CAQCD 16/05/2013

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Quark Confinement via Strongly-interacting non-Abelian Monopoles

Plan of the talk:

I. Confinement and XSB in QCD : nonAbelian monopoles?

II. Lessons from N=2 SQCD - singular SCFT and confinement

III. Recent developments

Argyres-Seiberg, Gaiotto-Seiberg-Tachikawa, Giacomelli

2009 2011 2012

IV. Singular SCFT and confinement

- New confinement picture -

Main part, based on Giacomelli-Konishi 2012, 2013

I. Quark confinement

- Abelian dual superconductor (dynamical Abelianization) ?

't Hooft, Nambu, Mandelstam

$$SU(3) \to U(1)^2 \to \mathbf{1}$$

$$\langle M \rangle \neq 0$$

☞ Doubling of the meson spectrum (*)

$$\Pi_1(U(1)^2) = \mathbf{Z} \times \mathbf{Z}$$

If confinement and XSB both induced by $\langle M^a_b \rangle = \delta^a_b \, \Lambda$

$$SU_L(N_F) \times SU_R(N_F) \to SU_V(N_F)$$

☞ Accidental SU(N_F^2) : too many NG bosons

- Non-Abelian monopole condensation ?

$$SU(3) \to SU(2) \times U(1) \to \mathbf{1}$$

☞ Problems (*) avoided but

$$\Pi_1(SU(2) \times U(1)) = \mathbf{Z}$$

Non-Abelian monopoles expected to be strongly coupled (no sign flip of b_0)

(difficulty for us, not for Nature...)

II. What $\mathcal{N}$=2 SQCD (softly broken by $\mu\Phi^2$) tell us

• Abelian dual superconductivity ✔

Seiberg, Witten

SU(2) with N_F = 0, 1,2,3

monopole condensation ⇒ confinement & dyn symm. breaking

Beautiful, but don't look like QCD

SU(N) $\mathcal{N}$ =2 SYM : SU(N) ⇒ $U(1)^{N-1}$

• Non-Abelian monopole condensation ✔

Argyres,Plesser,Seiberg,'96
Hanany-Oz, '96
Carlino-Konishi-Murayama '00
Shifman-Yung '10-'13

SU(N), N_F quarks

SU(N) ⇒ SU(r) x U(1) x U(1) x .... $r \le N_F/2$

r vacua are local, IR free theories

Beautiful, but don't look like QCD

• Non-Abelian monopoles interacting very strongly !

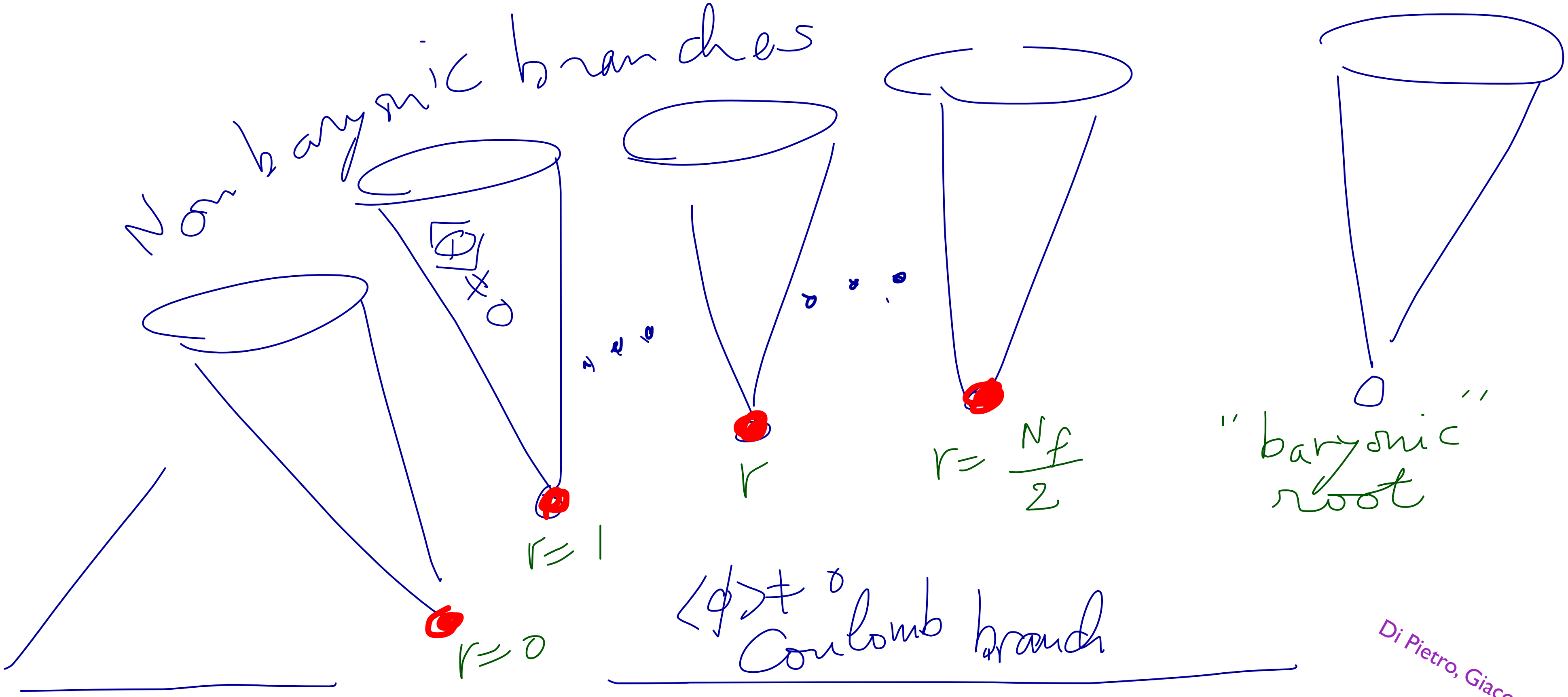
SCFT of highest criticality, EHIY points

