



Optimal Workforce Planning and Shift Scheduling for Snow and Ice Removal

Minnesota
Department of
Transportation

**RESEARCH
SERVICES**

Office of
Policy Analysis,
Research &
Innovation

Diwakar Gupta, Principal Investigator
Department of Mechanical Engineering
University of Minnesota

December 2010

Research Project
Final Report 2011-03



Your Destination... Our Priority



Technical Report Documentation Page

1. Report No. MN/RC 2011-03	2.	3. Recipients Accession No.	
4. Title and Subtitle Optimal Workforce Planning and Shift Scheduling for Snow and Ice Removal		5. Report Date December 2010	
		6.	
7. Author(s) Diwakar Gupta, Elif Tokar-Erdemir, Dustin Kuchera, Arun Kumar Mannava, Wei Xiong		8. Performing Organization Report No.	
9. Performing Organization Name and Address Industrial & Systems Engineering Program Department of Mechanical Engineering University of Minnesota		10. Project/Task/Work Unit No. CTS Project #2007097	
		11. Contract (C) or Grant (G) No. (c) 89795 (wo) 1	
12. Sponsoring Organization Name and Address Minnesota Department of Transportation Research Services Section 395 John Ireland Blvd., MS 330 St. Paul, MN 55155-1899		13. Type of Report and Period Covered Final Report	
		14. Sponsoring Agency Code	
15. Supplementary Notes http://www.lrrb.org/pdf/201103.pdf			
16. Abstract (Limit: 250 words) <p>Shrinking budgets and high equipment, fuel, and labor costs have raised the importance of workforce planning and efficient deployment of available workforce for county-level winter maintenance operations. This project focused on developing methodologies for the estimation workforce requirements, and economic evaluation of the impact of using contract employees, split shifts and staggered shifts. In order to achieve these goals, a fundamental question that needed to be addressed was the determination of the amount of work induced by different types of storms that occur in Saint Louis County. Researchers obtained relevant storm data from a variety of weather reporting sources and extracted parameters relevant for determining plow speeds and sand/salt consumption. These parameters were used to determine optimal workforce deployment strategies that balance overtime and delay costs, which in turn provided estimates of the amount of plowing time needed for the goal of clearing roads within 24 hours after the end of snow fall. Plowing time calculations were subject to rules concerning when call outs can occur during off-shift hours. Plow time estimates were subsequently used to develop efficient algorithms to calculate workforce requirements.</p>			
17. Document Analysis/Descriptors Winter maintenance, Workforce requirements planning, Plow route optimization, Value of flexible shift scheduling, Data-driven workforce optimization		18. Availability Statement No restrictions. Document available from: National Technical Information Services, Alexandria, Virginia 22312	
19. Security Class (this report) Unclassified	20. Security Class (this page) Unclassified	21. No. of Pages 155	22. Price

Optimal Workforce Planning and Shift Scheduling for Snow and Ice Removal

Final Report

Prepared by:

Diwakar Gupta
Elif Tokar-Erdemir
Dustin Kuchera
Arun Kumar Mannava
Wei Xiong

Industrial and Systems Engineering Program
Department of Mechanical Engineering
University of Minnesota

December 2010

Published by:

Minnesota Department of Transportation
Research Services Section
395 John Ireland Boulevard, MS 330
St. Paul, Minnesota 55155-1899

This report represents the results of research conducted by the authors and does not necessarily represent the views or policies of the Minnesota Department of Transportation or the University of Minnesota. This report does not contain a standard or specified technique.

The authors, the Minnesota Department of Transportation, and the University of Minnesota do not endorse products or manufacturers. Any trade or manufacturers' names that may appear herein do so solely because they are considered essential to this report.

Acknowledgements

The research team is grateful to Saint Louis County for supporting this research effort. In particular, researchers wish to acknowledge Mr. Marcus Hall and Mr. James Foldesi, St. Louis County Public Works directors whose tenure overlapped with this project, Mr. Ron Garden, St. Louis County Public Works deputy director, and Mr. Patrick McCarthy. The research team is indebted to them for valuable advice, help in obtaining data needed for this project, and insights concerning the relevance of model assumptions. The researchers are also grateful to Linda Taylor, Steve Haider, Steven Lund, and Curt Pape from Mn/DOT's Central Office who attended technical advisory panel meetings and gave their expert opinion during the course of this project. Finally, researchers are indebted to Dr. Alan Rindels, this project's administrative liaison, who helped the research team at every stage of the project and played a key role in helping bring this project to a successful completion.

Table of Contents

Chapter 1	Introduction.....	1
Chapter 2	Project Scope and Model Inputs	3
2.1	Snowplow Routing Data	3
2.1.1	AADT Counts	3
2.1.2	Snowplow Routes and Road Lengths	3
2.1.3	Sand/Salt Application Rates	5
2.1.4	Snowplow Speed.....	5
2.1.5	Equipment and Site Information.....	5
2.1.6	Weather Data: Estimating Typical Storm Scenarios	6
2.2	Model Development.....	7
2.2.1	Snowplow Route Network.....	7
2.2.2	Solution Approach	8
2.2.3	Data Matrices	12
2.2.4	Sample Route	12
2.3	Results	13
2.4	Conclusions	14
Chapter 3	Optimal Workforce Deployment	15
3.1	Preliminaries.....	15
3.2	Workforce Deployment Optimization Models.....	16
3.2.1	Scheduling Single Route: Plowing to Completion	16
3.2.2	Scheduling Multiple Passes of Different Routes	20
3.3	Conclusions	36
Chapter 4	Robust Crew Requirements	37
4.1	Introduction	37
4.2	Model Formulation.....	39
4.2.1	Phase I: Cluster Formulation	42
4.2.2	Phase II: Cluster Scheduling.....	44
4.3	Examples	47
4.4	Concluding Remarks	56

Chapter 5	Workforce Requirements Planning.....	57
5.1	Introduction	57
5.2	Model Description.....	58
5.3	Solution Methodology.....	60
5.4	Data Analysis and Default Parameter Settings	63
5.4.1	Monte-Carlo Simulation	63
5.4.2	Clustering Analysis.....	63
5.4.3	Default Parameter Settings	64
5.5	Results	65
5.6	Conclusions	69
Chapter 6	Quantifying Value of Flexibility.....	71
6.1	Graphical User Interface (GUI) for K-star Algorithm	72
6.1.1	Inputs and Outputs	72
6.1.2	Graphical User Interface (GUI)	73
6.2	Contract Workers	76
6.2.1	Inputs and Outputs	76
6.2.2	User Guide	77
6.3	Split Shifts.....	78
6.3.1	Inputs and Outputs	78
6.3.2	User Guide	79
6.4	Staggered Shifts.....	79
6.4.1	Inputs and Outputs	79
6.4.2	User Guide	80
6.5	Comparison of Different Workforce Deployment Strategies	80
6.5.1	Results.....	81
6.5.2	Analysis.....	82
References	85
Appendix A	Input Parameter Estimation	
Appendix B	User Guide: Calculations of the Rough-cut Capacity Requirements	

- Appendix C User Guide: Single Route Scheduling – Plowing to Completion (Chapter 3: MODEL 1)
- Appendix D User Guide: Scheduling Multiple Passes of Different Routes (Chapter 3: MODEL 2)
- Appendix E Two-phase Formulation
- Appendix F Delay Cost Multipliers by Road Type
- Appendix G OT and Delay Cost Functions
- Appendix H Proofs of Theoretical Results in Chapter 6
- Appendix I User Guide: Workforce Requirements Planning Under Different Scenarios
- Appendix J Analytical Results for Simplifications of Calculations of Workforce Deployment
- Appendix K The K-star Algorithm

List of Figures

Figure 2.1 The optimization of a snowplow route: (a) a snowplow route in SLC (displayed by the nautical line) and depot (displayed by \otimes), (b) the road segments selected for the first pass – solid lines represent roads to be actively traveled and dashed lines represent roads to be inactively traveled, (c) the second pass of the route contains sub-tours, and (d) the completed second pass with sub-tours removed.....	4
Figure 2.2 Route network: (a) network schematic, and (b) road matrix.....	8
Figure 2.3 The snowplow route is shown by dark lines. The circle includes a loop (a)-(b)-(c)-(d)-(e), which is nested among other loops.....	11
Figure 3.1 Illustration of problem parameters	25
Figure 3.2 Best plowing schedule for overtime block.....	28
Figure 3.3 Assignment of sorted first passes	29
Figure 3.4 Reordering of employees from shortest workload to longest workload.....	29
Figure 3.5 Assignment of sorted second passes.....	30
Figure 3.6 Assignment of sorted third passes and final schedule for three employees	30
Figure 3.7 Best schedule for storm type A; $t_0 = 2\text{am}$	32
Figure 3.8 Best schedule for storm type C; $t_0 = 2\text{am}$	32
Figure 3.9 Best schedule for storm type D; $t_0 = 2\text{am}$	33
Figure 3.10 Final schedule under storm type D (low intensity) and $t_0 = 6\text{pm}$	34
Figure 3.11 Final schedule under storm type D (low intensity) and $t_0 = 9\text{pm}$	35
Figure 3.12 Final schedule under storm type D (low intensity) and $t_0 = 4\text{am}$	35
Figure 4.1 Decisions hierarchy and information flow	38
Figure 4.2 Our solution approach including the two-phased inner loop.....	40
Figure 4.3 Example road network with plowing requirements data.....	47
Figure 4.4 Optimal plowing schedule under low storm intensity ($k = 2$)	50
Figure 4.5 Optimal plowing schedule under medium storm intensity ($k = 2$)	50
Figure 4.6 Optimal plowing schedule under high storm intensity ($k = 2$)	51
Figure 4.7 Example based on District 5 road segments.....	52
Figure 5.1. Flow chart of overall approach.....	58
Figure 5.2 Graphical representation of total costs	60
Figure 5.3 Depot 1 extra plowing costs, yearly plowing wages, and total costs versus k.....	67
Figure 5.4 Depot 1 distribution of extra plowing costs (OT + delay costs) over 10 simulated years	67
Figure 6.1 First window	74
Figure 6.2 Main Window	75
Figure 6.3 Default options window	75
Figure 6.4 Main window after running optimal k star	76
Figure 6.5 Options window for contract mode	77
Figure 6.6 Contract mode results	78

Figure 6.7 Results for split shift.....	79
Figure 6.8 Results of staggered shift.....	80
Figure D.1 Sample storm_type_t0.txt.....	D-1
Figure D.2 Sample passes.txt.....	D-2
Figure D.3 Sample unit_delay_costs.txt.....	D-2
Figure D.4 Sample regular_shift_times.txt.....	D-3
Figure D.5 Sample delay_cost_interval.txt (partial).....	D-4
Figure D.6 Sample pass_completion_times.txt.....	D-5
Figure D.7 Sample OT_cost_parameters.txt.....	D-6
Figure D.8 Sample numpasses_maxnumemp_passperemp.txt.....	D-6
Figure D.9 Sample wagerate_benefitpercentage.txt.....	D-7
Figure D.10 Sample results.txt.....	D-7
Figure D.11 Sample cost_results.txt.....	D-8
Figure D.12 Workforce deployment optimization program: Sample run screen.....	D-9
Figure G.1 Graphical representation of total costs.....	G-2
Figure I.1 Sample program screen of optimal_k_task4.exe.....	I-3
Figure I.2 Sample program screen of yearly_costs_kstar.exe.....	I-4

List of Tables

Table 2.1 Sand Application Rates Based on Road Temperature	5
Table 2.2 Clustered Storm Scenarios for SLC.....	7
Table 2.3 Snowplow Route Estimation for Route 501, 503, 508, 511, 515	14
Table 3.1 Completion Times of the Passes to Be Scheduled.....	31
Table 3.2 Delay Costs (\$) for Storm Type A (Medium Intensity).....	31
Table 3.3 Delay Costs (\$) for Storm Type C (High Intensity)	31
Table 3.4 Delay Costs (\$) for Storm Type D (Low Intensity).....	31
Table 4.1 Information Requirements for the Two-phase Solution Approach	41
Table 4.2 Phase I Terms: Default Priority Levels and Associated Weights.....	43
Table 4.3 Phase II Terms: Default Priority Levels and Associated Weights.....	46
Table 4.4 Results of Cluster Formation Algorithm	48
Table 4.5 Total Costs (TC), %Delay Costs (%D), and %Overtime Costs (%OT), $t_{\text{start}} = 9 \text{ PM}$	49
Table 4.6 Plowing Schedules under Each Storm Type ($k = 2$) (*TCT: Total Completion Time, TCTF: Total Completion Time under Fixed Schedule).....	50
Table 4.7 Total Costs for a Fixed Plowing Schedule, $t_{\text{start}} = 9 \text{ PM}$	51
Table 4.8 Data for Selected Road Segments.....	53
Table 4.9 Parameters for Each Storm Type	54
Table 4.10 Cluster Formation Results (Same for All Three Storm Types)	54
Table 4.11 Cluster Scores	55
Table 4.12 Total Costs (TC), %Delay (%D) and %Overtime (%OT), $t_{\text{start}} = 9 \text{ PM}$	55
Table 5.1 Optimal Number of Plow Operators for Each Depot in District 5 with Default Parameter Settings	66
Table 5.2 Current Number of Plow Operators in Four Depots of District 5	66
Table 5.3 Results under Modified Discretionary Parameter Values.....	69
Table 6.1 Depot 1 Results Comparison	81
Table 6.2 Depot 2 Results Comparison	81
Table 6.3 Depot 3 Results Comparison	82
Table 6.4 Depot 4 Results Comparison	82
Table A.1 Material Application Rates (from Wilson, Dadie-Amoah, and Zhang 2003).....	A-2
Table A.2 Application Rates from the Salt Institute.....	A-2
Table A.3 Proposed Sand/Salt Application Rates.....	A-2
Table A.4 Sample RWIS Data	A-4
Table A.5 Snowplow Speeds	A-4
Table A.6 Snowplow Speeds Based on Traffic Volumes.....	A-5
Table D.1 Delay Costs (\$) per 15-minutes for Each Pass Type and Traffic Density Period.....	D-3

Executive Summary

Shrinking budgets and high equipment and fuel costs have raised the importance of workforce planning and efficient deployment of available workforce for county-level winter maintenance operations. Workforce management practices need to be consistent with the county's level-of-service goals. This project developed a methodology for efficient deployment of available crew, estimation of workforce requirements, and economic evaluation of the impact of using contract employees, split shifts and staggered shifts. In order to achieve these goals, a fundamental question that needed to be addressed first was the determination of the amount of work induced by different types of storms that occur in Saint Louis County. Researchers obtained relevant storm data from a variety of weather reporting sources and extracted parameters relevant for determining plow speeds and sand/salt consumption. These parameters were used to determine optimal workforce deployment strategies that balance overtime and delay costs, which in turn provided estimates of the amount of plowing time needed for the goal of clearing roads within 24 hours after the end of snow fall. Plowing time calculations were subject to rules concerning when call outs can occur during off shift hours. Plow time estimates were subsequently used to develop efficient algorithms to calculate workforce requirements. The project was completed in five tasks and for each task, the underlying algorithms were embedded in software to support decisions made by county engineers.

Chapter 1 Introduction

This report contains the results of a study that examined the problem of workforce planning for winter maintenance operations for Saint Louis County (SLC) in Minnesota. The project has three major themes. In the first theme, the researchers worked with SLC managers to obtain relevant data and analyzed this data to obtain model inputs. Examples of data included time to clear objectives, paved/gravel road thresholds for operator call outs, weather data, road network data, plow routes and depot data, work rules, employee wage data, and average annual daily traffic count (AADT) data. As part of this theme, the researchers also developed a model to calculate the best way to group road segments belonging to each plow route into passes such that high priority road segments were in the same group, and each group represented a single pass of the plow that could be plowed and sanded with a single payload. All operations were assumed to occur in regular time mode. That is, it was not necessary to consider wage costs in this formulation. The results from this theme of the project are presented in Chapter 2. This work resulted in the development of a variety of decision support tools for extracting model inputs from data available from road weather information sites (RWIS), national weather service, and other sources of weather information, and displaying the results of route optimization. The results of this theme were used to compare route plow times predicted by the model and the actual route plow times.

The second theme focused on operations during overtime mode. The researchers solved three versions of the crew deployment problem. In the first two instances, it was assumed that the storm conditions were known and the maintenance supervisor had decided to plow either all remaining passes (obtained from results of Theme 1) of each route or a selected subset of passes across all routes. In the third setting, a robust approach to crew deployment was developed, given residual uncertainty in storm conditions at the time of making call out decisions. The results from this theme of the project are presented in Chapters 3 and 4. As before, the algorithms were converted into a series of decision support tools. The robust crew deployment problem was found to be computationally intensive and its solution required the use of commercial optimization software called CPLEX. It would be possible to develop a stand-alone computer code, based on heuristic approaches for solving the underlying discrete optimization problems, in a follow up study. At that point in time, data input could be standardized to make this approach useful to many different counties in Minnesota.

The third theme of the project concerned the determination of an optimal crew size and the evaluation of various flexibility enhancing strategies. Optimal crew size minimizes total cost, which is the sum of crew salaries (regular and overtime), costs of delays in clearing road segments, and the implicit costs of not meeting snow-removal objectives. The overall objective (e.g. clear roads within 24 hours) could be changed to consider its impact on total costs. The formulation of workforce requirements planning model also included a comparison of costs and

benefits resulting from the use of contract employees, split shifts and staggered shifts. The results of this effort are presented in Chapters 5 and 6 of the report. The algorithms developed to determine crew size and evaluate different deployment strategies were coded into a decision support tool with a graphical user interface.

Workforce planning for winter maintenance operations is a difficult problem because of uncertainty in storm conditions, work rules, nonlinear overtime costs, and complexity of determining optimal plow routes with varying constraints. This research explored a number of critical problems that arise in this arena. It provides the blueprint of a methodology that can be used to realize superior workforce decisions. However, this project focused on research and not on the development of professional software. That is, the continued use of algorithms developed by the researchers requires specialized knowledge. Developing user-friendly software that would interact with enterprise-level software to provide an automated decision support tool can facilitate the implementation of the research described in this report. That could be undertaken as the next phase of this project or farmed out to a commercial outfit using an appropriate intellectual property contract.

Chapter 2 Project Scope and Model Inputs

Given a forecast of storm, or characteristics of a storm that has recently ended, the key question that this research tries to answer is the following: How many man hours (or shifts) are needed to achieve the desired level of service (usually bare pavement) after the storm ends? The answer to this question depends on a variety of factors – weather conditions, storm parameters, traffic conditions, plow capacities (salt/sand payload) and deployment decisions made by the depot managers.

The key weather and storm parameters that affect plow speeds and sand/salt application rates are the air and pavement temperatures, moisture content, and wind speed. Plow speeds are also affected by traffic, which is largely determined by the AADT (average annual daily traffic) counts and time of day. In particular, speed is significantly lower during rush hours on high AADT roads. Payload capacities of the plows and sand/salt application rates affect the number of miles that a plow can travel before it is necessary to return to the refill depot. Weather conditions also determine the number of passes that may be necessary to achieve bare pavement.

This chapter focuses on analyzing and understanding a variety of data that can be used to determine manpower requirements for different storm scenarios. We develop a systematic methodology to extract/estimate key input parameters that allow Saint Louis County and other Minnesota counties to estimate manpower requirements for different storm types. The results of this chapter provide inputs for models reported in later chapters in which we explore the impact of different staffing levels on the costs of winter maintenance operations.

2.1 Snowplow Routing Data

The key data needed in this research are AADT counts, road-segment distances, sand/salt application rates, snowplow speeds, depot and equipment capabilities, and weather information. In order to clearly communicate how this approach may be replicated in other implementations, sections 2.1.1 through 2.1.4 present three types of information for each major data category: 1) the data type; 2) the source of the data and; 3) how relevant information may be extracted from the data. Complete details about data sources and estimation of input parameter are available in Appendix A. Additional data is presented in Sections 2.1.5 and 2.1.6.

2.1.1 AADT Counts

The AADT counts for roads in Minnesota are available on the web (Minnesota Department of Transportation, Transportation Data). The majority of AADT counts are available from 2003, and the AADT counts for major highways are available from 2006. These are located on road maps in PDF files and can be recorded manually for each relevant road. Some roads may not have AADT counts but those can be estimated by available AADT counts indicated on the surrounding/connecting roads.

2.1.2 Snowplow Routes and Road Lengths

The road network maps are obtained from a geographic information system (GIS) that can be viewed in ArcView (commercial GIS software). The GIS maps are manually converted into

network road maps as shown in the example in Figure 2.1. Routes can be extracted from ArcView for visual display. The lengths of road segments are also available from the GIS maps.

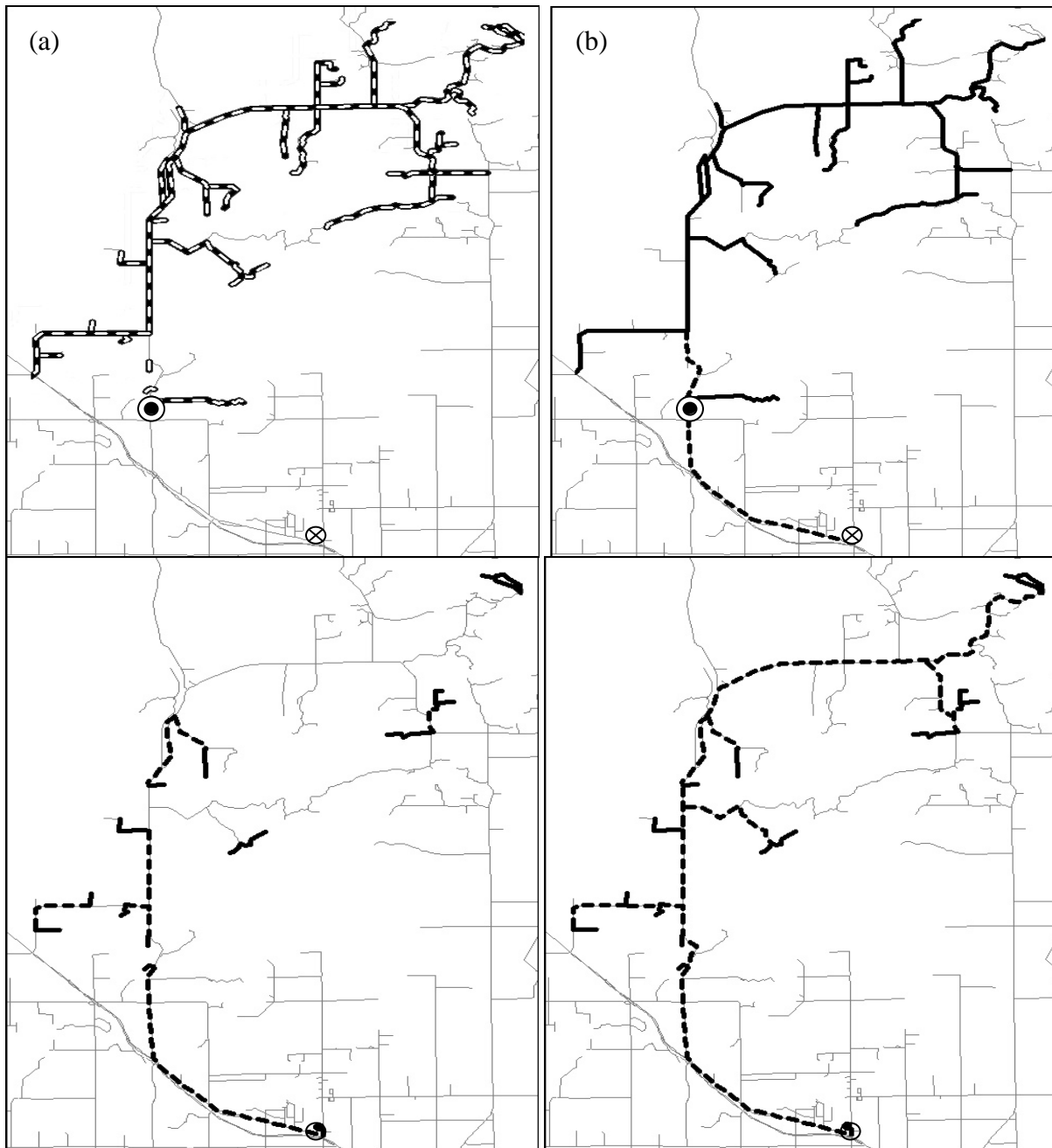


Figure 2.1 The optimization of a snowplow route: (a) a snowplow route in SLC (displayed by the nautical line) and depot (displayed by \otimes), (b) the road segments selected for the first pass – solid lines represent roads to be actively traveled and dashed lines represent roads to be inactively traveled, (c) the second pass of the route contains sub-tours, and (d) the completed second pass with sub-tours removed.

2.1.3 Sand/Salt Application Rates

SLC uses 90-percent-sand/10-percent-salt mixture. The sand/salt application rates were calculated based on three sources – (i) a previous study (Wilson et al., 2003), (ii) experts’ opinion, and (iii) the Salt Institute. For solving the optimization problem, the application rate data was divided into four categories based on the road temperature. The road temperature is one of the five parameters used to characterize storm scenarios and is the key factor that determines application rates. Table 2.1 displays the estimated application rates. Note, application rate is the same for all road segments for a given storm scenario.

Table 2.1 Sand Application Rates Based on Road Temperature

Pavement Temperature (F)	Sand Pounds per Lane Mile
Near 30	500
Below 30	450
Below 20	500
Below 10	800

2.1.4 Snowplow Speed

Average snowplow speeds were used in the optimization problem for both active and inactive mode of travel. Average snowplow speeds while in the active mode are based on storm parameters, see Wilson (2003) for details. The speeds range from 8 to 25 mph, and speeds above 20 mph decrease 5-10 mph from 7 AM – 7 PM in 1000-count or higher AADT areas. Each road is assigned a different active speed based on moisture content and accumulation rate. For sake of simplicity, roads traveled in inactive mode are assigned a speed of 30mph as recommended by experts from SLC. However, the formulation presented in this and later chapters allows for inactive speeds to be adjusted for each road segment as well.

The moisture content is a key factor for determining snowplow speeds. It is calculated from the air temperature, accumulation amounts, and the “New Snowfall to Estimated Melt Water Conversion Table” from the National Weather Service (2008). The moisture content is the snowfall depth divided by liquid equivalent. Moisture content between 1:1 and 9:1 is considered high, 9:1 and 15:1 is considered medium, and 15:1 to 100:1 is considered low (Roebber, 2003).

2.1.5 Equipment and Site Information

To illustrate the type of data we need, we mention equipment and site information for one depot in District 5 of SLC. In particular, this information pertains to the Pike Lake Depot. This depot has twelve tandem axle plow/sand trucks with a nine-yard sand capacity, one single axle plow/sand truck with a six-yard sand capacity, two single axle plow trucks, and four tandem axle plow trucks. The depot is capable of refilling both sand and fuel. For the sake of this study, all trucks were assumed to be tandem axle plow/sand trucks with a capacity of nine yards of sand/salt. The weight of the sand/salt mixture varies depending on the amount of moisture contained in the mix. However, on average a cubic yard of sand/salt weighs 1.05 tons. Thus a

snowplow can carry a maximum of 18900 lbs. The snowplows can operate for 10-12 hours per tank of gas. Therefore, a fuel constraint was not needed to solve the route optimization problem using SLC data because the sand and time constraints became active before the fuel constraint. Snowplows take approximately 3 minutes to turnaround.

2.1.6 Weather Data: Estimating Typical Storm Scenarios

Weather data were clustered into a small number of typical storm scenarios. Each storm instance is described by the following parameters:

- (1) Air temperature
- (2) Rate of snowfall
- (3) Total snowfall
- (4) Storm duration
- (5) Pavement temperature

The chosen storm parameters have a direct impact on winter operations. For example, air temperature and rate of snowfall are factors that affect plow speed, pavement temperature determines the sand/salt application rate, and storm duration affects the required crew size.

The National Weather Service's (NWS) climatic data for Duluth is found on the web (Minnesota Climatology Working Group, 2008). This source of data includes daily averages on temperature and total snowfall. Storm data is listed by day. This information was used primarily to identify the start and end of each storm event.

The Road and Weather Information System (RWIS) is a network of recording sites that measure and track weather data. Multiple recording sites are found in SLC. The RWIS information is located on the web (Minnesota Department of Transportation, SCAN Web 2008) and contains more detailed information than the NWS website. It includes current and historical data on air temperature, storm length, road temperature, total precipitation, and precipitation accumulation rates. The amount of data available can vary from one RWIS site to another based on the specific site's capabilities.

Precipitation can be converted to snow depth upon dividing by the moisture content. Given an air temperature (°F) and liquid precipitation depth (inches), the "New Snowfall to Estimated Melt Water Conversion Table" (National Weather Service, 2008) outputs a snowfall depth (in inches).

The storm data from the RWIS site was extracted and converted into Excel files. To efficiently analyze the data, a macro was created in Excel that converts the large amounts of weather data to a more compact form containing only the desired key storm parameters. The researchers were able to extract information on approximately fifty storms. Instructions on how to use the macros are included in Appendix B.

K-means clustering is a statistical tool that the researchers used to identify the typical storm scenarios. Note that K-means clustering requires the user to select the number of clusters and the number and type of variables to be clustered. The researchers chose to have five storm clusters. The number of clusters selected is equivalent to the number of times that the problem

must be solved for each route, so care must be exercised when selecting the number of clusters. Having more clusters results in a significantly greater effort in building the decision support tool. Similarly, the clustering variables must be selected carefully because too few variables can cause storms within each cluster to be quite different and too many variables can lead to less distinct clusters. Each component of storm intensity was scaled so individual components had equal weight when performing K-means clustering. The five storm clusters identified after analyzing SLC data are given in Table 2.2.

Table 2.2 Clustered Storm Scenarios for SLC

Variable	Cluster 1	Cluster 2	Cluster 3	Cluster 4	Cluster 5
Air Temp (°F)	27.1	27.4	21.4	18	10.8
Pavement Temp (°F)	30.6	31.2	23.3	24.4	17.8
Snowfall Total (inch)	12.1	2.4	8.8	4.2	2
Snowfall Rate (inch/hr)	0.55	0.5	1.31	0.27	0.51
Storm Duration (hrs)	22.3	5.7	9.5	17.3	5

The most frequently occurring storm type in SLC data was in Cluster 2. That is, a typical storm causes about 2.5 inches of snow, lasts 6 hours and occurs when the pavement temperature is just below freezing.

2.2 Model Development

2.2.1 Snowplow Route Network

The first step in solving the snowplow route optimization problem presented in this research is to build a route network. In a route network, each node represents a road-intersection, and connections between nodes (i.e. arcs) represent roads segments. Thus a route network is a set of nodes and connecting arcs that represent an actual snowplow route. Consider the route shown in Figure 2.2 (a). It consists of six nodes. Node 1 is the depot and each node represents a major road-intersection at which the snowplow can turn around. Two arcs (arrows) connect some nodes and represent two sides of the road (one lane each side). The route network is converted into a matrix to input data into a model. See the example road matrix given in Figure 2.2 (b). In this matrix, r_{12} is 1 because node 1 is connected to node 2, and r_{13} is 0 because node 1 is not connected to node 3.

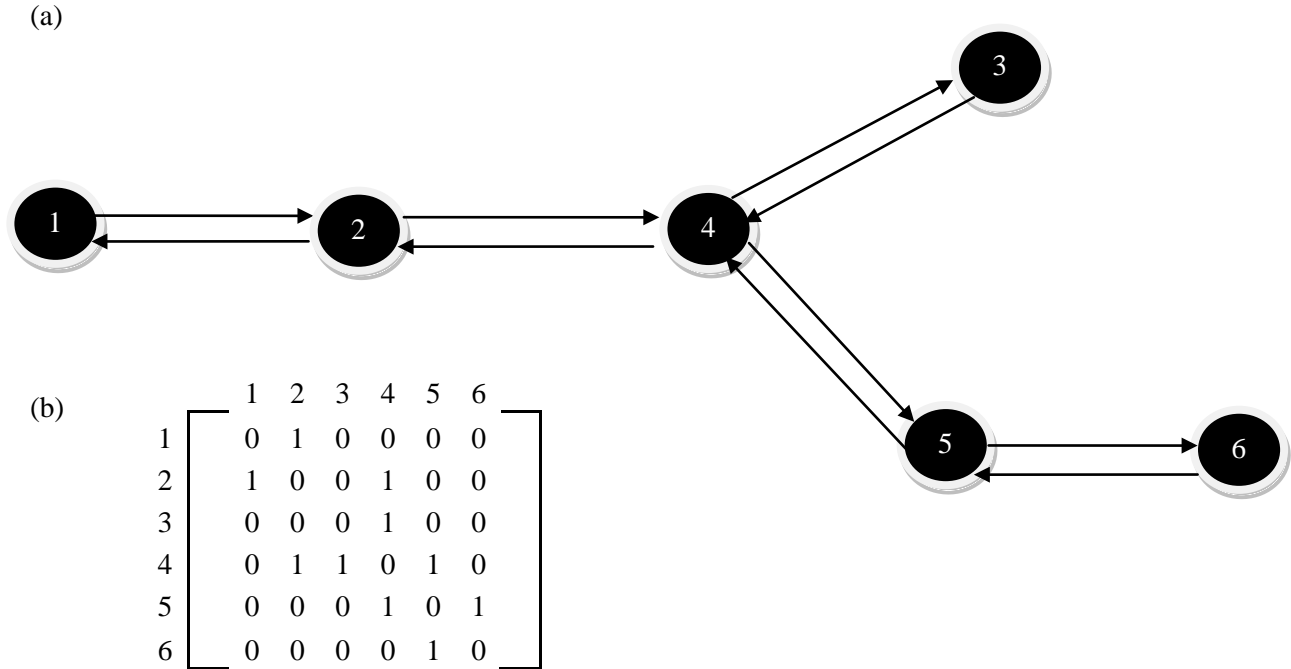


Figure 2.2 Route network: (a) network schematic, and (b) road matrix

2.2.2 Solution Approach

A set-partitioning formulation is used to select the arcs that a snowplow should actively traverse in each pass until all arcs have been actively traveled. The arcs selected in each pass are chosen because they yield the highest reward. The number of arcs that a snowplow can traverse is constrained by two factors: 1) the total time the snowplow can be away from the depot and, 2) the maximum amount of sand/salt that a snowplow can carry. If a pass contains sub-tours, a heuristic is used to eliminate them by treating each sub-tour as a node and solving the underlying Traveling Salesman Problem (TSP). Details of our approach are presented next.

Parameters

δ = average amount of time needed to turnaround

W = maximum vehicle payload in pounds (lbs)

K = maximum duration of a pass in hours

c = cost associated with turning a vehicle around

w = salt/sand application rate in lbs per lane mile

d_{ij} = distance of arc $i \rightarrow j$ in miles (mi.)

a_{ij} = AADT count of arc $i \rightarrow j$

v_{ij} = speed of snowplow on arc $i \rightarrow j$ traveling in active mode

α_{ij} = speed of a snowplow on arc $i \rightarrow j$ traveling in inactive mode

$r_{ij} = 1$ if arc $i \rightarrow j$ is an existing road segment, 0 otherwise

Decision Variables

$x_{ij} = 1$ if a truck actively traverses arc $i \rightarrow j$, 0 otherwise

$y_{ij} = 1$ if a truck inactively traverses arc $i \rightarrow j$, 0 otherwise

$t_{ij} = 1$ if a truck turns around at node j due to a dead end or a constraint that forces the truck to turnaround, 0 otherwise

$b_{ij} = 1$ if a truck turns around at node j due to a three node loop, 0 otherwise

Objective Function:

Choose sets $s_1 \dots s_p$ sequentially to

$$\text{Maximize } \sum_{i,j \in s_q} d_{ij} \alpha_{ij} x_{ij} - c \times t_{ij} - \frac{1}{6} c \times b_{ij} + c, \text{ for each pass } q, \quad (1)$$

where p is the maximum number of passes needed to cover all road segments.

Subject to:

$$\sum_{ij} x_{ij} \frac{d_{ij}}{v_{ij}} + y_{ij} \frac{d_{ij}}{\alpha} + t_{ij} \times \delta + \frac{1}{6} \delta \times b_{ij} - \delta \leq K \quad (2)$$

$$t_{ij} \leq x_{ij} + y_{ij} \text{ for all } i, j \in s_q \quad (3)$$

$$t_{ij} \leq x_{ji} + y_{ji} \text{ for all } i, j \in s_q \quad (4)$$

$$(x_{ij} + y_{ij}) - \sum_k r_{jk} \times (x_{jk} + y_{jk}) \leq t_{ij} \text{ for all } i, j \in s_q \quad (5)$$

$$t_{ij} + t_{ij} \leq 1 \text{ for all } i, j \in s_q \quad (6)$$

$$z_{ijk} - 0.8 \leq b_{ij} \text{ for all } i, j, k \in s_q \quad (7)$$

$$\sum_j x_{ij} + y_{ij} = \sum_j x_{ji} + y_{ji} \text{ for all } i \in s_q \quad (8)$$

$$\sum_{ij} d_{ij} \times w \times x_{ij} \leq W \text{ for all } i, j \in s_q \quad (9)$$

$$x_{ij} + y_{ij} \leq 1 \text{ for all } i, j \in s_q \quad (10)$$

$$x_{ij} = x_{ji} \text{ for all } i, j \in s_q \quad (11)$$

$$x_{ii}, y_{ii}, t_{ii} = 0 \text{ for all } i \in s_q \quad (12)$$

$$x_{0m} + y_{0m} = 1 \quad (13)$$

$$x_{m0} + y_{m0} = 1 \quad (14)$$

Here $z_{ijk} = \frac{1}{6}[(x_{ij} + y_{ij}) + (x_{ji} + y_{ji}) + (x_{jk} + y_{jk}) + (x_{kj} + y_{kj}) + (x_{ki} + y_{ki}) + (x_{ik} + y_{ik})]$. Also, note $s_q \subseteq \{S - s_1 - \dots - s_{q-1}\}$ and the procedure is applied until all segments in set S are actively traversed.

Objective function (1) prioritizes roads by their AADT counts and road lengths. That is, long roads with high AADT counts are actively traversed first. Turnarounds take time and add no benefit, so they are penalized. The third term in (1) penalizes three-node loop turnarounds, and it is multiplied by $\frac{1}{6}$ because there are six values of b_{ij} that are set equal to 1 for each three-node loop in which a turnaround occurs. This becomes clearer in the ensuing discussion. The final term in (1) removes the turnaround penalty counted by the formulation due to the snowplow beginning and ending at the depot (see Constraints 3 and 4).

Constraints 2 enforce time limitations because drivers need rest breaks. Constraints 3-7 account for turnarounds (see next paragraph for detailed explanation). Constraints 8 ensure that if a snowplow enters node i , then it must also exit node i . Constraints 9 enforce payload limits because snowplows can only carry a limited amount of sand/salt and must return to the depot to refill. Constraints 10 ensure that a snowplow can either traverse a segment actively or inactively, but not both. Constraints 11 make sure that if one side of the street is plowed during a pass, then the other side of the street must be plowed as well. Constraints 12 prevent a snowplow from traveling from a node to itself. Constraints 13 and 14 ensure that the snowplow travels from the depot (node 0) to the beginning of the route (node m) and returns to the depot along this same road segment. All the routes in SLC have a node that marks the beginning of a route. It is preselected to allow for the most efficient travel from the depot to a route.

Many streets in SLC are dead ends that require additional time to turnaround and plow, which is why turnarounds must be counted. Constraints 3-4 state that a snowplow must travel from node i to node j and back to node i for a turnaround to be counted. Constraints 5 state that if a snowplow travels from node i to node j , and has no other option at node j except to return to node i , a turnaround is counted. Constraints 6 avoid single road sub-tours as they do not allow turnarounds to occur at each end of a single road segment.

More complicated situations arise in turnaround accounting for loops (see Figure 2.3). Constraints 7 state if a snowplow actively traverses one street lane in a three-node loop, it must turnaround and actively traverse the opposite street lane. The formulation shown above assumes that no loops contain more than three nodes. It can be extended to larger loops by inputting constraints similar to those of Constraints 7. However, the process is manual. At the present time, the researchers do not have a general-purpose methodology for dealing with nested loops. However, upon examining each route and identifying nested loops, specific constraints that perform accurate turnaround accounting can be engineered.

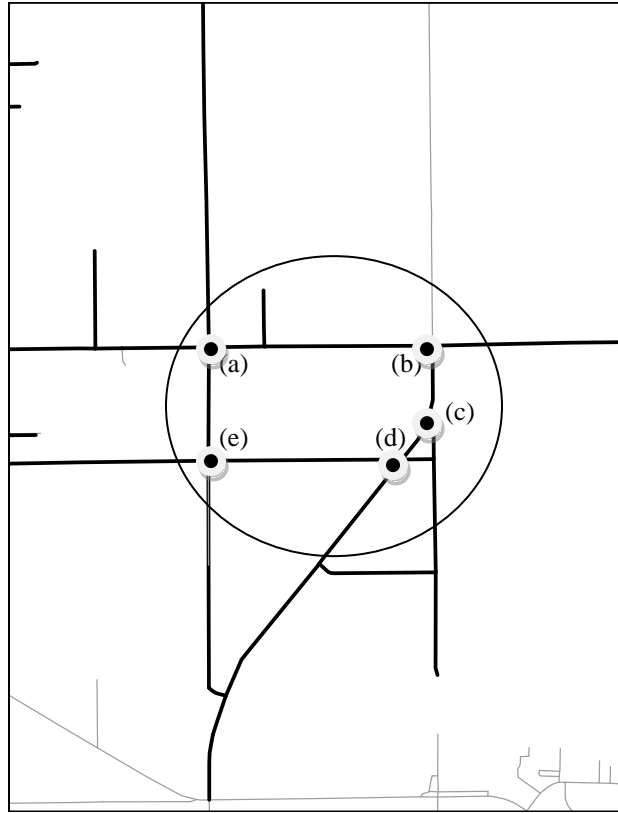


Figure 2.3 The snowplow route is shown by dark lines. The circle includes a loop (a)-(b)-(c)-(d)-(e), which is nested among other loops

Multiple lanes on the same road segment need additional effort because the road network must account for each road lane. Thus, if a road has two lanes in each direction, four arcs and one node are needed to distinguish the second lane. Details can be found in Kuchera (2008).

