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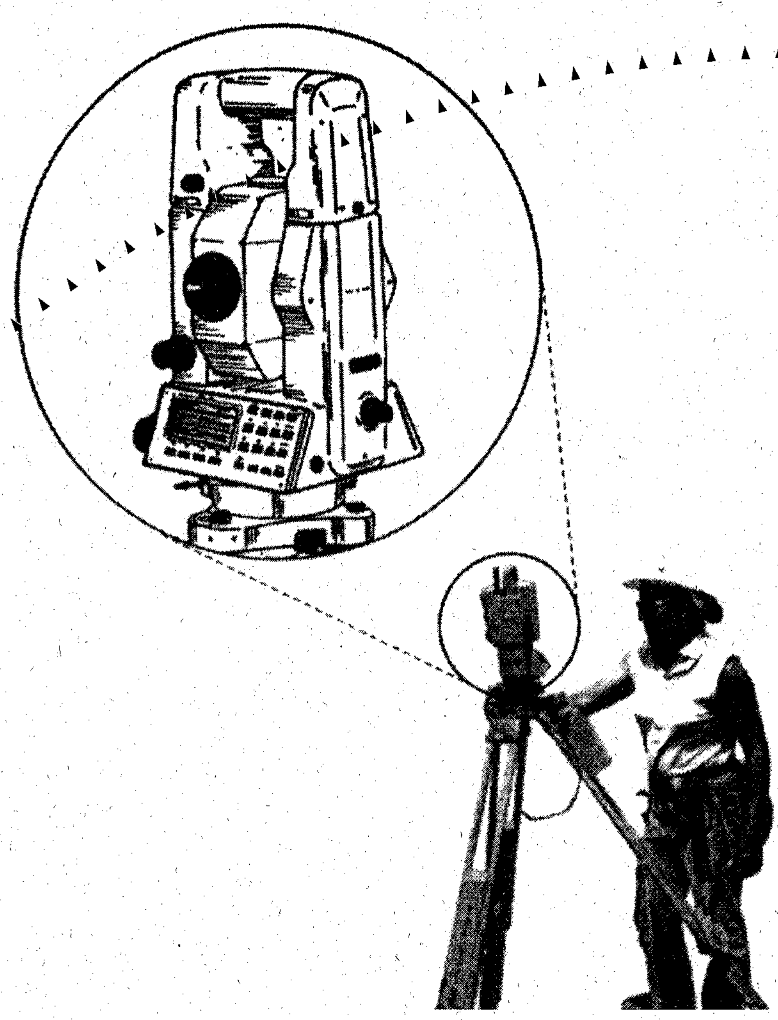

Development of a Three-Dimensional Least Squares Adjustment Program

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In the past horizontal position (X and Y coordinates) and vertical position (elevation) have been measured and computed in separate surveying operations. In order to take full advantage of the three-dimensional measurement capability of their "total station" surveying instruments, Mn/DOT wanted a user friendly least squares adjustment program that would simultaneously incorporate all three measurements - distance, horizontal angle and vertical angle in a common adjustment procedure. The only practical way to do this is to use a least squares adjustment program that can determine both horizontal and vertical components in the same adjustment procedure. There are commercial and government adjustment programs available that will do a three dimensional least squares adjustment, and the new program is not unique in that sense. It is unique, however, in that it satisfies special Mn/DOT requirements, particularly with respect to input and output format which is the SDMS (Survey Data Management System) format. In addition the program incorporates a simplified weighting procedure which allows the user to alter the weighting parameters if that should be necessary.

Although not a part of the original agreement, it became apparent during program testing that a field test facility would benefit the project and also provide a site for on-going equipment testing and performance checking. Thus the idea for a permanent three-dimension test site came into being.

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DEVELOPMENT OF A THREE-DIMENSIONAL LEAST SQUARES ADJUSTMENT PROGRAM

Final Report

Prepared By

Professor Gerald W. Johnson

Department of Civil Engineering
University of Minnesota

December 1994

Submitted to

Minnesota Department of Transportation
Office of Research Administration
200 Ford Building,, 117 University Avenue
ST. Paul, MN 55155

EXECUTIVE SUMMARY

Survey measurements are never exact, and in order that the results satisfy certain geometric constraints (ie., the sum of the angles of a triangle must total 180 degrees, or the final coordinates of a point are the same, regardless of which measurements are used to compute them), the measured field data is adjusted so that any and all constraints are satisfied. Many adjustment procedures have been developed and used over the years, and while different procedures lead to slightly different results, as a whole they all produced satisfactory results.