Beautiful, interesting and difficult

Argyres,Plesser,Seiberg,Witten,
Eguchi-Hori-Ito-Yang, '96

QMS OF N=2 SU(N) SQCD, Nf quarks

Argyres,Plesser,Seiberg,'96



Di Pietro, Giacomelli, '11

$\mu \Phi^2$ pert. $\Rightarrow \begin{cases} 0 = \text{non confining} \\ \bullet = \text{Confining } r\text{-vacua} \end{cases}$

$$m \to m^{cr} = \frac{2N-N_f}{N} \Lambda \, \omega^k$$

$$\omega = e^{2\pi i / 2N-N_f}$$

w/ bare quark masses $m_i = m$

NEXT SLIDE

Back to p.25

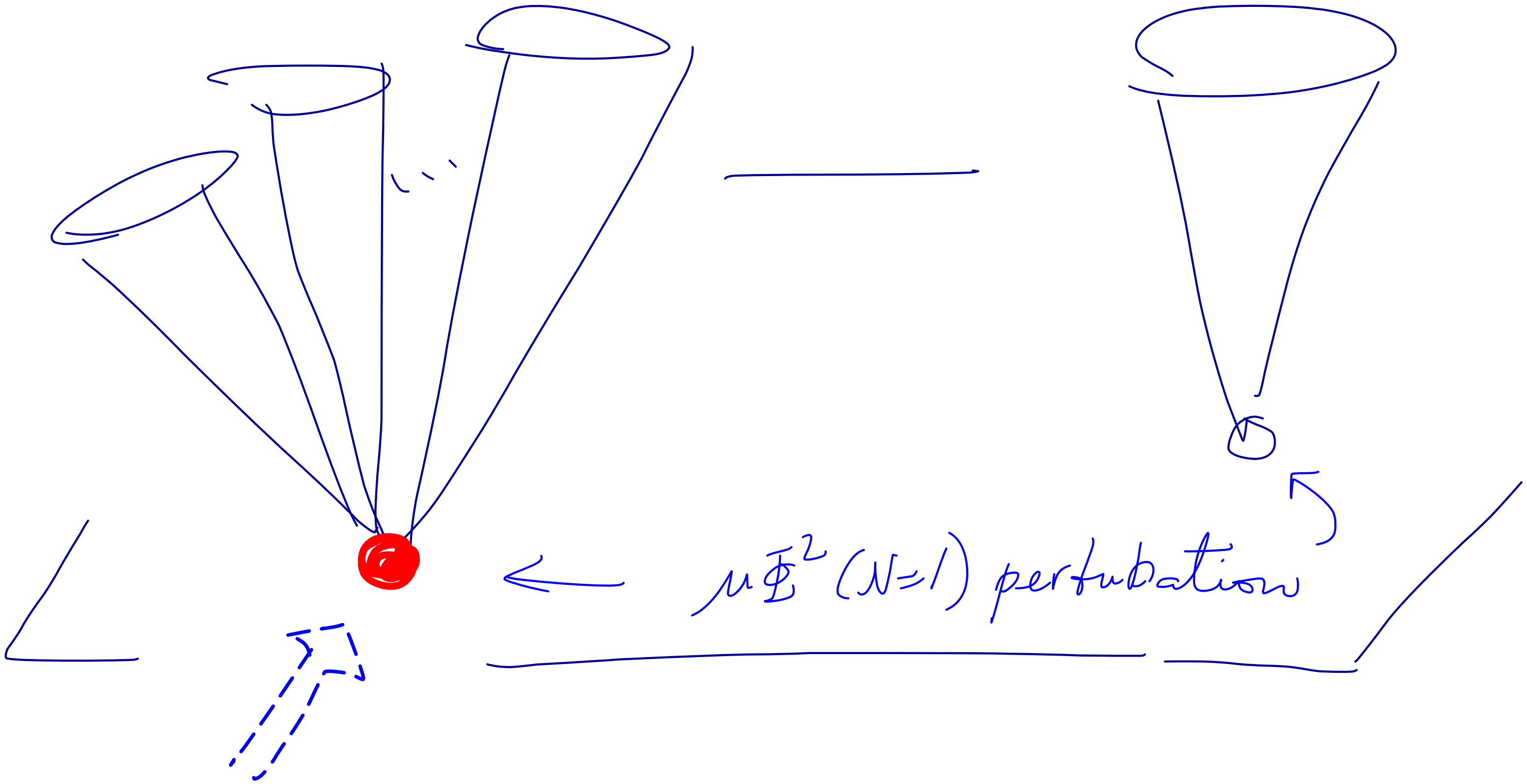
QMS of N=2, USp(2N) theory with Nf massless quarks

(or SO(N))

previous slide

$m \neq 0$

Eguchi-Hori-Ito-Yang



SCFT of highest criticality (EHIY point): non-Lagrangian theory

Our main interest

Note

● = N=1 confining vacua

○ = non confining

Back to p.15

Effective degrees of freedom in the quantum r vacua of softly broken N=2 SQCD ($r \le N_f / 2$)

Seiberg-Witten '94
Argyres,Plesser,Seiberg,'96
Hanany-Oz, '96

Carlino-Konishi-Murayama '00

Shifman-Yung '10-'13

	$SU(r)$	$U(1)_0$	$U(1)_1$	$\dots$	$U(1)_{N-r-1}$	$U(1)_B$
$N_F \times \mathcal{M}$	$\underline{\mathbf{r}}$	1	0	$\dots$	0	0
M_1	$\underline{\mathbf{1}}$	0	1	$\dots$	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
M_{N-r-1}	$\underline{\mathbf{1}}$	0	0	$\dots$	1	0

The massless non-Abelian and Abelian monopoles and their charges at the r vacua

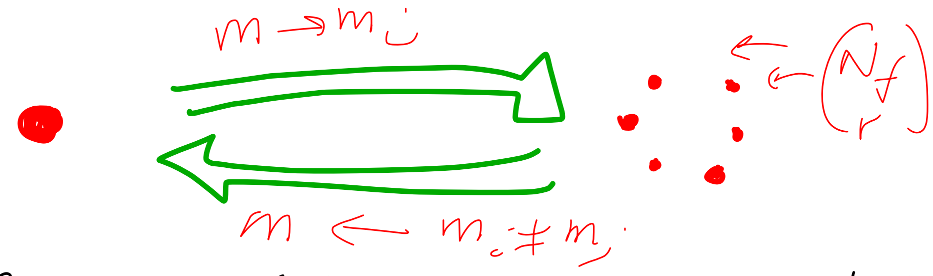
- "Colored dyons" do exist !!!
- they carry flavor q.n.

