.91	0.98	0.95	0.94	0.94
	0.1	30	0.97	0.88	0.95	0.96	0.96	0.87	0.92	0.92	0.97	0.89	0.95	0.97
		100	0.98	0.94	0.94	0.95	0.97	0.90	0.92	0.90	0.99	0.94	0.94	0.95
		200	0.97	0.95	0.94	0.94	0.97	0.92	0.92	0.88	0.98	0.95	0.94	0.94
	0.25	30	0.97	0.88	0.95	0.96	0.91	0.81	0.87	0.80	0.97	0.88	0.95	0.96
		100	0.98	0.94	0.94	0.95	0.94	0.84	0.87	0.80	0.99	0.94	0.94	0.96
		200	0.97	0.94	0.94	0.94	0.94	0.87	0.85	0.76	0.98	0.95	0.94	0.94
	0.5	30	0.97	0.90	0.96	0.96	0.80	0.73	0.77	0.64	0.96	0.89	0.95	0.97
		100	0.99	0.94	0.95	0.95	0.79	0.74	0.75	0.65	0.99	0.94	0.95	0.97
		200	0.97	0.93	0.94	0.95	0.77	0.75	0.76	0.65	0.97	0.95	0.94	0.95

Table H.2: Coverage of 95% CI around γ_{10} under MCAR

			MDT											
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	0.97	0.91	0.95	0.93	0.97	0.91	0.93	0.93	0.95	0.90	0.93	0.92
		100	0.97	0.94	0.94	0.94	0.98	0.93	0.92	0.94	0.96	0.93	0.92	0.93
		200	0.98	0.95	0.94	0.94	0.98	0.93	0.92	0.93	0.96	0.94	0.91	0.93
	0.1	30	0.97	0.93	0.94	0.94	0.96	0.90	0.91	0.94	0.94	0.91	0.92	0.94
		100	0.97	0.93	0.94	0.95	0.96	0.90	0.92	0.92	0.93	0.91	0.92	0.93
		200	0.98	0.94	0.94	0.94	0.97	0.92	0.91	0.93	0.94	0.92	0.90	0.93
	0.25	30	0.97	0.94	0.94	0.93	0.89	0.88	0.87	0.90	0.83	0.88	0.87	0.90
		100	0.97	0.93	0.95	0.95	0.88	0.86	0.87	0.92	0.82	0.86	0.85	0.92
		200	0.98	0.95	0.95	0.93	0.90	0.89	0.87	0.88	0.82	0.89	0.88	0.89
	0.5	30	0.97	0.93	0.93	0.93	0.75	0.86	0.84	0.88	0.54	0.84	0.84	0.89
		100	0.97	0.94	0.95	0.94	0.72	0.85	0.85	0.91	0.51	0.85	0.86	0.88
		200	0.98	0.95	0.94	0.95	0.72	0.86	0.84	0.87	0.55	0.85	0.84	0.88
0.1	0.05	30	0.98	0.93	0.94	0.94	0.97	0.91	0.92	0.92	0.97	0.91	0.92	0.93
		100	0.98	0.92	0.96	0.95	0.97	0.89	0.95	0.93	0.97	0.89	0.94	0.93
		200	0.98	0.94	0.94	0.96	0.97	0.92	0.91	0.94	0.97	0.92	0.91	0.94
	0.1	30	0.98	0.92	0.94	0.92	0.97	0.90	0.92	0.90	0.94	0.89	0.91	0.91
		100	0.98	0.92	0.96	0.93	0.96	0.88	0.94	0.90	0.95	0.88	0.92	0.91
		200	0.98	0.95	0.95	0.95	0.96	0.91	0.91	0.93	0.94	0.90	0.90	0.93
	0.25	30	0.98	0.93	0.95	0.94	0.93	0.87	0.85	0.87	0.85	0.87	0.87	0.87
		100	0.98	0.92	0.95	0.95	0.90	0.81	0.87	0.87	0.84	0.83	0.85	0.87
		200	0.98	0.97	0.96	0.96	0.91	0.87	0.86	0.87	0.83	0.86	0.83	0.89
	0.5	30	0.98	0.92	0.95	0.93	0.75	0.84	0.81	0.85	0.57	0.82	0.79	0.89
