Monopoles & instantons in confining gauge theories on R^3 X S^1 with Higgs

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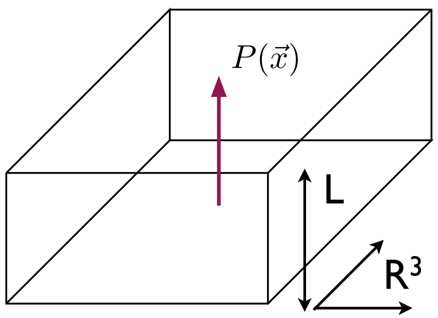


Collaboration with M. Ogilvie

Outline

- Introduction

  - Deformed Yang-Mills with adjoint scalar fields on R3 X S1

  - Small-L confined phase by deformations

- Semiclassical evaluations at small L

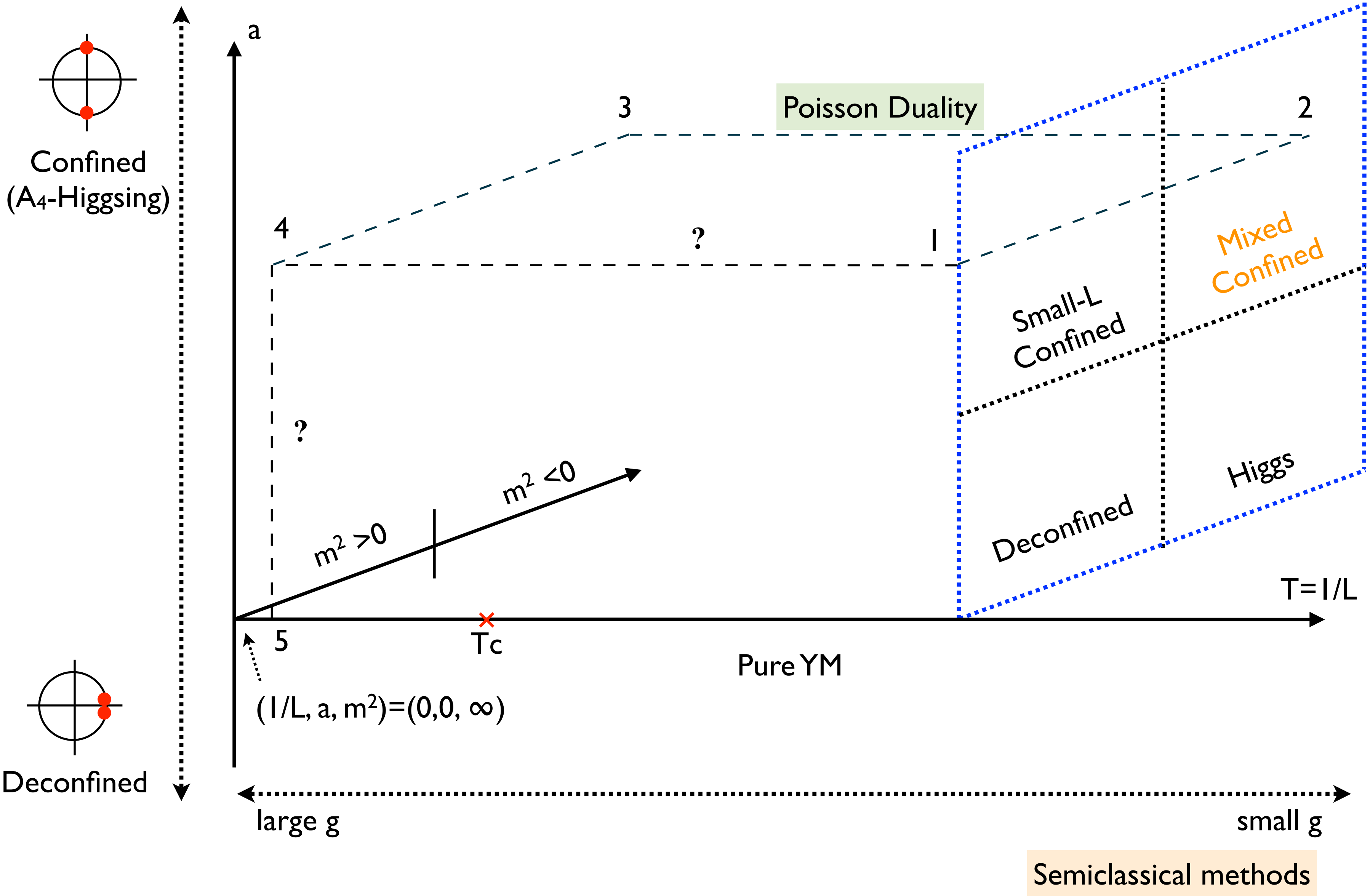
  - Effective potential of the Polyakov loop from perturbation theory