$\mu\Phi^2$ perturbation

- $\langle \mathcal{M}^i{}_\alpha \rangle = v\ \delta^i{}_\alpha \Rightarrow$ $U(N_f) \rightarrow U(r) \times U(N_f - r)$

Remarks

r - vacua



Local $SU(r) \times U(1)^{N-r}$ theory ⇄ Local $U(1)^{N-1}$ theory

massless nonAbelian monopoles | massless Abelian monopoles

- Flavor essential for generating the dual gauge symmetry (nonAbelian vortex and vortex-monopole complex)

- Monopoles in r also in N_F of flavor $SU(N_F)$

Multiple faces of flavor;
Solves the "too-many-Nambu-Goldstone bosons" puzzle

- THE LESSON:

  $SU(r) X U(1)^{N-r}$ contains richer physics (i.e., with larger degrees of freedom) than $U(1)^{N-1}$ effective theory

- Similarly, the singular SCFT enjoys larger degrees of freedom than component colliding r-vacua (the knowledge about the latter not sufficient)

III. Recent key developments

N=2 SCFT's

Gaiotto, Seiberg, Argyres, Tachikawa, Moore, Maruyoshi, ... '09 - '13

- S-duality for SCFT at $g = \infty$

Argyres-Seiberg '07

Gaiotto '09

- Argyres-Seiberg S-duality applied to the EHIY points

Gaiotto-Seiberg-Tachikawa '11

- GST duality for USp(2N), SO(N)

Giacomelli '12

- Colliding r-vacua and EHIY in SU(N)

Giacomelli, Di Pietro '11

Argyres-Seiberg's S duality

- SU(3) with N_F = 6 hypermultiplets ($Q_i, \tilde{Q}_i$ ' s) at infinite coupling

Minahan-Nemeschansky '96

$$SU(3)\ w/\ (6\cdot\mathbf{3}\oplus\bar{\mathbf{3}}) \quad = \quad SU(2)\ w/\ (2\cdot\mathbf{2}\oplus\mathrm{SCFT}_{E_6})$$

g = ∞   g = 0

$$SU(2)\times SU(6)\subset E_6$$

Flavor symmetry ~ SU(6) x U(1)

- USp(4) with N_F = 12 Q' s at infinite coupling

$$USp(4)\ w/\ 12\cdot\mathbf{4} = SU(2)\ w/\ SCFT_{E_7}$$

$$SU(2)\times SO(12)\subset E_7$$

SU(3) with $N_F = 6$

SCFT at
u=v=0

$$y^2 = (x^3 - ux - v)^2 - f(\tau)x^6,$$

$$y^2 = [(1 - \sqrt{f})\, x^3 - u\, x - v\,]\ [(1 + \sqrt{f})\, x^3 - u\, x - v\,]$$

$f \to 1$

means

$\tau \to 0$

or

$g \to \infty$

As $f \to 1$ the curve clearly degenerates to a genus one curve.

setting $u = 0$ $\quad y^2 = -v\ [\,2\, x^3 - v\,]\,, \qquad \partial\lambda/\partial v = dx\,/y$

With the redefinition

$$x \to -i\frac{x}{v}; \quad y \to 2\frac{y}{v}; \quad v \to 2iv,$$

we finally get

$$y^2 = x^3 - v^4; \quad \frac{\partial\lambda}{\partial v} = \frac{dx}{y},$$

E_6 Minahan-Nemeschansky theory

v=0 ⇨

$$y^2 = (x^2 - u)^2 - f x^4; \quad \frac{\partial\lambda_{SW}}{\partial u} = \frac{dx}{y},$$

But this is SCFT SU(2), $N_F = 4$ theory!

Gaiotto-Seiberg-Tachikawa (GST) (2011)

- Apply the basic idea of Argyres-Seiberg duality to the IR f.p. SCFT

- SU(N) with N_F = 2n :

$$y^2 = (x^N + u_1 x^{N-1} + u_2 x^{N-2} + \cdots + u_N)^2 - \Lambda^{2N-2n} \prod_{i=1}^{2n} (x + m_i)$$

At u = m=0,* $y^2 \sim x^{N+n}$ (EHIY point)

relatively non-local
massless monopoles and dyons

Note

* except for one u

Eguchi-Hori-Ito-Yang '96

- Straightforward treatment of fluctuations around u=m=0, gives incorrect scaling laws for the masses

❀ To keep the correct dimensions of masses, introduce two different scalings:

$$u_{N-n+2} \sim O(\epsilon_A^2), \quad u_{N-n+3} \sim O(\epsilon_A^3), \quad \ldots, \quad u_N \sim O(\epsilon_A^n).$$

