Using infrared observations of L1544, a dark cloud in the Taurus Molecular Complex, density maps were generated and the distribution was compared to that of a polytopic gas. A polytopic index of 3.5 was found to best fit the observed density distribution.

Introduction:

In certain areas of the sky, there are no visible stars at all. This is not because there are no stars there, but rather that the light from those stars is “blocked” by interstellar clouds of dust. As light from distant stars propagates through the interstellar medium, it interacts with the interstellar dust that inhabits the space between stars. This dust scatters and absorbs the light, diminishing the total flux that we observe here at Earth. These scattering and absorption effects, however, have efficiencies that vary with wavelength. Both processes are more efficient at diminishing light of shorter wavelengths. Because of this, light from stars that travels through regions of space that have high concentrations of this dust will appear “redder” to us than they are intrinsically, because the “bluer” light at shorter wavelengths has been reduced drastically, while the “redder” light has been reduced less. In the case of the dark clouds studied in this project, almost all of the visual light has been scattered or absorbed by blocking material. Some stars remain observable in the infrared, however.

In order to study the “color” of a star, we use the photometric system. This system uses filters to study a certain portion of the spectrum of light received at Earth from these distant sources. The most commonly used photometric filters are the U “ultraviolet,” B “blue,” and V “visual” bands. Instead, we use the J, H and K filters because we are studying infrared wavelengths. These wavelengths are able to pierce through the dark clouds better than shorter wavelengths, and as such we can use them to study the dark clouds that do not allow other wavelengths of light to pass through to reach us. By using a system of photometric filters in the infrared portion of the electromagnetic spectrum and comparing
observed values of the magnitudes of stars at these wavelengths, we can tell which stars have experienced interstellar reddening and thus study the dark clouds and their mass distributions.

Theory:

If there is $I_0$ intensity of light incident onto a cylindrical volume that contains some amount of blocking material, it will be decreased by scattering and absorption. This decrease is proportional to the relative “blocking area,” $dA$, of the material, which depends on the wavelength of the light, geometry of the dust, and other factors. The change in light is negative because light is being removed. In this project, the additional intensity due to emission of the dust is ignored.

$$\frac{dI}{I} = -\frac{dA}{A} \quad (1)$$

Figure 1: Depiction of how incident light is ‘blocked’ by scattering and absorption due to dust and gas. Each particle creates an effective cross-section that removes some portion of the incident light (Original Figure)

Figure 1 above shows incident intensity $I_0$ being decreased by some blocking material. The blocking area can be given by $dA = N \star \sigma = V \star n \star \sigma = A \star dl \star n \star \sigma$ where $N$ is the total number of blocking particles, $\sigma$ is the cross section for scattering and absorption, $V$ is the total volume, $n$ is the number
density of the dust particles, A is the cross-sectional area, and \( dl \) is the length of the cylinder.

Substituting this into equation 1 gives us:

\[
\frac{dl}{I} = -\frac{A*dl*n*\sigma}{A}
\]  

which with a few more steps of algebra and then integration gives us:

\[
\frac{dl}{I} = -n * \sigma * dl
\]

\[
\frac{dl}{dl} = -n * \sigma * I
\]

\[
I = I_0 e^{-nsl}
\]  

(3)

This expression gives the intensity of light remaining after it has propagated through a volume of length \( l \) containing material of constant \( n \) and \( \sigma \). In general, these will be functions of position and unknowns. However, \( n*l \) is a column density, or the total amount of material per area on the sky between us and the obscured object and \( \sigma \) can be theoretically obtained.

Due to the effects of scattering and absorption being greater at shorter wavelengths, it is useful to study the “colors” of stars. A color is a difference between two magnitudes taken in different filter bands. Since you cannot observe dark clouds in the visible, we used three infrared filterbands, J, H, and K. The J band was the shortest wavelength band at 1.25\( \mu m \), while H and K were at 1.65\( \mu m \) and 2.15\( \mu m \) respectively (1). Plotting two of these colors against each other (namely J-H and H-K) gives us what is creatively called a “color-color” diagram. A star will have a position on this diagram depending on its intrinsic colors, however the colors of stars will change if they experience reddening. Reddening will decrease the intensity in J more than in H, which in turn will be decreased more than in K. This then causes the corresponding magnitudes to increase, since a decrease in intensity is an increase in magnitude, as positive magnitudes are less bright than negative magnitudes. The magnitude in J will increase most, since it has the longest wavelength and thus light detected by this filter is more
efficiently scattered or absorbed by dust and gas. The number of magnitudes of increase in a particular filter band is known as the total extinction in that band, or $A(\lambda)$ where $\lambda$ is the wavelength of the band.

