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The SAFETRUCK program focuses on preventing accidents on rural highways, especially those associated with run-off-the-road incidents and driver fatigue, by giving the vehicle the ability to steer to the side of the road and come to a safe stop if the driver falls asleep or is otherwise incapacitated. Researchers have equipped a Navistar 9400 series class 8 truck tractor with the sensors and control computers necessary to perform this task.

Designing the controller that will steer the truck requires a mathematical model of the lateral response of the truck to steering inputs. In this project, researchers developed a lateral dynamic model by incorporating second order dynamics into the steering axle tires. Simulation of the resulting models indicated dynamic behavior that was close to the experimental data for speeds between 15 and 30 miles per hour. This is the first time that a lateral dynamic model of a truck has been experimentally verified. Both models, however, resulted in experimentally determined values for steering axle cornering stiffness that were considerably smaller than published values for the Goodyear G159 tires on the truck.

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A Lateral Dynamic Model of a Tractor-Trailer: Experimental Validation

Final Report

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EXECUTIVE SUMMARY

The SAFETRUCK program is a research project oriented towards preventing accidents on rural highways, especially those associated with run-off-the-road incidents and driver fatigue, by giving the vehicle the ability to steer to the side of the road and come to a safe stop if the driver falls asleep or is otherwise incapacitated. The University of Minnesota, in cooperation with the Minnesota Department of Transportation, is equipping a Navistar 9400 series class 8 truck tractor with the sensors and control computers necessary to perform this task. In order to design the controller that will steer the truck, a mathematical model of the lateral response of the truck to steering inputs is required. Using a Kalman filter, this model will also be used to determine a best estimate of the truck's state variables given noise in the sensors and the possibility of sensor drop out (e.g., loss of Global Positioning System signals.)

We developed a basic lateral dynamic model by adding one more axle (the rear tandem axle) to a standard automobile model [1], and then enhanced that basic model by incorporating second order dynamics into the steering axle tires. Probably the most important and most difficult part of modeling a highway vehicle is finding an accurate model of tires and their interaction with the road. Pneumatic tires in general are not mathematically tractable if extreme cornering forces are imposed, but since our present mission involves gently driving the truck over to the side of the road we felt that it was sufficient to use a simple linear tire model where the lateral force of the tire on the truck is directly proportional to the slip angle of the tire. The slip angle is the difference between the direction in which the tire is pointed and the direction in which it is actually moving. The constant of proportionality between the slip angle and the lateral force generated is called the cornering stiffness. In the enhanced model we modify the response of the front tires to include a slight delay (a second order lag) between the time at which the wheels are steered and when the resulting side force is generated.

The cornering stiffness of the tires, and various parameters of the second order enhanced model have widely varying values that depend on, among other things, the road surface and the load that the truck is carrying. This makes it unlikely that a handbook value for any of these parameters would correspond to the actual situation of our test vehicle. We experimentally determined values for these parameters by running a series of random steering tests [2] over a range of different forward speeds. This test consists of weaving the truck back and forth at a gradually increasing frequency while recording the steering wheel position and the resulting yaw rate. The time domain data recorded in the steer test is converted to the frequency domain using a Fast Fourier Transform (FFT). The ratio of the FFT of the yaw rate to the FFT of the steering angle results in an experimental transfer function. The parameters for both the basic and the enhanced models were varied until their transfer functions matched the experimental transfer function as closely as possible.

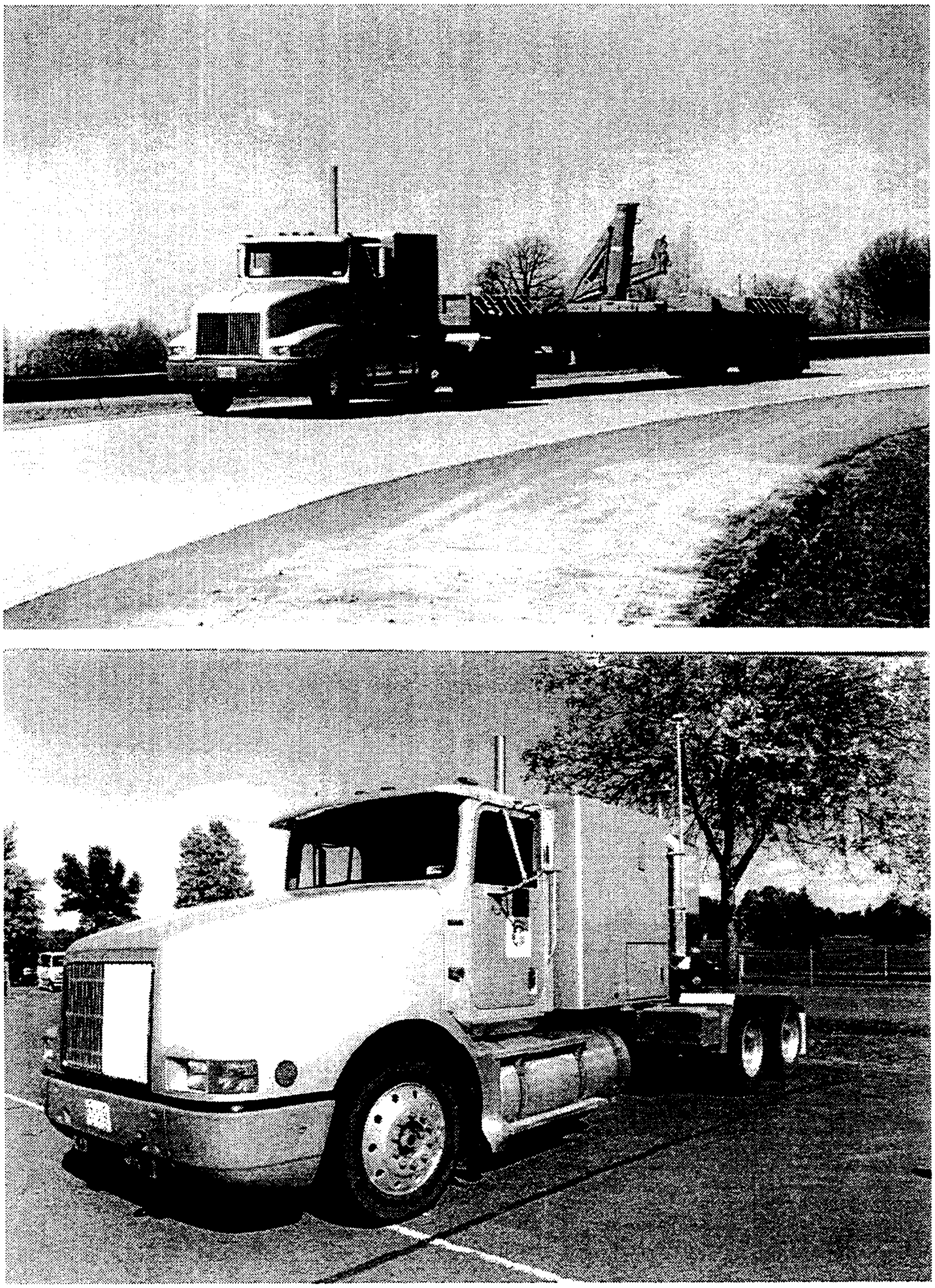
Simulation of the resulting models indicated dynamic behavior that was close to the experimental data for speeds between 15 and 30 mph. The MnROAD track configuration did not allow for experiments at higher speeds. The enhanced model was slightly more accurate at higher frequencies (when cycling the steering back and forth faster than once per second). Both models resulted in experimentally determined values for steering axle cornering stiffnesses that were considerably smaller than published values for the Goodyear G159 tires on the truck. This is due to the fact that the tires in the model had to account for the suspension and steering gear compliance that were not otherwise incorporated into the model.

