

General Oligopolistic Equilibrium, Trade
Liberalization and Competition Policy

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Dedication

To Xi Chen, for her unending love and support

Abstract

This thesis investigates the relationship between trade liberalization and competition level. The conventional wisdom tells us that with the removal or reduction of trade barriers, domestic firms will face more competition from outside and their market power will decrease. However, empirical evidence does not always support this argument. In chapter 2, based on Roeger (1995) and using U.S. manufacturing industries data from 1958 to 1996, I show that trade liberalization does not always decrease price cost margin, which is an indicator of competition level, for all 4-digit US manufacturing industries. Around one third 4-digit manufacturing industries in US have increasing price cost margins after trade liberalization.

Unlike traditional studies on competition level from partial equilibrium models, in chapter 3, this thesis builds a general oligopolistic equilibrium model with heterogeneous sectors, homogeneous firms and free entry based on Neary (2009) to explain why trade liberalization does not always bring more competition. In the model, a decrease in competition level shifts income from factor owners to recipients of profits, reallocates factors from low cost sectors to high cost sectors and decreases countries' welfare. In an open economy with trade liberalization, the previous findings in a closed economy will give countries incentives to adopt beggar-thy-neighbor competition policies, which decrease competition level in order to maximize its own welfare while sacrificing its trade partner's welfare. In an extreme case, both countries engaged in trade have incentives to decrease competition level and create a Prisoner's Dilemma. The above results also extend Neary (2003)'s analysis about competitive advantage and show that competition could be a disadvantage. In the end, this thesis further brings heterogeneous firms into

the model in chapter 4. Following firms' entry and exit strategy from Hopenhayn (1992), I show that previous findings are still valid under certain conditions when firms are heterogeneous.

Two general policy recommendations stand out from this thesis. First, lowering competition level may bring trade advantage and improves welfare for a country during trade liberalization; Second, an international agreement on competition policy could possibly increase both world welfare and trade.

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Chapter 1

Introduction

Does trade liberalization have any pro-competitive effect? The conventional wisdom tells us that with the removal or reduction of trade barriers, domestic firms will face more competition from outside and their market power will decrease. Tybout (2001) argues that, under the assumption of static profit maximization, the price set by a firm operating in an imperfectly competitive market as a ratio to marginal cost is a decreasing function of the elasticity of demand. As trade barriers are removed or reduced, the elasticity of demand will increase because of increased availability of imported goods and it will lead to an decrease in firms' price-cost margin, which is defined as the difference between price and marginal cost divided by price (also called Lerner Index) and indicates market power or competition level. Trade liberalization also makes cooperative behavior among firms less sustainable since trade liberalization changes the pay-off to defecting, or changes firms' ability to punish defectors or makes defection hard to detect, all of which also leads to more competition.

Although the theoretical explanations appear solid, the empirical evidence is

mixed. Some literature shows evidence against pro-competitive effect of trade liberalization, e.g. Domowitz, Hubbard and Petersen (1986), Konings, Van Cayseele and Warzynski (2002), Boulhol, Dobbelaere and Maioli (2011). The interaction between trade liberalization and competition policy, which affects the competition level in a country, has also been an important issue in the WTO. In 1996, as a result of the Ministerial Conference in Singapore, the WTO established a working group to study how domestic and international competition policy instruments such as antitrust or competition laws interact with international trade. Although, in July 2004, the General Council of the WTO decided that the interaction between trade and competition policy would no longer be discussed during the Doha round, the WTO working group left lots of research documents, and some of them also show evidence related to higher price-cost margins following trade liberalization in some sectors for some countries.

The aim of this thesis is to explain why trade liberalization does not always bring more competition. Different from most of traditional studies on competition policy from industry organization theory and partial equilibrium models, this thesis build a general equilibrium model, in which factor price is endogenous, to explain the relationship between trade liberalization and competition level in an economy. As stated in Horn and Francois (2006), partial equilibrium "may be appropriate when the concern is with a regulatory problem in a particular sector. But when the focus is on the establishment of a general competition policy stance, such as when formulating merger guidelines, it is clearly much less adequate." Besides that, the interaction between good markets and factor markets can be studied in a general equilibrium model. A regulatory problem in a particular sector will have cross-sector effects through factor markets, and it will bring some interesting insight into competition policy, that can not be found

in partial equilibrium models. The model in this thesis is a general oligopolistic equilibrium model with heterogeneous sectors, based on Neary (2009). Instead of allowing an arbitrary and identical number of firms in each sector as Neary (2009), this thesis assumes that the number of firms in each sector is determined by government competition policy. In the model, a decrease in competition level will not only shift income from factor owners to recipients of profits but also shift factors from low cost sectors to high cost sectors, which will also decrease countries' welfare in a closed economy. However, in an open economy with trade liberalization, the previous findings in a closed economy will lead countries to decrease their competition levels under some conditions in order to maximize their welfare. In an extreme case, both countries engaged in trade have incentives to decrease their competition level and create a Prisoner's Dilemma. This thesis also shows that under some other conditions, it is possible for countries engaged in trade to increase their competition level. This is consistent with mixed empirical evidence for pro-competitive effect of trade liberalization. This thesis also extends Neary (2003)'s analysis on "competitive advantage". Neary (2003) shows that the competition level affects countries' resource allocation, trade patterns and trade volumes but says nothing about welfare. In this thesis, I show that competition does not always bring advantage and increase countries' welfare. Its effect depends on trade partners and it is possible to have "competitive disadvantage" during trade liberalization.

Two general policy recommendations stand out from this thesis. First, under some conditions, lowering the industry competition level brings trade advantage and improves welfare for the whole economy. This may be especially meaningful for developing countries that lack advanced technology and need capital accumulation but face a more and more open world economy. However, it is not easy to

implement this strategy successfully and the key is income redistribution since this policy sacrifices workers' welfare while increasing country's total welfare. Countries should use redistribution tools to reimburse workers or invest gains from trade in public goods such education or infrastructure¹ . Otherwise, it will make the rich richer, the poor poorer, and the policy will be unsustainable. Second, an international agreement on competition policy could possibly increase both world welfare and trade. Putting competition policy into the framework of WTO may be a good choice.

This thesis makes contact with the following literature, which is related to both oligopolistic general equilibrium and competition policy. Konishi et al. (1990) studies a closed economy with two sectors producing two goods — one sector is perfectly competitive and the other follows oligopolistic competition, and two factors within the framework of general equilibrium and generalizes the excess entry theorem. Michihiro(1999) develops a tractable general equilibrium model of an economy with an arbitrary number of industries under increasing returns to scale and imperfect competition and finds that antitrust policy in an industry always improves the welfare of a country in autarkic long-run equilibrium. Horn and Francois (2006) study an economy which is a little bit different from that of Konishi et al. (1990). They still have two sectors, but both sectors produce homogeneous goods. In an open economy, Horn and Francois (2006) finds that the impact of imperfect competition is to redistribute income from factor owners to recipients of profits but aggregate welfare is unaffected in their model. They also claim that an international agreement on antitrust policy could increase world welfare and trade. Neary (2009) considers an oligopolistic general equilibrium model with infinite sectors and homogeneous firms. However, Neary (2009) focuses

¹ An ideal example for this policy is South Korea after 1970s.

on the effect of competition policy on the trade pattern and terms of trade of a country. This thesis provides a more general framework than Konishi et al. (1990) and Horn and Francois (2006), and also extends Neary (2009) to a model with free entry. Instead of focusing on trade pattern and terms of trade, the present thesis treats competition policy as a trade strategy, and a country chooses its best trade strategy to maximize its welfare.

For the explanation about ambiguous pro-competitive effect of trade liberalization, this thesis is related to Melitz and Ottaviano (2008) and Raff and Wagner (2010). Melitz and Ottaviano (2008) shows that an anti-competitive effect is possible if the removal of trade barriers leads to entry of relatively inefficient firms abroad, thereby raising prices of foreign exporters. Different from the monopolistic competition model in Melitz and Ottaviano (2008), Raff and Wagner (2010) uses an oligopoly model with heterogeneous firms to examine how an industry adjusts to rising import competition. Their The model predicts that in the short run the least efficient firms in the industry become inactive, surviving firms face a fall in output, mark-ups and profits, and the average productivity of survivors increases. These pro-competitive effects of import penetration on the domestic industry disappear in the long run. Although using different models, these two papers both argue that ambiguous pro-competitive effect of trade liberalization is caused by intra-industry adjustment and firms' entry and exit. However, in this thesis, the ambiguous pro-competitive effect of trade liberalization is caused by inter-industry adjustment and trade partners' marginal cost distribution.

The thesis is organized as follows:

- Chapter 2 documents some empirical evidence about increasing price-cost margins after trade liberalization.

- Chapter 3 first establishes an oligopolistic competition model for a closed economy. This thesis not only considers an economy with constant return to scale technology but also increasing return to scale technology. Since competition policy can act as a mechanism of income distribution, this thesis also consider a simple economy with heterogeneous consumers. After deriving some interesting properties of competition policy, an open economy is studied which gives some insight about the relationship between trade liberalization and competition level. In the end of chapter 3, a simple quantitative example is solved for an open economy.
- Chapter 4 introduces heterogeneous firms into the model in chapter 3. I show that properties in chapter 3 are still valid under certain conditions when firms are heterogeneous.
- Chapter 5 presents a final discussion of the analyses presented in the thesis.

Chapter 2

Motivating Evidence

In this chapter, I first briefly discuss two measurements for the price-cost margin (PCM), which indicates market power or competition level. I then summarize literature that shows increasing PCM. In the end, using 459 4-digit US manufacturing industry as an example, I analyze the change of PCM for US manufacturing industries and properties of those industries with increasing PCM.

2.1 PCM measurement

There are two widely used techniques. First, PCM can be directly measured by its definition. For example, in Paolo Epifani and Gino Gancia (2011), using US manufacturing industry data, PCMs are computed as the value of shipments (adjusted for inventory change) less the cost of labor, capital, materials and energy, divided by the value of shipments. There are several shortcomings with this measure and the biggest one is that it does not exclude the effect of productivity change.

The second measurement uses the production approach to recovering PCM in

the IO literature. This approach is different from the most popular approach in empirical IO, which relies on demand estimation. It only requires production data and puts no restrictions on underlying consumer demand. This approach is first developed by Hall (1988) and then improved by Roeger (1995), Klette (1999) etc. The most recent work is done by De Loecker and Warzynski (2010). De Loecker (2011) presents a good survey of this approach. This thesis will use the method in Roeger (1995) to test PCM change in U.S. manufacturing industries.

2.2 Increasing PCM

Empirical evidence for the effect of trade liberalization on PCM is mixed.¹ Although, for example, Esposito and Esposito (1971), Conyon and Machin (1991), Katics and Petersen (1994) and Kee and Hoekman (2007) all show decreasing PCM after trade liberalization, there also exists literature that does not support the pro-competitive effect of trade liberalization by different data or estimation methods. Based on the definition of PCM, Domowitz, Hubbard and Petersen (1986) tests 265 four digit manufacturing sectors in United States from 1958 to 1981 and find that PCM increases with a sector-fixed effect model while with an OLS model, PCM increases for least concentrated sectors and decreases for the most concentrated. Using Hall's method, Konings, Van Cayseele and Warzynski (2002) tests 4,700 manufacturing firms in Belgium and Netherlands from 1992 to 1997 and find that the import penetration ratio positively influences the PCM in the Netherlands, meaning that imports do not discipline the industry. They explain it by loose competition laws in the Netherlands. Boulhol, Dobbelaere and Maioli (2011) applies Hall's method to 9,820 firms in UK from 1988 to 2003

¹ See Table A.1: Survey of the pro-competitive effect of trade liberalization for details.

and finds pro-competitive effects when imports come from developed countries while the effect is not significant when imports come from developing countries. Using disaggregated data for EU manufacturing over the period 1989 to 1999, Chen, Imbs and Scott (2009) finds short run evidence that trade openness exerts a competitive effect. However, in the long run, trade liberalization effects are more ambiguous and may even be anti-competitive. Raff and Wagner (2010) finds a similar pattern of PCM change as Chen, Imbs and Scott (2009) for German clothing industry. In their model, they shows that trade liberalization that leads to greater import penetration has pro-competitive effects in the short run due to the selection effect. However, in the long run, these pro-competitive effects disappear or are even reversed.

Studies from developing countries as well as developed countries report increasing PCM for some manufacturing sectors after trade liberalization. Thanks to widely available industry-level and firm level data, the Indian trade reform in the 1990s gives economists a good opportunity to study the effect of trade liberalization. Krishna and Mitra (1998) studies four Indian industries and in the post-reform period markup declines significantly in three of the four industries. However, applying Hall's methodology, Balakrishnan, Pushpangadan, and Suresh Babu (2002) uses data from the Prowess database of the CMIE for 3,596 Indian firms for the period 1988 to 1998 and finds that the trade reform reduces the markup in certain industries (e.g., rubber, plastic and petroleum products, machinery, and transport equipment), while it raises the markup in certain other industries (e.g., food products, chemicals, basic metals, metal products, and non-metallic metal products). Goldar and Aggarwal (2005) uses Indian manufacturing industry panel data for 137 three-digit industries for 1980 to 1998 and finds that the lowering of tariffs and the removal of quantitative restrictions on imports of

manufactures in the 1990s has a significant pro-competitive effect on Indian industries while in the post-reform period PCM increases in most industries and aggregate manufacturing. They argue that it reflected a weakening of industrial labor's bargaining power. Beyond India, Yalcin (2000) and Aldaba (2008) report increasing PCM in manufacturing industries in Turkey and Phillipine.

2.3 U.S. manufacturing industry

In this part, I will show the following two findings for US manufacturing industry:

- (1) Trade liberalization does not always decreases PCM, which is an indicator of competition level, in US manufacturing industry.
- (2) Sectors with increasing PCMs and high import penetration ratios in US manufacturing industry tend to be labor intensive.

US manufacturing industry data are available from the NBER-CES Manufacturing Industry Database. The database covers all 4-digit manufacturing industries (459 industries) yearly from 1958 to 1996. The method to calculate the price-cost margin is from Roeger (1995), which can control the effect of productivity change and is therefore better than the direct application of the price-cost margin definition.

In Roeger (1995), the price-cost margin can be estimated from the difference between primary and dual productivity measures. Assume the production function for firm i at time t is $Y_{it} = A_{it} \times F^i(K_{it}, N_{it}, M_{it})$, where Y is output, A is productivity, K is capital input, N is labor input and M is material input. By first order differentiation,

$$\frac{\Delta Y_{it}}{Y_{it}} = \frac{\Delta A_{it}}{A_{it}} + \frac{F_K^i(K_{it}, N_{it}, M_{it})}{F^i(K_{it}, N_{it}, M_{it})} \Delta K_{it} + \frac{F_N^i(K_{it}, N_{it}, M_{it})}{F^i(K_{it}, N_{it}, M_{it})} \Delta N_{it} + \frac{F_M^i(K_{it}, N_{it}, M_{it})}{F^i(K_{it}, N_{it}, M_{it})} \Delta M_{it}$$

Let $\frac{\Delta A_{it}}{A_{it}} = \vartheta_{it}$ and $\alpha_{Jit} = \frac{P_{Jit}J_{it}}{P_{it}Y_{it}}$, where $J = K, N, M$. Assuming constant return to scale and perfect competition, the above equation becomes

$$\frac{\Delta Y_{it}}{Y_{it}} = \alpha_{Kit} \frac{\Delta K_{it}}{K_{it}} + \alpha_{Nit} \frac{\Delta N_{it}}{N_{it}} + \alpha_{Mit} \frac{\Delta M_{it}}{M_{it}} + \vartheta_{it}$$

Under imperfect competition, let $\mu = \frac{P}{c}$, which is the ratio of price to marginal cost,

$$\frac{\Delta Y_{it}}{Y_{it}} = \mu_{it} [\alpha_{Kit} \frac{\Delta K_{it}}{K_{it}} + \alpha_{Nit} \frac{\Delta N_{it}}{N_{it}} + \alpha_{Mit} \frac{\Delta M_{it}}{M_{it}}] + \vartheta_{it}$$

Let the price-cost margin be $\beta_{it} = \frac{(P_{it}-c_{it})}{P_{it}} = 1 - \frac{1}{\mu_{it}}$ and substitute it into the above equation,

$$\begin{aligned} & \frac{\Delta Y_{it}}{Y_{it}} - \alpha_{Nit} \frac{\Delta N_{it}}{N_{it}} - \alpha_{Mit} \frac{\Delta M_{it}}{M_{it}} - (1 - \alpha_{Nit} - \alpha_{Mit}) \frac{\Delta K_{it}}{K_{it}} \\ &= \beta_{it} \left(\frac{\Delta Y_{it}}{Y_{it}} - \frac{\Delta K_{it}}{K_{it}} \right) + (1 - \beta_{it}) \vartheta_{it} \end{aligned} \quad (2.1)$$

Equation (2.1) is related not only to the price-cost margin β_{it} but also to the change of productivity ϑ_{it} . Assuming P_{Jit} is the input price where $J = K, N, M$ and P_{it} is the output price, the dual productivity measure gives the following equation (2.2),

$$\begin{aligned} & \alpha_{Nit} \frac{\Delta P_{Nit}}{P_{Nit}} + \alpha_{Mit} \frac{\Delta P_{Mit}}{P_{Mit}} + (1 - \alpha_{Nit} - \alpha_{Mit}) \frac{\Delta P_{Kit}}{P_{Kit}} - \frac{\Delta P_{it}}{P_{it}} \\ &= -\beta_{it} \left(\frac{\Delta P_{it}}{P_{it}} - \frac{\Delta P_{Kit}}{P_{Kit}} \right) + (1 - \beta_{it}) \vartheta_{it} \end{aligned} \quad (2.2)$$

Combining equation (2.1) and equation (2.2), the effect of productivity is eliminated:

$$\begin{aligned} & \frac{\Delta P_{it}Y_{it}}{P_{it}Y_{it}} - \alpha_{Nit} \frac{\Delta P_{Nit}N_{it}}{P_{Nit}N_{it}} - \alpha_{Mit} \frac{\Delta P_{Mit}M_{it}}{P_{Mit}M_{it}} - (1 - \alpha_{Nit} - \alpha_{Mit}) \frac{\Delta P_{Kit}K_{it}}{P_{Kit}K_{it}} \\ &= \beta_{it} \left[\frac{\Delta P_{it}Y_{it}}{P_{it}Y_{it}} - \frac{\Delta P_{Kit}K_{it}}{P_{Kit}K_{it}} \right] \end{aligned} \quad (2.3)$$

Denote the left hand side of equation (2.3) by Δy_{it} and let $[\frac{\Delta P_{it}Y_{it}}{P_{it}Y_{it}} - \frac{\Delta P_{Kit}K_{it}}{P_{Kit}K_{it}}] = \Delta x_{it}$. Since the data available is US manufacturing industry-level data, only the average price-cost margin for industries can be estimated using equation $\Delta y_{it} = \beta_i \Delta x_{it} + \epsilon_{it}$, where i is the index for industry now. Based on the NBER-CES Manufacturing Industry Database, $P_{it}Y_{it}$ is total value of shipments, adjusted by inventories; $P_{Nit}N_{it}$ is total payroll; $P_{Mit}M_{it}$ is total cost of materials. The capital input $P_{Kit}K_{it}$ is calculated by total real capital stock at time $t - 1$ multiplied by real interest rate r and depreciation rate δ , which is set to 7% for all industries.

There are some potential problems with the above price-cost measurement. First, Roeger(1995) relies on the assumption of constant return to scale. Basu and Fernald (1997) find that not allowing for varying returns to scale may result in an upward or downward bias in the estimation of price-cost margin, according to returns to scale are decreasing or increasing. However, using U.S. manufacturing data, Basu and Fernald (1997) also find that firm level return to scale to be constant or slightly decreasing in US. With constant return to scale, the measurement of Roeger (1995) has no bias. With slight decreasing return to scale, since this focus on the change of price-cost margin, it seems not a big problem. Second, there is a possible problem of aggregation, since the estimation theory is from firm level decisions while the data is industry level. This problem can be only solved by better firm level data set. Third, there may be a problem with the measurement of the user's cost of capital.²

Since this thesis is interested in the effect of trade liberalization on industry PCM, the following F statistics is used to test the existence of possible structural breaks for equation $\Delta y_{it} = \beta_i \Delta x_{it} + \epsilon_{it}$. If there is one break point at time j ,

² The method by De Loecker Jan and Warzynski Frederic(2011) does not need to calculate user's cost of capital and also does not rely on the assumption of constant return to scale. However, their method is more suitable for firm level data.