  - Topological contents

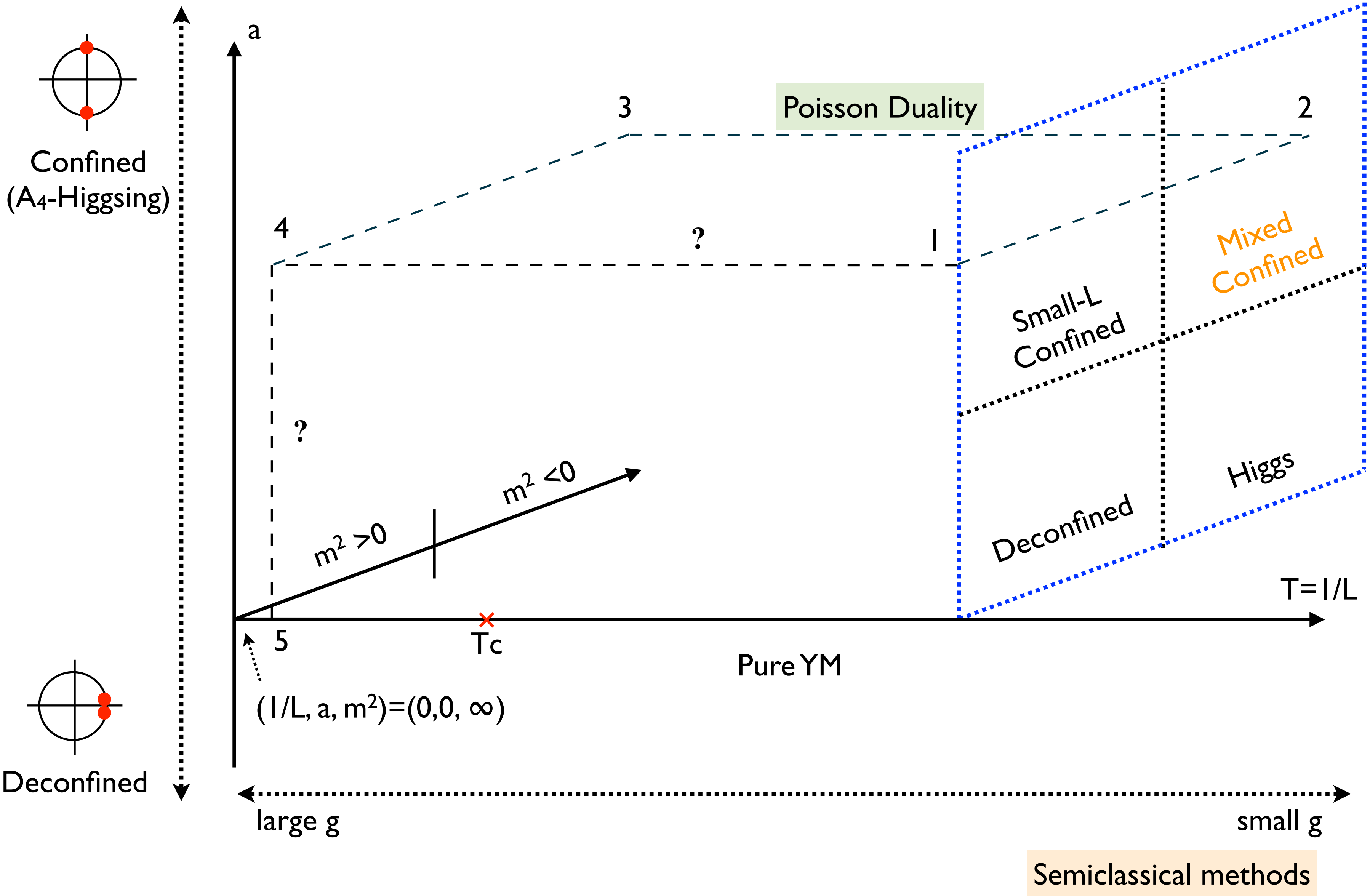
- Poisson resummation

- Conclusions

Deformed Yang-Mills with adjoint Higgs



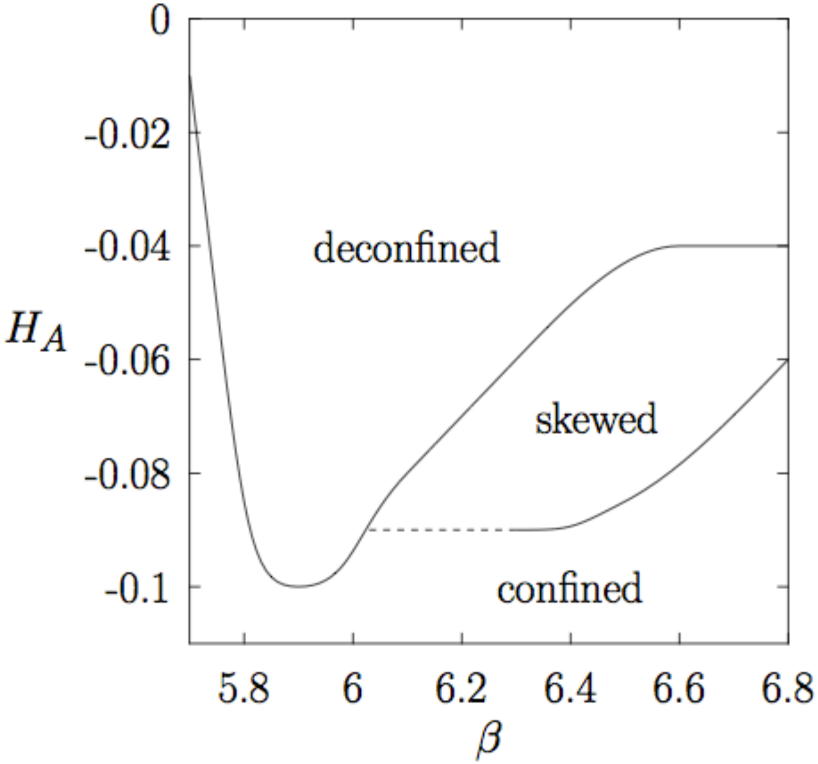
Deformed Yang-Mills with adjoint Higgs



Deformations

Lattice

<J. Myers and M. Ogilvie, PRD77, 2008>



$$S \to S - \int \frac{d^3x}{L^3} H_A \left|TrP\right|^2$$

- Small-L confined phase for SU(N)

- Rich phase diagrams: Partially confined phases

<M. Ogilvie, P Meisinger, J. Myers, Proc, Sci. LAT2007>

- Mass gap and string tension by A4 higgsing

- Large-N volume reduction (Eguchi-Kawai)

<M. Ünsal and L.Yaffe, PRD78, 2008>

<and many others>

Other "deformations": adjoint fermions

- QCD-like: adjoint fermions with periodic boundary conditions on S^1
  - One-loop argument (mass small & $1/2 < N_f < N_f^*$) <P. Kovtun, M. Ünsal and L. Yaffe, JHEP0706, 2007>

$$V_{eff}(P) = \frac{2}{L} Tr_A \int \frac{d^3k}{(2\pi)^3} \ln(1 - Pe^{-Lk}) - 4N_f\, Tr_A[\frac{1}{L} \int \frac{d^3k}{(2\pi)^3} \ln(1 - Pe^{-L\sqrt{k^2+m^2}})]$$

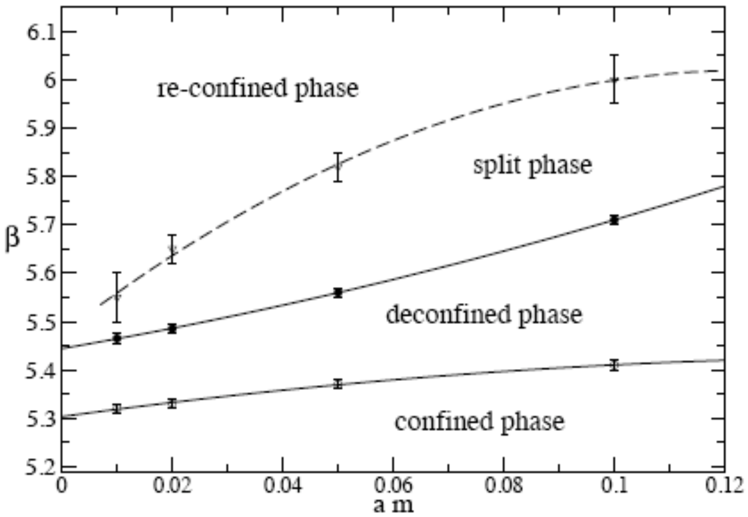
  - Rich phase diagrams for larger N

<J. Myers and M. Ogilvie, JHEP0907, 2009>

- Phase diagram of N_f=2 SU(3)

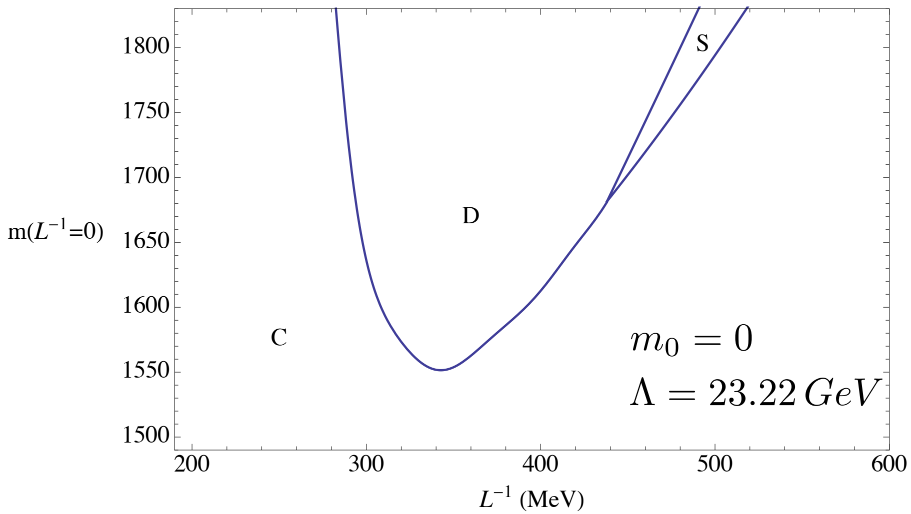
Lattice

<G. Cossu and M. D'Elia, JHEP0907, 2009>



PNJL

<HN and M. Ogilvie, PRD81, 2010>



Other "deformations": adjoint fermions

- Topological molecules (bions) <M. Ünsal, 2007>

  - New mechanism for confinement <M. Ünsal, PRL100 2008 PRD80 2009> <M. Shifman and M. Ünsal, PRD78 2008>
  <E. Poppitz, T. Schäfer, M. Ünsal, JHEP09 2009, JHEP12 2009>

  - Confinement vs conformality <E. Poppitz and M. Ünsal, JHEP09 2009, JHEP12 2009>

  - Renormalons and resurgence <P. Argyres and M. Ünsal, PRL109, 2012, JHEP08, 2012>
  <G. Dunne and M. Ünsal, JHEP11 2012 2012,PRD87 2013>

  - Much interesting work done by the speakers, Dunne, Shifman, Shuryak, Poppitz, Ünsal, and many others

- <u>In this talk, we treat the center-stabalizing potentials as deformations to the action.</u>

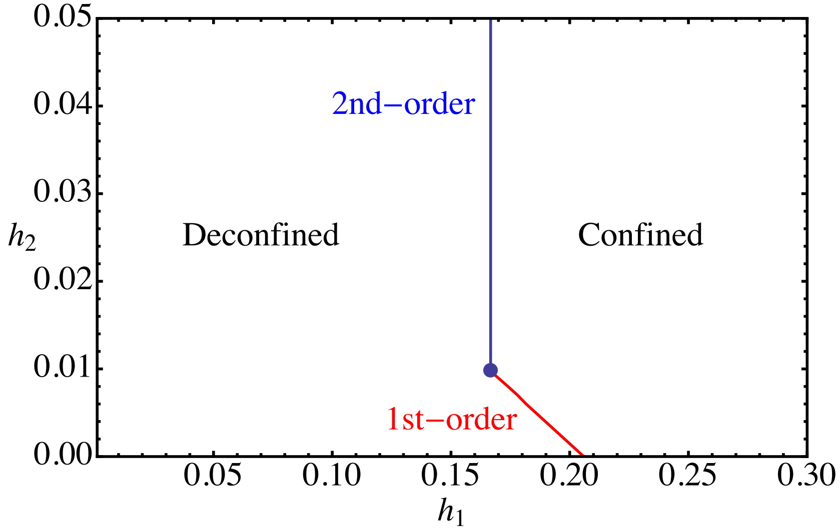
  - No dynamical fermions → it is easier to implement on the lattice.

