Photoleptonic decay of B meson beyond QCD factorization

Alexander Khodjamirian

UNIVERSITÄT SIEGEN
Theoretische Physik 1

DFG
RESEARCH UNIT

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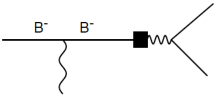
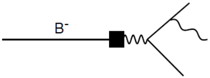
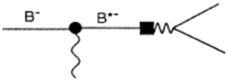
$B^- \to \gamma \ell \bar{\nu}_\ell$ ("photoleptonic") decay

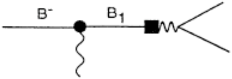
- involves $b\bar{u} \to W$ transition
- QED bremsstrahlung of γ (suppressed by m_μ),





- "structure-dependent" part: (no helicity-suppression)
  e.g., inserting heavy intermediate $B^*(1^-)$ and $B_1(1^+)$ states





[G.Burdman, T.Goldman, D.Wyler, PRD (1995)] (c.f. $\pi \to \ell \nu_\ell \gamma$)

Hadronic part of $B^- \to \gamma \ell \bar{\nu}_\ell$

- after factoring out γ and $\ell \bar{\nu}_\ell$:

$$-i \int d^4x\, e^{ipx} \langle 0 | T\{ j_\mu^{em}(x)\, \bar{u}(0)\gamma_\nu(1-\gamma_5) b(0)\} | B^-(p+q)\rangle$$
$$= \epsilon_{\mu\nu\tau\rho} p^\tau v^\rho F_V(q^2) + i\big[-g_{\mu\nu}(pv) + v_\mu p_\nu \big] F_A(q^2) + \dots .$$

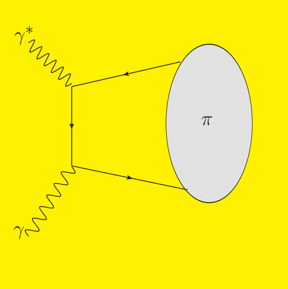
v- velocity of B , $(p+q) = m_B v$, q^2- invariant mass squared of $\ell \nu_\ell$

- $F_{V,A}(q^2)$ – two independent "$B \to \gamma$" form factors
  (B-meson characteristics, in addition to f_B)
- convenient variable: $E_\gamma = (m_B^2 - q^2)/(2m_B)$ (in B rest frame)
- consider $q^2 \ll m_b^2$, $m_b \to \infty$, $E_\gamma \sim m_b/2$ – large recoil
- the form factors allow factorization
- heavy-quark/ large recoil expansion
  (soft-collinear effective theory (SCET)),
- similar situation as for $\gamma\gamma^*(Q^2) \to \pi^0$ at $Q^2 \to \infty$

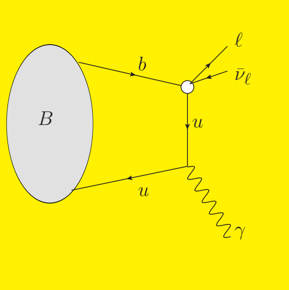
Two examples of factorization

$\boxed{B \to \gamma \ell \nu_\ell}$

$\boxed{\gamma\gamma^* \to \pi^0}$



at $Q^2 \to \infty$,
in terms of the pion
distribuiton amplitude (DA)



at $2E_\gamma \sim m_b \to \infty$,
in terms of B -meson
distribution amplitude (DA)

Distribution amplitudes of mesons

- pion DA: (twist 2)

$$\langle \pi(p)|\bar{u}(x)[x,0]\gamma_\mu\gamma_5 d(0)|0\rangle_{x^2=0} = -ip_\mu f_\pi \int_0^1 du\, e^{iup\cdot x}\varphi_\pi(u,\mu)\,,$$

$[x,0]$ – the gauge factor and $\mu \sim 1/\sqrt{|x^2|}$.

