Understanding chiral symmetry breaking and deconfinement phase transitions via the instanton-dyons

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(Continuous advances in QCD 2013)

in collaboration with
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Pietro Faccioli

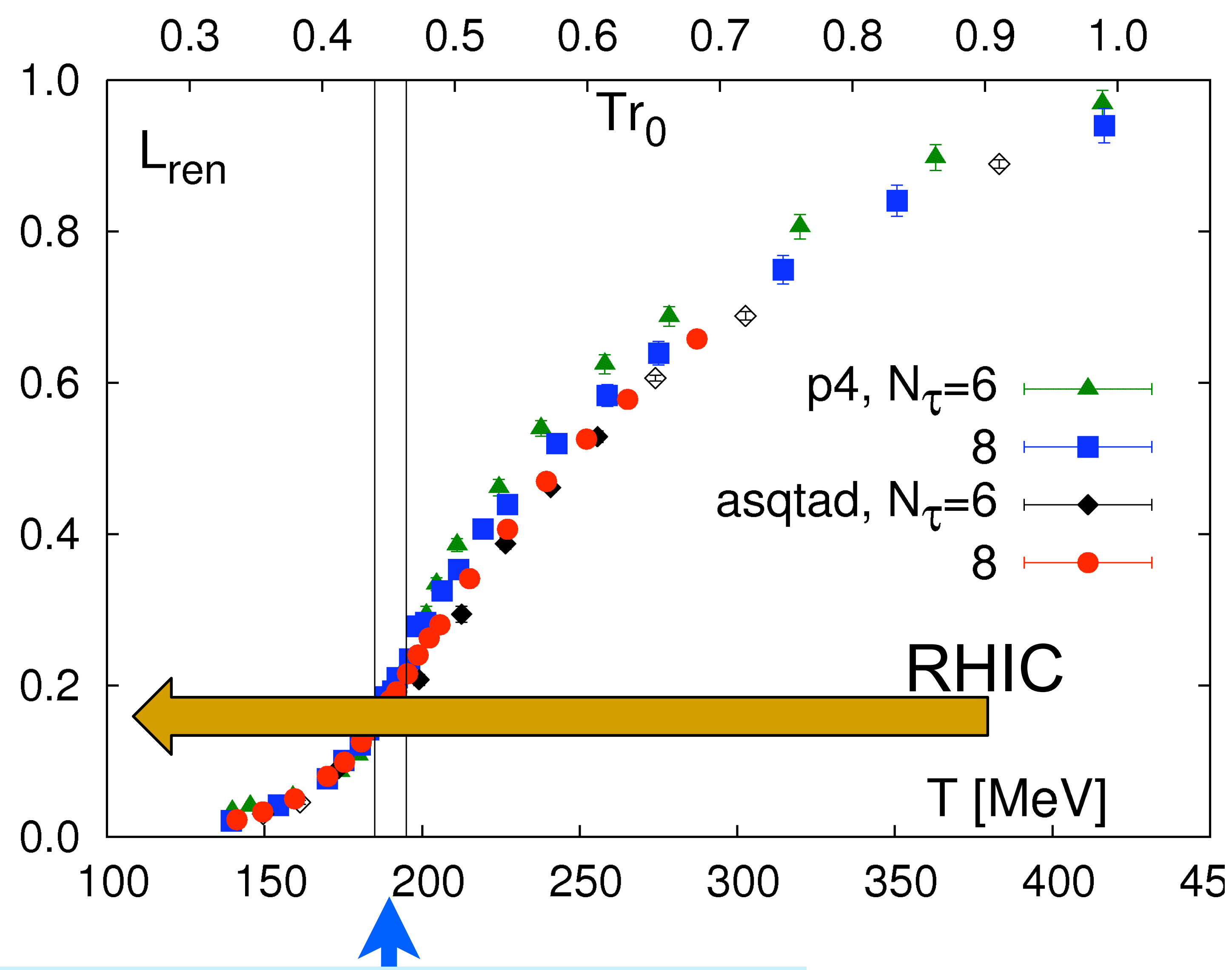
outline

- nonzero holonomy and instanton-dyons
- interactions: moduli,screening
- fermionic zero modes, molecules, chiral condensate ES and T.Sulejmanpasic,arXiv:1201.5624
- first numerical simulations
  P.Faccioli+ES,archive 1301.2523**Phys. Rev. D 87, 074009 (2013)**
- back reaction to holonomy,confinement
  ES and T.Sulejmanpasic,arXiv:1305.0796

holonomy and the onset of confinement

$$L = < P > = < \frac{1}{N_c} Tr P exp(i \int d\tau A_0) >$$

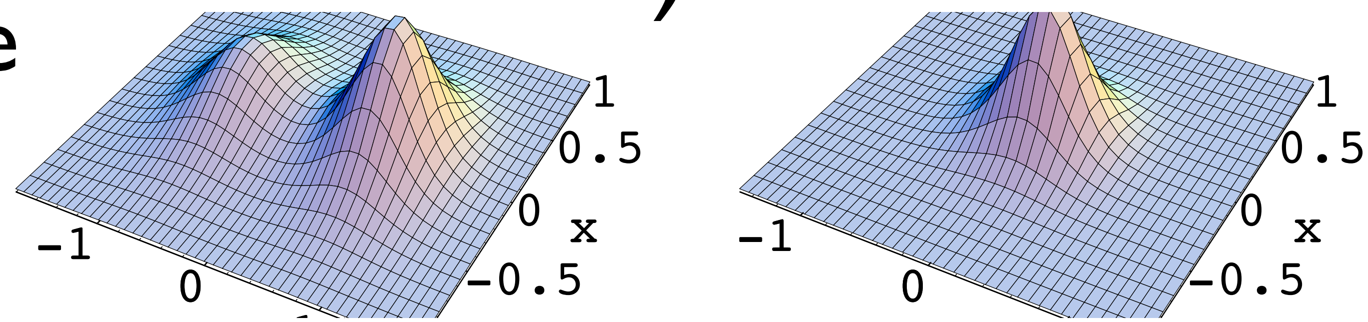
The Polyakov loop
L=1 => A0=0 high T full QGP
L=1/2 ``semi-QGP'' (Pisarski)
L=>0 no quarks or onset of confinement



The approximate width of the phase transition in thermodynamical quantities, energy and entropy

Instantons => Nc selfdual dyons (P.van Baal et al)

<P> nonzero Polyakov line
=> <A_4> nonzero
=> new solutions



Instanton liquid
4d+short range

Dyonic plasma
3+1d long range

instanton-dyons in SU(2)

name	E	M	mass
M	+	+	v
$\bar{M}$	+	-	v
L	-	-	$2\pi T - v$
$\bar{L}$	-	+	$2\pi T - v$

TABLE I: The charges and the mass (in units of $8\pi^2/e^2T$) for 4 SU(2) dyons.

calorons=M+L
are
E and M neutral

calorons (finite-T) were located on the lattice

Illgenfritz et al, Gattringer...

are the instanton-dyons semiclassical?