$\Delta y_{it} = \beta_i \Delta x_{it} + \epsilon_{it}$ is estimated together with an alternative segmented linear regression $\Delta y_{it} = \beta_{i0} \Delta x_{it} + \epsilon_{it}$ for the first j observations and $\Delta y_{it} = \beta_{i1} \Delta x_{it} + \epsilon_{it}$ for the remaining observations. Let n be the number of observations, \hat{u} be the OLS residuals for the unsegmented regression and $\hat{u}(j)$ be the OLS residuals for the segmented linear regression,

$$F_j = \frac{\hat{u}'\hat{u} - \hat{u}(j)'\hat{u}(j)}{\hat{u}(j)'\hat{u}(j)/(n-2)}$$

If F_j is largest and the hypothesis $\beta_i = \beta_{i1}$ is rejected, time j is chosen as the break point. Bai and Perron (1998,2003) extend this approach to F tests for 0 vs l breaks and l vs $l+1$ breaks respectively with arbitrary but fixed l . Therefore, not only breaking points but also the optimal number of breaking points can be obtained³.

(1) Increasing PCM in US manufacturing industry

The results from F statistics based on Bai and Perron (1998,2003) show that 165 out of 459 4-digit manufacturing industries in US have increasing price-cost margins from 1960s to 2000s, which is significant at the 0.1 level⁴. Almost every 2 digit sector has industries with increasing PCMs, except sector 29, which is petroleum refining and related industries. All of the 4-digit industries in sector 21, which is tobacco products, have increasing PCMs. The distributions of those industries by the number of 4-digit industries and average value of shipments are shown in Figure 2.1⁵:

The breaking years predicted by F statistics for those industries with increasing PCMs are summarized in the following graph, which shows a significant increase from the 1980s. To describe the openness of an industry, we use an openness ratio,

³ This procedure can be implemented by R package "strucchange".

⁴ PCMs for 10 industries decrease first and then increase.

⁵ See Table A.2 for 1987 SIC codes.

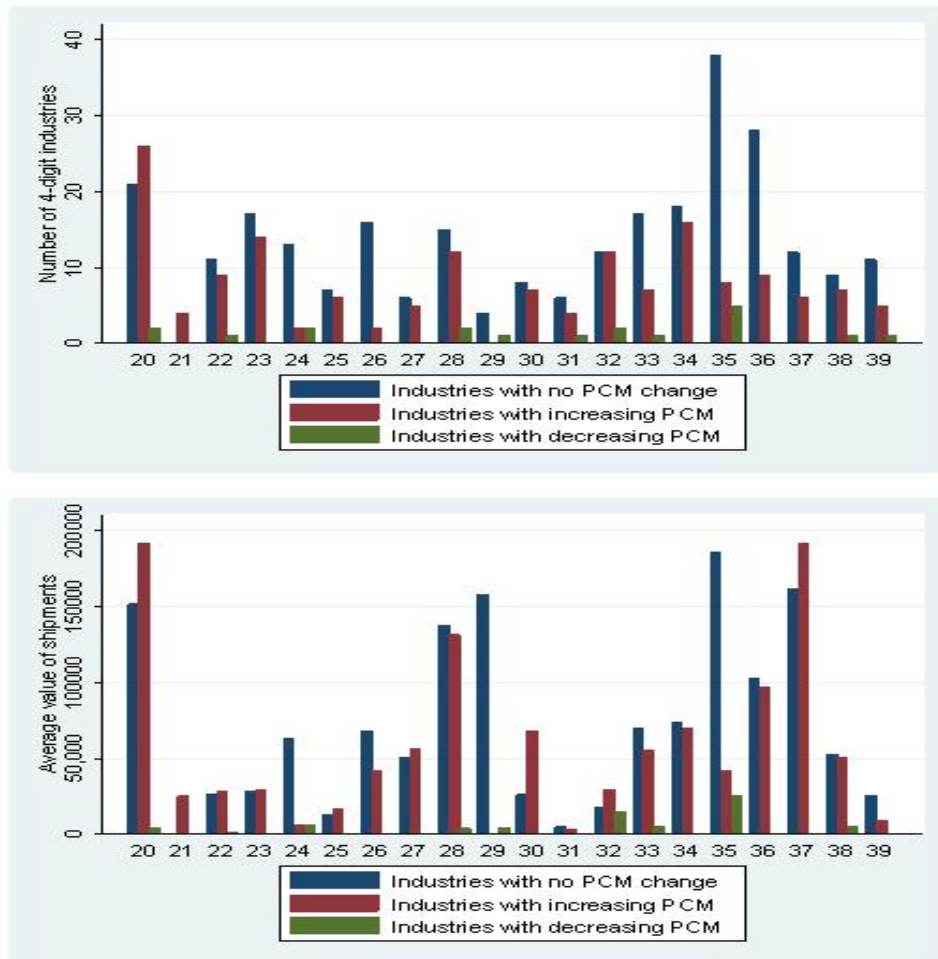


Figure 2.1: U.S. Manufacturing Industries and PCM Changes

which is calculated as exports plus imports divided by the value of shipments and an import penetration ratio, which is calculated as imports divided by the value of shipments. The data are from U.S. Manufacturing Imports and Exports by Peter K. Schott and the Center for International Data at UC Davis. From graphs below, there is a connection between trade liberalization and increasing PCM.

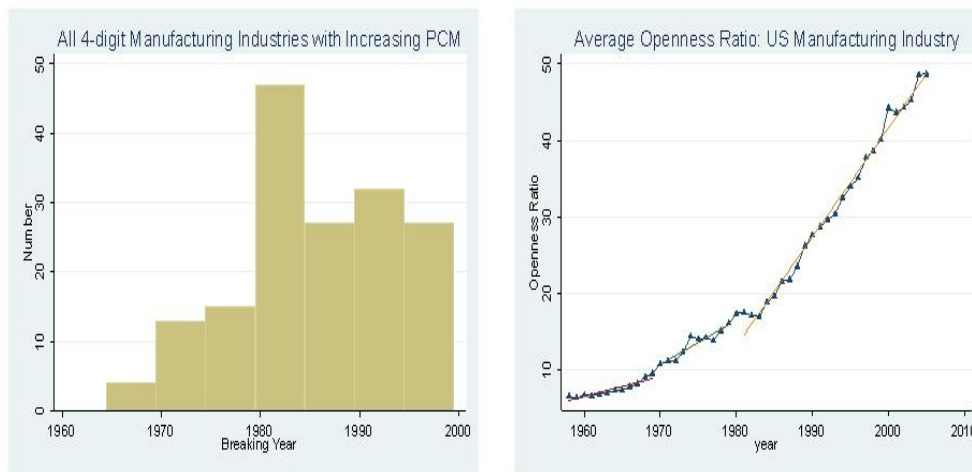


Figure 2.2: Trade Liberalization and Increasing PCM in U.S. Manufacturing Industries

(2) Labor Intensity and Increasing PCM

Besides the increasing PCM itself, another interesting question is what kinds of properties those industries with increasing PCM have. The following graphs illustrate the Labor-Capital ratio distribution of industries with increasing PCM. With an increase of the import penetration ratio, industries with increasing PCM tend to be more labor intensive.

If industries with large labor-capital ratios and sharply decreasing value of shipments, for example, sector 22(textile mill products), are excluded, as shown in the following graph, industries, which have increasing PCM and are affected

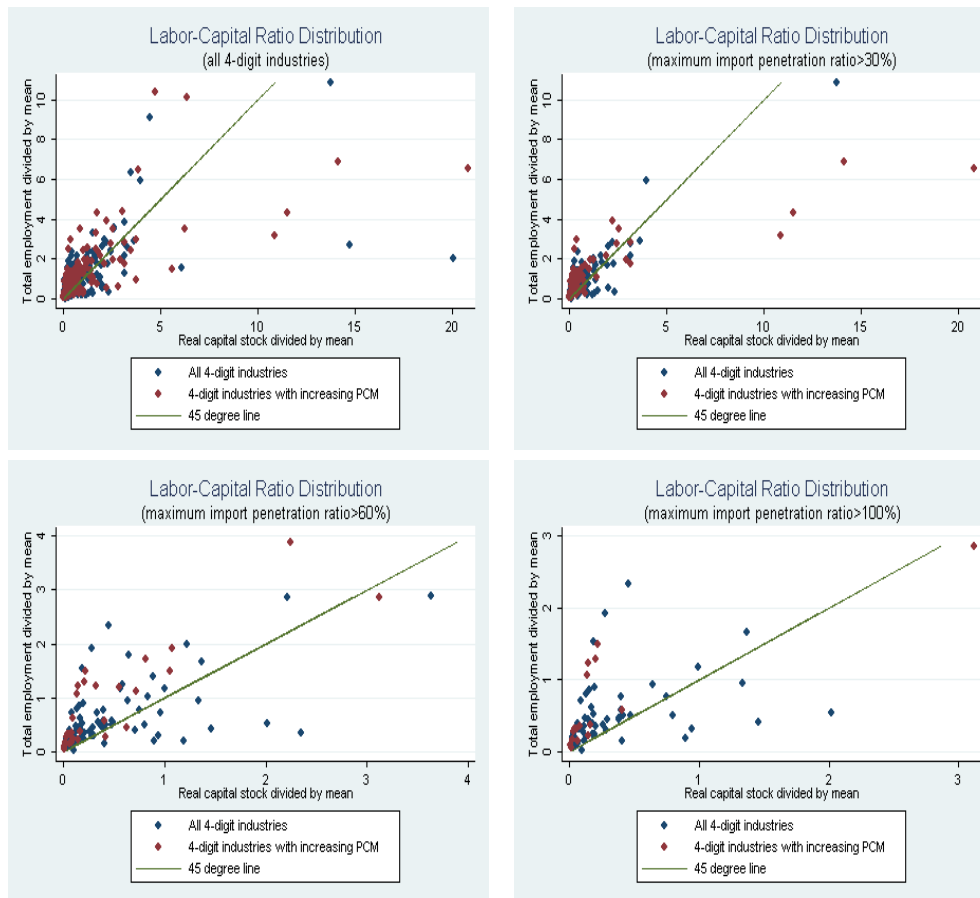


Figure 2.3: Labor Intensity and Increasing PCM: Part 1

by trade liberalization (for example, maximum import penetration ratio is greater than 60%) but still have relatively strong domestic production (for example, maximum import penetration ratio is less than 200%), are usually in the middle. In those industries, the U.S. does not completely have disadvantage while competitors from labor intensive countries do not have overwhelming advantages.

Since manufacturing industries in the U.S. that have increasing PCM and are affected by trade liberalization tend to be labor-intensive, and according to

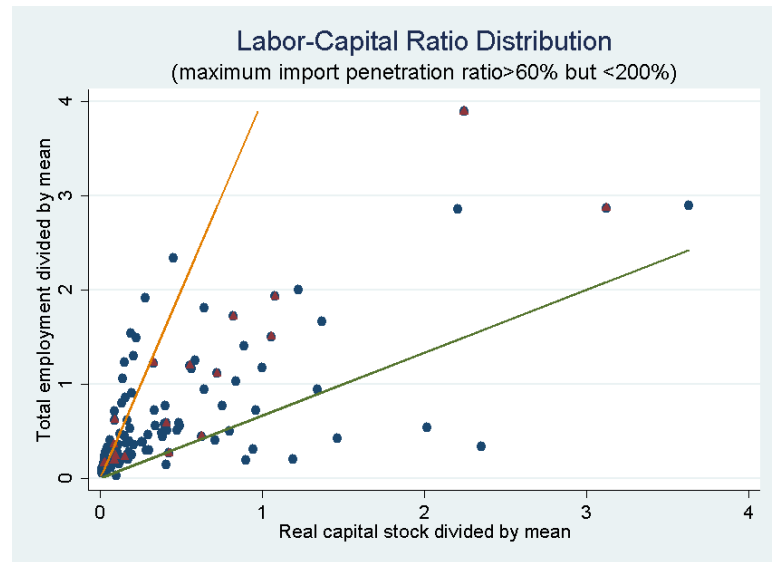


Figure 2.4: Labor Intensity and Increasing PCM: Part 2

comparative advantage theory, trade partners for those industries are mainly relatively labor intensive countries, I hypothesize that the change of PCM during trade liberalization is related to trade partners. PCM tends to increase when trade partners have different factor intensity.

In the following chapters, I build a model with heterogeneous sectors and show that the above hypothesis is true in the model. For simplicity, the model only has one factor. Therefore, the difference in factor intensity in the above analysis is changed to the difference in productivity distribution. I will show that countries have incentives to decrease their competition level under certain conditions while under other conditions, countries have incentives to increase their competition level. The model gives a possible explanation for why empirical evidence for the effect of trade liberalization on PCM are mixed. It also provides a possible explanation for the pattern of PCM changes in Indian manufacturing industries as stated in Goldar and Aggarwal (2005) and European countries' increasing PCM

in Chen, Imbs and Scott (2009).

Chapter 3

GOLE with Homogeneous Firms

3.1 General Model of Oligopolistic Competition

There is a continuum economy with an infinite number of types of goods $z \in [0, 1]$ and an infinite number of individuals $k \in [0, 1]$. Each type of good is produced in one sector, so there are also an infinite number of sectors $z \in [0, 1]$ in the economy.

Demand:

All the individuals have the same labor endowment L and the same quadratic utility function $u(x(z, k)) = ax(z, k) - \frac{b}{2}x(z, k)^2$, where $x(z, k)$ is individual k 's consumption of goods z and $a, b > 0$. The income of individuals is denoted by I_k , $k \in [0, 1]$ and has two parts. Individuals not only get wage income wL from the labor market but also get profit income $\pi(k)$ from shares of firms they own. The consumer's problem is as follows:

$$\max \int_0^1 [ax(z, k) - \frac{b}{2}x(z, k)^2] dz$$

$$s.t. \int_0^1 p(z)x(z, k)dz \leq I_k$$

Assume $\lambda(k)$ is the Lagrangian multiplier and from F.O.Cs of consumer's problem,

$$\begin{aligned} p(z) &= \frac{1}{\lambda(k)}[a - bx(z, k)] \\ \lambda(k) &= \frac{a \int_0^1 p(z)dz - bI_k}{\int_0^1 p(z)^2 dz} \end{aligned}$$

The aggregate demand for goods z is

$$x(z) = \int_0^1 x(z, k)dk = \int_0^1 \left(\frac{a}{b} - \frac{\lambda(k)p(z)}{b} \right) dk = \frac{a}{b} - \frac{p(z)}{b} \lambda \quad (3.1)$$

$$\lambda = \int_0^1 \lambda(k)dk = \frac{a \int_0^1 p(z)dz - b \int_0^1 I_k dk}{\int_0^1 p(z)^2 dz} \quad (3.2)$$

Supply:

For the supply side of the economy, the production function for sector z is $l(z) = \alpha(z)y(z)$, where $l(z)$ is labor input, $y(z)$ is output and $\alpha(z)$ is marginal labor input which is constant within each sector but is different across sectors. Markets for sectors are imperfectly competitive and firms in each sector are homogeneous and follow Cournot competition. Assuming there are $n(z)$ homogeneous firms with output $y(z)$ in sector z , from the property of Cournot competition,

$$p(z) - w\alpha(z) = y(z) / \left(- \frac{\partial x(z)}{\partial p(z)} \right)$$

There is a problem when calculating $\frac{\partial x(z)}{\partial p(z)}$, which is first highlighted by Gabszewicz and Vital (1972). The Lagrangian multiplier λ is related to $p(z)$ through I_k . In words, the fact that λ is related to $p(z)$ means large firms influence the cost of living and rational shareholders should take this into account when choosing the profit-maximizing level of output or price. To avoid this problem in mathematics,

I assume that every firm only has a finite number of shareholders, which means that the effect on the cost of living is very small. In math, the measure of a finite set is 0:

$$\lambda = \int_0^1 \lambda(k) dk = \int_{[0,1]/\text{shareholders of firms in sector } z} \lambda(k) dk$$

Therefore, under the above assumption, $\frac{\partial x(z)}{\partial p(z)} = -\frac{\lambda}{b}$. Hence, under Cournot competition,

$$p(z) - w\alpha(z) = by(z)/\lambda$$

Combing the above equation with aggregate demand equation $x(z) = \frac{a}{b} - \frac{p(z)}{b}\lambda = n(z)y(z)$ and normalizing $\lambda = 1$, firm's output $y(z) = \frac{a-w\alpha(z)}{b(n(z)+1)}$ and aggregate supply in sector z is $\frac{n(z)(a-w\alpha(z))}{b(n(z)+1)}$.

Entry&Exit of Firms and Competition Policy:

The entry and exit strategy of firms in this economy is a simple case of Hopenhayn (1992). Since firms are homogeneous, every firm will produce after it realizes its productivity. Instead of assuming a sunk cost, I assume there is a profit level Π which is decided by a competition policy in this economy. In each period, the profit level for every firm can not exceed this level. Hence, in the steady state, since there is free entry, firms in different sectors must have the same profit level Π . Otherwise, if the profit of firms in sector z_1 is lower than the profit of firms in other sectors, firms in sector z_1 will exit and enter other sectors, which increases profit in sector z_1 and decreases profit in other sectors, until all the firms have the same profit level.

When defining a competition policy, instead of profit level Π , the markup $m(z) = p(z) - w\alpha(z)$ is used to define the competition policy set by the government¹. Π and $m(z)$ are equivalent since $\Pi = \frac{m^2(z)}{b}$. The government competition

¹ The reason I use $m(z)$ is that the following model is first written in the form of $m(z)$,

policy is defined as follows:

Definition 1 *A competition policy in the economy is a threshold value of markup M . For each sector z , the markup $m(z) \leq M$.*

In the economy, the initial markup $m_0(z)$ for each sector and threshold markup M are given. With free entry, in the steady state, markup for every sector is equal to M and the number of firms $n(z) = \frac{a-w\alpha(z)}{M} - 1$. In the following part, I also show that when M increases, the price-cost margin in this economy increases and therefore the competition level in this economy decreases.

3.2 General Equilibrium for a Closed Economy

The following general equilibrium definition for a closed economy summarizes the setting described in the previous part.

Definition 2 *(normalize $\lambda = 1$) For a closed economy, a general equilibrium is a set of prices ($w, \{p(z)\}_{z \in [0,1]}$) and quantities ($\{x(z, k)\}_{z \in [0,1], k \in [0,1]}$, $\{y(z)\}_{z \in [0,1]}$, $\{n(z)\}_{z \in [0,1]}$) given ($L, \{m_0(z)\}_{z \in [0,1]}$, $\{\alpha(z)\}_{z \in [0,1]}$, M) such that:*

- *There are an infinite number of individuals $k \in [0, 1]$ in the economy. Taking w and $\{p(z)\}_{z \in [0,1]}$ as given, consumer maximizes its utility $u(x(k)) = \int_0^1 (ax(z, k) - \frac{b}{2}x(z, k)^2)dz$ subject to his budget constraint;*
- *There are an infinite number of sectors $z \in [0, 1]$ in the economy and each sector produces one type of good. In each sector z , the initial competition level is described by an initial markup $m_0(z)$, which is given and determines*

which is not only related to profit level Π but also the number of firms, $n(z)$.

the initial number of firms in each sector. Taking w as given, homogeneous firms maximize their profits under Cournot competition with production function $l(z) = \alpha(z)y(z)$;

- Goods markets and labor market clear: $x(z) = n(z)y(z)$ and $L = \int_0^1 n(z)l(z)dz$.
- There is a competition policy decided by the government, which requires the markup $m(z) \leq M$ for each sector z .

By market clearing conditions for labor market,

$$L = \int_0^1 n(z)l(z)dz = \int_0^1 n(z)\alpha(z)y(z)dz$$

Substituting $m(z) = M = by(z)$ and $n(z) = \frac{a-w\alpha(z)}{m(z)} - 1$ into the labor market clearing condition and denoting $\int_0^1 \alpha(z) = \mu_\alpha$ and $\int_0^1 \alpha^2(z)dz = \sigma_\alpha^2$, the wage rate is:

$$w = \frac{a\mu_\alpha - bL - M\mu_\alpha}{\sigma_\alpha^2} \quad (3.3)$$

After getting w , the number of firms $n(z)$ can be calculated from $n(z) = \frac{a-w\alpha(z)}{M} - 1$ and $p(z)$ from $p(z) = w\alpha(z) + M$. With the consumer's budget constraint, the market clearing conditions for the goods market and $\frac{a-bx(z_1)}{a-bx(z_2)} = \frac{p(z_1)}{p(z_2)}$, the consumption level of good z by individual k is

$$x(z, k) = (a \int_0^1 p(z)^2 dz - ap(z) \int_0^1 p(z) dz + bp(z)I_k) / b \int_0^1 p(z)^2 dz$$

In equation (4.3), the wage rate in the economy is not only decided by the labor endowment and distribution of marginal cost but also competition policy which decides the markup in each sector. The higher the markup, the lower the competition level in the economy and the lower the wage rate.

Theorem 3 *In the above closed economy,*

- If M increases, $\frac{w}{p(z)}$ will decrease.
- Let $\Pi = \int_0^1 n(z)\pi(z)dz$. When $n(z) \geq 1$, if M increases, $\frac{\Pi}{wL}$ will increase.
- If M increases, for sectors with $\alpha(z) \geq \frac{\sigma_\alpha^2}{\mu_\alpha}$, total output $Y(z)$ will increase. For sectors with $\alpha(z) < \frac{\sigma_\alpha^2}{\mu_\alpha}$, total output $Y(z)$ will decrease.