		100	0.97	0.93	0.95	0.94	0.68	0.77	0.78	0.88	0.50	0.75	0.82	0.86
		200	0.98	0.94	0.94	0.94	0.74	0.82	0.78	0.84	0.49	0.79	0.79	0.84
0.2	0.05	30	0.99	0.91	0.94	0.95	0.97	0.90	0.92	0.93	0.96	0.90	0.92	0.94
		100	0.98	0.93	0.95	0.94	0.98	0.91	0.94	0.92	0.97	0.92	0.93	0.92
		200	0.99	0.95	0.93	0.94	0.99	0.92	0.91	0.93	0.98	0.92	0.90	0.93
	0.1	30	0.99	0.91	0.94	0.95	0.96	0.88	0.90	0.92	0.93	0.88	0.89	0.93
		100	0.98	0.93	0.95	0.94	0.97	0.88	0.90	0.90	0.95	0.89	0.90	0.90
		200	0.99	0.94	0.94	0.94	0.97	0.91	0.89	0.89	0.96	0.91	0.89	0.90
	0.25	30	0.98	0.93	0.94	0.94	0.92	0.81	0.82	0.86	0.85	0.80	0.82	0.86
		100	0.98	0.93	0.94	0.94	0.90	0.82	0.82	0.83	0.83	0.81	0.80	0.84
		200	0.99	0.95	0.95	0.94	0.90	0.81	0.82	0.80	0.83	0.83	0.80	0.85
	0.5	30	0.98	0.93	0.93	0.94	0.72	0.76	0.75	0.80	0.53	0.74	0.73	0.80
		100	0.98	0.92	0.93	0.94	0.70	0.74	0.74	0.83	0.50	0.73	0.72	0.80
		200	0.99	0.95	0.94	0.94	0.73	0.76	0.76	0.79	0.53	0.73	0.72	0.77
0.5	0.05	30	0.96	0.92	0.95	0.93	0.95	0.90	0.92	0.92	0.94	0.91	0.92	0.91
		100	0.98	0.95	0.96	0.94	0.98	0.93	0.94	0.92	0.96	0.94	0.94	0.92
		200	0.99	0.93	0.95	0.94	0.98	0.91	0.93	0.87	0.98	0.91	0.92	0.90
	0.1	30	0.96	0.91	0.95	0.93	0.94	0.88	0.90	0.90	0.92	0.89	0.89	0.89
		100	0.98	0.95	0.96	0.94	0.95	0.90	0.92	0.87	0.94	0.91	0.92	0.89
		200	0.99	0.94	0.95	0.95	0.97	0.88	0.90	0.79	0.95	0.90	0.89	0.84
	0.25	30	0.96	0.92	0.95	0.95	0.89	0.81	0.80	0.80	0.81	0.80	0.75	0.80
		100	0.98	0.94	0.95	0.94	0.91	0.79	0.80	0.75	0.83	0.81	0.79	0.78
		200	0.99	0.94	0.96	0.94	0.90	0.78	0.78	0.62	0.84	0.81	0.77	0.71
	0.5	30	0.95	0.95	0.94	0.95	0.71	0.72	0.69	0.74	0.52	0.70	0.63	0.69
		100	0.98	0.95	0.97	0.94	0.77	0.72	0.71	0.74	0.57	0.71	0.65	0.68
		200	0.99	0.93	0.96	0.94	0.75	0.72	0.69	0.65	0.52	0.68	0.61	0.65

Table H.3: Coverage of 95% CI around γ_{20} under MCAR

			MDT											
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	0.98	0.93	0.94	0.97	0.97	0.92	0.93	0.96	0.98	0.93	0.93	0.97
		100	0.98	0.95	0.93	0.96	0.97	0.95	0.91	0.96	0.98	0.96	0.92	0.96
		200	0.98	0.95	0.95	0.95	0.98	0.95	0.94	0.95	0.98	0.96	0.95	0.95
	0.1	30	0.97	0.94	0.95	0.96	0.97	0.92	0.92	0.98	0.98	0.94	0.94	0.98
		100	0.98	0.95	0.93	0.95	0.96	0.93	0.91	0.96	0.98	0.95	0.93	0.97