CHAPTER 1

INTRODUCTION

This chapter describes the mathematical model of the Navistar 9400 truck tractor (see Figure 1.1) and Fruehauf semi-trailer that are being developed for use in the SAFETRUCK program. The purpose of this program is to develop a control system for a truck that will assist a driver if he or she has become incapacitated for any reason. By detecting the erratic behavior of a vehicle typically associated with drowsy drivers, various strategies can be implemented including assistance with lane keeping. Since driver alarms and warnings are typically unable to sufficiently arouse the driver, we focused on an aggressive intervention strategy, i.e., systems that can automatically steer the vehicle to the side of the road and bring it to a safe stop. This approach can be used not only if the driver falls asleep, but also when he or she is driving under the influence of alcohol, drugs or medication.

A mathematical model is needed for at least four reasons. First, the model and its precursors were used to design the control system that steers the truck. Second, the truck's Inertial Measurement Unit (IMU) incorporate a number of sensors that are integrated through the use of a Kalman filter. The Kalman filter requires a computerized model of the vehicle to help filter the noise out of the sensor array and arrive at a best estimate of the current position, orientation and velocity of the truck. Thirdly the model is used in the laboratory to simulate the moving truck in order to examine concepts such as the virtual bumper and allows us to experiment with different radar mounting locations on the truck. Furthermore the ability to run the model on the real-time computers on the truck while the truck is being steered, provides us with a means for comparing predicted with actual behavior and thus facilitates fault detection in the sensors and control systems - an additional safety mechanism. Since our current research primarily involves steering the truck to maintain its proper position in a traffic lane, the model we present here is for lateral control only. Future work will include a longitudinal model for braking and acceleration.



The Navistar 9400 used for this study.

CHAPTER 2

A MATHEMATICAL MODEL OF THE SAFETRUCK TRACTOR-TRAILER COMBINATION

SURVEY OF HEAVY VEHICLE MODEL LITERATURE

Our evaluation of the literature indicated that although there have been attempts at modeling the lateral dynamics of heavy vehicles, few if any of these models have been evaluated using an actual truck. When we attempted to verify these models on our truck, we found that they did not match up with the truck's actual dynamics. In this report we will document a model that we developed and tested that does provide adequate fidelity with the vehicle's true dynamic behavior. The tractor-trailer model we are using is similar to the standard automobile lateral model described in Wong [1] with extra axles and a "fifth wheel" hitch added to transform the two axle car model into a five axle truck tractor-semitrailer model. El-Gindy [3] presented a similar model along with a method for reducing the number of states required, by replacing the trailer with a mass located at the fifth wheel hitch. We found a number of errors in the El-Gindy article that we corrected before proceeding (El-Gindy confirmed these in response to our inquiry), but essentially this model was the basis upon which we added a number of additional features.

TIRES

In most lateral vehicle models including those in Wong [1] and El-Gindy [3], the vehicle is modeled as a rigid body acted on by forces generated by the interaction of its tires with the road surface. Pneumatic tires are quite difficult to model accurately. There have been a number of complex tire models developed over the years. Some of the most comprehensive work done recently includes the analytical models developed at the University of Arizona [4] and the "Magic Tire formula" [5] which uses experimentally determined parameters. Since we are going to use the model in real time and therefore require solutions that can be computed quickly we will initially use the same simplification that Wong and El-Gindy use and assume that the side force generated from a tire is directly proportional to its slip angle. We will call this our "basic" model. It is valid for small slip angles. The slip angle is the difference between the direction in which the tire is pointed and the direction in which it is actually moving. We will also make one enhancement to our basic model by modifying the front steering tire equation with a second order differential equation that will account for the fact that the side force is not generated instantaneously as the tire is turned to a new heading. This approach is reported by Heydinger, Garrot and Christos in [6]. We will call this our "enhanced" model.

The constant of proportionality between the slip angle and the lateral force generated is called the cornering stiffness. Any time a vehicle on pneumatic tires resists a side force, for instance due to the centrifugal force generated when cornering, a slip angle is generated by that force. The cornering stiffness of a tire is dependent on a variety of factors such as the type and condition of the road surface, the vertical load on the tire, the internal construction of the tire, and its inflation pressure. In the next chapter, we will describe the experimental procedure we used to find values for the cornering stiffness for each of the axles on the SAFETRUCK vehicle.

MODEL EQUATIONS

A schematic of the tractor-trailer is shown in Figure 2.1 and the various equations of motion and of the geometrical relationships follow.