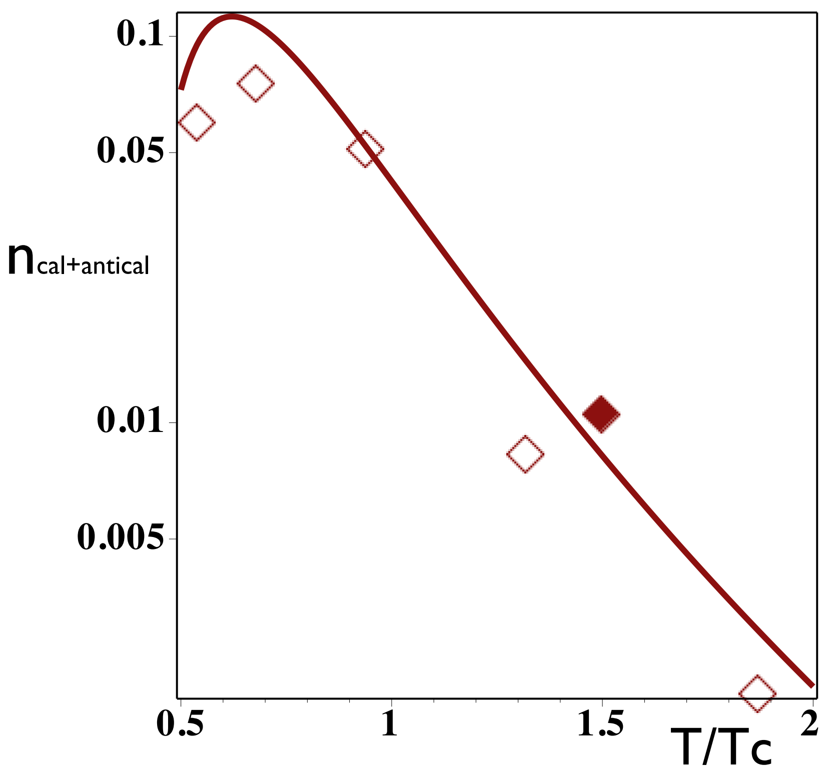


FIG. 1: Caloron density as a function of T/T_c. The solid curve is the semiclassical fit $n_{cal} = KS_{cal}^2 e^{-S_{cal}}$ in units of T with parameters $K = 0.024$, $S_{cal} = 8\pi^2/g^2(T)$, open (filled) points are the lattice data from [19] ([20]).

$$n_{cal+\bar{cal}} = KS_{cal}^4 e^{-S_{cal}}, \; S_{cal} = \frac{22}{3}\ln\left(\frac{T}{\Lambda}\right) \qquad (1)$$

with parameters[1] $K = 0.024, \Lambda/T_c = .36$. The caloron action at T_c is 7.50, so per dyon it makes $S_d = S_{cal}/2 =$

3.75, which gives an idea how semiclassical the discussed objects are. (SU(3) instantons have actions $S_{cal} \approx 12$ or $S_d = S_{cal}/3 \approx 4$, quite close in magnitude.) After those parameters are fixed, one knows semiclassical densities of the dyons and their pairs, as we explain in detail below.

M are lighter than L, both were identified on the lattice

The screening by the plasma (ES,Pisarski-Yaffe, Diakonov)

$$Z = \int \{dX_i\} e^{-S_c} \det G \; \det F_{zm} \; \frac{\det' F_{nzm}}{\sqrt{\det' B}}$$

(

The moduli space metric (Atiyah,Manton,Diakonov) includes E and M Coulomb

Fermionic determimant in zero mode approximation (ES.Sulejmanpasic), only for L dyons

In evaluating the fermionic determinant, the Dirac operator is approximated by retaining only the contribution evaluated on the sub-space of fermionic zero-modes of the individual pseudo-particles $|\phi_0^j\rangle$:

$$\mathrm{Det}(i\gamma_\mu D^\mu + im) \simeq \mathrm{Det}(\hat{T} + im), \qquad (22)$$

where

$$T_{ij} = \langle \phi_0^i | i\gamma_\mu D^\mu | \phi_0^j \rangle. \qquad (23)$$

This scheme was well tested in the frame-work of the instanton liquid model, where it corresponds to summing up all loop dia-grams created by 't Hooft effective Lagrangian.

$$V_{12} \sim \langle (A_4)^2 \rangle = \int d^3x \left| \frac{1}{r_L} - \frac{1}{r_M} \right|^2 = 4\pi \; r_{LM}$$

We further note that the form (14) can be ob-tained directly by the instanton screening term calculated by Pisarski and Yaffe [30] by recall-ing that the instanton size ρ and the $L - M$ separation are related by the expression

$$\pi \rho^2 T = r_{ML}. \qquad (17)$$

which relates the "4-d dipole" of the instanton field to the "3-d dipole" of the dyon LM pair made of opposite charges.

density

PHYSICAL REVIEW D **87,** 074009 (2013)

QCD topology at finite temperature: Statistical mechanics of self-dual dyons

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Topological phenomena in gauge theories have long been recognized as the driving force for chiral symmetry breaking and confinement. These phenomena can be conveniently investigated in the semiclassical picture, in which the topological charge is entirely carried by (anti-)self-dual gauge configurations. In such an approach, it has been shown that near the critical temperature, the nonzero expectation value of the Polyakov loop (holonomy) triggers the ‘‘Higgsing’’ of the color group, generating the splitting of instantons into N_c self-dual dyons. A number of lattice simulations have provided some evidence for such dyons, and traced their relation with specific observables, such as the Dirac eigenvalue spectrum. In this work, we formulate a model, based on one-loop partition function and including Coulomb interaction, screening and fermion zero modes. We then perform the first numerical Monte Carlo simulations of a statistical ensemble of self-dual dyons, as a function of their density, quark mass and the number of flavors. We study different dyonic two-point correlation functions and we compute the Dirac spectrum, as a function of the ensemble diluteness and the number of quark flavors.