		200	0.98	0.96	0.95	0.96	0.97	0.94	0.93	0.96	0.99	0.96	0.95	0.97
	0.25	30	0.98	0.95	0.93	0.96	0.92	0.91	0.91	0.96	0.97	0.94	0.94	0.98
		100	0.97	0.95	0.93	0.96	0.91	0.91	0.88	0.95	0.98	0.96	0.93	0.98
		200	0.98	0.93	0.95	0.94	0.93	0.89	0.92	0.95	0.98	0.95	0.95	0.97
	0.5	30	0.98	0.93	0.95	0.96	0.77	0.88	0.90	0.94	0.98	0.96	0.95	0.99
		100	0.97	0.94	0.94	0.96	0.76	0.88	0.83	0.92	0.97	0.95	0.95	0.98
		200	0.98	0.94	0.95	0.95	0.77	0.89	0.88	0.94	0.98	0.94	0.95	0.98
0.1	0.05	30	0.97	0.91	0.93	0.95	0.97	0.90	0.92	0.94	0.98	0.91	0.92	0.96
		100	0.98	0.94	0.95	0.95	0.97	0.92	0.93	0.95	0.98	0.95	0.94	0.96
		200	0.97	0.95	0.97	0.95	0.96	0.94	0.96	0.94	0.97	0.95	0.97	0.96
	0.1	30	0.97	0.91	0.94	0.95	0.97	0.88	0.91	0.94	0.97	0.91	0.93	0.96
		100	0.98	0.94	0.94	0.96	0.96	0.92	0.92	0.95	0.98	0.94	0.94	0.97
		200	0.97	0.94	0.98	0.96	0.96	0.93	0.95	0.95	0.97	0.95	0.97	0.97
	0.25	30	0.97	0.90	0.94	0.93	0.93	0.86	0.89	0.92	0.97	0.91	0.94	0.98
		100	0.98	0.95	0.95	0.97	0.93	0.88	0.88	0.94	0.98	0.95	0.94	0.98
		200	0.97	0.95	0.97	0.95	0.92	0.90	0.92	0.95	0.97	0.96	0.97	0.98
	0.5	30	0.97	0.91	0.95	0.97	0.78	0.81	0.82	0.94	0.97	0.93	0.94	0.99
		100	0.98	0.96	0.94	0.96	0.79	0.84	0.80	0.93	0.98	0.95	0.94	0.99
		200	0.97	0.95	0.96	0.96	0.77	0.84	0.83	0.93	0.97	0.95	0.96	0.97
0.2	0.05	30	0.97	0.91	0.93	0.95	0.96	0.90	0.92	0.96	0.97	0.91	0.93	0.97
		100	0.97	0.94	0.93	0.97	0.98	0.93	0.92	0.96	0.98	0.94	0.93	0.98
		200	0.98	0.95	0.95	0.97	0.97	0.94	0.95	0.97	0.98	0.95	0.95	0.97
	0.1	30	0.97	0.91	0.93	0.95	0.96	0.89	0.92	0.95	0.98	0.91	0.94	0.97
		100	0.98	0.94	0.92	0.96	0.97	0.92	0.91	0.95	0.98	0.94	0.93	0.97
		200	0.98	0.94	0.95	0.97	0.97	0.92	0.92	0.95	0.98	0.95	0.94	0.98
	0.25	30	0.98	0.91	0.95	0.96	0.92	0.86	0.88	0.96	0.97	0.92	0.94	0.99
		100	0.97	0.92	0.92	0.95	0.92	0.88	0.85	0.92	0.98	0.93	0.93	0.98
		200	0.98	0.95	0.95	0.96	0.94	0.88	0.89	0.95	0.98	0.95	0.95	0.98
	0.5	30	0.98	0.93	0.94	0.97	0.77	0.78	0.78	0.92	0.97	0.92	0.95	0.99
		100	0.98	0.94	0.94	0.95	0.74	0.80	0.78	0.91	0.97	0.94	0.93	0.99
		200	0.98	0.94	0.96	0.95	0.79	0.78	0.81	0.91	0.99	0.95	0.95	0.98
0.5	0.05	30	0.96	0.88	0.94	0.94	0.97	0.87	0.93	0.94	0.97	0.88	0.94	0.95
		100	0.96	0.93	0.94	0.96	0.96	0.91	0.93	0.96	0.96	0.93	0.94	0.98