  - No fermionic zero modes → topological objects are simpler.

More Deformations for SU(2)

Generalized double-trace deformation

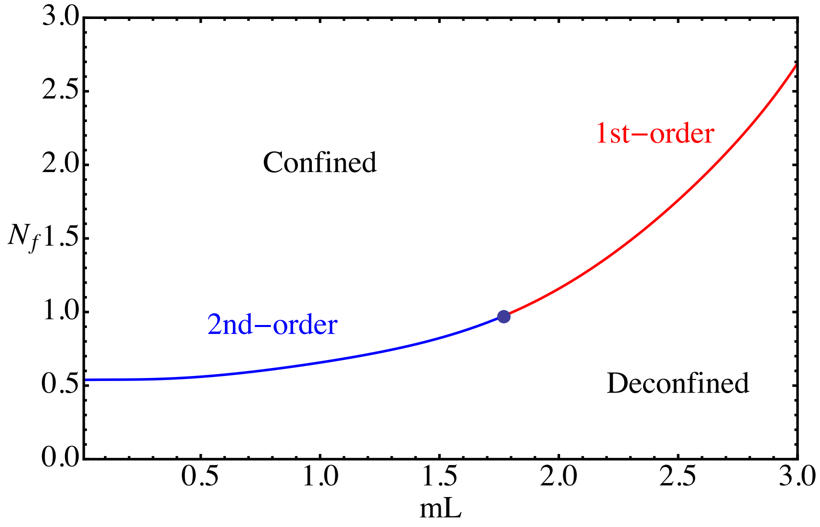
$$V_d = \left[h_1 \left(trP\right)^2 + h_2 \left(trP\right)^4\right] / L^4$$



2d Adjoint Fermions embedded in 4d

$$V_d = \frac{2mLN_f N_4^2}{\pi L^4} \sum_{n=1}^{\infty} \frac{K_1(nmL)\, Tr_A P^n}{n}$$

$$\xrightarrow{m \to 0} \boxed{\frac{4N_f N_4^2}{\pi L^4} \left(\theta - \pi/2\right)^2}$$



- The deformations can induce a 1st- or 2nd-order phase transition.
- Will work in 2nd-order region to simplify analysis of the phase diagram.

Higgs vs Confinement at small L

<HN and M. Ogilvie, PRD85, 2012>

- Symmetries and order parameters

$$L = \left[\frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{1}{2}\left(D_\mu\phi\right)^T \cdot D_\mu\phi + V\left(\phi\right) + V_d\right] \quad \text{where} \quad \begin{cases} V\left(\phi\right) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\left(\phi^2\right)^2 \\ V_d = \left(1+a\right)\left(\theta - \pi/2\right)^2/L^4 \end{cases}$$

The action has $Z(2)_C$ X $Z(2)_H$ symmetry:

- Center Symmetry $Z(2)_C$: P → - P
- Reflection Symmetry $Z(2)_H$: ϕ → - ϕ

(like a two-spin model)

Gauge-Invariant Order Parameters

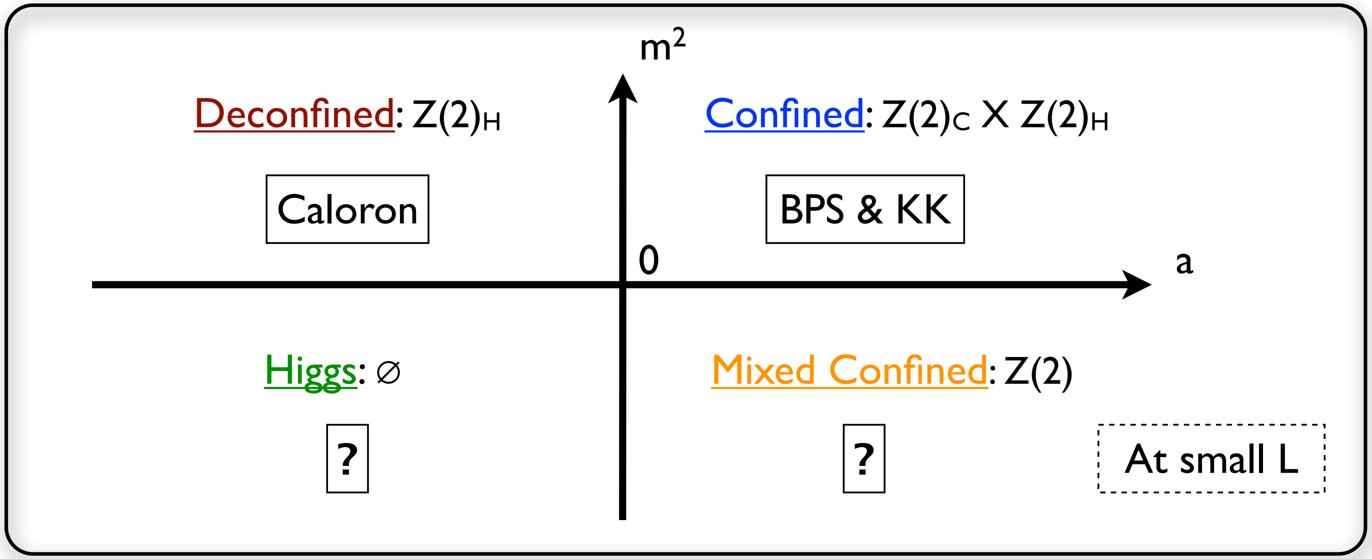
- <Tr P>: Transforms under $Z(2)_C$
- <Tr P^2ϕ >: Transforms under $Z(2)_H$
- <Tr Pϕ >: Transforms under $Z(2)_C$ X $Z(2)_H$

- Effective potential by the background field method.

Background fields: $\bar{A}_4 = \begin{pmatrix} w & 0 \\ 0 & -w \end{pmatrix} \quad \bar{\phi} = \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix} \quad \text{where} \quad w = 2\theta/gL$

$$U = \underbrace{\frac{1}{2}m^2(L)\,v^2 + \frac{1}{4}\lambda\left(L\right)v^4}_{\text{0-loop}} + \underbrace{\frac{2}{\pi^2L^4}\left[\left(\theta - \frac{\pi}{2}\right)^4 + \frac{\pi^2 a}{2}\left(\theta - \frac{\pi}{2}\right)^2\right] + O(L^{-2})}_{\text{1-loop}}$$

	Residual Symmetry	Phases	<TrP>	<TrP²ϕ>	<TrPϕ>	Parameters
✓	$Z(2)_C \times Z(2)_H$	Confined	0	0	0	$a > a_c, m^2 > 0$
✓	$Z(2)_H$	Deconfined	≠0	0	0	$a < a_c, m^2 > 0$
✓	∅	Higgs	≠0	≠0	≠0	$a < a_c, m^2 < 0$
✓	$Z(2)$	Mixed Confined	0	0	≠0	$a > a_c, m^2 < 0$
×	$Z(2)_C$	Higgs & Confined	0	≠0	0	N / A



- The Higgs and confined phases are not compatible. (Consistent with the ‘t Hooft’s argument.)
- A new phase: mixed confined phase.