The introduction of electronic surveying instruments, starting about twenty years ago, caused surveyors to look at the adjustment procedures they had been using. Electronic distance measuring (EDM) equipment allowed the surveyor to quickly and accurately take more measurements than the minimum number necessary for determining the position of survey points. This increased the accuracy of the results, but it also required more sophisticated adjustment procedures. This requirement, along with the advent of the computer, led to wider use of the least squares adjustment procedure. This was still only two dimensional, however, and the third dimension, elevation, continued to be a separate surveying and adjustment procedure.

The next development in surveying instrumentation brought the "total station" to the surveying profession. This instrument combines both horizontal and vertical angle measurements along with distance measurements in the same instrument, hence the name "total station". The original least squares adjustment programs were not capable of carrying out a three-dimensional adjustment, and thus the full capability of the new instrument could not be fully utilized.

The purpose of the research carried out under this contract was to develop a new survey adjustment program that would allow the Minnesota Department of Transportation (Mn/DOT) to take full advantage of the surveying capabilities available in the total stations that they were acquiring. There were two requirements for this adjustment program: first, it would obviously have to incorporate the least squares procedure, and second, it would have to satisfy Mn/DOT's data handling requirements. The final product of this agreement is a three-dimensional least squares survey adjustment program that satisfies the above two requirements. The coding has been field tested and is currently being implemented by Mn/DOT users.

As development proceeded, it became apparent that because this was a new procedure, existing survey data was not available to adequately test the program. It was therefore decided to develop a field test site that could be used to test the adjustment program. To this end, a permanent test facility was established at the rest stop off the west bound lane of I-94 midway between the St. Croix River and St. Paul. In addition to providing the data for testing the least squares adjustment program, the site has become a permanent facility for testing surveying equipment and procedures.

INTRODUCTION

For years the compass rule was used for the adjustment of simple survey loops that involved only angles and distances measured around the loop, but it was not capable of utilizing the angles and distances that could be measured across or through the loop. (Rather than just six distances and six angles measured around a six-sided loop, there could be fifteen or twenty angles and distances tying the points of the traverse to one another.) In addition the compass rule did not adjust distances and angles simultaneously. To take full advantage of the capabilities of new equipment, it has been necessary to develop adjustment procedures that incorporate all measurements in a simultaneous procedure.

This requirement for better adjustment procedures along with the advent of the computer has resulted in wide spread use of the method of least squares. The idea underlying a least squares adjustment can be stated as follows: the field measurements that are made during a survey, although not exact, are still the best data available, and the adjustment procedure should alter that data as little as possible. There is no other adjustment procedure that will alter the measured values to a lesser extent, in total, than a least squares adjustment. If you take the adjustments to all the measured values, square them, and then sum these squared values, this sum will be less than the sum of the squared adjustments of any other adjustment. (The squaring takes care of the algebraic sign of the adjustment, ie., plus or minus.) Thus the name "least squares" comes from the fact that the sum of the squares of the adjustments is the "least" sum that can be obtained from any adjustment, or to state it another way, there is no other adjustment that will alter the original measurements, in total, by a smaller amount. Because the field measurements are the best data available, they should be altered as little as possible in the adjustment procedure.

In theory the idea of the least squares adjustment is rather straight forward, and it has been in use for some fifty or sixty years at least. The problem was that the computational effort was so great and the procedures so complicated that only large government agencies ever used it. New kinds of survey equipment and the availability of the computer to ease the computations led to increasing use of least squares adjustments.

There were still some practical problems that needed to be addressed. The old compass rule adjusted angles in one step and distances in another step, but to have a "least" adjustment of all the measurements, the angles and distances, have to be adjusted simultaneously. In other words, "apples" and "oranges" have to be mixed together and treated as similar elements in the adjustment program. There are various ways of doing this, and this leads to differences in the programs that perform least squares adjustments.

To make full use of the three-dimensional capabilities of their total stations, Mn/DOT wanted a user friendly least squares adjustment program that would simultaneously incorporate the three measurements - distance, horizontal angle and vertical angle - in a common adjustment. If the output is separated into horizontal and vertical components and adjusted independently, much of the inherent capability of the instrument is lost. (The vertical angle and the distance influence each other in a solution for elevation. The horizontal angle and the distance influence each other in a solution for coordinates. If the coordinates are solved first, then a constraint is imposed on the distance when it is used to solve for the elevation. Conversely, if the initial solution is for elevation, then an artificial constraint is placed on the distance when it is used to solve for the coordinates.) Therefore to take full advantage of the total station, it is necessary to solve for both horizontal and vertical values simultaneously, and the only practical way to do this is to use a least squares adjustment that determines both in the same procedure.

Another factor in any least squares adjustment program is that of the weight of the observations. Not all measurements have the same "quality", some are better than others, and it is therefore desirable to give these a greater weight in the solution, ie., to alter them less in the final solution. Also how are angles of a given weight compared with a distance of a given weight? Again different programs handle this in different ways.

