

# Statewide Statistical Subgrade Characterization

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Final Report

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# Contents

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>The Data Set</b>	<b>3</b>
2.1	Data Extent . . . . .	3
2.2	Data Format . . . . .	3
2.3	Grouping of the Data Set . . . . .	5
<b>3</b>	<b>Computation of the Modulus</b>	<b>7</b>
3.1	Calculation Formulae . . . . .	7
3.2	Selection of the Appropriate Deflection . . . . .	7
<b>4</b>	<b>Statistical Characterization</b>	<b>9</b>
4.1	Statistical Summary . . . . .	9
4.2	Modulus Design-Value . . . . .	11
4.3	Design of the Sample Spacing . . . . .	12
<b>5</b>	<b>Software support</b>	<b>23</b>

# List of Tables

2.1	An example of a record from the input data . . . . .	5
2.2	Summary of special cases when a highway in a district is split for the purpose of statistical analysis . . . . .	6
3.1	Sensor spacings for each of the sensor spacing groups . . . . .	8
4.1	Average short scale variability, medium scale variability and correlation length for the each district . . . . .	11
4.2	Tolerated variability T in MPa for different acceptable errors and probabilities $\alpha$ . . . . .	14

# List of Figures

4.1	Number of samples for the project scale of 1 mile . . . . .	15
4.2	Number of samples for the project scale of 2 miles . . . . .	16
4.3	Number of samples for the project scale of 5 miles . . . . .	17
4.4	Number of samples for the project scale of 10 miles . . . . .	18
4.5	Number of samples for the project scale of 20 miles . . . . .	19
4.6	Number of samples for the project scale of 30 miles . . . . .	20
4.7	Number of samples for the project scale of 40 miles . . . . .	21
4.8	Number of samples for the project scale of 50 miles . . . . .	22



# Summary

This document presents a detailed statistical analysis and graphical presentation of more than 120,000 subgrade modulus values from Minnesota state roads and highways. These subgrade modulus values are based upon Falling Weight Deflectometer measurements collected by the Minnesota Department of Transportation between 1983 and 1993.

This document presents state-wide summaries, district summaries, and individual analysis for every state road in every district (for which data was available).

This document is meant to be used like an atlas. This *Minnesota Subgrade Modulus Atlas* was designed to assist highway engineers in the selection of design subgrade modulus values and as a guide during the design of future subgrade measurement campaigns.

In addition to this document, the *Minnesota Subgrade Modulus Atlas - Online* is a Microsoft Windows-based computer program that allows for the detailed analysis of the subgrade modulus along state roads and highways. This analysis can be carried out on a state-wide scale, on a small project scale of less than one mile, or any scale in between. This electronic form of the *Atlas* can be easily upgraded as additional measurements become available.



# Chapter 1

## Introduction

This document presents a detailed statistical analysis and graphical presentation of more than 120,000 subgrade modulus values from Minnesota state roads and highways. These subgrade modulus values are based upon Falling Weight Deflectometer (FWD) measurements collected by the Minnesota Department of Transportation between 1983 and 1993.

This document presents state-wide summaries, district summaries, and individual analysis for every state road in every district (for which data was available).

This document is meant to be used like an atlas. This *Minnesota Subgrade Modulus Atlas* was designed to assist highway engineers in the selection of design subgrade modulus values and as a guide during the design of future subgrade measurement campaigns.

In addition to this document, the *Minnesota Subgrade Modulus Atlas - Online* is a Microsoft Windows-based computer program that allows for the detailed analysis of the subgrade modulus along state roads and highways. This analysis can be carried out on a state-wide scale, on a small project scale of less than one mile, or any scale in between. This electronic form of the *Atlas* can be easily upgraded as additional measurements become available.

The most important part of this report are the appendices. They contain all the modulus plots and a number of statistical measures:

**Appendix A** contains the sample page. It explains the elements of a page from Appendix B, Appendix C, Appendix D and Appendix E, and the techniques used to obtain these elements.

**Appendix B** is the summary for the whole state and all years.

**Appendix C** shows the state summary for each year from 1983 to 1993.

**Appendix D** are district summaries for every district. All years are included.

**Appendix E** are district highway-summaries. Every highway in the each district is presented (if measurements exist). This appendix starts with district 1 and ends with district 9. Within each

district, highways are sorted in the ascending order. A highway in a district sometimes has more than one summary page: an explanation of these exceptions is provided in the next chapter.

**Appendix F** is the list of all records that indicate the modulus value below 50 MPa. This appendix is organized in the same way as the Appendix E.

**Appendix G** is the highway index: for the each highway, a list of pages where it appears in the Appendix E and Appendix F is provided.

In this report, the modulus is measured in MPa. One MPa equals 145 psi and one psi equals 6895 Pa.



# Chapter 2

## The Data Set

### 2.1 Data Extent

The Atlas is constructed from thirty Mn/DOT data files: d1i, d1u, d2u, d2m, d3i, d3u, d3upm, d3pm, d4u, d4m, d8u, d5i, d5u, d5m, d6i, d6u, d6m, d7i, d7u, d7m, d8m, d9i, d9u, d9m, d1m, fwd89inv.f, fwd90inv.f, fwd91inv.f, fwd92inv.f, fwd93inv.f. These thirty data files contained 204,380 data records spanning 1983 through 1993.

Of the initially available 204,380 data records, 59,020 were deemed inappropriate and were eliminated from further consideration. Most of the eliminated records were not FWD data (e.g. old Road Rater data). Some of the eliminated records contained unrecoverable typographic errors (such as a negative date, an incorrect sensor spacing group, incorrect data format).

A small number of the data had indicated subgrade modulus values in excess of 500 MPa. Of the 145,360 remaining data records, 22,300 were either duplicate records or records with the modulus in excess of 500 MPa. These records were also eliminated from further consideration, leaving a final record count to 123,130. (The upper threshold of 500 MPa was selected in consultation with the Mn/DOT Technical Panel.)

There are 180 state roads represented in the current database. Some of the roads have as many as 3,000 or 4,000 data records, other roads have as few as 10 or 20 data records.

### 2.2 Data Format

Each data record is available in the following format:

**Columns 1 to 2:** Highway System (1= Interstate, 2= U. S. Highway, 3= MN Highway)

**Columns 3 to 10:** Highway Number

**Columns 11 to 20:** Reference Post

**Columns 21 to 22:** Lane Designation (I= direction of increasing reference posts, D= direction of decreasing reference posts, 1= far right lane, 2= second lane from the right, etc.)

**Columns 23 to 28:** Date

**Columns 29 to 29:** District (1 to 9)

**Columns 30 to 31:** Load Type : FF or FN indicate FWD measurement

**Columns 32 to 35:** Impulse Load in kPa

**Columns 36 to 39:** Deflection #1 in  $\mu\text{m}$

**Columns 40 to 43:** Deflection #2 in  $\mu\text{m}$

**Columns 44 to 47:** Deflection #3 in  $\mu\text{m}$

**Columns 48 to 51:** Deflection #4 in  $\mu\text{m}$

**Columns 52 to 55:** Deflection #5 in  $\mu\text{m}$

**Columns 56 to 58:** Deflection #6 in  $\mu\text{m}$

**Columns 59 to 61:** Deflection #7 in  $\mu\text{m}$

**Columns 62 to 64:** Pavement Temperature

**Columns 65 to 66:** Sensor Spacing Group (1 to 5)

An example of a few records from file d1m is presented in Table 2.1.