Proof. See Appendix A.1 for proof. The first point shows that when M increases, the price-cost margin in this economy increases and therefore the competition level in this economy decreases. There is one interesting thing about the measurement of competition level from this model. In this economy, the competition level is measured by the price cost margin. However, high PCM does not necessarily mean that the number of firms decreases. The number of firms is not a good measurement for the competition level when cost is endogenous in general equilibrium. When the price cost margin decreases, the number of firms in high productivity sectors always decreases while the number of firms in low productivity sectors maybe increases. The third point is very obvious. Since $w = \frac{a\mu_\alpha - bL - M\mu_\alpha}{\sigma_\alpha^2}$,

$$p(z) = \frac{a\mu_\alpha\alpha(z) - bL\alpha(z)}{\sigma_\alpha^2} - M\left(\frac{\mu_\alpha\alpha(z)}{\sigma_\alpha^2} - 1\right)$$

If M increases, for sectors with $\alpha(z) \geq \frac{\sigma_\alpha^2}{\mu_\alpha}$, $p(z)$ will decrease. For sectors with $\alpha(z) < \frac{\sigma_\alpha^2}{\mu_\alpha}$, $p(z)$ will increase. Hence, ff M increases, for sectors with $\alpha(z) \geq \frac{\sigma_\alpha^2}{\mu_\alpha}$, total output $Y(z)$ will increase. For sectors with $\alpha(z) < \frac{\sigma_\alpha^2}{\mu_\alpha}$, total output $Y(z)$ will decrease. The intuition behind the proof of part three is as follows: In our economy, firms in each sector have the same profit. When M increases, profit increaseas since $\Pi = \frac{M^2}{b}$ and the change of profit is the same for all sectors. There are two ways to increase profit in our economy. One is a decrease in cost and the other is an increase in market power. We know the cost function is $w\alpha(z)$. The change of cost in a low cost sector is smaller that the change in a high cost sector.

Hence, to achieve the same increase in profit, low cost sectors need larger increase in market power or larger decrease in the number of firms. The output of firm in this economy is $\frac{M}{b}$, which has nothing to do with sector cost. Hence, the larger decrease in the number of firms, the larger decrease in the sector output and the larger decrease in the resource used by the sector. Therefore, the resource will move from sectors with low $\alpha(z)$ to sectors with high $\alpha(z)$. ■

From the above theorem, the markup M (or the competition level) determines two important characteristics of the economy: First, the ratio of wage income to profit income, which is related to income distribution and second, the ratio of output for sectors. An increase in M not only shifts income from wage income to profit income but also shifts factors from low-cost sectors to high-cost sectors.

If $\alpha(z) = \alpha$ for every sector in our economy, the second role of markup M will disappear. If all the individuals have the same income, it is not difficult to show that competition policy which controls markup M has no effect in the economy with $\alpha(z) = \alpha$ and market allocation is Pareto efficient, as Neary (2009) shows.

However, if individuals have different income and $\alpha(z) = \alpha$, the first role of markup M , which determines income distribution, will make competition policy important to social welfare. For simplicity, assume there are two types of individuals. For type I individuals $k \in [0, \theta]$, they only get wage income $I_k = wL$ while for type II individuals $k \in (\theta, 1]$, $I_k = wL + \int_{\text{the set of sectors invested}} \pi(z, k) dz$, which means that they not only have wage income but also have profit income from shares of many firms they own. To make the problem simple, I further assume that type II individuals have the same aggregate profit income, so type II individuals are all identical (so are type I individuals). The consumption levels

for type I and type II individuals are as follows:

$$\begin{aligned} x(z, k) &= x(1) = \frac{wL}{p} = \frac{1}{\mu_\alpha} \left(1 - \frac{M}{a - \frac{bL}{\mu_\alpha}}\right)L, \text{ if } k \in [0, \theta] \\ x(z, k) &= x(2) = \frac{1}{1 - \theta} \left[\frac{L}{\mu_\alpha} - \frac{\theta}{\mu_\alpha} \left(1 - \frac{M}{a - \frac{bL}{\mu_\alpha}}\right)L\right], \text{ if } k \in (\theta, 1] \end{aligned}$$

Assume that government maximize a weighted welfare function for above two groups and there should be a optimal level M which balances the two groups' interest.

Let ψ , which is smaller than θ , be the weight for the utility of type I individual in the social welfare function. The government social welfare maximization problem is

$$\begin{aligned} \max \quad & \psi(ax(1) - \frac{b}{2}x^2(1)) + (1 - \psi)(ax(2) - \frac{b}{2}x^2(2)) \\ \text{subject to} \quad & \theta x(1) + (1 - \theta)x(2) = \frac{L}{\mu_\alpha} \end{aligned}$$

Proposition 4 *When $\alpha(z) = \mu_\alpha$, the optimal competition level $M^* = \left(\frac{a^2}{\frac{bL}{\mu_\alpha}} + \frac{bL}{\mu_\alpha} - 2a\right) \frac{(\theta - \psi)}{\left(\frac{\theta^2}{1 - \theta} + \frac{1 - 2\theta}{1 - \theta} \psi\right)}$.*

In the economy with $\alpha(z) = \mu_\alpha$, the optimal competition level is related to ψ , the social welfare weight for type I people, L , the resource and μ_α , marginal cost. Since $\frac{bL}{\mu_\alpha} < a$, the larger L , the smaller M^* and the smaller μ_α , the smaller M^* . Also for ψ , the bigger the social welfare weight for type I people, the higher competition level in the economy. It can explain, for example, why smaller countries tend to adopt competition policy later, as illustrated in Forslid, Hackner and Muren (2011).

Another interesting problem is the effect of increasing returns to scale. Although increasing return to scale is an important argument for higher industry

concentration, in the above economy, it does not change two roles of Markup M very much.

Assuming that the production function for every sector is $l(z) = \beta + \alpha(z)y(z)$. The market clearing condition for labor market is

$$L = \int_0^1 n(z)l(z)dz = \int_0^1 n(z)\beta dz + \int_0^1 n(z)\alpha(z)y(z)dz$$

The wage rate with positive β is as follows:

$$w = \frac{\left(\frac{ab\beta}{M} - M \int_0^1 \alpha(z)dz + a \int_0^1 \alpha(z)dz - bL - b\beta\right)M}{b\beta \int_0^1 \alpha(z)dz + M \int_0^1 \alpha^2(z)dz}$$

Proposition 5 *If the production function is $l(z) = \beta + \alpha(z)y(z)$ for each sector,*

- If M increases, $\frac{w}{p(z)}$ will decrease.
- When $n(z) \geq 1$, if M increases, $\frac{\Pi}{wL}$ will increase.
- If M increases, for sectors with $\alpha(z) \geq \frac{\sigma_\alpha^2}{\mu_\alpha}$, total output $Y(z) = n(z)y(z)$ will increase. For sectors with $\alpha(z) \leq \frac{b\beta\mu_\alpha}{bL+b\beta+\frac{a}{\mu_\alpha}(\sigma_\alpha^2-\mu_\alpha^2)}$, total output $Y(z)$ will decrease;

Proof. See Appendix A.1 for proof. ■

The existence of β only slightly changes part three. Sectors with $\alpha(z) \geq \frac{\sigma_\alpha^2}{\mu_\alpha}$ still benefit from the increase of M while there also exists sectors which can only be harmed by an increase of M although increasing return to scale exists in the economy. Therefore, increasing returns to scale is not always a valid argument against competition policy and it depends on the position of sector's marginal cost on the cost distribution for the whole economy. $\frac{b\beta\mu_\alpha}{bL+b\beta+\frac{a}{\mu_\alpha}(\sigma_\alpha^2-\mu_\alpha^2)}$ is not an upper bound for sectors which will always suffer from an increase of M . Output of sectors between $\frac{b\beta\mu_\alpha}{bL+b\beta+\frac{a}{\mu_\alpha}(\sigma_\alpha^2-\mu_\alpha^2)}$ and $\frac{\sigma_\alpha^2}{\mu_\alpha}$ has two possibilities. For sectors with

low marginal cost, total output will decrease when M increases. For sectors with high marginal cost, when M increases, total output will increase first and then decrease.

3.3 General Equilibrium for an Open Economy

There are two countries, H and F . The economy in each country is as the same as the one described in the above section. The general equilibrium definition for an open economy with no trade barriers is as follows:

Definition 6 (normalize $\lambda^H = 1$) *For an open economy, a general equilibrium is a set of price ($w^i, \{p(z)\}_{z \in [0,1]}_{i=H,F}$) and quantities ($\{x^i(z, k)\}_{z \in [0,1], k \in [0,1]}$, $\{y^i(z)\}_{z \in [0,1]}$, $\{n^i(z)\}_{z \in [0,1]}_{i=H,F}$) given ($L^i, \{m_0^i(z)\}_{z \in [0,1]}$, $\{\alpha^i(z)\}_{z \in [0,1]}$, M^i) such that:*

- *There are an infinite number of individuals $k \in [0, 1]$ in country i where $i = H$ or F , which belong to two different types with different incomes. Taking w^i and $\{p(z)\}_{z \in [0,1]}$ as given, consumer maximizes its utility $u(x^i(k)) = \int_0^1 (ax^i(z, k) - \frac{b}{2}x^i(z, k)^2)dz$ subject to his budget constraint;*
- *There are an infinite number of sectors $z \in [0, 1]$ in country i where $i = H$ or F and each sector produces one type of goods. In each sector z , the initial competition level is described by an initial markup $m_0^i(z)$, which is given and determines the initial number of firms in each sector. Taking w^i as given, homogeneous firms maximize their profits under Cournot competition with production function $l^i(z) = \alpha^i(z)y^i(z)$;*
- *Goods markets and labor market clear: $x^H(z) + x^F(z) = n^H(z)y^H(z) + n^F(z)y^F(z)$ and $L^i = \int_0^1 n^i(z)l^i(z)dz$.*

- *In each country, the value of exports is equal to the value of imports.*
- *There is a competition policy decided by the government in country i where $i = H$ or F , which requires the markup $m^i(z) \leq M^i$ for each sector z .*

To determine the trade pattern for two countries, I assume that marginal cost distributions in two countries are monotone and only have one intersection point. Under this assumption, I only consider the case in which the trade pattern is complete specialization except for one sector. All the other trade patterns are neither possible nor interesting. Let's look at two arbitrary sectors z_1 and z_2 . In country H , by Cournot competition, I have prices $p(z_1) = w^H \alpha^H(z_1) + M^H$ and $p(z_2) = w^H \alpha^H(z_2) + M^H$. These prices guarantee that each sector has the same profit. If F also has production in these two sectors, there are similar prices $p(z_1) = w^F \alpha^F(z_1) + M^F$ and $p(z_2) = w^F \alpha^F(z_2) + M^F$. Since there are no trade barriers between two countries, prices must be the same for both countries. In country H , assume $\alpha^H(z_1) < \alpha^H(z_2)$ and therefore $p(z_1) < p(z_2)$. If $\alpha^F(z_1) > \alpha^F(z_2)$, $p(z_1) > p(z_2)$, there is a contradiction. If $\alpha^F(z_1) < \alpha^F(z_2)$, $w^H(\alpha^H(z_2) - \alpha^H(z_1)) = w^F(\alpha^F(z_2) - \alpha^F(z_1))$. The value for $\frac{\alpha^H(z_2) - \alpha^H(z_1)}{\alpha^H(z_2) - \alpha^H(z_1)}$ is different if cost distributions are non-linear. Even if cost distributions are linear, since this thesis considers the change of M , the ratio $\frac{w^F}{w^H}$ will change when M^H or M^F changes. Therefore, the case with linear cost distributions is uninteresting.

The sector of specialization for each country is determined by the following equation:

$$w^H \alpha^H(z^*) + M^H = w^F \alpha^F(z^*) + M^F \quad (3.4)$$

Country H will produce in sector $z \in [0, z^*]$ while Country F will produce in sector $z \in [z^*, 1]$. They both produce in sector z^* . However, the measure of z^* is zero.

From the previous section, the demand for goods z in each country is $x^i(z) = \frac{a}{b} - \frac{p(z)}{b}\lambda^i$, $i = H, M$. So the aggregate demand is

$$\bar{x}(z) = x^H(z) + x^F(z) = \frac{2a}{b} - \frac{p(z)}{b}(\lambda^H + \lambda^F)$$

Let $a' = \frac{2a}{\lambda^H + \lambda^F}$ and $b' = \frac{b}{\lambda^H + \lambda^F}$. By Cournot competition, $p(z) - w\alpha(z) = b'y(z)$ and the number of firms in sector z is $n(z) = \frac{a' - w\alpha(z)}{M} - 1$. Hence, by the market clear condition for labor market, in country H ,

$$w^H = \frac{a'\mu_\alpha^H - M^H\mu_\alpha^H - b'L}{\sigma_\alpha^{2H}} \quad (3.5)$$

In the above equation, $\mu_\alpha^H = \int_0^{z^*} \alpha^H(z)dz$ and $\sigma_\alpha^{2H} = \int_0^{z^*} [\alpha^H(z)]^2 dz$. Similarly, define $\mu_\alpha^F = \int_{z^*}^1 \alpha^F(z)dz$ and $\sigma_\alpha^{2F} = \int_{z^*}^1 [\alpha^F(z)]^2 dz$, and in country F ,

$$w^F = \frac{a'\mu_\alpha^F - M^F\mu_\alpha^F - b'L}{\sigma_\alpha^{2F}} \quad (3.6)$$

The last equation for the open economy general equilibrium is about the value of exports and the value of imports, which must be equal for each country.

$$\int_0^{z^*} p(z)\left(\frac{a}{b} - \frac{p(z)}{b}\lambda^F\right)dz = \int_{z^*}^1 p(z)\left(\frac{a}{b} - \frac{p(z)}{b}\lambda^H\right)dz \quad (3.7)$$

After normalizing $\lambda^H = 1$, $\{z^*, w^H, w^F, \lambda^F\}$ can be solved based on equation (4.4)-(4.7).

The calculation for the above model is complicated. For simplicity, I consider a special case, which also gives useful insights into the relationship between trade liberalization and competition policy. I consider a three-sector economy and each sector has a measure of 1/3. The marginal cost for H country is $\alpha(1)$, $\alpha(2)$ and $\alpha^H(3)$ and the marginal cost for F country is $\alpha^F(1)$, $\alpha(2)$ and $\alpha(3)$. Assuming $\alpha(1) < \alpha^F(1)$, $\alpha(3) < \alpha^H(3)$, country H specializes in the first sector while country F specializes in the third sector. Both countries produce in the second sector.

The same profit requires that in each country, every sector has the same markup, so the following price equations exist:

$$p(1) = w^H \alpha(1) + M^H$$

$$p(2) = w^H \alpha(2) + M^H = w^F \alpha(2) + M^F$$

$$p(3) = w^F \alpha(3) + M^F$$

In each sector, from Cournot competition firm output is as follows:

$$y^H(1) = \frac{a' - w^H \alpha(1)}{b'(n^H(1) + 1)} \text{ and } y^H(2) = \frac{a' - (n^F(2) + 1)w^H \alpha(2) + n^F(2)w^F \alpha(2)}{b'(n^H(2) + n^F(2) + 1)}$$

$$y^F(1) = \frac{a' - w^F \alpha(3)}{b'(n^F(3) + 1)} \text{ and } y^F(2) = \frac{a' - (n^H(2) + 1)w^F \alpha(2) + n^H(2)w^H \alpha(2)}{b'(n^H(2) + n^F(2) + 1)}$$

In the above equations, $n^H(2)$ is the number of firms from country H in the second sector and $n^F(2)$ is the number of firms from country F in the second sector. From the definition of markup and properties of Cournot competition, $y^H(1) = y^H(2) = \frac{M^H}{b'}$ and $y^F(1) = y^F(2) = \frac{M^F}{b'}$.

From the market clearing conditions for both countries' labor market:

$$L^H = \int_0^{1/3} n^H(1) \alpha(1) y^H(1) dz + \int_{1/3}^{2/3} n^H(2) \alpha(2) y^H(2) dz$$

$$L^F = \int_{1/3}^{2/3} n^F(2) \alpha(2) y^F(2) dz + \int_{2/3}^1 n^F(3) \alpha(3) y^F(3) dz$$

Substituting equations for n^H , y^H , n^F and y^F , two market clearing conditions for the labor market become,

$$L^H = \int_0^{1/3} \left(\frac{a' - w^H \alpha(1)}{M^H} - 1 \right) \alpha(1) \frac{M^H}{b'} dz + \int_{1/3}^{2/3} n^H(2) \alpha(2) \frac{M^H}{b'} dz \quad (3.8)$$

$$L^F = \int_{1/3}^{2/3} n^F(2)\alpha(2)\frac{M^F}{b'}dz + \int_{2/3}^1 \left(\frac{a' - w^F\alpha(3)}{M^F} - 1\right)\alpha(3)\frac{M^F}{b'}dz \quad (3.9)$$

In the second sector, $p(2) = w^H\alpha(2) + M^H = w^F\alpha(2) + M^F$. Hence,

$$(w^H - w^F)\alpha(2) = M^F - M^H \quad (3.10)$$

From aggregate demand, $n^F(2)y^F(2) + n^H(2)y^H(2) = \frac{a' - p(2)}{b'}$ for the second sector, so

$$n^F(2)M^F + n^H(2)M^H = a' - w^H\alpha(2) - M^H \quad (3.11)$$

From equation (4.7)-(4.10), wage rate in each country is as follows:

$$w^H = \frac{1}{\sum_i \alpha^2(i)} \left[a' \sum_i \alpha(i) - 3b'(L^H + L^F) - M^H \sum_i \alpha(i) + (M^H - M^F) \left(\frac{\alpha(2)\alpha(3) - \alpha^2(3)}{\alpha(2)} \right) \right]$$

$$w^F = \frac{1}{\sum_i \alpha^2(i)} \left[a' \sum_i \alpha(i) - 3b'(L^H + L^F) - M^F \sum_i \alpha(i) + (M^F - M^H) \left(\frac{\alpha(2)\alpha(1) - \alpha^2(1)}{\alpha(2)} \right) \right]$$

From the above two equations, country H 's wage rate is not only determined by its own competition level M^H but also influenced by foreign country's competition level M^F . Competition policy is integrated through trade for these two countries. In equations for wage rates, since I normalize $\lambda^H = 1$, the value for λ^F should be determined. For each country, the value of income should be equal to the value of expenditure and the value of exports should be equal to the value of imports. For country H ,

$$p(1)\left(\frac{a'}{b'} - \frac{p(1)}{b'}\right) + p(2)\left\{\frac{1}{\alpha(2)}[3L^H - \alpha(1)\left(\frac{a'}{b'} - \frac{p(1)}{b'}\right)]\right\} = \sum_i p(i)\left(\frac{a}{b} - \frac{p(i)}{b}\right)$$

Based on the above model, I consider the above two cases of marginal cost distribution and want to answer the following question: should a country decrease

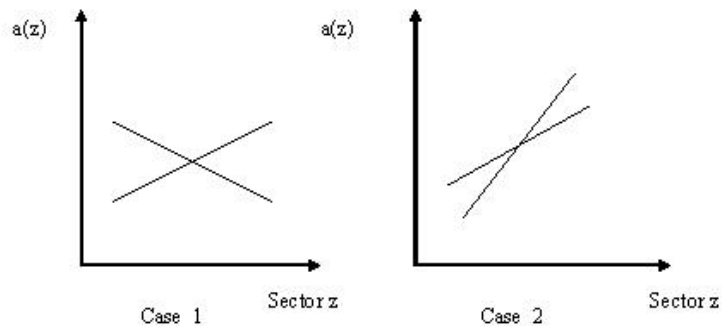


Figure 3.1: Two Different Cases

or increase its M after trade liberalization? I use country H in the model as an example and assume all the individuals are homogeneous. The welfare function for country H is

$$\begin{aligned} Welfare^H &= \int_0^1 (ax(z) - \frac{b}{2}x^2(z))dz \\ &= \frac{a^2}{2b} - \frac{1}{6b} \sum_i p^2(i) \end{aligned}$$

Theorem 7 Assume $L^H = L^F$, $\alpha(1) = \alpha(3) < \alpha(2)$ (case 1) and at the beginning $M^F = M^H$. After trade liberalization, if initial M^H is small, country H has an incentive to increase M^H .

Proof. The intuition is as follows. Since the change of M^H can shift resources from the low cost sector to the high cost sector, the increase in M^H will give country H more market share of good 2. However, it will also harm country H 's benefit in the good 1 market. When the benefit is larger than cost, country H will decrease its competition level. See appendix A.2 for the proof in detail. ■

In this economy, the wage rate is not only determined by M and L but also by the marginal cost distribution. In the proof of the above theorem, without additional assumptions for the marginal cost distribution, it is also possible that when the initial M^H is large, country H has an incentive to decrease M^H or increase the competition level after trade liberalization.