		200	0.98	0.91	0.93	0.95	0.99	0.90	0.92	0.95	0.99	0.91	0.93	0.96
	0.1	30	0.96	0.87	0.93	0.94	0.97	0.86	0.91	0.95	0.97	0.88	0.93	0.96
		100	0.96	0.93	0.94	0.96	0.96	0.90	0.92	0.97	0.96	0.93	0.94	0.99
		200	0.98	0.92	0.93	0.95	0.97	0.89	0.90	0.95	0.98	0.92	0.93	0.97
	0.25	30	0.96	0.88	0.93	0.92	0.93	0.80	0.85	0.94	0.97	0.89	0.94	0.97
		100	0.96	0.93	0.95	0.95	0.92	0.83	0.87	0.94	0.97	0.92	0.95	0.98
		200	0.98	0.92	0.94	0.96	0.93	0.84	0.84	0.93	0.99	0.92	0.94	0.98
	0.5	30	0.96	0.89	0.93	0.95	0.78	0.70	0.73	0.91	0.97	0.89	0.95	0.99
		100	0.96	0.92	0.94	0.96	0.76	0.71	0.78	0.90	0.97	0.93	0.93	1.00
		200	0.98	0.91	0.93	0.94	0.79	0.74	0.74	0.89	0.98	0.92	0.93	0.99

Table H.4: Coverage of 95% CI around γ_{11} under MCAR

								MDT						
				LD				MI				MLMI		
				Distribution				Distribution				Distribution		
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	0.93	0.94	0.95	0.94	0.94	0.96	0.96	0.94	0.93	0.96	0.95	0.95
		100	0.94	0.96	0.96	0.97	0.95	0.97	0.97	0.98	0.95	0.96	0.96	0.98
		200	0.95	0.94	0.96	0.93	0.96	0.95	0.96	0.95	0.95	0.95	0.97	0.94
	0.1	30	0.93	0.94	0.94	0.93	0.95	0.97	0.96	0.94	0.93	0.97	0.96	0.94
		100	0.94	0.96	0.95	0.97	0.96	0.98	0.97	0.98	0.95	0.98	0.97	0.98
		200	0.94	0.93	0.96	0.94	0.96	0.95	0.96	0.96	0.94	0.95	0.97	0.94
	0.25	30	0.92	0.93	0.92	0.94	0.96	0.99	0.98	0.99	0.94	0.98	0.97	0.98
		100	0.94	0.95	0.96	0.96	0.96	0.99	0.98	0.99	0.95	0.99	0.98	0.99
		200	0.95	0.94	0.95	0.94	0.97	0.98	0.99	0.95	0.96	0.99	0.99	0.95
	0.5	30	0.93	0.95	0.93	0.95	0.99	1.00	1.00	1.00	0.96	1.00	1.00	0.99
		100	0.94	0.94	0.93	0.94	0.99	1.00	1.00	0.99	0.97	1.00	1.00	0.99
		200	0.95	0.95	0.94	0.94	0.99	1.00	1.00	0.96	0.97	1.00	1.00	0.93
0.1	0.05	30	0.93	0.95	0.92	0.94	0.95	0.95	0.93	0.96	0.93	0.95	0.92	0.95
		100	0.95	0.94	0.95	0.96	0.95	0.94	0.95	0.97	0.95	0.95	0.95	0.96
		200	0.96	0.96	0.96	0.95	0.96	0.96	0.97	0.96	0.96	0.96	0.97	0.96
	0.1	30	0.93	0.95	0.92	0.95	0.96	0.96	0.93	0.96	0.93	0.96	0.92	0.96
		100	0.95	0.94	0.95	0.95	0.96	0.95	0.96	0.96	0.95	0.95	0.96	0.96
		200	0.96	0.96	0.95	0.96	0.96	0.96	0.97	0.96	0.96	0.96	0.97	0.97
	0.25	30	0.92	0.93	0.92	0.92	0.96	0.97	0.96	0.97	0.95	0.97	0.95	0.96
		100	0.95	0.95	0.95	0.95	0.97	0.98	0.98	0.97	0.95	0.98	0.98	0.96
		200	0.95	0.95	0.95	0.96	0.98	0.98	0.97	0.94	0.95	0.98	0.98	0.95