0300000001194+00.200I10924851FF 601 715 602 544 361 212129 90 5803
0300000001194+00.300I10924851FF 607 600 523 478 313 170 90 65 5803
0300000001194+00.400I10924851FF 615 709 597 537 351 203129 92 5903
0300000001194+00.500I10924851FF 604 933 745 656 398 225149102 5903
0300000001194+00.600I10924851FF 645 817 654 583 369 223130 83 5903
0300000001194+00.700I10924851FF 611 753 633 574 379 221140101 5903
0300000001194+00.800I10924851FF 606 887 736 655 413 233146101 5903
0300000001194+00.900I10924851FF 612 653 558 510 354 218139 97 5903
0300000001195+00.000I10924851FF 608 819 677 593 366 210132 99 5903
0300000001195+00.100I10924851FF 601 678 574 518 336 193120 87 5903

Table 2.1: An example of a record from the input data

### 2.3 Grouping of the Data Set

As a general rule, every highway in the each district is statistically analyzed from the smallest available reference post to the largest available reference post for that district. An exception to this rule is made if the highway encounters significantly different geologic materials along the route. The list of these exceptions is presented in the Table 2.2.

A few exceptions are made if there was a large gap (in terms of the reference post) in the data set (e.g. 20 miles or more) or if the modulus changes rapidly over a very short distance (e.g. Highway 2, District 2, reference post 135).

<b>District Number</b>	<b>Highway Number</b>	<b>Reference Post of the split</b>	<b>Geology before the Reference Post</b>	<b>Geology after the Reference Post</b>
1	1	250	Erskine Moraine Association	Vermilion Moraine Association
1	2	220	Association of Lake Sediment and Peat	Mix of Culver Moraine Association with Mille Lacs Highland Moraine Association
2	64	50	Alexandria Moraine Association	Itasca Moraine Association
3	6	25	Culver Moraine Association	St-Croix Moraine Association
3	6	50	St-Croix Moraine Association	Sugar Hills Moraine Association
4	9	140	Big Stone Moraine Association	Glacial Lake Sediment
4	78	20	Alexandria Moraine Association	Outwash
8	7	75	Glacial Lake Sediment	Altamont Moraine Association
8	23	145	Altamont Moraine Association	Alexandria Moraine Association
8	71	120	Altamont Moraine Association	Alexandria Moraine Association
9	95	85	Mix of Outwash with Pine City Moraine Association	Mix of St-Croix Moraine Association with Terraces

Table 2.2: Summary of special cases when a highway in a district is split for the purpose of statistical analysis

## Chapter 3

# Computation of the Modulus

### 3.1 Calculation Formulae

The subgrade modulus is calculated as:

$$M = \pi \frac{R^2 S_f I}{dr} \quad (3.1)$$

where  $R$  is the plate radius,  $S_f$  is a constant that depends on the Poisson ratio,  $I$  is the impulse load,  $d$  denotes the deflection and  $r$  is the distance from the center of the plate to the location where deflection is measured.

For the plate radius of 0.15 m used for FWD measurements and for the  $S_f$  value of 0.2792 (corresponds to the Poisson ratio of 0.45) the value  $\pi S_f R^2$  equals  $0.019735485 \text{m}^2$ .

For the impulse load  $I$  in kPa, distance  $r$  in mm and deflection  $d$  in  $\mu\text{m}$  the modulus  $M$  in MPa is calculated as:

$$M(\text{MPa}) = 19735.485 \frac{I(\text{kPa})}{d(\mu\text{m})r(\text{mm})} \quad (3.2)$$

### 3.2 Selection of the Appropriate Deflection

The AASTHO Guide for the Design of Pavement Structures (1986, Section III, p109) recommends that the sensor used to calculate the subgrade modulus should be about 4 feet or 1211 mm away from the center of the load plate. The locations of the sensors for each of the five sensor spacing groups are given in Table 3.1

For the purpose of modulus computation, the sensor closest to the 'ideal' distance is selected. Thus, the modulus is calculated using the distances in bold from the Table 3.1 and appropriate deflections.

Sensor	Sensor 1	Sensor 2	Sensor 3	Sensor 4	Sensor 5	Sensor 6	Sensor 7
Group	Distance	Distance	Distance	Distance	Distance	Distance	Distance
Number	mm	mm	mm	mm	mm	mm	mm
1	0	200	300	450	600	900	<b>1200</b>
2	0	200	300	500	800	<b>1200</b>	1600
3	0	190	260	570	930	<b>1180</b>	1640
4	0	300	500	800	<b>1200</b>	1600	2000
5	0	300	450	600	900	<b>1350</b>	1800

Table 3.1: Sensor spacings for each of the sensor spacing groups

# Chapter 4

## Statistical Characterization

### 4.1 Statistical Summary

Each page in Appendix B, Appendix C, Appendix D and Appendix E includes one or more of the following statistical measures:

- The mean value of the modulus and the mean based on the assumption that the modulus is lognormally distributed. Please see the discussion below.
- The standard deviation of the modulus and the standard deviation based on the assumption that the modulus is lognormally distributed.

The mean and the standard deviation of the assumed lognormally distributed variable are calculated as:

$$\mu = e^{\mu_y + \frac{\sigma_y^2}{2}} \quad (4.1)$$

$$\sigma = \mu \sqrt{e^{\sigma_y^2} - 1} \quad (4.2)$$

where:

$$\mu_y = \frac{1}{N} \sum_{i=1}^N \ln M_i, \quad (4.3)$$

$$\sigma_y^2 = \frac{1}{N} \sum_{i=1}^N (\ln M_i)^2 - \mu_y^2, \quad (4.4)$$

$M_i$  is a modulus value and  $N$  is the number of measurements.

- The correlation length is the maximum distance over which the significant correlation between measurements exist. In districts' summaries this is the average value for a given district. In the state summary this value is the state average.

- Short scale variability is the total variability realized below 0.1 mile scale. In districts' summaries this is the average value for a given district. In the state summary this value is the state average. The causes of this variability are: measurement repeatability, temporal variability and spatial variability on 0.1 mile scale.
- Medium scale variability is the total variability realized on the one mile scale. In districts' summaries this is the average value for a given district. In the state summary this value is the state average.

The average correlation length, short and medium scale variability for the each district are presented in Table 4.1.



<b>District</b>	<b>Short Scale Variability (MPa)</b>	<b>Medium Scale Variability (MPa)</b>	<b>Correlation Length (miles)</b>
1	45.3	50.8	7.2
2	34.7	39.8	11.5
3	42.5	50.0	2.5
4	34.4	40.3	4.8
5	31.0	37.0	2.3
6	41.4	49.0	2.8
7	27.1	32.0	2.1
8	30.1	35.1	2.9
9	45.7	55.4	1.2

Table 4.1: Average short scale variability, medium scale variability and correlation length for the each district

## 4.2 Modulus Design-Value

The common selection of the design value is the mean minus one standard deviation. This value is based on an assumed bell-shaped normal distribution. In that case, the mean minus one standard deviation is associated with the 16th percentile.