Solving the integer program 1-14 is time-consuming because the problem must be solved for each pass of the snowplow. In the first pass, all road segments are available to be selected for plowing and sanding. If a second pass is required, x_{ij} is set to 0 for those road segments that were already actively traversed in the first pass. If a third pass is required, x_{ij} is set to 0 for all those road segments that were already actively traversed in the first and second passes, and so on until all road segments have been actively traversed. Each instance was solved using ILOG software version 3.6.1. Solution time for each pass is minimal (less than one minute). The maximum number of passes needed for a given route in SLC was three.

The solution to the set-partitioning problem may contain sub-tours. This leads to additional complications when solving the snowplow route optimization problem. We used a heuristic approach to eliminate sub-tours. First the set of road segments were partitioned into passes using the above formulation. Then, in the event that sub-tours occurred, each sub-tour belonging to a pass was represented by a node in a TSP, and a TSP formulation was used with sub-tour elimination constraints, as explained in Laporte (1987), to obtain the path that the plow would travel.

This heuristic may not give a feasible solution for routes that spread over large distances. This is because the snowplow may not be able to travel to all sub-tours due to the time constraint. In order to obtain a solution, the time required to complete a route will need to be increased for a given pass, or road segments selected to be traversed in the active mode will need to be traversed in the inactive mode and subsequently traversed in the active mode in a future pass. The road segments selected in this way are chosen to minimize route-completion-time.

Some routes contain sidewalks that are plowed using the same equipment that is used to plow the roads. Sidewalks are left out of the mathematical formulation and are inserted at operators' discretion into the final pass because the impact of unplowed sidewalks on the community is less than that of roads. In some scenarios, both the last pass and second-to-last pass may contain sidewalks. This will occur only if all the road segments have been completed, and there is only enough time and/or sand to actively traverse a portion of the sidewalks. Thus, the snowplow will complete a portion of the sidewalks, and return to the depot to complete a final pass containing the remaining sidewalks. The sidewalks that are chosen for the second-to-last pass are selected to maximize snowplow coverage. The component of the decision support tool that displays passes includes the sidewalks so that snowplow operators can decide which sidewalks to include in which pass.

2.2.3 Data Matrices

Similar to the road matrix given in Figure 2.2 (b), we need to build AADT count, road length, and snowplow speed matrices. Care must be taken to model non-existent road segments and those that can only be traveled in inactive mode. The latter includes road segments that do not belong to the route. The following describes how the matrices are built to account for these unique road segments.

AADT counts are input into a matrix for existing road segments. Roads that cannot be traveled in active mode are assigned an AADT count of $-M$, where M is a large number (set equal to 10^6). Assigning $-M$ to these roads prevents them from being actively traveled because the objective function maximizes reward, thus only selects x_{ij} values with positive AADT counts. These roads can still be traversed in inactive mode because the AADT count has no impact on the y_{ij} decision variable.

Road segment lengths are input into a matrix. Non-existent roads are given a distance of M , which prevents them from being traveled due to maximum pass-duration constraints. All roads that can be traveled either actively or inactively are assigned a non-zero distance.

The snowplow speeds (mph) are also input into a matrix. Plow speeds may be different for each road segment. Non-existing roads are given a speed of 1. A speed of 1 is used rather than 0 because travel time is the distance divided by speed and dividing by 0 would result in error.

2.2.4 Sample Route

A sample route is optimized for a given storm scenario. This route contains multiple road segments displayed (see Figure 2.1 (a)). The depot is represented by \otimes and the beginning of the

route is represented by \odot . Recall that in our formulation the depot is denoted by subscript 0 and the beginning of a route is given by subscript m . For the first pass, the road segments that yield the highest reward are selected. These are displayed in Figure 2.1 (b). Notice that the road segments not selected are smaller, less traveled roads. After observing that the first pass contains no sub-tours, each x_{ij} chosen in this pass is set to 0 and the problem is solved for the second pass.

The second pass of the route contains sub-tours because multiple sets of roads do not connect back to the depot, as displayed in Figure 2.1 (c). A TSP with sub-tour elimination constraints is used to eliminate these sub-tours. Each sub-tour is treated as a node and travel distance is minimized to obtain a complete second pass. Figure 2.1(d) displays the completed second pass of the route with sub-tours removed. This pass is the last pass for this route as all road segments have been actively traversed.

2.3 Results

Several Excel spreadsheets and two macros were developed in this part of the project. The sheets contain basic data (such as road-segment lengths, speeds, AADT counts) for routes in District 5. The routing macro calculates the rough-cut capacity requirements – the number of plows needed for each target time to bare pavement. The weather analysis macro takes data from RWIS site and summarizes key weather metrics for each storm event. Instructions on how to use the macros are included in Appendix B.

Sample output obtained after analyzing different routes is summarized in Table 2.3. It shows estimates of plowing times for 5 arbitrarily chosen routes from District 5 for each of the 5 snow scenarios.

Table 2.3 Snowplow Route Estimation for Route 501, 503, 508, 511, 515

Route Number	Routing Information	Storm Senarios					
		<u>A</u>	<u>B</u>	<u>C</u>	<u>D</u>	<u>E</u>	
501	Number of Passes		2	2	2	1	2
	Time to Complete Entire Route	Plowing	4:17	4:17	5:26	3:13	3:35
		Sanding	4:42	4:42	6:16	3:13	4:19
	Sand/Salt Required (yard)		9.71	9.71	8.81	8.81	9.71
	Sand/Salt Application Rate (lbs/l _n mi)		500	500	450	450	500
	Snowplow Speed (MPH)		12	12	9	17	15
503	Number of Passes		2	2	2	2	2
	Time to Complete Entire Route	Plowing	6:52	6:52	8:00	5:53	6:12
		Sanding	7:27	7:27	8:27	6:47	6:58
	Sand/Salt Required (yard)		9.85	9.85	8.89	8.89	9.88
	Sand/Salt Application Rate (lbs/l _n mi)		500	500	450	450	500
	Snowplow Speed (MPH)		12	12	9	17	15
508	Number of Passes		2	2	2	2	2
	Time to Complete Entire Route	Plowing	5:22	5:22	6:37	4:16	4:37
		Sanding	5:52	5:52	7:12	4:45	5:01
	Sand/Salt Required (yard)		10.58	10.58	9.56	9.52	10.58
	Sand/Salt Application Rate (lbs/l _n mi)		500	500	450	450	500
	Snowplow Speed (MPH)		12	12	9	17	15
511	Number of Passes		2	2	3	2	2
	Time to Complete Entire Route	Plowing	7:04	7:04	8:51	5:30	6:00
		Sanding	7:57	7:57	10:50	6:34	6:58
	Sand/Salt Required (yard)		15.03	15.03	13.49	13.51	15.03
	Sand/Salt Application Rate (lbs/l _n mi)		500	500	450	450	500
	Snowplow Speed (MPH)		12	12	9	17	15
515	Number of Passes		1	1	2	1	1
	Time to Complete Entire Route	Plowing	4:04	4:04	4:59	3:15	3:31
		Sanding	4:04	4:04	5:39	3:15	3:31
	Sand/Salt Required (yard)		7.76	7.76	6.98	6.98	7.76
	Sand/Salt Application Rate (lbs/l _n mi)		500	500	450	450	500
	Snowplow Speed (MPH)		12	12	9	17	15

2.4 Conclusions

In this part of the project, we analyzed different types of data available, used data to develop estimates of manpower requirements, and studied the constraints on manpower planning and scheduling. These steps demonstrate that manpower requirements can be estimated from available data.

Chapter 3 Optimal Workforce Deployment

This chapter focuses on developing a decision support tool to optimize workforce deployment decisions under different storm conditions using the route and pass completion times from Chapter 2. The approach outlined here is suitable when a storm has already occurred, or storm conditions are highly predictable. We deal with the problem of robust crew deployment under storm uncertainty in Chapter 4.

Two different models are discussed. Each is appropriate under different plowing scenarios for SLC. The first model provides the optimum plowing start time for each route when the route needs to be plowed to completion. The second model is more flexible. It provides scheduling decisions for multiple passes of different routes and produces a complete solution for overtime period and regular shift hours. The key managerial inputs to the second model are the earliest time to call-out the employees (t_0) and the identity of passes that need to be plowed after t_0 . Given these input parameters, the questions that we answer in this chapter are the following: How many operators should be called-out? When should the employees start plowing? For how long should each operator work in the overtime mode? Which passes should be plowed during the next day's regular shift hours? What should be the plowing order of the passes that are assigned to each employee? The recommended schedule that answers these questions must have the minimum total cost, which is the sum of delay, overtime, and extra pay costs, among all possible schedules.

When scheduling passes, our approach gives priority to higher ranked passes, since they include the highest AADT road segments. The key factors that affect the final decision are the overtime cost structure (i.e. double-pay period and at least 4 hours of regular pay when an employee is called-out before the 4-hour period immediately preceding the shift start time) and delay costs (i.e. the cost of delaying the plowing operations, which depends on storm intensity and the time left till the start of regular shift). Upon analyzing problem instances with a variety of cost inputs, we found that if it is economical to do some of the passes in the overtime mode, then the best start time is either t_0 or a time that combines overtime block with the start of the next day's shift, so the employees continue working on the remaining portion of the passes during the regular shift hours.

3.1 Preliminaries

The key factors that determine the degrees of freedom that a county manager would have in calling plow operators either at the end of a storm or in anticipation of a storm can be divided into two categories — factors that define basic constraints, and rules that allow these constraints to be violated, at a cost, in accomplishing the winter maintenance work.

Basic Shift-Work Constraints

1. Operators work for a 7.5-hour shift, equivalently 37.5 hours per week, during their normal shift hours.
2. During the shift, workers are entitled to two 15-minute breaks and a 0.5-hour lunch break. Lunch break can occur any time during an hour before or an hour after mid shift.

3. Regular shifts currently begin at 7:30 AM and end at 3:30 PM. This includes the 0.5-hour lunch break.
4. Shifts must comprise of continuous 7.5 hours of work. Shift splitting is not permitted.
5. Workers whose shifts begin between 2 PM and 10 PM are paid an extra \$0.2 per hour.
6. Workers whose shifts begin between 10 PM and 5 AM are paid an extra \$0.25 per hour.

Rules and Costs of Constraint Violation

1. Workers are paid at least 1.5 times their regular pay if during a 24-hour period they work more than 7.5 hours, or if during a week they work more than 37.5 hours.
2. Workers are paid 2.0 times their regular pay if they are asked to work for more than 11 hours in a 24-hour period.
3. An employee may take compensatory time off (at 1.5 hours or 2.0 hours per hour of overtime worked according to the above rules) rather than overtime pay.
4. Employees may skip lunch and mandatory 15-minute break periods and receive 1.5 times their regular pay for that time period. This extra time worked cannot be taken as compensatory time off.
5. Employees may be called back to duty after their shift ends. If this happens, they must receive pay as follows.
Extra call back pay = $\max \{4 \text{ hours of regular pay, hours worked} \times 1.5 \text{ hourly rate}\}$.
That is, upon call back, a worker must receive at least 4 hours of regular pay.
6. Call backs cannot be credited as compensatory time off. They must be taken as overtime pay.
7. The call back rule above does not apply to the 4-hour period immediately preceding the start of a shift. That is, employees who are called to work earlier by no more than 4 hours before their regular shift start time receive 1.5 times their regular hourly pay for the number of extra hours worked.

3.2 Workforce Deployment Optimization Models

In this section, we describe two different models that address different instances of the workforce deployment optimization problem. The first model assumes that each route will be plowed to completion by the same employee. The second model builds on the first model and is developed by relaxing the assumption that routes should be plowed to completion. In the second model, SLC has the flexibility to select candidate passes of different routes that may be plowed in overtime mode in order to accommodate managerial discretion and the fact that certain passes may have been plowed during regular shift hours. In the following two sections, we present both models in detail.

3.2.1 Scheduling Single Route: Plowing to Completion

In this model, a simpler version of the problem is addressed. Specifically, we optimize the workforce deployment decisions for a given single route assuming that the route is plowed to completion (i.e. all the road segments of the selected route are plowed in the order determined in Chapter 2).

Model Formulation

We focus on the operation of a single depot and assume that the number of staff assigned to this depot equals the number of routes assigned to the depot. In this case, it is possible to assign a single operator to each route. We divide time into discrete time intervals, each 15-minute long. Thus, there are 96 time periods in a 24-hour period from 12:00:01 AM till 11:59:59 PM. Use of discrete time intervals helps reduce complexity of our formulation and it is also consistent with the unit of time used in calculating overtime pay. The price we pay for this simplification is that our approach is accurate only up to the nearest 15-minute interval. We focus on a single storm event. Here are the key assumptions we make in the model formulation.

1. Day 1 refers to the 24-hour period in which the earliest time to start plowing on a given route is t_0 . Because time is treated as discrete, t_0 can be one of $1, 2, \dots, 96$.
2. The storm is type k . The plow will require n_j passes to clear route j , $d_{l,j}$ is the number of 15-minute intervals needed to complete the l -th pass on route j , and $d_j = \sum_{\ell=1}^{n_j} d_{\ell,j}$ is the total route- j plowing time. See Kuchera and Gupta (2008) for a description of storm types, passes, and our overall methodology.
3. All employees assigned to the depot have the same shift start and end times. We assume these to be t_s and t_e respectively. Shift start and end times are also expressed in terms of the 15-minute intervals.
4. The regular shift consists of 32 intervals. If the lunch break is taken in the $[t_s + 13, t_s + 20]$ time interval, then there is no additional cost to SLC from the violation of lunch break constraint. The interval $[t_s + 13, t_s + 20]$ covers one hour before and one hour after mid shift.
5. If the length of a pass includes the interval $[t_s + 13, t_s + 20]$, then the operator is assumed to work through lunch for extra pay.
6. Regular time wages are $\$w$ per quarter-hour period. In reality wages depend on worker seniority. We note that regular time wages are sunk. We need wages per period to determine extra costs when an operator is asked to work outside of his/her regular shift. For that purpose, it is appropriate to use average wage rate per period, because SLC does not know which employee will agree to work overtime at the moment of making call-out/hold-back decision.
7. Once an operator starts to plow, (s)he continues to plow that route until it is completely clear. In the second model (see section 3.2.2), we relax this assumption and consider a variation where operators may plow high-reward passes first, and then turn to low-reward passes if plowing them in overtime mode is economical. It is also possible for SLC to plow only the high reward passes in overtime mode and have the plows return to complete the entire route during normal shift hours.
8. Cost of delay in the start of plowing is known and time-of-day dependent. Specifically, if SLC specifies the earliest time to start plowing operations as t_0 and if the county starts plowing operations at $t_0 + x_j$ on route j , then the cost of delay is $\sum_{\ell=0}^{x_j} c_{t_0+\ell,j}$, where the terms $c_{t_0+\ell,j}$ are assumed to be known. Moreover, these terms repeat after every 24 hours for each j and $c_{t_0,j} = 0$. In the second model, which allows plowing to be performed by pass (and not by route), we need to know the delay cost for each pass type separately.

We are now ready to provide a formulation of the problem of choosing x_j , the start of plowing operations on route j . Possible choices for x_j for day 1 are $1, \dots, 96 - t_0$. In some instances, it may not be economical to start plowing operations on day 1 (e.g. when the earliest plowing start time is specified close to midnight). In other instances, x_j may spill over to day 2 depending on route completion time. However, we do not allow plowing operations to be delayed by more than 24 hours in our model. Therefore, $x_j \in \{0, \dots, 192 - t_0 - d_j\}$, i.e. plowing operations must end by 192.

Given that plowing begins at $t_0 + x_j$, the plow will return to the depot at $t_0 + x_j + \sum_{i=1}^r d_{i,j}$ in the r -th pass, where $r = 1, \dots, n_j$. The total cost depends on which one of a number of events are associated with each choice of x_j . Since the number of choices is small and each pass can be solved independently in the simple version of our model, we will use a complete enumeration approach to solve for the best start time, i.e. workforce deployment strategy. The various events and related costs are listed below.

Event A occurs when the plow returns to the depot after completing a pass during an interval that would allow the operator to take a lunch break. If this happens, then lunch break does not result in an overtime cost. If not, then the county pays 1.5 times regular wages for two quarter-hour periods.

Event A \Leftrightarrow One of the following condition set holds:

- (1) for some $r \in \{1, \dots, n_j\}$, $(t_0 + x_j + \sum_{i=1}^r d_{i,j}) \in [t_s + 13, t_s + 20]$, or
- (2) for some $r \in \{1, \dots, n_j\}$, $(t_0 + x_j + \sum_{i=1}^r d_{i,j}) \in [t_s + 96 + 13, t_s + 96 + 20]$, or
- (3) $(t_0 + x_j + d_j) < (t_s + 13)$, or
- (4) $(t_s + 15) \leq (t_0 + x_j)$ and $(t_0 + x_j + d_j) < (t_s + 96 + 13)$, or
- (5) $(t_s + 96 + 15) \leq (t_0 + x_j)$,

then the lunch break occurs during the designated interval; otherwise the county's additional cost is $3w$. Recall that w is the regular wage rate per quarter-hour period.

Note that if Event A is true, $(t_0 + x_j + d_j)$ in the following conditions need to be replaced by $(t_0 + x_j + d_j + 2)$ to include the lunch break.

Event B occurs when plowing begins and ends during the hours of a regular shift. If event B happens, the county incurs no additional cost of plowing because the operators' regular wages are sunk.

Event B \Leftrightarrow

- If either $t_s \leq (t_0 + x_j) \leq (t_0 + x_j + d_j) \leq t_e$,
or $t_s + 96 \leq (t_0 + x_j) \leq (t_0 + x_j + d_j) \leq t_e + 96$,
then plowing occurs during regular shift hours.

Event C occurs when plowing starts before the regular shift start, but no more than 3.5 hours prior, and ends by the time that the shift ends. Because no route in our analysis so far takes more than 8 hours to plow, this would be possible to achieve for all routes.

Event C \Leftrightarrow

If either $t_s - 16 \leq (t_0 + x_j) < t_s \leq (t_0 + x_j + d_j) \leq t_e$,
or $t_s + 96 - 16 \leq (t_0 + x_j) < t_s + 96 \leq (t_0 + x_j + d_j) \leq t_e + 96$,
then the county pays for extra hours worked prior to the shift start. That is, county's
extra cost for route j is either $(1.5)(t_s - (t_0 + x_j))w$ or $(1.5)(t_s + 96 - (t_0 + x_j))w$.

Event D happens when plowing begins more than 3.5 hours prior to the start of a shift and ends before the end of the shift.

Event D \Leftrightarrow

If either $(t_0 + x_j) < t_s - 16 \leq t_s \leq (t_0 + x_j + d_j) \leq t_e$,
or $(t_0 + x_j) < t_s + 96 - 16 \leq t_s + 96 \leq (t_0 + x_j + d_j) \leq t_e + 96$,

then the county pays for extra hours worked prior to the shift start. That is, county's
extra cost for route j is either $(2.0)(t_s - 16 - (t_0 + x_j))w + (1.5)(16)w$ or $(2.0)(t_s + 96 - 16 - (t_0 + x_j))w + (1.5)(16)w$.

Event E occurs when plowing begins after the shift start time but extends after the shift end time for less than 3.5 extra hours.

Event E \Leftrightarrow

If either $t_s \leq (t_0 + x_j) \leq t_e < (t_0 + x_j + d_j) \leq t_e + 16$,
or $t_s + 96 \leq (t_0 + x_j) \leq t_e + 96 < (t_0 + x_j + d_j) \leq t_e + 96 + 16$,
then the county pays for extra hours worked after the shift end. That is, county's extra
cost for route j is either $(1.5)((t_0 + x_j + d_j) - t_e)w$ or $(1.5)((t_0 + x_j + d_j) - (t_e + 96))w$.

Event F occurs when plowing begins after the shift start time but extends more than 3.5 hours after the shift end time.

Event F \Leftrightarrow

If either $t_s \leq (t_0 + x_j) \leq t_e$ and $t_e + 16 \leq (t_0 + x_j + d_j)$,
or $t_s + 96 \leq (t_0 + x_j) \leq t_e + 96$ and $t_e + 96 + 16 \leq (t_0 + x_j + d_j)$,
then the county pays for extra hours worked after the shift end. That is, county's extra
cost for route j is either $(1.5)(16)w + (2.0)((t_0 + x_j + d_j) - (t_e + 16))w$ or
 $(1.5)(16)w + (2.0)((t_0 + x_j + d_j) - (t_e + 96 + 16))w$.

Event G occurs when plowing starts and ends outside the normal shift hours. In this case, the entire plowing time is paid on an overtime basis.

Event G \Leftrightarrow

If $[(t_0 + x_j), (t_0 + x_j + d_j)] \notin [t_s, t_e]$ and $[(t_0 + x_j), (t_0 + x_j + d_j)] \notin [t_s + 96, t_e + 96]$,

then the route is plowed outside the normal shift hours. In this case, the county incurs an additional cost that equals $\max \{16w, [(1.5)(\min(14, d_j)) + (2.0)(d_j - 14) + w]$.

With the above cost calculations in hand, we can write $h_j(x_j|t_0)$, the cost of plowing route j by starting plowing operations at x_j given that storm ends at time t_0 , for every possible choice of x_j . The optimal start time is the value of x_j that minimizes $h_j(x_j|t_0)$.

The model formulation is turned into an easy-to-use Excel-based decision support tool. The decision support tool has an additional feature for 2-inch/4-inch snow accumulation rule. According to SLC policy, if snow accumulation is below 2 inches on paved and 4 inches on gravel roads, then the employees should not be called out for overtime period. In that case, plowing operations for selected route occurs during regular shift hours. On the other hand, if snow accumulation is more than 2 inches on paved and 4 inches on gravel road segments of selected route, then above cost formulations apply. If SLC engineers/managers would like to call-out employees even if there is not enough snow accumulation, then the assignment feature based on snow accumulation can be turned off by changing the option for the 2-inch/4-inch rule in the drop-down menu. The decision support tool is included in CD that accompanies this report. Details on how to use the single route scheduling program are presented in Appendix C.

3.2.2 Scheduling Multiple Passes of Different Routes

In this model, we build on the previous modeling approach by considering the fact that routes may not be plowed to completion. In that case, different passes of the same route can be done at different times and by possibly different employees, which makes the new decision problem much harder as compared to the simpler version of assigning an entire route to a single employee. An advantage of this new modeling approach is that it exploits the flexibility in overtime work assignments to minimize labor costs while meeting service expectations. For example, when storm conditions dictate that plowing operations need to be carried out during overtime period, depot managers may call-out employees to do only first passes (i.e. high AADT road segments) at overtime rate and leave second and third passes for plowing during next day's regular shift hours.

We focus on the operation of a single depot. This is reasonable because plowing operations are indeed independent for each depot. The maximum number of employees assigned to this depot is assumed to be equal to the number of routes assigned to the depot. Time is divided into discrete time intervals, each of 15-minute length, and we focus on a single storm event with known characteristics.

In what follows, we provide a description of the modeling assumptions, the decisions that need to be made (outputs), and required parameters (inputs) needed to solve the model. We also present our solution approach.

Assumptions

1. The required number of employees is always available when either a call-out or hold-out event occurs.
2. If storm conditions require the start of plowing operations during non-shift hours, then we assume that the employees who are called-out start working at the same time (t_{start}).
3. The employees can be called-out or held back as early as time t_0 , which is equal to or after current day's shift end time (t_e) and before the next day's shift start time ($t_s + 96$), where 96 corresponds to the number of 15-minute intervals in a 24-hour day. Time epoch t_0 can be viewed as the time epoch at which overtime plowing decision is made. We assume that the passes that will be completed by t_0 are known. Since t_0 corresponds to a time within overtime period, we assume that the depot manager considers 2-inch/4-inch rule when selecting the passes to be plowed after t_0 . We assume that the depot manager knows t_0 based on storm predictions and managerial discretion. Below, we provide guidelines on selecting t_0 :
 - If storm occurs during regular shift hours, operators typically start plowing immediately. However, SLC managers need to decide when to start the plowing operations for those passes that cannot be completed during regular shift hours. In that case, set t_0 equal to the regular shift end time (t_e).
 - If storm occurs during overtime period, then set t_0 to be the anticipated snow storm end time.
 - If SLC managers are unsure about the anticipated snow storm end time, then set t_0 equal to either t_e or the current time, whichever is later.

In all the above three cases, our program will select the economically best start time during overtime period or decide on delaying the plowing operations to next day's regular shift. When selecting t_0 , SLC engineers/managers should also consider that since $t_{start} \geq t_0$, it must be feasible to start the plowing operations immediately at t_0 . We currently do not account for the need to re-plow certain passes due to additional snow fall after t_0 .

4. The completion time for each pass is converted to the number of 15-minute intervals needed to complete the pass for a given storm scenario.
5. All employees assigned to the depot have the same shift start (t_s) and end times (t_e).
6. Different employees may work on different passes of the same route during overtime and regular hours.
7. The regular shift consists of 32 intervals. Lunch break can be taken anytime between one hour before and one hour after mid shift (i.e. between 10:30am and 12:30pm for 7:30am start time), which corresponds to the interval $[t_s + 13, t_s + 20]$.
8. If the plowing time of a pass includes the lunch break interval, then the operator is assumed to work through lunch for extra pay. If several passes assigned to an operator end within the lunch break interval, then the operator is assumed to take lunch break at the end of his/her last pass that ends within the lunch break interval. This assumption minimizes delay cost. However, it is not central to our model and easily relaxed to reflect other modes of operation.

9. Average regular time wages are $\$w$ per quarter-hour period. See section 3.2.1 for reason why it is reasonable to ignore worker seniority for calculating wage costs of overtime work.
10. Cost of delaying start of plowing is assumed to be known and traffic (time-of-day) dependent. In our model, we propose three time-of-day dependent delay cost categories. These categories are (1) high traffic density ([6am, 10am] and [3pm, 8pm]), (2) medium traffic density ([10am, 3pm] and [8pm, 12am]), and (3) low traffic density ([12am, 6am]) periods.
11. Cost of delay also depends on pass type. We assume that all the first passes have the same delay costs. Similarly, all second passes have the same delay costs and all the third passes have the same delay costs. Since first passes consist of the highest reward road segments, cost of delaying plowing of first passes is higher than second passes, and the delay costs for second passes are higher than those of third passes.

Outputs (Decisions to Make)

Given the above modeling assumptions, we now list the outputs of our model. The following decisions will be made by the model with the objective of minimizing the sum of overtime costs, delay costs and extra costs. The extra cost consists of compensatory pay given to an employee who works during the lunch break.

1. How many employees to call-out or hold-back (k)?
2. When to call-out employees (t_{start}), i.e. the time at which employees should start working during overtime period?
3. How long to keep each employee for overtime work?
4. Which passes to assign to each employee who is called-out/held-back for overtime work and in what sequence should this employee work on the assigned pieces of work?
5. Which passes should be left to be plowed for the next day's regular shift hours and in what sequence they should be plowed?

Inputs:

The following are the input requirements, which need to be specified by the user before solving the model.

1. Average wage rate (in dollars per 15 minutes)
2. Delay costs, i.e. the cost of delaying the plowing operations after t_0 separately for three pass types and three time intervals during a day
3. Storm type
4. Earliest time to call-out or hold-back the employees (t_0). This time must lie in the interval $[t_e, t_s]$, which is currently 3:30pm – 7:30am.
5. Time to complete each pass under different storm conditions (obtained from Chapter 2)
6. Passes that are not done at t_0 , i.e. the passes that need to be scheduled.

In the above list, t_0 and delay costs are discretionary parameters chosen by SLC management. We provide guidelines on how to select t_0 in the Assumptions Section (item # 3). We provide guidelines for choosing delay costs in the next section.

Guidelines for selecting delay costs

Since the best schedule depends on the choice of unit delay costs, in this part we provide general guidelines to determine input delay cost values as a function of overtime wage rates. Delaying the start of plowing operations is not desirable due to inconvenience to public, especially during high traffic density periods, and also due to compaction of snow caused by traffic, which makes snow removal harder when delays occur.

Unit delay cost for a given pass type and traffic density interval is a measure of SLC's willingness to delay plowing operations under a given storm scenario for 15 minutes. Since there is a trade-off between providing an immediate plowing service and the overtime costs to be paid by the county, delay costs should be determined in relation to overtime pay rates. SLC engineers/managers should consider the following guidelines in setting the values for delay cost parameters:

1. Delay costs should be higher for higher traffic density time intervals. Therefore, delay costs for high traffic density interval $>$ delay costs for medium traffic density interval $>$ delay costs for low traffic density interval.
2. Unit delay costs should be higher for higher-ranked passes, because higher order passes include higher AADT road segments. Therefore, unit delay costs for first passes \geq unit delay costs for second passes \geq unit delay costs for third passes.
3. Delay costs should depend on expected storm conditions and should be higher for higher intensity storm types. When same delay costs are assumed for low and high-intensity storms, passes for higher intensity storm conditions are more likely to be delayed to regular shift hours due to their higher overtime costs. This happens because passes usually take longer to clear under higher intensity storm conditions.
4. For a given pass type, if we choose delay costs for traffic density intervals from t_0 to t_s , the shift start time, to be much greater than double-pay overtime wage rates, then plowing operations for these type of passes will be assigned to overtime period in the best schedule. If there are enough employees, then each pass will be assigned to a different employee and plowing will start at t_0 . If the number of employees is insufficient, then priority will be given to shorter passes of the same pass type. (NOTE: Currently, regular wage rate is \$35 per hour including fringe benefits and benefit rate is 62%. Therefore, overtime wage rate is $(1.5) \left(\frac{\$35}{4 \times 1.62} \right) = \8.1 per 15 minutes and during double-pay periods, this rate increases to \$10.8 per 15 minutes)
5. In contrast, if we choose delay costs for a given pass type for all traffic density intervals that lie in t_0 to t_s to be much less than regular overtime wage rates for 15 minutes, then that type of passes will be delayed to next day's shift.
6. If for a given pass type and traffic density interval type:
 - Delay cost $>$ regular overtime wage rate (currently \$8.1), then this type of pass will not be delayed during the traffic density interval provided the pass length is at most 2 hours 40 minutes.
 - Delay cost $>$ double-pay overtime wage rate (currently \$10.8), then this type of pass will not be delayed during the traffic density interval.
 - Delay cost $<$ regular overtime wage rate:

In this case, whether the pass will be delayed during the traffic density interval depends on two things: (1) the length of time between t_0 and the start of the traffic density interval or the start of the next day's shift (whichever comes earlier), and (2) the pass length. Therefore, the decision to delay a pass of given type cannot be explained easily on an intuitive level. If t_0 is close to next day's shift and the pass completion time is long, then we expect that the pass will be delayed to next day's shift. If t_0 is far from next day's shift start time or the pass length is not too long, then we expect that plowing of the pass will start during overtime period in the economically best schedule. To explore how the selection of unit delay costs affects the resulting schedules, SLC engineers/managers should run the program for a variety of unit delay cost settings. If SLC engineers/managers find that some of the passes that they would like to delay are assigned to overtime period, then the managers should decrease unit delay costs more and run the program again.

For example, if a pass is 3 hours long and the delay costs for this pass are \$6, \$4 and \$2 for 15-minutes in high, medium and low traffic density intervals respectively. The overtime cost for this pass is $3 * 4 * \$8.1 = \97.2 . Total delay costs until the start of next day's shift for $t_0 = 8\text{pm}$ is $16 * \$4 + 24 * \$2 + 6 * \$6 = \148 , which means that assigning the pass to overtime period (to start at t_0) is better as compared to delaying to next day even though all the unit delay costs are below regular overtime wage rate. Therefore, if SLC engineers/managers would like to delay some of the passes, they should decrease unit delay costs for these type of passes when $t_0 = 8\text{pm}$. On the other hand, if $t_0 = 1\text{am}$, then total delay costs until the start of next day's shift is $20 * \$2 + 6 * \$6 = \$76$, so in this case, delaying the pass to next day's shift is the best decision.

Solution Method

For each possible value of start time (t_{start}) during overtime period, overtime work block (b) and number of employees to call-out/hold-back (k), the solution method consists of two phases: (1) a *construction phase* to find a good initial schedule and (2) an *improvement phase* that looks for a better schedule by interchanging passes among employees given the initial solution. In the construction phase, our approach is to first assign first passes to the operators as long as they can be done within the given overtime work block. This process then continues with the second passes and finally the third passes. The decision to assign passes in this sequence can be justified by their relative delay costs. Figure 3.1 provides a schematic that explains the meaning of terms t_0 , t_{start} , b and k .

In the sequel, we describe the steps of our solution methodology.

Step 1. Determine the minimum and maximum values for k , t_{start} , and b . Set minimum cost as infinity, i.e. $\text{min_cost} = \infty$.

- $k : 0 \leq k \leq \text{number of routes}$
- $t_{\text{start}} : t_0 \leq t_{\text{start}} \leq \text{regular shift start time } (t_s)$

- o $b: 0 \leq b \leq t_s - t_{\text{start}}$

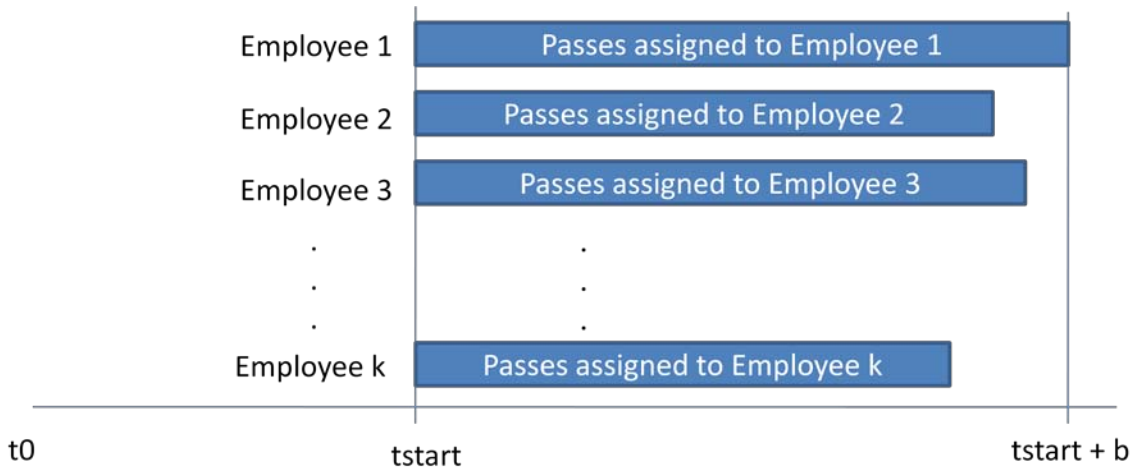


Figure 3.1 Illustration of problem parameters

Step 2. Sort first passes from the shortest completion time to the longest. Repeat the same sorting procedure for second and third passes.

Step 3. Start an iterative procedure for each combination of k , t_{start} , and b and apply the following steps:

Phase 1: Construction phase

1. Assign sorted passes to employees during overtime block
 - Start by assigning sorted first passes. The assignment procedure for the first k passes is from first employee to the last, and then for the second set of k first passes, the assignment is done in reverse order, i.e. from the last employee to the first one. The order is reversed for each new set of k passes until the number of first passes to be assigned is less than k , in which case the remaining passes will be assigned to the employees in the order of shortest work time to longest. The assignment procedure continues until there are no remaining first passes that can be assigned within the block size b .
 - The initial assignment procedure above attempts to minimize delay costs (by assigning shortest passes first) and also to keep overtime work hours for each employee as uniform as possible (by reversing the assignment order of sorted passes to employees). The latter minimizes the number of schedules that result in double-pay.
 - During the assignment procedure, if a pass cannot be assigned to a selected employee due to time limits, then we search for another employee who has less work and who could plow the pass.
 - If the overtime block is contiguous with the next day's shift, then block size b is not a constraint on pass assignment, so an employee may continue working on the passes from overtime block into next day's shift.

- After assigning first passes to k employees, we reorder the employees from shortest workload to longest workload and start assigning sorted second passes.
 - We repeat the procedure described above for second passes, and then for third passes.
2. Assign remaining sorted passes to the employees for next day's shift
 - At the end of overtime assignment procedure, the passes that are left to next day are determined. For the next day's shift, the number of employees to be considered is fixed and equal to the number of routes assigned to the depot in consideration.
 - We apply the same assignment procedure for the remaining sorted passes again starting from the remaining first passes.
 3. Calculate overtime employee costs
 - For each employee in current day's overtime period and next day's shift, we first calculate the number of 15-minutes worked (N) during overtime period.
 - For a given employee i ,
 - o If N is less than or equal to 14 (i.e. overtime ≤ 3.5 hours), then overtime cost (OC_i) of employee i is $(N)(1.5)w$.
 - o If N is greater than 3.5 hours, then $OC_i = (14)(1.5w) + (N - 14)(2.0)w$
 - o If t_{start} is after regular shift end time (t_e), which means employees are called back instead of being held over after their shift, and if t_{start} is before four hours prior to shift start time (i.e. $t_s - 16$), then OC_i should be at least $16w$ (i.e. four hours of regular pay). In such cases, if previous calculation for $OC_i < 16w$, then $OC_i = 16w$.
 - Then, total overtime costs are calculated as $OC = \sum_{i \in \text{all employees}} OC_i$
 4. Calculate delay costs
 - For each pass j , first we find its start time t_j .
 - Delay cost for pass j is then equal to $\sum_{t=t_0}^{t_j-1} DC_{j,t}$, where $DC_{j,t}$ is the cost of delaying pass j at time interval t .
 - Total delay costs for all the assigned passes j is then equal to $DC = \sum_{j \in \text{all assigned passes}} \sum_{t=t_0}^{t_j-1} DC_{j,t}$.
 5. Calculate extra costs due to lunch break
 - For each employee i assigned to next day's shift, we first find the end time of each pass j assigned to employee i .
 - If the employee finishes his work after the beginning of lunch break interval and if none of the passes assigned to employee i ends within the lunch break interval of [10:30am,12:30pm], then the extra cost incurred for employee i is $EC_i = 3w$, otherwise $EC_i = 0$.
 - Total extra costs equals $EC = \sum_{i \in \text{next day employees}} EC_i$
 6. Calculate total cost of current schedule for the given values of k, t_{start}, b .
 - Total cost (TC) is then equal to $TC = TC_1 = OC + DC + EC$.

Phase 2: Improvement phase

Try to improve the current schedule by interchanging passes between employees.

7. Interchange method 1: Pass insertion

- Temporarily remove one of the passes (j) from one of the employees' schedule (i_1) in overtime work block of construction phase solution
- Try inserting removed pass to another employee's schedule (i_2)
- If pass j can be inserted, then rearrange schedule i_2 to minimize delay costs
- Try inserting the remaining passes (i.e. the passes that were left to next day in construction phase schedule) to schedule i_1
- Assign remaining passes to next day's shift using the original assignment method from construction phase
- Calculate total cost of the new overtime and next day schedules (TC_2)
- If $TC_2 < TC_1$, then update the best schedule for k , t_{start} , and b and record the minimum cost so far (\min_TC_2)
- Repeat the pass insertion method for all employees and all passes in the overtime block of construction phase solution for given k , t_{start} , and b .

8. Interchange method 2: Pass exchange

- Consider the best schedule for k , t_{start} , and b as the starting solution
- Temporarily remove one of the passes (j_1) from one of the employees' schedule (i_1) and one of the passes (j_2) from another employees' schedule (i_2) in the overtime work block of the starting solution
- Try inserting pass (j_1) to schedule (i_2) and pass (j_2) to schedule (i_1)
- If both passes can be inserted, then rearrange schedules i_1 and i_2 as to minimize delay costs
- Try inserting the remaining passes (i.e. the ones that were left to next day in the starting solution of this interchanging method) to schedules i_1 and i_2
- Assign remaining passes to next day's shift using the original assignment method in construction phase
- Calculate total cost of the new overtime and next day schedules (TC_3)
- If $TC_3 < \min_TC_2$, then update the best schedule and record the minimum cost so far (\min_TC_3) for k , t_{start} , and b
- Repeat the pass exchange method for all employees and passes in the overtime block of starting solution for given k , t_{start} , and b .

9. Update minimum cost schedule.

- If $\min_TC_3 < \min_cost$, then update minimum cost schedule with current schedule and assign $\min_cost = \min_TC_3$.

10. If there is at least one combination of (k, t_{start}, b) , which is not considered, then repeat Steps 3-9 for the next combination. Otherwise, output overtime and next day scheduling decisions.

