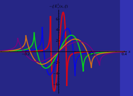

Singularities of Generalized Parton Distributions

A.V. Radyushkin

Physics Department, Old Dominion University
&
Theory Center, Jefferson Lab

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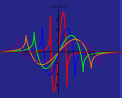


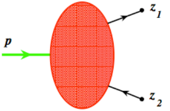
Hadrons in Terms of Quarks and Gluons

How to relate hadronic states $|p, s\rangle$

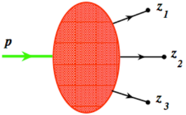
to quark and gluon fields $q(z_1)$, $q(z_2)$, $\ldots$?

Standard way: use matrix elements

$$\langle 0 \,|\, \bar{q}_\alpha(z_1)\, q_\beta(z_2) \,|\, M(p), s \,\rangle \;,\; \langle 0 \,|\, q_\alpha(z_1)\, q_\beta(z_2)\, q_\gamma(z_3) |\, B(p), s \,\rangle$$



Meson–quark matrix element



Baryon–quark matrix element

- Can be interpreted as hadronic wave functions

Phenomenological Functions

“Old” functions:

- Form Factors
- Usual Parton Densities
- Distribution Amplitudes

“New” functions:

Generalized Parton Distributions (GPDs)

GPDs = Hybrids of

Form Factors, Parton Densities and Distribution Amplitudes

“Old” functions

are limiting cases of “new” functions

Form Factors

Form factors are defined through matrix elements

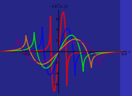
of electromagnetic and weak currents between hadronic states

Nucleon EM form factors:

$$\langle\, p',\, s' \,|\, J^{\mu}(0) \,|\, p,\, s \,\rangle = \bar{u}(p', s') \left[\gamma^{\mu} F_1(t) + \frac{\Delta^{\nu}\sigma^{\mu\nu}}{2m_N} F_2(t)\right] u(p, s)$$

$$(\Delta = p - p', t = \Delta^2)$$

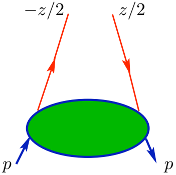
- Electromagnetic current
$$J^{\mu}(z) = \sum\nolimits_{\text{flavor}} e_f \bar{\psi}_f(z)\gamma^{\mu}\psi_f(z)$$
- Helicity non-flip form factor
$$F_1(t) = \sum\nolimits_f e_f F_{1f}(t)$$
- Helicity flip form factor
$$F_2(t) = \sum\nolimits_f e_f F_{2f}(t)$$



Usual Parton Densities

Parton Densities are defined through forward matrix elements

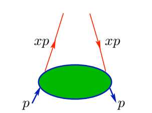
of quark/gluon fields separated by lightlike distances



Unpolarized quarks case:

$$\langle\, p \,|\, \bar{\psi}_a(-z/2)\gamma^\mu \psi_a(z/2) \,|\, p \,\rangle\big|_{z^2=0}$$
$$= 2p^\mu \int_0^1 \left[e^{-ix(pz)} f_a(x) - e^{ix(pz)} f_{\bar{a}}(x) \right] dx$$

Momentum space interpretation



$f_{a(\bar{a})}(x)$ is probability

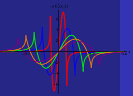
to find a $(\bar{a})$ quark with momentum xp

Local limit $z = 0$

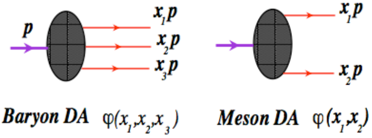
$\Rightarrow$ sum rule

$\int_0^1 [f_a(x) - f_{\bar{a}}(x)]\, dx = N_a$

for valence quark numbers



Distribution Amplitudes



DAs may be interpreted as

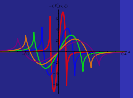
- LC wave functions integrated over transverse momentum
- Matrix elements $\langle 0|\mathcal{O}|p\rangle$ of LC operators