- *B*-meson distribution amplitude (DA), defined in HQET:

$$\langle 0|\bar{q}(x)[x,0]h_v(0)|\bar{B}_v\rangle \sim f_B m_B \int\limits_0^\infty d\omega e^{-i\omega v\cdot x}\Big\{\phi_+^B(\omega) + ...$$

Dirac-structure and $\phi_-^B(\omega)$ omitted for simplicity

Factorization in $B \to \gamma \ell \nu_\ell$

- after substituting the DA in the leading-order diagram:

$$F_V(E_\gamma) = F_A(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma} \int\limits_0^\infty \frac{d\omega}{\omega} \phi_+(\omega, \mu)$$

$\lim_{\omega \to \infty} \phi_+(\omega) \to 0$, effectively $\langle \omega \rangle \sim \Lambda_{QCD}$

- the key parameter: inverse moment of B meson DA:

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega} \phi_+(\omega, \mu)\,,$$

- important parameter for many factorization formulae: $B \to h$ form factors, $B \to hh$ etc.

  QCD sum rule estimate: $\lambda_B = 460 \pm 110$ MeV
  V.Braun, D.Ivanov, G.Korchemsky (2004)

  QCD fact. in nonleptonic B decays prefers $\lambda_B \sim 300$ MeV
  M.Beneke et al.,

$B \to \gamma \ell \nu_\ell$ in SCET

G.Korchemsky, D.Pirjol,TMYan (2000); S.Descotes-Genon,C.Sachrajda(2003);

E.Lunghi, D.Pirjol,D.Wyler(2003); S.Bosch,R.Hill, B.Lange, M.Neubert (2003)

a comprehensive update in [M.Beneke, J.Rohrwild (2011)]

- at $2E_\gamma \sim m_B$:

$$F_V(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma,\mu) + \left[\xi(E_\gamma) + \frac{e_b f_B m_B}{2E_\gamma m_b} + \frac{e_u f_B m_B}{(2E_\gamma)^2}\right],$$

$$F_A(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma,\mu) + \left[\xi(E_\gamma) - \frac{e_b f_B m_B}{2E_\gamma m_b} - \frac{e_u f_B m_B}{(2E_\gamma)^2}\right].$$

- $R(E_\gamma,\mu) = 1 \oplus \{\text{perturb. corr}\}$ (calculated to NLO),
- ξ - unknown, nonfactorizable soft contribution,
  long-distance photon emission from the light-spectator-quark
- [Beneke, Rohrwild (2011)] parameterization $\xi(E_\gamma) = C\, f_B/(2E_\gamma)$,
  "guesstimate": $-1 < C < 1$
- in what follows: calculation of ξ

Soft contribution in $B \to \gamma \ell \nu_\ell$: the method

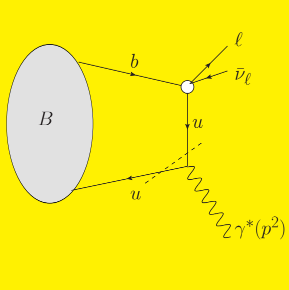
suggested for $\gamma\gamma^* \to \pi^0$ amplitude in [A. K., Eur. Phys. J. C6 (1999) 477],

see also [S. S. Agaev, V. M. Braun, N. Offen and F. A. Porkert, (2011)]

- Consider the same amplitude in unphysical region: $p^2 < 0$, highly-virtual (spacelike) photon ,

  $F_{V(A)}(q^2) \to F_{B\to\gamma^*}(q^2, p^2)$

- (unsubtracted) hadronic dispersion relation is valid:



$$F_{B\to\gamma^*}(q^2, p^2) = \frac{\sqrt{2} f_\rho F_{B\to\rho}(q^2)}{m_\rho^2 - p^2} + \frac{1}{\pi}\int_{s_0}^{\infty} ds\, \frac{\mathrm{Im} F_{B\to\gamma^*}(q^2, s)}{s - p^2},$$