Topological contents

- Two basic monopole solutions

$$L_E = \frac{1}{2}(B_i)^2 + \frac{1}{2}(D_i A_4)^2 + \frac{1}{2}(D_i \phi)^2 \longrightarrow \boxed{\begin{aligned} S_{BPS} &= \frac{4\pi}{g^2}\sqrt{4\theta^2 + g^2 L^2 v^2} \\ S_{KK} &= \frac{4\pi}{g^2}\sqrt{(2\pi - 2\theta)^2 + g^2 L^2 v^2} \end{aligned}}$$

Two Higgs (A_4 and ϕ) in the Lagrangian

Non-perturbative physics
supports mixed phase interpretation.

- Reproduce previous models

★ Unsal and Yaffe: no scalars (set v=0)

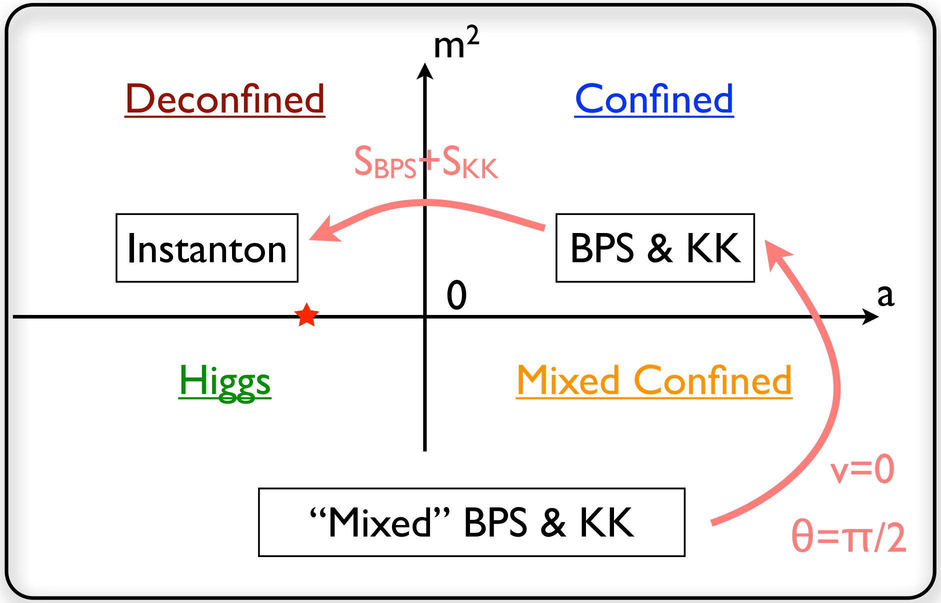
<M. Ünsal and L. Yaffe, PRD78, 2008>

$$S_{BPS} = S_{KK} = \frac{4\pi^2}{g^2}$$

BPS and KK are constituents of instantons.

<T. Kraan and P. van Baal, NPB533, 1998>

<K. Lee and C.-h Lu, PRD58, 1998>



Abelian duality

- Mass gap and 3D string tension from the Abelian duality.

<A. Polyakov, NPB120, 1977>
<M. Ünsal and L. Yaffe, PRD78, 2008>

<u>Dual Scalar Theory</u>

$$\int_{R^3} \left(\frac{1}{2Lq_M^2} (\partial_i \sigma)^2 - \sum_a \xi_a e^{iq_a \sigma} \right)$$

$$*d\sigma = \frac{Lq^2}{2\pi} F$$

<u>Effective U(1) Theory</u>

$$\int_{R^3} \frac{1}{4} (F_{ij})^2 + \text{Monopole gas}$$

(Distance >> M_W^{-1})

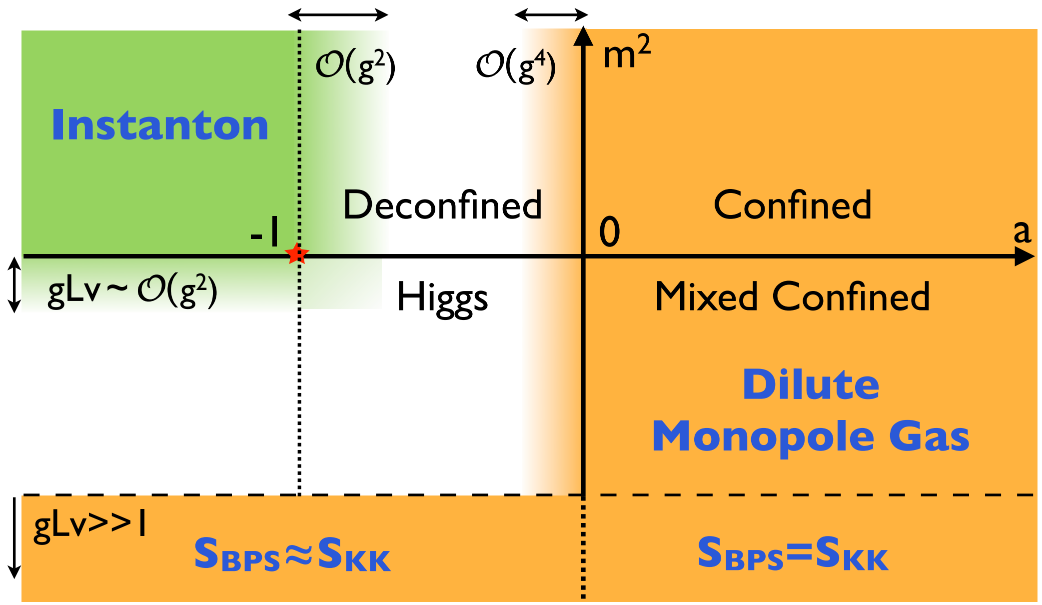
<u>Mass Gap:</u> $L \frac{16\pi^2}{g^2} \sum_a \xi_a$ <u>3d String Tension:</u> $\frac{2g}{\pi} \sqrt{\frac{1}{L} \sum_a \xi_a}$

