Evaluating the Performance of Two Competing Models Of School Suspension Under Simulation - The Zero-Inflated Negative Binomial and the Negative Binomial Hurdle

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Dedication

Til Kirsten - Takk for at du er så flemsekoselig.

Abstract

In many educational settings, count data arise that should not be considered realizations of the Poisson model. School days suspended represents an exemplary case of count data that may be zero-inflated and overdispersed relative to the Poisson model after controlling for explanatory variables. This study examined the performance of two models of school days suspended - the zero-inflated negative binomial and the negative binomial hurdle. This study aimed to understand whether the conditions considered would elicit comparable and/or disparate performance between these models. Additionally, this study aimed to understand the consequences of model misspecification when the data-generating mechanism was improperly specified. This study found that the negative binomial hurdle performed better in both simulation studies. Based on the conditions considered here, it is recommend that researchers consider the negative binomial hurdle model over the zero-inflated negative binomial model especially if the structural zero/zero parameters are to be treated as nuisance parameters or the presence of structural zeros is unknown. If structural zeros are expected, and interest is in these parameters, then the zero-inflated negative binomial should still be considered. Additionally, if interest is in the non-structural zero/count parameters, the results here suggest model misspecification has little effect on these parameters, and a researcher may select a model based on the parameters they are interested in interpreting.

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Chapter I: Introduction

Statement of the Problem

Count data frequently occur in educational research. They arise when measuring the number of schools days absent during an enrollment period, when measuring the number of items that a student endorses on an educational instrument, and when measuring the number of school days a student is suspended. The latter example, modeling school days suspended, presents a particularly interesting and challenging statistical problem. This problem is the focus of and motivation for this dissertation.

Proper understanding of factors and predictors of school suspensions is a general and salient concern for educators and society. Suspensions are associated with numerous outcomes including trouble with the law, school dropout, unemployment, substance abuse, and various other negative outcomes (Gregory, Skiba, & Noguera, 2010; Heckman, 2006; Krezmien, Leone, & Achilles, 2006). When models are developed to assess the relationship of covariates with school suspensions, in order to develop interventions and/or identify high risk students, it is imperative that these models are statistically valid and do not ignore the intricacies of the data. Otherwise, conclusions about the relative importance of the covariates and type I error rates may be greatly affected.

School suspension data are unique because most students are never suspended, several students are suspended a few days, and many chronic offenders are suspended for a week or more (Desjardins et al., 2013; Desjardins, 2012). Marginally, this data has an abundance of zeros and a variance that greatly exceeds its mean. The mean is pulled towards zeros by the students who are never suspended while the variance is inflated due to the presence of the chronic offenders. In the literature, these two resulting statistical problems are commonly referred to as zero-inflation and overdispersion and frequently occur in the modeling of count data (see Chapter 2).

Desjardins (2012) explored and modeled school suspension data from students in 8th grade in Minneapolis Public Schools (MPS) from the 2003 - 2004 through 2007 - 2008 school years. Data from 13,606 students were collected and information on several predictors were entered into the models. Initial exploration of the data involved constructing a marginal plot of school suspensions (see Figure 1). Of noticeable interest in Figure 1 is the abundance of students who were never suspended (about 74%), the range of school days suspended (59), and the sizable number of students that were suspended for more than a week (about 8%). This marginal distribution is often used to assess the plausibility of zero-inflation and overdispersion (Zeileis, Kleiber, & Jackman, 2008) and the salient pattern in Figure 1 would be indicative of the occurrence of both of these phenomena.

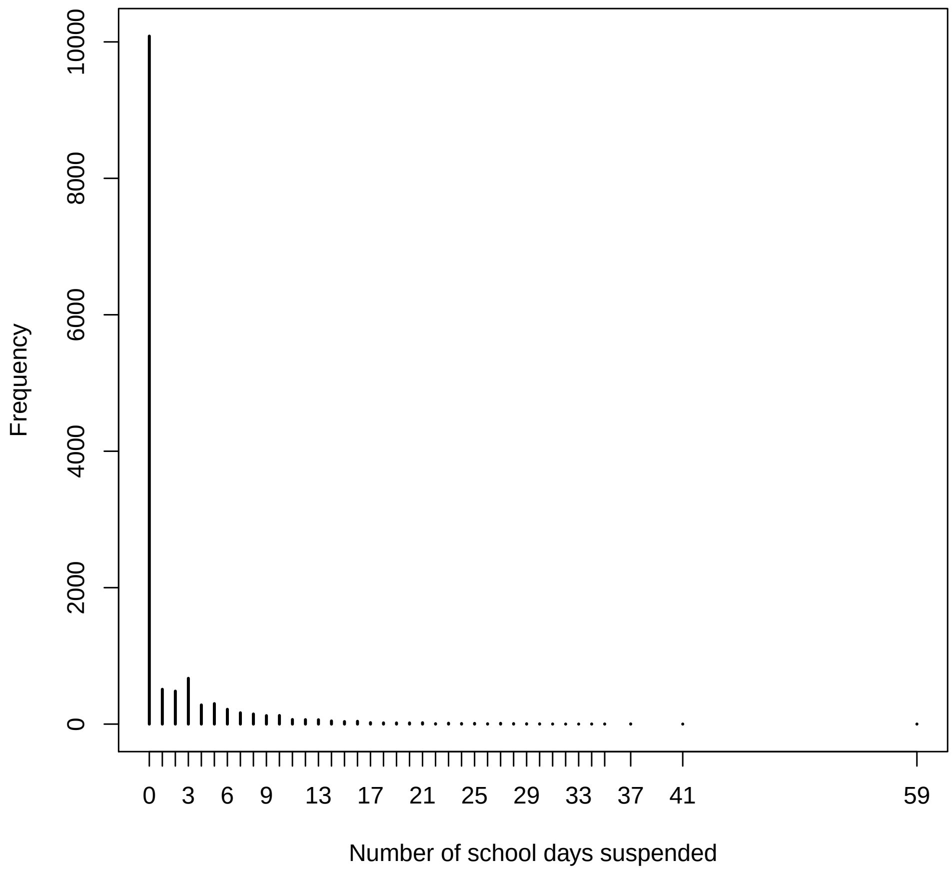

Figure 1: Marginal distribution of school suspensions for students in 8^{th} grade in Minneapolis Public Schools from 2003 - 2004 through 2007 - 2008.

In statistical modeling, we are concerned with conditional distributions and data that appear zero-inflated or overdispersed marginally may no longer be either after covariates are included in a model. Therefore, using Occam's razor as a guiding principle, Desjardins (2012) initially considered the data as realizations of the Poisson model (Agresti, 2002) and fit a Poisson model to the data. However, the Poisson model failed to provide adequate fit based on Pearson residuals. Therefore, a negative binomial model was considered. Based on the Akaike information criterion (AIC) and likelihood ratio test, the negative binomial did provide an improvement in fit over the Poisson. However, the Pearson residuals remained quite large (for both the training and testing data) and it was unclear to what extent the negative binomial adequately controlled for the zero-inflation and overdispersion present in Figure 1.

To remedy this, a negative binomial hurdle model was next considered (Mullahy, 1986). These models are described more completely in the next chapter and represent one class of models for handling zero-inflation and overdispersion. The decision to fit hurdle models resulted from the lack of belief of structural zeros (subjects who would always have a probability of zero of being suspended). In contrast, zero-inflated negative binomial models could have been fit that rely on the presence of a structural zero group (Ridout, Demétrio, & Hinde, 1998). The negative binomial hurdle model provided an improvement in fit over the Poisson and the negative binomial model based on the AIC and Vuong's statistic (Vuong, 1989). It was the final and most complex count model considered in Desjardins (2012). However, the negative binomial hurdle model still had large Pearson residuals and it remained unclear if this model was appropriate for the data.

The decision to choose hurdle models over zero-inflated models was based on the paucity of support for a structural zero group in this scenario. The plausibility of a class of students who could never be suspended seemed minute.

However, the rationale for choosing one model over another is often nebulous. While these two models differ in latent interpretation, they have been reported to have similar fit and predictive abilities in practice (Zeileis et al., 2008) and the ability to choose one model over another via AIC appears to be weak. These models often differ in parameter estimation and subsequently statistical inference and hypothesis testing may be greatly affected as each model is referring to entirely different parameters.

There is a dearth of literature exploring when and if these models do diverge, to what extent model misspecification may affect parameter interpretation, and when these models are inappropriate. There are very few simulation studies with hurdle and zero-inflated models (Lambert, 1992; Min & Agresti, 2005; Miller, 2007) and to my knowledge no simulations that compare the performance of negative binomial hurdle models to zero-inflated negative binomial models at recovering their parameters. Therefore, there are no studies investigating how sample size, multicollinearity, and overdispersion might affect estimation in the negative binomial hurdle and the zero-inflated negative binomial and particularly how parameter recovery, type I error, relative bias, and confidence interval coverage might be affected.

A tacit assumption of zero-inflated models is the presence of a structural zero group. However, no study has investigated whether these zero-inflated models can in fact predict membership into the structural zero group. If zero-inflated models are unable to predict structural group membership then their utility is limited and the decision to choose a hurdle model over a zero-inflated model becomes clear.

Finally, Rose, Martin, Wannemuehler, and Plikaytis (2006) argue the need for further research to understand the consequences of using a zero-inflated model rather then a hurdle model and vice versa when the data-generating mechanism is known. Only through a simulation study, where the population is known, will the consequences of selecting one model over another be understood and be able to be quantified. Desjardins (2012) assumed the non-existence of a structural zero class for the school suspension analysis and it would be illuminating to understand how his results could have changed if this assumption was false.

Significance of the Study

The purpose of this dissertation is to further our understanding of the differences between zero-inflated negative binomial and negative binomial hurdle models and to explore under simulation whether and when performance of these models might diverge. Specifically, this dissertation aims to answer the following research questions.

1. For a given set of simulated conditions, do the zero-inflated negative binomial (ZINB) and the negative binomial hurdle (NBH) models perform equally well in terms of parameter recovery and type I error rates? Under which of these conditions do they both perform poorly? And under which of these conditions does the performance between the two models differ?

2. What are the consequences, if any, of misspecifying the latent class for the zeros? In other words, given the existence (non-existence) of a structural

zero class, what are the consequences of fitting a model that does not (does) account for the structural zero class?

Additionally, this dissertation will generate data from the school suspension negative binomial hurdle model that was developed in Desjardins (2012). Building on that study, a zero-inflated negative binomial model will be fit to the school suspension data. These models will then serve as the populations for an additional empirically-derived condition. The aforementioned research questions and outcomes will then be explored within the context of this condition.

By exploring conditions under which the performance of these models differ, it will be possible to provide recommendations about the utility and appropriateness of these models. Additionally, it will be possible to quantify the consequences of model misspecification when the true model (i.e. whether a structural zero group is present) is known. Finally, by exploring to what extent zero-inflated models are actually able to predict structural zeros, it will be possible to comment on the appropriateness of these models for describing and predicting this class.

The literature on zero-inflated and overdispersed count data are discussed in Chapter 2. Chapter 3 describes the conditions, outcomes, and the models in the two simulation studies considered here. Chapter 4 presents the results of the simulation studies. Finally, Chapter 5 concludes with a discussion of the results and implications for educational and psychological modeling of zero-inflated and overdispersed count data.

Chapter II: Literature Review

In this chapter, the most popular methods for modeling count data that are overdispersed and zero-inflated are reviewed. This chapter begins by first reviewing the Poisson model. Issues within the Poisson model, specifically zero-inflation and overdispersion, are discussed and strategies to appropriately model these issues are then presented. Special emphasis is paid to the negative binomial model (NB), zero-inflation models, and hurdle models. Other parametric models (models that assume a distribution and are described by parameters) are then briefly discussed. Finally, statistical tests for model selection/comparison and limitations in the literature are presented.

Modeling count data as realizations of the Poisson model

As mentioned in Chapter 1, count data are frequently encountered in various settings in education and psychology. Generally, count data are considered realizations of the Poisson model (Agresti, 2002). The Poisson model represents the de facto model for handling count data and this model is completely described by one parameter, λ. The Poisson model, in general, is a flexible model that can accommodate a myriad of count data situations. Figure 2 shows several realizations of the Poisson model (the black bars) with different λs. Ignoring the gray bars (realizations from a point mass at zero), it is evident from Figure 2 that as λ increases the Poisson model approximates the normal model, that the ability of the Poisson to accommodate additional variability increases as λ increases, and that the proportion of expected zeros from the Poisson decreases as λ increases.

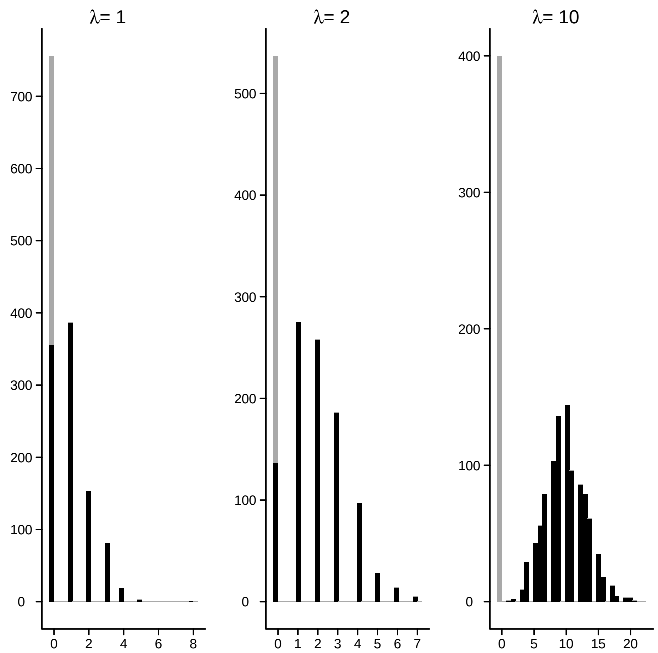

Figure 2: Random Sample of 1000 from Poisson Models with $\lambda = 1$, 2, and 10.

The probability mass function (pmf) for a random variable Y that follows a Poisson distribution is:

$$f(y; \lambda) = \frac{\exp(-\lambda)\lambda^y}{y!}\ I_{(y \geq 0)} \tag{1}$$

Where $I_{(y\geq 0)}$ represents an indicator function. The Poisson model is a member of the exponential family. Other members of the exponential family include the binomial, NB, normal, and chi-squared distributions. A distribution is in the exponential family if its density can be reexpressed as:

$$f(y|\theta) = h(y)\exp\left(y\theta - a(\theta)\right) \tag{2}$$

In Equation 2, $h(y)$ refers to a known function h of a random variable y, θ refers to the canonical parameter, and $a(\theta)$ to the cumulant function. The first derivative of the cumulant function corresponds to the expectation of the random variable and the second derivative of the cumulant function corresponds to the variance of the random variable. The canonical link function is the function that expresses the canonical parameter in terms of μ. The pmf of the Poisson model reexpressed as a member of the exponential family is:

$$\begin{aligned} f(y|\theta) &= \frac{\exp(-\lambda)\lambda^y}{y!}\, I_{(y\geq 0)} \\ &= \frac{1}{y!}\exp\log(\lambda^y\exp(-\lambda)) \\ &= \frac{1}{y!}\exp\{y\log\lambda - \lambda\} \quad \text{Let } \frac{1}{y!} = h(y), \theta = \log\lambda, \text{ and } \lambda = \exp\theta = a(\theta) \\ &= h(y)\exp\{y\theta - a(\theta)\} \end{aligned} \tag{3}$$

From Equation 3, we can see that the $\text{EY} = a'(\theta) = \exp\theta = \lambda$ and $\text{Var(Y)} = a''(\theta) = \exp\theta = \lambda$. In the Poisson model, the expectation is equal to the variance. The canonical link function is $\log\lambda$, which means that the mean function, if used with the canonical link, will be log linear.

Equation 4 expresses the mean function of the Poisson using the canonical link and shows how covariates may enter the model.

$$\log(\boldsymbol{\mu}) = \mathbf{X}\boldsymbol{\beta} \implies \begin{pmatrix} \log\mu_1 \\ \log\mu_2 \\ \vdots \\ \log\mu_n \end{pmatrix} = \begin{pmatrix} 1 & x_{12} & x_{13} & \dots & x_{1p} \\ 1 & x_{22} & x_{23} & \dots & x_{2p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{n2} & x_{n3} & \dots & x_{np} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{pmatrix} \tag{4}$$

Where,

$$\boldsymbol{\mu} = \exp(\mathbf{X}\boldsymbol{\beta}) \tag{5}$$

and $\boldsymbol{\mu}$ is an n x 1 response vector, $\mathbf{X}$ is an n x p covariate matrix, and $\boldsymbol{\beta}$ is a p x 1 vector of regression coefficients.

Problems with the Poisson Model

While the Poisson model is flexible, in practice count data often are not realizations from this model. The restriction that the E(Y) = Var(Y) is easily and often violated, even after covariates are entered into the model. Instead count data may need to be considered realizations of another count model (e.g. a NB or geometric model), a modified count model (e.g. a truncated Poisson), or a mixture (or two-part) model with components describing some or all of the zeros and the non-zero counts separately (e.g. zero-inflated or hurdle models).

Count data may not be thought of as realizations of a Poisson model, and will violate the mean-variance equality, because of two non-exclusive issues: inflation or deflation of the zero count and over- or underdispersion. The first issue occurs in the situation where there is an excess or dearth of zeros in the observed data after controlling for all relevant predictors and the observed zeros deviate substantially from what would be expected if the data were from a Poisson model. Returning again to Figure 2, this time notice the additional zeros contributed by the gray bars beyond those expected by the Poisson model. These gray bars correspond to excess zeros (400 additional zero counts) that would not be accounted for by a Poisson model. A random sample from a Poisson (again, the black bars in Figure 2) with $\lambda = 1$, $\lambda = 2$, and $\lambda = 10$ resulted in 35.6%, 13.7%, and 0 of all observed counts being zeros. It is clear that as the expectation of the Poisson increases (as well as the variance), that the occurrence of zeros from this model decreases.

There are numerous situations where excess zeros might arise in education and psychology. For example, even after controlling for socioeconomic status, ethnicity, gender, special education and english language learner status, and school effects, fewer students may still be suspended than would be expected under a Poisson model (Desjardins et al., 2013). When studying rare psychopathologies, many participants will endorse no items on a given instrument making model estimation difficult (Klimes-Dougan et al., in-press). Studies examining abuse and drug-use may also result in a higher occurrence of zeros given their low prevalence in the population (Atkins & Gallop, 2007). These are all examples of situations that may create zero-inflation regardless of the covariates included in the model.

In contrast, zero-deflation might occur when a researcher is measuring the number of items a student gets correct on a spelling test. Very few, if any, students would get all of the items incorrect and there are likely to be fewer students that get none correct than would be expected. Zero-deflation will not be covered in detail in this chapter. Instead this chapter focuses largely on zero-inflation as this is the more common scenario encountered in practice. However, zero-deflation will be briefly considered during the discussion of hurdle models.

In the situation where zero-inflation occurs, the mean will be pulled towards the zero count resulting in unreasonable fit for the zeros, unreasonable fit for the non-zero counts or both (Atkins & Gallop, 2007; Angers & Biswas, 2003; Böhning, 1998). Ignoring zero-inflation can lead to parameter estimates and standard errors that are biased, poor model fit, and undiagnosed zero-inflation can manifest itself as overdispersion (Lambert, 1992; Zuur, Ieno, Walker, Saveliev, & Smith, 2009). Predicted values will not closely mirror the observed data and will be especially poor for the zeros counts. Returning to the school suspension example, a Poisson model would be expected to greatly under predict the number of students without suspensions and because of the mean being pulled towards zero, there would likely be misfit for the other counts as well.

It has been suggested that the excess zeros arise from distinct latent classes in the data that are a result of unexplained population heterogeneity (Böhning, 1998; Ridout et al., 1998). Ridout et al. (1998) and others (e.g. Rose et al. (2006); Hu, Pavlicova, and Nunes (2011); Zuur et al. (2009)) make the distinction between zeros arising from two distinct latent classes: structural zeros and sampling zeros.[1] Structural zeros arise from a latent class where zeros are the only possible value and sampling zeros arise from a latent class where

[1]Lambert (1992) refers to these as the "perfect" state and the "not perfect" state

zeros occur as a function of sampling. The zeros associated with the gray bars in Figure 2 would be considered structural zeros and the black bars for the zero counts would correspond to sampling zeros. Returning to the example of school suspension, structural zeros would be associated with students who could never exhibit problem behaviors in school and would not be at risk to be suspended, whereas sampling zeros would be associated with students that might be expected to be suspended but are never observed to have a suspension during the study. However, zero-inflation can still occur when structural zeros are absent and the gray bars in Figure 2 could arise from a sampling zero source too. Nonetheless, it should be noted that an overabundance of zeros and heterogeneity in the data do not necessarily imply zero-inflation and thus a Poisson model may still be appropriate and should be tested once covariates are included in the model (Böhning, 1998).

The second problem, of which zero-inflation and zero-deflation may be considered special cases, occurs when there is greater or less variability in the data than would be expected by the Poisson model. When the observed variability is greater (less) than the observed mean then the data are said to be overdispersed (underdispersed). (For a review of overdispersion for categorical data in general, see Hinde and Demétrio (1998)). Formally, overdispersion may occur for any data-generating process when the ratio of Var(Y)/E(Y) exceeds that implied by the model (Mullahy, 1986). For example, in the Poisson model, the $\text{Var(Y)} = \text{E(Y)} = \lambda$ and this ratio becomes $\text{Var(Y)}/\text{E(Y)} = \lambda/\lambda = 1$. Therefore, when $\text{Var(Y)}/\text{E(Y)} > 1$ then there is overdispersion relative to what would be expected if the data were realizations of a Poisson. Underdispersion would occur when Var(Y)/E(Y) $<$ 1. The remainder of this review will focus on overdispersion and underdispersion will not be considered further.

Returning to the motivating school suspension example, while most students are rarely or never suspended, some students may be chronically suspended and have an extremely large number of suspensions. These students could cause greater variability than the model can handle. Again, because of the restriction that E(Y) = Var(Y), the data might not be considered as realizations of the Poisson model as this equality will likely be violated after controlling for predictors.

The problem with overdispersion is that the standard errors for the regression weights will be seriously underestimated (Hinde & Demétrio, 1998; Agresti, 2002) and statistical inference and hypothesis testing on the parameters will be inaccurate. Models that do not properly model overdispersion will have fitted values that do not appropriately reflect the observed data and will show lack-of-fit based on deviance tests (Atkins & Gallop, 2007; Hall, 2000; Lambert, 1992; Yau, Wang, & Lee, 2003). As with zero-inflation, overdispersion may not be present once covariates are included in a model and the presence of overdispersion may be a signal that important variables are omitted and that the model is misspecified (Berk & MacDonald, 2008).

There are a myriad of ways to combat overdispersion and zero-inflation in count data beyond adding more control variables. Perhaps the simplest approach to dealing with overdispersion involves using a modified Poisson (Hinde & Demétrio, 1998; Ridout et al., 1998). A modified Poisson is a Poisson with parameter μV, where V is a random variable with E(V) = 1 and Var(V) = α, where α corresponds to the unexplained heterogeneity. This model is estimated using quasi-likelihood. The expectation of the modified Poisson is μ and the variance is $\mu + \alpha\mu^2$ and thus the mean-variance equality is no longer required.

Random effects may be introduced to deal with overdispersion resulting in a Poisson-normal model (Hinde & Demétrio, 1998). In the case where overdispersion is present but zero-inflation is not then the data may also be considered realizations of the NB model. If zero-inflation is present either the zero-inflated negative binomial (ZINB) or negative binomial hurdle model (NBH) may be considered. Finally, if the data are not overdispersed but zero-inflated, then a zero-inflated Poisson (ZIP) or a Poisson hurdle (PH) could be used.

Figure 3 presents a flow chart that could be used to assist with selecting from one of the previously described models. The 'assumption' in Figure 3 refers to the mean-variance equality assumption of the Poisson model. The principle of parsimony, in the absence of theory and used concurrently with theory, should be used in selecting a model and a reasonable approach might involving starting with the Poisson and determining if a more complex model is needed. Model selection and fit will be discussed later.

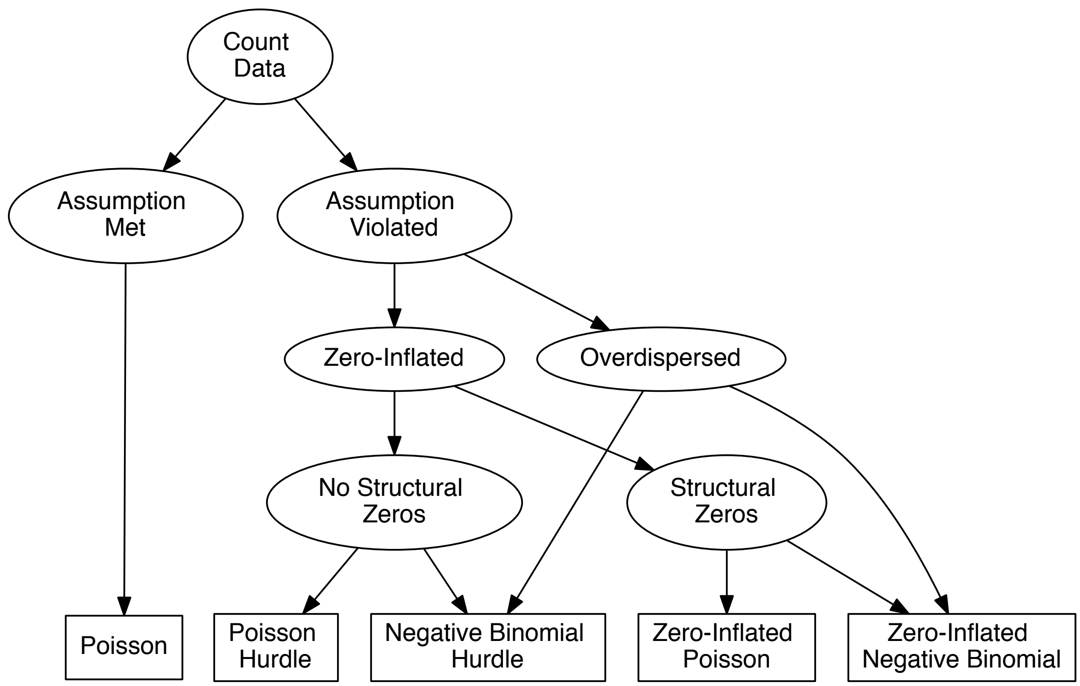

Figure 3: Flow Chart for Selecting a Zero-Inflated or Overdispersed Count Model.

The remainder of this literature review summarizes the various approaches that have been used to deal with zero-inflation and overdispersion in count data. The NB model is considered first. Then the zero-inflated and hurdle models are described. Then other approaches to modeling zero-inflation and overdispersion are briefly considered. Finally, model selection/comparison and the limitations in the extant literature are presented.

Negative Binomial Model

Data that do not fit the Poisson model and are overdispersed may be considered realizations of the NB model. The NB model is a more flexible model than the Poisson as it has an extra parameter to account for additional variability. It can be conceptualized as an extension of the Poisson model (Atkins & Gallop, 2007), specifically as a gamma mixture of Poisson distributions (Agresti, 2002). Conceptually, the NB works by essentially adding a fatter, longer tail to the Poisson model and may be more appropriate than the Poisson model when overdispersion in the data is present (Agresti, 2002). In the case of overdispersion, the Poisson and NB models will give similar parameter estimates but the standard errors, p-values, and confidence intervals for the Poisson model will be too small and narrow, while the NB model will be more appropriate (Atkins & Gallop, 2007).

The NB has been applied to overdispersed count data in psychology (Atkins & Gallop, 2007), manufacturing (Ghosh, Mukhopadhyay, & Lu, 2006; Lambert, 1992), agriculture (Hall, 2000), public health (Khan, Ullah, & Nitz, 2011), and medicine (Rose et al., 2006; Yau et al., 2003). The NB has been extended to the generalized linear mixed model (GLMM) framework (Agresti, 2002) and Bayesian models have been developed (Gelman, Carlin, Stern, & Rubin, 2004). Using Agresti (2002)'s notation, the NB pmf for a random variable Y can be expressed as:

$$f(y; k, \mu) = \frac{\Gamma(y+k)}{\Gamma(k)\Gamma(y+1)} \left(\frac{k}{\mu+k}\right)^k \left(1 - \frac{k}{\mu+k}\right)^y I_{(y \geq 0)} \tag{6}$$

This parameterization of the NB will have E(Y)$= \mu$ and Var(Y) $= \mu + \mu^2/k$. In Equation 6, k^{-1} is referred to as the dispersion parameter. The Poisson model may be considered a special case of the NB model. As the dispersion parameter approaches infinity, the Var(Y) $\rightarrow$ E(Y) and converges to a

Poisson with parameter μ (Agresti, 2002). When k is fixed the NB is considered a member of the exponential family and a generalized linear model (GLM) (Agresti, 2002). Similar to the Poisson model, the log linear link is usually used to model the mean function to facilitate comparisons with the Poisson model (see Equation 4). Furthermore, it is possible to test if the NB provides an improvement in fit to the Poisson by testing the dispersion parameter estimate divided by its standard error against a standard normal or using a likelihood ratio test (LRT) with a χ_1^2 distribution (a more conservative approach) (Atkins & Gallop, 2007) or using the more appropriate asymptotic distribution for the LRT statistic under the null hypothesis, which is a probability mass of 0.5 at 0 and $0.5\chi_1^2$ distribution above 0 (Loeys, Moerkerke, De Smet, & Buysse, 2012).

As was the case with the Poisson model, the NB has several limitations that impairs its ability to properly model zero-inflated and overdispersed data. First, modeling the dispersion parameter and the mean function with explanatory variables can be difficult and cumbersome and instead using a zero-inflated model (described next) may be easier to interpret (Agarwal, Gelfand, & Citron-Pousty, 2002). Second, the NB has been used with count data to handle overdispersion (Agresti, 2002), but it often performs poorly in the presence of zero-inflated data. In the studies with data that have been zero-inflated, the NB has continually performed poorly compared to more specialized mixture or two-part models (Desjardins, 2012; Atkins & Gallop, 2007; Ghosh et al., 2006; Hall, 2000; Hu et al., 2011; Lambert, 1992; Yau et al., 2003). Therefore, if data are either zero-inflated or overdispersed and zero-inflated then a NB alone may be insufficient. Instead mixture or two-part models such as zero-inflated models or hurdle models should be considered.

Zero-Inflated Models

Zero-inflated models refer to models that are designed to accommodate excess zeros in count data. They are also referred to as *added-zero* models (Heilbron, 1994). Zero-inflated models have been developed for the Poisson model (Lambert, 1992), the NB model (Ridout, Hinde, & Demétrio, 2001) and other models (e.g. the geometric and binomial models (Mullahy, 1986; Hall, 2000)). Zero-inflated models are able to handle zeros arising from both structural and sampling processes and models these processes separately. Because zeros may arise from both components and both components must be estimated simultaneously, these models are referred to as mixture models. This section describes the two most common zero-inflated models: the ZIP and the ZINB.

Zero-Inflated Poisson Model

The most commonly employed approach to dealing with heterogeneity associated with excess zeros, is to use a Poisson distribution that has been mixed with a point mass at zero to allow for the inclusion of additional structural zeros. Such a mixture model is called a *Poisson with zeroes*, *zero-modified Poisson*, *pseudo-contagious*, or more commonly *zero-inflated Poisson* model (Cohen, 1962; Dahiya & Gross, 1973; Heilbron, 1994; Johnson & Kotz, 1969; Lambert, 1992; Ridout et al., 1998). These models have been utilized in a myriad of disciplines including agriculture (Hall, 2000), biology (Angers & Biswas, 2003; Ridout et al., 1998), economics (Mullahy, 1986), manufacturing (Lambert, 1992; Ghosh et al., 2006; Xie, He, & Goh, 2001), medicine (Böhning, Dietz, Schlattmann, Mendonça, & Kirchner, 1999; Hu et al., 2011; Lee, Wang, Scott, Yau, & McLachlan, 2006; Neelon, O'Malley, & Normand, 2010; Rose et al., 2006), psychology (Atkins & Gallop, 2007), occupational injury (Yau & Lee, 2001), public health (Khan et al., 2011; Buu, Johnson, Li, & Tan, 2011), and ecology (Wenger & Freeman, 2008; Majumdar & Gries, 2010). They have been developed to be used within both the standard and mixed effects regression frameworks, with generalized estimating equations (Hall & Zhang, 2004), and expectation-maximization (E-M) algorithms are well developed (Lambert, 1992; Hall, 2000; Yau & Lee, 2001). Bayesian ZIP models have been proposed (Angers & Biswas, 2003; Ghosh et al., 2006; Neelon et al., 2010; Majumdar & Gries, 2010) and extensions of the ZIP model to an arbitrary number of components (i.e. more than 2) have been proposed (Böhning, 1998). Multivariate extensions have been developed (Majumdar & Gries, 2010) and the ZIP model has been extended to the generalized Poisson density allowing the thickness of a Poisson tail to be modified (Angers & Biswas, 2003).

The general ZIP regression is a mixture of two components: a point mass at zero (a Bernoulli component) with a probability of p and a Poisson component with probability of $1-p$. The pmf of a ZIP model can be expressed as:

$$Y \sim \begin{cases} Bern(p) & y = 0 \\ Poi(\lambda)^2 & \text{with probability } 1-p \text{ for } y = 0, 1, 2, \ldots \end{cases} \tag{7}$$

such that

$$\Pr(\mathrm{Y}=y)=\begin{cases} p+(1-p)\exp(-\lambda) & I_{(y=0)} \\ (1-p)\frac{\exp(-\lambda)\lambda^{y}}{y!} & I_{(y>0)} \end{cases} \tag{8}$$

Note that in the above equation that zeros arise from both the structural point mass component, p, and from the Poisson component, $(1-p)\exp(-\lambda)$.

In the ZIP model, the $\mathrm{E(Y)}=(1-p)\lambda=\mu$ and the $\mathrm{Var(Y)}=\mu+\frac{p}{1-p}\mu^2$ (Ridout et al., 1998). It can be easily seen from Equation 8 that when p is 0, that the ZIP model degenerates to the Poisson model. The probability of the point mass, $\mathbf{p}=(p_1,\ldots,p_n)^T$, may be modeled using a binomial logistic regression and $\boldsymbol{\lambda}=(\lambda_1,\ldots,\lambda_n)^T$ may be modeled using a Poisson log linear regression as shown in Equation 9 & Equation 10 below.

$$\begin{aligned}
\log\left(\frac{\boldsymbol{p}}{1-\boldsymbol{p}}\right)=\mathbf{G}\boldsymbol{\gamma} \implies \begin{pmatrix} \log\left(\frac{\boldsymbol{p_1}}{1-\boldsymbol{p_1}}\right) \\ \log\left(\frac{\boldsymbol{p_2}}{1-\boldsymbol{p_2}}\right) \\ \vdots \\ \log\left(\frac{\boldsymbol{p_n}}{1-\boldsymbol{p_n}}\right) \end{pmatrix} &= \begin{pmatrix} 1 & g_{12} & g_{13} & \ldots & g_{1p} \\ 1 & g_{22} & g_{23} & \ldots & g_{2p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & g_{n2} & g_{n3} & \ldots & g_{np} \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_p \end{pmatrix} \\
\log(\boldsymbol{\lambda})=\mathbf{B}\boldsymbol{\beta} \implies \begin{pmatrix} \log\mu_1 \\ \log\mu_2 \\ \vdots \\ \log\mu_n \end{pmatrix} &= \begin{pmatrix} 1 & b_{12} & b_{13} & \ldots & b_{1p} \\ 1 & b_{22} & b_{23} & \ldots & b_{2p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & b_{n2} & b_{n3} & \ldots & b_{np} \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_p \end{pmatrix}
\end{aligned} \tag{9}$$

[2] *Bern* refers to the Bernoulli distribution. Note that $\sum_{i=1}^{n} Y_i \sim Bin(n,p)$ where Bin refers to Binomial. *Poi* refers to the Poisson distribution. Note that the $\sum_{i=1}^{n} Y_i \sim Poi(\sum_{i=1}^{n}\lambda_i)$.

Where,

$$\mathbf{p} = \frac{\exp(\mathbf{G}\boldsymbol{\gamma})}{\exp(1 + \mathbf{G}\boldsymbol{\gamma})}$$
$$\boldsymbol{\lambda} = \exp(\mathbf{X}\boldsymbol{\beta}) \tag{10}$$

and $\mathbf{p}$ and $\boldsymbol{\lambda}$ are n x 1 response vectors corresponding to the probability of structural zeros and expected non-structural zero Poisson counts; $\mathbf{G}$ and $\mathbf{B}$ are covariate matrices; and $\boldsymbol{\gamma}$ and $\boldsymbol{\beta}$ are regression coefficients for the binomial logistic and Poisson log linear regressions, respectively. If the same covariates affect $\mathbf{p}$ and $\boldsymbol{\lambda}$ then Lambert's τ or Heilbron's zero-altered models may be used to describe their relationship and reduce the number of parameters estimated (Heilbron, 1994; Lambert, 1992).

Interpretation of a ZIP model can be difficult when the covariates affect λ, p, and the E(Y) differently (Lambert, 1992). To aid with interpretation various graphical devices have been proposed (Böhning et al., 1999; Lambert, 1992). One device involves plotting two separate graphs. The graphs involve plotting the coefficients of the logit(p) and log(λ) against the levels of a factor (Lambert, 1992). The purpose is to look for an overall effect of the covariate over both components of the model. Additionally, Lambert (1992) proposes plotting the average of $\hat{\mu}_i$ over all design points that share the same level of a factor, i.e. their marginal means, and comparing the averages. Using these devices can be especially useful for understanding when a covariate affects the prediction of the two components of the model differently and for understanding how it affects the E(Y). The other proposed device involves plotting the covariate coefficients of p

against the covariate coefficients of λ in a rectangular grid (Böhning et al., 1999). Again, this device can be useful for comparing the various parameters but it can also be useful for understanding variation associated with p and/or λ.

Lambert (1992) presents a motivating example from manufacturing in her seminal study on ZIP models. In her example, she examined components being mounted on printed wiring boards by soldering their leads onto the pads on a board. Her outcome was the number of leads improperly soldered. She had five independent variables: mask (5 levels, types of surface on the board), opening (3 levels, clearances in the mask around the pad), solder (2 levels, either thick or thin), pad (9 levels of different lengths and widths of the pads), and panel (3 levels, part of the board to be soldered).

Out of the 675 areas that had been soldered, 81% had no defects, 8% at least five, and 5.2% had at least nine. She argued that her data showed clear evidence of zero-inflation, and possibly overdispersion, and had theoretical arguments for the existence of structural zeros. She found that the most complex Poisson model examined performed poorly by both under predicting the number of zero defects and the number of areas with at least nine defects. More parsimonious Poisson models had substantially worse fit. She then fit several ZIP models and found that ZIP models (both with and without covariates for the binomial component) had substantially better fit (i.e. smaller log-likelihoods) and better modeled the observed counts than the Poisson models. She then fit the data to the NB model and found that the NB fit the data as poorly as the Poisson. This result is unsurprising given that the NB does not include a separate component for modeling zeros and may be inadequate in the presence of zero-inflation.

Additionally, Lambert (1992) proposed a ZIP model that allowed p and λ to be related through a shape parameter τ. Her motivation behind the use of τ was that it required fewer parameters to be estimated than a typical ZIP model and that it would be easier to interpret as the same coefficients describe the state with no defects, the state with defects, and the mean of Y. However, she found this model to have poorer fit and to underestimate the probability of no defects and at least nine defects. Additionally, compared to the ZIP model, the ZIP(τ) model had greater absolute Pearson residuals (indicative of lack-of-fit) than the ZIP model and different conclusions for some of the model parameters.

Lambert (1992) also presented a simulation study examining whether the asymptotic properties for the ZIP regression were appropriate for finite samples. She used 3 different sample sizes ($n = 25, 50, 100$) with the same covariates for both the binomial and the Poisson components with approximately 50% zeros. She found that regardless of the sample size, both ZIP and ZIP(τ) models were relatively easy to fit and that parameter recovery for the Poisson component did not differ by sample size whereas parameter recovery for the binomial component was a function of sample size (off by a factor of 4,500 when $n = 25$). Lambert argued that the poor performance for the binomial component was due to the infinite bias in finite samples for maximum likelihood estimates (MLE) in standard logistic regression. She also reported that the 95% likelihood ratio confidence intervals had better properties than the normal-theory confidence intervals for all parameters and at all sample sizes. This was likely because the log-likelihood was not approximately quadratic near the MLE. The relative bias for the estimated mean and the estimated probability of zero was lower for the ZIP(τ) model than for the ZIP model. It is worth noting that while other papers make reference to Lambert's ZIP(τ) model, none of the papers that were reviewed actually utilized this model.

Using Lambert (1992)'s manufacturing data, Hall (2000) used a ZIP mixed effects model to explore improvement in fit by including random effects. Hall (2000) found that the ZIP mixed effects model provided slightly better fit than Lambert's ZIP model. Improvement in fit was based on the reduction in the absolute value of the residuals, the Bayesian information criterion (BIC), and a test of significance of the variance component associated with the boards (the random effect).

The advantages of fitting the ZIP compared to the Poisson and NB model include better global fit (Böhning, 1998; Lambert, 1992), smaller Pearson residuals (Böhning, 1998), better fit to the zero counts (e.g. Atkins and Gallop (2007); Lambert (1992)) and ultimately better predictive performance. Böhning (1998) demonstrated the superiority of the ZIP model to the Poisson with zero-inflated data by reviewing and reanalyzing applications of the Poisson model to zero-inflated count data in various disciplines. In every situation, the Poisson model showed lack-of-fit (based on χ^2 test of deviance) and the ZIP model had adequate fit. The ZIP model always provides a substantial improvement in estimating the zeros over the Poisson in the case with zero-inflated or zero-inflated and overdispersed data.

The ZIP model is not always appropriate and generally can not handle overdispersion as well as the ZINB. Often the ZIP model will show lack-of-fit in scenarios with heavy overdispersion and have poorer fit than its NB counterparts (i.e. the NB, the NBH or the ZINB) (Hu et al., 2011; Rose et al., 2006; Wenger & Freeman, 2008; Yau et al., 2003). Ridout et al. (2001) fitted a ZIP model to data simulated from a ZINB and found biases of up to 30% for the estimated parameters. The ZINB model is now considered.

Zero-inflated Negative Binomial Model

The ZINB model is especially suited to dealing with both overdispersed and zero-inflated data. It is perhaps the best suited model for handling this type of data but it is also the least parsimonious. It is a widely used approach but still not as commonly used as the ZIP model. The ZINB may be more appropriate than the ZIP model when the nonzero counts are overdispersed relative to what would be expected under the Poisson. In these situations the ZIP model will often produce biased estimates (Yau et al., 2003; Ridout et al., 2001).

ZINB models have been used in psychology (Atkins & Gallop, 2007), medicine (Yau et al., 2003), ecology (Wenger & Freeman, 2008), manufacturing (Ghosh et al., 2006), and public health (Hu et al., 2011; Khan et al., 2011; Rose et al., 2006). It has been extended to mixed effects regression models (Yau et al., 2003), to generalized estimating equations (Hall & Zhang, 2004), and a Bayesian approach has been developed where the ZINB is treated as a member of the power series (Ghosh et al., 2006).

The ZINB consists of two components: a point mass at zero and a NB component (Ridout et al., 2001; Yau et al., 2003). Let Y be a realization of the

ZINB distribution, the ZINB distribution can be written as follows:

$$\Pr(\mathrm{Y} = y) = \begin{cases} p + (1-p)\left(\frac{k}{\mu+k}\right)^{k} & I_{(y=0)} \\ (1-p)\frac{\Gamma(y+k)}{\Gamma(k)\Gamma(y+1)}\left(\frac{k}{\mu+k}\right)^{k}\left(1-\frac{k}{\mu+k}\right)^{y} & I_{(y>0)} \end{cases} \tag{11}$$

Where μ is the mean of the NB defined in Equation 6. The mean and variance of the ZINB are: $\mathrm{E(Y)} = (1-p)\mu$ and $\mathrm{Var(Y)} = (1-p)\mu(1+\mu/k+p\mu)$. As $k^{-1} \to \infty$, the ZINB converges to the ZIP (Yau et al., 2003). Similar to the ZIP model, as $p \to 0$, the ZINB degenerates to the NB. Like Equation 10, the zero point mass for a ZINB may be modeled with a binomial logistic regression and the non-structural zero component with a NB log linear regression and both of these regressions can easily accept covariates.

Generally, the ZINB fits better than both the Poisson and NB (Hu et al., 2011; Rose et al., 2006; Yau et al., 2003) but not always better than the ZIP model (Atkins & Gallop, 2007). While the ZIP and ZINB models may both be considered contenders when data are zero-inflated and overdispersed they do not provide adequate fit when there is zero-deflation at any level of a factor (Min & Agresti, 2005). Theoretical justification for a ZIP or ZINB model also relies on the assumption that certain observations have a zero probability of a positive outcome.

Returning to the school suspension problem presented in Chapter 1, and presented briefly in this chapter, would any student really have a probability of zero of getting suspended? If not, then the appropriateness of these models in this scenario is questionable and interest in the parameters associated with the structural zero component would be minimal. Instead, hurdle models should be considered.

Hurdle Models

Hurdle models are models for count data that are either zero-inflated or zero-deflated. The hurdle model was originally proposed by Mullahy (1986) (and later by Welsh, Cunningham, Donnelly, and Lindenmayer (1996)). They have been referred to as *two-part models* (Heilbron, 1994), *zero-altered*, *conditional*, and *compatible* models (Zuur et al., 2009). Unlike the zero-inflated models, the hurdle models are not mixture models but instead are a two-part model and the parts may be fitted separately rather then simultaneously like a zero-inflated model. The first component is responsible for predicting all the zero counts and the second component is responsible for predicting the non-zero counts through the use of a truncated at-zero distribution. Formally, a hurdle model uses a binomial logistic regression model to assign a probability that will determine whether a count will be a zero or positive. (This will give parameter estimates that are identical to recoding the data and modeling just the presence or absence of a count (Desjardins, 2012).) If a positive count is realized, then the 'hurdle' is crossed and the counts are modeled by a truncated-at-zero count model (Mullahy, 1986). In the hurdle model then a zero may arise only from the binomial logistic regression component and not from the Poisson or NB component.

Both Poisson hurdle (PH) and NBH models have been used when the data are zero-inflated and overdispersed. The hurdle model has been used in medicine, within both the standard and mixed effects regression frameworks (Hu et al., 2011; Neelon et al., 2010; Rose et al., 2006; Min & Agresti, 2005), to examine substance abuse (Bandyopadhyay, DeSantis, Korte, & Brady, 2011), to model falling data (Khan et al., 2011), to examine self-harm, (Bethell, Rhodes, Bondy, Lou, & Guttmann, 2010), in economics (Mullahy, 1986), and in ecology (Zuur et al., 2009; Welsh et al., 1996; Dobbie & Welsh, 2001). Bayesian hurdle models have also been recently developed (Neelon et al., 2010) and they have been expanded to the generalized additive modeling (Barry & Welsh, 2002) and generalized estimating equation (Dobbie & Welsh, 2001) frameworks.

Min and Agresti (2005) express a generic hurdle model for a zero-inflated or zero-deflated problem as:

$$\mathrm{Pr(Y = y)} = \begin{cases} g_1(0) & I_{(y=0)} \\ (1 - g_1(0))\frac{g_2(y)}{1-g_2(0)} & I_{(y>0)} \end{cases} \tag{12}$$

where Y is the response variable; y is the response for subject i, $i = 1, \ldots, n$; g_1 is a probability mass function that governs the first stage of the hurdle for the zero responses; and g_2 is a truncated-at-zero probability mass function for the non-zero responses. This generalizes to models with explanatory variables. The explanatory variables may be different for each stage and be used in either component. If we let $\mathrm{P}(\mathrm{Y}_i > 0) = 1 - \pi_i$ and $\mathrm{P}(\mathrm{Y}_i = 0) = \pi_i$ then $1 - \pi_i$ is modeled with a binomial logistic regression and the mean, μ_i, of the untruncated g_2 distribution is modeled with a log linear model as shown in Equation 10.

Ridout et al. (1998) expresses the PH model as:

$$\Pr(\mathrm{Y} = y) = \begin{cases} \pi_0 & I_{(y=0)} \\ (1-\pi_0)\frac{\exp(-\lambda)\lambda^y}{(1-\exp(-\lambda)y!} & I_{(y>0)} \end{cases} \tag{13}$$

The first line of the pmf presented in Equation 13 corresponds to the probability of getting a zero. The second line of the pmf in Equation 13 corresponds to the probability of getting a non-zero count. First, note that the probability of getting a non-zero count is $(1 - \pi_0)$. Second, this probability is multiplied by the pmf of a truncated Poisson. It is truncated as the pmf of the Poisson, as noted in Equation 1, is divided by 1 minus the pmf of a Poisson when the pmf equals zero (i.e. $1 - \exp(-\lambda)$).

The PH model can be considered a reparameterization of the ZIP if you let $\pi_0 = p + (1 - p)\exp(-\lambda)$ and subsequently it could be considered nested within the ZIP model (Heilbron, 1994). It is worth noting that in regression, π_0 and p from the hurdle and the ZIP model are not equivalent and refer to entirely different parameters. Instead, the PH model may give similar regression estimates, standard errors, and p-values as the ZIP model for the count component. However, the regression estimates for the zero component in a PH could be quite different from those in a ZIP model (Zeileis et al., 2008).

In the PH model, the $\mathrm{E(Y)} = \frac{1-\pi_0}{1-e^{-\lambda}}\lambda$ and $\mathrm{Var(Y)} = \frac{1-\pi_0}{1-\exp(-\lambda)}(\lambda + \lambda^2) - \left(\frac{1-\pi_0}{1-\exp(-\lambda)}\lambda\right)^2$ (Zuur et al., 2009).

Similarly, a NBH model can be expressed as:

$$\Pr(\mathrm{Y}=y)=\begin{cases}\pi_1 & I_{(y=0)}\\ \frac{1-\pi_1}{1-\left(\frac{k}{\mu+k}\right)^k}\frac{\Gamma(y+k)}{\Gamma(k)\Gamma(y+1)}\left(\frac{k}{\mu+k}\right)^k\left(1-\frac{k}{\mu+k}\right)^y & I_{(y>0)}\end{cases} \tag{14}$$

If you let $\pi_1 = p + (1-p)\left(\frac{k}{\mu+k}\right)^k$ then this can also be considered a reparameterization of the ZINB model. The NBH will also have similar fit to the ZINB with the exception that the estimated parameters will not be identical and it will have a different latent interpretation. In the NBH model, the E(Y) = $\frac{1-\pi_1}{1-\left(\frac{k}{\mu+k}\right)^k}\mu$ and the Var(Y) = $\frac{(1-\pi_1)}{1-\left(\frac{k}{\mu+k}\right)^k}\left(\mu^2+\mu+\frac{\mu^2}{k}\right)-\left(\frac{(1-\pi_1)}{1-\left(\frac{k}{\mu+k}\right)^k}\mu\right)^2$ (Zuur et al., 2009).

Comparison of Hurdle and Zero-Inflated Models

The two major differences between hurdle and zero-inflated models involve the types of zeros that each are able to handle and their ability to model underdispersion of zero counts. As has been previously discussed, hurdle models do not make the distinction between structural and sampling zeros and handles them identically. Min and Agresti (2005) showed that the PH model performed on par with the ZIP model under the simulation scenarios considered in Lambert (1992). Furthermore, they found that the hurdle model outperformed the ZIP model for estimating the slope term in the logistic regression due to some simulated samples not having zero-inflation. They showed how Lambert's zero-inflated simulation conditions could be handled within a hurdle framework. Min and Agresti (2005) also considered a scenario where the data set was zero-inflated except for one level of a predictor for the log linear regression where it was zero-deflated. They found that the ZIP model gave highly unstable estimates while the hurdle model did not. The authors concluded that ZIP models may be unreliable for count data that are zero-deflated and instead suggested using the hurdle model in general. Based on their study, it would appear that hurdle models may be more general and robust than the zero-inflated models. (Jansakul and Hinde (2008) suggest that using an identity link for modeling the structural zeros instead of the canonical logit link would allow for the possibility of zero-deflation. However, this appears to be a very rare approach.)

The hurdle model is a less common approach compared to the zero-inflated models but has been gaining in usage recently (Bandyopadhyay et al., 2011; Khan et al., 2011; Bethell et al., 2010). Furthermore, it has been recommended that selection between these two models should be based on the data-generating mechanism of the zeros, whether zero-deflation is a possibility, and not through the use of model comparison. However, beyond the study by Min and Agresti (2005) and Miller (2007) (described below), no one has compared the performance of these two models and no one has considered the performance of these models using a NB extension to examine model performance and the effects of misspecification. A study of this nature would be very advantageous to this literature.

Other Count Models

The following section briefly describes alternative models for dealing with zero-inflated and overdispersed count data.

Geometric Models

Perhaps a more flexible way to conceptualize certain overdispersed data might be to consider them as a realization of the geometric distribution (Mullahy, 1986). Mullahy (1986) describes various geometric models including the standard geometric model, a truncated geometric model, and a zero-inflated geometric model. The benefit of the geometric model is that it offers more flexibility for dealing with the mean-variance structure but only in special cases. In the case where the data, $Y_1, \ldots, Y_n$, are independent and identically distributed and $Y_i \sim Geo(p)$[3] for $i = 1, \ldots, n$; this distribution may more appropriately model zero-inflation by allowing the variance, Var(Y)=$\left(\frac{1-p}{p^2}\right)$, to exceed the expectation, EY = $\left(\frac{1}{p}\right)$, but with the caveat that requires that the Pr(y) > Pr(y + 1) $\forall$ Y $\in 0, 1, 2, \ldots$ (Mullahy, 1986). In contrast, the Poisson model may be more appropriate for zero-inflated count data as it is not restricted solely to decay models.

Non-parametric and Cumulative Logit Models

This review focused on parametric models, that is models with a distribution and a finite number of parameters that describe these models. Non-parametric models, that do not rely on distributional assumptions, do exist

[3]Note: $Geo(p)$ refers to the geometric distribution with probability of success parameter, p with pmf: $f(y;p) = (1-p)^y p$ for $y = 0, 1, 2, \ldots$

(e.g. Yau et al. (2003); Hinde and Demétrio (1998)). These models have been found to have similar or better fit than the parametric models considered here (Yau et al., 2003). However, various shortcomings have been discussed, including their inability to allow for covariates in the zero function (Yau et al., 2003) and these methods are not routinely employed. A thorough discussion of these approaches is beyond the scope of this chapter.

Min and Agresti (2005) suggest using a cumulative logit model as an alternative to a zero-inflated or hurdle model. They found that a cumulative logit model had similar fit to the PH while requiring fewer parameters. The drawback of this approach, however, is that it requires truncating the larger counts into fewer bins and would ultimately involve throwing away information and variability. In situations where the large values could theoretically be binned this approach is promising. In these cases it is suggested to not bin into fewer than four categories to avoid loss in efficiency (Min & Agresti, 2005).

Model Selection

A researcher faced with the decision of how to select an appropriate model for count data that may exhibit zero-inflation and overdispersion has several decisions to make. First, if a theoretical reason exists for selecting one model over another (e.g. a Poisson over a NB or a zero-inflated model over a hurdle model) then this should trump a data-driven approach. However, it is imperative that a researcher still verify that they meet the necessary data conditions for this model otherwise inferences are likely to be incorrect. As mentioned earlier, the marginal appearance of zero-inflation and overdispersion are not necessarily indicative of these phenomena and once covariates are included in the model they may not be a concern. Additionally, the appearance of overdispersion conditional on covariates may indicate that a covariate is still missing (Berk & MacDonald, 2008) and if possible additional covariates related to the response should be included before moving from one probability model to another.

Second, if a theoretical reason does not exist then the following procedure could be used to determine an appropriate model. This involves thinking about the data-generating mechanism of the zeros; examining whether overdispersion is present and testing for it; examining relative model fit; and finally examining absolute model fit. Even if a theoretical reason exists for selecting a particular model over another, a careful consideration of the following approaches will ensure the statistical validity of the model selected.

Data-Generating Mechanism of the Zeros

First and foremost in determining whether the data are a realization of a Poisson, a hurdle, or a zero-inflated model, it is important to conceptualize the data-generating mechanism of the zeros (Rose et al., 2006). Returning back to the school suspension example, there might be a reason to believe that there are students who do not get suspended because they never engage in behavior that would get them suspended and students who do not get suspended during the timeframe of the study based solely on chance. These students are inherently different and zeros in the first case would be considered structural zeros and zeros in the latter case would be considered sampling zeros. Furthermore, if a researcher believes that there are both structural and sampling zeros in their data then the recommendation has been that they should only consider a zero-inflated mixture model with an untruncated probability distribution to handle the non-structural zeros and counts. Hurdle models do not make a distinction in zeros and therefore consider all zeros to arise from the same sampling source. If a researcher does not expect the structural zero class to exist then hurdle models should be considered. As previously discussed, if a researcher believes that there may be zero-deflation for any level of a factor then they should use a hurdle model instead of a zero-inflated model, as zero-inflated models struggle with zero-deflation.

Presence of Overdispersion

After a researcher has determined what the data-generating mechanism is for the zeros and whether zero-deflation is likely to be a factor, then they should consider whether their data are overdispersed. If the data are overdispersed then a ZINB model may be more appropriate than a ZIP model and if the zeros arise only from sampling then a NBH may be more appropriate than a PH. However, the presence of overdispersion should not exclude examination of ZIP and PH models as these models are less complex than their NB counterparts. Instead the relative and absolute fit of these models should be compared and tested.

Relative Fit Measures

There are a myriad of tests and selection criteria that have been developed for determining whether one model fits better than a competing model(s) and several of these methods are reviewed in detail in Mullahy (1986).

These tests are generally of three flavors: The likelihood ratio test, the score (Lagrange multiplier) test, or the Wald test. Here the most common approaches are presented.

Probably the most common approach for comparing competing nested models involves model comparison via a likelihood ratio test (LRT) (Böhning, 1998; Böhning et al., 1999; Hu et al., 2011; Khan et al., 2011; Rose et al., 2006; Yau & Lee, 2001). The LRT is defined as $2(\log(L_1) - \log(L_2))$ which follows a χ^2_{df}, where df, degrees of freedom, corresponds to the difference in number of estimated parameters between the models. A significant LRT favors the more complex model. Many of the models that have been previously described are inherently nested within one another and the LRT can be used. For example, the Poisson is nested within the NB, the PH, and the NBH (Zorn, 1996); the ZIP within the ZINB; and the PH within the NBH. However, all other comparsions such as a Poisson to a ZIP require a different statistical test.

If models are non-nested, then Vuong's test (Vuong, 1989), the Akaike information criterion (AIC) or BIC may be used (Burnham & Anderson, 2004; Hu et al., 2011; Wenger & Freeman, 2008). Vuong's test has been highly utilized for model selection in this literature. Vuong's statistic, like the AIC, is based on Kullback-Leibler information and is defined as $V \equiv n^{-1/2} LR_n(\hat{\theta}_n, \hat{\gamma}_n)/\hat{\omega}_n$ where $LR_n(\hat{\theta}_n, \hat{\gamma}) \equiv L_n^f(\hat{\theta}_n) - L_n^g(\hat{\gamma}_n) = \sum_{t=1}^{n} \log \frac{f(Y_t|X_t:\hat{\theta}_n)}{f(Y_t|X_t:\hat{\gamma}_n)}$; which is the ratio of two competing non-nested model's likelihoods; n is the sample size; and $\hat{\omega}_n$ is the standard error of the test statistic (see Vuong (1989) for the formula and derivations). Under the null hypothesis, Vuong's statistic follows a standard normal and one chooses a critical value, c, for a significance level (often 1.96).