For pion (π^+):

$$\langle\, 0 \,|\, \bar{\psi}_d(-z/2)\gamma_5\gamma^\mu\psi_u(z/2) \,|\, \pi^+(p)\,\rangle\Big|_{z^2=0}$$

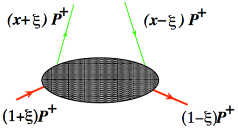
$$= ip^\mu f_\pi \int_{-1}^{1} e^{-i\alpha(pz)/2}\varphi_\pi(\alpha)\, d\alpha$$

with $\alpha = x_1 - x_2$ or $x_1 = (1+\alpha)/2,\ x_2 = (1-\alpha)/2$



Generalized Parton Distributions

Momentum fractions taken wrt average momentum $P = (p + p')/2$



4 functions of x, ξ, t:

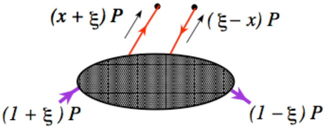
$H, E, \widetilde{H}, \widetilde{E}$

wrt hadron/parton helicity flip

$+/+, -/+, +/-, -/-$

- Skeweness $\xi \equiv \Delta^+/2P^+$ is $\xi = x_{Bj}/(2 - x_{Bj})$
- 3 regions:

$$
\begin{array}{lll}
\xi < x < 1 & \sim & \text{quark distribution} \\
-1 < x < -\xi & \sim & \text{antiquark distribution} \\
-\xi < x < \xi & \sim & \text{distribution amplitude for } N \to \bar{q}qN'
\end{array}
$$



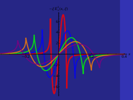
Definition of GPDs

- In scalar case, define GPD by

$$\langle P + r/2 | \psi(-z/2)\psi(z/2) | P - r/2 \rangle |_{z^2=0}$$
$$= \int_{-1}^{1} e^{-ix(Pz)} H(x,\xi;t)\, dx$$

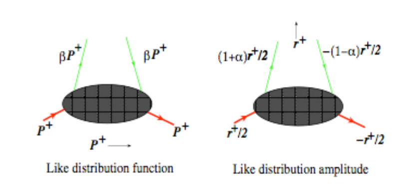
- Invariant momentum transfer $t = r^2$
- Skeweness $\xi = r^+/2P^+$
- $r = 0 \Rightarrow$ usual (forward) distribution

$$f(x) = H(x, \xi = 0; t = 0)$$

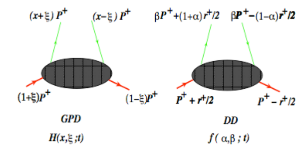


Double Distributions

"Superposition" of P^+ and r^+ momentum fluxes

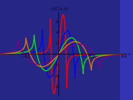


Connection with GPDs



Basic relation between fractions

$$x = \beta + \xi\alpha$$



Parton distributions and matrix elements

- For a scalar target, one may write

$$\langle P + r/2|\psi(0)\{\overleftrightarrow{\partial}_{\mu_1} \ldots \overleftrightarrow{\partial}_{\mu_n}\}\psi(0)|P - r/2\rangle$$
$$= A_{n0}\{P_{\mu_1} \ldots P_{\mu_n}\} + A_{nn}\{r_{\mu_1} \ldots r_{\mu_n}\}$$
$$+ \sum_{l=1}^{n-1} A_{nl}\{P_{\mu_1} \ldots P_{\mu_{n-l}} r_{\mu_{n-l+1}} \ldots r_{\mu_n}\}$$

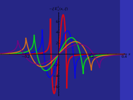
- $r = 0 \Rightarrow$ usual (forward) distribution $f(\beta)$ related to $l = 0$ moments

$$\int_{-1}^{1} f(\beta)\beta^n \, d\beta = A_{n0} \qquad (1)$$

- $P = 0 \Rightarrow$ D-term $D(\alpha)$ related to $l = n$ moments

$$\int_{-1}^{1} D(\alpha)(\alpha/2)^n \, d\alpha = A_{nn} \qquad (2)$$

- D comes with r_{μ_i} factors: it is invisible in DIS (then $r = 0$)