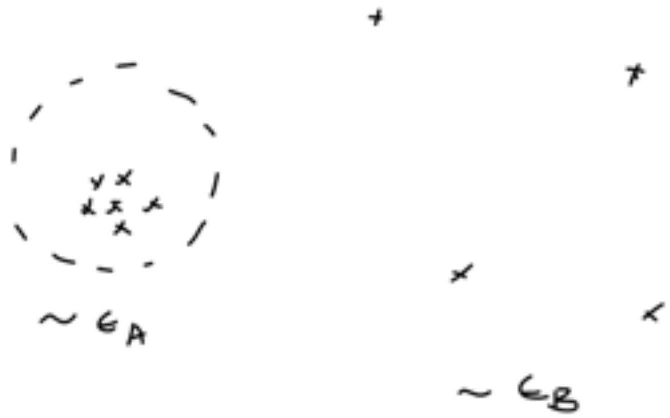
$$u_1 \sim O(\epsilon_B), \quad u_2 \sim O(\epsilon_B^2), \quad \ldots, \quad u_{N-n+2} \sim O(\epsilon_B^{N-n+2}).$$

$$\epsilon_A^2 = \epsilon_B^{N-n+2} \qquad \Longrightarrow$$

$$a_i = \oint_{\alpha_i} \lambda, \quad a_{D\,i} = \oint_{\beta_i} \lambda$$

$$\lambda \sim dx\, y/x^n$$

$$m_{(n_m,n_e,n_i)} = \sqrt{2}\,|n_m a_D + n_e a + n_i m_i|$$



$G_A \ll G_B$

Gaiotto-Seiberg-Tachikawa '11

- $U(1)^{N-n}$ gauge multiplets

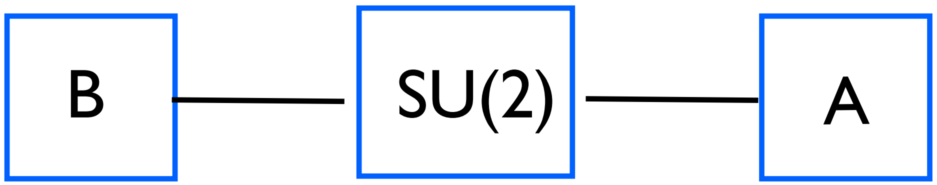
- SU(2) gauge multiplet (infrared free) coupled to the SU(2) flavor symmetry of the two SCFT's A & B

$$b_0 = \frac{N-n}{N-n+2}$$

- The A sector: the SCFT entering in the Argyres-Seiberg dual of SU(n), N_F =2n, having SU(2)xSU(2n) flavor symmetry

- The B sector: the maximally singular SCFT of the SU(N-n+1) theory with two flavors

Argyres-Plesser-Seiberg-Witten '95



where

A: 3 free <u>2</u>'s (n=2); E_6 of Minahan-Nemechansky (n=3), etc.

B: the maximally singular SCFT of SU(2), N_F =2 (Seinberg-Witten) for N=3, n=2 , etc.

- Analogous results for USp(2N), SO(N)

Giacomelli '12

IV. GST duals and confinement

USp(2N) theory w/ $N_F = 2n$

- Two types of Chebyshev* vacua ($\phi_1 = \phi_2 = \ldots = 0$; $\phi_n^{\,2} = \pm \Lambda^2$; ϕ_m det'd by Cheb. polynom.)

$$xy^2 \sim \left[x^n(x-\phi_n^2)\right]^2 - 4\Lambda^4 x^{2n} = x^{2n}(x-\phi_n^2-2\Lambda^2)(x-\phi_n^2+2\Lambda^2).$$

$y^2 \sim x^{2n}$ singular SCFT (EHIY point);
strongly interacting, relatively non-local monopoles and dyons

- A strategy: resolve the vacuum by adding small $m_i = m \neq 0$ (i.e., Vacua in confinement phase surviving N=1, $\mu\, \Phi^2$ perturbation)

Carlino-Konishi-Murayama '00

$$\Longrightarrow \quad \binom{N_f}{0} + \binom{N_f}{2} + \ldots \binom{N_f}{N_f} = 2^{N_f-1}$$

even r vacua, from one of the Chebyshev vacua

$$\binom{N_f}{1} + \binom{N_f}{3} + \ldots \binom{N_f}{N_f-1} = 2^{N_f-1}$$

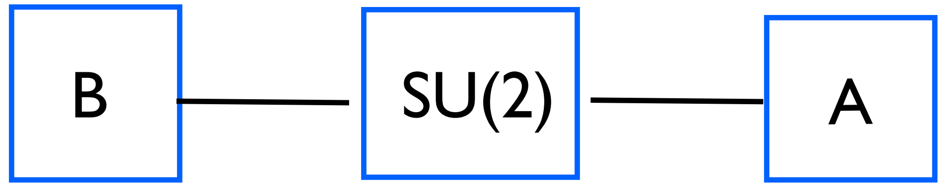
odd r vacua, from the other Chebyshev vacua

Figs.

GST dual for the Chebyshev point of USp(2N) (also SO(N))

Giacomelli '12

$$y^2 \sim x^{2n} \qquad \approx$$



- $U(1)^{N-n}$ gauge multiplets

- The A sector: a (in general) non-Lagrangian SCFT having SU(2)xSO(4n) flavor symmetry

- The B sector: a free doublet (coupled to U(1) gauge boson)

For N_F = 2n= 4, A sector ~ 4 free doublets

But this allows a direct description of IR physics !!

For $USp(2N), \; N_f = 4$

the GST dual is (both the A and B sectors are free doublets) :

Giacomelli, Konishi '12,'13

$$\boxed{1} - SU(2) - \boxed{4} \, .$$

U(1)

the effects of m_i and $\mu \, \Phi^2$ perturbation can be studied with the superpotential:

$$\sqrt{2}\, Q_0 A_D \tilde{Q}^0 + \sqrt{2}\, Q_0 \phi \tilde{Q}^0 + \sum_{i=1}^{4} \sqrt{2}\, Q_i \phi \tilde{Q}^i + \mu A_D \Lambda + \mu \, \mathrm{Tr}\phi^2 + \sum_{i=1}^{4} m_i \, Q_i \tilde{Q}^i \; ,$$

cfr. UV Lagrangian:

$$W = \mu \mathrm{Tr}\Phi^2 + \frac{1}{\sqrt{2}} Q_a^i \Phi_b^a Q_c^i \; J^{bc} + \frac{m_{ij}}{2} Q_a^i Q_b^j \; J^{ab}$$

