

Essays on International Macroeconomics

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Abstract

This dissertation consists of three essays. The first essay analyzes International Capital Controls and Financial Crises. This essay provides a theoretical support for a macroprudential regulation of capital flows into emerging markets. We study a model of a production economy where private market participants expand the stock of capital during booms and the price of capital rises, enabling them to take on more credit. During busts, the stock of capital becomes less valuable, and the collateral value declines. This leads to a feedback spiral of declining borrowing capacity, falling asset prices, and fire sales. We attribute collateral constraints to be a driving force behind current account reversals and domestic absorption. The paper analyzes the role for macro-prudential policies to lean against the wind when credit flows into the economy. These policies reduce excessive capital creation in booms, and increase social welfare by mitigating the need for fire sales, asset price decline and associated credit crunch in case of a bust. We assess quantitatively that the optimal tax rate on capital inflows should be 1.5%. We find that credit crunches have long-lasting detrimental effects on output due to slow recovery of investment. One implication is that the welfare costs of financial crises are particularly large since they stretch over many periods. This suggests that adding investment channel is important for modeling financial amplification effects.

In the second essay, I work on estimating term structure of interest rates using bond prices. Central banks have several reasons for extracting information from asset prices. Asset prices may embody more accurate and more up-to-date macroeconomic data than what is currently published or directly available to policy makers. Aberrations in some asset prices may indicate imperfections or manipulations relevant for banking and financial market surveillance. Especially, asset prices will reflect market participants' expectations about the future, which is the focus of this paper.

In the third essay, I study a model of sovereign defaults. I introduce fluctuations in trend growth to the model of Mendoza and Yue (QJE 2012), to achieve three objectives: 1) improve the model's fit of debt-to-GDP ratios in the data, 2) account for default spread dynamics in both emerging and developed economies by allowing for differences in trend growth volatility, as measured by Aguiar and Gopinath (JPE 2007), 3) build a state of the art model to match sovereign default dynamics in both emerging and developed economies and to analyze policy alternatives in the current economic environment, like for example, optimal IMF lending policy to sovereign borrowers.

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Chapter 1

International Capital Controls

1.1 Introduction

Recent financial crises over the last decades in the world economy seem to have a recurrent pattern: when one country is hit by a credit crunch, capital flows out in a panic to the next attractive destination. The recipient country experiences a boom, asset price inflation and rising leverage, making it vulnerable to future adverse shocks.

Korinek (2011*b*) analyzed this pattern of hot money and serial financial crises in a stylized model of an exchange economy. This paper extends his research to production economy to study the effects of investment channel on output during serial financial crises. The model consists of two countries borrowing from a global investor. Credit is collateralized by countries' stocks of capital valued at market prices.

In booms, asset prices and borrowing capacity are high. Countries accumulate debt and expand the stock of capital. The price of capital rises, enabling economies to take on more credit. In busts, negative TFP shock causes the Fisherian debt deflation. Deleveraging leads to fire sales of assets, falling consumption, investment, production and aggregate demand. As a result, the stock of capital becomes less valuable, and the collateral value declines. Borrowing capacity decreases, credit constraint becomes binding and leads to further credit tightening.

The world interest rate is increasing in the amount countries want to borrow. As countries experience financial crises, the demand for loanable funds decreases. Global investors find fewer lending opportunities and the world interest rates decline. Healthy unconstrained economies find new incentives to take on more debt and become vulnerable to future busts. These spillover effects lead to global financial fragility and increase the risk of serial financial crises.

We find that credit crunches have long-lasting detrimental effects on output due to slow recovery of investment. This implies that the welfare costs of financial crises are particularly large since they stretch over many periods. This suggests that adding investment channel is important for modelling financial amplification.

The financial feedback loop entails credit externality if borrowing constraint is binding next period. In a decentralized competitive equilibrium, agents do not internalize the negative effects of asset fire-sales on the value of other agents' assets. As a result, agents borrow "too much" ex ante, compared with a social planner who internalizes the effects of feedback spirals that deleveraging depresses asset prices and borrowing capacity. The social planner lets asset prices be determined by the free market and takes the global interest rates as given. Regulating capital inflows social planner takes on less debt leading to less severe future constraints, less volatility and financial fragility.

The paper analyzes the role for macro-prudential policies to lean against the wind when credit flows into the economy. These policies reduce excessive capital creation in booms, and increase social welfare by mitigating the need for fire sales, asset price decline and associated credit crunch in case of a bust. This paper finds that a planner finds optimal to impose a tax on foreign borrowing in the amount of 1.5%. This level of tax on foreign borrowing mitigates the effects of an adverse shock on consumption from 15.5% to 13.3% in the aftermath of a crisis in another region. Similarly, optimal taxation reduces the current account reversal from 5% to 3% of GDP. This provides a rationale for macro-prudential regulation of capital flows such as recent 2% capital inflow taxation in Brazil. Another contribution of this paper to the literature is that it develops a numerical solution algorithm which is an extension of endogenous grid method in two dimensions.

1.2 Related Literature

The starting point for the analysis of capital controls effectiveness is a body of well-established literature, which demonstrates that financial crises in emerging economies can be understood as episodes of financial amplification. Fisher (1933), Bernanke and Gertler (1989), Greenwald and Stiglitz (1993), Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1999), Aiyagari and Gertler (1999), Krugman (1999), and Aghion, Bacchetta and Banerjee (2000) were among the first to show analytically that adverse conditions in the real economy and in financial markets mutually reinforce each other, leading to a feedback loop that propagates the financial and macroeconomic downturn. Calibration attempts produced varying results pertaining to the extent of amplification depending on the timing of collateral value, which was used as a basis. Studies that used a model of credit constraints that depend on the future value of collateral, such as Kiyotaki and Moore (1997), Caballero and Krishnamurthy (2001), Paasche (2001), and Krishnamurthy (2003), showed that even small, temporary shocks to technology can generate large, persistent fluctuations in output and asset prices. These studies, however, were criticized for the use of non-standard assumptions, which might have created the large amplification effects. Using a model with more standard assumptions Cordoba and Ripoll (2004) found a small amplification effect because the impact of current shocks on the future value of collateral in their model was mitigated by increased consumption of borrowers, increased investment and rising interest rates. By contrast, later studies by Mendoza and Smith (2006), Mendoza (2010), and Jeanne and Korinek (2010) used a model of credit constraints that depend on the current value of collateral assets and found quantitatively significant amplification

effects. Our paper is similar to the latter papers in that it uses the model with the current value of collateral assets to model financial crisis and the effect of capital controls.

A number of authors, including Brunnermeier (2009), Adrian and Shin (2010) used financial amplification mechanism to explain the ongoing world-wide credit crisis. On the normative side, Caballero and Krishnamurthy (2003), Lorenzoni (2008), Jeanne and Korinek (2010), Korinek (2011*a*), Korinek (2012), and Stein (2012) analyzed the externalities of financial amplification. These normative studies used static three-period models and concentrated on qualitative effects. Therefore, the quantitative significance of pecuniary externalities, which is important for determining to what extent regulation is desirable for correcting this market failure, remained an open question. Bianchi (2011) considered a quantitative model of an emerging market economy with an infinite-horizon setup. In the model, which characterized optimal policy responses, real exchange rate depreciations could give rise to financial amplification. Korinek (2011*b*) considered an infinite-horizon setup, in which the deleveraging externality involved an asset price rather than the real exchange rate, which is applicable to both industrialized and emerging economies. Our paper continues the sequence of normative studies using a quantitative approach and asset-prices. However, unlike Korinek (2011*b*), who uses an exchange economy model, we are using a production economy model.

Mendoza and Bianchi (2010) also considered a normative production economy model, where borrowing and labor demand were constrained by asset prices. Their study used fixed interest rate, while our model uses linear upward sloping supply of loanable funds and allows us to model hot money similarly to Korinek (2011*b*) in the exchange economy model. In addition to that, Mendoza and Bianchi (2010) focused on a small open economy, whereas our study adds general equilibrium analysis of two countries and global investor. In our model countries may suffer from financial amplification and crisis at different times, allowing us to study the spillover effects of such episodes of financial amplification among countries.

Devereux and Yetman (2010) and Nguyen (2010) also develop multi-country models of financial amplification. In their models constraints are always binding, while our model exhibits infrequent binding constraints arising endogenously in case of a large adverse shock. This allows us to analyze capital controls imposed at the times when financial constraints are loose as a precaution against future binding constraints.

Our paper studies optimal macroprudential regulation to reduce the cost of financial crises ex-ante, i.e. before a financial crisis materializes. Benigno et al. (2011), Benigno et al. (2013), and Jeanne and Korinek (2013) concentrate on ex-post stimulus interventions to address financial crises. The general result of these papers is that policymakers would always want to engage in a mix of ex-ante prudential and ex-post stimulus measures when faced with the risk of financial crises that involve financial amplification.

1.3 Two-Country Model with Global Investor

Assume the world economy consists of two symmetric borrowing countries and a global investor.

1.3.1 Countries

There is a representative infinitely lived agent in each country $i = 1, 2$ producing a single tradable consumption-investment good using capital (k_t^i) and labor (h_t^i) inputs according to a Cobb-Douglas technology. The population is constant and is normalized to unity. Agents are endowed with one unit of time every period, which they allocate between market work and leisure. The agent maximizes the discounted utility of Greenwood-Hercowitz-Huffman form:

$$\max E_0 \sum_{t=0}^{\infty} \frac{\beta^t}{1-\sigma} \left(c_t^i - \theta \frac{h_t^{i1+\gamma}}{1+\gamma} \right)^{1-\sigma} \quad (1.1)$$

where c_t^i denotes consumption of country i in period t . This choice of the utility function removes the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor depend on labor only. This permits procyclical labour hours.

Countries have access to foreign risk-less bond market where they can borrow at the world risk free interest rate R_{t+1} . Country i faces the following budget constraint and law of motion for capital that features “adjustment costs” in investment:

$$c_t^i + x_t^i + b_t^i - \frac{b_{t+1}^i}{R_{t+1}} = y_t^i = z_t^i k_t^{i\alpha} h_t^{i1-\alpha} \quad (1.2)$$

$$k_{t+1}^i = (1-\delta)k_t^i + \Phi\left(\frac{x_t^i}{k_t^i}\right)k_t^i \quad (1.3)$$

where y_t^i , x_t^i and b_t^i are output, investment and external debt of country i at the start of period t ; and $\alpha \in (0, 1)$ is the factor share parameter. The stochastic technology level z_t^i is i.i.d. across time and countries. The adjustment cost function $\Phi(\cdot)$ is concave in investment to capture the difficulty of quickly changing the level of installed capital.

Let μ_t^i and $\mu_t^i q_t^i$ be the lagrange multipliers on the budget constraint (1.2) and capital accumulation equation (1.3) respectively. Following Hayashi (1982), since technology and adjustment costs have constant returns to scale, the marginal price of capital q_t^i is equal to the average price of installed capital (Tobin’s q). Therefore, the market value of capital assets (stock market price p_t^i) is equal the product of the shadow value of capital q_t^i and the quantity of capital k_{t+1}^i

$$p_t^i = q_t^i k_{t+1}^i \quad (1.4)$$

Agents in each country are the sole capital owners in the economy. They can not sell or rent capital to foreigners, but they can use capital as collateral to borrow from the rest of the world. The amount of external debt that the country can roll over can not exceed the fraction $\phi < 1$ of the market value of its capital assets. We assume the international debt market is perfectly enforceable so that countries never default. This implies the following collateralized borrowing constraint similar to that in Kiyotaki and Moore (1997).

$$\frac{b_{t+1}^i}{R_{t+1}} \leq \phi p_t^i \quad (1.5)$$

The optimization problem of the borrowing country i can be expressed as maximizing (1.1) subject to (1.2)-(1.5). Assigning the shadow price $\mu_t^i \lambda_t^i$ to the collateral constraint (1.5), the first-order conditions to the problem are

$$(1 - \lambda_t^i)u_c(c_t^i, h_t^i) = \beta R_{t+1} E_t[u_c(c_{t+1}^i, h_{t+1}^i)] \quad (1.6)$$

$$(1 - \phi \lambda_t^i) p_t^i u_c(c_t^i, h_t^i) = \beta E_t[u_c(c_{t+1}^i, h_{t+1}^i)(p_{t+1}^i + \alpha y_{t+1}^i - x_{t+1}^i)] \quad (1.7)$$

$$q_t^i \Phi' \left(\frac{x_t^i}{k_t^i} \right) = 1 \quad (1.8)$$

$$\theta h_t^{i1+\gamma} = (1 - \alpha) y_t^i \quad (1.9)$$

1.3.2 Global Investor

The global investor has a two period overlapping generations structure. Born at time t he smoothes the fall in his endowment income $e_1 > e_2$ by saving in non-contingent bond b_{t+1} at the interest rate R_{t+1} .

$$\max_{c_t, c_{t+1}, b_{t+1}} \log(c_t) + \beta \log(c_{t+1}) \quad \text{s.t.} \quad c_t + \frac{b_{t+1}}{R_{t+1}} = e_1, \quad c_{t+1} = e_2 + b_{t+1}$$

This leads to the following indirect supply of loanable funds function

$$R_{t+1}(b_{t+1}) = \frac{(1 + \beta)b_{t+1} + e_2}{\beta e_1} \quad (1.10)$$

Countries are linked only through international bond market:

$$b_{t+1} = b_{t+1}^1 + b_{t+1}^2 \quad (1.11)$$

1.4 Decentralized Equilibrium

In a given period, the policy and asset price functions of each country can be expressed in terms of their *beginning-of-period* external debt levels, (b_1, b_2) , capital stocks, (k_1, k_2) , and technology levels, (z_1, z_2) . Let $s = (s_1, s_2)$ denote the vector of states $s_i = (b_i, k_i, z_i)$. The dynamic programming problem of country i can be expressed as follows:

$$\begin{aligned} V_i(s) &= \max_{c_i, h_i, x_i, b'_i, k'_i} u(c_i, h_i) + \beta E[V_i(s')] \\ \text{s.t.} \quad c_i + x_i + b_i - \frac{b'_i}{R'(b'_1 + b'_2)} &= z_i k_i^\alpha h_i^{1-\alpha} & (\mu_i) \\ k'_i &= (1 - \delta)k_i + \Phi\left(\frac{x_i}{k_i}\right)k_i & (\mu_i q_i) \\ \frac{b'_i}{R'(b'_1 + b'_2)} &\leq \phi q_i k'_i & (\mu_i \lambda_i) \end{aligned}$$

Decentralized equilibrium for this economy is given by consumption, labor supply and investment decision rules for each country, $c_i(s)$, $h_i(s)$, $x_i(s)$; laws of motion for capital and debt for each

country, $k'_i(s)$, $b'_i(s)$; average capital asset price functions for each country $q_i(s)$; savings and interest rate functions for global investor, $b(s)$, $R(s)$; such that:

1. Given the pricing functions and the laws of motion, the value function and decision rules of each country solve that country's dynamic problem (1.1-1.5).
2. Savings and interest rate functions solve the global investor's problem (1.10).
3. International bond market clears (1.11).

The decentralized equilibrium can be expressed as recursive functions of the vector of the state variables s that solve the equilibrium conditions of two borrowing countries (1.2)-(1.9) and global investor (1.10)-(1.11). Appendix A.1 shows how to solve for these recursive functions numerically using endogenous grid method.

1.5 Social Planner

The decentralized equilibrium entails credit externality that arises during financial amplification when borrowing constraint becomes binding. The borrowers in each country take asset prices in their country as given. They do not internalize that their borrowing decisions during booms drive up asset prices and relax the collateral constraint for other borrowers in the economy. Similarly, deleveraging decisions during busts lead to fire sales of assets, depress asset prices and further tighten the credit for other agents in the economy. These borrowing decisions are not constrained efficient from the point of view of a social planner who can internalize these feedback effects and improve welfare in an economy.

This section shows that a planner can offset the distortion by imposing a Pigouvian tax on capital inflows. Compared with a social planner, agents in decentralized economy borrow "too much" ex ante. By regulating capital inflows, social planner takes on less debt leading to less severe future constraints, less volatility and financial fragility. This paper analyzes how the level of externalities and the optimal policy response in one country is affected by events in other parts of the world economy. It studies the global general equilibrium effects of macroprudential regulation.

Social planner lets asset prices be determined by free market and takes global interest rates $R(b)$ as given. Policymaker in country i views the borrowing constraint relevant to her problem as

$$\frac{b_{t+1}^i}{R_{t+1}} \leq \phi p_t^i(b_t^i) = \phi q_t^i(b_t^i) k_{t+1}^i \quad (1.12)$$

The optimization problem of the social planner in country i is to maximize (1.1) subject to the budget constraint (1.2), law of motion for capital (1.3) and the borrowing constrain (1.12). Assigning the shadow price $\mu_t^i \lambda_t^i$ to the borrowing constraint, the first order condition w.r.t. debt yields an Euler equation

$$(1 - \lambda_t^i) u_c(c_t^i, h_t^i) = \beta R_{t+1} E_t [u_c(c_{t+1}^i, h_{t+1}^i) (1 + \phi \lambda_{t+1}^i \frac{\partial p_{t+1}^i}{\partial b_{t+1}^i})] \quad (1.13)$$

Comparing to the decentralized Euler equation (1.6), the Euler equation in the planners equilibrium differs by the extra term $\phi\beta R_{t+1}E_t[u_c(c_{t+1}^i, h_{t+1}^i)\lambda_{t+1}^i\partial p_{t+1}^i/\partial b_{t+1}^i]$. This externality kernel can be interpreted as follows: $\partial p/\partial b$ captures an increase in asset price due to additional borrowing, ϕ reflects resulting relaxation in the borrowing constraint, and $u_c\lambda$ represents the utility cost of constraint.

1.5.1 Implementation

Optimal regulation can be implemented by a state-contingent tax τ_t^i that the social planner levies on foreign debt and rebates back as lump sum transfers:

$$c_t^i + x_t^i + b_t^i - (1 - \tau_t^i)\frac{b_{t+1}^i}{R_{t+1}} = y_t^i + T_t^i$$

The debt tax introduces a wedge in the Euler equation:

$$(1 - \lambda_t^i - \tau_t^i)u_c(c_t^i, h_t^i) = \beta R_{t+1}E_t[u_c(c_{t+1}^i, h_{t+1}^i)]$$

and replicates the constrained social optimum if it is set to

$$\tau_t^i = \frac{\phi\beta R_{t+1}E_t\left[u_c(c_{t+1}^i, h_{t+1}^i)\lambda_{t+1}^i\frac{\partial p_{t+1}^i}{\partial b_{t+1}^i}\right]}{u_c(c_t^i, h_t^i)}$$

1.6 Model Results

1.6.1 Parameterization

The model was calibrated using annual frequency with the discount rate $\beta = 0.96$, coefficient of relative risk aversion $\sigma = 2$, capital share $\alpha = 0.3$, and depreciation rate $\delta = 0.08$.

Under these parameters, capital to output ratio of borrowers is $\bar{p} = \bar{k} = 2.47$ in deterministic steady state with $\beta R = 1$. The leverage ratio ϕ was calibrated to 0.07 to target the external debt to output ratio of 20%. The GHH utility parameters $\gamma = 1$ and $\theta = 2.54$ were calibrated to target the Frisch elasticity to 1 and hours worked to 0.36 of available time.

The technology process in both countries is assumed to be i.i.d. across time and countries and follows a binomial distribution $z_t^i \in \{z_H, z_L\}$, where z_H and z_L capture booms and busts, with busts occurring on average three times a century ($\pi = 3\%$). The technology shock z_H was scaled to normalize output to one in booms, and z_L was scaled to match 6% decline in productivity during busts.