Proposition 8 *Assume $L^H = L^F$, $\alpha(1) = \alpha(3) < \alpha(2)$, $\alpha(2) - \alpha(1) < \frac{1}{6}\alpha(2)$ and at the beginning $M^F = M^H$. After trade liberalization, country H always has an incentive to increase M^H .*

Proof. See Appendix A.2 for proof. ■

In summary, a high initial M^H and a large gap between $\alpha(1)$ and $\alpha(2)$ increase a country's possibility to increase its competition level while a low initial M^H and a small gap between $\alpha(1)$ and $\alpha(2)$ increase a country's possibility to decrease its competition level after trade liberalization. When decreasing their competition levels, two countries face a Prisoner's dilemma. They both have the incentive to increase their M . If they increase M together, the welfare for two countries after the increase is less than the previous welfare since the welfare function is a decreasing function of M . If only one country increases its \dot{M} , the welfare for the country with larger \dot{M} will increase while the welfare for the country with smaller \dot{M} will decrease. It is obvious that two countries need cooperation when determining their competition policy to avoid the problem of Prisoner's dilemma.

Under some other conditions, trade liberalization helps countries increase their competition level.

Proposition 9 *Assume $L^H = L^F$, $\alpha(1) > \alpha(2) > \alpha(3)$ (case 2), $\alpha(1) - \alpha(2) = \alpha(2) - \alpha(3) < \frac{1}{4}\alpha(2)$ and at the beginning $M^F = M^H$. After trade liberalization, country H always has an incentive to decrease M^H .*

Proof. See Appendix A.2 for proof. ■

3.4 Simple quantitative exercise

Consider two identical countries with symmetric cost distribution. At the beginning, both countries are closed economies. After trade liberalization, country H increases its M and change its competition level while country F keeps its M constant. For parameters, let labor endowment for both countries $L^H = L^F = 200$. In country H, $\alpha^H(1) = 1$, $\alpha^H(2) = 1.5$ and $\alpha^H(3) = 2$ while in country F, $\alpha^F(1) = 2$, $\alpha^F(2) = 1.5$ and $\alpha^F(3) = 1$. Two countries have the same quadratic utility function with $a = 1000$ and $b = 2$. The initial $M = 50$ for both countries. Country H gradually increases its M to 60 after trade liberalization. Results are shown in the following two graphs.

After trade liberalization, total welfare in both countries increases sharply and PCM in sector 2 decreases², which shows pro-competitive effect of trade liberalization. After country H lowers its competition level, total welfare in country H increases while total welfare in country F decreases. Since country H increases its M , PCM in sector 2 for country H increases. PCM in sector 2 for country F also slightly increases. However, unlike country H, its PCM in sector 2 does not exceed the level before trade liberalization. The above example also shows that if two countries have different M , the country with higher M benefits more from trade liberalization. The PCM in the country with a low M will be affected by its trade partner and increase, which possibly makes the pro-competitive effect of trade liberalization ambiguous.

² Same for sector 1 and sector 3, which are less interesting than sector 2.

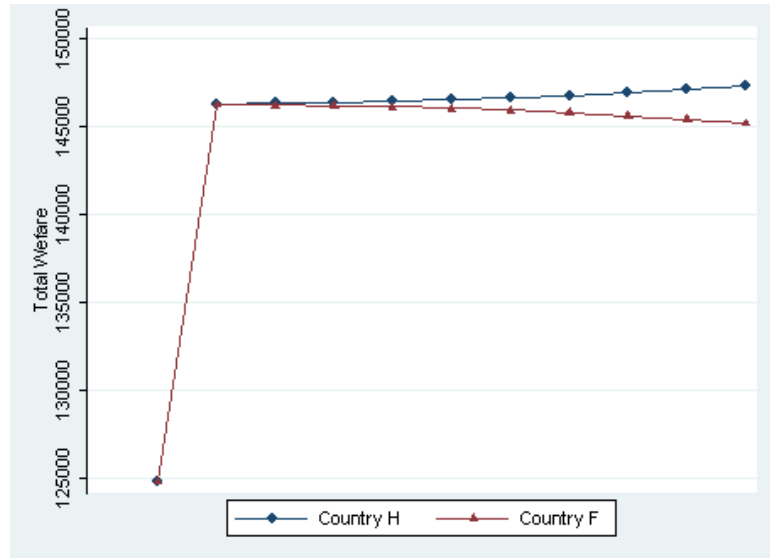


Figure 3.2: Numerical Example Result: Welfare Change

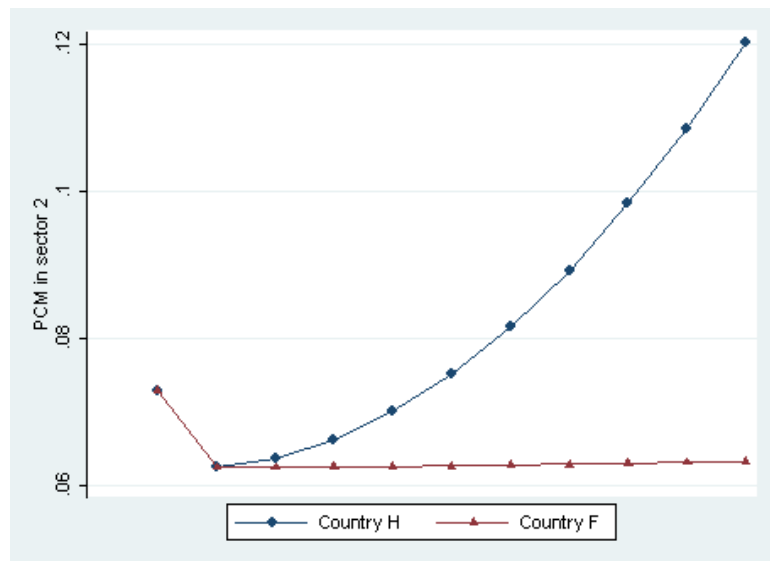


Figure 3.3: Numerical Example Result: PCM Change

Chapter 4

GOLE with Heterogeneous Firms

In this part, I follow the firm entry and exit strategy provided by Hopenhayn (1992) and Melitz and Ottaviano (2008) to introduce heterogeneous firms into the previous model. I want to show that a decrease in competition level will still reallocate a factor from high productivity sectors to low productivity sectors and this reallocation is so large that, in a model with three sectors, countries have incentives to decrease its competition level in trade liberalization.

4.1 Heterogeneous Firms in a Closed Economy

In our economy, there is still one factor L , which is the only input for a continuum of sectors $z \in [0, 1]$. I still use quadratic utility function $u(x(z)) = ax(z) - \frac{b}{2}x(z)^2$,¹

which gives us a linear demand function for each sector when I normalize $\lambda = 1$:

$$x(z) = \frac{a}{b} - \frac{p(z)}{b}$$

The production function for a firm in sector z is $l(z, \phi) = [\alpha(z) + \frac{1}{\phi}]y(z, \phi)$, where $\alpha(z)$ is the sector specific productivity and $\frac{1}{\phi}$ is the firm specific productivity. The parameter ϕ is draw from an ex-ante Pareto distribution $g(\phi) = \frac{\theta}{\phi^{\theta+1}}(\phi_0)^\theta$ for $\phi > \phi_0$ and $g(\phi) = 0$ for $\phi < \phi_0$ and the distribution is assumed to be the same for all the sectors. In each sector, firms follows Cournot competition, so

$$p(z) - [\alpha(z) + \frac{1}{\phi}]w = by(z, \phi)$$

For the firm entry and exit strategy, I slightly change the setting in Hopenhayn (1992) in order to eliminate any possible effect of increasing return to scale. After entry, there is a cutoff profit level Λ . Firms will only produce when their profit level is larger than or equal to Λ . Hence, there is a cutoff firm specific productivity ϕ^* , which satisfies $\pi(z, \phi^*) = \Lambda$. Before entry, there is an expected life time profit level M , which is the same for all sectors. Assuming the death rate for every firm in each period is δ , every firm want to achieve the expected life time profit level $M = \frac{1}{\delta} \int_{\phi^*}^{\infty} \pi(z, \phi)g(\phi)d\phi$.

From the cutoff profit condition and Cournot competition equation,

$$\begin{aligned} p(z) &= [\alpha(z) + \frac{1}{\phi^*}]w + \sqrt{b\Lambda} \\ y(z, \phi) &= \sqrt{\frac{\Lambda}{b}} + \frac{w}{b}(\frac{1}{\phi^*} - \frac{1}{\phi}) \end{aligned}$$

¹ The utility function in this economy is a special case of Melitz and Ottaviano (2008), in which, for one sector,

$$U = q_0^c + \alpha \int_{i \in \Omega} q_i^c di - \frac{\gamma}{2} \int_{i \in \Omega} (q_i^c)^2 di - \frac{\eta}{2} (\int_{i \in \Omega} q_i^c di)^2$$

There is horizontal differentiation in the above utility setting. Here, we do not have q_0^c and let $\gamma = 0$, which means in each sector we have different firms but they choose to product homogeneous goods (no product differentiation, not only horizontal but also vertical).

Substituting the second equation into the expected life time profit condition,

$$b\Lambda + 2\sqrt{b\Lambda} \frac{\theta}{\theta+1} \frac{w}{\phi^*} + \frac{2}{(\theta+1)(\theta+2)} \left(\frac{w}{\phi^*}\right)^2 = \frac{(\phi^*)^\theta}{(\phi_0)^\theta} b\delta M \quad (4.1)$$

One obvious result from equation (5.1) is that all the sectors have the same ϕ^* in this economy. Since some firms will not produce after entry and there is a cutoff firm specific productivity ϕ^* , the distribution of ϕ will change when the economy arrives it steady state. The ex-post distribution for ϕ can be calculated and is $\mu(\phi) = \frac{\theta}{\phi^{\theta+1}}(\phi^*)^\theta$ for $\phi > \phi^*$ and $\mu(\phi) = 0$ for $\phi < \phi^*$. Assume there are $n(z)$ firms in sector z , so the number of firms with productivity $\alpha(z) + \frac{1}{\phi}$ in sector z is $n(z)\mu(\phi)$. Since $y(z) = \int_{\phi^*}^{\infty} y(z, \phi)n(z)\mu(\phi)d\phi$ and $y(z) = \frac{a}{b} - \frac{p(z)}{b}$, the equation for the number of firms in sector z , $n(z)$ is

$$n(z) = \frac{a - \sqrt{b\Lambda} - [\alpha(z) + \frac{1}{\phi^*}]w}{\sqrt{b\Lambda} + \frac{1}{\theta+1} \frac{1}{\phi^*} w}$$

The resource constraint for this economy is $l(z) = \int_{\phi^*}^{\infty} [\alpha(z) + \frac{1}{\phi}]y(z, \phi)n(z)\mu(\phi)d\phi$. Denoting $\int_0^1 \alpha(z)dz = \mu_\alpha$ and $\int_0^1 \alpha^2(z)dz = \sigma_\alpha^2$ and Substituting equations for $y(z, \phi)$, $n(z)$ and $\mu(\phi)$, we have

$$\begin{aligned} bL = & [a - \sqrt{b\Lambda} - (\frac{1}{\phi^*} + \mu_\alpha)w](\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*}) \\ & - \frac{\theta}{(\theta+1)^2(\theta+2)} \left(\frac{1}{\phi^*}\right)^2 w \frac{a - \sqrt{b\Lambda} - (\frac{1}{\phi^*} + \mu_\alpha)w}{\sqrt{b\Lambda} + \frac{1}{\theta+1} \frac{1}{\phi^*} w} - (\sigma_\alpha^2 - \mu_\alpha^2)w \quad (4.2) \end{aligned}$$

Lemma 10 *When $\sqrt{b\Lambda} < a - \frac{bL}{\mu_\alpha}$, there is an unique equilibrium with a positive wage rate w for the above economy.*

Proof. The equilibrium can be solved through equation (5.1) and (5.2). Notice that in equation (5.1), the left hand side is a decreasing function of ϕ^* and the right hand side is an increasing function of ϕ^* when w is fixed, so there is an

unique solution to ϕ^* from equation (5.1). The only thing we need to do is to show that there is an unique solution to w from equation (5.2) when ϕ^* is given. See appendix B.1 for the proof detail. ■

I consider the following policy to decrease the competition level. The policy not only increases the cutoff profit Λ but also adjusts the expected profit level M to keep ϕ^* constant. Although an increase in Λ will decrease the number of firms in sectors, from equation (5.1), it will also increase the cutoff productivity level ϕ^* which is proved by Hopenhayn (1992). To eliminate this productivity selection effect, M is adjusted to maintain the value for ϕ^* .

Theorem 11 *When $\sqrt{b\Lambda} < \frac{1}{2}(a - \frac{bL}{\mu\alpha})$ and ϕ^* remains the same through the adjustment of M ,*

- If Λ increases, w and $\frac{w}{p}$ decrease.
- There exists an α^* and for $\alpha(z) + \frac{1}{\phi^*} > \alpha^*$, $p(z)$ decreases when Λ increases.

Proof. The first point says that a policy which increases Λ while keeping ϕ^* the same will decrease competition level, which is described by price cost margin, in our economy. Since $p(z) = [\alpha(z) + \frac{1}{\phi^*}]w + \sqrt{b\Lambda}$, $\frac{\partial p(z)}{\partial \sqrt{b\Lambda}} = [\alpha(z) + \frac{1}{\phi^*}] \frac{\partial w}{\partial \sqrt{b\Lambda}} + 1$. When Λ increases, w decreases and therefore $\frac{\partial w}{\partial \sqrt{b\Lambda}} < 0$. The increase of price is larger when $\alpha(z)$ is smaller, which means that a decrease in competition level will reallocate factor from high productivity sectors to low productivity sectors. The second point shows that, under certain conditions, this reallocation is so large that outputs of sectors with low productivity even increases when competition level decreases. See appendix B.1 for the proof detail. ■

The following table show the change of price when Λ changes from 40 to 44 based on a numerical example. Following Bernard, Redding and Schott (2007),

the death rate for firms is $\delta = 0.025$ and the expected life time profit $M = 800$, which makes Λ around 5% of the expected life time profit. For the underlying Pareto distribution of productivity within sectors, the shape parameter θ is set to be 3, instead of 3.4 in Bernard et al. (2007) to make calculation simple. Another parameter for the Pareto distribution ϕ_0 is set to be 100, which makes the model with heterogenous firms close to the model with homogeneous firms². I will change the value of ϕ_0 and show my previous results are still hold when there is big intra-sector difference among firms. For the aggregate demand in each sector, assume $a = 500$, $b = 1$ and the factor endowment $L = 100$. For simplicity, assume the productivity distribution across sectors $\alpha(z) = 0.25 + 0.05z$ and $z \in [0, 1]$.

Table 4.1: The Change of Price with Respect to the Change of Λ

$\alpha(z)$	0.25	0.26	0.27	0.28	0.29	0.30
$\Lambda=40$	130.3096	135.1424	139.9751	144.8078	149.6405	154.4732
$\Lambda=44$ w/o adj	130.2112	135.0307	139.8501	144.6695	149.4890	154.3084
$\Lambda=44$ w/ adj	130.3374	135.1592	139.9810	144.8027	149.6245	154.4463

From the above table, obviously, there is a productivity selection effect when Λ increases. For sectors with lower productivity, their output decreases when Λ increases while for sectors with higher productivity, their output increases when Λ increases. As in section 3, this property will give countries incentives to decrease their competition level in trade liberalization under certain conditions.

Denote $\alpha(z) = \alpha_0 + h \times z$. The above result is independent of α_0 and h . For example, let α_0 increase from 0.25 to 1 and keep h the same. The expected life time profit M is adjusted to keep the value for ϕ^* constant. Results are shown in the following table:

² Functions in the previous model with homogeneous firms are all continuous. Therefore, the previous results should still hold when there is small difference among firms.

Table 4.2: The Change of Price with respect to different α_0

$\alpha(z)$	α_0	$\alpha_0 + 0.1$	$\alpha_0 + 0.2$	$\alpha_0 + 0.3$	$\alpha_0 + 0.4$	$\alpha_0 + 0.5$
$\Lambda=40$	$\alpha_0=0.5$	296.8267	302.5633	308.2998	314.0364	319.7730
$\Lambda=44$	$\alpha_0=0.5$	296.8355	302.5662	308.2969	314.0275	319.7582
$\Lambda=40$	$\alpha_0=1$	393.3178	397.1618	401.0058	404.8499	408.6939
$\Lambda=44$	$\alpha_0=1$	393.3216	397.1626	401.0036	404.8446	408.6855

In the above example, the parameter ϕ_0 , which determines intra-sector difference is smaller than 0.05, which determines inter-sector difference. In the following table, I show that the inter-section resource reallocation result is still held when intra-sector difference is larger. The expected life time profit M is still adjusted to keep the value for ϕ^* constant.

Table 4.3: The Change of Price with respect to different ϕ_0

$\alpha(z)$	0.25	0.26	0.27	0.28	0.29	0.30
$\Lambda=40$	$\phi_0=25$	142.7971	147.8425	152.8879	157.9332	162.9786
$\Lambda=44$	$\phi_0=25$	142.8164	147.8511	152.8858	157.9204	162.9551
$\Lambda=40$	$\phi_0=20$	146.1727	151.2701	156.3675	161.4650	166.5624
$\Lambda=44$	$\phi_0=20$	146.1878	151.2746	156.3613	161.4480	166.5348
$\Lambda=40$	$\phi_0=15$	151.3164	156.4893	161.6622	166.8351	172.0080
$\Lambda=44$	$\phi_0=15$	151.3238	156.4859	161.6481	166.8102	171.9724

4.2 Heterogeneous Firms in an Open Economy

I maintain all the setting from the model with homogeneous firms. In addition, I assume that productivity difference within sectors is less than productivity difference across sectors, so that in a three sector economy, the previous trade pattern still holds. Since we have two countries now, the aggregate demand function becomes $p(z) = \frac{2a}{\lambda^H + \lambda^F} - \frac{b}{(\lambda^H + \lambda^F)} y(z)$ and the equation from Cournot competition is

as follows:

$$p(z) - [\alpha(z) + \frac{1}{\phi}]w = \frac{b}{(\lambda^H + \lambda^F)}y(z, \phi)$$

For sectors with specialization, since only one country produces, I drop the country index for all the variables except λ^F for convenience. From the cutoff profit condition,

$$\begin{aligned} p(z) &= [\alpha(z) + \frac{1}{\phi^*}]w + \sqrt{\frac{b}{(\lambda^H + \lambda^F)}}\Lambda \\ y(z, \phi) &= \sqrt{\frac{(\lambda^H + \lambda^F)}{b}}\Lambda + \frac{(\lambda^H + \lambda^F)}{b}w\left(\frac{1}{\phi^*} - \frac{1}{\phi}\right) \end{aligned}$$

From the life time expected profit condition,

$$\int_{\phi^*}^{\infty} \frac{b}{(\lambda^H + \lambda^F)}y(z, \phi)^2\mu(\phi)d\phi = \frac{\delta M}{(1 - G(\phi^*))}$$

Substitute the formula for $y(z, \phi)$ into the above equation, and we have

$$\frac{b}{(\lambda^H + \lambda^F)}\Lambda + 2\sqrt{\frac{b}{(\lambda^H + \lambda^F)}}\Lambda\frac{\theta}{\theta + 1}\frac{w}{\phi^*} + \frac{2}{(\theta + 1)(\theta + 2)}\left(\frac{w}{\phi^*}\right)^2 = \frac{(\phi^*)^\theta}{(\phi_0)^\theta} \frac{b}{(\lambda^H + \lambda^F)}\delta M$$

We also need to know the formula for $n(z)$. Since

$$\begin{aligned} y(z) &= \int_{\phi^*}^{\infty} y(z, \phi)n(z)\mu(\phi)d\phi \\ y(z) &= \frac{2a}{b} - \sqrt{\frac{\Lambda}{b}(\lambda^H + \lambda^F)} - [\alpha(z) + \frac{1}{\phi^*}]\frac{w}{b}(\lambda^H + \lambda^F) \\ \int_{\phi^*}^{\infty} y(z, \phi)\mu(\phi)d\phi &= \sqrt{\frac{\Lambda}{b}(\lambda^H + \lambda^F)} + \frac{1}{\theta + 1}\frac{1}{\phi^*}\frac{w}{b}(\lambda^H + \lambda^F) \end{aligned}$$

we have the equation for $n(z)$ as follows:

$$n(z) = \frac{\frac{2a}{(\lambda^H + \lambda^F)} - \sqrt{\frac{b}{(\lambda^H + \lambda^F)}}\Lambda - [\alpha(z) + \frac{1}{\phi^*}]w}{\sqrt{\frac{b}{(\lambda^H + \lambda^F)}}\Lambda + \frac{1}{\theta + 1}\frac{1}{\phi^*}w}$$