The basic mathematics of a least squares adjustment program are fairly straight forward and spelled out in most, if not all, college level surveying texts. (See for example: Wolf and Brinker, Elementary Surveying, Ninth Edition, published by Harper Collins; or Moffitt and Bouchard, Surveying, Eighth Edition, Harper and Row.)

The product of this agreement between Mn/DOT and the author is a computer program that performs a least squares adjustment of the three dimensional (distance and angles) measurements obtained with a "total station" surveying instrument. The program accepts and outputs data in a format proscribed by Mn/DOT. The program has built-in weight parameters which are based on current equipment standards. These may easily be altered to reflect future changes in the overall performance of equipment, or they may be altered to facilitate the mixing of measurements of different performance levels.

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PURPOSE

The purpose of the project was to develop a three-dimensional least squares adjustment program to be used by Mn/DOT personnel for adjusting survey data obtained from total station measurements. Such programs are available both from private vendors and through federal government agencies, and the intent of this project was to develop an adjustment program specifically tailored to Mn/DOT requirements (input and output formats, equipment and weight parameters, etc.)

Although not a part of the original agreement, it soon became apparent that a field test facility would benefit the project and also provide a site for on-going equipment testing and performance checking. Thus the idea of a permanent three-dimensional test site came into being.

3-DIMENSIONAL LEAST SQUARES ADJUSTMENT

There are commercial and government programs available that do a three dimensional adjustment and the Mn/DOT program is not unique in that sense. It was written to satisfy their requirements, particularly their input and output format which is the SDMS (Survey Data Management System) format. In addition it was desirable to incorporate a simplified weighting procedure if the user felt that it was necessary to alter the weighting parameters initially set in the program.

The weighting parameters are based on a concept that most surveyors inherently understand: the idea of the relative quality of "first", "second" and "third" order surveys. Thus weights are thought of in terms of a value of "one", "two" or "three"; "one" being best, "two" being second best, etc., similar to "first", "second" or "third" order surveys. There is no direct relation between these weights and published classifications and standards of accuracy, but it is this familiar concept of relative quality that is used.

(It should be noted that the program does not utilize the concept of using weights based on the standard errors of the angles and the distances to combine the two types of measurements, angles and distances, in the solution. The program establishes the relative equivalency of the two types of measurements by converting both to dimensionless values. This allows the weighting parameters for both angles and distances to be given in pure number values.)

The weights initially assigned to the three total station measurements are:

Distance: One

Horizontal Angle: One

Vertical Angle: Two

This gives distances and horizontal angles equal weight in the solution, but the reader will immediately note that vertical angles are given a lesser weight of "two". This is based on field testing (see following section) of Mn/DOT total stations which indicated that the standard errors of the vertical angles were about twice those of the horizontal angles.

For the total stations used at the test site, the actual values came out to about +/- 3 seconds for the horizontal angles (which is what the instrument specifications call for) and +/- 6 seconds for the vertical angles (twice what the instrument specifications call for). This difference is attributed to two factors: (1) vertical angles are dependent on the instrument being "perfectly" level, and even for instruments with automatic leveling compensation there is likely a residual vertical angle error from this source, and (2) vertical angles are subject to refraction errors.
Investigating the source of these larger vertical angle errors was beyond the scope of the present project, but because we know that they exist, they have been taken into account in the weighting parameters.

The above initial weights are assigned to all the measurements used in the adjustment. Thus all distances and horizontal angles are given a value of "one", and all vertical angles are given a value of "two". It is a simple matter to change these initial values at any time, and a user who felt that vertical angles should be weighted the same as the horizontal angles could change that parameter prior to running the program. Also the program has the capability of accepting weights for individual measurements. Thus if measurements are known to differ in quality (because they were made by different instruments, for example) they could be assigned appropriate weights.

TEST SITE

To evaluate the program as it was being developed it was decided that an actual three dimensional traverse with known parameters should be used. We were unable to find an existing traverse that had both horizontal and vertical data with the accuracy that was needed, and Mn/DOT, deciding that such a traverse would be useful for other test purposes also, established a permanent test facility.

Criteria for this facility were:

(1) It must be readily accessible by potential users.

(2) It should have one or two line segments of at least 1,000 feet in length.

(3) It must provide enough difference in elevation that vertical angles and slope distances could be evaluated.

This criteria led Mn/DOT to establish a traverse at the rest stop off the west bound lane of I-94 about half way between the St. Croix River and St. Paul. The traverse has six permanent control points, and is certainly one of the most accurately surveyed traverses in the State of Minnesota.

Involved in the survey were the following equipment and procedures:

(1) GPS receivers.

(2) T-3's for horizontal angles.

(3) Total stations for:

(a) Horizontal angles

(b) Vertical angles

(c) Distances.

(4) 2nd order, class 1, leveling for elevations.