When $V > c$, the statistic favors the model in the numerator, when $V < -c$, the statistic favors the model in the denominator, and when $V \in (-c, c)$ neither model is favored. Vuong's statistic can be used for comparing nested, overlapping, and non-nested models and Vuong (1989) described using this statistic to select between a linear regression (normal-based model) and a logistic regression (binomial-based model). This is one of the most common procedures reported in this literature (see Table 1).

The AIC is defined as $2k - 2\log(L)$ and BIC is defined as $-2\log(L) + k\log(n)$, where k is the number of parameters, n is the sample size, and L is the likelihood. Both of these criteria penalize a model based on the number of estimated parameters in the model. Models with the lowest AIC and BIC are favored. Burnham and Anderson (2004) suggest models differing by more than 10 units provide strong evidence in favor of the model with the smallest criterion and that the AIC should be used when many small effects may be present. Given that many of these models are non-nested, these information criteria have also been heavily utilized in this literature (Table 1).

There have been several tests developed specifically for testing zero-inflation (Böhning, 1998, 1994; Heilbron, 1994; Xiang & Teo, 2011; Xiang, Lee, Yau, & McLachlan, 2006; Xie et al., 2001). Heilbron (1994) proposed formally testing zero-inflation by considering a zero-altered Poisson (ZAP) and examining the γ parameter (Heilbron, 1994). A ZAP model is a restricted hurdle model that relates the parameters of the two-components through the parameter γ, for $\gamma \in (0, \infty)$. A small γ indicates zero-inflation and a large γ indicates zero-deflation, a $\gamma = 1$ indicates neither inflation nor deflation. An alternative way to test for zero-inflation with correlated count data (e.g. longitudinal data or hierarchical data) using the Wald and likelihood ratio tests have been proposed (Xiang & Teo, 2011). The Wald test is defined as $W \equiv \frac{(\hat{\gamma}-0)^2}{Var(\hat{\gamma})}$, where $\gamma \equiv \frac{p}{1-p}$ and W is asymptotically χ^2_r (see Xiang and Teo (2011) for discussion of the degrees of freedom, r). Large Wald statistics are indicative of zero-inflation. These tests under a variety of simulation conditions, have been found to have higher statistical power than the score test proposed in Xiang et al. (2006). Xie et al. (2001) reviewed several other tests for comparing the Poisson to the ZIP (e.g. score test, Cochran test, etc.) and found them to all be reasonably powerful. For brevity, all the tests of zero-inflation are not be reviewed here and the interested reader is encouraged to read Xie et al. (2001).

Other non-LRT based tests exists for model selection when interest is not in the ZIP model alone. A score test has been proposed for comparing a ZINB to a NB model (Jansakul & Hinde, 2008). A score test and Wald test statistic for examining whether a ZINB would provide an improvement in fit over the ZIP have also been proposed (Jansakul & Hinde, 2008; Ridout et al., 2001). The advantage of Ridout et al. (2001)'s score test over an LRT is that you only need to fit the ZINB.

Wenger and Freeman (2008) suggest cross-validation as a possible mode for model selection. In cross-validation part of the data are used to train the model and the remaining part are used to test it (Hastie, Tibshirani, & Friedman, 2009). Training the model involves selecting a subset of the data, e.g. 2/3 of the data, and using it to build your model. Testing the model involves using the model developed on the training set to see how well it fits the withheld

data. This provides a formal quantification of the validity and generalizability of the model. Splitting the data more than once, i.e. not using a holdout method but instead stratified k-fold cross-validation, typically 10-fold, is advised (Kohavi, 1995). R^2 and accuracy, the number of correct classifications, are common criterion used in cross-validation but other criteria exist.
Cross-validation is especially important when the purpose of model selection is to develop the model with the greatest predictive validity. Finally, least absolute shrinkage and selection operator (LASSO) and one-step smoothly clipped absolute deviation penalty (SCAD) procedures have recently been developed for ZIP models (Buu et al., 2011) allowing alternative variable selection procedures for the ZIP model.

Absolute Fit Measures

The simplest way to assess the adequacy of absolute model fit is to plot the fitted curve against the observed count for each level of the outcome (Atkins & Gallop, 2007). This is one of the most frequent methods reported (Ghosh et al., 2006; Hall, 2000; Lambert, 1992) and allows a nice comparison of models that are both nested and non-nested and allows comparison across statistical estimation techniques (e.g. Bayesian vs. ML). Hinde and Demétrio (1998) also suggest plotting residuals against fitted values and the use of half-normal plots with a simulation envelope.

Similarly, one may examine the Pearson residuals (Lambert, 1992; Yau & Lee, 2001). Large residuals are indicative of lack-of-fit (Agresti, 2002). Another approach involves comparison of the proposed alternative model to a fully saturated model (such as a Pearson's χ^2 test). This is a frequently reported test (Rose et al., 2006; Yau & Lee, 2001; Yau et al., 2003) but relies on cases in the data being grouped (Agresti, 2002). This test determines solely if a model has lack-of-fit (i.e. does it fit better than the saturated model).

Table 1 summarizes selected empirical studies that have had zero-inflated and/or overdispersed count data. The table includes the models examined, the relative and absolute fit criteria used, and any comments (e.g. was this a seminal paper) or conclusions that the authors made regarding whether one model had superior fit. In summary, Table 1 shows that the zero-inflated and hurdle models generally fit the best when considered and that authors used a variety of measures (described above) of absolute and relative fit both within and across studies.

Bayesian Fit Measures

Similarly, there are ways to detect absolute and relative fit in a Bayesian paradigm. To examine relative fit, simple comparison of deviance has been used to compare fit across models (Ghosh et al., 2006). The deviance information criterion (DIC), pseudo-marginal likelihoods (a cross-validation technique), and Bayes factors may all be used for model selection (Carlin & Louis, 2009; Neelon et al., 2010). Bayesian modeling averaging may be considered if the purpose is to amalgamate the models and for prediction (Hoeting, Madigan, Raftery, & Volinsky, 1999). To assess absolute model fit, Neelon et al. (2010) suggest various posterior predictive checks. This includes comparing the observed proportion of zeros, the overdispersion index, and sample skewness with their posterior counterparts and basing fit on Bayesian p-values.

Summary of Model Selection

In summary, the *modus operandi* for model selection involves contemplating the data-generating mechanism of zeros (and possibly zero-deflation); examining whether overdispersion is present; formally testing competing probability models; and finally examining absolute model fit.

Relationship to Prediction

Often after a best model has been selected, a researcher is interested in prediction. Assuming that your study is representative of the population as a whole, then the best fitting probability model should also have the highest predictive validity. For example, it is clear that if the data are overdispersed and zero-inflated using a Poisson model will not accurately predict the number of participants with zero counts and large counts. This will lead to poor predictions. Instead, a ZINB model, providing it fits the data best, should be used to predict future observations. Furthermore, obtaining the best fitting model, using global and relative model fit measures, and when fit between models is equal selecting the simpler model, is imperative for accurate forecasting. As previously mentioned, if the intention is primarily of prediction then cross-validation or a penalized regression technique (e.g. LASSO or SCAD) should be considered and posterior predictive checks if you are using a Bayesian framework.

Table 1: Summary of selected empirical studies involving zero-inflated and overdispersed count data.

Study	Models Examined						Fit Criteria	Comments/Conclusions
	1	2	3	4	5	6		
Agarwal et al. (2002)	X						None	Expanded Bayesian ZIP to spatial applications. Didn't compare models.
Angers and Biswas (2003)	X				X		Compared observed to fitted counts; χ^2 tests	Found generalized Poisson density (ZIP is a special case) had best fit. Expanded these models to the Bayesian framework.
Atkins and Gallop (2007)	X		X	X			Vuong's test (Vuong, 1989); deviance tests	ZIP model had superior fit to ZINB (based on deviance test) and NB models (based on Vuong's test).
Bandyopadhyay et al. (2011)		X	X				Vuong's test; AIC; BIC; Deviance; Compared observed to estimated proportions	Concluded that binomial hurdle model fit better than all other models and that hurdle models fit better than their standard counterparts.
Bethell et al. (2010)		X	X				AIC; BIC	Found NBH fit best and that PH and NB fit better than traditional Poisson.
Böhning et al. (1999)	X						Compared observed to estimated proportions; χ^2 tests	ZIP model had superior fit to Poisson, similar fit to general mixture models and more complex ZIP model.
Böhning (1998)	X						χ^2 tests	ZIP had model superior fit to the Poisson in all studies reviewed.
Ghosh et al. (2006)	X		X				Deviance; Compared observed to estimated proportions.	ZIP superior fit to Poisson and NB.

Hall (2000)	X		X			Compared observed to estimated proportions; examined residuals	ZIP superior fit to NB and Poisson. Developed first random effects extension to ZIP and the zero-inflated binomial (ZIB). ZIB and ZIP had similar fit.
Hu et al. (2011)	X	X	X	X		χ^2 tests; AIC; Vuong's test.	ZINB superior fit to other models and Poisson inferior fit compared to other models.
Khan et al. (2011)	X	X	X	X		Vuong's test; LRT; AIC; BIC; exact p-values via simulation for goodness-of-fit; compared observed to estimated proportions	The NB models had better fit than their Poisson counterpart; Vuong's tests found NB favored over ZINB and hurdle NB over NB and ZINB. Monte Carlo simulation showed all NB models with good fit.
Lambert (1992)	X		X			Compared observed to estimated proportions	ZIP superior fit to NB & ZIP(τ). Seminal ZIP paper and first to introduce covariates to ZIP
Neelon et al. (2010)	X	X			X	DIC; pseudo-marginal likelihoods; proportion of observations equal to zero; overdispersion index; sample skewness using Bayesian p-values; ZAP model	The ZIP showed superior fit to the Poisson, hurdle, and ZAP model.
Rose et al. (2006)	X	X	X	X		Compared observed to estimated proportions; AIC; BIC; Vuong's test; Pearson's χ^2 test of fit; LRT	ZINB and NBH models best fitting.
Wenger and Freeman (2008)	X			X		AIC	ZINB and NB models fit better than ZIP and Poisson models.

Yau and Lee (2001)	X						Deviance goodness-of-fit test; examined residuals	ZIP model had superior fit to the Poisson and there was no lack-of-fit based on the deviance test.
Yau et al. (2003)	X		X	X	X	X	Deviance goodness-of-fit test; Ridout et al. (2001)'s score test	Poisson, NB, & ZIP show lack-of-fit, ZINB shows adequate fit. 3-component Poisson mixture and 4-component non-parametric model fit better than ZINB.

Note. The models examined numbers correspond to: 1. zero-inflated Poisson (ZIP) ; 2. hurdle model; 3. negative binomial (NB); 4. zero-inflated negative binomial (ZINB); 5. Other - parametric model; 6. Other - non-parametric model.

Gaps and shortcomings in the zero-inflated and overdispersed literature

There are several gaps and shortcomings within the extant zero-inflated and overdispersed modeling literature. There are very few simulation studies examining hurdle and zero-inflated models and most of these involve the comparison of Bayesian to frequentist methods or various statistical tests (Majumdar & Gries, 2010; Ghosh et al., 2006; Jansakul & Hinde, 2008; Xiang & Teo, 2011; Xie et al., 2001; Zhang, Wu, & Johnson, 2010). Lambert (1992) examined the ZIP model under simulation primarily to demonstrate the validity of her proposed method. Min and Agresti (2005) compared a PH to a ZIP under simulation, to show that the ZIP model could not handle zero-deflation and that the hurdle model could handle Lambert (1992)'s conditions. Civettini and Hines (2005) examined the consequences of omitting a covariate in the structural zero component of a ZINB and including this covariate in the non-structural zero component. Civettini and Hines (2005) found that bias was captured by the structural zero intercept in the former case and by the non-structural zero intercept in the latter case. They concluded omission of a variable or inclusion in the wrong component had no effect on the other independent variables in the model.

More recently, Miller (2007) compared the performance of the Poisson, hurdle, and zero-inflated models under varying levels of skew and zero-inflation. Miller (2007) then used the AIC and estimated log-likelihoods to determine which model best fit the data. However, as indicated earlier, relative fit does not equal absolute fit and it remains unanswered under what levels of overdispersion and zero-inflation the zero-inflated and hurdle models would fail to provide adequate fit. In general, it remains unknown when inferences from the zero-inflation and hurdle models would converge and diverge and a study exploring these differences would be welcomed.

It is unclear what sample size is adequate to estimate the ZINB and NBH models. Lambert (1992) found issues with convergence for the simpler ZIP model for sample sizes of 25 and 50. She did not run into convergence issues with sample sizes of 100. She found parameter recovery and confidence interval performance to be reasonable regardless of sample size (after excluding simulates that did not converge). The empirical studies reviewed here represented a wide breadth of sample size to number of estimated parameters. These values ranged from 3.5 (Ghosh et al., 2006) to 294 (Zeileis et al., 2008), with many around 40 or 50 (Hu et al., 2011; Yau & Lee, 2001; Rose et al., 2006; Böhning et al., 1999; Loeys et al., 2012). Do ZINB models require a larger sample size than NBH models or vice versa and how might overdispersion affect this? There are no recommendations regarding this. It would be advantageous to this literature to examine how sample size may affect parameter recovery and the validity of the inferences (e.g. type I error rate, confidence interval converge, and bias) for these models.

Zero-inflated models involve the tacit assumption of structural zeros but no study has examined if these models can actually predict structural zeros. Answering whether these models can in fact accurately predict structural zeros would provide validity evidence for the zero-inflated models. In the event that these models are unable to explain this class, then concern over the occurrence of structural zeros in model selection would be rendered unimportant.

Rose et al. (2006) suggests that more research needs to be done to investigate the consequences of selecting a hurdle model rather than a zero-inflation model or vice versa when the data-generating mechanism is known. For example, how might parameter recovery or type I error be affected when selecting the wrong data-generating mechanism. Only a simulation study where the data-generating mechanism is known could answer this question.

Summary

This chapter described a variety of parametric models for dealing with zero-inflated and overdispersed data focusing primarily on zero-inflated and hurdle models. Tests of relative and absolute fit and model selection criteria were discussed. Finally, shortcomings in the extant literature were discussed. In the next chapter, the robustness of the ZINB and NBH will be considered under simulation and the methodology employed in the dissertation will be described.

Chapter III: Method

In Chapter 2, a review of the common approaches to modeling count data that were zero-inflated and overdispersed relative to a Poisson model were presented. At the conclusion of Chapter 2, various gaps in the extant literature were identified. This chapter examines two of these gaps through a simulation study and presents the methodology employed to investigate these gaps. The following topics are presented in this chapter: (a) summary of the research questions, (b) statistical models considered, (c) data-generating mechanism, (d) simulation conditions, and (e) evaluation of the simulation conditions.

Summary of the Research Questions

As initially stated in Chapter 1, the purpose of this dissertation is to answer the following research questions:

1. For a given set of simulated conditions, do the zero-inflated negative binomial (ZINB) and the negative binomial hurdle (NBH) models perform equally well in terms of parameter recovery and type I error rates? Under which of these conditions do they both perform poorly? And under which of these conditions does the performance between the two models differ?

2. What are the consequences, if any, of misspecifying the latent class for the zeros? In other words, given the existence (non-existence) of a structural zero class, what are the consequences of fitting a model that does not

(does) account for the structural zero class?

To answer the first research question, a simulation study was conducted where the true model was known. The conditions explored included sample size, dispersion, level of β_1 and γ_1, and level of correlation between categorical covariates within a component. The performance of the ZINB and NBH models under these scenarios was formally evaluated through examining parameter behavior (defined below), performance of 95% confidence intervals, type I error for $\alpha = .05$, bias, and proportion of correctly identified structural zeros for the ZINB model.

To answer the second research question, a second simulation study was conducted using a subset of the simulation conditions (when sample size equaled 500 only) to investigate the first research question. The second simulation study was evaluated using the same outcome measures except for proportion of correctly identified structural zeros. However, unlike the first simulation study, this study assessed the consequences of fitting a ZINB model to data generated from a NBH model and fitting a NBH model to data generated from a ZINB model.

Statistical Models

The ZINB and NBH models were examined in both simulation studies under all the simulation conditions but not for all the outcome responses. As evident from the review in Chapter 2, these models were selected as they represent the most frequent approaches to modeling count data that are concurrently zero-inflated and overdispersed relative to the Poisson model. A review of these models and their applications was presented in Chapter 2.

The cases model for the ZINB model assumed that $y_1, \ldots, y_n$ were a realization of $Y_1, \ldots, Y_n$ which are independent and

$$Y_i \sim \text{ZINB}(p_i, \lambda_i, k^{-1}), \quad i = 1, \ldots, n \tag{15}$$

Where

$$\begin{aligned} \log\left(\frac{p_i}{1-p_i}\right) &= \beta_0 + \beta_1 G_{1i} + \beta_2 G_{2i} \\ \log(\lambda_i) &= \gamma_0 + \gamma_1 B_{1i} + \gamma_2 B_{2i} \end{aligned} \tag{16}$$

In Equations 15 and 16, p_i corresponds to the probability of a structural zero; λ_i corresponds to the mean of the negative binomial; and k^{-1} refers to the dispersion parameter. The parameters in Equation 16, γ_0, γ_1, and γ_2 (collectively referred to as $\boldsymbol{\gamma}$); β_0, β_1, and β_2 (collectively referred to as $\boldsymbol{\beta}$); and n were not specified as they varied across simulation condition. In Equation 16, B_{ji} and G_{ji} are binary for all the simulation conditions.

Similarly, the cases model for the NBH model assumed that $y_1, \ldots, y_n$ were a realization of $Y_1, \ldots, Y_n$ which are independent and

$$Y_i \sim \text{NBH}(\pi_i, \mu_i, k^{-1}), \quad i = 1, \ldots, n \tag{17}$$

Where

$$\log\left(\frac{\pi_i}{1-\pi_i}\right) = \beta_0 + \beta_1 G_{1i} + \beta_2 G_{2i}$$
$$\log(\mu_i) = \gamma_0 + \gamma_1 B_{1i} + \gamma_2 B_{2i} \tag{18}$$

In Equations 17 and 18, π_i corresponds to the probability of a non-zero count and μ_i corresponds to the mean of the left-truncated at zero negative binomial. k^{-1}, B_{ji}, and G_{ji} were defined above. The parameters in Equation 18 are again not specified as they varied depending on the simulation condition considered.

Data-Generating Mechanism

Data were generated from the ZINB and the NBH models shown in Equation 15 through 18. Data for the ZINB were generated using a similar procedure as that described in Lambert (1992) for the zero-inflated Poisson model. For the ZINB model, the data-generating mechanism involved:

1. Calculating p_i and λ_i based on the specifications of the simulation condition (i.e. based on $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, and k^{-1}).

2. Generating a Uniform(0,1) random vector $\mathbf{U}$ of length n.

3. Assigning $y_i = 0$ if $U_i \leq p_i$ else $y_i \sim$ Negative Binomial(λ_i, k^{-1}).

For the NBH model, the data-generating mechanism involved:

1. Calculating π_i and μ_i based on the specifications of the simulation condition (i.e. based on $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, and k^{-1}).

2. Generating a realization, r_i, from a Bernoulli (π_i).

3. Assigning $y_i = 0$ if $r_i = 0$ else

   $y_i \sim$ Left-Truncated At Zero Negative Binomial(μ_i, k^{-1}).

4. Repeating steps 2 - 3 until there were n realizations.

All data were generated in `R` (R Core Team, 2012) using the `runif()`, `rbinom()`, and `rnbinom()` functions.

First Simulation Study - Model Performance and Recovery

The purpose of the first simulation study was to explore under what conditions the ZINB and NBH models had similar and dissimilar performance. For this study, data were generated under a ZINB model, then fit to a ZINB model, and the performance of the ZINB model was examined (described in more detail below). Similarly, data were generated under a NBH model, then fit to a NBH model, and the performance of the NBH model was examined.

To that extent, the following simulation conditions were considered in this study: model type, sample size, dispersion, varying levels of β_1 and γ_1, and varying levels of multicollinearity for two categorical covariates. In total, there were 2 levels of models (ZINB and NBH described earlier), 3 levels of sample size, 6 levels of dispersion, 2 levels of β_1 for the structural zero/zero component for the ZINB/NBH models, 2 levels of γ_1 for the non-structural zero/count component for the ZINB/NBH models, and 2 levels of multicollinearity resulting in a 2 X 3 X 6 X 2 X 2 X 2 factorial ANOVA design and a total of 288 conditions for each model for the first simulation study. Only second order interactions with model type as a factor were considered in the analysis (e.g. model type by dispersion interaction, model type by sample size, et cetera).

Sample Size

Sample size is one of the few conditions that a researcher generally has appreciable control over (depending upon resources). Thus it is an important and worthwhile condition to manipulate in a simulation study.

Initially, sample sizes of 25, 50, and 100 were considered to parallel those conditions considered for the ZIP in Lambert (1992). Similar to Lambert (1992), singularities and non-convergence were observed. Lambert (1992) for her simulation study with zero-inflated Poisson models found that the frequency of singularities when the sample size equaled 25 was 8% with fewer singularities occurring at sample size of 50 (less than 1%). Singularities occurred for both the NBH and ZINB models (less than 1% for both sample sizes of 25 and 50). Given this was a known issue with these models (at least for zero-inflated models) and that the principal focus of this dissertation was to examine conditions that a researcher was likely to encounter in practice (a researcher seems unlikely to fit these models with a sample size of 25 or 50 as this was never reported in the literature using maximum likelihood estimation), the sample sizes were increased to reflect the range of sample size to number of parameters estimated in the literature.

Singularities for sample sizes of 25 and 50 likely resulted from perfect or near perfect discrimination (Agresti, 2002). In other words, a hyperplane could be drawn in the predictor space that perfectly, or nearly perfectly, separates all the 0's and all the 1's such that all the 0's fall on one side of the plane and all the 1's on the other. The combination of small sample sizes with large effects (i.e. large β parameters) may have been the responsible for this occurring. Examination of several singular runs showed perfect discrimination (e.g. when $G_{1i} = 0$ and $G_{2i} = 0$ all $y_{ji} = 0$).

The sample sizes examined in this study were: 100, 250, and 500. These sample sizes were chosen as they represented the range of sample size to number of parameters estimated reported in the literature (see Chapter 2). These sample sizes resulted in ratios of approximately 14.3, 35.7, and 71.4 subjects per parameter estimated.

Dispersion

Dispersion is defined as the ratio of the variance of a random variable to the expectation of a random variable. Mathematically, let Y be a random variable, then dispersion, ϕ, is defined as $\phi \equiv \frac{\text{Var(Y)}}{\text{E(Y)}}$. Overdispersion occurs when ϕ is greater than that implied by the model.

The negative binomial model probability mass function for a random variable Y may be written as:

$$f(y;k,\mu) = \frac{\Gamma(y+k)}{\Gamma(k)\Gamma(y+1)} \left(\frac{k}{\mu+k}\right)^k \left(1-\frac{k}{\mu+k}\right)^y I_{(y\geq 0)} \tag{19}$$

Where Γ corresponds to the gamma function, k^{-1} to the dispersion parameter, μ to the mean of the negative binomial, and $I_{(y\geq 0)}$ refers to the indicator function. For the negative binomial model, E(Y) $= \mu$ and Var(Y) $=$

$\mu + \mu^2/k$. Therefore, dispersion in the negative binomial is defined as:

$$\phi = \frac{\mu + \mu^2/k}{\mu} = 1 + \frac{\mu}{k} \tag{20}$$

Therefore, using Equation 20, the Var(Y) and ϕ may be easily manipulated through their relationship with k. Henceforth, and for the remainder of this dissertation, θ will be referred to as the dispersion parameter, where $\theta = k$, to keep notation simpler.

In order to examine dispersion, the following values of θ were examined 1/4, 1/2, 1, 2, 5, and 10. These values represent a spectrum of overdispersion relative to the Poisson with 10 representing the lowest level of overdispersion (described in Chapter 2). When $\phi = 1$, the negative binomial model degenerates to Poisson model (Agresti, 2002) (i.e. as $\theta \to \infty$).

There were no literature-based recommendations regarding how much overdispersion relative to a Poisson could be considered problematic for either the ZINB or the NBH models. Few studies report the dispersion parameter. Therefore, a continuum was selected as a starting place for this literature. Additionally, the estimated dispersion parameter in the school suspension study corresponded to just above 1 and represented the fulcrum for this condition with values less than 1/2 reported in the literature as well (Zuur et al., 2009; Hu et al., 2011; Yau et al., 2003). The values of dispersion based on θ used in this study are reported below in Table 2.

Table 2: Conditions of the dispersion parameter, θ, dispersion, ϕ, and mean of the negative binomial, μ, used in both simulation studies for the ZINB and NBH.

θ	ϕ
1/4	$1 + 4\mu$
1/2	$1 + 2\mu$
1	$1 + 1\mu$
2	$1 + \mu/2$
5	$1 + \mu/5$
10	$1 + \mu/10$

Different Levels of β_1 and γ_1

The performance of the ZINB and NBH models at parameter recovery were examined using different parameter values for β_1 and γ_1. Specifically, the purpose for examining two different levels of β_1 and γ_1 was to understand how type I error rate might differ by model and by component type. Previous research has largely focused on power and comparing the Poisson to the ZIP (Xie et al., 2001), the ZINB to the NB (Jansakul & Hinde, 2008), the ZINB to the ZIP (Jansakul & Hinde, 2008; Ridout et al., 2001) and for detecting zero-inflation (Xiang et al., 2006) and not the NBH to the ZINB. Therefore, to my knowledge, this represents the first study examining how type I error rate might differ between these two models.

For the structural zero/zero component, β_1, the following parameter conditions were examined:

$$\log\left(\frac{\pi_i}{1-\pi_i}\right) = -1.5 + 0G_{1i} + 2G_{2i} \tag{21}$$

$$\log\left(\frac{\pi_i}{1-\pi_i}\right) = -1.5 + 0.5G_{1i} + 2G_{2i} \tag{22}$$

For the non-structural zero/count component, γ_1, the following parameter conditions were examined:

$$\log(\mu_i) = 1.5 + 0B_{1i} - 2B_{2i} \tag{23}$$

$$\log(\mu_i) = 1.5 + -0.5B_{1i} - 2B_{2i} \tag{24}$$

In Equations 21 and 22, p_i may be interchanged for π_i. By this it is meant that both of these conditions were used with both the structural zero component of the ZINB and the zero component of the NBH. Similarly, in Equations 23 and 24, λ_i may be substituted for μ_i. Therefore, the expected number of structural zeros in the ZINB will be identical to the expected number of zeros in the NBH model. However, the ZINB model will be expected to have more zeros because zeros may arise from a sampling and a structural source.

As previously indicated, the conditions in Equations 21 and 23 were selected as they allow examination of the type I error rate for these models when a parameter is set to zero (the de facto null hypothesis). The other parameter values selected in Equations 21 through 24 where chosen to be similar to Lambert (1992)'s values and represent potential parameters estimated in practice (note that these parameters were similar to those observed in the school suspension study).

Varying Levels of Covariate Correlation within a Component

Each component of each model contained three parameters: an intercept and two slope terms corresponding to the two dichotomous predictors shown in Equations 16 and 18. The level of correlation between $\mathbf{B_1} = (B_{11}, B_{12}, \ldots, B_{1n})^T$ and $\mathbf{B_2} = (B_{21}, B_{22}, \ldots, B_{2n})^T$ and between $\mathbf{G_1} = (G_{11}, G_{12}, \ldots, G_{1n})^T$ and $\mathbf{G_2} = (G_{21}, G_{22}, \ldots, G_{2n})^T$ were examined in two situations: when they were uncorrelated ($\rho = 0$) and moderately correlated ($\rho = .3$). The model matrices were uncorrelated across components within a model (i.e. $\mathbf{B_j}$ and $\mathbf{G_j}$ were orthogonal). These values of correlation were selected as they corresponded to the strongest correlation for the independent variables for the school suspension data (the correlation between the risk variable and ethnicity).

To generate the multicollinearity, the `rmvbin()` function in the `bindata` package (Leisch, Weingessel, & Hornik, 2011) in `R` was used. The `rmvbin()` function works by creating correlated multivariate Bernoulli random variables through thresholding a normal distribution. To do this, the user specifies the probability of success and the correlation between the vectors. The probability of success for $\mathbf{B_j}$ and $\mathbf{G_j}$ was set at 0.5 in order to have a balanced-design. This resulted in roughly 50% of $\mathbf{B_j}$ and $\mathbf{G_j}$ being coded as 1.

Number of Simulates, Iterations, and Software

Selecting the number of simulates is a crucial decision as too many simulates can be computationally taxing and too few could result in inaccurate estimates and not observing rare events (Bonate, 2001). Civettini and Hines (2005) used 1,000 simulates to investigate covariate omission and model misspecification with the ZINB model. Recently, Miller (2007) used 2,000 simulates to examine skew with zero-inflation and hurdle models in his doctoral dissertation. Using Lambert (1992)'s seminal work and Miller (2007) as a guide, the number of simulations used for each set of conditions was set to 2,000. The maximum number of iterations was set to 10,000, the default number of iterations specified by the software used in this study (Zeileis et al., 2008).

The `pscl` package (Zeileis et al., 2008) in `R` was used to fit all models. The `zeroinfl()` function was used to fit the ZINB models and the `hurdle()` function was used to fit the NBH models.

Second Simulation Study - Model Misspecification

It is not uncommon in the zero-inflation and overdispersed count data literature to see researchers evaluating both zero-inflated and hurdle models (e.g Hu et al. (2011); Rose et al. (2006); Khan et al. (2011)) even though these models have different underlying latent assumptions. It is believed that zero-inflated and hurdle models tend to have similar fit indices and predicted values (Loeys et al., 2012) but the interpretation of the parameters and their estimates are different. Researchers are strongly encouraged to select models based on a priori beliefs about the existence of structural zeros. However, there is a dearth of research exploring the consequences of using the wrong model when the true model is known (e.g. assuming there is a structural zero class when there is not one) as is the case in a simulation study and it has been argued in literature that a deeper understanding of model misspecification is necessary (Rose et al., 2006).

Specifically, for this simulation study, data were generated from a ZINB model and fit to a NBH model (and vice versa). In order to reduce the number of conditions examined, model misspecification was just considered for the $n = 500$ conditions. Therefore, there were a total of 48 conditions where a ZINB model was the data-generating mechanism and a NBH model was fit to the data and 48 conditions where a NBH model was the data-generating mechanism and a ZINB model was fit to the data.

Simulating Data from the School Suspension Model

In addition to the aforementioned conditions for the first and second simulation studies, data were simulated using the school suspension data and the performance of the ZINB and NBH was examined where the school suspension predicted models were treated as the true population models. To simulate data from the school suspension data, main-effects only ZINB and NBH models were fit, where risk, ethnicity, gender, english language learner status, and special education status were included as covariates. A breakdown of the covariates by risk are presented in Table 3 and additional information about these covariates and the data, in general, may be found in Desjardins (2012).

Table 3: Demographic characteristics by associated risk.

Group	N	Male	Ethnicity					Special Education	ELL
			AA	AI	AS	HI	WH		
HHM	1641	832	1154	154	125	104	104	510	426
FREE	8024	4140	4262	405	1193	1460	704	1681	3194
RED	578	323	250	31	86	51	160	92	136
GEN	3363	1761	654	104	185	210	2210	409	515

Note. AA refers to African American, AI refers to American Indian, AS refers to Asian, HI refers to Hispanic, WH refers to non-Hispanic White, and ELL refers to English language learner. HHM refers to homeless and highly mobile, FREE refers to eligible for free-price meals, RED refers to eligible for reduced-price meals, and GEN refers to none of the aforementioned groups.

After fitting a ZINB model to this data, the following predicted model was used as the population model:

$$\begin{aligned}\log\left(\frac{p_i}{1-p_i}\right) &= 1.41 - 1.08X_{1i} - 0.88X_{2i} + 0.76X_{3i} - 0.13X_{4i} - 0.33X_{5i}\\ &\quad + 0.06X_{6i} + 0.20X_{7i} + 0.46X_{8i} + 1.39X_{9i} - 0.63X_{10i}\\ \log\lambda_i &= 1.25 + 0.38X_{1i} + 0.38X_{2i} + 0.02X_{3i} + 0.08X_{4i} + 0.35X_{5i}\\ &\quad - 0.08X_{6i} - 0.12X_{7i} - 0.47X_{8i} - 0.55X_{9i} + 0.13X_{10i}\end{aligned} \tag{25}$$

Where X_{1i} to X_{10i} refer to dummy variables corresponding to African American ($X_{1i} = 1$), American Indian ($X_{2i} = 1$), Asian American ($X_{3i} = 1$), Hispanic ($X_{4i} = 1$), in special education ($X_{5i} = 1$), receiving ELL services ($X_{6i} = 1$), in the RED group ($X_{7i} = 1$), in the FREE group ($X_{8i} = 1$), in the GEN group ($X_{9i} = 1$), and male ($X_{10i} = 1$) students. Finally, $\theta_{ZINB} = 1.26$.

For the NBH model, the following predicted model was used as the population model:

$$
\begin{aligned}
\log\left(\frac{\pi_i}{1-\pi_i}\right) &= -1.67 + 1.10X_{1i} + 0.92X_{2i} - 0.72X_{3i} + 0.16X_{4i} + 0.39X_{5i} \\
&\quad - 0.08X_{6i} - 0.21X_{7i} - 0.55X_{8i} - 1.44X_{9i} + 0.60X_{10i} \\
\log \mu_i &= 1.25 + 0.38X_{1i} + 0.38X_{2i} + 0.02X_{3i} + 0.07X_{4i} + 0.35X_{5i} \\
&\quad - 0.08X_{6i} - 0.12X_{7i} - 0.47X_{8i} - 0.54X_{9i} + 0.13X_{10i}
\end{aligned}
\tag{26}
$$

Where the $X'_{ji}s$ are defined above and $\theta_{NBH} = 1.27$.

Two thousand samples of size 13,606 (the original sample size) were then drawn from the ZINB and NBH models. Each sample was then fit to their original data-generating mechanism to examine model recovery/performance and the competing model (i.e ZINB data were fit to an NBH model and NBH data were fit to a ZINB model) to examine model misspecification. The performance of these models was assessed on the criteria defined below except for the type I error criterion.

Validity of Simulation Conditions

To show that the simulated data possessed (approximately) the desired properties (e.g. with a pre-specified level of dispersion, multicollinearity, and parameters), a replicate of a large sample size (100,000) for a few simulation conditions was run to verify that the data-generating mechanism was functioning properly. Variability in the simulates within a simulation condition was expected, but the use of a large sample size should minimize sampling variability and provide empirical evidence that the generated data represented the desired model.

The use of this approach to test that the data-generating mechanism is correct rests on the assumption that the pseudorandom number generator in `R` functions properly. Therefore, this study assumed that the pseudorandom number generator in `R` was functioning properly. If the validation runs provided estimates close to the pre-specified values, then this was taken as evidence that the data-generating mechanism was correctly specified.

Specifically, 4 conditions for each model were examined to assess the validity of the data-generating mechanism. These conditions were:

1. $\theta = 1/4$, $\rho = 0$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2.0\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2.0\}$
2. $\theta = 1/4$, $\rho = .3$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2.0\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2.0\}$
3. $\theta = 10$, $\rho = 0$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2.0\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2.0\}$
4. $\theta = 10$, $\rho = .3$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2.0\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2.0\}$

Finally, given that the multicollinearity was created stochastically based on the method described above, correlation matrices were examined to verify that the data-generating mechanism produced the correct correlations.

Evaluation of the Simulation Studies

Criteria to Assess the Performance of the Models under the Different Conditions

In order to assess the performance of the ZINB and NBH model under the different simulation conditions, various measures related to parameter recovery and model performance were examined. Specifically, to assess recovery and performance, parameter behavior (i.e. how close were the estimated parameters to the true parameters), confidence interval coverage and width, correct identification of structural zeros, type I error rate, and bias for the E(Y|X) and the Pr(Y = 0) were examined.

Parameter Behavior

For each condition, the mean and median of the estimated parameters based on the 2000 simulates were calculated. These statistics were then aggregated based on the interaction of interest and the mean of the mean of the simulates and the mean of the median of the simulates were calculated (see Chapter 4). The mean and median of the simulates for every condition examined are reported in Appendix A.

In addition, the standard deviation of the estimated parameters of the 2000 simulates and the mean squared error were calculated providing measures of the precision of the estimates. As with the mean and median, the standard deviations and mean squared errors were aggregated depending upon the interaction of interest and the mean of the standard deviation and the mean of the mean squared error were calculated (see Chapter 4). The standard deviation of the stimulates and the mean squared error for every condition examined in the first simulation study are reported in Appendix A.

If the models are performing well, it is expected that the mean and median should both be close to the true parameters and that the standard deviation and the mean squared error of the estimated parameters should be small (i.e. have high precision). However, if the models have a difficult time recovering the parameters and there is a large amount of variability, then the mean and median may be quite different and the standard deviation and mean squared error of the estimated parameters will be quite large.

Confidence Interval Coverage and Width

For each simulate within a condition, 95% Wald confidence intervals for all the estimated parameters, except θ, were calculated. For each simulate, whether the 95% confidence interval included the true parameter was recorded. The percentage of confidence intervals out of the 2000 simulates that had 95% confidence intervals that included the true parameter was then calculated. A properly functioning 95% confidence interval should have approximately 95% of the 95% Wald confidence intervals for a condition capturing the true parameter value. Additionally, the width of each simulate's confidence interval was recorded. Shorter 95% confidence intervals imply a more precise estimate for a model or condition. As with the parameter behavior, the 95% confidence interval coverage and width were aggregated and the means reported depending upon the interaction of interest. The 95% confidence interval coverage and width for every condition examined are reported in Appendix B.

Correct Identification of Structural Zeros

One measure that sheds insight into the appropriateness of the ZINB model is its ability to correctly identify structural zeros. Furthermore, the ability of the ZINB to correctly identify structural zeros was monitored. Because structural zeros were known beforehand, it was possible to record the number of times that the ZINB correctly identified a structural zero. Low percentages of correct classification, and subsequently high percentages of misclassification, suggests that the ZINB is not appropriate and that the use of a model that does not make the distinction between the source of the zero is more appropriate.

The proportion of correctly identified structural zeros for each simulate was monitored and the mean for each condition was calculated and is reported in Appendix C. As with the other outcomes, the mean of the structural zeros was then aggregated depending upon the interaction of interest and the mean of the means of the structural zeros was calculated as appropriate.

Type I error

For each simulate within the conditions where either $\beta_1 = 0$ or $\gamma_1 = 0$, a Wald test of the null hypothesis of the respective parameter being zero was performed. The p-values from these tests were then saved. From these p-values, the proportion of the simulates out of all of the simulates that the true null hypothesis was rejected at $\alpha = .05$ (i.e. type I error) was recorded. Observed type I error rates for $\alpha = .05$ between .025 and .075 were considered acceptable, with values closest to .05 considered best.

Type I error rates were aggregated over conditions and the mean of the means of the type I error rates and the standard deviation of the means of the type I error rates were then computed. For each simulate, but not when aggregated to examine an interaction of interest, 95% Wald confidence intervals for $\alpha = 0.05$ were constructed. When the data were aggregated instead standard deviations bands (2 times the standard deviation) were calculated to give a measure of uncertainty. The type I error rate and the corresponding 95% confidence interval for each simulate are reported in Appendix D.

Relative Bias for the E(Y|X) and Pr(Y = 0)

Similar to Lambert (1992), the ability of the models to estimate the E(Y|X) and the Pr(Y = 0) was examined. To examine this ability, the relative bias for the E(Y|X) and Pr(Y = 0) were calculated. The relative bias for the E(Y|X) was defined as $[\hat{E}(Y|x) - E(Y|x)]/E(Y|x)$ and for Pr(Y = 0) was defined as $[\hat{Pr}(Y = 0) - Pr(Y = 0)]/Pr(Y = 0)$. The average relative bias for the E(Y|X) and Pr(Y = 0) for each simulate was calculated and the mean of the simulates was then calculated. Relative bias for the E(Y|X) and Pr(Y = 0) was then aggregated and the mean of the means of the relative biases were examined depending upon the interaction of interest. Relative bias for the E(Y|X) and Pr(Y = 0) for each simulate are reported in Appendix E.

Chapter IV: Results

This chapter presents the results of the simulation studies described in Chapter 3. Results are first presented to demonstrate the validity of the simulation studies, then results from the first simulation study are presented, and finally the results from the second simulation study are presented. Within each simulation study section, parameter behavior measures are first described followed by the confidence interval measures, structural zero recovery for the ZINB model (when appropriate), type I error rate (when appropriate), and finally bias. The empirically-derived condition is treated at the end of each section.

Validation of Simulation Conditions

Tables 4 and 5 show the correlation matrix of the covariates for the size 100,000 validation run for $\rho = 0$ and .3, respectively. These tables show the correlations to be close to their pre-specified values corroborating that the data-generating mechanism for the covariates was functioning properly.

Table 4: Correlation matrix for orthogonal covariates.

	G_1	G_2	B_1	B_2
G_1	1.000	0.002	-0.005	0.002
G_2	0.002	1.000	-0.005	-0.003
B_1	-0.005	-0.005	1.000	0.006
B_2	0.002	-0.003	0.006	1.000

Note. The true correlation G_1, G_2, B_1, and B_2 was 0.

Table 5: Correlation matrix for correlated covariates.

	G_1	G_2	B_1	B_2
G_1	1.000	0.301	0.002	-0.002
G_2	0.301	1.000	0.004	-0.001
B_1	0.002	0.004	1.000	0.298
B_2	-0.002	-0.001	0.298	1.000

Note. The true correlation between B_1 and B_2 was .3, G_1 and G_2 was .3, and B_j and G_j were uncorrelated.

Table 6 shows the estimated parameters from the size 100,000 validation run. This table shows that the estimated parameters for the structural zero/zero component (i.e. the βs), the non-structural zero/count component (i.e. the γs), and θ are all very close to their pre-specified values. This corroborates that the data-generating mechanism for the response for both models was functioning properly.

Table 6: Estimated parameters from the size 100,000 validation run.

Condition	β_0	β_1	β_2	γ_0	γ_1	γ_2	θ
			NBH				
1	-1.496	0.485	1.997	1.468	0.010	-1.993	0.240
2	-1.501	0.525	2.002	1.489	0.016	-1.985	0.249
3	-1.502	0.495	2.007	1.489	0.010	-1.999	10.321
4	-1.503	0.494	2.002	1.500	0.004	-2.013	10.059
			ZINB				
1	-1.490	0.536	1.974	1.497	0.034	-2.030	0.253
2	-1.407	0.408	1.981	1.525	-0.021	-1.990	0.255
3	-1.537	0.509	2.027	1.504	-0.009	-2.020	9.780
4	-1.498	0.483	2.011	1.500	-0.003	-2.005	9.727

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.5, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = 0$, and $\gamma_2 = -2$. For conditions 1 and 2, $\theta = 1/4$ and for conditions 3 and 4, $\theta = 10$.

First Simulation Study - Model Performance and Recovery

Parameter Behavior

Measures of parameter behavior are presented in Tables 38 - 85 in Appendix A. In these tables, the mean, median, standard error, and mean squared error of the simulates for $\beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1$, and γ_2 for each condition are presented. Tables 38 through 41 and 62 through 65 correspond to $\theta = 1/4$; Tables 42 through 45 and 66 through 69 correspond to $\theta = 1/2$; Tables 46 through 49 and 70 through 73 correspond to $\theta = 1$; Tables 50 through 53 and 74 through 77 to $\theta = 2$; Tables 54 through 57 and 78 through 81 to $\theta = 5$; and Tables 58 through 61 and 82 through 85 to $\theta = 10$. These tables each correspond to a different level of β_1, γ_1, and ρ. Tables 38 - 61 correspond to conditions where the covariates were orthogonal within and across components ($\rho = 0$) and Tables 62 - 85 correspond to conditions where the covariates within a component were correlated ($\rho = .3$) but orthogonal across component. For each table, the estimated parameters are presented by sample size (n = 100, 250, and 500) and model type (NBH and ZINB).

Table 7 shows the parameter behavior of the NBH and ZINB models at recovering parameters collapsed by sample size. Table 7, and subsequent parameter behavior tables in this chapter, include information on the mean of the means of the collapsed conditions, the mean of the medians of the collapsed conditions, the mean of the standard errors of the collapsed conditions, and the mean of the mean squared error of the collapsed conditions for each parameter of interest (i.e. all parameters except θ). In these tables, β_1 and γ_1 should be close to 0.25 and -0.25, because these tables are collapsed over the conditions where these parameters varied and 50% of the time they were set equal to 0 and 50% of the time they were set equal to 0.50 and -0.50, respectively.

In Table 7, with the exception of γ_0, the NBH model outperformed the ZINB with estimates closer to their true parameter values (both the mean and median) and smaller standard deviations and mean squared error for the structural zero/zero component parameters regardless of sample size. Parameter recovery was more similar with respect to the non-structural zero/count component parameters than the non-structural zero/count component. Given that the mean and median differed substantially for the ZINB's structural zero component implies that the ZINB model struggled to recover these parameters.

It is evident in Table 7 that with a sample size of 500, the ZINB model struggled to recover the structural zero component parameters (based on the mean) and except for γ_0, the NBH model performed reasonably well at recovering all the parameters even at a sample size of 100. The ZINB performed especially poorly at estimating β_0 and β_2 when the sample size was less than 500. The non-structural zero component of the ZINB performed quite well regardless of sample size with little gain in behavior after 250 (with the exception of the standard deviation and mean squared error decreasing).

Table 7: Parameter behavior of the NBH and ZINB coefficients by sample size, n, from the 2,000 simulates.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.552	0.249	2.080	1.326	-0.267	-2.039
	250	-1.523	0.253	2.031	1.433	-0.249	-2.009
	500	-1.511	0.251	2.015	1.467	-0.250	-2.000
ZINB	100	-2.672	0.325	3.292	1.509	-0.271	-2.033
	250	-2.236	0.270	2.762	1.503	-0.251	-2.016
	500	-2.054	0.260	2.567	1.504	-0.252	-2.007
				Median			
NBH	100	-1.520	0.252	2.060	1.515	-0.256	-1.976
	250	-1.512	0.254	2.021	1.507	-0.248	-1.993
	500	-1.506	0.251	2.012	1.503	-0.249	-1.994
ZINB	100	-1.360	0.264	2.093	1.526	-0.263	-2.012
	250	-1.425	0.255	2.021	1.511	-0.251	-2.008
	500	-1.454	0.254	2.009	1.508	-0.251	-2.003
				Standard Deviation			
NBH	100	0.425	0.499	0.505	0.839	0.455	0.709
	250	0.275	0.298	0.304	0.435	0.245	0.314
	500	0.185	0.210	0.213	0.270	0.177	0.212
ZINB	100	3.251	1.926	3.228	0.289	0.407	0.477
	250	2.056	0.591	1.930	0.206	0.240	0.286
	500	1.595	0.385	1.493	0.156	0.171	0.193
				Mean Squared Error			
NBH	100	0.365	0.498	0.516	2.574	0.441	1.082
	250	0.152	0.178	0.186	0.799	0.128	0.200
	500	0.069	0.088	0.091	0.300	0.067	0.091
ZINB	100	26.069	9.321	25.943	0.211	0.385	0.515
	250	13.103	0.802	11.517	0.109	0.133	0.175
	500	9.036	0.317	7.902	0.065	0.066	0.080

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.25, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = -.25$, and $\gamma_2 = -2$.

Tables 8 and 9 show the parameter behavior of the NBH and ZINB models at recovering the parameters by the dispersion parameter, θ, for $n = 100$. Parameter behavior tables for $n = 250$ and $n = 500$ are located in Appendix A in Tables 32 through 35. For the NBH model, dispersion appeared to be unrelated to ability to recover the zero component and the count component, with the possible exception of γ_0.

Table 8: Parameter behavior, mean and median, of the NBH and ZINB coefficients by dispersion parameter, θ, from the 2,000 simulates for $n = 100$.

	θ	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	1/4	-1.553	0.251	2.078	0.922	-0.262	-1.948
	1/2	-1.555	0.254	2.080	1.211	-0.267	-1.977
	1	-1.547	0.249	2.073	1.404	-0.267	-2.016
	2	-1.553	0.251	2.079	1.457	-0.264	-2.055
	5	-1.553	0.242	2.089	1.480	-0.272	-2.106
	10	-1.552	0.248	2.082	1.483	-0.269	-2.134
ZINB	1/4	-3.114	0.383	4.075	1.611	-0.288	-2.037
	1/2	-3.349	0.383	3.982	1.521	-0.274	-2.029
	1	-3.122	0.317	3.671	1.478	-0.268	-2.032
	2	-2.576	0.300	3.110	1.475	-0.265	-2.037
	5	-2.020	0.295	2.534	1.482	-0.264	-2.036
	10	-1.853	0.269	2.378	1.488	-0.268	-2.029
				Median			
NBH	1/4	-1.519	0.255	2.053	1.591	-0.261	-1.923
	1/2	-1.524	0.254	2.064	1.529	-0.258	-1.948
	1	-1.518	0.256	2.050	1.494	-0.254	-1.971
	2	-1.524	0.253	2.064	1.492	-0.248	-1.986
	5	-1.518	0.246	2.067	1.493	-0.259	-2.009
	10	-1.520	0.251	2.063	1.493	-0.259	-2.020
ZINB	1/4	-0.694	0.236	1.854	1.640	-0.274	-2.009
	1/2	-1.229	0.273	2.076	1.542	-0.269	-2.011
	1	-1.506	0.264	2.189	1.497	-0.260	-2.010
	2	-1.570	0.265	2.171	1.490	-0.257	-2.014
	5	-1.584	0.280	2.136	1.492	-0.256	-2.018
	10	-1.580	0.268	2.130	1.495	-0.261	-2.011

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.25, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = -.25$, and $\gamma_2 = -2$.

In contrast, the structural zero component of the ZINB appeared to be affected by dispersion with performance improving as dispersion decreased (i.e.

as θ increased). The estimates for the structural zero parameters were quite far from their true values until θ was greater than 1 based on the median and 10 based on the mean. The standard deviation remained twice as large as the NBH for the structural zero component compared to the zero component even when $\theta = 10$ and when $\theta = 1/4$, the standard deviation of the structural zero component of the ZINB model was about 13 times larger than the standard deviation of the zero component of the NBH model. This was even more pronounced when examining mean squared error.

For the non-structural zero component for the ZINB, as the dispersion decreased, the standard deviation decreased by a factor of 2 to 4. For the NBH model, the standard deviation of γ_0 decreased by a factor of 10 and γ_1 by a factor of 2. A similar pattern was observed for the mean squared error.

Table 9: Parameter behavior, standard deviation and mean squared error, of the NBH and ZINB coefficients by dispersion, θ, from the 2,000 simulates for $n = 100$.

		β_0	β_1	β_2	γ_0	γ_1	γ_2
Standard Deviation							
NBH	1/4	0.430	0.504	0.507	2.167	0.603	0.611
	1/2	0.424	0.499	0.503	1.448	0.542	0.579
	1	0.428	0.504	0.504	0.673	0.499	0.615
	2	0.423	0.494	0.503	0.363	0.426	0.709
	5	0.425	0.493	0.505	0.213	0.347	0.835
	10	0.421	0.499	0.507	0.170	0.314	0.907
ZINB	1/4	4.955	3.634	4.982	0.547	0.701	0.790
	1/2	4.351	2.573	4.210	0.390	0.511	0.561
	1	3.763	1.888	3.651	0.284	0.400	0.457
	2	2.973	1.451	2.946	0.218	0.325	0.381
	5	1.952	1.075	1.986	0.160	0.267	0.342
	10	1.512	0.938	1.598	0.136	0.239	0.329
Mean Squared Error							
NBH	1/4	0.370	0.505	0.531	3.669	0.535	0.776
	1/2	0.364	0.504	0.517	3.156	0.396	1.029
	1	0.359	0.486	0.506	0.223	0.247	0.778
	2	0.371	0.510	0.535	4.480	0.685	1.087
	5	0.368	0.504	0.517	3.548	0.475	1.615
	10	0.360	0.479	0.493	0.365	0.310	1.206
ZINB	1/4	33.752	10.665	34.477	0.280	0.453	0.501
	1/2	27.851	11.446	26.549	0.266	0.446	0.534
	1	13.042	3.789	12.448	0.090	0.185	0.274
	2	35.450	12.563	36.559	0.280	0.510	0.665
	5	30.045	12.747	29.894	0.256	0.484	0.752
	10	16.276	4.719	15.730	0.091	0.232	0.363

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.5, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = 0$, and $\gamma_2 = -2$.

Finally, Table 10 shows the parameter behavior of the NBH and ZINB models at recovering the parameters by covariate correlation within a component for $n = 100$. Tables 36 and 37 in Appendix A show parameter behavior for $n = 250$ and $n = 500$ by covariate correlation. Table 10 shows no evidence of an effect of covariate correlation for the conditions considered on parameter behavior and no evidence of a disparate covariate correlation effect by model. The means, medians, standard errors, and mean squared errors were similar for both models when the covariates within a component were orthogonal or with a small correlation.

Table 10: Parameter behavior of the NBH and ZINB coefficients by covariate correlation within a component, ρ, from the 2,000 simulates for $n = 100$.

	ρ	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	0	-1.548	0.255	2.075	1.319	-0.267	-2.045
	.3	-1.556	0.243	2.086	1.334	-0.266	-2.033
ZINB	0	-2.743	0.403	3.347	1.508	-0.265	-2.037
	.3	-2.602	0.246	3.237	1.511	-0.278	-2.030
				Median			
NBH	0	-1.518	0.253	2.057	1.515	-0.254	-1.975
	.3	-1.522	0.252	2.064	1.516	-0.259	-1.977
ZINB	0	-1.355	0.265	2.109	1.523	-0.258	-2.014
	.3	-1.366	0.264	2.076	1.529	-0.267	-2.010
				Standard Deviation			
NBH	0	0.408	0.495	0.498	0.875	0.466	0.752
	.3	0.443	0.503	0.512	0.803	0.444	0.667
ZINB	0	3.355	2.040	3.286	0.285	0.386	0.473
	.3	3.147	1.813	3.171	0.293	0.428	0.480
				Mean Squared Error			
NBH	0	0.335	0.490	0.501	2.698	0.464	1.204
	.3	0.396	0.507	0.532	2.449	0.419	0.960
ZINB	0	28.066	10.489	27.087	0.205	0.348	0.508
	.3	24.073	8.154	24.799	0.216	0.422	0.521

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.25, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = -.25$, and $\gamma_2 = -2$.

Confidence Interval Measures

Ninety-five percent Wald confidence interval coverage (Tables 96 through 111) and average width (Tables 112 through 127) are presented in Appendix B by sample size and dispersion parameter.

Table 11 shows the average 95% Wald confidence interval coverage and average width for the NBH and ZINB models by sample size. In general, as sample size increased the average Wald 95% confidence interval coverage approached the nominal 95% level for both models. Again, the NBH model generally had average Wald 95% confidence interval coverage closer to 95% than the ZINB model. For the NBH model, the coverage of the count component was more affected than the coverage of the zero component. At a sample size of 100, the coverage of the NBH's zero component was reasonably good and this component was largely unaffected by sample size. However, the coverage for the NBH's count component improved substantially from 100 to 500. For the ZINB model, both components' coverage improved as the sample size increased and approached 95%. However, β_0, β_2, and γ_0 remained below 95% when $n = 500$.

Focusing on average width in Table 11, the NBH's zero component greatly outperformed the ZINB's structural zero component being substantial smaller especially when the sample size was small (ranging from 1.73 to 116.2 times smaller). Performance was quite similar for the non-structural zero and count component, with the ZINB often having a smaller average interval width. While the ZINB's non-structural zero component had a smaller average 95% confidence interval width than the NBH's count component, the coverage remained better for the NBH model than for the ZINB model.

Tables 12 and 13 show the average 95% Wald confidence interval coverage and average width for the NBH and ZINB models by the dispersion parameter for $n = 100$. Tables 90 through 93 in Appendix B show coverage and width for $n = 250$ and $n = 500$ by θ. Again, coverage was generally better for the NBH model (or comparable) regardless of condition of θ. The average 95% Wald confidence interval coverage for the NBH model was largely unaffected by the dispersion parameter. The zero component outperformed the count component and was closer to 95%. For the ZINB model, the 95% Wald confidence interval coverage also appeared to be unrelated to dispersion.

Table 11: 95% confidence interval coverage and width for the ZINB and NBH coefficients by sample size. Standard errors are in parentheses.

	n	β_0	β_1	β_2
Average Coverage				
NBH	100	0.953 (0.005)	0.953 (0.005)	0.952 (0.005)
	250	0.950 (0.005)	0.951 (0.005)	0.950 (0.005)
	500	0.952 (0.005)	0.951 (0.005)	0.951 (0.005)
ZINB	100	0.884 (0.007)	0.971 (0.004)	0.950 (0.005)
	250	0.906 (0.007)	0.955 (0.005)	0.940 (0.005)
	500	0.915 (0.006)	0.951 (0.005)	0.938 (0.005)
Average Width				
NBH	100	1.601 (0.001)	1.901 (0.000)	1.914 (0.001)
	250	1.058 (0.000)	1.160 (0.000)	1.177 (0.000)
	500	0.722 (0.001)	0.819 (0.000)	0.834 (0.000)
ZINB	100	186.047 (9.328)	28.200 (0.955)	191.959 (9.439)
	250	37.370 (0.943)	2.765 (0.069)	36.576 (0.919)
	500	22.964 (0.620)	1.419 (0.007)	22.641 (0.611)
	n	γ_0	γ_1	γ_2
Average Coverage				
NBH	100	0.922 (0.006)	0.932 (0.006)	0.940 (0.005)
	250	0.940 (0.088)	0.942 (0.006)	0.945 (0.003)
	500	0.946 (0.005)	0.948 (0.005)	0.946 (0.005)
ZINB	100	0.905 (0.007)	0.923 (0.006)	0.940 (0.005)
	250	0.924 (0.006)	0.939 (0.005)	0.945 (0.005)
	500	0.926 (0.006)	0.945 (0.005)	0.946 (0.005)
Average Width				
NBH	100	9.076 (0.270)	1.644 (0.009)	2.743 (0.021)
	250	2.679 (0.088)	0.935 (0.006)	1.192 (0.003)
	500	1.198 (0.033)	0.690 (0.004)	0.818 (0.002)
ZINB	100	1.018 (0.011)	1.436 (0.012)	2.241 (0.046)
	250	0.767 (0.009)	0.903 (0.008)	1.082 (0.006)
	500	0.587 (0.007)	0.657 (0.005)	0.743 (0.004)

Looking at the average width in Tables 12 and 13, for both models, dispersion appeared to be unrelated to average confidence interval width. For the

NBH model, the average width of the count component was usually larger than for the zero component and the average width of the structural zero component was larger than the nonstructural zero component for the ZINB model.

Table 12: 95% confidence interval coverage and width for the ZINB and NBH structural zero/zero coefficients by dispersion (θ) for $n = 100$. Standard errors are in parentheses.