$\rho, \omega \oplus$ {hadronic continuum, eff. threshold s_0}

- dispersion representation for the calculated amplitude

$$F^{\mathrm{HQE}}_{B\to\gamma^*}(q^2, p^2) = \frac{1}{\pi}\int_{0}^{\infty} ds\, \frac{\mathrm{Im} F^{\mathrm{HQE}}_{B\to\gamma^*}(q^2, s)}{s - p^2},$$

Soft contribution in $B \to \gamma \ell \nu_\ell$: the method

- quark-hadron duality

$$\mathrm{Im} F_{B\to\gamma^*}(q^2, s) = \mathrm{Im} F^{HQE}_{B\to\gamma^*}(q^2, s) \qquad \text{for } \; s > s_0$$

- matching HQE result to the dispersion relation (at $p^2 < 0$, the validity region of HQE !), subtraction of continuum using duality, Borel transformation: $p^2 \to M^2$

$$\sqrt{2} f_\rho F_{B\to\rho}(q^2) = \frac{1}{\pi} \int_0^{s_0} ds\, e^{-(s-m_\rho^2)/M^2}\, \mathrm{Im} F^{\mathrm{HQE}}_{B\to\gamma^*}(q^2, s)\,.$$

  a QCD sum rule (light-cone sum rule) for $B \to \rho$ form factor,

  LCSRs with B DA's [A.K., N.Offen, Th.Mannel] (2005),

  LCSRs in SCET [F. de Fazio, Th.Feldmann, T.Hurth (2006)]

- insert the sum rule in the dispersion relation
- the final step: $p^2 \to 0$ to access the real photon

Soft contribution in $B \to \gamma \ell \nu_\ell$: the calculation

- LO diagram in HQE (γ-emission from spectator quark in B)

$$F^{(0)}_{B\to\gamma^*}(E_\gamma, p^2) = e_u f_B m_B \int_0^\infty d\omega \frac{\phi_+(\omega,\mu)}{2E_\gamma\omega - p^2}\,,$$

neglecting $\sim \omega/E_\gamma$ terms in the integrand,

two DA models used: $\phi_+(\omega) \sim \omega$ $(\omega \to 0)$, $\omega_{max} \sim \bar{\Lambda} \sim m_B - m_b$

- after converting into dispersion integral, the result:

$$F^{(0)}_{B\to\gamma}(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma}\Bigg[(2E_\gamma)\int\limits_0^{s_0/(2E_\gamma)} \frac{d\omega}{m_\rho^2}\,\phi_+(\omega,\mu)e^{-(2E_\gamma\omega - m_\rho^2)/M^2} + \int\limits_{s_0/(2E_\gamma)}^{\infty} \frac{d\omega}{\omega}\,\phi_+(\omega,\mu)\Bigg].$$

completing the second integral from 0 to ∞,
reproduce the HQE limit $\oplus$ soft correction:

Soft contribuiton to $B \to \gamma \ell \nu_\ell$: the result

$$F^{(0)}_{B\to\gamma}(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} + \xi^{(0)}(E_\gamma)$$

where

$$\xi^{(0)}(E_\gamma) = \frac{e_u f_B m_B}{2E_\gamma} \int\limits_0^{s_0/(2E_\gamma)} d\omega \left[\frac{2E_\gamma}{m_\rho^2} e^{-(2E_\gamma \omega - m_\rho^2)/M^2} - \frac{1}{\omega} \right] \phi_+(\omega, \mu)\,.$$

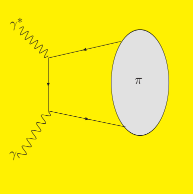
- the modification of the HQE expression only concerns $\omega < s_0/(2E_\gamma) \sim s_0/m_b$, hence, this is a soft contribution.
- the LO approximaiton can be improved further by adding calculable higher orders in α_s and/or HQE systematically, at $p^2 < 0$
- define a rescaled correction, to factor out f_B, λ_B,

$$\xi(E_\gamma) \equiv \left(\frac{e_u f_B m_B}{2E_\gamma \lambda_B(\mu)} \right) \frac{\widehat{\xi}(E_\gamma)}{2E_\gamma}$$