	0.5	30	0.93	0.92	0.94	0.94	0.98	1.00	1.00	0.99	0.97	0.99	1.00	0.99
		100	0.95	0.95	0.96	0.96	0.98	1.00	1.00	0.96	0.97	1.00	1.00	0.94
		200	0.96	0.96	0.94	0.96	0.99	1.00	1.00	0.89	0.98	1.00	1.00	0.87
0.2	0.05	30	0.93	0.91	0.93	0.93	0.94	0.92	0.94	0.93	0.93	0.92	0.94	0.93
		100	0.95	0.95	0.94	0.96	0.96	0.96	0.95	0.96	0.95	0.96	0.94	0.96
		200	0.95	0.94	0.96	0.94	0.96	0.96	0.96	0.94	0.96	0.95	0.95	0.94
	0.1	30	0.92	0.92	0.94	0.93	0.94	0.93	0.95	0.93	0.92	0.93	0.94	0.93
		100	0.95	0.96	0.96	0.97	0.96	0.97	0.96	0.96	0.95	0.97	0.95	0.96
		200	0.95	0.93	0.96	0.95	0.96	0.94	0.96	0.94	0.95	0.95	0.97	0.95
	0.25	30	0.93	0.92	0.93	0.93	0.97	0.97	0.98	0.95	0.94	0.96	0.97	0.94
		100	0.95	0.96	0.93	0.96	0.97	0.99	0.97	0.95	0.96	0.99	0.96	0.92
		200	0.95	0.94	0.96	0.94	0.98	0.97	0.98	0.93	0.95	0.95	0.98	0.91
	0.5	30	0.92	0.92	0.93	0.93	0.99	1.00	1.00	0.96	0.96	0.99	0.99	0.95
		100	0.95	0.95	0.96	0.96	0.98	1.00	1.00	0.89	0.98	1.00	0.99	0.85
		200	0.95	0.96	0.95	0.95	0.99	0.99	1.00	0.71	0.97	0.99	0.99	0.62
0.5	0.05	30	0.95	0.93	0.94	0.94	0.96	0.94	0.94	0.95	0.95	0.93	0.94	0.94
		100	0.95	0.93	0.96	0.96	0.95	0.94	0.96	0.96	0.95	0.93	0.97	0.95
		200	0.95	0.95	0.95	0.95	0.96	0.95	0.95	0.95	0.95	0.95	0.95	0.95
	0.1	30	0.95	0.93	0.94	0.95	0.96	0.94	0.94	0.96	0.95	0.94	0.94	0.96
		100	0.95	0.92	0.97	0.95	0.96	0.93	0.97	0.95	0.95	0.93	0.97	0.95
		200	0.95	0.95	0.95	0.95	0.96	0.95	0.95	0.94	0.95	0.95	0.95	0.93
	0.25	30	0.95	0.92	0.94	0.95	0.97	0.94	0.95	0.95	0.96	0.94	0.95	0.95
		100	0.95	0.93	0.97	0.96	0.96	0.95	0.98	0.94	0.95	0.94	0.97	0.91
		200	0.95	0.95	0.95	0.96	0.97	0.96	0.96	0.89	0.96	0.96	0.96	0.83
	0.5	30	0.95	0.94	0.93	0.96	0.99	0.98	0.98	0.92	0.97	0.97	0.97	0.87
		100	0.95	0.93	0.96	0.95	0.99	0.98	1.00	0.70	0.97	0.97	0.97	0.50
		200	0.95	0.95	0.95	0.95	0.99	0.98	0.99	0.45	0.98	0.96	0.98	0.19

Table H.5: Coverage of 95% CI around γ_{01} under MCAR

			MDT											
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	0.94	0.93	0.94	0.94	0.94	0.93	0.94	0.93	0.94	0.93	0.95	0.94
		100	0.95	0.95	0.94	0.95	0.94	0.94	0.92	0.94	0.95	0.94	0.93	0.95
		200	0.95	0.95	0.95	0.93	0.95	0.95	0.94	0.93	0.96	0.95	0.95	0.93
	0.1	30	0.93	0.94	0.95	0.93	0.93	0.92	0.93	0.91	0.95	0.93	0.94	0.95