	θ	β_0	β_1	β_2
		Average Coverage		
NBH	1/4	0.953 (0.005)	0.955 (0.005)	0.953 (0.005)
	1/2	0.954 (0.005)	0.952 (0.005)	0.954 (0.005)
	1	0.954 (0.005)	0.951 (0.005)	0.952 (0.005)
	2	0.954 (0.005)	0.953 (0.005)	0.951 (0.005)
	5	0.949 (0.005)	0.951 (0.005)	0.952 (0.005)
	10	0.952 (0.005)	0.954 (0.005)	0.953 (0.005)
ZINB	1/4	0.847 (0.008)	0.976 (0.003)	0.934 (0.006)
	1/2	0.867 (0.008)	0.969 (0.004)	0.947 (0.005)
	1	0.947 (0.005)	0.966 (0.004)	0.967 (0.004)
	2	0.835 (0.008)	0.977 (0.003)	0.932 (0.006)
	5	0.862 (0.008)	0.968 (0.004)	0.946 (0.005)
	10	0.946 (0.005)	0.969 (0.004)	0.970 (0.004)
		Average Width		
NBH	1/4	1.608 (0.002)	1.918 (0.000)	1.937 (0.001)
	1/2	1.601 (0.002)	1.901 (0.000)	1.914 (0.001)
	1	1.594 (0.001)	1.884 (0.000)	1.891 (0.000)
	2	1.608 (0.002)	1.919 (0.000)	1.937 (0.001)
	5	1.602 (0.002)	1.903 (0.000)	1.916 (0.001)
	10	1.592 (0.001)	1.884 (0.000)	1.890 (0.000)
ZINB	1/4	208.620 (6.439)	32.369 (0.665)	220.198 (6.583)
	1/2	186.129 (5.997)	35.328 (1.038)	191.195 (6.398)
	1	32.771 (0.727)	6.916 (0.093)	33.015 (0.730)
	2	423.835 (20.319)	50.717 (1.737)	430.122 (20.375)
	5	124.419 (2.980)	34.683 (0.863)	136.748 (3.562)
	10	140.507 (5.925)	9.187 (0.126)	140.476 (5.923)

Table 13: 95% confidence interval coverage and width for the ZINB and NBH non-structural zero/count coefficients by dispersion (θ) for $n = 100$. Standard errors are in parentheses.

	θ	γ_0	γ_1	γ_2
Average Coverage				
NBH	1/4	0.921 (0.006)	0.926 (0.006)	0.929 (0.006)
	1/2	0.924 (0.006)	0.933 (0.006)	0.940 (0.005)
	1	0.932 (0.006)	0.931 (0.006)	0.947 (0.006)
	2	0.909 (0.006)	0.929 (0.006)	0.930 (0.006)
	5	0.917 (0.006)	0.936 (0.005)	0.945 (0.005)
	10	0.931 (0.006)	0.936 (0.005)	0.949 (0.005)
ZINB	1/4	0.894 (0.007)	0.918 (0.006)	0.930 (0.006)
	1/2	0.901 (0.007)	0.919 (0.006)	0.937 (0.005)
	1	0.927 (0.006)	0.932 (0.006)	0.946 (0.005)
	2	0.888 (0.007)	0.917 (0.006)	0.936 (0.005)
	5	0.895 (0.007)	0.922 (0.006)	0.940 (0.005)
	10	0.925 (0.006)	0.930 (0.006)	0.949 (0.005)
Average Width				
NBH	1/4	12.706 (0.279)	1.848 (0.007)	2.305 (0.005)
	1/2	10.873 (0.282)	1.551 (0.011)	2.671 (0.020)
	1	1.220 (0.021)	1.265 (0.006)	2.215 (0.014)
	2	15.514 (0.341)	2.075 (0.008)	2.832 (0.017)
	5	12.423 (0.330)	1.708 (0.008)	3.563 (0.035)
	10	1.717 (0.036)	1.415 (0.005)	2.871 (0.015)
ZINB	1/4	1.226 (0.010)	1.607 (0.010)	1.742 (0.009)
	1/2	1.082 (0.014)	1.486 (0.017)	1.914 (0.025)
	1	0.739 (0.005)	1.085 (0.006)	1.364 (0.003)
	2	1.227 (0.009)	1.681 (0.010)	2.889 (0.039)
	5	1.086 (0.014)	1.552 (0.015)	3.714 (0.097)
	10	0.745 (0.005)	1.205 (0.005)	1.822 (0.023)

Table 14 shows the average 95% Wald confidence interval coverage and average width for the NBH and ZINB models by covariate correlation for $n = 100$. Tables 94 and 95 in Appendix B show coverage and width for $n = 250$ and

$n = 500$ by θ. For both models, the levels of covariate correlation considered here did not appear to affect the average 95% confidence interval coverage. For the NBH model, the average width of the 95% confidence interval appeared to be unrelated to whether or not the covariates were correlated. However, the structural zero component of the ZINB model appeared to be affected by the covariate correlation. The average width when the covariate correlation was .3, was always larger than when the covariates were uncorrelated for the ZINB structural zero component. Coverage for the ZINB models, in contrast, appeared to be unrelated to covariate correlation.

Table 14: 95% confidence interval coverage and width for the ZINB and NBH structural zero/zero coefficients by correlation (ρ) for $n = 100$. Standard errors are in parentheses.

	ρ	β_0	β_1	β_2
Average Coverage				
NBH	0	0.952 (0.005)	0.953 (0.005)	0.953 (0.005)
	.3	0.954 (0.005)	0.953 (0.005)	0.951 (0.005)
ZINB	0	0.880 (0.007)	0.972 (0.004)	0.951 (0.005)
	.3	0.889 (0.007)	0.969 (0.004)	0.948 (0.005)
Average Width				
NBH	0	1.537 (0.000)	1.893 (0.000)	1.898 (0.000)
	.3	1.665 (0.000)	1.910 (0.000)	1.930 (0.001)
ZINB	0	82.591 (1.691)	24.828 (0.653)	84.406 (1.669)
	.3	289.502 (12.797)	31.572 (1.194)	299.512 (12.932)
	ρ	γ_0	γ_1	γ_2
Average Coverage				
NBH	0	0.920 (0.006)	0.931 (0.006)	0.939 (0.005)
	.3	0.925 (0.006)	0.933 (0.006)	0.941 (0.005)
ZINB	0	0.908 (0.006)	0.925 (0.006)	0.940 (0.005)
	.3	0.901 (0.007)	0.921 (0.006)	0.939 (0.005)
Average Width				
NBH	0	9.602 (0.279)	1.682 (0.010)	2.949 (0.022)
	.3	8.549 (0.266)	1.606 (0.009)	2.536 (0.020)
ZINB	0	1.008 (0.011)	1.367 (0.011)	2.290 (0.055)
	.3	1.027 (0.011)	1.506 (0.012)	2.192 (0.036)

Correct Identification of Structural Zeros

Results for the correct identification of structural zeros are presented in Figures 11 through 18 in Appendix C. These figures show the mean proportion of correctly identified structural zeros by sample size and dispersion for each condition considered in the simulation study.

The proportion of correctly identified structural zeros by dispersion, sample size, and covariate correlation, along with their standard errors, are presented in Table 15 by sample size. The mean proportion of correctly identified structural zeros ranged from 0.62 to 0.74 with a grand mean of 0.71 (ignoring sample size). In Table 15, as dispersion decreased, the mean proportion of correctly identified structural zeros increased and the standard error of this estimate decreased. When θ was equal to 1, the proportion of correctly identified structural zeros appeared to start to level off at all sample sizes considered. Increasing the sample size appeared to have little effect on the proportion of correctly identified structural zeros. In Table 15, it can also be seen that covariate correlation appeared to have no effect on the proportion of correctly identified structural zeros as well.

Table 15: Proportion of correctly identified structural zeros for the ZINB model. Standard errors are in parentheses.

		Sample Size		
		100	250	500
Grand		0.709 (0.010)	0.712 (0.010)	0.719 (0.010)
θ	1/4	0.634 (0.011)	0.672 (0.010)	0.695 (0.010)
	1/2	0.699 (0.010)	0.710 (0.010)	0.718 (0.010)
	1	0.723 (0.010)	0.719 (0.010)	0.723 (0.010)
	2	0.731 (0.010)	0.722 (0.010)	0.725 (0.010)
	5	0.734 (0.010)	0.723 (0.010)	0.726 (0.010)
	10	0.734 (0.010)	0.725 (0.010)	0.726 (0.010)
ρ	0	0.713 (0.010)	0.711 (0.010)	0.711 (0.010)
	.30	0.705 (0.010)	0.713 (0.010)	0.727 (0.010)

Figure 4 shows the average proportion of correctly identified structural zeros by dispersion and sample size collapsed over the other conditions. The hollow points correspond to the estimate and the black bars to plus or minus two times the standard error of the mean proportion of structural zeros. Two salient observations can be made from this figure. First, with the exception of $\theta = 1/4$ and $n = 100$ and $\theta = 1/4$ and $n = 250$, the points all have standard errors bars that overlap implying that while the point estimates differed there was no evidence to suggest that there was a true difference between the estimates. Second, when $\theta = 1/4$ and $n = 100$, the mean proportion of structural zeros was lowest and different from the other conditions. This suggests a possible interaction between dispersion and sample size and that the effect of high dispersion on the mean proportion of correctly identified structural zeros may be lessened when the sample size was 250 or greater and possibly exacerbated when the sample size was 100 or less. Third, while the average proportion of correctly identified structural zero for ZINB was greater than chance (i.e. .50), it was still relatively low overall (approximately 70%).

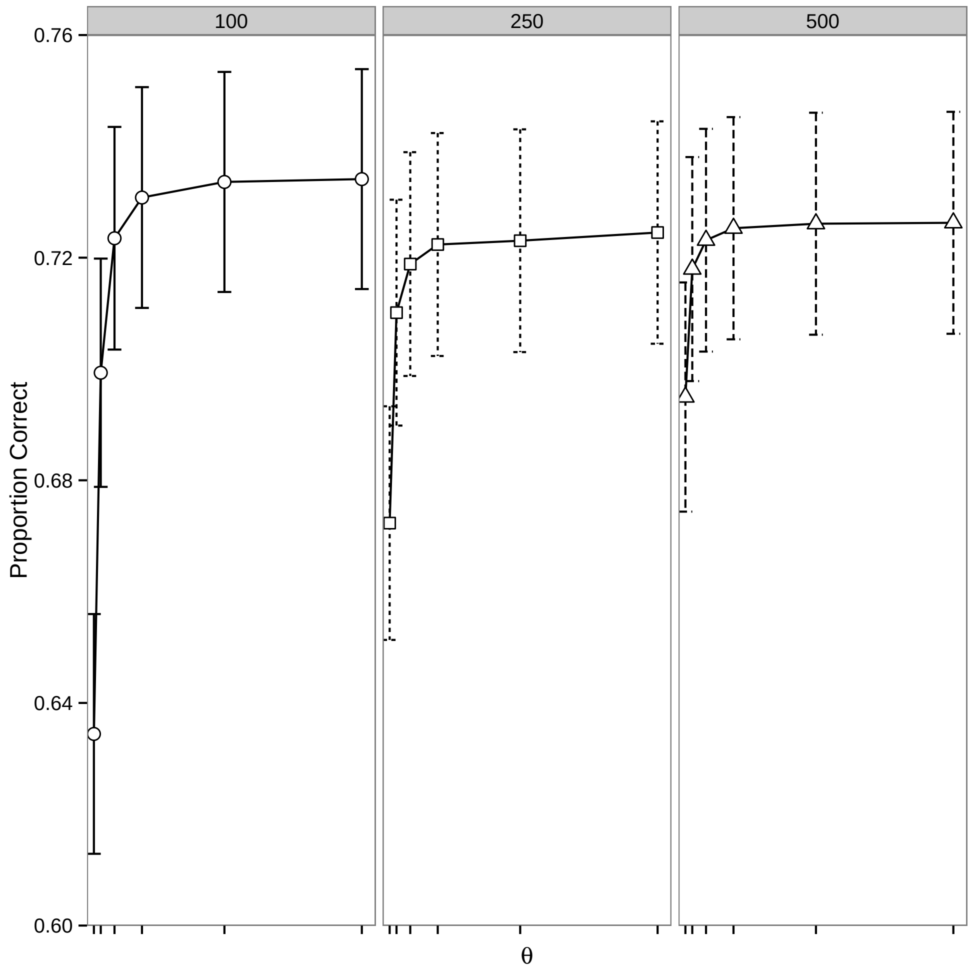

Figure 4: The average proportion of correctly identified structural zeros for the ZINB model by dispersion, θ, and sample size. The tick marks (from left to right) correspond to $\theta = 1/4, 1/2, 1, 2, 5$, and 10 and the solid circles, dotted squares, and dashed triangles to sample sizes of 100, 250, and 500, respectively. The black bars correspond to 2 times the standard error of the mean proportion of the structural zeros.

Type I Error

Type I error rate for all the conditions considered are presented in Tables 136 through 139 in Appendix D. In Appendix D, type I error rates (and associated 95% Wald confidence intervals) are presented by model type and when β_1 or γ_1 was equal to zero.

The overall type I error rate when $\gamma_1 = 0$ for the NBH model was .070 (SE = 0.006), 0.059 (SE = 0.005), and 0.053 (SE = 0.005) and for the ZINB model was .077 (SE = 0.006), .059 (SE = 0.005), and .059 (SE = 0.005) for $n = 100$, $n = 250$, and $n = 500$, respectively. When the sample size was small, both models had type I error rates whose confidence intervals would not include $\alpha = .05$ but were acceptable based on the criteria defined in Chapter 3. The overall type I error rate when $\beta_1 = 0$ for the NBH model was 0.046 (SE = 0.005), 0.048 (SE = 0.005), and 0.048 (SE = 0.005) and for the ZINB model was 0.028 (SE = 0.004), 0.045 (SE = 0.005), 0.047 (SE = 0.005) for $n = 100$, $n = 250$, and $n = 500$, respectively. For $\beta_1 = 0$, the ZINB model when $n = 100$ had a type I error rate whose confidence interval would not include $\alpha = .05$, however, its type I error rate would again be considered acceptable with the NBH type I error rate being considered better.

Figure 5 shows the overall type I error rate by model and sample size. In Figure 5, the circle and solid black bars correspond to the NBH model and the triangle and dashed black bars correspond to the ZINB model. The length of the black bars (standard error bars) corresponds to 2 times the standard error of the means of the type I error for each model. Figure 5 shows that while the overall type I error rate for the NBH model was closer to $\alpha = .05$ than the ZINB model, there was no real difference in the models or their components except for when n = 100 as described above.

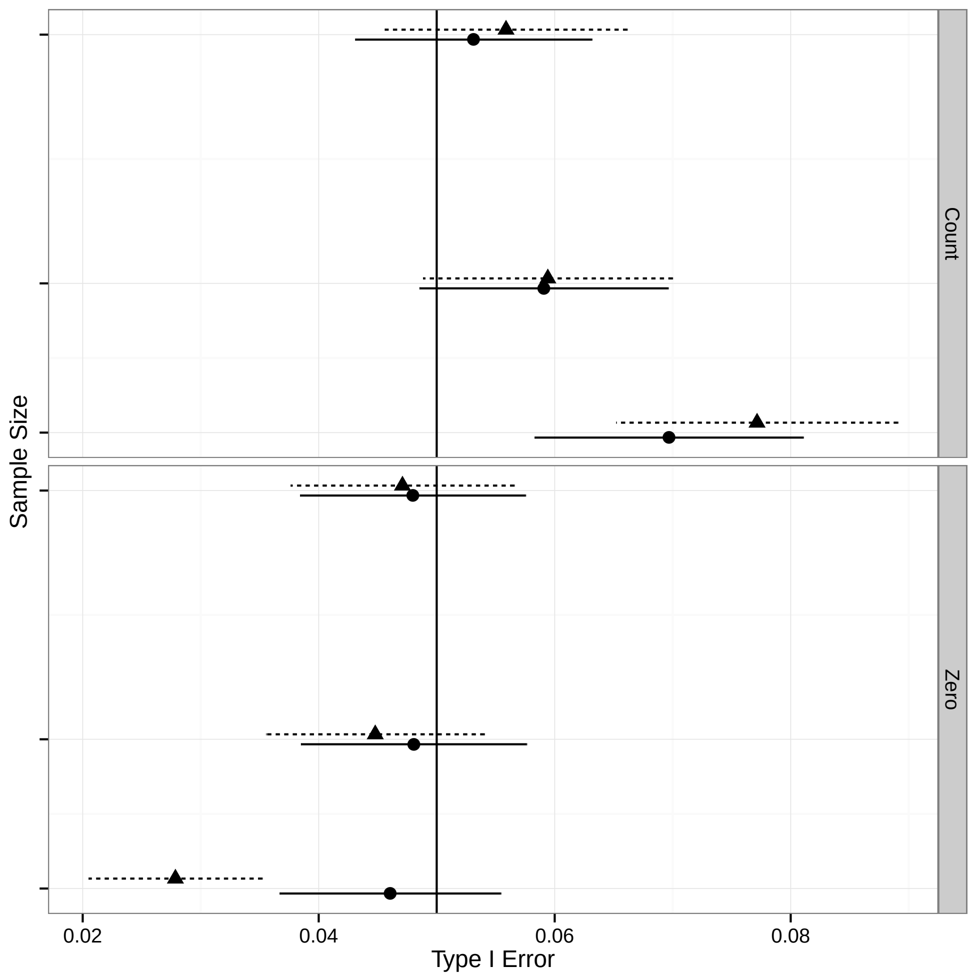

Figure 5: The mean type I error rate for $\alpha = .05$ for the NBH (circle, solid bands) and ZINB (triangle, dashed bands) model, respectively. The black bands around the point estimates corresponds to the mean +/- 2 standard errors. Count refers to γ_1 and zero to β_1. The horizontal bar corresponds to $\alpha = .05$

Figure 6 shows the type I error rate by dispersion, component type, and sample size. In general, the mean type I error rate for the NBH model was closer to the nominal $\alpha = .05$ level than the ZINB. For $\beta_1 = 0$ and $n = 100$, $\alpha = .05$ was always in the interval for the NBH, whereas for the ZINB, it was only in the interval when $\theta = 10$. For $\gamma_1 = 0$ and $n = 100$, $\alpha = .05$ was never in the interval for the ZINB and was only in the interval for the NBH when $\theta = 10$. For the other sample sizes and dispersion levels, $\alpha = .05$ was generally in the interval. Again, all intervals were within the acceptable range defined in Chapter 3.

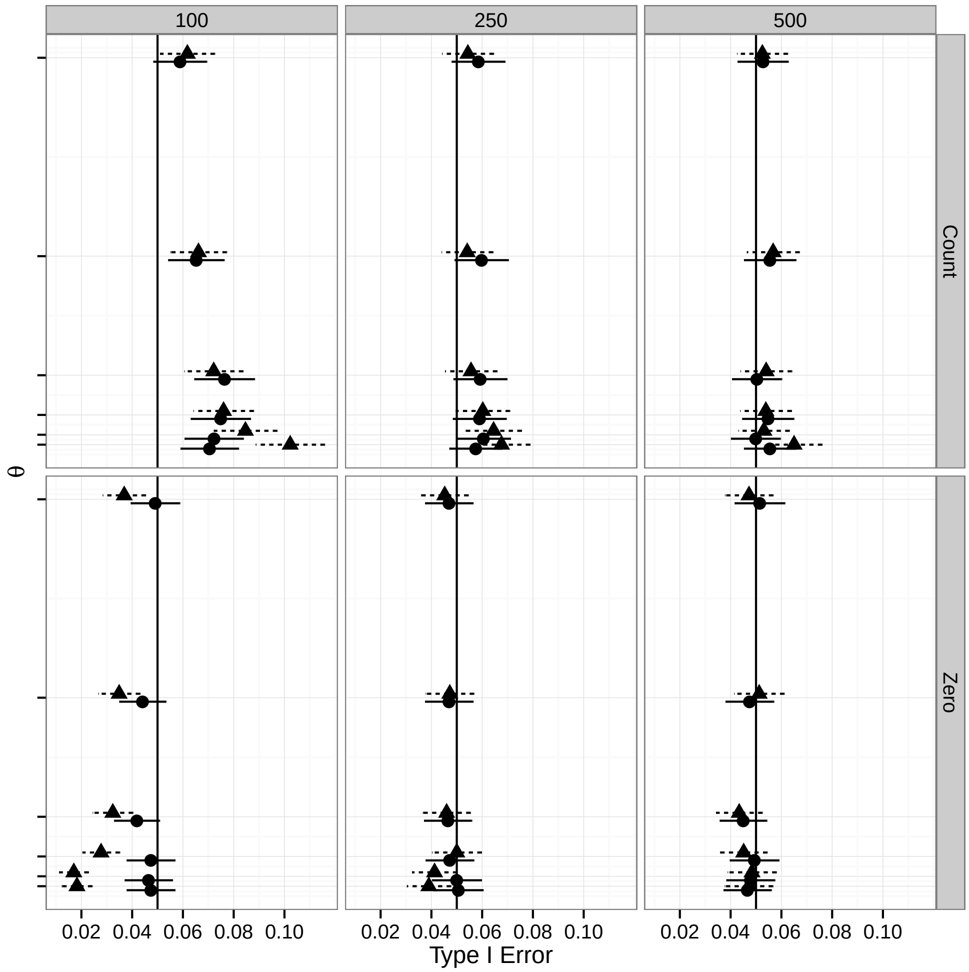

Figure 6: The mean type I error rate for $\alpha = .05$ for the NBH (circle, solid bars) and ZINB (triangle, dashed bars) model by dispersion (θ). The tick marks (from bottom to top) correspond to $\theta = 1/4$, 1/2, 1, 2, 5, and 10. The bands around the point estimates corresponds to the mean +/- 2 standard errors. Count refers to γ_1 and zero to β_1. The vertical bar corresponds to $\alpha = .05$

Figure 7 shows the type I error rate by covariate correlation. As before, the NBH model had an estimated value closer to $\alpha = .05$ than the ZINB but the standard error bars included $\alpha = .05$ for both models at both levels of correlation regardless of component examined except for $n = 100$ as noted above. Additionally, there appeared to be no relationship between covariate correlation and length of the error bars for either the NBH or the ZINB model.

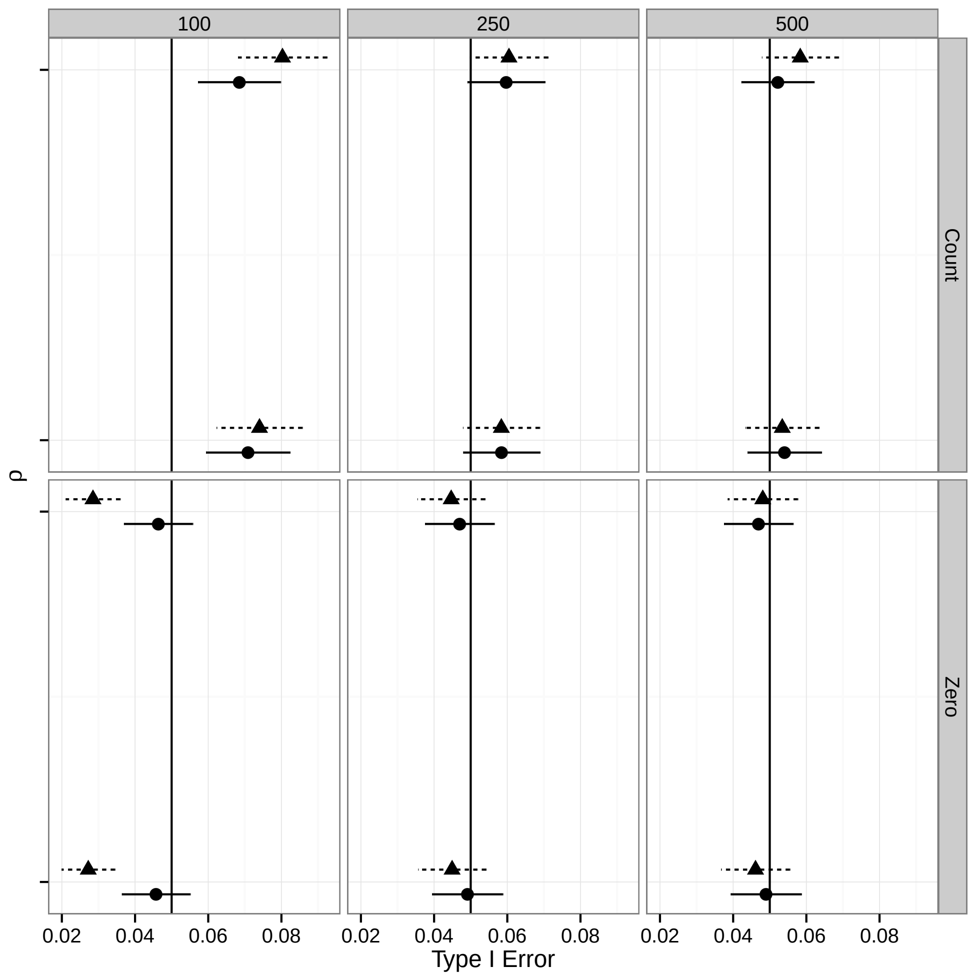

Figure 7: The mean type I error rate for $\alpha = .05$ for the NBH (circle, solid bars) and ZINB (triangle, dashed bars) model by by covariate correlation within a component. The tick marks (from bottom to top) correspond to $\rho = 0$ and .3. The bands around the point estimates corresponds to the mean +/- 2 standard errors. Count refers to γ_1 and zero to β_1. The vertical bar corresponds to $\alpha = .05$

Bias

Results examining relative bias for E(Y|X) and Pr(Y = 0) are presented in Tables 144 through 151 in Appendix E. Each table presents relative bias by sample size and dispersion for a specific level of β_1, γ_1, and covariate correlation.

The mean relative bias ignoring all the conditions for E(Y|X) for the NBH model were 0.004 (SE = 0.000), 0.000 (SE = 0.000), and 0.000 (SE = 0.000) and for the ZINB model were 0.179 (SE = 0.027), 0.001 (SE = 0.000), 0.001 (SE = 0.000) for $n = 100$, $n = 250$, and $n = 500$, respectively. The mean relative bias ignoring all the conditions for Pr(Y = 0) was essentially zero for the NBH and ZINB models regardless of sample size.

Table 16: Relative bias for the E(Y|X) for the ZINB and NBH models. Standard errors are in parentheses.

		Sample Size		
		100	250	500
		NBH		
Grand		0.004 (0.000)	0.000 (0.000)	0.000 (0.000)
θ	1/4	0.009 (0.000)	0.002 (0.000)	0.001 (0.000)
	1/2	0.006 (0.000)	0.000 (0.000)	0.000 (0.000)
	1	0.004 (0.000)	0.000 (0.000)	0.000 (0.000)
	2	0.003 (0.000)	0.001 (0.000)	-0.001 (0.000)
	5	0.001 (0.000)	-0.001 (0.000)	0.000 (0.000)
	10	0.001 (0.000)	0.001 (0.000)	-0.001 (0.000)
ρ	0	0.005 (0.000)	0.000 (0.000)	0.000 (0.000)
	.3	0.003 (0.000)	0.000 (0.000)	0.000 (0.000)
		ZINB		
Grand		0.179 (0.027)	0.001 (0.000)	0.001 (0.000)
θ	1/4	1.057 (0.066)	0.002 (0.000)	0.003 (0.000)
	1/2	0.012 (0.000)	0.003 (0.000)	0.002 (0.000)
	1	0.004 (0.000)	0.000 (0.000)	0.001 (0.000)
	2	0.000 (0.000)	-0.001 (0.000)	0.000 (0.000)
	5	0.000 (0.000)	0.003 (0.000)	0.001 (0.000)
	10	0.002 (0.000)	-0.001 (0.000)	-0.001 (0.000)
ρ	0	0.006 (0.000)	0.000 (0.000)	0.001 (0.000)
	.3	0.353 (0.000)	0.002 (0.000)	0.001 (0.000)

Tables 16 and 17 shows the relative bias overall, by dispersion, and by covariate correlation within a component for each sample size. Focusing on sample size, in general, the relative bias for the E(Y|X) and Pr(Y = 0) decreased for both the NBH and ZINB models as the sample size increased with the largest relative bias for the ZINB model occurring for the E(Y|X) when the sample size was equal to 100 (approximately 18%). The relative bias for E(Y|X) for the ZINB model decreased substantially when the sample size increased to

250 and when the sample size was 250 or greater both models had essentially zero bias for the E(Y|X) and Pr(Y = 0).

Table 17: Relative bias for the Pr(Y = 0) for the ZINB and NBH models. Standard errors are in parentheses.

		Sample Size		
		100	250	500
			NBH	
Grand		-0.001 (0.000)	0.000 (0.000)	0.000 (0.000)
θ	1/4	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
	1/2	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
	1	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
	2	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
	5	-0.001 (0.000)	0.000 (0.000)	0.000 (0.000)
	10	-0.001 (0.000)	0.000 (0.000)	0.000 (0.000)
ρ	0	-0.001 (0.000)	0.000 (0.000)	0.000 (0.000)
	.3	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
			ZINB	
Grand		0.002 (0.000)	0.000 (0.000)	0.000 (0.000)
θ	1/4	0.002 (0.001)	0.000 (0.000)	0.000 (0.000)
	1/2	0.002 (0.000)	0.001 (0.000)	0.000 (0.000)
	1	0.002 (0.000)	0.001 (0.000)	-0.001 (0.000)
	2	0.002 (0.000)	0.000 (0.000)	0.000 (0.000)
	5	0.000 (0.000)	0.000 (0.000)	0.000 (0.000)
	10	0.001 (0.000)	0.000 (0.000)	0.000 (0.000)
ρ	0	0.001 (0.000)	0.001 (0.000)	0.000 (0.000)
	.3	0.002 (0.000)	0.000 (0.000)	0.000 (0.000)

Focusing on dispersion, in general, as θ increased the relative bias for the E(Y|X) decreased while the relative bias for Pr(Y = 0) appeared to be largely unaffected as it was essentially zero. The largest relative bias was again observed for the ZINB model for the E(Y|X) with a mean relative bias of 105.7% when $\theta = 1/4$ and $n = 100$. Once θ was 1/2 or larger, the relative bias for E(Y|X) was essentially zero for the ZINB and the relative bias for Pr(Y = 0) was essentially zero regardless of the model, dispersion condition, or sample size considered.

Finally, examining covariate correlation within a component, the relative bias for the NBH model was essentially identical regardless of correlation for both E(Y|X) and Pr(Y = 0) and was equal to zero. However, for the ZINB, the relative bias for E(Y|X) when the covariates were orthogonal was 0 and increased to about 35% when they were correlated for $n = 100$. For the Pr(Y = 0), the relative bias for the ZINB model was unaffected by covariate correlation and was essentially zero.

Empirically-Derived Simulation Condition

Measures of parameter behavior for the empirically-derived simulation condition are presented in Table 18. Comparing the mean and median of the estimated parameters to their true parameters in Equations 25 and 26, both the ZINB and NBH models were easily able to recover their parameters with a sample size of 13,606 with relatively high precision (i.e. the standard errors of the simulates were quite small) and small mean squared error. Based on parameter behavior, both models performed quite well.

Table 19 shows the Wald 95% confidence interval coverage and average width for the NBH and ZINB models. Both models had 95% confidence intervals that had coverage close to 95%, with both models having coverage within 1% of 95% for all parameters. For the structural zero/zero component, the average 95% confidence interval width was smaller for the NBH than the ZINB though not substantially. This difference was typically a couple hundredths. For the non-structural zero/count component, the average 95% confidence interval widths were nearly identical for the two models.

Table 18: Parameter behavior of the NBH and ZINB coefficients for the empirically-derived condition.

	Mean		Median		SD		MSE	
	NBH	ZINB	NBH	ZINB	NBH	ZINB	NBH	ZINB
β_0	-1.674	1.405	-1.673	1.408	0.093	0.100	0.017	0.020
β_1	1.109	-1.081	1.108	-1.083	0.073	0.080	0.011	0.013
β_2	0.925	-0.875	0.925	-0.874	0.106	0.120	0.023	0.029
β_3	-0.717	0.770	-0.713	0.767	0.116	0.125	0.027	0.032
β_4	0.164	-0.131	0.166	-0.134	0.098	0.103	0.019	0.021
β_5	0.391	-0.334	0.390	-0.334	0.050	0.056	0.005	0.006
β_6	-0.081	0.062	-0.083	0.062	0.052	0.057	0.005	0.007
β_7	-0.204	0.199	-0.203	0.200	0.060	0.066	0.007	0.009
β_8	-0.549	0.466	-0.550	0.467	0.120	0.139	0.029	0.039
β_9	-1.437	1.390	-1.435	1.389	0.090	0.097	0.016	0.019
β_{10}	0.602	-0.629	0.603	-0.629	0.043	0.051	0.004	0.005
γ_0	1.249	1.246	1.250	1.247	0.081	0.080	0.013	0.013
γ_1	0.382	0.383	0.380	0.382	0.069	0.069	0.009	0.009
γ_2	0.379	0.380	0.379	0.379	0.091	0.093	0.017	0.017
γ_3	0.018	0.017	0.020	0.015	0.116	0.114	0.027	0.026
γ_4	0.073	0.075	0.073	0.074	0.091	0.091	0.017	0.017
γ_5	0.346	0.345	0.346	0.345	0.038	0.037	0.003	0.003
γ_6	-0.085	-0.085	-0.085	-0.085	0.042	0.043	0.004	0.004
γ_7	-0.120	-0.119	-0.120	-0.119	0.043	0.044	0.004	0.004
γ_8	-0.477	-0.471	-0.474	-0.471	0.104	0.102	0.022	0.021
γ_9	-0.543	-0.543	-0.541	-0.544	0.079	0.084	0.013	0.014
γ_{10}	0.128	0.130	0.129	0.130	0.035	0.036	0.002	0.003

Note. The true parameters for the NBH are β_0 = -1.67, β_1 = 1.10, β_2 = 0.92, β_3 = -0.72, β_4 = 0.16, β_5 = 0.39, β_6 = -0.08, β_7 = -0.21, β_8 = -0.55, β_9 = -1.44, β_{10} = 0.60, γ_0 = 1.25, γ_1 = 0.38, γ_2 = 0.38, γ_3 = 0.02, γ_4 = 0.07, γ_5 = 0.35, γ_6 = -0.08, γ_7 = -0.12, γ_8 = -0.47, γ_9 = -0.54, and γ_{10} = 0.13. The true parameters for the ZINB are β_0 = 1.41, β_1 = -1.08, β_2 = -0.88, β_3 = 0.76, β_4 = -0.13, β_5 = -0.33, β_6 = 0.06, β_7 = 0.20, β_8 = 0.46, β_9 = 1.39, β_{10} = -0.63, γ_0 = 1.25, γ_1 = 0.38, γ_2 = 0.38, γ_3 = 0.02, γ_4 = 0.08, γ_5 = 0.35, γ_6 = -0.08, γ_7 = -0.12, γ_8 = -0.47, γ_9 = -0.55, γ_{10} = 0.13. SD and MSE refer to the standard error and mean squared error, respectively.

Table 19: 95% confidence interval coverage and average width for the NBH and ZINB coefficients for the empirically-derived condition. Standard errors are in parentheses.

	Average Coverage		Average Width	
	NBH	ZINB	NBH	ZINB
β_0	0.944 (0.005)	0.951(0.005)	0.356 (0.000)	0.397 (0.000)
β_1	0.948 (0.005)	0.952 (0.005)	0.286 (0.000)	0.313 (0.000)
β_2	0.954 (0.005)	0.946 (0.005)	0.419 (0.000)	0.462 (0.000)
β_3	0.958 (0.005)	0.950 (0.005)	0.453 (0.000)	0.488 (0.000)
β_4	0.944 (0.005)	0.955 (0.005)	0.375 (0.000)	0.411 (0.000)
β_5	0.951 (0.005)	0.946 (0.005)	0.195 (0.000)	0.218 (0.000)
β_6	0.941 (0.005)	0.947 (0.005)	0.200 (0.000)	0.224 (0.000)
β_7	0.946 (0.005)	0.954 (0.005)	0.232 (0.000)	0.262 (0.000)
β_8	0.946 (0.005)	0.944 (0.005)	0.462 (0.000)	0.529 (0.000)
β_9	0.947 (0.005)	0.951 (0.005)	0.349 (0.000)	0.384 (0.000)
β_{10}	0.948 (0.005)	0.941 (0.005)	0.170 (0.000)	0.190 (0.000)
γ_0	0.949 (0.005)	0.955 (0.005)	0.319 (0.000)	0.319 (0.000)
γ_1	0.949 (0.005)	0.948 (0.005)	0.270 (0.000)	0.270 (0.000)
γ_2	0.951 (0.005)	0.951 (0.005)	0.363 (0.000)	0.363 (0.000)
γ_3	0.945 (0.005)	0.951 (0.005)	0.446 (0.000)	0.447 (0.000)
γ_4	0.949 (0.005)	0.946 (0.005)	0.353 (0.000)	0.354 (0.000)
γ_5	0.941 (0.005)	0.958 (0.004)	0.147 (0.000)	0.148 (0.000)
γ_6	0.950 (0.005)	0.943 (0.005)	0.164 (0.000)	0.164 (0.000)
γ_7	0.958 (0.005)	0.949 (0.005)	0.173 (0.000)	0.173 (0.000)
γ_8	0.945 (0.005)	0.954 (0.005)	0.399 (0.000)	0.399 (0.000)
γ_9	0.954 (0.005)	0.941 (0.005)	0.317 (0.000)	0.316 (0.000)
γ_{10}	0.956 (0.005)	0.954 (0.005)	0.141 (0.000)	0.141 (0.000)

The average proportion of structural zeros correctly identified for the ZINB model was .72 with a standard error of .004. This was similar to that reported for the other conditions for the first simulation study.

The relative bias for the E(Y|X) was .0007 and .0004 for the NBH and ZINB models, respectively. The relative bias for the Pr(Y = 0) was .0002 for both the NBH and ZINB models. This means that the relative bias for the E(Y|X) and the Pr(Y = 0) was essentially zero for both the NBH and ZINB models for the empirically-derived condition.

Second Simulation Study - Model Misspecification

Parameter Behavior

Parameter behavior for the second simulation study where the ZINB data were fit to the NBH model and the NBH data were fit to the ZINB model are presented in Appendix A in Tables 86 through 89. Table 20 shows the overall behavior of the NBH and ZINB models at recovering the parameters, collapsed over all the conditions, when the models were misspecified. This table includes the mean of the means of the collapsed conditions, the mean of the medians of the collapsed conditions, the mean of the standard errors of the collapsed conditions, and the mean of the mean squared errors for all estimated parameters. As with the first simulation study, β_1 and γ_1 values close to 0.25 and -0.25 indicate that these parameters were successfully recovered.

Focusing on the structural zero/zero components, i.e. β_0, β_1, and β_2, neither the mean nor the median were close to the true parameter for either model. The NBH model was consistently closer to the true value than the ZINB for this component based on the mean and median. For β_1, the estimated parameter for the NBH was closer to the true value than other structural zero/zero parameters, however, this parameter was in the opposite direction and the distance was 0.35.

For the non-structural zero/count components, i.e. γ_0, γ_1, and γ_2, both models performed substantially better than they did for the structural zero/zero components. The NBH model outperformed the ZINB at recovering these parameters with the exception of γ_0, where based on the mean the ZINB outperformed the NBH and based on the median the NBH outperformed the ZINB. γ_2 was substantially underestimated by the ZINB model with NBH data. The mean of the standard errors and mean squared error for the ZINB model were smaller for the non-structural zero/count component than the NBH model and the opposite was observed for the structural zero/zero component.

Table 20: Parameter behavior of the NBH model with the ZINB data (ZNBH) and the ZINB model with the NBH data (NZINB) from the 2,000 simulates for n = 500.

	β_0	β_1	β_2	γ_0	γ_1	γ_2
			Mean			
ZNBH	-0.123	-0.167	-1.253	1.455	-0.253	-2.018
NZINB	1.118	-0.316	-3.913	1.527	-0.172	-1.341
			Median			
ZNBH	-0.122	-0.166	-1.249	1.509	-0.251	-2.002
NZINB	1.116	-0.318	-3.551	1.531	-0.172	-1.343
			Standard Deviation			
ZNBH	0.143	0.198	0.206	0.323	0.200	0.292
NZINB	0.234	0.319	1.654	0.133	0.137	0.134
			Mean Squared Error			
ZNBH	2.089	0.417	10.684	0.551	0.094	0.176
NZINB	6.981	0.858	48.446	0.052	0.052	0.478

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.25, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = -0.25$, and $\gamma_2 = -2$.

Tables 21 and 22 show the behavior of the ZINB and NBH models at recovering the parameters when the models were misspecified by θ. For the zero component of the NBH model, as θ increased, parameter behavior worsened. For the structural zero component of the ZINB model, as θ increased, parameter behavior worsened for β_0 and improved for β_1 and β_2.

In contrast, for the non-structural zero/count component, as θ increased both models approached their true values. For γ_0 and $\theta = 1/4$, the performance of the NBH count component was quite poor. For the other NBH count parameters at $\theta = 1/4$, the estimated parameters were close to their true values.

For the ZINB model's non-structural zero component, as θ increased performance improved, however, all the parameters were underestimated especially γ_2 based on the mean. The performance was worst when $\theta = 1/4$ for the ZINB as well.

Table 21: Parameter behavior, mean and median, of the NBH model with the ZINB data (ZNBH) and the ZINB model with the NBH data (NZINB) from the 2,000 simulates by θ for $n = 500$.

		β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
ZNBH	1/4	-0.793	-0.136	-1.074	1.281	-0.261	-2.002
	1/2	-0.403	-0.149	-1.160	1.472	-0.252	-2.012
	1	-0.104	-0.166	-1.247	1.489	-0.253	-2.013
	2	0.087	-0.176	-1.308	1.495	-0.252	-2.021
	5	0.217	-0.185	-1.356	1.496	-0.249	-2.029
	10	0.260	-0.188	-1.375	1.498	-0.252	-2.030
NZINB	1/4	0.947	-0.339	-7.613	1.754	-0.157	-1.345
	1/2	1.015	-0.344	-5.290	1.545	-0.155	-1.309
	1	1.106	-0.335	-3.285	1.467	-0.161	-1.315
	2	1.179	-0.309	-2.585	1.457	-0.174	-1.337
	5	1.225	-0.288	-2.374	1.467	-0.187	-1.365
	10	1.239	-0.279	-2.330	1.472	-0.196	-1.378
				Median			
ZNBH	1/4	-0.791	-0.134	-1.066	1.548	-0.259	-1.993
	1/2	-0.400	-0.151	-1.154	1.507	-0.250	-1.999
	1	-0.103	-0.163	-1.244	1.499	-0.251	-1.997
	2	0.086	-0.178	-1.305	1.499	-0.249	-2.003
	5	0.216	-0.184	-1.350	1.498	-0.248	-2.011
	10	0.258	-0.187	-1.374	1.500	-0.251	-2.010
NZINB	1/4	0.944	-0.346	-7.734	1.757	-0.157	-1.345
	1/2	1.017	-0.348	-3.539	1.545	-0.154	-1.310
	1	1.106	-0.334	-2.822	1.477	-0.162	-1.319
	2	1.174	-0.308	-2.523	1.464	-0.173	-1.339
	5	1.221	-0.289	-2.364	1.470	-0.188	-1.366
	10	1.233	-0.281	-2.323	1.473	-0.196	-1.379

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.25, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = -0.25$, and $\gamma_2 = -2$.

Table 22: Parameter behavior, standard deviation and mean squared error, of the NBH model with the ZINB data (ZNBH) and the ZINB model with the NBH data (NZINB) from the 2,000 simulates by θ for $n = 500$.

		β_0	β_1	β_2	γ_0	γ_1	γ_2
Standard Deviation							
ZNBH	1/4	0.161	0.229	0.240	1.210	0.346	0.382
	1/2	0.148	0.205	0.215	0.314	0.259	0.318
	1	0.141	0.195	0.203	0.158	0.202	0.278
	2	0.138	0.189	0.195	0.109	0.158	0.264
	5	0.136	0.187	0.192	0.078	0.124	0.256
	10	0.136	0.184	0.191	0.067	0.109	0.252
NZINB	1/4	0.270	0.399	3.725	0.183	0.184	0.186
	1/2	0.265	0.378	3.348	0.176	0.163	0.165
	1	0.241	0.334	1.741	0.154	0.144	0.142
	2	0.217	0.288	0.578	0.119	0.126	0.120
	5	0.207	0.261	0.273	0.088	0.109	0.100
	10	0.203	0.255	0.259	0.077	0.099	0.092
Mean Squared Error							
ZNBH	1/4	0.559	0.395	9.565	3.008	0.240	0.293
	1/2	1.256	0.394	10.083	0.202	0.134	0.204
	1	2.002	0.411	10.632	0.050	0.082	0.155
	2	2.570	0.427	11.020	0.024	0.050	0.141
	5	3.002	0.437	11.338	0.012	0.031	0.134
	10	3.149	0.439	11.466	0.009	0.024	0.130
NZINB	1/4	6.138	1.004	120.513	0.134	0.084	0.500
	1/2	6.471	0.985	75.855	0.066	0.070	0.535
	1	6.912	0.918	34.347	0.050	0.057	0.515
	2	7.270	0.806	21.789	0.031	0.043	0.475
	5	7.511	0.728	19.287	0.017	0.031	0.429
	10	7.585	0.708	18.884	0.013	0.026	0.412

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.25, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = -0.25$, and $\gamma_2 = -2$.

Table 23 shows the behavior of the ZINB and NBH models at recovering the parameters when the models were misspecified by covariate correlation. For the structural zero/zero component, as the correlation moved from 0 to .3, there was a slight decrease in performance for both models and for all βs. Similarly, for the non-structural zero/count component, as the correlation changed from 0 to .3, there was a slight decrease in performance, except for γ_1 and γ_2 for the ZINB model, with the estimated parameters a slightly greater distance from their true values. However, this difference was slight and in the case of the structural zero/zero component, neither condition of covariate correlation resulted in estimated parameters that were close to the truth. For the non-structural zero/count component, the estimated parameters for both conditions were reasonable for the NBH and for the ZINB except for γ_2.

Table 23: Parameter behavior of the NBH model with the ZINB data (ZNBH) and the ZINB model with the NBH data (NZINB) from the 2,000 simulates by ρ for $n = 500$.

		β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
ZNBH	0	-0.194	-0.071	-1.252	1.451	-0.251	-2.015
	.3	-0.051	-0.263	-1.254	1.460	-0.255	-2.021
NZINB	0	1.094	-0.273	-3.947	1.520	-0.168	-1.345
	.3	1.143	-0.358	-3.879	1.534	-0.175	-1.337
Median							
ZNBH	0	-0.194	-0.071	-1.248	1.508	-0.250	-2.002
	.3	-0.050	-0.262	-1.250	1.510	-0.253	-2.003
NZINB	0	1.092	-0.273	-3.592	1.524	-0.169	-1.347
	.3	1.139	-0.362	-3.509	1.538	-0.174	-1.339
Standard Deviation							
ZNBH	0	0.151	0.196	0.202	0.344	0.201	0.283
	.3	0.136	0.201	0.210	0.301	0.198	0.300
NZINB	0	0.249	0.320	1.665	0.139	0.133	0.132
	.3	0.218	0.318	1.643	0.126	0.142	0.136
Mean Squared Error							
ZNBH	0	1.887	0.336	10.673	0.618	0.095	0.165
	.3	2.292	0.498	10.695	0.484	0.092	0.187
NZINB	0	6.867	0.812	48.705	0.055	0.050	0.471
	.3	7.095	0.904	48.187	0.049	0.054	0.485

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.25, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = -0.25$, and $\gamma_2 = -2$.

Confidence Interval Measures

Average 95% confidence interval coverage and average width when the NBH data were fit to the ZINB model and the ZINB data were fit to the NBH model is presented in Appendix B in Tables 128 through 135.

Table 24 shows the average 95% confidence interval and average width for the ZINB and NBH models when the models were misspecified and collapsed over all conditions. For the structural zero/zero component, the average 95% confidence interval coverage was abysmal. For both models, the average 95% confidence interval coverage for β_0 was essentially zero. Performance was moderately better for β_1, where the proportion of 95% confidence intervals containing the true parameter was approximately 50%. Coverage for β_2 for the NBH model was zero and 17% for the ZINB model. These results corroborate the relative poor performance of both of these models described above in the parameter behavior section.

For the non-structural zero/count component, the performance for the NBH model was greatly improved and moderately improved for the ZINB model. The observed coverage for the NBH models was very close to the nominal 95% level for all parameters. For the ZINB model, the 95% confidence interval coverage was low (approximately 85% for γ_0 and γ_1) and for γ_2 coverage was approximately 1%. For both models, the average 95% confidence interval coverage was closest to 95% for γ_1.

The average width for the 95% intervals were smaller for the NBH model than for the ZINB for the structural zero/zero component and this was reversed for the non-structural zero/count component. However, the average width for the structural zero/zero component was a relatively useless measure given the extremely poor performance for both of these models when the model was misspecified. Similarly, while the ZINB may have an average 95% confidence interval width that is shorter for the non-structural zero/count component, the coverage of the NBH model was substantially better than the ZINB rendering this metric somewhat futile.

Table 24: 95% confidence interval coverage and width for the NBH model with the ZINB data (ZNBH) and the ZINB model with the NBH data (NZINB) for n = 500. Standard errors are in parentheses.

	β_0	β_1	β_2
	Average Coverage		
ZNBH	0.005 (0.002)	0.489 (0.011)	0.000 (0.000)
NZINB	0.000 (0.000)	0.505 (0.011)	0.174 (0.008)
	Average Width		
ZNBH	0.604 (0.001)	0.816 (0.001)	0.838 (0.001)
NZINB	0.904 (0.002)	1.226 (0.004)	44.077 (1.394)
	γ_0	γ_1	γ_2
	Average Coverage		
ZNBH	0.943 (0.005)	0.946 (0.005)	0.948 (0.005)
NZINB	0.841 (0.008)	0.862 (0.008)	0.012 (0.002)
	Average Width		
ZNBH	1.921 (0.071)	0.771 (0.007)	1.110 (0.004)
NZINB	0.448 (0.003)	0.522 (0.002)	0.545 (0.002)

Average 95% confidence interval coverage and width when the NBH data were fit to the ZINB model and the ZINB data were fit to the NBH model by θ is presented in Tables 25 and 26. For the structural zero/zero component, as θ increased the average 95% confidence interval coverage decreased for all parameters for both models except for γ_2 for the ZINB model. Similarly, the average 95% confidence interval width decreased as θ increased. For the non-structural zero/count component, as θ increased, in general, the average 95% confidence interval coverage approached 95% for all parameters and both models. For both models, performance was best when θ was equal to 5 and 10. Additionally, as θ increased, the average 95% confidence interval width decreased for the non-structural zero/count component.

For the structural zero/zero component, the average coverage was typically better for the ZINB model than the NBH model at all levels of θ except where performance was identical. In contrast, for the non-structural/count component the average coverage was always better for the NBH model for all parameters at all levels of θ. Again, the average width of structural zero/zero component was smaller for the NBH than for the ZINB model at all levels of θ and the reverse was observed for the non-structural zero/count component.

Table 25: 95% confidence interval coverage and width for the NBH model with the ZINB data (ZNBH) and the ZINB model with the NBH data (NZINB) for the structural zero/zero coefficients by dispersion (θ) for $n = 500$. Standard errors are in parentheses.

	θ	β_0	β_1	β_2
Average Coverage				
ZNBH	1/4	0.031 (0.004)	0.525 (0.011)	0.000 (0.000)
	1/2	0.000 (0.000)	0.498 (0.011)	0.000 (0.000)
	1	0.000 (0.000)	0.483 (0.011)	0.000 (0.000)
	2	0.000 (0.000)	0.479 (0.011)	0.000 (0.000)
	5	0.000 (0.000)	0.475 (0.011)	0.000 (0.000)
	10	0.000 (0.000)	0.472 (0.011)	0.000 (0.000)
NZINB	1/4	0.000 (0.000)	0.543 (0.011)	0.626 (0.011)
	1/2	0.000 (0.000)	0.528 (0.011)	0.339 (0.011)
	1	0.000 (0.000)	0.504 (0.011)	0.073 (0.006)
	2	0.000 (0.000)	0.491 (0.011)	0.005 (0.002)
	5	0.000 (0.000)	0.484 (0.011)	0.000 (0.000)
	10	0.000 (0.000)	0.483 (0.011)	0.000 (0.000)
Average Width				
ZNBH	1/4	0.647 (0.001)	0.904 (0.001)	0.948 (0.001)
	1/2	0.608 (0.001)	0.836 (0.000)	0.866 (0.001)
	1	0.593 (0.001)	0.803 (0.000)	0.824 (0.001)
	2	0.590 (0.001)	0.789 (0.000)	0.804 (0.001)
	5	0.591 (0.001)	0.783 (0.000)	0.795 (0.001)
	10	0.592 (0.001)	0.782 (0.000)	0.792 (0.001)
NZINB	1/4	1.032 (0.002)	1.499 (0.001)	158.392 (0.986)
	1/2	1.004 (0.002)	1.420 (0.001)	82.583 (0.502)
	1	0.926 (0.001)	1.271 (0.001)	18.652 (0.201)
	2	0.852 (0.001)	1.127 (0.001)	2.727 (0.035)
	5	0.811 (0.001)	1.033 (0.001)	1.079 (0.001)
	10	0.800 (0.001)	1.008 (0.001)	1.030 (0.001)

Table 26: 95% confidence interval coverage and width for the NBH model with the ZINB data (ZNBH) and the ZINB model with the NBH data (NZINB) for the non-structural zero/count coefficients by dispersion (θ) for $n = 500$. Standard errors are in parentheses.

	θ	γ_0	γ_1	γ_2
		Average Coverage		
ZNBH	1/4	0.916 (0.006)	0.941 (0.005)	0.942 (0.005)
	1/2	0.946 (0.005)	0.947 (0.005)	0.947 (0.005)
	1	0.953 (0.005)	0.945 (0.005)	0.948 (0.005)
	2	0.946 (0.005)	0.949 (0.005)	0.950 (0.005)
	5	0.949 (0.005)	0.947 (0.005)	0.952 (0.005)
	10	0.945 (0.005)	0.950 (0.005)	0.953 (0.005)
NZINB	1/4	0.570 (0.011)	0.857 (0.008)	0.052 (0.005)
	1/2	0.853 (0.008)	0.850 (0.008)	0.015 (0.003)
	1	0.894 (0.007)	0.846 (0.008)	0.004 (0.001)
	2	0.909 (0.006)	0.864 (0.008)	0.001 (0.001)
	5	0.911 (0.006)	0.870 (0.008)	0.001 (0.001)
	10	0.912 (0.006)	0.886 (0.007)	0.000 (0.000)
		Average Width		
ZNBH	1/4	8.763 (0.042)	1.315 (0.001)	1.452 (0.002)
	1/2	1.167 (0.004)	1.009 (0.001)	1.213 (0.002)
	1	0.611 (0.001)	0.781 (0.001)	1.073 (0.002)
	2	0.419 (0.001)	0.616 (0.001)	1.003 (0.002)
	5	0.303 (0.001)	0.481 (0.001)	0.966 (0.003)
	10	0.261 (0.001)	0.425 (0.001)	0.956 (0.003)
NZINB	1/4	0.599 (0.001)	0.665 (0.001)	0.676 (0.001)
	1/2	0.580 (0.001)	0.608 (0.001)	0.626 (0.001)
	1	0.512 (0.001)	0.550 (0.001)	0.569 (0.001)
	2	0.408 (0.001)	0.490 (0.001)	0.509 (0.001)
	5	0.313 (0.000)	0.425 (0.001)	0.456 (0.001)
	10	0.277 (0.000)	0.394 (0.001)	0.436 (0.001)

Average 95% confidence interval coverage and width when the NBH data were fit to the ZINB model and the ZINB data were fit to the NBH model by covariate correlation is presented in Table 27. Again, poor performance of the structural zero/zero component was observed across both levels of covariate correlation. Performance when the covariates within a component were orthogonal was slightly better for the structural zero/zero component than when they were moderately correlated. For two of the three structural zero/zero parameters, as the correlation increased the average width of the 95% confidence intervals decreased.

For the non-structural zero/count component, the performance again was much better than the structural zero/count component. The NBH model always outperformed the ZINB model. For the NBH model, as the correlation increased, the average 95% confidence interval was closer to the nominal 95% confidence interval level. However, the average increase was approximately .01%. For the ZINB, for one of the three parameters, there was a decrease in the performance of the 95% confidence interval coverage. Suggesting there may not be an overall effect of correlation (at the levels considered here) on either 95% confidence interval coverage for either model when it is misspecified.

Examining average width the same pattern emerges: average width was smaller for the NBH model for the structural zero/zero component and smaller for the ZINB model for the non-structural zero/count component.

Table 27: 95% confidence interval coverage and width for the NBH model with the ZINB data (ZNBH) and the ZINB model with the NBH data (NZINB) by covariate correlation (ρ) for $n = 500$. Standard errors are in parentheses.

	ρ	β_0	β_1	β_2
Average Coverage				
ZNBH	0	0.009 (0.002)	0.508 (0.011)	0.000 (0.000)
	.3	0.001 (0.001)	0.469 (0.011)	0.000 (0.000)
NZINB	0	0.000 (0.000)	0.512 (0.010)	0.178 (0.006)
	.3	0.000 (0.000)	0.499 (0.010)	0.170 (0.005)
Average Width				
ZNBH	0	0.633 (0.001)	0.808 (0.001)	0.821 (0.001)
	.3	0.575 (0.000)	0.824 (0.001)	0.856 (0.001)
NZINB	0	0.960 (0.002)	1.229 (0.004)	41.910 (1.265)
	.3	0.848 (0.002)	1.224 (0.004)	46.244 (1.538)
	ρ	γ_0	γ_1	γ_2
Average Coverage				
ZNBH	0	0.942 (0.005)	0.946 (0.005)	0.948 (0.005)
	.3	0.944 (0.005)	0.947 (0.005)	0.949 (0.005)
NZINB	0	0.844 (0.003)	0.848 (0.002)	0.009 (0.000)
	.3	0.839 (0.003)	0.876 (0.002)	0.016 (0.001)
Average Width				
ZNBH	0	2.112 (0.080)	0.773 (0.007)	1.078 (0.004)
	.3	1.729 (0.063)	0.769 (0.007)	1.143 (0.005)
NZINB	0	0.466 (0.003)	0.502 (0.002)	0.522 (0.002)
	.3	0.430 (0.003)	0.542 (0.002)	0.569 (0.002)

Average 95% confidence interval coverage and width when the NBH data were fit to the ZINB model and the ZINB data were fit to the NBH model by β_1 is presented in Table 28. Examination of Table 28 shows that when $\beta_1 = 0$, for both the NBH and the ZINB models, that the confidence interval coverage for β_1 was near 95% with the ZINB model slightly outperforming the NBH model. When $\beta_1 = 0.5$, both models struggled to locate the parameter and had actual coverage around 5%. Focusing on average width, there appeared to be no relationship between the average β_1 value and average width.

Table 28: 95% confidence interval coverage and width and by β_1 for the NBH model with ZINB data (ZNBH) and the ZINB model with NBH data (NZINB) for β_1 only for $n = 500$. Standard errors are in parentheses.

		β_1
Average Coverage		
ZNBH	0	0.934 (0.006)
	.5	0.043 (0.005)
NZINB	0	0.949 (0.000)
	.5	0.062 (0.001)
Average Width		
ZNBH	0	0.804 (0.001)
	.5	0.829 (0.001)
NZINB	0	1.234 (0.005)
	.5	1.218 (0.004)

Type I Error

Type I error rate for all the conditions when the NBH data were fit to the ZINB model and the ZINB data were fit to the NBH model is presented in Tables 140 to 143 in Appendix D.

