Essays in Macroeconomics and Financial Markets

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Ariel Zetlin-Jones

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Varadarajan V. Chari
Larry E. Jones

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Abstract

In this dissertation, we study the interaction of financial markets and the macroeconomy.

In Chapter 1, we examine the quantitative importance of financial market shocks in accounting for business cycle fluctuations. We emphasize the role financial markets play in reallocating funds from cash-rich, low productivity firms to cash-poor, high productivity firms. Using evidence on financial flows at the firm level, we find that for publicly traded firms (in Compustat), almost all investment is financed internally. However, using an alternative data source (Amadeus), we find that most investment by privately held firms is financed through borrowing. Motivated by these observations, we build a quantitative model featuring publicly and privately held firms that face collateral constraints and idiosyncratic risk over productivity as well as non-financial linkages. In our calibrated model, we find that a shock to the collateral constraints which generates a one standard deviation decline in the debt-to-asset ratio leads to a 0.5% decline in aggregate output on impact, roughly comparable to the effect of a one standard deviation shock to aggregate productivity in a standard real business cycle model. In this sense, we find that disturbances in financial markets are a promising source of business cycle fluctuations when non-financial linkages across firms are sufficiently strong.

In Chapter 2, we analyze the causes of financial crises and policies designed to mitigate their effects. we provide new evidence that the capital structure of financial institutions is significantly more illiquid than that of non-financial businesses. We develop a theory in which such differences in capital structure arise from the differences in information lenders have about the assets of financial and non-financial businesses. We use the theory to show that the illiquid capital structure used by financial institutions leads such institutions to be inherently fragile and that government interventions during a crisis, such as bailouts, are not desirable.

In Chapter 3, we study policies intended to remedy collapses in secondary loan markets. Loan originators often securitize some loans in secondary loan markets and hold on to others. New issuances in such secondary markets collapse abruptly on occasion, typically when collateral values used to secure the underlying loans fall and these collapses are viewed by policymakers as inefficient. We develop a dynamic adverse selection model in which small reductions in collateral values can generate abrupt inefficient collapses in new issuances in the secondary loan market by affecting reputational incentives. We find that a variety of policies intended to remedy market inefficiencies do not help resolve the adverse selection problem.

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Chapter 1

Introduction

What are the nature and consequences of financial shocks? How should governments respond to financial shocks? This dissertation pursues a quantitative and theoretical study of these questions. In the three chapters that follow, we evaluate the quantitative role that financial shocks play over the business cycle, the theoretical role that fragile capital structures play in bank finance, and the role of policy in responding to collapses in secondary loan markets.

In Chapter 2, based on joint work with Ali Shourideh, we evaluate the quantitative effects of financial market shocks on the macroeconomy over the business cycle. In particular, we analyze the role financial markets play in reallocating funds from cash-rich, low productivity firms to cash-poor, high productivity firms. Toward this end, we obtain evidence on financial flows to analyze the importance of this role of financial markets. At the firm level, we find that for publicly traded firms (in Compustat), almost all investment is financed internally while, using a alternative data source (Amadeus), we find that most investment by privately held firms is financed through borrowing. These observations suggest that the quantitative impact of financial market shocks depend both on the sensitivity of investment and output of privately held firms to such shocks and on the extent to which the investment and output of publicly held firms respond to the actions of privately held firms.

Motivated by these observations, we build a quantitative model featuring publicly and privately held firms that face collateral constraints and idiosyncratic risk over productivity. We model financial market shocks as shocks to the collateral constraints. In our model, each firm has a monopoly in producing a differentiated good and uses the goods produced by other firms as an input for production – features that create non-financial linkages between publicly and privately held firms. In our calibrated model, we find that a shock to the collateral constraints which generates a one standard deviation decline in the debt-to-asset ratio leads to a 0.5% decline in aggregate output on impact, roughly comparable to the effect of a one standard deviation shock to aggregate productivity in our model. In this sense, we find that disturbances in financial markets are a promising source of business cycle fluctuations when non-financial linkages across firms are sufficiently strong.

In Chapter 3, we study the sources of financial crises, such as banking crises, and the consequences of alternative policies intended to mitigate the effects of financial crises. We argue that the effects of policies aimed at mitigating financial crises cannot be understood without a coherent theory of the sources of such crises. Following the literature on financial crises, we argue that the origins of these crises can be traced back to the heavy reliance of financial institutions on debt that they must repay or roll over within a short time horizon, also known as short-term debt.