The solution method is coded using programming language C and turned into an executable computer program. Details on how to use the program are presented in Appendix D.

Illustration of solution approach: Small example

The following small example illustrates the implementation of the first step of the construction phase in Step3 for specific values of k and b . This example also shows that our solution method can obtain the best possible schedule in the construction phase. In the case that the final schedule of construction phase is not the best schedule, then a better schedule is identified during the improvement phase.

Assume that $k = 3, b = 14$, number of first passes = 2 with sorted lengths of (12, 16), number of second passes = 3 with sorted lengths of (8, 10, 12), and number of third passes = 3 with sorted lengths of (6, 8, 10). We assume that cost of delaying a pass to next day is greater than the cost of assigning the pass to overtime block for this example. We also assume that delay costs have the following relation, first pass > second pass > third pass. After considering all feasible assignments, the best assignment for the overtime block to minimize total of overtime and delay costs is shown below in Figure 3.2.

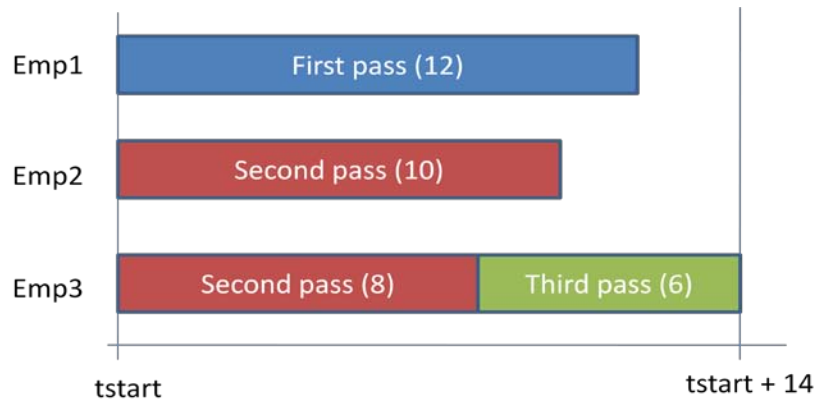


Figure 3.2 Best plowing schedule for overtime block

Based on the assignment above, four of the initial passes (i.e. one first pass (16), one second pass (12) and two third passes (8, 10)) are left to next day's regular shift period.

We now show the implementation of our solution method on this example. We start by assigning first passes in sorted order starting from the employee with the shortest workload (see Figure 3.3). Since the block size is 14, first pass of length 16 cannot fit into the overtime block and it must be plowed during next day's shift.

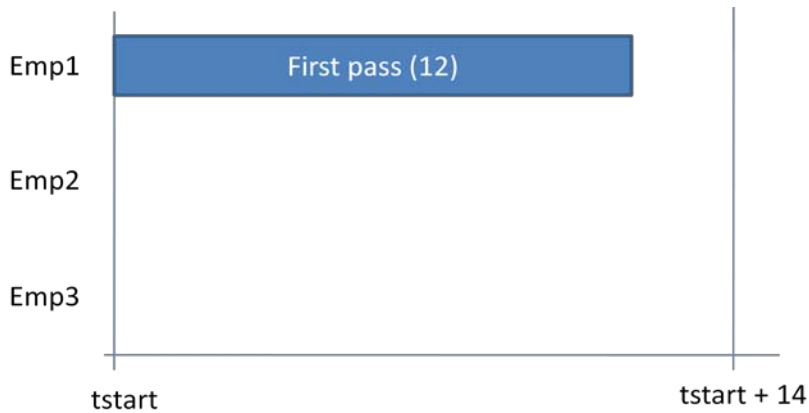


Figure 3.3 Assignment of sorted first passes

Then, we reorder the employees from shortest workload to longest as shown in Figure 3.4 before assigning second passes.

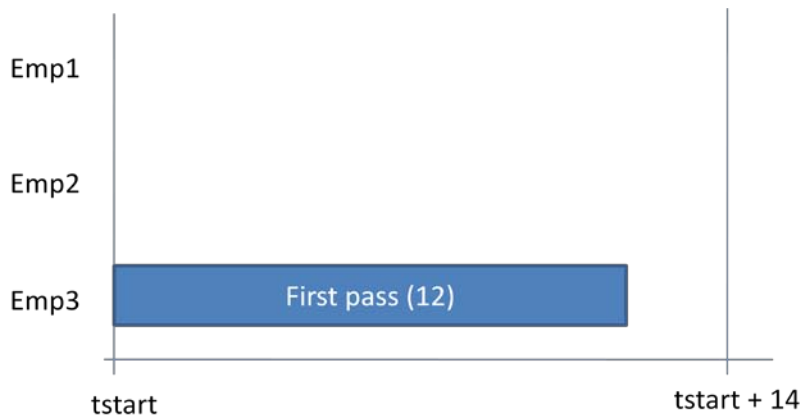


Figure 3.4 Reordering of employees from shortest workload to longest workload

We continue by assigning sorted second passes starting from first employee until less than k second passes left to be assigned. Therefore, we assign the shortest second pass to first employee and then assign remaining second passes each time to the employee with the shortest workload. Figure 3.5 shows the schedule after the assignment of second passes. Since there is not enough time remaining in the block, second pass of length 12 is left to next day's shift.

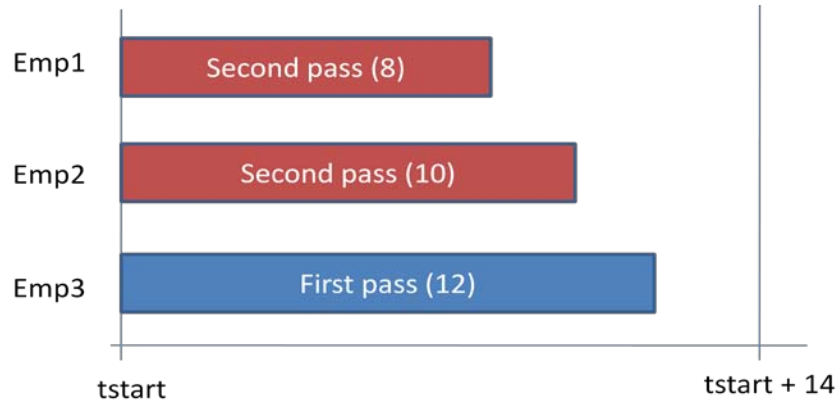


Figure 3.5 Assignment of sorted second passes

As we did after assigning all the first passes, we reorder employees from the shortest workload to the longest, which gives the schedule shown in Figure 3.5. Finally, we assign third passes starting from the first employee until $k - 1$ of the third passes are left, which will then be assigned to shortest workload employee. Two of the third passes are not assigned and left to next day's shift. Figure 3.6 shows the final schedule.

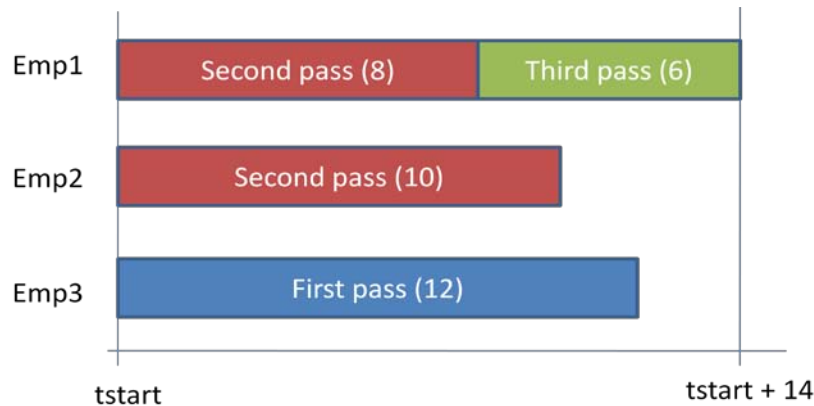


Figure 3.6 Assignment of sorted third passes and final schedule for three employees

The final schedule is actually the best possible schedule that can be obtained (Figure 3.2) for the given values of k and b .

Realistic Examples

In the following examples, we consider scheduling a few passes of SLC's current routes using real plowing times under different storm scenarios (obtained from the procedure described in Chapter 2).

Assume that SLC needs to schedule passes given in Table 3.1 below. Completion times of each pass are obtained from the approach presented in Chapter 2. These completion times are converted and then rounded up to the closest 15-minute interval.

Table 3.1 Completion Times of the Passes to Be Scheduled

Route	Pass	Completion Times		
		Storm A (medium intensity)	Storm C (high intensity)	Storm D (low intensity)
504	1	3:00	3:45	2:15
504	2	3:00	3:45	2:15
505	1	4:00	4:30	3:15
505	2	2:30	3:15	2:00
506	1	4:00	4:30	3:15
506	2	4:45	4:30	4:30
506	3	3:30	4:30	0:30

Assume that $t_0 = 2\text{am}$ and wage rate (w) = \$8.75 per 15-minutes. The regular wage rate includes fringe benefits. Current fringe rate is 62% of the wage. Thus, regular overtime wage rate corresponds to $\left(\frac{8.75}{1.62}\right) * 1.5 = \8.1 , and double-pay period rate is $\left(\frac{8.75}{1.62}\right) * 2.0 = \10.8 per 15 minutes. Table 3.2, Table 3.3 and Table 3.4 show examples of assumed delay costs per 15-minutes for storm types A, C and D, respectively.

Table 3.2 Delay Costs (\$) for Storm Type A (Medium Intensity)

	High traffic density (6am-10am,3pm-8pm)	Medium traffic density (10am-3pm,8pm-12am)	Low traffic density (12am-6am)
First pass	10.5	8.5	5
Second pass	8.5	5	2.5
Third pass	6.25	2.5	1.25

Table 3.3 Delay Costs (\$) for Storm Type C (High Intensity)

	High traffic density (6am-10am,3pm-8pm)	Medium traffic density (10am-3pm,8pm-12am)	Low traffic density (12am-6am)
First pass	18.75	10.5	8.5
Second pass	12.5	8.5	1.25
Third pass	6.25	2.5	0.25

Table 3.4 Delay Costs (\$) for Storm Type D (Low Intensity)

	High traffic density (6am-10am,3pm-8pm)	Medium traffic density (10am-3pm,8pm-12am)	Low traffic density (12am-6am)
First pass	10	5	2.5
Second pass	8.5	2.5	1.5
Third pass	5	1.5	0.5

Note that we have specified different delay costs for different storm types. Since delaying the plowing operations under high intensity storm conditions may be less preferable as compared to lower intensity storm scenarios, delay costs are increasing with the storm intensity. The choices of these delay costs are consistent with the guidelines provided earlier in this section 3.2.2. What we plan to show is that, the actual decision concerning whether to delay passes to next day depends in a non-intuitive way on the time left until the start of next day's shift and the lengths of the passes.

When we consider storm scenario A (medium intensity storm) and $t_0 = 2\text{am}$, the best schedule identified by our algorithm is as shown in Figure 3.7. Please see Appendix D for a detailed description of the output format and each item shown in Figure 3.7.

```

Number of employees to call-out/hold = 4
Start time of employees (OT period) (24-hour format)= 2:00
Total cost = 693.20
Overtime costs = 453.70
Extra costs (lunch break) = 0.00
Delay costs = 239.50

Schedule for Each Employee (Overtime)
-----
505_2 (2hr 30min), OT work time = 2hr 30min
504_1 (3hr 0min), OT work time = 3hr 0min
505_1 (4hr 0min), OT work time = 4hr 0min
506_1 (4hr 0min), OT work time = 4hr 0min

Schedule for Each Employee (Next Day's shift)
-----
(start time = 7:30) 506_3 (3hr 30min), work time = 3hr 30min
(start time = 7:30) 504_2 (3hr 0min), work time = 3hr 0min
(start time = 7:30) 506_2 (4hr 45min), work time = 4hr 45min

```

Figure 3.7 Best schedule for storm type A; $t_0 = 2\text{am}$

For storm scenario C (high intensity storm), Figure 3.8 shows the best schedule obtained by our solution method.

```

Number of employees to call-out/hold = 3
Start time of employees (OT period) (24-hour format)= 2:00
Total cost = 749.79
Overtime costs = 599.54
Extra costs (lunch break) = 0.00
Delay costs = 150.25

Schedule for Each Employee (Overtime)
-----
504_1 (3hr 45min), 505_2 (3hr 15min), OT work time = 5hr 30min
505_1 (4hr 30min), 504_2 (3hr 45min), OT work time = 5hr 30min
506_1 (4hr 30min), 506_2 (4hr 30min), OT work time = 5hr 30min

Schedule for Each Employee (Next Day's shift)
-----
(start time = 7:30) 506_3 (4hr 30min), work time = 4hr 30min
work time = 1hr 30min
work time = 2hr 45min
work time = 3hr 30min

```

Figure 3.8 Best schedule for storm type C; $t_0 = 2\text{am}$

For storm scenario D (low intensity storm), we obtain the schedule in Figure 3.9.

Number of employees to call-out/hold = 1 Start time of employees (OT period) (24-hour format)= 2:00 Total cost = 518.62 Overtime costs = 89.12 Extra costs (lunch break) = 0.00 Delay costs = 429.50 Schedule for Each Employee (Overtime) ----- 504_1 (2hr 15min), 506_3 (0hr 30min), OT work time = 2hr 45min Schedule for Each Employee (Next Day's shift) ----- (Start time = 7:30) 505_2 (2hr 0min), work time = 2hr 0min (Start time = 7:30) 504_2 (2hr 15min), work time = 2hr 15min (Start time = 7:30) 505_1 (3hr 15min), work time = 3hr 15min (Start time = 7:30) 506_1 (3hr 15min), work time = 3hr 15min (Start time = 7:30) 506_2 (4hr 30min), work time = 4hr 30min

Figure 3.9 Best schedule for storm type D; $t_0 = 2\text{am}$

When we compare the three schedules above, we see that the optimal assignments are different for each storm type. This is due to different pass completion times under different storm types and different delay cost specifications. The number of passes that should be done during overtime interval increases with the storm intensity because delaying the start of plowing is less desirable under high intensity storm conditions.

For storm type A, four employees must be called out to do all the first passes and one of the second passes. The threshold working time in overtime mode is 2 hours and 40 minutes, which corresponds to 4 hours of regular working time wage rate. Since the first pass lengths in overtime block are more than the threshold time and the second pass length is very close to the threshold value, all the passes in overtime mode are assigned to a different employee. In this case, overtime costs for a pass do not change from one employee to the other. However, delay costs are lower when a pass is assigned to a new employee. Longer second passes are delayed till the start of the next day's shift, since total of overtime wages for each of the delayed second passes is greater than the cost of delaying them up to shift start time. Same argument also applies to the only third pass, which is also delayed to next day's shift.

When we compare the final schedules for storm types A and C, we see that even though the start times stay the same in overtime mode, the overtime block becomes contiguous with the next day's shift for storm type C. All second passes start in overtime mode at high traffic density time period and continue during next day's shift. Since second passes have long completion times under high intensity storm condition C, they are not assigned to new employees because of their high overtime costs as compared to the delay costs in low traffic density times. In addition, delaying the second passes to next day is not desirable for storm type C. Therefore, in the best schedule, a portion of the second passes are done in overtime mode and the remaining part that is plowed during regular shift hours does not incur additional cost to the county. The only third pass is left to the start of next day's shift because its delay cost is low.

The comparison between storm type D (low intensity) and higher storm types show that many passes for storm type D are delayed to next day's shift including two of the first passes. Only the shortest first pass is done in overtime mode since its overtime employee cost stays below the cost of delaying the pass until the shift start time. Another interesting point is that the only third pass is also assigned to overtime period. Since the threshold time to pay four hours of regular pay to a called-out employee is 2 hours 40 minutes, and the first pass assigned to the employee in overtime block takes only 2 hours 15 minutes, SLC incurs almost no additional cost for also assigning the 30 minutes long third pass to the same employee. But it can lower delay costs for the third pass as compared to assigning it to the next day's shift. Given that there are enough employees during regular shift hours, each delayed pass is assigned to a different employee because that minimizes delay costs.

We now present three more examples to show the effect of a different earliest start time (t_0) on the final schedule. We consider changing t_0 to 6pm, 9pm and 4am for storm type D. The final schedules that minimize total costs for each t_0 value are shown in Figure 3.10, Figure 3.11 and Figure 3.12 respectively.

```

Number of employees to call-out/hold = 6
Start time of employees (OT period) (24-hour format)= 18:00
Total cost = 666.54
Overtime costs = 626.54
Extra costs (lunch break) = 0.00
Delay costs = 40.00

Schedule for Each Employee (Overtime)
-----
505_2 (2hr 0min), 506_3 (0hr 30min), OT work time = 2hr 30min
504_2 (2hr 15min), OT work time = 2hr 15min
504_1 (2hr 15min), OT work time = 2hr 15min
506_2 (4hr 30min), OT work time = 4hr 30min
506_1 (3hr 15min), OT work time = 3hr 15min
505_1 (3hr 15min), OT work time = 3hr 15min

Schedule for Each Employee (Next Day's Shift)
-----

```

Figure 3.10 Final schedule under storm type D (low intensity) and $t_0 = 6pm$

```

Number of employees to call-out/hold = 4
Start time of employees (OT period) (24-hour format)= 21:00
Total cost = 596.10
Overtime costs = 445.60
Extra costs (lunch break) = 0.00
Delay costs = 150.50

Schedule for Each Employee (Overtime)
-----
504_1 (2hr 15min), 506_3 (0hr 30min), OT work time = 2hr 45min
505_1 (3hr 15min), OT work time = 3hr 15min
506_1 (3hr 15min), OT work time = 3hr 15min
505_2 (2hr 0min), 504_2 (2hr 15min), OT work time = 4hr 15min

Schedule for Each Employee (Next Day's Shift)
-----
(Start time = 7:30) 506_2 (4hr 30min), work time = 4hr 30min

```

Figure 3.11 Final schedule under storm type D (low intensity) and $t_0 = 9\text{pm}$

```

Number of employees to call-out/hold = 6
Start time of employees (OT period) (24-hour format)= 6:00
Total cost = 421.67
Overtime costs = 291.67
Extra costs (lunch break) = 0.00
Delay costs = 130.00

Schedule for Each Employee (Overtime)
-----
505_2 (2hr 0min), OT work time = 1hr 30min
504_2 (2hr 15min), OT work time = 1hr 30min
506_2 (4hr 30min), OT work time = 1hr 30min
504_1 (2hr 15min), OT work time = 1hr 30min
505_1 (3hr 15min), OT work time = 1hr 30min
506_1 (3hr 15min), OT work time = 1hr 30min

Schedule for Each Employee (Next Day's Shift)
-----
(Start time = 7:30) 506_3 (0hr 30min), work time = 0hr 30min
work time = 0hr 30min
work time = 0hr 45min
work time = 0hr 45min
work time = 1hr 45min
work time = 1hr 45min
work time = 3hr 0min

```

Figure 3.12 Final schedule under storm type D (low intensity) and $t_0 = 4\text{am}$

When we compare the final schedules with different earliest call-out times, we see that the overtime and next day assignments differ. Additionally, even if the storm conditions remain the same, the best decision is not always to start plowing operations immediately at t_0 . When $t_0 = 6\text{pm}$, since there is a long time until the start of next day's shift, all the passes are assigned to overtime block and priority is given to first and second passes. The only third pass does not incur an additional cost because a called-out employee must be paid for at least 2 hours 40 minutes. In this case, two of the called-out employees must be paid for 2 hours and 40 minutes even though they work only 2 hours 15 minutes. However, it is better to pay for the idle 25 minutes as compared to delaying the two 2 hours 15 minutes long passes and assigning them to one of the already called-out employees.

When $t_0 = 9\text{pm}$, the employees should again start immediately at 9pm. However, now the longest second pass (4 hours 30 minutes) could be delayed to next day. Since high traffic density period is now over, the delay costs for a second pass is much lower and for the 4 hour 30 minutes long second pass, the delay costs are lower than its overtime costs. The reason for assigning only the third pass to overtime period is again to utilize the called-out employee for at least 2 hours 40 minutes, in which case the third pass does not incur any additional costs to the county.

When $t_0 = 4\text{am}$, since 4am is closer to the start of next day's shift as compared to previous t_0 values, the county may delay the start of plowing operations for a few hours with the purpose of decreasing overtime employee costs. In this case, the best start time for the employees is 6am, which is the start of high traffic density period. In addition, all of the employees are 100% utilized until the shift start time. The only third pass is delayed to shift start time, because if a new employee is called-out at 6am to do the third pass, then the county should have to pay at least 1 hour and 30 minutes at overtime rate for the 30-minute long pass. This is not desirable, since third passes have the lowest priority.

3.3 Conclusions

We developed a decision support system based on an efficient solution method to identify the best schedule that minimizes the sum of SLC's overtime, delay and extra costs. Delay costs are introduced into the model in order to provide a desired service level under a given storm scenario. To utilize our approach, the broader input parameters that SLC engineers/managers should decide are the earliest possible start time of the plowing operations during overtime period (t_0), one of the storm types as categorized in Chapter 2, the passes to schedule after t_0 , and the cost of delaying each pass type for 15-minutes during high, medium and low traffic intensity periods. Given the input parameters, our decision support tool will help SLC engineers/managers arrive at a minimum cost schedule by considering the potential assignments within overtime period and regular shift hours.

Chapter 4 Robust Crew Requirements

4.1 Introduction

The model described in this chapter adds the following new features to the approach presented in Chapter 3: (1) uncertainty in storm conditions, (2) multiple and possibly competing priorities of maintenance supervisors, and (3) user selection of which road segments to plow based on either an AADT threshold, or road surface type, or direct input. The workforce management problem addressed in this chapter comprises of two decision stages. The two stages relate to how uncertainty is resolved over time. At the first stage, which occurs at an arbitrary time epoch denoted by t_c , our model (acting on behalf of maintenance supervisors) picks a *deployment strategy*. The deployment strategy consists of two parameters – the number k of operators to call out, and the time t_{start} at which operators must be ready to start plowing. Our approach assumes that there may be significant uncertainty about snowfall conditions at t_c . Therefore, the deployment strategy must be robust with respect to all possible realizations of storm intensity.

The second stage occurs at t_{start} . At that time, more information is usually available about realized storm conditions. The second stage decisions are collectively referred to as a *plowing schedule*. Our model develops a detailed plowing schedule at time t_c for each possible storm type. The schedule specifies which road segments will be plowed by each plow and in what sequence. Note, some of these road segments may be designated for plowing in the next regular shift. The maintenance supervisor can then pick a particular schedule based on the realized storm conditions at t_{start} . That is, a plowing schedule is not picked at t_c , although all possible plowing schedules are known at t_c , which makes it easy to implement the appropriate schedule at t_{start} .

We allow the user to specify an earliest start time t_0 such that $t_{\text{start}} \geq t_0$ must hold. This adds additional level of flexibility in choosing desired solutions and also incorporates the minimum necessary time needed to call out operators, if required. Put differently, a default value of the parameter t_0 in our formulation is either t_c or the end of the shift, t_e , whichever occurs later. However, our approach allows a maintenance supervisor to choose a different value as needed. Figure 4.1 illustrates the decision stages of workforce management problem on a time line.

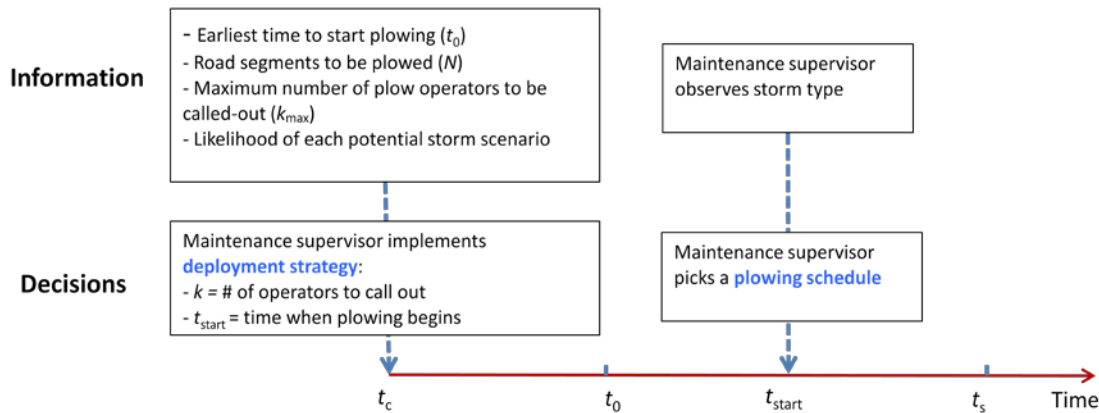


Figure 4.1 Decisions hierarchy and information flow

The model presented in Chapter 3 identified two types of costs that must be balanced by a maintenance supervisor – cost of delays in clearing roads of snow and ice, and cost of overtime. Chapter 3 also identified the difficulty of monetizing the cost of delays. To address concerns about priorities that may change over time, we incorporated multiple objectives in the optimization approach presented in this chapter. Each objective is represented by a different term in an overall objective function. Our approach allows county managers to specify relative priorities of these terms and obtain alternate solutions by doing so. Removing one or more terms is also allowed. This can be achieved by assigning a priority level of zero to the corresponding term in the workforce management problem. We illustrate this idea with an example below.

Consider the problem of identifying a plowing schedule for a particular storm scenario. County managers may want that each time a plow leaves the depot, it should plow as many road segments as possible before returning to the depot to reload. At the same time, they may want to minimize unnecessary travel (deadheading) and cluster road segments with similar AADT counts for earlier plowing. That is, it may not be desirable to include a low priority road segment in the plowing sequence when higher priority segments remain snow covered, even though the former may reduce deadheading or lead to a more efficient use of a plow's payload. Therefore, each of these objectives is represented by a different term in our formulation and their relative importance (priority level) can be varied.

The workforce management decision support tool presented here allows a maintenance supervisor to pick which road segments should be considered for plowing. Our approach in Chapter 3 had pre-determined clusters of road segments, based on plowing efficiency and AADT counts, such that each cluster belonged to a single route, which was reasonable when either all road segments of a route were plowed or the entire route was put aside to be plowed during the next day's regular shift. We have modified our approach so that the input to our model is an arbitrary set of road segments, which may belong to different routes. These segments can be selected based on either an AADT threshold, or road surface type, or direct input.

The addition of the features described in this report resulted in a complex optimization problem, which is difficult to solve optimally in the short amount of time that is typically available to a maintenance supervisor when making call-out decisions. Therefore, we developed a two-phased heuristic approach to solve this problem. While this approach does not guarantee

an optimal solution, it can be solved in a significantly shorter time than a formulation that does not decompose the problem as we do. We tested our approach in numerical examples and found it to perform well.

The remainder of this chapter is organized as follows. We describe the two phases of our approach in Section 4.2. Results obtained from testing this approach on example problems can be found in Section 4.3, and concluding remarks in Section 4.4.

4.2 Model Formulation

The problem of choosing a robust workforce management plan is difficult on account of the following reasons:

1. Storm conditions are highly variable, which requires that maintenance supervisors be able to specify for each storm event which road segments need to be considered for plowing.
2. Overtime costs are nonlinear.
3. Delay costs are both time-of-day and AADT dependent.
4. Storm conditions affect both the plow speeds and the sand/salt usage rate.

In addition, priorities may change over time and multiple priorities may exist at any given time. For example, if maintenance budget is close to being spent, maintenance supervisors may assign a higher weight to minimizing plowing completion time and smaller weight to all other terms in the objective function.

Even upon fixing the number of employees to call out and plowing start time, and without multiple competing priorities and nonlinear constraints, the problem of determining an optimal plowing schedule is a combinatorial problem. To appreciate the complexity, suppose there are 40 road segments that are selected for plowing and 5 employees are called out. Then, there are approximately 658 thousand possible ways of assigning road segments to employees, and for each such assignment, the problem of finding an optimal sequence in which road segments should be plowed is also a hard problem.

For the reasons explained above, it is necessary to simplify the manpower planning problem, which can be done in a variety of different ways. We describe an approach that first solves the plow scheduling problem for each fixed deployment strategy. The plow scheduling problem is solved by decomposing it into two phases, which are solved sequentially. In what follows, we refer to Phase I as the Cluster Formation phase, and Phase II as the Cluster Scheduling phase. Later in this report, we describe each phase in detail in a separate section. Knowing how to schedule plows, we identify an optimal deployment strategy by an exhaustive search over all possible values of t_{start} and $k \in 1, \dots, k_{\text{max}}$. We also call the combination of Phases I and II as the inner loop and the enumerative approach for obtaining optimal t_{start} and k as the outer loop. Figure 4.2 shows a flow chart of decisions and information flow for our overall manpower optimization approach.

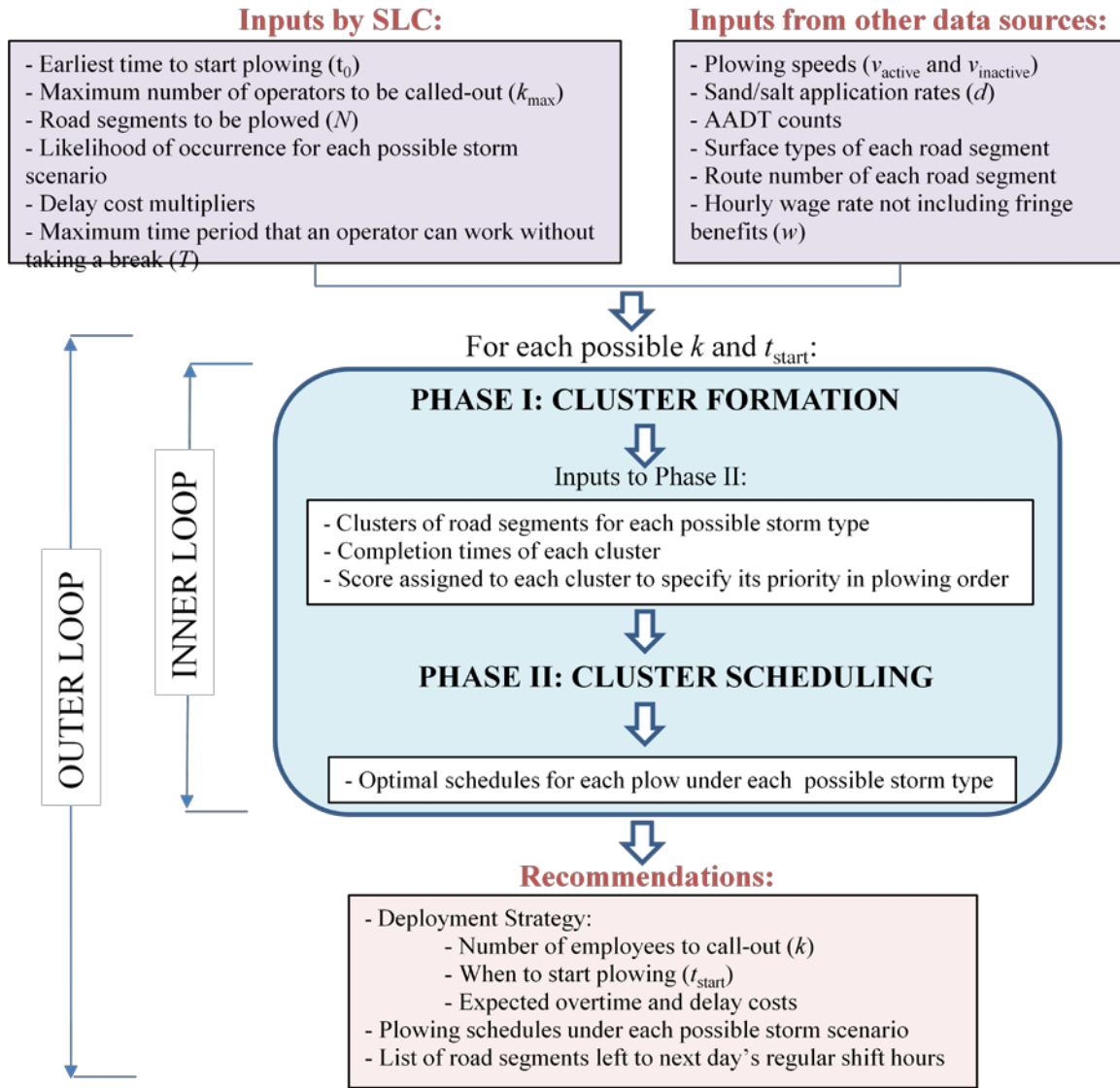


Figure 4.2 Our solution approach including the two-phased inner loop

Next, we provide a list below of the information that is required as input to the two-phase solution approach (inner loop). For each piece of information, we also identify whether it comes from data sources identified in earlier chapters, or direct input from a maintenance supervisor. If maintenance supervisors prefer not to provide specific values for some of the direct input parameters and default values are specified, then our model will use default values. Default values may be changed by SLC managers.

Table 4.1 Information Requirements for the Two-phase Solution Approach

Description	Parameter	Source	Default Values
Maximum number of operators available	k_{\max}	Direct Input	Number of plows
Earliest start time	t_0	Direct Input	$\max \{t_c, t_e\}$
Road segments to be plowed	N	Direct Input	None
Likelihood of each storm scenario ξ	$p(\xi)$	Direct Input	$\frac{1}{5}$ [5 sotrm types] $\frac{1}{5} \frac{\text{AADT} \times \text{Length}}{\text{AADT} \times \text{Length}}$
Delay cost multipliers (road)	c_{road}	Direct Input	$\frac{\text{Avg}(\text{AADT} \times \text{Length})}{\text{AADT} \times \text{Length}}$
Delay cost multipliers (storm)	c_{storm}	Direct Input	1, 2, 3, 4, 5
Delay cost multipliers (traffic)	c_{traffic}	Direct Input	1, 2, 3
Hourly wage rate (not including fringe)	W	Labor Contract	\$21.6 (i.e. \$35/1.62)
Maximum work time without a break	T	Direct Input	5 hours
Snowplow speed (active mode) (mph)	$v_{\text{active}}(\xi)$	<i>Wilson et al., (2003)</i>	17, 12, 15, 12, 9
Snowplow speed (inactive mode) (mph)	$v_{\text{inactive}}(\xi)$	Direct Input	22, 17, 20, 17, 14
Sand/Salt application rates (lbs/lm mi)	$d(\xi)$	<i>Wilson et al., (2003)</i>	450, 500, 500, 500, 450
AADT counts	AADT	SLC GIS maps	N/A
Road surface types (paved or gravel)	P or G	SLC GIS maps	N/A
Route numbers	R	SLC GIS maps	N/A

Storm conditions are classified into five groups in Chapter 2. If no other information is available, the five storm scenarios are assumed to have an equal chance of occurrence, i.e. each has a likelihood of 0.2. Delay cost multiplier represents the relative magnitude of delaying the plowing operations as a multiple of regular wage rate (not including fringe). Delay cost multipliers are specified separately by road segment, storm type and traffic intensity interval type. For road segments, we assume that AADT count and length of a road segment are the two factors that affect its relative importance in the plowing schedule. Therefore, default value for the delay cost multiplier of a road segment is selected as the ratio $\frac{\text{AADT count} \times \text{Length}}{\text{Average}(\text{AADT count} \times \text{Length})}$. The denominator of this ratio is calculated as the average of AADT count \times Length of each selected road segment. For storm types and traffic intensity interval types, default delay cost multipliers start from 1 for the lowest intensity and increase by 1 for higher storm and traffic intensity levels. The overall multiplier for a road segment under specific storm and traffic conditions is determined as the average of the three delay cost multiplier values. For example, if delay cost multiplier of a road segment is 3, then under highest intensity storm type (i.e. multiplier 5) and lowest traffic intensity (i.e. multiplier 1), the final value of delay cost multiplier for that road segment is $\frac{3+5+1}{3} = 3$. It is possible to use other methods of aggregating individual delay cost multipliers if deemed appropriate by county managers.

In the two sections that follow, we describe the two phases of the inner loop. For each phase, we list terms in the objective function, default values of weights assigned to these terms, and key constraints. We also summarize relevant literature.

4.2.1 Phase I: Cluster Formulation

Based on discussions with SLC staff, we developed a list of terms to be included in Phase-I objectives function. SLC managers can choose relative priorities for these terms, including placing a zero weight for one or more terms, as needed.

Term 1: Form clusters that have similar unit delay costs (i.e. similar AADT and length)

Term 2: Minimize total travel time of plows when forming clusters

Term 3: Minimize the number of clusters

Term 4: Minimize the number of clusters with total completion times > 3.5 hours

The first term groups road segments with similar unit delay costs together in the same cluster. Because each cluster is plowed in a single tour by the snowplow, this term can help delay the plowing of lower AADT count road segments when higher priority segments are present. In Phase II, clusters that contain segments with higher delay costs are scheduled first. If there are many available plows, then SLC managers may decrease the weight given to this term, since different plows could work on high priority road segments in parallel.

The second term minimizes the total completion time because that reduces overtime costs, deadheading, and delay costs. Inefficient use of available sand/salt payload may not be desirable because it creates frequent travel to the depot and leads to an increase in delay and overtime costs. Therefore, the third term discourages frequent travel to the depot by minimizing the number of clusters. This term also ensures that each sand/salt payload will be utilized as much as possible.

The final term minimizes the number of clusters whose completion times exceeds 3.5 hours. Employee work rules state that if an employee works for more than 11 hours in a day (i.e. more than 3.5 hours in overtime period because the regular shift is 7.5 hours), then the employee should be paid twice the regular rate. Therefore, overtime exceeding 3.5 hours per employee is more costly. This term penalizes each instance of double pay, proportional to the length of the corresponding double-pay period. It applies only when the employees are called out more than 3.5 hours before the start of the regular shift t_s because if an employee's overtime work continues into the regular shift, then no extra charges are incurred by the county. Put differently, term 4 is relevant only when $t_{\text{start}} < t_s - 3.5$ hours.

For each term, SLC managers should specify a relative priority level. A priority level of 0 will have the effect of removing that term from the objective. Priority levels, whose default values are shown in Table 4.1, need to be converted into appropriate weights. These weights are different for different terms, even when priority levels are the same, because the size of each term is different. For example, the first term is sum of absolute differences in delay costs between each pair of road segments included in a cluster, whereas the second term is the sum of travel times, in order of plowing, of all segments in a cluster. These terms have different units of measurement and magnitude.

Table 4.2 Phase I Terms: Default Priority Levels and Associated Weights

Term	Weight parameter	Default priority	Default weights
1	α_1	1	Avg. travel time between two road segments
2	α_2	1	Avg. delay cost multiplier difference between two road segments
3	α_3	1	1
4	α_4	1	Avg. travel time between two road segments
			1

In Table 4.2, we provide the default priorities of each term and their associated weights. These priorities are relative and the priority level of any one of the terms can be fixed arbitrarily. We set the priority level for Term 2 to 1. If SLC managers prefer to change the default priorities, then the priority levels of the terms should be set in relation to Term 2. For example, if Term 1 is more desirable than Term 2, then priority level of Term 1 should be less than 1, but not 0. If SLC managers modify priority levels, then our program will automatically update the weights based on new priority levels relative to the ratio of previous weight to the new weight. For example, if priority level of a term is lowered from 1 to 2, then its new relative weight will be half of its previous weight.

Next, we describe the constraints imposed on the cluster formation algorithm. The first constraint ensures that a plow can apply sand/salt to all road segments in a cluster that are actively traveled with a single load of sand/salt. The second constraint is needed because the completion time of a cluster should be less than the maximum interval of time that a plow operator should work without taking a break.

Constraint 1: Total sand/salt requirement of road segments in a cluster should not exceed sand/salt capacity of a single plow.

Constraint 2: Total completion time of a cluster cannot exceed a given threshold value T .

There are additional technical constraints in Phase I, which are not presented in this section. These additional constraints ensure the desired relationships between parameters and decision variables. Appendix E includes a complete mathematical formulation of Phase I together with explanations of all decision variables, parameters, and constraints.

When the objective function consists only of Term 2 and T is large (i.e. Constraint 2 is not binding), Phase I problem reduces to the Capacitated Arc Routing Problem (CARP), in which a fleet of identical plows with limited sand/salt capacity should service each selected road segment such that the total travel time is minimized. The CARP is an NP-Hard problem, which means that there are no known polynomial-time algorithms to solve instances of CARP optimally. Existing exact methods for solving this problem are based on branch-and-bound algorithms and work only for small-sized problems (Hirabayashi *et al.*, 1992; Longo *et al.*, 2006). Therefore, researchers have developed heuristic methods to solve larger instances of CARP. Golden and Wong (1981) introduced Augment-Merge method, which constructively forms clusters of arcs. Later, Golden *et al.* (1983) developed a path-scanning heuristic, which is

based on a greedy adding approach to determine the arc to be visited next on the current vehicle route. More recently, Santos *et al.* (2009) developed a new path-scanning heuristic. At each iteration of their algorithm, routes are constructed by adding a single arc, which is the nearest arc to the last added one on the same route. However, when vehicle capacity is close to its limit, then an ellipse-rule is applied. In this method, only the arcs closer to the shortest path between last arc of the route and the depot (i.e. arcs within an ellipse) could be added. In addition to constructive methods, meta heuristics are also developed to solve CARP. Some of these methods are based on tabu search heuristics (Hertz *et al.*, 2000; Greistorfer, 2003), hybrid genetic algorithms (Lacomme *et al.*, 2004), and ant colony optimization (Lacomme, 2004). We refer to the paper by Santos *et al.* (2009) for a review of heuristic methods developed for the solution of CARP.