The overall type I error rate when $\gamma_1 = 0$ was .058 and .057 for the NBH and ZINB models, respectively. The overall type I error rate when $\beta_1 = 0$ was .066 and .051 for the NBH and ZINB model. Ignoring all conditions, type I error rates were similar between the models with the exception that for $\beta_1 = 0$, the NBH model was larger than ZINB but still within an acceptable level.

Figure 8 shows the estimated type I error rates for the NBH model (black circle, solid lines) and the ZINB model (black triangle, dashed lines) by parameter type. Again, the length of the standard error bands corresponded to 2 times the standard error. For the $\gamma_1 = 0$ and the NBH model, the interval does not include $\alpha = .05$, but again this interval contained acceptable values. The estimated type I error rate for the ZINB model, ignoring all conditions, was closest to $\alpha = .05$ (the solid black horizontal line) and included $\alpha = .05$ in both of its intervals.

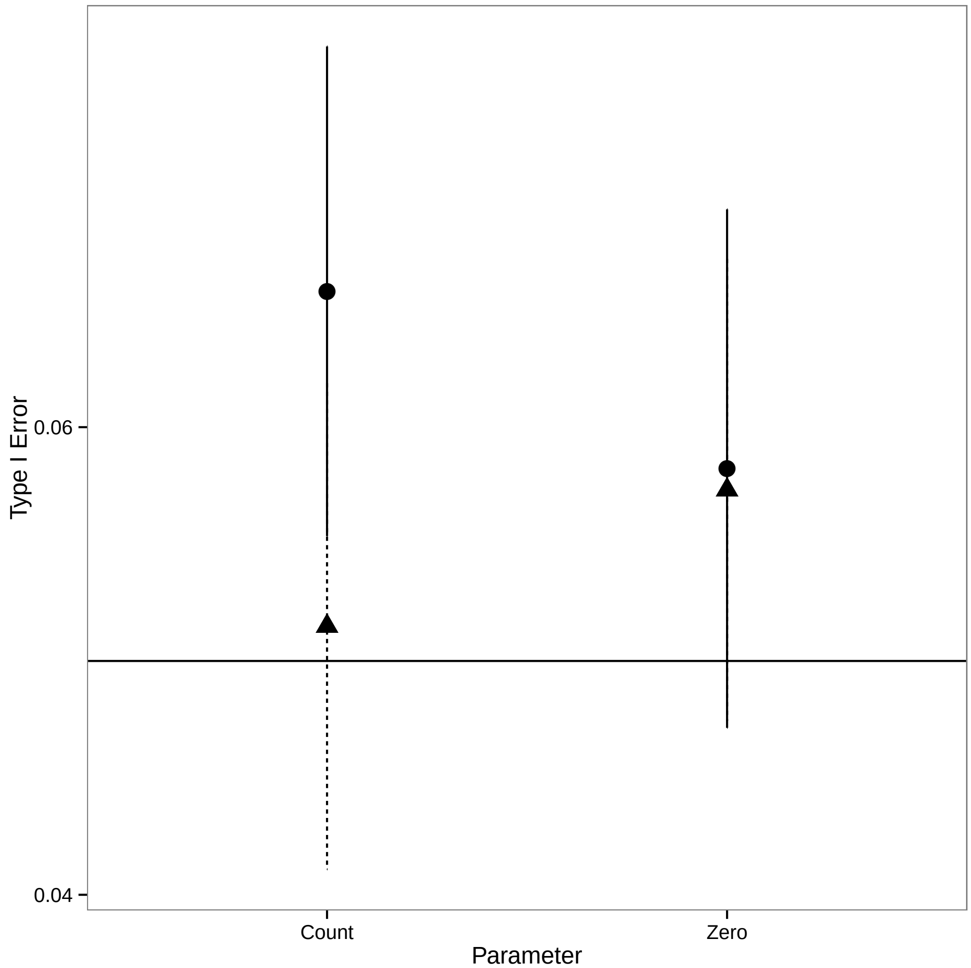

Figure 8: The mean type I error rate for $\alpha = .05$ for the NBH (circle, solid lines) and ZINB (triangle dashed lines) model, respectively. The bands around the point estimates corresponds to the mean +/- 2 standard errors. Count refers to γ_1 and zero to β_1.The vertical black bar corresponds to $\alpha = .05$

Figure 9 shows the estimated type I error rates for the NBH model and the ZINB model by parameter type and θ. The NBH model again had intervals that did not include $\alpha = .05$ for $\beta_1 = 0$, but were considered acceptable. In contrast, the ZINB model had intervals that included $\alpha = .05$ except for $\gamma_1 = 0$

when $\theta = 1/4$ and $1/2$, but were considered acceptable. The NBH model, regardless of θ, included $\alpha = .05$ in its intervals for $\gamma_1 = 0$.

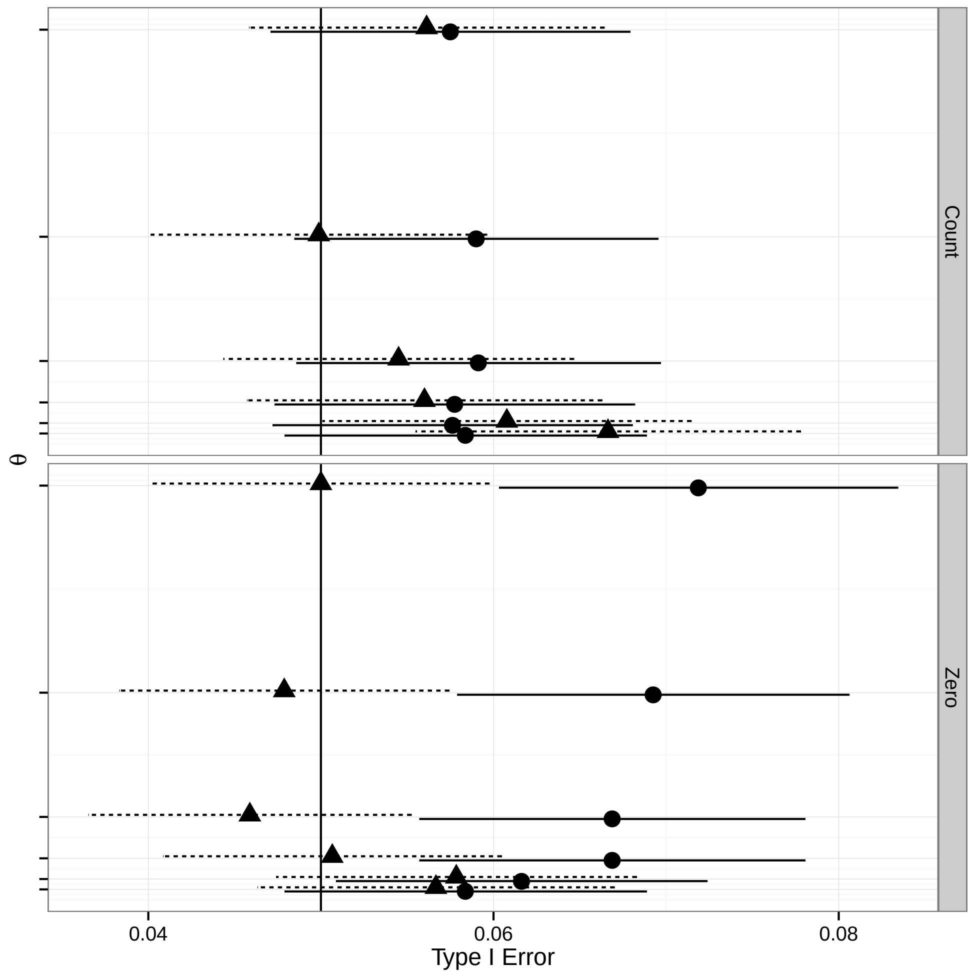

Figure 9: The mean type I error rate for $\alpha = .05$ for the NBH (circle, solid lines) and ZINB (triangle, dashed lines) model, respectively. The tick marks (from bottom to top) correspond to $\theta = 1/4$, $1/2$, 1, 2, 5, and 10. The bands around the point estimates corresponds to the mean +/- 2 standard errors. Count refers to γ_1 and zero to β_1. The vertical black bar corresponds to $\alpha = .05$

Figure 10 shows the estimated type I error rates for the NBH model and the ZINB model by parameter type and covariate correlation. For the NBH model and $\beta_1 = 0$, when the covariates were orthogonal, $\alpha = .05$ was included in the interval but not when they were correlated. When they were correlated, the plausible values were still considered acceptable. In contrast, the ZINB always included $\alpha = .05$ in its intervals.

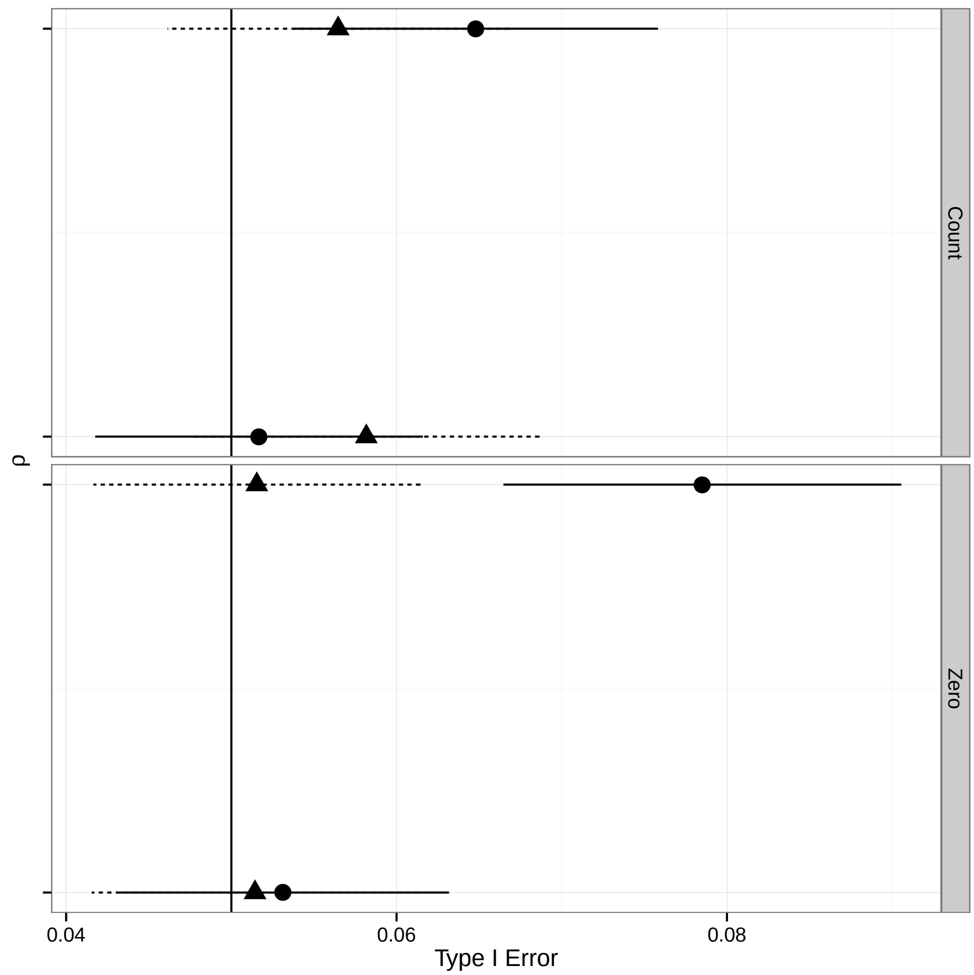

Figure 10: The mean type I error rate for $\alpha = .05$ for the NBH (circle, solid lines) and ZINB (triangle, dashed lines) model, respectively. The tick marks (from bottom to top) correspond to $\rho = 0$ and .3. The bands around the point estimates corresponds to the mean +/- 2 standard errors. Count refers to γ_1 and zero to β_1.The vertical black bar corresponds to $\alpha = .05$

Bias

Results examining relative bias for E(Y|X) and Pr(Y = 0) when the NBH data were fit to the ZINB model and the ZINB data were fit to the NBH model are presented in Appendix E in Tables 152 through 159. Table 29 below presents a summary of overall bias, by dispersion, and by covariate correlation. First, we see that the overall bias (labeled as grand in the table), was greater for the NBH model than for the ZINB model. The overall bias for the E(Y|X) for the NBH model when ZINB data were fit to the model was 17% and for the ZINB model when NBH data were fit to the model was 4%. Similarly, the relative bias for the Pr(Y = 0) was approximately 10% for the NBH and 6% for the ZINB model. These relative biases are substantially higher than those observed when the data were fit to the correct data-generating model.

Table 29 shows that as θ increased for both the NBH and ZINB models that the relative biases of both the E(Y|X) and Pr(Y = 0) increased except for the ZINB for Pr(Y = 0). This was the opposite pattern to that observed when the data were fit to the correct data-generating model. For both models relative bias was greatest when $\theta = 5$ or 10. For the NBH model, the Pr(Y = 0) appeared to be much more greatly affected than for the ZINB model. When $\theta = 1/4$, relative bias was approximately 2%, lower than for the ZINB model, and increased to approximately 18% when $\theta = 10$. When θ was greater than or equal to 1 for the Pr(Y = 0), the ZINB model had lower bias than the NBH. Overall, the effect of θ on the relative bias of Pr(Y = 0) was greatest for the NBH model.

Focusing on covariate correlation in Table 29, for both models and for both E(Y|X) and Pr(Y = 0), as the covariate correlation increased, the relative bias increased except for the ZINB for Pr(Y = 0). The effect was most dramatic for the NBH model, where the relative bias for the Pr(Y = 0) when the covariates were uncorrelated was 4% and increased to 15% when the covariates were correlated. The relative bias for E(Y|X) increased from 15% to 19% for the NBH and from 3% to 4% for the ZINB model.

Table 29: Relative bias for E(Y|X) and Pr(Y = 0) for the NBH model with the ZINB data (ZNBH) and the ZINB model with the NBH data (NZINB) for $n = 500$. Standard errors are in parentheses.

		E(Y|X)		Pr(Y = 0)	
		ZNBH	NZINB	ZNBH	NZINB
Grand		0.171 (0.009)	0.035 (0.002)	0.096 (0.006)	0.061 (0.005)
θ	1/4	0.126 (0.008)	0.027 (0.001)	0.015 (0.002)	0.059 (0.005)
	1/2	0.154 (0.009)	0.028 (0.002)	0.037 (0.003)	0.060 (0.005)
	1	0.176 (0.010)	0.033 (0.002)	0.073 (0.005)	0.060 (0.005)
	2	0.188 (0.010)	0.037 (0.002)	0.117 (0.007)	0.061 (0.005)
	5	0.193 (0.010)	0.043 (0.003)	0.160 (0.008)	0.061 (0.005)
	10	0.191 (0.010)	0.044 (0.003)	0.177 (0.009)	0.061 (0.005)
ρ	0	0.154 (0.003)	0.032 (0.001)	0.107 (0.004)	0.056 (0.002)
	.3	0.188 (0.003)	0.039 (0.001)	0.086 (0.004)	0.065 (0.002)

Empirically-Derived Simulation Condition

Parameter behavior for the empirically-derived simulation condition where the ZINB data were fit to the NBH model and the NBH data were fit to the NBH model are presented in Table 30. First, the mean and median parameter values for the zero component of the NBH model with the ZINB data were very similar to the count parameters in Equation 26. Similarly, the estimated mean and median parameters for the structural zero component of the ZINB model with NBH data corresponded to the structural zero parameters in Equation 25. The non-structural/count parameters were very similar to those reported in Equations 25 and 26 corresponding to their appropriate parameter values. The Standard Deviation was quite low and similar across the models and components. The mean squared error, however, was much larger for the structural zero/zero component parameters than for the non-structural zero/count component parameters.

The average proportion of 95% confidence intervals containing the true parameter and their average width for the empirically-derived condition where the ZINB data were fit to the NBH model and the NBH data were fit to the NBH model is shown in Table 31. The non-structural zero/count parameters had average coverage that was close to 95%. The structural zero/zero parameters, however, had coverage that was essentially zero except for β_4 and β_6. The average width of these intervals were similar across models and components.

The relative bias for the NBH with ZINB data and for the ZINB model with NBH data for both the E(Y|X) and Pr(Y = 0) was approximately 0. This suggest that model misspecification for this condition did not affect bias for either E(Y|X) or Pr(Y = 0).

Table 30: Parameter behavior for the NBH model with the ZINB data (ZNBH) and the ZINB model with the NBH data (NZINB) for the empirically-derived simulation.

	Mean		Median		SD		MSE	
	ZNBH	NZINB	ZNBH	NZINB	ZNBH	NZINB	ZNBH	NZINB
β_0	-1.673	1.408	-1.676	1.409	0.090	0.103	9.493	9.476
β_1	1.110	-1.078	1.112	-1.078	0.073	0.079	4.814	4.777
β_2	0.923	-0.877	0.923	-0.876	0.109	0.117	3.267	3.263
β_3	-0.724	0.762	-0.725	0.759	0.117	0.124	2.243	2.223
β_4	0.155	-0.142	0.154	-0.143	0.093	0.106	0.101	0.112
β_5	0.390	-0.336	0.389	-0.336	0.050	0.056	0.528	0.534
β_6	-0.078	0.066	-0.079	0.067	0.051	0.058	0.024	0.028
β_7	-0.208	0.194	-0.209	0.193	0.059	0.067	0.171	0.168
β_8	-0.552	0.461	-0.552	0.464	0.122	0.138	1.060	1.055
β_9	-1.444	1.380	-1.442	1.380	0.087	0.099	8.023	7.975
β_{10}	0.611	-0.618	0.611	-0.619	0.045	0.048	1.539	1.494
γ_0	1.246	1.250	1.247	1.251	0.080	0.081	0.013	0.013
γ_1	0.383	0.381	0.382	0.380	0.068	0.069	0.009	0.009
γ_2	0.380	0.379	0.378	0.378	0.093	0.091	0.017	0.017
γ_3	0.017	0.018	0.014	0.020	0.114	0.116	0.026	0.027
γ_4	0.076	0.072	0.074	0.072	0.091	0.091	0.016	0.017
γ_5	0.345	0.346	0.344	0.346	0.037	0.038	0.003	0.003
γ_6	-0.085	-0.085	-0.086	-0.085	0.043	0.042	0.004	0.004
γ_7	-0.119	-0.120	-0.120	-0.120	0.044	0.043	0.004	0.004
γ_8	-0.471	-0.477	-0.470	-0.473	0.101	0.104	0.021	0.022
γ_9	-0.543	-0.542	-0.544	-0.540	0.084	0.079	0.014	0.013
γ_{10}	0.130	0.128	0.130	0.129	0.036	0.035	0.003	0.002

Note. The true parameters for the ZNBH are β_0 = 1.41, β_1 = -1.08, β_2 = -0.88, β_3 = 0.76, β_4 = -0.13, β_5 = -0.33, β_6 = 0.06, β_7 = 0.20, β_8 = 0.46, β_9 = 1.39, β_{10} = -0.63, γ_0 = 1.25, γ_1 = 0.38, γ_2 = 0.38, γ_3 = 0.02, γ_4 = 0.08, γ_5 = 0.35, γ_6 = -0.08, γ_7 = -0.12, γ_8 = -0.47, γ_9 = -0.55, γ_{10} = 0.13. The true parameters for the NZINB are β_0 = -1.67, β_1 = 1.10, β_2 = 0.92, β_3 = -0.72, β_4 = 0.16, β_5 = 0.39, β_6 = -0.08, β_7 = -0.21, β_8 = -0.55, β_9 = -1.44, β_10 = 0.60, γ_0 = 1.25, γ_1 = 0.38, γ_2 = 0.38, γ_3 = 0.02, γ_4 = 0.07, γ_5 = 0.35, γ_6 = -0.08, γ_7 = -0.12, γ_8 = -0.47, γ_9 = -0.54, and γ_{10} = 0.13.

Table 31: 95% confidence interval coverage and width for the NBH model with the ZINB data (ZNBH) and the ZINB model with the NBH data (NZINB) for the empirically-derived simulation. Standard errors are in parentheses.

	Average Coverage		Average Width	
	ZNBH	NZINB	ZNBH	NZINB
β_0	0.000 (0.000)	0.000 (0.000)	0.356 (0.000)	0.396 (0.000)
β_1	0.000 (0.000)	0.000 (0.000)	0.286 (0.000)	0.313 (0.000)
β_2	0.000 (0.000)	0.000 (0.000)	0.419 (0.000)	0.461 (0.000)
β_3	0.000 (0.000)	0.000 (0.000)	0.453 (0.000)	0.487 (0.000)
β_4	0.146 (0.008)	0.186 (0.009)	0.376 (0.000)	0.411 (0.000)
β_5	0.000 (0.000)	0.000 (0.000)	0.195 (0.000)	0.217 (0.000)
β_6	0.220 (0.009)	0.279 (0.010)	0.200 (0.000)	0.223 (0.000)
β_7	0.000 (0.000)	0.000 (0.000)	0.232 (0.000)	0.261 (0.000)
β_8	0.000 (0.000)	0.000 (0.000)	0.462 (0.000)	0.528 (0.000)
β_9	0.000 (0.000)	0.000 (0.000)	0.349 (0.000)	0.383 (0.000)
β_{10}	0.000 (0.000)	0.000 (0.000)	0.170 (0.000)	0.190 (0.000)
γ_0	0.956 (0.005)	0.947 (0.005)	0.319 (0.000)	0.319 (0.000)
γ_1	0.950 (0.005)	0.951 (0.005)	0.270 (0.000)	0.270 (0.000)
γ_2	0.950 (0.005)	0.952 (0.005)	0.363 (0.000)	0.363 (0.000)
γ_3	0.950 (0.005)	0.944 (0.005)	0.447 (0.000)	0.446 (0.000)
γ_4	0.949 (0.005)	0.949 (0.005)	0.354 (0.000)	0.353 (0.000)
γ_5	0.957 (0.005)	0.942 (0.005)	0.148 (0.000)	0.147 (0.000)
γ_6	0.942 (0.005)	0.950 (0.005)	0.164 (0.000)	0.164 (0.000)
γ_7	0.949 (0.005)	0.957 (0.005)	0.173 (0.000)	0.172 (0.000)
γ_8	0.954 (0.005)	0.946 (0.005)	0.399 (0.000)	0.399 (0.000)
γ_9	0.941 (0.005)	0.955 (0.005)	0.316 (0.000)	0.317 (0.000)
γ_{10}	0.955 (0.005)	0.957 (0.005)	0.141 (0.000)	0.141 (0.000)

Summary of Findings

The main findings of the first simulation study can be summarized as follows:

- The NBH generally outperformed the ZINB regardless of the condition considered. The was seen in parameter recovery, 95% confidence interval coverage, and bias. This was especially evident for the comparison of the recovery of the zero component parameters to the structural zero component parameters.

- The percentage of correctly identified structural zeros hovered around 71% regardless of the condition considered.

- There was an effect of sample size and dispersion parameter on parameter recovery.

- Type I error rates were inflated for the ZINB and NBH model for $\gamma_1 = 0$ when $n = 100$ and were deflated for the ZINB for $\beta_1 = 0$ when $n = 100$. However, they were still within the acceptable range.

- Finally, for the empirically-derived condition, the performance between the two models was essentially equivalent.

The main findings of the second simulation study can be summarized as follows:

- The data-generating mechanism of the zeros had a large effect on the structural zero/zero parameters. When the data-generating mechanism was improperly specified, both models struggled at recovering the structural zero/zero parameters.

- For the non-structural zero/count parameters, the NBH generally outperformed the ZINB regardless of the condition considered. This was especially evident in the comparison of the recovery of these parameters and their observed 95% confidence interval coverage.

- Type I error rates were inflated for the NBH model for $\beta_1 = 0$ but were within the acceptable range.

- Bias was generally increased for both model when the models were misspecified.

Chapter V: Conclusion

As discussed in Chapters 1 and 2, zero-inflated negative binomial (ZINB) and negative binomial hurdle models (NBH) are frequently utilized in the count literature to accommodate overdispersion and zero-inflation. Zero-inflated and hurdle models have generally been used interchangeably and the decision to choose one model over another has been argued based on the data-generating mechanism of the zeros (Bethell et al., 2010; Rose et al., 2006) and rarely by a fit measure (Hu et al., 2011; Neelon et al., 2010). The use of fit measures to compare zero-inflated and hurdle models has generally been advised against (Zeileis et al., 2008; Loeys et al., 2012). While researchers in this field argue that model examination should be based on theory regarding the presence of structural zeros, there have been no studies investigating whether the zero-inflated models can properly classify structural zeros and how model misspecification might affect parameter recovery and behavior.

The comparative performance of these models under simulation has rarely been considered (Miller, 2007; Min & Agresti, 2005) and examination of potential disparities in performance when the data-generating mechanism is known are lacking. This dissertation aimed to address this knowledge gap by quantifying the performance and recovery of these models under known conditions. Specifically, the purpose of this dissertation was to answer the following research questions:

1. For a given set of simulated conditions, do the ZINB and the NBH models perform equally well in terms of parameter recovery and type I error rates? Under which of these conditions do they both perform poorly? And under which of these conditions does the performance between the two models differ?

2. What are the consequences, if any, of misspecifying the latent class for the zeros? In other words, given the existence (non-existence) of a structural zero class, what are the consequences of fitting a model that does not (does) account for the structural zero class?

To answer these research questions, two simulation studies were performed. These simulation studies aimed to provide practical recommendations to researchers in scenarios mirroring, as closely as possible, those described in the extant literature.

First Simulation Study - Model Performance and Recovery

The purpose of the first simulation study was to answer the first research question. To that extent, performance of these models under varying sample sizes, dispersion parameters, parameter values for both the structural zero/zero and non-structural zero/count components, and correlation between covariates within a component were considered. Performance was assessed by exploring parameter behavior, 95% confidence interval coverage and width, type I error when one of the dichotomous covariates on each component was set equal to zero, the ability of the ZINB model to predict structural zeros, and the relative bias for the E(Y|X) and Pr(Y = 0).

Parameter recovery for the zero component of the NBH was better than the structural zero component of the ZINB with comparable performance between the count component of the NBH and the non-structural zero component of the ZINB. Ninety-five percent confidence interval coverage was closer to the nominal level for the NBH regardless of the parameter considered except for γ_2. Coverage for β_0 and γ_0 in the ZINB was quite poor. Type I error rate was comparable between the two models when n was larger than 100. Relative bias for $\Pr(\mathrm{Y} = 0)$ was approximately 0 for both models and relative bias for $\mathrm{E}(\mathrm{Y}|\mathrm{X})$ was similar between the models except when $n = 100$ and $\theta = 1/4$. Finally, the percent of correctly identified structural zeros was roughly 71%. For the empirically-derived condition, performance was essentially equivalent between the two models.

Based on this study, the NBH model appears to be the better performing model. In the situation where little a priori theory dictates the data-generating mechanism of the zeros, the results from this simulation study strongly suggest the use of the NBH given its superior performance in known situations. Given that researchers in this field have occasionally used model comparison to select between zero-inflated and hurdle models, this simulation study strongly suggests that the researcher consider just the NBH.

With any simulation study, it is unknown to what extent these findings would generalize beyond the conditions considered or if they would generalize to ZIP and PH models. Previous simulation studies agree with the findings here and suggest that the NBH model, and perhaps the hurdle, in general, may be the more robust model (Min & Agresti, 2005). However, while the NBH performed better at the conditions considered here, parameter recovery and performance was essentially equivalent for the empirically-derived condition. This suggests that performance between the two models may converge at a larger sample size or when more covariates are included in the model.

The bias associated with the prediction of the expected count and the probability of a zero was roughly similar between the two models under most conditions (although the NBH did slightly better). Therefore, if the principal purpose of a study is prediction and not parameter estimation/interpretation, then either model might be sufficient given that the data-generating mechanism of the zeros was properly specified. However, researchers are often interested in interpretation and given that the zero-inflated parameters are occasionally interpreted incorrectly (Hu et al., 2011) and are generally difficult to interpret (Lambert, 1992; Loeys et al., 2012; Böhning et al., 1999; Welsh et al., 1996), the fact that bias is largely equivalent between the two models, again suggest that researchers use the NBH models given its more parsimonious interpretation.

Sample size affected parameter behavior, confidence interval coverage, and bias. For both models, as the sample size increased, these outcome measures improved. Generally, the ZINB was more affected by sample size than the NBH and this was most evident for the structural zero component. Lambert (1992) reported poorer performance of the ZIP model with smaller samples sizes ($n =$ 25 and 50) and convergence issues. However, unlike Lambert (1992), it was found that the ZINB's structural zero component struggled to recover its parameters adequately even with larger sample sizes than Lambert considered. This could have resulted from the additional complexity caused by the use of the negative binomial extension and the estimation of the dispersion parameter.

This is the first study to suggest that performance differs between NBH and ZINB models based on sample size. The results here suggest that NBH models could be used with high confidence when the ratio of sample size to number of estimated parameters is as small as 14. If concern is primarily in the count components, then parameter behavior and confidence interval coverage were found to, in general, increase with sample size. Results here suggest that confidence interval coverage close to 95% for all parameters can be obtained when this ratio is approximately 36. This ratio is similar to that reported already in the literature for these models (Hu et al., 2011; Yau & Lee, 2001; Rose et al., 2006; Böhning et al., 1999; Loeys et al., 2012) suggesting that 95% confidence interval constructed around the estimated parameters in these studies should have roughly 95% coverage.

For the ZINB model, the findings here suggest that the sample size to estimated number of parameters should be at least 71 and to ensure approximately 95% this ratio should be even larger. These results parallel Miller (2007)'s findings. This implies that studies with a ratio of sample size to estimated number of parameters smaller than 71 and using the ZINB model, are likely untrustworthy even if they are properly specified. Given that many papers reviewed here used ZINB models with ratios smaller than this (e.g. Hu et al. (2011); Atkins and Gallop (2007)), it is plausible that inferences about the parameters in these studies may be incorrect and that confidence interval coverage may be less than the nominal value.

There was also evidence that dispersion affected parameter behavior for the ZINB structural zero component, proportion of correctly identified structural zeros, and the relative bias for E(Y|X) for the ZINB model. When the dispersion parameter was small, the parameter behavior for the ZINB structural zero component was poor while the NBH model performed well. The results here suggest that when there is greater amounts of dispersion that the NBH model can still perform reasonably well and is superior to the ZINB model. Again, these findings echo Miller (2007)'s dissertation that found the hurdle models to outperform the zero-inflated models with positively-skewed data mixed with a point mass at zero. While parameter behavior was closer for the NBH model than the ZINB model, parameter behavior was in the correct direction for the ZINB and the intercept terms (β_0 and γ_0) were most affected.

This is the first study to examine the effects of altering levels of dispersion in these models and to show that the ZINB and NBH have disparate performance depending upon this parameter's values. Small dispersion parameters (less than or around 1/2) are commonly reported in the literature (Zuur et al., 2009; Hu et al., 2011; Yau et al., 2003). Results from this study suggest that the estimated structural zero parameters in those studies could be biased and are likely untrustworthy. Studies with larger dispersion parameters (e.g. Arulampalam and Booth (1997); Ghosh et al. (2006)) would have parameter estimates that are less biased. Unfortunately, many researchers using these models do not report the estimated dispersion parameter. Based on this simulation study, it is strongly recommended that researchers include the estimated dispersion parameter for the ZINB as this may shed insight into the validity of the estimated structural zero parameters.

The levels of correlation between covariates within a component considered in this dissertation had minimal effect in the first simulation study with the exception that the average 95% confidence interval width when $\rho = .3$ was substantially larger for the ZINB structural zero parameters than when the covariates were orthogonal. It is unclear why this would be the case especially given similar mean standard deviations of the simulates when examining the parameter behavior of the structural zero parameters. It still remains unclear if covariate correlation between predictors within a component, or even across components, could affect the two models differently.

The percentage of structural zeros correctly identified by the ZINB model hovered around 71% regardless of the condition considered in either the main conditions or the empirically-derived condition. This suggests a possible plateau and warrants further investigation. While this percentage was greater than chance alone, only the context of the research question can identify if this percent agreement is suitable.

The proportion of correctly identified structural zeros could be conceptualized as representing sensitivity (Altman & Bland, 1994), true positive structural zeros, for the ZINB model. In the context of medicine, this level of sensitivity would be considered low. For example, Deeks (2001) reported a pooled estimate of sensitivity of 0.96 (95% confidence interval 0.93 to 0.99) from a meta-analysis of 20 studies of diagnostic accuracy of detecting endometrial cancer with endovaginal ultrasonography. Gould et al. (2003), in a meta-analysis of the diagnostic accuracy of two techniques for mediastinal staging in patients with nonsmall-cell lung cancer, reported a median sensitivity for the best performing technique of 85% marginally and 100% conditional on observing enlarged lymph nodes. While these applications are quite different, as one instance is comparing a diagnostic test and the other one a model, these applications of sensitivity comment on overall validity. Seventy-one percent agreement might be too low to argue that this model is statistically valid and instead supports the use of the hurdle model. In the context of educational or psychological applications (e.g. school suspensions or psychopathologies), perhaps this level of sensitivity might be acceptable as the ramifications of misidentification are not as high in the social sciences as medicine.

Sensitivity could also be conceptualized as the power for the ZINB at classifying structural zeros. The de facto standard for power in education and psychology is .80. The power of the ZINB model would fall below this. Both conceptualizations, as sensitivity or power, argue that the proportion of structural zeros correctly identified should be higher to be useful.

Furthermore, and more importantly, if a researcher is principally concerned with classifying structural zeros then the ZINB may not be sensitive enough. If there is a high-stakes decision based on this classification then the ZINB model may be inappropriate and an alternative method should be considered. If concern is just about predicting whether a subject is likely to have a zero, then this study suggests, based on the relative bias of $\Pr(\mathrm{Y} = 0)$, that both models perform reasonably well at predicting zeros regardless of the simulated conditions and that a practitioner could consider either model.

Returning to the original research question, the NBH model and the ZINB model had comparable performance when the sample size was large and the dispersion parameter was large, regardless of covariate correlation within a component. The NBH model outperformed the ZINB model when the sample size was small and the dispersion parameter was small. The ZINB model never outperformed the NBH model. For the NBH model, researchers can feel confident using this model in similar conditions to those considered here. However, when the sample size was large, as was the case with the empirically-derived condition, then these models will likely both perform well. It is unclear what might happen with an extremely large sample, like that considered in the empirically-derived condition, and a small dispersion parameter. Might the ZINB model perform reasonably well under this condition? It is unclear.

Second Simulation Study - Model Misspecification

The main purpose of the second simulation study was to answer the second research question. Data generated from a ZINB model were fit to the NBH model and vice versa with a subset of criteria and conditions used in the first simulation study (see Chapters 3 and 4).

In general, both models performed much more poorly when misspecified especially for the structural zero/zero components, parameter recovery was very poor. The distance between the observed and the true value for the structural zero/zero components was always large and often in the opposite direction (i.e. the model would estimate the magnitude of the parameter as negative if it was positive). Neither the ZINB model with the NBH data nor the NBH model with the ZINB data were able to recover these parameters. However, both models performed reasonably well at recovering the non-structural zero/count components with the NBH outperforming the ZINB model.

Parameter recovery improved as the dispersion parameter increased for both the non-structural zero/count component parameters. However, as the dispersion parameter increased, parameter behavior improved for some of the ZINB structural zero parameters and not for the NBH zero parameters. Therefore, there did not appear to be an overall effect of the dispersion parameter on these components when the model was misspecified.

Similarly, both models had horrible coverage for the 95% confidence interval for the structural zero/zero component and coverage for the non-structural zero/count component was generally poorer in the second simulation study than the first. Coverage for β_0 and β_2 was approximately 0 for the NBH model and 0 and 17% for the ZINB model overall. When $\beta_1 = 0$, coverage was approximately 93.4% for the NBH and 94.9% for the ZINB model and when $\beta_1 = 0.5$, coverage was around 5% for both models. This suggests that when a parameter is unrelated to the model for the structural zero/zero component, even if there is misspecification of the data-generating mechanism, that these models are still able to identify this. This was also evident in the type I error rates generally overlapping with $\alpha = .05$. Otherwise, 95% confidence interval coverage will likely be abysmal for the structural zero/zero component when the model is misspecified.

Coverage for γ_0, γ_1, and γ_2 was substantially better for the NBH model than for the ZINB and approached the nominal 95% confidence interval level. This likely result from the zero-deflation for the non-structural zero component as the NBH has fewer expected zeros than the ZINB model. This suggests that the NBH model, even with the misspecified data-generating mechanism, performs reasonable well at recovering the non-structural zero parameters and has comparable performance to the ZINB model when the data-generating mechanism is properly specified.

Biases of up to 19% for the E(Y|X) were found for the NBH model when fit with the ZINB data and up to 6% for the Pr(Y = 0) for the ZINB model fit to the NBH data. Higher bias likely resulted from the extra zeros, as the ZINB model with the same parameters has a greater expected number of zeros than the NBH model. The fact that biases were higher for the NBH model with the ZINB data suggests that the consequences of ignoring structural zeros may be more important than assuming their existence if prediction is the principal end goal. This study is the first study to report that bias in prediction could be greatly affected by model misspecification for the NBH and ZINB models.

Covariate correlation did not seem to be related to performance when the models were misspecified. There was similar parameter behavior, type I error, and 95% coverage and width. Relative bias did increase when the correlation increased and was most pronounced for the NBH model. It is unclear why correlation would have a more pronounced effect on bias and not the estimated parameters. Similar to the findings in the first simulation study, more work remains to be done to understand how multicollinearity might affect these models.

Returning to the second research question, what are the consequences of misspecifying the latent class? This is the first study to heed Rose et al. (2006)'s call to examine the consequences of misspecification of the zeros and attempt to quantify the consequences. The consequences can be summed up as follows.

First, the estimated parameters are likely to be quite far, and possibly in the wrong direction, from their true values for the structural zero/zero component and there could be inflated type I error rates. This is largely unsurprising given that the binomial logistic regression for the ZINB corresponds to the probability of a structural zero and the binomial logistic regression for the NBH corresponds to the probability of a non-zero. Interpretation and hypothesis testing will be wrong and decisions made based on these parameters could be quite costly.

The non-structural zero/count components will be similar to their true values especially for the NBH model. The 95% confidence intervals for these parameters, however, will have lower coverage particularly for the ZINB model. Researchers interested only in these parameters can be largely assured that these parameters are mostly unaffected by misspecification of the data-generating mechanism of the zeros and they could select either model based on the inferences they are interested in making.

Second, prediction appears to be greatly affected by model misspecification. However, given that this finding is in contradiction to the general belief that prediction should be similar across models (Loeys et al., 2012; Zeileis et al., 2008; Zuur et al., 2009), this suggests that more work should be done in this area to replicate the findings here.

It is also widely believed that these models will have similar fit indices but at least one applied study found that the AICs can be different (Hu et al.,

2011) and Miller (2007) used the AIC to select between these models in his simulation study. In the simulation studies considered here, the AIC was correct nearly 100% of the time in selecting the correct data-generating mechanism in the main simulation conditions (i.e. if the data-generating mechanism was NBH, then the NBH had a smaller AIC than the ZINB fit with the NBH data and vice versa). However, for the empirically-derived condition, the AIC was correct 62% of the time when the data-generating mechanism was NBH and 64% of the time when the data-generating mechanism was ZINB. It is clear that more work needs to be done to better understand prediction and fit in these models.

These findings, under a known situation of model misspecification, again corroborate the recommendations of selecting between these models based on theory about the presence of structural zeros (Rose et al., 2006; Miller, 2007; Loeys et al., 2012; Zeileis et al., 2008). Therefore, it is strongly recommended that a researcher put considerable care and thought into the potential data-generating mechanism of the zeros. This is especially important if interest is in the structural zero/zero component. If these parameters are not of interest, then findings here suggest a NBH should be used.

The complexity of the ZINB also favors the NBH model in situations where the data-generating mechanism of the zeros is unknown. The ZINB model, being a zero-inflated model, is an inherently more difficult model to estimate because its likelihood function can not be factored into separate likelihood functions and thus these two components must be estimated simultaneously (Min & Agresti, 2005). This often involves the introduction of a third, latent variable (Lambert, 1992) and the solution to these models may not be unique (i.e. maximum likelihood estimates based on local maxima) (Agarwal et al., 2002). In contrast, the NBH model, being a hurdle model, is more easily estimated as its likelihood function can be factored into separate likelihood functions that can be maximized separately (Min & Agresti, 2005). Furthermore, the NBH model suffers only from the same issues that binomial logistic regression and negative binomial loglinear models suffer from.

Educational and Psychological Significance

Hurdle and zero-inflated models are essentially absent from the education literature. Miller (2007)'s simulation work represents the one exception and there has been no applied work using these models in education. It is evident that data that are zero-inflated and overdispersed occur in education. School days suspended, the motivating problem for this dissertation, represents one such example. Other examples include grade retention and substance abuse (Miller, 2007).

Zero-inflation and overdispersion may occur because of model misspecification and omitted variables (Berk & MacDonald, 2008) and the use of extant data sources could result in either of these phenomena. For example, Desjardins (2012)'s study, examining school suspensions in Minneapolis Public Schools, used their annually collected data. This greatly reduced the number of covariates available for modeling and restricted the models to variables tracked for other reasons. Given that many empirical studies often use existing school district data (e.g. Obradovic et al. (2009); Cutuli et al. (2012); Desjardins et al. (2013)), omission of covariates might be quite common.

It is unclear to what extent structural zeros would exist in educational research, whereas structural zeros likely abound in psychology. For example, many studies in psychology will have exclusion criteria. These exclusion criteria often amount to removing structural zeros from the study. If exclusion criteria were used, then this suggests any zero-inflation should be handled with hurdle models. However, if exclusion criteria is not used, it seems reasonable that structural zeros could exist, especially in certain rare psychological phenomena, and that these models should be considered.

Packages that implement zero-inflated and hurdle models are readily available for most major statistical languages. For users of `R`, there is the `zeroinfl` package (Zeileis et al., 2008) (used in the simulation studies here) that is capable of fitting ZIP, ZINB, PH, and NBH models. If there is hierarchical ordering to the data, users of `R` could use the `glmmADMB` package (Fournier et al., 2012) and if there is interest in fitting Bayesian multilevel models, the `MCMCglmm` package (Hadfield, 2010) is capable of fitting ZIP and PH models. The `glmmADMB` package, however, does not allow predictors to enter the structural zero component of zero-inflated models. For users of `SAS`, there is zero-inflated and hurdle software available via `PROC GENMOD`. `Mplus` also implements ZIP models (Muthén & Muthén, 2010) and `SPSS` has routines to handle these models as well. Given the variety of available options and platforms, there are no computational hurdles to practitioners in these areas to employing these models and they should be made of aware of the availability of these techniques and software.

Future Work and Limitations

Future theory work could be done to generalize the work of Loeys et al. (2012) and Min and Agresti (2005) to allow conversion of the parameters from a ZINB to a NBH regardless of the number of covariates in the model. This future work would be useful for informing the conditions considered here and determining whether the ZINB correctly predicted the ZINB parameters given the NBH model and the NBH correctly predicted the NBH parameters given the ZINB model. In other words, knowing the general relationship, if one exists, between the two models would allow a more objective evaluation of parameter behavior during model misspecification. Additionally, knowing this information would be useful if the data-generating mechanism of the zeros becomes known as it would allow easy conversion of the parameters without needing to refit the model.

While no simulate of a condition was observed to be singular, it was possible that there was false convergence. Given the sheer number of conditions and simulates considered here, it is not feasible to examine output from each model. The large standard deviations for the simulates seen in Tables 7 and 9 for the ZINB's structural zero component, especially when the sample size was 100 and θ was less than or equal to 1, suggest that while parameter estimates may have been provided by the software, false convergence could be an issue for these parameters. When the sample size was small this likely decreased the variability for the binomial logistic regression model and could have resulted in situations where near perfect discrimination occurred (Agresti, 2002). Similarly, when $\theta = 1/4$, $1/2$, and 1, on average, the proportion of zeros was 75%, 68%, and 61% with 58%, 64%, and 70% of the zeros coming as structural zeros, respectively. While these are not especially high numbers of zeros marginally, conditional on a particular structural zero covariate it is possible that only a few observations were observed to have a zero or a one.

In contrast, the ZINB's non-structural zero component and both components of the NBH model had relatively small standard deviations suggesting stabler estimates. Lambert (1992) reported that even when the structural zero component failed to converge that the estimates for the non-structural zero component for the ZIP model were still reasonable. She also argued that singular runs can still provide insight about bias. Furthermore, while the ZINB model could have had undiagnosed issues with convergence, particularly when θ was small and the sample size was small, this again argues for the NBH model as the more general model. Issues with fitting the ZINB model elsewhere have been reported (Lambert, 1992; Famoye & Singh, 2006).

Given the potential limitations with the ZINB reported here, future work could compare the NBH model to the zero-inflated generalized Poisson model (ZIGP) (Famoye & Singh, 2006). ZIGP combines a point mass at zero with a generalized Poisson model. The generalized Poisson model includes an additional parameter to model dispersion (α). The ZIGP has been found to be more easily estimated than the ZINB, however, it is unknown under what conditions the ZIGP might perform better than the ZINB (Famoye & Singh, 2006). Comparison of the ZIGP, ZINB, and NBH models could be a fruitful contribution to this literature. Additionally, given that many researchers use the same covariates for both the structural zero/zero and non-structural zero/count components (Atkins & Gallop, 2007; Bandyopadhyay et al., 2011; Hu et al., 2011; Rose et al., 2006), a sensible extension of this study would be examination of the ZIP(τ) or zero-altered model to reduce the number of estimated parameters (Lambert, 1992; Heilbron, 1994).

More work needs to be done to understand if the misspecification of the models, as reported here, really affects prediction. The results here show that prediction is affected by model misspecification but this needs to be corroborated in future simulation work as this disagrees with the general belief about these models. Similarly, simulation work needs to be done to see if the proportion of correctly identified structural zeros can be made to be larger than that observed here.

Future work could consider purposefully manipulate the proportion of zeros. This could be done in the ZINB model through manipulating both the structural and non-structural zero parameters. For the NBH model this would be done only through manipulating the zero parameters. Careful selection of the parameters would need to be done to assure that the models are generating roughly the same number of zeros. In this study, the proportion of zeros, on average, ranged from 53 to 76% for the ZINB model and from 53 to 60% for the NBH model. Therefore, use of the same parameters for both models, as done here, would not ensure that the expected proportion of zeros would be similar across models.

Given that covariate correlation had seemingly no major effect on model performance, future work could be done in this area. It would be very useful to understand when two variables are correlated, at what level of correlation will this affect model estimation and if it will affect the models differently. Given that many important predictors are correlated with one another, understanding and quantifying consequences of covariate correlation, within and across components, at a multitude of levels could be very helpful to an applied researcher using these models.

Final Thoughts

This study, along with the work of Min and Agresti (2005) and Miller (2007), suggest that hurdle models may be more versatile models than zero-inflated models. Based on the conditions considered, the NBH performed as well or better than the ZINB model in both studies. This study highlighted the importance of model misspecification and the importance of careful consideration of the data-generating mechanism of the zeros, as this can have dramatic implications for the estimated structural zero/zero parameters, confidence interval coverage, and bias in prediction. In the absence of a strong a priori theory about zeros, hurdle models should be considered, given their better overall performance.

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Appendix A: Summary Statistics

This appendix includes 58 tables. The the first 54 tables correspond to the first simulation study and last 4 tables to the second simulation study. The first 54 tables contain the mean, median, standard deviation, and mean squared error for a given set of conditions for $\beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1$, and γ_2 with the first 6 corresponding to specific values of sample size. The final 4 tables contain the mean of the estimated parameter for both correlated and orthogonal covariates and each data-generating mechanism. Throughout this appendix θ refers to the dispersion parameter.

First Simulation Study - Model Performance and Recovery

Table 32: Parameter behavior (mean and median) of the NBH and ZINB coefficients by dispersion parameter, θ, from the 2,000 simulates for $n = 250$.

	θ	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	1/4	-1.521	0.253	2.028	1.222	-0.247	-1.985
	1/2	-1.524	0.252	2.033	1.426	-0.245	-1.998
	1	-1.525	0.255	2.031	1.481	-0.253	-2.007
	2	-1.520	0.251	2.029	1.485	-0.249	-2.010
	5	-1.529	0.255	2.035	1.492	-0.251	-2.019
	10	-1.521	0.253	2.028	1.493	-0.250	-2.037
ZINB	1/4	-3.154	0.301	3.773	1.560	-0.254	-2.018
	1/2	-2.933	0.281	3.461	1.501	-0.248	-2.013
	1	-2.362	0.271	2.858	1.483	-0.250	-2.016
	2	-1.807	0.266	2.307	1.487	-0.254	-2.016
	5	-1.586	0.248	2.091	1.493	-0.251	-2.013
	10	-1.576	0.257	2.082	1.494	-0.251	-2.018
Median							
NBH	1/4	-1.510	0.253	2.017	1.542	-0.248	-1.977
	1/2	-1.511	0.254	2.022	1.505	-0.243	-1.988
	1	-1.514	0.257	2.021	1.502	-0.251	-1.993
	2	-1.506	0.251	2.021	1.495	-0.249	-1.990
	5	-1.520	0.254	2.026	1.497	-0.250	-1.995
	10	-1.511	0.252	2.019	1.498	-0.248	-2.012
ZINB	1/4	-1.126	0.255	1.916	1.569	-0.256	-2.015
	1/2	-1.367	0.257	2.013	1.514	-0.245	-2.006
	1	-1.476	0.258	2.054	1.494	-0.250	-2.006
	2	-1.524	0.261	2.057	1.493	-0.253	-2.008
	5	-1.524	0.245	2.037	1.497	-0.251	-2.003
	10	-1.533	0.255	2.050	1.498	-0.250	-2.009

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.25, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = -.25$, and $\gamma_2 = -2$.

Table 33: Parameter behavior of the NBH and ZINB coefficients by dispersion (standard deviations and mean squared error), θ, from the 2,000 simulates for n = 250.

		β_0	β_1	β_2	γ_0	γ_1	γ_2
Standard Deviation of the Simulates							
NBH	1/4	0.275	0.300	0.303	1.362	0.340	0.356
	1/2	0.277	0.298	0.306	0.599	0.304	0.332
	1	0.275	0.299	0.305	0.240	0.261	0.311
	2	0.274	0.299	0.304	0.171	0.221	0.301
	5	0.275	0.297	0.304	0.129	0.180	0.292
	10	0.273	0.297	0.305	0.111	0.162	0.293
ZINB	1/4	3.980	1.034	3.621	0.404	0.404	0.424
	1/2	3.401	0.683	3.176	0.282	0.307	0.330
	1	2.525	0.553	2.392	0.203	0.241	0.276
	2	1.365	0.465	1.309	0.147	0.194	0.245
	5	0.598	0.413	0.600	0.107	0.156	0.225
	10	0.466	0.398	0.483	0.091	0.139	0.216
Mean Squared Error							
NBH	1/4	0.153	0.177	0.192	1.076	0.168	0.214
	1/2	0.152	0.179	0.185	0.899	0.122	0.189
	1	0.152	0.178	0.186	0.052	0.078	0.148
	2	0.156	0.180	0.191	1.427	0.175	0.237
	5	0.150	0.179	0.185	1.287	0.134	0.221
	10	0.147	0.177	0.179	0.056	0.090	0.189
ZINB	1/4	19.297	0.888	17.252	0.142	0.161	0.191
	1/2	13.914	0.875	11.986	0.135	0.152	0.184
	1	3.106	0.378	2.796	0.042	0.066	0.112
	2	21.998	1.123	19.619	0.151	0.177	0.209
	5	15.612	1.080	13.247	0.143	0.161	0.216
	10	4.692	0.466	4.203	0.044	0.079	0.134

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.5, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = 0$, and $\gamma_2 = -2$.

Table 34: Parameter behavior (mean and median) of the NBH and ZINB coefficients by dispersion parameter, θ, from the 2,000 simulates for $n = 500$.

	θ	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	1/4	-1.511	0.253	2.014	1.364	-0.246	-1.988
	1/2	-1.512	0.249	2.016	1.467	-0.247	-1.992
	1	-1.512	0.251	2.014	1.489	-0.251	-1.998
	2	-1.512	0.250	2.016	1.492	-0.252	-2.001
	5	-1.511	0.252	2.015	1.495	-0.250	-2.008
	10	-1.511	0.250	2.015	1.497	-0.253	-2.012
ZINB	1/4	-3.024	0.269	3.604	1.549	-0.257	-2.006
	1/2	-2.607	0.267	3.110	1.498	-0.250	-2.007
	1	-2.014	0.261	2.505	1.487	-0.251	-2.006
	2	-1.603	0.256	2.103	1.494	-0.251	-2.007
	5	-1.543	0.253	2.043	1.496	-0.249	-2.009
	10	-1.535	0.254	2.039	1.498	-0.251	-2.008
				Median			
NBH	1/4	-1.505	0.253	2.010	1.520	-0.246	-1.987
	1/2	-1.507	0.249	2.013	1.501	-0.249	-1.990
	1	-1.505	0.250	2.010	1.500	-0.251	-1.993
	2	-1.506	0.251	2.013	1.498	-0.250	-1.992
	5	-1.508	0.255	2.012	1.498	-0.249	-1.998
	10	-1.507	0.251	2.011	1.499	-0.251	-2.005
ZINB	1/4	-1.250	0.246	1.928	1.550	-0.260	-2.001
	1/2	-1.429	0.258	2.005	1.506	-0.248	-2.004
	1	-1.498	0.257	2.045	1.498	-0.250	-2.002
	2	-1.508	0.256	2.021	1.499	-0.250	-2.003
	5	-1.520	0.253	2.025	1.498	-0.248	-2.006
	10	-1.516	0.254	2.028	1.500	-0.251	-2.005

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.25, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = -.25$, and $\gamma_2 = -2$.

Table 35: Parameter behavior of the NBH and ZINB coefficients by dispersion (standard deviations and mean squared error), θ, from the 2,000 simulates for n = 500.

		β_0	β_1	β_2	γ_0	γ_1	γ_2
Standard Deviation of the Simulates							
NBH	1/4	0.185	0.210	0.214	0.855	0.241	0.254
	1/2	0.185	0.210	0.214	0.316	0.215	0.230
	1	0.185	0.211	0.211	0.164	0.191	0.210
	2	0.183	0.210	0.213	0.118	0.162	0.200
	5	0.185	0.210	0.213	0.089	0.135	0.190
	10	0.185	0.210	0.214	0.078	0.121	0.187
ZINB	1/4	3.581	0.565	3.281	0.316	0.285	0.286
	1/2	2.843	0.451	2.652	0.219	0.214	0.225
	1	1.852	0.383	1.741	0.151	0.172	0.187
	2	0.681	0.325	0.641	0.106	0.140	0.166
	5	0.319	0.301	0.334	0.076	0.114	0.149
	10	0.292	0.289	0.309	0.066	0.102	0.144
Mean Squared Error							
NBH	1/4	0.069	0.089	0.095	0.445	0.085	0.103
	1/2	0.068	0.089	0.092	0.356	0.064	0.088
	1	0.068	0.088	0.088	0.026	0.042	0.065
	2	0.069	0.089	0.094	0.506	0.092	0.111
	5	0.069	0.089	0.091	0.441	0.069	0.101
	10	0.068	0.088	0.087	0.027	0.048	0.080
ZINB	1/4	14.105	0.367	12.631	0.085	0.082	0.090
	1/2	10.048	0.337	8.621	0.080	0.076	0.086
	1	1.399	0.198	1.235	0.022	0.035	0.051
	2	16.113	0.407	14.241	0.091	0.085	0.096
	5	10.504	0.367	8.928	0.087	0.080	0.095
	10	2.046	0.228	1.754	0.024	0.040	0.060

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.5, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = 0$, and $\gamma_2 = -2$.

Table 36: Parameter behavior of the NBH and ZINB coefficients by covariate correlation within a component, ρ, from the 2,000 simulates for $n = 250$.

	ρ	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	0	-1.524	0.255	2.032	1.437	-0.247	-2.012
	.3	-1.523	0.251	2.029	1.430	-0.251	-2.007
ZINB	0	-2.213	0.280	2.737	1.501	-0.246	-2.017
	.3	-2.260	0.261	2.787	1.505	-0.257	-2.014
Median							
NBH	0	-1.512	0.254	2.022	1.507	-0.247	-1.993
	.3	-1.512	0.253	2.021	1.506	-0.249	-1.992
ZINB	0	-1.426	0.262	2.022	1.509	-0.246	-2.008
	.3	-1.424	0.248	2.020	1.512	-0.255	-2.007
Standard Deviation of the Simulates							
NBH	0	0.277	0.296	0.303	0.435	0.243	0.321
	.3	0.273	0.300	0.306	0.435	0.246	0.308
ZINB	0	2.014	0.572	1.883	0.212	0.231	0.284
	.3	2.097	0.610	1.977	0.200	0.250	0.288
Mean Squared Error							
NBH	0	0.154	0.176	0.185	0.772	0.127	0.208
	.3	0.150	0.180	0.188	0.827	0.129	0.192
ZINB	0	12.659	0.748	11.073	0.115	0.123	0.172
	.3	13.548	0.856	11.962	0.104	0.142	0.177

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.25, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = -.25$, and $\gamma_2 = -2$.

Table 37: Parameter behavior of the NBH and ZINB coefficients by covariate correlation within a component, ρ, from the 2,000 simulates for $n = 500$.

	ρ	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	0	-1.512	0.252	2.015	1.467	-0.249	-1.998
	.3	-1.511	0.250	2.015	1.468	-0.251	-2.002
ZINB	0	-2.062	0.265	2.573	1.504	-0.250	-2.007
	.3	-2.046	0.256	2.561	1.504	-0.253	-2.007
Median							
NBH	0	-1.507	0.251	2.012	1.502	-0.249	-1.994
	.3	-1.505	0.252	2.011	1.503	-0.249	-1.995
ZINB	0	-1.460	0.257	2.006	1.508	-0.249	-2.004
	.3	-1.447	0.252	2.012	1.509	-0.253	-2.003
Standard Deviation of the Simulates							
NBH	0	0.408	0.495	0.498	0.875	0.466	0.752
	.3	0.443	0.503	0.512	0.803	0.444	0.667
ZINB	0	3.355	2.040	3.286	0.285	0.386	0.473
	.3	3.147	1.813	3.171	0.293	0.428	0.480
Mean Squared Error							
NBH	0	0.335	0.490	0.501	2.698	0.464	1.204
	.3	0.396	0.507	0.532	2.449	0.419	0.960
ZINB	0	28.066	10.489	27.087	0.205	0.348	0.508
	.3	24.073	8.154	24.799	0.216	0.422	0.521

Note. The true parameters are $\beta_0 = -1.5, \beta_1 = 0.25, \beta_2 = 2, \gamma_0 = 1.5, \gamma_1 = -.25$, and $\gamma_2 = -2$.