Therefore

$$n(1) = \frac{\frac{2a}{(\lambda^H + \lambda^F)} - \sqrt{\frac{b}{(\lambda^H + \lambda^F)} \Lambda^H} - [\alpha(z) + \frac{1}{\phi_H^*}] w^H}{\sqrt{\frac{b}{(\lambda^H + \lambda^F)} \Lambda^H} + \frac{1}{\theta+1} \frac{1}{\phi_H^*} w^H} \quad (4.3)$$

$$n(3) = \frac{\frac{2a}{(\lambda^H + \lambda^F)} - \sqrt{\frac{b}{(\lambda^H + \lambda^F)} \Lambda^F} - [\alpha(z) + \frac{1}{\phi_F^*}] w^F}{\sqrt{\frac{b}{(\lambda^H + \lambda^F)} \Lambda^F} + \frac{1}{\theta+1} \frac{1}{\phi_F^*} w^F} \quad (4.4)$$

For sector 2 in which both countries produce, let $i = H$ and F and from the cutoff profit condition $\pi(z, \phi_i^*) = \Lambda^i$,

$$p(z) = [\alpha(z) + \frac{1}{\phi_i^*}] w^i + \sqrt{\frac{b}{(\lambda^H + \lambda^F)} \Lambda^i}$$

$$y^i(z, \phi) = \sqrt{\frac{(\lambda^H + \lambda^F)}{b} \Lambda^i} + \frac{(\lambda^H + \lambda^F)}{b} w^i \left(\frac{1}{\phi^*} - \frac{1}{\phi} \right)$$

Since the price in sector 2 must be the same for two countries, we have the following equation

$$[\alpha(2) + \frac{1}{\phi_H^*(2)}] w^H + \sqrt{\frac{b}{(\lambda^H + \lambda^F)} \Lambda^H} = [\alpha(2) + \frac{1}{\phi_F^*(2)}] w^F + \sqrt{\frac{b}{(\lambda^H + \lambda^F)} \Lambda^F}$$

From the expected life time profit condition, we have similar equations in sector 2 for both countries

$$\Lambda^H + 2\sqrt{\frac{(\lambda^H + \lambda^F)}{b} \Lambda^H} \frac{\theta}{\theta+1} \frac{w^H}{\phi_H^*} + \frac{(\lambda^H + \lambda^F)}{b} \frac{2}{(\theta+1)(\theta+2)} \left(\frac{w^H}{\phi_H^*} \right)^2 = \frac{(\phi_H^*)^\theta}{(\phi_0)^\theta} \delta M \quad (4.5)$$

$$\Lambda^F + 2\sqrt{\frac{(\lambda^H + \lambda^F)}{b} \Lambda^F} \frac{\theta}{\theta+1} \frac{w^F}{\phi_F^*} + \frac{(\lambda^H + \lambda^F)}{b} \frac{2}{(\theta+1)(\theta+2)} \left(\frac{w^F}{\phi_F^*} \right)^2 = \frac{(\phi_F^*)^\theta}{(\phi_0)^\theta} \delta M \quad (4.6)$$

We need to calculate $n^H(2)$ and $n^F(2)$. We know

$$p(z) = \frac{2a}{\lambda^H + \lambda^F} - \frac{b}{(\lambda^H + \lambda^F)} y(z)$$

$$y(z) = \int_{\phi_H^*}^{\infty} y^H(z, \phi) n^H(z) \mu^H(\phi) d\phi + \int_{\phi_F^*}^{\infty} y^F(z, \phi) n^F(z) \mu^F(\phi) d\phi$$

Substitute formulas for $p(z)$, $y^H(z, \phi)$ and $y^F(z, \phi)$ and we have

$$\begin{aligned}
[\alpha(2) + \frac{1}{\phi_H^*}]w^H + \sqrt{\frac{b}{(\lambda^H + \lambda^F)}}\Lambda^H &= -n^F(z)\left\{\sqrt{\frac{b}{(\lambda^H + \lambda^F)}}\Lambda^F + w^F\frac{1}{\theta + 1}\frac{1}{\phi_F^*}\right\} \\
&\quad -n^H(z)\left\{\sqrt{\frac{b}{(\lambda^H + \lambda^F)}}\Lambda^H + w^H\frac{1}{\theta + 1}\frac{1}{\phi_H^*}\right\} \\
&\quad + \frac{2a}{\lambda^H + \lambda^F}
\end{aligned} \tag{4.7}$$

From resources constraints for two countries, we can get additional two equations for $n^H(2)$ and $n^F(2)$.

$$n^H(2) = \frac{3L^H - \int_{\phi_H^*}^{\infty} [\alpha(1) + \frac{1}{\phi}]y^H(1, \phi)n^H(1)\mu^H(\phi)d\phi}{\int_{\phi_H^*}^{\infty} [\alpha(2) + \frac{1}{\phi}]y^H(2, \phi)\mu^H(\phi)d\phi} \tag{4.8}$$

$$n^F(2) = \frac{3L^F - \int_{\phi_F^*}^{\infty} [\alpha(3) + \frac{1}{\phi}]y^F(1, \phi)n^F(3)\mu^F(\phi)d\phi}{\int_{\phi_F^*}^{\infty} [\alpha(2) + \frac{1}{\phi}]y^F(2, \phi)\mu^H(\phi)d\phi} \tag{4.9}$$

The last equation for the open economy equilibrium is about the value of exports and the value of imports, which must be equal for each country. For country H,

$$\sum_{i=1}^3 p(i)\left(\frac{a}{b} - \frac{p(i)}{b}\lambda^H\right) = p(1)\left(\frac{2a}{b} - \frac{p(1)}{b}(\lambda^H + \lambda^F)\right) + p(2) \int_{\phi_H^*}^{\infty} y^H(2, \phi)n^H(2)\mu^H(\phi)d\phi \tag{4.10}$$

From equation (17) to (24), we can solve the equilibrium $\{w^H, w^F, \phi_H^*, \phi_F^*, \lambda^F\}$ for the above open economy when normalizing $\lambda^H = 1$. However, it is not easy to solve the equilibrium for the above open economy. I use the following numerical example to show that under certain conditions, countries have incentives to decrease its competition level in trade liberalization or in other words, trade liberalization does not necessarily increase countries' competition level.

Consider two identical countries with symmetric cost distribution. At the beginning, both countries are closed economies. After trade liberalization, country

H increases its Λ^H and adjust its M to keep ϕ_H^* constant while country F does nothing. For the aggregate demand in each sector, still assume $a = 500$ and $b = 1$. For the cost distribution across sectors, in country H, $\alpha^H(1) = 0.75$, $\alpha^H(2) = 1$ and $\alpha^H(3) = 1.25$ while in country F, $\alpha^F(1) = 1.25$, $\alpha^F(2) = 1$ and $\alpha^F(3) = 0.75$. For the underlying Pareto distribution of cost within sectors, let $\phi_0 = 100$, which makes the model with heterogenous firms close to the model with homogeneous firms and $\theta = 3$. The factor endowment for this economy $L = 100$. For the expected life time profit $M = 800$ as before and the death rare for firms is $\delta = 0.025$. I assume after trade liberalization, country H increases its Λ^H from 40 to 100. Before trade liberalization, the welfare for both countries is 47943. After trade liberalization and before country H increases its Λ^H , both countries' welfare increases to 53953. When country H increases its Λ^H from 40 to 100, country H's welfare increases to 53959 while country F's welfare decreases to 53945³

The above result depends on values of $\alpha(1)$, $\alpha(2)$ and $\alpha(3)$. When they are relatively large, the increase of welfare for country H is relatively significant. For example, when $\alpha(1) = 1$, $\alpha(2) = 1.25$ and $\alpha(3) = 1.5$, country H's welfare increases from 42694 to 42702 while country F's welfare decreases to 42685 if country H increases its Λ^H from 40 to 100. When $\alpha(1) = 0.6$, $\alpha(2) = 0.8$ and $\alpha(3) = 1$, country H's welfare increases from 64588 to 64592 while country F's welfare decreases to 64580 if country H increases its Λ^H from 40 to 100. However, when $\alpha(1) = 0.3$, $\alpha(2) = 0.4$ and $\alpha(3) = 0.5$, both countries' welfare decrease if country H increases its Λ^H from 40 to 100. For the parameter ϕ_0 , the above result still holds when ϕ_0 decreases from 100 to 25. When ϕ_0 is less than 25 and country H increases its Λ^H , both countries' welfare decrease. In the above example, to make country

³ The change is not big since the change of competition level depends on $\sqrt{\Lambda}$, which increases from 6.32 to 10.

H's welfare increase, inter-sector difference must be larger than intra-sector difference. For another example with $\alpha(1) = 0.95$, $\alpha(2) = 1$ and $\alpha(3) = 1.05$, when ϕ_0 is larger than 50, which makes intra-sector difference less than 0.02, the above result holds.

In summary, countries still have incentives to decrease its competition level during trade liberalization in order to get higher welfare under certain conditions when firms are heterogeneous within sectors although the result is not as strong as the result with homogeneous firms. There are several reasons for this weaker result. One possible reason is the way the competition level is changed in the model. To change the competition level in the economy, I not only change Λ but also adjust the expected life time profit M . Although it eliminates the selection effect, it adds noise to the system by changing M . Another reason is the way the change of competition level is modeled. The change of competition level does not only effect profit level but also the shape of firm size distribution.

Chapter 5

Conclusion and Discussion

This thesis extends Neary (2003, 2009). Instead of assuming a fixed number of firms in each sector and changing competition level through changing the number of firms¹, the number of firms in this thesis is determined by the level of price cost margin, which is controlled by the competition policy in the economy. In the model, a decrease in competition level reallocates factor from low cost sectors to high cost sectors and decreases countries' welfare. In an open economy with trade liberalization, under certain conditions, the previous reallocation will give countries incentives to adopt beggar-thy-neighbor competition policies, which decrease competition level in order to maximize its own welfare while sacrificing its trade partner's welfare. It also means under certain conditions competition could be a disadvantage during trade liberalization. These results are not only valid when firms in each sector are homogeneous but also when firms in each sector are heterogeneous.

The model predicts, as shown in the numerical example in chapter 3, trade

¹ If there is no barrier to entry, firms in every sector should have the same profit level and the number of firms in every sector should be different. I can also show that more firms does not necessarily mean higher competition.

liberalization can increase competition level. However, after trade liberalization, the pro-competitive effect of trade liberalization depends on trade partners. When trade partner's marginal cost distribution is similar, countries have incentives to increase competition level while when trade partner's marginal cost distribution is different, countries have incentives to decrease competition level. It can explain why trade liberalization does not always bring more competition and empirical evidence in Chen, Imbs and Scott (2009) for European countries and Goldar and Aggarwal (2005) for India, which shows strong pro-competitive effect of trade liberalization in the short run while ambiguous or even anti-competitive effect of trade liberalization in the long run. Since the pro-competitive effect of trade liberalization depends on trade partners, it can also explain empirical evidence in Boulhol, Dobbelaere and Maioli (2011) which applies Hall's method to 9,820 firms in UK from 1988 to 2003 and finds pro-competitive effect when imports come from developed countries while the effect is not significant when imports come from developing countries² .

For future study, one interesting thing is to bring product horizontal differentiation into the model. The model with heterogeneous firms in the thesis is an extreme case of Melitz and Ottaviano (2008) with a continuum of sectors. Product horizontal differentiation will be a big improvement to the model. The other interesting thing is about firms' entry and exit strategies. In this paper, I simply use the entry and exit strategy of firms from Hopenhayn (1992), which makes the case with heterogeneous firms very complicated.

² In this thesis, I also show that sectors with increasing PCMs and high import penetration ratios in US manufacturing industry tend to be labor intensive, which can be explained by the model.

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Appendix A

Appendix to Chapter 2

Table A.1: Survey of the pro-competitive effect of trade liberalization

Study	Country	Aggregation level	Period	Measurement	Main results
Esposito and Esposito (1971)	USA	77 industries	1963-1965	Profit rate	Pro-competitive effect
Domowitz, Hubbard and Petersen (1986)	USA	265 4-digit sectors	1958-1981	PCM	FE: positive impact; OLS: positive for the least concentrated sectors.
Canyon and Machin (1991)	UK	3-digit	1983-1986	PCM	Pro-competitive effect

Katics and Petersen (1994)	and USA	131 4-digit	1964-1986	PCM	Pro-competitive effect	
Chen, and (2009)	Imbs and Scott	7 euro countries	10 2-digit	1988-2000	PCM	Pro-competitive effect in the short run; In the long run, anti or ambiguous.
Raff and Wagner (2010)	and Germany	Firms in clothing industry	2000-2006	PCM	Pro-competitive effect in the short run; In the long run, anti or ambiguous.	
Kee and Hoekman (2007)	and 42 countries	28 3-digit	1981-1998	Hall	Pro-competitive effect	
Konings, Van Cayseele and Warzynski (2002)	Van Belgium and Netherlands	4700 firms	1992-1997	Hall	Not significant in Belgium and anti in Netherlands.	

Boulhol, Dobbelaere and Maioli (2011)	UK	9820 firms	1988- 2003	Hall	Pro-competitive effect when imports come from developed countries; not significant from developing
Krishna and Mitra (1998)	India	460 firms	1986- 1993	Hall	Pro in 3 sectors and anti in one sector.
Balakrishnan, Pushpan- gadan, and Suresh Babu (2002)	India	3596 firms	1988- 1998	Hall	Pro in some sec- tors and anti in other sectors.
Goldar and Aggarwal (2005)	India	137 3-digit	1980- 1998	PCM	Pro-competitive effect in the short run; In the long run, PCM increases.
Yalcin (2000)	Turkey	4-digit	1983- 1994	PCM	Pro in some sec- tors and anti in other sectors.

Aldaba (2008)	Philippine 18 firms	cement	1994- 2003	PCM	Trade liberaliza- tion is not a sub- stitute for com- petition policy.
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Table A.2: 1987 SIC codes for U.S. manufacturing industry

SIC code	Industry description
20	FOOD AND KINDRED PRODUCTS
21	TOBACCO PRODUCTS
22	TEXTILE MILL PRODUCTS
23	APPAREL AND OTHER FINISHED PRODUCTS MADE FROM FABRICS AND SIMILAR MATERIALS
24	LUMBER AND WOOD PRODUCTS, EXCEPT FURNITURE
25	FURNITURE AND FIXTURES
26	PAPER AND ALLIED PRODUCTS
27	PRINTING, PUBLISHING, AND ALLIED INDUSTRIES
28	CHEMICALS AND ALLIED PRODUCTS
29	PETROLEUM REFINING AND RELATED INDUSTRIES
30	RUBBER AND MISCELLANEOUS PLASTICS PRODUCTS
31	LEATHER AND LEATHER PRODUCTS
32	STONE, CLAY, GLASS, AND CONCRETE PRODUCTS
33	PRIMARY METAL INDUSTRIES
34	FABRICATED METAL PRODUCTS, EXCEPT MACHINERY AND TRANSPORTATION EQUIPMENT
35	INDUSTRIAL AND COMMERCIAL MACHINERY AND COMPUTER EQUIPMENT
36	ELECTRONIC AND OTHER ELECTRICAL EQUIPMENT AND COMPONENTS, EXCEPT COMPUTER EQUIPMENT
37	TRANSPORTATION EQUIPMENT
38	MEASURING, ANALYZING AND CONTROLLING INSTRUMENTS; PHOTOGRAPHIC, MEDICAL AND OPTICAL GOODS; WATCHES AND CLOCKS
39	JEWELRY, SILVERWARE, AND PLATED WARE

Appendix B

Appendix to Chapter 3

B.1 Closed Economy

If I can prove the case with $\beta > 0$, the case with $\beta = 0$ is just a special case.

Proof. When $l(z) = \beta + \alpha(z)y(z)$, By market clear conditions for labor market, the wage rate is as follows:

$$w = \frac{(\frac{ab\beta}{M} - M\mu_\alpha + a\mu_\alpha - bL - b\beta)M}{b\beta\mu_\alpha + M\sigma_\alpha^2}$$
$$\Rightarrow w = \left(\frac{a}{\mu_\alpha} - \frac{M}{\mu_\alpha}\right)\left(1 - \frac{\sigma_\alpha^2 - \mu_\alpha^2}{\frac{b\beta\mu_\alpha}{M} + \sigma_\alpha^2}\right) - \frac{bL}{\frac{b\beta\mu_\alpha}{M} + \sigma_\alpha^2}$$

Since $\sigma_\alpha^2 > \mu_\alpha^2$ and $M = by(z) < a$ from equation (1), wage rate decreases as M increases.

Notice that $p(z) = w\alpha(z) + M$, so $\frac{p(z)}{w} = \alpha(z) + \frac{M}{w}$ and it is obvious that when M decreases, $\frac{p(z)}{w}$ decreases.

$$p(z) = \alpha(z)\left(\frac{a}{\mu_\alpha} - \frac{M}{\mu_\alpha}\right)\left(1 - \frac{\sigma_\alpha^2 - \mu_\alpha^2}{\frac{b\beta\mu_\alpha}{M} + \sigma_\alpha^2}\right) - \frac{\alpha(z)bL}{\frac{b\beta\mu_\alpha}{M} + \sigma_\alpha^2} + M$$
$$p(z) = \alpha(z)\frac{a}{\mu_\alpha}\left(1 - \frac{\sigma_\alpha^2 - \mu_\alpha^2}{\frac{b\beta\mu_\alpha}{M} + \sigma_\alpha^2}\right) - \frac{\alpha(z)bL + b\beta\alpha(z)\left(1 - \frac{\mu_\alpha^2}{\sigma_\alpha^2}\right)}{\frac{b\beta\mu_\alpha}{M} + \sigma_\alpha^2} - M\left(\frac{\mu_\alpha\alpha(z)}{\sigma_\alpha^2} - 1\right)$$

Therefore if $\alpha(z) \geq \frac{\sigma_\alpha^2}{\mu_\alpha}$, $p(z)$ decreases as M increases. From equation (1), the total output in each sector $Y(z) = n(z)y(z)$ decreases.

If $\alpha(z) < \frac{\sigma_\alpha^2}{\mu_\alpha}$, we rearrange the above equation for $p(z)$,

$$p(z) = \alpha(z) \frac{a}{\mu_\alpha} - \frac{\alpha(z)bL + (\frac{a}{\mu_\alpha} + \frac{b\beta}{\sigma_\alpha^2})\alpha(z)(\sigma_\alpha^2 - \mu_\alpha^2)}{\frac{b\beta\mu_\alpha}{M} + \sigma_\alpha^2} + M(1 - \frac{\mu_\alpha\alpha(z)}{\sigma_\alpha^2})$$

Split $p(z)$ into two part. Let $f(M) = \alpha(z) \frac{a}{\mu_\alpha} + M(1 - \frac{\mu_\alpha\alpha(z)}{\sigma_\alpha^2})$ and $g(M) = \frac{\alpha(z)bL + (\frac{a}{\mu_\alpha} + \frac{b\beta}{\sigma_\alpha^2})\alpha(z)(\sigma_\alpha^2 - \mu_\alpha^2)}{\frac{b\beta\mu_\alpha}{M} + \sigma_\alpha^2}$. Therefore $p(z) = f(M) - g(M)$. $f(M)$ is a upward sloping line with $f(M) = \alpha(z) \frac{a}{\mu_\alpha}$ as $M = 0$ while $g(M)$ is concave curve with $g(M) = 0$ as $M = 0$. For some $\alpha(z)$, when $M = 0$, the slope of $f(M)$ is always larger than the slope of $g(M)$. That is,

$$\begin{aligned} & \frac{\alpha(z)bL + (\frac{a}{\mu_\alpha} + \frac{b\beta}{\sigma_\alpha^2})\alpha(z)(\sigma_\alpha^2 - \mu_\alpha^2)}{b\beta\mu_\alpha} \leq 1 - \frac{\mu_\alpha\alpha(z)}{\sigma_\alpha^2} \\ \Rightarrow & \frac{\alpha(z)bL + \frac{a}{\mu_\alpha}\alpha(z)(\sigma_\alpha^2 - \mu_\alpha^2)}{b\beta\mu_\alpha} + \frac{b\beta\alpha(z)(1 - \frac{\mu_\alpha^2}{\sigma_\alpha^2})}{b\beta\mu_\alpha} \leq 1 - \frac{\mu_\alpha\alpha(z)}{\sigma_\alpha^2} \\ \Rightarrow & \frac{\alpha(z)bL + \frac{a}{\mu_\alpha}\alpha(z)(\sigma_\alpha^2 - \mu_\alpha^2)}{b\beta\mu_\alpha} + \frac{\alpha(z)}{\mu_\alpha} - \frac{\mu_\alpha\alpha(z)}{\sigma_\alpha^2} \leq 1 - \frac{\mu_\alpha\alpha(z)}{\sigma_\alpha^2} \\ \Rightarrow & \alpha(z) \leq \frac{b\beta\mu_\alpha}{bL + b\beta + \frac{a}{\mu_\alpha}(\sigma_\alpha^2 - \mu_\alpha^2)} \end{aligned}$$

It is obvious that $\frac{\sigma_\alpha^2}{\mu_\alpha} > \frac{b\beta\mu_\alpha}{bL + b\beta + \frac{a}{\mu_\alpha}(\sigma_\alpha^2 - \mu_\alpha^2)}$. Hence when $\alpha(z) \leq \frac{b\beta\mu_\alpha}{bL + b\beta + \frac{a}{\mu_\alpha}(\sigma_\alpha^2 - \mu_\alpha^2)}$, $p(z)$ increases as M increases. For $\alpha(z)$ between these two values, when M increases, $p(z)$ will decrease first for sure. It is also possible for some $\alpha(z)$, $p(z)$ will increase in the end and follows a "U" shape curve. Actually, I can further show that if $\alpha(z) \geq \frac{(b\beta\mu_\alpha + a\sigma_\alpha^2)^2}{\frac{\mu_\alpha}{\sigma_\alpha^2}(b\beta\mu_\alpha + a\sigma_\alpha^2)^2 + [bL + (\frac{a}{\mu_\alpha} + \frac{b\beta}{\sigma_\alpha^2})(\sigma_\alpha^2 - \mu_\alpha^2)]b\beta\mu_\alpha}$, $p(z)$ will decrease if M increases.