Since CARP is a special case of our problem (with a single term in the objective and no time constraint), Phase I formulation is also NP-hard. In addition, none of the above techniques can be directly applied to our model because Phase I has additional constraints and terms in the objective function. Moreover, some of these terms are in conflict with the goal of minimizing completion time. For the examples included in this report, we use the commercial optimization software CPLEX to test Phase-I formulation.

4.2.2 Phase II: Cluster Scheduling

Phase II assigns the clusters formed in Phase I to available plows. We begin by assigning a score to each cluster to indicate its priority in scheduling. Each cluster's score is the ratio of the sum of unit delay costs for road segments that belong to the cluster divided by the total completion time of the cluster. A higher score implies higher priority in scheduling that cluster – see Term 1 of the Phase II objective function below. In other words, clusters of road segments with higher delay costs and shorter completion times are scheduled first because this helps decrease total delay costs. We identify the minimum score among all clusters by \min_{score} .

Term 1: Prioritize clusters by their scores when selecting their plowing order in a plow's schedule

Term 2: Penalize assignments with more than 3.5 hours of work in overtime mode

Term 3: Penalize the idle time of an employee when the next shift starts within a 4-hour period from the start of overtime work

Term 4: Penalize not assigning a cluster to any plow in overtime period (penalty equals the maximum delay cost until the start of next day's regular shift hours)

Term 5: Minimize differences in total work times of plows

Constraint: Total continuous work time of an operator cannot exceed a given threshold value T .

Term 2 penalizes cluster assignments that lead to total work time exceeding 3.5 hours because of the requirement to pay double. This is similar to Term 4 of Phase I with the difference that it is now applied to the total time that the plow works. As before, this term is relevant only when $t_{\text{start}} < t_s - 3.5$ hours.

Another SLC work rule is that when an employee starts overtime work within a 4-hour period before the start of regular shift, then he/she should be paid until the start of the shift at overtime rate. It is desirable to utilize these employees with plowing operations until the start of the shift, so any idle time should be discouraged in the formulation. Therefore, Term 3 penalizes the idle time of an employee when he/she starts working within a 4-hour period before the start of next day's shift. If a cluster of road segments is not assigned to any of the called-out operators, then all the road segments in the cluster would be delayed to the start of the next day's shift. Therefore, Term 4 introduces a penalty for each cluster that is not assigned and this penalty is the total cost of delaying all road segments in that cluster until the start of the next day's shift.

In addition to assigning higher priority to clusters with higher scores, it is also important to choose plowing start times that minimize delay costs. This could be achieved by minimizing differences in total work time of any pair of plows. In Term 5, we encourage assignment of clusters to plows in a manner that each cluster is started at the earliest possible time.

We illustrate next with an example that simply knowing scores of clusters is not enough to obtain an optimal plowing schedule. This happens because maintenance supervisors also need to pay attention to other terms in the objective function. Suppose there are two plows and three clusters. Clusters are indexed such that Cluster 1 has the highest score and Cluster 3 the lowest. If assignments are based only on scores, then it will be optimal to assign Cluster 1 to Plow 1, Cluster 2 to Plow 2 and Cluster 3 to either one of the plows. However, if completion times of Clusters 1 and 2 are less than that of Cluster 3, then assigning Cluster 2 to Plow 1 (i.e. just after Cluster 1) and Cluster 3 to Plow 2 may be an overall better solution. This may consume less overtime either by utilizing each plow close to 2 hour 40 minutes minimum pay period, or by eliminating double pay for the operator of Plow 1. Therefore, Phase II formulation considers not only clusters scores but also other critical objectives. This adds to the difficulty of solving Phase II, as we explain in more detail later in this section.

If an employee is called-out, then he/she should be paid for at least 4 hours of regular work time, which corresponds to 2 hours and 40 minutes in overtime mode. This issue is not explicitly addressed in Phase II because the number of called-out employees k is fixed (i.e. minimum overtime pay is sunk). However, when calculating the overtime costs for each possible k and t_{start} in the outer loop, the minimum pay is included in the actual cost of selected deployment strategy. Note that, the minimum pay rule applies only when $t_{\text{start}} < t_s - 4$ hours and $t_{\text{start}} \neq t_e$, where t_e is the end time of regular shift hours.

The constraint in Phase II formulation limits the work time between breaks for each plow operator to a threshold value T . Similar to Phase I formulation, there are several other technical constraints needed to complete the formulation of Phase II, which are not presented in this section. These additional constraints ensure the desired relationships between parameters and decision variables. Appendix E includes a complete mathematical formulation of Phase II together with explanations of all decision variables, parameters, and constraints.

Different terms in Phase II objective are assigned relative priorities. These priority levels are then converted into weights (similar to Phase I) because the units of measurement and the magnitudes of different terms are different. If desired, some of these terms could be removed by specifying a priority level of 0. Table 4.3 provides default priority levels (set to 1) for each term

and their corresponding default weights. Because priority levels are relative, we recommend setting the priority level of Term 1 equal to 1 and then determining the priorities of other terms in relation to Term 1. Moreover, we recommend that Term 1 should be assigned the highest priority because it significantly affects the level of service, as explained below.

Table 4.3 Phase II Terms: Default Priority Levels and Associated Weights

Term	Weight parameter	Default priority	Default weight
1	β_1	1	$\frac{\max \{(T - 3.5) \times 2 \times w, 3.5 \times 1.5 \times w + 0.5 \times 2 \times w\}}{\min_{\text{score}}}$
2	β_2	1	1
3	β_3	1	1
4	β_4	1	1
5	β_5	1	$\frac{\beta_1 \times \min_{\text{score}}}{T}$

If Term 1 is removed from the objective function, then the only concern will be to minimize overtime payments. In that case, road segments with high AADT counts would not necessarily be plowed first in the optimal plowing schedules. Therefore, the default weight of Term 1 is derived by equating minimum value of Term 1 to the maximum value of other terms. This is a suggested default value and could be changed if desired by SLC managers. As in Phase I, if a default priority level is changed by SLC managers, then the relative weights of all terms will be proportionally updated.

When we consider only Term 1 and when the total work time constraint is not binding, then Phase II formulation reduces to a parallel machine scheduling problem with sequence dependent costs, which is an NP-hard problem. Parallel machines correspond to called out operators, jobs correspond to clusters, and costs are sequence dependent because of nonlinear overtime/delay cost rates. Kang *et al.* (1999) developed an optimization method based on column generation and branch-and-bound techniques and heuristically adapted their approach to test real-life problem instances.

When only Term 5 is considered, then Phase II problem reduces to parallel machine scheduling problem in which the goal is to minimize makespan. Graham (1966) developed a greedy heuristic for this problem, called LIST. In the worst-case, the LIST method yields an objective value of $\left(2 - \frac{1}{\text{number of machines}}\right) \times (\text{optimal solution})$. LIST obtains the optimal solution when number of machines is equal to 2 or 3. For an arbitrary number of machines, Bartal *et al.*, (1992) succeeded to obtain the worst-case ratio of $2 - \epsilon$ of their heuristic objective value to the optimal objective. Note, ϵ denotes a small number whose magnitude depends on the number of iterations of the heuristic approach. That is, in the worst case, the heuristic solution is guaranteed to be no worse than twice as bad as an optimal solution. This performance bound was later improved by Karger *et al.*, (1994) and Albers (1997).

Clearly, Phase II in our setting is at least as difficult to solve as the parallel machine scheduling problem. In fact, because it has additional terms in the objective function and additional constraints, earlier solution methods cannot be directly applied to solve Phase II. Similar to Phase I, for the examples reported in this report, we use the commercial optimization software CPLEX to test Phase-II formulation.

4.3 Examples

In this section, we present two examples. The first example applies the two-phase approach to a hypothetical road network shown in Figure 4.3. This road network has 8 road segments that need to be plowed (shown by solid arrows). Circled numbers are the end points of the road segments (called nodes). Node 0 is the depot. Three sets of data, labeled as L, M and H, are shown alongside each road segment. These data are the sand/salt requirements, active travel time, and inactive travel time for low, medium and high storm intensities, respectively. The data are identical for both directions of each road segment. The values alongside the dashed road segments correspond to travel times to and from the depot under three storm types.

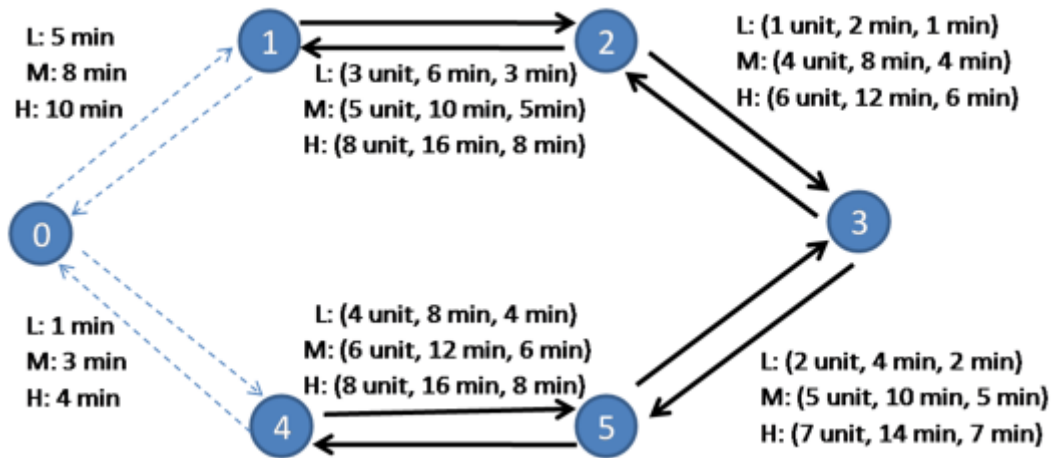


Figure 4.3 Example road network with plowing requirements data

The sand/salt capacity of a single plow is assumed to be 15 units (a unit can be in hundreds or thousands of pounds). The maximum time that an operator can work without taking a break is limited to 5 hours. The maximum number of plows that may be available in the overtime mode is assumed to be 3. Wage rate per hour during regular shift is \$35 including fringe benefits and the benefits are 62% of the wage rate. Hourly delay costs are specified as multiples of regular hourly wage rate (not including fringe). The nominal multipliers for road segments $\{(1,2), (2,1), (2,3), (3,2), (3,5), (5,3), (4,5), (5,4)\}$ are (1, 1, 0.5, 0.5, 1, 1, 1.5, 1.5), respectively. In addition, we also specify multipliers for storm type and traffic intensity for each road segment. Multipliers for storm types are 3, 2, and 1 for high, medium, and low storm intensities, and multipliers for traffic intensities are also 3, 2, and 1 for high, medium, and low traffic intensities, respectively. That is, for road segment (2,1), the hourly delay cost during a high intensity storm and a low traffic intensity period, with the above specification of multipliers, will be $\left(\frac{\$35}{1.62}\right) \times \left(\frac{1+3+1}{3}\right)$.

To obtain a workforce management strategy, we assume the following managerial inputs: $t_0 = 9$ PM, high, medium and low storm types are equally likely to occur, no more than three employees (equivalently plows) are available for overtime duty, and each term in the objective function of Phase I and Phase II formulations is assigned default priorities. In this example, the delay costs for road segments (4,5) and (5,4) are at least as much as overtime wage rate even when storm and traffic intensities are low. Therefore, it makes sense that some segments be plowed starting at 9 PM. For this reason, the results shown for this example fix plowing start time $t_{\text{start}} = 9$ PM.

Table 4.4 Results of Cluster Formation Algorithm

Cluster No.	Low		Medium			High			
	1	2	1	2	3	1	2	3	4
Road Segments to plow	(1,2),(2,3), (3,5),(5,3), (3,2),(2,1)	(4,5),(5,4)	(1,2),(2,3), (3,5)	(5,3),(3,2), (2,1)	(4,5),(5,4)	(4,5),(5,3)	(1,2),(2,3)	(3,2),(2,1)	(3,5),(5,4)
Path of the Plow	0→1→2 →3→5→3 →2→1→0	0→4→5 →4→0	0→1→2 →3→5 →4→0	0→4→5 →3→2 →1→0	0→4→5 →4→0 →4→0	0→4→5 →3→5 →4→0	0→1→2 →3→5 →4→0	0→4→5 →3→2 →1→0	0→4→5 →3→5 →4→0
Completion time (min)	34	18	45	45	30	53	57	57	53
Sand/Salt req. (unit)	12	8	14	14	12	15	14	14	15
Score	0.147	0.167	0.056	0.056	0.100	0.047	0.026	0.026	0.047

Given the managerial inputs, Table 4.4 shows the clusters of road segments formed in Phase I. Specifically, Table 4.4 shows for each storm type, the road segments that constitute different clusters, and their completion times, sand/salt requirements, and assigned scores. These scores are used as input to Phase II. The clusters are different for the three storm types due to differences in input parameters, such as plowing times and sand/salt requirements. For example, for high storm intensity, the number of road segments in a cluster (i.e. a single tour of a plow) is less than that of a low storm intensity case due to higher sand/salt application rate and less travel speed under high intensity storm conditions. We also observe that the clusters under each storm type are formed in a way that road segments with similar delay cost multipliers belong to the same cluster. In addition, for each cluster the sand/salt usage is close to the capacity of a plow.

In order to show the impact of storm intensities, we have tabulated total costs along with the percentage of total that come from delay and overtime components in Table 4.5 when either 1, 2 or 3 operators are called out. For each value of k , the plowing schedule depends on the realized storm intensity. The first three columns show costs associated with Low, Medium and High storm intensities, the last column shows the expected cost assuming that plowing schedule is chosen after storm intensity is realized. Minimum cost solutions for each storm intensity and the overall minimum-cost deployment strategy are shown in bold font. For low intensity storms, it is optimal to call out 1 employee ($k = 1$). Similarly, the maintenance supervisor should call out 2

operators for medium intensity and 3 operators for high intensity storms. Because there is uncertainty in storm type at the time of calling out employees, a robust decision is to call-out two employees to start plowing at 9 PM. This solution is dependent on input parameters described earlier.

Table 4.5 Total Costs (TC), %Delay Costs (%D), and %Overtime Costs (%OT), $t_{\text{start}} = 9 \text{ PM}$

	Low			Medium			High			Expected
	TC	%D	%OT	TC	%D	%OT	TC	%D	%OT	
$k = 1$	194	55.4	44.6	502	82.8	17.2	1036	88.4	11.6	577
$k = 2$	248	30.4	69.6	377	54.2	45.8	598	71.1	28.9	408
$k = 3$	335	22.6	77.4	379	31.6	68.4	575	54.9	45.1	430

In order to obtain the results reported in Table 4.5, one needs to solve for the optimal plowing schedule for each storm scenario. We describe those results next when $k = 2$, i.e. an optimal deployment strategy is chosen. But first, note that for each fixed storm scenario as the number of called-out employees increases, delay costs decrease and overtime costs increase. The reason is that when more employees are called-out, road segments could be plowed in parallel by different plows, reducing delay costs. However, each plow operator is paid at least 2 hours and 40 minutes overtime pay, which increases overtime cost.

Table 4.6 shows the optimal plowing schedules and total completion times of road segments assigned to each employee under each storm type. Under low storm intensity, road segments are clustered into two groups and each cluster is assigned to a different employee. Under medium storm intensity, there are three clusters, two of which are assigned to the first employee and one to the second employee. Finally, if the storm intensity is high, road segments are grouped into four clusters and each plow works on two clusters. Clusters are listed in their plowing order. For example, first employee should plow road segments in Cluster 3 first and then continue with road segments in Cluster 1 when the realized storm intensity is medium. When determining these schedules, higher priority is given to clusters with higher delay costs.

Table 4.6 displays completion times for each operator (denoted by TCT). The time to clear all assigned road segments is the maximum TCT for all employees. That is, it will take 34 minutes from call out time to clear all assigned road segments if storm intensity is low, 79 minutes if storm intensity if medium, and 110 minutes otherwise. This table also shows what would happen if a plowing schedule is picked at time t_c . This is called a fixed plowing schedule and the completion times are denoted by TCTF. In particular, the table illustrates these comparisons when maintenance supervisor picks the plowing schedule for high storm intensity. Although the time to clear all segments are not significantly different, certain clusters are cleared much faster when plowing schedules are chosen based on more accurate information about realized storm intensity, which reduces overall delay costs. Figure 4.4, Figure 4.5 and Figure 4.6 show the optimal plowing schedules presented in Table 4.6 when $k = 2$ for low, medium, and high storm intensities, respectively.

Table 4.6 Plowing Schedules under Each Storm Type ($k = 2$) (*TCT: Total Completion Time, TCTF: Total Completion Time under Fixed Schedule)

	Low			Medium			High		
	Clusters	TCT*	TCTF*	Clusters	TCT	TCTF	Clusters	TCT	TCTF
Employee 1	1	34	40	3,1	75	79	1,3	110	110
Employee 2	2	18	40	2	45	79	4,2	110	110

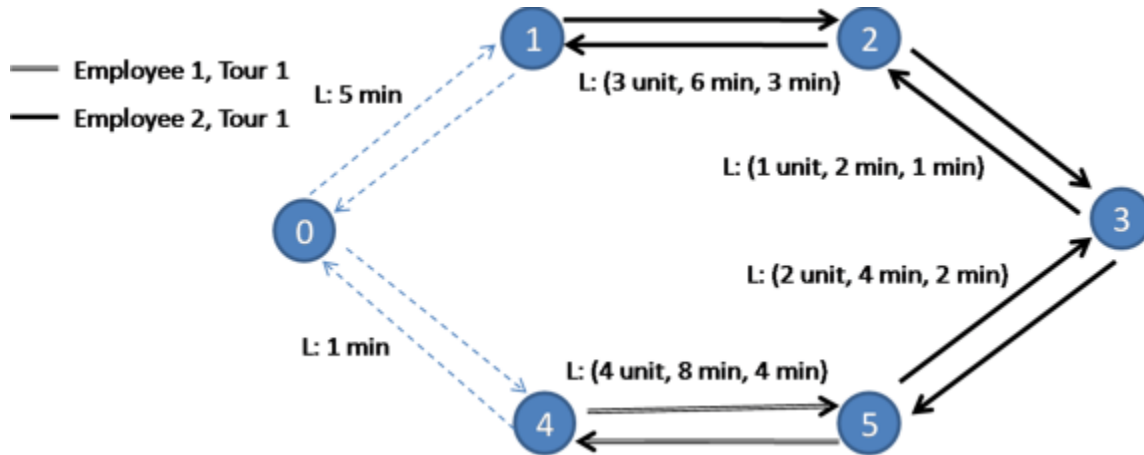


Figure 4.4 Optimal plowing schedule under low storm intensity ($k = 2$)

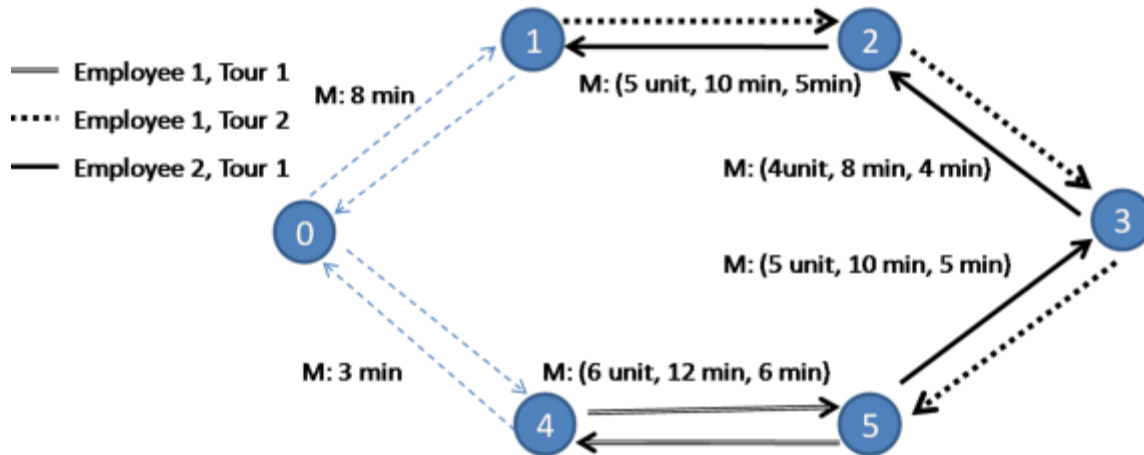


Figure 4.5 Optimal plowing schedule under medium storm intensity ($k = 2$)

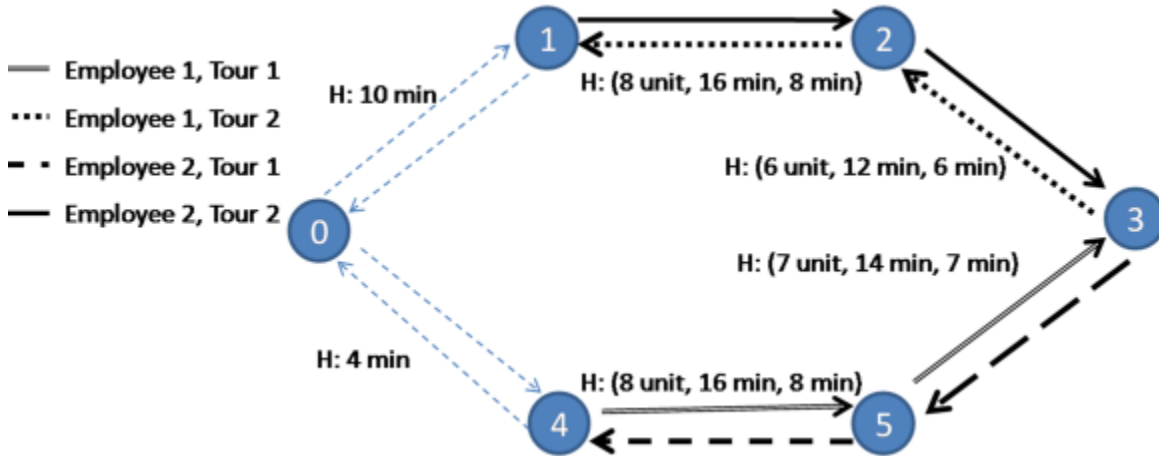


Figure 4.6 Optimal plowing schedule under high storm intensity ($k = 2$)

To further emphasize the role of utilizing more accurate storm intensity information, Table 4.7 shows total costs for different values of k if the optimal plowing schedule for high storm intensity is used under medium and low storm intensity cases. This means that a fixed plowing schedule is selected at t_c , which is the high storm intensity schedule. The realized storm type information is not utilized at t_{start} . A comparison of total costs in Table 4.5 and Table 4.7 show that total costs would be higher if low or medium type of storm intensity is realized and a fixed plowing schedule is implemented.

Table 4.7 Total Costs for a Fixed Plowing Schedule, $t_{start} = 9$ PM

	Low	Medium	High
$k = 1$	311	642	1036
$k = 2$	277	431	598
$k = 3$	335	447	575

In a second example, we implemented our approach on test data from SLC. Figure 4.7 shows a road network from District 5 of SLC. Different routes of the county are represented with a different line type. On this road network, we assigned numbers to some of the intersections to indicate end points of the selected road segments to be plowed. Node 0 represents the depot. Table 4.8 lists the selected road segments together with their start and end points, lengths, AADT counts, and delay cost multipliers. The road-segment dependent delay cost multipliers are set at their default values, i.e. equal to $\frac{AADT \times Length}{Avg.(AADT \times Length)}$. The average value in the denominator is calculated over all possible pairs of selected road segments. The base rate for the delay costs is determined as the regular wage rate without including the fringe benefits, which is equal to \$21.6/hour. Delay costs for each road segment are also assumed to change based on the storm type. Storm-intensity based delay cost multipliers are selected to be 2, 4, and 6, respectively for low, medium and high storm intensities. These values are assumed for illustration only. SLC managers have the flexibility to modify delay cost multipliers in any desired way and the multipliers could be updated for each run of the model.

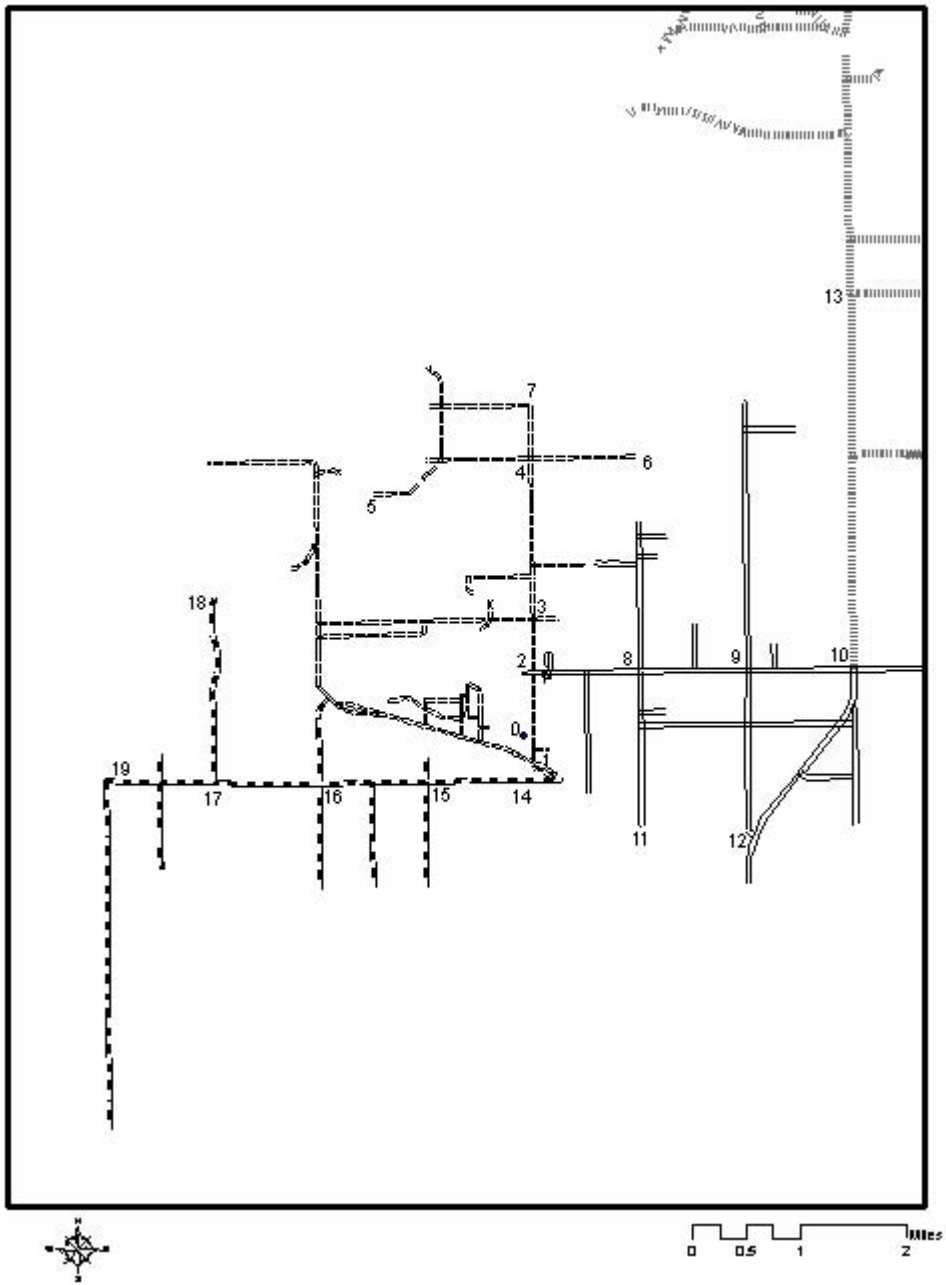


Figure 4.7 Example based on District 5 road segments

Table 4.8 Data for Selected Road Segments

From Node	To Node	Length (miles)	AADT	Delay cost multiplier
1	2	0.86	5238	2.29
2	3	0.51	2703	0.70
3	4	1.50	1633	1.25
4	5	1.60	732	0.60
4	6	1.01	44	0.02
4	7	0.50	283	0.07
2	8	1.02	3492	1.81
8	9	1.00	3492	1.78
9	10	0.99	3492	1.76
8	11	1.46	417	0.31
9	12	1.59	282	0.23
10	13	3.52	3066	5.49
14	15	1.02	496	0.26
15	16	0.98	227	0.11
16	17	1.00	227	0.12
17	18	1.81	98	0.09
17	19	1.01	227	0.12

In total, 17 road segments are selected to be plowed from four different routes of District 5. These road segments have a variety of AADT counts. The earliest time to start plowing operations t_0 is assumed to be 9 PM. We consider three storm types with low, medium and high intensities and assume that each storm type has an equal probability of occurrence. Table 4.9 lists the snowfall rate, snowplow speed, and sand/salt application rate under each storm scenario. Snowplow speeds in Table 4.9 are for the active mode when the snowplows are plowing the road segments. During inactive mode, snowplow speeds are assumed to be 35, 30, and 25 miles/hr, respectively, under low, medium and high storm intensities. Total sand/salt capacity of each plow is assumed to be 4000 lbs. The sand/salt capacity of a plow in this example is smaller than the real capacity. This is deliberate. If the real capacity of 18900 lbs is used, then all the selected road segments could be served by a single plow, since the maximum total sand/salt requirement is approximately 10600 lbs for all the selected road segments. The maximum number of plows to be called-out is assumed to be 5 and each plow operator is assumed to work for up to 5 hours without taking a break.

Table 4.9 Parameters for Each Storm Type

Variable	Storm intensity		
	Low	Medium	High
Snowfall rate (inch/hr)	0.27	0.55	1.31
Snowplow speed	17	12	9
Sand/salt application rate (lbs/mile)	450	500	450

For the selected road segments and input parameters, we solved Phase I and Phase II formulations repeatedly for each possible start time and number of employees. In Phase I, Term 3 is assigned a priority level of 2 because in this problem instance, it may be good to have more clusters, each with a small number of road segments, in order to allow plowing in parallel by available plows. Table 4.10 shows the clusters of road segments formed in Phase I. Note that, the clusters contain road segments from multiple routes and road segments with similar unit delay costs are grouped together. For all the storm types, the optimal set of clusters turn out to be same. Because delay cost multipliers are high, it is easy to see that the best start time is at 9 PM, i.e. we set $t_{\text{start}} = 9 \text{ PM}$.

Table 4.10 Cluster Formation Results (Same for All Three Storm Types)

Cluster	Road segments to plow	Path of the plow	Completion time (min)			Sand/salt requirement (lbs)		
			Low	Medium	High	Low	Medium	High
1	(10,13)	0→2→8→9 →10→13→10→9 →8→2→0	30.77	38.98	49.13	1583	1759	1583
2	(8,11),(9,12), (14,15)	0→2→8→11 →8→9→12→14 →15→14→0	25.72	33.18	43	1834	2038	1834
3	(2,3),(3,4), (4,5)	0→2→3→4 →5→4→3 →2→0	20.95	27.63	35.57	1624	1804	1624
4	(1,2),(2,8), (8,9),(9,10)	0→1→2→8 →9→10→9 →8→2→0	17.8	23.43	30.13	1738	1931	1738
5	(15,16),(16,17), (17,19),(17,18), (4,7),(4,6)	0→14→15→16 →17→19→17→18 →4→7→4→6 →4→3→2→0	42.8	55.52	70.85	2844	3161	3844

After forming clusters in Phase I, we next assign these clusters to available plows by solving Phase II formulation. In Phase II, default priority levels of terms in the objective function are assumed, so that the highest weight is given to Term 1. Table 4.11 shows the score of each cluster for the three storm types. In each case, the highest priority is given to Cluster 4, which has the road segments with maximum AADT counts and the lowest priority is given to Cluster 5, which includes road segments with lowest AADT counts.

Table 4.11 Cluster Scores

Cluster	Scores		
	Low	Medium	High
1	0.178	0.141	0.112
2	0.031	0.024	0.019
3	0.122	0.092	0.072
4	0.429	0.326	0.254
5	0.012	0.010	0.007

Table 4.12 shows optimal total costs and their composition in terms of percentage of delay and overtime components under the three storm types. Even though the optimal clusters of road segments are the same under each possible storm type, cost values are not equal. This is due to different snowplow speeds under different storm types, which leads to different completion times of the clusters and to different delay and overtime costs.

Table 4.12 Total Costs (TC), %Delay (%D) and %Overtime (%OT), $t_{\text{start}} = 9 \text{ PM}$

	Low			Medium			High			Expected
	TC	%D	%OT	TC	%D	%OT	TC	%D	%OT	
$k = 1$	1035	91.6	8.4	1888	94.9	5.1	3124	95.9	4.1	2016
$k = 2$	636	72.8	27.2	1057	83.6	16.4	1664	89.6	10.4	1119
$k = 3$	547	52.6	47.4	812	68.1	31.9	1194	78.3	21.7	851
$k = 4$	582	40.6	59.4	797	56.6	43.4	1108	68.8	31.2	829
$k = 5$	583	25.9	74.1	717	39.8	60.2	910	52.5	47.5	737

Table 4.12 shows that if the storm type is known, then the optimal strategy is to call-out three employees under low storm intensity, five employees under medium storm intensity, and five employees under high storm intensity. In reality, there is uncertainty about which storm type will occur (we assume equal likelihood in this example) with the result that a robust decision is to call-out five employees and to start plowing at 9 PM. This deployment strategy has the minimum expected total cost among all the alternatives. At t_{start} more precise information will be obtained and a plowing schedule will be picked. In this example, clusters do not change with storm type and their completion times increase proportionally in storm type. Therefore, the plowing schedule is the same for all storm types. In particular, each cluster is assigned to a different employee, which allows the plowing of highest priority clusters in parallel.

4.4 Concluding Remarks

In this chapter, we focused on modeling SLC's workforce management problem in the presence of uncertainty about storm conditions. Our models allow maintenance supervisors to specify which road segments need to be plowed after the end of the regular shift. The selection of road segments could be based on an AADT threshold, road surface type, or direct input.

We decomposed the problem into two parts – deployment strategy and plowing schedule. A deployment strategy is picked at the time of making the call-out decision, but the plowing schedule is implemented after uncertainty about the storm intensity is realized. For the latter, we developed a two-phased model formulation, which has a Cluster-Formation and a Cluster-Scheduling phase. The model formulation allows multiple and possibly competing priorities of SLC managers to be considered. The broader information requirements of the two-phased model that maintenance supervisors should input are the earliest start time of the plowing operations, road segments to be plowed, the maximum number of employees to be called-out, and delay cost multipliers.

At the time when SLC managers need to decide the number of operators to call out and the plowing start time, there is uncertainty in the expected storm conditions. Therefore, optimal plowing schedules are obtained for each combination of the number of employees to call out and the plowing start time. We obtain the best combination of these parameters by enumerating all possible solutions and choosing the minimum cost alternative. Effort involved in performing this step can be significantly reduced by pre-processing problem data to rule out ranges that do not contain an optimal solution.

Chapter 5 Workforce Requirements Planning

5.1 Introduction

This chapter describes an approach to determine optimal winter maintenance workforce requirements for SLC, which minimizes total costs. The formulation considers tradeoffs between regular time wages and overtime wages as well as cost of not meeting snow removal objectives through delay costs. In this section, we present model assumptions and our overall approach.

The optimal number of plow operators to have in the workforce depends on the time-to-clear goals of the county. Tighter time windows increase the requirement of the number of employees. Based on the suggestions of SLC's maintenance supervisors, the model currently assumes that all the designated road segments should be cleared within 24 hours after the storm ends. The advantage of introducing a time-to-clear goal into the model is that it provides a minimum number of employee requirements and also allows the user to affect relative use of regular time (RT) and overtime (OT) mode of operations.

Another modeling assumption is that each road segment belongs to a type, such that delay cost rate of segments of a particular type are the same. Groups of road segments are based on one of two criteria. In the first method, statistical clustering method is used to form road-type categories. These clusters are based on AADT counts and length of roads. In the second method, each road type is a separate category. In particular, the road types are county-state aid roads, county roads, township roads, municipal state-aid streets, municipal streets, and private jurisdiction roads, in order of their priority in plowing. Given The aggregate minimum plowing time of each group is calculated under each storm realization and then inflated by an average deadheading percentage. Plowing occurs in the order of priority with the highest priority road types plowed first. To reduce analytical complexity, each group of road segments is treated as a single task, which is completed in the priority order without the need to make any routing decisions. The length of a group is the sum of its road segment lengths and delay cost multiplier is the sum of the delay cost multipliers in the group. Detailed description on the formulation and validity of these assumptions is presented in Appendix F.

Figure 5.1 illustrates the overall approach as a flow chart. The algorithm is the black-box decision support system. Its details are presented in Section 5.2. Inputs of the algorithm are either discretionary, which need to be specified by maintenance supervisors, or from data sources. The main output of the algorithm is the optimal number of employees (k^*) for the considered depot. The decision support system also provides a graph of the change in costs with respect to each possible number of employees (k). This graph helps SLC managers to visualize the robustness of costs around k^* . In addition, total costs of the county may vary from year to year with k^* employees due to uncertainty in yearly storm conditions. The decision support system also provides a distribution of extra plowing costs due to OT and delay for several years of storm scenarios based on Monte-Carlo simulation. These graphs can be used to develop a contingency budget for labor cost portion of winter maintenance costs. Technical details of these simulations are presented in Section 5.2.

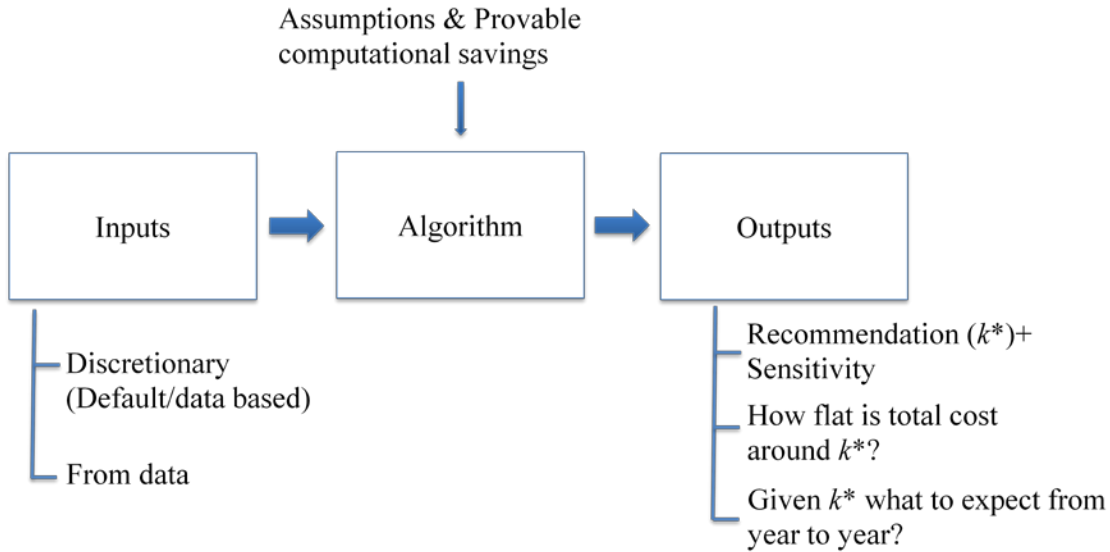


Figure 5.1. Flow chart of overall approach

5.2 Model Description

In this section, we provide input requirements of the model and present our approach for calculating k^* .

As mentioned in Section 5.1, there are two input types; discretionary and from data sources. Discretionary inputs of the model are as follows:

- 1) Inflation factor due to deadheading (θ): This is the amount by which minimum plowing time should be multiplied to obtain expected actual plowing time, i.e. actual plowing time = $\theta \times$ minimum plowing time.
- 2) Average number of passes for each storm in the historical data: This is the number of times that a plow goes over the same road segment to achieve bare pavement in each storm scenario.
- 3) Decrease in plowing times in subsequent passes over the same road segments: Since road conditions get better, total completion times are expected to decrease in a subsequent pass over the same road segments.
- 4) Proportion of annual wages charged to snow removal budget (α)
- 5) Maximum work time limit in a day: Due to safety considerations, employees are restricted to work for a specified number of hours out of 24 hours in a day.
- 6) Hourly wage rate
- 7) Percentage of fringe benefits in wages
- 8) Maximum number of employees to at each depot
- 9) Shift start and end times: This is the same for all employees.

Inputs to the algorithm from other data sources are as follows:

- 1) AADT count of road segments (Source: GIS maps of SLC)
- 2) Lengths of road segments (Source: GIS maps of SLC)

- 3) Road types as county state-aid, township etc. (Source: GIS maps of SLC)
- 4) Number of lanes for each road segment (Source: Microsoft Access database records of SLC)
- 5) Storm end times (Source: RWIS data from Duluth International Airport)
- 6) Plow speeds under each storm scenario (Source: Wilson, Dadie-Amoah, and Zhang (2003) as reported in Chapter 2)

Given the assumptions and input parameters, we first developed an explicit cost function for overtime costs and delay costs for a given storm scenario. In this formulation, the number of plow operators in workforce (k), the number of employees to call-out (k_{OT}), start time of plowing operations in overtime period (t_{start}), and the length of overtime duty for each called-out employee (W_{OT}) are decision variables. For each k , the best combination of k_{OT} , t_{start} and W_{OT} are selected, and the minimum cost $C(\xi, k)$ of k employees as total of OT and delay costs under given storm realization ξ is computed. The optimal number of employees (k^*) is then determined as follows:

$$k^* = \arg \min_k \{E_\xi[C(\xi, k)] + \alpha \times W \times k\} \quad (1)$$

where W is the total yearly wages of an employee and $E_\xi[C(\xi, k)]$ is the expected extra plowing costs (OT + delay) given an average year's storm data.