Definition of DDs

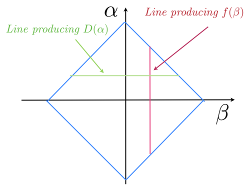
- Define Double Distribution (DD)

$$\frac{n!}{(n-l)!\,l!\,2^l}\int_\Omega F(\beta,\alpha)\beta^{n-l}\alpha^l\,d\beta\,d\alpha = A_{nl}$$

- Support region Ω is given by rhombus $|\alpha|+|\beta|\le 1$
- "DD parameterization" of the matrix element

$$\left\langle P-\frac{r}{2}\right|\psi(-z/2)\psi(z/2)\left|P+\frac{r}{2}\right\rangle\Big|_{z^2=0} = \int_\Omega F(\beta,\alpha)\,e^{-i\beta(Pz)-i\alpha(rz)/2}\,d\beta\,d\alpha$$

Getting PDF and D-term from DDs

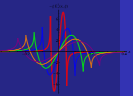


- Usual (forward) distribution

$$f(\beta) = \int_{-1+|\beta|}^{1-|\beta|} F(\beta,\alpha)\,d\alpha$$

- D-term

$$D(\alpha) = \int_{-1+|\alpha|}^{1-|\alpha|} F(\beta,\alpha)\,d\beta$$



Isolating D-term

- Using $e^{-i\beta(Pz)} = [e^{-i\beta(Pz)} - 1] + 1$
- split DD-integral into "plus" part

$$\int_\Omega [F(\beta,\alpha)]_+ \, e^{-i\beta(Pz) - i\alpha(rz)/2} \, d\beta \, d\alpha$$

- and D-term part

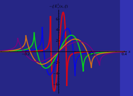
$$\int_{-1}^{1} D(\alpha) \, e^{-i\alpha(rz)/2} \, d\alpha$$

- with

$$[F(\beta,\alpha)]_+ = F(\beta,\alpha) - \delta(\beta) \int_{-1+|\alpha|}^{1-|\alpha|} F(\gamma,\alpha) \, d\gamma$$

- "Plus" "+" D representation:

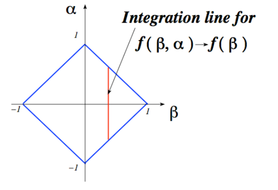
$$F(\beta,\alpha) = [F(\beta,\alpha)]_+ + \delta(\beta) D(\alpha)$$



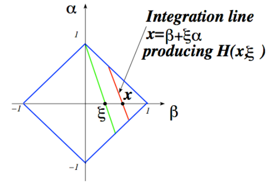
Getting GPDs from DDs

$$H(x,\xi) = \int_{\Omega} F(\beta,\alpha)\,\delta(x-\beta-\xi\alpha)\,d\beta\,d\alpha$$

DDs live on rhombus $|\alpha| + |\beta| \le 1$



Converting DDs into GPDs



"Munich" symmetry:

$$f_a(\beta,\alpha;t) = f_a(\beta,-\alpha;t)$$

GPDs $H(x,\xi)$ are obtained from DDs $f(\beta,\alpha)$

by scanning DDs
at ξ-dependent angles

$\Rightarrow$ DD-tomography

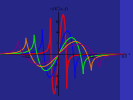


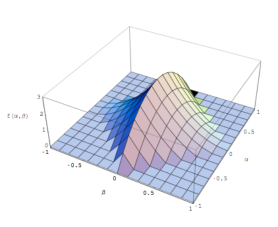
Illustration of DD→GPD conversion

Factorized model for DDs:

($\sim$ usual parton density in β-direction) $\otimes$
($\sim$ distribution amplitude in α-direction)