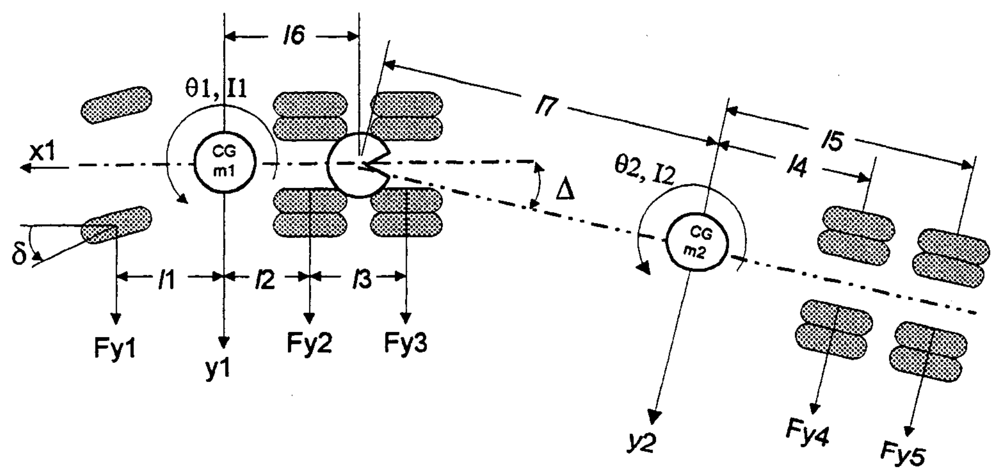


Figure 2.1 Schematic of the SAFETRUCK model showing forces and dimensions used in the model equations.

Parameters to be determined experimentally:

C_n	Sum of cornering stiffness for all tires on axle n, newtons/radian
ω_n	Natural path frequency of front axle tires, radians/meter
ξ	Damping ratio of front axle tires

List of known constants (measurements from the actual vehicle):

m_1	Mass of the Navistar 9400 tractor - 9053 kg
I_1	Yaw moment of inertia of the tractor - 52161 kg m^2
m_2	Mass of the trailer in the 80,000 lb. configuration - 27361 kg
I_2	Yaw moment of inertia of loaded trailer - 767667 kg m^2
l_1	Distance from tractor CG to steering axle - 2.59 meters
l_2	Distance from tractor CG to front tandem axle - 2.70 meters
l_3	Distance from tractor CG to rear tandem axle - 4.02 meters
l_4	Distance from trailer CG to rear trailer axle - 4.17 meters
l_5	Distance from trailer CG to rear trailer axle - 5.41 meters
l_6	Distance from tractor CG to 5th wheel hitch - 3.36 meters
l_7	Distance from trailer CG to 5th wheel hitch - 6.32 meters

Masses were measured at MnDOT's Lakeville truck scale. Moments of inertia were calculated from UMTRI measurements of a similar truck augmented by information from Navistar.

Other variables in the equations:

$\dot{x}_1$	Forward velocity of the tractor, meter/sec
$\dot{y}_1$	Lateral velocity of the tractor, meter/sec
$\ddot{y}_1$	Lateral acceleration of the tractor, meter/sec^2
$\dot{x}_2$	Forward velocity of the trailer, meter/sec
$\dot{y}_2$	Lateral velocity of the trailer, meter/sec
$\ddot{y}_2$	Lateral acceleration of the trailer, meter/sec^2
α_{axn}	Slip angle for tires on axle n, radians
$\dot{\alpha}_{ax1}$	First derivative of front tire slip angle, radians/sec
$\ddot{\alpha}_{ax1}$	Second derivative of front tire slip angle, radians/sec^2
F_{axn}	Lateral force at axle n, newtons
$\dot{\phi}_1$	Yaw rate of the tractor, radians/sec

$\ddot{\phi}_1$	Yaw acceleration of the tractor, radians/sec^2
$\dot{\phi}_2$	Yaw rate of the trailer, radians/sec
$\ddot{\phi}_2$	Yaw acceleration of the trailer, radians/sec^2
δ	Steering angle of front tires, radians
Δ	Articulation angle between tractor and trailer, radians

Note that since the system is nonholonomic it may not be considered strictly correct to use the dot notation which implies the ability to integrate the dotted quantities..

The equations of motion for the tractor are:

Summing forces in the lateral direction:

$$m_1\ddot{y}_1 + m_1\dot{x}_1\dot{\phi}_1 = F_{ax1} + F_{ax2} + F_{ax3} - F_{hitch}$$

Summing moments around the center of mass:

$$I\ddot{\phi}_1 = l_1F_{ax1} - l_2F_{ax2} - l_3F_{ax3} + l_6F_{hitch}$$

The equations of motion for the trailer are:

Summing forces in the lateral direction:

$$m_2\ddot{y}_2 + m_2\dot{x}_2\dot{\phi}_2 = F_{ax4} + F_{ax5} + F_{hitch}$$

Summing moments around the center of mass:

$$I_2\phi_2 = -l_4F_{ax4} - l_5F_{ax5} + l_7F_{hitch}$$

The hitch coupling equations are (assuming that the angle between the tractor and the trailer is small - i.e., a maximum of about 7 degrees when negotiating the 275 ft (84 meter) radius loops at the ends of the MnROAD track):

$$\dot{y}_2 + l_7\dot{\phi}_2 = \dot{y}_1 - l_6\dot{\phi} + \dot{x}_1\Delta$$

$$\ddot{y}_2 + \dot{x}_2\dot{\phi}_2 + l_7\ddot{\phi}_2 = \ddot{y}_1 + \dot{x}_1\dot{\phi}_1 - l_6\ddot{\phi}_1$$

The tire slip angles for the front (discussed here for both the basic and the enhanced models) and for the rear axles of the tractor and for the trailer axles are:

Tractor front axle - basic model:

$$\alpha_{ax1} = \delta - \frac{\dot{y}_1 + l_2\dot{\phi}_1}{\dot{x}_1}$$

Tractor front axle - enhanced model:

$$\ddot{\alpha}_{ax1} = -2\xi\dot{x}_1\omega_n\dot{\alpha}_{ax1} - \dot{x}_1^2\omega_n^2\alpha_{ax1} + \dot{x}_1^2\omega_n^2\left(\delta - \frac{\dot{y}_1 + l_1\dot{\phi}_1}{\dot{x}_1}\right)$$

Where ω_n is a spacial frequency term in units of radians/meter [6].