Graphical representation of OT and delay costs are as following. Their explicit formulae is provided in Appendix G .

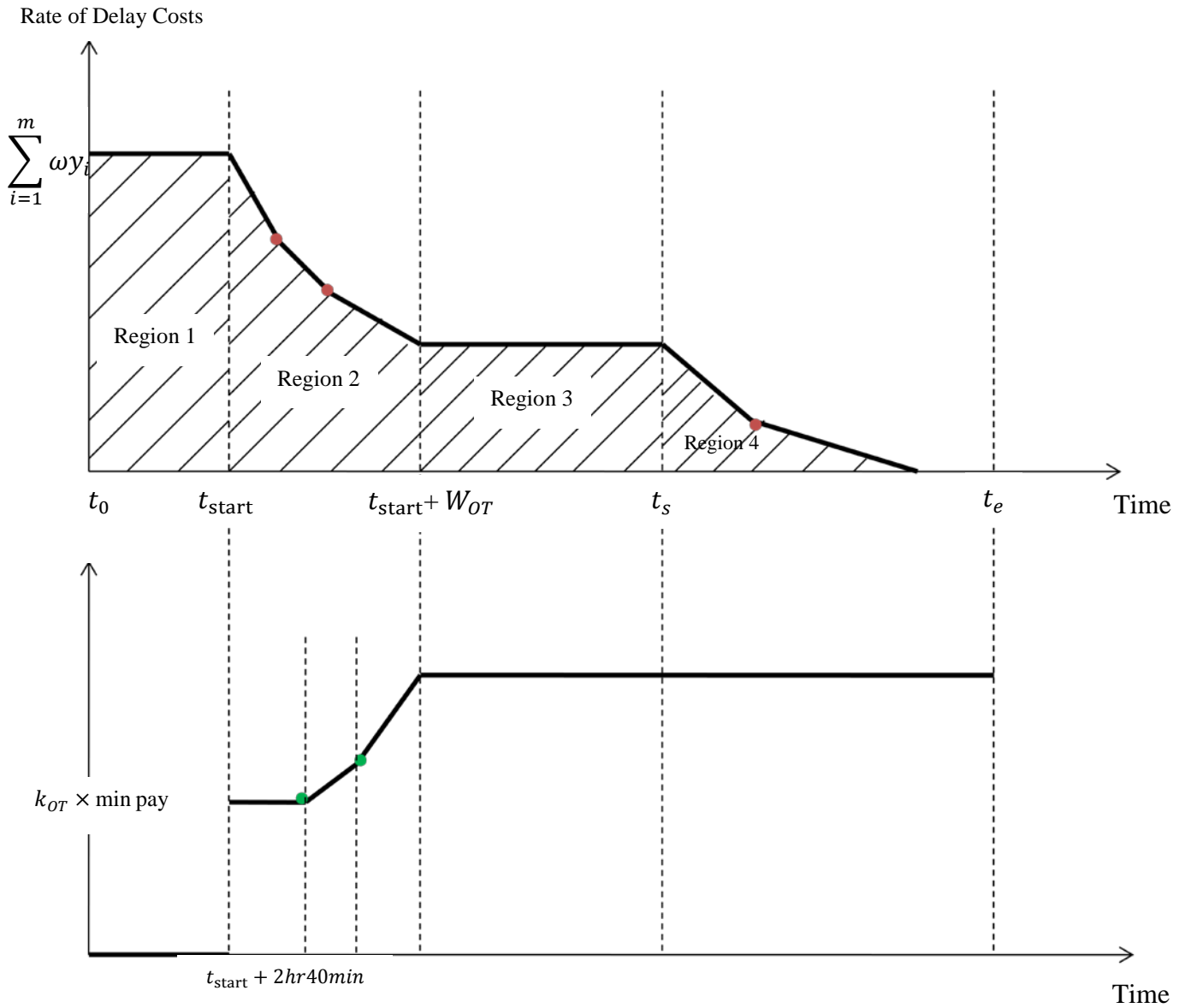


Figure 5.2 Graphical representation of total costs

5.3 Solution Methodology

The sum of total OT and delay cost functions is not well-behaved, meaning that the best values of the decision variables cannot be obtained analytically. Therefore, we developed an intelligent enumeration procedure by identifying ranges of optimal decisions. The procedure outputs the optimal number of employees (k^*), which has the least sum of expected extra plowing costs (OT + delay) and yearly wages for snow removal operations as given in Equation (1) in Section 5.2.

Before presenting the enumeration procedure, we summarize our findings on the potential ranges of decision variables k , k_{OT} , t_{start} and W_{OT} . These analyses help to decrease computational complexity of the enumeration procedure. Detailed explanation of theorems and proofs are presented in Figure H.

Theorem 1: The minimum value of k is equal to $\min k = \frac{t(\xi)}{\min(\text{max work time limit, time-to-clear goal})}$, where $t(\xi)$ is the minimum plowing time under a storm realization ξ .

As the time-to-clear goal is set at higher values, the maximum work time limit also changes. For example, its default value for 24 hours to clear is 16 hours. We expect this to double if the time-to-clear goal is set at 48 hours.

Theorem 2: Extra plowing costs as sum of delay and OT costs is convex with respect to k , when $k_{OT}, W_{OT}, t_{\text{start}}$ are fixed.

Theorem 3: Optimal start time of plowing operations during overtime period (t_{start}^*) is either equal to t_0 (i.e. the earliest call-out time) or within 4-hour interval before the start of regular shift hours, i.e. $[t_s - 4 \text{ hrs}, t_s]$.

Theorem 4: Given values of k and k_{OT} , $t_{\text{start}}^* \in \{t_0\} \cup [t_s - 4 \text{ hrs}, t_s - \min W_{OT}]$, if $\min W_{OT} \leq 4 \text{ hrs}$; $t_{\text{start}}^* = \emptyset$, if $\min W_{OT} > t_s - t_0$; $t_{\text{start}}^* = t_0$, otherwise.

$\min W_{OT}$ is the minimum work time in the first overtime period after the end of snow storm, and it is equal to

$$\min W_{OT} = \frac{t(\xi) - (7.5 \text{ hours} + t_0 - t_e) \times k}{k_{OT}}$$

Theorem 5: Optimal work time length during the first overtime period after the storm ends (W_{OT}^*) equals to $t_s - t_{\text{start}}$ if $t_{\text{start}} \in [t_s - 4 \text{ hr}, t_s]$ when $t_{\text{start}}, k, k_{OT}$ are fixed.

Theorem 6: $W_{OT}^* \in [2 \text{ hr } 40 \text{ min}, t_s - t_{\text{start}}]$ if $t_{\text{start}} \in (t_e, t_s - 4 \text{ hr})$ and total extra plowing costs due to OT and delay is convex with respect to W_{OT} in this interval of t_{start} .

Theorem 7: $W_{OT}^* \in (0, t_s - t_{\text{start}}]$ if $t_{\text{start}} = t_e$ and total extra plowing costs due to OT and delay is convex with respect to W_{OT} in this interval of t_{start} .

By utilizing above results, we developed the following enumeration procedure to obtain k^* .

$\xi \in$ storm realizations

$$k \in \{\min k, \dots, k_{\max}\}$$

$$\min C(\xi, k) = \infty$$

$$k_{OT} \in \{1, \dots, k\}$$

Calculate $\min W_{OT}$

$$t_{\text{start}} \in \{t_0 \cup [t_s - 4 \text{ hrs}, t_s - \min W_{OT}]\}$$

If $t_0 > t_s - \min W_{OT}$, break (i.e. there is no feasible t_{start})

If $t_{\text{start}} \in [t_s - 4 \text{ hrs}, t_s]$, then $W_{OT} = t_s - t_{\text{start}}$

Compute cost $C(\xi, k, k_{OT}, t_{\text{start}}, W_{OT})$

If $C(\xi, k, k_{OT}, t_{\text{start}}, W_{OT}) < \min C(\xi, k)$

$$\min C(\xi, k) = C(\xi, k, k_{OT}, t_{\text{start}}, W_{OT})$$

If $t_{\text{start}} \in (t_e, t_s - 4 \text{ hrs})$,

$W_{OT} \in [\max\{2\text{hrs } 40\text{mn}, \min W_{OT}\}, t_s - t_{\text{start}}]$ (utilize convexity)

Compute cost $C(\xi, k, k_{OT}, t_{\text{start}}, W_{OT})$

If $C(\xi, k, k_{OT}, t_{\text{start}}, W_{OT}) < \min C(\xi, k)$

$$\min C(\xi, k) = C(\xi, k, k_{OT}, t_{\text{start}}, W_{OT})$$

If $t_{\text{start}} = t_e$

$W_{OT} \in [\min W_{OT}, t_s - t_{\text{start}}]$ (utilize convexity)

Compute cost $C(\xi, k, k_{OT}, t_{\text{start}}, W_{OT})$

If $C(\xi, k, k_{OT}, t_{\text{start}}, W_{OT}) < \min C(\xi, k)$

$$\min C(\xi, k) = C(\xi, k, k_{OT}, t_{\text{start}}, W_{OT})$$

$$C(\xi, k) = \min C(\xi, k)$$

$$k^* = \arg \min_k \{E_\xi [C(\xi, k)] + \alpha(k) \times W \times k\}$$

5.4 Data Analysis and Default Parameter Settings

5.4.1 Monte-Carlo Simulation

Since storm conditions vary from year to year, extra plowing costs incurred by the county are also expected to vary. Such variations require that SLC have a contingency budget to absorb fluctuations in labor cost component of total costs of winter maintenance. Therefore, the decision support system also provides distribution of expected extra plowing costs over the years with k^* employees using Monte-Carlo simulation.

Analysis of storm data from the years 2005 to 2008 showed that 12 storms occur on average in SLC in a year. By randomly sampling from the 43 recorded storms from these years, we generated 10 years of representative storm realizations. Number of storms in each year varies from 11 to 14 with an average of 12. Decision support system reads the generated years' storm data and calculates the extra OT and delay costs for each year considering that there are k^* employees in the workforce. Cost figures are displayed in the form of a box plot to easily visualize the range, median and critical percentiles of the expected costs.

5.4.2 Clustering Analysis

As mentioned in Section 5.1, road segments are grouped into types such that delay cost rate of road segments of the same type is the same. There are two reasons for clustering road segments into groups instead of considering them separately:

- 1) Ease of setting priority levels. The data is needed to specify relative priorities for a small number of road types.
- 2) Computational savings. This allows maintenance supervisors to run the program many times under different parameter specifications and quickly observe the changes in results.

Clustering road segments into groups can be done in two ways based on 1) statistical clustering methods, or 2) road type / who pays for the road. The decision support system is flexible to consider both clustering methods and user can specify which method to use.

Statistical clustering method is known as K-means clustering. In this method, best number of groups for the designated road segments is determined based on their AADT counts and lengths. Using road segments from District 5 of SLC, K-means clustering forms 5 clusters as the best number of clusters for each depot.

Another method to group road segments is based on road types and who pays for the road. In this method, 6 groups are formed for each depot of District 5 and listed as follows in order of their priority in plowing:

- 1) County state-aid roads
- 2) County roads
- 3) Township roads
- 4) Municipal state-aid streets
- 5) Municipal streets
- 6) Private jurisdiction roads

In both clustering methods, delay cost rates of road segments connected to a given depot are determined as $\frac{AADT \times \text{Length}}{\text{Average AADT} \times \text{Length in that depot}}$.

5.4.3 Default Parameter Settings

All the default parameter settings in the decision support system are based on data from District 5 of SLC. Below we summarize the default parameter settings and how we obtained these values for each discretionary input parameter.

- 1) *Inflation factor due to deadheading (θ)*: The excel tool submitted with Task 1 provides the actual completion times of the 15 routes in District 5 as well as total deadheading times and number of turnarounds in each tour of the plow. Each turnaround takes 3 minutes on the average. Using this data, we calculated the inflation factor due to deadheading ($\theta_{t,r,\xi}$) for each tour t of the plow in a given route r under a given storm scenario ξ as follows:

$$\theta_{t,r,\xi} = \frac{\text{total completion time}}{\text{total completion time} - (\text{deadheading time} + \text{turnaround time})}$$

Then, default value of θ is obtained as 1.28 as the average of $\theta_{t,r,\xi}$. When calculating the default, only first tours of the plows are considered. The reason for not using second and third tours of the plows is that in Task 1 deadheading times are reported by considering that plows perform only a single pass over the same road segments. However, SLC usually performs multiple passes over the same road segments until all snow and ice is cleared, which decreases deadheading time in subsequent tours.

- 2) *Average number of passes for each storm in the historical data*: Summary reports from Microsoft Access database show that SLC performs two passes on average over the same road segments. Therefore, a default value of 2 passes is used for each historical storm realization. However, maintenance supervisors may modify the number of passes for each individual storm in the historical data.
- 3) *Decrease in plowing times in subsequent passes over the same road segments*: A default value of 10% decrease in total completion times is used.
- 4) *Proportion of annual wages charged to snow removal budget (α)*: The proportion of annual wages spent on plowing operations is based on hours worked as stated by SLC managers. On average, 12 storms occur in a year in SLC based on RWIS data obtained from Duluth International Airport. Most of the main road segments are required to be cleared within 24 hours after the end of snow storm. An additional 24 hours is usually spent to clear private roads and driveways. This corresponds to 24 days in total out of $52 \times 5 = 210$ weekdays in a year. Therefore, the default value of α is set to $(24/210) \times 100 = 9.2\%$.
- 5) *Maximum work time limit in a day*: The default value is set to 16 hours and could be modified by maintenance supervisors.
- 6) *Hourly wage rate*: Default value of hourly wage rate is set to \$35 as stated by SLC managers.
- 7) *Percentage of fringe benefits in wages*: As stated by SLC managers, default value is set to 62%.

- 8) *Maximum number of employees to keep in workforce for the considered depot*: Default value is set to 40 employees for each depot, which is more than twice the current number of employees in a depot.
- 9) *Shift start and end times*: Default is 7:30 AM to 3:30 PM.

5.5 Results

In this section, we provide results on k^* values for the four depots of District 5 based on default parameter settings. We also present sensitivity of results based on changes in several discretionary input parameters. Sample graphical results are provided for Depot 1 (Jean Duluth), to visualize behavior of costs with respect to k and distribution of costs over the years with k^* employees. A user guide, provided in Appendix I, can be used to evaluate a variety of other what-if scenarios.

Table 5.1 shows k^* values and Table 5.2 shows the current number of plow operators for the four depots in District 5. Summary reports from Microsoft Access database records of SLC show that on average road segments have two lanes (i.e. one lane at each side of the road). Therefore, Table 5.1 provides k^* values considering a single lane on each side of the road and assuming that on average each lane is plowed twice. In reality, there is one road segment with four lanes and there are several road segments with three lanes. Some road segments require more than one pass for each lane to complete plowing due to wide or paved shoulders. Data provided to the researchers show that the road segments in Access database and GIS maps of the county do not match. The number of lanes for each road segment is not provided in GIS maps of the county. This explains the choice of default values in terms of number of lanes and number of passes required to achieve bare pavement.

Although we do not input the exact number of lanes for each road segment, and the precise number of passes needed, the decision support system developed in this chapter is able to accept such data when in the future the number of lanes are recorded in county GIS maps. We also perform sensitivity analysis by considering more than one lane for county state-aid roads.

The decision support system assumes that the time-to-clear goal is 24 hours and that both sides of the road segments are plowed. Therefore, all the road segments in GIS maps connected to the depots need to be cleared in 24 hours after all storm realizations using storm data of the year 2006. The k^* values do not change based on which road-type clustering method is used. The optimal number of employees is expected to be lower if the time-to-clear goal is more than 24 hours for all the road segments or if only the high priority road segments are required to be cleared within 24 hours.

Table 5.1 Optimal Number of Plow Operators for Each Depot in District 5 with Default Parameter Settings

	Optimal Number of Plow Operators (k^*)
Jean Duluth (Depot 1)	13
Nopeming (Depot 2)	2
Brookston (Depot 3)	6
Pike Lake (Depot 4)	11

Table 5.2 Current Number of Plow Operators in Four Depots of District 5

	Current Number of Plow Operators
Jean Duluth	14
Nopeming	N/A
Brookston	7
Pike Lake	18

Figure 5.3 shows the graphical output of the decision support system for Depot 1 under default parameter settings. The graph shows the changes in extra plowing costs due to OT and delay, yearly wages spent on plowing operations, and total costs with respect to k . The minimum cost is obtained at $k^* = 13$ employees. The x -axis of the graph starts from 13 employees because with less than 13 employees all the road segments cannot be cleared within 24 hours after the end of a snow storm. For the same depot and under default parameter settings, Figure 5.4 shows the distribution of extra plowing costs as the sum of delay and OT costs over 10 years of simulated storm conditions in the form of a box plot. The red line in the middle represents the median of the costs, whereas the upper and lower edges of the box correspond to 75th and 25th percentile of the costs respectively. For this example, there is one outlier value, which is shown with a plus sign. In this year, the costs are expected to be unusually high (more than 25th quartile plus three times the inter-quartile range). The graph shows that the county can expect high costs in a few years that are substantially different from the normal pattern.

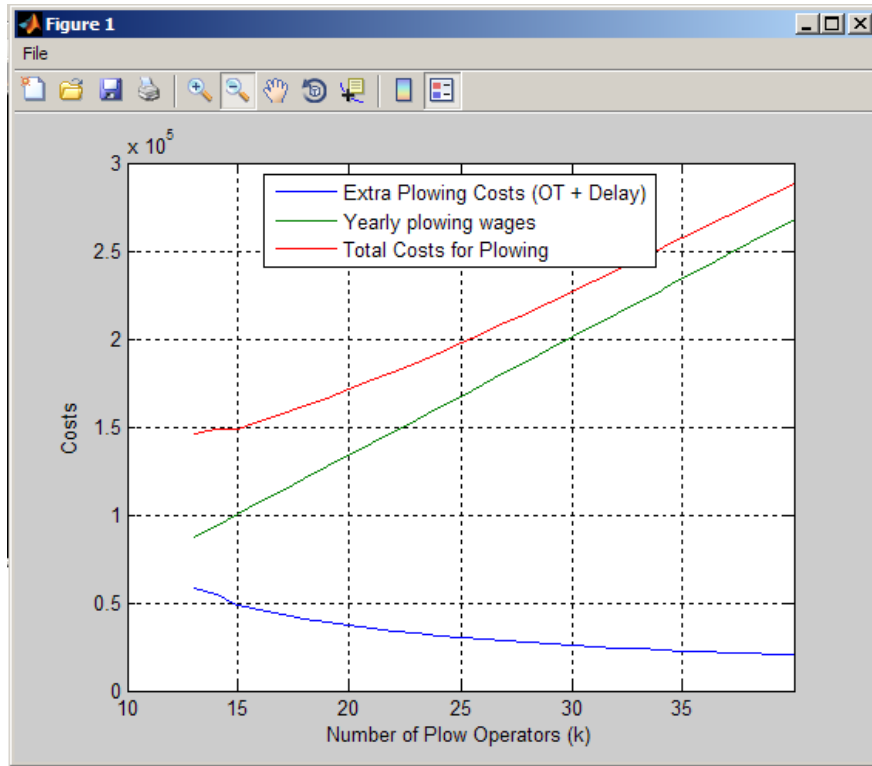


Figure 5.3 Depot 1 extra plowing costs, yearly plowing wages, and total costs versus k

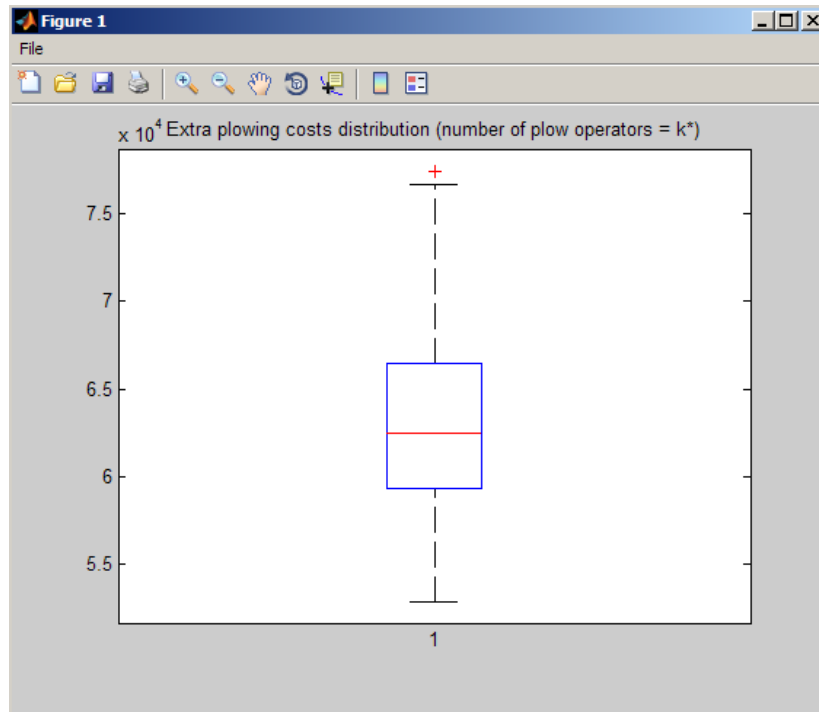


Figure 5.4 Depot 1 distribution of extra plowing costs (OT + delay costs) over 10 simulated years

Table 5.3 provides k^* values for all four depots under different discretionary parameter settings. Default value of the corresponding parameter is shown in bold font. For the values of α greater than 9.2%, yearly plowing wages are dominant over extra plowing costs. Therefore, when α is greater than or equal to 9.2%, the value of k^* stays the same for all depots, where k^* is the first feasible number of employees to clear all the road segments within 24 hours after the end of snow storms. When $\alpha = 4.6\%$, only important road segments are plowed within 24 hours. The plowing wages decrease and no longer dominate extra plowing cost. In this case, in order to decrease OT and delay costs, k^* values increase in three depots.

Modifying deadheading inflation factor (θ) by 25% changes the optimal number of employees by 19.7% on average. Therefore, efficiency of plowing operations significantly impacts the optimal number of employees. Similarly, if employees are allowed to work up to 20 hours, then k^* decreases by 15% on average, whereas if the maximum work time limit is 12 hours, then 35% more employees are needed on average. Additionally, if the percent decrease in total completion times is 15% more than the default value of 10%, then k^* values decrease by 1 in Depot 1 and Depot 4. When number of passes for each storm in the historical data is assumed to be 1, then 5 less employees are needed in Depot 1 and Depot 4, 2 fewer employees are needed for Depot 3 and the required number of employees stays the same for Depot 2.

Since the exact number of lanes for each road segment in GIS maps is currently unknown, in addition to default value of average number of two lanes (i.e. one lane at each side), we obtained results for the case in which all the county state-aid road segment have two lanes on each side whereas all the other roads have a single lane at each side. Since county state-aid road segments have the highest priority and they form a significant percentage of all the road segments, the optimal number of employees increases at all depots. In this case, the average percent increase is about 40%.

Table 5.3 Results under Modified Discretionary Parameter Values

		Optimal Number of Plow Operators (k^*)			
		Depot 1	Depot 2	Depot 3	Depot 4
Proportion of wages spent on plowing operations (α)	13.8%	13	2	6	11
	9.2%	13	2	6	11
	4.6%	15	3	7	11
Deadheading inflation factor (θ)	1.03	10	2	5	9
	1.28	13	2	6	11
	1.53	15	3	7	13
Maximum work time limit	12 hours	17	3	8	14
	16 hours	13	2	6	11
	20 hours	10	2	5	9
Percent decrease in subsequent passes	5%	13	2	6	11
	10%	13	2	6	11
	25%	12	2	6	10
Number of passes for each storm	1	8	2	4	6
	2	13	2	6	11
	3	18	3	9	15
Number of lanes at each side of the road	All Roads (1)	13	2	6	11
	All County-State Aid Roads (2), Other Roads (1)	18	3	8	15

5.6 Conclusions

In this chapter, we described an approach to help SLC to determine an optimal workforce size for snow and ice removal operations. An optimal workforce size has the smallest total sum of yearly wages spent on plowing operations, expected extra plowing cost due to OT, and cost of delay in plowing start times.

The approach is based on an intelligent enumeration procedure, which quickly obtains the best decisions on the number of employees to call-out (k_{OT}), time to start plowing in overtime period (t_{start}), and length of duty in overtime period (W_{OT}) for each value of number of employees (k) and a storm realization (ξ). In this approach, an optimal number of employees depend on time-to-clear goals of the county as well as several discretionary inputs such as proportion of wages spent on snow removal operations and maximum daily work time limit of an employee. Default values for all the discretionary inputs are identified based on real data from four depots of District 5 of SLC. The users may change these default values and run a variety of what-if analyses to understand how these parameters affect workforce requirements.

In addition to providing an optimal number of employees for a selected depot, the decision support system has the following features:

- 1) It clusters road segments into types according to one of two criteria – either a statistical clustering methods, or road type.
- 2) It provides a graphical representation of costs with respect to number of employees in workforce.
- 3) Given an optimal number of employees, it calculates the range of possible extra plowing costs over 10 years of storm realizations.

The use of this approach can help SLC decision makers in a number of ways. First, decision makers can quantify the effect of different factors on workforce requirements. Second, its graphical outputs will help visualize the robustness of different workforce sizing decisions on total costs. Third, significant differences in optimal number of employees for a set of depots may help SLC decision makers to reorganize plow route assignments to depots in order to balance workload at each depot.

Chapter 6 Quantifying Value of Flexibility

This chapter describes research that led to the creation of a computer program for evaluating the economic benefits of three workforce deployment strategies. These strategies involve use of contract employees, split shifts and staggered shifts. Note that the use of common pool employees from other counties is not different from the use of contract employees. Therefore, that option is not analyzed independently. The computer program also provides a graphical user interface (GUI) for the algorithm developed as part of Chapter 5. The computer program is referred to as the *Workforce Deployment Tool* in this report. This tool has a user friendly interface. Users can click on an executable file to open the user interface window. GUI allows users to enter various discretionary parameters and run the program in different modes.

Default parameters are provided along with the program, but users can enter a variety of discretionary inputs. Discretionary inputs are as follows: inflation factor due to deadheading, average number of passes for each storm in the historical data, decrease in plowing times in subsequent passes over the same road segments, proportion of annual wages charged to snow removal budget, maximum work time limit in a day, hourly wage rate, percentage of fringe benefits in wages, maximum number of employees to keep in workforce, and shift start and end times. Default values for all of these parameters are obtained from SLC data and are presented in the Chapter 5. However, it is easy to change these default values.

The three strategies investigated in this report are (1) the use of contract employees, (2) split shifts, and (3) staggered shifts. Contract employees are those that are not part of the regular crew. It is assumed that SLC has access to contract employees and that a small fixed cost (called retainer) is incurred for having each contract employee available. A variant of the K-star algorithm (this is the name we give to the algorithm developed as part of Chapter 5) is used to find the optimal number of contract employees. Users can specify the number of regular employees that should be called before calling contract employees. This helps to accommodate any union restrictions on the use of contract employees. Note that the details of all algorithms used in the creation of the Workforce Deployment Tool can be found in Appendix K. Also, all mathematical results that help obtain or simplify computations are presented in Appendix J. Results show that overtime cost and delay costs can be reduced upon using contract employees. Total cost including the retainer is smaller when contract employees are used as compared to the total cost that was obtained without assuming availability of contract employees.

Splitting shifts, the second alternative, gives SLC the flexibility to call employees based on the end time of a storm. This helps complete more plowing work in regular shift hours and reduces extra plowing cost. An algorithm to evaluate the economic impact of staggered shifts is developed and incorporated in the Workforce Deployment Tool. Our analysis shows that there is a decrease in delay cost as well as overall cost with this mode of operation. Thus, we are able to quantify the extent of savings from having split shifts to help SLC decide if this option may be worthwhile to pursue in the future.

Staggering shifts, the third strategy, creates two shifts – an AM shift and a PM shift. The AM shift goes from 4:00 AM to 12:00 PM and evening shift goes from 8:00 AM to 4:00 PM. Employees are split over these two shifts. An algorithm is designed to find the best way to deploy employees in staggered shifts. We also quantify the benefit of using staggered shifts.

6.1 Graphical User Interface (GUI) for K-star Algorithm

In Chapter 5, an algorithm was developed to find the optimal number of employees required to minimize annual plowing and delay costs over representative storm realizations. We call that algorithm the K-star algorithm. This section describes the development of a graphical user interface (GUI) for the K-star algorithm. Earlier, users were required to open several different windows to provide input. With the GUI, all inputs can be provided via a graphical interface and default values are clearly indicated for the users to view. This GUI also works with the three alternative strategies analyzed in this report. We begin by specifying the inputs and outputs for the GUI for the K-star algorithm.

6.1.1 Inputs and Outputs

There are three types of inputs. These are:

1. Required inputs
2. Default inputs
3. Inputs from data sources

In the sequel, we discuss each of these types of inputs separately.

Required inputs: The following inputs must be provided to execute the algorithm.

- a. **Clustering type:** User is asked to select from one of the two clustering approaches – statistical clustering and road type clustering.
- b. **Depot number:** Users need to select the depot number for which the algorithm is to be run. There are four depots in SLC district 1. The program can be extended to allow both district and depot to be selected. This will require data from other districts.

Default inputs: Each of these inputs has a default value, but the defaults can be changed easily. We list the inputs and their current default values below.

- a. **Inflation factor due to deadheading (θ):** This is the amount by which minimum plowing time is inflated to account for deadheading. In particular,
$$\text{Actual Plowing Time} = [1 + \theta/100] \times \text{Minimum Plowing Time},$$

and the default value of θ is 40.
- b. **Average number of passes for each storm in the historical data:** The number of times that a plow needs to go over the same road segment to achieve bare pavement in each storm scenario. The default value is 2.
- c. **Decrease in plowing times in subsequent passes over same road segments:** As road conditions get better, total completion times are expected to decrease in a subsequent pass over the same road segments. Default value is a 10 % reduction.
- d. **Proportion of annual wages charged to snow removal budget (α):** Default value is 9.2. This means that 9.2% of employees' annual wages are charged to the winter maintenance budget.

- e. **Maximum work time limit in a day:** Due to safety considerations, employees are not allowed to work more than a certain number of hours in each 24-hour period. Default value is 16 hours.
- f. **Hourly wage rate:** Default value for hourly wage is \$35.
- g. **Percentage of fringe benefits in wages:** Fringe rate is 62% of base wage rate.
- h. **Maximum number of employees:** At each depot, the maximum number of employees is limited to 40 by default. This helps reduce computational effort when calculating the optimal number of employees. From a practical viewpoint, this is not a serious limitation because it is uneconomical to have more employees than plows and the number of plows is well below 40 in all depots.
- i. **Number of plows:** This is the number of plows available with the depot. This number is by default set to 25.

Inputs from Data Sources: Inputs to the algorithm from other data sources are as follows

- a. AADT count of road segments (Source: GIS maps of SLC)
- b. Lengths of road segments (Source: GIS maps of SLC)
- c. Road types as county state-aid, township etc. (Source: GIS maps of SLC)
- d. Number of lanes for each road segment (Source: Microsoft Access database records of SLC)
- e. Storm end times (Source: RWIS data from Duluth International Airport)
- f. Plow speeds under each storm scenario (Source: Wilson, Dadie-Amoah, and Zhang 2003, as reported in Chapter 2).

Outputs:

The output from the algorithm is the optimal number of employees that minimize total cost – i.e. the sum of plowing and delay costs. Outputs for each mode of operation or each algorithm are displayed in separate text files, which can be opened and viewed from the GUI.

6.1.2 Graphical User Interface (GUI)

In order to analyze different workforce deployment strategies, users start by clicking on the Task5Tool.exe file. This opens the window shown below.

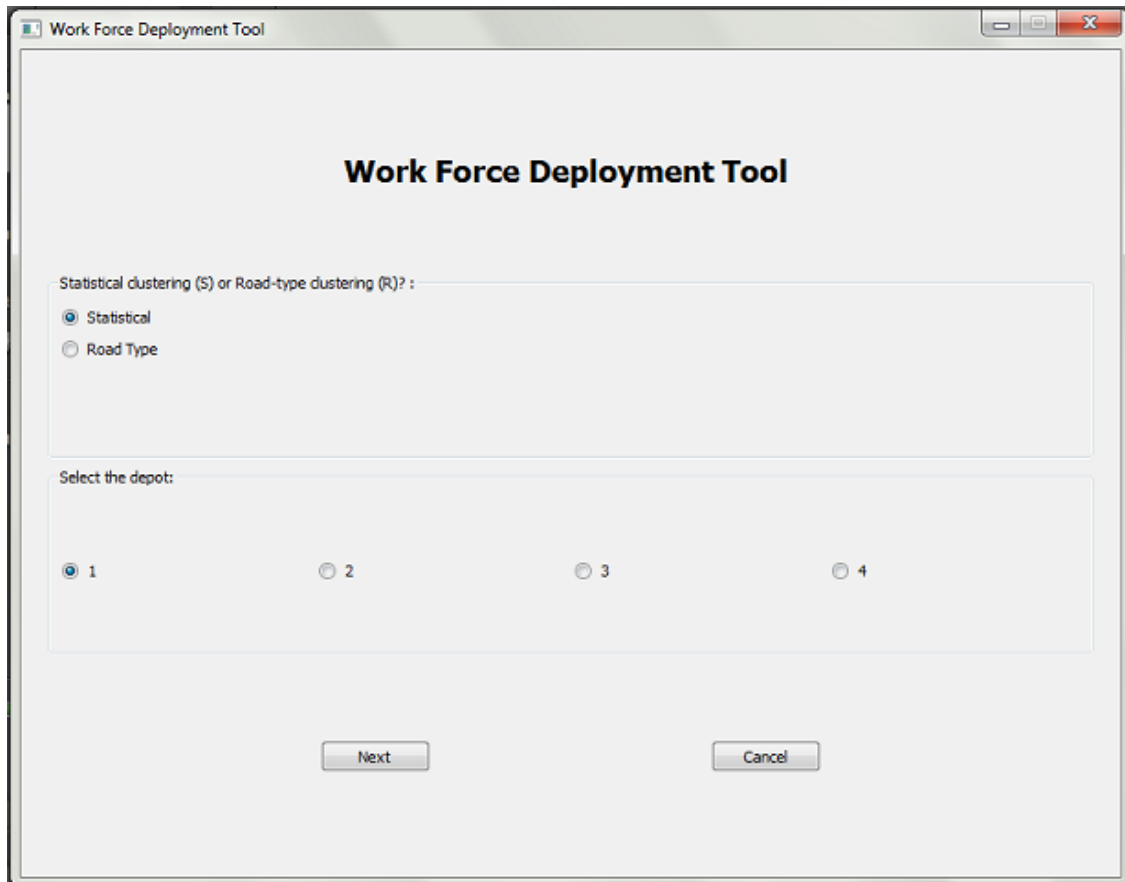


Figure 6.1 First window

Next, the user selects clustering type – either statistical or road type (default statistical). Then there is the option to select the depot number for which deployment strategies need to be compared (default = 1). Then, the user clicks ‘Next’ to go to the next screen. As soon as ‘Next’ is clicked the cursor will change to busy and will stay so for few minutes which indicates that the clustering algorithm is being run. Once clustering is done users are prompted to the next screen. Note, users can exit the GUI by clicking on the ‘Cancel’ button.

Once the user clicks on the Next button the following screen is displayed. Key options at this stage are either to edit the default values or run the K-star algorithm. In the first option, the user can first edit default values and then run K-star algorithm, whereas in the latter option, the K-star algorithm is run with default values. Any subset or all of the default values may be changed.

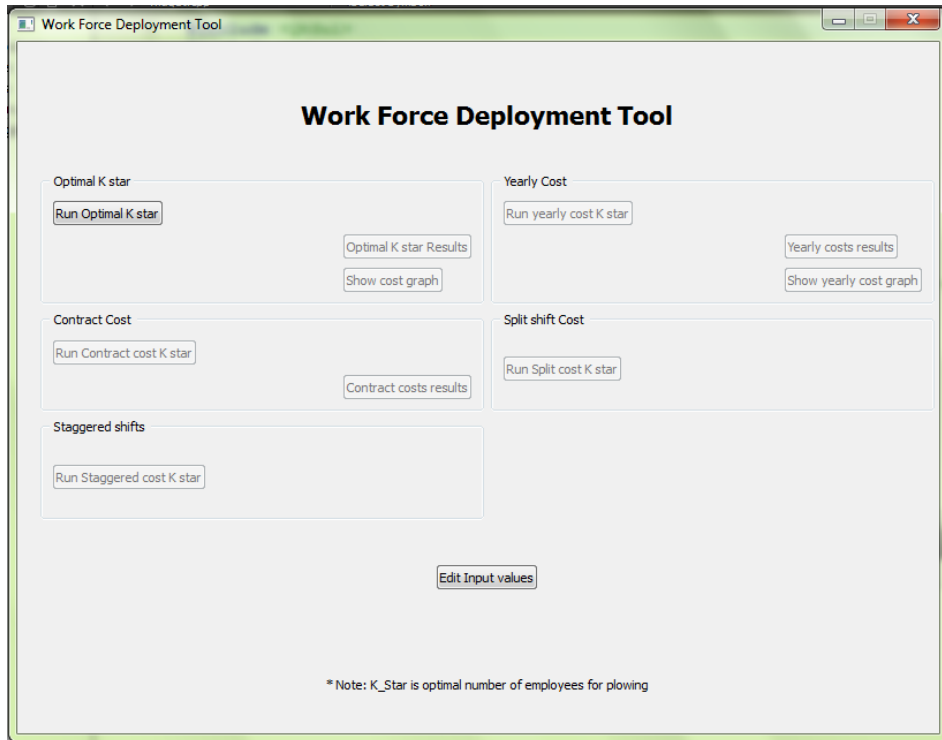


Figure 6.2 Main window

Following figure shows the window through which default inputs may be provided.

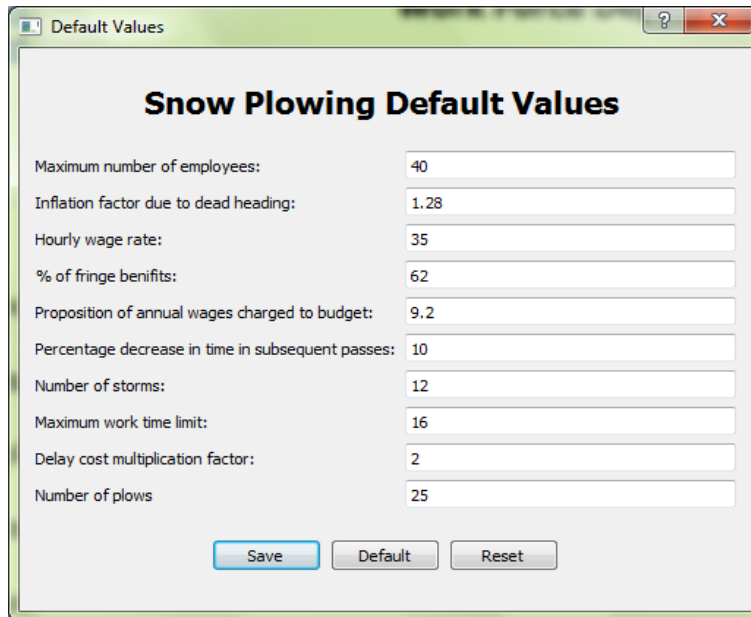


Figure 6.3 Default options window

Once 'Run Optimal K_star' is clicked and the operation finishes, user has access to all other modes of operation of the GUI, which we describe in the following sections.