$$m = -i\sigma_2 \otimes \mathrm{diag}\,(m_1, m_2, \ldots, m_{n_f})\, .$$

Correct flavor symmetry for all {m}

- $m_i = m$: SU(4) x U(1) ;
- $m_i = 0$: SO(8) ; etc.,

vacuum equations

$$\sqrt{2}\, Q_0 \tilde{Q}_0 + \mu\Lambda = 0 \;;$$

$$(\sqrt{2}\,\phi + A_D)\tilde{Q}_0 = Q_0\,(\sqrt{2}\,\phi + A_D) = 0 \;;$$

$$\sqrt{2}\;\left[\frac{1}{2}\sum_{i=1}^{4} Q_i^a \tilde{Q}_b^i - \frac{1}{4}\delta_b^a Q_i \tilde{Q}^i + \frac{1}{2} Q_0^a \tilde{Q}_b^0 - \frac{1}{4}\delta_b^a Q_0 \tilde{Q}^0\right] + \mu\,\phi_b^a = 0 \;;$$

$$(\sqrt{2}\,\phi + m_i)\,\tilde{Q}^i = Q_i\,(\sqrt{2}\,\phi + m_i) = 0, \qquad \forall i \;.$$

Solutions

$$Q_0 = \tilde{Q}_0 = \begin{pmatrix} 2^{-1/4}\sqrt{-\mu\Lambda} \\ 0 \end{pmatrix}$$

four solutions

$$a = -\frac{m_i}{\sqrt{2}}, \qquad Q_i = \tilde{Q}_i = \begin{pmatrix} f_i \\ 0 \end{pmatrix}; \qquad Q_j = \tilde{Q}_j = 0, \quad j \neq i.$$

four more solutions

$$a = +\frac{m_i}{\sqrt{2}}, \qquad Q_i = \tilde{Q}_i = \begin{pmatrix} 0 \\ g_i \end{pmatrix}; \qquad Q_j = \tilde{Q}_j = 0, \quad j \neq i.$$

$$f_i^2 = \frac{\mu\Lambda - 4\,a}{\sqrt{2}} = \mu(\frac{\Lambda}{\sqrt{2}} + 2m_i).$$

$$g_i^2 = \frac{-\mu\Lambda + 4\,a}{\sqrt{2}} = -\mu(\frac{\Lambda}{\sqrt{2}} - 2m_i).$$

They are 4 + 4 , r=1 vacua ! (p. -3)

But where are the even r-vacua (r=0,2) ???

Answer: in the second Chebyshev vacuum:

$$\sqrt{2}\,Q_0 A_D \tilde{Q}^0 + \sqrt{2}\,Q_0 \phi \tilde{Q}^0 + \sum_{i=1}^{4} \sqrt{2}\,Q_i \phi \tilde{Q}^i + \mu A_D \Lambda + \mu\,\mathrm{Tr}\phi^2 + \sum_{i=1}^{4} \tilde{m}_i\,Q_i \tilde{Q}^i \ ,$$

with

$$\tilde{m}_1 = \frac{1}{4}(m_1 + m_2 - m_3 - m_4) \ ;$$
$$\tilde{m}_2 = \frac{1}{4}(m_1 - m_2 + m_3 - m_4) \ ;$$
$$\tilde{m}_3 = \frac{1}{4}(m_1 - m_2 - m_3 + m_4) \ ;$$
$$\tilde{m}_4 = \frac{1}{4}(m_1 + m_2 + m_3 + m_4) \ .$$

Flavor symmetry OK in all cases:

m_i	$\tilde{m}_i$	Symmetry in UV	Symmetry in IR
$m_i = 0$	$\tilde{m}_i = 0$	$SO(8)$	$SO(8)$
$m_i = m \neq 0$	$\tilde{m}_4,\ \tilde{m}_1 = \tilde{m}_2 = \tilde{m}_3 = 0$	$U(1) \times SU(4)$	$U(1) \times SO(6)$
$m_1 = m_2,\ m_3,\ m_4$, generic	$\tilde{m}_2 = -\tilde{m}_3,\ \tilde{m}_4, \tilde{m}_1$ generic	$U(1) \times U(1) \times U(2)$	$U(1) \times U(1) \times U(2)$
$m_1 = m_2,\ m_3 = m_4,\ m_1 \neq m_3$	$\tilde{m}_2 = \tilde{m}_3 = 0,\ \tilde{m}_4,\ \tilde{m}_1$, generic	$SU(2) \times U(1) \times SU(2) \times U(1)$	$SO(4) \times U(1) \times U(1)$
$\ldots$	$\ldots$	$\ldots$	$\ldots$

Solutions similar to the previous case but: 1 + 1 + 6 in the $m_i \to m$

To recapitulate:

- Perturbation in m_i of the EHIY (SCFT) singularity $\Longrightarrow$
  the resolution of the Chebyshev vacua into the sum of the r-vacua
  (local Lagrangian theories with $SU(r)xU(1)^{N-r}$ gauge symmetry)

  Carlino-Konishi-Murayama '00

  and the consequent identification of the N=1 vacua surviving $\mu\,\Phi^2$ perturbation

  correctly reproduced by GST duals after $\mu\,\Phi^2$ perturbation.

• Physics was unclear in $m_i \to 0$ limit (i.e., the limit of colliding r-vacua) - strongly-coupled monopoles and dyons; nonlocal theory

☞

- Now the limit $m \to 0$ can be taken smoothly in the GST description

Physics of USp(2N), $N_F = 4$ theory at m=0 :

$$\sqrt{2}\, Q_0 A_D \tilde{Q}^0 + \sqrt{2}\, Q_0 \phi \tilde{Q}^0 + \sum_{i=1}^{4} \sqrt{2}\, Q_i \phi \tilde{Q}^i + \mu A_D \Lambda + \mu \,\mathrm{Tr} \phi^2 \ .$$