We build on that literature by developing a new set of statistics which shows that the capital structure of banks and financial institutions differs significantly from that of non-financial corporate businesses. We develop a theory in which such differences in capital structure arise from differences in the kind and quality of information investors have about the balance sheets of financial and non-financial businesses. We argue that these differences in information arise naturally from the underlying characteristics of these businesses. We show that the greater reliance on short-term debt of financial institutions leads such institutions to be inherently more fragile than non-financial businesses. We use the theory to show that while government interventions, such as bailouts, intended to mitigate the consequences of financial crises may appear to be desirable during a crisis, they will have detrimental long run effects on the economy.

Finally in Chapter 4, based on joint work with V. V. Chari and Ali Shourideh, we study policies that remedy ineffeciencies arising from collapses of trade in secondary loan markets. We begin by studying the determinants of the decision of a bank of whether to hold or to sell its loans. Secondary loan markets are often argued to suffer from adverse selection problems when originators of loans are better informed than potential purchasers regarding the quality of the loans.

We then, analyze the role of reputation in mitigating such adverse selection problems. We demonstrate that reputation that reputational incentives lead to multiplicity of equilibria and fragile outcomes in the volume of trade in these markets. In one of these equilibria, reputational forces help mitigate the adverse selection problem while in the other reputational forces actually worsen the adverse selection problem. We use a refinement adapted from the global games literature which leads to a unique equilibrium. This equilibrium is fragile in the sense that small fluctuations in fundamentals can lead to large changes in the volume of loans sold in the secondary market. Our model is consistent with the collapse in the volume of loans sold in the secondary market in the United States that occured from 2007 to 2009. We analyze a variety of policies that were proposed by the U.S. government to resolve adverse selection problems in the secondary loan market. We find that many such policies do not help resolve this problem and, indeed, worsen the allocative efficiency of the secondary loan market.

Chapter 2

External Financing and the Role of Financial Frictions over the Business Cycle: Measurement and Theory

2.1 Introduction

One role of financial markets is to reallocate capital from cash-rich, low productivity firms to cash-poor, high productivity firms. In this chapter, we examine the importance of shocks to the ability of financial markets to reallocate capital for the magnitude and duration of business cycles. In particular, we focus on the role of financial markets in funding investment. In the aggregate, data from the flow of funds shows that internal funds generated by firms is more than adequate to finance investment. This observation might lead an observer to think financial market shocks would have very modest business cycle effects. We argue, however, looking only at the aggregates may be misleading. New evidence on privately held firms (in the Amadeus data set) shows that almost all of investment by such firms is financed through borrowing. This data suggests that disruptions to the reallocative role of financial markets could have a large impact on the investment and output of privately held firms and, through spillovers on other firms, on aggregate economic activity. The size of the impact depends critically both on the sensitivity of investment and output of privately held firms to shocks to financial markets and on the extent to which there are spillovers from the actions of privately held firms to the investment and output decisions of publicly held firms.

To analyze the quantitative importance of financial market shocks, we develop a model in which firms are subject to idiosyncratic shocks so that it is desirable to reallocate capital from low productivity firms to high productivity firms, but financial frictions impede this reallocation. We model these frictions as collateral constraints and disturbances in the ability of financial markets to reallocate capital as shocks to collateral constraints. Our approach follows a substantial literature that has examined the role of financial market frictions over the business cycle (see [Bernanke et al., 1999], [Kiyotaki and Moore, 2008], and [Jermann and Quadrini, 2009] to cite just three examples). We discipline our quantitative exercise by using data on financial flows.

Our model differs from those in the existing literature by incorporating three key ingredients. First, data on financial flows shows that at the aggregate level, firms generate internal funds substantially in excess of what they need to finance operations and investments. Thus, if financial market frictions affect investment over the business cycle, they must affect the reallocation of funds among firms rather than impeding the ability of firms to obtain investment funds from households. To allow for reallocation problems among firms, our model features heterogeneous firms in the sense that each individual firm faces idiosyncratic risk and incomplete markets.

Second, the evidence from Compustat shows that for publicly traded firms, almost all capital expenditures can be financed using internal funds. We show, however, that the bulk of capital expenditures by privately held firms is financed through borrowing. To be consistent with this large observed difference between the two types of firms in the data, our model features both publicly held and privately held firms, each of which face potentially binding collateral constraints. The difference between these types of firms is that publicly held firms are owned by households that can better insure themselves against the idiosyncratic risk affecting a particular firm than can owners of privately held firms.