In the end, I prove that If $n(z) \geq 1$ and M increases, $\frac{\Pi}{wL}$ will increase. I want to show that If M increases, Π will increase and therefore $\frac{\Pi}{wL}$ will increase.

$$\Pi = \int_0^1 n(z)y(z)p(z)dz - wL = \frac{1}{b} \int_0^1 p(z)(a - p(z))dz - wL$$

From the labor market clear condition,

$$L = \int_0^1 n(z)l(z)dz = \int_0^1 n(z)\beta dz + \int_0^1 \left(\frac{a}{b} - \frac{p(z)}{b}\right)\alpha(z)dz$$

Since $\alpha(z) = \frac{p(z)-M}{w}$,

$$wL = w \int_0^1 n(z)\beta dz + \frac{1}{b} \int_0^1 (p(z) - M)(a - p(z))dz$$

Hence, total profit is

$$\Pi = \frac{1}{b} \int_0^1 M(a - p(z))dz - w \int_0^1 n(z)\beta dz$$

I have already known that when M increases, w decreases. It is also not difficult to show that $\int_0^1 n(z)dz$ decreases as M increases.

$$\begin{aligned} \int_0^1 n(z)dz &= \frac{a}{M} - \frac{w\mu_\alpha}{M} - 1 \\ &= \frac{a\sigma_\alpha^2 + M\mu_\alpha^2 - a\mu_\alpha^2 + (bL + b\beta)\mu_\alpha}{b\beta\mu_\alpha + M\sigma_\alpha^2} - 1 \\ &= \frac{\mu_\alpha bL + a(\sigma_\alpha^2 - \mu_\alpha^2) - M(\sigma_\alpha^2 - \mu_\alpha^2)}{b\beta\mu_\alpha + M\sigma_\alpha^2} \end{aligned}$$

I only need to show that $\int_0^1 M(a - p(z))dz$ increases as M increases. Let $y(M) = \int_0^1 M(a - p(z))dz$,

$$\begin{aligned} y(M) &= \frac{\mu_\alpha bL + (a - M)(\sigma_\alpha^2 - \mu_\alpha^2)}{\frac{b\beta\mu_\alpha}{M} + \sigma_\alpha^2} M \\ &= \frac{[\mu_\alpha bL + a(\sigma_\alpha^2 - \mu_\alpha^2) - M(\sigma_\alpha^2 - \mu_\alpha^2)]M^2}{b\beta\mu_\alpha + \sigma_\alpha^2 M} \end{aligned}$$

From $M = \frac{a-w\alpha(z)}{n(z)+1}$, for $n(z) \geq 1$, $0 < M < \frac{a}{2}$. I want to show that if $0 < M < \frac{a}{2}$, $\frac{\partial y(M)}{\partial M} > 0$.

$$\begin{aligned} \frac{\partial y(M)}{\partial M} &= \frac{M[-2M^2\sigma_\alpha^2(\sigma_\alpha^2 - \mu_\alpha^2) + 2b\beta\mu_\alpha(\mu_\alpha bL + a(\sigma_\alpha^2 - \mu_\alpha^2))]}{(b\beta\mu_\alpha + \sigma_\alpha^2 M)^2} \\ &\quad + \frac{M[M(\sigma_\alpha^2(\mu_\alpha bL + a(\sigma_\alpha^2 - \mu_\alpha^2)) - 3b\beta\mu_\alpha(\sigma_\alpha^2 - \mu_\alpha^2))]}{(b\beta\mu_\alpha + \sigma_\alpha^2 M)^2} \end{aligned}$$

It is easy to show that for $0 < M < \frac{a}{2}$, $\frac{\partial y(M)}{\partial M} > 0$. Therefore, when $n(z) \geq 1$, if M increases, $\frac{\Pi}{wL}$ will increase. ■

B.2 Open Economy

First, calculate the income and expenditure equation for country H :

$$p(1)\left(\frac{a'}{b'} - \frac{p(1)}{b'}\right) + p(2)\left\{\frac{1}{\alpha(2)}[3L^H - \alpha(1)\left(\frac{a'}{b'} - \frac{p(1)}{b'}\right)]\right\} = \sum_i p(i)\left(\frac{a}{b} - \frac{p(i)}{b}\right)$$

$$\Rightarrow 2aM^H\left(1 - \frac{\alpha(1)}{\alpha(2)}\right) - M^H(1 + \lambda^F)\left(1 - \frac{\alpha(1)}{\alpha(2)}\right)p(1) + p(2)\frac{3bL^H}{\alpha(2)} = a\sum_i p(i) - \sum_i p^2(i)$$

For the right hand side of the above equation, I calculated $\sum_i p(i)$ and $\sum_i p^2(i)$. I also notice that $\sum_i p^2(i)$ is also in the country H 's welfare function. For $\sum_i p(i)$,

$$\begin{aligned} \sum_i p(i) &= \frac{\sum_i \alpha(i)}{\sum_i \alpha^2(i)} [2a \sum_i \alpha(i) - 3b(L^H + L^F)] \frac{1}{1 + \lambda^F} \\ &\quad - M^F \frac{\alpha(3) - \alpha(2)}{\alpha(2) \sum_i \alpha^2(i)} \left\{ \sum_i \alpha^2(i) - \alpha(3) \sum_i \alpha(i) \right\} \\ &\quad + M^H \frac{\alpha(2) - \alpha(1)}{\alpha(2) \sum_i \alpha^2(i)} [\alpha^2(2) + \alpha^2(3) - \alpha(1)\alpha(2) - \alpha(1)\alpha(3)] \end{aligned}$$

For $\sum_i p^2(i)$,

$$\begin{aligned} \sum_i p^2(i) &= \frac{1}{\sum_i \alpha^2(i)} (2a \sum_i \alpha(i) - 3b(L^H + L^F))^2 \frac{1}{(1 + \lambda^F)^2} \\ &\quad + (M^H)^2 \frac{[\alpha(2) - \alpha(1)]^2 [\alpha^2(2) + \alpha^2(3)]}{\alpha^2(2) \sum_i \alpha^2(i)} \\ &\quad + (M^F)^2 \frac{[\alpha(3) - \alpha(2)]^2 [\alpha^2(1) + \alpha^2(2)]}{\alpha^2(2) \sum_i \alpha^2(i)} \\ &\quad - 2M^H M^F \frac{1}{\sum_i \alpha^2(i)} \left[\frac{\alpha(2)\alpha(1) - \alpha^2(1)}{\alpha(2)} \right] \left[\frac{\alpha(2)\alpha(3) - \alpha^2(3)}{\alpha(2)} \right] \end{aligned}$$

The left hand side of the income and expenditure equation is

$$\begin{aligned}
Left = & 2aM^H \frac{[\alpha(2) - \alpha(1)]}{\alpha(2) \sum_i \alpha^2(i)} \left[\sum_i \alpha^2(i) - \alpha(1) \sum_i \alpha(i) \right] \\
& - (M^H)^2 (1 + \lambda^F) \frac{[\alpha(2) - \alpha(1)]^2 [\alpha^2(2) + \alpha^2(3)]}{\alpha^2(2) \sum_i \alpha^2(i)} \\
& + \frac{3bL^F M^H}{\sum_i \alpha^2(i)} \left[\frac{\alpha(2)\alpha(1) - \alpha^2(1)}{\alpha(2)} \right] - \frac{3bL^H M^F}{\sum_i \alpha^2(i)} \left[\frac{\alpha(2)\alpha(3) - \alpha^2(3)}{\alpha(2)} \right] \\
& + \frac{3bL^H}{\sum_i \alpha^2(i)} \left[2a \sum_i \alpha(i) - 3b(L^H + L^F) \right] \frac{1}{1 + \lambda^F} \\
& + M^H M^F (1 + \lambda^F) \frac{1}{\sum_i \alpha^2(i)} \left[\frac{\alpha(2)\alpha(1) - \alpha^2(1)}{\alpha(2)} \right] \left[\frac{\alpha(2)\alpha(3) - \alpha^2(3)}{\alpha(2)} \right]
\end{aligned}$$

Integrate the left and right hand side of the equation and multiply both sides by $\sum_i \alpha^2(i)$.

$$\begin{aligned}
& aM^H \left[\frac{\alpha(2) - \alpha(1)}{\alpha(2)} \right] \left[\sum_i \alpha^2(i) - \alpha(1) \sum_i \alpha(i) \right] + aM^F \left[\frac{\alpha(3) - \alpha(2)}{\alpha(2)} \right] \left[\sum_i \alpha^2(i) - \alpha(3) \sum_i \alpha(i) \right] \\
& + 3bL^F M^H \left[\frac{\alpha(2)\alpha(1) - \alpha^2(1)}{\alpha(2)} \right] - 3bL^H M^F \left[\frac{\alpha(2)\alpha(3) - \alpha^2(3)}{\alpha(2)} \right] + M^H M^F (\lambda^F - \\
& \quad 1) \left[\frac{\alpha(2)\alpha(1) - \alpha^2(1)}{\alpha(2)} \right] \left[\frac{\alpha(2)\alpha(3) - \alpha^2(3)}{\alpha(2)} \right] \\
& + [3bL^H - a \sum_i \alpha(i)] \left[2a \sum_i \alpha(i) - 3b(L^H + L^F) \right] \frac{1}{1 + \lambda^F} + (2a \sum_i \alpha(i) - 3b(L^H + \\
& \quad L^F))^2 \frac{1}{(1 + \lambda^F)^2} \\
& - (M^H)^2 \frac{[\alpha(2) - \alpha(1)]^2 [\alpha^2(2) + \alpha^2(3)]}{\alpha^2(2)} \lambda^F + (M^F)^2 \frac{[\alpha(3) - \alpha(2)]^2 [\alpha^2(1) + \alpha^2(2)]}{\alpha^2(2)} = 0
\end{aligned}$$

In the following analysis, to make things simple, I always assume that $L^H = L^F$

Case 1: Consider fully symmetric case $M^{H^*} = M^F$ and $\lambda^{F^*} = 1$. also $\alpha(1) = \alpha(3)$

Differentiate the above equation by M^H and λ^F :

$$a \left[\frac{\alpha(2) - \alpha(1)}{\alpha(2)} \right] \left[\sum_i \alpha^2(i) - \alpha(1) \sum_i \alpha(i) \right] \Delta^M + 3bL \left[\frac{\alpha(2)\alpha(1) - \alpha^2(1)}{\alpha(2)} \right] \Delta^M$$

$$\begin{aligned}
& -(M^{H^*})^2 \frac{[\alpha(2)-\alpha(1)]^2[\alpha^2(2)+\alpha^2(3)]}{\alpha^2(2)} \Delta^\lambda - 2M^{H^*} \Delta^M \frac{[\alpha(2)-\alpha(1)]^2[\alpha^2(2)+\alpha^2(3)]}{\alpha^2(2)} \lambda^{F^*} \\
& - [3bL - a \sum_i \alpha(i)] [2a \sum_i \alpha(i) - 6bL] \frac{1}{(1+\lambda^{F^*})^2} \Delta^\lambda - (2a \sum_i \alpha(i) - 6bL)^2 \frac{2}{(1+\lambda^{F^*})^3} \Delta^\lambda \\
& + M^F (\lambda^{F^*} - 1) \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] \left[\frac{\alpha(2)\alpha(3)-\alpha^2(3)}{\alpha(2)} \right] \Delta^M + \\
& M^{H^*} M^F \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] \left[\frac{\alpha(2)\alpha(3)-\alpha^2(3)}{\alpha(2)} \right] \Delta^\lambda = 0
\end{aligned}$$

Since $M^{H^*} = M^F$ and $\lambda^{F^*} = 1$,

$$\begin{aligned}
& a \left[\frac{\alpha(2)-\alpha(1)}{\alpha(2)} \right] [\sum_i \alpha^2(i) - \alpha(1) \sum_i \alpha(i)] \Delta^M + 3bL \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] \Delta^M \\
& - (M^{H^*})^2 \frac{[\alpha(2)-\alpha(1)]^2[\alpha^2(2)+\alpha^2(3)]}{\alpha^2(2)} \Delta^\lambda - 2M^{H^*} \Delta^M \frac{[\alpha(2)-\alpha(1)]^2[\alpha^2(2)+\alpha^2(3)]}{\alpha^2(2)} \\
& - \frac{1}{2} (a \sum_i \alpha(i) - 3bL)^2 \Delta^\lambda + (M^{H^*})^2 \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] \left[\frac{\alpha(2)\alpha(3)-\alpha^2(3)}{\alpha(2)} \right] \Delta^\lambda = 0
\end{aligned}$$

By $\alpha(1) = \alpha(3)$,

$$\begin{aligned}
& \left\{ a \left[\frac{\alpha(2)-\alpha(1)}{\alpha(2)} \right] [\sum_i \alpha^2(i) - \alpha(1) \sum_i \alpha(i)] - 2M^{H^*} \frac{[\alpha(2)-\alpha(1)]^2[\alpha^2(2)+\alpha^2(3)]}{\alpha^2(2)} \right\} \Delta^M \\
& + 3bL \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] = \left\{ \frac{1}{2} (a \sum_i \alpha(i) - 3bL)^2 + (M^{H^*})^2 [\alpha(2) - \alpha(1)]^2 \right\} \Delta^\lambda
\end{aligned}$$

To calculate the change of country H 's welfare, only the change of $\sum_i p^2(i)$ matters.

$$\begin{aligned}
\Delta \{ \sum_i p^2(i) \} &= - \frac{1}{\sum_i \alpha^2(i)} (2a \sum_i \alpha(i) - 3b(L^H + L^F))^2 \frac{2}{(1+\lambda^{F^*})^3} \Delta^\lambda \\
& + 2M^{H^*} \frac{[\alpha(2)-\alpha(1)]^2[\alpha^2(2)+\alpha^2(3)]}{\alpha^2(2) \sum_i \alpha^2(i)} \Delta^M - 2M^F \frac{1}{\sum_i \alpha^2(i)} \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] \left[\frac{\alpha(2)\alpha(3)-\alpha^2(3)}{\alpha(2)} \right] \Delta^M
\end{aligned}$$

Since $M^{H^*} = M^F$, $\lambda^{F^*} = 1$ and $\alpha(1) = \alpha(3)$,

$$\sum_i \alpha^2(i) \Delta \{ \sum_i p^2(i) \} = -(a \sum_i \alpha(i) - 3bL)^2 \Delta^\lambda + 2M^{H^*} [\alpha(2) - \alpha(1)]^2 \Delta^M$$

Substitute the differentiated income and expenditure equation,

$$\begin{aligned}
\frac{\sum_i \alpha^2(i) \Delta \{\sum_i p^2(i)\}}{\Delta^M} &= -(a \sum_i \alpha(i) - 3bL)^2 \left\{ a \left[\frac{\alpha(2) - \alpha(1)}{\alpha(2)} \right] [\sum_i \alpha^2(i) - \alpha(1) \sum_i \alpha(i)] \right. \\
&\quad \left. + 3bL \left[\frac{\alpha(2)\alpha(1) - \alpha^2(1)}{\alpha(2)} \right] - 2M^{H^*} \frac{[\alpha(2) - \alpha(1)]^2 [\alpha^2(2) + \alpha^2(3)]}{\alpha^2(2)} \right\} \\
&\quad + \left\{ \frac{1}{2} (a \sum_i \alpha(i) - 3bL)^2 + (M^{H^*})^2 [\alpha(2) - \alpha(1)]^2 \right\} 2M^{H^*} [\alpha(2) - \alpha(1)]^2 \\
\Rightarrow \frac{\sum_i \alpha^2(i) \Delta \{\sum_i p^2(i)\}}{\Delta^M} &= -(a \sum_i \alpha(i) - 3bL)^2 \left\{ a \left[\frac{\alpha(2) - \alpha(1)}{\alpha(2)} \right] \alpha^2(2) + 3bL \left[\frac{\alpha(2)\alpha(1) - \alpha^2(1)}{\alpha(2)} \right] \right. \\
&\quad \left. - 2M^{H^*} \frac{[\alpha(2) - \alpha(1)]^2 [\alpha^2(2) + \alpha^2(3)]}{\alpha^2(2)} - M^{H^*} [\alpha(2) - \alpha(1)]^2 \right\} + 2(M^{H^*})^3 [\alpha(2) - \alpha(1)]^4
\end{aligned}$$

When M^{H^*} is small, the sign for the above equation is definitely negative. Since $Welfare^H = \frac{a^2}{2b} - \frac{1}{6b} \sum_i p^2(i)$, when M^{H^*} is small, the welfare for country H will increase if M^H increases.

Assume $\alpha(2) - \alpha(1) = \alpha(2) - \alpha(3) = x$, we know that $a \sum_i \alpha(i) - 3bL - M^{H^*} \sum_i \alpha(i) > 0$, so the above equation is less than the following equation:

$$\begin{aligned}
&-(a \sum_i \alpha(i) - 3bL)^2 \left\{ a \left[\frac{\alpha(2) - \alpha(1)}{\alpha(2)} \right] \alpha^2(2) + 3bL \left[\frac{\alpha(2)\alpha(1) - \alpha^2(1)}{\alpha(2)} \right] \right. \\
&\quad \left. - 2M^{H^*} \frac{[\alpha(2) - \alpha(1)]^2 [\alpha^2(2) + \alpha^2(3)]}{\alpha^2(2)} - M^{H^*} [\alpha(2) - \alpha(1)]^2 \right\} \\
&\quad + 2(M^{H^*}) (a \sum_i \alpha(i) - 3bL)^2 \frac{[\alpha(2) - \alpha(1)]^4}{(\sum_i \alpha(i))^2} \\
&= -(a \sum_i \alpha(i) - 3bL)^2 \left\{ a \left[\frac{\alpha(2) - \alpha(1)}{\alpha(2)} \right] \alpha^2(2) + 3bL \left[\frac{\alpha(2)\alpha(1) - \alpha^2(1)}{\alpha(2)} \right] \right. \\
&\quad \left. - 2M^{H^*} \frac{[\alpha(2) - \alpha(1)]^2 [\alpha^2(2) + \alpha^2(3)]}{\alpha^2(2)} - M^{H^*} [\alpha(2) - \alpha(1)]^2 - 2M^{H^*} \frac{[\alpha(2) - \alpha(1)]^4}{(\sum_i \alpha(i))^2} \right\}
\end{aligned}$$

Since $M^{H^*} < a$, it is not difficult to show that when x is small, for example $x < \frac{1}{6}\alpha(2)$, the above equation is always negative, which means that if tech in full specialization sector is close to the tech in diversification sector, country H always has an incentive to increase M . If x is large and M^{H^*} is large enough, it is also possible that welfare for country H increases if M^H decreases.

Two country now is in the condition of Prisoner's dilemma. To see it, when $M^{H^*} = M^F$ and $\lambda^{F^*} = 1$

$$\begin{aligned}
\sum_i p^2(i) &= \frac{1}{\sum_i \alpha^2(i)} (2a \sum_i \alpha(i) - 3b(L^H + L^F))^2 \frac{1}{(1+\lambda^F)^2} \\
&+ (M^H)^2 \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(2)+\alpha^2(3)]}{\alpha^2(2) \sum_i \alpha^2(i)} + (M^F)^2 \frac{[\alpha(3)-\alpha(2)]^2 [\alpha^2(1)+\alpha^2(2)]}{\alpha^2(2) \sum_i \alpha^2(i)} \\
&- 2M^H M^F \frac{1}{\sum_i \alpha^2(i)} \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] \left[\frac{\alpha(2)\alpha(3)-\alpha^2(3)}{\alpha(2)} \right] \\
\Rightarrow \sum_i p^2(i) &= \frac{1}{\sum_i \alpha^2(i)} (a \sum_i \alpha(i) - 3bL)^2 + 2(M^H)^2 \frac{[\alpha(2)-\alpha(1)]^2}{\sum_i \alpha^2(i)}
\end{aligned}$$

When both countries' M increases, the welfare for both countries decreases. When only one country's M increases, the country with larger M will have higher welfare while the country lower M will have less welfare. Two countries need cooperation in competition policy to avoid this dilemma.

In the following part, at least additional assumptions for tech is needed to make sure that trade pattern is valid.