➜ $$Q_0 = \tilde{Q}_0 = \begin{pmatrix} 2^{-1/4}\sqrt{-\mu\Lambda} \\ 0 \end{pmatrix}$$

$$(Q_1)^1 = (\tilde{Q}^1)_1 = 2^{-1/4}\sqrt{\mu\Lambda}\ , \qquad Q_i = \tilde{Q}_i = 0, \quad i = 2, 3, 4.$$

$$\phi = 0, \quad A_D = 0\ .$$

➜ XSB

$$SO(8) \to U(1) \times SO(6) = U(1) \times SU(4) = U(4),$$

OK with results at $\mu, m \gg \Lambda$

➜ Confinement

UV: $\Pi_1(USp(2N)) = \mathbf{1}$

IR: $\mathcal{L}_{GST} = SU(2) \times U(1), \qquad \Pi_1(SU(2) \times U(1)) = \mathbf{Z}$

Higgsed at low energies; the vortex = the unique (N=n) confining string

Cfr. Abelianization implies $U(1)^N$ low energy theory

multiplication of the meson spectrum

- The confining string is Abelian cfr. non-Abelian vortex of r-vacua
- Confining order parameter (Q_0) triggers XSB order parameter (Q_i)

Some other systems studied

Giacomelli, Konishi
'13

- Colliding r-vacua of SU(3), N_f =4 : G_f = U(4) unbroken
- Singular r=2 -vacua of SU(4), N_f =4 : G_f = U(4) $\Rightarrow$ U(2)xU(2)
- SO(2N+1), N_f =1 : G_f = USp(2) =SU(2) $\Rightarrow$ U(1)
- SO(2N), N_f =2 : G_f = USp(4) $\Rightarrow$ U(2)

In all cases the GST description gives the correct number of the vacua and the symmetry breaking pattern known from the large μ analysis

SU(3) theory w/ $N_F = 4$: colliding r-vacua at $m \to m^{cr}$ Fig

GST dual is:

$$D_3 - SU(2) - \boxed{3}\,. \qquad (\&)$$

D_3 is an AD point of SU(2), $\tilde{N}_F = 2$ theory (at $u=m^2$; $m= \Lambda$):☐

difficult to analyze $\Longrightarrow$ replace the above GST system with

$$SU(2) \overset{P}{-} SU(2) - \boxed{3}\,. \qquad (\%)$$

where P is a bifundamental hyper. The left SU(2) is asymptotically free and becomes strong, and flows into D_3 . As (%) is a local theory, can minimize the superpotential to get the vacuum properties. Notes

- In the UV theory $m_i = m_j \to m^{cr}$ the collision of r=0,1,2 $\Rightarrow$ U(4) global symm; was difficult to analyse
- The IR theory (%) ~ (&) correctly reproduces all these.

Summary

- Certain degeneration limits in N=2 SQCD:

$U(1)^{N-1} \quad \Longrightarrow \quad SU(r) \times U(1)^{N-1} \quad \Longrightarrow \quad$ EHIY (IRFP SCFT)

#d.f < #d.f < #d.f

Abelian monopoles — nonAbelian monopoles — strongly coupled monopoles and dyons

- Gaiotto-Seiberg-Tachikawa duals allow to describe (confining) systems near singular IRFP SCFT

- In some simple cases (e.g., USp(2N), $N_F = 4$) the GST dual is a quiver like, local theory: can be, and have been analyzed in detail

- More general cases involve nonlocal (non Lagrangian) systems : small steps forward made

Some thoughts:

- Features different from the naive dual superconductor picture.

Confinement order parameter triggers the order parameter of the global symmetry breaking

- Sometimes gauge systems seem to generate dynamically richer IR effective systems, e.g. with a larger degrees of freedom, to prevent some "awkward" situation from occurring

- Ordinary (N=0) Q C D:

$U(1)^2$	or	$SU(2) \times U(1)$
#d.f	$<$	#d.f
Abelian monopoles		strongly-coupled nonAbelian monopoles

A new venue ?

The End

Non-Abelian monopoles

$$G \xrightarrow{\langle \phi \rangle \neq 0} H$$

$$SU(3) \xrightarrow{\langle \phi \rangle} \frac{SU(2) \times U(1)}{\mathbb{Z}_2}, \qquad \langle \phi \rangle = \begin{pmatrix} v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & -2v \end{pmatrix}$$

H: non-Abelian

Goddard-Nuyts-Olive, E.Weinberg, Lee,Yi, Bais, Schroer, ... '77-80

$$F_{ij} = \epsilon_{ijk} \frac{r_k}{r^3} (\beta \cdot \mathrm{T}), \qquad 2\,\beta \cdot \alpha \in \mathbf{Z}$$

cfr. (Dirac)

$2\, m \cdot e \in Z$

"Monopoles are multiplets of $\tilde{\mathrm{H}}$ (GNO)"

$\tilde{\mathrm{H}}$ generated by $\quad \alpha^* \equiv \dfrac{\alpha}{\alpha \cdot \alpha}.$

$$\langle \Phi \rangle = v_I = h \cdot T$$

H	$\tilde{\mathrm{H}}$
U(N)	U(N)
SU(N)	SU(N)/Z_N
SO(2N)	$\widetilde{\mathrm{SO(2N)}}$
SO(2N+1)	USp(2N)

$$A_i(\mathrm{r}) = A_i^a(\mathrm{r}, \mathrm{h} \cdot \alpha)\, S_a; \quad \phi(\mathrm{r}) = \chi^a(\mathrm{r}, \mathrm{h} \cdot \alpha)\, S_a + [\, \mathrm{h} - (\mathrm{h} \cdot \alpha)\, \alpha^*] \cdot \mathrm{T},$$

$$S_1 = \frac{1}{\sqrt{2\alpha^2}} (E_\alpha + E_{-\alpha}); \quad S_2 = -\frac{i}{\sqrt{2\alpha^2}} (E_\alpha - E_{-\alpha}); \quad S_3 = \alpha^* \cdot \mathrm{T},$$

Difficulties

① Topological obstructions (Abouelsaad et.al. '83)

Φ = diag(v,v,-2v)

e.g., SU(3) → SU(2)×U(1),

∄ monopoles ~ (2, 1*)

"No colored dyons exist" (Coleman, et.al. '84)

② Non-normalizable gauge zero modes: (Dorey, et.al. '96)

Monopoles are not multiplets of H

cfr.
Jackiw-Rebbi
Flavor Q.N. of monopoles
via
fermion zeromodes

The real issue:
how do they transform under $\tilde{H}$?