The first statistical simulations

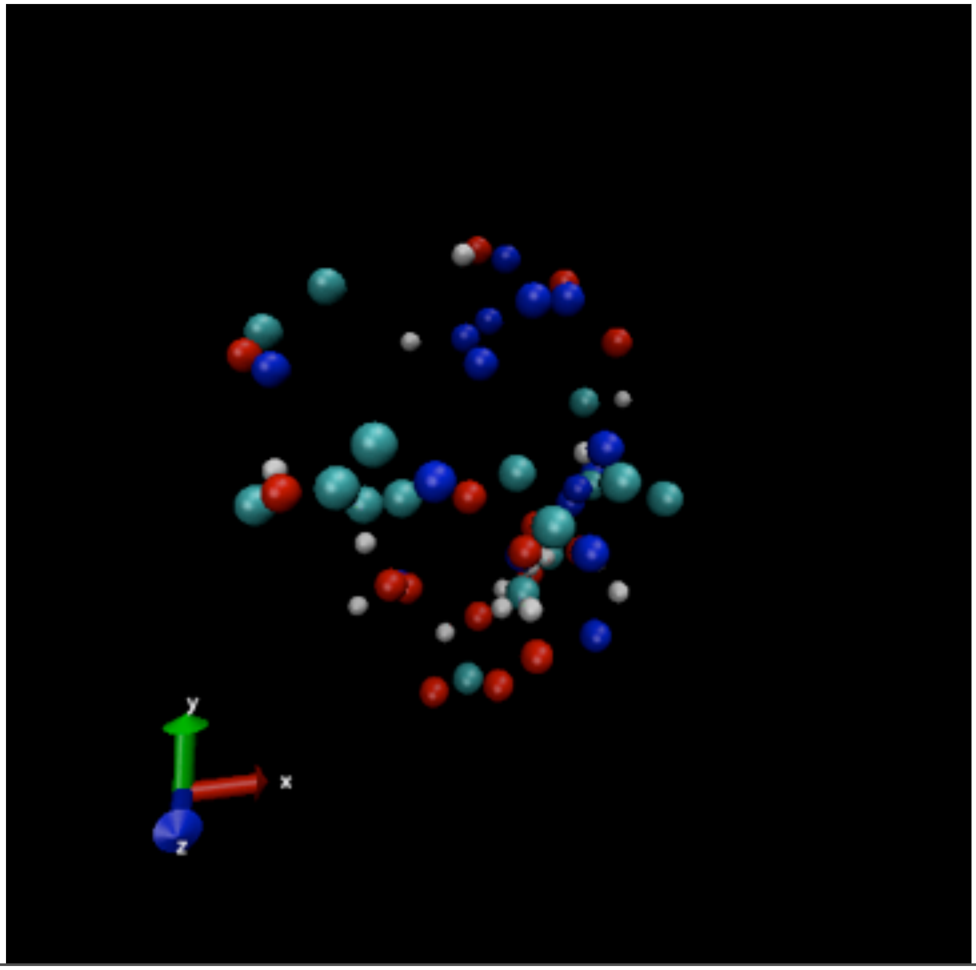
Coulomb++

64 dyons on S^3, Faccioli+ES

Coulomb +-

Nf

fermions



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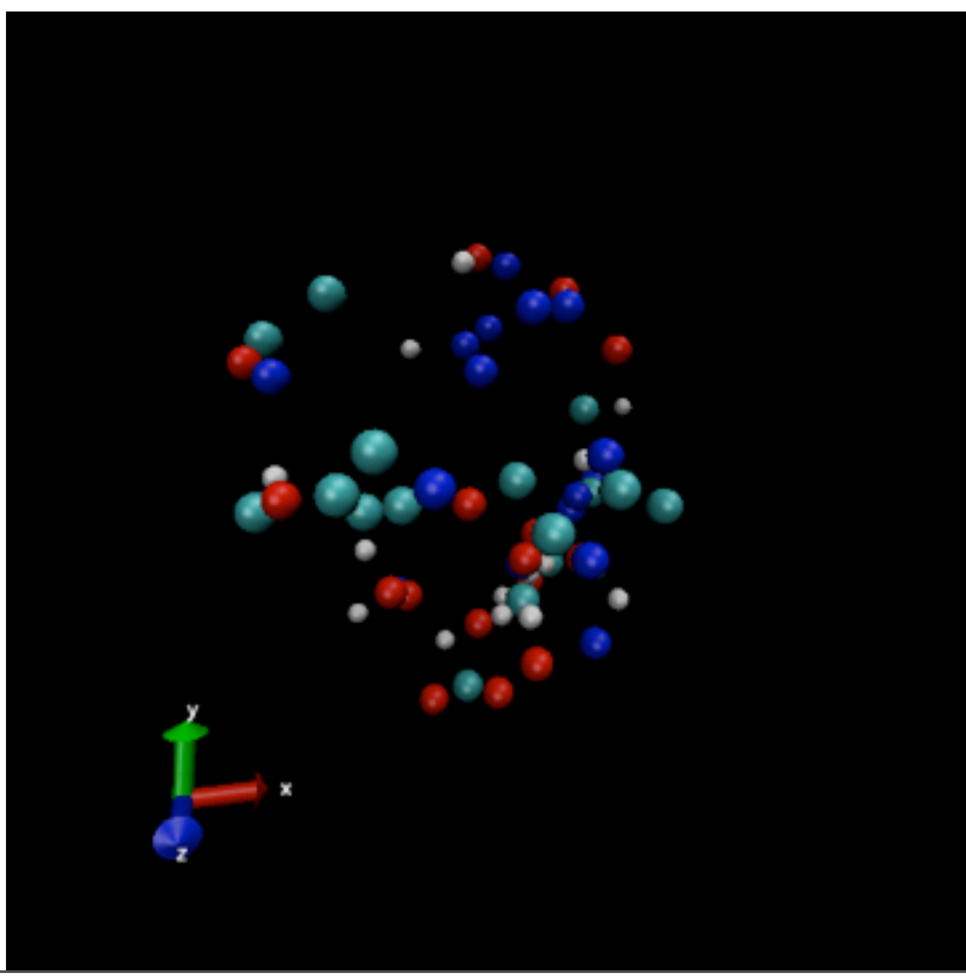