For many roads presented in this atlas, the distribution is not normal. However, the differences between the true distribution and the normal distribution are much less important than the fact that the modulus changes in space. One can, for example, use the moving average minus one moving standard

deviation as a design value for a specific project. If these curves change significantly over the project area, the engineer has to make a decision based on the spatial distribution of the modulus.

The moving average curves are presented whenever the amount of data necessary to perform the analysis was available. A simple mean minus one standard deviation rule is rigid, especially now that so much information is available.

#### Example

If an engineer has to choose the modulus design value for a 5 mile project on Highway 18 in District 3 between reference post 20 and reference post 25, the design value of 120 MPa seems like a reasonable choice (page E-78).

Even though there are no data available for that section of the road, the selection, based on the modulus distribution in the neighboring sections of the same road, seems possible.

### 4.3 Design of the Sample Spacing

The selection of the sample spacing for future FWD measurements depends on the spatial variability of the data and required precision of estimation. The variability of the data is only available for the existing FWD records. If a decision about sample spacing has to be made for a new road, it is the best to use the information from the neighboring roads, or the district average if nothing else is available. Of course, the best is to have the variability information for that particular road in that district, and, quite often, an engineer can find this information in the Atlas.

The criterion used for the design of sample spacing is:

$$\text{Probability [ | true mean - mean estimated as arithmetic average | } \geq \text{ acceptable error ] } \leq \alpha$$

The first step of the design is to select the precision of the estimation by selecting the acceptable error in the estimation of the mean modulus value (in MPa) and probability  $\alpha$  that the 'true' error is larger than the acceptable one. Smaller probability  $\alpha$  and smaller acceptable errors will result in a stricter sampling design (more samples per mile are needed). Based on these two decisions, one obtains the tolerated variability T from Table 4.2.

The next step is based on the tolerated variability T, and short and medium scale variability that apply for the specific project. Using one of the figures (4.1) to (4.8) (depending on the project length) one obtains the total number of samples required to achieve requested precision. The number of samples per mile is then obtained by dividing the total number of samples with the project length.

Note, the two graphs on each of the figures (4.1) to (4.8) differ only in the scale.

#### Example

If an engineer has to choose the modulus design value for a 5 mile project on Highway 18 in District 3 between reference post 20 and reference post 25, the design value of 120 MPa seems like a reasonable choice.

However, because there are no data available for that section, the decision was made to perform new FWD measurements to obtain the modulus distribution. To find the spacing between FWD measurements he selects the acceptable error equal to 10 MPa and  $\alpha$  (probability that the 'true' error is larger than the acceptable error) equal to 0.1. From the Table 4.2, the tolerated variability T is 6.1 MPa.

In the Atlas on page E-78 he finds that the short scale variability for this road in this district is 36.1 MPa and medium scale variability is 37.1 MPa. Because the project length is 5 miles, the appropriate figure for selection of the number of samples is Figure (4.3). The values that are required for the Figure (4.3) are:  $36.1/6.1 = 5.9$  and  $37.1/6.1 = 6.1$ . From Figure (4.3) the minimum number of samples equals 35. The number of samples per mile is 7. Therefore, the approximate sample spacing is 0.15 miles.

$\alpha$	Acceptable Error (MPa)							
	5	10	15	20	25	30	35	40
0.01	1.9	3.9	5.8	7.8	9.7	11.6	13.6	15.5
0.02	2.1	4.3	6.4	8.6	10.7	12.9	15.0	17.2
0.05	2.6	5.1	7.7	10.2	12.8	15.3	17.9	20.4
0.10	3.0	6.1	9.1	12.2	15.2	18.2	21.3	24.3
0.15	3.5	7.0	10.4	13.9	17.4	20.9	24.3	27.8
0.20	3.9	7.8	11.7	15.6	19.5	23.4	27.3	31.2
0.25	4.4	8.7	13.1	17.4	21.8	26.1	30.5	34.8
0.30	4.8	9.7	14.5	19.3	24.2	29.0	33.8	38.7
0.35	5.4	10.7	16.1	21.5	26.8	32.2	37.5	42.9
0.40	6.0	11.9	17.9	23.8	29.8	35.8	41.7	47.7
0.45	6.6	13.3	19.9	26.6	33.2	39.9	46.5	53.1
0.50	7.4	14.9	22.3	29.8	37.2	44.7	52.1	59.5

Table 4.2: Tolerated variability  $T$  in MPa for different acceptable errors and probabilities  $\alpha$

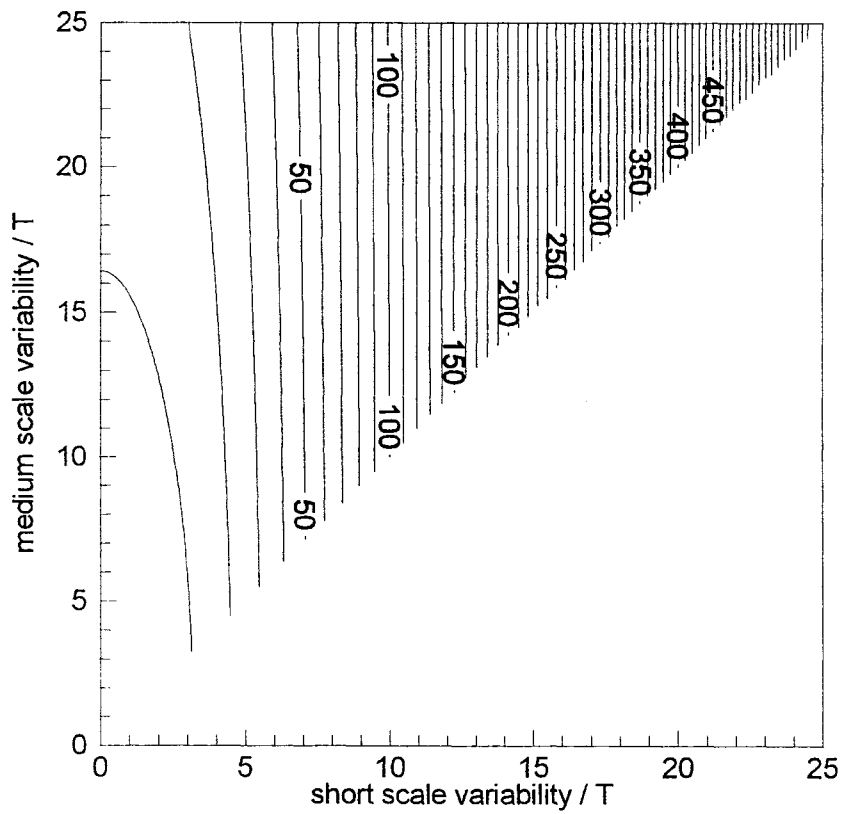
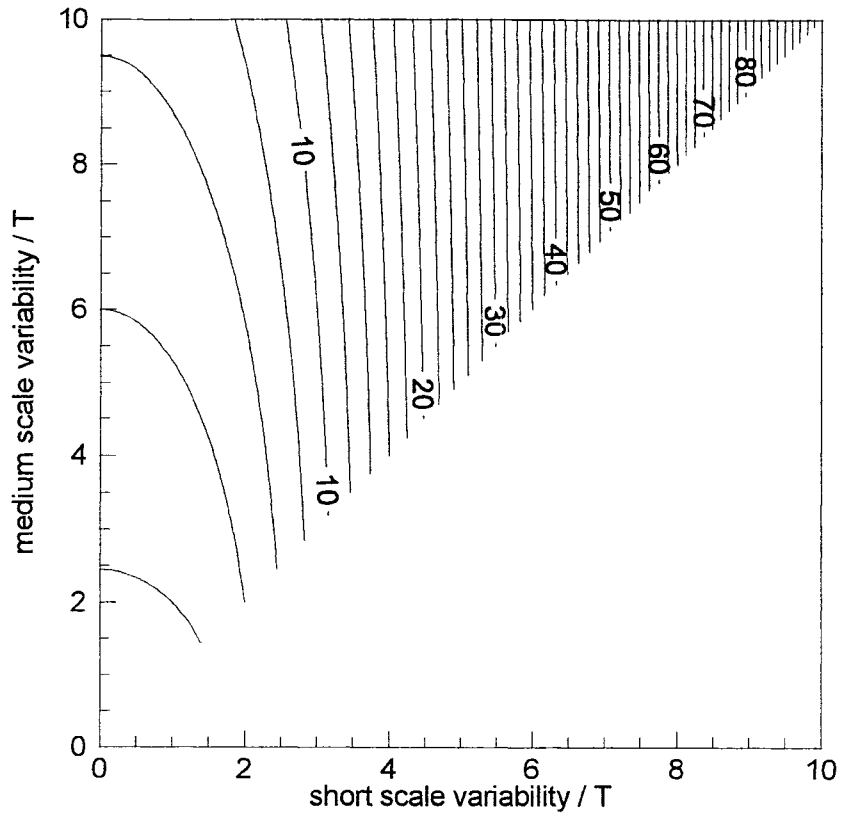