Table 38: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = \frac{1}{4}$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.550	-0.009	2.071	0.955	-0.017	-1.943
NBH	250	-1.527	0.003	2.031	1.301	0.009	-1.976
	500	-1.506	0.003	2.004	1.373	0.004	-1.996
	100	-3.166	0.032	4.029	1.588	-0.015	-2.010
ZINB	250	-3.146	-0.025	3.806	1.552	0.017	-2.017
	500	-3.116	0.000	3.706	1.546	-0.003	-2.011
				Median			
	100	-1.512	-0.012	2.067	1.584	-0.005	-1.924
NBH	250	-1.511	0.007	2.016	1.537	0.004	-1.973
	500	-1.498	0.006	2.002	1.521	-0.002	-1.989
	100	-0.689	-0.020	1.875	1.608	-0.012	-2.012
ZINB	250	-1.071	-0.012	1.855	1.555	0.019	-2.015
	500	-1.215	-0.008	1.898	1.547	-0.004	-2.001
				Standard Deviation			
	100	0.396	0.493	0.491	2.100	0.620	0.635
NBH	250	0.277	0.291	0.308	1.138	0.356	0.355
	500	0.193	0.201	0.212	0.822	0.238	0.247
	100	4.890	3.439	4.754	0.532	0.632	0.656
ZINB	250	3.936	0.935	3.626	0.393	0.366	0.402
	500	3.608	0.525	3.331	0.311	0.271	0.276
				Mean Squared Error			
	100	0.316	0.486	0.488	9.111	0.769	0.810
NBH	250	0.155	0.169	0.191	2.629	0.253	0.253
	500	0.075	0.081	0.090	1.369	0.113	0.122
	100	50.579	23.651	49.304	0.573	0.800	0.860
ZINB	250	33.687	1.749	29.547	0.312	0.267	0.323
	500	28.638	0.551	25.102	0.195	0.147	0.152

Table 39: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = \frac{1}{4}$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.556	0.545	2.078	0.958	-0.018	-1.956
NBH	250	-1.523	0.513	2.030	1.263	0.005	-1.985
	500	-1.510	0.508	2.015	1.372	0.003	-1.980
	100	-3.674	1.247	4.455	1.596	-0.051	-2.018
ZINB	250	-3.062	0.629	3.644	1.544	-0.001	-2.009
	500	-2.827	0.571	3.377	1.536	0.001	-2.013
				Median			
	100	-1.525	0.543	2.067	1.589	-0.037	-1.925
NBH	250	-1.516	0.511	2.020	1.519	-0.000	-1.978
	500	-1.509	0.507	2.009	1.517	0.003	-1.981
	100	-0.770	0.498	1.950	1.631	-0.027	-1.993
ZINB	250	-1.214	0.531	2.001	1.564	0.004	-2.004
	500	-1.371	0.540	1.982	1.537	0.003	-2.007
				Standard Deviation			
	100	0.411	0.503	0.505	2.099	0.592	0.588
NBH	250	0.278	0.299	0.300	1.246	0.328	0.347
	500	0.188	0.206	0.211	0.778	0.224	0.242
	100	5.633	4.138	5.333	0.560	0.670	0.692
ZINB	250	3.857	0.929	3.473	0.421	0.401	0.409
	500	3.367	0.544	3.063	0.337	0.295	0.279
				Mean Squared Error			
	100	0.341	0.508	0.516	9.103	0.701	0.693
NBH	250	0.155	0.179	0.181	3.160	0.215	0.241
	500	0.071	0.085	0.089	1.225	0.100	0.117
	100	68.173	34.790	62.904	0.637	0.899	0.959
ZINB	250	32.187	1.743	26.819	0.356	0.322	0.334
	500	24.429	0.598	20.659	0.229	0.173	0.156

Table 40: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = \frac{1}{4}$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.550	-0.015	2.077	0.850	-0.508	-1.961
	250	-1.515	0.003	2.024	1.218	-0.490	-1.993
	500	-1.510	-0.001	2.016	1.331	-0.496	-1.977
ZINB	100	-2.897	-0.096	4.006	1.624	-0.519	-2.058
	250	-3.092	-0.001	3.754	1.565	-0.477	-2.021
	500	-3.225	-0.006	3.828	1.567	-0.515	-2.013
				Median			
NBH	100	-1.522	-0.010	2.025	1.604	-0.494	-1.938
	250	-1.513	0.003	2.016	1.564	-0.484	-1.987
	500	-1.504	-0.005	2.014	1.521	-0.493	-1.973
ZINB	100	-0.650	-0.004	1.836	1.654	-0.501	-2.029
	250	-0.988	0.016	1.879	1.586	-0.482	-2.012
	500	-1.231	-0.015	1.894	1.560	-0.522	-2.008
				Standard Deviation			
NBH	100	0.410	0.496	0.495	2.327	0.654	0.653
	250	0.281	0.299	0.308	1.440	0.359	0.373
	500	0.190	0.207	0.217	0.937	0.239	0.253
ZINB	100	4.765	3.712	4.907	0.527	0.676	0.698
	250	3.974	0.979	3.620	0.404	0.395	0.407
	500	3.752	0.540	3.440	0.329	0.276	0.270
				Mean Squared Error			
NBH	100	0.339	0.492	0.495	11.246	0.854	0.854
	250	0.158	0.179	0.191	4.224	0.258	0.279
	500	0.072	0.086	0.095	1.784	0.115	0.129
ZINB	100	47.357	27.561	52.164	0.571	0.914	0.976
	250	34.105	1.918	29.275	0.331	0.312	0.332
	500	31.125	0.583	27.006	0.221	0.152	0.146

Table 41: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = \frac{1}{4}$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.548	0.510	2.077	0.919	-0.519	-1.933
NBH	250	-1.522	0.503	2.030	1.178	-0.501	-1.995
	500	-1.515	0.504	2.015	1.366	-0.493	-1.986
	100	-3.439	1.259	4.288	1.615	-0.527	-2.064
ZINB	250	-3.126	0.620	3.734	1.567	-0.497	-2.045
	500	-2.969	0.531	3.544	1.548	-0.499	-2.002
				Median			
	100	-1.516	0.497	2.048	1.596	-0.497	-1.900
NBH	250	-1.512	0.493	2.013	1.559	-0.497	-1.977
	500	-1.509	0.496	2.014	1.514	-0.498	-1.989
	100	-0.697	0.536	1.869	1.648	-0.520	-2.020
ZINB	250	-1.180	0.518	1.968	1.579	-0.493	-2.036
	500	-1.298	0.486	1.961	1.546	-0.502	-2.000
				Standard Deviation			
	100	0.412	0.484	0.506	2.198	0.584	0.606
NBH	250	0.269	0.295	0.294	1.495	0.336	0.355
	500	0.190	0.204	0.203	0.839	0.229	0.245
	100	5.483	4.086	5.417	0.538	0.713	1.031
ZINB	250	4.027	1.117	3.604	0.437	0.404	0.449
	500	3.558	0.564	3.218	0.350	0.294	0.296
				Mean Squared Error			
	100	0.342	0.468	0.517	10.001	0.682	0.739
NBH	250	0.145	0.174	0.174	4.573	0.226	0.252
	500	0.072	0.083	0.083	1.425	0.105	0.120
	100	63.875	33.965	63.897	0.591	1.018	2.129
ZINB	250	35.063	2.510	28.980	0.386	0.326	0.405
	500	27.472	0.638	23.086	0.247	0.173	0.175

Table 42: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = \frac{1}{2}$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.555	0.008	2.081	1.173	-0.026	-1.963
NBH	250	-1.520	-0.005	2.022	1.425	0.017	-2.002
	500	-1.517	-0.004	2.027	1.458	0.005	-1.983
	100	-3.429	0.024	4.024	1.510	-0.010	-2.019
ZINB	250	-2.950	0.014	3.483	1.503	0.003	-2.018
	500	-2.738	0.011	3.254	1.505	-0.003	-2.010
				Median			
	100	-1.521	0.008	2.055	1.535	-0.025	-1.935
NBH	250	-1.496	0.003	2.009	1.496	0.026	-1.993
	500	-1.507	-0.004	2.028	1.489	0.003	-1.981
	100	-1.199	0.018	2.030	1.518	-0.010	-2.015
ZINB	250	-1.387	0.023	2.005	1.510	0.009	-2.006
	500	-1.435	0.006	2.005	1.520	-0.000	-2.004
				Standard Deviation			
	100	0.402	0.491	0.496	1.601	0.544	0.585
NBH	250	0.282	0.295	0.309	0.602	0.300	0.330
	500	0.193	0.206	0.209	0.333	0.212	0.225
	100	4.329	2.420	4.170	0.379	0.473	0.497
ZINB	250	3.354	0.662	3.159	0.271	0.286	0.311
	500	2.977	0.425	2.813	0.224	0.206	0.210
				Mean Squared Error			
	100	0.327	0.482	0.499	5.230	0.592	0.686
NBH	250	0.159	0.174	0.191	0.731	0.180	0.217
	500	0.074	0.085	0.088	0.224	0.089	0.102
	100	41.184	11.709	38.863	0.287	0.447	0.494
ZINB	250	24.601	0.876	22.153	0.146	0.163	0.194
	500	19.254	0.361	17.390	0.100	0.085	0.089

Table 43: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = \frac{1}{2}$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.548	0.522	2.066	1.200	-0.027	-1.971
NBH	250	-1.526	0.502	2.044	1.441	0.009	-2.007
	500	-1.503	0.496	2.009	1.477	0.003	-1.996
	100	-3.376	0.905	3.950	1.497	0.003	-2.009
ZINB	250	-2.633	0.587	3.131	1.485	0.012	-2.016
	500	-2.377	0.523	2.879	1.498	-0.006	-2.000
				Median			
	100	-1.513	0.517	2.056	1.504	-0.026	-1.930
NBH	250	-1.517	0.503	2.039	1.502	0.008	-2.001
	500	-1.495	0.497	2.002	1.500	0.001	-1.998
	100	-1.199	0.545	2.081	1.530	0.006	-1.994
ZINB	250	-1.411	0.545	2.024	1.503	0.007	-2.009
	500	-1.429	0.504	1.987	1.511	-0.002	-2.001
				Standard Deviation			
	100	0.406	0.499	0.498	1.447	0.533	0.572
NBH	250	0.275	0.298	0.301	0.540	0.287	0.330
	500	0.192	0.201	0.208	0.272	0.199	0.214
	100	4.524	2.921	4.336	0.403	0.490	0.526
ZINB	250	3.055	0.657	2.831	0.303	0.304	0.326
	500	2.581	0.418	2.381	0.237	0.215	0.227
				Mean Squared Error			
	100	0.332	0.498	0.499	4.274	0.570	0.654
NBH	250	0.152	0.177	0.184	0.586	0.165	0.217
	500	0.074	0.081	0.086	0.149	0.079	0.091
	100	44.450	17.230	41.390	0.325	0.479	0.553
ZINB	250	19.945	0.871	17.302	0.184	0.184	0.213
	500	14.085	0.350	12.106	0.113	0.092	0.103

Table 44: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = \frac{1}{2}$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.549	-0.007	2.081	1.217	-0.524	-1.974
NBH	250	-1.518	-0.004	2.032	1.420	-0.500	-2.007
	500	-1.515	0.001	2.016	1.474	-0.501	-1.992
	100	-3.468	0.063	4.115	1.531	-0.512	-2.061
ZINB	250	-3.079	0.002	3.627	1.515	-0.504	-2.011
	500	-2.933	0.015	3.431	1.499	-0.498	-2.008
				Median			
	100	-1.510	0.001	2.071	1.531	-0.497	-1.941
NBH	250	-1.506	-0.009	2.019	1.513	-0.499	-1.989
	500	-1.514	0.005	2.004	1.509	-0.496	-1.991
	100	-1.196	0.043	2.099	1.551	-0.502	-2.024
ZINB	250	-1.349	0.002	1.968	1.523	-0.507	-2.001
	500	-1.434	0.015	1.968	1.503	-0.497	-2.006
				Standard Deviation			
	100	0.411	0.512	0.510	1.476	0.573	0.590
NBH	250	0.284	0.298	0.310	0.691	0.317	0.351
	500	0.189	0.201	0.213	0.292	0.221	0.232
	100	4.433	2.632	4.241	0.380	0.478	0.650
ZINB	250	3.541	0.700	3.325	0.283	0.293	0.331
	500	3.193	0.435	2.975	0.226	0.213	0.224
				Mean Squared Error			
	100	0.340	0.525	0.527	4.436	0.657	0.697
NBH	250	0.161	0.178	0.194	0.962	0.201	0.246
	500	0.072	0.081	0.091	0.171	0.098	0.108
	100	43.166	13.852	40.440	0.290	0.457	0.849
ZINB	250	27.568	0.980	24.750	0.160	0.172	0.219
	500	22.432	0.379	19.743	0.103	0.091	0.100

Table 45: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = \frac{1}{2}$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	100	-1.552	0.514	2.077	1.178	-0.506	-1.968
	250	-1.528	0.511	2.038	1.436	-0.495	-2.005
	500	-1.510	0.503	2.012	1.466	-0.496	-1.998
ZINB	100	-3.687	1.063	4.253	1.520	-0.524	-2.053
	250	-2.874	0.579	3.382	1.496	-0.495	-2.020
	500	-2.399	0.548	2.889	1.497	-0.494	-2.006
Median							
NBH	100	-1.522	0.505	2.065	1.531	-0.485	-1.944
	250	-1.515	0.507	2.021	1.509	-0.492	-1.997
	500	-1.510	0.496	2.008	1.502	-0.496	-1.997
ZINB	100	-1.288	0.554	2.186	1.542	-0.512	-2.011
	250	-1.389	0.531	2.032	1.505	-0.494	-2.010
	500	-1.445	0.521	2.010	1.500	-0.487	-2.010
Standard Deviation							
NBH	100	0.411	0.494	0.487	1.523	0.539	0.543
	250	0.275	0.298	0.299	0.567	0.297	0.333
	500	0.191	0.208	0.205	0.358	0.207	0.224
ZINB	100	4.776	3.131	4.510	0.384	0.509	0.549
	250	3.377	0.650	3.133	0.293	0.304	0.343
	500	2.635	0.433	2.416	0.242	0.220	0.233
Mean Squared Error							
NBH	100	0.341	0.488	0.480	4.741	0.581	0.590
	250	0.152	0.177	0.180	0.646	0.177	0.222
	500	0.073	0.086	0.084	0.257	0.085	0.101
ZINB	100	50.399	19.922	45.744	0.295	0.519	0.606
	250	24.686	0.852	21.542	0.172	0.185	0.236
	500	14.689	0.377	12.461	0.117	0.097	0.108

Table 46: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 1$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.534	0.002	2.054	1.401	-0.024	-2.007
NBH	250	-1.521	-0.005	2.040	1.472	0.001	-2.011
	500	-1.517	0.001	2.018	1.487	-0.001	-1.993
	100	-3.230	0.024	3.782	1.479	-0.020	-2.027
ZINB	250	-2.332	0.014	2.835	1.483	0.009	-2.020
	500	-2.061	-0.004	2.552	1.485	0.001	-2.009
				Median			
	100	-1.506	0.011	2.023	1.494	-0.014	-1.971
NBH	250	-1.506	-0.007	2.021	1.496	-0.002	-1.992
	500	-1.505	-0.001	2.014	1.498	0.003	-1.993
	100	-1.483	-0.003	2.188	1.493	-0.016	-2.015
ZINB	250	-1.461	0.008	2.029	1.493	0.006	-2.016
	500	-1.506	0.015	2.038	1.490	0.006	-2.001
				Standard Deviation			
	100	0.425	0.509	0.509	0.721	0.486	0.697
NBH	250	0.279	0.297	0.305	0.287	0.261	0.318
	500	0.199	0.202	0.221	0.165	0.185	0.203
	100	3.813	1.606	3.630	0.269	0.357	0.405
ZINB	250	2.497	0.507	2.374	0.205	0.224	0.252
	500	1.888	0.351	1.809	0.155	0.162	0.172
				Mean Squared Error			
	100	0.362	0.519	0.521	1.049	0.473	0.972
NBH	250	0.156	0.176	0.187	0.165	0.136	0.203
	500	0.080	0.082	0.098	0.055	0.068	0.082
	100	32.069	5.157	29.515	0.145	0.255	0.328
ZINB	250	13.161	0.514	11.968	0.084	0.101	0.128
	500	7.443	0.247	6.850	0.048	0.052	0.059

Table 47: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 1$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.542	0.504	2.063	1.415	-0.023	-2.017
NBH	250	-1.531	0.509	2.034	1.487	-0.012	-2.016
	500	-1.511	0.508	2.009	1.487	-0.005	-1.988
	100	-3.070	0.785	3.534	1.453	0.009	-2.026
ZINB	250	-2.073	0.542	2.556	1.472	0.014	-2.017
	500	-1.843	0.531	2.336	1.487	-0.001	-2.001
				Median			
	100	-1.502	0.518	2.037	1.489	-0.017	-1.965
NBH	250	-1.513	0.502	2.021	1.499	-0.011	-2.001
	500	-1.508	0.506	2.011	1.497	-0.007	-1.986
	100	-1.586	0.542	2.275	1.473	0.014	-1.986
ZINB	250	-1.515	0.527	2.050	1.484	0.010	-2.005
	500	-1.490	0.507	2.027	1.501	0.001	-1.998
				Standard Deviation			
	100	0.420	0.501	0.502	0.641	0.448	0.596
NBH	250	0.284	0.300	0.317	0.220	0.251	0.288
	500	0.183	0.204	0.202	0.165	0.175	0.189
	100	3.723	2.009	3.523	0.291	0.383	0.433
ZINB	250	2.027	0.507	1.916	0.211	0.233	0.269
	500	1.507	0.365	1.406	0.158	0.172	0.183
				Mean Squared Error			
	100	0.354	0.502	0.507	0.830	0.402	0.710
NBH	250	0.162	0.180	0.202	0.097	0.126	0.166
	500	0.067	0.083	0.082	0.054	0.061	0.072
	100	30.183	8.151	27.164	0.171	0.293	0.376
ZINB	250	8.545	0.515	7.652	0.090	0.108	0.145
	500	4.660	0.267	4.067	0.050	0.059	0.067

Table 48: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 1$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.542	-0.001	2.072	1.374	-0.520	-2.051
	250	-1.528	0.010	2.033	1.485	-0.500	-2.016
	500	-1.519	0.003	2.018	1.487	-0.500	-1.996
ZINB	100	-3.257	-0.026	3.806	1.484	-0.515	-2.029
	250	-2.727	0.001	3.221	1.476	-0.494	-2.018
	500	-2.236	-0.015	2.744	1.489	-0.501	-1.998
				Median			
NBH	100	-1.515	0.000	2.049	1.488	-0.479	-1.984
	250	-1.514	0.010	2.019	1.510	-0.505	-1.999
	500	-1.514	0.003	2.008	1.504	-0.502	-1.992
ZINB	100	-1.472	-0.044	2.204	1.498	-0.513	-2.003
	250	-1.488	-0.011	2.078	1.488	-0.491	-2.008
	500	-1.479	-0.016	2.028	1.499	-0.503	-1.990
				Standard Deviation			
NBH	100	0.421	0.513	0.509	0.798	0.705	0.824
	250	0.278	0.302	0.305	0.255	0.266	0.341
	500	0.191	0.203	0.213	0.183	0.199	0.215
ZINB	100	3.868	1.909	3.753	0.278	0.375	0.440
	250	2.965	0.573	2.817	0.205	0.237	0.280
	500	2.323	0.378	2.213	0.162	0.175	0.186
				Mean Squared Error			
NBH	100	0.356	0.526	0.522	1.288	0.993	1.361
	250	0.156	0.182	0.187	0.130	0.142	0.233
	500	0.073	0.083	0.091	0.067	0.079	0.092
ZINB	100	32.996	7.284	31.417	0.155	0.282	0.387
	250	19.080	0.656	17.354	0.085	0.113	0.157
	500	11.334	0.287	10.345	0.053	0.061	0.069

Table 49: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 1$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
	100	-1.542	0.517	2.074	1.368	-0.514	-2.003
NBH	250	-1.533	0.513	2.036	1.477	-0.495	-2.005
	500	-1.510	0.502	2.016	1.489	-0.500	-2.001
	100	-3.210	0.744	3.781	1.490	-0.521	-2.068
ZINB	250	-2.244	0.557	2.730	1.483	-0.499	-2.018
	500	-1.916	0.531	2.406	1.489	-0.497	-2.010
Median							
	100	-1.522	0.505	2.067	1.493	-0.502	-1.954
NBH	250	-1.526	0.515	2.033	1.496	-0.492	-1.993
	500	-1.508	0.497	2.017	1.500	-0.498	-1.993
	100	-1.483	0.514	2.216	1.505	-0.518	-2.044
ZINB	250	-1.456	0.527	2.064	1.493	-0.504	-2.009
	500	-1.492	0.523	2.053	1.500	-0.500	-2.010
Standard Deviation							
	100	0.400	0.495	0.483	0.816	0.493	0.603
NBH	250	0.275	0.294	0.304	0.242	0.260	0.317
	500	0.192	0.211	0.207	0.172	0.185	0.201
	100	3.954	2.307	3.850	0.285	0.401	0.602
ZINB	250	2.404	0.568	2.250	0.216	0.240	0.289
	500	1.683	0.383	1.544	0.168	0.182	0.195
Mean Squared Error							
	100	0.322	0.490	0.472	1.348	0.486	0.728
NBH	250	0.153	0.173	0.186	0.117	0.135	0.201
	500	0.074	0.089	0.086	0.059	0.068	0.081
	100	34.191	10.704	32.812	0.162	0.322	0.730
ZINB	250	12.106	0.648	10.654	0.093	0.115	0.167
	500	5.839	0.295	4.930	0.056	0.066	0.076

Table 50: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 2$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	100	-1.554	-0.015	2.071	1.452	-0.013	-2.034
	250	-1.528	0.006	2.037	1.485	0.001	-2.007
	500	-1.510	-0.004	2.016	1.491	0.002	-2.007
ZINB	100	-2.559	-0.035	3.116	1.477	-0.008	-2.042
	250	-1.730	0.001	2.232	1.490	-0.002	-2.010
	500	-1.609	0.001	2.102	1.491	0.001	-1.999
Median							
NBH	100	-1.526	-0.010	2.041	1.503	-0.014	-1.969
	250	-1.513	0.006	2.028	1.494	0.006	-1.984
	500	-1.503	-0.002	2.015	1.497	0.002	-2.002
ZINB	100	-1.559	-0.034	2.195	1.487	-0.015	-2.020
	250	-1.534	0.007	2.061	1.496	-0.001	-2.009
	500	-1.513	-0.001	2.023	1.495	0.004	-1.992
Standard Deviation							
NBH	100	0.410	0.499	0.494	0.511	0.418	0.670
	250	0.272	0.302	0.302	0.180	0.215	0.305
	500	0.188	0.208	0.214	0.125	0.157	0.190
ZINB	100	2.926	1.321	2.844	0.212	0.293	0.348
	250	1.133	0.431	1.096	0.143	0.177	0.229
	500	0.776	0.314	0.738	0.110	0.132	0.155
Mean Squared Error							
NBH	100	0.339	0.499	0.493	0.524	0.350	0.900
	250	0.149	0.183	0.184	0.065	0.093	0.186
	500	0.071	0.086	0.092	0.031	0.049	0.072
ZINB	100	18.242	3.490	17.421	0.091	0.171	0.244
	250	2.621	0.372	2.456	0.041	0.063	0.105
	500	1.217	0.197	1.098	0.024	0.035	0.048

Table 51: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 2$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.551	0.522	2.076	1.466	-0.012	-2.037
NBH	250	-1.510	0.503	2.030	1.489	-0.003	-1.999
	500	-1.512	0.501	2.019	1.491	0.001	-1.996
	100	-2.502	0.742	3.031	1.473	-0.013	-2.040
ZINB	250	-1.656	0.549	2.142	1.484	0.007	-2.013
	500	-1.566	0.521	2.066	1.493	-0.001	-2.006
				Median			
	100	-1.527	0.500	2.067	1.495	-0.003	-1.999
NBH	250	-1.496	0.500	2.022	1.501	-0.002	-1.990
	500	-1.507	0.500	2.017	1.499	-0.001	-1.992
	100	-1.569	0.570	2.190	1.482	-0.011	-2.012
ZINB	250	-1.518	0.542	2.044	1.490	0.004	-2.005
	500	-1.489	0.521	2.011	1.500	-0.002	-2.007
				Standard Deviation			
	100	0.403	0.478	0.496	0.286	0.384	0.535
NBH	250	0.275	0.297	0.298	0.176	0.210	0.283
	500	0.193	0.205	0.210	0.123	0.151	0.179
	100	2.985	1.736	2.923	0.216	0.294	0.379
ZINB	250	0.835	0.429	0.767	0.150	0.176	0.242
	500	0.529	0.302	0.482	0.114	0.137	0.163
				Mean Squared Error			
	100	0.328	0.457	0.497	0.165	0.295	0.574
NBH	250	0.151	0.176	0.178	0.062	0.088	0.160
	500	0.075	0.084	0.089	0.030	0.046	0.064
	100	18.823	6.087	18.141	0.094	0.173	0.288
ZINB	250	1.419	0.370	1.197	0.045	0.062	0.117
	500	0.563	0.183	0.468	0.026	0.037	0.053

Table 52: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 2$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.549	-0.014	2.078	1.433	-0.514	-2.091
NBH	250	-1.515	-0.003	2.018	1.486	-0.495	-2.012
	500	-1.518	-0.001	2.018	1.489	-0.501	-2.001
	100	-2.879	-0.013	3.428	1.478	-0.517	-2.031
ZINB	250	-1.926	0.004	2.422	1.485	-0.502	-2.016
	500	-1.679	0.002	2.175	1.494	-0.498	-2.009
				Median			
	100	-1.524	-0.005	2.081	1.485	-0.481	-1.991
NBH	250	-1.496	0.004	2.005	1.498	-0.494	-1.976
	500	-1.515	-0.002	2.017	1.497	-0.500	-1.987
	100	-1.604	-0.011	2.226	1.489	-0.504	-2.008
ZINB	250	-1.504	0.002	2.060	1.487	-0.503	-2.010
	500	-1.506	0.002	2.009	1.497	-0.492	-2.002
				Standard Deviation			
	100	0.403	0.496	0.500	0.478	0.505	0.904
NBH	250	0.278	0.290	0.307	0.182	0.228	0.332
	500	0.190	0.202	0.210	0.130	0.169	0.212
	100	3.326	1.432	3.266	0.211	0.312	0.384
ZINB	250	1.722	0.469	1.661	0.152	0.193	0.254
	500	0.991	0.323	0.956	0.113	0.143	0.168
				Mean Squared Error			
	100	0.327	0.493	0.505	0.462	0.510	1.643
NBH	250	0.155	0.169	0.189	0.066	0.104	0.220
	500	0.073	0.081	0.088	0.034	0.057	0.090
	100	24.021	4.101	23.362	0.090	0.195	0.296
ZINB	250	6.109	0.440	5.694	0.047	0.075	0.129
	500	1.997	0.208	1.859	0.026	0.041	0.056

Table 53: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 2$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	100	-1.538	0.513	2.072	1.458	-0.506	-2.093
	250	-1.523	0.500	2.038	1.485	-0.489	-2.015
	500	-1.512	0.500	2.016	1.496	-0.502	-1.997
ZINB	100	-2.648	0.680	3.156	1.474	-0.515	-2.051
	250	-1.818	0.563	2.306	1.484	-0.495	-2.021
	500	-1.623	0.512	2.112	1.493	-0.502	-2.010
Median							
NBH	100	-1.514	0.505	2.057	1.491	-0.487	-1.992
	250	-1.513	0.495	2.027	1.494	-0.494	-1.992
	500	-1.509	0.500	2.009	1.500	-0.503	-1.991
ZINB	100	-1.556	0.537	2.185	1.491	-0.503	-2.022
	250	-1.525	0.540	2.049	1.492	-0.489	-2.007
	500	-1.525	0.506	2.026	1.501	-0.504	-2.003
Standard Deviation							
NBH	100	0.405	0.493	0.482	0.356	0.419	0.879
	250	0.274	0.300	0.299	0.174	0.225	0.316
	500	0.192	0.206	0.210	0.123	0.161	0.198
ZINB	100	3.257	1.715	3.160	0.216	0.328	0.403
	250	1.398	0.469	1.314	0.158	0.191	0.259
	500	0.712	0.330	0.645	0.123	0.145	0.170
Mean Squared Error							
NBH	100	0.329	0.486	0.470	0.255	0.351	1.554
	250	0.151	0.180	0.180	0.061	0.101	0.200
	500	0.074	0.085	0.088	0.030	0.052	0.078
ZINB	100	22.535	5.913	21.307	0.094	0.216	0.327
	250	4.011	0.445	3.547	0.050	0.073	0.134
	500	1.030	0.218	0.845	0.030	0.042	0.058

Table 54: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 5$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
	100	-1.557	-0.008	2.083	1.494	-0.031	-2.100
NBH	250	-1.535	0.008	2.035	1.493	0.000	-2.022
	500	-1.511	0.004	2.011	1.493	-0.002	-2.002
	100	-1.897	0.011	2.417	1.487	-0.011	-2.024
ZINB	250	-1.573	-0.001	2.078	1.495	-0.002	-2.008
	500	-1.545	0.010	2.034	1.496	-0.000	-2.009
Median							
	100	-1.529	0.006	2.083	1.507	-0.024	-2.012
NBH	250	-1.527	0.001	2.029	1.501	-0.002	-2.007
	500	-1.514	0.013	2.007	1.493	-0.001	-1.992
	100	-1.557	0.008	2.136	1.491	-0.000	-2.013
ZINB	250	-1.514	-0.002	2.045	1.498	-0.004	-1.998
	500	-1.517	0.002	2.009	1.498	-0.000	-2.007
Standard Deviation							
	100	0.407	0.483	0.502	0.207	0.330	0.809
NBH	250	0.277	0.299	0.300	0.136	0.172	0.291
	500	0.193	0.205	0.213	0.097	0.129	0.177
	100	1.711	0.958	1.695	0.150	0.229	0.307
ZINB	250	0.576	0.383	0.574	0.109	0.135	0.208
	500	0.324	0.281	0.322	0.080	0.107	0.135
Mean Squared Error							
	100	0.334	0.467	0.512	0.086	0.219	1.318
NBH	250	0.154	0.179	0.181	0.037	0.059	0.170
	500	0.075	0.084	0.091	0.019	0.033	0.063
	100	6.011	1.836	5.916	0.045	0.105	0.189
ZINB	250	0.670	0.293	0.666	0.024	0.036	0.086
	500	0.212	0.158	0.209	0.013	0.023	0.037

Table 55: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 5$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	100	-1.540	0.506	2.078	1.479	0.002	-2.086
	250	-1.532	0.510	2.039	1.489	0.003	-2.006
	500	-1.511	0.500	2.019	1.500	-0.003	-2.002
ZINB	100	-1.939	0.606	2.461	1.478	0.000	-2.041
	250	-1.563	0.521	2.065	1.489	0.005	-2.014
	500	-1.542	0.512	2.045	1.495	0.004	-2.013
Median							
NBH	100	-1.512	0.496	2.081	1.486	0.009	-2.006
	250	-1.521	0.515	2.020	1.491	0.004	-1.977
	500	-1.508	0.497	2.018	1.502	-0.002	-1.994
ZINB	100	-1.585	0.523	2.140	1.489	-0.002	-2.026
	250	-1.526	0.504	2.048	1.495	0.003	-2.001
	500	-1.525	0.520	2.036	1.499	0.005	-2.008
Standard Deviation							
NBH	100	0.403	0.485	0.481	0.201	0.304	0.728
	250	0.282	0.296	0.308	0.131	0.163	0.278
	500	0.192	0.204	0.207	0.090	0.122	0.165
ZINB	100	1.874	1.242	1.872	0.161	0.233	0.332
	250	0.421	0.392	0.421	0.113	0.142	0.213
	500	0.304	0.287	0.298	0.085	0.111	0.146
Mean Squared Error							
NBH	100	0.326	0.470	0.468	0.081	0.184	1.067
	250	0.160	0.176	0.191	0.034	0.053	0.155
	500	0.074	0.083	0.086	0.016	0.030	0.054
ZINB	100	7.217	3.097	7.222	0.053	0.109	0.222
	250	0.358	0.308	0.359	0.026	0.040	0.091
	500	0.187	0.165	0.180	0.014	0.025	0.043

Table 56: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 5$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	100	-1.556	0.012	2.077	1.470	-0.533	-2.162
	250	-1.534	0.013	2.042	1.491	-0.505	-2.023
	500	-1.514	0.007	2.012	1.491	-0.496	-2.009
ZINB	100	-2.103	0.008	2.631	1.483	-0.505	-2.044
	250	-1.614	-0.015	2.116	1.495	-0.501	-2.018
	500	-1.560	-0.001	2.058	1.496	-0.498	-2.004
Median							
NBH	100	-1.530	-0.001	2.050	1.493	-0.502	-2.024
	250	-1.526	0.017	2.039	1.497	-0.503	-1.982
	500	-1.514	0.010	2.011	1.494	-0.497	-2.003
ZINB	100	-1.537	0.026	2.117	1.492	-0.498	-2.019
	250	-1.512	-0.023	2.013	1.500	-0.505	-2.007
	500	-1.525	-0.002	2.021	1.499	-0.495	-2.001
Standard Deviation							
NBH	100	0.403	0.482	0.499	0.290	0.473	1.143
	250	0.270	0.297	0.295	0.140	0.189	0.342
	500	0.194	0.213	0.210	0.101	0.142	0.190
ZINB	100	2.213	1.085	2.268	0.159	0.259	0.350
	250	0.851	0.421	0.841	0.113	0.151	0.231
	500	0.421	0.298	0.413	0.082	0.117	0.147
Mean Squared Error							
NBH	100	0.328	0.464	0.504	0.169	0.448	2.639
	250	0.147	0.177	0.176	0.039	0.071	0.234
	500	0.075	0.091	0.088	0.020	0.040	0.072
ZINB	100	10.158	2.353	10.682	0.051	0.134	0.247
	250	1.460	0.354	1.428	0.025	0.046	0.107
	500	0.358	0.177	0.344	0.013	0.027	0.043

Table 57: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 5$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	100	-1.543	0.489	2.076	1.476	-0.511	-2.115
	250	-1.527	0.520	2.027	1.493	-0.499	-2.020
	500	-1.508	0.509	2.016	1.497	-0.497	-2.017
ZINB	100	-2.086	0.581	2.601	1.483	-0.517	-2.040
	250	-1.602	0.516	2.107	1.490	-0.499	-2.015
	500	-1.557	0.517	2.049	1.495	-0.496	-2.011
Median							
NBH	100	-1.510	0.489	2.037	1.493	-0.501	-2.013
	250	-1.521	0.511	2.020	1.502	-0.501	-1.997
	500	-1.499	0.508	2.015	1.502	-0.496	-2.010
ZINB	100	-1.558	0.520	2.117	1.493	-0.502	-2.023
	250	-1.543	0.507	2.054	1.496	-0.501	-2.004
	500	-1.533	0.513	2.035	1.497	-0.494	-2.008
Standard Deviation							
NBH	100	0.409	0.485	0.503	0.208	0.352	0.836
	250	0.276	0.291	0.301	0.135	0.179	0.304
	500	0.194	0.204	0.211	0.094	0.137	0.191
ZINB	100	2.166	1.326	2.230	0.162	0.273	0.375
	250	0.593	0.422	0.566	0.115	0.161	0.245
	500	0.357	0.299	0.342	0.086	0.122	0.157
Mean Squared Error							
NBH	100	0.336	0.471	0.511	0.087	0.248	1.410
	250	0.153	0.169	0.182	0.037	0.064	0.186
	500	0.076	0.083	0.089	0.018	0.038	0.073
ZINB	100	9.724	3.522	10.306	0.053	0.149	0.283
	250	0.712	0.357	0.652	0.027	0.052	0.120
	500	0.258	0.179	0.236	0.015	0.030	0.049

Table 58: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 10$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.540	0.005	2.070	1.475	-0.003	-2.148
NBH	250	-1.514	0.000	2.023	1.493	0.003	-2.040
	500	-1.515	-0.003	2.017	1.497	0.000	-2.011
	100	-1.741	-0.046	2.307	1.491	-0.009	-2.016
ZINB	250	-1.558	0.009	2.070	1.497	-0.002	-2.020
	500	-1.533	0.006	2.035	1.494	0.002	-2.002
				Median			
	100	-1.512	-0.009	2.065	1.480	0.004	-2.009
NBH	250	-1.504	-0.000	2.009	1.498	0.006	-2.018
	500	-1.511	-0.010	2.013	1.500	-0.000	-2.003
	100	-1.559	-0.018	2.096	1.499	-0.009	-1.997
ZINB	250	-1.524	0.003	2.056	1.501	-0.002	-2.012
	500	-1.505	0.008	2.021	1.495	0.001	-2.000
				Standard Deviation			
	100	0.402	0.497	0.507	0.181	0.293	1.016
NBH	250	0.276	0.297	0.306	0.118	0.152	0.284
	500	0.195	0.207	0.213	0.085	0.113	0.176
	100	1.296	0.944	1.469	0.130	0.203	0.305
ZINB	250	0.382	0.370	0.403	0.093	0.119	0.202
	500	0.298	0.273	0.309	0.069	0.093	0.131
				Mean Squared Error			
	100	0.325	0.493	0.518	0.066	0.171	2.086
NBH	250	0.152	0.176	0.188	0.028	0.046	0.163
	500	0.076	0.086	0.091	0.015	0.026	0.062
	100	3.419	1.784	4.410	0.034	0.082	0.187
ZINB	250	0.294	0.274	0.329	0.017	0.028	0.082
	500	0.179	0.149	0.192	0.010	0.017	0.034

Table 59: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 10$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.546	0.514	2.067	1.478	-0.004	-2.137
NBH	250	-1.526	0.512	2.030	1.494	0.003	-2.029
	500	-1.508	0.505	2.007	1.497	-0.002	-2.003
	100	-1.725	0.554	2.258	1.489	-0.011	-2.028
ZINB	250	-1.557	0.534	2.054	1.491	0.003	-2.014
	500	-1.524	0.509	2.033	1.500	-0.003	-2.007
				Median			
	100	-1.513	0.509	2.038	1.489	-0.004	-2.019
NBH	250	-1.513	0.514	2.030	1.504	0.003	-2.008
	500	-1.498	0.506	2.000	1.498	-0.002	-1.997
	100	-1.573	0.530	2.115	1.496	-0.010	-2.015
ZINB	250	-1.532	0.527	2.039	1.494	0.004	-2.005
	500	-1.507	0.500	2.023	1.503	-0.003	-2.007
				Standard Deviation			
	100	0.405	0.486	0.503	0.172	0.268	0.954
NBH	250	0.276	0.295	0.306	0.116	0.145	0.270
	500	0.189	0.203	0.209	0.081	0.109	0.166
	100	1.161	0.988	1.232	0.131	0.206	0.315
ZINB	250	0.370	0.368	0.388	0.098	0.128	0.212
	500	0.287	0.273	0.286	0.075	0.099	0.137
				Mean Squared Error			
	100	0.330	0.473	0.510	0.060	0.143	1.839
NBH	250	0.153	0.174	0.188	0.027	0.042	0.147
	500	0.071	0.082	0.087	0.013	0.024	0.055
	100	2.746	1.956	3.103	0.034	0.085	0.200
ZINB	250	0.277	0.272	0.303	0.019	0.033	0.090
	500	0.166	0.149	0.165	0.011	0.020	0.038

Table 60: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 10$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.557	-0.012	2.089	1.483	-0.535	-2.179
NBH	250	-1.526	-0.001	2.032	1.490	-0.499	-2.050
	500	-1.517	-0.002	2.019	1.497	-0.504	-2.011
	100	-1.980	-0.025	2.512	1.492	-0.528	-2.046
ZINB	250	-1.600	-0.008	2.120	1.491	-0.497	-2.014
	500	-1.554	0.002	2.054	1.497	-0.499	-2.013
				Median			
	100	-1.530	-0.006	2.065	1.499	-0.512	-2.037
NBH	250	-1.509	-0.006	2.025	1.497	-0.498	-2.027
	500	-1.514	-0.002	2.018	1.499	-0.499	-1.998
	100	-1.597	0.001	2.182	1.496	-0.527	-2.028
ZINB	250	-1.545	-0.014	2.067	1.495	-0.498	-1.999
	500	-1.523	-0.000	2.038	1.498	-0.497	-2.011
				Standard Deviation			
	100	0.407	0.506	0.493	0.195	0.352	1.103
NBH	250	0.276	0.288	0.297	0.122	0.173	0.333
	500	0.196	0.203	0.216	0.087	0.130	0.190
	100	1.776	0.898	1.851	0.135	0.243	0.333
ZINB	250	0.592	0.402	0.584	0.094	0.133	0.223
	500	0.337	0.297	0.332	0.070	0.106	0.146
				Mean Squared Error			
	100	0.334	0.511	0.495	0.076	0.249	2.464
NBH	250	0.153	0.165	0.178	0.030	0.060	0.225
	500	0.077	0.083	0.094	0.015	0.034	0.073
	100	6.534	1.612	7.116	0.037	0.119	0.224
ZINB	250	0.711	0.324	0.696	0.018	0.036	0.099
	500	0.231	0.177	0.223	0.010	0.022	0.043

Table 61: Parameter behavior for the NBH with ZINB data for orthogonal covariates within a component, $\theta = 10$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.553	0.519	2.083	1.481	-0.522	-2.149
	250	-1.516	0.509	2.023	1.492	-0.502	-2.038
	500	-1.507	0.500	2.010	1.498	-0.500	-2.015
ZINB	100	-1.869	0.582	2.378	1.487	-0.517	-2.038
	250	-1.572	0.517	2.074	1.491	-0.500	-2.019
	500	-1.556	0.522	2.055	1.498	-0.502	-2.013
				Median			
NBH	100	-1.522	0.514	2.066	1.497	-0.507	-2.017
	250	-1.510	0.502	2.015	1.494	-0.500	-1.999
	500	-1.504	0.502	2.009	1.499	-0.500	-2.012
ZINB	100	-1.559	0.528	2.110	1.493	-0.506	-2.015
	250	-1.533	0.501	2.043	1.496	-0.498	-2.010
	500	-1.541	0.520	2.030	1.499	-0.501	-2.009
				Standard Deviation			
NBH	100	0.402	0.491	0.497	0.176	0.320	0.971
	250	0.270	0.295	0.296	0.115	0.163	0.324
	500	0.192	0.203	0.208	0.080	0.119	0.181
ZINB	100	1.642	1.004	1.634	0.139	0.245	0.348
	250	0.460	0.393	0.455	0.095	0.142	0.233
	500	0.330	0.295	0.323	0.076	0.111	0.153
				Mean Squared Error			
NBH	100	0.326	0.483	0.502	0.062	0.206	1.907
	250	0.146	0.174	0.175	0.027	0.053	0.212
	500	0.073	0.082	0.087	0.013	0.028	0.066
ZINB	100	5.527	2.021	5.480	0.039	0.121	0.244
	250	0.428	0.309	0.419	0.018	0.040	0.109
	500	0.221	0.175	0.212	0.011	0.025	0.047

Table 62: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = \frac{1}{4}$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.570	-0.013	2.100	0.962	0.005	-1.932
NBH	250	-1.517	-0.002	2.020	1.189	0.005	-1.973
	500	-1.509	-0.003	2.016	1.374	0.001	-1.995
	100	-2.795	-0.346	3.889	1.611	-0.034	-2.021
ZINB	250	-3.154	-0.004	3.808	1.575	-0.020	-2.006
	500	-3.166	-0.011	3.763	1.544	-0.016	-1.999
				Median			
	100	-1.535	-0.012	2.064	1.585	-0.011	-1.914
NBH	250	-1.508	-0.001	2.008	1.534	-0.002	-1.964
	500	-1.505	0.000	2.010	1.537	0.007	-2.000
	100	-0.695	-0.039	1.765	1.650	-0.016	-2.008
ZINB	250	-1.029	-0.022	1.833	1.580	-0.022	-2.012
	500	-1.238	-0.001	1.925	1.544	-0.025	-1.992
				Standard Deviation			
	100	0.463	0.519	0.541	2.081	0.600	0.616
NBH	250	0.274	0.302	0.309	1.435	0.353	0.371
	500	0.181	0.216	0.221	0.890	0.256	0.271
	100	4.432	3.209	4.831	0.545	0.680	0.687
ZINB	250	3.984	0.950	3.668	0.385	0.402	0.406
	500	3.638	0.571	3.393	0.294	0.279	0.280
				Mean Squared Error			
	100	0.433	0.539	0.595	8.949	0.721	0.764
NBH	250	0.150	0.182	0.191	4.211	1/4	0.277
	500	0.065	0.094	0.098	1.598	0.131	0.147
	100	40.952	20.713	50.228	0.606	0.924	0.945
ZINB	250	34.478	1.806	30.169	0.303	0.324	0.330
	500	29.236	0.652	26.132	0.175	0.156	0.157

Table 63: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = \frac{1}{4}$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.549	0.505	2.064	1.002	-0.039	-1.938
	250	-1.522	0.502	2.030	1.302	0.000	-1.981
	500	-1.516	0.507	2.014	1.380	0.008	-1.990
ZINB	100	-3.212	0.689	4.125	1.588	-0.013	-2.039
	250	-3.256	0.583	3.811	1.523	-0.019	-1.997
	500	-3.015	0.535	3.570	1.540	-0.014	-2.004
				Median			
NBH	100	-1.518	0.523	2.032	1.582	-0.037	-1.903
	250	-1.511	0.508	2.022	1.548	-0.003	-1.976
	500	-1.506	0.507	2.010	1.508	0.006	-1.999
ZINB	100	-0.826	0.485	1.997	1.620	-0.007	-1.991
	250	-1.317	0.517	2.000	1.516	-0.024	-1.998
	500	-1.301	0.499	1.993	1.539	-0.015	-2.007
				Standard Deviation			
NBH	100	0.444	0.511	0.498	2.033	0.558	0.568
	250	0.264	0.295	0.298	1.167	0.311	0.335
	500	0.178	0.213	0.211	0.803	0.243	0.259
ZINB	100	4.995	3.448	4.838	0.575	0.774	0.851
	250	3.925	1.060	3.594	0.389	0.423	0.436
	500	3.551	0.583	3.249	0.298	0.292	0.295
				Mean Squared Error			
NBH	100	0.396	0.523	0.499	8.516	0.623	0.649
	250	0.140	0.174	0.178	2.763	0.194	0.225
	500	0.063	0.091	0.089	1.305	0.118	0.134
ZINB	100	52.816	23.812	51.310	0.668	1.199	1.450
	250	33.892	2.254	29.105	0.304	0.359	0.379
	500	27.509	0.682	23.575	0.179	0.171	0.174

Table 64: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = \frac{1}{4}$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.548	-0.018	2.079	0.831	-0.485	-1.937
	250	-1.517	-0.008	2.029	1.144	-0.496	-1.983
	500	-1.510	-0.000	2.017	1.354	-0.491	-1.994
ZINB	100	-2.750	-0.286	3.794	1.637	-0.566	-2.032
	250	-3.198	-0.002	3.856	1.589	-0.529	-2.012
	500	-3.079	-0.004	3.677	1.557	-0.511	-2.010
				Median			
NBH	100	-1.509	0.003	2.058	1.575	-0.495	-1.911
	250	-1.504	-0.006	2.021	1.544	-0.491	-1.981
	500	-1.503	0.005	2.007	1.529	-0.491	-1.986
ZINB	100	-0.605	0.012	1.714	1.663	-0.531	-2.004
	250	-1.044	-0.017	1.877	1.592	-0.531	-2.009
	500	-1.144	-0.009	1.877	1.567	-0.505	-2.008
				Standard Deviation			
NBH	100	0.456	0.504	0.528	2.310	0.629	0.634
	250	0.280	0.308	0.303	1.557	0.350	0.368
	500	0.185	0.222	0.223	0.913	0.263	0.263
ZINB	100	4.526	3.414	4.769	0.532	0.723	0.803
	250	4.063	1.137	3.740	0.390	0.409	0.417
	500	3.693	0.596	3.401	0.299	0.277	0.287
				Mean Squared Error			
NBH	100	0.418	0.507	0.563	11.120	0.790	0.808
	250	0.157	0.190	0.184	4.971	0.245	0.271
	500	0.069	0.099	0.100	1.689	0.138	0.139
ZINB	100	42.523	23.392	48.698	0.585	1.048	1.290
	250	35.894	2.586	31.414	0.312	0.335	0.348
	500	29.756	0.711	25.940	0.182	0.154	0.165

Table 65: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = \frac{1}{4}$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.551	0.505	2.077	0.901	-0.512	-1.981
	250	-1.524	0.508	2.029	1.185	-0.511	-1.992
	500	-1.508	0.503	2.018	1.360	-0.505	-1.987
ZINB	100	-2.982	0.562	4.016	1.628	-0.576	-2.051
	250	-3.200	0.606	3.774	1.561	-0.508	-2.036
	500	-2.794	0.539	3.365	1.551	-0.502	-1.995
				Median			
NBH	100	-1.517	0.505	2.066	1.611	-0.515	-1.965
	250	-1.504	0.510	2.021	1.531	-0.508	-1.984
	500	-1.506	0.505	2.018	1.516	-0.504	-1.981
ZINB	100	-0.622	0.421	1.827	1.649	-0.581	-2.010
	250	-1.162	0.513	1.918	1.578	-0.515	-2.030
	500	-1.202	0.481	1.891	1.561	-0.509	-1.988
				Standard Deviation			
NBH	100	0.452	0.521	0.490	2.185	0.588	0.591
	250	0.274	0.308	0.307	1.422	0.325	0.345
	500	0.179	0.211	0.217	0.860	0.237	0.255
ZINB	100	4.913	3.622	5.005	0.569	0.743	0.905
	250	4.077	1.167	3.640	0.411	0.429	0.464
	500	3.482	0.597	3.155	0.313	0.294	0.304
				Mean Squared Error			
NBH	100	0.412	0.542	0.486	9.904	0.691	0.698
	250	0.151	0.190	0.189	4.140	0.212	0.239
	500	0.064	0.089	0.094	1.500	0.112	0.130
ZINB	100	50.462	26.242	54.162	0.664	1.111	1.641
	250	36.120	2.733	29.647	0.342	0.368	0.431
	500	25.924	0.714	21.771	0.199	0.172	0.185

Table 66: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = \frac{1}{2}$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	100	-1.563	-0.020	2.084	1.243	-0.029	-1.981
	250	-1.529	-0.005	2.032	1.437	0.003	-1.989
	500	-1.513	0.000	2.011	1.462	0.006	-1.992
ZINB	100	-3.316	-0.161	4.059	1.521	-0.014	-2.017
	250	-3.163	0.010	3.686	1.504	-0.002	-2.002
	500	-2.740	-0.016	3.252	1.495	0.001	-2.001
Median							
NBH	100	-1.540	-0.034	2.059	1.535	-0.029	-1.969
	250	-1.518	0.001	2.014	1.518	0.004	-1.981
	500	-1.510	-0.002	2.012	1.497	0.006	-1.993
ZINB	100	-1.210	-0.025	2.035	1.536	-0.024	-2.010
	250	-1.373	0.002	2.032	1.511	0.004	-2.003
	500	-1.417	-0.013	2.007	1.499	0.004	-1.998
Standard Deviation							
NBH	100	0.444	0.498	0.522	1.335	0.551	0.634
	250	0.275	0.297	0.315	0.567	0.318	0.342
	500	0.180	0.216	0.221	0.323	0.227	0.244
ZINB	100	4.199	2.361	4.298	0.380	0.508	0.511
	250	3.563	0.625	3.336	0.264	0.301	0.319
	500	2.985	0.468	2.822	0.199	0.208	0.218
Mean Squared Error							
NBH	100	0.398	0.497	0.551	3.628	0.608	0.804
	250	0.152	0.176	0.199	0.647	0.202	0.233
	500	0.065	0.093	0.098	0.210	0.103	0.119
ZINB	100	38.542	11.174	41.183	0.290	0.516	0.523
	250	28.150	0.780	25.097	0.140	0.181	0.203
	500	19.352	0.438	17.490	0.079	0.087	0.095

Table 67: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = \frac{1}{2}$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.555	0.519	2.070	1.293	-0.021	-1.977
	250	-1.527	0.515	2.032	1.441	-0.000	-1.988
	500	-1.512	0.504	2.013	1.472	0.004	-1.988
ZINB	100	-3.180	0.565	3.794	1.514	-0.041	-2.016
	250	-2.779	0.529	3.302	1.496	0.006	-2.011
	500	-2.323	0.524	2.832	1.500	-0.008	-2.005
				Median			
NBH	100	-1.524	0.526	2.061	1.547	-0.016	-1.943
	250	-1.516	0.513	2.022	1.512	-0.002	-1.981
	500	-1.510	0.500	2.014	1.503	0.000	-1.981
ZINB	100	-1.318	0.516	2.111	1.544	-0.028	-2.008
	250	-1.353	0.480	2.016	1.516	0.018	-2.005
	500	-1.413	0.509	2.013	1.502	-0.009	-2.003
				Standard Deviation			
NBH	100	0.434	0.492	0.491	1.233	0.511	0.531
	250	0.273	0.302	0.298	0.531	0.301	0.313
	500	0.177	0.216	0.211	0.238	0.215	0.221
ZINB	100	4.097	2.120	3.946	0.407	0.527	0.524
	250	3.287	0.671	3.062	0.281	0.319	0.331
	500	2.550	0.446	2.360	0.205	0.216	0.227
				Mean Squared Error			
NBH	100	0.380	0.485	0.486	3.084	0.523	0.563
	250	0.150	0.182	0.178	0.567	0.182	0.196
	500	0.063	0.093	0.090	0.114	0.092	0.098
ZINB	100	36.382	8.992	34.357	0.331	0.558	0.549
	250	23.233	0.902	20.443	0.158	0.203	0.219
	500	13.677	0.399	11.829	0.084	0.093	0.103

Table 68: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = \frac{1}{2}$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.567	-0.003	2.089	1.118	-0.502	-1.997
	250	-1.521	0.002	2.028	1.400	-0.497	-1.995
	500	-1.510	-0.004	2.020	1.472	-0.497	-1.996
ZINB	100	-3.208	-0.120	3.967	1.539	-0.532	-2.027
	250	-3.139	-0.021	3.704	1.504	-0.502	-2.007
	500	-2.910	0.002	3.415	1.493	-0.496	-2.017
				Median			
NBH	100	-1.535	0.003	2.064	1.523	-0.499	-1.969
	250	-1.509	0.014	2.023	1.493	-0.495	-1.976
	500	-1.506	-0.005	2.014	1.512	-0.501	-1.992
ZINB	100	-1.158	-0.011	2.019	1.555	-0.522	-1.994
	250	-1.324	-0.021	2.029	1.521	-0.495	-2.001
	500	-1.428	0.000	2.025	1.502	-0.493	-2.011
				Standard Deviation			
NBH	100	0.443	0.505	0.518	1.685	0.568	0.574
	250	0.280	0.300	0.314	0.661	0.309	0.339
	500	0.183	0.214	0.225	0.335	0.227	0.244
ZINB	100	4.226	2.450	4.212	0.383	0.540	0.645
	250	3.619	0.761	3.410	0.278	0.316	0.327
	500	3.162	0.499	2.981	0.207	0.210	0.228
				Mean Squared Error			
NBH	100	0.397	0.509	0.544	5.824	0.644	0.658
	250	0.157	0.180	0.197	0.883	0.191	0.230
	500	0.067	0.092	0.101	0.226	0.103	0.119
ZINB	100	38.628	12.014	39.345	0.295	0.583	0.832
	250	28.879	1.157	26.156	0.154	0.199	0.214
	500	21.979	0.499	19.767	0.086	0.088	0.105

Table 69: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = \frac{1}{2}$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	100	-1.554	0.499	2.092	1.265	-0.499	-1.988
	250	-1.523	0.499	2.036	1.403	-0.495	-1.994
	500	-1.515	0.499	2.020	1.455	-0.504	-1.994
ZINB	100	-3.126	0.725	3.691	1.535	-0.565	-2.031
	250	-2.851	0.545	3.371	1.502	-0.500	-2.020
	500	-2.436	0.532	2.930	1.497	-0.499	-2.007
Median							
NBH	100	-1.524	0.502	2.079	1.524	-0.482	-1.952
	250	-1.509	0.500	2.030	1.501	-0.491	-1.990
	500	-1.507	0.500	2.017	1.494	-0.505	-1.989
ZINB	100	-1.262	0.544	2.049	1.557	-0.560	-2.030
	250	-1.352	0.494	1.994	1.521	-0.496	-2.015
	500	-1.434	0.524	2.029	1.508	-0.502	-1.999
Standard Deviation							
NBH	100	0.441	0.503	0.507	1.285	0.520	0.601
	250	0.272	0.298	0.300	0.633	0.301	0.320
	500	0.176	0.214	0.215	0.377	0.213	0.239
ZINB	100	4.222	2.546	3.964	0.404	0.564	0.582
	250	3.412	0.742	3.154	0.285	0.331	0.351
	500	2.660	0.480	2.470	0.213	0.224	0.233
Mean Squared Error							
NBH	100	0.392	0.506	0.523	3.359	0.541	0.722
	250	0.148	0.178	0.181	0.811	0.181	0.205
	500	0.062	0.092	0.093	0.286	0.091	0.114
ZINB	100	38.288	13.014	34.272	0.327	0.640	0.678
	250	25.104	1.101	21.771	0.163	0.219	0.247
	500	15.022	0.461	13.067	0.091	0.101	0.109

Table 70: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 1$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	100	-1.552	-0.012	2.070	1.428	-0.025	-2.000
	250	-1.528	-0.000	2.036	1.488	-0.005	-2.003
	500	-1.507	-0.009	2.018	1.485	-0.001	-2.002
ZINB	100	-3.119	-0.059	3.717	1.488	-0.023	-2.020
	250	-2.422	-0.020	2.942	1.494	-0.018	-2.010
	500	-1.999	-0.004	2.498	1.487	0.000	-2.006
Median							
NBH	100	-1.540	0.003	2.036	1.503	-0.018	-1.953
	250	-1.520	0.014	2.031	1.509	0.002	-1.990
	500	-1.495	-0.012	2.010	1.497	-0.007	-2.001
ZINB	100	-1.509	0.014	2.145	1.505	-0.011	-2.003
	250	-1.478	-0.013	2.047	1.505	-0.015	-1.998
	500	-1.504	0.004	2.046	1.492	-0.001	-2.007
Standard Deviation							
NBH	100	0.441	0.512	0.513	0.601	0.477	0.528
	250	0.276	0.299	0.310	0.222	0.262	0.302
	500	0.177	0.218	0.217	0.158	0.195	0.218
ZINB	100	3.660	1.725	3.671	0.282	0.402	0.420
	250	2.595	0.544	2.496	0.194	0.241	0.270
	500	1.831	0.379	1.742	0.136	0.165	0.183
Mean Squared Error							
NBH	100	0.392	0.524	0.530	0.728	0.456	0.557
	250	0.153	0.179	0.194	0.099	0.138	0.182
	500	0.062	0.095	0.094	0.050	0.076	0.095
ZINB	100	29.401	5.951	29.893	0.159	0.324	0.353
	250	14.320	0.593	13.342	0.075	0.117	0.146
	500	6.951	0.288	6.317	0.037	0.054	0.067