Case 2: Consider the case where $\alpha(1) > \alpha(2) > \alpha(3)$. assume that $\alpha(2) - \alpha(1) = \alpha(3) - \alpha(2)$. Also assume that at the beginning M level is the same for both countries. $M^{H*} = M^F$ but we also should tell whether $\lambda^{F*} > 1$ or $\lambda^{F*} < 1$.

a) Welfare related part:

$$\begin{aligned}
\Delta\{\sum_i p^2(i)\} &= -\frac{1}{\sum_i \alpha^2(i)} (2a \sum_i \alpha(i) - 3b(L^H + L^F))^2 \frac{2}{(1+\lambda^{F*})^3} \Delta^\lambda \\
&+ 2M^{H*} \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(2)+\alpha^2(3)]}{\alpha^2(2) \sum_i \alpha^2(i)} \Delta^M - 2M^F \frac{1}{\sum_i \alpha^2(i)} \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] \left[\frac{\alpha(2)\alpha(3)-\alpha^2(3)}{\alpha(2)} \right] \Delta^M
\end{aligned}$$

Simplify the above equation by assumptions:

$$\begin{aligned}
\sum_i \alpha^2(i) \Delta\{\sum_i p^2(i)\} &= -(a \sum_i \alpha(i) - 3bL)^2 \frac{8}{(1+\lambda^{F*})^3} \Delta^\lambda \\
&+ 2M^{H*} \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(2)+\alpha^2(3)+\alpha(1)\alpha(3)]}{\alpha^2(2)} \Delta^M
\end{aligned}$$

b) income and expenditure equation:

$$\begin{aligned}
& aM^H \left[\frac{\alpha(2)-\alpha(1)}{\alpha(2)} \right] [\sum_i \alpha^2(i) - \alpha(1) \sum_i \alpha(i)] + aM^F \left[\frac{\alpha(3)-\alpha(2)}{\alpha(2)} \right] [\sum_i \alpha^2(i) - \alpha(3) \sum_i \alpha(i)] \\
& \quad + 3bL^F M^H \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] - 3bL^H M^F \left[\frac{\alpha(2)\alpha(3)-\alpha^2(3)}{\alpha(2)} \right] \\
& \quad - (M^H)^2 \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(2)+\alpha^2(3)]}{\alpha^2(2)} \lambda^F + (M^F)^2 \frac{[\alpha(3)-\alpha(2)]^2 [\alpha^2(1)+\alpha^2(2)]}{\alpha^2(2)} \\
& \quad + [3bL^H - a \sum_i \alpha(i)] [2a \sum_i \alpha(i) - 3b(L^H + L^F)] \frac{1}{1+\lambda^F} + (2a \sum_i \alpha(i) - 3b(L^H + \\
& \quad \quad \quad L^F))^2 \frac{1}{(1+\lambda^F)^2} \\
& \quad + M^H M^F (\lambda^F - 1) \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] \left[\frac{\alpha(2)\alpha(3)-\alpha^2(3)}{\alpha(2)} \right] = 0
\end{aligned}$$

Simplify the above equation by assumptions:

$$\begin{aligned}
& aM^{H*} \left[\frac{\alpha(2)-\alpha(1)}{\alpha(2)} \right] [2 \sum_i \alpha^2(i) - (\alpha(1) + \alpha(3)) \sum_i \alpha(i)] + 3bLM^{H*} \frac{(\alpha(1)+\alpha(3))(\alpha(2)-\alpha(1))}{\alpha(2)} \\
& \quad - (a \sum_i \alpha(i) - 3bL)^2 \frac{2}{1+\lambda^F} + (a \sum_i \alpha(i) - 3bL)^2 \frac{4}{(1+\lambda^F)^2} \\
& \quad + (M^{H*})^2 \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(1)-\alpha^2(3)]}{\alpha^2(2)} - (M^{H*})^2 (\lambda^F - 1) \frac{[\alpha(2)-\alpha(1)]^2 \alpha(1)\alpha(3)}{\alpha^2(2)} = 0
\end{aligned}$$

If $\lambda^{F*} = 1$, we have

$$\begin{aligned}
& aM^{H*} \left[\frac{\alpha(2)-\alpha(1)}{\alpha(2)} \right] [2 \sum_i \alpha^2(i) - (\alpha(1) + \alpha(3)) \sum_i \alpha(i)] + 3bLM^{H*} \frac{(\alpha(1)+\alpha(3))(\alpha(2)-\alpha(1))}{\alpha(2)} \\
& \quad + (M^{H*})^2 \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(1)-\alpha^2(3)]}{\alpha^2(2)} = \\
& \quad M^{H*} \left[\frac{\alpha(2)-\alpha(1)}{\alpha(2)} \right] \{ a [2 \sum_i \alpha^2(i) - (\alpha(1) + \alpha(3)) \sum_i \alpha(i)] \\
& \quad \quad + 3bL(\alpha(1) + \alpha(3)) + M^{H*} \frac{[\alpha(2)-\alpha(1)] [\alpha^2(1)-\alpha^2(3)]}{\alpha(2)} \}
\end{aligned}$$

let $\alpha(1) = \alpha(2) + x$ and $\alpha(3) = \alpha(2) - x$, $x > 0$ we have the above equation is negative.

$$\begin{aligned}
& -(a \sum_i \alpha(i) - 3bL)^2 \frac{2}{1+\lambda^F} + (a \sum_i \alpha(i) - 3bL)^2 \frac{4}{(1+\lambda^F)^2} - (M^{H^*})^2 (\lambda^F - 1) \frac{[\alpha(2)-\alpha(1)]^2 \alpha(1)\alpha(3)}{\alpha^2(2)} \\
& = (a \sum_i \alpha(i) - 3bL)^2 \frac{2}{1+\lambda^F} \left(\frac{2}{1+\lambda^F} - 1 \right) + (M^{H^*})^2 (1 - \lambda^F) \frac{[\alpha(2)-\alpha(1)]^2 \alpha(1)\alpha(3)}{\alpha^2(2)} \\
& = (a \sum_i \alpha(i) - 3bL)^2 \frac{2}{1+\lambda^F} \left(\frac{1-\lambda^F}{1+\lambda^F} \right) + (M^{H^*})^2 (1 - \lambda^F) \frac{[\alpha(2)-\alpha(1)]^2 \alpha(1)\alpha(3)}{\alpha^2(2)}
\end{aligned}$$

so $\lambda^{F^*} < 1$.

c) income and expenditure equation differentiation:

$$\begin{aligned}
& a \left[\frac{\alpha(2)-\alpha(1)}{\alpha(2)} \right] [\sum_i \alpha^2(i) - \alpha(1) \sum_i \alpha(i)] \Delta^M + 3bL \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] \Delta^M \\
& - (M^{H^*})^2 \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(2)+\alpha^2(3)]}{\alpha^2(2)} \Delta^\lambda - 2M^{H^*} \Delta^M \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(2)+\alpha^2(3)]}{\alpha^2(2)} \lambda^{F^*} \\
& + (a \sum_i \alpha(i) - 3bL)^2 \frac{2}{(1+\lambda^{F^*})^2} \Delta^\lambda - (a \sum_i \alpha(i) - 3bL)^2 \frac{8}{(1+\lambda^{F^*})^3} \Delta^\lambda \\
& + M^F (\lambda^{F^*} - 1) \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] \left[\frac{\alpha(2)\alpha(3)-\alpha^2(3)}{\alpha(2)} \right] \Delta^M \\
& + M^{H^*} M^F \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] \left[\frac{\alpha(2)\alpha(3)-\alpha^2(3)}{\alpha(2)} \right] \Delta^\lambda = 0
\end{aligned}$$

Simplify the above equation by assumptions:

$$\begin{aligned}
& a \left[\frac{\alpha(2)-\alpha(1)}{\alpha(2)} \right] [\sum_i \alpha^2(i) - \alpha(1) \sum_i \alpha(i)] \Delta^M + 3bL \left[\frac{\alpha(2)\alpha(1)-\alpha^2(1)}{\alpha(2)} \right] \Delta^M \\
& - 2M^{H^*} \lambda^{F^*} \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(2)+\alpha^2(3)]}{\alpha^2(2)} \Delta^M - M^{H^*} (\lambda^{F^*} - 1) \Delta^M \frac{[\alpha(2)-\alpha(1)]^2 \alpha(1)\alpha(3)}{\alpha^2(2)} \\
& = (a \sum_i \alpha(i) - 3bL)^2 \frac{8}{(1+\lambda^{F^*})^3} \Delta^\lambda - (a \sum_i \alpha(i) - 3bL)^2 \frac{2}{(1+\lambda^{F^*})^2} \Delta^\lambda \\
& + (M^{H^*})^2 \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(2)+\alpha^2(3)+\alpha(1)\alpha(3)]}{\alpha^2(2)} \Delta^\lambda \\
& \Rightarrow \Delta^M \left[\frac{\alpha(2)-\alpha(1)}{\alpha(2)} \right] \{ a [\sum_i \alpha^2(i) - \alpha(1) \sum_i \alpha(i)] + 3bL\alpha(1) \\
& + 2M^{H^*} \lambda^{F^*} \frac{[\alpha(1)-\alpha(2)] [\alpha^2(2)+\alpha^2(3)]}{\alpha(2)} - M^{H^*} (1 - \lambda^{F^*}) \frac{[\alpha(1)-\alpha(2)] \alpha(1)\alpha(3)}{\alpha(2)} \} \\
& = \Delta^\lambda \{ (a \sum_i \alpha(i) - 3bL)^2 \frac{8}{(1+\lambda^{F^*})^3} - (a \sum_i \alpha(i) - 3bL)^2 \frac{2}{(1+\lambda^{F^*})^2} \\
& + (M^{H^*})^2 \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(2)+\alpha^2(3)+\alpha(1)\alpha(3)]}{\alpha^2(2)} \}
\end{aligned}$$

For the welfare function, we also know that

$$\begin{aligned} \sum_i \alpha^2(i) \Delta \{ \sum_i p^2(i) \} &= -(a \sum_i \alpha(i) - 3bL)^2 \frac{8}{(1+\lambda^{F^*})^3} \Delta^\lambda \\ &+ 2M^{H^*} \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(2)+\alpha^2(3)+\alpha(1)\alpha(3)]}{\alpha^2(2)} \Delta M \end{aligned}$$

Hence we have the sign of $\sum_i \alpha^2(i) \Delta \{ \sum_i p^2(i) \}$ is as the same as the sign of the following equation:

$$\begin{aligned} &-(a \sum_i \alpha(i) - 3bL)^2 \frac{8}{(1+\lambda^{F^*})^3} \left[\frac{\alpha(2)-\alpha(1)}{\alpha(2)} \right] \{ a [\sum_i \alpha^2(i) - \alpha(1) \sum_i \alpha(i)] \\ &+ 3bL\alpha(1) + 2M^{H^*} \lambda^{F^*} \frac{[\alpha(1)-\alpha(2)] [\alpha^2(2)+\alpha^2(3)]}{\alpha(2)} - M^{H^*} (1 - \lambda^{F^*}) \frac{[\alpha(1)-\alpha(2)] \alpha(1)\alpha(3)}{\alpha(2)} \} \\ &+ 2M^{H^*} \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(2)+\alpha^2(3)+\alpha(1)\alpha(3)]}{\alpha^2(2)} \{ (a \sum_i \alpha(i) - 3bL)^2 \frac{8}{(1+\lambda^{F^*})^3} \\ &- (a \sum_i \alpha(i) - 3bL)^2 \frac{2}{(1+\lambda^{F^*})^2} + (M^{H^*})^2 \frac{[\alpha(2)-\alpha(1)]^2 [\alpha^2(2)+\alpha^2(3)+\alpha(1)\alpha(3)]}{\alpha^2(2)} \} \end{aligned}$$

$x < \frac{\alpha}{4}$ and M^{H^*} is not big, which guarantee that the trade pattern is the pattern we consider in the paper. The above equation is negative. This case is difficult to handle since the trade pattern will change if x is large.

Appendix C

Appendix to Chapter 4

C.1 Proof of Theorem 11

To show that there is an unique equilibrium for the close economy with heterogeneous firm, we first show steps to derive equation (16). We know that

$$\begin{aligned}l(z) &= \alpha(z)n(z) \int_{\phi^*}^{\infty} y(z, \phi)\mu(\phi)d\phi + n(z) \int_{\phi^*}^{\infty} \frac{1}{\phi}y(z, \phi)\mu(\phi)d\phi \\ \int_{\phi^*}^{\infty} y(z, \phi)\mu(\phi)d\phi &= \sqrt{\frac{\Lambda}{b}} + \frac{1}{\theta+1} \frac{1}{\phi^*} \frac{w}{b} \\ n(z) &= \frac{a - \sqrt{b\Lambda} - [\alpha(z) + \frac{1}{\phi^*}]w}{\sqrt{b\Lambda} + \frac{1}{\theta+1} \frac{1}{\phi^*} w}\end{aligned}$$

The only thing we need to calculate is $\int_{\phi^*}^{\infty} \frac{1}{\phi}y(z, \phi)\mu(\phi)d\phi = \int_{\phi^*}^{\infty} \frac{1}{\phi}[\sqrt{\frac{\Lambda}{b}} + \frac{w}{b}(\frac{1}{\phi^*} - \frac{1}{\phi})]\mu(\phi)d\phi$. After some transformation, we have

$$\int_{\phi^*}^{\infty} \frac{1}{\phi}y(z, \phi)\mu(\phi)d\phi = \frac{\theta}{\theta+1} \frac{1}{\phi^*} \left\{ \sqrt{\frac{\Lambda}{b}} + \frac{w}{b} \frac{1}{\theta+1} \frac{1}{\phi^*} - \frac{w}{b} \frac{1}{(\theta+1)(\theta+2)} \frac{1}{\phi^*} \right\}$$

Substitute the above equation into the resource constraint and we will have

equation (16), which is a quadratic equation with respect to w . Let

$$\begin{aligned} A &= -\frac{1}{\theta+1} \frac{1}{\phi^*} \left[\frac{1}{(\theta+2)} \left(\frac{1}{\phi^*}\right)^2 + \frac{\theta+3}{(\theta+2)} \frac{1}{\phi^*} \mu_\alpha + \sigma_\alpha^2 \right] \\ B &= -bL \frac{1}{\theta+1} \frac{1}{\phi^*} + a \left\{ \frac{1}{\theta+1} \frac{1}{\phi^*} \mu_\alpha + \frac{\theta}{(\theta+1)(\theta+2)} \left(\frac{1}{\phi^*}\right)^2 \right\} \\ &\quad - \sqrt{b\Lambda} \left\{ \sigma_\alpha^2 + 2\mu_\alpha \frac{1}{\phi^*} + \frac{\theta(\theta+3)}{(\theta+1)(\theta+2)} \left(\frac{1}{\phi^*}\right)^2 \right\} \\ C &= \sqrt{b\Lambda} \left\{ \left[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*} \right] (a - \sqrt{b\Lambda}) - bL \right\} \end{aligned}$$

The equation (16) can be expressed as $Aw^2 + Bw + C = 0$. Since A is negative and when $\mu_\alpha(a - \sqrt{b\Lambda}) > bL$ or $\sqrt{b\Lambda} < a - \frac{bL}{\mu_\alpha}$, C is a positive number equation (16) has two real solutions to w when ϕ^* is given. One solution is positive and the other solution is negative. In summary, there is an unique equilibrium.

To show that $\frac{w}{p}$ decreases when $\sqrt{\Lambda}$ increases, we know that $p(z) = (\alpha(z) + \frac{1}{\phi^*})w + \sqrt{b\Lambda}$. Hence $\frac{p}{w} = (\alpha(z) + \frac{1}{\phi^*}) + \frac{1}{\sqrt{b\Lambda}}$. If I can show that $\frac{w}{\sqrt{b\Lambda}}$ is a decreasing function of $\sqrt{\Lambda}$, we can show $\frac{w}{p}$ decreases when $\sqrt{\Lambda}$ increases. The positive root for $Aw^2 + Bw + C = 0$ is $w = \frac{B + \sqrt{B^2 - 4AC}}{-2A}$, so

$$\frac{w}{\sqrt{b\Lambda}} = \frac{2C}{\sqrt{b\Lambda}} \frac{1}{\sqrt{B^2 - 4AC} - B}$$

In the above equation, $\frac{2C}{\sqrt{b\Lambda}}$ is a decreasing function in $\sqrt{\Lambda}$ and $-B$ is a increasing function in $\sqrt{\Lambda}$. If $B^2 - 4AC$ is a increasing function in $\sqrt{\Lambda}$, I get the results. We just need to show that $-4AC$ is a increasing function in $\sqrt{\Lambda}$. It is true when $\sqrt{b\Lambda} < \frac{1}{2}(a - \frac{bL}{\mu_\alpha})$. In summary, when $\sqrt{b\Lambda} < \frac{1}{2}(a - \frac{bL}{\mu_\alpha})$, $\frac{w}{p}$ decreases when $\sqrt{\Lambda}$ increases and competition level which described by price cost margin decreases.

Next, I want to show that w decreases when $\sqrt{b\Lambda}$ increases. Since w is determined by $Aw^2 + Bw + C = 0$ and A is unrelated to $\sqrt{b\Lambda}$, I have

$$2Aw \frac{\partial w}{\partial \sqrt{b\Lambda}} + B \frac{\partial w}{\partial \sqrt{b\Lambda}} + \frac{\partial B}{\partial \sqrt{b\Lambda}} w + \frac{\partial C}{\partial \sqrt{b\Lambda}} = 0$$

From the expression of A, B and C, I have

$$2Aw \frac{\partial w}{\partial \sqrt{b\Lambda}} + B \frac{\partial w}{\partial \sqrt{b\Lambda}} - \left\{ \sigma_\alpha^2 + 2\mu_\alpha \frac{1}{\phi^*} + \frac{\theta(\theta+3)}{(\theta+1)(\theta+2)} \left(\frac{1}{\phi^*} \right)^2 \right\} w + \left[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*} \right] a - bL - 2\sqrt{b\Lambda} \left[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*} \right] = 0$$

We can solve w from the above equation

$$\frac{\partial w}{\partial \sqrt{b\Lambda}} = \frac{-\left\{ \sigma_\alpha^2 + 2\mu_\alpha \frac{1}{\phi^*} + \frac{\theta(\theta+3)}{(\theta+1)(\theta+2)} \left(\frac{1}{\phi^*} \right)^2 \right\} w + \left[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*} \right] a - bL - 2\sqrt{b\Lambda} \left[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*} \right]}{-2Aw - B}$$

Since we have $w = \frac{B + \sqrt{B^2 - 4AC}}{-2A}$, $-2Aw - B = \sqrt{B^2 - 4AC}$.

$$\frac{\partial w}{\partial \sqrt{b\Lambda}} = \frac{-\left\{ \sigma_\alpha^2 + 2\mu_\alpha \frac{1}{\phi^*} + \frac{\theta(\theta+3)}{(\theta+1)(\theta+2)} \left(\frac{1}{\phi^*} \right)^2 \right\} w + \left[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*} \right] a - bL - 2\sqrt{b\Lambda} \left[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*} \right]}{\sqrt{B^2 - 4AC}}$$

We want to show that $\frac{\partial w}{\partial \sqrt{b\Lambda}} < 0$. Since $\sqrt{B^2 - 4AC} > 0$,

$$-\left\{ \sigma_\alpha^2 + 2\mu_\alpha \frac{1}{\phi^*} + \frac{\theta(\theta+3)}{(\theta+1)(\theta+2)} \left(\frac{1}{\phi^*} \right)^2 \right\} w + \left[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*} \right] a - bL - 2\sqrt{b\Lambda} \left[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*} \right] < 0$$

Solving the above inequality, I have

$$w > \frac{\left[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*} \right] a - bL - 2\sqrt{b\Lambda} \left[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*} \right]}{\left\{ \sigma_\alpha^2 + 2\mu_\alpha \frac{1}{\phi^*} + \frac{\theta(\theta+3)}{(\theta+1)(\theta+2)} \left(\frac{1}{\phi^*} \right)^2 \right\}}$$

For the equation $Aw^2 + Bw + C = 0$, I have two solution. One is positive and the other is negative. Therefore, $w > -\frac{B}{A}$, which is as follows,

$$w > \frac{-bL + a \left\{ \mu_\alpha + \frac{\theta}{(\theta+2)} \left(\frac{1}{\phi^*} \right) \right\} - \sqrt{b\Lambda} \left\{ \sigma_\alpha^2 (\theta+1) \phi^* + 2\mu_\alpha (\theta+1) + \frac{\theta(\theta+3)}{(\theta+2)} \left(\frac{1}{\phi^*} \right) \right\}}{\left[\frac{1}{(\theta+2)} \left(\frac{1}{\phi^*} \right)^2 + \frac{\theta+3}{(\theta+2)} \frac{1}{\phi^*} \mu_\alpha + \sigma_\alpha^2 \right]}$$

It is very easy to show the following two inequalities:

$$\begin{aligned} & \left[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*} \right] a - bL - 2\sqrt{b\Lambda} \left[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*} \right] > \\ & -bL + a \left\{ \mu_\alpha + \frac{\theta}{(\theta+2)} \left(\frac{1}{\phi^*} \right) \right\} - \sqrt{b\Lambda} \left\{ \sigma_\alpha^2 (\theta+1) \phi^* + 2\mu_\alpha (\theta+1) + \frac{\theta(\theta+3)}{(\theta+2)} \left(\frac{1}{\phi^*} \right) \right\} \\ & \left[\frac{1}{(\theta+2)} \left(\frac{1}{\phi^*} \right)^2 + \frac{\theta+3}{(\theta+2)} \frac{1}{\phi^*} \mu_\alpha + \sigma_\alpha^2 \right] < \left\{ \sigma_\alpha^2 + 2\mu_\alpha \frac{1}{\phi^*} + \frac{\theta(\theta+3)}{(\theta+1)(\theta+2)} \left(\frac{1}{\phi^*} \right)^2 \right\} \end{aligned}$$

So, the positive solution from $Aw^2 + Bw + C = 0$ will make the following inequality valid,

$$-\{\sigma_\alpha^2 + 2\mu_\alpha \frac{1}{\phi^*} + \frac{\theta(\theta+3)}{(\theta+1)(\theta+2)} (\frac{1}{\phi^*})^2\}w + [\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*}]a - bL - 2\sqrt{b\Lambda}[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*}] < 0$$

And therefore we will have $\frac{\partial w}{\partial \sqrt{b\Lambda}} < 0$.