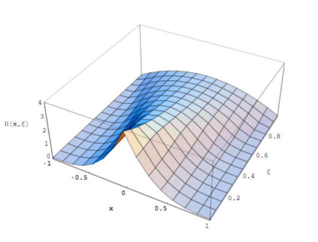
Toy model for double distribution

$$f(\beta,\alpha) = 3[(1-|\beta|)^2 - \alpha^2]\,\theta(|\alpha|+|\beta| \le 1)$$



- Corresponds to toy "forward" distribution $f(\beta) = (1-|\beta|)^3$

GPD $H(x,\xi)$ resulting from toy DD



- For $\xi = 0$ reduces to usual parton density
- For $\xi = 1$ has shape like meson distribution amplitude

"DD plus D" Model for GPDs

- Factorized Ansatz for DDs:

$$F(\beta, \alpha) = f(\beta) h_a(\beta, \alpha)$$

Normalization

$$\int_{-1}^{1} d\alpha \, h(\beta, \alpha) = 1$$

Guarantees forward limit

$$\int_{-1}^{1} d\alpha \, f(\beta, \alpha) = f(\beta)$$

- DD modeling misses terms invisible in the forward limit:
  - Meson exchange contributions
  - D-term, which can be interpreted as σ exchange
- Inclusion of D-term induces contribution confined to $|x| < \xi$ region

$$H_D(x, \xi) = \frac{1}{|\xi|} D(x/\xi)$$

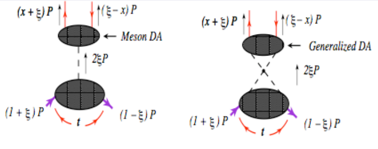
Model for GPDs based on DDs

- DD+D Ansatz: $F(\beta,\alpha) = f(\beta)h_a(\beta,\alpha) + \delta(\beta)D(\alpha)$
- General form of model profile

$$h(\beta,\alpha) = \frac{\Gamma(2+2b)}{2^{2b+1}\Gamma^2(1+b)} \frac{[(1-|\beta|)^2 - \alpha^2]^b}{(1-|\beta|)^{2b+1}}$$

- Power b is parameter of the model
- $b = \infty$ gives $h(\beta,\alpha) = \delta(\alpha)$ and $H(x,\xi) = f(x) + D(x/\xi)/|\xi|$

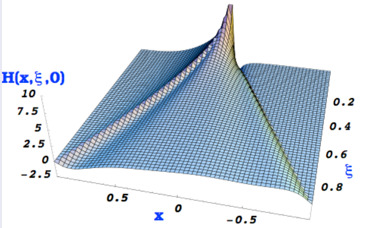
Meson and D-term terms

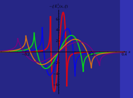


Meson exchange contribution

Structure of D−term contribution

DD + D-term model

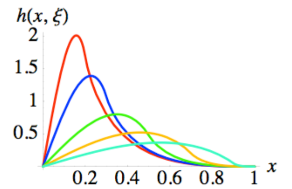




Model with Regge behavior of $f(\beta)$

- PDFs $f(\beta)$ are known to be singular for small β
- $f(\beta) \sim \beta^{-a}(1-\beta)^3$
- $x_+ = (x+\xi)/(1+\xi)$
- $x_- = (x-\xi)/(1-\xi)$
- $\sim |x-\xi|^{2-a}$+const behavior for $x \sim \xi$

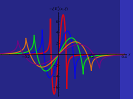
b=1 DD with Regge PDFs



$\xi = 0.2, 0.3, 0.5, 0.7, 0.9$

- Model $H(x,\xi) = \int_\Omega d\beta\, f(\beta)\, h_b(\beta,\alpha)\, \delta(x-\beta-\xi\alpha)$ with $b=1$

$$
\begin{aligned}
H(x,\xi)|_{|x|\geq\xi} &= \frac{1}{\xi^3}\left(1-\frac{a}{4}\right)\left\{\left[(2-a)\xi(1-x)(x_+^{2-a}+x_-^{2-a})\right.\right. \\
&+ \left.\left.(\xi^2-x)(x_+^{2-a}-x_-^{2-a})\right]\,\theta(x) - (x\to -x)\right\} \\
H(x,\xi)|_{|x|\leq\xi} &= \frac{1}{\xi^3}\left(1-\frac{a}{4}\right)\left\{x_+^{2-a}[(2-a)\xi(1-x)+(\xi^2-x)]\right. \\
&- \left.(x\to -x)\right\}
\end{aligned}
$$