When plotting colors, this effect creates a “reddening line” along which stars move as they experience more and more reddening due to interstellar material. A J-H, H-K plot is shown below in figure 2, along with reddening lines with slopes of $1.7 \frac{J-H}{H-K}$, a value which comes from known absolute extinction laws, $\frac{A(\lambda)}{A(V)}$, which compare extinction at one wavelength to the extinction in the visual range (2).

Figure 2: A J-H, H-K color-color diagram for L1544. Data source: University of Montréal (Original Figure)

If we know where a star should be intrinsically, with no reddening, on this color-color diagram, its observed colors can tell us about how much extinction has taken place. The difference between the
observed color of a star and its intrinsic color is called its color excess. This excess is related to the total extinction present by:

\[ E_{H-K} = A_H - A_K = \left( \frac{A_H}{A_K} - 1 \right) A_K = \left( \frac{0.175}{0.112} - 1 \right) A_K = 0.563 A_K \] (4)

where \( E_{H-K} \) is the excess in H-K, \( A_H \) and \( A_K \) are total extinctions in H and K respectively, and the values come from the absolute extinction law (2). Since for our purposes, the observable is the excess, we arrange equation 4 like this:

\[ A_K = 1.776 E_{H-K} \] (5)

A relation between the total extinction in a filter band and the two dimensional projected column density of the blocking molecular and gaseous hydrogen is known to be

\[ N_H = 1.9 \times 10^{25} \frac{\text{Hydrogen atoms}}{\text{m}^2 \cdot \text{mag}} \times A_V \] (3)

Which after applying absolute extinction values and equation 5 becomes

\[ N_H = 3.01 \times 10^{26} \frac{\text{Hydrogen atoms}}{\text{m}^2 \cdot \text{mag}} \times E_{H-K} \] (6)

This allows us to find a surface density from observations.

Once we have an observed surface density profile, we can compare this with a theoretical model of the dust cloud. We might expect the cloud to obey a polytrope model in hydrostatic equilibrium. This means that pressure is proportional to the density to some power.

\[ P \propto \rho^{\frac{n+1}{n}} \] (7)

where \( P \) is the pressure, \( \rho \) is the density, and \( n \) is known as the polytropic index. Starting from equation 7 and Hydrostatic equilibrium:

\[ \frac{\partial P}{\partial r} = -\frac{GM}{r^2} \rho \] (8)

we can derive the Lane-Emden equation

\[ \frac{1}{\xi^2} \frac{\partial}{\partial \xi} \left( \xi^2 \frac{\partial \theta}{\partial \xi} \right) = -\theta^n \] (9)
where $\xi$ is a scaled radius and $\theta(\xi)$ is a solution such that $\rho(\xi) = \rho_c \theta^n$, where $\rho_c$ is the central density. Boundary conditions are assumed to be $\theta(0) \equiv 1$ and $\frac{\partial \theta}{\partial \xi} (\xi = 0) \equiv 0$, so that the density continuously increases to the central density at $\xi=0$. By projecting the resulting density distribution into two dimensions for different values of the polytropic index, we can find the polytope model that best fits our observations. This will give us a good model of the structure of the dark cloud.

**Methods and Results:**

All data used in this project was provided by Professor Pierre Bastien from the Observatoire du Mont-Mégantic at the University of Montréal. Observations were taken in the infrared filters J, H, and K. Information about the instruments is provided in table 1 below.

<table>
<thead>
<tr>
<th>Basic instrument characteristics</th>
<th>Available filters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field of view (OHMS)</td>
<td>50x50</td>
</tr>
<tr>
<td>Pixel scale (OHMS)</td>
<td>0.45&quot; (pixel)</td>
</tr>
<tr>
<td>Detector</td>
<td>2048 x 2048 pixels, Hawaii II</td>
</tr>
<tr>
<td>Minimum integration time</td>
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<tr>
<td>Integration time is always a multiple of 1.15 s</td>
<td>3 s</td>
</tr>
<tr>
<td>Overhead per exposure</td>
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<tr>
<td>Overhead per dither (35&quot;)</td>
<td>5-10 s</td>
</tr>
<tr>
<td>Overhead for dither=35&quot;</td>
<td>20-45 s</td>
</tr>
<tr>
<td>Median Full Width at Half-Maximum (FWHM) at the OHMS</td>
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</tr>
<tr>
<td>Readout noise</td>
<td>10 electrons</td>
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<tr>
<td>Linear write</td>
<td>~30 000 ADU/second</td>
</tr>
<tr>
<td>Gain</td>
<td>~2.5 e-ADU/ADU</td>
</tr>
<tr>
<td>Transmission</td>
<td>30-25%</td>
</tr>
</tbody>
</table>