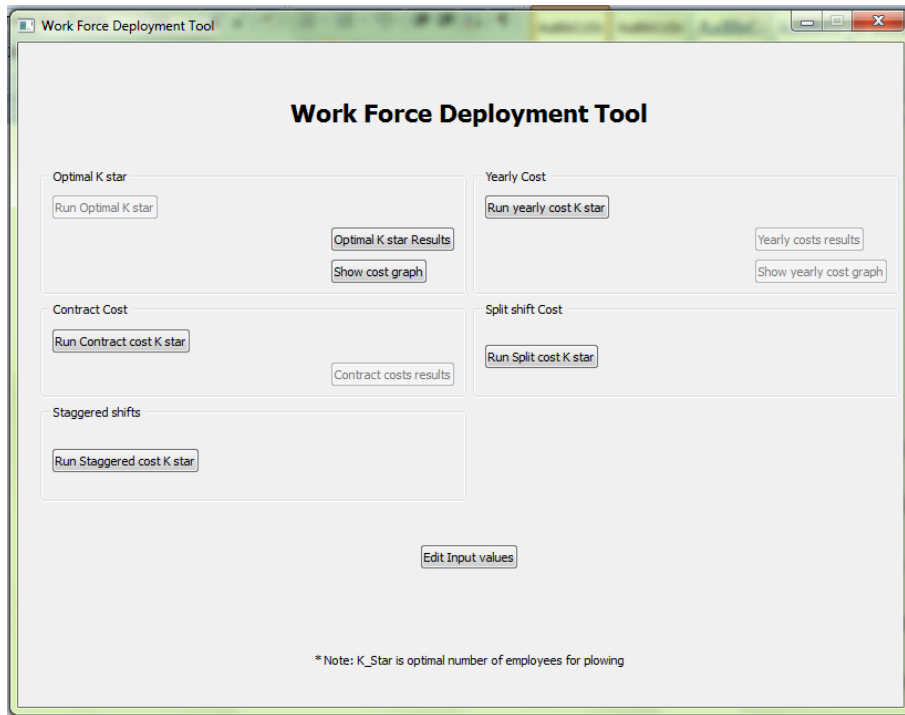


Figure 6.4 Main window after running optimal k star

6.2 Contract Workers

This option assumes that SLC has access to some contract workers. Use of contract employees incurs a fixed cost (e.g. paperwork, retainer), which can be set by the user (default \$1500 per employee). We also assume that the contract employees have a default on call pay of 1.5 times the hourly wage of regular employees and no fringe, but this can be changed. Contract workers are called at the end of a storm, if overtime work is deemed necessary. They work alongside regular employees to complete the overtime work. If overtime work spreads to a subsequent overtime period, then these contract workers are assumed to continue working until their share of the overtime work is complete. This ensures that all work is done as early as possible. Such a strategy makes sense for contract employees because their wage rate does not depend on how many hours they work in overtime mode, whereas regular time employees are paid twice the regular wage rate after completing 4 hours of overtime work. In order to accommodate union concerns, the program allows the user to select the number of regular employees who must be called first before calling any contract employees. Our algorithm assumes that employees who are offered overtime work do not refuse the offer. The maximum number of sum of regular and contract employees is limited by the availability of the plows.

6.2.1 Inputs and Outputs

When the user clicks 'Run Contract K-star button' a window opens and asks the user to input the following values:

Min number of regular employees: The number of regular employees to be called before contract employees may be used. This number should be less than the value of k star, the optimal

number of employees, which we get upon using the K-star algorithm. We impose this restriction because it would not be practical to have more regular workers than the optimal number of regular employees needed to complete the work.

Retainer: The amount that is required to have each contract worker available on an as-needed basis. Default value of the retainer is \$1,500.

On call pay: This amount is the rate at which the contract employees are paid. Default value is 1.5 times the regular wage rate. This can be set as needed.

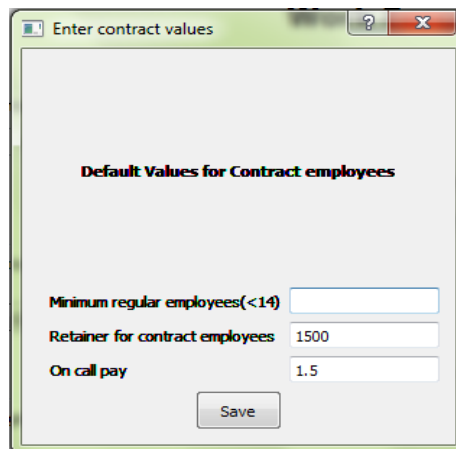
The outputs from running the program are as follows.

1. Number of Regular employees
2. Number of contract employees
3. Extra plowing cost
4. Regular wages
5. Total cost of plowing

6.2.2 User Guide

In order to run the tool in the 'Contract operators' mode follow the steps below.

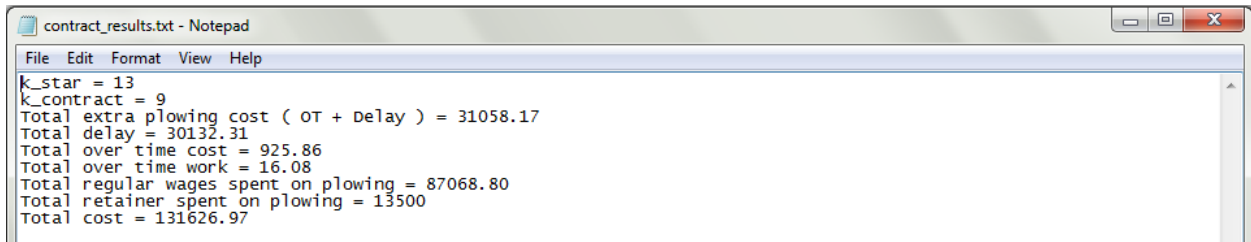
1. Run the K-star algorithm (see Section 6.1.1 for details).
2. Click the 'Run Contract cost K star' button. It will open a window that will ask the user to enter default values.



The screenshot shows a dialog box titled "Enter contract values". Inside the dialog, the text "Default Values for Contract employees" is centered. Below this, there are three rows of input fields. The first row is labeled "Minimum regular employees(<14)" and has an empty text box. The second row is labeled "Retainer for contract employees" and has a text box containing the value "1500". The third row is labeled "On call pay" and has a text box containing the value "1.5". At the bottom center of the dialog is a "Save" button.

Figure 6.5 Options window for contract mode

3. Once the user enters the default values and presses save button, the 'Run contract cost k star' button will be deactivated and the 'Contract costs result' button will get activated.
4. Click on the 'Contract costs result' button and a notepad will show the results of running the algorithm in contract cost mode.

A screenshot of a Notepad window titled 'contract_results.txt - Notepad'. The window contains the following text:

```
k_star = 13
k_contract = 9
Total extra plowing cost ( OT + Delay ) = 31058.17
Total delay = 30132.31
Total over time cost = 925.86
Total over time work = 16.08
Total regular wages spent on plowing = 87068.80
Total retainer spent on plowing = 13500
Total cost = 131626.97
```

Figure 6.6 Contract mode results

Running the tool in ‘Contract’ mode resulted in the following changes.

- Decrease in delay cost: This decrease happens because more employees are now available to work in the overtime mode, causing high priority road segments to be plowed faster.
- Decrease in over time cost: Contract workers will not have fringe benefits which are the major part of the pay and they don’t even have the 4 hour pay restriction. As the number of contract workers increases, the amount of overtime work each person does is reduced and hence the overtime cost is decreased.
- Decrease in total cost: The decrease in both the overtime cost and delay cost, resulted in a lower total cost.

6.3 Split Shifts

If split shifts option is available, then this means that SLC can call employees to work for any continuous 8-hour period between 4:00 AM to 4:00 PM. Clearly, the optimal strategy is to call employees when the storm ends so that as much as possible, plowing work can be completed in regular time work period. We do not account for the incentive pay that SLC may have to provide to its employees to have them participate in the proposed shift schedule. The purpose of this analysis is to quantify the economic benefit to SLC of having split shifts, which can be used to compute an upper bound on the total incentive pay.

6.3.1 Inputs and Outputs

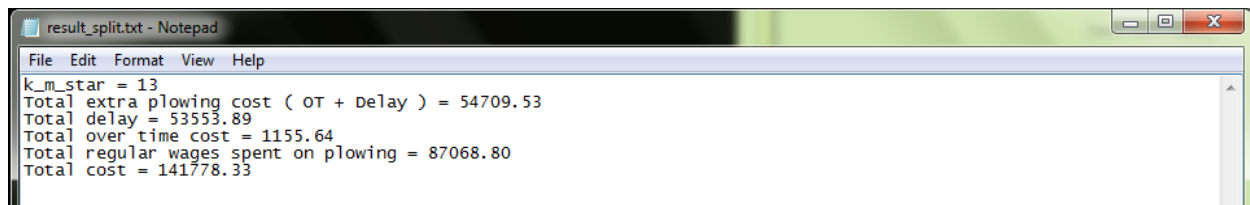
Inputs for the split shift case are same as that for the standard K-star algorithm. The only modification is to the t_0 file, i.e. the file which stores the end times of storms. Instead of considering 3:30 PM as the shift end time, 4:00 PM is now considered as the shift end time. So, if a storm ends at 3:45 PM, then the value of t_0 is still 0. If a storm ends at 5:00 PM then the value of t_0 will be 16 (i.e. two periods of 15-minute length). The user does not need to make these changes. The changes are automatically made to the appropriate input files.

The outputs are as follows.

- 1 Number of Regular employees
- 2 Number of contract employees
- 3 Extra plowing cost
- 4 Regular wages
- 5 Total cost of plowing

6.3.2 User Guide

In order to run the GUI in the split-shift mode of operation, click on the 'Run split shift k star' button. This will run the split shift algorithm and will display the results in a text file.



```
result_split.txt - Notepad
File Edit Format View Help
k_m_star = 13
Total extra plowing cost ( OT + Delay ) = 54709.53
Total delay = 53553.89
Total over time cost = 1155.64
Total regular wages spent on plowing = 87068.80
Total cost = 141778.33
```

Figure 6.7 Results for split shift

Use of split shifts results in the following changes to costs.

- Reduction in delay cost: Delay costs decrease because more plowing work is completed as soon as the storm ends.
- Decrease in over time cost: When a storm ends in regular shift time, employees start working on it immediately. Now, more storms end in the regular shift time. In fact, we know from the historical data that in a typical year, end times of 5 out of 12 storms fall between 4:00 AM to 4:00 PM. So, most of the storms fall in regular time decreasing the amount of work that need to be done in overtime mode.
- Decrease in total cost: As both over time cost and delay cost decreased there is decrease in the total cost .

6.4 Staggered Shifts

This option allows two shifts, an AM shift that starts at 4:00 AM and ends at 12:00 PM, and a PM shift that starts at 8:00 AM and ends at 4:00 PM. We also call these shifts morning and evening shifts. The staggered shift algorithm finds the optimal value of morning and evening shift employees. Having employees in two shifts means that at least some plowing can occur in regular time mode for all storms that end somewhere between 4:00 AM and 4:00 PM. This helps reduce total costs.

6.4.1 Inputs and Outputs

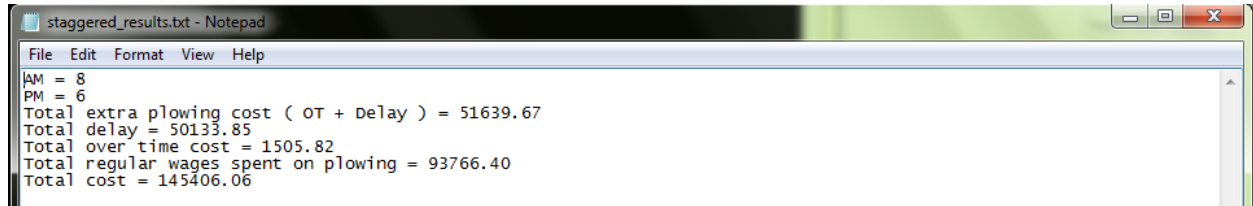
There are no special inputs for this mode of operation. All inputs are the same as described in Section 6.1.1.

The outputs from running the algorithm are as follows.

1. Number of morning employees
2. Number of evening employees
3. Extra plowing cost
4. Regular wages
5. Total cost of plowing

6.4.2 User Guide

The user can run the tool in staggered-shift mode of operation by clicking on the 'Run staggered cost K star' button. This will run the above algorithm and open a notepad which contains the results.



```
staggered_results.txt - Notepad
File Edit Format View Help
AM = 8
PM = 6
Total extra plowing cost ( OT + Delay ) = 51639.67
Total delay = 50133.85
Total over time cost = 1505.82
Total regular wages spent on plowing = 93766.40
Total cost = 145406.06
```

Figure 6.8 Results of staggered shift

This mode of operation decreases total cost relative to the base case because PM shift employees can provide 4 extra hours of work in the regular time mode from 12:00 PM to 4:00 PM and 5 out of 12 storms in a typical year end between 4:00 AM and 4:00 PM. Key differences between staggered shifts and the base case are as follows.

- Decrease in delay cost: As explained above, some regular workers are available from 4:00 AM and 4:00 PM, and 5 out of 12 storms occur in this interval. This lowers delay costs.
- Decrease in over time cost: Now the time 12:00 PM to 4:00 PM is not overtime period and hence results in reduced overtime cost. Previously this 4-hour period was counted as overtime and employees were paid 1.5 times the regular pay.
- Decrease in total cost: Total costs decrease because both the delay cost and the overtime cost decrease.

6.5 Comparison of Different Workforce Deployment Strategies

Previous sections presented how different techniques for deploying the available workforce affect plowing and delay costs, and calculate the optimal number of employees required for each strategy. In this section we present a cost comparison of the three strategies: (1) Contract employees (2) Split shifts and (3) Staggered shifts. In each case, the chosen strategy is compared with the default option, which is based on the optimal K-star algorithm. The K-star result is considered as the base case for comparison because it is the current workforce deployment strategy used by SLC.

6.5.1 Results

Depot 1:

Table 6.1 Depot 1 Results Comparison

	K star	Contract employees	Split shifts	Staggered shifts
K(number of employees)	14	13	13	AM = 8 PM = 6
Other employees	0	9 (Contract)	0	
Delay cost	51,714.56	30,132.31	53,553.89	50,133.85
Over time cost	1,644.68	925.86	1,155.64	1,505.82
Regular wages	93,766.40	87,068.80	87,068.80	93,766.40
Other cost	0	13,500 (Retainer)	0	
Total cost	147,125.63	131,626.97	141,778.33	145,406.06

Depot 2:

Table 6.2 Depot 2 Results Comparison

	K star	Contract employees	Split shifts	Staggered shifts
K(number of employees)	2	2	2	AM = 1 PM = 1
Other employees	0	2 (Contract)	0	
Delay cost	14,959.73	6,387.62	12,708.35	14,680.53
Over time cost	238.55	191.74	188.59	259.26
Regular wages	13,395.20	13,395.20	13,395.20	13,395.20
Other cost	0	3,000 (Retainer)	0	
Total cost	28,593.48	22,974.56	26,292.15	28,334.99

Depot 3:**Table 6.3 Depot 3 Results Comparison**

	K star	Contract employees	Split shifts	Staggered shifts
K (number of employees)	6	6	6	AM = 3 PM = 3
Other employees	0	4 (Contract)	0	
Delay cost	27,833.09	14,049.65	23,170.78	26,365.49
Over time cost	777.78	605.39	602.24	757.52
Regular wages	40,185.60	40,185.60	40,185.60	40,185.60
Other cost	0	6,000 (Retainer)	0	
Total cost	68,796.47	60,840.64	63,958.62	67,308.61

Depot 4:**Table 6.4 Depot 4 Results Comparison**

	K star	Contract employees	Split shifts	Staggered shifts
K (number of employees)	11	11	11	AM = 5 PM = 6
Other employees	0	5 (Contract)	0	
Delay cost	43,901.39	26,930.82	38,754.92	45,810.41
Over time cost	1,366.51	1,091.05	948.14	1,565.68
Regular wages	73,673.60	73,673.60	73,673.60	73,673.60
Other cost	0	7,500 (Retainer)	0	
Total cost	118,941.50	10,9195.47	11,3376.66	12,1049.70

6.5.2 Analysis

As seen in the four tables above, the use of contract employees results in the lowest total cost. The ordering of different strategies, from least to most costly, is as follows:

1. Contract employees
2. Split shifts

3. Staggered shifts

We discuss each strategy separately next.

1. **Contract employees:**

Having contract employees is the least costly option for the following reasons.

1. With a small retainer of 1500 (the default value), SLC can have more number of workers in the overtime shift, which reduces the delay cost.
2. The number of regular employees is equal to the optimal number obtained in the K-star algorithm. Savings in costs come from the fact that contract workers are cheaper to use than regular workers in the overtime mode.

If the retainer were greater, the total cost would increase and eventually the use of contract employees may not be better than using permanent employees.

2. **Split shifts:**

Split shift mode of operation has the second least total cost. This is due to the following reasons.

- 1 In this mode there is flexibility to change the employees' report time such that they may be required to work any continuous 8-hour interval time between 4:00 AM to 4:00 PM. As a consequence, employees are available to start plowing immediately if the storm ends sometime in that time interval.
- 2 Over time cost is less in split shift as regular shift extends from 4:00 AM to 4:00 PM. Even though this is also true with staggered shift, costs are even lower here because in the latter instance, all regular employees may not be available even if the storm ends during 4:00 AM to 4:00 PM.
- 3 Delay costs are lower with split shifts than both normal and staggered deployment strategies. This is again due to greater employee availability during a 12-hour period.

3. **Staggered shift:**

Staggered shift mode of operation is similar to having split shifts. The key difference is that some employees are locked into the AM shift schedule and the rest are locked into the PM schedule. This approach to shift work may be easier for employees. It turns out to be quite close to the use of split shifts in terms of the cost savings that it produces. This makes it a worthwhile option to evaluate.

1. Regular employees are available from 4:00 AM to 4:00 PM, but all of them are not available during some part of this 12-hour interval. If the storm ends between 4:00 AM and 8:00 AM or between 12:00 PM and 4:00 PM, then SLC will have either morning shift employees only or evening shift employees only in the regular work mode.
2. Staggered shifts have more overtime cost than split shifts because all the employees are not available at the same time when storm ends between 4:00 AM and 4:00 PM.

3. Delay cost of staggered shift is greater than split shift and smaller than that normal shift. The former occurs because the rate at which delay costs can be reduced is greater with split shift use. The latter happens because some employees are available to plow important road segments immediately after 12:00 PM.

References

- [1] S. Albers, 1997. “Better bounds for on-line scheduling”. In *Proceedings of the 29th ACM Symposium on Theory of Computing*, El Paso, TX.
- [2] Y. Bartal, A. Fiat, H. Karloff, R. Vohra, 1992. “New algorithms for an ancient scheduling problem”. In *Proceedings of the 24th ACM Symposium on Theory of Computing*, Victoria, B.C., Canada, 51-58.
- [3] B.L. Golden, J. DeArmon, E.K. Baker, 1983. “Computational experiments with algorithms for a class of routing problems”. *Computers and Operations Research* **10**, 47-59.
- [4] R.L. Graham, 1966. “Bounds for certain multiprocessing anomalies”. *Bell System Technical Journal* **45**, 1563-1581.
- [5] P. Greistorfer, 2003. “A tabu scatter search metaheuristic for the capacitated arc routing problem”. *Computers and Industrial Engineering* **44** (2), 249-266.
- [6] A. Hertz, G. Laporte, M. Mittaz, 2000. “A tabu search heuristic for the capacitated arc routing problem”. *Operations Research* **48**, 129-135.
- [7] R. Hirabayashi, Y. Saruwarari, N. Nishida, 1992. “Tour Construction Algorithm for the capacitated arc routing problem”. *Asia Pacific Journal of Operations Research* **9** (2), 155-175.
- [8] S. Kang, K. Malik, J.L. Thomas, 1999. “Lotsizing and scheduling on parallel machines with sequence-dependent setup costs”. *Management Science* **45** (2), 273-289.
- [9] D. Karger, S. Phillips, E. Torng, 1996. “A better algorithm for an ancient scheduling problem”. *Journal of Algorithms* **20** (2), 400-430.
- [10] D. Kuchera, 2008. *A decision support tool for optimizing county-level winter maintenance operations*. University of Minnesota Plan B Report. Department of Mechanical Engineering, Minneapolis, MN.
- [11] P. Lacomme, C. Prins, W. Ramdane-Chérif, 2004. “Competitive memetic algorithms for arc routing problems”. *Annals of Operations Research* **131**, 159-185.
- [12] P. Lacomme, C. Prins, A. Tanguy, 2004. “First competitive ant colony scheme for the CARP”. In *Ant Colony, Optimization and Swarm Intelligence*, Dorigo, M., M. Birattari, C. Blum, M. L. Gambardella, F. Mondada, and T. Stützle, Editors, **3172**, 426-427, Springer Berlin/Heidelberg.
- [13] G. Laporte, 1987. “Generalized Subtour Elimination Constraints and Connectivity Constraints”. *Journal of the Operational Research Society* **37** (5), 509-514.
- [14] H. Longo, M.P. Aragão, E. Uchoa, 2006. “Solving capacitated arc routing problems using a transformation to the CVRP”. *Computers and Operations Research* **33**, 1823-1837.
- [15] L. Santos, J. Coutinho-Rodriguez, J.R. Current, 2009. “An improved heuristic for the capacitated arc routing problem”. *Computers and Operations Research* **36**, 2632-37.
- [16] M.C. Wilson, K. Dadie-Amoah, Y. Zhang, 2003. *Snowplow operations and resource management*, Report No. CTS 04-09. MN Department of Transportation, St. Paul, MN.

Appendix A Input Parameter Estimation

For each different type of data that we propose to use to estimate model parameters, we describe the following: data type; source, information to be extracted; and the model parameter to be estimated.

1. **AADT Counts:** The AADT (average annual daily traffic) counts for roads are available on the web at <http://www.dot.state.mn.us/traffic/data/maps/thecountymapdex.html>. The majority of local county road AADT counts are available from 2003, and the AADT counts for major highways are available from 2006. These are located on road maps in PDF files and can be recorded manually for each relevant road. Some local county roads on the current routes do not have AADT counts. We estimated missing counts by averaging counts from adjacent or connecting roads for which counts were available. AADT counts were used in the estimation of snowplow speeds – speeds on high AADT count roads are affected significantly during rush hours.
2. **Route Maps:** The snowplow route maps can be obtained from county GIS maps. The latter can be viewed in ArcView and include the snowplow routes, roads, sidewalks, and maintenance maps. We use the GIS maps to develop a graphical representation of each plow route. In this representation, each major intersection along a route is called a node and sequentially labeled. The graphs also contain information on the length of each road segment (see item 3 below). Routes obtained from ArcView can be imported into a Microsoft Word document for creating the graphical representation. Alternatively, graphs of routes can be created manually.
3. **Route-Segment Lengths:** The length of each segment of a snowplow route can be obtained from the county GIS maps in ArcView. The segment lengths are found on the routing map by clicking on a segment with the information pointer, which displays an information box that contains the segment length data under category ‘MILEAGE_MI’. This information is recorded manually on the route maps. Along with snowplow speed, this is used to calculate the amount of time it takes a plow to traverse each route segment.
4. **Sand/Salt Application:** Application rates determine how long a snowplow can treat roads between refills. We obtained application rates from three different sources – Wilson, Dadie-Amoah, and Zhang (2003), the Salt Institute and personal communication with experts. Table A.1 shows the application rates as a function of weather conditions developed for snowplow operations for various routes in Virginia, MN (Wilson, Dadie-Amoah, & Zhang 2003). Table A.2 contains similar information obtained from the Salt Institute. Finally, we consulted Ms. Linda Taylor from MnDOT Central Office who described the range of application rate in 200-500 lbs/lane mile with an average rate of 300 lbs/lane mile. Based on all three information sources, we plan to use the application rates shown in Table A.3.

St. Louis County uses a 90% salt - 10% sand mixture. The plows hold 8 cu/yds of salt/sand. On average, there are 1.05 tons per yard (from the Snowplow Router Program). This weight needs to be verified due to changes in content. We also need to know the maximum weight that sand/salt trucks can hold.

Table A.1 Material Application Rates (from Wilson, Dadie-Amoah, and Zhang 2003)

Pavement Temperature (F)	Weather Conditions	Pounds/Lane Mile		Application Frequency
		100% Salt	50% Salt	
Above 30	Snow	100-200	200-400	as needed
	Freezing Rain	100	200	as needed
25-30	Wet Snow	200-250	400-500	as needed
	Freezing	150	300	initial
	Snow	100	200	repeat
20-25	Wet Snow	250-300	500-600	initial
		125	250	repeat
	Freezing Snow	200	400	initial
		150	300	repeat
15-20	Dry Snow	200	400	sand hazard area
	Wet Snow	300-400	600-800	sand as needed
Below 15	Dry Snow			sand hazard area

Table A.2 Application Rates from the Salt Institute

Temperature (F)	Road Surface	Salt/Sand Lbs/Lane Mile
Near 30	Wet	250
Below 30	Falling Wet or Sticky	200 – 250
Below 20	Falling Dry	250 – 300
Below 20	Wet	200
Below 10	Packed Ice or Snow	400

Table A.3 Proposed Sand/Salt Application Rates

Road Temperature (F)	Sand Lbs/Lane Mile
Near 30	250
Below 30	225
Below 20	250
Below 10	400

5. **Weather:** We compiled weather data to develop scenarios corresponding to typical storms of different intensity. Storm intensity is measured by (1) air temperature, (2) rate of snowfall, (3) wind speed, (4) moisture content, (5) length of the storm, and (6) pavement temperature. Storm intensity affects snowplow speeds and sand/salt application rates, which in turn determine the length of time it would take to clear a route of snow.

The National Weather Service's (NWS) climatic data for Duluth is found on the web at http://climate.umn.edu/doc/prelim_lcd_dlh.htm. This source of data includes daily averages on temperature, wind speed, and total snowfall. Select the desired month, and storm data is listed by day. Column eight lists total daily snowfall. We use this data as a quick reference to find days during which snow storm events occurred.

The RWIS data can be found on the web at <http://www.rwis.dot.state.mn.us/> and contains more detailed information than the NWS website. It includes current and historical data on the air temperature, wind speed and direction, storm length, pavement temperature, total precipitation, and precipitation accumulation rates. Precipitation can be converted to snow depth by dividing this number by the moisture content (see below). The amount of data available can vary from one RWIS site to another based on the specific site's capabilities. The moisture content is defined as the liquid-to-snow ratio. We received fifty years of historical data on the daily liquid-to-snow ratios measured at Duluth International Airport. The liquid measurements were made at midnight and the snow measurements were made the next morning at approximately 7AM. The moisture content values were sent via Email. A Microsoft Excel data table has been created to be used as an input in this project.

The RWIS sites that collect storm information include Blatnik Bridge - South Abutment, located in the southeast corner of the county near Duluth; TH 53 @ MP 129.21 - TH 53 South Bound, located in the Northwest corner of the county; and TH 65 @ MP 145.18 - TH 65 South Bound, located just Southwest of the county. Data from this RWIS site is found in daily tables, which can be extracted and converted into Excel files. A sample of an Excel file is shown in Table A.4 below. The table has been split in two to fit on a single page.

Table A.4 Sample RWIS Data

Time (CST)	AirTemp (F)	RH (%)	Dewpoint	Barometric Pressure (in)
10:53 PM	22	98	22	28.3
10:53 PM	22	98	22	28.3
10:48 PM	22	98	22	28.3
10:43 PM	22	98	22	28.3
10:43 PM	22	98	22	28.3
10:38 PM	22	98	22	28.3

AvgWind Speed (mph)	GustWind Speed (mph)	Wind Direction	Precip Type	Precip Intensity	Precip Accumulation (inch)	PrecipRate (iph)
4	10	NE	Snow	Slight	4.3	0.1
4	10	NE	Snow	Slight	4.3	0.1
7	10	NE	Snow	Slight	4.29	0.2
4	12	NE	Snow	Slight	4.27	0.3
4	12	NE	Snow	Slight	4.26	0.2
4	12	NE	Snow	Slight	4.24	0.2

6. **Snowplow Speeds:** An average speed chart based on storm conditions is displayed in Table A.5 below. These rates come from a study by Wilson, Dadie-Amoah, and Zhang (2003).

Table A.5 Snowplow Speeds

MOISTURE CONTENT	ACCUMULATION RATE (INCH/HR)	PLOW SPEED (MI/HR)
Low	0.0-0.25	25
Low	0.25-0.5	17
Low	0.5-1.0	15
Low	Above 1.0	10
Medium	0.0-0.25	20
Medium	0.25-0.5	15
Medium	0.5-1.0	12
Medium	Above 1.0	9
High	0.0-0.25	15
High	0.25-0.5	12
High	0.5-1.0	10
High	Above 1.0	8

SLC managers estimated snowplow speeds at 20-25MPH. The speeds decrease by approximately 5-10 MPH from 7AM – 7PM in 1000 count or higher AADT areas. The traffic decreases are found in Table A.6.

Table A.6 Snowplow Speeds Based on Traffic Volumes

AADT Count	Typical Speed	Speed with Traffic
>1000	25	20
>1000	20	15
>3000	25	15
>3000	20	10

7. **Employee Compensation:** All employee compensation information comes from the Employees Union Agreement. Information about breaks and plowing times comes from Ron Garden. This information is necessary for building a model that will determine the optimal workforce deployment strategies for different types of storms.
- 1) All employees are County employees. Temporary staff and contract agencies are not used for snow plowing in the St. Louis County.
 - 2) Operators must be called one hour before their services are required.
 - 3) Operators’ regular work hours are from 7:30AM - 3:30PM Monday through Friday. The cost of plowing during this period is sunk.
 - 4) If operators are called in during 3:30AM - 7:30AM, they receive pay and a half for the extra hours worked and get to work their normal shift until 3:30PM. For example, if operators are called at 4:30 AM, then they receive time and half pay for 3 hours and regular pay for this regular shift which begins at 7:30 AM.
 - 5) After 3:30PM, operators earn time and a half if they have already worked their full shift (7 ½ hours). After operators have worked eleven consecutive hours, they receive double pay.
 - 6) Between 3:30PM and 3:30AM, operators who are sent home and are later asked to return to work receive pay and a half for each hour worked or regular pay for four consecutive hours, whichever is greater.
 - 7) All breaks can be taken along the route. Break times can vary. Lunch is typically from 12 – 12:30PM and can be skipped for pay and a half for ½ hour. It is important to have a spot to stop for the lunch break.
 - 8) High priority is given to clean roads for both the morning and the evening commute times.
 - 9) After 6PM, plowing can stop until the next morning except for major roads.
 - 10) Compensation rates:
 - Equipment Operator Junior: \$15.3510
 - Equipment Operator Senior: \$16.4738
8. **Depot Information:** The locations of the depots are found on a large map titled “St. Louis County Public Works Maintenance Districts”. The depot locations are also found in a GIS

map. All the depots are capable of refilling sand/salt. The Pike Lake depot is the only refueling depot. Depot locations are used for determining optimal tours for snowplows, which in turn determine workforce requirements. The number of employees at each depot was also obtained from SLC managers.

9. **General Information:** The following information was given by Ron Garden, Deputy Public Works Director, St. Louis County.
 - 1) The plow units can go 10-12 hours per tank of gas.
 - 2) It takes 5-10 minutes for a snowplow to turn around.

Appendix B User Guide: Calculations of the Rough-cut Capacity Requirements

This section explains how to retrieve weather information, how the weather information is analyzed in Excel, and how the Routing Macro works.

Retrieving Weather Information: A ‘measurement’ is defined as one measurement the RWIS station makes at a specific time that contains date/time, temperature, wind, and snow accumulation data. Here is a step by step guide to access the historical weather information:

- 1) One should know the date of a storm in order to find historical data on the storm. The National Weather Service’s climatic data for Duluth is found on the web at http://climate.umn.edu/doc/prelim_lcd_dlh.htm. Select the desired month, and storm data is listed by day. Column eight lists total daily snowfall. Use this tool to identify storm episodes.
- 2) Once a storm date is selected, go to the following RWIS website: <http://www.rwis.dot.state.mn.us/scanweb/SWFrame.asp?Pageid=Summary&groupid=3&units=English&DisplayClass=Java&SenType=All>. Select the weather station you’d like to copy historical storm information from by clicking ‘History’ in the far right column (an example site with precipitation rates is station I-35 @ MP 181.)
- 3) On the bottom of the new page, under ‘Scale’, enter the end date and time of the storm and click ‘Update’ (an example date is 12/01/07 at 23:59).
- 4) At the top of the new page, click on ‘Atmospheric History’ to get air temperature, wind direction and speed, and precipitation rates.
- 5) The data must be exported for analysis. Click on ‘export’, highlight the weather data, and copy the data to notepad.
- 6) Save the notepad file with a ‘.csv’ (comma-separated values) extension. A csv file allows for the data to be properly imported into Excel for analysis.
- 7) Open the .csv file in Microsoft Excel.
- 8) Select and copy only the daily data that contains storm information. The daily data that contains storm data has ‘snow’ listed under precipitation type.
- 9) Copy the data into the Weather Analysis Macro on the first row, first column of sheet two.
- 10) Return to the RWIS website; road surface data is needed.
- 11) At the top of the page, click on ‘Surface/Precip. History’ to get road surface data.
- 12) The data must be exported for analysis. Click on ‘export’, highlight the data, and copy the data to notepad.
- 13) Save the notepad file with a ‘.csv’ (comma-separated values) extension.
- 14) Open the .csv file in Microsoft Excel.
- 15) Find the column titled ‘SFTemp’, and select and copy the surface temperature data for the same measurement times that are being used for the weather data.
- 16) Paste the surface temperature data into the last column of the Weather Analysis Macro.
- 17) Run the Weather Analysis Macro to automatically have the data analyzed.

Weather Analysis Macro: The following steps are performed in the Weather Analysis Macro in Excel.

Step 1: On Sheet 2, the user has input the data and presses the ‘Adjust Cells’ button. Further calculations are made on the data.

- 1) Every row is compared to the previous. If they are the same, one row is deleted because occasionally two measurements are taken at the same time.
- 2) Two columns are inserted after column A for additional time analysis.
- 3) The time interval between measurements is calculated. Column B finds the difference between two measurement times by converting the date and time from column A to minutes using the 'minute()' function. Periodically, the difference will be negative because of a new hour. In these situations, an 'if' statement is used to add the current minute to the difference between sixty and the previous minute.
- 4) The total time accumulation is calculated in column C. The previous row's total time value is added to the time interval in column B for each row with a measurement.
- 5) The number format of columns B and C are converted to 'general'.
- 6) A column is inserted after column N to input the actual rate of snowfall. The actual rate of snowfall is calculated by taking the difference between two consecutive snow accumulation measurements and dividing that value by the time interval. Multiplying by sixty gives the rate in inches per hour.

Step 2: After the cells have been adjusted, on Sheet 1 the user presses the 'Calculate' button to fill in the chart and data tables.

- 1) The total accumulation rate (inches per hour) is calculated by dividing total snow accumulation by the total storm time and multiplying this value by sixty.
- 2) The mean is calculated for the following storm parameters: air temperature, road temperature, wind speed, and wind gust speed.
- 3) The ranges are calculated by finding the minimum and maximum values for the following storm parameters: accumulation rate, air temperature, road temperature, wind speed, and wind gust speed.
- 4) The medium road temperature is calculated.
- 5) The directions in which the wind travels are calculated as percentage of time the wind is traveling in each direction on the compass, eight total. A count for a respective direction is made for each measurement, and every count is divided by the total count of measurements to get the percentages. The program features a pie chart to best display the wind directions.
- 6) The total snow accumulation is displayed by copying the value found in cell 2 from column 'Precipitation Accumulation' on Sheet 2.
- 7) The storm duration is copied from cell 2, column 'Cumulative Storm Time' on Sheet 2, and divided by sixty to convert to hours.
- 8) The wind affects snow plowing when it is above ten miles per hour. Therefore, the percent of wind gust speeds above and below 10 mph is calculated for each wind direction. Each wind direction uses an 'if' statement to tally the number of gusts above and below 10 mph. The tally is divided by the total number of measurements to get the percentage.
- 9) The data is transferred to another data table to copy to a future program for further analysis.

Routing Macro: The routing macro finds the maximum amount of time the snowplow will spend on a given route based on weather conditions and traffic. The following steps are performed in the Excel-based Routing Macro.

- 1) On the first sheet, the user selects various routing parameters including snow accumulation rate, moisture content, and traffic. The user inputs the time to bare-pavement and the number of snowplows to use on the route.
- 2) On the second sheet, routing information is listed for each road segment. These include AADT count, distance, number of lanes, sidewalks, snowplow speed, and time to complete.
- 3) On the first sheet, once the user has chosen the parameters they press the 'Press to Calculate' button.
- 4) The snowplow speed for each road segment (determined by Table A.6 in Appendix A) is calculated based on the AADT count and the distance.
- 5) If the 'rush hour' option is select, those road segments with speeds over twenty (20) mph and AADT counts over one-thousand (1000) are reduced 5-10 mph (determined by Table 6 in Appendix A).
- 6) The time-to-complete each road segment is calculated by dividing road distance by snowplow velocity. The total time to complete the route is the sum of the time-to-complete values.
- 7) An output table is displayed on the first sheet.
- 8) The output table lists the drive time of the entire route, a rough estimate of time spent on stops and refills, total time (drive time + stops and refills), total time per snowplow, and the number of passes per snowplow. If not enough snowplows are input, the output in the chart will read 'Need more Snowplows'.

**Appendix C User Guide: Single Route Scheduling - Plowing to Completion (Chapter
3: MODEL 1)**

The single route scheduling decision tool is provided as an excel file, named `single_route_scheduling.xls`. The user enters required input parameters to 'Main' sheet of the file. The outputs are also summarized in the 'Main' sheet.

Inputs

The cells in 'Main' sheet with blue fill color are the inputs to be entered by the user. All the input time values should be entered in 24-hour format and as an integer. For example, for 9 pm (21:00), the user should enter 2100.

t0: The earliest time to start the plowing operations on the selected route for the selected storm type. t0 can be anytime from 12:01am (i.e. 00:01, enter as 0001 or 1) to 11:59pm (i.e. 23:59, enter as 2359).

ts: Shift start time

te: Shift end time

w: Wage rate per 15 minutes during regular shift hours (including fringe benefits)

Route: This is the route to be scheduled. The user selects the route from drop down menu, which currently includes the routes from 501 to 515.

Storm: The user selects the storm type from drop down menu, which includes storm types from A to E as determined in Task 1.

2-inch snow on paved and 4-inch snow on gravel roads: The user selects 'Yes' option from drop down menu, if the snow accumulation on the selected route is more than 2-inches on paved and 4-inches on gravel roads. Otherwise, the user selects 'No'. When the user selects 'No', meaning that there is not enough snow accumulation, the employees are not called-out during overtime period. The user has the flexibility to turn off this feature, to be able to make overtime assignments even if snow accumulation is low, by always selecting 'Yes' from drop-down menu.

Delay cost: There are three entries for delay costs corresponding to high, medium and low traffic density periods. Delay cost of a route corresponds to the county's willingness to delay the plowing operations for the entire route during the corresponding traffic density interval as compared to paying employees during overtime period. Therefore, delay costs should be determined based on overtime wage rates. Guidelines on how to select delay costs are given in section 3.2.2 when scheduling multiple passes. The same guidelines also apply to single route scheduling problems where an entire route will replace a single pass in the guidelines provided. Delaying plowing operations is not desirable due to public inconvenience especially during high traffic density periods and also due to compaction of snow caused by traffic, which makes snow removal and achieving of bare pavement harder as time passes.

The other input parameters, which should not be modified by the user, are pass completion times for the selected route and storm type.

Outputs

The outputs are listed under 'Results' section of 'Main' sheet. In addition, the events that occur for the optimal schedule are given with a value of 1 under the corresponding event. Event descriptions are provided under section 3.2.1 of the report.

Optimal x : Corresponds to optimal number of 15-minutes to delay the start time of plowing operations on the selected route.

Optimal start time: This is the optimal start time of plowing operations on the selected route.

Optimal end time: This is the optimal end time of plowing operations on the selected route, which also takes into account the lunch break.

Delay cost: Total delay costs incurred up to optimal start time since t_0 .

Overtime cost: Total overtime costs incurred from optimal start time to optimal end time.

Total extra cost: Total of overtime, delay and additional costs due to lunch break work time for the optimal schedule.

'Cost Profile' sheet of the file provides a graph of total extra cost versus x , where x represents a possible value of plowing start time. Therefore, this graph visualizes the change in cost for each 15-minute of delaying the plowing operations after t_0 . The optimal x has the lowest extra cost among all possible x values.

**Appendix D User Guide: Scheduling Multiple Passes of Different Routes (Chapter 3:
MODEL 2)**

This section explains how to use the executable program to schedule the chosen passes from a variety of different routes. The user guide provides the following information in order of presentation:

- 1) Input text files: Explains the required data for each of the input text files and specifies the input parameters that the user should consider updating at each run of the workforce deployment program.
- 2) Output text files: Explains the output format and how to read the output, which is provided as a single text file by the workforce deployment program.
- 3) Running the program: Describes how to run the executable program file.

The above information is presented together with a sample run of the program on an example using realistic data from SLC.

1. Input text files

The input parameters for the program should be entered into text files before running the program. There are only three input files that may require modification at each run of the program, which are presented as follows:

Input file 1: storm_type_t0.txt

This file has two entries. The two lines that should be entered into the text file are as follows:

Line 1: Storm type – The value that is entered for storm type should be either 1, 2, 3, 4, or 5 corresponding to storm types A, B, C, D and E respectively from Chapter 2.

Line 2: The earliest start time during overtime period (t_0) – The value that is entered for t_0 should correspond to a time outside of regular shift hours; however it can be the shift end time. The format for the time value is 24-hours and should be entered as an integer. Some examples are as follows:

$t_0 = 9\text{pm} = 21:00 \rightarrow$ enter 2100

$t_0 = 4\text{am} = 4:00 \rightarrow$ enter 400

$t_0 = 3:30\text{pm} = 15:30 \rightarrow$ enter 1530

$t_0 = 12\text{am} = 00:00 \rightarrow$ enter 0000 (or 0, or 2400)

$t_0 = 12:15\text{am} = 00:15 \rightarrow$ enter 0015 (or 15)

A sample input file of storm_type_t0.txt is shown in Figure D.1 for storm type 4 (D) and $t_0 = 9\text{pm}$.