Figure 4.1: Number of samples for the project scale of 1 mile

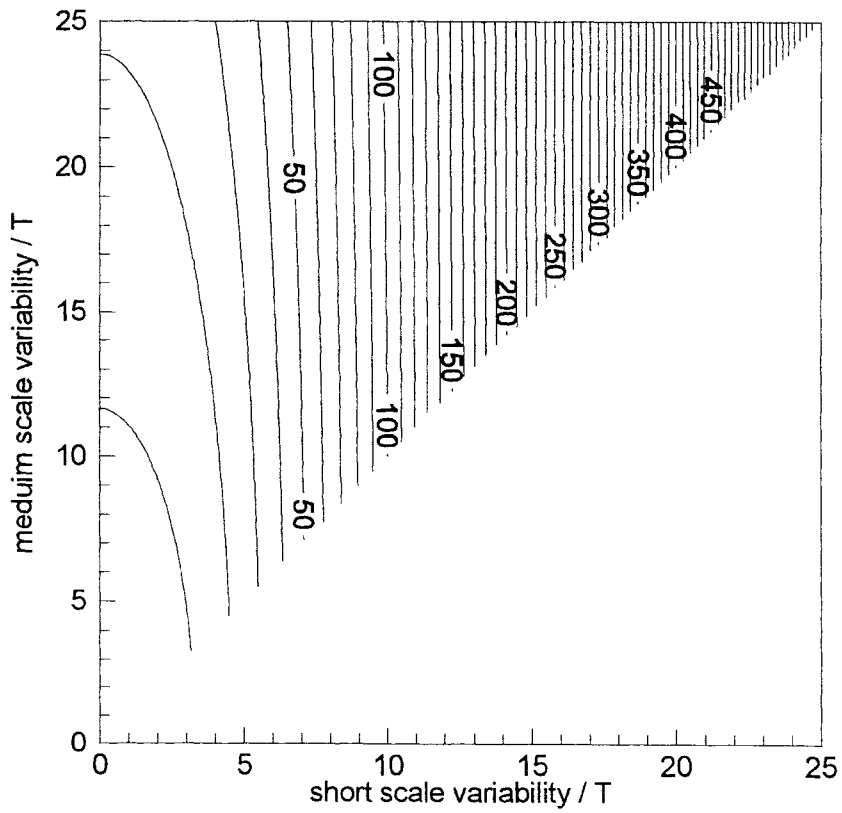
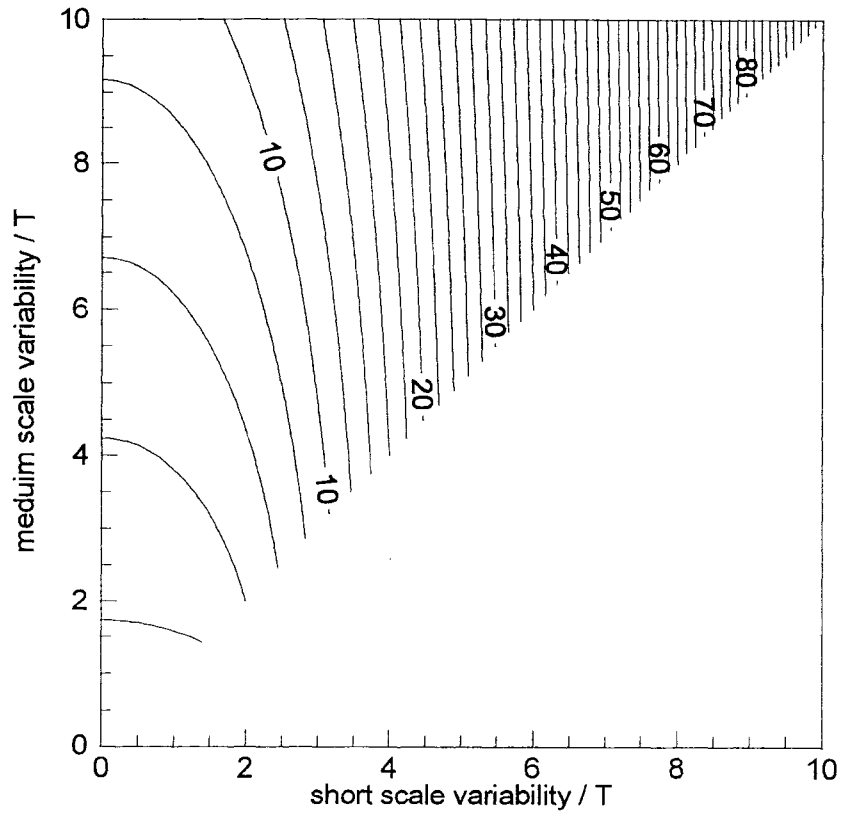