Tractor rear axles:

$$\alpha_{ax2} = -\frac{\dot{y}_1 - l_2\dot{\phi}_1}{\dot{x}_1} \qquad \alpha_{ax3} = -\frac{\dot{y}_1 - l_3\dot{\phi}_1}{\dot{x}_1}$$

Trailer axles:

$$\alpha_{ax4} = -\frac{\dot{y}_2 - l_4\dot{\phi}_2}{\dot{x}_2} \qquad \alpha_{ax5} = -\frac{\dot{y}_2 - l_5\dot{\phi}_2}{\dot{x}_2}$$

The lateral forces applied by the road to each axle are:

Tractor:

$$F_{ax1} = C_1\alpha_{ax1} \qquad F_2 = C_2\alpha_{ax2} \qquad F_{ax3} = C_3\alpha_{ax3}$$

Trailer:

$$F_{ax4} = C_4\alpha_{ax4} \qquad F_{ax5} = C_5\alpha_{ax5}$$

Rearranging the equations of motion for the basic model into matrix form results in:

$$M\dot{x} = Ax + Bu$$
$$z = Cx$$

Where:

$$M = \begin{bmatrix} m_1 & 0 & m_2 & 0 \\ 0 & I_1 & -l_6 m_2 & 0 \\ 0 & 0 & -l_7 m_2 & I_2 \\ 1 & -l_6 & -1 & -l_6 \end{bmatrix} \qquad x = \begin{bmatrix} \dot{y}_1 \\ \dot{\phi}_1 \\ \dot{y}_2 \\ \dot{\phi}_2 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{-C_1 - C_2 - C_3}{\dot{x}_1} & \frac{-C_1 l_1 + C_2 l_2 + C_3 l_3}{\dot{x}_1} - m_1 \dot{x}_1 & \frac{-C_4 - C_5}{\dot{x}_1} & \frac{C_4 l_4 + C_5 l_5}{\dot{x}_1} - m_2 \dot{x}_1 \\ \frac{-C_1 l_1 + C_2 l_2 + C_3 l_3}{\dot{x}_1} & \frac{-C_1 l_1^2 - C_2 l_2^2 - C_3 l_3^2}{\dot{x}_1} & \frac{-C_4 l_6 - C_5 l_6}{\dot{x}_1} & l_6 \left(\frac{-C_4 l_4 - C_5 l_5}{\dot{x}_1} + m_2 \dot{x}_1 \right) \\ 0 & 0 & \frac{C_4 l_7 + C_5 l_7 + C_4 l_4 + C_5 l_5}{\dot{x}_1} & \frac{-C_4 l_4 l_7 - C_5 l_5 l_7 - C_4 l_4^2 - C_5 l_5^2}{\dot{x}_1} + l_7 m_2 \dot{x}_1 \\ 0 & -\dot{x}_1 & 0 & \dot{x}_1 \end{bmatrix}$$

$$B = \begin{bmatrix} C_1 \\ C_1 l_1 \\ 0 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

By premultiplying both sides of the first equation by the inverse of the left 4x4 matrix **M**, one can form the final state space representation of the system. We used MATLAB™ to do this numerically. Note that the state vector x does not contain the forward velocity $\dot{x}_1$ since that would make the system nonlinear.

Rearranging the equations of motion for the enhanced model into matrix form results in:

$$M\dot{x} = Ax + Bu$$
$$z = Cx$$

Where:

$$M = \begin{bmatrix} 0 & \frac{I_1}{l_6} & 0 & 0 & 0 & -\frac{I_2}{l_7} \\ m_1 & 0 & 0 & 0 & m_2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & l_6 & 0 & 0 & 1 & l_7 \\ m_1 & 0 & 0 & 0 & 0 & \frac{I_2}{l_7} \end{bmatrix} \qquad x = \begin{bmatrix} \dot{y}_1 \\ \dot{\phi}_1 \\ \alpha_{ax1} \\ \dot{\alpha}_{ax1} \\ \dot{y}_2 \\ \dot{\phi}_2 \end{bmatrix}$$

$$A = \begin{bmatrix} \frac{l_2c_2 + l_3c_3}{l_6\dot{x}_1} & -\frac{l_2^2c_2 + l_3^2c_3}{l_6\dot{x}_1} & \frac{l_1c_1}{l_6} & 0 & \frac{l_4c_4 + l_5c_5}{l_7\dot{x}_1} & \frac{l_4^2c_4 + l_5^2c_5}{l_7\dot{x}_1} \\ -\frac{c_2 + c_3}{\dot{x}_1} & \frac{l_2c_2 + l_3c_3}{\dot{x}_1} - m_1\dot{x}_1 & c_1 & 0 & -\frac{c_4 + c_5}{\dot{x}_1} & \frac{l_4c_4 + l_5c_5}{\dot{x}_1} - m_2\dot{x}_2 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -\dot{x}_1\omega_n^2 & -l_1\dot{x}_1\omega_n^2 & -\dot{x}_1\omega_n^2 & -2\xi\dot{x}_1\omega_n & 0 & 0 \\ 0 & \dot{x}_1 & 0 & 0 & 1 & -\dot{x}_2 \\ -\frac{c_2 + c_3}{\dot{x}_1} & \frac{l_2c_2 + l_3c_3}{\dot{x}_1} - m_1\dot{x}_{11} & c_1 & 0 & \frac{l_4c_4 + l_5c_5}{l_7\dot{x}_1} & -\frac{l_4^2c_4 + l_5^2c_5}{l_7\dot{x}_1} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \dot{x}_1\omega_n^2 \\ 0 \\ 0 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad u = [\delta] \qquad z = \begin{bmatrix} \dot{y}_1 \\ \dot{\phi}_1 \end{bmatrix}$$

Once again the final state space model is formed by premultiplying both sides of the first equation by the inverse of the left 6x6 matrix **M**.

CHAPTER 3

FITTING PARAMETERS TO A MATHEMATICAL MODEL OF A SEMI-TRUCK USING A RANDOM STEERING TEST

The mathematical models of the truck that were developed in the previous chapter have several parameters that must be experimentally determined. Both the basic and the enhanced models need values for the cornering stiffness at each axle. The enhanced model also requires a damping coefficient and a natural frequency for the front steering axle. There are two reasons why these parameters must be determined experimentally. First, they have widely varying values that depend on the actual conditions under which the vehicle is operating, so it is not likely that a published "handbook" value would be appropriate for our purposes. Second, since the models themselves are not exact, it is possible that a parameter value somewhat different from its actual real world value might be able to compensate for approximations in the model and make the model's response better fit the response of the real vehicle. The experiment that we performed to determine these parameters is called a Random Steering Test. It is widely used in the automobile industry to characterize the dynamics of vehicles under development [2]. Using the data from the steering test as a target, we computed values for our model's parameters that made the model respond as closely as possible to the actual truck. The steering test and the parameter fit are described in the next two parts of this chapter.

RANDOM STEERING TEST

Procedure

This part of the experiment involves driving the truck down a straight section of the test track at a constant speed while weaving back and forth across the lane using a steering input approximating a sine wave that varies in frequency from close to zero to as fast as the driver can move the wheel. The position of the steering wheel was recorded using a potentiometer connected to an analog to digital converter card in a computer based data acquisition system. The yaw rate of the truck tractor was measured with a fiber optic rate gyro mounted in the cab of the truck and connected to the same computer. Several runs were made at speeds ranging from 15 to 45 mph with the trailer loaded to the 80,000 lb. configuration.