Table 71: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 1$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.556	0.523	2.071	1.452	0.000	-2.016
	250	-1.509	0.495	2.025	1.476	0.006	-1.992
	500	-1.508	0.502	2.013	1.494	-0.002	-2.001
ZINB	100	-2.878	0.584	3.405	1.470	-0.017	-2.023
	250	-2.134	0.524	2.633	1.481	-0.001	-2.015
	500	-1.873	0.532	2.356	1.491	-0.002	-2.006
				Median			
NBH	100	-1.525	0.537	2.064	1.506	0.005	-1.983
	250	-1.510	0.499	2.020	1.495	0.014	-1.984
	500	-1.499	0.504	2.007	1.501	0.001	-2.000
ZINB	100	-1.496	0.551	2.171	1.494	-0.005	-2.009
	250	-1.470	0.504	2.048	1.488	0.003	-2.006
	500	-1.491	0.516	2.040	1.503	-0.001	-2.007
				Standard Deviation			
NBH	100	0.438	0.498	0.497	0.457	0.422	0.473
	250	0.270	0.299	0.298	0.220	0.250	0.285
	500	0.179	0.210	0.209	0.150	0.188	0.200
ZINB	100	3.454	1.638	3.338	0.294	0.408	0.441
	250	2.245	0.519	2.115	0.198	0.249	0.282
	500	1.560	0.382	1.451	0.139	0.176	0.188
				Mean Squared Error			
NBH	100	0.387	0.496	0.499	0.420	0.356	0.447
	250	0.146	0.178	0.178	0.098	0.125	0.163
	500	0.064	0.089	0.087	0.045	0.071	0.080
ZINB	100	25.757	5.369	24.250	0.174	0.332	0.389
	250	10.482	0.539	9.350	0.079	0.124	0.159
	500	5.007	0.292	4.334	0.039	0.062	0.070

Table 72: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 1$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.554	-0.029	2.087	1.391	-0.526	-2.017
NBH	250	-1.528	0.006	2.032	1.481	-0.510	-2.007
	500	-1.511	-0.001	2.007	1.492	-0.502	-2.007
	100	-3.259	-0.126	3.871	1.485	-0.530	-2.018
ZINB	250	-2.633	0.008	3.123	1.484	-0.503	-2.021
	500	-2.159	-0.002	2.651	1.486	-0.499	-2.013
				Median			
	100	-1.519	-0.016	2.057	1.496	-0.518	-1.974
NBH	250	-1.512	0.009	2.021	1.500	-0.506	-1.989
	500	-1.501	-0.003	2.002	1.503	-0.498	-2.002
	100	-1.476	-0.009	2.143	1.509	-0.513	-2.004
ZINB	250	-1.481	0.011	2.061	1.497	-0.499	-2.008
	500	-1.518	-0.006	2.072	1.497	-0.501	-2.011
				Standard Deviation			
	100	0.438	0.509	0.526	0.746	0.504	0.671
NBH	250	0.273	0.300	0.311	0.241	0.273	0.326
	500	0.180	0.222	0.217	0.161	0.207	0.235
	100	3.937	1.986	3.923	0.282	0.423	0.434
ZINB	250	2.917	0.607	2.767	0.198	0.248	0.278
	500	2.098	0.416	1.986	0.139	0.168	0.189
				Mean Squared Error			
	100	0.387	0.519	0.560	1.126	0.509	0.900
NBH	250	0.150	0.180	0.195	0.116	0.150	0.213
	500	0.065	0.099	0.094	0.052	0.085	0.111
	100	34.080	7.901	34.268	0.159	0.358	0.376
ZINB	250	18.300	0.736	16.565	0.079	0.123	0.155
	500	9.240	0.346	8.309	0.039	0.056	0.072

Table 73: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 1$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.558	0.484	2.093	1.404	-0.502	-2.015
	250	-1.518	0.513	2.015	1.481	-0.507	-2.003
	500	-1.512	0.505	2.014	1.491	-0.497	-1.999
ZINB	100	-2.955	0.613	3.473	1.474	-0.528	-2.043
	250	-2.333	0.539	2.818	1.488	-0.509	-2.010
	500	-2.022	0.524	2.502	1.486	-0.506	-2.005
				Median			
NBH	100	-1.516	0.494	2.069	1.481	-0.490	-1.986
	250	-1.512	0.517	2.005	1.508	-0.508	-1.993
	500	-1.510	0.506	2.014	1.499	-0.496	-1.980
ZINB	100	-1.539	0.551	2.170	1.498	-0.516	-2.018
	250	-1.463	0.508	2.057	1.505	-0.506	-1.999
	500	-1.502	0.515	2.052	1.499	-0.502	-1.992
				Standard Deviation			
NBH	100	0.443	0.495	0.491	0.605	0.459	0.528
	250	0.264	0.298	0.286	0.234	0.261	0.310
	500	0.179	0.220	0.206	0.154	0.192	0.219
ZINB	100	3.694	1.929	3.518	0.295	0.454	0.480
	250	2.551	0.597	2.403	0.201	0.259	0.288
	500	1.924	0.410	1.778	0.147	0.174	0.198
				Mean Squared Error			
NBH	100	0.395	0.490	0.492	0.740	0.421	0.558
	250	0.140	0.178	0.164	0.110	0.136	0.192
	500	0.064	0.097	0.085	0.048	0.074	0.096
ZINB	100	29.396	7.450	26.923	0.175	0.414	0.462
	250	13.704	0.714	12.216	0.081	0.134	0.166
	500	7.675	0.336	6.569	0.044	0.060	0.078

Table 74: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 2$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.561	-0.008	2.083	1.465	-0.019	-2.021
	250	-1.515	-0.001	2.026	1.477	0.006	-2.010
	500	-1.513	-0.001	2.020	1.493	-0.003	-1.994
ZINB	100	-2.614	-0.058	3.182	1.482	-0.021	-2.025
	250	-1.765	-0.024	2.293	1.490	-0.008	-2.010
	500	-1.587	-0.008	2.096	1.494	0.001	-2.009
				Median			
NBH	100	-1.520	0.016	2.056	1.487	-0.007	-1.976
	250	-1.504	-0.004	2.031	1.488	0.006	-1.995
	500	-1.506	0.001	2.018	1.501	-0.003	-1.984
ZINB	100	-1.554	0.001	2.146	1.499	-0.017	-1.996
	250	-1.509	-0.016	2.043	1.498	-0.009	-2.004
	500	-1.512	0.003	2.034	1.497	0.003	-2.003
				Standard Deviation			
NBH	100	0.440	0.498	0.530	0.259	0.392	0.599
	250	0.273	0.296	0.315	0.167	0.219	0.282
	500	0.177	0.216	0.222	0.112	0.161	0.201
ZINB	100	2.984	1.318	3.002	0.212	0.306	0.362
	250	1.282	0.454	1.264	0.137	0.193	0.228
	500	0.609	0.320	0.576	0.096	0.136	0.157
				Mean Squared Error			
NBH	100	0.391	0.497	0.569	0.135	0.307	0.717
	250	0.150	0.175	0.199	0.056	0.096	0.159
	500	0.063	0.094	0.099	0.025	0.052	0.081
ZINB	100	19.046	3.475	19.412	0.090	0.188	0.263
	250	3.359	0.413	3.281	0.038	0.075	0.104
	500	0.749	0.205	0.672	0.018	0.037	0.049

Table 75: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 2$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.569	0.513	2.088	1.469	-0.014	-2.025
	250	-1.530	0.501	2.031	1.490	-0.003	-2.006
	500	-1.511	0.502	2.015	1.490	-0.002	-1.991
ZINB	100	-2.216	0.573	2.708	1.476	-0.013	-2.030
	250	-1.747	0.526	2.237	1.485	-0.005	-2.013
	500	-1.552	0.510	2.053	1.495	0.002	-2.014
				Median			
NBH	100	-1.541	0.523	2.077	1.492	-0.004	-1.979
	250	-1.517	0.501	2.025	1.496	-0.003	-1.992
	500	-1.506	0.503	2.011	1.492	0.003	-1.984
ZINB	100	-1.547	0.521	2.105	1.491	0.001	-2.023
	250	-1.545	0.518	2.038	1.489	-0.005	-2.004
	500	-1.510	0.512	2.022	1.498	0.000	-2.010
				Standard Deviation			
NBH	100	0.436	0.506	0.504	0.243	0.375	0.531
	250	0.275	0.300	0.305	0.155	0.211	0.268
	500	0.177	0.214	0.211	0.106	0.153	0.189
ZINB	100	2.407	1.223	2.386	0.219	0.343	0.375
	250	1.235	0.451	1.172	0.147	0.196	0.238
	500	0.411	0.320	0.399	0.097	0.140	0.164
				Mean Squared Error			
NBH	100	0.385	0.511	0.515	0.119	0.282	0.564
	250	0.152	0.180	0.187	0.048	0.089	0.143
	500	0.063	0.092	0.089	0.023	0.047	0.071
ZINB	100	12.092	2.996	11.885	0.096	0.235	0.283
	250	3.111	0.408	2.802	0.043	0.076	0.114
	500	0.340	0.204	0.322	0.019	0.039	0.054

Table 76: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 2$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.559	-0.009	2.084	1.454	-0.521	-2.082
	250	-1.525	-0.005	2.031	1.477	-0.492	-2.013
	500	-1.513	-0.001	2.017	1.494	-0.509	-2.010
ZINB	100	-2.685	-0.105	3.282	1.475	-0.523	-2.037
	250	-1.936	-0.017	2.450	1.490	-0.507	-2.018
	500	-1.608	-0.022	2.122	1.497	-0.507	-2.006
				Median			
NBH	100	-1.526	0.003	2.064	1.490	-0.490	-1.984
	250	-1.512	0.001	2.018	1.486	-0.493	-2.000
	500	-1.509	0.005	2.020	1.502	-0.503	-2.004
ZINB	100	-1.554	-0.034	2.132	1.492	-0.503	-2.017
	250	-1.520	-0.009	2.088	1.502	-0.505	-2.007
	500	-1.494	-0.017	2.025	1.501	-0.507	-2.004
				Standard Deviation			
NBH	100	0.447	0.503	0.527	0.409	0.512	0.940
	250	0.275	0.301	0.312	0.172	0.238	0.320
	500	0.175	0.213	0.217	0.117	0.178	0.224
ZINB	100	3.117	1.481	3.251	0.218	0.349	0.393
	250	1.711	0.507	1.665	0.143	0.204	0.244
	500	0.716	0.347	0.687	0.096	0.141	0.171
				Mean Squared Error			
NBH	100	0.403	0.505	0.561	0.337	0.525	1.775
	250	0.152	0.181	0.195	0.060	0.113	0.205
	500	0.061	0.091	0.095	0.027	0.064	0.100
ZINB	100	20.826	4.396	22.779	0.096	0.244	0.311
	250	6.045	0.514	5.746	0.041	0.083	0.119
	500	1.037	0.241	0.957	0.018	0.040	0.058

Table 77: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 2$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.543	0.508	2.079	1.462	-0.511	-2.055
	250	-1.518	0.508	2.020	1.494	-0.516	-2.016
	500	-1.506	0.502	2.008	1.496	-0.503	-2.007
ZINB	100	-2.501	0.619	2.979	1.468	-0.510	-2.038
	250	-1.881	0.522	2.372	1.487	-0.517	-2.023
	500	-1.603	0.528	2.093	1.494	-0.503	-2.004
				Median			
NBH	100	-1.511	0.494	2.072	1.495	-0.495	-1.999
	250	-1.501	0.509	2.013	1.503	-0.514	-1.993
	500	-1.495	0.504	2.001	1.498	-0.498	-1.996
ZINB	100	-1.618	0.568	2.185	1.491	-0.506	-2.015
	250	-1.536	0.509	2.073	1.494	-0.515	-2.016
	500	-1.519	0.522	2.022	1.502	-0.502	-2.001
				Standard Deviation			
NBH	100	0.438	0.478	0.494	0.364	0.403	0.617
	250	0.271	0.302	0.298	0.157	0.224	0.303
	500	0.172	0.216	0.210	0.107	0.164	0.205
ZINB	100	2.786	1.381	2.733	0.236	0.372	0.404
	250	1.602	0.505	1.531	0.148	0.225	0.263
	500	0.701	0.342	0.645	0.102	0.146	0.182
				Mean Squared Error			
NBH	100	0.385	0.456	0.494	0.266	0.325	0.764
	250	0.148	0.182	0.178	0.050	0.101	0.183
	500	0.059	0.094	0.088	0.023	0.054	0.084
ZINB	100	16.517	3.829	15.888	0.113	0.277	0.328
	250	5.279	0.511	4.825	0.044	0.101	0.139
	500	0.993	0.234	0.840	0.021	0.043	0.066

Table 78: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 5$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.572	-0.022	2.100	1.486	-0.021	-2.075
	250	-1.526	0.000	2.030	1.491	0.000	-2.023
	500	-1.513	-0.009	2.022	1.494	0.004	-2.004
ZINB	100	-1.959	-0.009	2.484	1.487	-0.014	-2.038
	250	-1.595	-0.014	2.099	1.495	-0.004	-2.015
	500	-1.532	-0.006	2.035	1.498	0.001	-2.012
				Median			
NBH	100	-1.526	-0.007	2.086	1.498	-0.010	-1.983
	250	-1.515	0.001	2.020	1.498	-0.005	-2.002
	500	-1.508	-0.005	2.014	1.499	0.002	-1.994
ZINB	100	-1.580	-0.005	2.120	1.500	-0.003	-2.024
	250	-1.541	-0.005	2.037	1.498	-0.007	-2.006
	500	-1.517	-0.001	2.023	1.498	0.002	-2.010
				Standard Deviation			
NBH	100	0.448	0.513	0.522	0.185	0.315	0.767
	250	0.283	0.297	0.313	0.125	0.179	0.273
	500	0.176	0.215	0.224	0.084	0.135	0.191
ZINB	100	1.810	0.870	1.774	0.155	0.263	0.320
	250	0.591	0.411	0.617	0.100	0.151	0.203
	500	0.277	0.312	0.324	0.068	0.108	0.144
				Mean Squared Error			
NBH	100	0.407	0.527	0.556	0.068	0.199	1.183
	250	0.161	0.176	0.197	0.031	0.064	0.150
	500	0.062	0.092	0.100	0.014	0.037	0.073
ZINB	100	6.758	1.515	6.525	0.048	0.139	0.206
	250	0.707	0.338	0.770	0.020	0.046	0.083
	500	0.155	0.195	0.211	0.009	0.023	0.042

Table 79: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 5$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.562	0.520	2.087	1.489	-0.020	-2.064
	250	-1.524	0.498	2.025	1.492	-0.001	-2.014
	500	-1.506	0.500	2.009	1.495	-0.003	-2.010
ZINB	100	-1.917	0.595	2.401	1.475	-0.010	-2.021
	250	-1.548	0.507	2.051	1.496	-0.003	-2.016
	500	-1.525	0.499	2.037	1.497	0.002	-2.008
				Median			
NBH	100	-1.519	0.523	2.042	1.490	-0.009	-2.007
	250	-1.511	0.490	2.017	1.495	0.001	-1.996
	500	-1.501	0.507	2.004	1.496	-0.003	-2.001
ZINB	100	-1.606	0.567	2.160	1.487	-0.009	-2.007
	250	-1.519	0.501	2.025	1.500	0.002	-2.010
	500	-1.512	0.500	2.031	1.501	-0.001	-2.004
				Standard Deviation			
NBH	100	0.446	0.502	0.513	0.176	0.291	0.567
	250	0.270	0.297	0.300	0.118	0.171	0.255
	500	0.181	0.220	0.217	0.081	0.124	0.178
ZINB	100	1.543	0.831	1.527	0.168	0.266	0.333
	250	0.419	0.402	0.424	0.101	0.153	0.221
	500	0.264	0.293	0.294	0.070	0.108	0.145
				Mean Squared Error			
NBH	100	0.401	0.504	0.533	0.062	0.169	0.647
	250	0.147	0.176	0.181	0.028	0.059	0.130
	500	0.066	0.097	0.094	0.013	0.031	0.063
ZINB	100	4.932	1.391	4.825	0.057	0.142	0.222
	250	0.353	0.323	0.362	0.020	0.047	0.098
	500	0.140	0.172	0.175	0.010	0.023	0.042

Table 80: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 5$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	100	-1.554	-0.046	2.114	1.476	-0.528	-2.139
	250	-1.529	-0.008	2.044	1.492	-0.505	-2.021
	500	-1.507	0.000	2.012	1.493	-0.500	-2.013
ZINB	100	-2.267	-0.043	2.829	1.490	-0.530	-2.053
	250	-1.613	-0.018	2.122	1.490	-0.500	-2.015
	500	-1.539	-0.016	2.052	1.497	-0.503	-2.006
Median							
NBH	100	-1.517	-0.034	2.076	1.492	-0.515	-2.010
	250	-1.525	-0.002	2.031	1.498	-0.496	-1.999
	500	-1.504	0.005	2.009	1.498	-0.497	-1.992
ZINB	100	-1.614	0.011	2.168	1.499	-0.524	-2.027
	250	-1.517	-0.011	2.037	1.495	-0.494	-2.007
	500	-1.514	-0.006	2.030	1.497	-0.501	-2.006
Standard Deviation							
NBH	100	0.453	0.509	0.527	0.198	0.366	1.032
	250	0.278	0.305	0.315	0.127	0.200	0.308
	500	0.176	0.213	0.219	0.083	0.151	0.222
ZINB	100	2.433	1.210	2.605	0.159	0.305	0.351
	250	0.734	0.444	0.754	0.100	0.172	0.231
	500	0.301	0.317	0.355	0.069	0.115	0.153
Mean Squared Error							
NBH	100	0.413	0.520	0.567	0.079	0.269	2.147
	250	0.155	0.186	0.201	0.032	0.080	0.190
	500	0.062	0.091	0.096	0.014	0.046	0.099
ZINB	100	12.423	2.931	14.251	0.050	0.187	0.249
	250	1.091	0.395	1.151	0.020	0.059	0.107
	500	0.183	0.201	0.254	0.009	0.027	0.047

Table 81: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 5$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.537	0.488	2.094	1.475	-0.529	-2.105
	250	-1.526	0.501	2.036	1.493	-0.503	-2.022
	500	-1.517	0.503	2.015	1.499	-0.504	-2.011
ZINB	100	-1.991	0.614	2.451	1.477	-0.525	-2.031
	250	-1.581	0.491	2.092	1.491	-0.509	-2.006
	500	-1.541	0.506	2.035	1.495	-0.504	-2.006
				Median			
NBH	100	-1.501	0.497	2.079	1.485	-0.517	-2.015
	250	-1.514	0.499	2.030	1.494	-0.495	-2.000
	500	-1.514	0.503	2.020	1.502	-0.499	-1.997
ZINB	100	-1.638	0.590	2.125	1.487	-0.508	-2.003
	250	-1.519	0.493	2.040	1.494	-0.506	-1.988
	500	-1.514	0.502	2.011	1.497	-0.503	-2.002
				Standard Deviation			
NBH	100	0.435	0.488	0.493	0.236	0.345	0.798
	250	0.266	0.299	0.299	0.118	0.185	0.285
	500	0.177	0.207	0.206	0.082	0.140	0.205
ZINB	100	1.868	1.076	1.918	0.164	0.311	0.369
	250	0.599	0.429	0.599	0.102	0.180	0.247
	500	0.304	0.316	0.325	0.071	0.121	0.167
				Mean Squared Error			
NBH	100	0.380	0.476	0.495	0.112	0.238	1.285
	250	0.142	0.178	0.180	0.028	0.069	0.163
	500	0.063	0.086	0.085	0.013	0.039	0.084
ZINB	100	7.216	2.326	7.557	0.054	0.194	0.273
	250	0.725	0.368	0.727	0.021	0.065	0.122
	500	0.186	0.200	0.212	0.010	0.029	0.056

Table 82: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 10$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
	100	-1.555	-0.020	2.086	1.488	-0.011	-2.096
NBH	250	-1.524	-0.002	2.026	1.496	-0.002	-2.026
	500	-1.513	0.003	2.015	1.498	-0.003	-2.008
	100	-1.829	-0.046	2.376	1.486	-0.013	-2.018
ZINB	250	-1.584	0.000	2.084	1.493	-0.001	-2.012
	500	-1.532	-0.008	2.039	1.496	-0.001	-2.000
Median							
	100	-1.516	-0.012	2.064	1.499	-0.010	-2.023
NBH	250	-1.517	0.003	2.021	1.498	-0.003	-2.001
	500	-1.507	0.004	2.009	1.499	-0.001	-1.992
	100	-1.558	-0.005	2.111	1.497	-0.009	-2.007
ZINB	250	-1.541	0.012	2.043	1.496	-0.002	-2.006
	500	-1.520	-0.010	2.035	1.499	-0.002	-1.997
Standard Deviation							
	100	0.444	0.516	0.522	0.164	0.286	0.733
NBH	250	0.277	0.304	0.311	0.107	0.163	0.273
	500	0.178	0.219	0.222	0.074	0.118	0.187
	100	1.532	0.897	1.649	0.137	0.228	0.300
ZINB	250	0.433	0.404	0.472	0.086	0.135	0.194
	500	0.264	0.289	0.304	0.059	0.090	0.136
Mean Squared Error							
	100	0.397	0.533	0.552	0.054	0.164	1.083
NBH	250	0.154	0.185	0.194	0.023	0.053	0.149
	500	0.063	0.096	0.098	0.011	0.028	0.070
	100	4.803	1.610	5.579	0.038	0.105	0.180
ZINB	250	0.382	0.326	0.452	0.015	0.037	0.076
	500	0.140	0.167	0.186	0.007	0.016	0.037

Table 83: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 10$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
NBH	100	-1.553	0.508	2.083	1.486	-0.015	-2.029
	250	-1.519	0.498	2.032	1.493	0.004	-2.031
	500	-1.509	0.500	2.020	1.497	-0.005	-2.017
ZINB	100	-1.720	0.530	2.242	1.489	-0.012	-2.023
	250	-1.559	0.510	2.060	1.500	-0.007	-2.021
	500	-1.518	0.504	2.027	1.500	-0.005	-2.006
				Median			
NBH	100	-1.535	0.526	2.069	1.495	-0.019	-1.988
	250	-1.508	0.500	2.021	1.496	0.007	-2.013
	500	-1.507	0.502	2.020	1.500	-0.007	-2.014
ZINB	100	-1.556	0.517	2.110	1.497	0.004	-2.010
	250	-1.531	0.514	2.034	1.503	-0.003	-2.012
	500	-1.506	0.503	2.025	1.502	-0.003	-2.000
				Standard Deviation			
NBH	100	0.426	0.489	0.505	0.155	0.268	0.434
	250	0.271	0.300	0.303	0.101	0.147	0.247
	500	0.178	0.214	0.213	0.071	0.113	0.177
ZINB	100	1.126	0.796	1.211	0.143	0.233	0.325
	250	0.387	0.381	0.411	0.087	0.134	0.206
	500	0.254	0.271	0.291	0.059	0.097	0.142
				Mean Squared Error			
NBH	100	0.365	0.477	0.517	0.048	0.144	0.377
	250	0.147	0.180	0.185	0.020	0.043	0.123
	500	0.063	0.092	0.091	0.010	0.025	0.063
ZINB	100	2.585	1.266	2.993	0.041	0.109	0.212
	250	0.302	0.291	0.341	0.015	0.036	0.086
	500	0.129	0.147	0.170	0.007	0.019	0.040

Table 84: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 10$, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				Mean			
	100	-1.552	-0.031	2.088	1.486	-0.534	-2.198
NBH	250	-1.520	-0.001	2.029	1.497	-0.509	-2.038
	500	-1.510	-0.010	2.020	1.495	-0.501	-2.018
	100	-2.068	0.018	2.563	1.490	-0.529	-2.040
ZINB	250	-1.593	-0.006	2.101	1.497	-0.504	-2.022
	500	-1.534	-0.008	2.037	1.499	-0.503	-2.012
				Median			
	100	-1.513	-0.016	2.059	1.496	-0.511	-2.041
NBH	250	-1.512	-0.004	2.015	1.503	-0.503	-2.006
	500	-1.507	-0.003	2.017	1.494	-0.498	-2.009
	100	-1.650	0.038	2.172	1.496	-0.513	-2.021
ZINB	250	-1.537	0.010	2.053	1.499	-0.500	-2.012
	500	-1.519	-0.002	2.030	1.500	-0.502	-2.006
				Standard Deviation			
	100	0.438	0.516	0.526	0.163	0.411	1.200
NBH	250	0.272	0.302	0.314	0.105	0.184	0.318
	500	0.179	0.218	0.219	0.074	0.136	0.220
	100	1.987	0.984	2.087	0.129	0.266	0.347
ZINB	250	0.571	0.429	0.612	0.085	0.159	0.220
	500	0.277	0.305	0.326	0.055	0.106	0.152
				Mean Squared Error			
	100	0.386	0.533	0.561	0.053	0.339	2.918
NBH	250	0.148	0.182	0.198	0.022	0.068	0.203
	500	0.064	0.095	0.096	0.011	0.037	0.097
	100	8.218	1.938	9.026	0.033	0.143	0.243
ZINB	250	0.661	0.369	0.758	0.014	0.050	0.097
	500	0.154	0.187	0.214	0.006	0.023	0.046

Table 85: Parameter behavior for the NBH with ZINB data for correlated covariates within a component ($\rho = 0.3$), $\theta = 10$, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
Mean							
NBH	100	-1.559	0.501	2.091	1.485	-0.530	-2.136
	250	-1.521	0.507	2.025	1.492	-0.498	-2.045
	500	-1.512	0.508	2.009	1.499	-0.506	-2.018
ZINB	100	-1.893	0.585	2.387	1.480	-0.526	-2.021
	250	-1.582	0.499	2.095	1.495	-0.504	-2.025
	500	-1.528	0.503	2.035	1.499	-0.499	-2.013
Median							
NBH	100	-1.515	0.502	2.077	1.493	-0.513	-2.029
	250	-1.511	0.508	2.017	1.496	-0.497	-2.024
	500	-1.506	0.512	1.999	1.499	-0.500	-2.013
ZINB	100	-1.585	0.553	2.139	1.485	-0.517	-1.995
	250	-1.525	0.490	2.065	1.497	-0.503	-2.013
	500	-1.506	0.514	2.024	1.501	-0.500	-2.009
Standard Deviation							
NBH	100	0.447	0.491	0.501	0.157	0.317	0.844
	250	0.270	0.299	0.304	0.102	0.172	0.293
	500	0.177	0.209	0.209	0.069	0.128	0.199
ZINB	100	1.573	0.996	1.646	0.142	0.288	0.355
	250	0.533	0.437	0.537	0.089	0.163	0.238
	500	0.286	0.307	0.305	0.061	0.110	0.159
Mean Squared Error							
NBH	100	0.403	0.481	0.510	0.049	0.201	1.443
	250	0.146	0.179	0.185	0.021	0.059	0.174
	500	0.063	0.088	0.088	0.010	0.033	0.080
ZINB	100	5.101	1.991	5.567	0.041	0.167	0.253
	250	0.575	0.381	0.586	0.016	0.053	0.114
	500	0.164	0.188	0.187	0.007	0.024	0.051

Second Simulation Study - Model Misspecification

Table 86: Estimated parameter behavior (mean) for the NBH with ZINB data from the 2,000 simulates for orthogonal covariates for $n = 500$.

θ	level β_1	level γ_1	β_0	β_1	β_2	γ_0	γ_1	γ_2
1/4	0	0	-0.804	0.048	-1.031	1.308	-0.006	-2.003
1/4	0.5	0	-0.765	-0.208	-1.133	1.307	-0.007	-2.009
1/4	0	-0.5	-0.922	0.069	-1.022	1.203	-0.524	-2.000
1/4	0.5	-0.5	-0.885	-0.169	-1.118	1.214	-0.495	-1.989
1/2	0	0	-0.408	0.056	-1.132	1.488	-0.005	-2.008
1/2	0.5	0	-0.376	-0.203	-1.223	1.478	-0.006	-1.994
1/2	0	-0.5	-0.553	0.070	-1.095	1.463	-0.500	-2.021
1/2	0.5	-0.5	-0.517	-0.192	-1.193	1.458	-0.491	-2.009
1	0	0	-0.114	0.066	-1.216	1.485	0.003	-2.012
1	0.5	0	-0.074	-0.230	-1.315	1.489	-0.003	-2.012
1	0	-0.5	-0.279	0.087	-1.181	1.490	-0.503	-2.001
1	0.5	-0.5	-0.241	-0.197	-1.275	1.489	-0.500	-2.018
2	0	0	0.073	0.075	-1.278	1.492	0.003	-2.003
2	0.5	0	0.111	-0.244	-1.374	1.494	-0.002	-2.011
2	0	-0.5	-0.096	0.088	-1.237	1.496	-0.500	-2.018
2	0.5	-0.5	-0.059	-0.201	-1.327	1.495	-0.503	-2.030
5	0	0	0.203	0.070	-1.329	1.496	0.001	-2.019
5	0.5	0	0.242	-0.255	-1.427	1.495	0.006	-2.026
5	0	-0.5	0.032	0.094	-1.285	1.495	-0.495	-2.022
5	0.5	-0.5	0.063	-0.211	-1.369	1.495	-0.495	-2.028
10	0	0	0.247	0.069	-1.355	1.494	0.003	-2.014
10	0.5	0	0.277	-0.255	-1.439	1.501	-0.004	-2.026
10	0	-0.5	0.071	0.098	-1.304	1.498	-0.501	-2.037
10	0.5	-0.5	0.109	-0.221	-1.394	1.499	-0.502	-2.041

Table 87: Estimated parameter behavior (mean) for the NBH with ZINB data from the 2,000 simulates for correlated covariates for $n = 500$.

θ	level β_1	level γ_1	β_0	β_1	β_2	γ_0	γ_1	γ_2
1/4	0	0	-0.694	-0.084	-1.026	1.330	-0.020	-2.000
1/4	0.5	0	-0.673	-0.315	-1.150	1.284	-0.006	-2.001
1/4	0	-0.5	-0.816	-0.100	-0.990	1.301	-0.517	-2.021
1/4	0.5	-0.5	-0.783	-0.329	-1.119	1.300	-0.511	-1.993
1/2	0	0	-0.292	-0.086	-1.114	1.481	-0.000	-2.007
1/2	0.5	0	-0.263	-0.351	-1.243	1.477	-0.003	-2.005
1/2	0	-0.5	-0.422	-0.121	-1.078	1.467	-0.501	-2.035
1/2	0.5	-0.5	-0.393	-0.369	-1.202	1.469	-0.505	-2.015
1	0	0	0.029	-0.110	-1.214	1.491	-0.004	-2.005
1	0.5	0	0.066	-0.399	-1.335	1.495	-0.007	-2.008
1	0	-0.5	-0.122	-0.138	-1.161	1.488	-0.502	-2.023
1	0.5	-0.5	-0.093	-0.404	-1.284	1.488	-0.508	-2.024
2	0	0	0.236	-0.117	-1.290	1.496	-0.001	-2.022
2	0.5	0	0.264	-0.427	-1.398	1.497	-0.001	-2.025
2	0	-0.5	0.063	-0.139	-1.220	1.498	-0.506	-2.027
2	0.5	-0.5	0.102	-0.445	-1.338	1.496	-0.505	-2.036
5	0	0	0.372	-0.126	-1.335	1.498	0.002	-2.028
5	0.5	0	0.404	-0.445	-1.457	1.498	-0.000	-2.022
5	0	-0.5	0.193	-0.148	-1.265	1.497	-0.502	-2.040
5	0.5	-0.5	0.229	-0.455	-1.377	1.496	-0.506	-2.043
10	0	0	0.421	-0.127	-1.360	1.497	-0.001	-2.010
10	0.5	0	0.445	-0.452	-1.469	1.501	-0.006	-2.017
10	0	-0.5	0.238	-0.156	-1.277	1.499	-0.504	-2.044
10	0.5	-0.5	0.269	-0.458	-1.401	1.499	-0.500	-2.049

Table 88: Estimated parameter behavior (mean) for the ZINB with NBH data from the 2,000 simulates for orthogonal covariates for $n = 500$.

θ	level β_1	level γ_1	β_0	β_1	β_2	γ_0	γ_1	γ_2
1/4	0	0	0.857	0.045	-7.775	1.727	0.004	-1.384
1/4	0.5	0	0.996	-0.623	-6.906	1.802	0.010	-1.379
1/4	0	-0.5	0.850	0.045	-8.338	1.680	-0.317	-1.294
1/4	0.5	-0.5	0.982	-0.612	-7.262	1.767	-0.310	-1.312
1/2	0	0	0.955	0.053	-5.198	1.531	0.003	-1.353
1/2	0.5	0	1.057	-0.631	-4.721	1.596	0.010	-1.364
1/2	0	-0.5	0.919	0.039	-6.108	1.478	-0.313	-1.257
1/2	0.5	-0.5	1.025	-0.617	-5.379	1.545	-0.305	-1.264
1	0	0	1.073	0.048	-2.990	1.488	0.003	-1.389
1	0.5	0	1.152	-0.639	-3.067	1.505	0.001	-1.368
1	0	-0.5	1.008	0.059	-3.693	1.403	-0.319	-1.257
1	0.5	-0.5	1.100	-0.626	-3.704	1.437	-0.317	-1.247
2	0	0	1.146	0.053	-2.465	1.487	0.003	-1.431
2	0.5	0	1.207	-0.584	-2.560	1.487	0.005	-1.402
2	0	-0.5	1.098	0.054	-2.713	1.405	-0.343	-1.281
2	0.5	-0.5	1.167	-0.603	-2.796	1.420	-0.342	-1.260
5	0	0	1.202	0.040	-2.320	1.487	0.000	-1.459
5	0.5	0	1.238	-0.539	-2.382	1.494	-0.000	-1.438
5	0	-0.5	1.158	0.041	-2.417	1.428	-0.371	-1.310
5	0.5	-0.5	1.201	-0.572	-2.502	1.434	-0.370	-1.291
10	0	0	1.222	0.047	-2.294	1.493	0.001	-1.478
10	0.5	0	1.245	-0.534	-2.323	1.492	0.000	-1.450
10	0	-0.5	1.178	0.052	-2.377	1.440	-0.389	-1.324
10	0.5	-0.5	1.211	-0.547	-2.430	1.441	-0.384	-1.301

Table 89: Estimated parameter behavior (mean) for the ZINB with NBH data from the 2,000 simulates for correlated covariates for $n = 500$.

θ	level β_1	level γ_1	β_0	β_1	β_2	γ_0	γ_1	γ_2
1/4	0	0	0.913	-0.059	-8.080	1.731	-0.001	-1.387
1/4	0.5	0	1.048	-0.713	-7.031	1.815	0.013	-1.388
1/4	0	-0.5	0.897	-0.066	-8.400	1.701	-0.326	-1.305
1/4	0.5	-0.5	1.030	-0.728	-7.115	1.805	-0.329	-1.309
1/2	0	0	1.000	-0.054	-5.054	1.534	0.004	-1.363
1/2	0.5	0	1.109	-0.736	-4.846	1.597	0.008	-1.355
1/2	0	-0.5	0.975	-0.071	-5.627	1.507	-0.322	-1.263
1/2	0.5	-0.5	1.081	-0.733	-5.383	1.569	-0.320	-1.253
1	0	0	1.109	-0.021	-2.972	1.489	-0.002	-1.393
1	0.5	0	1.181	-0.706	-3.113	1.509	0.005	-1.372
1	0	-0.5	1.076	-0.059	-3.271	1.443	-0.339	-1.257
1	0.5	-0.5	1.151	-0.735	-3.467	1.465	-0.325	-1.234
2	0	0	1.205	-0.017	-2.425	1.489	-0.001	-1.424
2	0.5	0	1.242	-0.651	-2.533	1.484	0.003	-1.391
2	0	-0.5	1.162	-0.044	-2.538	1.442	-0.365	-1.267
2	0.5	-0.5	1.201	-0.683	-2.651	1.445	-0.350	-1.238
5	0	0	1.257	-0.006	-2.293	1.491	0.003	-1.459
5	0.5	0	1.274	-0.601	-2.330	1.489	0.001	-1.430
5	0	-0.5	1.213	-0.034	-2.340	1.452	-0.383	-1.282
5	0.5	-0.5	1.254	-0.634	-2.408	1.457	-0.380	-1.254
10	0	0	1.273	-0.019	-2.256	1.494	-0.002	-1.473
10	0.5	0	1.289	-0.589	-2.300	1.492	-0.002	-1.445
10	0	-0.5	1.233	-0.018	-2.306	1.460	-0.396	-1.289
10	0.5	-0.5	1.260	-0.625	-2.351	1.461	-0.394	-1.262

Appendix B: Confidence Intervals Measures

This appendix includes 46 tables, the first 38 corresponding to the first simulation study and the last 8 corresponding to the second simulation study. Each table contains information on the proportion of 95% confidence intervals containing the true parameter or the mean of the width of the 95% confidence intervals. The first 8 tables contain both the NBH and ZINB models and includes either all levels of dispersion or correlation and a specific sample size. The remaining tables contain both the NBH and ZINB models and include all levels of dispersion, θ, and samples size for a given set of conditions.

First Simulation Study - Model Performance and Recovery

Table 90: 95% confidence interval coverage and width (standard errors) for the ZINB and NBH structural zero/zero coefficients by dispersion (θ) for $n = 250$.

	θ	β_0	β_1	β_2
		Average Coverage		
NBH	1/4	0.951 (0.005)	0.952 (0.005)	0.952 (0.005)
	1/2	0.949 (0.005)	0.951 (0.005)	0.951 (0.005)
	1	0.950 (0.005)	0.951 (0.005)	0.946 (0.005)
	2	0.948 (0.005)	0.951 (0.005)	0.949 (0.005)
	5	0.950 (0.005)	0.951 (0.005)	0.952 (0.005)
	10	0.953 (0.005)	0.949 (0.005)	0.952 (0.005)
ZINB	1/4	0.879 (0.007)	0.956 (0.005)	0.929 (0.006)
	1/2	0.899 (0.007)	0.955 (0.005)	0.935 (0.005)
	1	0.951 (0.005)	0.954 (0.005)	0.960 (0.004)
	2	0.869 (0.008)	0.956 (0.005)	0.919 (0.006)
	5	0.890 (0.007)	0.955 (0.005)	0.934 (0.006)
	10	0.948 (0.005)	0.952 (0.005)	0.962 (0.004)
		Average Width		
NBH	1/4	1.063 (0.000)	1.163 (0.000)	1.192 (0.000)
	1/2	1.059 (0.000)	1.160 (0.000)	1.177 (0.000)
	1	1.054 (0.000)	1.157 (0.000)	1.163 (0.000)
	2	1.063 (0.000)	1.162 (0.000)	1.191 (0.000)
	5	1.059 (0.000)	1.160 (0.000)	1.178 (0.000)
	10	1.053 (0.000)	1.157 (0.000)	1.163 (0.000)
ZINB	1/4	53.968 (0.856)	2.557 (0.026)	53.446 (0.847)
	1/2	37.047 (0.900)	2.801 (0.051)	36.749 (0.894)
	1	7.501 (0.200)	1.635 (0.004)	7.381 (0.196)
	2	70.373 (1.237)	2.916 (0.032)	69.682 (1.223)
	5	44.178 (1.038)	4.911 (0.153)	41.228 (0.943)
	10	11.152 (0.272)	1.768 (0.005)	10.969 (0.267)

Table 91: 95% confidence interval coverage and width (standard errors) for the ZINB and NBH non-structural zero/count coefficients by dispersion (θ) for $n = 250$.

	θ	γ_0	γ_1	γ_2
Average Coverage				
NBH	1/4	0.940 (0.005)	0.940 (0.005)	0.942 (0.005)
	1/2	0.939 (0.005)	0.941 (0.005)	0.945 (0.005)
	1	0.943 (0.005)	0.941 (0.005)	0.946 (0.005)
	2	0.938 (0.005)	0.941 (0.005)	0.945 (0.005)
	5	0.937 (0.005)	0.944 (0.005)	0.946 (0.005)
	10	0.941 (0.005)	0.942 (0.005)	0.947 (0.005)
ZINB	1/4	0.915 (0.006)	0.936 (0.005)	0.943 (0.005)
	1/2	0.919 (0.006)	0.938 (0.005)	0.942 (0.005)
	1	0.942 (0.005)	0.947 (0.005)	0.951 (0.005)
	2	0.910 (0.006)	0.933 (0.006)	0.947 (0.005)
	5	0.915 (0.006)	0.938 (0.005)	0.942 (0.005)
	10	0.941 (0.005)	0.942 (0.005)	0.949 (0.005)
Average Width				
NBH	1/4	3.570 (0.095)	1.087 (0.004)	1.237 (0.003)
	1/2	2.850 (0.079)	0.908 (0.007)	1.160 (0.003)
	1	0.586 (0.004)	0.733 (0.004)	1.035 (0.001)
	2	4.456 (0.122)	1.124 (0.004)	1.308 (0.002)
	5	4.007 (0.119)	0.962 (0.006)	1.256 (0.001)
	10	0.603 (0.005)	0.796 (0.003)	1.156 (0.001)
ZINB	1/4	0.933 (0.008)	1.030 (0.007)	1.138 (0.006)
	1/2	0.820 (0.011)	0.926 (0.010)	1.094 (0.008)
	1	0.525 (0.004)	0.681 (0.004)	0.902 (0.002)
	2	0.956 (0.008)	1.066 (0.006)	1.203 (0.005)
	5	0.834 (0.011)	0.976 (0.010)	1.169 (0.008)
	10	0.536 (0.004)	0.742 (0.004)	0.988 (0.002)

Table 92: 95% confidence interval coverage and width (standard errors) for the ZINB and NBH structural zero/zero coefficients by dispersion (θ) for $n = 500$.

	θ	β_0	β_1	β_2
Average Coverage				
NBH	1/4	0.953 (0.005)	0.953 (0.005)	0.950 (0.005)
	1/2	0.954 (0.005)	0.949 (0.005)	0.950 (0.005)
	1	0.950 (0.005)	0.949 (0.005)	0.951 (0.005)
	2	0.953 (0.005)	0.953 (0.005)	0.951 (0.005)
	5	0.953 (0.005)	0.950 (0.005)	0.950 (0.005)
	10	0.951 (0.005)	0.949 (0.005)	0.953 (0.005)
ZINB	1/4	0.899 (0.007)	0.953 (0.005)	0.926 (0.006)
	1/2	0.900 (0.007)	0.947 (0.005)	0.933 (0.006)
	1	0.950 (0.005)	0.951 (0.005)	0.959 (0.004)
	2	0.890 (0.007)	0.955 (0.005)	0.921 (0.006)
	5	0.902 (0.007)	0.949 (0.005)	0.931 (0.006)
	10	0.951 (0.005)	0.948 (0.005)	0.959 (0.004)
Average Width				
NBH	1/4	0.726 (0.001)	0.823 (0.001)	0.846 (0.000)
	1/2	0.722 (0.001)	0.819 (0.000)	0.834 (0.000)
	1	0.718 (0.001)	0.816 (0.000)	0.823 (0.000)
	2	0.726 (0.001)	0.823 (0.001)	0.846 (0.000)
	5	0.723 (0.001)	0.819 (0.000)	0.835 (0.000)
	10	0.719 (0.001)	0.815 (0.000)	0.823 (0.000)
ZINB	1/4	36.387 (0.688)	1.507 (0.006)	35.952 (0.680)
	1/2	24.702 (0.635)	1.431 (0.008)	24.317 (0.624)
	1	3.543 (0.093)	1.186 (0.003)	3.467 (0.089)
	2	42.102 (0.728)	1.599 (0.006)	41.620 (0.719)
	5	26.009 (0.653)	1.529 (0.009)	25.569 (0.641)
	10	5.039 (0.138)	1.265 (0.003)	4.919 (0.134)

Table 93: 95% confidence interval coverage and width (standard errors) for the ZINB and NBH non-structural zero/count coefficients by dispersion (θ) for $n = 500$.

	θ	γ_0	γ_1	γ_2
Average Coverage				
NBH	1/4	0.948 (0.005)	0.948 (0.005)	0.941 (0.005)
	1/2	0.946 (0.005)	0.947 (0.005)	0.943 (0.005)
	1	0.947 (0.005)	0.946 (0.005)	0.948 (0.005)
	2	0.944 (0.005)	0.947 (0.005)	0.944 (0.005)
	5	0.942 (0.005)	0.950 (0.005)	0.948 (0.005)
	10	0.947 (0.005)	0.949 (0.005)	0.951 (0.005)
ZINB	1/4	0.919 (0.006)	0.944 (0.005)	0.943 (0.005)
	1/2	0.920 (0.006)	0.942 (0.005)	0.947 (0.005)
	1	0.944 (0.005)	0.946 (0.005)	0.947 (0.005)
	2	0.913 (0.006)	0.945 (0.005)	0.944 (0.005)
	5	0.917 (0.006)	0.945 (0.005)	0.948 (0.005)
	10	0.943 (0.005)	0.950 (0.005)	0.946 (0.005)
Average Width				
NBH	1/4	1.592 (0.036)	0.794 (0.003)	0.866 (0.002)
	1/2	1.307 (0.032)	0.667 (0.005)	0.799 (0.003)
	1	0.415 (0.003)	0.546 (0.003)	0.701 (0.001)
	2	1.897 (0.047)	0.826 (0.003)	0.908 (0.002)
	5	1.548 (0.038)	0.710 (0.004)	0.858 (0.002)
	10	0.426 (0.003)	0.598 (0.002)	0.776 (0.001)
ZINB	1/4	0.718 (0.007)	0.745 (0.005)	0.790 (0.004)
	1/2	0.631 (0.009)	0.671 (0.007)	0.756 (0.006)
	1	0.383 (0.003)	0.498 (0.003)	0.615 (0.002)
	2	0.744 (0.007)	0.776 (0.005)	0.829 (0.004)
	5	0.650 (0.010)	0.710 (0.007)	0.798 (0.005)
	10	0.395 (0.003)	0.545 (0.002)	0.667 (0.002)

Table 94: 95% confidence interval coverage and width (standard errors) for the ZINB and NBH structural zero/zero coefficients by correlation (ρ) for $n = 250$

	ρ	β_0	β_1	β_2
Average Coverage				
NBH	0	0.950 (0.005)	0.951 (0.005)	0.949 (0.005)
	.3	0.950 (0.005)	0.950 (0.005)	0.952 (0.005)
ZINB	0	0.908 (0.006)	0.954 (0.005)	0.939 (0.005)
	.3	0.905 (0.007)	0.955 (0.005)	0.941 (0.005)
Average Width				
NBH	0	1.064 (0.000)	1.152 (0.000)	1.166 (0.000)
	.3	1.053 (0.000)	1.168 (0.000)	1.188 (0.000)
ZINB	0	34.287 (0.831)	2.945 (0.091)	33.148 (0.790)
	.3	40.452 (1.057)	2.585 (0.038)	40.003 (1.044)
	ρ	γ_0	γ_1	γ_2
Average Coverage				
NBH	0	0.940 (0.005)	0.940 (0.005)	0.946 (0.005)
	.3	0.939 (0.005)	0.943 (0.005)	0.945 (0.005)
ZINB	0	0.927 (0.006)	0.940 (0.005)	0.947 (0.005)
	.3	0.921 (0.006)	0.938 (0.005)	0.944 (0.005)
Average Width				
NBH	0	2.605 (0.084)	0.927 (0.006)	1.218 (0.002)
	.3	2.752 (0.094)	0.944 (0.006)	1.166 (0.003)
ZINB	0	0.790 (0.009)	0.869 (0.007)	1.081 (0.006)
	.3	0.744 (0.009)	0.937 (0.008)	1.084 (0.006)

Table 95: 95% confidence interval coverage and width (standard errors) for the ZINB and NBH structural zero/zero coefficients by correlation (ρ) for $n = 500$

	ρ	β_0	β_1	β_2
Average Coverage				
NBH	0	0.953 (0.005)	0.951 (0.005)	0.950 (0.005)
	.3	0.952 (0.005)	0.951 (0.005)	0.951 (0.005)
ZINB	0	0.915 (0.006)	0.951 (0.005)	0.936 (0.005)
	.3	0.915 (0.006)	0.951 (0.005)	0.940 (0.005)
Average Width				
NBH	0	0.749 (0.000)	0.801 (0.000)	0.824 (0.000)
	.3	0.696 (0.000)	0.837 (0.000)	0.845 (0.000)
ZINB	0	23.334 (0.629)	1.381 (0.006)	22.941 (0.621)
	.3	22.594 (0.624)	1.458 (0.007)	22.340 (0.615)
	ρ	γ_0	γ_1	γ_2
Average Coverage				
NBH	0	0.945 (0.005)	0.947 (0.005)	0.944 (0.005)
	.3	0.946 (0.005)	0.949 (0.005)	0.948 (0.005)
ZINB	0	0.927 (0.006)	0.945 (0.005)	0.945 (0.005)
	.3	0.926 (0.006)	0.945 (0.005)	0.946 (0.005)
Average Width				
NBH	0	1.209 (0.033)	0.672 (0.004)	0.786 (0.002)
	.3	1.186 (0.033)	0.709 (0.004)	0.850 (0.002)
ZINB	0	0.626 (0.008)	0.656 (0.005)	0.729 (0.004)
	.3	0.548 (0.007)	0.659 (0.005)	0.756 (0.004)

Table 96: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients with orthogonal covariates within a component, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta = 1/4$, $1/2$, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
NBH	100	0.961	0.958	0.962	0.897	0.922	0.911
	250	0.949	0.950	0.943	0.928	0.934	0.947
	500	0.954	0.955	0.950	0.940	0.940	0.944
ZINB	100	0.691	0.983	0.882	0.861	0.911	0.915
	250	0.777	0.957	0.870	0.886	0.940	0.935
	500	0.818	0.955	0.853	0.893	0.941	0.933
				$\theta = 1/2$			
NBH	100	0.954	0.958	0.959	0.921	0.931	0.919
	250	0.948	0.945	0.950	0.942	0.948	0.950
	500	0.961	0.949	0.957	0.952	0.945	0.938
ZINB	100	0.825	0.981	0.932	0.892	0.913	0.929
	250	0.867	0.954	0.926	0.927	0.933	0.944
	500	0.876	0.951	0.916	0.910	0.944	0.944
				$\theta = 1$			
NBH	100	0.936	0.954	0.951	0.939	0.926	0.923
	250	0.948	0.952	0.957	0.953	0.943	0.946
	500	0.944	0.954	0.940	0.954	0.949	0.939
ZINB	100	0.899	0.975	0.959	0.925	0.935	0.944
	250	0.931	0.957	0.960	0.921	0.938	0.955
	500	0.941	0.954	0.957	0.940	0.954	0.954

Table 97: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients with orthogonal covariates within a component, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta = 2$, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 2$			
NBH	100	0.952	0.957	0.958	0.915	0.922	0.950
	250	0.951	0.949	0.956	0.939	0.939	0.938
	500	0.955	0.947	0.948	0.954	0.951	0.941
ZINB	100	0.939	0.964	0.969	0.925	0.930	0.950
	250	0.955	0.950	0.969	0.954	0.941	0.947
	500	0.954	0.950	0.965	0.949	0.951	0.939
				$\theta = 5$			
NBH	100	0.951	0.961	0.957	0.931	0.934	0.946
	250	0.951	0.951	0.957	0.943	0.938	0.950
	500	0.951	0.956	0.953	0.943	0.934	0.940
ZINB	100	0.969	0.969	0.970	0.941	0.935	0.951
	250	0.962	0.957	0.957	0.945	0.950	0.944
	500	0.963	0.953	0.966	0.954	0.944	0.952
				$\theta = 10$			
NBH	100	0.953	0.949	0.952	0.939	0.942	0.960
	250	0.951	0.953	0.945	0.948	0.940	0.955
	500	0.950	0.944	0.951	0.948	0.954	0.948
ZINB	100	0.969	0.958	0.970	0.946	0.939	0.947
	250	0.969	0.959	0.959	0.952	0.952	0.952
	500	0.959	0.960	0.956	0.947	0.948	0.946

Table 98: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients with orthogonal covariates within a component, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta = 1/4$, $1/2$, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
	100	0.953	0.952	0.951	0.904	0.925	0.925
NBH	250	0.945	0.946	0.949	0.933	0.943	0.942
	500	0.955	0.949	0.940	0.939	0.946	0.929
	100	0.703	0.974	0.909	0.850	0.894	0.917
ZINB	250	0.794	0.952	0.892	0.880	0.924	0.941
	500	0.807	0.931	0.869	0.875	0.934	0.947
				$\theta = 1/2$			
	100	0.952	0.948	0.948	0.920	0.923	0.925
NBH	250	0.953	0.953	0.947	0.947	0.944	0.931
	500	0.957	0.952	0.953	0.949	0.953	0.938
	100	0.821	0.979	0.939	0.881	0.914	0.930
ZINB	250	0.875	0.953	0.930	0.908	0.934	0.941
	500	0.880	0.946	0.936	0.906	0.950	0.942
				$\theta = 1$			
	100	0.948	0.944	0.951	0.933	0.921	0.927
NBH	250	0.952	0.952	0.933	0.951	0.940	0.949
	500	0.958	0.955	0.964	0.958	0.943	0.948
	100	0.901	0.972	0.959	0.911	0.915	0.938
ZINB	250	0.930	0.949	0.961	0.938	0.949	0.949
	500	0.934	0.946	0.959	0.937	0.949	0.945

Table 99: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients with orthogonal covariates within a component, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta = 2$, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 2$			
	100	0.954	0.961	0.957	0.914	0.924	0.946
NBH	250	0.952	0.952	0.948	0.933	0.942	0.937
	500	0.941	0.949	0.950	0.941	0.946	0.941
	100	0.954	0.976	0.977	0.927	0.938	0.939
ZINB	250	0.961	0.958	0.969	0.946	0.952	0.951
	500	0.954	0.957	0.967	0.950	0.953	0.940
				$\theta = 5$			
	100	0.954	0.947	0.959	0.929	0.932	0.957
NBH	250	0.949	0.952	0.940	0.943	0.943	0.950
	500	0.952	0.948	0.948	0.947	0.951	0.950
	100	0.964	0.965	0.975	0.937	0.942	0.950
ZINB	250	0.958	0.949	0.955	0.944	0.943	0.960
	500	0.965	0.948	0.962	0.947	0.947	0.949
				$\theta = 10$			
	100	0.956	0.953	0.949	0.942	0.947	0.967
NBH	250	0.955	0.952	0.947	0.938	0.946	0.949
	500	0.957	0.953	0.950	0.944	0.942	0.942
	100	0.971	0.962	0.968	0.949	0.945	0.949
ZINB	250	0.965	0.948	0.950	0.945	0.944	0.955
	500	0.951	0.951	0.957	0.938	0.947	0.953

Table 100: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients with orthogonal covariates within a component, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for $\theta = 1/4$, $1/2$, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
$\theta = 1/4$							
NBH	100	0.954	0.953	0.955	0.874	0.924	0.912
	250	0.947	0.947	0.949	0.912	0.929	0.940
	500	0.958	0.952	0.950	0.935	0.948	0.940
ZINB	100	0.679	0.982	0.885	0.851	0.891	0.914
	250	0.774	0.971	0.849	0.883	0.927	0.942
	500	0.802	0.950	0.842	0.865	0.938	0.950
$\theta = 1/2$							
NBH	100	0.956	0.947	0.953	0.897	0.933	0.922
	250	0.945	0.951	0.947	0.944	0.942	0.940
	500	0.955	0.955	0.947	0.940	0.946	0.938
ZINB	100	0.795	0.988	0.933	0.879	0.921	0.937
	250	0.847	0.958	0.913	0.912	0.931	0.945
	500	0.879	0.957	0.913	0.909	0.939	0.940
$\theta = 1$							
NBH	100	0.948	0.948	0.947	0.925	0.929	0.938
	250	0.950	0.950	0.948	0.943	0.944	0.949
	500	0.957	0.952	0.952	0.948	0.948	0.944
ZINB	100	0.890	0.972	0.955	0.908	0.935	0.941
	250	0.916	0.947	0.944	0.927	0.932	0.953
	500	0.914	0.955	0.940	0.932	0.942	0.946

Table 101: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients with orthogonal covariates within a component, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for $\theta = 2$, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
			$\theta = 2$				
NBH	100	0.955	0.957	0.949	0.920	0.934	0.943
	250	0.956	0.955	0.948	0.946	0.940	0.953
	500	0.954	0.957	0.954	0.952	0.943	0.933
ZINB	100	0.940	0.970	0.969	0.922	0.939	0.948
	250	0.955	0.954	0.964	0.940	0.936	0.948
	500	0.951	0.963	0.966	0.949	0.947	0.937
			$\theta = 5$				
NBH	100	0.947	0.964	0.955	0.919	0.942	0.961
	250	0.952	0.954	0.958	0.944	0.939	0.950
	500	0.954	0.940	0.948	0.940	0.948	0.958
ZINB	100	0.960	0.967	0.978	0.927	0.931	0.957
	250	0.956	0.948	0.962	0.942	0.940	0.950
	500	0.959	0.958	0.962	0.949	0.946	0.952
			$\theta = 10$				
NBH	100	0.952	0.949	0.961	0.942	0.944	0.942
	250	0.949	0.956	0.958	0.952	0.941	0.952
	500	0.952	0.955	0.948	0.944	0.952	0.944
ZINB	100	0.971	0.966	0.968	0.929	0.930	0.957
	250	0.972	0.953	0.968	0.946	0.948	0.950
	500	0.968	0.942	0.969	0.942	0.950	0.949

Table 102: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients with orthogonal covariates within a component, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for $\theta = 1/4$, $1/2$, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
NBH	100	0.946	0.958	0.946	0.890	0.931	0.910
	250	0.957	0.950	0.952	0.917	0.945	0.938
	500	0.958	0.953	0.952	0.928	0.949	0.940
ZINB	100	0.681	0.977	0.901	0.845	0.881	0.925
	250	0.776	0.957	0.872	0.867	0.934	0.927
	500	0.798	0.927	0.857	0.870	0.935	0.939
				$\theta = 1/2$			
NBH	100	0.948	0.949	0.955	0.909	0.926	0.935
	250	0.950	0.952	0.950	0.940	0.940	0.942
	500	0.946	0.943	0.953	0.951	0.947	0.942
ZINB	100	0.821	0.973	0.939	0.888	0.927	0.936
	250	0.866	0.958	0.931	0.920	0.938	0.944
	500	0.884	0.959	0.926	0.906	0.945	0.943
				$\theta = 1$			
NBH	100	0.953	0.953	0.956	0.933	0.921	0.926
	250	0.955	0.958	0.946	0.948	0.941	0.942
	500	0.949	0.942	0.957	0.950	0.945	0.954
ZINB	100	0.909	0.973	0.964	0.912	0.931	0.942
	250	0.912	0.946	0.950	0.923	0.943	0.950
	500	0.935	0.944	0.954	0.941	0.943	0.947

Table 103: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients with orthogonal covariates within a component, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for $\theta = 2$, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 2$			
NBH	100	0.956	0.951	0.959	0.927	0.928	0.951
	250	0.947	0.947	0.954	0.945	0.931	0.943
	500	0.951	0.956	0.944	0.950	0.946	0.944
ZINB	100	0.936	0.968	0.974	0.930	0.923	0.950
	250	0.946	0.957	0.966	0.940	0.947	0.955
	500	0.956	0.950	0.964	0.939	0.953	0.945
				$\theta = 5$			
NBH	100	0.953	0.952	0.951	0.917	0.935	0.957
	250	0.951	0.955	0.946	0.935	0.943	0.956
	500	0.949	0.948	0.950	0.937	0.945	0.950
ZINB	100	0.964	0.969	0.972	0.926	0.934	0.942
	250	0.958	0.951	0.968	0.941	0.938	0.945
	500	0.965	0.954	0.957	0.948	0.946	0.945
				$\theta = 10$			
NBH	100	0.957	0.952	0.947	0.931	0.950	0.959
	250	0.954	0.955	0.956	0.945	0.941	0.949
	500	0.949	0.955	0.950	0.944	0.952	0.949
ZINB	100	0.962	0.965	0.977	0.938	0.943	0.959
	250	0.962	0.954	0.963	0.951	0.948	0.953
	500	0.959	0.952	0.952	0.944	0.949	0.946