Figure 4.2: Number of samples for the project scale of 2 miles

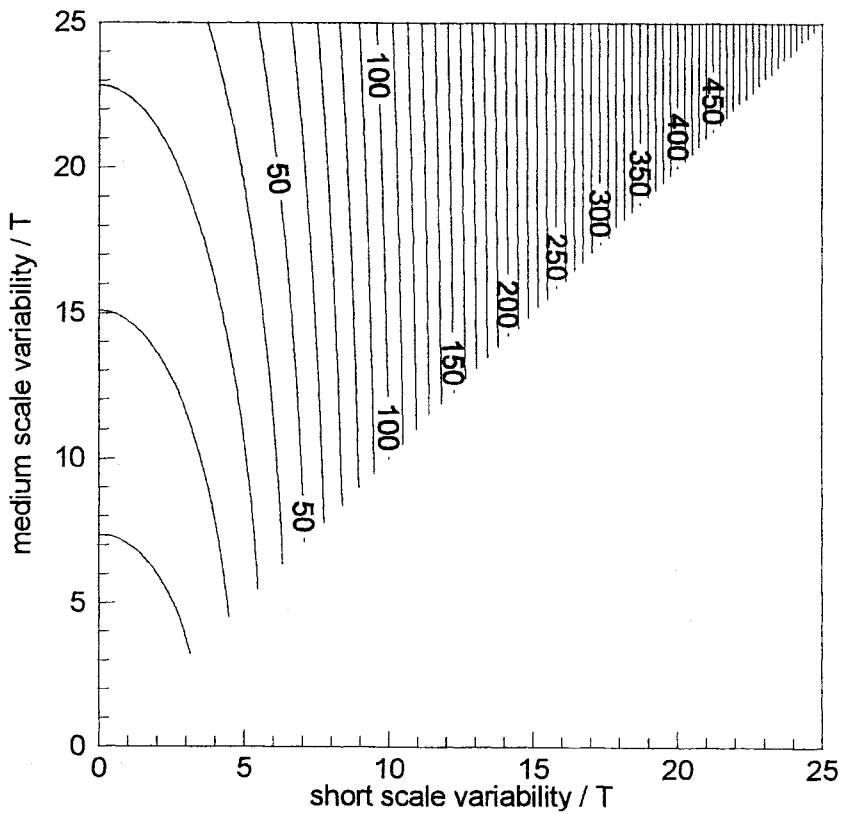
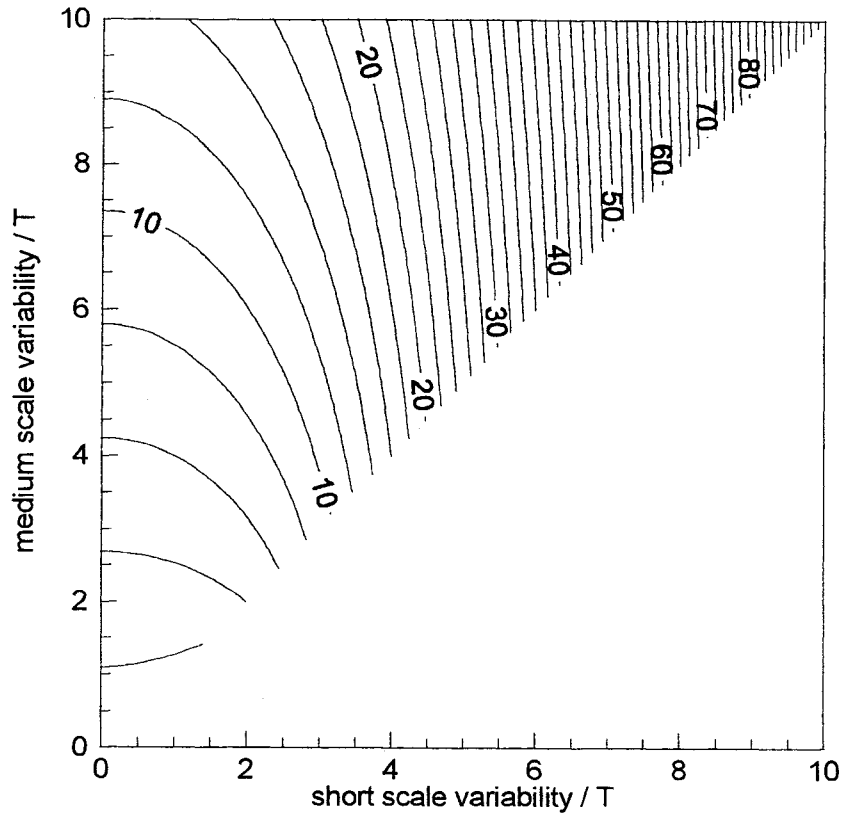


Figure 4.3: Number of samples for the project scale of 5 miles

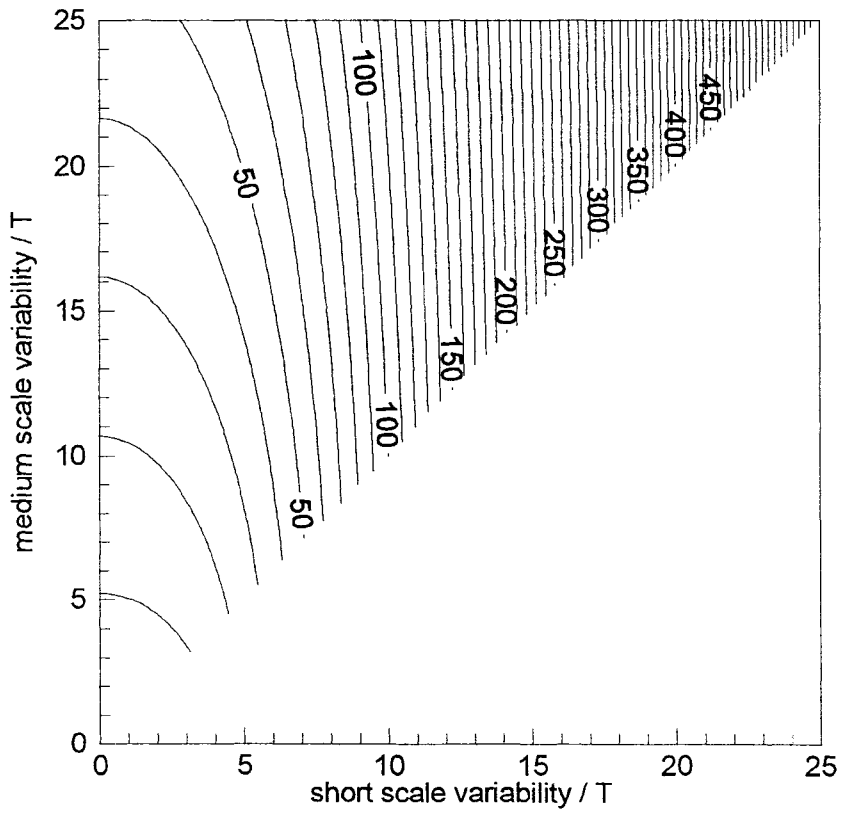
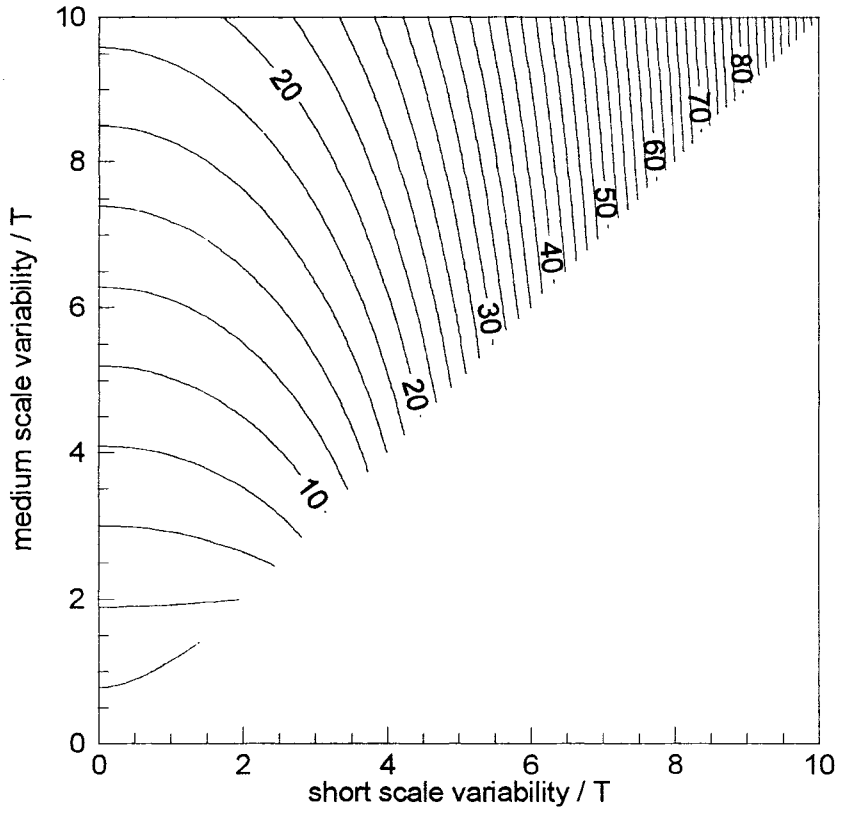