In the end, let's look at the change of $p(z)$ when $\sqrt{\Lambda}$ changes. When $\frac{1}{\phi^*}$ goes to zero, $A = 0$, $B = -\sqrt{b\Lambda}\sigma_\alpha^2$ and $C = \sqrt{b\Lambda}\{\mu_\alpha(a - \sqrt{b\Lambda}) - bL\}$. Wage rate now is $w = \frac{\mu_\alpha(a - \sqrt{b\Lambda}) - bL}{\sigma_\alpha^2}$, which is identical to the case with homogeneous firms. Let $w = w(\frac{1}{\phi^*}, \sqrt{\Lambda})$ and we have

$$w(\frac{1}{\phi^*}, \sqrt{\Lambda}) = \frac{\mu_\alpha(a - \sqrt{b\Lambda}) - bL}{\sigma_\alpha^2} + \frac{\partial w(\xi, \sqrt{\Lambda})}{\partial \frac{1}{\phi^*}} \frac{1}{\phi^*}$$

In the above equation, $0 < \xi < \frac{1}{\phi^*}$. Since $p(z) = (\alpha(z) + \frac{1}{\phi^*})w + \sqrt{b\Lambda}$, $\frac{\partial p(z)}{\partial \sqrt{b\Lambda}} = (\alpha(z) + \frac{1}{\phi^*})\frac{\partial w}{\partial \sqrt{b\Lambda}} + 1$. We also know that $\frac{\partial w}{\partial \sqrt{b\Lambda}} = -\frac{\mu_\alpha}{\sigma_\alpha^2} + \frac{\partial^2 w(\xi, \sqrt{\Lambda})}{\partial \frac{1}{\phi^*} \partial \sqrt{b\Lambda}} \frac{1}{\phi^*}$, so

$$\frac{\partial p(z)}{\partial \sqrt{b\Lambda}} = (\alpha(z) + \frac{1}{\phi^*})\left(-\frac{\mu_\alpha}{\sigma_\alpha^2} + \frac{\partial^2 w(\xi, \sqrt{\Lambda})}{\partial \frac{1}{\phi^*} \partial \sqrt{b\Lambda}} \frac{1}{\phi^*}\right) + 1$$

For the case $\frac{\partial p(z)}{\partial \sqrt{b\Lambda}} < 0$, we have

$$\alpha(z) > \frac{1}{\frac{\mu_\alpha}{\sigma_\alpha^2} - \frac{\partial^2 w(\xi, \sqrt{\Lambda})}{\partial \frac{1}{\phi^*} \partial \sqrt{b\Lambda}} \frac{1}{\phi^*}} - \frac{1}{\phi^*}$$

For $0 < \eta < \frac{\partial^2 w(\xi, \sqrt{\Lambda})}{\partial \frac{1}{\phi^*} \partial \sqrt{b\Lambda}} \frac{1}{\phi^*}$,

$$\alpha(z) > \frac{\sigma_\alpha^2}{\mu_\alpha} + \frac{1}{(\frac{\mu_\alpha}{\sigma_\alpha^2} - \eta)^2} \frac{\partial^2 w(\xi, \sqrt{\Lambda})}{\partial \frac{1}{\phi^*} \partial \sqrt{b\Lambda}} \frac{1}{\phi^*} - \frac{1}{\phi^*}$$

Since all the functions are continuous, at least when $\frac{1}{\phi^*}$ is small, there exists an α^* and for $\alpha(z) > \alpha^*$, $p(z)$ decreases when Λ increases.

To get a possible expression for α^* , since $[\alpha(z) + \frac{1}{\phi^*}] \frac{\partial w}{\partial \sqrt{b\Lambda}} + 1 < 0$, from the expression for $\frac{\partial w}{\partial \sqrt{b\Lambda}}$,

$$\alpha(z) + \frac{1}{\phi^*} > \frac{\sqrt{B^2 - 4AC}}{\{\sigma_\alpha^2 + 2\mu_\alpha \frac{1}{\phi^*} + \frac{\theta(\theta+3)}{(\theta+1)(\theta+2)} (\frac{1}{\phi^*})^2\} w - [\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*}] a - bL - 2\sqrt{b\Lambda} [\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*}]}$$

Since $w > -\frac{B}{A} = \frac{-bL + a\{\mu_\alpha + \frac{\theta}{(\theta+2)} (\frac{1}{\phi^*})\} - \sqrt{b\Lambda}\{\sigma_\alpha^2(\theta+1)\phi^* + 2\mu_\alpha(\theta+1) + \frac{\theta(\theta+3)}{(\theta+2)} (\frac{1}{\phi^*})\}}{[\frac{1}{(\theta+2)} (\frac{1}{\phi^*})^2 + \frac{\theta+3}{(\theta+2)} \frac{1}{\phi^*} \mu_\alpha + \sigma_\alpha^2]}$, I have

$$\alpha(z) + \frac{1}{\phi^*} > \frac{\sqrt{B^2 - 4AC}}{\{\sigma_\alpha^2 + 2\mu_\alpha \frac{1}{\phi^*} + \frac{\theta(\theta+3)}{(\theta+1)(\theta+2)} (\frac{1}{\phi^*})^2\} (-\frac{B}{A}) - [\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*}] a - bL - 2\sqrt{b\Lambda} [\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*}]}$$

Since we know $[\frac{1}{(\theta+2)} (\frac{1}{\phi^*})^2 + \frac{\theta+3}{(\theta+2)} \frac{1}{\phi^*} \mu_\alpha + \sigma_\alpha^2] < \{\sigma_\alpha^2 + 2\mu_\alpha \frac{1}{\phi^*} + \frac{\theta(\theta+3)}{(\theta+1)(\theta+2)} (\frac{1}{\phi^*})^2\}$ and $[\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*}] a - bL - 2\sqrt{b\Lambda} [\mu_\alpha + \frac{\theta}{\theta+1} \frac{1}{\phi^*}] > B$,

$$\alpha(z) + \frac{1}{\phi^*} > \frac{\sqrt{B^2 - 4AC}}{B} \frac{[\frac{1}{(\theta+2)} (\frac{1}{\phi^*})^2 + \frac{\theta+3}{(\theta+2)} \frac{1}{\phi^*} \mu_\alpha + \sigma_\alpha^2]}{\frac{\theta+1}{\theta+2} \mu_\alpha \frac{1}{\phi^*} + \frac{\theta^2+2\theta-1}{(\theta+1)(\theta+2)} (\frac{1}{\phi^*})^2}$$

The cutoff α^* can be defined as $\frac{\sqrt{B^2-4AC}}{B} \frac{[\frac{1}{(\theta+2)} (\frac{1}{\phi^*})^2 + \frac{\theta+3}{(\theta+2)} \frac{1}{\phi^*} \mu_\alpha + \sigma_\alpha^2]}{\frac{\theta+1}{\theta+2} \mu_\alpha \frac{1}{\phi^*} + \frac{\theta^2+2\theta-1}{(\theta+1)(\theta+2)} (\frac{1}{\phi^*})^2}$.

C.2 Matlab Code

The following is the main body of matlab code to calculate the change of welfare in an open economy when firms are heterogeneous:

```
clear
clc
digits(10);
%Now in an open economy
%Model with heterogeneous firms
%Define demand side p(z)=a-bx(z)
a=500;
```

```
b=1;
%Define the pareto productivity distribution. Free to change
phai_0=100;
theta=3;

%Define the alpha(z) part. Assume alpha(z)=(a_0-x,a_0,a_0+x)
a_0=0.5;
x=0.05;
miu_alpha=a_0;
sigma_alpha=a_0^2+(2/3)*x^2;
alpha_z=zeros(1,3);
alpha_z(1,1)=a_0-x;
alpha_z(1,2)=a_0;
alpha_z(1,3)=a_0+x;

%Labor endowment
L=100;

%Threshold expected profit
lamda=40;
M=50;
delta=0.75;

%phai_v=zeros(1,5);
%wage_v=zeros(1,5);
%count_v=zeros(1,5);
```



```

%pricev=zeros(5,length((0:0.1:1)));

%initialize phai_star
w_0=(a*miu_alpha-b*L-(lamda)^0.5*miu_alpha)/sigma_alpha;
poly1=zeros(1,theta+3);
poly1(1,1)=b*delta*M/(phai_0^theta);
poly1(1,theta+1)=-b*lamda;
poly1(1,theta+2)=-2*(b*lamda)^0.5*w_0*theta/(theta+1);
poly1(1,theta+3)=-2*(w_0)^2/((theta+1)*(theta+2));
w=0;
count=0;
while (abs(w_0-w)>0.000000001)
    x=roots(poly1);
    w_0=w;
    for i=1:length(x)
        if (isreal(x(i,1)))&&(x(i,1)>0)
            phai=x(i,1);
        end
    end
    t=1/phai;
    poly2=zeros(1,3);
    poly2(1,1)=-(t^2/(theta+2)+miu_alpha*t*(theta+3) ...
    /(theta+2)+sigma_alpha)*t/(theta+1);
    poly2(1,2)=-b*L*t/(theta+1)+a*(t*miu_alpha/(theta+1) ...
    +t^2*theta/((theta+1)*(theta+2)))-(b*lamda)^0.5*(sigma_alpha ...
    +2*t*miu_alpha+t^2*theta*(theta+3)/((theta+1)*(theta+2)));

```

```

poly2(1,3)=(b*lamda)^0.5*((miu_alpha+t*theta/(theta+1)) ...
*(a-(b*lamda)^0.5)-b*L);
y=roots(poly2);
for i=1:length(y)
    if (isreal(y(i,1)))&&(y(i,1)>0)
        w=y(i,1);
    end
end
poly1(1,1)=b*delta*M/(phai_0^theta);
poly1(1,theta+1)=-b*lamda;
poly1(1,theta+2)=-2*(b*lamda)^0.5*w*theta/(theta+1);
poly1(1,theta+3)=-2*(w)^2/((theta+1)*(theta+2));
count=count+1;
end
wage_close=w;
phai_close=phai;
price_close=(t+alpha_z)*w+(b*lamda)^0.5;
number_close=(1/((b*lamda)^0.5+t*w/(theta+1))) ...
*(a-(b*lamda)^0.5-t*w-w*alpha_z);
output_close=a/b-1/b*price_close;

% above gives our solution to a closed economy.
% Now move to open economy
%we already know that lamdaf=1
lamdaf=1;
mh=40;
mf=40;

```

```

a_0=0.5;
x=0.05;
wf_new=wage_close;
wf=0;
while (abs(wf-wf_new)>0.000001)
wf=wf_new;
%calculate phaif_new using expected profit equation
poly1=zeros(1,theta+3);
poly1(1,1)=(b/(1+lamdaf))*delta*M/(phai_0^theta);
poly1(1,theta+1)=-b*mf/(1+lamdaf);
poly1(1,theta+2)=-2*(b*mf/(1+lamdaf))^0.5*wf*theta/(theta+1);
poly1(1,theta+3)=-2*(wf)^2/((theta+1)*(theta+2));
s=roots(poly1);
    for i=1:length(s)
        if (isreal(s(i,1)))&&(s(i,1)>0)
            phaif=s(i,1);
        end
    end
    phaih=phaif;
syms who
left=3*L*((b*mh/(1+lamdaf))^0.5+who/(phaih*(theta+1)));
rightA=2*a/(1+lamdaf)-(b*mh/(1+lamdaf))^0.5-(a_0-x+1/phaih)*who;
rightB=2*a/(1+lamdaf)-(b*mh/(1+lamdaf))^0.5-(a_0+1/phaih)*who;
rightC=(a_0-x+theta/(theta+1)*(1/phaih))*((mh*(1+lamdaf)/b)^0.5 ...
+who*(1+lamdaf)/(b*phaih*(theta+1)))-theta*who*(1+lamdaf)/ ...
(b*phaih^2*(theta+1)^2*(theta+2));

```

```

rightD=(a_0+theta/(theta+1)*(1/phaih))*((mh*(1+lamdaf)/b) ...
^0.5+who*(1+lamdaf)/(b*phaih*(theta+1))-theta*who*(1+lamdaf) ...
/(b*phaih^2*(theta+1)^2*(theta+2));
right=rightA*rightC+0.5*rightB*rightD;
number2=rightA/((b*mh/(1+lamdaf))^0.5+who/(phaih*(theta+1)));
whoeq=simplify(left-right);
whoA=zeros(3,3);
whoB=zeros(3,1);
for i=1:3
    whoA(i,1)=i^2;
    whoA(i,2)=i^1;
    whoA(i,3)=1;
    whoB(i,1)=subs(whoeq,who,i);
end
whopoly=(inv(whoA)*whoB);
whosolv=roots(whopoly);
    for i=1:length(whosolv)
        if (isreal(whosolv(i,1)))&(whosolv(i,1)>0)
            wf_new=whosolv(i,1);
        end
    end
    %phaif
end
phai_open=phaif;
wage_open=wf_new;

%now we chang mh and make phaih constant and see the welfare change

```

```

lamdaf_new=1;
lamdaf=0;
wh_new=wage_open;
mh=45;
mf=40;
a_0=0.5;
x=0.05;
for ss=1:2
    wh=wh_new;
    lamdaf=0;
while(abs(lamdaf-lamdaf_new)>0.001)

lamdaf=lamdaf_new;
%calculate new phaif

syms phais
left=phais^theta*b*delta*M*(a_0*phais+1)^2/((1+lamdaf)*phai_0^theta);

rightA=b*mf*(a_0*phais+1)^2/(1+lamdaf);
rightB=2*(b*mf/(1+lamdaf))^0.5*(theta/(1+theta))*(a_0*phais+1) ...
*((a_0+1/phaih)*wh+(b*mh/(1+lamdaf))^0.5-(b*mf/(1+lamdaf))^0.5);
rightC=2/((theta+1)*(theta+2))*((a_0+1/phaih)*wh+(b*mh/(1+lamdaf)) ...
^0.5-(b*mf/(1+lamdaf))^0.5)^2;
right=rightA+rightB+rightC;

phaieq=simplify(left-right);

```

```

%We fix the number of theta here
phaiA=zeros(theta+3,theta+3);
phaiB=zeros(theta+3,1);
for i=1:(theta+3)
    phaiA(i,1)=i^(theta+2);
    phaiA(i,2)=i^(theta+1);
    phaiA(i,3)=i^theta;
    phaiA(i,4)=i^2;
    phaiA(i,5)=i^1;
    phaiA(i,6)=1;
    phaiB(i,1)=subs(phaieq,phais,i);
end
phaipoly=inv(phaiA)*phaiB;
phaisolv=roots(phaipoly);
    for i=1:length(phaisolv)
        if (isreal(phaisolv(i,1)))&(phaisolv(i,1)>0)
            phaif=phaisolv(i,1);
        end
    end
end

%calculate new lamdaf
syms t
%left hand side
p1=(a_0-x+1/phaih)*wh+mh^0.5*t;
p2=(a_0+1/phaih)*wh+mh^0.5*t;

```

```

p3=(a_0-x+1/phaif)*(a_0+1/phaih)*wh/(a_0+1/phaif)+ ...
((a_0-x+1/phaif)*(mh^0.5-mf^0.5)/(a_0+1/phaif)+mf^0.5)*t;
leftA=(p1*(a-p1)+p2*(a-p2)+p3*(a-p3))*t^2/b-p1*(2*a*t^2/b-p1);
leftB=(a_0+theta/(1+theta)*(1/phaih))*(mh^0.5*t+wh/ ...
((1+theta)*phaih))-theta*wh/((theta+1)^2*(theta+2)*phaih^2);
left=leftA*leftB;
%right hand side
rightA=3*L*t^2*(mh^0.5*t+wh/((1+theta)*phaih));
rightB=2*a*t^2/b-mh^0.5*t-(a_0-x+1/phaih)*wh;
rightC=(a_0-x+theta/((1+theta)*phaih))*(mh^0.5*t+wh/ ...
((1+theta)*phaih))-theta*wh/((theta+1)^2*(theta+2)*phaih^2);
right=p2*(rightA-rightB*rightC);
f=simplify(left-right);
tA=zeros(6,6);
tB=zeros(6,1);
for i=1:6
    tA(i,1)=i^5;
    tA(i,2)=i^4;
    tA(i,3)=i^3;
    tA(i,4)=i^2;
    tA(i,5)=i^1;
    tA(i,6)=1;
    tB(i,1)=subs(f,t,i);
end
tpoly=inv(tA)*tB;
lamdafsolv=roots(tpoly);

```

```

for i=1:length(lamdafsolv)
    if (isreal(lamdafsolv(i,1))&(lamdafsolv(i,1)>0)
        lamdaf_tt=lamdafsolv(i,1);
    end
end

lamdaf_new=b/lamdaf_tt^2-1;
end
lamdaf=lamdaf_new;
%find new wh through aggregate output function in sector 2
syms whs
wf=((a_0+1/phaih)*whs+(b*mh/(1+lamdaf))^0.5-(b*mf/(1+lamdaf)) ...
^0.5)/(a_0+1/phaif);
rightwhAh=(a_0+theta/(theta+1)*(1/phaih))*((mh*(1+lamdaf)/b)^0.5 ...
+whs*(1+lamdaf)/(b*phaih*(theta+1)))-theta*whs*(1+lamdaf) ...
/(b*phaih^2*(theta+1)^2*(theta+2));
rightwhAf=(a_0+theta/(theta+1)*(1/phaif))*((mf*(1+lamdaf)/b)^0.5 ...
+wf*(1+lamdaf)/(b*phaif*(theta+1)))-theta*wf*(1+lamdaf)/ ...
(b*phaif^2*(theta+1)^2*(theta+2));
rightwhB=2*a/(1+lamdaf)-(b*mh/(1+lamdaf))^0.5-(a_0+1/phaih)*whs;
rightwh=rightwhAf*rightwhAh*rightwhB;

leftwhAh=2*a/(1+lamdaf)-(b*mh/(1+lamdaf))^0.5-(a_0-x+1/phaih)*whs;
leftwhBh=(a_0-x+theta/(theta+1)*(1/phaih))*((mh*(1+lamdaf)/b)^0.5 ...
+whs*(1+lamdaf)/(b*phaih*(theta+1)))-theta*whs*(1+lamdaf) ...
/(b*phaih^2*(theta+1)^2*(theta+2));

```



```

leftwhCh=3*L*((b*mh/(1+lamdaf))^0.5+whs/(phaih*(theta+1)));
leftwhh=(leftwhCh-leftwhAh*leftwhBh)*rightwhAf;

leftwhAf=2*a/(1+lamdaf)-(b*mf/(1+lamdaf))^0.5-(a_0-x+1/phaif)*wf;
leftwhBf=(a_0-x+theta/(theta+1)*(1/phaif))*((mf*(1+lamdaf)/b)^0.5 ...
+wf*(1+lamdaf)/(b*phaif*(theta+1)))-theta*wf*(1+lamdaf) ...
/(b*phaif^2*(theta+1)^2*(theta+2));
leftwhCf=3*L*((b*mf/(1+lamdaf))^0.5+wf/(phaif*(theta+1)));
leftwhf=(leftwhCf-leftwhAf*leftwhBf)*rightwhAh;
leftwh=leftwhh+leftwhf;
g=simplify(leftwh-rightwh);
whA=zeros(4,4);
whB=zeros(4,1);
for i=1:4
    whA(i,1)=i^3;
    whA(i,2)=i^2;
    whA(i,3)=i^1;
    whA(i,4)=1;
    whB(i,1)=subs(g,whs,i);
end
whpoly=(inv(whA)*whB);
whsolv=roots(whpoly);
for i=1:length(whsolv)
    if (isreal(whsolv(i,1)))&(whsolv(i,1)>0)
        wh_new=whsolv(i,1);
    end
end

```

```
end  
vv=[lamdaf wh_new phaif]  
end
```