Equipment

The rate gyro we used was an Andrews 3ARG-D AUTOGYRO™ with digital output. This unit is a single axis interferometric fiber optic gyroscope designed to be used in land based navigation systems. It produces a digital output of the incremental angular rotation every tenth of a second. Our sampling rate for the test was therefore limited to 10 Hz. Since we are interested in the response of the system to steering inputs approaching 3 Hz (the practical limit with a human doing the steering) there is not much margin between our maximum frequency and the Nyquist limit of 5 Hz or half of the gyro's output rate.

The potentiometer was connected to the steering column with two small pulleys and a string kept in tension by a spring. This setup limited the amount of recordable steering input to about 100 degrees on either side of straight ahead since the spring could not negotiate the pulleys at either end of the mechanism.

Results

A sample of the data collected in one run of the experiments is graphed in figure 3.1. The results show that, for a forward velocity of 35 mph, the response of the tractor-trailer (the small signal in the center of the graph) is fairly constant until a steering frequency of approximately 2 Hz is reached. The response then rolls off to nearly 0 by the time the steering input reaches 2.5 Hz. Therefore there is a definite attenuation of the response when the forcing frequency is within the range that we can reach with manual steering input. Variations in the steering wheel input amplitude result from the driver attempting to stay on the test track during the experiment, and at the high frequency end, by his limited ability to move the steering wheel far enough at those frequencies.

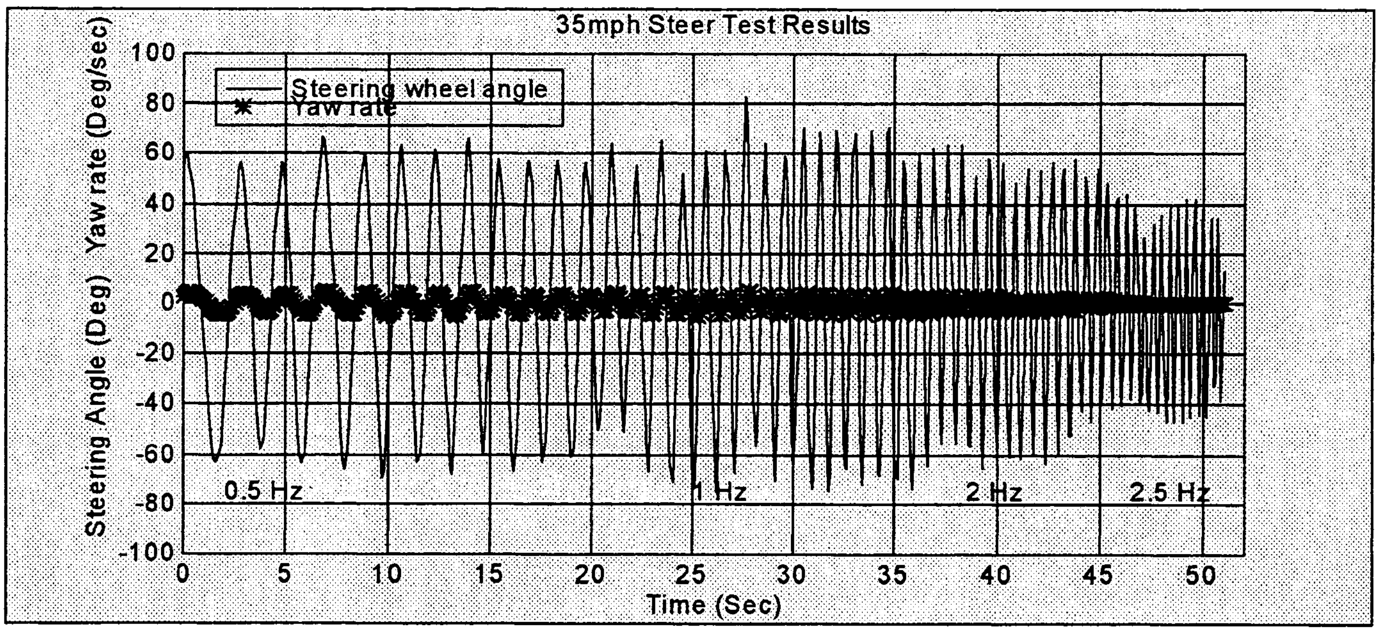


Figure 3.1 Plot of raw data from a 35 mph steer test.

PARAMETER FIT

Experimental Transfer Function

The Fourier transform of a signal is a means of looking at its frequency content or at how much of each particular frequency is present in the signal. The Fourier transforms of the input (steer angle) and output (yaw rate) are graphed in figure 3.2. The ratio of the Fourier transform of the yaw rate to the Fourier transform of the steering wheel position gives the experimental transfer function of our system. We developed a transfer function for each speed since the model is non-linear with respect to speed. The objective of the parameter fit is to find values for cornering stiffness and other model variables such that the Bode plot of the model is a close fit with the Bode plot of the experimental transfer function. To do this we concentrated on the Bode amplitude plot as shown in figure 3.3. The Bode amplitude plot graphs the gain of the system (output/input) over a range of input frequencies. We used an optimization routine (described in the next section) to search for parameter values that minimized the difference between the model and the experimentally determined amplitude response.