**Table 1: Information on the instruments used at the Observatoire du Mont-Mégantic in Montréal (1)**

Data was provided in large excel spreadsheets with information about the position (in right ascension and declination) and J, H, and K magnitudes and their uncertainties for thousands of stars observed in multiple dark clouds. The cloud studied in this project was L1544 in the Taurus Molecular Complex, approximately 140 parsecs away from Earth (4). The data set originally contained 4756 objects. Data with uncertainties in the magnitude measurement larger than 0.25 magnitudes were removed leaving 2846 objects studied. These objects were then plotted on a J-H, H-K color-color diagram, shown in figure 2. The theoretical reddening line was also plotted.
From Lee 1970 (5) and Frogel 1978 (6), values for the intrinsic colors of different types of main sequence and giant stars were obtained. An average value for H-K color was assumed to be 0.15, since we do not know what star was which type. This introduces a source of uncertainty to all further calculations. Assuming this intrinsic value of H-K, values for the H-K color excess were found and plotted with their spatial coordinates. This created a two dimensional density distribution, since the excess is directly proportional to the 2D column density. Since these quantities are proportional, they are used fairly interchangeably in this report. In the very center of the plotted density distribution, there were no objects, because of the limiting magnitude of those observations. Another data set of objects taken at a larger limiting magnitude (dimmer objects) from the center of the cloud was then added, adding an additional 74 objects. These objects only had H and K observations, so they were not plotted on the J-H, H-K color-color plot, but they could be added to the density distribution plot. One issue remained, however: the new objects did not have right ascension and declination positions associated with them. Rather, they had x and y coordinates on the viewing image. This required the exact position of the viewing image in the sky to be located. Using fits files and SAOImage DS9, the brightest stars in the image were matched to their sky locations from the fits files, as shown in figure 3 below.

Figure 3: Locating the central core region of L1544. The three bright stars were used to locate the area. The vertical and horizontal directions are both reversed on the image given on the left, while the SAOImage DS9 plot is given on the right.
Thus the right ascension and declination of the objects in the image were obtained. With this new information, the objects near the central core of the cloud were added to the 2D column density map.

The final map is shown in figure 4 below.

![2D column density map](image)

**Figure 4:** 2 dimensional plot of the excesses, which are in turn proportional to the column density. (Original Figure)

From this plot, contours of constant excess (or column density) were drawn by hand. These contours are shown in figure 5 below and inside each contour the excess (density) was assumed some constant average value of the excesses (densities) contained.
Then by simple numerical integration (rectangle rule style) the total mass of the cloud was found, given a known distance of 140 parsec to the cloud to convert an angular size to a total area. The total mass was found to be $47.3M_\odot$. This was compared to a paper that found the mass of L1544 by CO spectra and emission temperatures to be about $28M_\odot$. (7) Because of differences of techniques and difficulties in deciding exactly where to stop integration, these values are not taken to be in disagreement.

To find a good three dimensional model for the structure of the cloud, a polytrope model was used. This involved assuming spherical symmetry. Using a numerical differential equation solver in Matlab, solutions for the Lane-Emden equation were generated for different values of the polytropic index. Then, the resulting 3D density distributions were then projected into 2D using a program which integrated through the cloud numerically and summed up the total mass in each column along the line...
of sight. This effectively removed one dimension and projected the 3D distribution into 2 dimensions. The resulting 2D plot was compared to the observed 2D density distribution in figures 4 and 5. The observed plots had to be azimuthally averaged in order to be compared, since they were not symmetric. This meant finding the distance from the center to each contour line at a given angle from the vertical. These values were averaged for many angles, creating an average value to compare to the generated solutions. A few polytrope solutions are plotted against the azimuthal average in figure 6 below.

![Figure 6: Comparison of observed and azimuthally averaged density with projected polytropic distributions. The observation fits best with a n=3.5 polytrope, however it looks closer to 4 at small radii and closer to 3 at larger radii.](image)

As shown in figure 6, the distribution best matches a polytropic index of $n = 3.5 \pm 0.5$. This is more centrally condensed than an ideal gas, which would have a lower polytopic index.

**Conclusion:**

Understanding the structure of dark clouds can help us to better understand how the very early stages of stellar formation might occur. This project concluded in the calculation of a polytropic index for
L1544 in the Taurus Molecular Complex. This value was $n = 3.5 \pm 0.5$. This gives us an idea of the structure of the cloud and helps us to understand what is happening inside.

References