The functional form for Φ is specified as $a_1(x_t/k_t)^{1-1/\xi} + a_2$, as in Jermann (1998), where a_1 and a_2 are constants chosen such that the steady state level of capital is invariant to ξ . The curvature parameter ξ determines the severity of adjustment costs. As ξ approaches infinity, Φ becomes linear, and investment is converted into capital one for one (frictionless economy limit). At the other extreme, as ξ approaches zero, Φ becomes a constant function, and the capital stock remains

constant regardless of the investment level (exchange economy limit). I set $\xi = 0.4$, which is broadly consistent with the values reported in the empirical literature.

To calibrate the parameters of global investors, we use the deterministic steady state and $\beta R = 1$. In this case, investors enjoy constant consumption of $c_t = c_{t+1} = \bar{c} = \frac{e_1 + \beta e_2}{1 + \beta}$ and save constant amount $b_{t+1} = 2\bar{b} = \frac{e_1 - e_2}{1 + \beta}$ where $\bar{b} = \phi \bar{p}$ is the debt of each country. This implies that endowments are $e_1 = \bar{c} + 2\beta \bar{b}$ and $e_2 = \bar{c} - 2\bar{b}$. The parameter \bar{c} determines how interest rate responds to changes in credit demand. Higher values of \bar{c} lead to smaller fluctuations in interest rate. We set $\bar{c} = 3$ to target a decline in the interest rate to zero if one of the countries experiences a bust.

Table 1.1 summarizes the parameter choices.

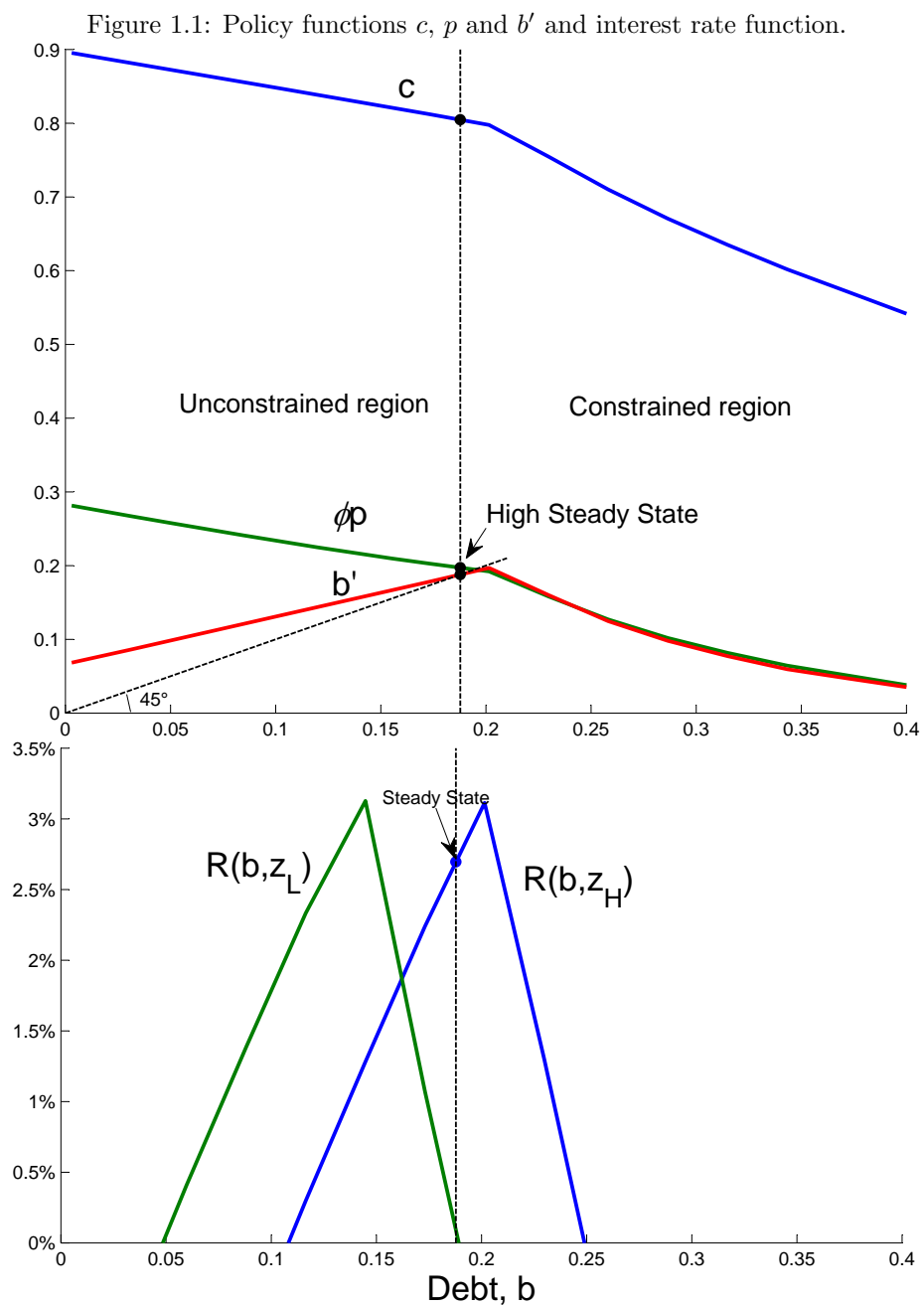
Table 1.1: Parameterization

Parameter		Value
β	Time discount rate	0.96
σ	Relative risk aversion	2
α	Capital share	0.3
ϕ	Leverage ratio	0.015
$1/\gamma$	Frisch elasticity	1
θ	Labor supply coefficient	2.54
ξ	Adjustment cost coefficient	0.4
δ	Depreciation rate	0.08
y_H	Output in booms	1
z_L/z_H	Productivity decline during crisis	0.94
π	Probability of crisis	3%
\bar{c}	Interest rate response to credit demand	3

1.6.2 Results

Figure 1.1 depicts the policy functions for b' , ϕp and c of a borrowing country i experiencing positive shock z_h^i as a function of its debt b^i while keeping the level of capital and all states of the other country j at their steady state values. All three policy functions exhibit a kink when the region switches from the unconstrained to the unconstrained region, which occurs at $b = 0.197$. To the right of this threshold, consumption and the asset price respond considerably more to changes in debt level than to the left of the threshold since borrowing constraints are binding and the economy experiences financial amplification. For unconstrained values of b , the policy function b' is increasing as the agent optimally smoothes consumption and carries less wealth in the future by borrowing more today. For constrained values of b , the policy function b' is declining since greater indebtedness implies a lower asset price and a lower borrowing limit.

The dashed vertical line indicates the level of debt that is reached if the economy has been in the boom state for a long number of periods. For simplicity, we call this debt level the high steady



The upper panel of the figure depicts the policy functions c , p and b' as a function of the debt b of a representative agent in the borrowing country. The domestic capital and all states in the other country are kept at their steady state levels. The lower panel shows the resulting world interest rate R .

state.

The bottom panel of the figure shows the two world interest rates R during booms z_H and busts z_L as a function of debt b while keeping the capital in the domestic country and all states in the foreign country at their steady state levels. The interest rate is a scaled image of the policy function b' – the more agent borrows, the higher the interest rate that international investors demand. The interest rate drops to zero in the event of a bust, due to calibration of \bar{c} .

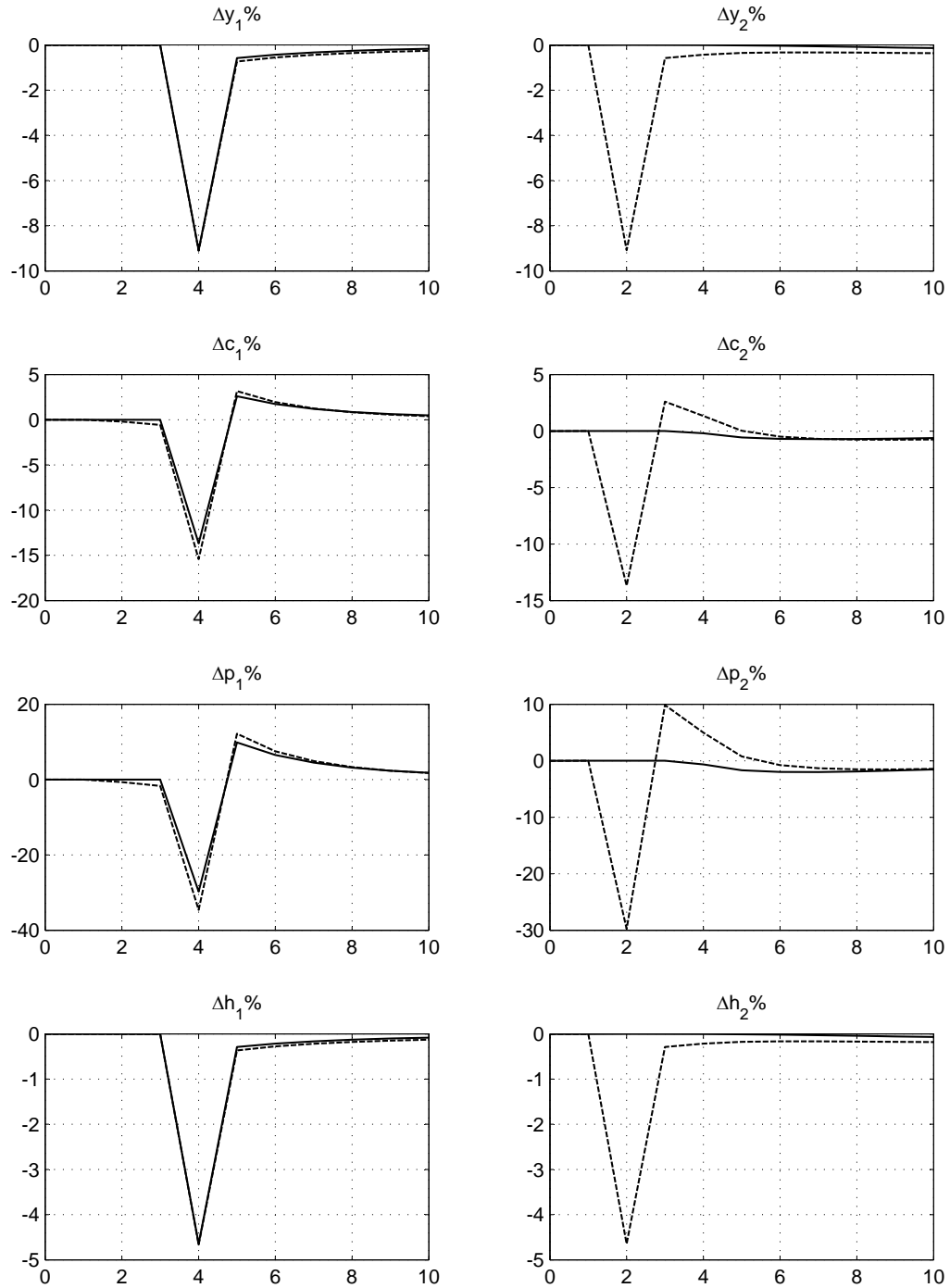
In figures 1.2 and 1.3 we depict a sample simulation of the world economy with two countries. They show percentage deviations of output, consumption, asset prices, hours worked, investment, capital and global interest rate from the high steady state values for both countries. The top two panels of figure 1.3 also show the current account reversals as changes in debt as a percentage of GDP. There are two simulations given by the solid and dashed line. Under both scenarios country 1 experiences a single bust in period 4. Under the solid line scenario, country 2 does not experience any busts. Whereas under the dashed line scenario, country 2 experiences a bust in period 2.

When a region is in its high steady state and experiences a bust shock $z_t = z_L$, it deleverages, i.e. its debt goes down. Under the benchmark calibration, the economy's debt level declines by 5% of GDP, which equals the magnitude of the economy's current account reversal. Domestic consumption falls by 13.7%, and the asset price collapses by 30%. In each such episode, consumption declines more strongly than output because falling asset prices and falling borrowing capacity reinforce the effects of the initial output shock. Following a number of positive shocks z_H , the economy slowly returns to the steady state debt level \bar{b} .

Adding investment channel proved to be important for financial amplification. Investment falls by 12.9% during a credit crunch and cannot be fully made up for during the ensuing recovery. In the aftermath of the crisis, investment recovered only up to 2.9% above pre-crisis levels. This has long-lasting detrimental effect on output which falls by 9.1% during a crisis and is still 1.7% below steady state in period 10. This implies that the welfare costs of financial crises are particularly large since they stretch over many periods.

Figures 1.2 and 1.3 also illustrate the spillover effect of financial crises in one region to the other region. If a country is hit by a negative output shock after its debt level has just gone up, then it is more vulnerable to financial crises and experiences more severe amplification effects. This situation is depicted by the dashed line. Under this scenario country 2 suffers a financial crisis at period 2, becomes financially constrained and is forced to deleverage. Given the lower world demand for capital, the world interest rate declines. This induces hot money flows to country 1, which takes advantage of the cheap credit by borrowing temporarily. If country 1 continues to experience positive shocks, both countries converge back to the high steady state. But if country 1 suffers an adverse shock at period 4 (dashed line), then it will experience a crisis that is significantly larger than the one under solid line episode. Table 1.2 reports the effects on all variables in country 1 at period 4 under two scenarios: in absence and presence of global financial fragility in the form of a prior crisis in country 2 at period 2:

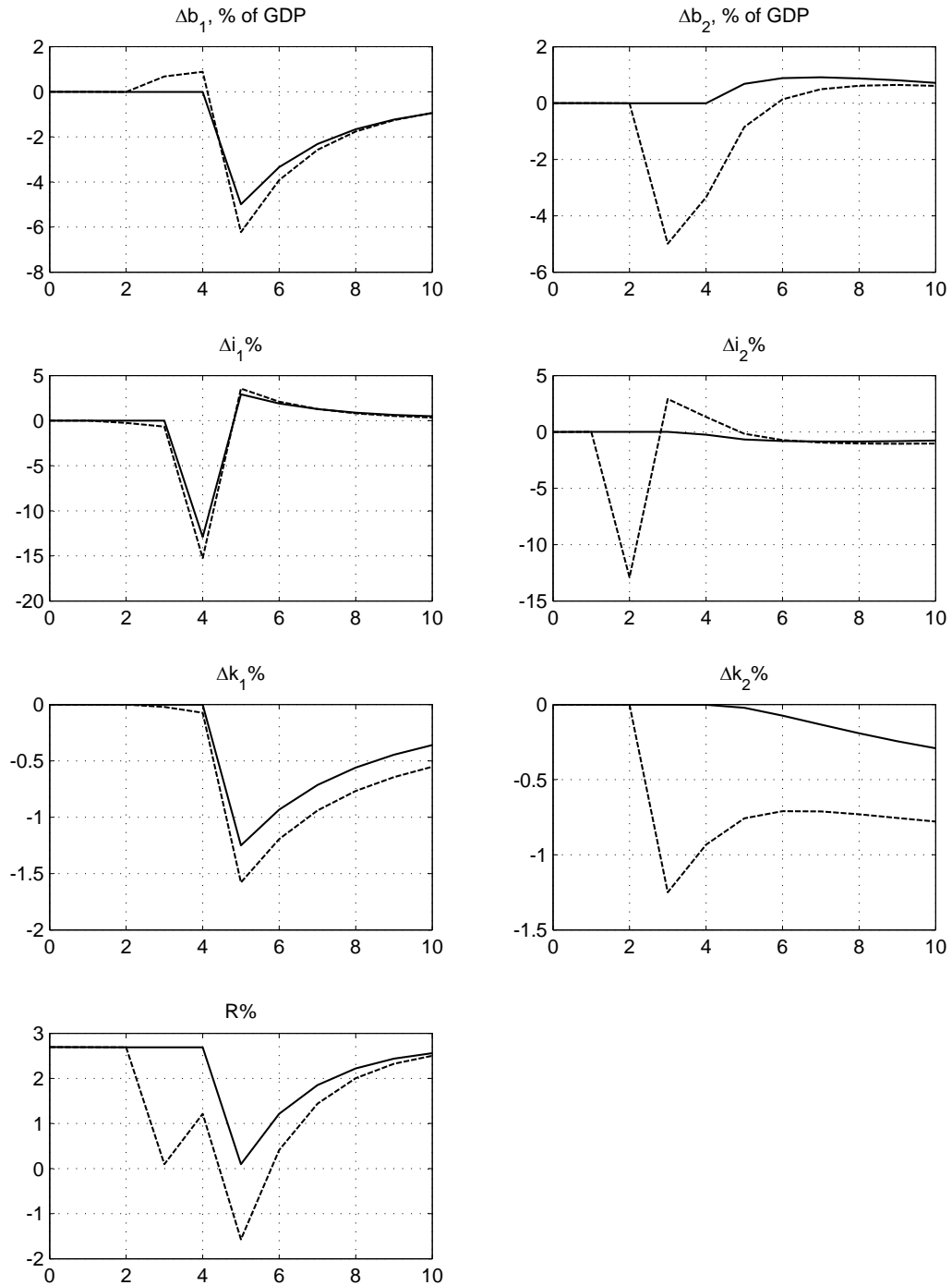
Figure 1.2: Simulated sample path of output, consumption, asset prices and labour hours



This figure illustrates a simulation of the world economy over 10 periods. The solid line represents the case when country 1 experiences shock in period 4 without country 2 experiencing any shocks.

The dashed line represents the case when beside the shock in country 1 in period 4, there is a shock in country 2 in period 2.

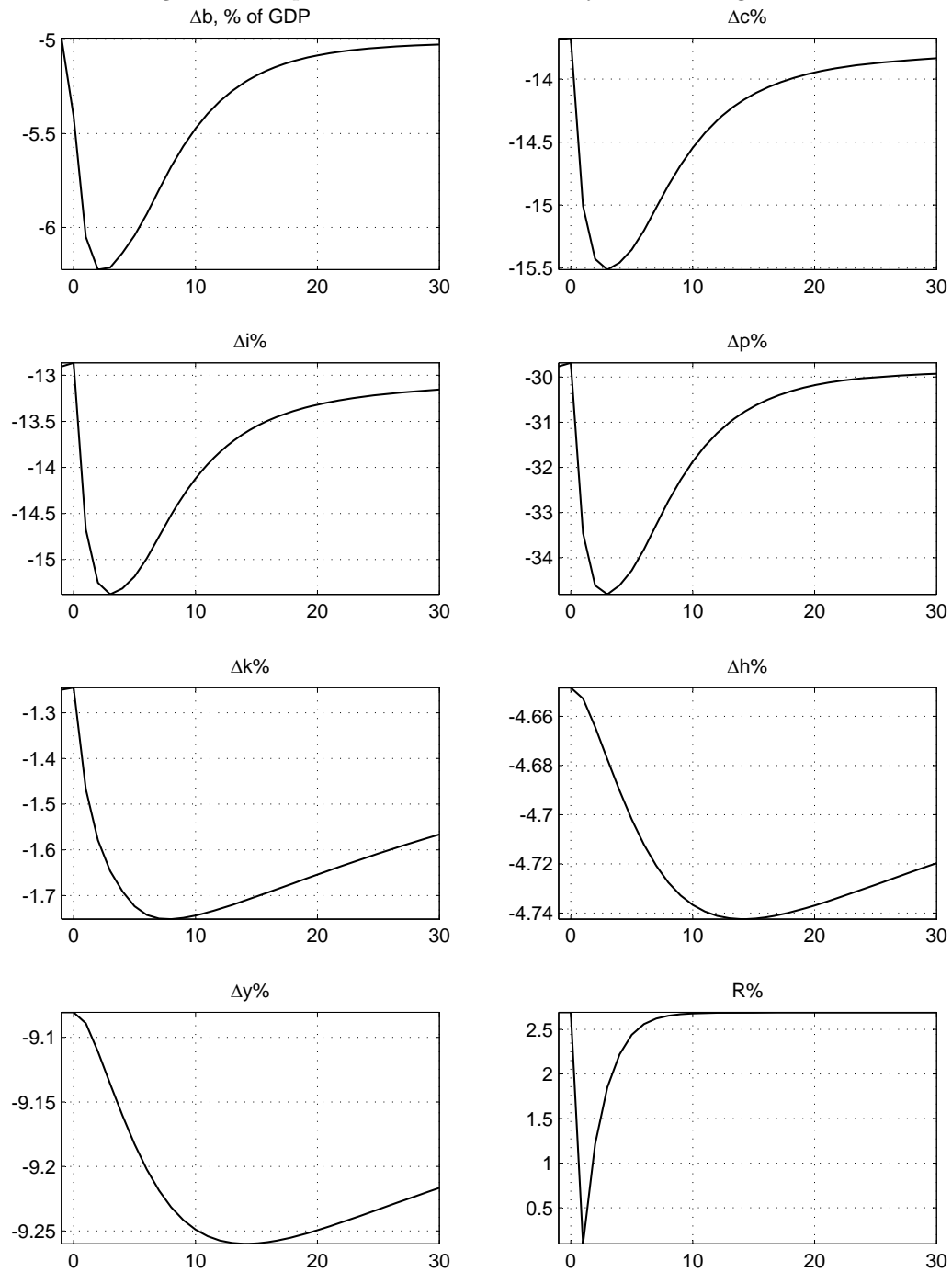
Figure 1.3: Simulated sample path of debt, investment, capital and global interest rate



This figure illustrates a simulation of the world economy over 10 periods. The solid line represents the case when country 1 experiences shock in period 4 without country 2 experiencing any shocks.