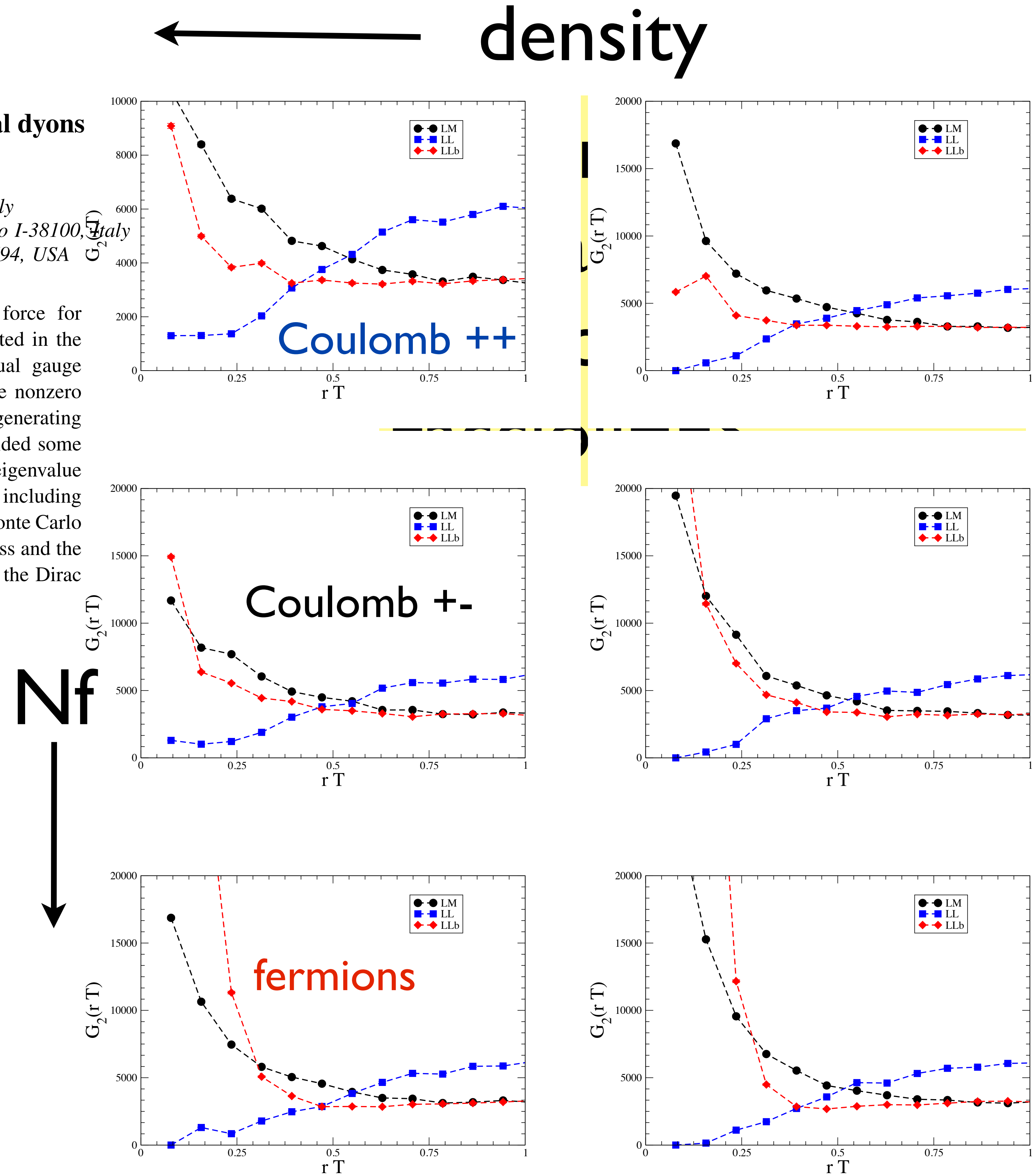
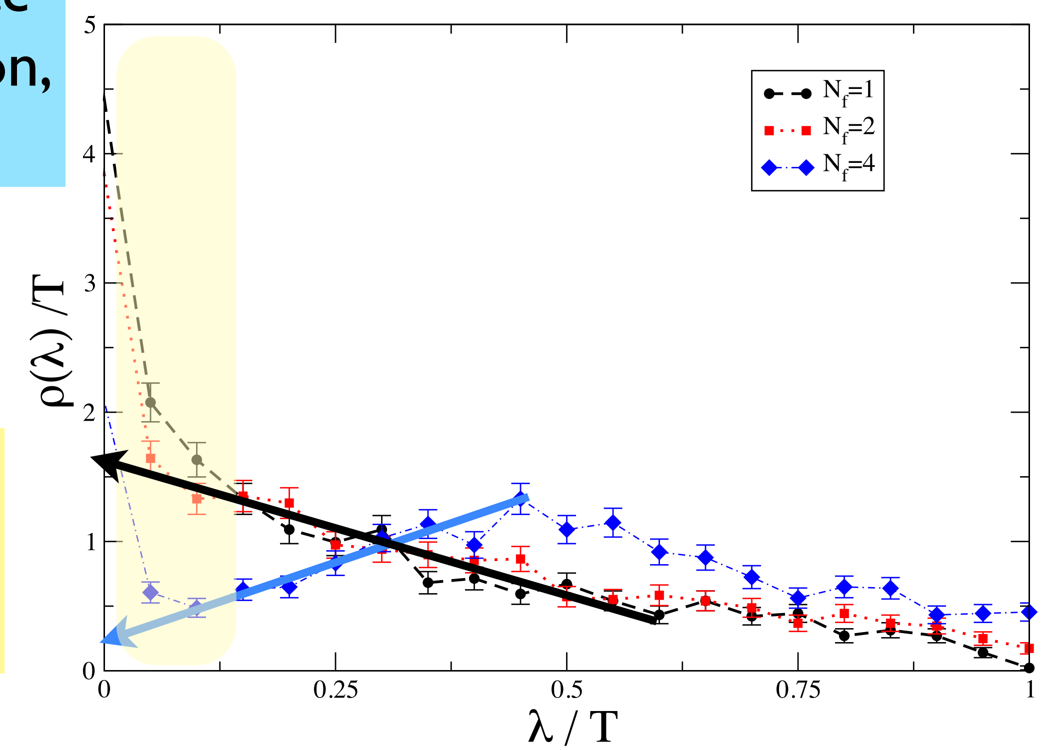


FIG. 2: The correlation function for LM, LL and $L\bar{L}$ dyons versus distance, normalized to the volume available. From top to bottom we show $N_f = 1, 2, 4$, respectively. Left/right columns are for the volumes per dyon $VT^3 = 0.31, 1.04$.

dyons	$R(S^3)T$	$VT^3/dyon$
64	4.5	28.
64	3.0	8.3
64	2.5	4.8
64	2.2	3.28
64	1.5	1.04
64	1.2	0.53
64	1.	0.31

Example of the Dirac eigenvalue distribution, high dyon density

at small Dirac eigenvalues one finds known finite volume effects



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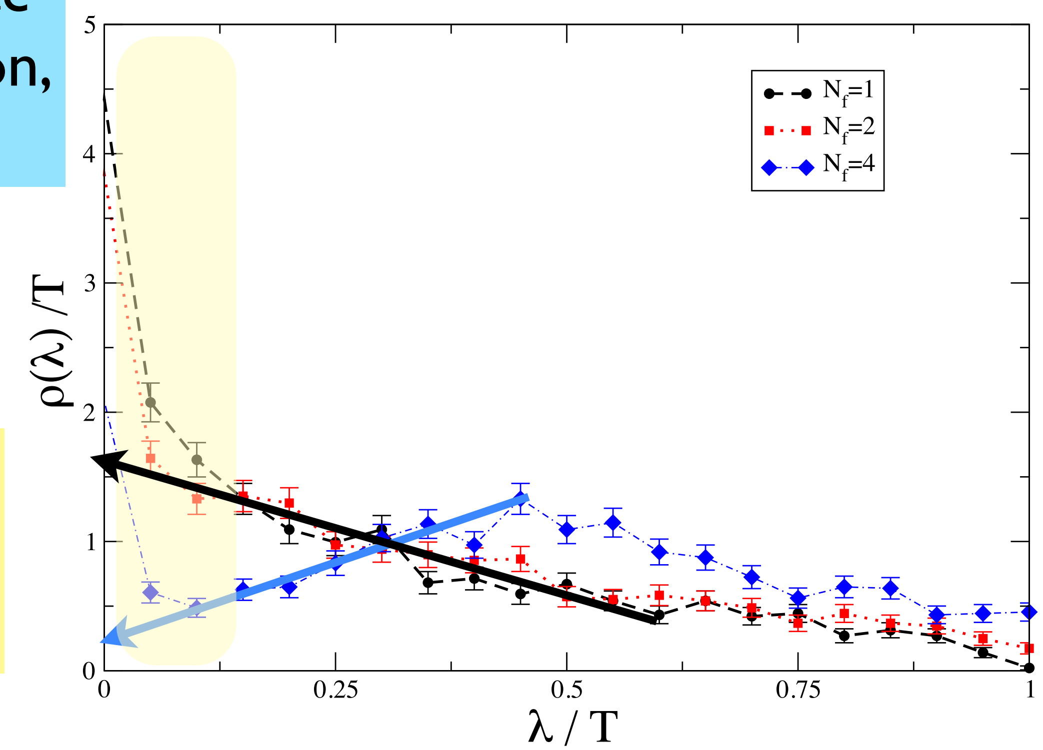