Spin-1/2 quarks: two-DD representation

- For a (pseudo)scalar target

$$\langle P - r/2|\bar\psi(-z/2)\gamma_\mu\psi(z/2)|P + r/2\rangle|_{\mathrm{twist}-2}$$
$$= 2P_\mu f\big((Pz),(rz),z^2\big) + r_\mu g\big((Pz),(rz),z^2\big)$$

- Two-DD parametrization

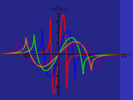
$$z^\mu\langle P - r/2|\bar\psi(-z/2)\gamma_\mu\psi(z/2)|P + r/2\rangle\Big|_{z^2=0}$$
$$= \int_\Omega e^{-i\beta(Pz)-i\alpha(rz)/2}\Big[2(Pz)F(\beta,\alpha) + (rz)G(\beta,\alpha)\Big]\,d\beta\,d\alpha$$
$$= \frac{2}{i}\int_\Omega e^{-i\beta(Pz)-i\alpha(rz)/2}\left[\frac{\partial F(\beta,\alpha)}{\partial\beta} + \frac{\partial G(\beta,\alpha)}{\partial\alpha}\right]d\beta\,d\alpha$$

- Not unique: invariant under transformation

$$F(\beta,\alpha) \to F(\beta,\alpha) + \partial\chi(\beta,\alpha)/\partial\alpha\ ,$$
$$G(\beta,\alpha) \to G(\beta,\alpha) - \partial\chi(\beta,\alpha)/\partial\beta\ ,$$

- “DD+D” form corresponds to “gauge” in which one has

$$2(Pz)F_D(\beta,\alpha) + (rz)\delta(\beta)D(\alpha)$$



Spin-1/2 quarks: one-DD representation

- Note: in local twist-2 operators $\bar{\psi}\{\gamma_\mu \overleftrightarrow{\partial}_{\mu_1} \ldots \overleftrightarrow{\partial}_{\mu_n}\}\psi$ index μ is symmetrized with μ_i indices that produce $\beta P_{\mu_i} + \alpha r_{\mu_i}/2$
- $\Rightarrow$ μ also produces $\beta P_\mu + \alpha r_\mu/2$, i.e.

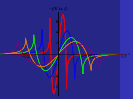
$$2(Pz)F(\beta,\alpha) + (rz)G(\beta,\alpha) = [2\beta(Pz) + \alpha(rz)]f(\beta,\alpha)$$

- Or $F(\beta,\alpha) = \beta f(\beta,\alpha)$ and $G(\beta,\alpha) = \alpha f(\beta,\alpha)$
- GPD in two-DD parametrization

$$H(x,\xi) = \int_\Omega \left[F(\beta,\alpha) + \xi G(\beta,\alpha)\right] \delta(x - \beta - \xi\alpha)\, d\beta\, d\alpha$$

- GPD in one-DD formulation

$$\begin{aligned} H(x,\xi) &= \int_\Omega (\beta + \xi\alpha) f(\beta,\alpha)\, \delta(x - \beta - \xi\alpha)\, d\beta\, d\alpha \\ &= x \int_\Omega f(\beta,\alpha)\, \delta(x - \beta - \xi\alpha)\, d\beta\, d\alpha \end{aligned}$$



One-DD formulation

- D-term in the one-DD case

$$D(\alpha) = \alpha \int_{-1+|\alpha|}^{1-|\alpha|} f(\beta, \alpha)\, d\beta$$

- Separating D-term

$$f(\beta, \alpha) = [f(\beta, \alpha)]_{+} + \delta(\beta) \frac{D(\alpha)}{\alpha} \tag{3}$$