Soft contribution to $\gamma\gamma^* \to \pi^0$ form factor

$$T_{\mu\nu}(p,q) = -i\int d^4x\, e^{ipx}\langle\pi^0(p+q)|T\{j_\mu^{em}(x)\, j_\nu^{em}(0)\}|0\rangle$$

$$= \epsilon_{\mu\nu\tau\rho}p^\tau q^\rho F_{\gamma\gamma^*\to\pi}(Q^2)$$



- consider $p^2 \neq 0$, i.e., $\gamma^*\gamma^* \to \pi^0$
- with the same method, in the same LO approximation:

$$Q^2 F_{\gamma^*\gamma\to\pi}(Q^2) = \frac{\sqrt{2}f_\pi}{3}\Bigg\{\int_0^1 \frac{dx}{x}\phi_\pi(x) \leftarrow \text{LO term}$$

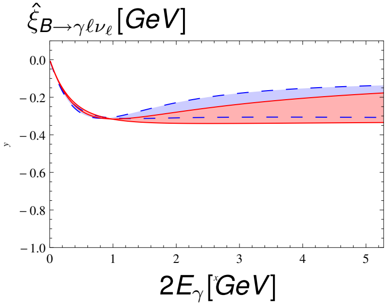
$$+\int_0^{x_0} dx\left[\frac{Q^2}{\bar{x}m_\rho^2}e^{(\bar{x}m_\rho^2 - xQ^2)/(\bar{x}M^2)} - \frac{1}{x}\right]\phi_\pi(x) \leftarrow \text{soft corr.}\Bigg\}$$

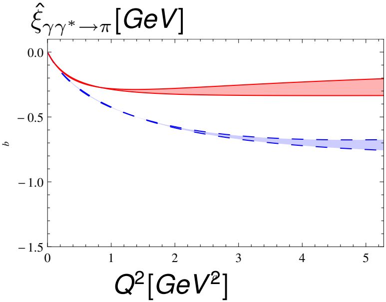
$$\equiv \frac{\sqrt{2}f_\pi}{3}\left(\int_0^1 \frac{dx}{x}\phi_\pi(x)\right)\left[1 + \frac{\widehat{\xi}_{\gamma^*\gamma\to\pi}(Q^2)}{Q^2}\right]$$

$x_0 = s_0/(s_0+Q^2)$, for simplicity asymptotic pion DA $\phi_\pi(x) = 6x(1-x)$:

Soft contributions to $B \to \gamma \ell \nu_\ell$ and to $\gamma\gamma^* \to \pi$

- numerical illustration: lower (upper) curves: $M^2 = 1.0(1.5)$ GeV^2; solid, dashed - two different models of $\phi_+^B(\omega)$, $\lambda_B = 500$ MeV





- the predictions reliable at sufficiently large E_γ (Q^2) both curves essentially flat for $2E_\gamma > 1$ GeV and $Q^2 > 1$ GeV^2, similarity is visible

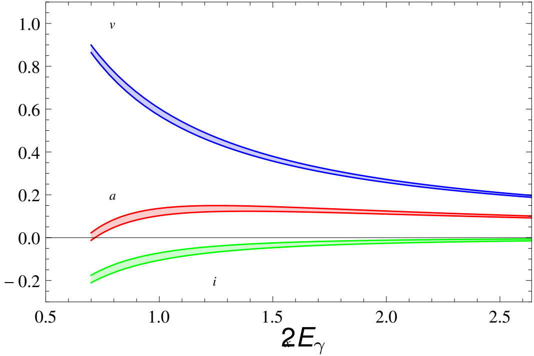
Improved $B \to \gamma \ell \nu_\ell$ form factors