The dashed line represents the case when beside the shock in country 1 in period 4, there is a shock in country 2 in period 2.

Figure 1.4: Impact of domestic shock delayed after foreign shock



This figure shows the impact of adverse output shocks on debt, consumption, investment, asset prices, capital stock, hours worked, output and interest rate if the domestic shock is delayed t periods after the other country experienced a financial crisis.

Table 1.2: Global financial fragility

Percentage change from steady state in country 1	No prior crisis in country 2	After crisis in country 2
Current account reversal, % of GDP	-5.0%	-6.2%
Output	-9.1%	-9.1%
Consumption	-13.7%	-15.4%
Asset price	-29.7%	-34.6%
Labor hours	-4.6%	-4.7%
Investment	-12.9%	-15.2%
Capital	-1.2%	-1.6%
Interest rate	0.1%	-1.6%

Figure 1.4 shows the impact of an adverse shock z_L on the level of debt, consumption, investment, asset prices, capital stock, hours worked, output and global interest rate t periods after a financial crisis in the other country. The decline in borrowing Δb can be interpreted as the extent of deleveraging in the economy and will materialize in the form of a current account reversal. The baseline – an adverse shock without a preceding crisis in the other region – is depicted for $t < 0$ and consists of decline in borrowing capacity of 5.0% of GDP and a 13.7% decline in consumption from the high steady state. If the shock hits in the aftermath of crises in other parts of the world economy, the decline in borrowing capacity is up to 6.2% of GDP, and the decline in consumption up to 15.4%.

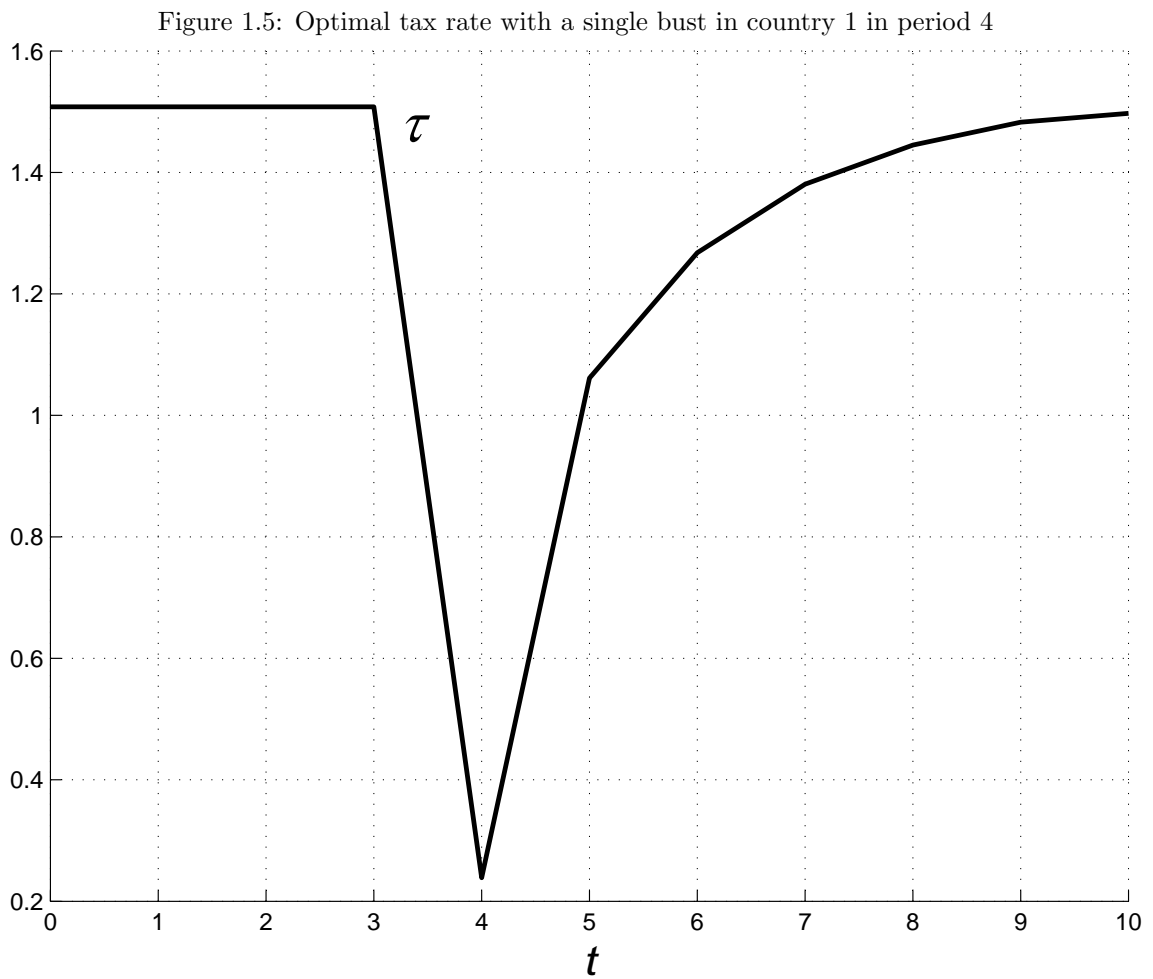
1.6.3 Optimal Policy

Given the risk of financial amplification effects and the associated externalities, policymakers in the described economies find it optimal to impose Pigouvian taxes on foreign borrowing in good times so as to mitigate the crises that occur in response to adverse shocks.

Under the benchmark calibration, a planner finds it optimal to impose a tax on foreign borrowing in the amount of 1.5% in the high steady state. For example, if a borrower took on \$100 in foreign credit, the planner would impose a Pigouvian tax of \$1.5 per year. This magnitude of inflow taxation is within the range of policy measure that have recently been enacted by Brazil and other emerging economies.

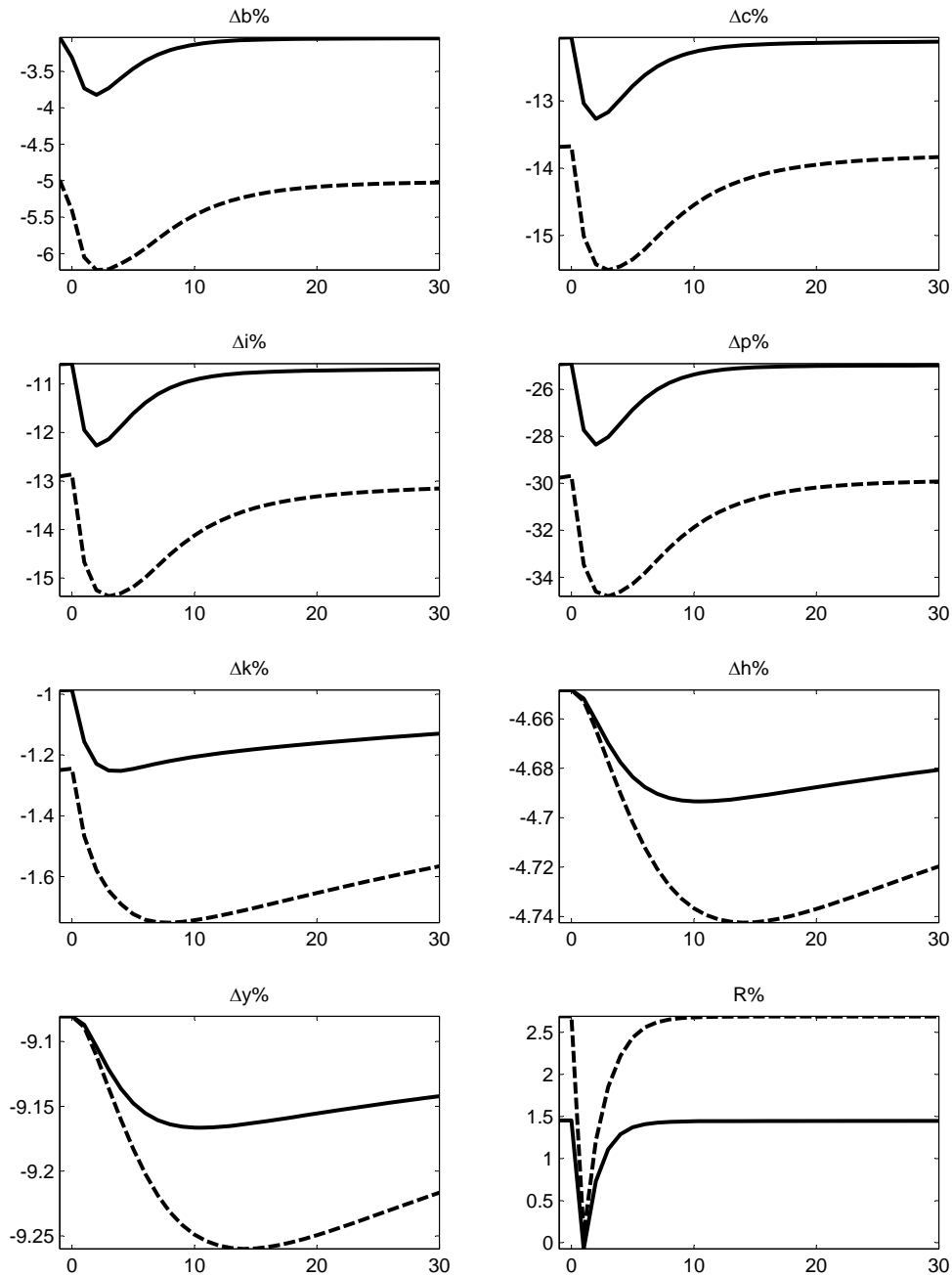
If financial crisis occurs in the domestic economy, it is desirable to lower the tax in order to encourage investors to keep capital in the country. Figure 1.5 depicts the optimal macroprudential tax when a crisis has occurred in the country 1 in period 4. In the period the crisis occurs, the optimal level of the tax drops to zero percent. Over the ensuing periods, it rises progressively back to the steady state level.

Figure 1.6 replicates the results of figure 1.4 given the optimal level of macroprudential taxation. The figure shows the impact of an adverse shock in one country t period after an adverse shock has



This figure reports the optimal level of macroprudential taxation in one country experiencing financial crisis in period 4 without any output shocks in the other country.

Figure 1.6: Optimal macroprudential taxation and impact of output shocks



This figure reports the impact of adverse output shocks on debt, consumption, investment, asset prices, capital stock, hours worked, output and interest rate if the domestic shock is delayed t periods after the other country experienced a financial crisis. The solid line represents the impact given the optimal policy intervention. For comparison, the dashed lines show the impact in the decentralized equilibrium.

occurred in the other country. The dashed line represents the impact in the decentralized equilibrium and the solid line under a planner's optimal intervention. The tax on foreign borrowing mitigates the effects of an adverse shock on consumption from 13.7% to 12% in the high steady state of the world economy. When the country has experienced inflows of hot money in the aftermath of a crisis in another country, the tax reduces the impact of an adverse shock on consumption from 15.4% to 13.3%. Similarly, optimal taxation reduces the current account reversal from 5.0% to 3.0% in the high steady state of the world economy, and from 6.2% to 3.8% when a country has experienced inflows of hot money in the aftermath of a financial crisis somewhere else.

1.7 Conclusions

We have studied the consequences of serial financial crises in a two-country model with production and collateralized borrowing. Countries take on excessive debt and later suffer severe financial crisis with large current account reversals, decline in consumption, investment, output and asset prices. Private borrowers do not internalize that their decisions expose their country to such financial amplification effects creating credit externalities on other borrowers. A social planner can induce market participants to internalize these effects by imposing macroprudential regulation on capital inflows. This reduces macroeconomic volatility, financial fragility and improves welfare.

The model highlights a feature of international bond market when crises in one country push hot money into other countries making them more vulnerable to future adverse shocks. For example, one of the results is that an adverse shock that normally leads to 13.7% fall in consumption will cause 15.4% fall in consumption if a country has just experienced inflows of hot money. By imposing a capital inflow tax of 1.5%, these magnitudes can be reduced to 12.1% and 13.3% respectively.

Our findings suggest that adding investment channel is important for financial amplification. Investment lost during a credit crunch cannot be fully made up for during the ensuing recovery and hence has long-lasting detrimental effects on output. This implies that the welfare costs of financial crises are particularly large since they stretch over many periods.

A limitation of the present model is that it abstracts from long-run growth. Essentially I assume the booming economy is in the high steady state. Incorporating an endogenous growth would be a valuable extension.

Another avenue for further research could be modeling the foreign exchange rates to address policymaking concerns about appreciation during booms and sudden depreciation in busts when capital flows reverse. This mechanism is particularly important for emerging market economies who have taken excessive debts denominated in foreign currencies.

Chapter 2

Yield Curve Estimation

Central banks have several reasons for extracting information from asset prices. Asset prices may embody more accurate and more up-to-date macroeconomic data than what is currently published or directly available to policy makers. Aberrations in some asset prices may indicate imperfections or manipulations relevant for banking and financial market surveillance. Especially, asset prices will reflect market participants' expectations about the future, which is the focus of this paper.

In fixed exchange rate regimes, central banks have an obvious interest in assessing the credibility of the regime and the likelihood of future speculative attacks. For this purpose, differentials between domestic and foreign interest rates, forward exchange rates, and prices on exchange rates and option prices are used to estimate expectations of realignments and other regime changes.

With floating exchange rates and inflation targeting, central bank also has an obvious interest in assessing the credibility of the regime and market expectations of future monetary policy. This paper is a selective survey of new or recent methods to extract information about market expectations from asset prices for monetary policy purposes. There are several commonly used techniques for estimation of the term structure of interest rates: regression analysis with cubic splines by Litzenberger and Rolfo (1984), binomial functions by Schaefer (1981), and parameterized methods utilizing parsimonious exponential functions by Nelson and Siegel (1987), Soderlind and Svensson (1997) and Bliss (1996).

In linear programming and regression approaches to term structure estimation, the estimator is a solution to an optimization problem. In linear programming, the estimator is the maximizer of the present value of a prescribed sequence of cash flows, subject to the present value of each bond being less than or equal to its price. In regression approaches, the estimator is the minimizer of the sum of squared deviations of the bond prices from their present values.

This paper considers a general framework in which I try to apply the regression approach and the linear programming approach to real term structure estimation for different bond data sets and to report on my experiments. Section 1 explains the methodology by presenting one discrete and four continuous approximation methods of term structure estimation. In Section 2, I give the results of my calculations and compare the estimated term structures across various types of bonds. The

results show that Schaefer's approach produces results that are much more stable than results from other methods. I also give my conclusions in this section. Section 3 raises implementation issues and serves to provide documentation on MATLAB codes that I have written. Here I describe the technical details of how my programs accomplished different programming tasks such as importing bond data from text files, constructing cash flows matrix, computing tenor periods and imposing monotonicity constraints on quadratic optimization.

2.1 Methodology

2.1.1 Discrete Approximation

The process of using a discrete approximation to estimate the term structure from a given sample of M bonds can be broken down into four steps:

- construct $N \times 1$ vector \mathbf{t} containing unique and monotonically increasing dates at which any coupon or principal payment is made,
- construct $M \times N$ matrix \mathbf{A} in which each row represents the cash flow structure of a particular bond mapped to corresponding dates in vector \mathbf{t} ,
- construct $M \times 1$ vector \mathbf{p} containing the bonds' cash prices by adding accrued interests to quoted prices,
- solve the *least squares problem* of finding the $N \times 1$ vector of discount factors \mathbf{d} , which would minimize the norm $\|\mathbf{A}\mathbf{d} - \mathbf{p}\|^2$, or,
- add monotonicity constraints on the discount factors and solve *quadratic programming* problem to find the closest fit to minimize the norm $\|\mathbf{A}\mathbf{d} - \mathbf{p}\|^2$, or
- add monotonicity constraints on the discount factors and solve a *linear programming* problem by maximizing the present value of future cash flows subject to the present value of each bond being less than or equal to its actual price.

I note that my experiments with these three methods showed that the solution to linear programming problem is much more computationally efficient and usually more accurate than solution to quadratic programming or least squares problems.

2.1.2 Continuous Approximation

Let's first give definition to discount function, yield curve and forward interest rate curve. Discount function, denoted as $d(t)$, is equal to present value of discount (zero-coupon) risk-free bond paying one dollar at time t . Continuously compounding interest rate on this type of bond is called spot rate and is denoted as $r(t)$. By changing maturity date t , one can obtain a plot of spot interest rates or a (spot) yield curve. Let $f(t)$ denote instantaneous forward rate, i.e. rate on forward contract with

maturity date equal to settlement date. Then, we have the following relationships between $d(t)$, $r(t)$ and $f(t)$:

$$d(t) = \exp(-r(t)t) = \exp\left(-\int_{s=0}^{s=t} f(s) ds\right) \quad (2.1)$$

$$r(t) = -\frac{\ln(d(t))}{t} = \frac{1}{t} \int_{s=0}^{s=t} f(s) ds \quad (2.2)$$

$$f(t) = -\frac{d(t)'}{d(t)} = r(t) + r(t)'t \quad (2.3)$$

The main drawback of the discrete approximation is that it estimates the discount rates only at the particular time values t_1, t_2, \dots, t_N . Continuous approximations avoid this by assuming that the discount rate takes a continuous functional form such as:

$$d(t) \approx \sum_{k=0}^K x_k b_k(t), \quad (2.4)$$

where $b_k(t)$ — are specified component functions of t .