Figure 4.4: Number of samples for the project scale of 10 miles



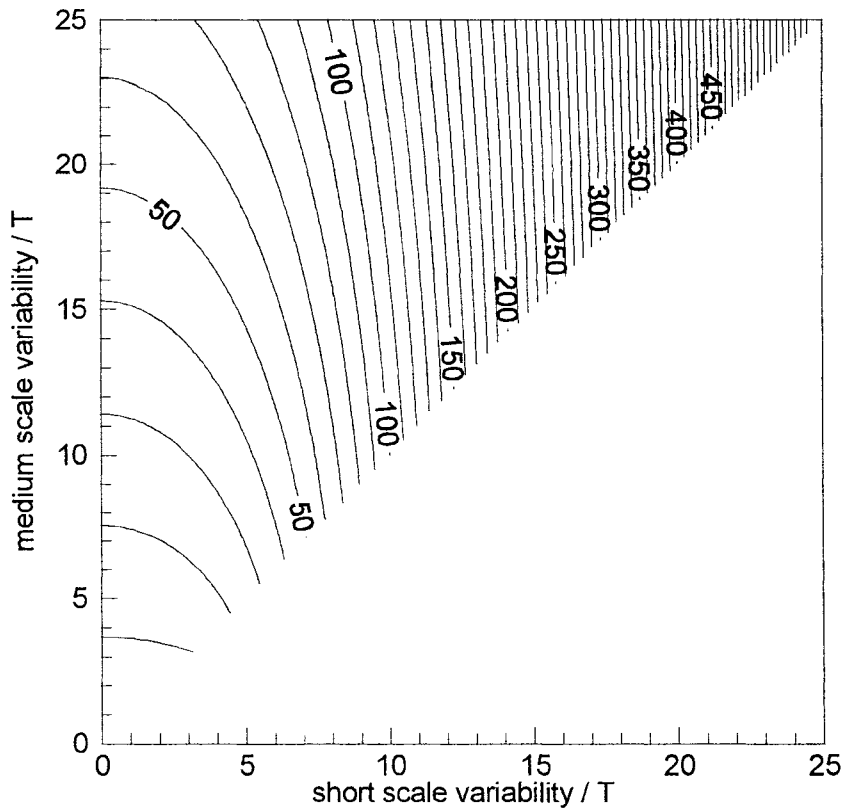
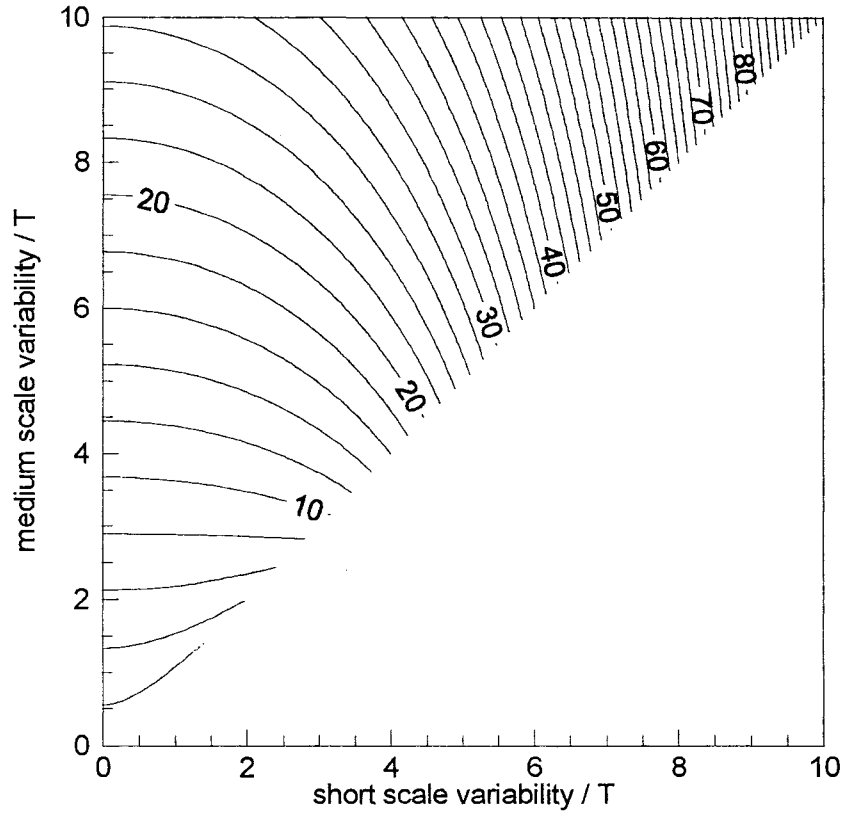