- We use the HQE expression from [Beneke,Rohrwild (2011)] with the calculated soft correction:

F_V (solid)

F_A, (dashed)

$\xi(E_\gamma)$ (solid)



$\lambda_B = 500$ MeV; short-dashed - II model of $\phi^B(\omega)$

- form factors $\Rightarrow$ the partial width at large E_γ

$B \to \gamma \ell \nu_\ell$ decay width

- the photon energy spectrum

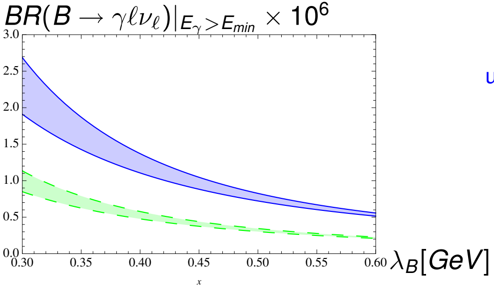
$$\frac{d\Gamma}{dE_\gamma} = \frac{\alpha_{\rm em} G_F^2 |V_{ub}|^2}{6\pi^2} m_B E_\gamma^3 \left(1 - \frac{2E_\gamma}{m_B}\right) \left(|F_V|^2 + |\widetilde{F}_A|^2\right),$$

with the contact term added, $\widetilde{F}_A = F_A + e_\ell f_B / E_\gamma$

- measuring dBR/dE_γ with energetic photons is feasible

- so far only the upper limit published by BABAR (2009): $BR(B \to \gamma \ell \nu_\ell) < 14 \cdot 10^{-6}$ for $E_\gamma > 1.0$ GeV

Predicted $B \to \gamma \ell \nu_\ell$ width



upper (lower) $E_{min} = 1.0\ (1.7)$ GeV

$f_B = 195$ MeV, $|V_{ub}| = 3.5 \times 10^{-3}$

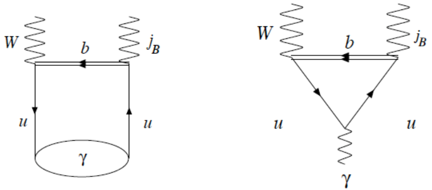
- the partial width: a promising source of λ_B determination
- relation to the leptonic width ($|V_{ub}|$ and f_B cancel !)

$$\frac{BR(B \to \gamma \mu \nu_\mu)_{E_\gamma > E_{min}}}{BR(B \to \tau \nu_\tau)} = \frac{4\alpha_{em}}{3\pi m_\tau^2 (1 - m_\tau^2 / m_B^2)} \int\limits_{E_{min}}^{m_B/2} \left(1 - \frac{2E_\gamma}{m_B}\right) E_\gamma^3 \left[\frac{|F_V|^2 + |\widetilde{F}_A|^2}{f_B^2}\right],$$

- Belle collaboration plans to detect this decay

Application of QCD light-cone sum rules

- a different method, not using $m_b \to \infty$ and HQET
- calculating correlation functions with photon DA's and interpolating B-meson with the quark current:



$\oplus \;\; O(\alpha_s) \oplus \bar{q}Gq$ photon DA's

AK, D.Wyler, G.Stoll (1995); G. Eilam, I.Halperin, G.Mendel (1995); P.Ball, E.Kou (2003)

- the interplay of two approaches: a future task

(using photon DA's worked out in P.Ball, V.Braun, N.Kivel (2002))

Conclusions

- soft contributions to $B \to \gamma \ell \nu_\ell$ amplitude at large E_γ can be calculated within the HQE/SCET framework, making use of hadronic dispersion relation and duality
- outlook: adding corrections to the LO HQE spectral density
- the method /results very similar to $\gamma\gamma^* \to \pi^0$
- combination/interplay with LCSR involving photon DA's
- the measurement of the photoleptonic B-decay will allow us to determine/constrain the inverse moment λ_B the key parameter of B meson "partonic" structure