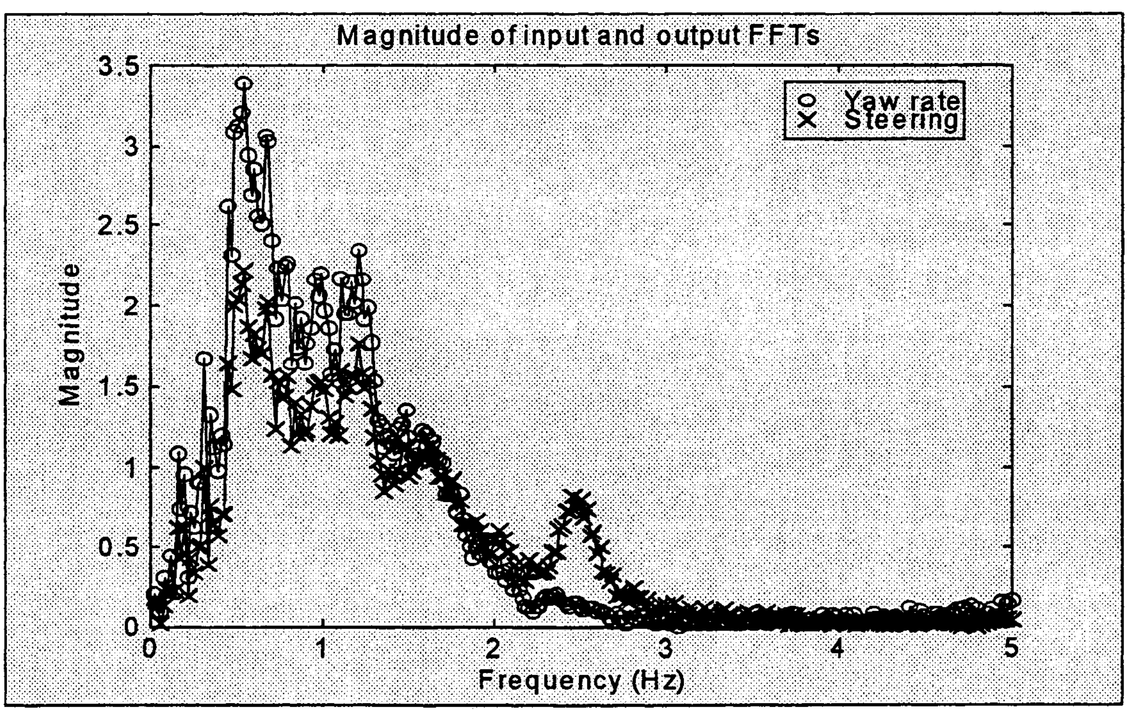


Figure 3.2 Fourier transforms of 35 mph steer test data.

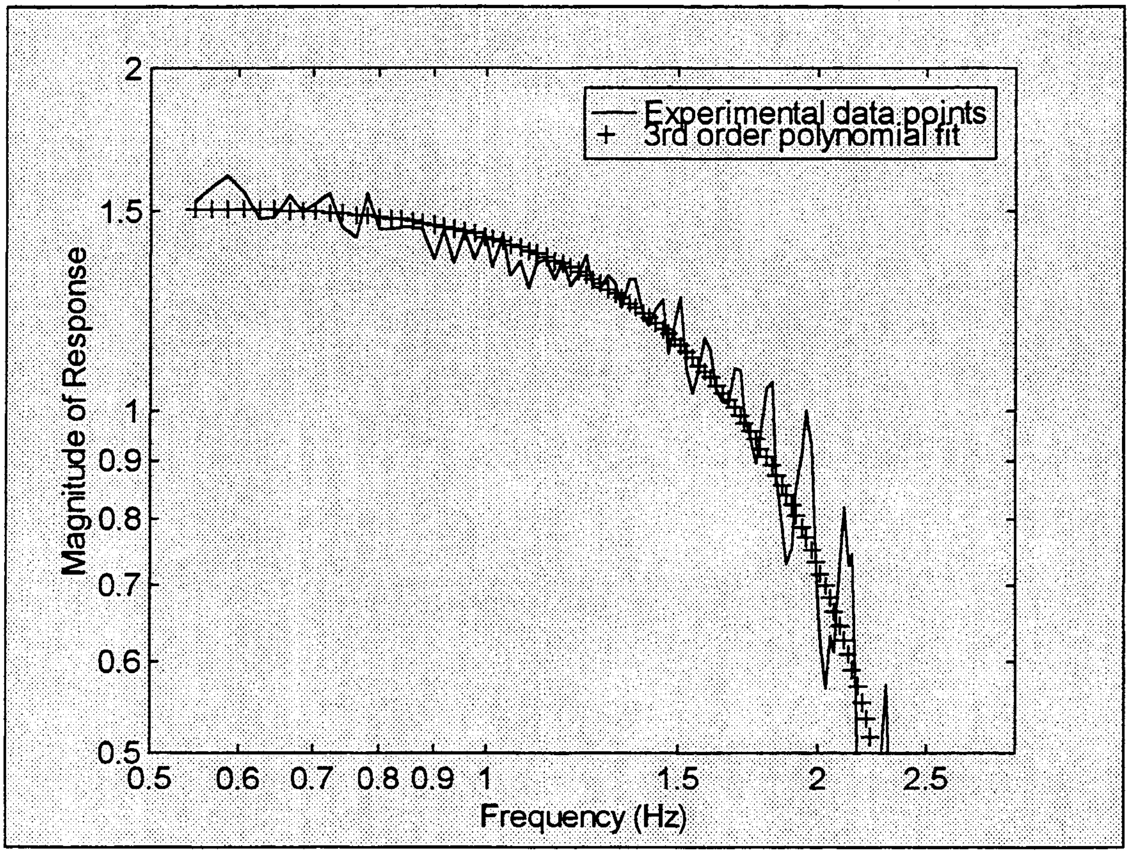


Figure 3.3 3rd order polynomial fit to experimental transfer function data for 35 mph.

The data represented by the line in figure 3.3 is the ratio of yaw rate response (system output) to steering wheel angle (system input) for the 35 mph data shown in figure 3.2. A third order polynomial, shown by the + symbols in figure 3.3, was then fit to this data. The third order polynomial was used to fit the data at higher frequencies where the response was rolling off and a straight horizontal line was used at lower frequencies where the response was constant. Steer test data for 15, 20, 25 and 30 mph were processed in the same way. The resulting experimental transfer functions for each of the tested speeds are graphed in figure 3.4. As expected the response to a steering input is greater at higher speeds.