for Nf=4 the chiral symmetry is broken only at very high density => low T

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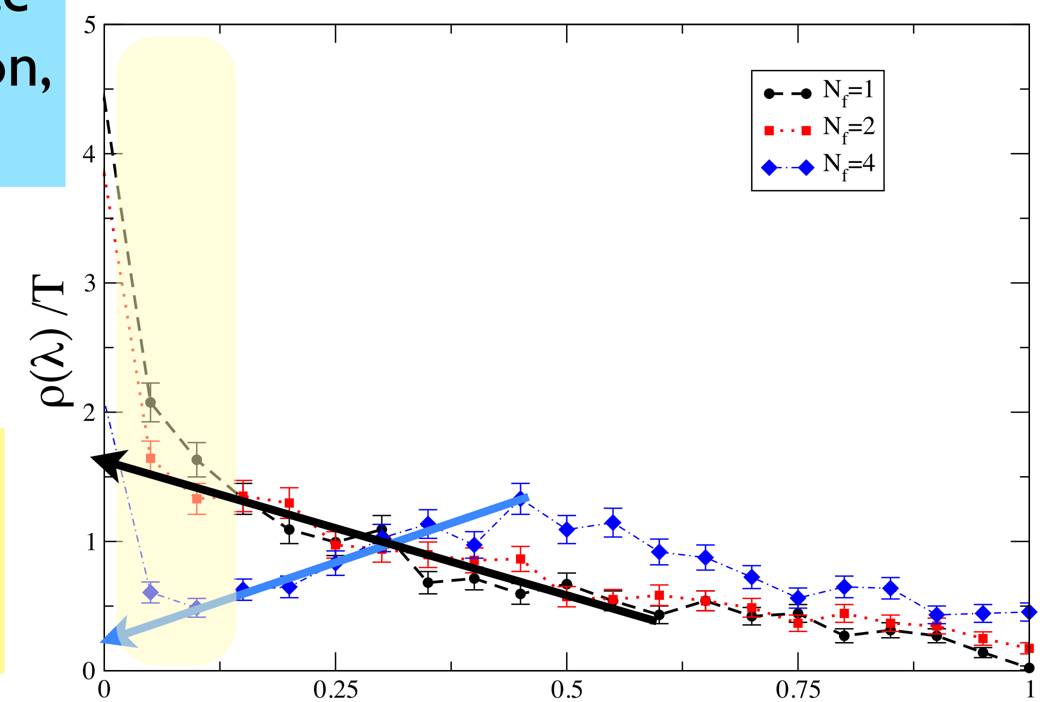

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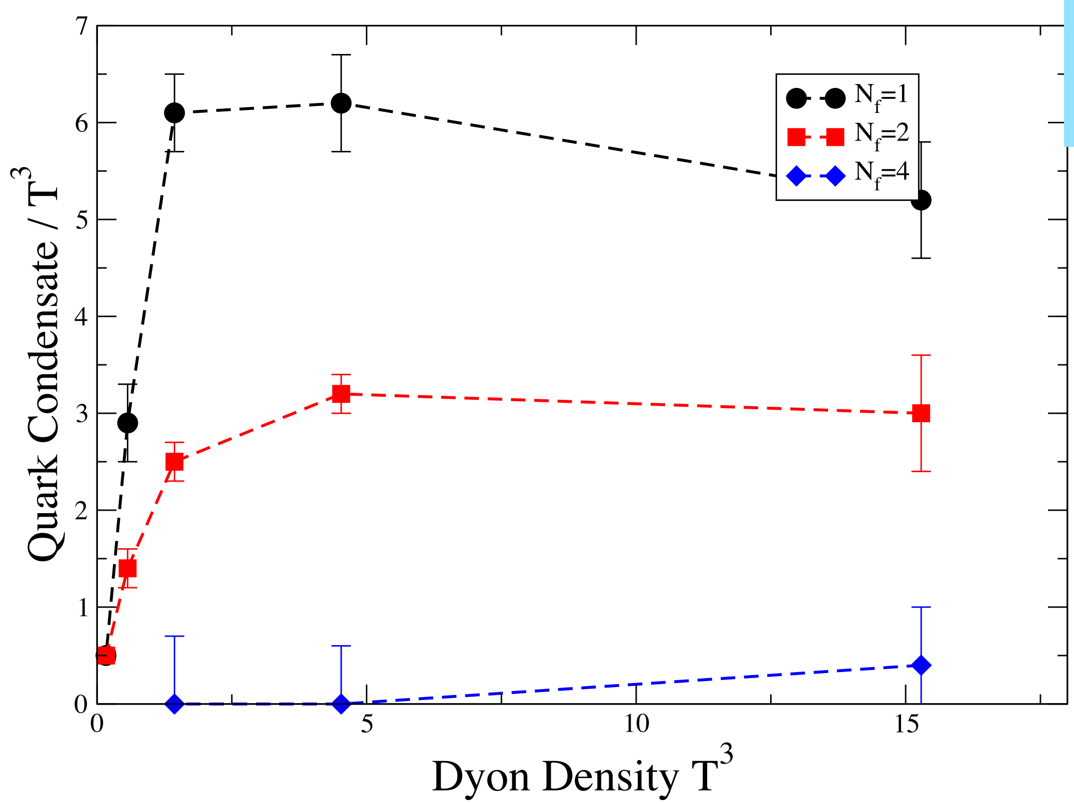

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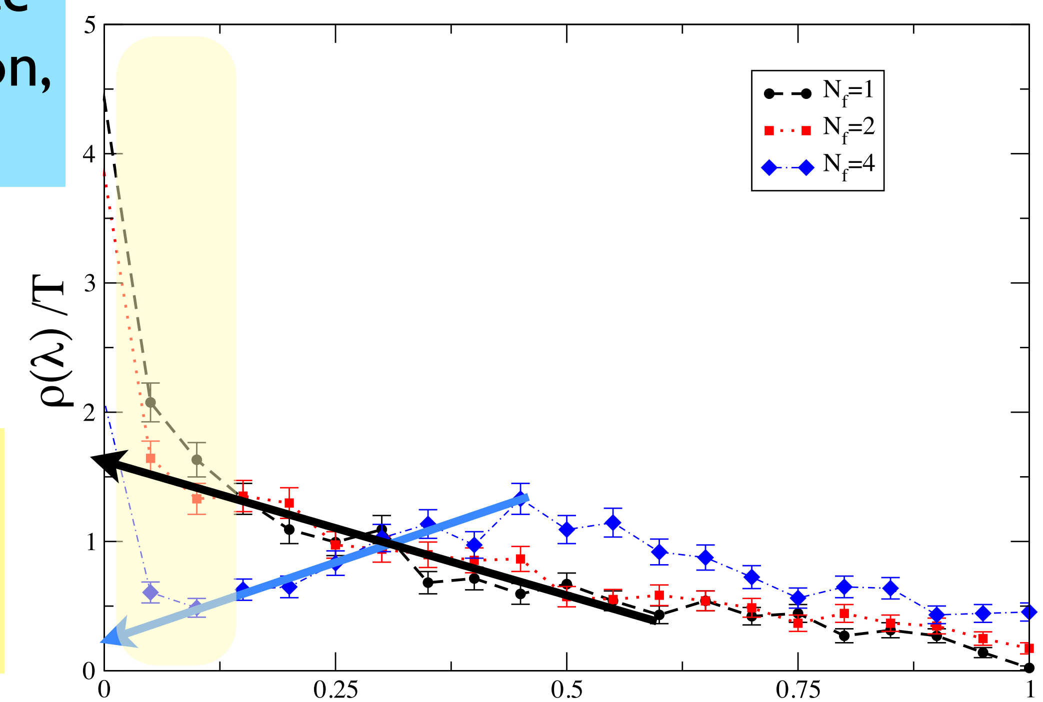