Figure 4.5: Number of samples for the project scale of 20 miles

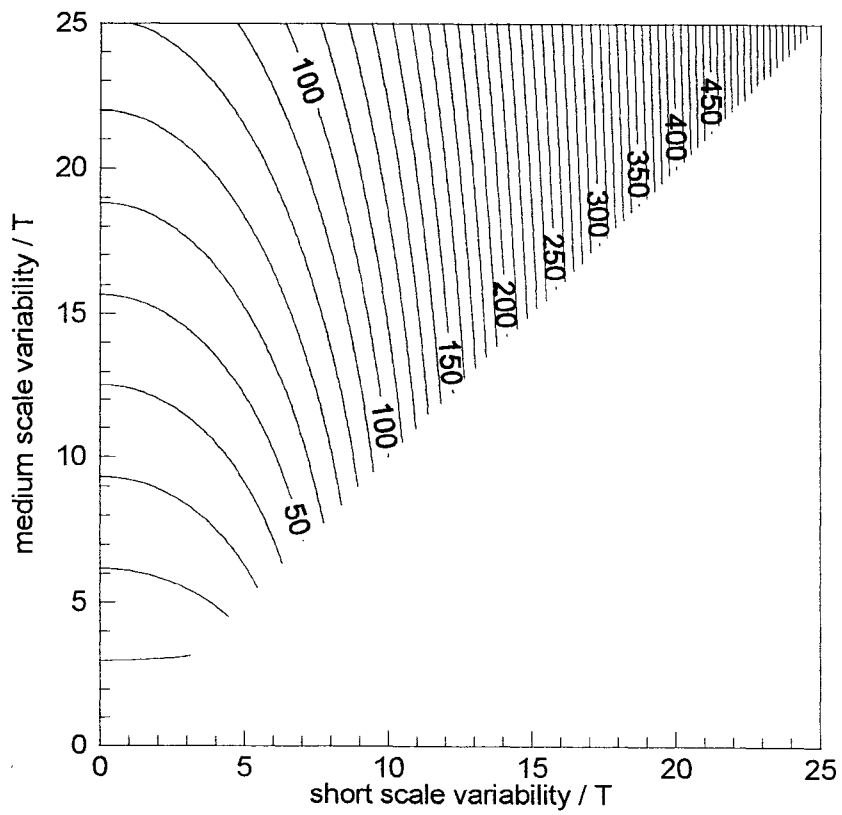
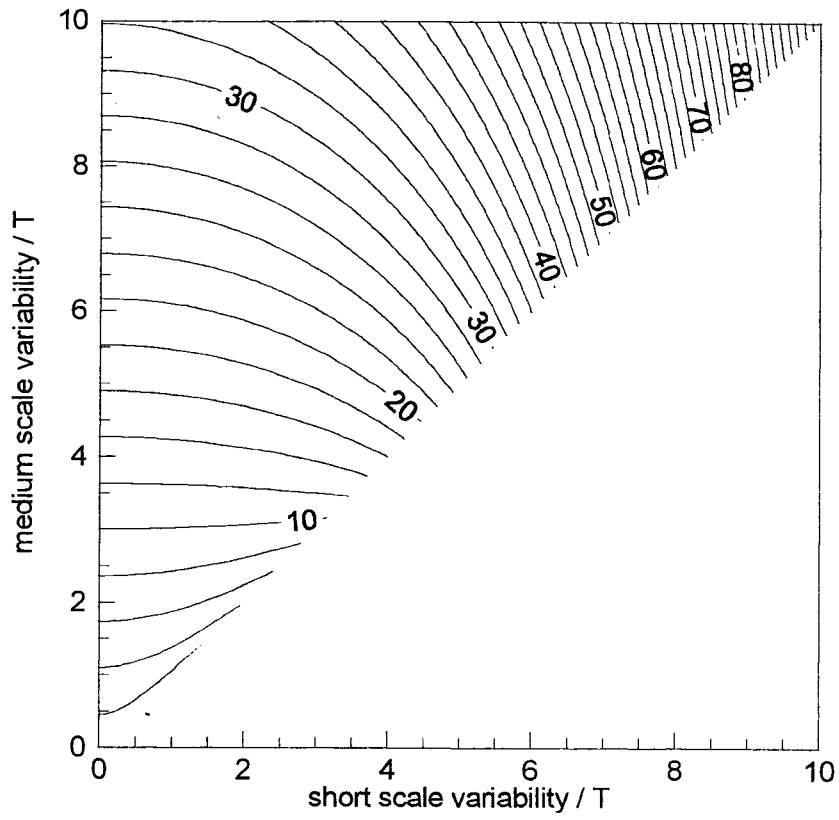


Figure 4.6: Number of samples for the project scale of 30 miles

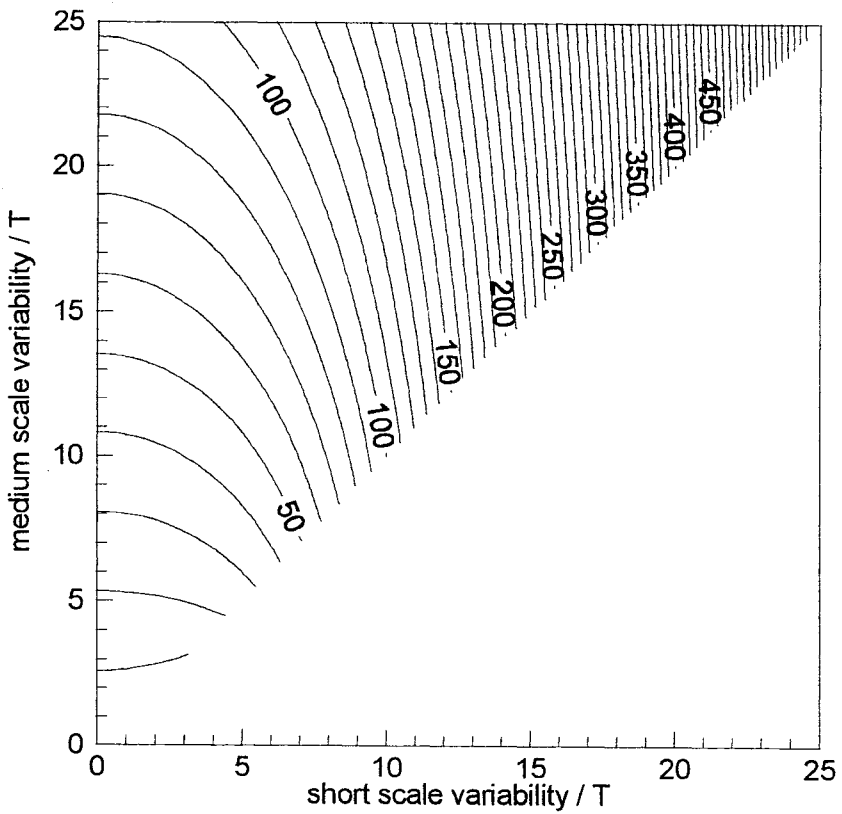
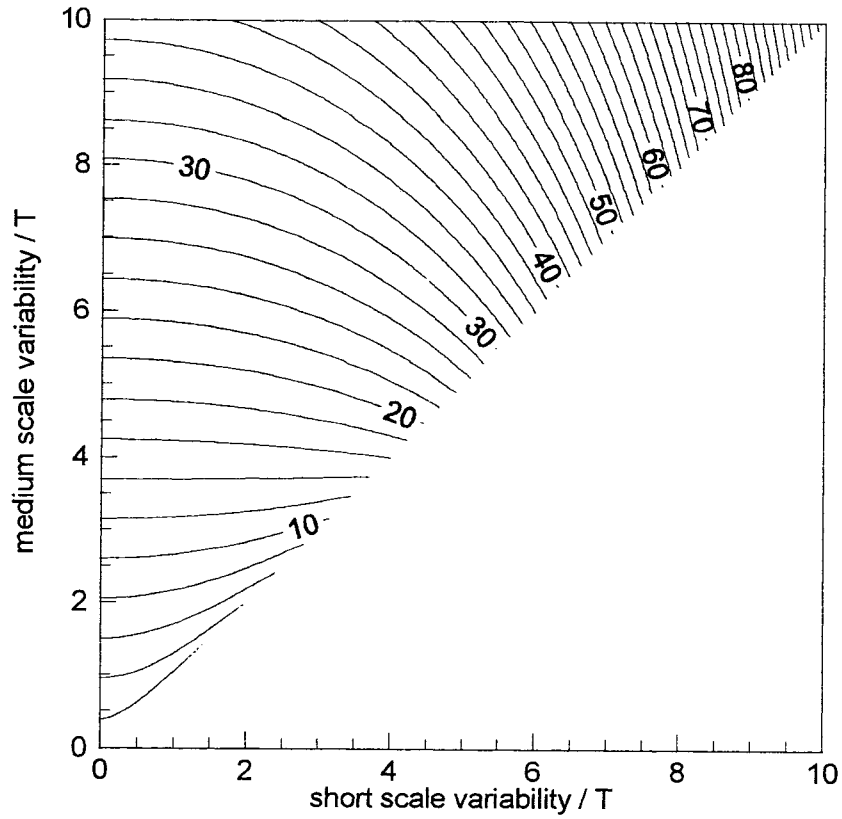