		100	0.95	0.94	0.93	0.95	0.93	0.92	0.91	0.93	0.96	0.95	0.93	0.96
		200	0.94	0.96	0.94	0.95	0.94	0.94	0.94	0.92	0.96	0.95	0.94	0.95
	0.25	30	0.94	0.95	0.95	0.92	0.89	0.92	0.90	0.89	0.94	0.95	0.94	0.95
		100	0.95	0.95	0.93	0.97	0.89	0.92	0.87	0.92	0.96	0.94	0.94	0.98
		200	0.95	0.96	0.95	0.94	0.91	0.92	0.90	0.89	0.95	0.95	0.94	0.95
	0.5	30	0.94	0.95	0.93	0.94	0.84	0.91	0.87	0.86	0.95	0.94	0.94	0.96
		100	0.95	0.95	0.94	0.96	0.81	0.88	0.85	0.86	0.95	0.95	0.93	0.97
		200	0.95	0.93	0.94	0.95	0.84	0.86	0.89	0.85	0.97	0.92	0.94	0.96
0.1	0.05	30	0.94	0.94	0.94	0.96	0.93	0.94	0.94	0.94	0.94	0.94	0.94	0.96
		100	0.97	0.94	0.95	0.95	0.96	0.93	0.95	0.94	0.98	0.95	0.96	0.95
		200	0.95	0.95	0.95	0.93	0.95	0.94	0.94	0.91	0.95	0.96	0.95	0.93
	0.1	30	0.93	0.95	0.94	0.95	0.92	0.92	0.93	0.92	0.94	0.94	0.94	0.95
		100	0.97	0.94	0.95	0.97	0.94	0.92	0.93	0.93	0.97	0.94	0.95	0.96
		200	0.95	0.96	0.95	0.93	0.94	0.93	0.93	0.90	0.95	0.95	0.95	0.93
	0.25	30	0.94	0.95	0.96	0.95	0.90	0.91	0.89	0.89	0.94	0.95	0.95	0.96
		100	0.97	0.95	0.95	0.96	0.90	0.90	0.90	0.89	0.97	0.95	0.95	0.97
		200	0.95	0.95	0.95	0.93	0.91	0.88	0.90	0.84	0.96	0.95	0.96	0.93
	0.5	30	0.93	0.95	0.94	0.94	0.81	0.87	0.83	0.80	0.93	0.93	0.93	0.96
		100	0.97	0.94	0.95	0.94	0.82	0.85	0.82	0.83	0.97	0.94	0.95	0.96
		200	0.95	0.95	0.96	0.93	0.81	0.84	0.84	0.79	0.95	0.95	0.96	0.95
0.2	0.05	30	0.94	0.92	0.95	0.94	0.95	0.91	0.93	0.92	0.95	0.91	0.94	0.94
		100	0.91	0.94	0.93	0.93	0.92	0.93	0.92	0.91	0.93	0.94	0.93	0.94
		200	0.94	0.95	0.95	0.96	0.94	0.94	0.94	0.94	0.95	0.95	0.96	0.96
	0.1	30	0.94	0.92	0.94	0.94	0.94	0.90	0.92	0.90	0.95	0.93	0.95	0.94
		100	0.91	0.94	0.94	0.94	0.91	0.92	0.92	0.89	0.93	0.94	0.94	0.94
		200	0.94	0.94	0.95	0.96	0.93	0.92	0.93	0.91	0.95	0.94	0.96	0.95
	0.25	30	0.94	0.92	0.95	0.94	0.90	0.86	0.88	0.85	0.95	0.93	0.94	0.95
		100	0.91	0.95	0.94	0.94	0.87	0.86	0.88	0.83	0.93	0.94	0.95	0.94
		200	0.94	0.95	0.95	0.96	0.90	0.89	0.88	0.84	0.95	0.95	0.95	0.95
	0.5	30	0.94	0.93	0.94	0.95	0.83	0.77	0.83	0.77	0.95	0.92	0.95	0.97
		100	0.92	0.94	0.95	0.94	0.80	0.78	0.80	0.72	0.93	0.94	0.94	0.95
		200	0.94	0.95	0.96	0.95	0.81	0.82	0.80	0.74	0.96	0.94	0.96	0.96
0.5	0.05	30	0.94	0.96	0.95	0.94	0.95	0.95	0.93	0.93	0.96	0.96	0.94	0.94