Then the estimation problem consists of computing the $K + 1$ parameters x_k , such that $d(t)$ is consistent with market prices. This can be done by minimizing $\|\mathbf{A}\mathbf{B}\mathbf{x} - \mathbf{p}\|^2$ with respect to \mathbf{x} , where

A — $M \times N$ matrix of cash flows described above,

B — functional $N \times (K + 1)$ matrix, whose i -th row is given by

$$b(t_i) = [b_0(t_i), b_1(t_i), \dots, b_K(t_i)], \quad (2.5)$$

\mathbf{x} — decision vector $[x_0, x_1, \dots, x_K]'$.

Cubic Splines Regression Model by Litzenberger and Rolfo

Among the earlier methods of estimating term structure are those of Litzenberg and Rolfo. These authors utilize regression methodology to find the present value factors, or, equivalently, the term structure of spot interest rates, that best "explain" the observed prices of coupon bonds. Using multivariate linear regression analysis, the difference between a bond's price and its present value is minimized. This regression analysis incorporates smoothing of the term structure with cubic splines.

In the cubic spline method, $\{\tau_k\}_{k=0}^m \in [0, T]$ denotes a set of knot points over the interval in which coupon payments are made for which $0 \equiv \tau_0 < \tau_1 < \dots < \tau_m \equiv T$. The knots are placed such that an equal number of payment dates falls into each subinterval. The value for m is taken to be the integer closest to the square root of the total number of bonds in the marketplace ($m \approx \sqrt{M}$). The value of r may be interpreted as the sample size but should not be less than 10. The cubic splines approximate $d(t)$ by

$$d(t) \approx 1 + x_1 t + x_2 t^2 + x_3 t^3 + \sum_{k=1}^m x_{k+3} (t - \tau_k)^3 \mathbf{I}_{t \geq \tau_k}, \quad (2.6)$$

where $\mathbf{I}_{t \geq \tau_k} = 1$ when $t \geq \tau_k$ and $\mathbf{I}_{t \geq \tau_k} = 0$ when $t < \tau_k$.

For each realized payment date t_i , let $b(t_i)$ denote the vector

$$b(t_i) = [1, t_i, t_i^2, t_i^3, (t_i - \tau_1)^3 \mathbf{I}_{t_i \geq \tau_1}, (t_i - \tau_2)^3 \mathbf{I}_{t_i \geq \tau_2}, \dots, (t_i - \tau_m)^3 \mathbf{I}_{t_i \geq \tau_m}], \quad (2.7)$$

and \mathbf{B} denotes the $N \times (4 + m)$ matrix whose i -th row is given by $b(t_i)$. The estimation problem is then to minimize the norm $\|(\mathbf{AB})\mathbf{x} - \mathbf{p}\|^2$ for $\mathbf{x} = [1, x_1, x_2, \dots, x_{3+m}]'$.

The cubic splines are designed to be continuous and have both continuous first and second derivatives. Therefore we can derive the following forward rate curve from discount function of cubic spline method:

$$f(t) = -\frac{d(t)'}{d(t)} \approx -\frac{x_1 + 2x_2t + 3x_3t^2 + 3\sum_{k=1}^m x_{k+3}(t - \tau_k)^2 \mathbf{I}_{t \geq \tau_k}}{1 + x_1t + x_2t^2 + x_3t^3 + \sum_{k=1}^m x_{k+3}(t - \tau_k)^3 \mathbf{I}_{t \geq \tau_k}} \quad (2.8)$$

However, cubic spline is not guaranteed to be a monotonically decreasing function. To ensure monotonicity, I estimated the term structure using Schafer's approach.

Schaefer's Approach

In order to implement Schafer's approach, one must normalize time vector \mathbf{t} , i.e. we must scale each payment date by the longest one so that time is measured on the interval $[0, 1]$. The component functions $b_k(t)$ are defined as follows:

$$b_0(t) \equiv 1, \quad (2.9)$$

and for each $k = 1, 2, \dots, K$,

$$b_k(t) = -\int_0^t u^{k-1}(1-u)^{K-k} du = \sum_{j=0}^{K-k} (-1)^{j+1} C_j^{K-k} \left(\frac{t^{k+j}}{k+j} \right) \quad (2.10)$$

where $C_n^k = \frac{n!}{k!(n-k)!}$ and K is usually taken to be 25.

Each x_k is constrained to be non-negative, and each $b_k(t)$ is a monotonically decreasing nonpositive function on the interval $[0, 1]$. Therefore, Schafer's Approach ensures that $d(t)$ is a monotonically decreasing function. To ensure that $d(t)$ are nonnegative, one also adds the constraint

$$\sum_{k=0}^K x_k b_k(1) \geq 0. \quad (2.11)$$

and the parameters are estimated as they were before.

Nelson-Siegel Exponential Functions Method

Rather than explicitly modeling the term structure, one may want to approximate it by a flexible functional form. There are several parsimonious exponential models, proposed by Nelson and Siegel (1987), Soderlind and Svensson (1997) and Bliss (1996).

The Nelson-Siegel approximation is derived from the assumption that the spot rates follow a second-order differential equation and that forward rates, which are the predicted spot rates, are the solution to this differential equation with equal roots. Let's assume that the equation for instantaneous forward rate is given in the following form:

$$f(t) = \beta_0 + \beta_1 \exp\left(-\frac{t}{\lambda}\right) + \beta_2 \frac{t}{\lambda} \exp\left(-\frac{t}{\lambda}\right) \quad (2.12)$$

Then the yield curve can be computed using formula 2.2 to obtain:

$$r(t) = \beta_0 + (\beta_1 + \beta_2) \left[\frac{1 - \exp\left(-\frac{t}{\lambda}\right)}{\frac{t}{\lambda}} \right] - \beta_2 \exp\left(-\frac{t}{\lambda}\right). \quad (2.13)$$

Soderlind and Svensson (1997) improved the Nelson-Siegel model by formulating forward rates and spot rates as follows:

$$f(t) = \beta_0 + \beta_1 \exp\left(-\frac{t}{\lambda_1}\right) + \beta_2 \frac{t}{\lambda_1} \exp\left(-\frac{t}{\lambda_1}\right) + \beta_3 \frac{t}{\lambda_2} \exp\left(-\frac{t}{\lambda_2}\right) \quad (2.14)$$

$$r(t) = \beta_0 + (\beta_1 + \beta_2) \left[\frac{1 - \exp\left(-\frac{t}{\lambda_1}\right)}{\frac{t}{\lambda_1}} \right] - \beta_2 \exp\left(-\frac{t}{\lambda_1}\right) + \beta_3 \left[\frac{1 - \exp\left(-\frac{t}{\lambda_2}\right)}{\frac{t}{\lambda_2}} - \exp\left(-\frac{t}{\lambda_2}\right) \right] \quad (2.15)$$

The Nelson-Siegel method also took further development by Bliss (1996). His improved approximation is given as:

$$f(t) = \beta_0 + \beta_1 \exp\left(-\frac{t}{\lambda_1}\right) + \beta_2 \frac{t}{\lambda_2} \exp\left(-\frac{t}{\lambda_2}\right) \quad (2.16)$$

$$r(t) = \beta_0 + \beta_1 \left[\frac{1 - \exp\left(-\frac{t}{\lambda_1}\right)}{\frac{t}{\lambda_1}} \right] + \beta_2 \left[\frac{1 - \exp\left(-\frac{t}{\lambda_2}\right)}{\frac{t}{\lambda_2}} - \exp\left(-\frac{t}{\lambda_2}\right) \right] \quad (2.17)$$

In each of the above models, parameters $\beta_0, \lambda, \lambda_1$ and λ_2 must be positive. The parameter $\frac{1}{\lambda}$ governs the exponential decay rate; small values of $\frac{1}{\lambda}$ produce slow decay and can better fit the curve at long maturities, while large values of $\frac{1}{\lambda}$ produce fast decay and can better fit the curve at short maturities. We can interpret $\beta_0, \beta_1, \beta_2$ as three latent factors. The loading on β_0 is a constant that does not decay to zero in the limit; thus, it may be viewed as a long-term factor. The loading on β_1 is $\left[1 - \exp\left(-\frac{t}{\lambda_1}\right)\right] \frac{\lambda_1}{t}$, which starts at 1 but decays quickly and monotonically to 0; hence, β_1 may be viewed as a short term factor. The loading on β_2 is $\left[1 - \exp\left(-\frac{t}{\lambda_2}\right)\right] \frac{\lambda_2}{t} - \exp\left(-\frac{t}{\lambda_2}\right)$, which starts at 0 (and is thus not short-term), increases, and then decays to zero (and thus is not long term); hence, β_2 can be interpreted as a medium term factor.

β_0, β_1 and β_2 can also be interpreted in terms of the aspect of the curve that they govern: level, slope, and curvature. The long-term factor β_0 governs the yield curve level. In particular, $r(\infty) = \beta_0$. Alternatively, note that an increase in β_0 augments all yields equally, as the loading is

identical at all maturities. The short-term factor β_1 is equal to the yield curve slope, $r(\infty) - r(0)$. Note that an increase in β_1 augments short yields more than long yields because the short rates load on β_1 more heavily, thereby changing the slope of the yield curve. Finally, β_2 is closely related to yield curvature: an increase in β_2 will have very little effect on very short or very long yields, which load minimally on it, but will increase medium-term yields, which load more heavily on it, thereby increasing yield curve curvature.

Let θ denote the set of five parameters discussed above, i.e.

$$\theta = \{\beta_0, \beta_1, \beta_2, \lambda_1, \lambda_2\}. \quad (2.18)$$

The parameters can be estimated using nonlinear least-squares data fitting by the Gauss-Newton method, implemented in `nlinfit` function from MATLAB Statistics Toolbox, i.e.

$$\hat{\theta} = \arg \min \sum_{i=1}^N \varepsilon_i^2, \quad (2.19)$$

where ε_i is the difference between the actual market yields and theoretical fitted yields on i -th bond at time $t = 0$ in both methods.

2.2 Results and Discussion

2.2.1 Comparison of Approximation Methods

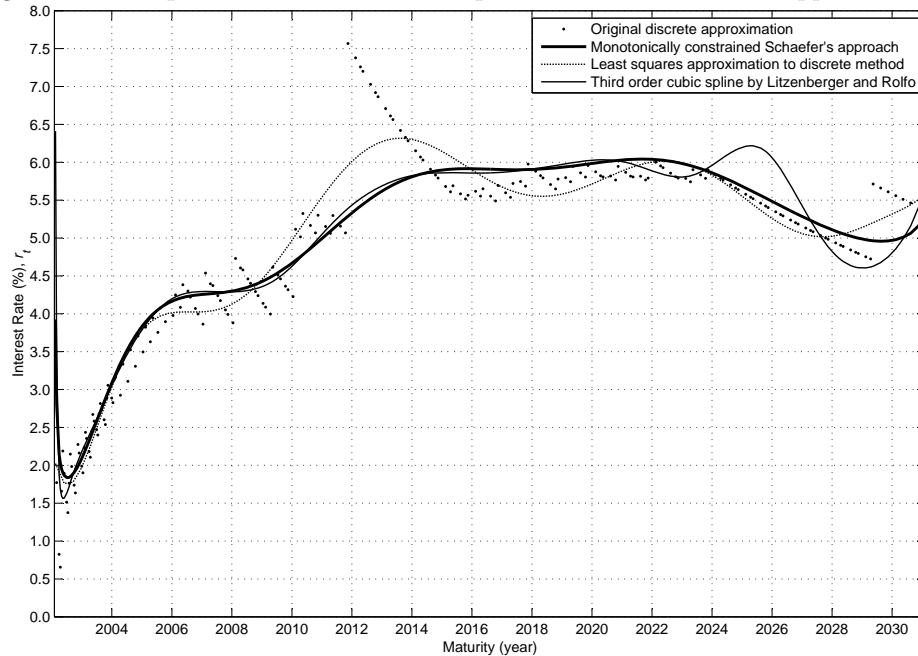
In this paper I use five methods to estimate the term structure from different kinds of bonds - one discrete approximation and four continuous approximations. For the discrete approximation, the objective function is given in terms of discount factors $d(t)$ on discrete dates t . I simply find the vector \mathbf{d} such that the sum of the squared errors $\|\mathbf{A}\mathbf{d} - \mathbf{p}\|^2$ is minimized. The model, therefore, is easy to formulate. However, the quality of the solutions is not likely to be satisfying since discrete approximations rarely give smoothed curves.

Let's now consider the first two continuous approaches. The first one is the cubic spline method, which defines a new set of variables, x_i . This method tries to fit the values of each x_i such that the sum of the squared errors $\|(\mathbf{A}\mathbf{B})\mathbf{x} - \mathbf{p}\|^2$ is minimized. To formulate this model, we have to construct a new matrix, \mathbf{B} , where \mathbf{d} is estimated to equal to $\mathbf{B}\mathbf{x}$. This method produces better solutions than those from discrete method.

In the second continuous approximation, Schaefer's approach, we also have to construct a new matrix, \mathbf{B} , and a set of variables \mathbf{x} . The objective of this method is the same as the former two; that is, to minimize the sum of the squared errors. However, in this method, unlike in the former two, time is scaled such that it is measured on the interval $[0, 1]$.

I found that the curves produced using Schaefer's approach almost always look smoother than those of the other two methods. This method also gives me the monotonically decreasing function of $d(t)$ by simply setting the lower bound of the variables to zero. In practice, it takes a short time to

Figure 2.1: Comparison of discrete, least squares, Schaefer and cubic approximations



This figure shows how noise in discrete method can be smoothed by using least squares approximation from MATLAB splines toolbox. Cubic splines produce more volatile solution compared to the Schaefer's method.

solve for the optimal solution while the discrete approximation with a set of monotonic constraints takes a much longer time to solve for a set of solutions.

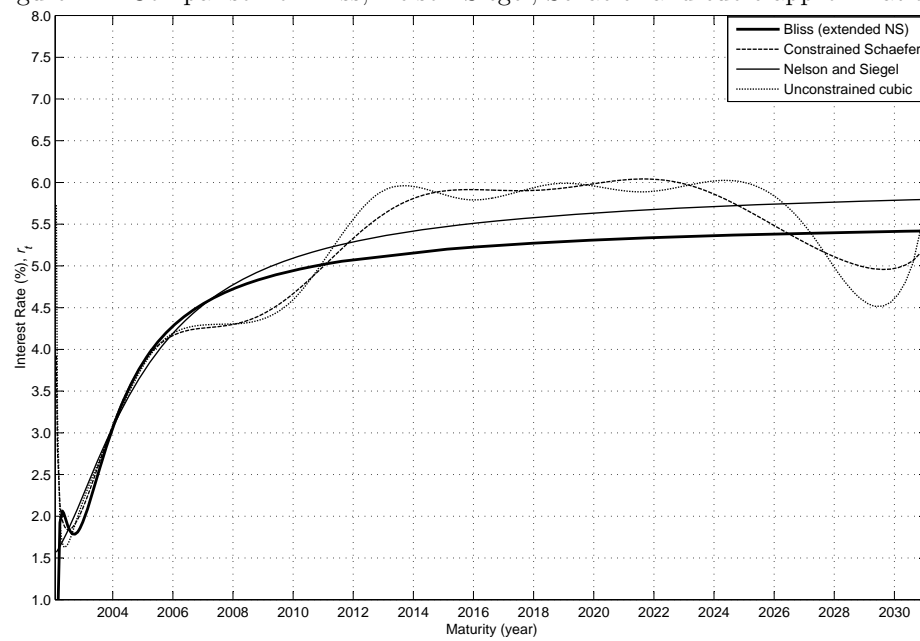
Figure 2.1 illustrates the treasury yield curves obtained using each method. I found that the curve from Schaefer's method gave me the smoothest curve while the discrete approximation produced the worst curve. The continuous methods also do better in avoiding the unreasonable fluctuations in the curve.

Figure 2.2 compares Nelson-Siegel, Bliss, cubic and Schaefer approximations. The exponential function approximation proposed by Nelson and Siegel and its further development by Robert Bliss resemble the embedded exponential shape. As we expected, the exponential form is much less volatile compared to cubic splines and Schaefer's approximation and thus is most preferred in practice.

2.2.2 Estimated Term Structure Across Various Bond Types

My estimates for the term structure using a discrete approximation, cubic splines and Schaefer's approach were calculated using data from treasury coupon securities, zero coupon bonds (STRIPS), AA government bonds, and AAA corporate bonds. Figure 2.3 shows the term structure as estimated by Schaefer's approach with the constraints $d(t) \geq 0$ (non-negativity) and $d_1 \geq d_2 \geq \dots \geq d_N$ (monotonicity) for each of the aforementioned types of bonds.

Figure 2.2: Comparison of Bliss, Nelson-Siegel, Schaefer and cubic approximations



This figure shows that the smooth exponential functional approximation proposed by Nelson-Siegel and Bliss is the preferred method to estimate yield curves compared with volatile cubic splines and Schaefer approximations.

Figure 2.3: Term structure for various types of bond by constrained Schaefer method

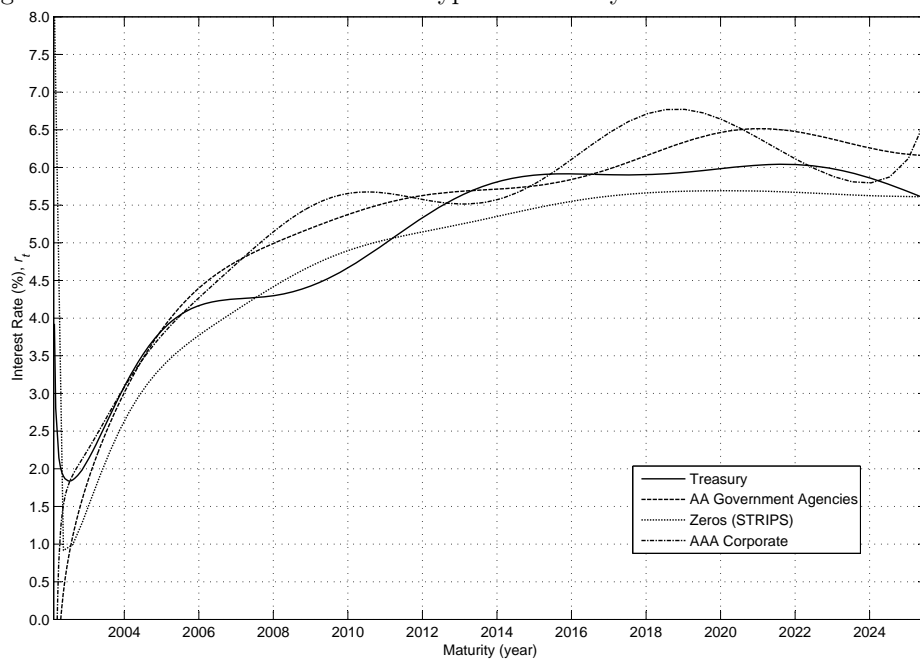
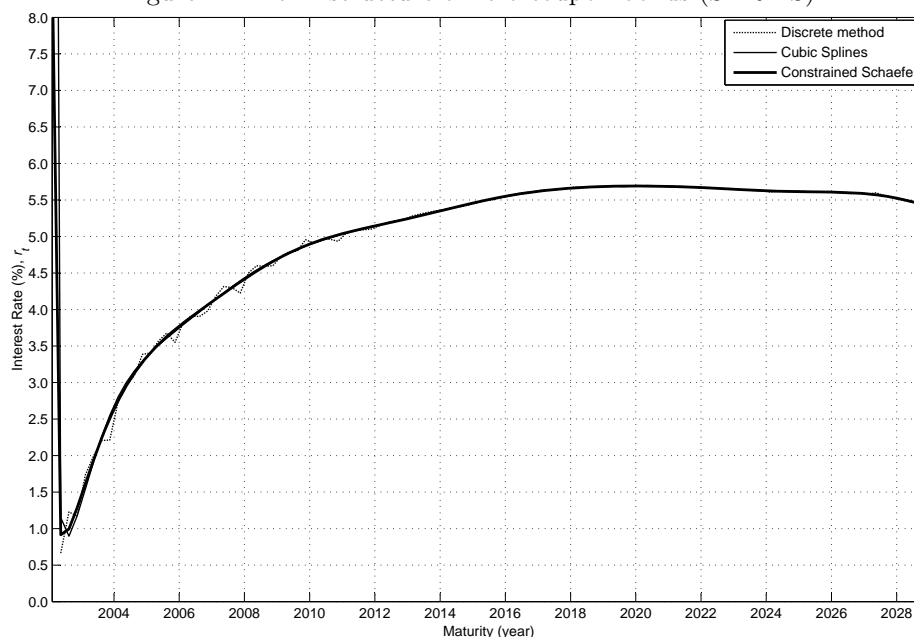


Figure 2.4: Term structure of zero-coupon bonds (STRIPS)



In case of STRIPS, all of the considered methods produce very close solutions due to the fact that zero coupon bonds do not bear intermediate coupons payments that create volatility in estimating term structure.