Figure 4.7: Number of samples for the project scale of 40 miles

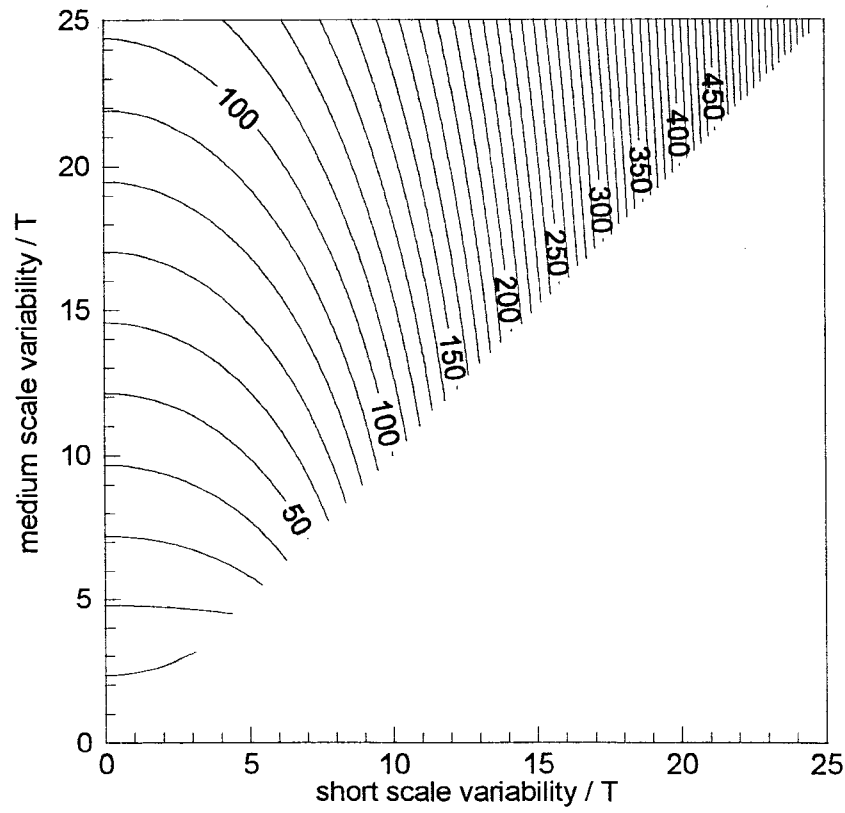
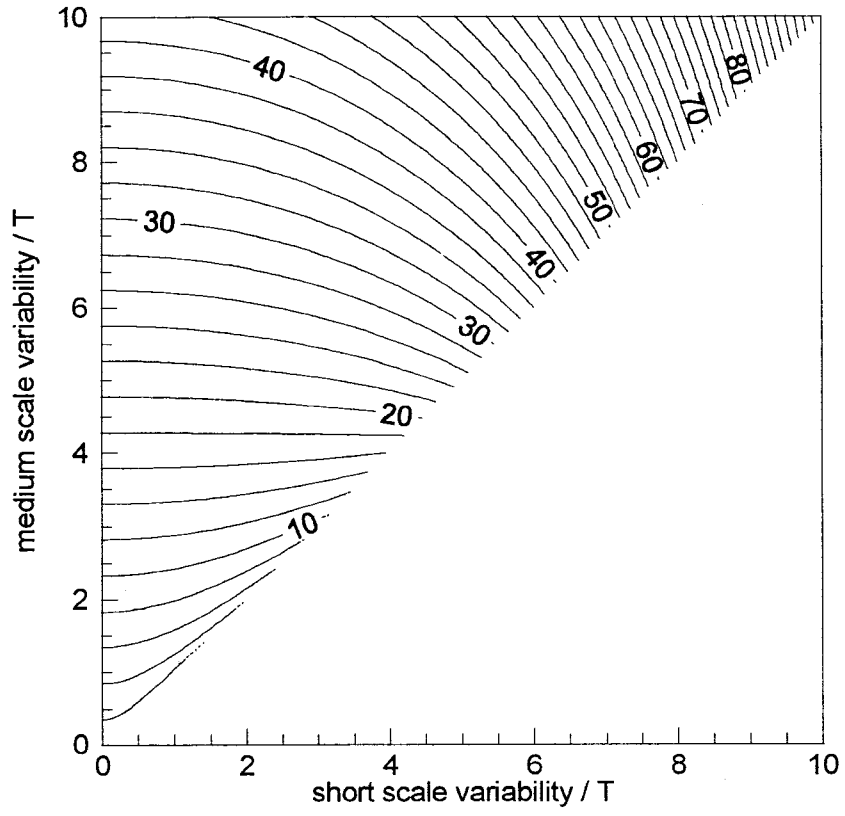


Figure 4.8: Number of samples for the project scale of 50 miles

## Chapter 5

# Software support

Besides the hard copy of the Atlas (this document), three computer programs were also developed and are available for general use. These are:

- *Minnesota Subgrade Modulus - Database Manager* converts FWD records to the modulus database file used in the other programs. This management code screens the data files for possible typographic errors and computes the modulus. The modulus is then written in the modulus database file. The database manager is able to process FWD records from 1983 to 1999 and can be used for the future updates of the database.
- *Minnesota Subgrade Modulus - Page Generator* is a MS/DOS program is used to convert the modulus database to the printed version of the Atlas. This code was used to produce all the appendices of the Atlas.
- *Minnesota Subgrade Modulus Atlas - Online* is a Microsoft Windows 3.1 program that can be used to statistically analyze and visualize the available subgrade modulus data for approximately 180 Minnesota roads and highways. The user specifies the state highway by number, the Minnesota Department of Transportation (Mn/DOT) District or Districts of interest, the years and months of interest, and the reference post interval (from/to) of interest. The program then computes and presents a statistical summary of the selected data and associated plots. This code and the manual are available upon request from Mn/DOT.