Table 104: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients for correlated covariates within a component ($\rho = 0.3$), $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta = 1/4$, $1/2$, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
NBH	100	0.951	0.947	0.944	0.908	0.934	0.919
	250	0.957	0.954	0.949	0.926	0.940	0.937
	500	0.946	0.958	0.954	0.923	0.944	0.941
ZINB	100	0.712	0.982	0.879	0.842	0.893	0.906
	250	0.771	0.956	0.859	0.868	0.932	0.936
	500	0.813	0.951	0.860	0.876	0.935	0.944
				$\theta = 1/2$			
NBH	100	0.958	0.958	0.952	0.927	0.929	0.925
	250	0.953	0.954	0.958	0.945	0.937	0.928
	500	0.951	0.955	0.950	0.959	0.953	0.941
ZINB	100	0.838	0.981	0.926	0.884	0.918	0.924
	250	0.857	0.968	0.923	0.909	0.939	0.939
	500	0.894	0.948	0.925	0.901	0.952	0.940
				$\theta = 1$			
NBH	100	0.953	0.954	0.955	0.922	0.921	0.935
	250	0.957	0.955	0.952	0.947	0.942	0.941
	500	0.956	0.950	0.955	0.950	0.951	0.945
ZINB	100	0.922	0.971	0.954	0.909	0.916	0.931
	250	0.920	0.954	0.960	0.913	0.931	0.940
	500	0.930	0.958	0.958	0.939	0.944	0.941

Table 105: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients for correlated covariates within a component ($\rho = 0.3$), $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta = 2$, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 2$			
NBH	100	0.956	0.959	0.946	0.934	0.923	0.948
	250	0.950	0.956	0.951	0.945	0.942	0.951
	500	0.955	0.955	0.951	0.951	0.954	0.944
ZINB	100	0.951	0.969	0.972	0.911	0.930	0.944
	250	0.951	0.957	0.967	0.940	0.935	0.947
	500	0.963	0.960	0.972	0.940	0.936	0.950
				$\theta = 5$			
NBH	100	0.959	0.948	0.958	0.950	0.938	0.960
	250	0.947	0.958	0.954	0.937	0.943	0.957
	500	0.957	0.957	0.946	0.956	0.940	0.944
ZINB	100	0.961	0.960	0.979	0.938	0.924	0.956
	250	0.966	0.958	0.963	0.942	0.941	0.946
	500	0.955	0.940	0.954	0.950	0.938	0.947
				$\theta = 10$			
NBH	100	0.957	0.956	0.955	0.945	0.940	0.959
	250	0.945	0.953	0.956	0.943	0.940	0.949
	500	0.958	0.946	0.948	0.948	0.950	0.954
ZINB	100	0.962	0.964	0.972	0.935	0.933	0.953
	250	0.962	0.958	0.962	0.941	0.935	0.942
	500	0.950	0.955	0.958	0.940	0.955	0.947

Table 106: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients for correlated covariates within a component ($\rho = 0.3$), $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta = 1/4$, $1/2$, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
	100	0.956	0.948	0.955	0.881	0.936	0.918
NBH	250	0.955	0.949	0.953	0.918	0.953	0.943
	500	0.956	0.948	0.950	0.933	0.949	0.938
	100	0.723	0.967	0.909	0.837	0.893	0.909
ZINB	250	0.812	0.957	0.897	0.881	0.933	0.934
	500	0.819	0.938	0.884	0.880	0.930	0.940
				$\theta = 1/2$			
	100	0.950	0.955	0.957	0.920	0.927	0.924
NBH	250	0.950	0.943	0.950	0.945	0.929	0.933
	500	0.953	0.941	0.957	0.955	0.950	0.955
	100	0.832	0.979	0.932	0.877	0.915	0.934
ZINB	250	0.855	0.949	0.924	0.900	0.935	0.941
	500	0.871	0.952	0.940	0.910	0.942	0.949
				$\theta = 1$			
	100	0.956	0.952	0.953	0.935	0.931	0.939
NBH	250	0.950	0.953	0.948	0.950	0.940	0.940
	500	0.950	0.951	0.956	0.945	0.939	0.956
	100	0.914	0.966	0.962	0.908	0.929	0.937
ZINB	250	0.916	0.952	0.959	0.931	0.941	0.938
	500	0.936	0.945	0.960	0.936	0.938	0.945

Table 107: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients for correlated covariates within a component ($\rho = 0.3$), $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta = 2$, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
			$\theta = 2$				
	100	0.956	0.951	0.950	0.941	0.925	0.938
NBH	250	0.945	0.947	0.948	0.950	0.941	0.944
	500	0.950	0.947	0.948	0.950	0.949	0.947
	100	0.949	0.972	0.967	0.922	0.913	0.947
ZINB	250	0.953	0.961	0.966	0.938	0.949	0.952
	500	0.959	0.953	0.965	0.947	0.945	0.944
			$\theta = 5$				
	100	0.953	0.951	0.945	0.929	0.935	0.952
NBH	250	0.947	0.952	0.954	0.936	0.938	0.942
	500	0.945	0.943	0.946	0.949	0.953	0.948
	100	0.966	0.957	0.969	0.924	0.934	0.955
ZINB	250	0.958	0.955	0.960	0.946	0.951	0.943
	500	0.951	0.947	0.957	0.955	0.946	0.954
			$\theta = 10$				
	100	0.960	0.954	0.949	0.933	0.936	0.949
NBH	250	0.948	0.952	0.947	0.945	0.941	0.956
	500	0.951	0.949	0.945	0.944	0.944	0.950
	100	0.960	0.961	0.961	0.935	0.936	0.952
ZINB	250	0.968	0.962	0.962	0.949	0.952	0.960
	500	0.948	0.960	0.947	0.947	0.941	0.945

Table 108: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients for correlated covariates within a component ($\rho = 0.3$), $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for $\theta = 1/4$, $1/2$, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
	100	0.951	0.954	0.950	0.890	0.921	0.916
NBH	250	0.942	0.947	0.956	0.915	0.942	0.943
	500	0.942	0.949	0.952	0.931	0.946	0.952
	100	0.699	0.980	0.878	0.839	0.895	0.909
ZINB	250	0.757	0.961	0.851	0.869	0.929	0.938
	500	0.800	0.955	0.850	0.870	0.943	0.932
				$\theta = 1/2$			
	100	0.956	0.952	0.958	0.908	0.928	0.939
NBH	250	0.944	0.951	0.951	0.947	0.947	0.942
	500	0.944	0.951	0.945	0.944	0.949	0.947
	100	0.834	0.982	0.928	0.883	0.901	0.936
ZINB	250	0.837	0.956	0.911	0.892	0.927	0.944
	500	0.883	0.950	0.926	0.894	0.946	0.944
				$\theta = 1$			
	100	0.961	0.956	0.947	0.932	0.937	0.928
NBH	250	0.952	0.955	0.950	0.945	0.947	0.950
	500	0.957	0.948	0.954	0.957	0.947	0.949
	100	0.898	0.972	0.950	0.905	0.923	0.953
ZINB	250	0.919	0.943	0.959	0.919	0.939	0.950
	500	0.933	0.952	0.960	0.935	0.954	0.948

Table 109: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients for correlated covariates within a component ($\rho = 0.3$), $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for $\theta = 2$, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 2$			
NBH	100	0.954	0.960	0.951	0.927	0.926	0.945
	250	0.951	0.955	0.947	0.954	0.936	0.947
	500	0.957	0.962	0.956	0.948	0.952	0.952
ZINB	100	0.943	0.969	0.961	0.917	0.927	0.950
	250	0.948	0.956	0.966	0.940	0.945	0.953
	500	0.956	0.955	0.970	0.949	0.950	0.954
				$\theta = 5$			
NBH	100	0.949	0.952	0.947	0.938	0.947	0.962
	250	0.946	0.950	0.952	0.941	0.952	0.947
	500	0.952	0.958	0.954	0.954	0.949	0.950
ZINB	100	0.963	0.965	0.968	0.929	0.930	0.957
	250	0.955	0.949	0.959	0.954	0.948	0.944
	500	0.971	0.945	0.955	0.950	0.952	0.957
				$\theta = 10$			
NBH	100	0.959	0.951	0.948	0.945	0.946	0.967
	250	0.955	0.951	0.955	0.945	0.948	0.954
	500	0.958	0.950	0.953	0.944	0.943	0.950
ZINB	100	0.965	0.965	0.968	0.948	0.945	0.952
	250	0.965	0.951	0.969	0.942	0.934	0.954
	500	0.963	0.956	0.962	0.963	0.949	0.955

Table 110: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients for correlated covariates within a component ($\rho = 0.3$), $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for $\theta = 1/4$, $1/2$, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
NBH	100	0.946	0.941	0.953	0.883	0.932	0.926
	250	0.947	0.948	0.946	0.913	0.946	0.938
	500	0.950	0.954	0.942	0.929	0.954	0.940
ZINB	100	0.702	0.961	0.910	0.822	0.906	0.916
	250	0.777	0.962	0.879	0.862	0.927	0.927
	500	0.799	0.948	0.885	0.860	0.943	0.942
				$\theta = 1/2$			
NBH	100	0.947	0.949	0.948	0.911	0.919	0.923
	250	0.949	0.948	0.947	0.943	0.939	0.944
	500	0.957	0.952	0.949	0.948	0.957	0.946
ZINB	100	0.834	0.969	0.938	0.870	0.923	0.920
	250	0.857	0.961	0.934	0.890	0.933	0.938
	500	0.876	0.962	0.932	0.900	0.942	0.947
				$\theta = 1$			
NBH	100	0.950	0.951	0.954	0.931	0.930	0.932
	250	0.956	0.943	0.961	0.942	0.948	0.932
	500	0.952	0.942	0.959	0.946	0.948	0.948
ZINB	100	0.917	0.976	0.963	0.907	0.922	0.940
	250	0.921	0.961	0.964	0.931	0.940	0.950
	500	0.929	0.953	0.958	0.923	0.948	0.951

Table 111: Proportion of 95% confidence intervals that contain the true parameter for the NBH and ZINB coefficients for correlated covariates within a component ($\rho = 0.3$), $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for $\theta = 2$, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 2$			
	100	0.951	0.962	0.955	0.936	0.935	0.948
NBH	250	0.955	0.942	0.955	0.944	0.942	0.951
	500	0.962	0.944	0.955	0.957	0.958	0.956
	100	0.954	0.966	0.971	0.910	0.918	0.949
ZINB	250	0.954	0.948	0.968	0.940	0.932	0.950
	500	0.957	0.947	0.966	0.951	0.958	0.940
				$\theta = 5$			
	100	0.948	0.954	0.955	0.931	0.933	0.955
NBH	250	0.954	0.949	0.954	0.932	0.950	0.952
	500	0.947	0.961	0.960	0.943	0.948	0.951
	100	0.963	0.967	0.966	0.936	0.934	0.955
ZINB	250	0.969	0.956	0.959	0.955	0.941	0.946
	500	0.959	0.943	0.959	0.954	0.952	0.947
				$\theta = 10$			
	100	0.951	0.956	0.950	0.945	0.952	0.963
NBH	250	0.953	0.948	0.950	0.941	0.945	0.954
	500	0.951	0.948	0.950	0.949	0.956	0.960
	100	0.966	0.968	0.971	0.943	0.936	0.956
ZINB	250	0.963	0.942	0.960	0.948	0.945	0.943
	500	0.947	0.945	0.960	0.947	0.951	0.953

Table 112: Average 95% confidence interval width for the NBH and ZINB coefficients with orthogonal covariates, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta = 1/4$, $1/2$, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
NBH	100	1.539	1.909	1.908	31.947	2.252	2.300
	250	1.069	1.151	1.175	8.332	1.331	1.394
	500	0.752	0.800	0.832	3.850	0.925	0.964
ZINB	100	131.054	73.642	142.944	1.812	2.162	2.260
	250	96.860	4.566	95.899	1.470	1.405	1.483
	500	77.817	1.783	76.840	1.199	1.027	1.035
				$\theta = 1/2$			
NBH	100	1.542	1.909	1.909	17.442	2.002	2.103
	250	1.067	1.150	1.174	2.614	1.174	1.289
	500	0.754	0.802	0.835	1.185	0.825	0.865
ZINB	100	111.760	24.357	119.291	1.327	1.659	1.782
	250	68.166	2.396	67.558	1.051	1.076	1.180
	500	48.219	1.526	47.593	0.833	0.794	0.814
				$\theta = 1$			
NBH	100	1.535	1.905	1.905	4.108	1.715	2.649
	250	1.068	1.152	1.175	1.009	0.996	1.197
	500	0.755	0.801	0.834	0.660	0.718	0.782
ZINB	100	79.978	8.489	80.822	0.987	1.307	1.487
	250	30.938	1.834	30.644	0.757	0.842	0.991
	500	18.284	1.334	17.963	0.586	0.632	0.675

Table 113: Average 95% confidence interval width for the NBH and ZINB coefficients with orthogonal covariates, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta =$ 2, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 2$			
NBH	100	1.543	1.911	1.911	2.066	1.467	2.508
	250	1.069	1.152	1.175	0.682	0.824	1.134
	500	0.753	0.801	0.833	0.493	0.611	0.724
ZINB	100	43.483	7.507	44.153	0.757	1.064	1.315
	250	7.222	1.630	7.144	0.562	0.674	0.876
	500	3.468	1.183	3.372	0.422	0.513	0.590
				$\theta = 5$			
NBH	100	1.543	1.911	1.911	0.763	1.182	3.110
	250	1.070	1.152	1.176	0.518	0.658	1.101
	500	0.753	0.801	0.833	0.373	0.496	0.680
ZINB	100	14.175	3.901	14.441	0.567	0.846	1.187
	250	2.055	1.487	2.106	0.415	0.528	0.800
	500	1.251	1.099	1.261	0.312	0.409	0.534
				$\theta = 10$			
NBH	100	1.535	1.907	1.905	0.679	1.072	4.626
	250	1.065	1.150	1.173	0.453	0.582	1.096
	500	0.754	0.801	0.834	0.326	0.442	0.667
ZINB	100	7.175	3.664	8.380	0.494	0.753	1.141
	250	1.480	1.450	1.557	0.360	0.466	0.777
	500	1.157	1.074	1.189	0.270	0.362	0.513

Table 114: Average 95% confidence interval width for the NBH and ZINB coefficients with orthogonal covariates, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for θ = 1/4, 1/2, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
NBH	100	1.537	1.879	1.892	30.617	2.126	2.178
	250	1.061	1.153	1.158	8.756	1.271	1.329
	500	0.745	0.802	0.814	3.593	0.876	0.913
ZINB	100	308.139	108.660	281.581	1.846	2.257	2.371
	250	83.307	3.569	81.748	1.504	1.463	1.547
	500	60.811	1.849	59.691	1.237	1.074	1.082
				$\theta = 1/2$			
NBH	100	1.533	1.875	1.887	15.009	1.912	2.020
	250	1.062	1.154	1.160	2.107	1.113	1.220
	500	0.744	0.801	0.813	1.026	0.781	0.820
ZINB	100	121.143	32.806	127.245	1.367	1.720	1.860
	250	48.686	2.286	47.845	1.086	1.119	1.232
	500	32.380	1.550	31.642	0.871	0.830	0.851
				$\theta = 1$			
NBH	100	1.532	1.875	1.886	3.174	1.652	2.167
	250	1.063	1.154	1.159	0.878	0.947	1.136
	500	0.745	0.801	0.813	0.627	0.680	0.741
ZINB	100	80.013	14.855	79.124	1.031	1.363	1.557
	250	19.180	1.862	18.763	0.790	0.881	1.038
	500	10.169	1.330	9.781	0.604	0.659	0.704

Table 115: Average 95% confidence interval width for the NBH and ZINB coefficients with orthogonal covariates, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for θ = 2, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 2$			
NBH	100	1.534	1.875	1.888	1.036	1.377	1.978
	250	1.057	1.152	1.157	0.651	0.783	1.067
	500	0.745	0.802	0.814	0.464	0.576	0.684
ZINB	100	35.186	9.682	35.716	0.781	1.108	1.379
	250	3.696	1.631	3.545	0.580	0.701	0.917
	500	1.939	1.179	1.794	0.438	0.535	0.619
				$\theta = 5$			
NBH	100	1.530	1.874	1.887	0.736	1.113	2.875
	250	1.063	1.154	1.160	0.494	0.623	1.034
	500	0.745	0.802	0.814	0.351	0.467	0.643
ZINB	100	10.952	5.192	11.140	0.591	0.882	1.249
	250	1.566	1.485	1.578	0.430	0.549	0.840
	500	1.202	1.096	1.178	0.325	0.426	0.560
				$\theta = 10$			
NBH	100	1.532	1.874	1.886	0.652	1.007	3.397
	250	1.061	1.153	1.159	0.431	0.551	1.034
	500	0.744	0.801	0.813	0.308	0.417	0.631
ZINB	100	5.537	4.110	6.292	0.507	0.779	1.199
	250	1.445	1.443	1.490	0.371	0.482	0.813
	500	1.125	1.070	1.126	0.283	0.379	0.539

Table 116: Average 95% confidence interval width for the NBH and ZINB coefficients with orthogonal covariates, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for θ = 1/4, 1/2, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
NBH	100	1.541	1.910	1.911	38.862	2.290	2.329
	250	1.065	1.150	1.173	11.620	1.343	1.424
	500	0.753	0.801	0.833	5.486	0.937	0.974
ZINB	100	130.828	70.618	157.947	1.810	2.220	2.350
	250	99.907	4.991	98.718	1.515	1.426	1.538
	500	85.931	1.880	84.904	1.236	1.053	1.061
				$\theta = 1/2$			
NBH	100	1.541	1.912	1.912	15.594	2.068	2.171
	250	1.067	1.151	1.174	3.137	1.202	1.347
	500	0.754	0.801	0.834	1.121	0.853	0.896
ZINB	100	110.733	26.021	111.374	1.333	1.708	4.542
	250	76.477	2.499	75.830	1.072	1.097	1.242
	500	57.434	1.626	56.749	0.876	0.826	0.848
				$\theta = 1$			
NBH	100	1.539	1.909	1.909	5.266	2.742	3.332
	250	1.069	1.151	1.175	0.976	1.036	1.289
	500	0.755	0.801	0.834	0.704	0.759	0.833
ZINB	100	84.855	14.440	83.646	1.003	1.377	1.590
	250	47.393	1.995	47.026	0.781	0.878	1.069
	500	27.823	1.407	27.455	0.609	0.666	0.713

Table 117: Average 95% confidence interval width for the NBH and ZINB coefficients with orthogonal covariates, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for θ = 2, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 2$			
NBH	100	1.540	1.911	1.911	2.183	1.616	3.989
	250	1.065	1.149	1.173	0.695	0.872	1.253
	500	0.754	0.801	0.834	0.508	0.661	0.790
ZINB	100	57.587	7.930	58.385	0.764	1.147	1.423
	250	13.859	1.757	13.713	0.579	0.717	0.963
	500	5.178	1.266	5.023	0.439	0.552	0.633
				$\theta = 5$			
NBH	100	1.542	1.908	1.908	1.004	1.639	5.461
	250	1.070	1.152	1.176	0.525	0.724	1.240
	500	0.753	0.801	0.833	0.380	0.556	0.759
ZINB	100	24.874	4.397	25.860	0.574	0.959	1.324
	250	3.854	1.595	3.873	0.421	0.580	0.891
	500	1.653	1.169	1.631	0.318	0.455	0.577
				$\theta = 10$			
NBH	100	1.543	1.914	1.913	0.714	1.297	4.476
	250	1.068	1.151	1.175	0.462	0.660	1.255
	500	0.754	0.801	0.835	0.330	0.507	0.749
ZINB	100	15.974	3.552	16.565	0.499	0.884	1.283
	250	2.203	1.559	2.259	0.365	0.526	0.866
	500	1.308	1.142	1.316	0.274	0.414	0.559

Table 118: Average 95% confidence interval width for the NBH and ZINB coefficients with orthogonal covariates, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for $\theta = 1/4$, $1/2$, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
NBH	100	1.534	1.876	1.888	34.723	2.140	2.184
	250	1.060	1.152	1.158	13.203	1.284	1.355
	500	0.746	0.802	0.814	4.179	0.888	0.924
ZINB	100	198.577	67.550	229.581	1.841	2.271	13.096
	250	109.289	21.439	90.505	1.517	1.484	1.607
	500	69.556	1.924	68.423	1.259	1.097	1.106
				$\theta = 1/2$			
NBH	100	1.535	1.876	1.888	16.026	1.965	2.065
	250	1.062	1.154	1.159	2.442	1.141	1.279
	500	0.745	0.802	0.814	1.245	0.809	0.851
ZINB	100	210.735	62.262	189.690	1.369	1.780	1.958
	250	64.290	2.307	63.490	1.122	1.149	1.301
	500	34.292	1.651	33.444	0.914	0.863	0.887
				$\theta = 1$			
NBH	100	1.530	1.874	1.886	5.148	1.732	2.202
	250	1.063	1.154	1.159	0.928	0.984	1.222
	500	0.745	0.802	0.814	0.658	0.717	0.789
ZINB	100	76.674	18.890	77.608	1.034	1.438	4.373
	250	28.068	1.983	27.582	0.809	0.913	1.116
	500	13.359	1.427	12.876	0.638	0.696	0.747

Table 119: Average 95% confidence interval width for the NBH and ZINB coefficients with orthogonal covariates, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for $\theta = 2$, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 2$			
NBH	100	1.529	1.873	1.886	1.298	1.508	3.655
	250	1.061	1.153	1.159	0.669	0.832	1.187
	500	0.745	0.802	0.814	0.477	0.622	0.745
ZINB	100	92.639	12.923	92.517	0.788	1.199	1.504
	250	8.879	1.769	8.646	0.600	0.750	1.009
	500	3.017	1.267	2.797	0.457	0.577	0.662
				$\theta = 5$			
NBH	100	1.531	1.876	1.886	0.743	1.289	3.544
	250	1.061	1.152	1.158	0.502	0.687	1.171
	500	0.745	0.802	0.814	0.356	0.522	0.719
ZINB	100	19.864	6.401	20.594	0.590	0.998	1.381
	250	2.321	1.600	2.277	0.438	0.608	0.934
	500	1.374	1.169	1.303	0.332	0.476	0.606
				$\theta = 10$			
NBH	100	1.535	1.878	1.889	0.659	1.200	3.465
	250	1.058	1.151	1.157	0.436	0.621	1.182
	500	0.744	0.801	0.813	0.311	0.477	0.710
ZINB	100	10.258	4.032	10.845	0.517	0.918	1.346
	250	1.753	1.540	1.767	0.376	0.548	0.909
	500	1.261	1.140	1.230	0.287	0.434	0.586

Table 120: Average 95% confidence interval width for the NBH and ZINB coefficients with correlated covariates, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta = 1/4$, $1/2$, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
NBH	100	1.684	1.932	1.972	29.426	2.215	2.276
	250	1.057	1.173	1.207	11.951	1.341	1.388
	500	0.699	0.845	0.858	4.403	0.998	1.032
ZINB	100	899.019	52.201	915.273	1.829	2.326	2.368
	250	103.310	4.130	101.957	1.411	1.485	1.520
	500	77.880	1.926	77.040	1.078	1.043	1.064
				$\theta = 1/2$			
NBH	100	1.678	1.928	1.966	12.675	1.999	2.305
	250	1.061	1.175	1.210	2.479	1.178	1.257
	500	0.699	0.845	0.858	1.095	0.884	0.933
ZINB	100	269.919	74.197	321.987	1.336	1.775	1.844
	250	80.201	2.254	79.608	0.988	1.134	1.192
	500	46.121	1.635	45.618	0.748	0.805	0.840
				$\theta = 1$			
NBH	100	1.672	1.925	1.960	3.017	1.706	1.920
	250	1.060	1.175	1.210	0.866	1.007	1.151
	500	0.698	0.846	0.859	0.610	0.760	0.848
ZINB	100	84.857	12.290	86.465	1.007	1.418	1.535
	250	36.793	1.928	36.537	0.706	0.897	0.993
	500	17.054	1.418	16.869	0.516	0.634	0.696

Table 121: Average 95% confidence interval width for the NBH and ZINB coefficients with correlated covariates, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta =$ 2, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 2$			
NBH	100	1.676	1.927	1.963	0.971	1.429	2.377
	250	1.057	1.174	1.206	0.630	0.847	1.083
	500	0.699	0.845	0.859	0.439	0.634	0.780
ZINB	100	48.890	6.267	50.648	0.756	1.149	1.346
	250	8.251	1.720	8.224	0.518	0.723	0.870
	500	2.255	1.248	2.317	0.362	0.510	0.608
				$\theta = 5$			
NBH	100	1.682	1.932	1.971	0.708	1.168	2.890
	250	1.060	1.174	1.209	0.471	0.680	1.034
	500	0.699	0.846	0.859	0.328	0.512	0.736
ZINB	100	16.138	3.195	16.236	0.571	0.928	1.223
	250	2.271	1.574	2.379	0.378	0.572	0.785
	500	1.077	1.154	1.247	0.263	0.403	0.548
				$\theta = 10$			
NBH	100	1.673	1.928	1.963	0.618	1.046	2.477
	250	1.059	1.174	1.208	0.408	0.603	1.018
	500	0.699	0.845	0.858	0.285	0.454	0.723
ZINB	100	11.151	3.790	11.628	0.497	0.832	1.172
	250	1.594	1.538	1.726	0.326	0.505	0.754
	500	1.003	1.127	1.192	0.227	0.356	0.525

Table 122: Average 95% confidence interval width for the NBH and ZINB coefficients with correlated covariates, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta = 1/4$, $1/2$, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
			$\theta = 1/4$				
NBH	100	1.652	1.890	1.891	27.863	2.058	2.115
	250	1.046	1.161	1.168	8.091	1.252	1.298
	500	0.693	0.829	0.832	3.624	0.942	0.974
ZINB	100	787.650	108.745	848.722	1.903	2.699	4.424
	250	96.921	8.112	97.447	1.453	1.560	1.604
	500	69.807	1.963	68.839	1.093	1.093	1.116
			$\theta = 1/2$				
NBH	100	1.654	1.889	1.890	10.732	1.847	1.950
	250	1.047	1.162	1.168	1.998	1.108	1.180
	500	0.693	0.829	0.831	0.902	0.832	0.878
ZINB	100	223.457	17.867	221.327	1.409	1.855	1.930
	250	60.061	2.393	59.182	1.036	1.190	1.257
	500	30.131	1.629	29.478	0.773	0.839	0.880
			$\theta = 1$				
NBH	100	1.655	1.890	1.892	2.043	1.580	1.783
	250	1.043	1.160	1.167	0.823	0.954	1.081
	500	0.692	0.829	0.831	0.570	0.713	0.795
ZINB	100	84.991	9.810	86.677	1.060	1.485	1.611
	250	23.801	1.907	23.394	0.744	0.943	1.050
	500	10.341	1.388	10.051	0.527	0.661	0.729

Table 123: Average 95% confidence interval width for the NBH and ZINB coefficients with correlated covariates, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, 0, -2\}$ for $\theta =$ 2, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 2$			
	100	1.660	1.895	1.896	0.897	1.340	1.895
NBH	250	1.048	1.162	1.169	0.589	0.799	1.013
	500	0.692	0.829	0.831	0.416	0.600	0.735
	100	28.397	5.632	28.356	0.815	1.221	1.417
ZINB	250	7.183	1.702	7.029	0.547	0.763	0.922
	500	1.541	1.212	1.517	0.376	0.533	0.640
				$\theta = 5$			
	100	1.659	1.894	1.896	0.653	1.081	2.054
NBH	250	1.047	1.161	1.168	0.437	0.639	0.965
	500	0.692	0.829	0.831	0.312	0.484	0.695
	100	11.446	3.113	11.144	0.599	0.971	1.273
ZINB	250	1.625	1.540	1.662	0.394	0.598	0.832
	500	1.051	1.119	1.165	0.274	0.422	0.578
				$\theta = 10$			
	100	1.653	1.892	1.893	0.570	0.970	1.573
NBH	250	1.046	1.162	1.168	0.384	0.569	0.953
	500	0.692	0.830	0.832	0.270	0.428	0.682
	100	5.650	2.937	5.672	0.524	0.872	1.228
ZINB	250	1.516	1.506	1.587	0.340	0.528	0.801
	500	0.980	1.092	1.120	0.236	0.372	0.555

Table 124: Average 95% confidence interval width for the NBH and ZINB coefficients with correlated covariates, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for θ = 1/4, 1/2, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
NBH	100	1.672	1.925	1.961	37.125	2.240	2.298
	250	1.058	1.174	1.208	14.468	1.349	1.403
	500	0.699	0.845	0.858	5.115	1.005	1.041
ZINB	100	2671.429	236.407	2683.461	1.800	2.379	5.703
	250	183.560	5.279	181.577	1.413	1.504	1.553
	500	81.524	2.015	80.633	1.110	1.067	1.095
				$\theta = 1/2$			
NBH	100	1.678	1.928	1.963	19.062	2.044	2.163
	250	1.058	1.174	1.207	3.166	1.216	1.313
	500	0.699	0.845	0.859	1.164	0.901	0.959
ZINB	100	116.853	24.833	122.961	1.342	1.850	4.433
	250	82.328	2.872	81.569	1.020	1.177	1.251
	500	54.803	1.752	54.261	0.774	0.830	0.878
				$\theta = 1$			
NBH	100	1.673	1.929	1.966	4.401	1.816	2.487
	250	1.060	1.174	1.208	0.932	1.061	1.237
	500	0.699	0.845	0.858	0.631	0.798	0.903
ZINB	100	148.316	16.610	151.669	0.996	1.498	1.615
	250	45.826	2.087	45.486	0.733	0.948	1.061
	500	21.196	1.515	20.970	0.533	0.666	0.741

Table 125: Average 95% confidence interval width for the NBH and ZINB coefficients fwith correlated covariates, $\boldsymbol{\beta} = \{-1.5, 0, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for θ = 2, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 2$			
NBH	100	1.676	1.927	1.964	1.616	1.787	3.891
	250	1.059	1.175	1.209	0.650	0.916	1.198
	500	0.699	0.845	0.858	0.451	0.693	0.868
ZINB	100	70.076	8.876	71.533	0.771	1.269	1.457
	250	13.633	1.849	13.535	0.533	0.783	0.946
	500	2.924	1.327	2.964	0.371	0.550	0.660
				$\theta = 5$			
NBH	100	1.677	1.934	1.974	0.723	1.360	4.351
	250	1.061	1.177	1.211	0.475	0.766	1.178
	500	0.698	0.845	0.858	0.333	0.583	0.846
ZINB	100	31.574	5.333	33.242	0.577	1.072	1.341
	250	3.144	1.674	3.227	0.384	0.648	0.872
	500	1.196	1.227	1.363	0.268	0.457	0.608
				$\theta = 10$			
NBH	100	1.672	1.930	1.966	0.629	1.270	5.598
	250	1.058	1.174	1.208	0.412	0.701	1.179
	500	0.699	0.846	0.859	0.288	0.534	0.842
ZINB	100	22.911	4.409	23.266	0.501	0.986	1.290
	250	2.473	1.627	2.597	0.329	0.591	0.847
	500	1.084	1.193	1.279	0.229	0.415	0.589

Table 126: Average 95% confidence interval width for the NBH and ZINB coefficients with correlated covariates, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for $\theta = 1/4$, 1/2, and 1.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
				$\theta = 1/4$			
NBH	100	1.655	1.893	1.895	34.192	2.106	2.159
	250	1.047	1.162	1.168	11.755	1.276	1.326
	500	0.692	0.829	0.832	4.294	0.948	0.981
ZINB	100	389.425	104.197	475.258	1.895	2.508	7.397
	250	98.772	6.251	95.516	1.462	1.591	1.650
	500	64.744	2.177	63.575	1.138	1.110	1.143
				$\theta = 1/2$			
NBH	100	1.656	1.895	1.897	11.370	1.890	2.207
	250	1.047	1.162	1.169	2.780	1.147	1.233
	500	0.693	0.830	0.832	1.336	0.856	0.909
ZINB	100	101.284	25.764	100.526	1.431	1.954	2.022
	250	69.402	2.836	68.360	1.070	1.238	1.321
	500	34.236	1.745	33.524	0.796	0.866	0.918
				$\theta = 1$			
NBH	100	1.657	1.894	1.897	3.059	1.687	1.919
	250	1.044	1.159	1.165	0.857	0.998	1.158
	500	0.693	0.829	0.831	0.594	0.748	0.847
ZINB	100	78.972	15.054	77.478	1.067	1.584	1.706
	250	31.643	2.171	31.077	0.760	0.991	1.116
	500	16.440	1.486	16.108	0.547	0.694	0.776

Table 127: Average 95% confidence interval width for the NBH and ZINB coefficients with correlated covariates, $\boldsymbol{\beta} = \{-1.5, 0.5, 2\}$, and $\boldsymbol{\gamma} = \{1.5, -0.5, -2\}$ for $\theta = 2$, 5, and 10.

	n	β_0	β_1	β_2	γ_0	γ_1	γ_2
			$\theta = 2$				
NBH	100	1.650	1.888	1.889	1.394	1.461	2.276
	250	1.045	1.160	1.167	0.601	0.861	1.122
	500	0.691	0.829	0.831	0.426	0.650	0.813
ZINB	100	39.553	7.007	38.758	0.816	1.334	1.520
	250	12.165	1.834	11.963	0.557	0.825	1.004
	500	2.636	1.290	2.576	0.386	0.576	0.694
			$\theta = 5$				
NBH	100	1.648	1.892	1.893	0.849	1.263	2.889
	250	1.047	1.162	1.169	0.445	0.724	1.100
	500	0.693	0.829	0.832	0.313	0.547	0.794
ZINB	100	791.491	4.853	791.339	0.614	1.125	1.394
	250	2.370	1.645	2.371	0.401	0.680	0.921
	500	1.163	1.188	1.261	0.277	0.477	0.639
			$\theta = 10$				
NBH	100	1.658	1.893	1.897	0.588	1.180	3.018
	250	1.046	1.161	1.167	0.385	0.661	1.107
	500	0.692	0.829	0.831	0.273	0.502	0.791
ZINB	100	14.606	4.333	14.668	0.533	1.047	1.350
	250	2.015	1.603	2.066	0.345	0.622	0.898
	500	1.060	1.156	1.199	0.238	0.434	0.622

Second Simulation Study - Model Misspecification

Table 128: Proportion of 95% confidence intervals that contained the true parameter for the NBH model with ZINB data with orthogonal covariates for $n = 500$.

θ	level β_1	level γ_1	β_0	β_1	β_2	γ_0	γ_1	γ_2
1/4	0	0	0.028	0.957	0	0.926	0.947	0.942
1/4	0.5	0	0.021	0.130	0	0.913	0.931	0.944
1/4	0	-0.5	0.094	0.944	0	0.908	0.941	0.949
1/4	0.5	-0.5	0.081	0.174	0	0.910	0.934	0.932
1/2	0	0	0	0.953	0	0.936	0.951	0.951
1/2	0.5	0	0	0.086	0	0.948	0.941	0.942
1/2	0	-0.5	0.001	0.946	0	0.952	0.943	0.950
1/2	0.5	-0.5	0	0.098	0	0.948	0.951	0.941
1	0	0	0	0.954	0	0.954	0.942	0.949
1	0.5	0	0	0.042	0	0.951	0.943	0.951
1	0	-0.5	0	0.934	0	0.954	0.941	0.953
1	0.5	-0.5	0	0.076	0	0.953	0.949	0.940
2	0	0	0	0.938	0	0.947	0.949	0.950
2	0.5	0	0	0.032	0	0.946	0.953	0.948
2	0	-0.5	0	0.948	0	0.953	0.947	0.949
2	0.5	-0.5	0	0.056	0	0.940	0.954	0.952
5	0	0	0	0.954	0	0.950	0.946	0.952
5	0.5	0	0	0.032	0	0.946	0.947	0.950
5	0	-0.5	0	0.944	0	0.947	0.954	0.955
5	0.5	-0.5	0	0.044	0	0.947	0.947	0.948
10	0	0	0	0.960	0	0.948	0.943	0.951
10	0.5	0	0	0.018	0	0.934	0.951	0.948
10	0	-0.5	0	0.933	0	0.944	0.955	0.954
10	0.5	-0.5	0	0.044	0	0.947	0.953	0.957

Table 129: Proportion of 95% confidence intervals that contained the true parameter for the NBH model with ZINB data with correlated covariates for $n = 500$.

θ	level β_1	level γ_1	β_0	β_1	β_2	γ_0	γ_1	γ_2
1/4	0	0	0.001	0.938	0	0.920	0.947	0.946
1/4	0.5	0	0.002	0.067	0	0.926	0.940	0.941
1/4	0	-0.5	0.015	0.928	0	0.919	0.944	0.944
1/4	0.5	-0.5	0.009	0.063	0	0.904	0.945	0.936
1/2	0	0	0	0.935	0	0.949	0.951	0.947
1/2	0.5	0	0	0.021	0	0.942	0.940	0.946
1/2	0	-0.5	0	0.919	0	0.946	0.950	0.949
1/2	0.5	-0.5	0	0.021	0	0.948	0.951	0.950
1	0	0	0	0.929	0	0.956	0.945	0.945
1	0.5	0	0	0.007	0	0.950	0.943	0.947
1	0	-0.5	0	0.915	0	0.958	0.951	0.952
1	0.5	-0.5	0	0.006	0	0.953	0.947	0.947
2	0	0	0	0.929	0	0.942	0.940	0.943
2	0.5	0	0	0.006	0	0.946	0.945	0.945
2	0	-0.5	0	0.918	0	0.949	0.952	0.958
2	0.5	-0.5	0	0.004	0	0.946	0.950	0.954
5	0	0	0	0.913	0	0.952	0.937	0.950
5	0.5	0	0	0.003	0	0.956	0.950	0.957
5	0	-0.5	0	0.912	0	0.950	0.951	0.951
5	0.5	-0.5	0	0.002	0	0.949	0.945	0.954
10	0	0	0	0.912	0	0.939	0.955	0.954
10	0.5	0	0	0.001	0	0.946	0.948	0.952
10	0	-0.5	0	0.908	0	0.959	0.946	0.954
10	0.5	-0.5	0	0.002	0	0.945	0.953	0.955

Table 130: Average 95% confidence interval width for the NBH model with ZINB data with orthogonal covariates for $n = 500$.

θ	level β_1	level γ_1	β_0	β_1	β_2	γ_0	γ_1	γ_2
1/4	0	0	0.674	0.867	0.891	7.541	1.249	1.348
1/4	0.5	0	0.677	0.904	0.935	7.729	1.304	1.407
1/4	0	-0.5	0.691	0.891	0.918	11.938	1.325	1.445
1/4	0.5	-0.5	0.694	0.927	0.963	11.487	1.372	1.498
1/2	0	0	0.632	0.805	0.819	1.153	0.952	1.103
1/2	0.5	0	0.636	0.833	0.853	1.116	0.993	1.148
1/2	0	-0.5	0.644	0.823	0.840	1.208	1.030	1.221
1/2	0.5	-0.5	0.648	0.854	0.878	1.588	1.069	1.268
1	0	0	0.616	0.776	0.783	0.631	0.737	0.961
1	0.5	0	0.621	0.800	0.812	0.654	0.766	0.998
1	0	-0.5	0.623	0.790	0.801	0.660	0.802	1.082
1	0.5	-0.5	0.628	0.816	0.833	0.694	0.837	1.134
2	0	0	0.613	0.764	0.768	0.442	0.575	0.875
2	0.5	0	0.619	0.786	0.793	0.458	0.599	0.917
2	0	-0.5	0.616	0.775	0.782	0.452	0.639	1.019
2	0.5	-0.5	0.621	0.797	0.809	0.472	0.667	1.066
5	0	0	0.614	0.760	0.761	0.323	0.441	0.834
5	0.5	0	0.620	0.779	0.784	0.336	0.459	0.873
5	0	-0.5	0.613	0.767	0.772	0.326	0.505	0.980
5	0.5	-0.5	0.619	0.788	0.797	0.340	0.527	1.028
10	0	0	0.615	0.759	0.760	0.278	0.385	0.818
10	0.5	0	0.620	0.778	0.782	0.291	0.403	0.856
10	0	-0.5	0.613	0.766	0.770	0.280	0.452	0.979
10	0.5	-0.5	0.619	0.786	0.795	0.293	0.473	1.022

Table 131: Average 95% confidence interval width for the NBH model with ZINB data with correlated covariates for $n = 500$.

θ	level β_1	level γ_1	β_0	β_1	β_2	γ_0	γ_1	γ_2
1/4	0	0	0.602	0.879	0.925	6.838	1.258	1.393
1/4	0.5	0	0.604	0.917	0.982	8.441	1.317	1.458
1/4	0	-0.5	0.616	0.904	0.954	7.941	1.324	1.505
1/4	0.5	-0.5	0.617	0.944	1.013	8.186	1.372	1.563
1/2	0	0	0.570	0.819	0.850	0.941	0.954	1.142
1/2	0.5	0	0.573	0.848	0.895	1.082	0.992	1.195
1/2	0	-0.5	0.578	0.838	0.874	1.113	1.022	1.290
1/2	0.5	-0.5	0.581	0.869	0.921	1.135	1.061	1.339
1	0	0	0.561	0.793	0.813	0.541	0.727	0.992
1	0.5	0	0.565	0.815	0.848	0.559	0.756	1.035
1	0	-0.5	0.564	0.805	0.831	0.563	0.796	1.166
1	0.5	-0.5	0.567	0.831	0.871	0.583	0.828	1.217
2	0	0	0.563	0.784	0.798	0.372	0.564	0.913
2	0.5	0	0.567	0.802	0.828	0.386	0.587	0.957
2	0	-0.5	0.561	0.791	0.811	0.377	0.634	1.111
2	0.5	-0.5	0.565	0.814	0.847	0.394	0.663	1.166
5	0	0	0.567	0.781	0.791	0.268	0.430	0.866
5	0.5	0	0.571	0.797	0.818	0.279	0.450	0.905
5	0	-0.5	0.562	0.785	0.801	0.272	0.508	1.093
5	0.5	-0.5	0.566	0.805	0.832	0.282	0.529	1.147
10	0	0	0.569	0.780	0.790	0.230	0.375	0.843
10	0.5	0	0.573	0.796	0.815	0.239	0.391	0.888
10	0	-0.5	0.563	0.783	0.798	0.232	0.454	1.091
10	0.5	-0.5	0.567	0.804	0.830	0.241	0.473	1.148

Table 132: Proportion of 95% confidence intervals that contained the true parameter for the ZINB model with NBH data with orthogonal covariates.

θ	level β_1	level γ_1	β_0	β_1	β_2	γ_0	γ_1	γ_2
1/4	0	0	0	0.947	0.676	0.700	0.925	0.077
1/4	0.5	0	0	0.168	0.538	0.464	0.923	0.040
1/4	0	-0.5	0	0.943	0.731	0.733	0.793	0.024
1/4	0.5	-0.5	0.001	0.186	0.578	0.529	0.739	0.015
1/2	0	0	0	0.941	0.348	0.863	0.938	0.025
1/2	0.5	0	0	0.129	0.258	0.842	0.933	0.013
1/2	0	-0.5	0	0.942	0.454	0.851	0.771	0.008
1/2	0.5	-0.5	0	0.143	0.341	0.870	0.706	0
1	0	0	0	0.954	0.047	0.915	0.940	0.008
1	0.5	0	0	0.052	0.038	0.906	0.936	0
1	0	-0.5	0	0.952	0.127	0.862	0.734	0
1	0.5	-0.5	0	0.091	0.105	0.880	0.700	0
2	0	0	0	0.948	0.001	0.936	0.948	0.005
2	0.5	0	0	0.026	0.001	0.912	0.938	0
2	0	-0.5	0	0.956	0.017	0.883	0.772	0
2	0.5	-0.5	0	0.042	0.012	0.878	0.733	0
5	0	0	0	0.950	0	0.931	0.940	0.002
5	0.5	0	0	0.013	0	0.936	0.946	0
5	0	-0.5	0	0.947	0	0.866	0.787	0
5	0.5	-0.5	0	0.024	0	0.867	0.750	0
10	0	0	0	0.948	0	0.934	0.951	0.002
10	0.5	0	0	0.014	0	0.938	0.939	0
10	0	-0.5	0	0.955	0	0.884	0.823	0
10	0.5	-0.5	0	0.019	0	0.870	0.794	0

Table 133: Proportion of 95% confidence intervals that contained the true parameter for the ZINB model with NBH data with correlated covariates for $n = 500$.

θ	level β_1	level γ_1	β_0	β_1	β_2	γ_0	γ_1	γ_2
1/4	0	0	0	0.944	0.679	0.668	0.938	0.120
1/4	0.5	0	0	0.111	0.553	0.386	0.919	0.071
1/4	0	-0.5	0	0.939	0.701	0.690	0.818	0.043
1/4	0.5	-0.5	0	0.106	0.556	0.389	0.797	0.023
1/2	0	0	0	0.942	0.316	0.867	0.934	0.046
1/2	0.5	0	0	0.088	0.273	0.824	0.940	0.015
1/2	0	-0.5	0.001	0.943	0.383	0.851	0.805	0.008
1/2	0.5	-0.5	0	0.091	0.338	0.855	0.774	0.003
1	0	0	0	0.948	0.048	0.899	0.943	0.021
1	0.5	0	0	0.041	0.048	0.897	0.939	0.004
1	0	-0.5	0	0.943	0.086	0.896	0.818	0.003
1	0.5	-0.5	0	0.052	0.084	0.893	0.762	0
2	0	0	0	0.952	0.001	0.930	0.952	0.005
2	0.5	0	0	0.016	0.001	0.929	0.944	0
2	0	-0.5	0	0.960	0.005	0.898	0.839	0
2	0.5	-0.5	0	0.029	0.004	0.905	0.786	0
5	0	0	0	0.958	0	0.938	0.939	0.004
5	0.5	0	0	0.016	0	0.933	0.948	0.001
5	0	-0.5	0	0.955	0	0.914	0.832	0
5	0.5	-0.5	0	0.013	0	0.899	0.818	0
10	0	0	0	0.948	0	0.934	0.952	0.002
10	0.5	0	0	0.015	0	0.926	0.941	0
10	0	-0.5	0	0.950	0	0.902	0.860	0
10	0.5	-0.5	0	0.012	0	0.910	0.829	0

Table 134: Average 95% confidence interval width for the ZINB model with NBH data with orthogonal covariates for $n = 500$.

θ	level β_1	level γ_1	β_0	β_1	β_2	γ_0	γ_1	γ_2
1/4	0	0	1.112	1.540	156.745	0.671	0.672	0.687
1/4	0.5	0	1.083	1.459	114.621	0.591	0.616	0.625
1/4	0	-0.5	1.112	1.555	174.222	0.642	0.652	0.665
1/4	0.5	-0.5	1.099	1.483	126.294	0.574	0.601	0.611
1/2	0	0	1.071	1.437	83.970	0.649	0.608	0.632
1/2	0.5	0	1.046	1.379	52.582	0.560	0.562	0.577
1/2	0	-0.5	1.092	1.475	119.527	0.635	0.600	0.623
1/2	0.5	-0.5	1.065	1.409	73.756	0.545	0.554	0.567
1	0	0	0.964	1.227	12.044	0.550	0.545	0.563
1	0.5	0	0.960	1.243	9.008	0.490	0.508	0.522
1	0	-0.5	1.017	1.323	33.307	0.584	0.550	0.573
1	0.5	-0.5	1.001	1.308	28.194	0.499	0.508	0.524
2	0	0	0.877	1.072	1.629	0.427	0.481	0.495
2	0.5	0	0.888	1.105	1.409	0.396	0.450	0.464
2	0	-0.5	0.924	1.158	5.742	0.464	0.497	0.513
2	0.5	-0.5	0.926	1.180	4.378	0.419	0.463	0.477
5	0	0	0.839	0.991	1.006	0.332	0.410	0.441
5	0.5	0	0.847	1.011	1.040	0.311	0.385	0.414
5	0	-0.5	0.865	1.047	1.099	0.351	0.440	0.459
5	0.5	-0.5	0.876	1.076	1.173	0.327	0.411	0.430
10	0	0	0.831	0.972	0.979	0.295	0.376	0.422
10	0.5	0	0.835	0.985	0.996	0.278	0.354	0.395
10	0	-0.5	0.852	1.020	1.041	0.310	0.414	0.440
10	0.5	-0.5	0.860	1.040	1.084	0.290	0.388	0.413

Table 135: Average 95% confidence interval width for the ZINB model with NBH data with correlated covariates for $n = 500$.

θ	level β_1	level γ_1	β_0	β_1	β_2	γ_0	γ_1	γ_2
1/4	0	0	0.982	1.524	220.370	0.633	0.736	0.750
1/4	0.5	0	0.941	1.440	121.596	0.544	0.668	0.676
1/4	0	-0.5	0.981	1.537	224.360	0.607	0.722	0.735
1/4	0.5	-0.5	0.944	1.454	128.925	0.527	0.654	0.663
1/2	0	0	0.957	1.430	85.504	0.623	0.667	0.690
1/2	0.5	0	0.916	1.375	57.953	0.516	0.608	0.623
1/2	0	-0.5	0.963	1.452	106.228	0.606	0.659	0.680
1/2	0.5	-0.5	0.923	1.400	81.146	0.502	0.603	0.617
1	0	0	0.862	1.240	13.498	0.520	0.593	0.615
1	0.5	0	0.849	1.245	9.898	0.457	0.546	0.565
1	0	-0.5	0.887	1.293	21.348	0.539	0.599	0.623
1	0.5	-0.5	0.868	1.291	21.918	0.459	0.549	0.567
2	0	0	0.787	1.089	1.682	0.392	0.515	0.540
2	0.5	0	0.791	1.107	1.589	0.368	0.482	0.505
2	0	-0.5	0.810	1.145	2.636	0.415	0.536	0.559
2	0.5	-0.5	0.809	1.161	2.749	0.382	0.496	0.518
5	0	0	0.756	1.017	1.032	0.300	0.435	0.483
5	0.5	0	0.758	1.014	1.059	0.284	0.409	0.451
5	0	-0.5	0.769	1.053	1.087	0.311	0.470	0.504
5	0.5	-0.5	0.774	1.055	1.139	0.292	0.437	0.468
10	0	0	0.750	0.998	1.005	0.265	0.397	0.461
10	0.5	0	0.751	0.992	1.016	0.250	0.372	0.428
10	0	-0.5	0.760	1.030	1.044	0.272	0.440	0.483
10	0.5	-0.5	0.764	1.028	1.075	0.257	0.410	0.449

Appendix C: Proportion of Correctly Identified Structural Zeros

This appendix includes 8 figures, each containing information on the mean proportion of correctly identified structural zeros. Each table contains all levels of sample size, dispersion parameter (θ), the specified values for β_1 and γ_1, and whether the covariates are orthogonal or correlated.

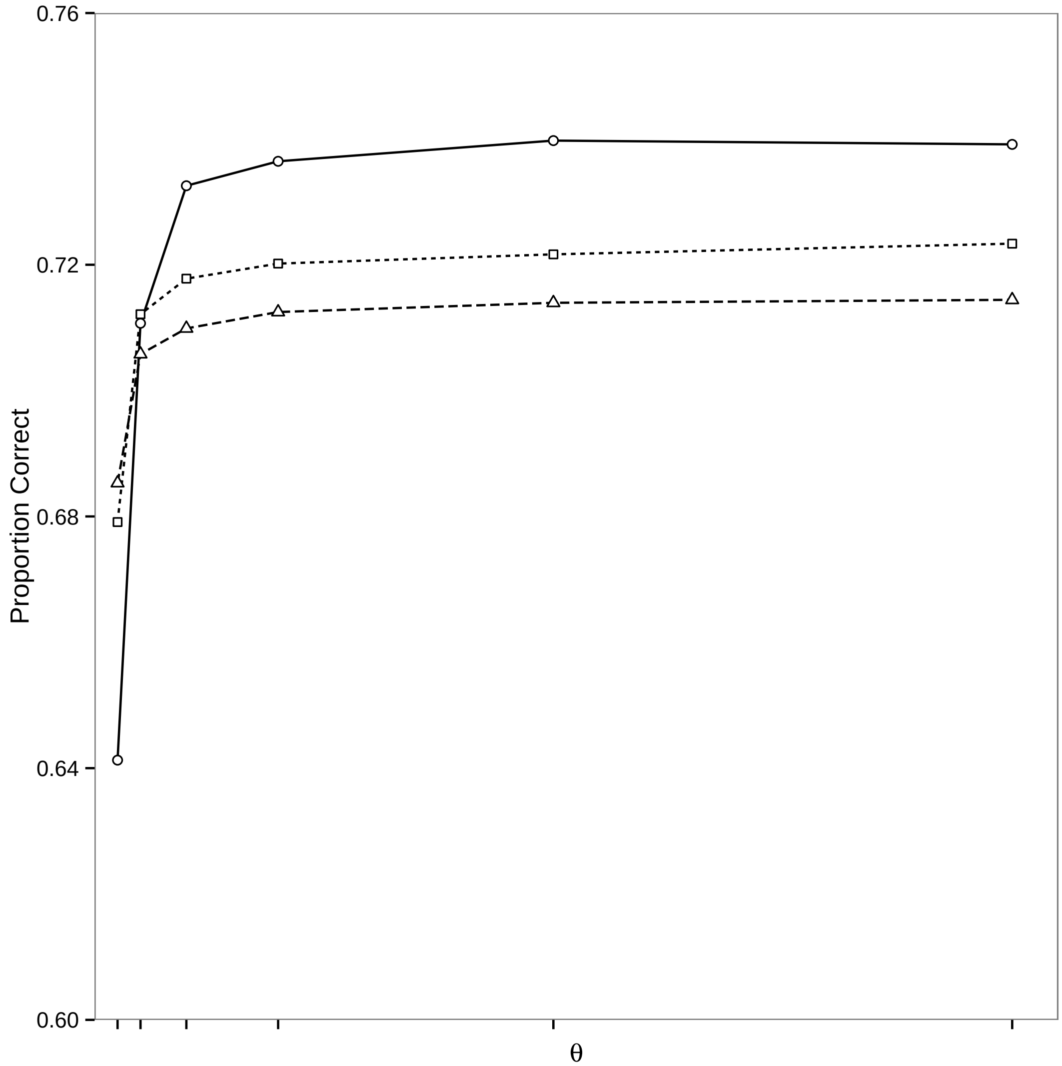

Figure 11: The average proportion of correctly identified structural zeros for various levels of θ for sample size of 100 (solid, circles), 250 (dotted, squares) and 500 (dashed, triangle) when $\beta_1 = 0$, $\gamma_1 = 0$, and the covariates are orthogonal. The tick marks corresponds to $\theta = 1/4, 1/2, 1, 2, 5$, and 10, respectively.

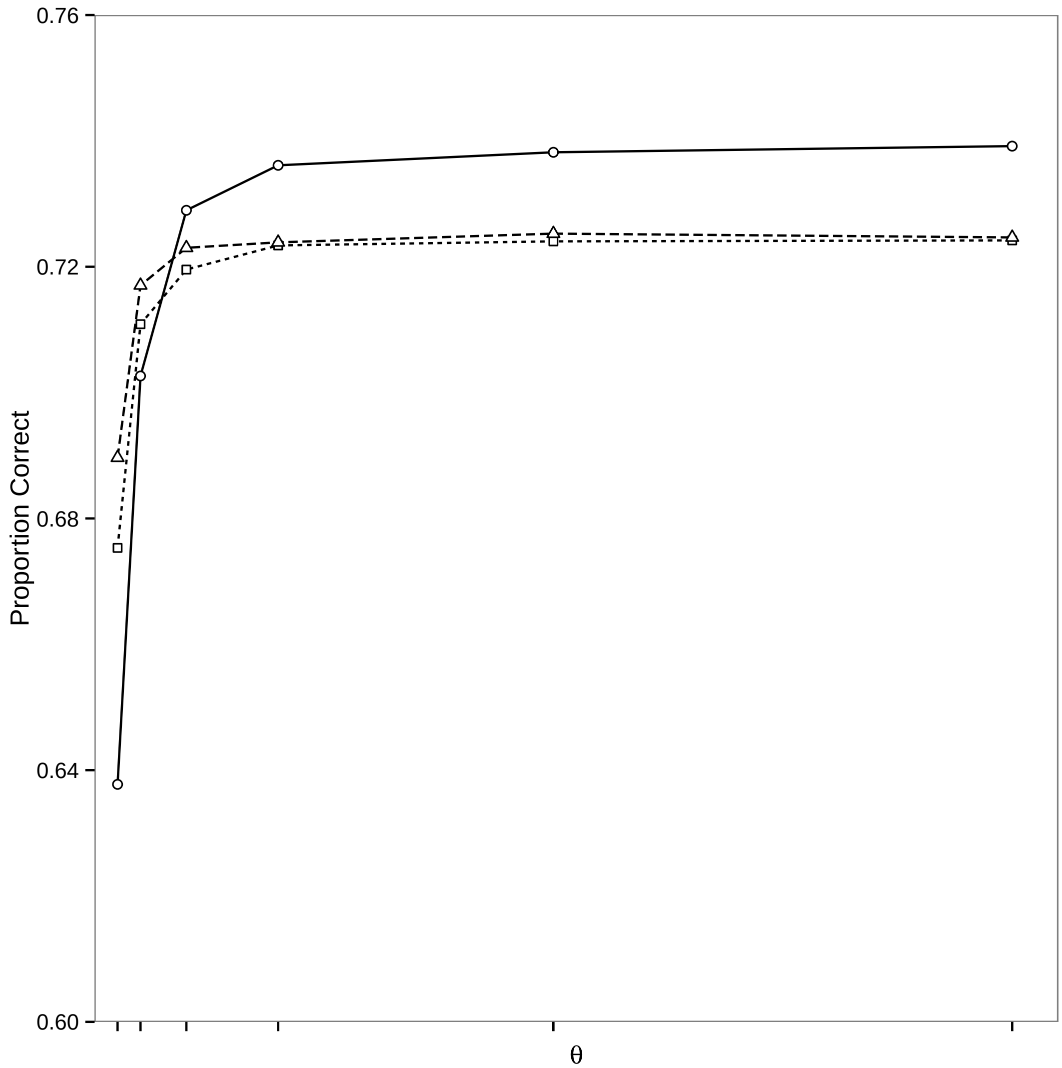

Figure 12: The average proportion of correctly identified structural zeros for various levels of θ for sample size of 100 (solid, circles), 250 (dotted, squares) and 500 (dashed, triangle) when $\beta_1 = 0.5$, $\gamma_1 = 0$, and the covariates are orthogonal.The tick marks corresponds to $\theta = 1/4, 1/2, 1, 2, 5$, and 10, respectively.