Third, a key feature of the data is that firms use intermediate goods produced by other firms as well as capital and labor to produce gross output. This observation implies that firms are naturally connected with each other through trade linkages. We model these connections by assuming that each firm in our model has a monopoly in producing a differentiated good and uses the goods produced by other firms as an intermediate good in production. We show that under reasonable parameter assumptions regarding the fraction of gross output that represents use of intermediate goods and on the size of markups, these linkages across firms can generate co-movement of output by publicly held and privately held firms in response to financial market shocks. These linkages also play an important role in amplifying the quantitative effects of financial shocks.

In our calibrated model, we find that a shock to the collateral constraints which generates a one standard deviation decline in the aggregate debt-to-asset ratio leads to a 0.5% decline in aggregate output on impact. This decline is roughly comparable to the effect of a one standard deviation shock to aggregate productivity in a standard real business cycle model. A collateral constraint shock also leads to a sizable decline in consumption, investment and employment. We also find that shocks to collateral constraints induce persistent effects on aggregates. In this sense, we find that disturbances in financial markets are a promising source of business cycle fluctuations.

Firm heterogeneity plays a central role in generating our results. In our model firms face idiosyncratic shocks to productivity and collateral constraints in accumulating capital. We argue that a representative firm version of our model in which the collateral constraint binds is inconsistent with aggregate data on financial flows in both the United States and Europe. Specifically, in both the U.S. and the U.K., after paying for interest, taxes, payments to labor and other businesses for materials, the non-financial business sector in the aggregate generates funds, which we call *Available Funds*, substantially in excess of funds used for investment. This observation implies that it is difficult for macroeconomic models with a representative firm subject to collateral constraints to produce large fluctuations in investment and output in response to a tightening of collateral constraints. One reason is that shareholders of firms have strong incentives to accept delayed payments in dividends so as to relax current collateral constraints.

Our quantitative results are disciplined by data on firm level use of external funds for investment. Disaggregated data on publicly held firms in the U.S. and the U.K. and on privately held firms in the U.K. suggests that it is important to distinguish between these subsets of firms. We develop a statistic to measure how much a subset of firms rely on external financing. In particular, our statistic measures the amount of external funds used for investment by firms. We find that in both the U.S. and the U.K., on average (over time) roughly 20% of investment by publicly held firms is externally financed so that 80 % of investment by firms is finaced by available funds. In the U.K., on average roughly 90% of investment by privately held firms is externally financed so that only 10% of investment is financed by available funds.

This observation motivates us to introduce a second type of heterogeneity in our model. Specifically, we partition firms into those that are owned by diversified households, and therefore better able to insure themselves against idiosyncratic risk and those that are owned by entrepreneurs who cannot insure themselves against idiosyncratic risk. The observation that privately held firms use external funds for investment to a much greater extent than do publicly held firms suggests that the role played by financial markets in reallocating capital for investment is more important for privately held firms than it is for publicly held firms. In this sense, privately held firms are more likely to be affected by disturbances to their collateral constraints than are publicly traded firms.

In our model, collateral constraints are present for both types of firms. Both public and private firms can borrow up to a level proportional to their asset holding. Both types of firms are limited in their access to financial markets, in the sense that they must borrow using debt. We assume that, in addition to productivity risk, firms are subject to bankruptcy risk so that the effective discount rate is higher than the interest rate for privately held firms. However, because publicly held firms are owned by diversified households, the bankruptcy risk does not raise the effective discount rate of publicly held firms. These differences in effective discount factors imply that, in a stationary equilibrium, publicly held firms have sufficient asset holdings so that the collateral constraint does not bind, whereas privately held firms face occasionally binding constraints. The differences in the extent to which collateral constraints bind implies that privately held firms are much more reliant on external funds for investment than are publicly held firms.

Trade linkages across firms ensure that output and investment of different types of firms move together. We assume that each firm, both public and private, produces a differentiated good and there is monopolistic competition between different firms a la Dixit-Stiglitz. Moreover, we assume that each firm uses a bundle of goods produced by other firms as an input as well as capital and labor. These elements capture observed markups and the observation that gross output is roughly twice as large as value added.