		100	0.94	0.95	0.95	0.92	0.94	0.94	0.94	0.90	0.95	0.95	0.95	0.91
		200	0.96	0.96	0.97	0.97	0.96	0.95	0.95	0.93	0.96	0.96	0.97	0.97
	0.1	30	0.94	0.96	0.94	0.94	0.94	0.92	0.91	0.91	0.95	0.96	0.94	0.95
		100	0.94	0.94	0.96	0.91	0.93	0.91	0.93	0.88	0.95	0.94	0.96	0.91
		200	0.96	0.96	0.97	0.96	0.95	0.93	0.94	0.91	0.96	0.96	0.97	0.96
	0.25	30	0.95	0.96	0.96	0.94	0.90	0.87	0.87	0.82	0.95	0.95	0.94	0.95
		100	0.94	0.94	0.96	0.91	0.90	0.88	0.88	0.78	0.95	0.95	0.95	0.92
		200	0.96	0.96	0.97	0.96	0.91	0.87	0.89	0.82	0.95	0.96	0.97	0.97
	0.5	30	0.95	0.95	0.95	0.94	0.81	0.80	0.80	0.66	0.96	0.95	0.94	0.95
		100	0.95	0.95	0.95	0.91	0.79	0.77	0.74	0.65	0.95	0.94	0.94	0.93
		200	0.96	0.93	0.96	0.97	0.82	0.76	0.77	0.65	0.95	0.94	0.96	0.96

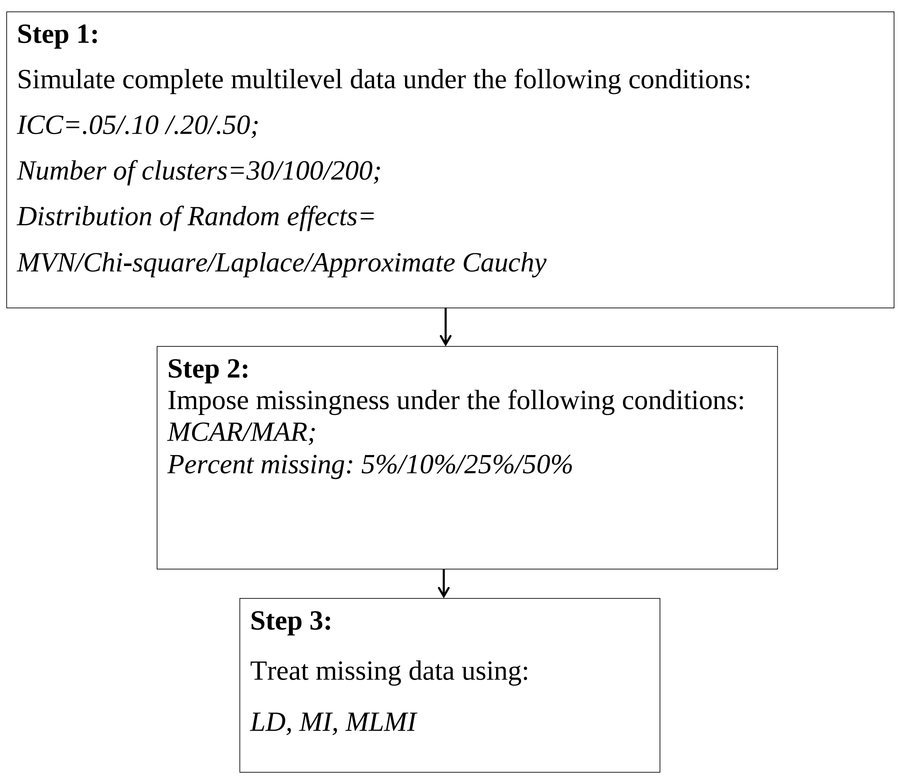
Table H.6: Coverage of 95% CI around γ_{21} under MCAR

			MDT											
			LD				MI				MLMI			
			Distribution				Distribution				Distribution			
ICC	Missing	Size	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn	cauchy	chi-square	laplace	mvn
0.05	0.05	30	0.94	0.95	0.92	0.97	0.96	0.96	0.93	0.98	0.95	0.96	0.92	0.97
		100	0.95	0.94	0.96	0.96	0.96	0.95	0.96	0.96	0.96	0.94	0.96	0.96
		200	0.94	0.94	0.94	0.97	0.95	0.95	0.95	0.97	0.94	0.94	0.95	0.97
	0.1	30	0.95	0.95	0.92	0.97	0.95	0.96	0.94	0.98	0.95	0.95	0.92	0.97
		100	0.95	0.94	0.96	0.94	0.96	0.95	0.97	0.97	0.96	0.95	0.96	0.96
		200	0.94	0.94	0.95	0.95	0.94	0.95	0.96	0.95	0.94	0.94	0.95	0.97
	0.25	30	0.95	0.96	0.93	0.96	0.95	0.98	0.97	0.99	0.95	0.95	0.93	0.98
		100	0.95	0.96	0.95	0.97	0.97	0.97	0.97	0.96	0.96	0.95	0.95	0.98
		200	0.93	0.94	0.94	0.96	0.95	0.96	0.96	0.90	0.94	0.93	0.95	0.97
	0.5	30	0.95	0.95	0.94	0.96	0.97	1.00	0.99	0.99	0.94	0.94	0.94	0.99
		100	0.96	0.95	0.95	0.96	0.98	0.99	0.98	0.88	0.95	0.95	0.94	0.97
		200	0.94	0.96	0.94	0.94	0.97	0.98	0.99	0.66	0.95	0.94	0.95	0.97
0.1	0.05	30	0.93	0.95	0.94	0.95	0.94	0.95	0.95	0.96	0.94	0.94	0.95	0.96
		100	0.95	0.96	0.97	0.95	0.96	0.95	0.97	0.96	0.96	0.96	0.97	0.96
		200	0.96	0.94	0.93	0.96	0.96	0.95	0.94	0.97	0.97	0.95	0.94	0.97
	0.1	30	0.93	0.94	0.95	0.95	0.95	0.94	0.96	0.97	0.94	0.94	0.95	0.97
		100	0.95	0.96	0.97	0.96	0.96	0.97	0.97	0.95	0.96	0.95	0.98	0.97
		200	0.96	0.93	0.94	0.96	0.97	0.94	0.94	0.93	0.96	0.93	0.94	0.97