N.B. : H and $\tilde{H}$ relatively nonlocal

light flavors crucial

The phase: $\tilde{H}$ in conf. phase ⇔ H in higgs phase

Colliding r vacua of SU(3), N_F =4 theory

The GST dual is now:

$$D_3 - SU(2) - \boxed{3}$$

where D_3 is the most singular SCFT of the $\mathcal{N}=2$ SU(2), N_F =2, theory, and 3 is three free doublets of SU(2). D_3 is a nonlocal theory,* it not easy to analyze.

We replace the system by

$$SU(2) \overset{P}{-} SU(2) - \boxed{3}$$

* arises from the collision of a doublet vacuum and a singlet vacuum

where P is a bifundamental field. The superpotential is:

$$\sum_{i=1}^{3} \sqrt{2} Q_i \Phi \tilde{Q}^i + \sum_{i=1}^{3} \tilde{m}_i \, Q_i \tilde{Q}^i + \mu \Phi^2 + \sqrt{2} P \Phi \tilde{P} + \sqrt{2} \tilde{P} \chi P + \mu \, \chi^2 + m' \, \tilde{P} P,$$

The first SU(2) is AF: its dynamics is not affected by the second SU(2). But to exract the D_3 point, need to keep $m' \simeq \pm \Lambda'$, but not exactly equal. The system Abelianizes →

Doublet vacuum (of the new, strong SU(2) N_F =2 theory)

$$\sum_{i=1}^{3}\sqrt{2}Q_i\Phi\tilde{Q}^i+\sum_{i=1}^{3}\tilde{m}_i\,Q_i\tilde{Q}^i+\mu\Phi^2+\sqrt{2}M\Phi\tilde{M}+\sqrt{2}\tilde{M}A_\chi M+\mu\,A_\chi\Lambda^{'},$$

with m

$$\begin{aligned}\tilde{m}_1 &= \frac{1}{4}(m_1+m_2-m_3-m_4)\ ;\\ \tilde{m}_2 &= \frac{1}{4}(m_1-m_2+m_3-m_4)\ ;\\ \tilde{m}_3 &= \frac{1}{4}(m_1-m_2-m_3+m_4)\ ,\end{aligned}$$

→ correct symmetry

for all m_i :

→ six solutions (r=2 vacua)

SU(3), N_F =4 theory has r=0,1,2 vacua: where are the r=0,1 vacua? Answer:

Singlet vacuum (of the new, strong SU(2) N_F =2 theory)

$$\sum_{i=1}^{3}\sqrt{2}Q_i\Phi\tilde{Q}^i+\sum_{i=1}^{3}\tilde{m}_i\,Q_i\tilde{Q}^i+\mu\Phi^2+\sqrt{2}\tilde{N}AN+\mu\,A\Lambda^{'}+m^{'}\,\tilde{N}N.$$

AF: becomes strongly coupled. → 4 + 1 vacua of

SW SU(2) N_F =3 theory! → r=1 (4) and r=0 (1) vacua

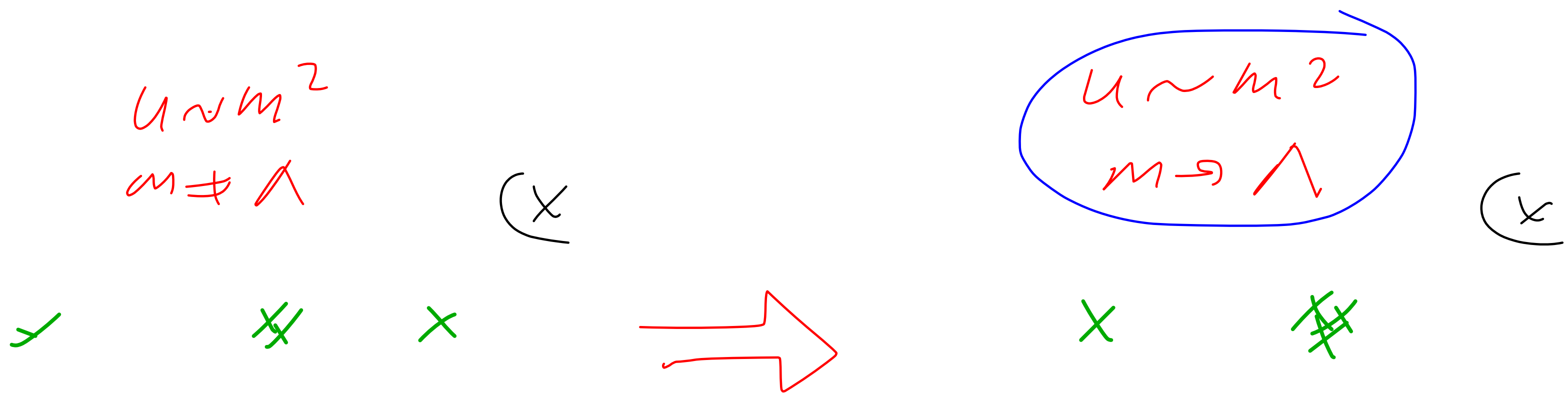
Vacuum structure OK

D_3 = AD point of $SU(2)$, $N_f = 2$,
realized at $m = \Lambda$

SW curve: $y^2 = (x^2 - u)^2 - 4\Lambda^2 (x+m)^2$

$$u = m^2 \Rightarrow y^2 = (x+m)^2 (x - m + 2\Lambda)(x - m - 2\Lambda)$$

$$\xrightarrow[m \to \Lambda]{} (x+m)^3 \quad \text{(AD !)}$$



* Remarks: fate of N=2 SCFT

Tachikawa, Wecht '09

- N=2 SCFT's in UV flow into N=1 SCFT, upon N=1, $\mu\,\Phi^2$ perturbation (27/32)

- Some of them survive and brought into confinement phase; r-vacua, (r=0,1,2,...) upon N=1, $\mu\,\Phi^2$ perturbation