In our quantitative model, under our assumed parameter values, a tightening of collateral constraints lead to a decline in output. The economic mechanism that drives this result is that for privately held firms a tightening of the collateral constraint decreases the demand for capital by firms for whom the collateral constraint binds. For such firms, the decrease in capital demand leads to a decrease in demand both for labor and for intermediate goods to be used in production. As a result, the rental rate of capital and the wage rate both decline, tending to raise the output of unconstrained firms. However, the fall in the wage and capital rental rate results in a negative wealth shock to households and privately held firms, causing a decline in demand for the final good – both for consumption and as an intermediate input in production. Because goods are not perfect substitutes, demand for goods produced by all firms falls, tending to lower output of unconstrained firms. Thus, a tightening of collateral constraints has competing effects on the production of firms for whom the collateral constraint does not bind. The effect of the shock on aggregate demand and intermediate good production provide incentives for unconstrained firms to decrease production while the effect of the shock on wages and interest rates provide incentives for unconstrained firms to increase production. We show that under our reasonable parameter assumptions, the aggregate demand and intermediate good effects dominate so that a tightening of collateral constraints leads unconstrained firms to decrease their production.

We calibrate our quantitative model with both publicly and privately firms to be consistent with four key facts from the financial flows data. We require the model to replicate the share of gross output accounted for by privately held firms, the aggregate debt-to-total assets ratio for all firms, the dispersion of the firm level debt-to-total assets ratio among privately held firms, and the amount of external funds used for investment by privately held firms. We show that these facts in the data help determine the quantitative importance of disturbances to the collateral constraints in the model.

We then perform an impulse response analysis from the steady state of our calibrated model. Specifically, we shock the economy with a partially persistent disturbance to the collateral constraint which generates a one standard deviation decline in the debt-to-total assets ratio of privately held firms on impact. We find that such an impulse generates a 0.5% decline in aggregate output on impact. Although sales of publicly held firms initially rise on impact, partially off-setting the initial decline in sales by privately held firms, within two periods they fall below the steady state level. The non-financial linkages we introduce play an important role, both in dampening the initial response of publicly held firms and in generating co-movement in sales after the second period of the shock.

Related Literature. The work in this chapter is related to an extensive literature on the effect of financial frictions in macroeconomics, starting from [Bernanke and Gertler, 1989], [Carlstrom and Fuerst, 1997], [Kiyotaki and Moore, 1997], [Bernanke et al., 1999] and more recently [Kiyotaki and Moore, 2008] and [Jermann and Quadrini, 2009]. The common goal is to identify and understand the channels through which financial market disruptions affect economic activity and their quantitative importance. Our goal in this chapter is the same while our approach differs in that we use data on external financing to discipline the importance of these channels.

Our empirical work on external financing needs is related to a literature in corporate finance which attempts to identify the extent to which firms face constraints in financing their investment (see [Fazzari et al., 1988], and [Gilchrist and Himmelberg, 1995] among many others). The approach in this literature is to test the implications of models with financing constraints, namely that Tobin's Q as well as cash-flow would have a significant effect on investment. Note that [Kaplan and Zingales, 1997] question the validity of this approach. Our approach, however, differs from theirs in that our measurement approach emphasizes the role of financial markets in firms' financing decision, i.e., how much of their investment is financed using external funds. In this regard, our approach is closer to the one taken by [Rajan and Zingales, 1998] and [Buera et al., 2011]. Additionally, this measurement approach allows us to abstract from which firms in particular face binding financing constraints while disciplining the importance of financial markets for investment.

From a modeling perspective, our model of financial frictions is a natural extension of [Evans and Jovanovic, 1989] to dynamic environments. The basic structure of the model is very similar to [Gomes, 2001]. While he models financial frictions as additional costs to external financing, we follow [Evans and Jovanovic, 1989] and assume that investment is bound by a factor of net worth. Our model is also related to an extensive series of papers on the effect of idiosyncratic investment risk on firm dynamics and its financial structure including [Cooley and Quadrini, 2001], [Hennessy and Whited, 2005], and [Angeletos, 2007] to name a few.

Our modelling approach is also similar to an extensive literature that analyzes the effects of financial frictions on misallocation and Total Factor Productivity, such as [Midrigan and Xu, 2010], [Buera et al., 2011], and [Moll, 2011]. While our basic model is very similar, aside from inclusion of monopolistic competition and intermediate goods as inputs, we focus on short-term dynamics of the model. Moreover, because of our focus on business cycle frequency fluctuations and the importance of the role played by financial markets, our calibration is somewhat different. In particular, we calibrate the model to match evidence on external financing as well as the variance of debt to asset ratios while the papers mentioned above focus on the dynamics of firm size. While these measures are correlated, our focus on financial flows and external financing makes our work here different from theirs. Moreover, our firm level employment dynamics closely resembles the evidence documented by [Davis et al., 2007].