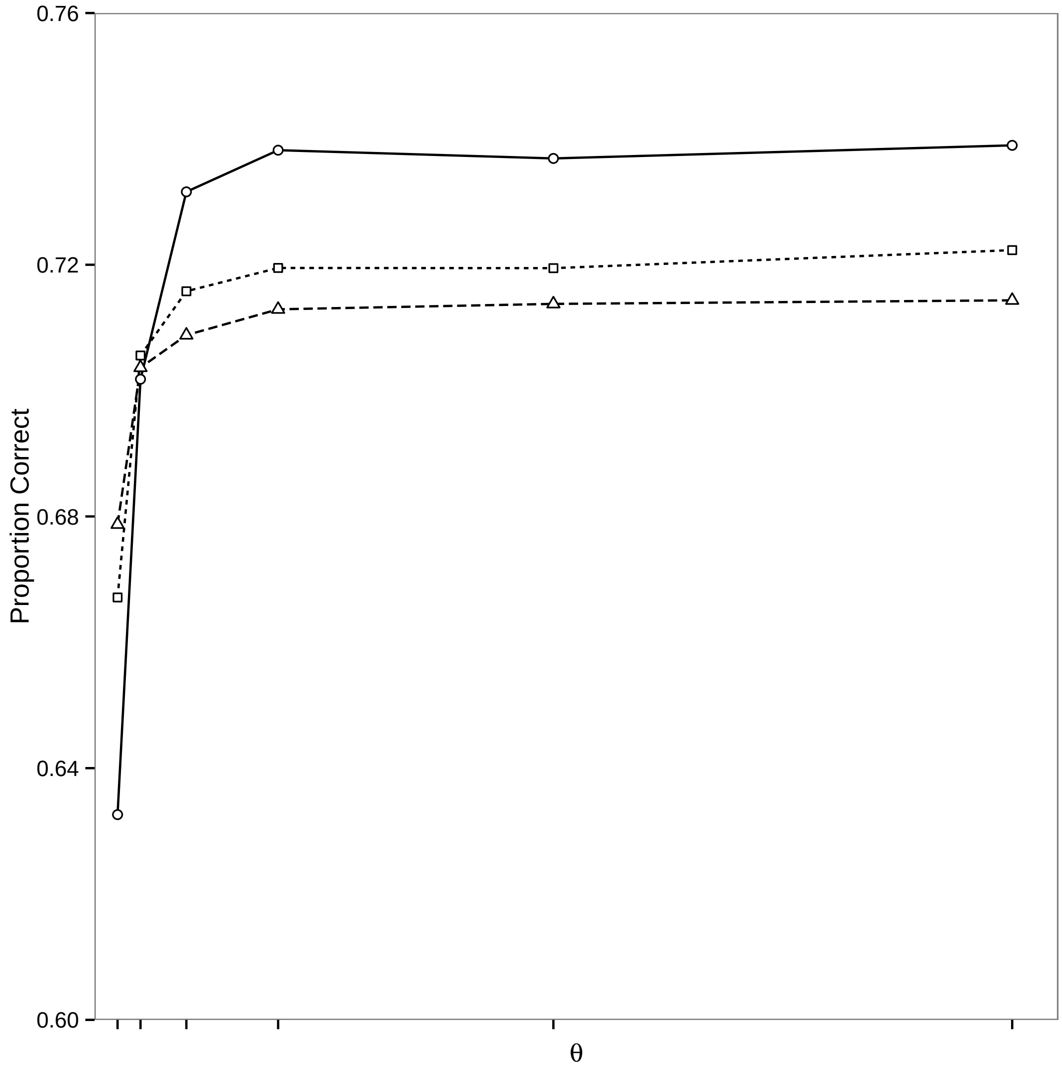

Figure 13: The average proportion of correctly identified structural zeros for various levels of θ for sample size of 100 (solid, circles), 250 (dotted, squares) and 500 (dashed, triangle) when $\beta_1 = 0$, $\gamma_1 = -0.5$, and the covariates are orthogonal. The tick marks corresponds to $\theta = 1/4, 1/2, 1, 2, 5$, and 10, respectively.

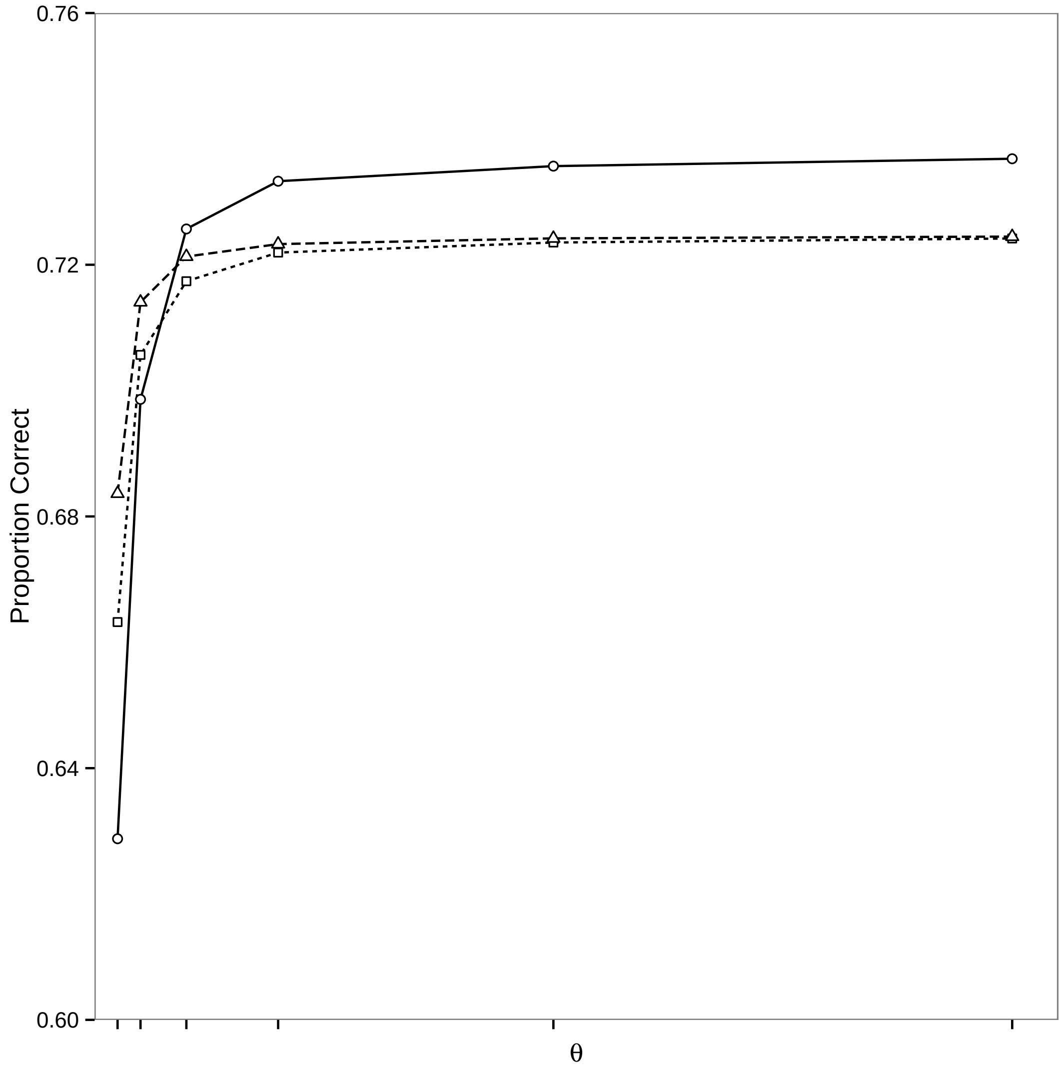

Figure 14: The average proportion of correctly identified structural zeros for various levels of θ for sample size of 100 (solid, circles), 250 (dotted, squares) and 500 (dashed, triangle) when $\beta_1 = 0.5$, $\gamma_1 = -0.5$, and the covariates are correlated. The tick marks corresponds to $\theta = 1/4, 1/2, 1, 2, 5,$ and 10, respectively.

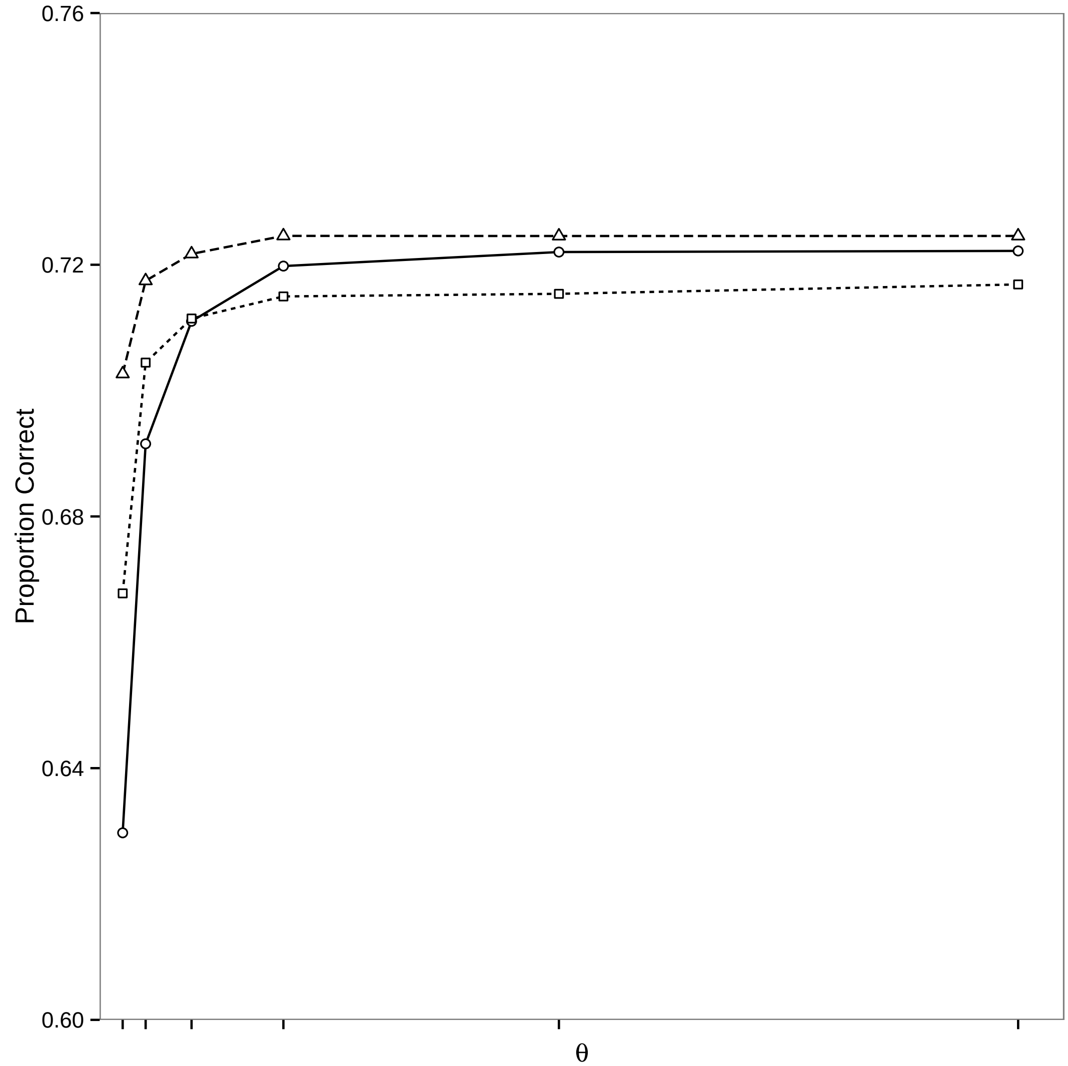

Figure 15: The average proportion of correctly identified structural zeros for various levels of θ for sample size of 100 (solid, circles), 250 (dotted, squares) and 500 (dashed, triangle) when $\beta_1 = 0$, $\gamma_1 = 0$, and the covariates are correlated. The tick marks corresponds to $\theta = 1/4, 1/2, 1, 2, 5$, and 10, respectively.

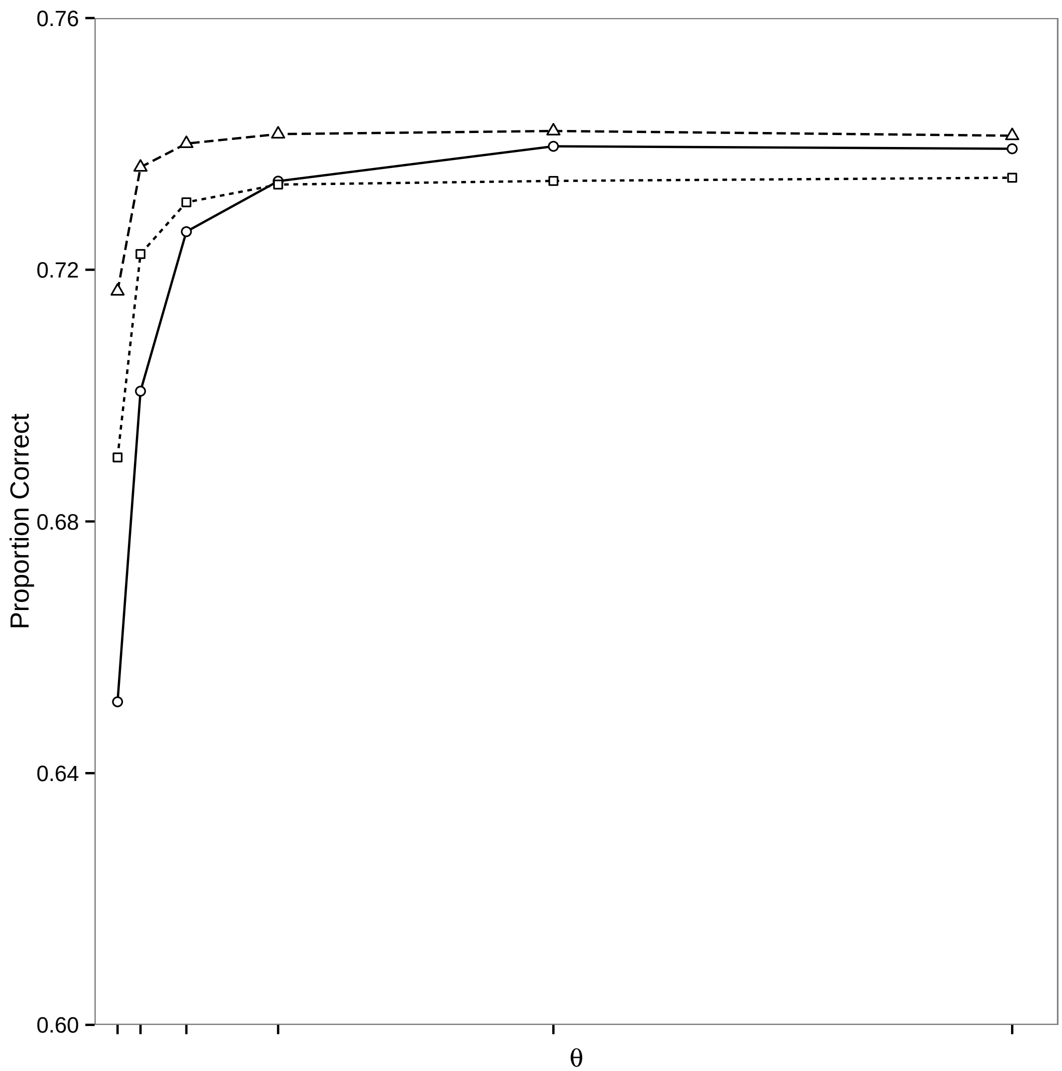

Figure 16: The average proportion of correctly identified structural zeros for various levels of θ for sample size of 100 (solid, circles), 250 (dotted, squares) and 500 (dashed, triangle) when $\beta_1 = 0.5$, $\gamma_1 = 0$, and the covariates are correlated. The tick marks corresponds to $\theta = 1/4, 1/2, 1, 2, 5$, and 10, respectively.

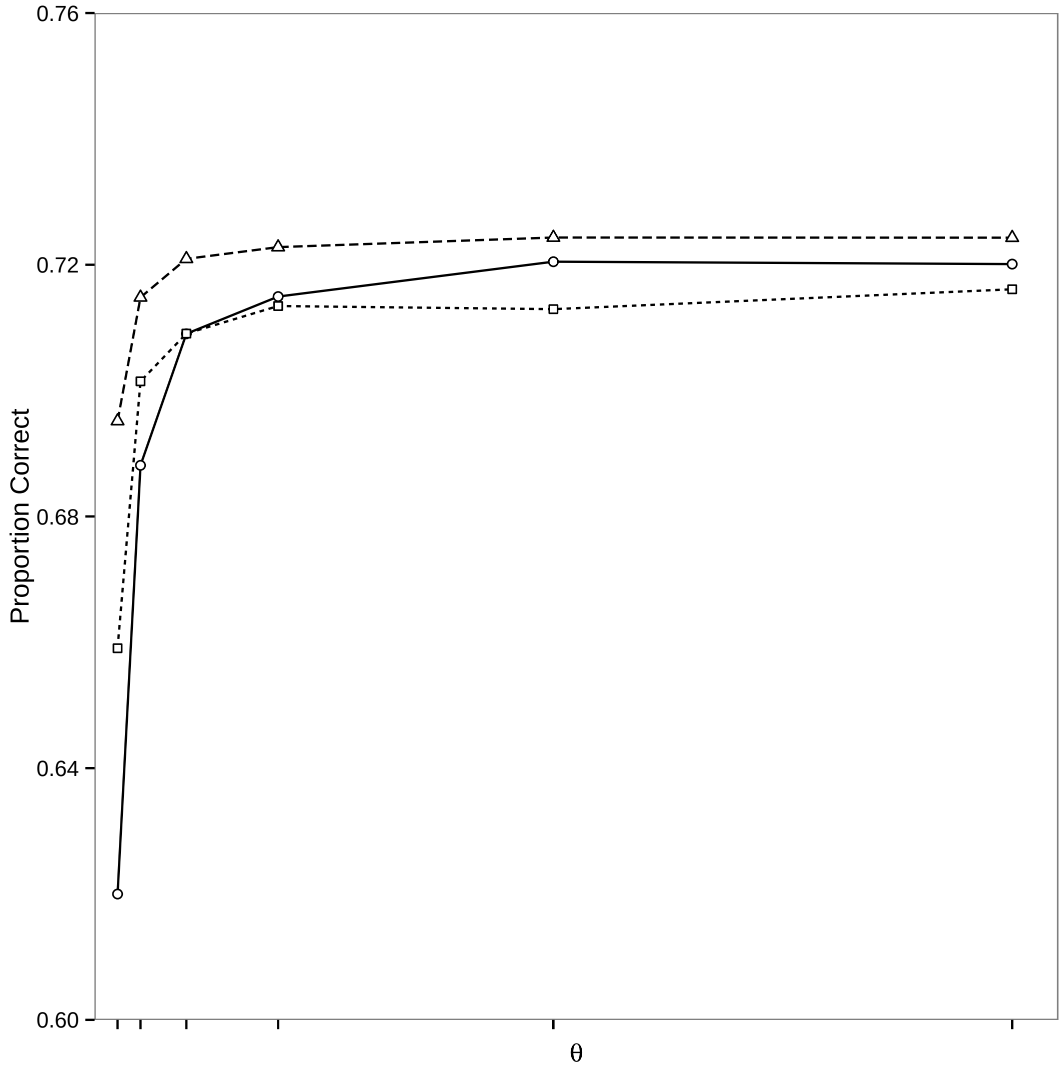

Figure 17: The average proportion of correctly identified structural zeros for various levels of θ for sample size of 100 (solid, circles), 250 (dotted, squares) and 500 (dashed, triangle) when $\beta_1 = 0$, $\gamma_1 = -0.5$, and the covariates are correlated. The tick marks corresponds to $\theta = 1/4, 1/2, 1, 2, 5$, and 10, respectively.

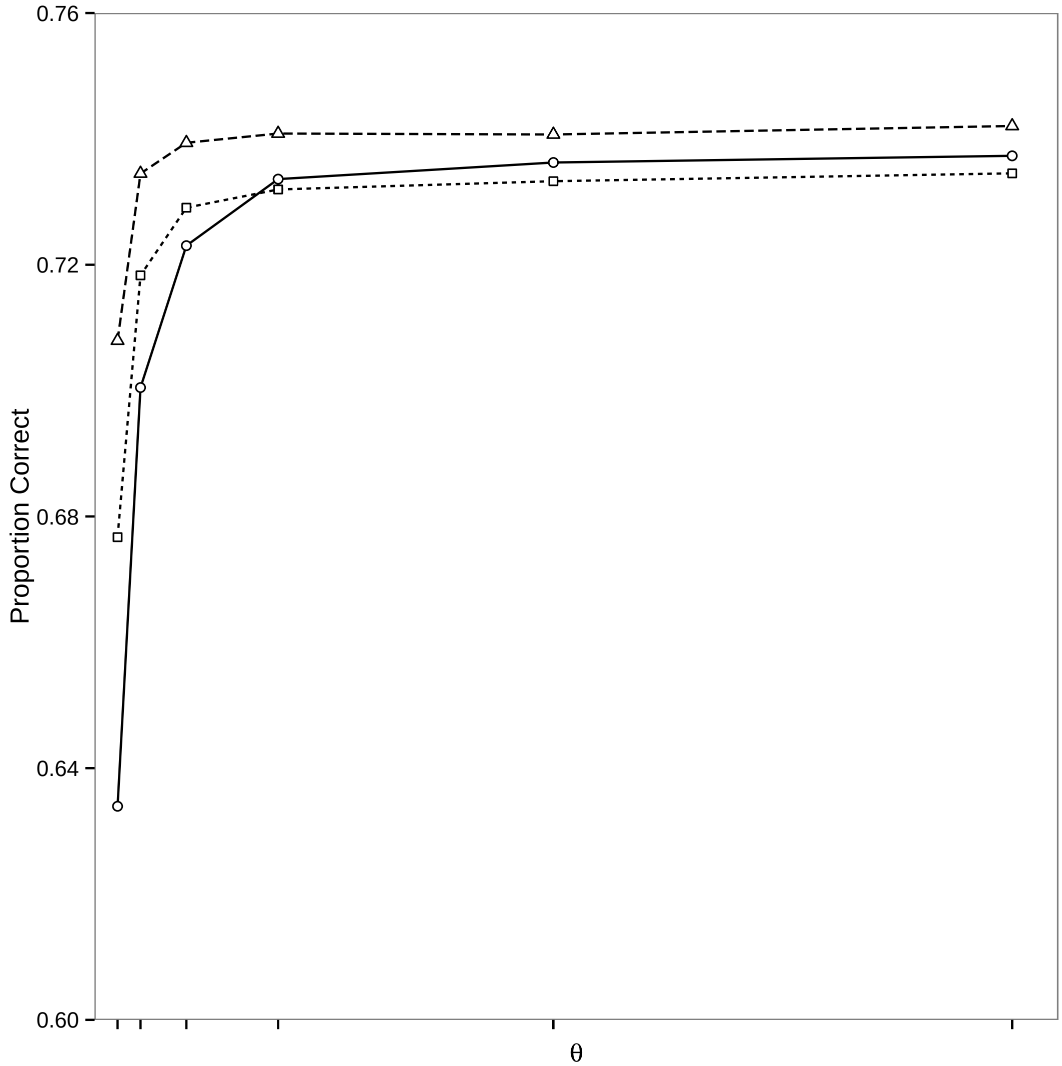

Figure 18: The average proportion of correctly identified structural zeros for various levels of θ for sample size of 100 (solid, circles), 250 (dotted, squares) and 500 (dashed, triangle) when $\beta_1 = 0.5$, $\gamma_1 = -0.5$, and the covariates are correlated. The tick marks corresponds to $\theta = 1/4, 1/2, 1, 2, 5$, and 10, respectively.

Appendix D: Type I Error

This appendix includes 8 tables, each containing information on the type I error rate and its associated 95% Wald confidence interval. The first 4 tables correspond to the first simulation study and the last 4 tables correspond to the second simulation study. Each table contains all levels of the dispersion parameter, θ, and covariate correlation, ρ.

First Simulation Study - Model Performance and Recovery

Table 136: Type I error rate (with 95% Wald confidence interval) for dispersion, varying levels of β_1, covariate correlation, and sample size (100, 250, and 500) for the NBH model when $\gamma_1 = 0$.

θ	β_1	ρ	100	250	500
1/4	0	0	.078 (.066, .090)	.066 (.055, .076)	.060 (.050, .070)
1/4	.5	0	.074 (.063, .086)	.057 (.047, .067)	.054 (.045, .064)
1/2	0	0	.069 (.058, .080)	.052 (.043, .062)	.055 (.045, .065)
1/2	.5	0	.077 (.065, .089)	.056 (.046, .066)	.047 (.038, .056)
1	0	0	.073 (.062, .085)	.058 (.047, .068)	.051 (.042, .061)
1	.5	0	.079 (.067, .091)	.060 (.050, .070)	.057 (.047, .067)
2	0	0	.078 (.066, .090)	.061 (.051, .072)	.050 (.040, .059)
2	.5	0	.076 (.064, .088)	.058 (.048, .068)	.054 (.045, .064)
5	0	0	.066 (.055, .077)	.062 (.051, .073)	.066 (.055, .076)
5	.5	0	.068 (.057, .079)	.057 (.047, .067)	.049 (.040, .058)
10	0	0	.058 (.048, .068)	.060 (.050, .071)	.046 (.037, .056)
10	.5	0	.053 (.044, .063)	.054 (.044, .064)	.059 (.048, .069)
1/4	0	.3	.066 (.055, .077)	.060 (.050, .071)	.057 (.046, .067)
1/4	.5	.3	.064 (.053, .074)	.047 (.038, .056)	.051 (.041, .061)
1/2	0	.3	.070 (059, .082)	.063 (.052, .074)	.048 (.038, .057)
1/2	.5	.3	.072 (.061, .084)	.070 (.059, .082)	.050 (.040, .060)
1	0	.3	.079 (.067, .090)	.058 (.048, .068)	.050 (.040, .059)
1	.5	.3	.069 (.057, .080)	.060 (.050, .071)	.061 (.051, .071)
2	0	.3	.076 (.065, .088)	.058 (.048, .068)	.046 (.037, .056)
2	.5	.3	.075 (.063, .087)	.059 (.049, .070)	.051 (.041, .061)
5	0	.3	.062 (.051, .073)	.058 (.047, .068)	.060 (.050, .071)
5	.5	.3	.065 (.054, .076)	.062 (.052, .073)	.047 (.038, .056)
10	0	.3	.060 (.050, .070)	.060 (.050, .071)	.050 (.040, .060)
10	.5	.3	.064 (.053, .075)	.059 (.049, .069)	.056 (.046, .066)

Table 137: Type I error rate (with 95% Wald confidence interval) for dispersion, varying levels of β_1, covariate correlation, and sample size (100, 250, and 500) for the ZINB model when $\gamma_1 = 0$.

θ	β_1	ρ	100	250	500
1/4	0	0	.089 (.077, .102)	.060 (.050, .070)	.059 (.049, .070)
1/4	.50	0	.106 (.093, .119)	.075 (.064, .087)	.066 (.055, .077)
1/2	0	0	.086 (.074, .099)	.067 (.056, .077)	.056 (.046, .066)
1/2	.50	0	.086 (.073, .098)	.066 (.055, .077)	.050 (.040, .060)
1	0	0	.065 (.054, .075)	.062 (.051, .073)	.046 (.037, .055)
1	.50	0	.085 (.073, .097)	.051 (.041, .061)	.051 (.041, .061)
2	0	0	.070 (.059, .081)	.059 (.049, .069)	.050 (.040, .059)
2	.50	0	.062 (.051, .073)	.048 (.039, .057)	.048 (.038, .057)
5	0	0	.065 (.054, .075)	.051 (.041, .060)	.057 (.046, .067)
5	.50	0	.059 (.048, .069)	.058 (.047, .068)	.053 (.044, .063)
10	0	0	.061 (.051, .071)	.049 (.039, .058)	.052 (.043, .062)
10	.50	0	.056 (.045, .066)	.056 (.046, .066)	.053 (.043, .063)
1/4	0	.3	.107 (.093, .121)	.068 (.057, .079)	.065 (.054, .076)
1/4	.50	.3	.107 (.093, .121)	.067 (.056, .078)	.070 (.058, .081)
1/2	0	.3	.082 (.070, .094)	.061 (.051, .071)	.049 (.039, .058)
1/2	.50	.3	.085 (.072, .097)	.065 (.054, .075)	.058 (.048, .068)
1	0	.3	.084 (.072, .096)	.069 (.057, .080)	.057 (.046, .067)
1	.50	.3	.070 (.059, .082)	.059 (.049, .070)	.062 (.051, .073)
2	0	.3	.070 (.059, .081)	.065 (.054, .075)	.064 (.053, .074)
2	.50	.3	.086 (.074, .099)	.051 (.041, .061)	.056 (.045, .066)
5	0	.3	.076 (.064, .088)	.059 (.049, .070)	.062 (.052, .073)
5	.50	.3	.066 (.055, .076)	.049 (.040, .058)	.054 (.045, .064)
10	0	.3	.067 (.056, .077)	.065 (.054 ,.075)	.045 (.036, .055)
10	.50	.3	.064 (.053, .075)	.049 (.039, .058)	.059 (.049, .069)

Table 138: Type I error rate (with 95% Wald confidence interval) for dispersion, varying levels of γ_1, covariate correlation, and sample size (100, 250, and 500) for the NBH model when $\beta_1 = 0$.

θ	γ_1	ρ	100	250	500
1/4	0	0	.043 (.034, .051)	.050 (.040, .060)	.045 (.036, .054)
1/4	-.50	0	.048 (.038, .057)	.053 (.043, .063)	.048 (.039, .057)
1/2	0	0	.042 (.033, .051)	.056 (.045, .066)	.051 (.042, .061)
1/2	-.50	0	.053 (.043, .063)	.049 (.040, .058)	.045 (.036, .055)
1	0	0	.046 (.037, .056)	.048 (.039, .057)	.046 (.037, .056)
1	-.50	0	.052 (.043, .062)	.051 (.041, .060)	.048 (.039, .057)
2	0	0	.043 (.034, .052)	.051 (.042, .061)	.053 (.043, .063)
2	-.50	0	.043 (.034, .052)	.045 (.036, .055)	.043 (.035, .052)
5	0	0	.040 (.031, .048)	.049 (.040, .058)	.044 (.035, .054)
5	-.50	0	.036 (.028, .045)	.046 (.037, .055)	.060 (.050, .071)
10	0	0	.051 (.042, .061)	.047 (.038, .056)	.057 (.046, .067)
10	-.50	0	.051 (.042, .061)	.044 (.035, .054)	.045 (.036, .055)
1/4	0	.3	.053 (.043, .063)	.046 (.037, .055)	.043 (.034, .051)
1/4	-.50	.3	.046 (.037, .056)	.053 (.044, .063)	.051 (.041, .061)
1/2	0	.3	.043 (.034, .051)	.046 (.037, .055)	.045 (.036, .054)
1/2	-.50	.3	.049 (.039, .058)	.050 (.040, .059)	.050 (.040, .059)
1	0	.3	.046 (.037, .056)	.045 (.036, .055)	.051 (.041, .060)
1	-.50	.3	.044 (.035, .053)	.045 (.036, .054)	.052 (.043, .062)
2	0	.3	.041 (.032, .050)	.044 (.035, .053)	.045 (.036, .055)
2	-.50	.3	.041 (.032, .049)	.045 (.036, .054)	.038 (.030, .046)
5	0	.3	.052 (.043, .062)	.043 (.034, .051)	.043 (.034, .052)
5	-.50	.3	.048 (.039, .057)	.051 (.041, .060)	.042 (.033, .051)
10	0	.3	.044 (.035, .054)	.047 (.038, .056)	.054 (.044, .064)
10	-.50	.3	.049 (.040, .058)	.050 (.040, .059)	.050 (.040, .060)

Table 139: Type I error rate (with 95% Wald confidence interval) for dispersion, varying levels of γ_1, covariate correlation, and sample size (100, 250, and 500) for the ZINB model when $\beta_1 = 0$.

θ	γ_1	ρ	100	250	500
1/4	0	0	.017 (.011, .022)	.043 (.035, .052)	.045 (.036, .054)
1/4	-.50	0	.018 (.013, .024)	.029 (.022, .036)	.050 (.040, .059)
1/2	0	0	.019 (.013, .025)	.046 (.037, .055)	.049 (.040, .059)
1/2	-.50	0	.012 (.007, .017)	.043 (.034, .051)	.043 (.034, .051)
1	0	0	.025 (.018, .032)	.043 (.035, .052)	.046 (.036, .055)
1	-.50	0	.028 (.021, .035)	.053 (.043, .063)	.045 (.036, .054)
2	0	0	.036 (.028, .045)	.051 (.041, .060)	.051 (.041, .060)
2	-.50	0	.030 (.023, .038)	.046 (.037, .056)	.037 (.029, .046)
5	0	0	.032 (.024, .039)	.043 (.035, .052)	.048 (.038, .057)
5	-.50	0	.033 (.025, .041)	.052 (.042, .062)	.042 (.033, .051)
10	0	0	.042 (.033, .051)	.042 (.033, .050)	.041 (.032, .049)
10	-.50	0	.034 (.026, .042)	.048 (.038, .057)	.059 (.048, .069)
1/4	0	.3	.018 (.012, .024)	.044 (.035, .053)	.049 (.040, .058)
1/4	-.50	.3	.020 (.014, .026)	.040 (.031, .048)	.045 (.036, .054)
1/2	0	.3	.019 (.013, .025)	.033 (.025, .040)	.052 (.042, .062)
1/2	-.50	.3	.018 (.012, .024)	.044 (.035, .053)	.050 (.040, .060)
1	0	.3	.029 (.022, .037)	.046 (.037, .056)	.042 (.033, .051)
1	-.50	.3	.029 (.021, .036)	.057 (.047, .067)	.048 (.039, .057)
2	0	.3	.032 (.024, .039)	.043 (.034, .052)	.040 (.031, .049)
2	-.50	.3	.031 (.023, .039)	.044 (.035, .053)	.045 (.036, .055)
5	0	.3	.040 (.031, .049)	.042 (.033, .051)	.060 (.050, .071)
5	-.50	.3	.035 (.027, .043)	.051 (.042, .061)	.055 (.045, .065)
10	0	.3	.036 (.028, .045)	.043 (.034, .051)	.045 (.036, .055)
10	-.50	.3	.035 (.027, .043)	.050 (.040, .059)	.044 (.035, .054)

Second Simulation Study - Model Misspecification

Table 140: Type I error rate (with 95% Wald confidence interval) for dispersion, varying levels of γ_1, covariate correlation for the NBH model with ZINB data when $\beta_1 = 0$ for $n = 500$.

θ	γ_1	ρ	Estimated Type I Error
1/4	0	0	0.043 (0.034, 0.052)
1/4	-0.5	0	0.057 (0.046, 0.067)
1/2	0	0	0.047 (0.038, 0.056)
1/2	-0.5	0	0.054 (0.045, 0.064)
1	0	0	0.046 (0.037, 0.056)
1	-0.5	0	0.066 (0.055, 0.076)
2	0	0	0.062 (0.052, 0.073)
2	-0.5	0	0.052 (0.042, 0.062)
5	0	0	0.046 (0.037, 0.056)
5	-0.5	0	0.056 (0.046, 0.066)
10	0	0	0.041 (0.032, 0.049)
10	-0.5	0	0.067 (0.056, 0.078)
1/4	0	.3	0.062 (0.051, 0.073)
1/4	-0.5	.3	0.072 (0.061, 0.083)
1/2	0	.3	0.065 (0.054, 0.075)
1/2	-0.5	.3	0.081 (0.069, 0.092)
1	0	.3	0.070 (0.059, 0.082)
1	-0.5	.3	0.085 (0.073, 0.097)
2	0	.3	0.071 (0.060, 0.082)
2	-0.5	.3	0.082 (0.070, 0.094)
5	0	.3	0.086 (0.074, 0.099)
5	-0.5	.3	0.088 (0.076, 0.100)
10	0	.3	0.088 (0.076, 0.100)
10	-0.5	.3	0.092 (0.079, 0.105)

Table 141: Type I error rate (with 95% Wald confidence interval) for dispersion, varying levels of γ_1, covariate correlation for the ZINB model with NBH data when $\beta_1 = 0$ for $n = 500$.

θ	γ_1	ρ	Estimated Type I Error
1/4	0	0	0.053 (0.044, 0.063)
1/4	-0.5	0	0.057 (0.046, 0.067)
1/2	0	0	0.059 (0.048, 0.069)
1/2	-0.5	0	0.058 (0.048, 0.068)
1	0	0	0.046 (0.037, 0.055)
1	-0.5	0	0.048 (0.038, 0.057)
2	0	0	0.052 (0.042, 0.062)
2	-0.5	0	0.044 (0.035, 0.052)
5	0	0	0.051 (0.041, 0.060)
5	-0.5	0	0.053 (0.044, 0.063)
10	0	0	0.052 (0.042, 0.062)
10	-0.5	0	0.045 (0.036, 0.055)
1/4	0	.3	0.056 (0.045, 0.066)
1/4	-0.5	.3	0.061 (0.051, 0.072)
1/2	0	.3	0.058 (0.047, 0.068)
1/2	-0.5	.3	0.057 (0.047, 0.067)
1	0	.3	0.052 (0.043, 0.062)
1	-0.5	.3	0.057 (0.046, 0.067)
2	0	.3	0.048 (0.039, 0.057)
2	-0.5	.3	0.040 (0.031, 0.049)
5	0	.3	0.042 (0.033, 0.051)
5	-0.5	.3	0.045 (0.036, 0.055)
10	0	.3	0.052 (0.042, 0.062)
10	-0.5	.3	0.051 (0.041, 0.060)

Table 142: Type I error rate (with 95% Wald confidence interval) for dispersion, varying levels of β_1, covariate correlation for the NBH model with ZINB data when $\gamma_1 = 0$ for $n = 500$.

θ	β_1	ρ	Estimated Type I Error
1/4	0	0	0.043 (0.034, 0.052)
1/4	0.5	0	0.069 (0.057, 0.080)
1/2	0	0	0.047 (0.038, 0.056)
1/2	0.5	0	0.059 (0.049, 0.069)
1	0	0	0.046 (0.037, 0.056)
1	0.5	0	0.057 (0.047, 0.067)
2	0	0	0.062 (0.052, 0.073)
2	0.5	0	0.048 (0.038, 0.057)
5	0	0	0.046 (0.037, 0.056)
5	0.5	0	0.053 (0.043, 0.063)
10	0	0	0.041(0.032, 0.049)
10	0.5	0	0.049 (0.040, 0.058)
1/4	0	.3	0.062 (0.051, 0.073)
1/4	0.5	.3	0.060 (0.050, 0.070)
1/2	0	.3	0.065 (0.054, 0.075)
1/2	0.5	.3	0.060 (0.050, 0.070)
1	0	.3	0.070 (0.059, 0.082)
1	0.5	.3	0.057 (0.047, 0.067)
2	0	.3	0.071 (0.060, 0.082)
2	0.5	.3	0.056 (0.045, 0.066)
5	0	.3	0.086 (0.074, 0.099)
5	0.5	.3	0.050 (0.040, 0.060)
10	0	.3	0.088 (0.076, 0.100)
10	0.5	.3	0.052 (0.043, 0.062)

Table 143: Type I error rate (with 95% Wald confidence interval) for dispersion, varying levels of β_1, covariate correlation for the ZINB model with NBH data when $\gamma_1 = 0$ for $n = 500$.

θ	β_1	ρ	Estimated Type I Error
1/4	0	0	0.053 (0.044, 0.063)
1/4	0.5	0	0.077 (0.065, 0.089)
1/2	0	0	0.059 (0.048, 0.069)
1/2	0.5	0	0.067 (0.056, 0.078)
1	0	0	0.046 (0.037, 0.055)
1	0.5	0	0.064 (0.053, 0.075)
2	0	0	0.052 (0.042, 0.062)
2	0.5	0	0.062 (0.051, 0.073)
5	0	0	0.051 (0.041, 0.060)
5	0.5	0	0.054 (0.045, 0.064)
10	0	0	0.052 (0.042, 0.062)
10	0.5	0	0.061 (0.051, 0.071)
1/4	0	.3	0.056 (0.045, 0.066)
1/4	0.5	.3	0.081 (0.069, 0.092)
1/2	0	.3	0.058 (0.047, 0.068)
1/2	0.5	.3	0.060 (0.050, 0.070)
1	0	.3	0.052 (0.043, 0.062)
1	0.5	.3	0.061 (0.051, 0.072)
2	0	.3	0.048 (0.039, 0.057)
2	0.5	.3	0.056 (0.046, 0.066)
5	0	.3	0.042 (0.033, 0.051)
5	0.5	.3	0.052 (0.043, 0.062)
10	0	.3	0.052 (0.042, 0.062)
10	0.5	.3	0.059 (0.049, 0.070)

Appendix E: Bias

This appendix includes 16 tables, each containing information on the relative bias for the E(Y|X) and/or the Pr(Y = 0) for the NBH and ZINB models. The first 8 tables correspond to the first simulation study and each table contains information on the relative bias of E(Y|X) and Pr(Y = 0). The last 8 tables correspond to the second simulation study and each table contains information on the relative bias of E(Y|X) or Pr(Y = 0) . Each table contains all levels of sample size, θ, and the specified values for β_1, γ_1, and whether the covariates were orthogonal or correlated.

First Simulation Study - Model Performance and Recovery

Table 144: Relative bias for E(Y|X) and the Pr(Y = 0) by sample size (n), dispersion (θ), with orthogonal covariates for $\beta_1 = 0$ and $\gamma_1 = 0$.

		E(Y|X)		Pr(Y = 0)	
n	θ	NBH	ZINB	NBH	ZINB
100	1/4	0.008	0.021	0.001	0.002
250	1/4	0.001	0.000	0.000	0.002
500	1/4	0.005	-0.002	0.001	0.001
100	1/2	0.006	0.014	-0.003	0.003
250	1/2	-0.001	0.001	0.002	0.002
500	1/2	-0.005	0.001	-0.001	0.000
100	1	0.008	-0.003	-0.002	0.002
250	1	-0.001	-0.003	-0.003	0.002
500	1	-0.002	-0.001	0.001	-0.001
100	2	0.004	-0.008	0.004	0.003
250	2	0.000	0.004	-0.001	-0.002
500	2	0.000	0.003	-0.001	-0.000
100	5	0.003	0.002	-0.000	-0.000
250	5	-0.002	0.004	0.002	0.000
500	5	-0.001	0.000	0.000	-0.000
100	10	0.008	0.006	-0.005	0.001
250	10	0.004	-0.005	-0.001	0.002
500	10	-0.002	-0.001	0.001	0.001

Table 145: Relative bias for E(Y|X) and the Pr(Y = 0) by sample size (n), dispersion (θ), with correlated covariates for $\beta_1 = 0$ and $\gamma_1 = 0$.

		E(Y|X)		Pr(Y = 0)	
n	θ	NBH	ZINB	NBH	ZINB
100	1/4	0.007	0.027	-0.000	0.003
250	1/4	0.004	0.007	0.001	0.002
500	1/4	0.005	0.001	-0.001	-0.000
100	1/2	0.003	0.016	0.004	0.002
250	1/2	-0.001	0.012	0.002	-0.002
500	1/2	-0.002	0.006	0.002	-0.000
100	1	0.007	0.007	0.002	0.004
250	1	0.001	0.001	0.000	0.001
500	1	-0.001	-0.000	-0.001	-0.000
100	2	-0.001	0.002	0.001	0.001
250	2	0.001	-0.004	-0.002	0.001
500	2	0.000	-0.002	-0.001	0.000
100	5	-0.006	-0.002	0.003	0.001
250	5	-0.001	0.003	0.001	-0.002
500	5	-0.001	0.000	-0.000	0.000
100	10	0.003	0.002	-0.000	0.002
250	10	0.000	0.002	0.002	-0.003
500	10	0.000	0.002	0.000	-0.002

Table 146: Relative bias for E(Y|X) and the Pr(Y = 0) by sample size (n), dispersion (θ), with orthogonal covariates for $\beta_1 = 0$ and $\gamma_1 = -0.5$.

		E(Y|X)		Pr(Y = 0)	
n	θ	NBH	ZINB	NBH	ZINB
100	1/4	0.004	0.005	0.000	0.003
250	1/4	0.003	-0.002	-0.002	0.002
500	1/4	-0.001	0.003	-0.001	0.000
100	1/2	0.006	0.005	-0.002	0.002
250	1/2	0.004	0.007	-0.001	-0.000
500	1/2	0.000	0.002	0.001	0.000
100	1	0.010	0.010	-0.004	0.002
250	1	0.003	-0.001	-0.001	0.001
500	1	-0.000	0.002	0.001	0.001
100	2	0.002	0.008	-0.000	-0.004
250	2	0.001	-0.001	0.001	0.001
500	2	-0.004	0.002	0.002	-0.000
100	5	0.004	0.001	-0.002	0.004
250	5	-0.001	0.004	-0.002	0.001
500	5	-0.000	0.004	0.000	-0.002
100	10	-0.001	-0.004	-0.001	-0.001
250	10	-0.003	-0.001	0.001	-0.001
500	10	-0.002	-0.001	0.001	-0.001

Table 147: Relative bias for E(Y|X) and the Pr(Y = 0) by sample size (n), dispersion (θ), with correlated covariates for $\beta_1 = 0$ and $\gamma_1 = -0.5$.

		E(Y|X)		Pr(Y = 0)	
n	θ	NBH	ZINB	NBH	ZINB
100	1/4	0.009	0.019	-0.001	0.002
250	1/4	0.006	0.005	-0.001	0.001
500	1/4	-0.000	0.000	-0.001	0.001
100	1/2	0.003	0.022	0.001	0.001
250	1/2	0.003	-0.002	-0.001	0.003
500	1/2	0.002	-0.003	-0.001	0.001
100	1	0.001	0.008	0.001	0.002
250	1	0.002	0.000	-0.000	0.001
500	1	0.001	-0.000	0.002	-0.001
100	2	0.003	-0.003	0.000	0.007
250	2	-0.002	-0.000	0.001	-0.001
500	2	-0.001	-0.000	0.000	-0.000
100	5	-0.002	-0.005	-0.003	0.004
250	5	-0.002	0.001	-0.000	0.001
500	5	0.002	0.000	-0.001	0.000
100	10	0.001	0.005	0.001	-0.003
250	10	0.003	-0.000	-0.001	0.001
500	10	-0.001	0.001	-0.000	-0.000

Table 148: Relative bias for E(Y|X) and the Pr(Y = 0) by sample size (n), dispersion (θ), with orthogonal covariates for $\beta_1 = 0.5$ and $\gamma_1 = 0$.

		E(Y|X)		Pr(Y = 0)	
n	θ	NBH	ZINB	NBH	ZINB
100	1/4	0.013	0.018	-0.004	0.004
250	1/4	0.002	0.006	-0.001	0.001
500	1/4	0.002	-0.004	-0.002	0.001
100	1/2	0.007	0.019	0.001	0.003
250	1/2	-0.001	-0.002	-0.002	0.001
500	1/2	0.002	0.004	0.000	-0.000
100	1	-0.000	-0.000	0.004	0.001
250	1	-0.005	0.001	0.003	0.001
500	1	-0.000	-0.001	0.001	-0.000
100	2	0.003	-0.013	-0.001	0.002
250	2	0.007	0.000	-0.004	-0.001
500	2	-0.001	-0.003	-0.001	0.001
100	5	0.007	-0.004	-0.003	-0.001
250	5	-0.002	-0.001	0.001	0.000
500	5	0.001	-0.002	-0.001	-0.000
100	10	-0.001	0.000	0.001	0.000
250	10	-0.000	-0.002	0.001	0.001
500	10	0.001	-0.002	0.001	0.002

Table 149: Relative bias for E(Y|X) and the Pr(Y = 0) by sample size (n), dispersion (θ), with correlated covariates for $\beta_1 = 0.5$ and $\gamma_1 = 0$.

		E(Y|X)		Pr(Y = 0)	
n	θ	NBH	ZINB	NBH	ZINB
100	1/4	0.011	8.320	0.003	0.002
250	1/4	0.003	0.001	0.001	0.000
500	1/4	-0.001	0.007	0.001	-0.000
100	1/2	0.008	0.008	-0.000	0.002
250	1/2	-0.001	0.008	-0.001	-0.001
500	1/2	0.000	-0.001	0.001	0.000
100	1	0.010	0.001	-0.001	0.002
250	1	0.006	-0.003	-0.001	0.002
500	1	0.002	0.004	-0.000	-0.003
100	2	-0.006	0.006	0.002	0.002
250	2	-0.003	-0.001	0.004	0.001
500	2	-0.001	-0.002	0.000	0.001
100	5	0.001	-0.002	-0.002	-0.003
250	5	-0.004	0.002	0.004	0.001
500	5	-0.001	-0.001	0.001	0.001
100	10	0.003	0.004	-0.002	0.003
250	10	-0.000	0.001	-0.000	-0.001
500	10	-0.001	-0.002	-0.002	0.002

Table 150: Relative bias for E(Y|X) and the Pr(Y = 0) by sample size (n), dispersion (θ), with orthogonal covariates for $\beta_1 = 0.5$ and $\gamma_1 = -0.5$.

		E(Y|X)		Pr(Y = 0)	
n	θ	NBH	ZINB	NBH	ZINB
100	1/4	0.005	0.031	0.000	0.001
250	1/4	0.000	0.001	0.001	0.001
500	1/4	-0.004	0.008	0.001	-0.001
100	1/2	0.007	0.004	0.000	0.002
250	1/2	0.000	0.001	-0.001	0.001
500	1/2	0.002	0.003	0.001	0.000
100	1	0.002	0.009	-0.003	0.001
250	1	-0.003	-0.001	0.001	0.002
500	1	0.001	-0.000	-0.000	0.000
100	2	0.009	-0.001	-0.003	0.001
250	2	0.000	-0.004	-0.001	0.001
500	2	0.000	0.003	0.001	-0.002
100	5	-0.002	0.009	0.003	-0.001
250	5	0.002	0.002	-0.000	-0.001
500	5	0.004	0.002	-0.003	-0.002
100	10	0.002	0.002	-0.002	0.002
250	10	0.002	-0.000	-0.001	-0.000
500	10	0.000	-0.002	0.001	-0.001

Table 151: Relative bias for E(Y|X) and the Pr(Y = 0) by sample size (n), dispersion (θ), with orthogonal covariates for $\beta_1 = 0.5$ and $\gamma_1 = -0.5$.

		E(Y|X)		Pr(Y = 0)	
n	θ	NBH	ZINB	NBH	ZINB
100	1/4	0.011	0.014	-0.000	0.003
250	1/4	-0.001	0.000	0.000	0.002
500	1/4	0.004	0.010	-0.002	-0.001
100	1/2	0.006	0.012	-0.002	0.003
250	1/2	-0.001	0.003	0.000	0.001
500	1/2	-0.002	0.005	0.001	-0.001
100	1	-0.006	0.002	0.003	0.001
250	1	0.001	0.007	0.001	-0.001
500	1	0.003	0.003	-0.000	-0.002
100	2	0.010	0.006	-0.005	-0.000
250	2	0.000	-0.003	0.001	-0.000
500	2	0.003	0.002	0.000	0.000
100	5	0.003	0.003	-0.007	-0.000
250	5	-0.001	0.005	0.001	-0.001
500	5	-0.002	0.003	0.002	-0.001
100	10	-0.004	0.002	0.000	0.001
250	10	-0.001	-0.001	0.001	0.000
500	10	-0.000	-0.001	0.001	0.001

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Table 152: Relative bias for E(Y|X) for the NBH model with ZINB data with orthogonal covariates.

θ	ρ	level of β_1	level of γ_1	E(Y|X)
1/4	0	0	0	0.084
1/4	0	1/2	0	0.086
1/4	0	0	-0.5	0.132
1/4	0	0.5	-0.5	0.143
1/2	0	0	0	0.107
1/2	0	1/2	0	0.116
1/2	0	0	-0.5	0.162
1/2	0	0.5	-0.5	0.169
1	0	0	0	0.121
1	0	1/2	0	0.123
1	0	0	-0.5	0.192
1	0	0.5	-0.5	0.194
2	0	0	0	0.131
2	0	1/2	0	0.128
2	0	0	-0.5	0.210
2	0	0.5	-0.5	0.215
5	0	0	0	0.128
5	0	1/2	0	0.128
5	0	0	-0.5	0.222
5	0	0.5	-0.5	0.226
10	0	0	0	0.124
10	0	1/2	0	0.126
10	0	0	-0.5	0.218
10	0	0.5	-0.5	0.221

Table 153: Relative bias for E(Y|X) for the NBH model with ZINB data with correlated covariates.

θ	ρ	level of β_1	level of γ_1	E(Y|X)
1/4	.3	0	0	0.124
1/4	.3	1/2	0	0.118
1/4	.3	0	-0.5	0.162
1/4	.3	0.5	-0.5	0.159
1/2	.3	0	0	0.154
1/2	.3	1/2	0	0.134
1/2	.3	0	-0.5	0.198
1/2	.3	0.5	-0.5	0.192
1	.3	0	0	0.167
1	.3	1/2	0	0.156
1	.3	0	-0.5	0.235
1	.3	0.5	-0.5	0.221
2	.3	0	0	0.170
2	.3	1/2	0	0.158
2	.3	0	-0.5	0.252
2	.3	0.5	-0.5	0.237
5	.3	0	0	0.174
5	.3	1/2	0	0.160
5	.3	0	-0.5	0.260
5	.3	0.5	-0.5	0.248
10	.3	0	0	0.175
10	.3	1/2	0	0.158
10	.3	0	-0.5	0.263
10	.3	0.5	-0.5	0.245

Table 154: Relative bias for Pr(Y = 0) for the NBH model with ZINB data with orthogonal covariates.

θ	ρ	level of β_1	level of γ_1	Pr(Y = 0)
1/4	0	0	0	0.021
1/4	0	1/2	0	0.020
1/4	0	0	-0.5	0.020
1/4	0	0.5	-0.5	0.017
1/2	0	0	0	0.048
1/2	0	1/2	0	0.041
1/2	0	0	-0.5	0.045
1/2	0	0.5	-0.5	0.039
1	0	0	0	0.092
1	0	1/2	0	0.077
1	0	0	-0.5	0.087
1	0	0.5	-0.5	0.074
2	0	0	0	0.146
2	0	1/2	0	0.121
2	0	0	-0.5	0.137
2	0	0.5	-0.5	0.111
5	0	0	0	0.199
5	0	1/2	0	0.158
5	0	0	-0.5	0.187
5	0	0.5	-0.5	0.151
10	0	0	0	0.218
10	0	1/2	0	0.173
10	0	0	-0.5	0.210
10	0	0.5	-0.5	0.168

Table 155: Relative bias for Pr(Y = 0) for the NBH model with ZINB data with correlated covariates.

θ	ρ	level of β_1	level of γ_1	Pr(Y = 0)
1/4	.3	0	0	0.011
1/4	.3	1/2	0	0.011
1/4	.3	0	-0.5	0.012
1/4	.3	0.5	-0.5	0.011
1/2	.3	0	0	0.031
1/2	.3	1/2	0	0.029
1/2	.3	0	-0.5	0.033
1/2	.3	0.5	-0.5	0.029
1	.3	0	0	0.070
1	.3	1/2	0	0.057
1	.3	0	-0.5	0.069
1	.3	0.5	-0.5	0.057
2	.3	0	0	0.117
2	.3	1/2	0	0.094
2	.3	0	-0.5	0.116
2	.3	0.5	-0.5	0.093
5	.3	0	0	0.163
5	.3	1/2	0	0.125
5	.3	0	-0.5	0.166
5	.3	0.5	-0.5	0.128
10	.3	0	0	0.179
10	.3	1/2	0	0.138
10	.3	0	-0.5	0.186
10	.3	0.5	-0.5	0.144

Table 156: Relative bias for E(Y|X) for the ZINB model with NBH data with orthogonal covariates.

θ	ρ	level of β_1	level of γ_1	E(Y|X)
1/4	0	0	0	0.021
1/4	0	1/2	0	0.024
1/4	0	0	-0.5	0.017
1/4	0	0.5	-0.5	0.020
1/2	0	0	0	0.015
1/2	0	1/2	0	0.027
1/2	0	0	-0.5	0.022
1/2	0	0.5	-0.5	0.028
1	0	0	0	0.026
1	0	1/2	0	0.028
1	0	0	-0.5	0.027
1	0	0.5	-0.5	0.030
2	0	0	0	0.037
2	0	1/2	0	0.034
2	0	0	-0.5	0.031
2	0	0.5	-0.5	0.035
5	0	0	0	0.041
5	0	1/2	0	0.041
5	0	0	-0.5	0.043
5	0	0.5	-0.5	0.045
10	0	0	0	0.042
10	0	1/2	0	0.043
10	0	0	-0.5	0.044
10	0	0.5	-0.5	0.044

Table 157: Relative bias for E(Y|X) for the ZINB model with NBH data with correlated covariates.

θ	ρ	level of β_1	level of γ_1	E(Y|X)
1/4	.3	0	0	0.020
1/4	.3	1/2	0	0.031
1/4	.3	0	-0.5	0.030
1/4	.3	0.5	-0.5	0.052
1/2	.3	0	0	0.014
1/2	.3	1/2	0	0.031
1/2	.3	0	-0.5	0.037
1/2	.3	0.5	-0.5	0.049
1	.3	0	0	0.018
1	.3	1/2	0	0.033
1	.3	0	-0.5	0.043
1	.3	0.5	-0.5	0.056
2	.3	0	0	0.024
2	.3	1/2	0	0.029
2	.3	0	-0.5	0.049
2	.3	0.5	-0.5	0.060
5	.3	0	0	0.026
5	.3	1/2	0	0.032
5	.3	0	-0.5	0.058
5	.3	0.5	-0.5	0.059
10	.3	0	0	0.029
10	.3	1/2	0	0.032
10	.3	0	-0.5	0.057
10	.3	0.5	-0.5	0.063

Table 158: Relative bias for Pr(Y = 0) for the ZINB model with NBH data with orthogonal covariates.

θ	ρ	level of β_1	level of γ_1	Pr(Y = 0)
1/4	0	0	0	0.033
1/4	0	1/2	0	0.072
1/4	0	0	-0.5	0.037
1/4	0	0.5	-0.5	0.083
1/2	0	0	0	0.031
1/2	0	1/2	0	0.070
1/2	0	0	-0.5	0.039
1/2	0	0.5	-0.5	0.082
1	0	0	0	0.035
1	0	1/2	0	0.069
1	0	0	-0.5	0.041
1	0	0.5	-0.5	0.078
2	0	0	0	0.037
2	0	1/2	0	0.067
2	0	0	-0.5	0.043
2	0	0.5	-0.5	0.078
5	0	0	0	0.037
5	0	1/2	0	0.066
5	0	0	-0.5	0.043
5	0	0.5	-0.5	0.075
10	0	0	0	0.038
10	0	1/2	0	0.067
10	0	0	-0.5	0.044
10	0	0.5	-0.5	0.078

Table 159: Relative bias for Pr(Y = 0) for the ZINB model with NBH data with correlated covariates.

θ	ρ	level of β_1	level of γ_1	Pr(Y = 0)
1/4	.3	0	0	0.045
1/4	.3	1/2	0	0.074
1/4	.3	0	-0.5	0.051
1/4	.3	0.5	-0.5	0.080
1/2	.3	0	0	0.048
1/2	.3	1/2	0	0.073
1/2	.3	0	-0.5	0.051
1/2	.3	0.5	-0.5	0.083
1	.3	0	0	0.049
1	.3	1/2	0	0.074
1	.3	0	-0.5	0.056
1	.3	0.5	-0.5	0.082
2	.3	0	0	0.051
2	.3	1/2	0	0.077
2	.3	0	-0.5	0.055
2	.3	0.5	-0.5	0.083
5	.3	0	0	0.052
5	.3	1/2	0	0.080
5	.3	0	-0.5	0.053
5	.3	0.5	-0.5	0.084
10	.3	0	0	0.052
10	.3	1/2	0	0.077
10	.3	0	-0.5	0.053
10	.3	0.5	-0.5	0.082