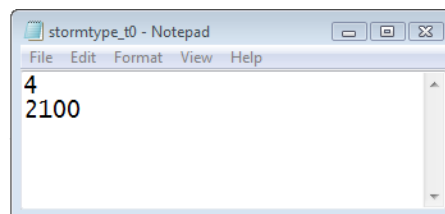


Figure D.1 Sample storm_type_t0.txt

Input file 2: `passes.txt`

This file contains all the passes from all depots for all possible storm scenarios. The user selects the passes to be scheduled by entering 1 next to these passes (i.e. to third column of the file). First column in the file corresponds to route ID and the second column shows the rank of the pass in the corresponding route.

A sample input file of `passes.txt` is given in Figure D.2 to schedule first and second passes of route 504, first and second passes of route 505, and first, second and third passes of route 506. When selecting the passes to schedule among all passes in this file, the user should be careful that the selected pass exists for the selected storm type.

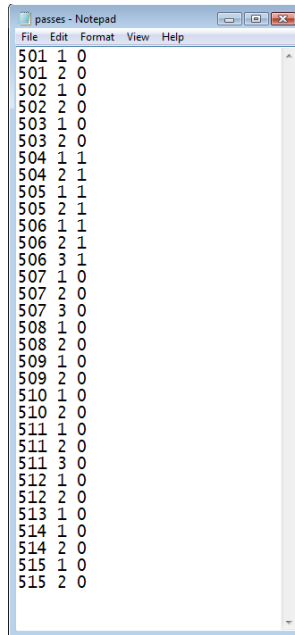


Figure D.2 Sample `passes.txt`

Input file 3: `unit_delay_costs.txt`

This file has nine entries corresponding to delay costs in dollars per 15-minutes for first, second and third passes during high, medium and low traffic density time intervals. Figure D.3 shows a sample input file for `unit_delay_costs.txt`. Table D.1 shows the corresponding pass type and traffic density interval for each entry in the text file. Guidelines for selecting delay costs are given in 3.2.2.

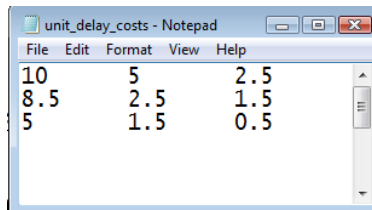


Figure D.3 Sample `unit_delay_costs.txt`

Table D.1 Delay Costs (\$) per 15-minutes for Each Pass Type and Traffic Density Period

	High traffic density (6am-10am,3pm-8pm)	Medium traffic density (10am-3pm,8pm-12am)	Low traffic density (12am-6am)
First pass	10	5	2.5
Second pass	8.5	2.5	1.5
Third pass	5	1.5	0.5

The input files that do not require modification before each run of the program, but may be considered for modification when a change occurs in SLC's operations are as follows:

Input file 4: `regular_shift_times.txt`

This file stores information on regular shift start and end times. The two entries in the text file are as follows:

Line 1: Shift start time – Should be entered in 24-hour format and as an integer. For example, if shift start time is 7:30am, then enter 730.

Line 2: Shift end time – Should be entered in 24-hour format and as an integer. For example, if shift end time is 3:30pm (15:30), then enter 1530.

A sample `regular_shift_times.txt`, which is also the default version of the input file, is given in Figure D.4.

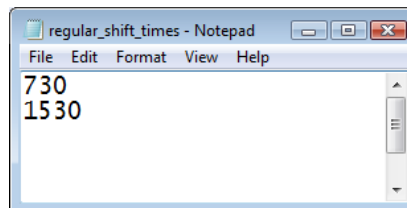


Figure D.4 Sample regular_shift_times.txt

Input file 5: `delay_cost_interval.txt`

This file stores information on the corresponding traffic density interval types for each 15 minute interval within two days, starting from the interval [12am, 12:15am) of Day 1 and ending at the interval [11:45pm, 12am) of Day 2. There is a unique number corresponding for each 15-minute interval to represent the priority of traffic density interval type that the time interval belongs to. Therefore, there are 192 entries in the file and the entries are from 1 (high traffic density) to 3 (low traffic density). High traffic density period (value = 1) contains the 15-minute intervals within [6am, 10am] or [3pm, 8pm], medium traffic density period (value = 2) contains the 15-minute intervals within [10am, 3pm] or [8pm, 12am], and low traffic density period contains the 15-minute intervals within [12am, 6am]. A sample `delay_cost_interval.txt`, which is also the default version of the input file, is given in Figure D.5. Figure D.5 shows only a part of the input file for the time period between 12am to 11am of Day 1. Traffic density priority values for remaining 15-minute intervals should be entered in the same format.

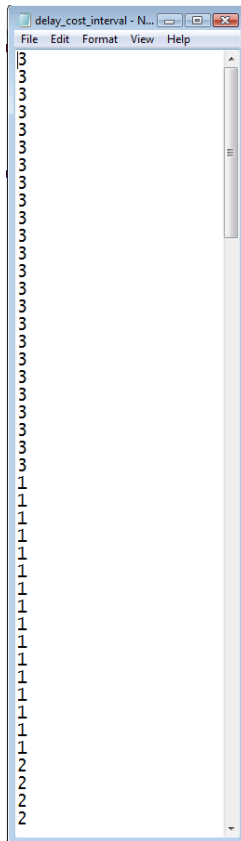


Figure D.5 Sample delay_cost_interval.txt (partial)

Input file 6: pass_completion_times.txt

This file stores information on pass completion times for all routes and all passes under all possible storm scenarios. The pass completion times are obtained from Task 1. The route and pass IDs in this file should match with the route and pass IDs in `passes.txt` file. Therefore, both files contain same passes. There are 7 columns (fields) in the file:

Column 1: Route ID – e.g. 502, 505, 512

Column 2: Pass ID – This corresponds to the rank of pass within the corresponding route. The values are 1 for first pass, 2 for second pass and 3 for third pass.

Columns 3, 4, 5, 6 and 7: Pass completion times under storm types A (1), B (2), C (3), D (4) and E (5) respectively. The format for completion times is same with t_0 and regular shift start and end times.

A sample `pass_completion_times.txt`, which is the default version of the file, is given in Figure D.6. For example, for third pass of route 507, the completion time is 33 minutes under storm type 1, 33 minutes under storm type 2, and 1 hour 54 minutes under storm type C. Under storm types D and E third pass does not exist, therefore completion time is entered as 0.

```

pass_completion_times - Notepad
File Edit Format View Help
501 1 328 328 429 313 310
501 2 113 113 146 0 109
502 1 339 339 425 240 302
502 2 106 106 136 46 100
503 1 428 428 428 428 427
503 2 258 258 358 219 231
504 1 249 249 336 208 221
504 2 249 249 336 208 221
505 1 351 351 430 310 319
505 2 223 223 306 148 215
506 1 354 354 430 307 312
506 2 431 431 427 429 430
506 3 322 322 425 24 249
507 1 343 343 436 306 300
507 2 431 431 429 339 412
507 3 33 33 154 0 0
508 1 411 411 428 340 334
508 2 141 141 244 105 127
509 1 435 435 430 341 358
509 2 435 435 140 341 358
510 1 348 348 427 310 310
510 2 219 219 304 149 206
511 1 349 349 428 320 318
511 2 407 407 428 313 340
511 3 0 154 0 0 0
512 1 318 318 429 240 240
512 2 112 112 107 35 105
513 1 337 337 424 227 308
514 1 406 406 423 425 432
514 2 223 223 337 128 135
515 1 404 404 429 315 331
515 2 0 0 110 0 0

```

Figure D.6 Sample pass_completion_times.txt

Input file 7: OT_cost_parameters.txt

This file stores the information on multipliers for regular and extra overtime wage rates and the time parameters that determine overtime pay rates. There are five entries in the file.

- Line 1: Multiplier for regular overtime pay – Currently, when the employees are called out for overtime work, they are paid 1.5 times their regular wages for the number of hours worked up to double-pay period. Therefore, the default entry for this line is 1.5.
- Line 2: Multiplier for extra overtime pay – Currently, the called-out employees start to get paid 2 times their regular wage rates for the number of hours worked above 3.5 hours. Therefore, the default entry for this line is 2.
- Line 3: Number of 15 minute intervals for regular overtime work up to extra pay period – Currently, the number of hours that called-out employees are paid at regular overtime wage rate (i.e. 1.5 times regular shift wages) is 3.5 hours, since it adds up to 11 hours of work within a day after 7.5 hours of regular shift work. Therefore, the default entry for this line is 14, since 3.5 hours correspond to 14 15-minute intervals.
- Line 4: The maximum number of 15-minute intervals before the start of the shift to start an employee early as oppose to name the event as a “call-out” – Currently, when an employee starts working within 4-hour period immediately preceding the start of the shift, then the employee is not considered to be called-out and is not eligible for at least a constant regular pay. Therefore, the default entry for this line is 16, corresponding to number of 15-minute intervals within four hours.

Line 5: Constant pay to a called-out employee in terms of number of 15-minute intervals – Currently, a called-out employee receives at least four hours of regular pay. Therefore, the default entry for this line is 16, corresponding to number of 15-minute intervals within four hours.

A sample `OT_cost_parameters.txt`, which is the default version of the file, is given in Figure D.7.

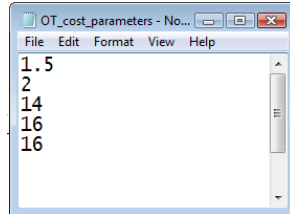


Figure D.7 Sample `OT_cost_parameters.txt`

Input file 8: `numpasses_maxnumemp_passperemp.txt`

This file stores the information on total number of passes (as listed in `pass_completion_times.txt` and `passes.txt`), maximum number of employees (operators) to start working at the same type, and maximum number of passes that can be consecutively assigned to the same employee. Three entries of the file are as follows:

- Line 1: Total number of passes – Currently, there are 32 passes, which should be done for at least one of storm types. Therefore, the default entry for this field is 32.
- Line 2: Maximum number of employees to start working at the same time – The number to be entered in this field should be the maximum value among number of available employees or the number of available plows. Currently, there are 15 routes for which the passes are determined. Assuming that there are at least 15 employees who can start working at the same time, and also assuming that there is a plow assigned to each route, the default entry of this field is 15.
- Line 3: Maximum number of passes for each employee – This field limits the number of passes that can be assigned to the same employee consecutively. In order to explore all the possible schedules to minimize costs, currently a large number is assigned for this field. The default entry is 15.

A sample `numpasses_maxnumemp_passperemp.txt`, which is the default version of the file, is given in Figure D.8.

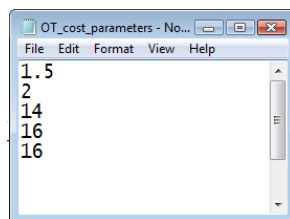


Figure D.8 Sample `numpasses_maxnumemp_passperemp.txt`

Input file 9: `wagerate_benefitpercentage.txt`

This file stores the information on regular wage rate per 15 minutes including fringe benefits (first line) and the percentage of benefits in regular wage rate (second line). A sample `wagerate_benefitpercentage.txt`, which is the default version of the file, is given in Figure D.9. Currently, regular wage rate is \$35/hour, which corresponds to \$8.75 per 15 minutes, and 62% of the regular wage rate is for fringe benefits.

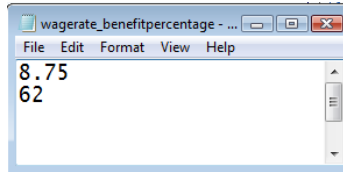


Figure D.9 Sample `wagerate_benefitpercentage.txt`

2. Output text files

The workforce deployment optimization program provides the main output in a single file, which is called `results.txt`. The program also provides an additional output file, which is called `cost_results.txt`, which lists the overtime, delay, extra and total costs for each combination of parameters k , b and t_{start} . The resulting schedule for overtime and next day periods correspond to the best combination of k , b and t_{start} , which yields minimum total cost. Both files are described in detail as follows.

Main output file: `results.txt`

The main output file provides the best schedule for overtime period and next day's shift. Figure D.10 shows a sample `results.txt` file, which is also the output file for all the input files presented in previous section.

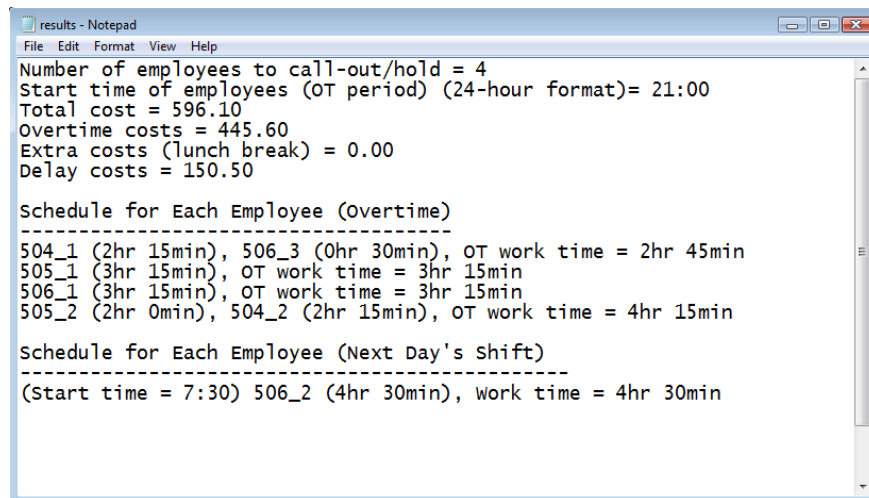


Figure D.10 Sample `results.txt`

Figure D.10 shows that in the best schedule, the number of employees to call-out is 4 and the called-out employees should start at 9pm (which is equal to t_0 from Figure B.1). Total cost of the overtime and next day schedules is \$596.10, whereas overtime costs are \$445.60, delay costs are \$150.50, and there is no extra pay because lunch break occurs during the desired time interval. Then, the file provides overtime schedule. Each line corresponds to a different employee and an employee's schedule ends by listing his/her total work time during overtime period (i.e. OT work time). For example, the first employee has two pass assignments, which are the first pass of route 504 (i.e. 504_1) and the third pass of route 506 (i.e. 506_3), which takes 2 hours 15 minutes and 30 minutes respectively. Total work time of the first employee during overtime period is 2 hours 45 minutes. For the next day schedule, start time of first passes that are assigned to each employee is recorded first, since some of the employees may continue with remaining passes from overtime period and then start plowing for next day assignments. In the example of Figure D.10, the only employee, who is assigned to next day's shift starts his/her first pass, which is second pass of route 506, at 7:30am (i.e. shift start time). Similar to overtime period, each employee's total work time is listed at the end of their schedule for next day's shift.

Additional output file: `cost_results.txt`

The additional output file lists the scheduling cost for each combination of parameters k (i.e. number of called-out employees), t_{start} (i.e. start time of employees during overtime period corresponding to the 15-minute interval starting at t_0 , where 1 corresponds to 12am of Day1), and b (i.e. overtime block size in terms of 15-minute intervals). Since the file size is large, only the starting two lines and the part that contains the minimum cost parameter combination are given as a sample in Figure D.11.

k	tstart	b	OT_emp_cost	OT_delay_cost	ND_extra_cost	ND_delay_cost	ND_OT_emp_cost	total_cost
0	85	2	0.000000	0.000000	0.000000	951.000000	0.000000	951.000000
0	85	3	0.000000	0.000000	0.000000	951.000000	0.000000	951.000000
4	85	4	345.679012	0.000000	0.000000	891.000000	0.000000	1236.679012
4	85	5	345.679012	0.000000	0.000000	891.000000	0.000000	1236.679012
4	85	6	345.679012	0.000000	0.000000	891.000000	0.000000	1236.679012
4	85	7	345.679012	0.000000	0.000000	891.000000	0.000000	1236.679012
4	85	8	345.679012	0.000000	0.000000	774.000000	0.000000	1119.679012
4	85	9	345.679012	0.000000	0.000000	477.000000	0.000000	822.679012
4	85	10	345.679012	0.000000	0.000000	477.000000	0.000000	822.679012
4	85	11	345.679012	0.000000	0.000000	477.000000	0.000000	822.679012
4	85	12	345.679012	0.000000	0.000000	477.000000	0.000000	822.679012
4	85	13	383.487654	12.000000	0.000000	234.000000	0.000000	629.487654
4	85	14	383.487654	12.000000	0.000000	234.000000	0.000000	629.487654
4	85	15	383.487654	12.000000	0.000000	234.000000	0.000000	629.487654
4	85	16	383.487654	12.000000	0.000000	234.000000	0.000000	629.487654
4	85	17	445.601852	33.500000	0.000000	117.000000	0.000000	596.101852
4	85	18	445.601852	33.500000	0.000000	117.000000	0.000000	596.101852

Figure D.11 Sample cost_results.txt

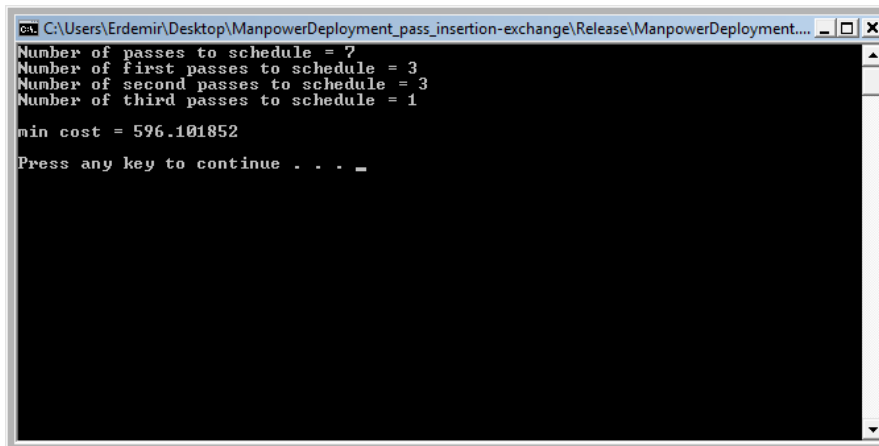
Figure D.10 and Figure D.11 are the results of the same example and they are obtained as the outputs of all the sample input files in figures Figure D.1 through Figure D.9. The minimum total cost is \$596.10, which corresponds to $k = 4$, $t_{start} = 85 = 9pm$, $b = 17 = 4$ hours 15 minutes.

We do not expect the user to examine `cost_results.txt` except when testing/investigating different parameter values.

3. Running the program

Before running the workforce deployment optimization program, the user should prepare the input files. Default versions of all the input files, which are presented as sample files in figures Figure D.1 through Figure D.9, are provided to SLC along with the program.

The program is provided as an application file with a name `ManpowerDeployment.exe`. All the input files and `ManpowerDeployment.exe` should be in the same folder. To start the program, double click on `ManpowerDeployment.exe`. Then the program starts to run and a black screen appears as given in Figure D.12.



```
C:\Users\Erdemir\Desktop\ManpowerDeployment_pass_insertion-exchange\Release\ManpowerDeployment...
Number of passes to schedule = 7
Number of first passes to schedule = 3
Number of second passes to schedule = 3
Number of third passes to schedule = 1
min cost = 596.101852
Press any key to continue . . . _
```

Figure D.12 Workforce deployment optimization program: Sample run screen

The screen first lists the number of passes to schedule as requested by the user in `passes.txt` file. Then, the program provides the minimum cost for scheduling different passes. When the run is completed, the program prompts the user to continue by pressing any key on the keyboard. After that the screen closes and the output files are ready to open and view. Output files are stored in the same folder where the program and input reside.

Appendix E Two-phase Formulation

Outer Loop

for $k \in \{1, \dots, \text{maximum number of plows}\}$

for $t_{\text{start}} \in t_0 \cup \{[\max\{t_0, t_s - T\}, t_s]\}$

for $\xi \in \{\text{possible storm scenarios}\}$

solve formulation P1 (Phase I)

solve formulation P2 (Phase II)

calculate objective function value $z^*(k, t_{\text{start}}, \xi)$ based on P2 solution

calculate $z^*(k, t_{\text{start}}) = E[z^*(k, t_{\text{start}}, \xi)] = \sum_{\xi} p(\xi) \cdot z^*(k, t_{\text{start}}, \xi)$

calculate $z^*(k^*, t_{\text{start}}^*) = \min_{k, t_{\text{start}}} z^*(k, t_{\text{start}})$

return $z^*(k^*, t_{\text{start}}^*), k^*, t_{\text{start}}^*, z^*(k^*, t_{\text{start}}^*, \xi)$ and complete solution for $\forall \xi$

Inner Loop: Phase I

Parameters:

$c_i(\xi)$: unit delay cost multiplier for road segment i under storm scenario ξ

$t_{i,j}(\xi)$: travel time from road segment i to j . Each road segment is directed and travel time from road segment i to j is measured using the distance from the end point of road segment i to the beginning point of road segment j

$w_i(\xi)$: sand/salt requirement of road segment i under storm scenario ξ

$d_i(\xi)$: plowing time of road segment i under storm scenario ξ

Sets:

E : set of selected road segments. Road segments are indexed by either i or j in the formulation

E_0 : set of selected road segments (including depot location or starting point (denoted by index 0))

C : set of clusters

Decision Variables:

$x_{i,q}$: 1, if road segment i is assigned to cluster q , 0 otherwise

$s_{i,j,q}$: 1, if road segment i immediately precedes road segment j in cluster q

$u_{i,q}$: subtour elimination variable to represent road segment i 's relative position in cluster q

$y_{i,j,q}$: 1, if road segments i and j are both assigned to cluster q , 0, otherwise

μ_q : time required to complete cluster q over 3.5 hours if total completion time is greater than 3.5 hours, 0, otherwise

v_q : 1, if there is at least one road segment assigned to cluster q , 0, otherwise

Formulation P1:

$$\text{Minimize } \sum_{q \in C} \sum_{i \in E} \sum_{\substack{j \in E \\ j > i}} y_{i,j,q} \cdot |c_i(\xi) - c_{ij}(\xi)| \quad (1)$$

$$\text{Minimize } \sum_{q \in C} \sum_{i \in E_0} \sum_{\substack{j \in E_0 \\ j \neq i}} s_{i,j,q} \cdot t_{i,j}(\xi) \quad (2)$$

$$\text{Minimize } \sum_{q \in C} v_q \quad (3)$$

$$\text{Minimize } \sum_{q \in C} \mu_q \quad (4)$$

S.t.

$$\sum_{i \in E} w_i(\xi) \cdot x_{i,q} \leq W \quad \forall q \in C \quad (5)$$

$$\sum_{i \in E_0} \sum_{\substack{j \in E_0 \\ j \neq i}} s_{i,j,q} \cdot t_{i,j}(\xi) + \sum_{i \in E} x_{i,q} \cdot d_i(\xi) \leq T \quad \forall q \in C \quad (6)$$

$$\sum_{i \in E} x_{i,q} \leq N \cdot v_q \quad \forall q \in C \quad (7)$$

$$\sum_{i \in E_0} \sum_{\substack{j \in E_0 \\ j \neq i}} s_{i,j,q} \cdot t_{i,j}(\xi) + \sum_{i \in E} x_{i,q} \cdot d_i(\xi) \leq 3.5 \text{ hours} + \mu_q \quad \forall q \in C \quad (8)$$

$$x_{0,q} = 1 \quad \forall q \in C \quad (9)$$

$$\sum_{q \in C} x_{i,q} = 1 \quad \forall i \in E \quad (10)$$

$$x_{i,q} = \sum_{\substack{j \in E_0 \\ j \neq i}} s_{i,j,q} \quad \forall i \in E, q \in C \quad (11)$$

$$\sum_{\substack{i \in E_0 \\ j \neq i}} s_{i,j,q} - \sum_{k \in E_0} s_{j,k,q} = 0 \quad \forall j \in E_0, q \in C \quad (12)$$

$$u_{i,q} \geq 2 \cdot x_{i,q} \quad \forall i \in E, q \in C \quad (13)$$

$$u_{i,q} \leq (N + 1) \cdot x_{i,q} \quad \forall i \in E, q \in C \quad (14)$$

$$u_{i,q} - u_{j,q} + 1 \leq N \cdot (1 - s_{i,j,q}) \quad \forall i, j \in E, i \neq j, q \in C \quad (15)$$

$$x_{i,q} - x_{j,q} + 1 \leq y_{i,j,q} \quad \forall i, j \in E, i < j, q \in C \quad (16)$$

$$y_{i,j,q} \geq 0 \quad \forall i, j \in E, i < j, q \in C \quad (17)$$

$$\mu_q \geq 0 \quad \forall q \in C \quad (18)$$

$$x_{i,q} \in \{0,1\} \quad \forall i \in E, q \in C \quad (19)$$

$$s_{i,j,q} \in \{0,1\} \quad \forall i, j \in E_0, q \in C, i \neq j \quad (20)$$

$$v_q \in \{0,1\} \quad \forall q \in C \quad (21)$$

Description of Constraints:

- (1) minimizes the difference between unit delay cost multipliers of road segments in a cluster (see Term 1 in Section 4.2.1),
- (2) minimizes the total travel time of the formed clusters (see Term 2 in Section 4.2.1)
- (3) minimizes the number of clusters to favor clusters using a single load of sand/salt efficiently (see Term 3 in Section 4.2.1),
- (4) penalizes clusters with total completion times greater than 3.5 hours (i.e. going over to double pay period) (see Term 4 in Section 4.2.1),
- (5) ensures that sand/salt requirement in a cluster does not exceed sand/salt capacity of a single plow (see Constraint 1 in Section 4.2.1),
- (6) ensures that total completion time of a cluster does not exceed total work time limit of a plow operator without taking a break (see Constraint 2 in Section 4.2.1),
- (7) ensures that v_q is 0, when there are no road segments assigned to cluster q . Otherwise, v_q is 1,
- (8) defines variable μ_q ,
- (9) ensures that depot is assigned to each cluster,
- (10) states that each road segment should be assigned to exactly one cluster (except starting point),
- (11) states that if road segment i is assigned to cluster q , then road segment i should be an immediate predecessor of a road segment j in cluster q ,
- (12) ensures that if road segment j is assigned to cluster q and if there are other road segments in the cluster, then road segment j should be connected to two other road segments (ingoing and outgoing) in the cluster,
- (13) – (15) are the subtour elimination constraints for each cluster q ,
- (16) is the linearization constraint for $y_{i,jq} = x_{i,q} \cdot x_{j,q}$,
- (17) – (21) define variable types and relevant ranges.

Inner Loop: Phase II

Parameters:

$s_i = \frac{\sum_i c_i}{t_i}$: score assigned to cluster i

t_i : total completion time of cluster i

k : total number of plows

L : maximum number of clusters to be assigned to a single plow

$D_i(t_{\text{start}})$: sum of total delay costs of road segments in cluster i , when cluster i is delayed to next day's shift

Sets:

C : set of clusters

Decision Variables:

$x_i^{p,l}$: 1, if cluster i is assigned to plow p in order l , 0 otherwise

u_p : idle time of a plow p , when $t_{\text{start}} \geq t_s - 4$ hours

y_p : 0, if there are not any clusters assigned to plow p , 1, otherwise

v_p : extra time required for plow p to complete all the clusters assigned to the plow above 3.5 hours, 0, if total work time is less than 3.5 hours

τ_p : total work time of plow p

$l_{p_1 p_2}$: represents the absolute difference between total work times of two plows p_1 and p_2

Formulation P2:

$$\text{Minimize } \sum_{p=1}^k \sum_{l=1}^L \sum_{i \in C} l \cdot s_i \cdot x_i^{p,l} \quad (22)$$

$$\text{Minimize } \sum_{p=1}^k v_p \quad (23)$$

$$\text{Minimize } \sum_{p=1}^k u_p \quad (24)$$

$$\text{Maximize } \sum_{i \in C} D_i(t_{\text{start}}) \cdot \sum_{p=1}^k \sum_{l=1}^L x_i^{p,l} \quad (25)$$

$$\text{Minimize } \sum_{p_1=1}^k \sum_{\substack{p_2=1 \\ p_2 > p_1}}^k l_{p_1 p_2} \quad (26)$$

S.t.

$$\tau_p = \sum_{l=1}^L \sum_{i \in C} t_i \cdot x_i^{p,l} \quad \forall p \in \{1, \dots, k\} \quad (27)$$

$$l_{p_1 p_2} \geq \tau_{p_2} - \tau_{p_1} \quad \forall p_1, p_2 \in \{1, \dots, k\} \quad (28)$$

$$l_{p_1 p_2} \geq \tau_{p_1} - \tau_{p_2} \quad \forall p_1, p_2 \in \{1, \dots, k\} \quad (29)$$

$$\tau_p \leq T \cdot y_p \quad \forall p \in \{1, \dots, k\} \quad (30)$$

$$u_p \geq (t_s - t_{\text{start}}) - \tau_p \quad \forall p \in \{1, \dots, k\} \quad (31)$$

$$u_p \geq 0 \quad \forall p \in \{1, \dots, k\} \quad (32)$$

$$v_p \geq \tau_p - (3.5 \text{ hours}) \quad \forall p \in \{1, \dots, k\} \quad (33)$$

$$v_p \geq 0 \quad \forall p \in \{1, \dots, k\} \quad (34)$$

$$\sum_{p=1}^k \sum_{l=1}^L x_i^{p,l} \leq 1 \quad \forall i \in C \quad (35)$$

$$\sum_{i \in C} x_i^{p,l} \leq 1 \quad \forall p \in \{1, \dots, k\}, l \in \{1, \dots, k\} \quad (36)$$

$$x_i^{p,l} \in \{0,1\} \quad \forall p \in \{1, \dots, k\}, l \in \{1, \dots, k\}, i \in C \quad (37)$$

$$y_p \in \{0,1\} \quad \forall p \in \{1, \dots, k\} \quad (38)$$

Description of Constraints:

- (22) prioritizes clusters with highest scores (see Term 1 in Section 4.2.2),
- (23) penalizes total work time of plows over 3 hour 30 minutes (see Term 2 in Section 4.2.2),
- (24) penalizes idle time of employees when the next shift starts within a 4-hour period from the start of overtime work (see Term 3 in Section 4.2.2),
- (25) incurs a delay cost penalty for not assigning a cluster to any of the plows (see Term 4 in Section 4.2.2),
- (26) penalizes total work time difference between plows (see Term 5 in Section 4.2.2),
- (27) states that total work time of a plow is equal to sum of total completion times of clusters assigned to the plow,
- (28) – (29) are needed to define variable $l_{p_1 p_2}$ as the absolute difference between τ_{p_1} and τ_{p_2} ,
- (30) ensures that each plow works at most T time units (Constraint in Section 4.2.2),
- (31) – (32) defines variable u_p as the idle time of an employee when overtime work starts within 4-hour period preceding the start of regular shift,
- (33) defines variable v_p as [total work time of plow $p - 3 \text{ hours } 30 \text{ minutes}]^+$,
- (34) ensures that v_p is non-negative,
- (35) ensures that each cluster i is assigned to exactly one plow in a unique order,
- (36) ensures that at most one cluster can be assigned to a given order of a plow,
- (37) – (38) define binary variables.

Appendix F Delay Cost Multipliers by Road Type

All road segments are grouped into types such that delay cost rate of segments of a particular type are the same. Delay cost rate is defined as the cost per 15 minutes of not plowing a road segment. Delay cost rate is relative to the cost of regular wages per 15 minutes and it is incurred only until such time that plowing a road segment is started.

Consider type- i road segments indexed $j = 1, \dots, n_i$ with plowing times $t_{j,i}$ for j^{th} segment. Road segments are indexed such that segments with smallest plowing time have the smallest index. The delay cost rates are y_i for each road segment. Let $t_i(m) = \sum_{j=1}^m t_{j,i}$ denote the plowing times of first m road segments. Given our conception of how delay cost is incurred, it makes sense for the county to plow smallest segments first within each class. Then, if plowing starts at time 0 and all type- i segments are plowed, the delay cost rate is $n_i y_i$ during $[0, t_{1,i})$, $(n_i - 1)y_i$ during $[t_i(1), t_i(2))$ and so on. Define $t_i = t_i(n_i)$ and $t_i(0) = 0$. Then, the delay cost incurred over $[0, t_i]$ is

$$\sum_{j=1}^{n_i} y_i [n_i - (j - 1)] [t_i(j) - t_i(j - 1)].$$

The total delay cost has a step-function profile. We approximate it by a smooth function. If k_{OT} operators work simultaneously on type- i segments, the total time it takes is t_i/k_{OT} , during which time, we assume that the delay cost rate decreases uniformly from $n_i y_i$ to zero. Therefore, if type- i segments are plowed to completion, the approximate total cost is

$$(1/2)[t_i/k_{OT}]n_i y_i$$

Now, we try to figure out what happens when plowing does not complete all type- i segments. Suppose plowing is stopped at time t . Then, we can use triangular inequality to find the equivalent delay cost rate at time t . Let us denote the latter by $y(t)$. We get

$$\frac{n_i y_i}{t_i/k_{OT}} = \frac{y(t)}{t},$$

which simplifies to

$$y(t) = \frac{n_i y_i t}{t_i/k_{OT}}.$$

Appendix G OT and Delay Cost Functions

Parameters

$y_i(\xi)$: Sum of delay cost multipliers of road segments type i under storm realization ξ . We assume that all the road segments of same type have the same delay cost multipliers. Therefore, if there are n_i road segments of type i , and each one has a delay cost multiplier $d_i(\xi)$, then $y_i(\xi) = n_i \times d_i(\xi)$.

$t_i(\xi)$: Minimum plowing time of road segments of type i under storm realization ξ

$t(\xi)$: Minimum plowing time of all the road segments under storm realization ξ

m : Total number of distinct road segment types

t_s : Start time of regular shift hours

t_e : End time of regular shift hours

ω : Regular wage rate per minute

α : Proportion of annual wages spent on plowing operations

W : Annual wage rate for full-time plow operators

For simplification, y_i and t_i will be used in the formulas instead of $y_i(\xi)$ and $t_i(\xi)$.

Decision variables

W_{OT} : Length of overtime duty

j : All the road segments with type j are assumed to be cleared in the overtime period and the plows are plowing road segments with type $j + 1$ when overtime duty ends.

k_{OT} : Number of plow operators working in overtime period

t_{start} : Start time of plowing operations in overtime period

k : Number of employees in workforce

$C(\xi, k)$: Minimum overtime and delay costs under storm realization ξ and when there are k employees in workforce

Graphical Representation of Total Costs

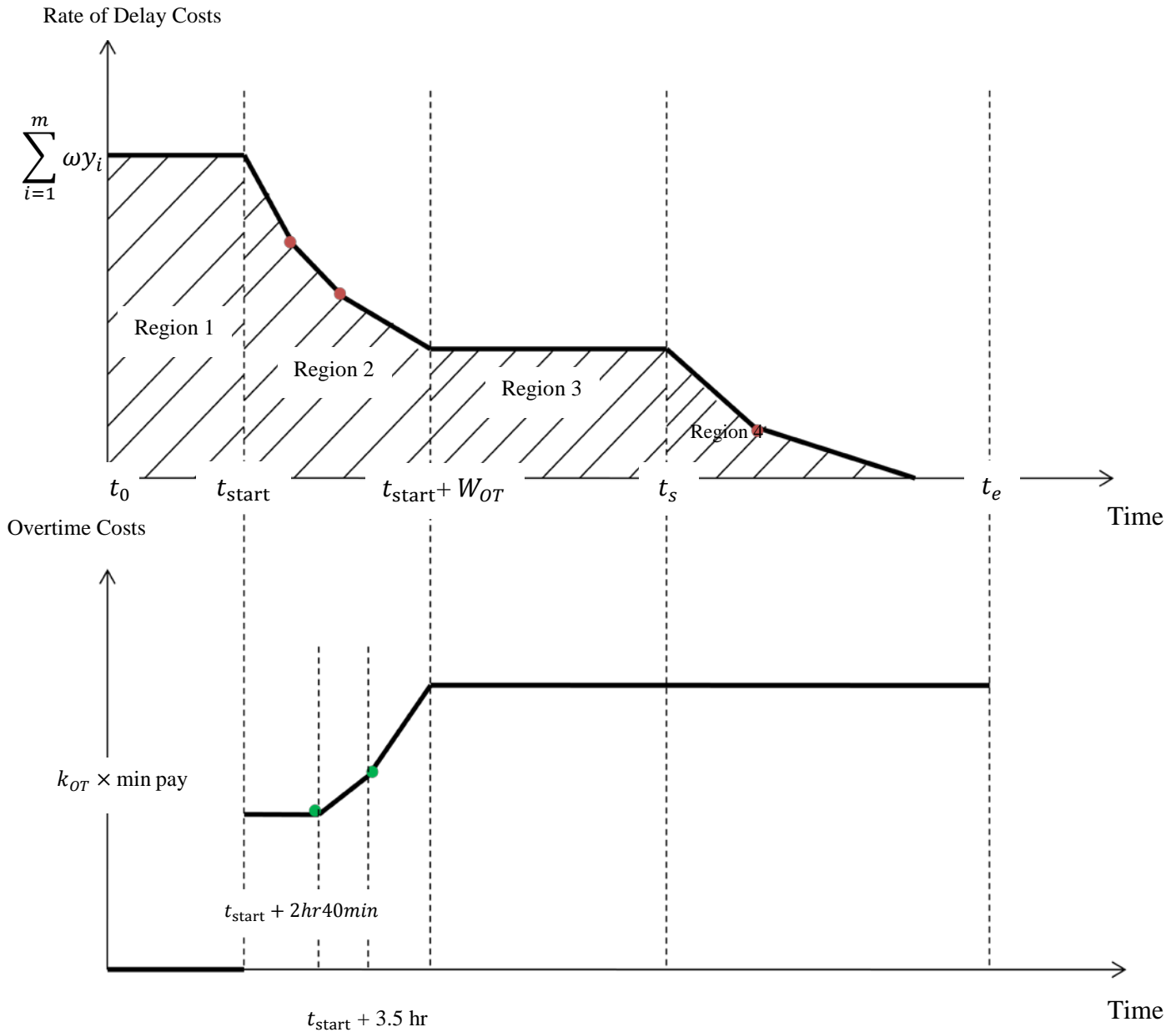


Figure G.1 Graphical representation of total costs

In the above graph for rate of delay costs, shaded area gives the total delay costs, which is the sum of four regions in the graph. Overtime costs are illustrated in the above graph for the case that work time in the overtime period exceeds 3.5 hours and $t_{\text{start}} < t_s - 4$ hours. Explicit formula of both cost functions are provided in the next section. For overtime costs, there are three different formulas based on t_{start} .

Delay Cost Function (f_D) and Overtime Cost Function (f_{OT})

Define $\tilde{f}_D = \frac{f_D}{\omega}$ and $\tilde{f}_{OT} = \frac{f_{OT}}{\omega}$.

Delay Costs

$$\tilde{f}_D = \text{Area (Region 1 + Region 2 + Region 3 + Region 4)}$$

$$\text{Area (Region 1)} = (t_{start} - t_0) \times \sum_{i=1}^m y_i$$

$$\text{Area (Region 2)} = \left(\frac{t_1}{k_{OT}} \times \sum_{i=2}^m y_i \right) + \frac{t_1}{2k_{OT}} \times y_1$$

$$+ \left(\frac{t_2}{k_{OT}} \times \sum_{i=3}^m y_i \right) + \frac{t_2}{2k_{OT}} \times y_2$$

+ ...

$$+ \left(\frac{t_j}{k_{OT}} \times \sum_{i=j+1}^m y_i \right) + \frac{t_j}{2k_{OT}} \times y_j$$

$$+ \frac{\left[\sum_{i=j+1}^m y_i + \left(\sum_{i=j+1}^m y_i - \frac{W_{OT} - \sum_{i=1}^j \frac{t_i}{k_{OT}}}{\frac{t_{j+1}}{k_{OT}}} \times y_{j+1} \right) \right] \times \left(W_{OT} - \sum_{i=1}^j \frac{t_i}{k_{OT}} \right)}{2}$$

$$= \frac{1}{2k_{OT}} \times \sum_{i=1}^j y_i \times t_i + \frac{\sum_{i=j+1}^m y_i}{k_{OT}} \times \left(t_j + t_{j-1} \times y_j + t_{j-2} \times \sum_{i=j-1}^j y_i + \dots + t_1 \times \right. \\ \left. i=2jy_i \right)$$

$$+ \left(W_{OT} - \sum_{i=1}^j \frac{t_i}{k_{OT}} \right) \times \sum_{i=j+1}^m y_i - \frac{\left(W_{OT} - \sum_{i=1}^j \frac{t_i}{k_{OT}} \right)^2 \times y_{j+1}}{2 \frac{t_{j+1}}{k_{OT}}}$$

$$\text{Area (Region 3)} = \left(\sum_{i=j+1}^m y_i - \frac{W_{OT} - \sum_{i=1}^j \frac{t_i}{k_{OT}}}{\frac{t_{j+1}}{k_{OT}}} \times y_{j+1} \right) \times (t_s - t_{start} - W_{OT})$$

$$\text{Area (Region 4)} = \frac{\left[\sum_{i=j+2}^m y_i + \left(\sum_{i=j+1}^m y_i - \frac{W_{OT} - \sum_{i=1}^j \frac{t_i}{k_{OT}}}{\frac{t_{j+1}}{k_{OT}}} \times y_{j+1} \right) \right] \times \left(\sum_{i=1}^{j+1} \frac{t_i}{k_{OT}} - W_{OT} \right) \times \frac{k_{OT}}{k}}{2}$$

$$+ \left(\frac{t_{j+2}}{k} \times \sum_{i=j+3}^m y_i \right) + \frac{t_{j+2}}{2k} \times y_{j+2}$$

+ ...

$$+ \frac{t_m}{2k} \times y_m$$

$$= \frac{1}{2k} \times \sum_{i=j+2}^m y_i \times t_i + \frac{1}{k} \times \left(t_{j+2} \times \sum_{i=j+3}^m y_i + \dots + t_{m-1} \times \sum_{i=m}^m y_m \right)$$

$$+ \left(\sum_{i=1}^{j+1} \frac{t_i}{k_{OT}} - W_{OT} \right) \times \frac{k_{OT}}{k} \times \left[\sum_{i=j+2}^m y_i + \frac{y_{j+1} - \frac{W_{OT} - \sum_{i=1}^j \frac{t_i}{k_{OT}}}{\frac{t_{j+1}}{k_{OT}}} \times y_{j+1}}{2} \right]$$

Overtime Costs

$$\tilde{f}_{OT} = \begin{cases} k_{OT} \times 1.5 \times 2\text{hr}40\text{mn} + k_{OT} \times 1.5 \times \min\{50\text{mn}, W_{OT} - 2\text{hr}40\text{mn}\}^+ + k_{OT} \times 2 \times \{W_{OT} - 3.5\text{hr}\}^+, & t_{start} \in (t_e, t_s - 4\text{hr}) \\ k_{OT} \times 1.5 \times \min\{W_{OT}, 3.5\text{hr}\}^+ + k_{OT} \times 2 \times \{W_{OT} - 3.5\text{hr}\}^+, & t_{start} = t_e \\ k_{OT} \times 1.5 \times \min\{t_s - t_{start}, 3.5\text{hr}\}^+ + k_{OT} \times 2 \times \{t_s - t_{start} - 3.5\text{hr}\}^+, & t_{start} \in [t_s - 4\text{hr}, t_s) \end{cases}$$

Appendix H Proofs of Theoretical Results in Chapter 6

Theorem 1: The minimum value of k is equal to $\min k = \frac{t(\xi)}{\min(\text{max work time limit, time-to-clear goal})}$.