$$\xi_a = c\mu^{7/2} (NL)^{1/2} S_a^2 e^{-S_a}$$

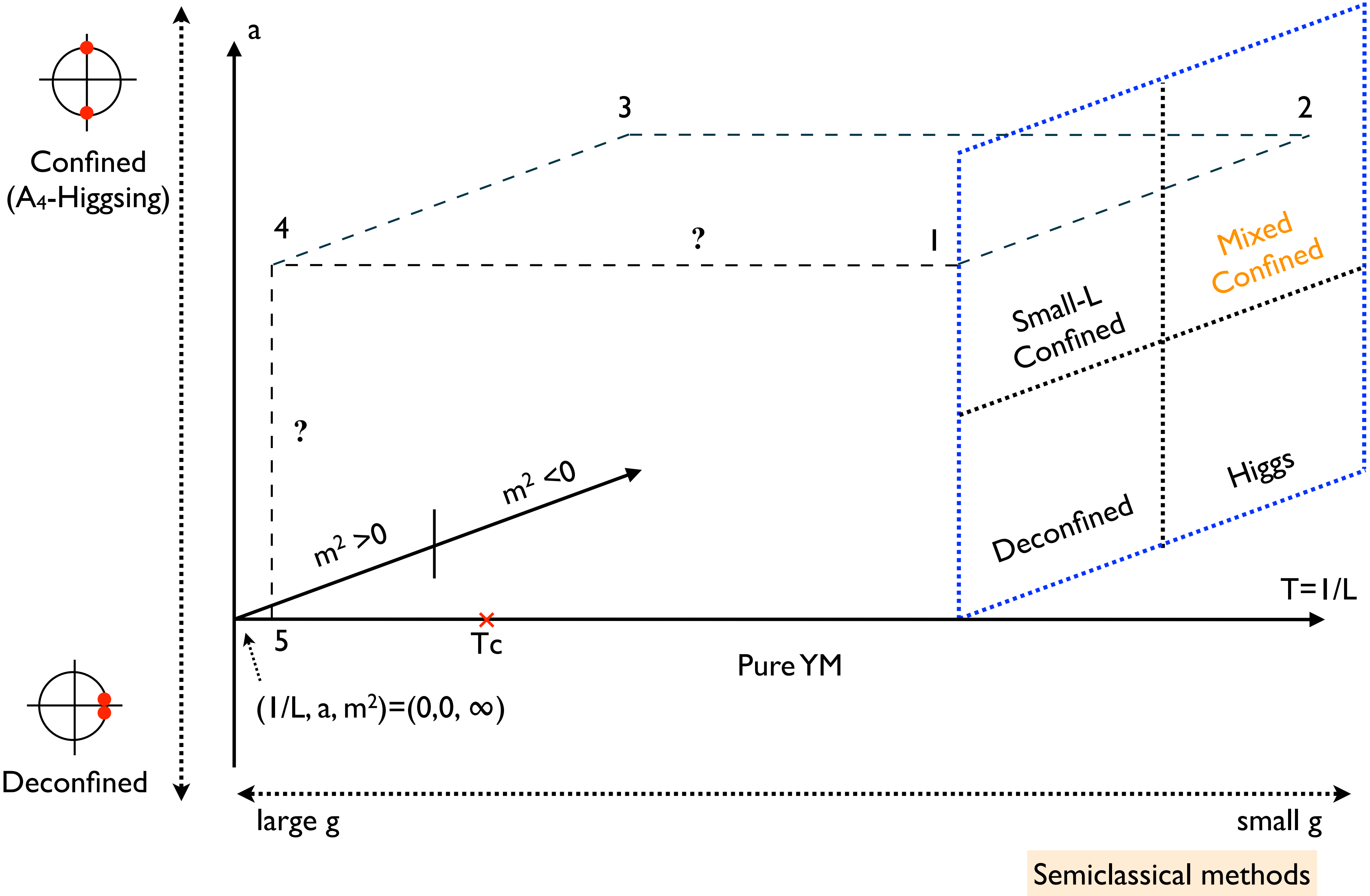
- Validity of gas approximation

- Dilute monopole-gas approximation is valid in the orange region where $e^{-S} << 1$.

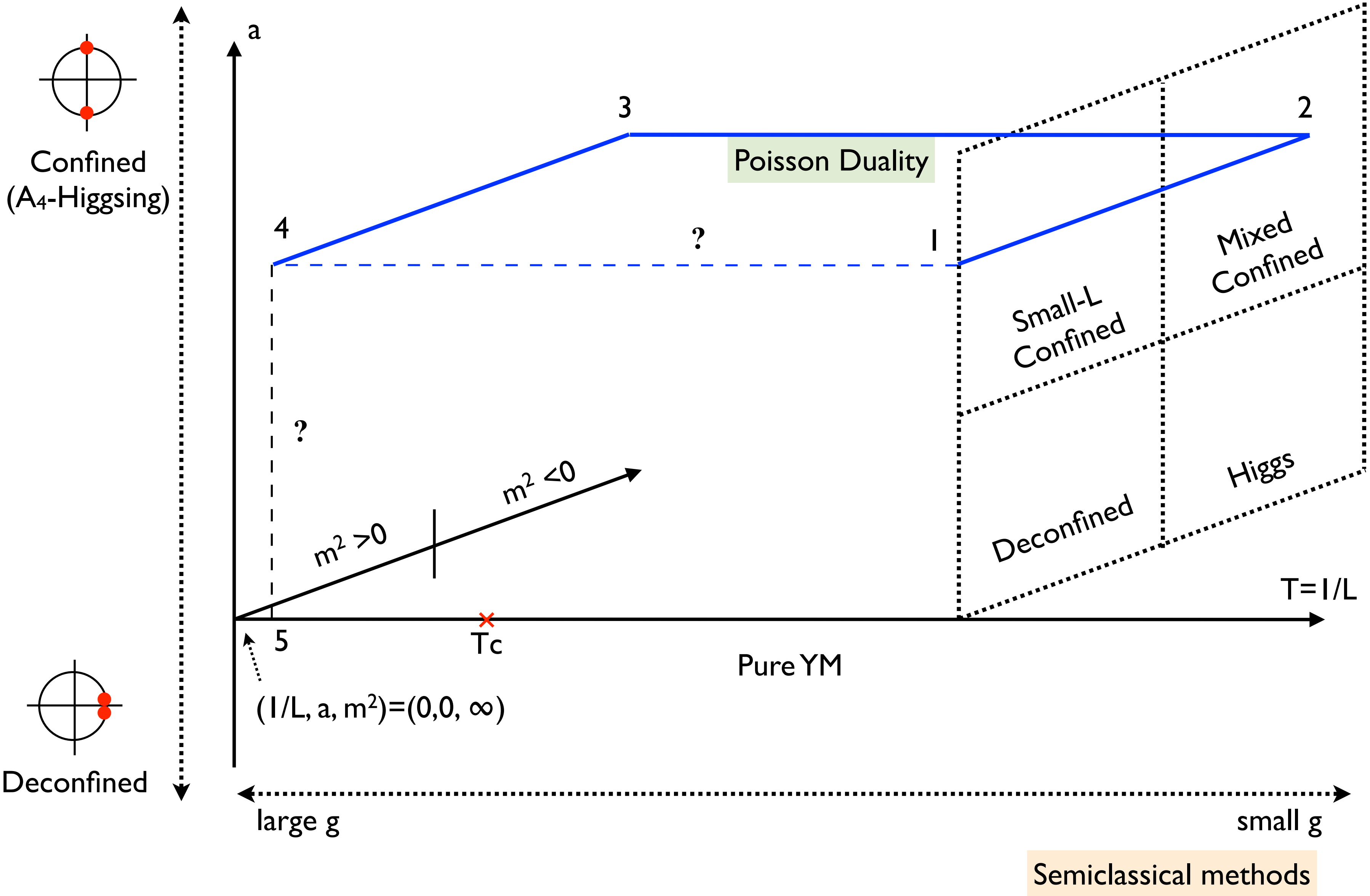
- String tension and mass gap can be computed analytically in portions of all phases.



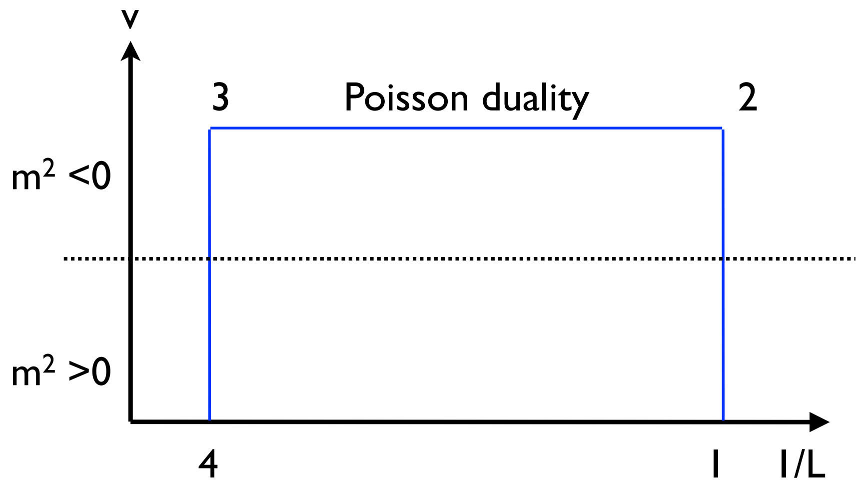
Deformed Yang-Mills with adjoint Higgs



Deformed Yang-Mills with adjoint Higgs

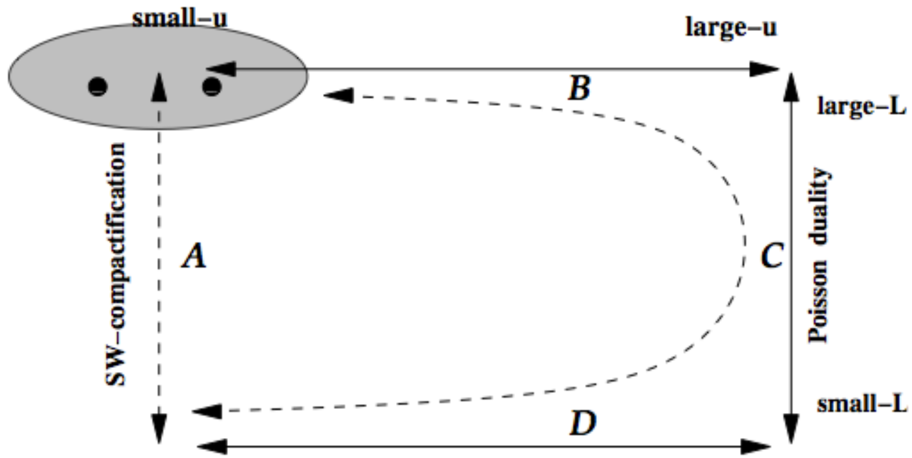


Poisson duality



Deformed SU(2) with adjoint Higgs
on $R^3 \times S^1$

Mass deformed Seiberg-Witten theory
on $R^3 \times S^1$



<E Poppitz and M. Ünsal, JHEP,07 2011>

- Long range force is by sigma only because:

  - Not strictly in BPS limit:

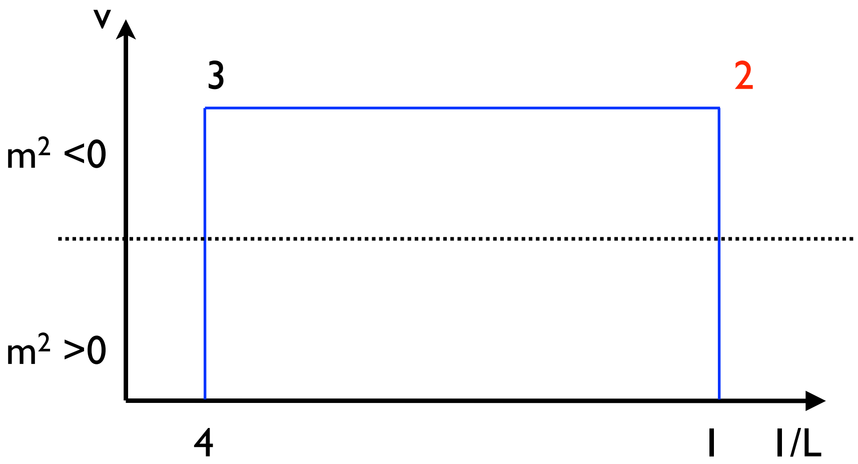
    → no Φ force $\quad S = S_{BPS}C(\sqrt{\lambda}/g) \quad \text{where} \quad 1 < C < 1.787$

  - The deformation is large so that mass for the Polyakov loop is nonzero:

    → no A_4 force

$$S_{eff} = \int_{R^3} \left(\frac{1}{2Lq_M^2} (\partial_i \sigma)^2 + V_{np} \right)$$

- We now compute V_{np} at 2 (in the mixed confined phase)



$$\bar{A}_4 = \begin{pmatrix} w & 0 \\ 0 & -w \end{pmatrix} \qquad \text{where} \quad w = 2\theta/gL$$

$$\bar{\phi} = \begin{pmatrix} v & 0 \\ 0 & -v \end{pmatrix}$$

At 2 ($L\Lambda$<<1): following Poppitz and Ünsal <E Poppitz and M. Ünsal, JHEP,07 2011>

$$V_{np,2} \quad = \quad -\sum_{n_m=\pm 1}\sum_{n_\omega\in\mathbb{Z}} \xi e^{-\frac{4\pi}{g^2}|n_m|\sqrt{(2\pi n_\omega+\omega)^2+g^2L^2v^2}} e^{in_m\sigma}$$

- Only n_m =1 and -1 contribute because magnetic monopoles with higher charges are heavier.
- They add infinite sum of Kaluza-Klein tower to make the potential periodic in ω.

  (n_ω = 0 and -1 correspond to the BPS and KK monopoles, respectively)

$$V_{np,2} = -\sum_{n_m=\pm 1}\sum_{n_e\in\mathbb{Z}} \frac{2Lv^2\xi}{M\left(n_m,n_e\right)} K_1(LM\left(n_m,n_e\right)) e^{in_m\sigma+i2n_e\theta}$$

$$\text{where} \quad M\left(n_m,n_e\right) = v\sqrt{\left(\frac{4\pi}{g}n_m\right)^2+\left(gn_e\right)^2}$$

- Electric charge, n_e and winding number, n_ω, are dual variables.
- $M(n_m, n_e)$ is the mass of the Julia-Zee dyon.

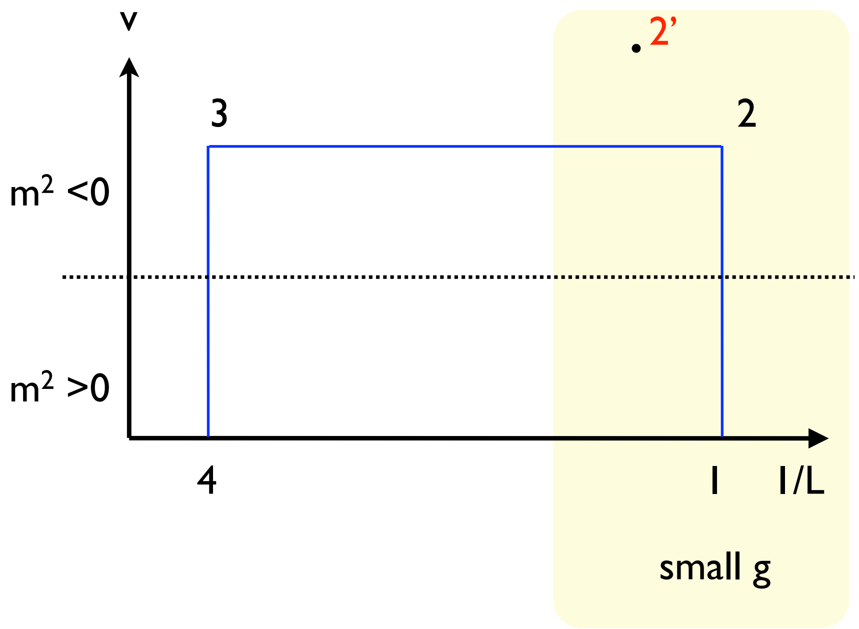
At 2 (LΛ<<1): another approach <HN and M. Ogilvie, PRD85, 2012>

Using $\sum_{k=0}^{N-1} f(\theta - \frac{2\pi k}{N}) = \sum_{n_e \in \mathbb{Z}} N\tilde{f}(Nn_e)e^{in_e N\theta}$, and setting the potential to be periodic, we have

$$\left(\xi(\theta)e^{-S_{BPS}} + \xi(\pi-\theta)e^{-S_{KK}}\right)e^{in_m\sigma} = -\sum_{n_m=\pm1}\sum_{n_\omega=0,-1}\xi e^{-\frac{4\pi}{g^2}|n_m|\sqrt{(2\pi n_\omega+\omega)^2+g^2L^2v^2}}e^{in_m\sigma}$$

$$\underset{g<<1}{\simeq} -\sum_{n_m=\pm1}\sum_{n_e\in\mathbb{Z}}\frac{2Lv^2\xi}{M(n_m,n_e)}K_1(LM(n_m,n_e))e^{in_m\sigma+i2n_e\theta}$$

- At small g, the most contribution of the Kaluza-Klein tower comes from BPS and KK (n_ω=0 and 1).



$$S_{eff} = \int_{R^3} \left(\frac{1}{2Lq_M^2} \left(\partial_i \sigma\right)^2 + V_{np} \right)$$