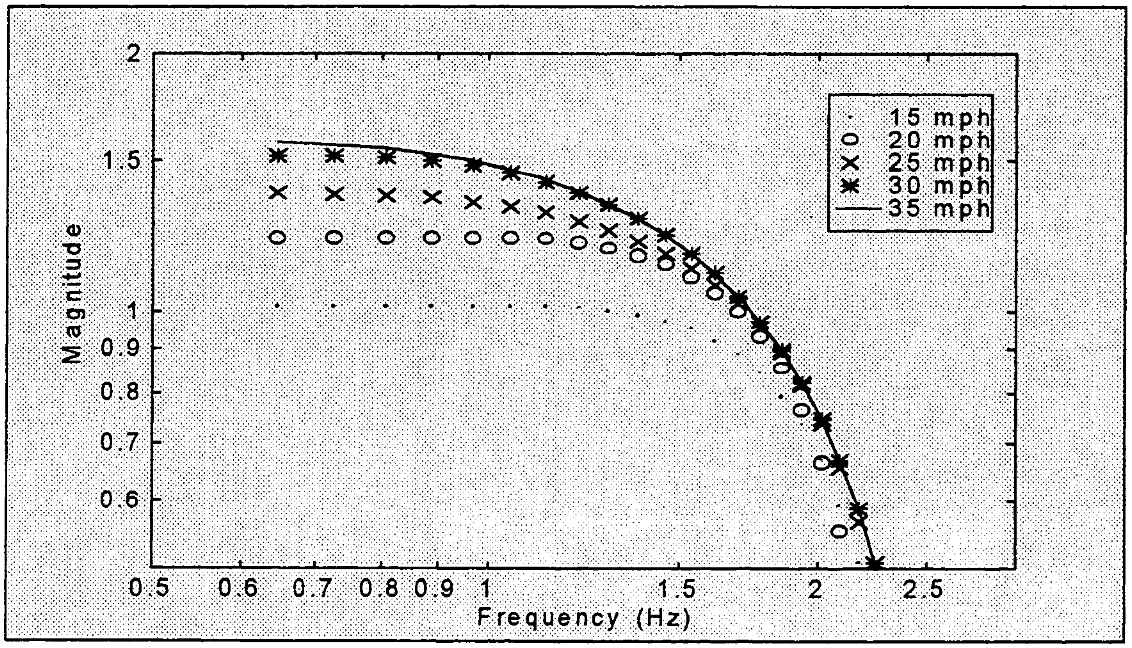


Figure 3.4 Experimental transfer functions for a range of different speeds.

Optimization Routine

To search for the best fit between the model and the experiment, we used a constrained optimization routine from the MATLAB™ Optimization Toolbox. This routine uses a Sequential Quadratic Programming method [7] that lets us constrain the search to within reasonable limits (keeping the tire stiffnesses greater than zero for instance.) The objective function that we minimized was the area between the model predictions and experimental results on a log-log plot of amplitude versus frequency (the Bode amplitude plot.) Figures 3.5 and 3.6 show the best fit that the optimization routine found after a number of runs starting at different seed values for the two models considered. Both models provide reasonable fits to the experimental data at frequencies below 1 Hz, but neither one works very well at higher frequencies. We originally believed that the higher order enhanced model would be significantly more accurate at higher frequencies and in fact it does drop off faster (30 dB/decade versus 18 dB/decade for the basic model). The experimental data however drops off at a rate of approximately 250 dB/decade, equivalent to a 12th order lateral dynamics mode (best estimate given noise). Both models therefore are of limited use at higher frequencies.

A higher order (more complicated) model would be able to fit the data better, with a steeper slope at higher frequencies, the tradeoff being a longer computation time when the model is being used. Figure 3.7 compares the experimental data to the bode plot of the 4th order transfer function:

$$G(s) = \frac{0.88}{0.0001s^4 + 0.0003s^3 + 0.012s^2 + 0.11s + .5}$$

This transfer function is a significantly better fit at high frequencies. The disadvantage of this type of transfer function model is that there are no parameters related to the actual truck (mass, wheelbase, cornering coefficients, etc.) that we can change in simulations as we experiment with different controllers in the lab.

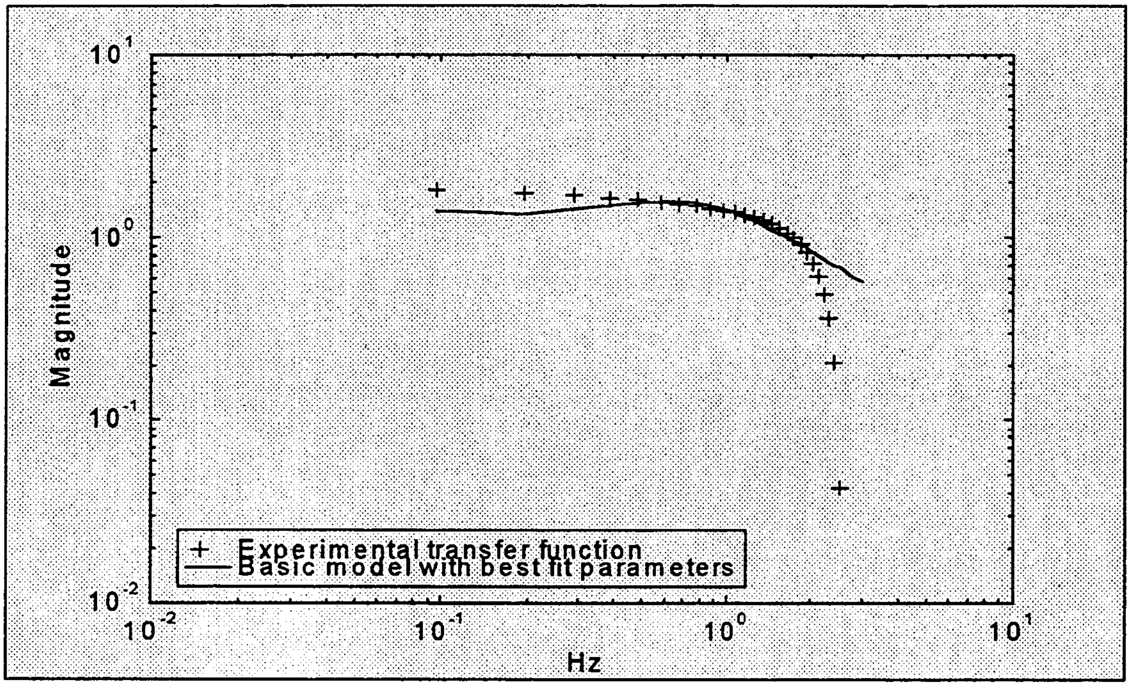


Figure 3.5. Amplitude Response vs. Frequency for the basic model at 35 mph with best fit values for cornering stiffness plotted over the experimental response from the steer test.