As ‘zeroes’ only pay the principals at maturities and do not make periodic coupon payments, they produce the ‘smoothest’ term structure curve as can be seen by the lack of oscillations. Only the short rates between maturity dates, when the principal is paid to the investor, need be considered when estimating the term structure with zeroes. Note that a smooth yield curve is always produced when the term structure is estimated from STRIPS regardless of whether a discrete or continuous approximation is used (figure 2.4).

Furthermore, zeroes and treasuries produce the lowest yield curves - i.e., they predict lower yields to maturity compared to the estimates made using other bonds. This is because zeroes and treasuries are backed by the full faith and credit of the U.S. government and thus have no credit risk; hence, they have lower risk premiums (smaller yields) than other types of bonds.

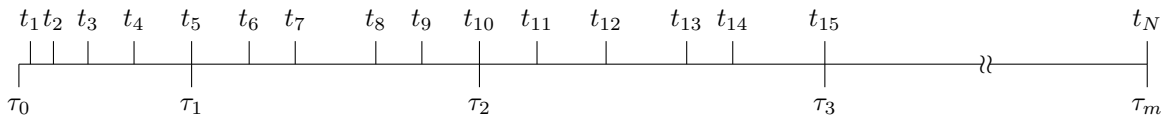
Bonds issued by government sponsored enterprises (GSEs) such as the Federal Home Loan Mortgage Corporation and the Federal National Mortgage Association are privately owned and publicly chartered entities; they carry slightly more credit risk than do treasuries.

Corporate bonds, of course, carry the most credit risk since the ability of a firm to make timely principal and coupon payments depends on how successful the firm is, which varies from fiscal year to fiscal year. These varying susceptibilities to credit risk are in accordance with figure 2.3, in which the corporate bond yield curve tends to be higher than the AA government bond curve, which, in turn, is higher than the yield curve estimated from zeroes.

Treasuries, GSE securities, and corporate bonds are all equally susceptible to interest rate risk, inflation risk, and reinvestment risk. Although the U.S. government backs both zeroes and treasuries, since zeroes do not pay coupons, they are not susceptible to reinvestment risk, which may explain why they tend to predict lower yields to maturity than do treasury coupon securities.

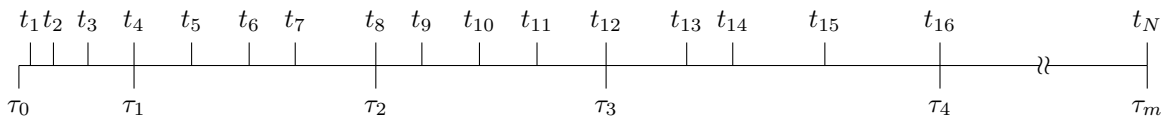
2.2.3 Effect of Including More Terms in the Continuous Approximation

For continuous approximations, I used the cubic splines and Schaefer's approach. In the cubic spline method, I define the knot points $\tau_0, \tau_1, \dots, \tau_m$ such that an equal number of payment dates falls into each subinterval:



We know that $d(t) \approx 1 + x_1t + x_2t^2 + x_3t^3 + \sum_{k=1}^m x_{k+3}(t - \tau_k)^3 I_{t \geq \tau_k}$, where $I_{t \geq \tau_k} = 1$ when $t \geq \tau_k$ and $I_{t \geq \tau_k} = 0$ when $t < \tau_k$. If we closely look at this condition, we will see that each $d(t)$ will be made up of different numbers of variables x_i , i.e. in the example above, $d(t_1), d(t_2), d(t_3), d(t_4), d(t_5)$ are each the linear combination of three variables x_1, x_2 and x_3 , while each of $d(t_6), d(t_7), d(t_8), d(t_9)$ and $d(t_{10})$ is the linear combination of four variables x_1, x_2, x_3 and x_4 , and each of $d(t_{11}), d(t_{12}), d(t_{13}), d(t_{14}), d(t_{15})$ is the linear combination of five variables x_1, x_2, x_3, x_4 and x_5 , and so on.

In each subinterval, the values of $d(t)$ will not be much different since they are estimated from the same set of variables. Nevertheless, when we add more terms in the approximation, i.e. when we increase m , the number of payment dates in each subinterval will decrease. This, therefore, leads to the decreasing of smoothness of $d(t)$.



Top panel of figure 2.5 illustrates the effect of including more conditioned terms in the cubic spline approximation 2.6. I found that the best solution is obtained if I set $m = 10$ as it was recommended integer closest to the square root of the number of bonds (135 for treasury securities). When I increased the value of m to 20, the curve swung slightly higher than it did when when m was equal to 15. For m equal to 25, the interest rate curve deviated wildly. I found that the curve did not oscillate so frequently if m was decreased to 10 or 5. The number of terms can be interpreted as our sample size. Including more terms in cubic spline approximation increases volatility. This is evident in case of 25 terms. However, the allowable number of terms should be at least 10. Otherwise, the term structure would be overly smoothed (as in the case of 5 terms).

I also considered the effect of including more unconditioned terms, i.e. those of the from $x_i t^i$ in cubic spline approximation 2.6. Bottom panel of figure 2.5 shows term structures as estimated with treasuries using the cubic splines with 3, 4, 5, 6, and 9 unconditioned terms. Including more than

three unconditioned terms in cubic spline approximation results in higher volatility and hence not recommended. When 4, 5, or 6 terms are used, the yield curve estimation oscillated noticeably. The 3-term curve and 9-term curve seemed to produce the best fit.

In Schaefer's approach, I found the same results; that is, the more I increased the number of terms K , the more the interest rate curves fluctuated (figure 2.6). In this project I illustrated the effect of including more terms in Schaefer's approximation on the term structure of treasury securities. I varied the number of terms K from 15 to 45. I found that the discount rate curve corresponding to $K = 15$ resembled the one corresponding to $K = 25$. However, when I increased the value of K to 30 and to 35, the oscillations worsened. A wildly fluctuating curve resulted when I increased the value of K to 45.

Let me speculate as to why these oscillations occur. I estimate the discount rates, $d(t)$, by construction of the vector $\mathbf{x} = [x_0, x_1, \dots, x_K]'$. Each $d(t)$ is equal to the linear combination of $x_k, k = 0, 1, \dots, K$. When we increase the value of K , we increase the number of terms in the linear combination for each $d(t)$. Consider what would happen if we continuously increase the value of K until it equals M , the total number of bonds. This optimization problem will try to estimate a set of M variables such that they minimize the sum of the squared errors from M sets of data. From a statistical point of view, it is meaningless to do so. This reason can be used to explain the fluctuations in the cubic spline approximation as well.

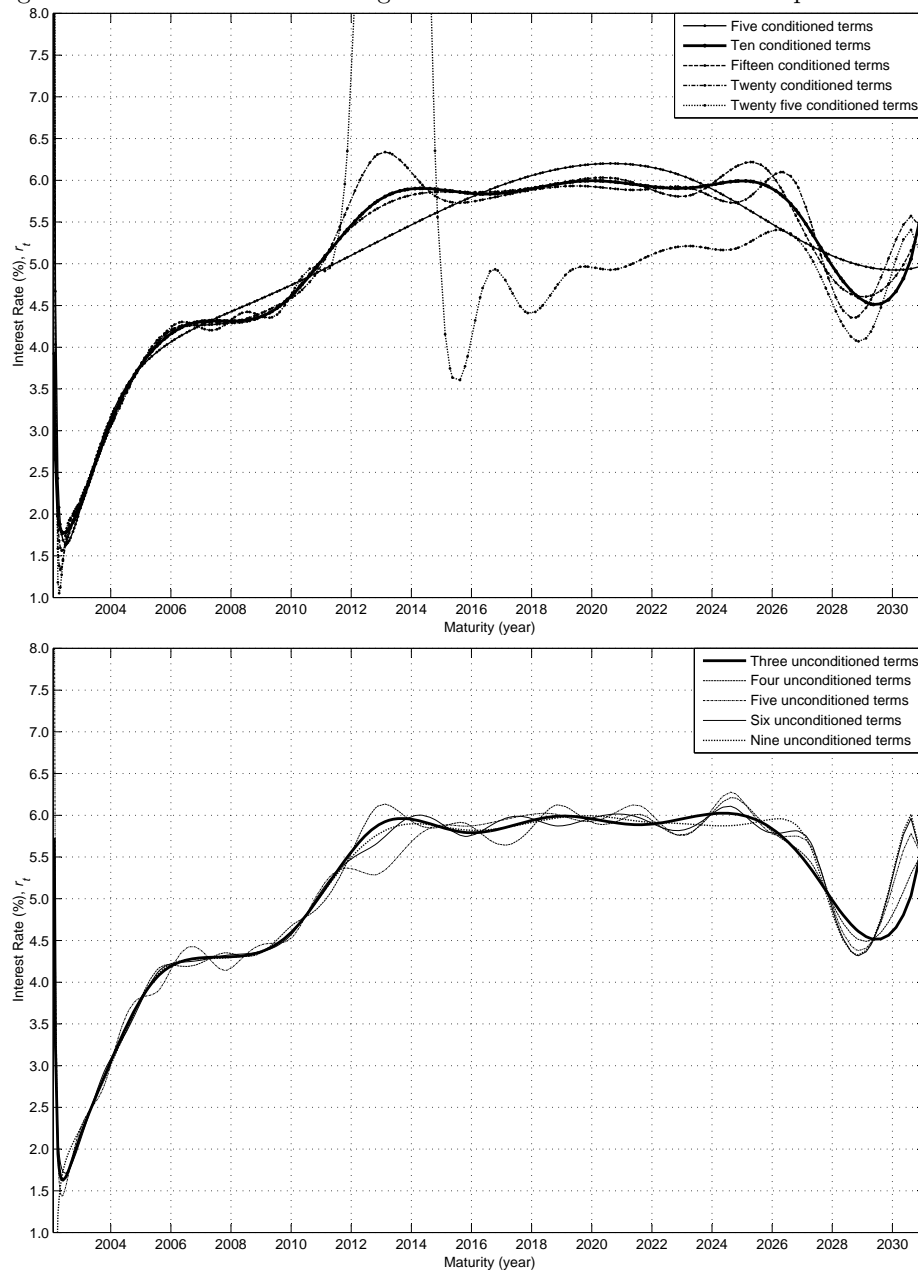
2.2.4 Effect of Monotonicity Constraints

In theory, it should be the case that $d_1 \geq d_2 \geq \dots \geq d_N$, i.e. the discount rates should be declining. Otherwise, a negative interest rate exists between two payment dates. Schaefer's approach guarantees that the discount rates will be declining. However, for the discrete approximation, the monotonicity constraints may or may not be enforced. I examined the discrete approximation estimates of the term structure with and without monotonicity constraints on $d(t)$. Figure 2.7 shows that both estimates produce discount factors that trend downward with increasing maturity, but, as expected, only when the constraints are enforced is $d(t)$ monotonically decreasing.

Figure 2.7 also compares the constrained and unconstrained discrete approximations to the yield curves. For early maturities - those before 2006 - the unconstrained approximation is better; it produces a smooth yield curve. In contrast, the monotonicity constraints force a series of zigzags to appear in the early part of the yield curve: when $d(t_i) = d(t_{i+1})$, then $r(t_i) > r(t_{i+1})$, which produces the downward slopes of each peak, and when $d(t_i) > d(t_{i+1})$, then $r(t_i) < r(t_{i+1})$, resulting in the small upward jumps. However, for later maturities, when yields are more difficult to predict, the unconstrained discrete approximation produces a rather noisy yield curve. The constrained solution, on the other hand, produces a yield curve with smaller fluctuations for later maturities.

Similar results were observed in Schaefer method. Figure 2.8 shows that the term structure of treasury securities obtained from the constrained method is smoother than the curve obtained from the unconstrained method. The unconstrained discount rates do not decrease monotonically at longer maturities which destabilizes long term interest rates.

Figure 2.5: The effect of including more conditioned terms in cubic spline method



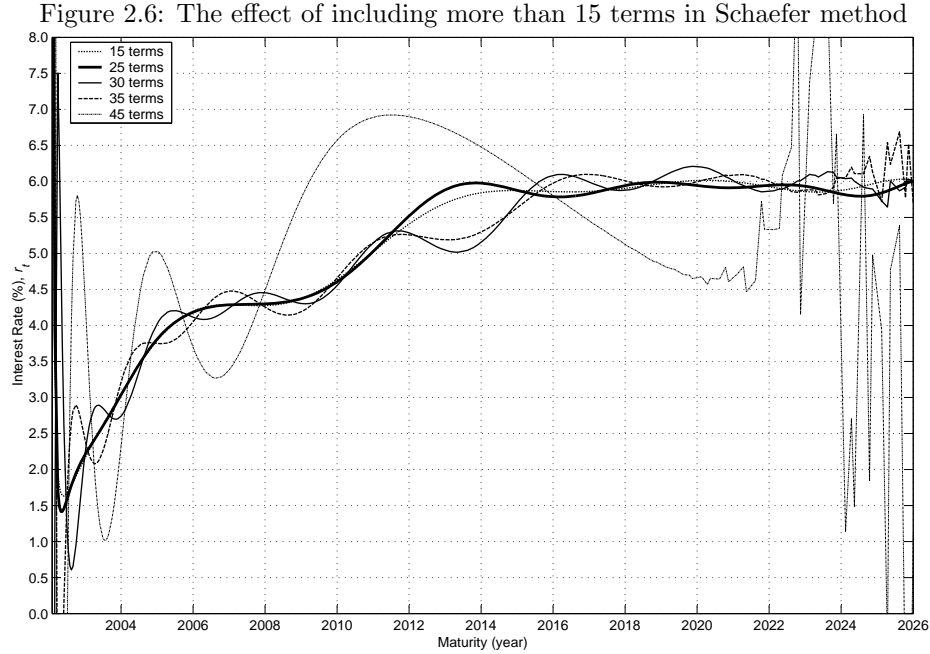


Figure 2.9 demonstrates that adding monotonicity constraints to Schaefer’s approximation is important for estimation of term structures of corporate bonds and government agency bonds. Without monotonicity constraints, the term structure of these bonds is useful only for short maturities.

I also compared constrained and unconstrained versions of the cubic approximations. I decided to impose constraints on the cubic approximation in order to remove the upward slope that resulted at the end of the discount factor curve $d(t)$ when the constraints were ignored. However, the constrained cubic approximation was difficult to implement. The effect of adding monotonicity constraints on discount factors was a rapid decline of $d(t)$ in the short term (figure 2.10). Enforcing monotonicity in the cubic approximation produces an unrealistic, nearly vertical yield curve and is thus unusable.

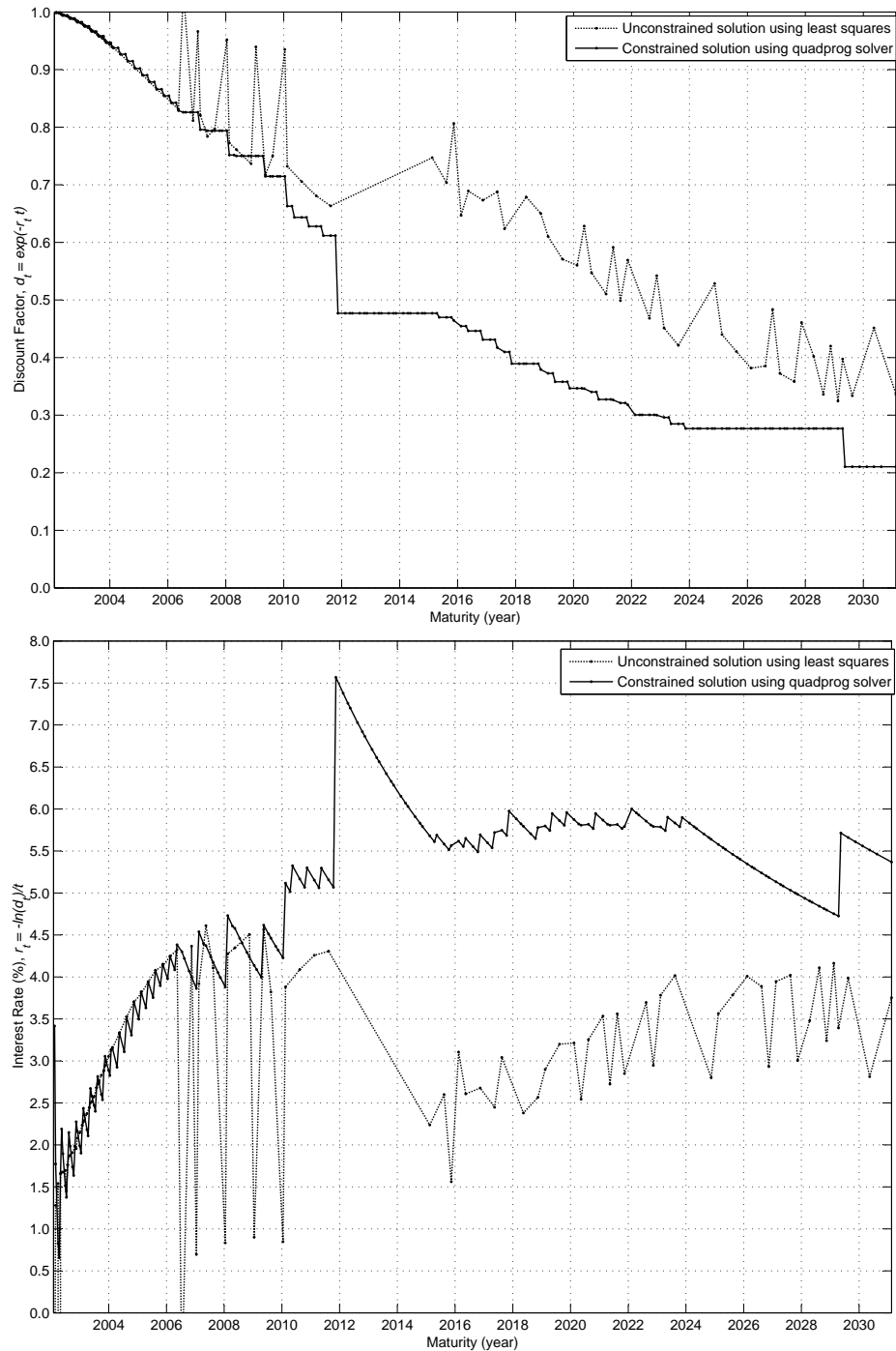
In terms of optimization effort, it is obvious that adding some constraints to the model will require more effort to solve for a set of solutions. In order to minimize the norm $\|\mathbf{A}\mathbf{d} - \mathbf{p}\|^2$, I simply use the linear least squares method. To apply this method, given a dependent variable, y , and a set of independent variables, x_1, x_2, \dots, x_N , I try to find a linear relation by determining the set of parameters, b_0, b_1, \dots, b_N , such that the sum of the squared errors is minimized. To do this, I construct a vector of dependent variables y_i , say \mathbf{y} , and a matrix of independent variables x_i , say \mathbf{X} , from the historical data and set the relations as follows:

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \varepsilon, \quad (2.20)$$

where \mathbf{b} is the vector of parameters b_i , and ε is the vector of errors. Using the least squares method to minimize $\varepsilon'\varepsilon$ (the sum of the squared errors), we may calculate the solution from

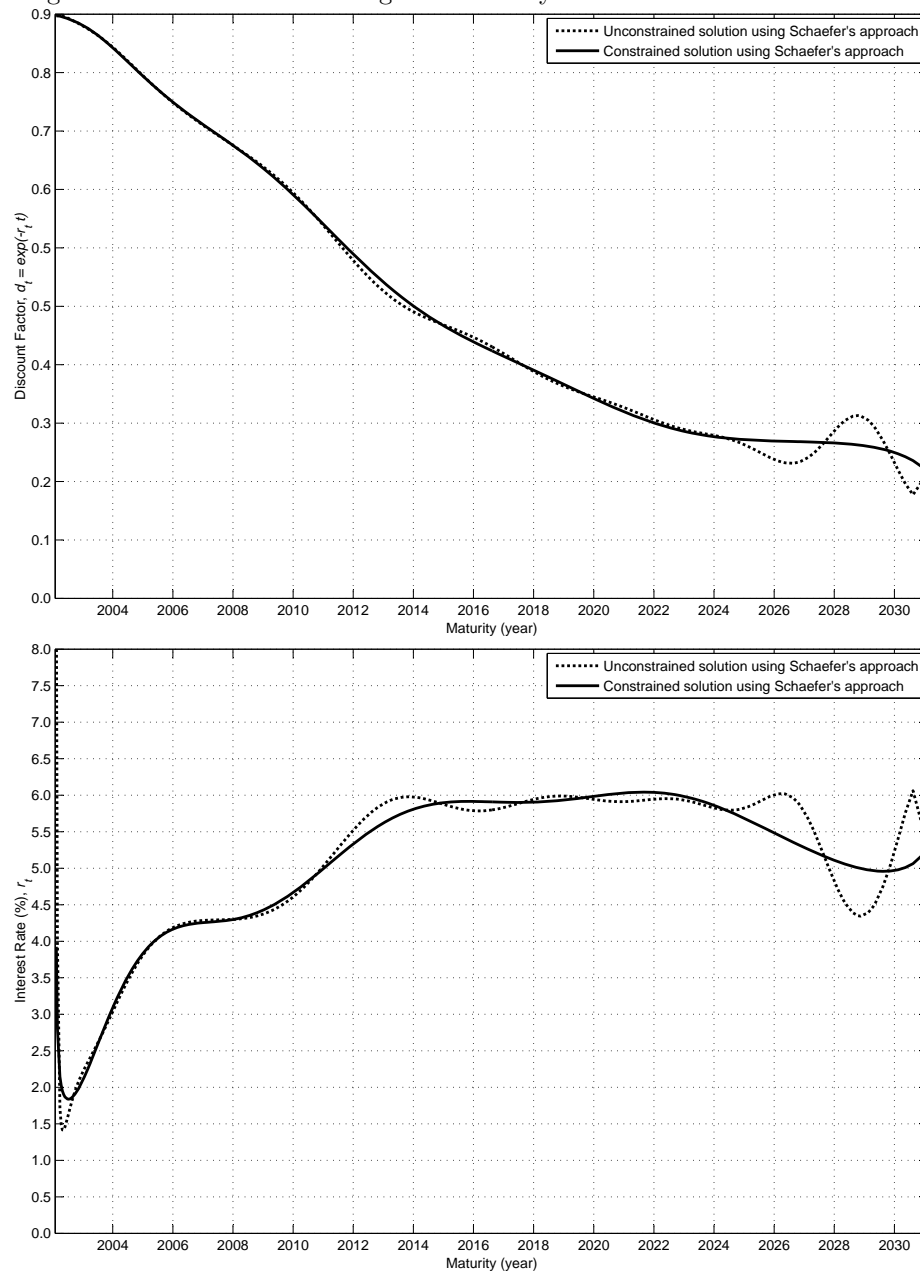
$$\mathbf{b}^* = (\mathbf{X}'\mathbf{X})^{-1} \mathbf{X}'\mathbf{y}. \quad (2.21)$$

Figure 2.7: The effect of adding monotonicity constraints in discrete approximation



The top panel of this figure shows how monotonicity constraints in discrete approximation create L-shaped patterns in discount curve when there is no data available. This explains large deviations in term structure in the bottom panel which also shows that constrained yield curve is smoother than unconstrained one in periods when data is available.

Figure 2.8: The effect of adding monotonicity constraints in Schaefer method



Adding monotonicity constraints in Schaefer's method further reduces volatility of discount rates and interest rates. In contrast with discrete method, adding constraints to the optimization process in Schaefer's method did not require much more computational effort because of less number of variables (26 in case of cubic splines compared to 211 in discrete method).

Figure 2.9: Constrained Schaefer method for corporate and government agency bonds

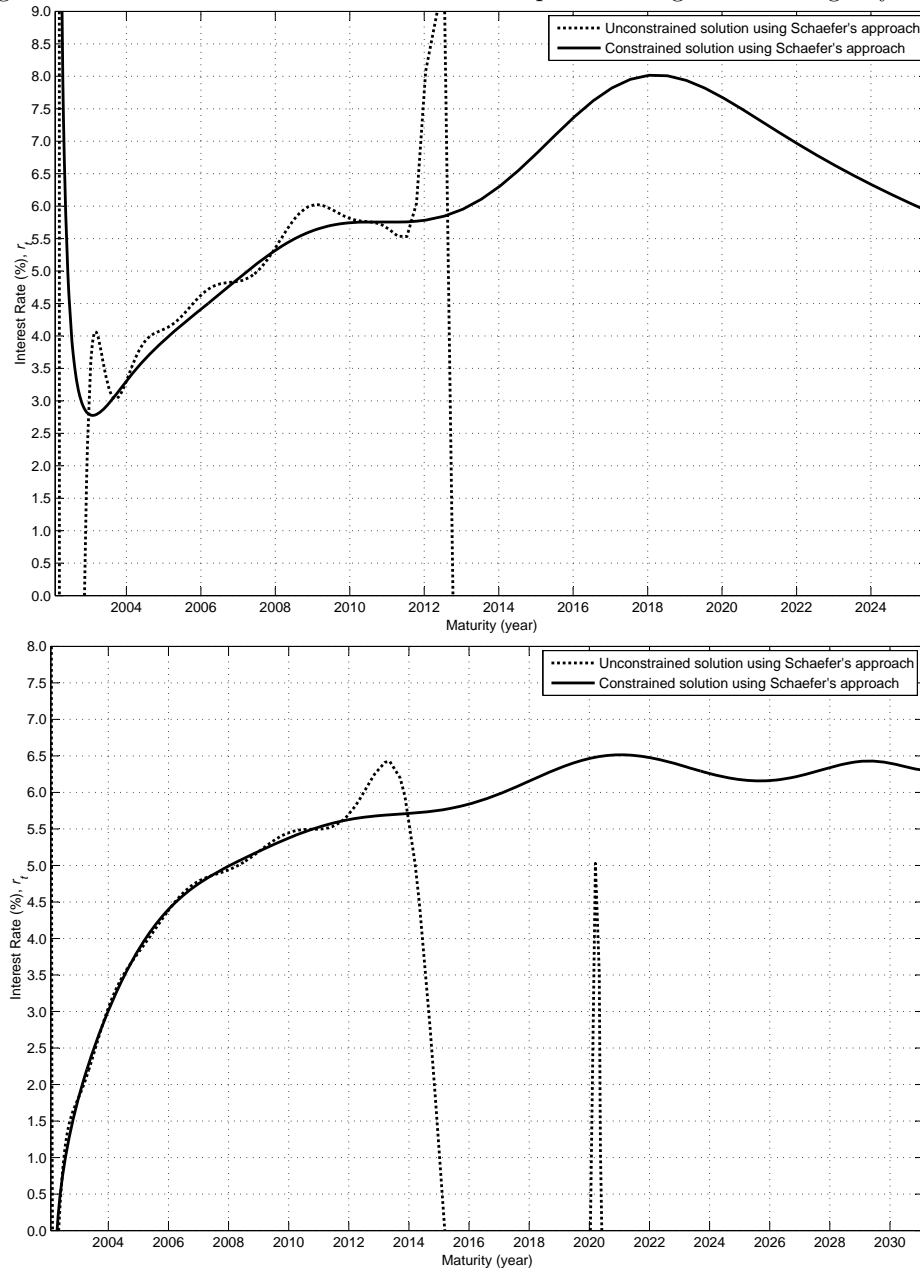
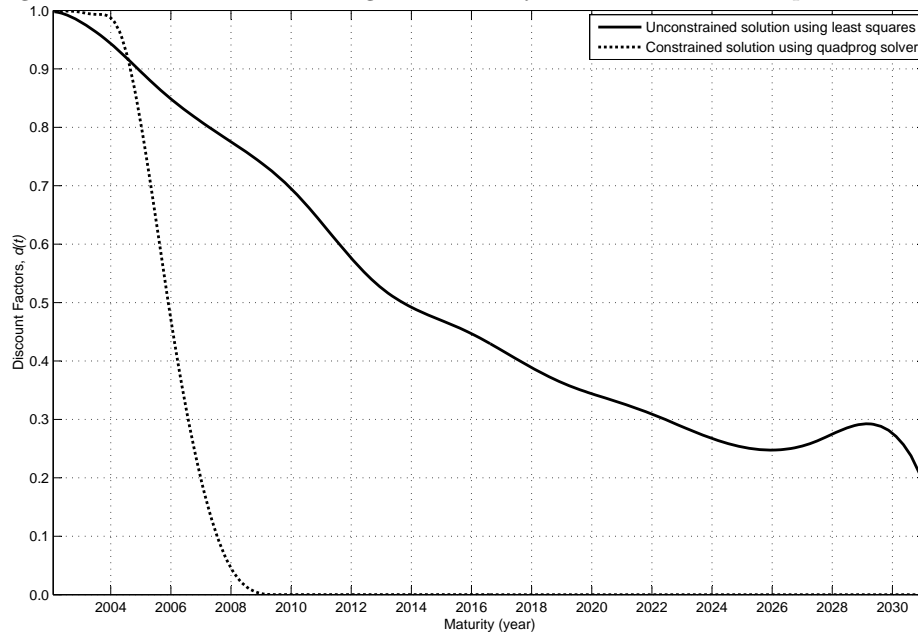


Figure 2.10: The effect of adding monotonicity constraints in cubic spline method



This figure shows that we should not constrain discount factors in cubic spline method because discount factors would drop rapidly to zero.

In discrete model, I defined a vector of prices \mathbf{p} and a matrix of coupon payments \mathbf{A} . The vector of discount rates that minimize the norm $\|\mathbf{A}\mathbf{d} - \mathbf{p}\|^2$ is $\mathbf{d}^* = (\mathbf{A}'\mathbf{A})^{-1} \mathbf{A}'\mathbf{p}$. For continuous models, I simply replace matrix \mathbf{A} with \mathbf{AB} , and we have $\mathbf{x}^* = (\mathbf{B}'\mathbf{A}'\mathbf{AB})^{-1} \mathbf{B}'\mathbf{A}'\mathbf{p}$.

Although calculating the inverses of the matrices may take a long time, many software packages such as MATLAB use the interior-reflective Newton method in which each iteration involves the approximate solution of a large linear system using the method of preconditioned conjugate gradients (PCG). In practice, we solve this kind of problem in a few seconds.

When we add a set of constraints to the models, we cannot use the same formula to find the solutions. In MATLAB, when we add the inequality constraints, the problem can be solved using quadratic programming. This program uses an active set method which finds an initial feasible solution by first solving a linear programming problem. At each major iteration, a positive definite quasi-Newton approximation of the Hessian of the Lagrangian function is calculated using the BFGS method. In the case of discrete approximation, where we have a large number of variables, the time required to find monotonically constrained solutions reached the order of several minutes. However, in the case of continuous approximations, the computation time was negligible due to the small number of monotonically constrained variables.

2.2.5 Conclusions

In this project I experimented with various methods for approximating the term structure from different types of bonds. My results showed that the constrained Schaefer's approximation produces smoother and more stable curves than does unconstrained Schaefer's approximation, cubic splines, or discrete approximation. Moreover, the effect of adding monotonicity constraints in Schaefer's method does not significantly increase in the optimization effort as it does in the discrete approximation. My experiments showed the unreasonable results of adding monotonicity constraints on cubic spline method. Finally, I experimented with three well know parsimonious functional approximations of the term structure, the Nelson-Siegel, Svensson and Bliss exponential functions.

Chapter 3

Sovereign Defaults

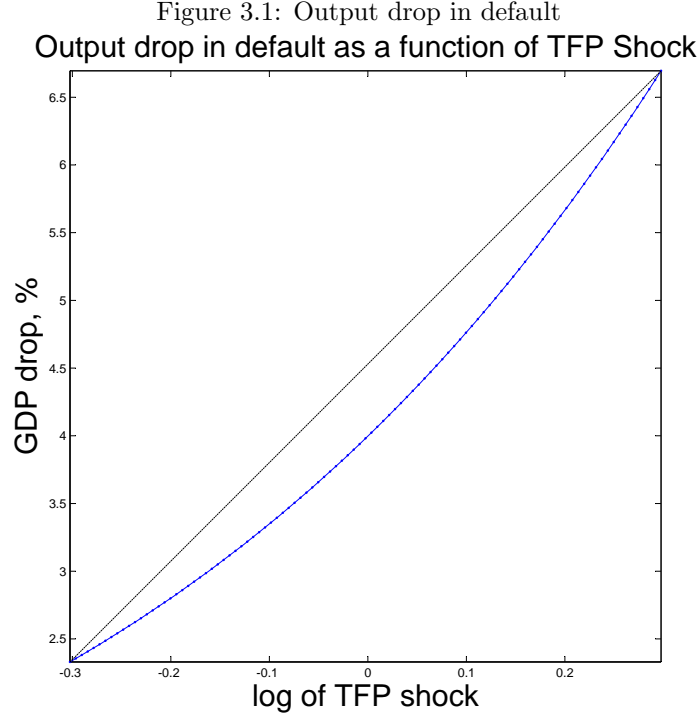
Two recent papers advance frontiers of sovereign default modeling. First, Aguiar and Gopinath (2006) highlight the importance of fluctuations in long-term productivity growth for sovereign default dynamics in emerging markets. Lately, Mendoza and Yue (2012) build a quantitative model in which both sovereign default spreads and output costs of default are determined endogenously. The output costs of default arise because domestic producers lose access to trade credit and are forced to substitute away from foreign intermediate inputs. Both papers substantially increase debt-to-GDP ratios of defaulting countries, relative to those attained in earlier sovereign default models. Nevertheless, debt-to-GDP ratios in both papers still fall short of those ratios observed in the data (23 percent in the models, versus 35 to 71 percent in the data). Both papers are calibrated to Argentinean data and have little to say about sovereign defaults in developed countries.

In this chapter we investigate sovereign defaults in the presence of temporary and permanent shocks to firms' total factor productivity. We introduce fluctuations in trend growth to the model of Mendoza and Yue (2012) to achieve three objectives. First, we want to improve the model's fit of debt-to-GDP ratios in the data. Second, the model will account for default spread dynamics in both emerging and developed economies by allowing for differences in trend growth volatility, as measured by Aguiar and Gopinath (2007). Lastly, we would like to build a state of the art model to match sovereign default dynamics in both emerging and developed economies and to analyze policy alternatives in the current economic environment, like for example, optimal IMF lending policy to sovereign borrowers.

3.1 Two Models with Stable Trend and Growth Shocks

We are considering two extensions of the baseline model of Mendoza and Yue (2012). In addition to transitory TFP shocks, models 1 and 2 introduce stable and volatile trend to TFP respectively.

There are four agents: households, firms, sovereign government and risk neutral foreign lenders. There are two sectors: final f and intermediate m goods. Final goods producers borrow working capital at fixed interest rate r^* from abroad to pay for subset of imported intermediate inputs.



Sovereign default excludes both firms and government from world credit markets for a period of time as a punishment in a standard framework of Eaton and Gersovitz (1981). Default causes final good firms to incur efficiency loss due to imperfect substitution of imported intermediate goods. Default cost (output drop) is an increasing convex function of TFP determined endogenously. Figure 3.1 depicts the drop in GDP at the same period of default as a function of the TFP shock.

The TFP process ε consists of two components: transitory shock z and permanent shock Γ . Both models add permanent growth to transitory z TFP shocks ε through

$$\begin{aligned}\varepsilon_t &= \exp(z_t + \Gamma_t) \\ z_t &= \mu_z(1 - \rho_z) + \rho_z z_{t-1} + \varepsilon_{zt} \\ \Gamma_t &= \mu_\Gamma + \Gamma_{t-1} + g_t \\ g_t &= \mu_g(1 - \rho_g) + \rho_g g_{t-1} + \varepsilon_{gt}\end{aligned}$$

where ε_{zt} and ε_{gt} are i.i.d. normal random variables with zero mean and standard deviations σ_z and σ_g respectively. In order $E[\hat{\varepsilon}] = 1$, we set $\mu_z = -\frac{\sigma_z^2}{2(1-\rho_z^2)}$ and $\mu_g = -\frac{\sigma_g^2}{2(1-\rho_g^2)}$. In model 1, growth is stable with $\sigma_g = 0$. Model 2 features additional shock g to permanent growth μ_Γ by allowing $\sigma_g > 0$.

For labour hours to remain stationary in household's problem, we need to use Cobb-Douglas

preferences or adjust GHH preferences with permanent productivity shocks on disutility of labor:

$$\begin{aligned} \max_{c_t, L_t} \quad & E_0 \sum_{t=0}^{\infty} \frac{\beta^t}{1-\sigma} \left(c_t - \exp(\mu_\Gamma + \Gamma_{t-1}) \frac{L_t^\omega}{\omega} \right)^{1-\sigma} \\ \text{s.t.} \quad & c_t = w_t L_t + \pi_t^f + \pi_t^m + T_t \end{aligned}$$

where w_t is the wage rate, π_t^f, π_t^m are profits paid by firms in f and m sectors, and T_t is government transfers.

Detrending by $\exp(\mu_\Gamma + \Gamma_{t-1})$ yields

$$\begin{aligned} \max_{\hat{c}_t, \hat{L}_t} \quad & E_0 \sum_{t=0}^{\infty} \frac{\prod_{\tau=0}^t \hat{\beta}_\tau}{1-\sigma} \left(\hat{c}_t - \frac{\hat{L}_t^\omega}{\omega} \right)^{1-\sigma} \\ \text{s.t.} \quad & \hat{c}_t = \hat{w}_t \hat{L}_t + \hat{\pi}_t^f + \hat{\pi}_t^m + \hat{T}_t \end{aligned}$$

where $\hat{\beta}_t = \beta \exp[(1-\sigma)(\mu_\Gamma + g_t)]$.