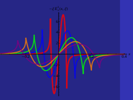
- Forward distribution

$$f(x) = \int_{-1+|x|}^{1-|x|} F(x, \alpha)\, d\alpha = x \int_{-1+|x|}^{1-|x|} f(x, \alpha)\, d\alpha$$

- Suggests factorized model

$$f(\beta, \alpha) = \frac{f(\beta)}{\beta} h(\beta, \alpha)$$

- $\Rightarrow$ Reconstructing DDs/GPDs from $f(x)/x$: very singular $\sim x^{-\alpha(0)-1}$ for small x !



GPDs in one-DD representation

- "DD$_+$+ D" separation corresponds to the representation

$$H(x,\xi) \equiv H_+(x,\xi) + \mathrm{sgn}(\xi) D(x/\xi) \ ,$$

- "Plus" part of GPD

$$H_+(x,\xi) \equiv \int_\Omega (\beta+\xi\alpha) f(\beta,\alpha) \Big[\delta(x-\beta-\xi\alpha) - \delta(x-\xi\alpha)\Big] \, d\beta \, d\alpha \ .$$

- Using $f(\beta,\alpha) = F(\beta,\alpha)/\beta$ we may rewrite

$$H_+(x,\xi) = \int_\Omega F(\beta,\alpha) \, \delta(x-\beta-\xi\alpha) \, d\beta \, d\alpha$$
$$+ \, \xi \int_\Omega \frac{\alpha F(\beta,\alpha)}{\beta} \Big[\delta(x-\beta-\xi\alpha) - \delta(x-\xi\alpha)\Big] \, d\beta \, d\alpha$$

- GPD constructed from DD $F(\beta,\alpha)$ by "classic" formula

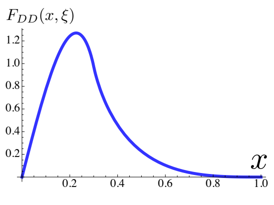
$$F_{DD}(x,\xi) = \int_\Omega F(\beta,\alpha) \, \delta(x-\beta-\xi\alpha) \, d\beta \, d\alpha$$

- GPD built from the "plus" part of the DD $\alpha F(\beta,\alpha)/\beta = G(\beta,\alpha)$.

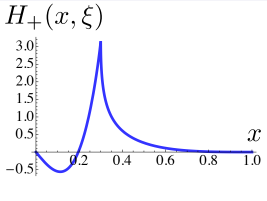
$$F^1_+(x,\xi) \equiv \int_\Omega \left(\frac{\alpha}{\beta} F(\beta,\alpha)\right)_+ \delta(x-\beta-\xi\alpha) \, d\beta \, d\alpha$$

Pion GPDs for $n = 1$ profile $\sim (1-\beta)^2 - \alpha^2$

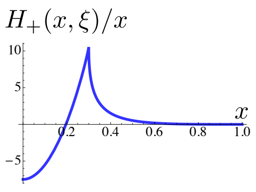
GPD $F_{DD}(x,\xi)$ for $\xi = 0.3$



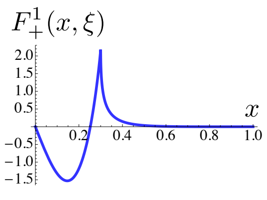
Pion GPD $H_+(x,\xi)$ for $\xi = 0.3$

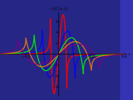


GPD $H_+(x,\xi)/x$ for $\xi = 0.3$



Function $\xi F_+^1(x,\xi)$ for $\xi = 0.3$





Definitions of Nucleon DDs and GPDs

- In nucleon case for unpolarized target, one can parametrize

$$\langle p'|\bar{\psi}(-z/2)\not{z}\,\psi(z/2)|p\rangle|_{\text{twist}-2}$$
$$= \int_\Omega e^{-i\beta(Pz)-i\alpha(rz)/2}\left[\bar{u}(p')\not{z}\,u(p)\,a(\beta,\alpha)\right.$$
$$\left.+\frac{\bar{u}(p')u(p)}{2M_N}\left[2\beta(Pz)+\alpha(rz)\right]b(\beta,\alpha)\right]d\beta\,d\alpha$$