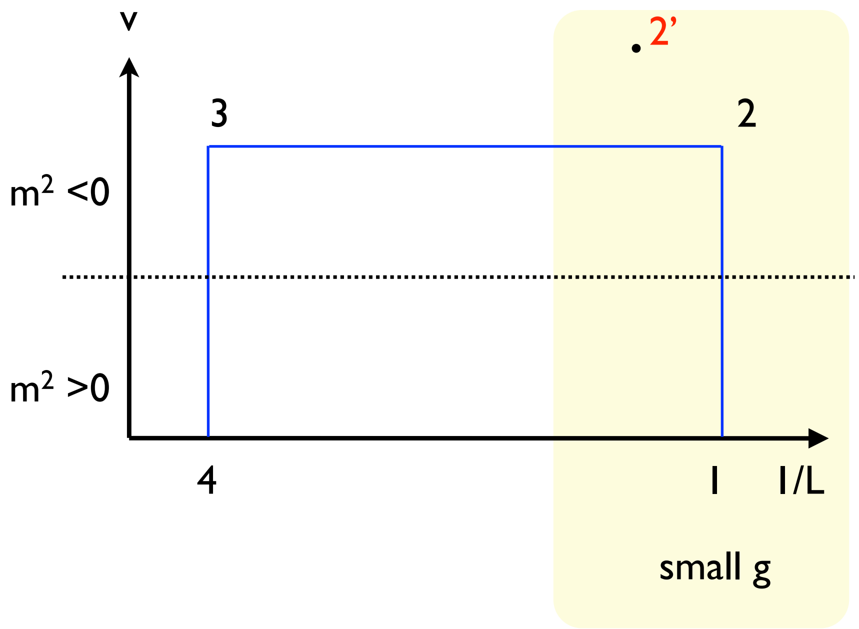
At 2:

$$- \sum_{n_m=\pm 1} \sum_{n_e \in \mathbb{Z}} \frac{2Lv^2\xi}{M\left(n_m, n_e\right)} K_1(LM\left(n_m, n_e\right)) e^{in_m\sigma + i2n_e\theta}$$

At 2' (when LM>>1 and g<<1):

$$V_{np,2'} = - \sum_{n_m=\pm 1} \sum_{n_e \in \mathbb{Z}} \frac{\sqrt{2\pi}Lv^2\xi}{M^{3/2}\left(n_m, n_e\right)} \boxed{e^{-LM(n_m,n_e)+i2n_e\theta}} e^{in_m\sigma}$$

- Finite sum over BPS and KK, which are constituents of instantons, is (almost) equivalent to an infinute sum of Julia-Zee dyons, each carrying a Polyakov loop factor appropriate to its charge.



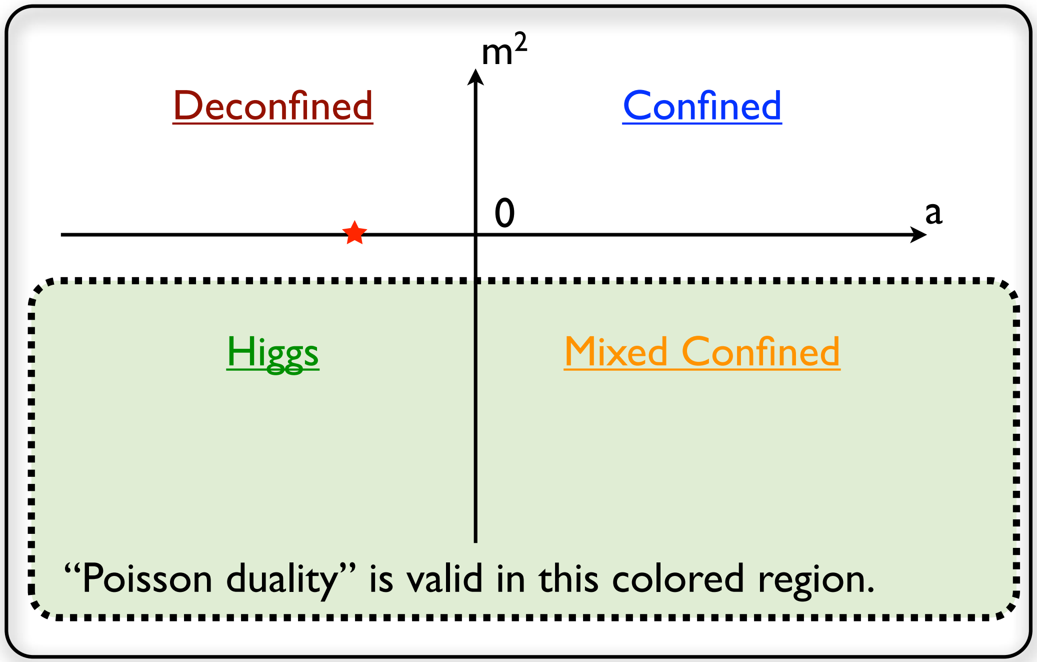
$$S_{eff} = \int_{R^3} \left(\frac{1}{2Lq_M^2} \left(\partial_i \sigma\right)^2 + V_{np} \right)$$

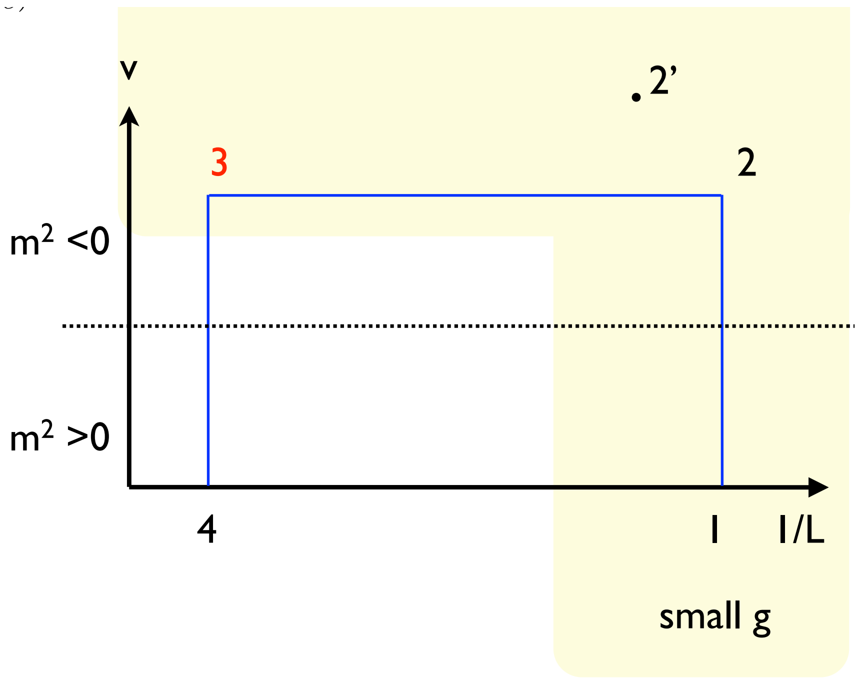
At 2:

$$- \sum_{n_m = \pm 1} \sum_{n_e \in \mathbb{Z}} \frac{2Lv^2 \xi}{M\left(n_m, n_e\right)} K_1(LM\left(n_m, n_e\right)) e^{i n_m \sigma + i 2 n_e \theta}$$

- The interpretation is valid most of the Higgs and mixed confined phases, except in the region near m^2=0, where M_0 is small.

- In this framework, the only difference between mixed confined and Higgs phases is whether θ=π/2 or not.





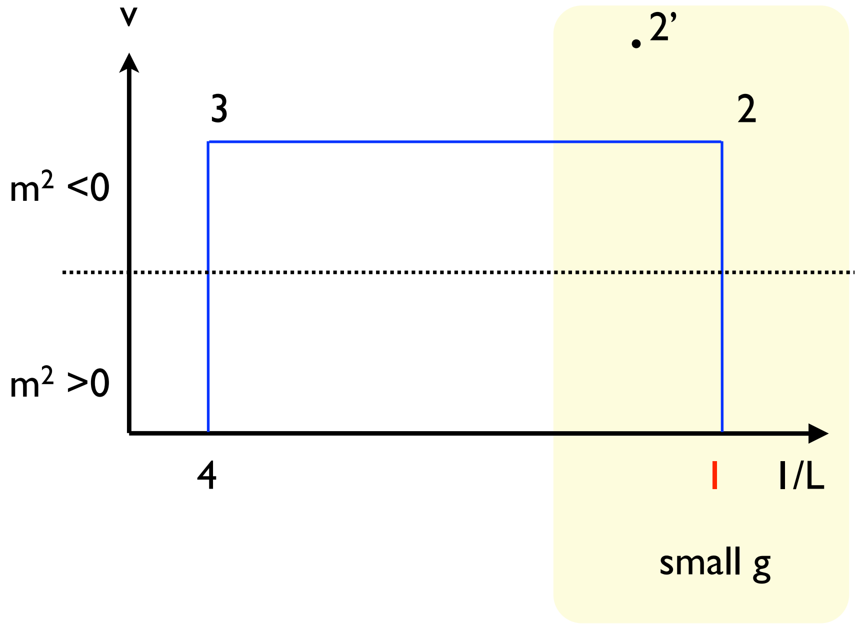
$$S_{eff} = \int_{R^3} \left(\frac{1}{2Lq_M^2} (\partial_i \sigma)^2 + V_{np} \right)$$