The firm's production function is Cobb-Douglas with CES Armington aggregator of intermediate goods. Final goods producers solve

$$\begin{aligned} \max_{L_t^f, m_t^d, m_t^*} \quad & \hat{\pi}_t^f = \hat{y}_t - \hat{w}_t L_t^f - \hat{p}_t^m m_t^d - \hat{p}_t^* m_t^* \\ \text{s.t.} \quad & \hat{y}_t = \hat{\varepsilon}_t (M(m_t^d, m_t^*))^{\alpha_M} (L_t^f)^{\alpha_L} \\ & M(m_t^d, m_t^*) = [\lambda (m_t^d)^\eta + (1-\lambda)(m_t^*)^\eta]^{\frac{1}{\eta}} \\ & \hat{\varepsilon}_t = \exp(z_t + g_t) \end{aligned}$$

where m_t^* is Dixit-Stiglitz aggregator of continuum of differentiated imported inputs $m_j^*, j \in [0, 1]$. Subset θ of imported inputs requires final goods producers to obtain working capital financing at interest rate r_t^* :

$$\begin{aligned} \max_{\{m_{jt}^*\}_{j \in [0,1]}} \quad & \hat{p}_t^* m_t^* - \int_0^1 \hat{p}_j^* m_{jt}^* dj - r_t^* \int_0^\theta \hat{p}_j^* m_{jt}^* dj \\ \text{s.t.} \quad & m_t^* = \left[\int_0^1 (m_{jt}^*)^\nu dj \right]^{\frac{1}{\nu}} \end{aligned}$$

In default, $r_t^* = \infty$. Assuming $\hat{p}_j^* = 1, \forall j$, we get

$$\hat{p}_t^* = [\theta(1+r_t^*)^{\frac{\nu}{\nu-1}} + 1 - \theta]^{\frac{\nu-1}{\nu}} = (1-\theta)^{\frac{\nu-1}{\nu}}$$

GDP is defined as $\widehat{gdp}_t = \hat{y}_t - \hat{p}_t^* m_t^*$.

Intermediate goods producers solve

$$\begin{aligned} \max_{L_t^m} \quad & \hat{\pi}_t^m = \hat{p}_t^m m_t^d - \hat{w}_t L_t^m \\ \text{s.t.} \quad & m_t^d = A(L_t^m)^\gamma \end{aligned}$$

Dynamic Problem of Sovereign Government is identical to Eaton and Gersovitz (1981) with endogenous link between sovereign default and private economics activity.

$$\begin{aligned}
\hat{V}(\hat{a}_t, \hat{\varepsilon}_t) &= \max \left\{ \hat{V}^G(\hat{a}_t, \hat{\varepsilon}_t), \hat{V}^B(\hat{\varepsilon}_t) \right\} \\
\hat{V}^G(\hat{a}_t, \hat{\varepsilon}_t) &= \max_{\hat{c}_t, \hat{a}_{t+1}} U\left(\hat{c}_t - \frac{L_t^\omega}{\omega}\right) + \hat{\beta}_t E[\hat{V}(\hat{a}_{t+1}, \hat{\varepsilon}_{t+1})] \\
&\text{s.t. } \hat{c}_t = \hat{y}_t - \hat{p}_t^* m_t^* + \hat{a}_t - \exp(\mu_\Gamma + g_t) \hat{q}(\hat{a}_{t+1}, \hat{\varepsilon}_t) \hat{a}_{t+1} \\
\hat{V}^B(\hat{\varepsilon}_t) &= \max_{\hat{c}_t} U\left(\hat{c}_t - \frac{L_t^\omega}{\omega}\right) + \hat{\beta}_t E[\phi \hat{V}(0, \hat{\varepsilon}_{t+1}) + (1 - \phi) \hat{V}^B(\hat{\varepsilon}_{t+1})] \\
&\text{s.t. } \hat{c}_t = \hat{y}_t - \hat{p}_t^* m_t^* \\
\hat{q}(\hat{a}_{t+1}, \hat{\varepsilon}_t) &= \frac{1}{1 + r_t^*} Pr(\hat{V}^G(\hat{a}_{t+1}, \hat{\varepsilon}_{t+1}) > \hat{V}^B(\hat{\varepsilon}_{t+1}))
\end{aligned}$$

3.2 Solution Algorithm

State space consists of credit history h , debt $a > 0$, and TFP ε . Algorithm is very similar to the rest of the quantitative literature on sovereign debt, except we solve for the recursive equilibrium in two steps:

1. Static problem of private sector is solved first to determine equilibrium production plans: factor allocations, domestic and imported intermediate inputs, prices and final output as functions of TFP only.
2. Dynamic problem of sovereign government is solved for the optimal default decisions in debt market equilibrium.

The static problem of private sector and dynamic problem of government can be solved separately because

1. GHH preferences allow pro-cyclical labor hours and remove the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor depend on labor only: $\hat{w}_t = L_t^{\omega-1}$. This allows production plans depend on TFP only. In case of Cobb-Douglas utility these would be functions of two state variables: TFP and debt.
2. Global interest rate on working capital loans is fixed. As an extension we could model foreign lenders risk averse to create upward sloping interest rate schedule on working capital loans.

If either of these two assumptions were relaxed, dynamic problem would have to be solved simultaneously with static problem.

3.3 Calibration

Calibration is identical to Mendoza and Yue (2012).

$$\begin{array}{lll}
 \alpha_M = 0.43 & \alpha_k = 0.17 & \alpha_L = 0.40 \\
 \beta = 0.88 & \sigma = 2 & \omega = 1.455 \\
 r^* = 1\% & \nu = 0.59 & \theta = 0.7 \\
 A = 0.31 & \gamma = 0.7 & \phi = 0.083 \\
 \lambda = 0.62 & \eta = 0.65 &
 \end{array}$$

In addition to transitory TFP shocks, models 1 and 2 introduce stable and volatile trend to TFP respectively:

$$\begin{array}{llllll}
 \text{Model 1:} & \rho_z = 0.95 & \sigma_z = 1.7\% & \rho_g = 0 & \sigma_g = 0 & \mu_\Gamma = 2\% \\
 \text{Model 2:} & \rho_z = 0 & \sigma_z = 0\% & \rho_g = 0 & \sigma_g = 1.7\% & \mu_\Gamma = 2\%
 \end{array}$$

3.4 Results

Table 3.1 summarizes the results of a sensitivity analysis of model 1 to changes in the growth rate of TFP μ_Γ . It evaluates the robustness of model 1 predictions of debt to GDP ratio and quarterly default frequency. Row (1) reports the statistics from the Argentine data. Rows (2)-(5) report results varying the value of μ_Γ . Row (2) removes TFP trend altogether by setting $\mu_\Gamma = 0$, whereas rows(3), (4) and (5) set μ_Γ to 1%, 2% and 3% respectively. Without TFP trend, the model is identical to baseline Mendoza and Yue (2012). Adding stable quarterly growth rate to TFP increases debt to GDP ratio to 33% and matches the debt ratio from the data while *at the same time* keeping the 0.14% default frequency. Row (6) reports that replacing persistent transitory shocks by i.i.d. trend shocks with the same standard deviation 1.7% further increases the debt ratio to 39%, but lowers default frequency to .10%. Simulation statistics in table 3.2 illustrates improved correlation of macro variables with output.

Table 3.1: Sensitivity Analysis

	Output drop at default	Mean debt/GDP ratio	Default frequency	Mean spread	Std. dev. of spread	GDP corr. with spread default	
(1) Data	13%	35%	0.69%	1.86%	0.78%	-0.62	-0.11
Model 1							
(2) $\mu_\Gamma = 0\%$	14.3%	21.9%	0.13%	0.57%	0.29%	-0.31	-0.11
(3) $\mu_\Gamma = 1\%$	14.3%	25.8%	0.14%	0.59%	0.29%	-0.27	-0.11
(4) $\mu_\Gamma = 2\%$	14.4%	32.7%	0.14%	0.57%	0.27%	-0.23	-0.10
(5) $\mu_\Gamma = 3\%$	15.2%	61.3%	0.07%	0.27%	0.12%	-0.03	-0.11
(6) Model 2	14.1%	39.1%	0.10%	0.42%	0.21%	-0.61	-0.17

Table 3.2: Simulation Statistics

Statistics	Data	Model 1	Model 2
Average debt/GDP ratio	35%	32.7%	39.1%
Average bond spreads	1.86%	0.57%	0.42%
GDP autocorrelation	0.82	0.71	0.29
Std. dev. of GDP	4.7%	4.53%	3.26%
Std. dev. of trade balance		1.10%	0.84%
Std. dev. of bond spreads	0.78%	0.27%	0.21%
Consumption std.dev./GDP std.dev.	1.44	1.10	1.15
Correlations with GDP			
bond spreads	-0.62	-0.23	-0.61
trade balance	-0.87	-0.34	-0.63
labor	0.39	0.55	0.74
intermediate goods	0.90	0.96	0.74
Correlations with bond spreads			
trade balance	0.82	0.37	0.58
labor	-0.42	-0.18	-0.77
intermediate goods	-0.39	-0.23	-0.72
Historical default-output co-movements			
correlation between default and GDP	-0.11	-0.11	-0.17
frac.of def. with GDP below trend	61.5%	99.5%	100.0%
frac.of def. with large recessions	32.0%	50.7%	97.0%
Default frequency	0.69%	0.13%	0.10%
Output drop in default	13%	14.3%	14.1%

Figure 3.2: Solution to optimal default problem

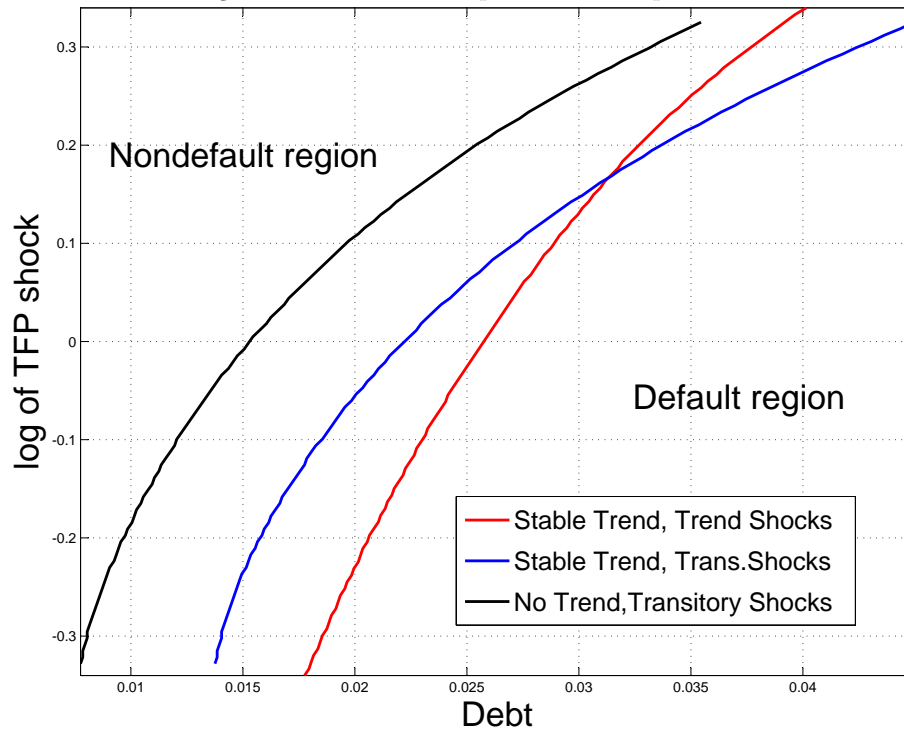
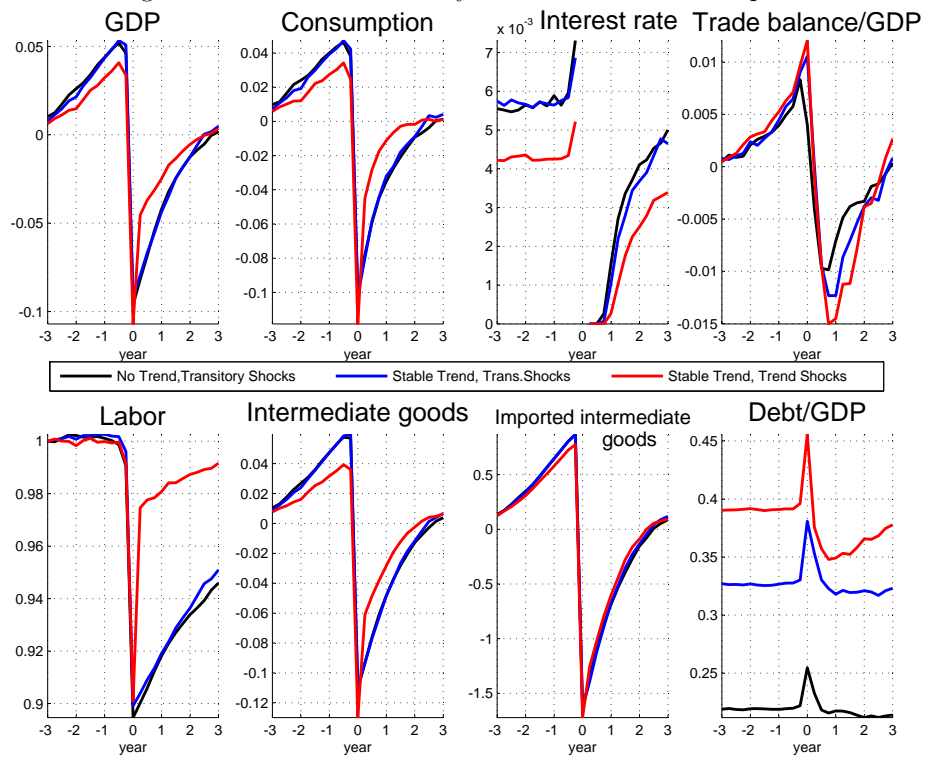


Figure 3.3: Macroeconomic dynamics around default episodes



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Appendix A

Computational Appendix

This computational appendix lists the numerical methods used throughout the thesis.

A.1 Numerical Solution Method

Our numerical solution method is a two dimensional extension of the endogenous grid method of Korinek (2011*b*). Denote the vector of states of country i as $s_i = (b_i, k_i, z_i)$. The interest rate is a function of aggregate worldwide borrowing $R = R(b_1 + b_2)$ as given by (1.10). The problem is to obtain policy functions $c(s_1, s_2)$, $x(s_1, s_2)$, $p(s_1, s_2)$, $\lambda(s_1, s_2)$, $k'(s_1, s_2)$ and $b'_1(s_1, s_2)$. By symmetry, the latter function is identical to $b'_2(s_2, s_1)$.

The algorithm requires two nested loops to take full advantage of the endogenous grid method.

A.1.1 Outer Loop

At the beginning of iteration J in the outer loop, we start with the initial guesses $b'_J(s_1, s_2)$ and $k'_J(s_1, s_2)$ of policy functions $b'_1(s_1, s_2) = b'_2(s_2, s_1)$ and $k'_1(s_1, s_2) = k'_2(s_2, s_1)$. (The initial values can be set arbitrarily.)

A.1.2 Inner Loop

Each inner loop starts with the initial guess of policy functions $c_j(s_1, b'_2, k'_2)$, $x_j(s_1, b'_2, k'_2)$, $p_j(s_1, b'_2, k'_2)$ and $\lambda_j(s_1, b'_2, k'_2)$. (The initial values can be set arbitrarily or taken from the previous iteration of the outer loop). We calculate $\hat{c}(s_1, s_2) = c_j(s_1, b'_j(s_2, s_1), k'_j(s_2, s_1))$ and similarly for \hat{x} , \hat{p} and $\hat{\lambda}$ by interpolation. In order to take advantage of the endogenous grid method, it is useful to perform our iterations over the grid (b'_1, k'_1, b'_2, k'_2) (since z'_1 and z'_2 are i.i.d.). For any pair (b'_1, b'_2) , we calculate the world interest rate $R'(b'_1, b'_2) = R'(b'_1 + b'_2)$, which is obtained from lenders' optimality condition (1.10). Then we define

$$\mathbb{P}(b'_1, k'_1, b'_2, k'_2) = \beta E_{z'_1, z'_2} \left\{ \left[\hat{c}(s'_1, s'_2) - \frac{\theta}{1 + \gamma} h(s'_1)^{1 + \gamma} \right]^{-\sigma} \cdot [\hat{p}(s'_1, s'_2) + \alpha y(s'_1) - \hat{x}(s'_1, s'_2)] \right\}$$

where labor supply and output are defined as

$$\begin{aligned} h(s'_1) &= \left(\frac{1-\alpha}{\theta} z'_1 k_1'^\alpha \right)^{\frac{1}{\alpha+\gamma}} \\ y(s'_1) &= z'_1 k_1'^\alpha h(s'_1)^{1-\alpha} = \frac{\theta}{1-\alpha} h(s'_1)^{1+\gamma} \end{aligned}$$

Then we solve the system of optimality conditions first under the assumption that the borrowing constraint is loose.

$$\begin{aligned} \mathbb{C}^{\text{unc}}(b'_1, k'_1, b'_2, k'_2) &= \beta R'(b'_1 + b'_2) E_{z'_1, z'_2} \left\{ [\hat{c}(s'_1, s'_2) - \frac{\theta}{1+\gamma} h(s'_1)^{1+\gamma}]^{-\sigma} \right\} \\ p^{\text{unc}}(b'_1, k'_1, b'_2, k'_2) &= \frac{\mathbb{P}(b'_1, k'_1, b'_2, k'_2)}{\mathbb{C}^{\text{unc}}(b'_1, k'_1, b'_2, k'_2)} \\ \lambda^{\text{unc}}(b'_1, k'_1, b'_2, k'_2) &= 0 \end{aligned}$$

In the same way, we can solve for the constrained branch of the system under the assumption that the borrowing constraint is binding in the current period as

$$\begin{aligned} p^{\text{con}}(b'_1, k'_1, b'_2, k'_2) &= \frac{b'_1}{\phi R'(b'_1 + b'_2)} \\ \mathbb{C}^{\text{con}}(b'_1, k'_1, b'_2, k'_2) &= \frac{1}{1-\phi} \left[\frac{\mathbb{P}(b'_1, k'_1, b'_2, k'_2)}{p^{\text{con}}(b'_1, k'_1, b'_2, k'_2)} - \phi \mathbb{C}^{\text{unc}}(b'_1, k'_1, b'_2, k'_2) \right] \\ \lambda^{\text{con}}(b'_1, k'_1, b'_2, k'_2) &= 1 - \frac{\mathbb{C}^{\text{unc}}(b'_1, k'_1, b'_2, k'_2)}{\mathbb{C}^{\text{con}}(b'_1, k'_1, b'_2, k'_2)} > 0 \end{aligned}$$

The other variables in unconstrained and constrained branches can be found as follows

$$\begin{aligned} q^{\text{unc|con}}(b'_1, k'_1, b'_2, k'_2) &= \frac{p^{\text{unc|con}}(b'_1, k'_1, b'_2, k'_2)}{k'_1} \\ k^{\text{unc|con}}(b'_1, k'_1, b'_2, k'_2) &= \frac{k'_1}{1 + \frac{\delta}{1-\frac{1}{\xi}} \{ [q^{\text{unc|con}}(b'_1, k'_1, b'_2, k'_2)]^{\xi-1} - 1 \}} \\ x^{\text{unc|con}}(b'_1, k'_1, b'_2, k'_2) &= \delta k^{\text{unc|con}}(b'_1, k'_1, b'_2, k'_2) [q^{\text{unc|con}}(b'_1, k'_1, b'_2, k'_2)]^\xi \\ h^{\text{unc|con}}(z_1, b'_1, k'_1, b'_2, k'_2) &= \left[\frac{1-\alpha}{\theta} z_1 [k^{\text{unc|con}}(b'_1, k'_1, b'_2, k'_2)]^\alpha \right]^{\frac{1}{\alpha+\gamma}} \\ y^{\text{unc|con}}(z_1, b'_1, k'_1, b'_2, k'_2) &= \frac{\theta}{1-\alpha} [h^{\text{unc|con}}(z_1, b'_1, k'_1, b'_2, k'_2)]^{1+\gamma} \\ c^{\text{unc|con}}(z_1, b'_1, k'_1, b'_2, k'_2) &= [\mathbb{C}^{\text{unc|con}}(b'_1, k'_1, b'_2, k'_2)]^{-\frac{1}{\sigma}} + \frac{1-\alpha}{1+\gamma} y^{\text{unc|con}}(z_1, b'_1, k'_1, b'_2, k'_2) \\ b^{\text{unc|con}}(z_1, b'_1, k'_1, b'_2, k'_2) &= y^{\text{unc|con}}(z_1, b'_1, k'_1, b'_2, k'_2) - c^{\text{unc|con}}(z_1, b'_1, k'_1, b'_2, k'_2) \\ &\quad - x^{\text{unc|con}}(b'_1, k'_1, b'_2, k'_2) + \frac{b'_1}{R'(b'_1 + b'_2)} \end{aligned}$$

Concatenating constrained and unconstrained results allows us to obtain policy functions $c_{j+1}(s_1, b'_2, k'_2)$, $x_{j+1}(s_1, b'_2, k'_2)$, $p_{j+1}(s_1, b'_2, k'_2)$ and $\lambda_{j+1}(s_1, b'_2, k'_2)$. The steps are iterated until convergence is reached.