Proof: The minimum number of employees is obtained when all the k employees are called-out to work in overtime period. In order to complete all the work by time-to-clear goal, minimum number of employees is obtained when each employee works at least $\min(\text{max work time limit, time-to-clear goal})$. Therefore,

$$\min k = \frac{t(\xi)}{\min(\text{max work time limit, time-to-clear goal})} \quad \blacksquare$$

Theorem 2: $\tilde{f}_D + \tilde{f}_{OT}$ is convex with respect to k , when $k_{OT}, W_{OT}, t_{start}$ are fixed.

Proof: Define $\tilde{f}_C = \tilde{f}_D + \tilde{f}_{OT}$. Then, we need to show that $\tilde{f}_C(k+1) - \tilde{f}_C(k) \geq \tilde{f}_C(k) - \tilde{f}_C(k-1)$ for all $k \in \{2, \dots, k_{max}\}$.

$$\tilde{f}_C(k+1) - \tilde{f}_C(k) = \left(\frac{1}{k+1} - \frac{1}{k} \right) \times A$$

$$\text{where } A = \frac{\sum_{i=j+2}^m y_i \times t_i}{2} + \left(t_{j+2} \times \sum_{i=j+3}^m y_i + \dots + t_{m-1} \times \sum_{i=m}^m y_m \right) + \left(\sum_{i=1}^{j+1} \frac{t_i}{k_{OT}} - W_{OT} \right) \times k_{OT} \times \left(\sum_{i=j+2}^m y_i + \frac{y_{j+1} - \frac{W_{OT} - \sum_{i=1}^j \frac{t_i}{k_{OT}} \times y_{j+1}}{\frac{t_{j+1}}{k_{OT}}}}{2} \right).$$

Then,

$$\tilde{f}_C(k) - \tilde{f}_C(k-1) = \left(\frac{1}{k} - \frac{1}{k-1} \right) \times A$$

$$[\tilde{f}_C(k+1) - \tilde{f}_C(k)] - [\tilde{f}_C(k) - \tilde{f}_C(k-1)] = \left(\frac{1}{k \times (k-1)} - \frac{1}{k \times (k+1)} \right) \times A.$$

$$\frac{1}{k \times (k-1)} - \frac{1}{k \times (k+1)} \geq 0 \text{ and } A \geq 0, \text{ since } \sum_{i=1}^{j+1} \frac{t_i}{k_{OT}} \geq W_{OT} \text{ and } W_{OT} \geq \sum_{i=1}^j \frac{t_i}{k_{OT}}.$$

Therefore, $[\tilde{f}_C(k+1) - \tilde{f}_C(k)] - [\tilde{f}_C(k) - \tilde{f}_C(k-1)] \geq 0 \quad \blacksquare$

Theorem 3: $t_{start}^* \in \{t_0\} \cup [t_s - 4 \text{ hrs}, t_s]$.

Proof:

Case 1. $t_{start} \in [t_e, t_s - 4 \text{ hr}]$

Compare the costs of $t_{start} = t_0$ (i.e. Cost1) and $t_{start} = t_0 + x$ (i.e. Cost2) where $x < t_s - 4 \text{ hr} - t_0$ when k, k_{OT} and W_{OT} are fixed. Therefore, we know that this comparison is valid for $W_{OT} \leq t_s - (t_0 + x)$.

$$\begin{aligned}
\tilde{f}_D(t_0) - \tilde{f}_D(t_0 + x) &= (t_0 - t_0 - t_0 - x + t_0) \times \sum_{i=1}^m y_i \\
&\quad + \left(\sum_{i=j+1}^m y_i - \frac{W_{OT} - \sum_{i=1}^j \frac{t_i}{k_{OT}}}{\frac{t_{j+1}}{k_{OT}}} \times y_{j+1} \right) \times (t_s - t_0 - W_{OT} - t_s + t_0 + x + \\
&W_{OT} \\
&= x \times \left(\sum_{i=j+1}^m y_i - \frac{W_{OT} - \sum_{i=1}^j \frac{t_i}{k_{OT}}}{\frac{t_{j+1}}{k_{OT}}} \times y_{j+1} - \sum_{i=1}^m y_i \right) < 0,
\end{aligned}$$

since $x > 0$ and $\sum_{i=j+1}^m y_i - \frac{W_{OT} - \sum_{i=1}^j \frac{t_i}{k_{OT}}}{\frac{t_{j+1}}{k_{OT}}} \times y_{j+1} - \sum_{i=1}^m y_i < 0$.

In addition, $\tilde{f}_{OT}(t_0) - \tilde{f}_{OT}(t_0 + x) = 0$. Therefore, $\text{Cost1} < \text{Cost2}$ for any $0 < x < t_s - 4 \text{ hr} - t_0$.

Given this result, using sample path approach, we conclude that

$\tilde{f}_C(t_0, W_{OT}^*(t_0)) \leq \tilde{f}_C(t_0, W_{OT}^*(t)) \leq \tilde{f}_C(t, W_{OT}^*(t))$, where the first inequality is known by optimality property and the second inequality holds by the above result that for a fixed W_{OT} , total costs under $t_{\text{start}} = t_0$ is less than total costs under $t_{\text{start}} = t = t_0 + x$.

Since the range of W_{OT} values under $t_{\text{start}} = t_0$ includes the range of W_{OT} values under $t_{\text{start}} = t_0 + x$, $t_{\text{start}} = t_0$ in this interval.

Case 2. $t_{\text{start}} \in [t_s - 4 \text{ hrs}, t_s - 3.5 \text{ hrs})$

Compare the costs of $t_{\text{start}} = t_s - 4 \text{ hrs}$ (i.e. Cost1) and $t_{\text{start}} = t_s - 4 \text{ hrs} + x$ (i.e. Cost2) where $x < 0.5 \text{ hrs}$ when k, k_{OT} and W_{OT} are fixed. Therefore, this comparison is valid for $W_{OT} \leq 4 \text{ hrs} - x$.

$$\begin{aligned}
&\tilde{f}_D(t_s - 4 \text{ hrs}) - \tilde{f}_D(t_s - 4 \text{ hrs} + x) = \\
&= (t_s - 4 \text{ hrs} - t_0 - t_s + 4 \text{ hrs} - x + t_0) \times \sum_{i=1}^m y_i \\
&\quad + \left(\sum_{i=j+1}^m y_i - \frac{W_{OT} - \sum_{i=1}^j \frac{t_i}{k_{OT}}}{\frac{t_{j+1}}{k_{OT}}} \times y_{j+1} \right) \\
&\quad \quad \quad \times (t_s - t_s + 4 \text{ hrs} - W_{OT} - t_s + t_s - 4 \text{ hrs} + x + W_{OT}) \\
&= x \times \left(\sum_{i=j+1}^m y_i - \frac{W_{OT} - \sum_{i=1}^j \frac{t_i}{k_{OT}}}{\frac{t_{j+1}}{k_{OT}}} \times y_{j+1} - \sum_{i=1}^m y_i \right)
\end{aligned}$$

< 0 ,

since $x > 0$ and $\sum_{i=j+1}^m y_i - \frac{W_{OT} - \sum_{i=1}^j k_{OT} t_i}{t_{j+1} k_{OT}} \times y_{j+1} - \sum_{i=1}^m y_i < 0$.

$$\begin{aligned} \tilde{f}_{OT}(t_s - 4 \text{ hrs}) - \tilde{f}_{OT}(t_s - 4 \text{ hrs} + x) &= k_{OT} \times 2 \times (t_s - t_s + 4 \text{ hrs} - t_s + t_s - 4 \text{ hrs} + x) \\ &= k_{OT} \times 2 \times x > 0 \end{aligned}$$

$$Cost\ 1 - Cost\ 2 = x \times \left(\sum_{i=j+1}^m y_i - \frac{W_{OT} - \sum_{i=1}^j k_{OT} t_i}{t_{j+1} k_{OT}} \times y_{j+1} - \sum_{i=1}^m y_i + k_{OT} \times 2 \right).$$

The sign of $Cost\ 1 - Cost\ 2$ is unknown. However, total costs are either increasing or decreasing linearly in x , when t_{start} increases from $t_s - 4$ hrs to $t_s - 3.5$ hrs for a constant W_{OT}, k and k_{OT} . Therefore, two end points of the interval are the candidates for t_{start} . Since, right end point of the t_{start} interval depends on W_{OT} , $t_{start} \in \{t_s - 4 \text{ hrs}, t_s - \max\{W_{OT}, 3.5 \text{ hrs}\}\}$. Since W_{OT} varies in the interval $[0, 4 \text{ hrs}]$, for this interval of t_{start} , we conclude that

$$t_{start} \in [t_s - 4 \text{ hrs}, t_s - 3.5 \text{ hrs})$$

This means that there is no reduction in initial t_{start} interval size.

Case 3. $t_{start} \in [t_s - 3.5 \text{ hr}, t_s]$

The proof for this case is very similar to Case 2. The only difference is that the unit decrease in OT costs is 1.5 when t_{start} increases by one unit instead of 2.

For this interval of t_{start} , we conclude that

$$t_{start} \in [t_s - 3.5 \text{ hrs}, t_s]$$

This means that there is no reduction in initial t_{start} interval size. ■

Theorem 4: Given values of k and k_{OT} , $t_{start}^* \in \{t_0\} \cup [t_s - 4 \text{ hrs}, t_s - \min W_{OT}]$, if $\min W_{OT} \leq 4 \text{ hrs}$;

$$t_{start}^* = \emptyset, \text{ if } \min W_{OT} > t_s - t_0;$$

$$t_{start}^* = t_0, \text{ otherwise.}$$

Proof: By proof of Theorem 3, we know that $t_{start}^* \in \{t_0\} \cup [t_s - 4 \text{ hrs}, t_s]$. In addition, to be able to finish all the plowing work by the end of next day's regular work hours, the employees should work at least $\min W_{OT}$ time units in overtime period.

$$\min W_{OT} = \left\lceil \frac{t(\xi) - \{7.5 \text{ hrs} + (t_0 - t_e)\} \times k}{k_{OT}} \right\rceil^+, \text{ when time to clear goal is 24 hours.}$$

Therefore, if $\min W_{OT} \leq 4$ hrs, then $t_{\text{start}}^* \in \{t_0\} \cup [t_s - 4 \text{ hrs}, t_s - \min W_{OT}]$.

If $\min W_{OT} > t_s - t_0$, then $t_{\text{start}}^* = \emptyset$. Otherwise, $t_{\text{start}}^* = t_0$. ■

Theorem 5: $W_{OT}^* = t_s - t_{\text{start}}$ if $t_{\text{start}} \in [t_s - 4 \text{ hr}, t_s]$ when $t_{\text{start}}, k, k_{OT}$ are fixed.

Proof: In this interval, $\frac{\partial \tilde{f}_{OT}}{\partial W_{OT}} = 0$. Total delay costs are decreasing when W_{OT} increases.

Therefore, W_{OT} should have its maximum value in this interval, i.e. $W_{OT}^* = t_s - t_{\text{start}}$. ■

Theorem 6: $W_{OT}^* \in [2 \text{ hr } 40 \text{ mn}, t_s - t_{\text{start}}]$ if $t_{\text{start}} \in (t_e, t_s - 4 \text{ hr})$ and \tilde{f}_c is convex with respect to W_{OT} in this interval of t_{start} .

Proof: If $W_{OT} \leq 2 \text{ hr } 40 \text{ mn}$, then $\frac{\partial \tilde{f}_{OT}}{\partial W_{OT}} = 0$ and total delay costs are decreasing (by the proof of Theorem 5 – can be shown graphically). Therefore, $W_{OT}^* = 2 \text{ hr } 40 \text{ mn}$ when $W_{OT} \leq 2 \text{ hr } 40 \text{ mn}$.

When $W_{OT} > 2 \text{ hr } 40 \text{ mn}$, then $\frac{\partial \tilde{f}_{OT}}{\partial W_{OT}} \in \{1.5, 2\} > 0$. Since total delay costs are decreasing when W_{OT} increases, it is not known with certainty that \tilde{f}_c is increasing or decreasing with W_{OT} . Therefore, $W_{OT}^* \in [2 \text{ hr } 40 \text{ mn}, t_s - t_{\text{start}}]$ if $t_{\text{start}} \in (t_e, t_s - 4 \text{ hr})$.

However, in the interval $[2 \text{ hr } 40 \text{ mn}, t_s - t_{\text{start}}]$, $\tilde{f}_c = \tilde{f}_{OT} + \tilde{f}_D$ is convex in W_{OT} for fixed $t_{\text{start}}, k, k_{OT}$. In the following, we show that both \tilde{f}_{OT} and \tilde{f}_D is convex in this interval, therefore, \tilde{f}_c is convex as sum of two convex functions.

Since $\frac{\partial \tilde{f}_{OT}}{\partial W_{OT}} \in \{1.5, 2\}$ and $\frac{\partial^2 \tilde{f}_{OT}}{\partial W_{OT}^2} = 0$, \tilde{f}_{OT} is convex in W_{OT} .

$$\begin{aligned}
& \tilde{f}_D(W_{OT} + 1) - \tilde{f}_D(W_{OT}) \\
&= (W_{OT} + 1) \times \sum_{i=j+1}^m y_i - W_{OT} \times \sum_{i=j+1}^m y_i \\
&\quad - \frac{\left[(W_{OT} + 1)^2 - 2(W_{OT} + 1) \times \sum_{i=1}^j \frac{t_i}{k_{OT}} \right] \times y_{j+1}}{2 \frac{t_{j+1}}{k_{OT}}} \\
&\quad + \frac{\left[W_{OT}^2 - 2W_{OT} \times \sum_{i=1}^j \frac{t_i}{k_{OT}} \right] \times y_{j+1}}{2 \frac{t_{j+1}}{k_{OT}}} - (W_{OT} + 1) \times \sum_{i=j+1}^m y_i + W_{OT} \times \sum_{i=j+1}^m y_i \\
&\quad - \frac{(W_{OT} + 1) \times (t_s - t_{start}) \times y_{j+1}}{\frac{t_{j+1}}{k_{OT}}} + \frac{W_{OT} \times (t_s - t_{start}) \times y_{j+1}}{\frac{t_{j+1}}{k_{OT}}} \\
&\quad + \frac{(W_{OT} + 1)^2 \times y_{j+1}}{\frac{t_{j+1}}{k_{OT}}} - \frac{(W_{OT} + 1) \times y_{j+1} \times \sum_{i=1}^j \frac{t_i}{k_{OT}}}{\frac{t_{j+1}}{k_{OT}}} - \frac{W_{OT}^2 \times y_{j+1}}{\frac{t_{j+1}}{k_{OT}}} \\
&\quad + \frac{W_{OT} \times y_{j+1} \times \sum_{i=1}^j \frac{t_i}{k_{OT}}}{\frac{t_{j+1}}{k_{OT}}} - (W_{OT} + 1) \times \frac{k_{OT}}{k} \times \sum_{i=j+2}^m y_i + W_{OT} \times \frac{k_{OT}}{k} \\
&\quad \times \sum_{i=j+2}^m y_i - (W_{OT} + 1) \times \frac{k_{OT}}{k} \times \frac{y_{j+1}}{2} + W_{OT} \times \frac{k_{OT}}{k} \times \frac{y_{j+1}}{2} \\
&\quad + \frac{(W_{OT} + 1)^2 \times k_{OT} \times y_{j+1}}{2k \times \frac{t_{j+1}}{k_{OT}}} - \frac{W_{OT}^2 \times k_{OT} \times y_{j+1}}{2k \times \frac{t_{j+1}}{k_{OT}}} \\
&\quad - \frac{(W_{OT} + 1) \times k_{OT} \times y_{j+1} \times \sum_{i=1}^j \frac{t_i}{k_{OT}}}{2k \times \frac{t_{j+1}}{k_{OT}}} + \frac{W_{OT} \times k_{OT} \times y_{j+1} \times \sum_{i=1}^j \frac{t_i}{k_{OT}}}{2k \times \frac{t_{j+1}}{k_{OT}}} \\
&= \frac{(2W_{OT} + 1) \times y_{j+1} \times \left(1 + \frac{k_{OT}}{k}\right) + \frac{k_{OT}}{k} \times y_{j+1} \times \sum_{i=1}^j \frac{t_i}{k_{OT}} - 2(t_s - t_{start}) \times y_{j+1}}{2 \times \frac{t_{j+1}}{k_{OT}}} \\
&\quad - \frac{k_{OT}}{k} \times \left(\sum_{i=j+2}^m y_i - \frac{y_{j+1}}{2} \right)
\end{aligned}$$

$[\tilde{f}_D(W_{OT} + 1) - \tilde{f}_D(W_{OT})] - [\tilde{f}_D(W_{OT}) - \tilde{f}_D(W_{OT} - 1)] = y_{j+1} \times \left(1 + \frac{k_{OT}}{k}\right) \geq 0$. Therefore, \tilde{f}_D is convex. ■

Theorem 7: $W_{OT}^* \in (0, t_s - t_{\text{start}}]$ if $t_{\text{start}} = t_e$ and \tilde{f}_c is convex with respect to W_{OT} in this interval of t_{start} .

Proof: $(0, t_s - t_{\text{start}}]$ contains the entire range of values for W_{OT} . When $t_{\text{start}} = t_e$, $\frac{\partial \tilde{f}_{OT}}{\partial W_{OT}} \in \{1.5, 2\}$ and $\frac{\partial^2 \tilde{f}_{OT}}{\partial W_{OT}^2} = 0$. Then, the proof of convexity follows by the proof of Theorem 6. ■

Appendix I User Guide: Workforce Requirements Planning Under Different Scenarios

Input files

Some of the discretionary parameter settings are not expected to be changed frequently. Therefore, these parameters are written into text files. The input files for discretionary parameter settings are as follows:

`daily_max_work_time.txt`: Records maximum work time limit in a day in terms of hours. Default value in that file is 16.

`hourly_wage_rate.txt`: Records hourly wage rate for plow operators in dollars. Default value in that file is 35.

`k_max.txt`: Records maximum number of employees to be hired at the considered depot. Default value in that file is 40.

`num_passes.txt`: This file records the number of passes performed over the same road segments for each storm realization. Each row in that file corresponds to a single storm realization from base year of 2006. Default value is 2 for all the storm realizations. Storm data will be provided to SLC together with the program; therefore, maintenance supervisors could modify the number of passes specific to storm parameters.

`num_yearly_passes.txt`: This file records the number of passes performed over the same road segments for each storm realization for 10 years of storm simulations as well as the base year of 2006. Each column corresponds to a year and the last column is for the base year of 2006. Each row is for a single storm realization in the corresponding year. Default values for all the storms are 2. By looking at the storm parameters in the excel file that contains all the generated years' storm data, maintenance supervisors could modify the number of passes in that file. The number of rows in the file equals to maximum number of storms observed in any one of the years, which is 14. Therefore, for the years, which contain less number of storms, a value of -1 is inserted to the storm rows for which the corresponding storm does not exist.

`perc_decr.txt`: This file records the percent decrease in total completion times in subsequent passes of the plows over the same road segments. Default value is 10.

`percent_fringe.txt`: This file records the percentage of fringe benefits in hourly wage rate. Default value is 62.

Other input files, which contain records from other data sources, are as follows:

`depot_data.txt`: This file has four columns and each row corresponds to a single road segment connected to the considered depot. First column records the length of the road segment in miles, second column records the AADT count of the road segment (if AADT count is unknown, enter 5), third column records the number of lanes at each side of the road segment, and fourth column records the road type (1: county state-aid road, 2: county road, 3: township road, 4: municipal state-aid street, 5: municipal street, 6: private jurisdiction street). If any given lane requires more than one pass to clear due to wide or paved shoulder, then number of lanes could be increased for the corresponding road segment. For example, if a road segment has a total of four lanes (i.e. two at each side) and one of these lanes at each side require two passes to clear, then the input in third column should be 3, since one lane at each side is like two lanes.

`max_num_storms_in_a_year.txt`: This file records maximum number of storms in generated 10 years of storms. Maximum number of storms in one of these years is 14.

`num_storms.txt`: This file records the number of storms in the base year of 2006. In 2006, there are 12 storms.

`num_yearly_storms.txt`: This file records the number of storms in each of the 10 generated years. 11th row corresponds to the base year of 2006.

`num_years.txt`: This file records the number of years generated by Monte-Carlo simulation plus the base year. Therefore, stored value is 11.

`plow_speeds.txt`: This file records the plow speeds under each storm realization of the base year of 2006.

`yearly_plow_speeds.txt`: This file records the plow speeds under each storm realization of the 10 generated years. Each column corresponds to a different year and each row is for a different storm realization in the corresponding year. A value of -1 is entered if the corresponding storm does not exist in the corresponding year.

`t_0.txt`: This file records the t_0 values for each storm in the base year of 2006. t_0 is the earliest call-out time after the end of each snow storm and it is recorded in terms of 15-minute intervals passed after the end of regular shift hours. For example, if snow storm ends at 1:00 PM, then t_0 is 3:30 PM, which is recorded as 0 in the file. If snow storm ends at 4:00 PM, then t_0 is 2.

`yearly_t_0.txt`: This file records the t_0 values for each storm in the 10 simulated years. 11th column corresponds to the base year of 2006. Calculation of t_0 values is same as in `t_0.txt`. The structure of the file in terms of rows and columns is same with `yearly_plow_speeds.txt`.

All the input files for each of the four depots in District 5 will be provided to SLC together with the decision support system.

Running the program files and reading the outputs

Currently, there are 5 different executable program files as components of the decision support system. The input files and the executable program files should be in the same folder. To run the executable programs, the user should install MCRInstaller.exe to their computer. MCRInstaller.exe will be included within the CD that contains the program files.

When all the input files are ready and MCRInstaller.exe is run and installed, SLC maintenance supervisors should run the programs in the following order:

1. `clustering.exe`: Double click on the `clustering.exe` file. The user is then asked about the preferred clustering method, statistical clustering (S) or road-type clustering (R). Then, press enter. The program outputs the information on road segment types, which is then used as an input to subsequent programs.
2. `optimal_k_task4.exe`: This is the main program and it outputs k^* . Double click on the `optimal_k_task4.exe`. Then, enter the values for the discretionary parameters asked on the screen and press enter after each entry. After entering each value, the program provides the k^* value and corresponding costs in dollars. A sample program screen is shown in Figure I.1 with default parameter settings.

```
C:\Documents and Settings\Elif Tokar Erdemir\Desktop\DESKTOP\Manpower SLC Project\Task 4\COD...
Inflation factor due to deadheading = 1.28

Percentage of yearly wages spent on plowing operations (alpha) = 9.2

Percent decrease in plowing times in subsequent passes = 10

k_star = 25
Total cost = 204108.03
Total extra plowing cost (OT + Delay) = 36668.03
Total regular wages spent on plowing = 167440.00
Press any key to continue . . . _
```

Figure I.1 Sample program screen of optimal_k_task4.exe

3. `graph1.exe`: This program provides graphical representation of total costs, extra plowing costs due to delay and OT, and yearly wages spent on plowing operations with respect to k . It also helps SLC to visualize the robustness of costs around k^* . To run the program, double click on `graph1.exe`. The graphic screen also allows the user to zoom in and out.
4. `yearly_costs_kstar.exe`: This program takes 11 years of storm data including the base year of 2006 and outputs expected extra plowing costs for each year when there are k^* employees in the workforce. To run the program, double click on `yearly_costs_kstar.exe`. Then enter k^* value and press enter. Then the results will be displayed in the screen. A sample program screen is shown in Figure I.2 for k^* value of Figure I.2.


```
C:\Documents and Settings\Elif Tokar Erdemir\Desktop\DESKTOP\Manpower SLC Project\Task 4\COD...
k_star = 25

min_cost_kstar[year] = 11 = 53552.229954
min_cost_kstar[year] = 21 = 37223.191949
min_cost_kstar[year] = 31 = 38474.538876
min_cost_kstar[year] = 41 = 41108.630280
min_cost_kstar[year] = 51 = 31793.470187
min_cost_kstar[year] = 61 = 36443.389061
min_cost_kstar[year] = 71 = 35856.962510
min_cost_kstar[year] = 81 = 52473.433269
min_cost_kstar[year] = 91 = 41589.491499
min_cost_kstar[year] = 101 = 37431.170956
min_cost_kstar[year] = 111 = 36668.029210
Press any key to continue . . . _
```

Figure I.2 Sample program screen of yearly_costs_kstar.exe

5. graph3.exe: This program provides graphical representation of extra plowing costs as outputted by yearly_costs_kstar.exe. The graph is in the form of a box plot. To run the program, double click on graph3.exe. The graphic screen also allows the user to zoom in and out.

Appendix J Analytical Results for Simplifications of Calculations of Workforce Deployment

J.1 Formulation of Workforce Requirement Problem

Notation:

1. $j = 1 \dots n$ road segments.
2. w_j = total work content of road segments of type j . Work content is measured in the amount of time it would take a single employee to complete plowing road segments of type j .
3. Delay costs are $y_j = y_j(0)$ per unit time when none of type- j road segments are plowed and decrease smoothly during plowing operations until all plowing done.
4. Suppose that road segments of type j are plowed in m different modes, where in mode i , there are k_i employees working on this type of road segments.
5. The length of the mode- i plowing interval is t_i . We also define $t_0 = 0$.
6. $w_j(i)$ = remaining work content at the start of mode i . Clearly, $w_j(i) = w_j - \sum_{l=1}^{i-1} k_l t_l$. We can calculate $w_j(i)$ for each i before calculating delay costs.
7. $r_i(k) = \frac{w_j(i)}{k}$ = total remaining plowing time if $k \geq 1$ employees were to work continuously on the remaining work content from the start of mode i .

Consider the interval $[0, t_1]$ with k_1 employees. During this time, the delay cost rate will decrease from $y_j(0)$ to $y_j(1)$ where $y_j(1)$ can be determined from the following triangular inequality

$$\frac{y_j(0)}{r_1(k_1)} = \frac{(y_j(0) - y_j(1))}{t_1}$$

This simplifies to yield

$$y_j(1) = y_j(0) * \left(\frac{r_1(k_1) - t_1}{r_1(k_1)} \right).$$

From here, we know that the delay costs incurred in $[0, t_1]$ are

$$\begin{aligned} C_d(1) &= t_1 y_j(1) + \left(\frac{1}{2} \right) (t_1) (y_j(0) - y_j(1)) \\ &= 0.5 t_1 [y_j(0) + y_j(1)] \\ &= 0.5 t_1 y_j(0) [2r_1(k_1) - t_1] / r_1(k_1) \end{aligned}$$

Next, consider plowing mode 2 which goes from t_1 to $(t_1 + t_2)$ and during which time there are k_2 employees plowing. The completion time if these k_2 employees were to continue until all road segments of type- j are plowed is

$$r_2(k_2) = \frac{w_j(1)}{k_2} = \frac{w_j - k_1 t_1}{k_2}.$$

The delay cost rate now reduces from $y_j(1)$ to $y_j(2)$, where $y_j(2)$ is determined as follows.

$$y_j(2) = y_j(1) \left(\frac{r_2(k_2) - t_2}{r_2} \right).$$

From here, it is straight forward to see that the delay costs in the time interval $[t_1, t_1 + t_2]$ are

$$\begin{aligned} C_d(2) &= 0.5t_2y_j(1)[2r_2(k_2) - t_2] \\ &= 0.5t_2y_j(0) \left(\frac{r_1(k_1) - t_1}{r_1(k_1)} \right) \left(\frac{2r_2(k_2) - t_2}{r_2(k_2)} \right). \end{aligned}$$

Continuing in this fashion, we can recursively calculate delay costs in each plowing mode. In fact, it is possible to write out an expression for the $l - th$ mode, where $l < m$, as follows.

$$C_d(l) = 0.5 t_l y_j(0) \left[\prod_{p=1}^{l-1} \left(\frac{r_p(k_p) - t_p}{r_p(k_p)} \right) \right] \left(\frac{2r_l(k_l) - t_l}{r_l(k_l)} \right)$$

We have to be careful in the last period during which the delay cost rate reduces to zero. In that case, the delay cost expression is as follows.

$$\begin{aligned} C_d(m) &= \left(\frac{1}{2} \right) (t_m) y_j(m - 1) \\ &= 0.5t_m y_j(0) \left[\prod_{p=1}^{m-1} \left(\frac{r_p(k_p) - t_p}{r_p(k_p)} \right) \right]. \end{aligned}$$

Putting it all together, we have the total delay costs as $TD = \sum_{l=1}^m C_d(l)$. The total delay cost can be calculated by the following expression:

$$TD = \sum_{l=1}^{m-1} 0.5t_l y_j(0) \left[\prod_{p=1}^{l-1} \left(\frac{r_p(k_p) - t_p}{r_p(k_p)} \right) \right] \left(\frac{2r_l(k_l) - t_l}{r_l(k_l)} \right) + 0.5t_m y_j(0) \left[\prod_{p=1}^{m-1} \left(\frac{r_p(k_p) - t_p}{r_p(k_p)} \right) \right]$$

Note that for any given sequence of k_i 's and t_i 's, we can calculate the above expression easily by writing a simple subroutine to do so. In actual implementation, we will know k_i 's because for any given amount of work done in overtime, we will aggressively schedule the maximum number of employees in the overtime mode so long as we do not incur double overtime pay.

If it were necessary to schedule a hiatus at the end of mode l (say), then all we have to do is to add cost $y_l(l)$ times the length of the hiatus to the above expression. The hiatus will take care of the delays during the period when plowing is shut down.

J.2 Theorem: If a storm ends between 7:30 AM and 11:30 AM, then aggressive strategy of calling the evening employees immediately is better than a timid strategy in which we don't call them immediately and use them after their shift.

Proof: We compare the two strategies when SLC commits to using the same amount of OT effort in both cases. Let the overtime work SLC commits to do in both the cases is W_{OT} . We can complete this work in many combinations.

In the aggressive strategy, SLC's effort reduces incomplete plowing work at rate W_{OT}/K_e in the morning period, either for the entire duration until evening shift comes on (11:30 AM), or for a period of time that allows us to expend all effort. All this effort helps to plow the highest delay cost road segments. If all of W_{OT} is not spent until 11:30 AM then additional overtime work is done after 3:30 PM and the rate of work is W_{OT}/k_m at that point in time. If W_{OT} is still not completed at 7:30 PM, then the additional work is done by utilizing some number of employees, and noting that some of these employees will be cheaper if they have not already worked 4 hours. This may also involve a break until the next day morning to complete plowing.

In the timid strategy, the same W_{OT} is expended starting 3:30 PM. This can be expended at rate W_{OT}/K_m until 7:30 and at rate W_{OT}/k thereafter. Now, all of the morning employees (if asked to work overtime) will be paid double pay but evening employees will be paid 1.5 times regular pay. As before, we can also break until the next day morning to do some of the plowing. However, given the same W_{OT} , k_e and k_m , for each deployment strategy in the timid approach, there is a corresponding deployment strategy in the aggressive approach that incurs the same total overtime cost. The issue is simply one of when the OT cost is incurred. We can change the number of employees called to match the total OT cost in the two approaches. It is worth noting that when we are paying 1.5 times regular wages, it is best in both strategies to use all available employees, until the limit W_{OT} , to reduce delay costs faster. Given that we can match the OT costs, it is straightforward to see that aggressive strategy is better because it plows higher costing road segments earlier. #

Above, we show that for each choice of W_{OT} effort, the aggressive strategy is better. This argument therefore also extends when we use optimal W_{OT} under each strategy. Finally, a similar argument can be used to show that any strategy that lies somewhere between aggressive and timid cannot be better than aggressive. Therefore, we need to pay attention to the aggressive strategy only.

Appendix K The K-star Algorithm

K.1 Contract Workers

$\xi \in$ storm realizations

if ($\min_k \geq \min_regular_employees$)

$k = \min_k$;

else

$k = \min_regular_employees$;

$k \in \{\min k, \dots, k_{\max}\}$

$k_c \in \{0, \dots, k_{\max} - k\}$

$\min C(\xi, k) = \infty$

$K_{OT} \in \{0, \dots, k + k_c\}$

Calculate $\min W_{OT}$

$t_{start} \in \{t_0 \cup [t_s - 4 \text{ hrs}, t_s - \min W_{OT}]\}$

If $t_0 > t_s - \min W_{OT}$, break (i.e. there is no feasible t_{start})

If $t_{start} \in [t_s - 4 \text{ hrs}, t_s]$, then $W_{OT} = t_s - t_{start}$

Compute cost $C(\xi, k, k_{OT}, t_{start}, W_{OT})$

If $C(\xi, k, k_{OT}, t_{start}, W_{OT}) < \min C(\xi, k)$

$\min C(\xi, k) = C(\xi, k, k_{OT}, t_{start}, W_{OT})$

If $t_{start} \in (t_e, t_s - 4 \text{ hrs})$,

$W_{OT} \in [\max\{2\text{hrs } 40\text{mn}, \min W_{OT}\}, t_s - t_{start}]$ (utilize convexity)

Compute cost $C(\xi, k, k_{OT}, t_{start}, W_{OT})$

If $C(\xi, k, k_{OT}, t_{start}, W_{OT}) < \min C(\xi, k)$

$\min C(\xi, k) = C(\xi, k, k_{OT}, t_{start}, W_{OT})$

If $t_{start} = t_e$

$W_{OT} \in [\min W_{OT}, t_s - t_{start}]$ (utilize convexity)

Compute cost $C(\xi, k, k_{OT}, t_{start}, W_{OT})$

If $C(\xi, k, k_{OT}, t_{start}, W_{OT}) < \min C(\xi, k)$

$\min C(\xi, k) = C(\xi, k, k_{OT}, t_{start}, W_{OT})$

$C(\xi, k) = \min C(\xi, k)$

$(k^*, k_c^*) = \arg \min_k \{E_\xi[C(\xi, k)] + \alpha(k) \times W \times k\}$

K.2 Split Shifts

$\xi \in$ storm realizations

$$k \in \{\min k, \dots, k_{\max}\}$$

$$\min C(\xi, k) = \infty$$

$$k_{OT} \in \{1, \dots, k\}$$

Calculate $\min W_{OT}$

$$t_{start} \in \{t_0 \cup [t_s - 4 \text{ hrs}, t_s - \min W_{OT}]\}$$

If $t_0 > t_s - \min W_{OT}$, break (i.e. there is no feasible t_{start})

If $t_{start} \in [t_s - 4 \text{ hrs}, t_s]$, then $W_{OT} = t_s - t_{start}$

 Compute cost $C(\xi, k, k_{OT}, t_{start}, W_{OT})$

 If $C(\xi, k, k_{OT}, t_{start}, W_{OT}) < \min C(\xi, k)$

$\min C(\xi, k) = C(\xi, k, k_{OT}, t_{start}, W_{OT})$

If $t_{start} \in (t_e, t_s - 4 \text{ hrs})$,

$W_{OT} \in [\max\{2\text{hrs } 40\text{mn}, \min W_{OT}\}, t_s - t_{start}]$ (utilize convexity)

 Compute cost $C(\xi, k, k_{OT}, t_{start}, W_{OT})$

 If $C(\xi, k, k_{OT}, t_{start}, W_{OT}) < \min C(\xi, k)$

$\min C(\xi, k) = C(\xi, k, k_{OT}, t_{start}, W_{OT})$

If $t_{start} = t_e$

$W_{OT} \in [\min W_{OT}, t_s - t_{start}]$ (utilize convexity)

 Compute cost $C(\xi, k, k_{OT}, t_{start}, W_{OT})$

 If $C(\xi, k, k_{OT}, t_{start}, W_{OT}) < \min C(\xi, k)$

$\min C(\xi, k) = C(\xi, k, k_{OT}, t_{start}, W_{OT})$

$$C(\xi, k) = \min C(\xi, k)$$

$$k^* = \arg \min_k \{E_\xi[C(\xi, k)] + \alpha(k) \times W \times k\}$$

K.3 Staggered Shifts

$\xi \in$ storm realizations

$$k_m \in \left\{ \frac{k_{min}}{2} + 1, \dots, k_{max} \right\}$$

$$k_e \in \left\{ \frac{k_{min}}{2} + 1, \dots, k_{max} - k_m \right\}$$

Calculate $\min W_{OT}$

$$W_{OT1} = 16 * (k_m + k_e);$$

$$W_{OT2} = 0;$$

If $W_{OT1} > \min W_{OT}$ then do

$$W_{OT1} = \min W_{OT};$$

$$W_{OT2} = 0;$$

If $t_0 \in [0, 16]$ then

Assign OT_start and OT_end times to k_m & k_e

Delay cost ($k_m, k_e, OT_start, OT_end, t_start$)

If $C(\xi, k_m, k_e, t_start, W_{OT}) < \min C(\xi, k_m, k_e)$

$$\min C(\xi, k_m, k_e) = C(\xi, k_e, k_e, t_start, W_{OT})$$

If $t_0 \in [16, 32]$ then

Assign OT_start and OT_end times to k_m & k_e

Delay cost ($k_m, k_e, OT_start, OT_end, t_start$)

If $C(\xi, k_m, k_e, t_start, W_{OT}) < \min C(\xi, k_m, k_e)$

$$\min C(\xi, k_m, k_e) = C(\xi, k_e, k_e, t_start, W_{OT})$$

If $t_0 \in [32, 48]$ then

Assign OT_start and OT_end times to k_m & k_e

Delay cost ($k_m, k_e, OT_start, OT_end, t_start$)

If $C(\xi, k_m, k_e, t_start, W_{OT}) < \min C(\xi, k_m, k_e)$

$$\min C(\xi, k_m, k_e) = C(\xi, k_e, k_e, t_start, W_{OT})$$

If $t_0 > 48$ then

$$t_{start} \in \{t_0 \cup [t_s - 4 \text{ hrs}, t_s - \min W_{OT}]\}$$

If $t_0 > t_s - \min W_{OT}$, break (i.e. there is no feasible t_{start})

If $t_{start} \in [t_s - 4 \text{ hrs}, t_s]$, then $W_{OT} = t_s - t_{start}$

Delay cost ($k_m, k_e, OT_start, OT_end, t_{start}$)

If $C(\xi, k_m, k_e, t_{start}, W_{OT}) < \min C(\xi, k_m, k_e)$

$$\min C(\xi, k_m, k_e) = C(\xi, k_e, k_e, t_{start}, W_{OT})$$

If $t_{start} \in (t_e, t_s - 4 \text{ hrs})$,

$$W_{OT} \in [\max\{2\text{hrs } 40\text{mn}, \min W_{OT}\}, t_s - t_{start}]$$

Delay cost ($k_m, k_e, OT_start, OT_end, t_{start}$)

If $C(\xi, k_m, k_e, t_{start}, W_{OT}) < \min C(\xi, k_m, k_e)$

$$\min C(\xi, k_m, k_e) = C(\xi, k_e, k_e, t_{start}, W_{OT})$$

If $t_{start} = t_e$

$$W_{OT} \in [\min W_{OT}, t_s - t_{start}] \text{ (utilize convexity)}$$

Delay cost ($k_m, k_e, OT_start, OT_end, t_{start}$)

If $C(\xi, k_m, k_e, t_{start}, W_{OT}) < \min C(\xi, k_m, k_e)$

$$\min C(\xi, k_m, k_e) = C(\xi, k_e, k_e, t_{start}, W_{OT})$$

$$(k_m^*, k_e^*) = \arg \min_k \{E_\xi[C(\xi, k)] + \alpha(k) \times W \times k\}$$