At 2:

$$-\sum_{n_m=\pm 1} \sum_{n_e \in \mathbb{Z}} \frac{2Lv^2\xi}{M\,(n_m, n_e)} K_1(LM\,(n_m, n_e)) e^{in_m\sigma + i2n_e\theta}$$

At 3 (when LM>>1 and g<<1): g is still small if v>>Λ

$$V_{np,2'} = -\sum_{n_m=\pm 1} \sum_{n_e \in \mathbb{Z}} \frac{\sqrt{2\pi}Lv^2\xi}{M^{3/2}\,(n_m, n_e)} e^{-LM(n_m,n_e)+i2n_e\theta} e^{in_m\sigma}$$



$$S_{eff} = \int_{R^3} \left(\frac{1}{2Lq_M^2} \left(\partial_i \sigma\right)^2 + V_{np} \right)$$

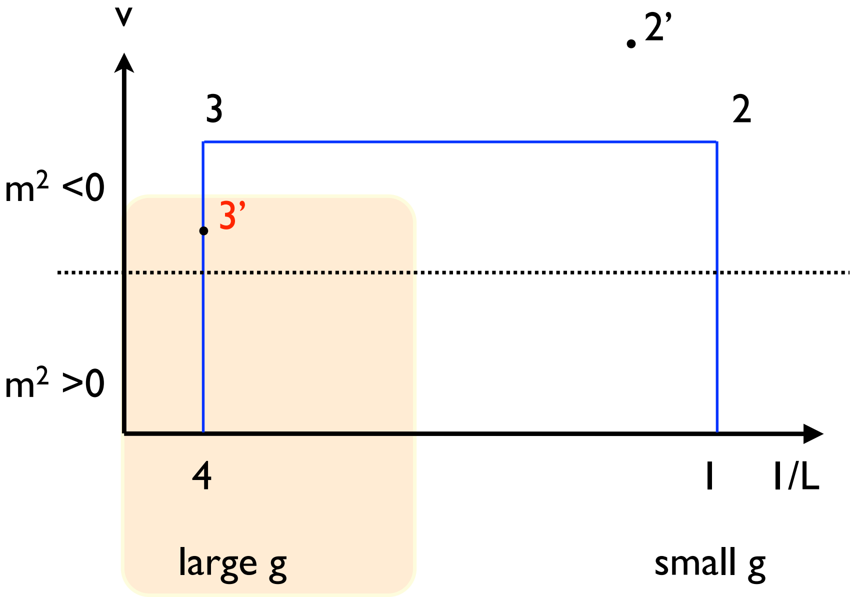
At 2:

$$- \sum_{n_m = \pm 1} \sum_{n_e \in \mathbb{Z}} \frac{2Lv^2 \xi}{M\left(n_m, n_e\right)} K_1 (LM\left(n_m, n_e\right)) e^{i n_m \sigma + i 2 n_e \theta}$$

At 1 (when M→0): taking only BPS and KK

$$V_{np,1} = - \sum_{n_m = \pm 1} e^{i n_m \sigma} \xi \left(e^{-\frac{8\pi}{g^2}\theta} + e^{-\frac{8\pi}{g^2}(\pi - \theta)} \right) \simeq - \sum_{n_m = \pm 1} e^{i n_m \sigma} \xi \left(\sum_{n_e \in \mathbb{Z}} \frac{4g^2}{16\pi^2 + g^4 n_e^2} e^{i 2 n_e \theta} \right)$$

The left-hand side is physical, but the right-hand side is hard to interpret.



in progress (hand-wavy)

$$S_{eff} = \int_{R^3} \left(\frac{1}{2Lq_M^2} (\partial_i \sigma)^2 + V_{np} \right)$$

At 2:

$$- \sum_{n_m=\pm1} \sum_{n_e \in \mathbb{Z}} \frac{2Lv^2\xi}{M(n_m,n_e)} K_1(LM(n_m,n_e)) e^{in_m\sigma + i2n_e\theta}$$

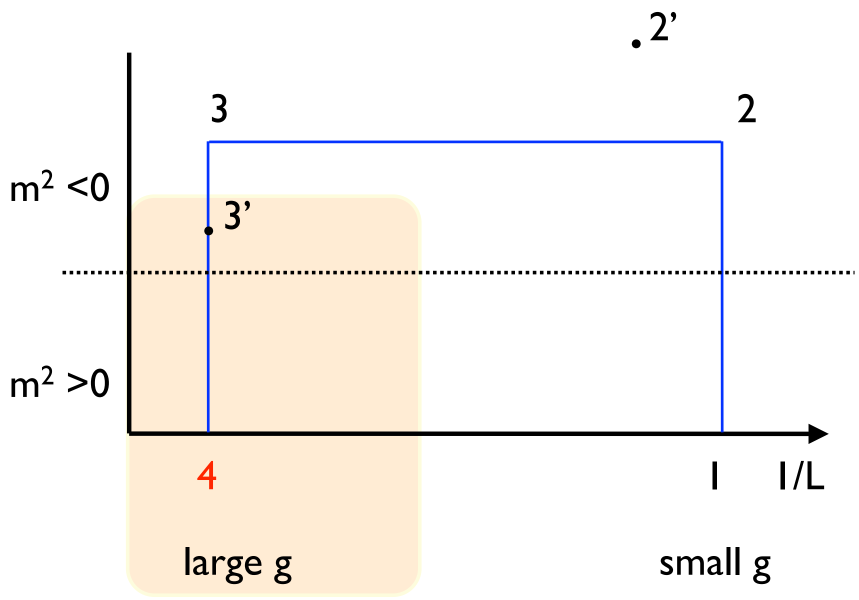
At 3' (g>>1):

$$V_{np,3'} = - \sum_{n_m \in \mathbb{Z}} \sum_{n_e \in \mathbb{Z}} \frac{\sqrt{2\pi}Lv^2\xi}{M^{3/2}(n_m,n_e)} e^{-LM(n_m,n_e)+i2n_e\theta} e^{in_m\sigma} \quad \text{where} \quad M(n_m,n_e) = v\sqrt{\left(\frac{4\pi}{g}n_m\right)^2 + (gn_e)^2}$$

- Role of electric and magnetic charges change when g is large.
- We have to sum over all magnetic charge. The leading contribution is when n_e=0:

$$V_{np,3'} = - \sum_{n_m \neq 0} \frac{\sqrt{2\pi}Lv^2\xi}{M^{3/2}(n_m,0)} e^{-LM(n_m,0)} e^{in_m\sigma}$$

- But what is ξ?



in progress (hand-wavy)

$$S_{eff} = \int_{R^3} \left(\frac{1}{2Lq_M^2} \left(\partial_i \sigma\right)^2 + V_{np} \right)$$

At 2:

$$-\sum_{n_m=\pm 1} \sum_{n_e \in \mathbb{Z}} \frac{2Lv^2 \xi}{M\left(n_m, n_e\right)} K_1(LM\left(n_m, n_e\right)) e^{in_m\sigma + i2n_e\theta}$$

At 4 (g>>1):

$$V_{np,4} = -\sum_{n_m \neq 0} \frac{2\xi}{\left(\frac{4\pi}{g}\right)^2 n_m^2} e^{in_m\sigma}$$

- Minimum at sigma =0.
- If ξ does not depend on n_m, then it is convergent.

Conclusions

- Can find a phase diagram at small-L using the effective potential of the Polyakov loop.
- The Higgs and the confined phases are not compatible.
- A new phase (mixed confined phase) in the deformed SU(2) with adjoint Higgs.
- Generalization of BPS and KK monopoles.
- BPS +KK monopoles ≃ Infinite sum of Julia-Zee dyons in the mixed confined phase.

Things to do

- Explore the Poisson duality further into the strong-coupling region.
- Thermodynamic properties (free energy, etc) in the confining gauge theories.