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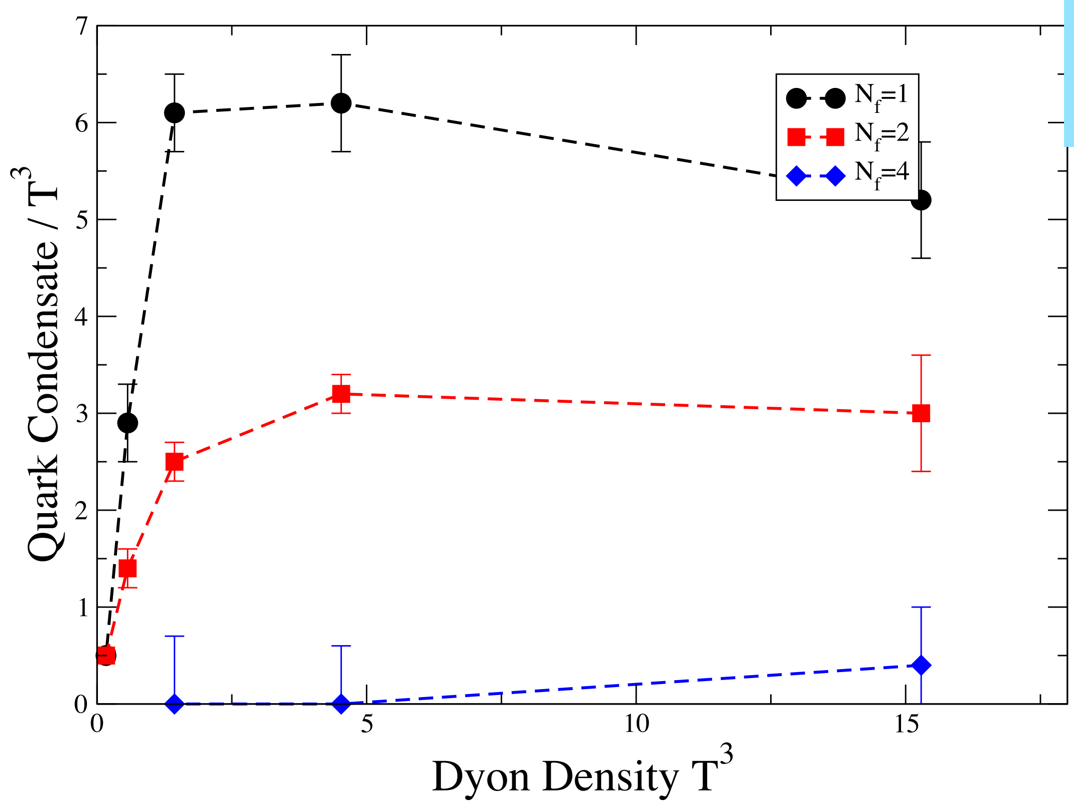

at small density => high T, chiral symmetry gets restored

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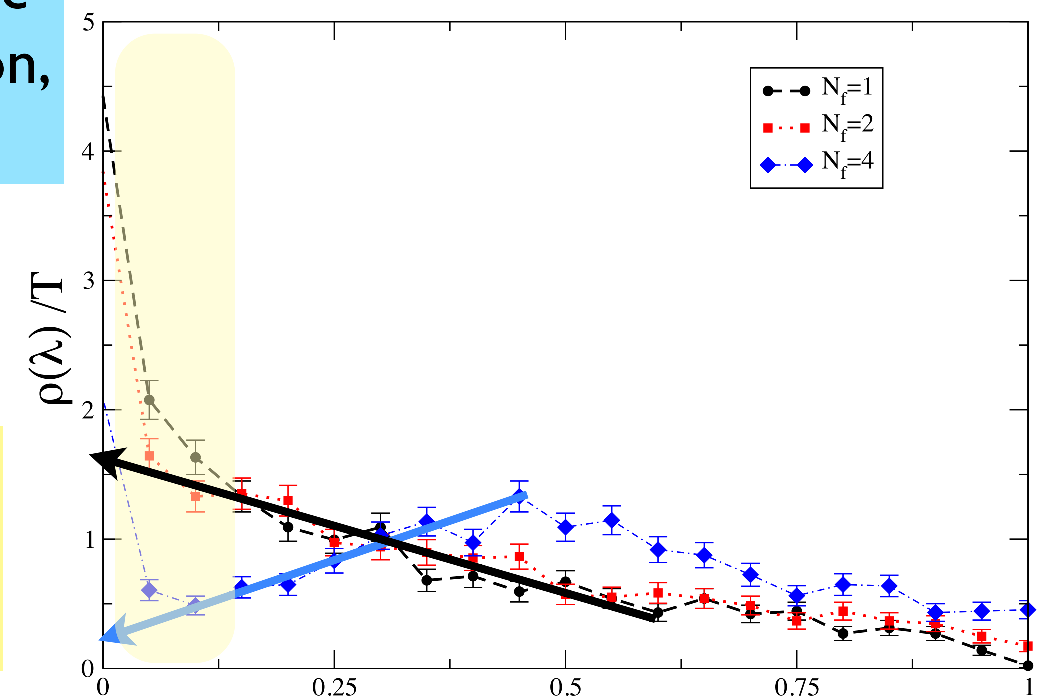