These data were all incorporated into a final adjustment with more than 100 observations to establish the X and Y coordinates and elevations for each of he six points.

The results of this adjustment indicate that the traverse points have an internal consistency of:

(1) +/- 3 mm (0.010 ft.) in horizontal coordinates.

(2) +/- 1 mm (0.003 ft.) in elevation.

(3) +/- 2 sec. in angular relationships.

The longest distance in the traverse is approximately 1250 feet and the greatest difference in elevation is approximately 26 feet. The points are all permanently monumented with brass discs mounted on rods driven to refusal and marked with witness posts.

The coordinate system is a local plane coordinate system, and the area that it covers is limited such that it is not necessary for the user to take into account any geodetic conversions (i.e., convergence of the meridians, ellipsoid/geoid/ground distance conversions, scale factors, etc.).

SUMMARY

The adjustment program is currently operational and is being implemented within Mn/DOT. The source code is not proprietary and is available for use by other government agencies by contacting:

Mark Wright

Mn/DOT Right of Way

1500 West County Road B2

Roseville, MN 55113

(612) 582-1023

However, because it might be construed as unfair competition and because there is no mechanism for providing program updates outside Mn/DOT, the program cannot formally be given to private users. The I-94 Test Traverse is available to anyone who can benefit from its use. By contacting the Mn/DOT Geodetic Unit potential users can obtain a photo map of the site showing the location of the six control points and a list of the coordinates and elevations for the six points. Thus the site can be used for any survey equipment testing that might be necessary. Given the known elevations and coordinates for the points, the user can measure and evaluate horizontal angles, vertical angles, slope distances and differences in elevation with total stations, EDM's, theodolites, transits, etc.

APPENDIX A

BLUNDER/MISTAKE ANALYSIS USING LEAST SQUARES ADJUSTMENT PROGRAM

It is important that the users of a least squares adjustment not just accept the computed coordinates, but that they also look at the "quality" of the results as expressed in the standard errors of the coordinates and the measurements. The program has the capability to indicate the presence of blunders or mistakes. Along with the adjusted values (coordinates and elevations), program output provides standard errors for each of the adjusted values and standard errors for the measurements (distances and angles) themselves. If these standard errors are not within range of expected norms, there is the strong possibility that a blunder or mistake has found its way into the measurements and needs to be identified and eliminated. Often the standard errors will give a strong indication of the source of the blunder/ mistake, and the program will in fact give the erroneous measurement a higher than expected correction.

Example: The table below lists the standard errors for a set of coordinates and measurements taken from the an adjustment of the test traverse. Following the initial adjustment (which contained no blunders or mistakes) blunders were introduced into a distance measurement (one foot), into a horizontal angle measurement (one minute), and finally into a vertical angle measurement (one minute). The table lists the standard errors that resulted from each of these blunders.

Coordinates	Standard Errors			
	No Mistakes	One Foot Mistake in a Distance	One Minute Mistake in a Hor. Ang.	One Minute Mistake in a Vert. Ang.
X	+/- 0.012	0.235	0.018	0.017
Y	+/- 0.014	0.269	0.021	0.019
Z	+/- 0.010	0.199	0.016	0.014
Measurements				
Distances	+/- 0.005	0.240	0.005	0.005
H. Angles	+/- 1.5 sec	35.4 sec	11.6 sec	1.3 sec
V. Angles	+/- 6.9 sec	6.8 sec	6.9 sec	10.1 sec

A distance mistake is immediately apparent when the standard errors of the resulting coordinates are examined (note that the errors have jumped from approximately +/- 0.012 to +/- 0.235 in the example). Also the overall standard error for the distances has increased by the same magnitude. If the corrections to the distances were also examined, it would be found the particular distance that contained the error would have received the largest correction.

An angle mistake (either horizontal or vertical) is not quite so apparent when examining the standard errors of the coordinates. In this case the errors have increased from only +/- 0.012 to +/- 0.018, and this might go unnoticed. However, the overall error for the measured angles has increased significantly, from +/- 1.5 sec to +/- 11.6 sec in the case of the horizontal angle mistake, and from +/- 6.9 sec to +/- 10.1 sec in the case of the vertical angle mistake. Again, a look at the corrections to the individual measurements would more than like point out the location of the mistake.

A study of the results of the above example point out a comment made earlier in this report, namely that the standard errors of the vertical angles are greater than those of the horizontal angles. The example indicates a relative accuracy of +/- 1.5 sec to +/- 6.9 sec between the two. A more thorough study would show the relative accuracy of the two to be about +/- 3 sec to +/- 6 sec. As discussed in the report, this is more than likely due to refraction and the self leveling capability of the instrument. Offsetting this larger standard error in the vertical angles is the fact that these angles have less influence on the results than either the distances or the horizontal angles.