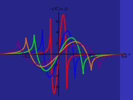
- DDs a, b correspond to $A = H + E$ and $B = -E$ of usual H and E
- A is given by simple “classic” DD representation

$$A(x,\xi) = \int_\Omega a(\beta,\alpha)\,\delta(x-\beta-\xi\alpha)\,d\beta\,d\alpha \qquad (4)$$

- B is given by one-DD representation

$$B(x,\xi) = x\int_\Omega b(\beta,\alpha)\,\delta(x-\beta-\xi\alpha)\,d\beta\,d\alpha\,. \qquad (5)$$

- Since $H = A + B$, it is given by combination of both types of DD-representation



Modeling a and b

- In the forward limit, we have for A

$$A(x,0) = H(x,0) + E(x,0) = f(x) + e(x)$$

- and for B

$$B(x,0) = -E(x,0) = -e(x)$$

- Suggest model representation for a

$$a(\beta,\alpha) = f(\beta,\alpha) + e(\beta,\alpha)$$

- and for b

$$b(\beta,\alpha) = -\frac{e(\beta,\alpha)}{\beta}$$

- Possible singularity of $e(\beta,\alpha)/\beta$ at $\beta = 0$, demands “DD$_+ + D$”

$$b(\beta,\alpha) = -\left(\frac{e(\beta,\alpha)}{\beta}\right)_+ + \delta(\beta)\frac{D(\alpha)}{\alpha}$$

- Here $D(\alpha)$ is the D-term

Start modeling E and H

- For H GPD:

$$\begin{aligned} H(x,\xi) &= A(x,\xi) + B(x,\xi) \\ &= \int_\Omega [f(\beta,\alpha) + e(\beta,\alpha)]\,\delta(x-\beta-\xi\alpha)\,d\beta\,d\alpha \\ &- x\int_\Omega \left[\left(\frac{e(\beta,\alpha)}{\beta}\right)_+ - \delta(\beta)\frac{D(\alpha)}{\alpha}\right]\delta(x-\beta-\xi\alpha)\,d\beta\,d\alpha \\ &= F_{DD}(x,\xi) + E_{DD}(x,\xi) - E_+(x,\xi) + \mathrm{sgn}(\xi)\,D(x/\xi)\ , \end{aligned}$$

- Terms constructed using the simplest DD formula

$$\begin{aligned} F_{DD}(x,\xi) &= \int_\Omega f(\beta,\alpha)\,\delta(x-\beta-\xi\alpha)\,d\beta\,d\alpha \\ E_{DD}(x,\xi) &= \int_\Omega e(\beta,\alpha)\,\delta(x-\beta-\xi\alpha)\,d\beta\,d\alpha \end{aligned}$$

- "Plus" part of E/x GPD:

$$\frac{E_+(x,\xi)}{x} = \int_\Omega \frac{e(\beta,\alpha)}{\beta}\Big[\delta(x-\beta-\xi\alpha) - \delta(x-\xi\alpha)\Big]\,d\beta\,d\alpha$$