at small Dirac eigenvalues one finds known finite volume effects

for Nf=4 the chiral symmetry is broken only at very high density => low T

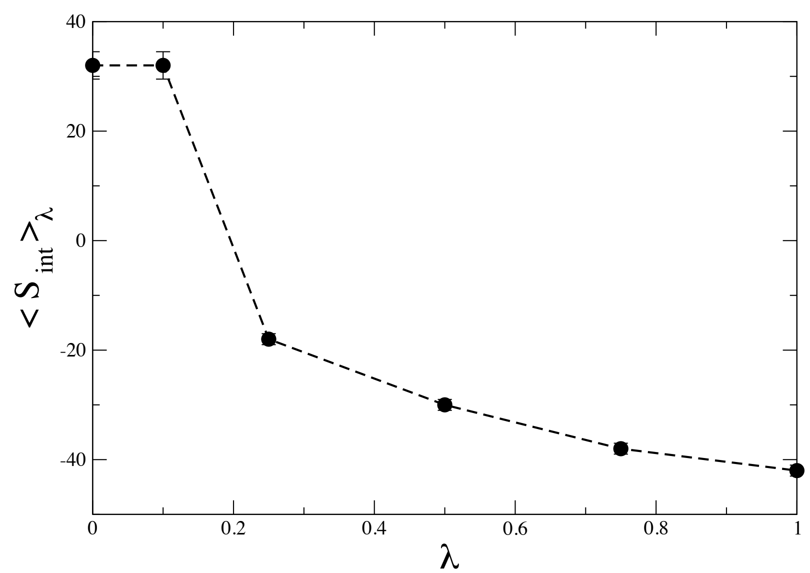
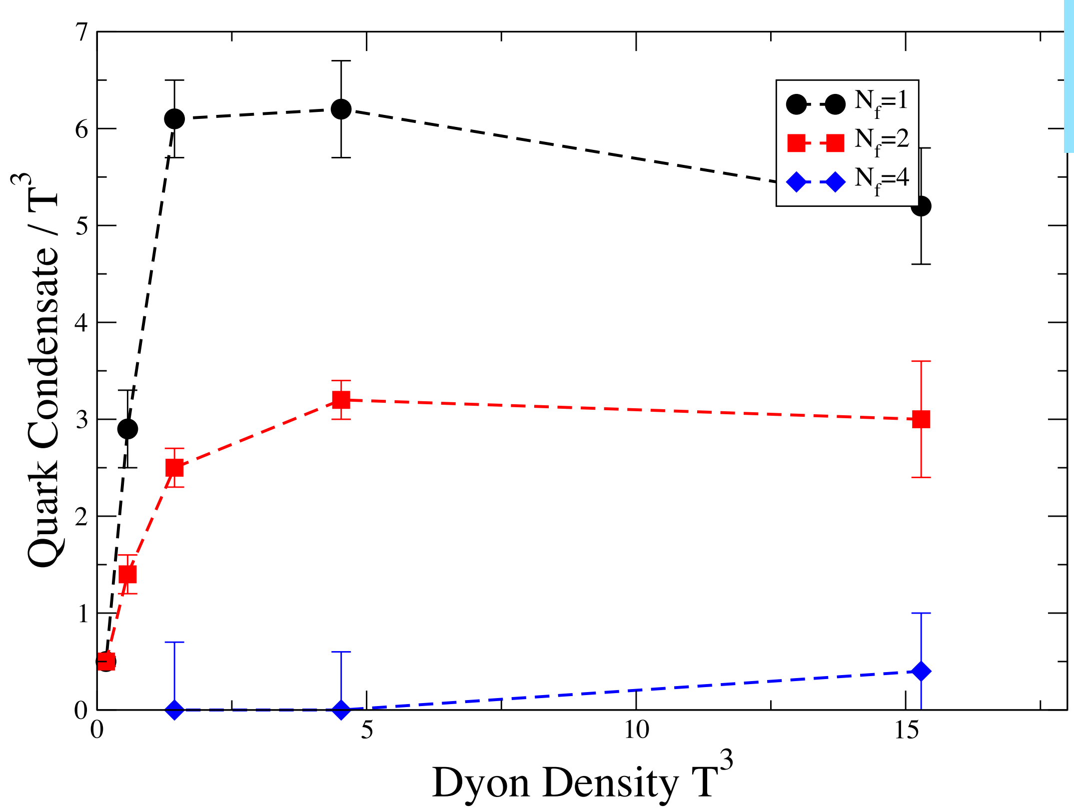


FIG. 7: The dependence of the average interaction free energy on the dimensionless adiabatic parameter λ.



at small density => high T, chiral symmetry gets restored

Holonomy potential and confinement from a simple model of the gauge topology

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Close $\bar{I}I$ pairs correspond to weak fields, which cannot be treated semiclassically and should be subtracted from the semiclassical configurations. This physical idea has been implemented in the Instanton Liquid Model via an "excluded volume", which generates a repulsive core and stabilizes the density.

In a few important cases, in which the partition function is independently known, such subtraction can be performed exactly, *without* any parameters. The $\bar{I}I$ pair contribution to the partition function in QM instanton problem has been done via the analytic continuation in the coupling constant $g^2 \to -g^2$ by Bogomolny [14] and Zinn-Justin [15] (BZJ), who verified it via known semiclassical series. Another analytic continuation has been used by Balitsky and Yung [13] for supersymmetric quantum mechanics.

Recently Poppitz, Schäfer and Ünsal (PSU) [16, 17] used BZJ approach in the $N = 1$ Super-Yang-Mills theory on $R^3 \times S^1$, observing that the result obtained matches exactly the result derived via supersymmetry [18]. PSU papers are the most relevant for this work, as they focus on the instanton-dyons (referred to as

v=<A0> is Higgs VEV
shifted and rescaled,
v=0 trivial limit (high T)
b=0 confining (T<Tc)

$$\frac{1}{2}\mathrm{Tr}\,P(x) = \cos\left(\frac{v(x)}{2T}\right)\ , \quad b = \frac{4\pi^2}{g^2}\left(\frac{v}{\pi T} - 1\right)$$

Similarly to electric holonomy
Polyakov introduced magnetic
one <C0>=sigma

Poppitz+Schafer+Unsal idea:
If the quadratic term is repulsive,
it can lead to confinement

$$V_{eff} = 2n_d \left(-2\cos\sigma \cosh b + n_d \mathcal{A} \cosh(2b)\right) \ .$$

4 dyon amplitudes

$$\mathcal{M} \sim e^{-b+i\sigma-S_d}\,, \qquad \bar{\mathcal{M}} \sim e^{-b-i\sigma-S_d}$$
$$\mathcal{L} \sim e^{b-i\sigma-S_d}\,, \qquad \bar{\mathcal{L}} \sim e^{b+i\sigma-S_d}\,.$$

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$$V_{pert} = \frac{\pi^2}{12}\left(1 - \frac{b^2}{S_d^2}\right)^2$$

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Yet, unlike in the SUSY
setting PSU discussed,
there is also the perturbative
holonomy potential to be
overcomed!

In fact the excluded volume model works well for SU(2) YM

the density is deduced from calorons
n(dyons)=(n(calorons))^(1/Nc)
and is large enough to make second-order in density term do its work

the only parameter A is fixed from known Tc value
and has a reasonable size (including Coulomb enhancement)

electric and magnetic screening masses are even factor 2 **too large** as compared to those from lattice propagators:

their ratio ME/MM is well reproduced

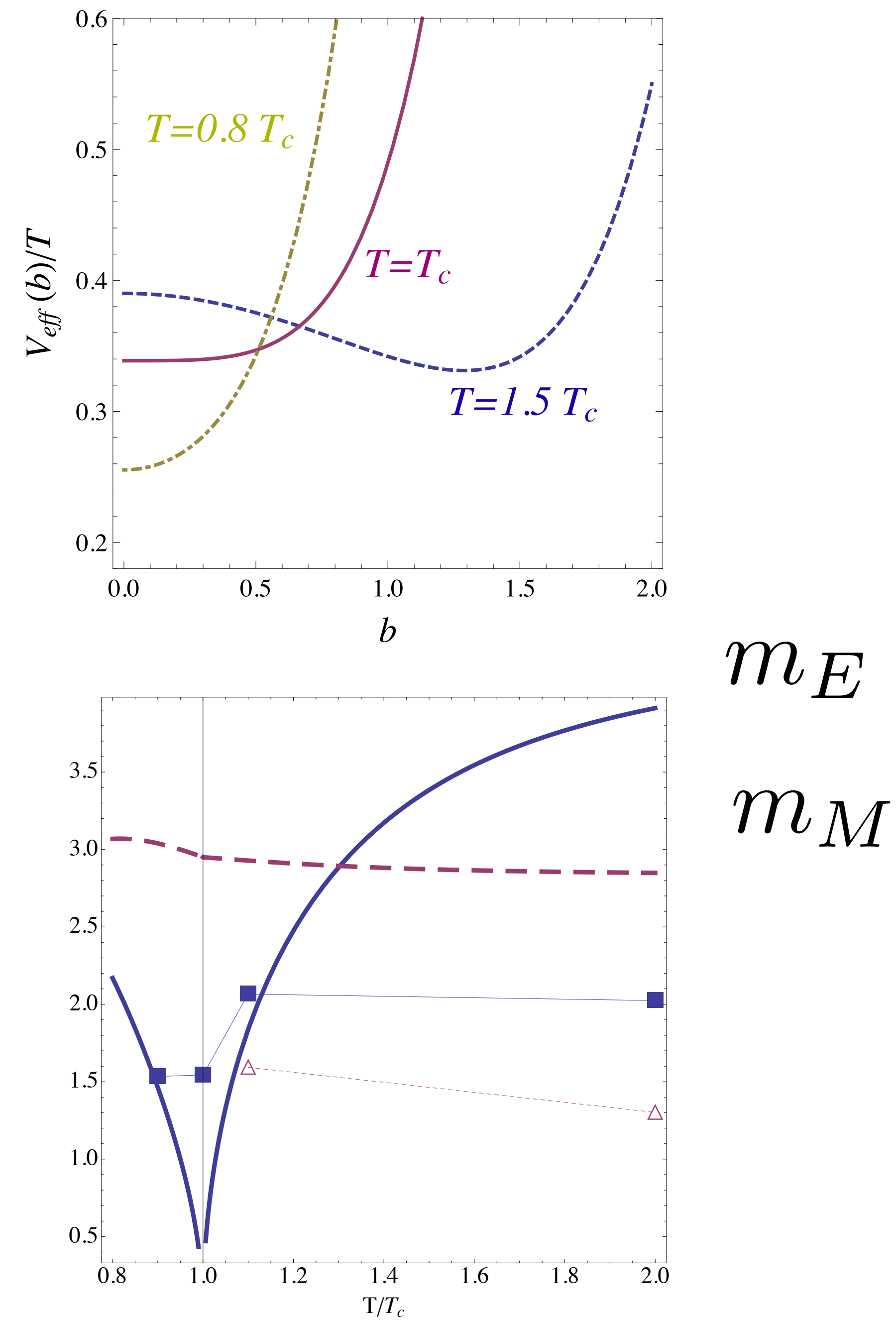

FIG. 2: The upper plot shows the effective potential $V_{eft}(b)/T$ (13) for $T/T_c = 0.8, 1, 1.5$ shown by the dashed,solid and dot-dashed lines, respectively. The plot shows electric m_E/T and magnetic m_M/T screening masses versus temperature, indicated by the solid and dashed lines, respectively. Thick lines are our model, the data points are from lattice propagators [26], the lines connecting data points are shown simply for their identification.

predictions: definite densities of the M and L dyons

crosses: "unidentified topological objects", an upper limit

circles: identified M

L dyon size is very small and measuring <P> at its center is hard, as well as E and M charges: not done yet

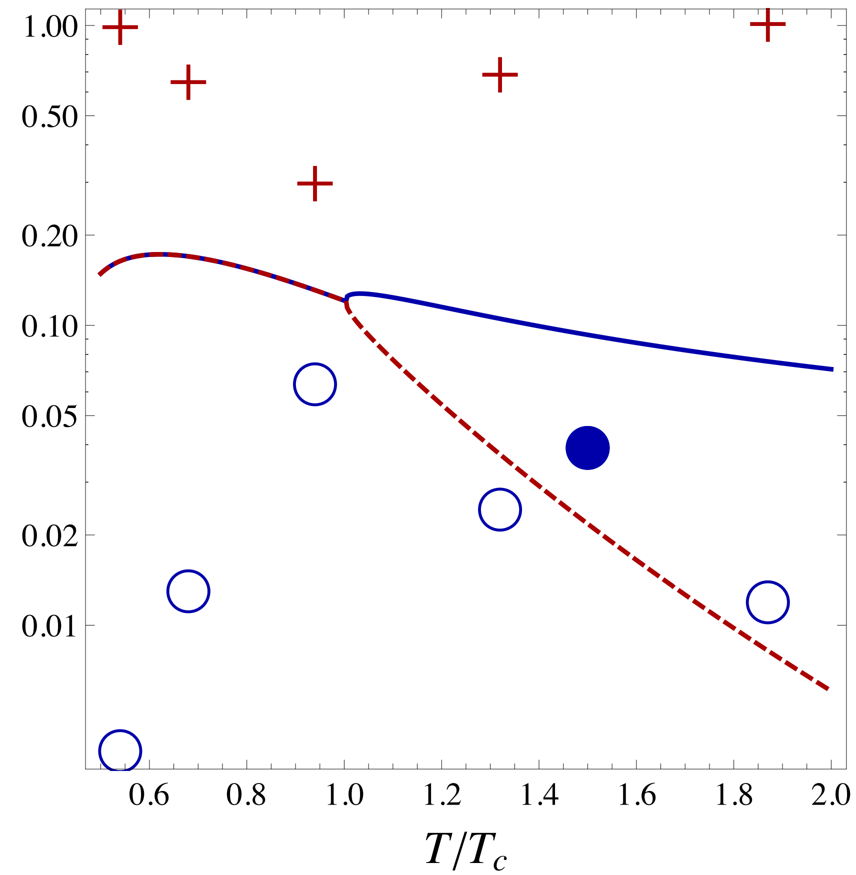

FIG. 3: Prediction of the model for the temperature dependence of the density of the instanton-dyons are shown by the lines, those with solid and dashed lines are for M, L type dyons, respectively. Open (filled) circles show identified M-type dyons from ref. [19] ([20]). The crosses show "unidentified topological objects" from [19]. Circles and crosses provide the lower and the upper bound for the dyon density.

Summary

- Nonzero <Polyakov loop>: instantons split into Nc instanton-dyons
- electric and magnetic charges=> Coulomb plasma
- screening: linear potentials
- fermionic zero modes are different for different fermions => explains multiple lattice puzzles
- first simulation done: **chiral Tc(Nf) found**
- back reaction on holonomy: **repulsion can lead to confinement**

Nambu-Jona-Lasinio versus the instanton liquid model

- NJL (1961) introduced hypothetical 4-fermion interaction (chirally symmetric)
- 2 parameters, G and cutoff Lambda (about 1 GeV)
- Good chiral physics, pions etc (no confinement)
- Eta' also massless
- Other particles like sigma, rho,N = 2 const.quarks
- Higher orders undefined

The Instanton Liquid Model ES (1982) also has 2 parameters

$$n \approx 1fm^{-4}, \rho \approx 1/3fm, n\rho^4 \sim 10^{-2}$$

It also describes all the chiral physics correctly

It can be and was solved to all orders
Rho and nucleon are bound and many correlators are well described

eta' is now correct (repulsive 4-fermi term from 't Hooft makes it heavy)