	0.25	30	0.93	0.94	0.95	0.95	0.96	0.96	0.96	0.97	0.94	0.95	0.95	0.97
		100	0.95	0.95	0.96	0.95	0.97	0.97	0.97	0.89	0.95	0.95	0.96	0.98
		200	0.96	0.96	0.94	0.93	0.98	0.96	0.96	0.72	0.96	0.95	0.94	0.97
	0.5	30	0.94	0.94	0.93	0.97	0.97	0.98	0.99	0.95	0.94	0.94	0.94	0.99
		100	0.96	0.95	0.95	0.97	0.98	0.99	1.00	0.61	0.95	0.95	0.96	0.98
		200	0.96	0.96	0.94	0.94	0.99	0.97	0.98	0.21	0.96	0.94	0.94	0.99
0.2	0.05	30	0.95	0.93	0.94	0.96	0.96	0.93	0.94	0.96	0.95	0.92	0.94	0.96
		100	0.94	0.95	0.94	0.95	0.96	0.94	0.94	0.95	0.94	0.95	0.94	0.96
		200	0.96	0.95	0.95	0.94	0.96	0.96	0.95	0.94	0.96	0.96	0.95	0.96
	0.1	30	0.94	0.93	0.94	0.95	0.96	0.92	0.95	0.95	0.95	0.92	0.94	0.98
		100	0.94	0.95	0.94	0.94	0.95	0.95	0.96	0.93	0.95	0.95	0.95	0.97
		200	0.96	0.94	0.95	0.95	0.97	0.94	0.95	0.83	0.96	0.95	0.95	0.98
	0.25	30	0.94	0.92	0.94	0.96	0.97	0.95	0.96	0.94	0.95	0.93	0.94	0.99
		100	0.94	0.95	0.95	0.96	0.96	0.96	0.97	0.73	0.94	0.95	0.94	0.99
		200	0.95	0.95	0.95	0.94	0.97	0.96	0.96	0.33	0.96	0.94	0.95	0.98
	0.5	30	0.94	0.94	0.93	0.94	0.98	0.97	0.97	0.85	0.95	0.93	0.93	0.99
		100	0.95	0.95	0.94	0.97	0.97	0.98	0.98	0.17	0.95	0.96	0.95	0.99
		200	0.95	0.94	0.96	0.95	0.99	0.97	0.98	0.00	0.96	0.94	0.95	0.99
0.5	0.05	30	0.93	0.95	0.93	0.95	0.94	0.95	0.93	0.96	0.94	0.94	0.93	0.97
		100	0.94	0.94	0.95	0.94	0.95	0.94	0.95	0.93	0.95	0.94	0.95	0.96
		200	0.95	0.94	0.96	0.96	0.95	0.95	0.96	0.88	0.96	0.94	0.96	0.97
	0.1	30	0.93	0.95	0.93	0.95	0.93	0.95	0.93	0.95	0.94	0.94	0.93	0.96
		100	0.94	0.95	0.95	0.95	0.95	0.95	0.95	0.81	0.95	0.95	0.95	0.97
		200	0.95	0.94	0.95	0.95	0.96	0.95	0.97	0.60	0.96	0.95	0.96	0.96
	0.25	30	0.93	0.95	0.93	0.94	0.95	0.95	0.92	0.81	0.93	0.94	0.93	0.98
		100	0.94	0.95	0.96	0.94	0.96	0.96	0.96	0.21	0.95	0.94	0.96	0.99
		200	0.95	0.94	0.95	0.94	0.95	0.96	0.97	0.02	0.95	0.96	0.96	0.98
	0.5	30	0.93	0.95	0.92	0.94	0.97	0.97	0.95	0.37	0.94	0.95	0.93	1.00
		100	0.94	0.94	0.94	0.95	0.98	0.96	0.97	0.01	0.95	0.94	0.95	1.00
		200	0.95	0.95	0.97	0.96	0.98	0.96	0.96	0.00	0.95	0.93	0.97	0.99

Appendix I

Data simulation process

Figure I.1: Steps used in simulating data within each cell of factorial design of study



Note: Details of these steps are provided in section 3.2.