Continue modeling E and H

- Function $E_+(x,\xi)$ is similar to $H_+(x,\xi)$ of pion case

$$
\begin{aligned}
E_+(x,\xi) &= \int_\Omega \frac{e(\beta,\alpha)}{\beta}(\beta+\xi\alpha)\left[\,\delta(x-\beta-\xi\alpha)-\delta(x-\xi\alpha)\right]\,d\beta\,d\alpha \\
&= \int_\Omega e(\beta,\alpha)\,\delta(x-\beta-\xi\alpha)\,d\beta\,d\alpha \\
&+ \xi\int_\Omega \frac{\alpha}{\beta}\,e(\beta,\alpha)\,\left[\delta(x-\beta-\xi\alpha)-\delta(x-\xi\alpha)\right]\,d\beta\,d\alpha \\
&= E_{DD}(x,\xi)+\xi\int_\Omega\left(\frac{\alpha}{\beta}\,e(\beta,\alpha)\right)_+\,\delta(x-\beta-\xi\alpha)\,d\beta\,d\alpha \\
&\equiv E_{DD}(x,\xi)+\xi E_+^1(x,\xi)
\end{aligned}
$$

- Important function

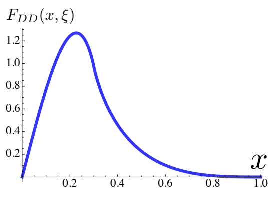
$$
E_+^1(x,\xi)\equiv\int_\Omega\left(\frac{\alpha}{\beta}\,e(\beta,\alpha)\right)_+\,\delta(x-\beta-\xi\alpha)\,d\beta\,d\alpha
$$

- Modifies “DD+D” construction to

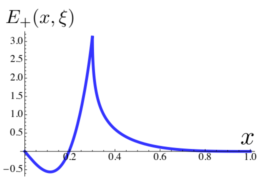
$$
H(x,\xi)=F_{DD}(x,\xi)-\xi E_+^1(x,\xi)+\mathrm{sgn}(\xi)\,D(x/\xi)
$$

Nucleon GPDs for $n = 1$ profile $\sim (1-\beta)^2 - \alpha^2$

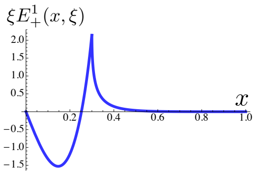
GPD $F_{DD}(x,\xi)$ for $\xi = 0.3$



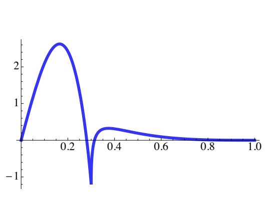
GPD $E_{+}(x,\xi)$ for $\xi = 0.3$



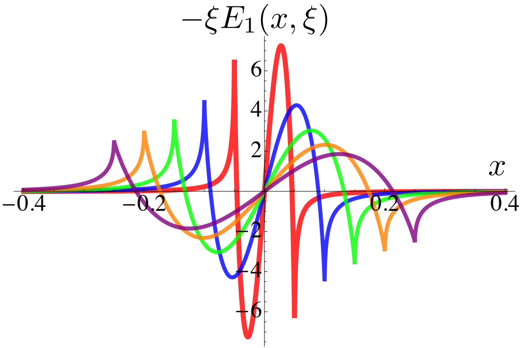
Function $\xi E_{+}^{1}(x,\xi)$ for $\xi = 0.3$



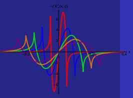
Nucleon GPD $H(x,\xi)/x$ without D-term for $\xi = 0.3$



What is added on top of D term



- “DD plus D” model is substituted by “DD $-\xi E_+^1(x,\xi) + \mathrm{sgn}(\xi) D(x/\xi)$”
- Important differences between $E_+^1(x,\xi)$ and $D(x/\xi)$:
- Support region of $E_+^1(x,\xi)$ is not restricted to $|x| \le \xi$
- $E_+^1(x,\xi)$ does not vanish at border points $|x| = \xi$



Conclusions

- Singular Regge behavior of usual PDFs implies singular structure of double distributions generating GPDs
- DD for E GPD reduces to $e(x)/x$ in forward limit – very strong singularity
- Formal expression for D-term diverges: need for renormalization
- Old “DD plus D” construction for GPD H is modified by extra non-monotonic term related to GPD E
- New term does not vanish at border point $x = \xi$
- New phenomenology for GPD modeling

Summary