- Not all N=2 singular SCFT's survives N=1, $\mu\,\Phi^2$ perturbation (e.g., Argyres-Douglas point in pure SU(3))

- N=2 IRFP SCFT's can survive and brought into confinement phase; e.g., Colliding r-vacua of SU(N) theories, m=0, USp(2N) theory (Chebyshev vacua); m=0, SO(N) theory

r-vacua

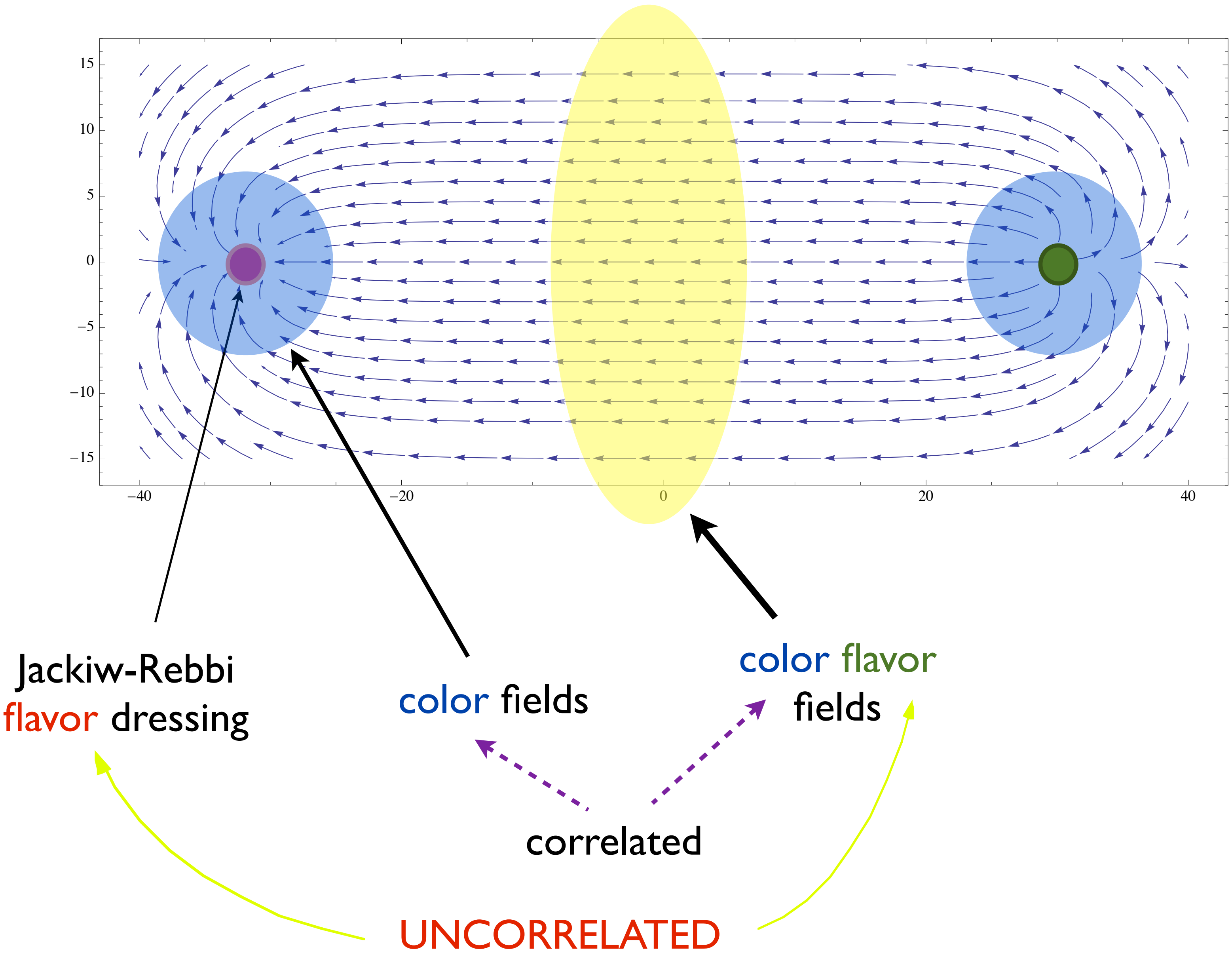
$$y^2 = \prod_a^N (x-\phi_a)^2 - \Lambda^{2N-2n} \prod_{i=1}^{2n} (x+m_i)$$

$$\phi_a = (-m_1, -m_2, \ldots, -m_r, \phi_{r+1}, \ldots)$$

$m_i \to m,$

$$y^2 = (x+m)^{2r} \prod_{b=1}^{N-r} (x-\alpha_b)^2 \, (x-\gamma)(x-\delta)$$

This describes SU(r) x U(1)$^{N-r}$ theory


15
10
5
0
−5
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40
Jackiw-Rebbi
flavor dressing
color fields
color flavor
fields
correlated
UNCORRELATED

SU(3) with $N_F = 6$

SCFT at
u=v=0

$$y^2 = (x^3 - ux - v)^2 - f(\tau)x^6,$$

f $\rightarrow$ 1 $\Rightarrow$ the curve degenerates into a genus one curve:

f $\rightarrow$ 1
means
$\tau \rightarrow 0$
or
$g \rightarrow \infty$

u=0 $\Rightarrow$

$$y^2 = -v(2x^3 - v); \quad \frac{\partial\lambda}{\partial v} = \frac{dx}{y}.$$

With the redefinition

$$x \to -i\frac{x}{v}; \quad y \to 2\frac{y}{v}; \quad v \to 2iv,$$

we finally get

$$y^2 = x^3 - v^4; \quad \frac{\partial\lambda}{\partial v} = \frac{dx}{y},$$

E_6 Minahan-Nemeschansky theory

v=0 $\Rightarrow$

$$y^2 = (x^2 - u)^2 - fx^4; \quad \frac{\partial\lambda_{SW}}{\partial u} = \frac{dx}{y},$$

But this is SCFT SU(2), $N_F = 4$ theory!