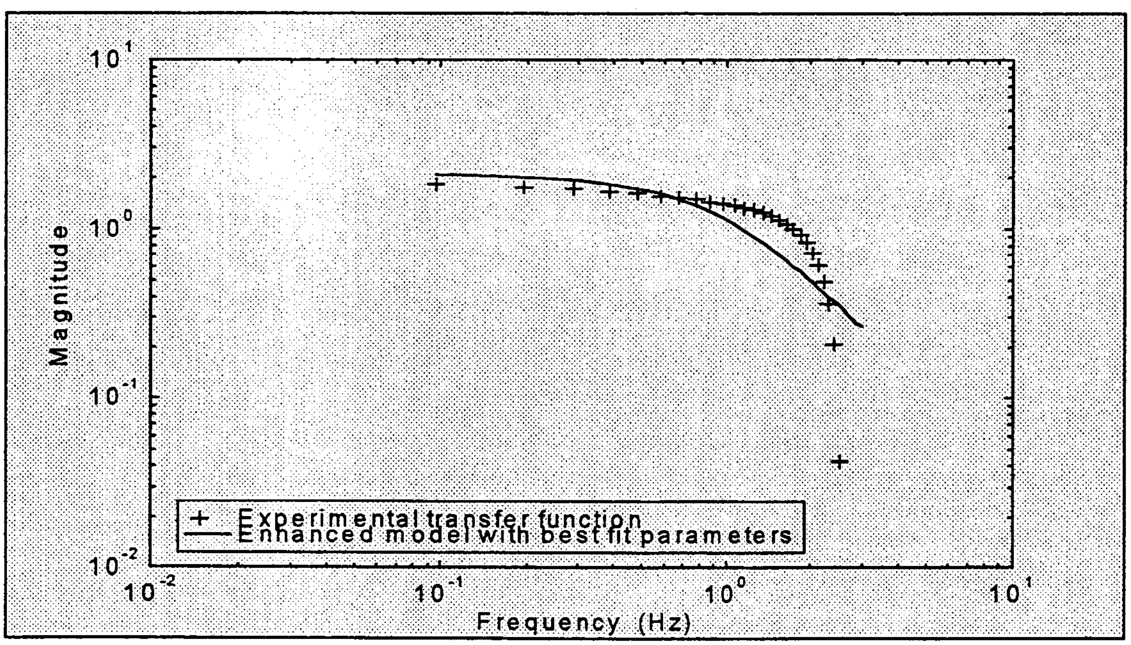


Figure 3.6 Amplitude Response vs. Frequency for the enhanced model at 35 mph with best fit values for cornering stiffness plotted over the experimental response from the steer test.

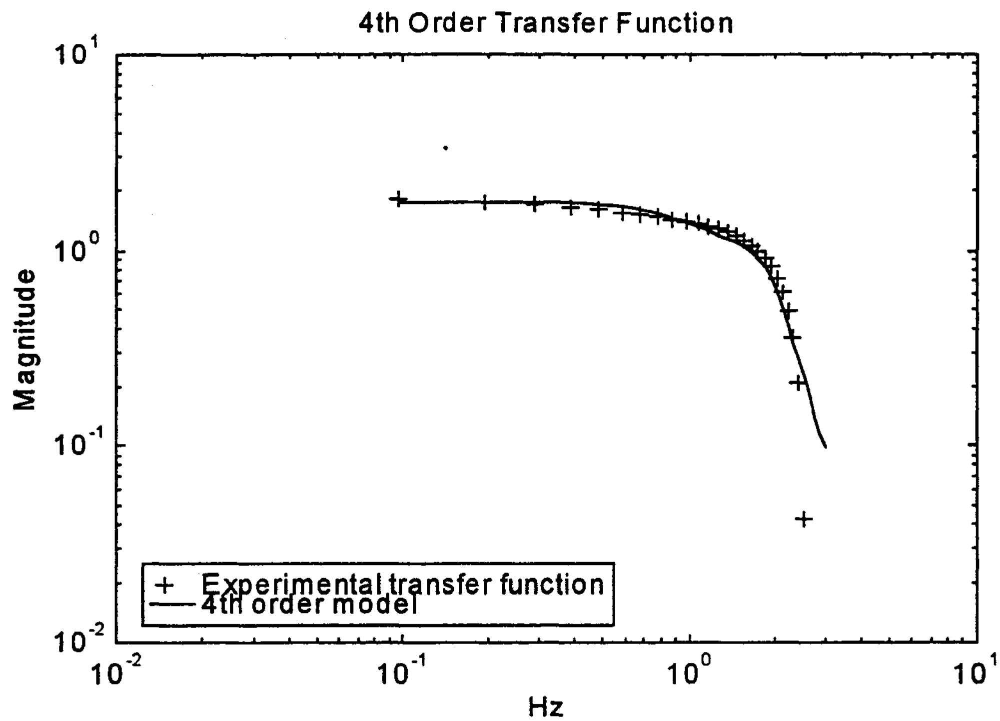


Figure 3.7 Amplitude Response vs. Frequency for the transfer function :
0.88 / ($0.0001s^4+.0003s^3+0.012s^2+.11s+.5$) plotted over the experimental response from the steer test at 35 mph.

Resulting Tire Stiffnesses

Front tires

The cornering stiffness for the model's front tires calculated using the procedure discussed above turned out to be considerably less than what would be expected for a real truck tire. The basic model fit returned a value of 47,000 newtons per radian and the enhanced model returned 14,500 newtons per radian. Both models were initially optimized using experimental transfer functions created from the 35 mph data. The optimization routine was then performed again for other speeds using the 35 mph results as the seed values. No significant differences in tire stiffnesses were found. The University of Michigan [8] has published data leading us to expect a value on the order of 150,000 newtons per radian for the Goodyear G159 tires under the conditions that we tested on the Navistar. The lower than expected values we obtained for the front axle cornering stiffness are due to simplifications inherent in our models (all the real world steering gear and suspension compliance's were neglected in order to create a mathematically tractable model that will run on a computer in real time.) The optimization therefore gives the front tires a correspondingly lower stiffness to compensate for the "give" in the actual suspension that was not explicitly included in the model. The procedure returned a value of 0.4 for the damping ratio and 1.35 cycles per meter for the path frequency in the enhanced model.

Rear Tires

The cornering stiffness calculated for the tires on the tractor's two rear axles and the two trailer axles were 121,000 newtons per radian for the basic model and 237,500 for the enhanced model. These values were closer to the expected real world values that are on the order of 120,000 newtons/radian. Since the two tandem axles on the rear of the tractor tend to roll in a straight line due to the longitudinal distance between them, they counteract the steering tires. Therefore a higher than expected stiffness is found for these axles for the same reason that a lower than expected value is found for the front axle.

Conclusions

Both the basic and the enhanced models give accurate results at frequencies below 1 Hz so they should perform acceptably in routine driving maneuvers such as lane changing and the negotiation of highway curves. The basic model is now being used successfully in laboratory simulations. With some additional modifications, the enhanced model is being incorporated into the Kalman filter for the truck's navigation system. Neither the basic nor the enhanced model gave accurate results at high frequencies, as such, they would be less useful for simulating emergency maneuvers. We will continue to refine these models as required, but currently they appear to be adequate for our present purposes.

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