The root of the following equation yields $b'_1(s_1, b'_2, k'_2)$

$$c(s_1, b'_2, k'_2) + x(s_1, b'_2, k'_2) + b_1 - \frac{b'_1(s_1, b'_2, k'_2)}{R'(b'_1 + b'_2)} = y(s_1)$$

Once the loop is completed, we observe that the two functions $b'_1(s_1, b'_2, k'_2)$ and the symmetric $b'_2(s_2, b'_1, k'_1)$ can be combined to

$$b' = b_1(s_1, b'_2(s_2, b'_1, k'_1), k'_2)$$

Finding the root of this equation yields new functions $b'_{J+1}(s_1, s_2), k'_{J+1}(s_1, s_2)$.

A.2 Yield Curve Estimation in MATLAB

My program had to accomplish a number of technical tasks such as scanning text files to import bonds' data, computing the cash flow payment matrix \mathbf{A} , vector \mathbf{t} of unique and monotonically increasing payment dates, vector \mathbf{p} of cash prices. Next sections describe and document the technical features of my implementation of these functions in MATLAB.

A.2.1 Importing Bond Data From Text File

Function `tstrscan.m` is responsible for scanning text files for necessary bond data. Assuming that the input file is in the proper format, the function can import raw data on settlement date, quoted prices, coupon rates, maturity dates, coupon frequencies, and even day-count conventions for different types of bonds. Then it can compute the cash flows matrix using the specified day-count convention methodology for different types of bonds.

The following is an example of readable input text file in the proper format:

```
----- begin of treasury.txt -----
Settle
02/15/2002
Coupon      Maturity      Price  Period  Basis
  5.625      12-31-2002P    103.108    2      0
  7.500       Feb-15-2005    111.131    12     1
  6.125        8-15-2007     108.239    6      2
 10.625   August-15-2015  150.485    4      3
  7.625      15/Feb-2025C   126.104    1      0
  ...           ...           ...      ...    ...
----- end of treasury.txt -----
```

To call this function we use the following syntax:

```
[Settle, Maturity, QuotedPrice, CouponRate, Period, Basis] =
tstrscan('treasury.txt')
```

Note that the format of dates in my program can vary. You may want to abbreviate names of months as in 'Feb-15-2005', use a full date format with spaces such as 'February 15, 2002', or even use the letters 'C' or 'P' to distinguish between callable and puttable bonds as in '15-Feb-2002C' or '15-Feb-2002P'. The only requirement is that if you use spaces then you need to use another delimiter instead of spaces. For example, you could use asterisks or commas.

Another important feature of my program is that it automatically skips columns that it does not recognize as necessary for importing. You may abbreviate column names to the first three letters. The program does not distinguish between lowercase or uppercase letters. The following are the names of columns that are considered necessary for importing:

Column name	Abbreviation
Settlement Date	set
Coupon Rate	cou
Maturity Date	mat
Quoted Price	pri
Coupon Frequency	per
Day-count basis	bas

If the file contains extraneous columns aside from those indicated in the table, then they are ignored. If a special delimiter is used, then it needs to be specified as an additional parameter in call of the function `tstrscan.m`. For example, let's assume that we have the following file:

```
----- begin of test.txt -----
Settlement Date
February 15, 2002
Type of Issue * Size * Cou * Maturity Date * Price
T-NOTE * 200 * 5.625 * December 31, 2002 * 103.108
T-BOND * 200 * 7.500 * Feb 15, 2005 * 111.131
T-NOTE (5YR) * 200 * 6.125 * August 8 2007P * 108.239
T-NOTE * 200 * 10.625 * Aug 15 2015 * 150.485
... * ... * ... * ... * ...
----- end of test.txt -----
```

It contains two extra columns, column names contain spaces, and delimiter is asterisk. To import this file we need to call the function `tstrscan.m` by specifying the delimiter '*':

```
[Settle,Maturity,QuotedPrice,CouponRate,Period,Basis] =
tstrscan('test.txt','*')
```

The first two columns will not be imported because the program does not consider them necessary for subsequent analysis.

As an experiment, I imported large csv file of 4462 municipals bonds, information on which was download from www.bondsonline.com:

```

----- begin of municipal.csv -----
set
2/15/02
State Moody S&P Qty Issue Coupon Maturity Yield LY Price
NY A3 -- 55 Beaver Riv Cent Sch 4.750 3/15/2002 --4.63 NC 102.217
CA -- BBB+ 300 Emeryville Calif Pub 6.350 05--01--2002P --.583 NC 103.354
MI A3 -- 475 Emmet Cnty Mich Bldg 3.100 5/1/2002 --.148 NC 100.632
WI Aa3 AA 50 Wisconsin St G.O. Bd 5.750 5/1/2002 --.684 NC 101.255
OR -- AA-- 85 Tualatin Valley Ore 4.750 6/1/2002 --.041 NC 101.331
FL -- A 7550 Hillsborough Cnty FL 6.625 07--01--2002P 0.359 NC 104.256
MA Baa2 BBB+ 55 Massachusetts Mun Wh 6.125 07--01--2002P 0.540 NC 104.007
NY A3 BBB 95 New York St Dorm Aut 5.000 7/1/2002 0.361 NC 101.672
NY A3 AA-- 80 New York St Dorm Aut 5.000 7/1/2002 0.361 NC 101.672
NY -- AA-- 40 New York St Dorm Aut 4.450 7/1/2002 0.189 NC 101.537
PR Aaa AAA 200 Puerto Rico Comwlth 6.000 07--01--2002P 0.767 NC 103.378
PA Aaa AAA 160 Westmoreland Cnty Pa 2.000 7/1/2002 0.563 NC 100.517
MA Aaa AA+ 25 Massachusetts St Wtr 6.000 8/1/2002 0.597 NC 102.394
TX Ba1 -- 5 Weston Mun Util Dist 7.600 08--01--2002C 4.006 0 101.562
DC Aaa AAA 100 District Columbia Ho 6.500 8/15/2002 0.890 NC 102.699
PA Aaa AAA 25 Harrisburg Pa Auth W 6.400 08--15--2002P 0.587 NC 102.801
IL Aaa AAA 100 Illinois Health Facs 5.100 8/15/2002 0.885 NC 102.028
AL Aaa AAA 30 Mobile Ala G.O. Ref 0.000 08--15--2002P 1.228 PRE 102.205
CA -- -- 50 Kingsburg Calif Hosp 6.400 9/1/2002 8.126 NC 99.122
NC Aa2 AA 25 North Carolina Hsg F 4.850 9/1/2002 1.260 NC 101.882
WA Aa1 AA+ 100 Washington St G.O. R 5.750 9/1/2002 0.802 NC 102.600
... ..
----- end of municipal.csv -----

```

Here is the result of the import of file `municipal.csv` using function `tstrscan.m`:

```

>> tic;[Settle,Maturity,Price,CRate,Period,Basis] = tstrscan('municipal.csv', ',');toc
Missing period column. Using two coupons per year.
Missing basis column. Using actual/actual basis.

```

CRate	Maturity	Price	Period	Basis
4.75	15-Mar-2002	102.217	2	0
6.35	01-May-2002	103.354	2	0
3.1	01-May-2002	100.632	2	0
5.75	01-May-2002	101.255	2	0
4.75	01-Jun-2002	101.331	2	0
6.625	01-Jul-2002	104.256	2	0
6.125	01-Jul-2002	104.007	2	0
7.5	01-Jul-2002	70.375	2	0
5	01-Jul-2002	101.672	2	0
5	01-Jul-2002	101.672	2	0
4.45	01-Jul-2002	101.537	2	0
6	01-Jul-2002	103.378	2	0
2	01-Jul-2002	100.517	2	0
6	01-Aug-2002	102.394	2	0
7.6	01-Aug-2002	101.562	2	0
6.5	15-Aug-2002	102.699	2	0
6.4	15-Aug-2002	102.801	2	0
5.1	15-Aug-2002	102.028	2	0
0	15-Aug-2002	102.205	2	0
6.4	01-Sep-2002	99.122	2	0
4.85	01-Sep-2002	101.882	2	0
5.75	01-Sep-2002	102.6	2	0
...

```

elapsed_time = 87.82

```

As you can see, the scanning program `tstrscan.m` successfully identified necessary columns of

data and imported them. Now we can begin constructing cash flow matrix using this data.

A.2.2 Constructing Cash Flows Matrix

One very important tool for estimating term structures is a program that does all the necessary cash flow and time mapping given bond parameters. I resorted to the `cfamounts` function from MATLAB Financial Toolbox for its functionality and ability to work with different types of bonds and day-count conventions. Once we know the exact time dates and corresponding cash flow amounts, we can proceed to construct the payment matrix **A**.

In previous section, I described how to use `tstrscan.m` function to import bond data from a text file. Now we use the result of this import as parameters for the `tstrprep.m` function:

```
[Settle,Maturity,QuotedPrice,CouponRate,Period,Basis] = tstrscan(FileName);
[p,A,s,t] = tstrprep(Settle,Maturity,QuotedPrice,CouponRate,Period,Basis);
```

The `tstrprep.m` function returns a vector of cash prices (**p**), a matrix of cash flows (**A**), a vector of settlement dates in serial date number format (*s*), and a vector of cash flow dates in serial date number format (*t*).

Input parameters are as follows:

Settle Settlement date (must be earlier than or equal to Maturity).

Maturity A vector of bonds' maturity dates in serial date number format.

QuotedPrice A vector of quoted (clean) prices.

CouponRate (Optional) A vector of percentage numbers indicating the annual percentage rate used to determine the coupons payable on a bond. Default is vector of zeros.

Period (Optional) Coupons per year of the bond. A vector of integers (0, 1, 2, 3, 4, 6, and 12). Default is vector of 2's (two coupons per year).

Basis (Optional) Day-count basis of the bond. A vector of integers: 0 = actual/actual, 1=30/360, 2=actual/360, 3=actual/365. Default is vector of zeros (actual/actual).

Note that `CouponRate` must be specified in percentages, not in decimals.

Continuing the example from the previous section, let's construct cash flows payment matrix **A** for a large set of 4462 municipal bonds using function `tstrprep.m`:

```
>> [P,A,S,T] = tstrprep(Settle,Maturity,QuotedPrice,CouponRate,Period,Basis);
>> whos
```

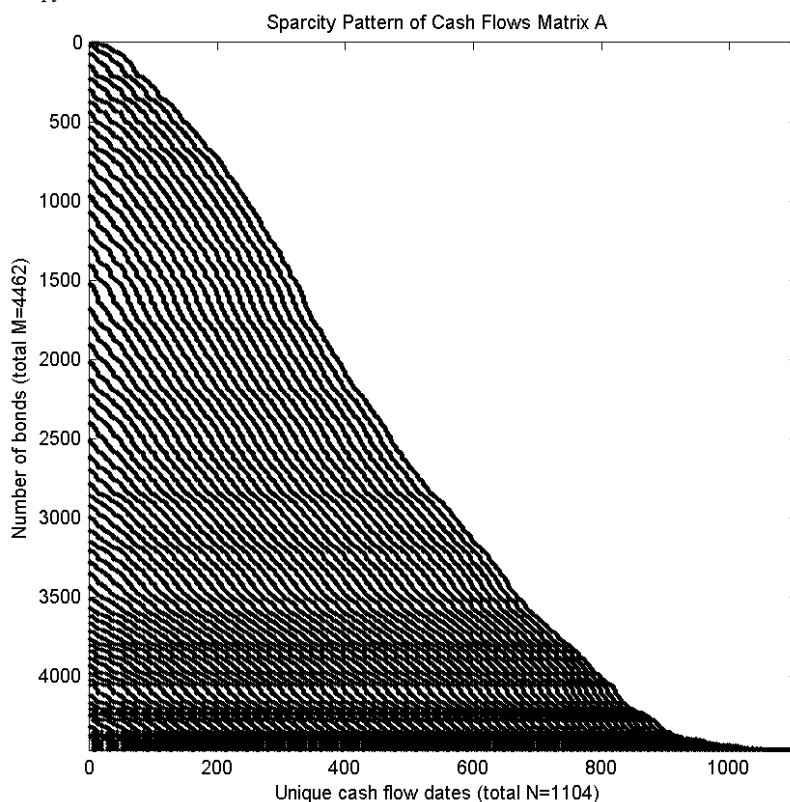
Name	Size	Bytes	Class
Settle	1x1	8	double array
Maturity	4462x1	35696	double array
QuotedPrice	4462x1	35696	double array
CouponRate	4462x1	35696	double array
Period	4462x1	35696	double array
Basis	4462x1	35696	double array
p	4462x1	35696	double array
A	4462x1104	1457152	sparse array
s	1x1	8	double array
t	1104x1	8832	double array

As you can see, large matrix **A** contains cash flows for 4462 bonds and 1104 time dates. It occupies 1.457 Mb of memory space while the full A matrix would occupy 39.408 Mb:

```
>> A_full = full(A); whos
Name      Size      Bytes    Class
A         4462x1104 1457152  sparse array
A_full    4462x1104 39408384 double array
```

Thus, I use a sparse matrix instead of a full matrix. To visualize the sparsity pattern of matrix **A**, I use the spy command:

```
>> spy(A)
```



The cash flows matrix conforms to the expected sparsity pattern: as the total number of bonds increases, the total number of unique cash flow dates also increases. Note the increasing effect of adding longer-term maturity bonds on total number of unique dates.

A.2.3 Quadratic Programming Issues

In order to add monotonicity constraints and solve quadratic programming problem, we have to convert the parameters in each model to match the quadratic programming function in MATLAB. The quadratic programming function `quadprog` in MATLAB requires parameters in the following form:

$$\begin{aligned} \min_{\mathbf{x}} \quad & \frac{1}{2} \mathbf{x}' \mathbf{H} \mathbf{x} + \mathbf{f}' \mathbf{x} \\ \text{s.t.} \quad & \mathbf{M} \mathbf{x} \leq \mathbf{z} \end{aligned} \tag{A.1}$$

where \mathbf{H} and \mathbf{M} — matrices, \mathbf{f} , \mathbf{z} and \mathbf{x} — vectors.

Therefore we need to put matrices and vectors of the models in the above format; namely, we have to define matrices \mathbf{H} and \mathbf{M} and vectors \mathbf{f} and \mathbf{z} .

Let's put objective function $\|\mathbf{A}\mathbf{d} - \mathbf{p}\|^2$ of discrete model in the following form:

$$\begin{aligned} \|\mathbf{A}\mathbf{d} - \mathbf{p}\|^2 &= (\mathbf{A}\mathbf{d} - \mathbf{p})'(\mathbf{A}\mathbf{d} - \mathbf{p}) = (\mathbf{d}'\mathbf{A}' - \mathbf{p}')(\mathbf{A}\mathbf{d} - \mathbf{p}) \\ &= \mathbf{d}'\mathbf{A}'\mathbf{A}\mathbf{d} - \mathbf{d}'\mathbf{A}'\mathbf{p} - \mathbf{p}'\mathbf{A}\mathbf{d} + \mathbf{p}'\mathbf{p} \\ &= \mathbf{d}'(\mathbf{A}'\mathbf{A})\mathbf{d} - 2\mathbf{p}'\mathbf{A}\mathbf{d} + \mathbf{p}'\mathbf{p} \\ &= \frac{1}{2}\mathbf{d}'(2\mathbf{A}'\mathbf{A})\mathbf{d} + (-2\mathbf{A}'\mathbf{p})'\mathbf{d} + \mathbf{p}'\mathbf{p} \end{aligned}$$

Therefore, $\mathbf{H} = 2\mathbf{A}'\mathbf{A}$ and $\mathbf{f} = -2\mathbf{A}'\mathbf{p}$.

For the continuous approximation, we want to minimize $\|\mathbf{A}\mathbf{d} - \mathbf{p}\|^2 = \|\mathbf{A}\mathbf{B}\mathbf{x} - \mathbf{p}\|^2$. Hence we simply replace the matrix \mathbf{A} with the product of matrices \mathbf{A} and \mathbf{B} and have the following results: $\mathbf{H} = 2(\mathbf{A}\mathbf{B})'\mathbf{A}\mathbf{B}$ and $\mathbf{f} = -2(\mathbf{A}\mathbf{B})'\mathbf{p}$. Then we set the variables vector \mathbf{x} equal to vector \mathbf{d} , which completes the transformation.

To add monotonicity constraints on decreasing discount factors $\mathbf{d} = [d_1, d_2, d_3, \dots, d_N]'$ in discrete approximation, i.e. $d_1 \geq d_2 \geq d_3 \geq \dots \geq d_N$, we construct such $(N-1) \times N$ matrix \mathbf{M} and $(N-1) \times 1$ vector \mathbf{z} that $\mathbf{M}\mathbf{d} \leq \mathbf{z}$:

$$\mathbf{M} = \begin{pmatrix} -1 & 1 & 0 & 0 & \dots & 0 \\ 0 & -1 & 1 & 0 & \dots & 0 \\ 0 & 0 & -1 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix} \text{ and } \mathbf{z} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

Similarly, we can add monotonicity constraints on $d(t)$ in the cubic splines model. Since we estimate $\mathbf{d} = \mathbf{B}\mathbf{x}$, the matrix \mathbf{M} is equal to its product with matrix \mathbf{B} from the model. Vector \mathbf{z} is a vector of zeros.

For Schaefer's approach, we need the nonnegative $d(t)$ constraints, while their monotonous decay is guaranteed by model specification. Therefore, the only constraint I impose is $d(1) = \mathbf{b}(1)\mathbf{x} \geq 0$, where $\mathbf{b}(1)$ — last N -th row of matrix \mathbf{B} (see formula 2.11). Therefore, in Schaefer's method matrix $\mathbf{M} = -\mathbf{b}(1)$, and vector \mathbf{z} is zero.