Our quantitative exercise is most closely related to [Jermann and Quadrini, 2009] where shocks to financial constraints cause fluctuations in economic activity. Our analysis, however, is substantially different from theirs. They focus on a model with a representative firm that is financially constrained and which faces exogenously specified cost of reducing dividends to finance investment. One way of thinking about this chapter is that we develop a model in which the cost of reducing dividends are endogenous, in the sense that for privately held firms, dividend reductions affect the marginal utility of consumption of entrepreneurs. We think of their paper as a first step toward developing a workhorse model for analysis of shocks to financial markets and the effect of such shocks on real activity. In this chatper, we take the next step and show that there is a great deal of heterogeneity in firms' reliance on external financing. This helps us impose further discipline on our model of financial frictions and firm heterogeneity and the mechanisms that translate financial shocks to the real economy. This chapter is also related to a number of studies that focus on how presence of financial frictions amplify and propagate the effect of productivity shocks to the economy such as [Khan and Thomas, 2011] and [Nezafat and Slavık, 2011]. Furthermore, our transitional dynamic exercise is very similar to [Guerrieri and Lorenzoni, 2011], however, they focus on changes in household's borrowing opportunities while we focus on the production side of the economy.

The remainder of this chapter is organized as follows: in section 2.2 we provide evidence on firms' external financing behavior, in section 2.3 contains our model and its theoretical analysis, section 2.4 contains our quantitative exercise, section 2.5 concludes.

2.2 Evidence on External Financing Needs

In this section, we present evidence on the use of external funds for investment in the aggregate and at the firm level in the United States and in the United Kingdom. We show that in the aggregate, firms can self-finance the entirety of their investments. We also show that privately held firms externally finance a significant fraction of their investment while publicly held firms externally finance much less of their investment. We conclude by presenting evidence that the difference in the reliance on external financing between publicly held and privately held firms holds even when we focus on firms of approximately the same size.

We begin this section by discussing the conceptual objects we intend to study. We then describe our data sources and how we measure the use of external funds for investment in our different data sources. We then outline the key facts on external financing.

2.2.1 Conceptual Measurement of Financial Flows

We construct an empirical measure of the amount external funds that a firm requires to fund its investment. One way to measure this external financing need is to measure net *financial* inflows or outflows for firms. By definition, if a firm's investment in any period is greater than the amount of funds generated from operations after paying for interest, legal obligations, payments to labor and other businesses for materials, then the firm must receive net financial inflows. To the extent that available funds are fixed, one less dollar of financial inflows would lead necessarily to a one dollar decline in investment. On the other hand, if a firm's amount of investment is less than its fund generated from operations, then the firm has the ability to self finance the entirety of its investment if it so chooses.

It is easiest to understand these definitions by considering the budget constraint of a firm that manages its capital stock, may issue equity and debt, and may hold financial assets. We can write this budget constraints as

$$s_t d_t + q_t(a_{t+1} - a_t) + x_t \leq \pi_t(k_t, l_t) + a_t e_t - r_t b_t + b_{t+1} - b_t + p_t(s_{t+1} - s_t)$$

where s represents the stock of outstanding equity shares which have price p, d represents dividends paid per share, a is the stock of the firm's financial assets which have price q and pay period t dividends e, x is physical investment, π is gross profit of the firm (net of factor payments and taxes), r is the net interest rate on debt b. We rearrange this express as

$$s_t d_t + q_t(a_{t+1} - a_t) - (b_{t+1} - b_t) - p_t(s_{t+1} - s_t) \leq \pi_t(k_t, l_t) + a_t e_t - r_t b_t - x_t. \quad (2.1)$$

The left hand side of equation (2.1) represents the net financial flow into or out of the firm. If the right hand side of equation (2.1) is positive so that the firm's investment is less than the cash generated by the firm in period t, then funds flow out of the firm. If the right hand side of equation (2.1) is negative, then funds must flow into the firm as the firm's investment is greater than the cash it generated. Implicitly, by netting interest payments, $r_t b_t$, out of available funds, but leaving dividend payments, $s_t d_t$, on the left hand side, we assume that dividends are not set in advance while interest payments cannot be re-negotiated.

Our aim, then, is to measure available funds defined as

$$AF_t = \pi_t + a_t e_t - r_t b_t$$

and physical investment, x_t, as these measures are sufficient to determine whether a given firm receives inflows of cash or makes cash outlays.

Our preferred statistic of how individual firms rely on external funds for investment is given by

$$\frac{1}{T} \sum_{t=1}^{T} \frac{\sum_i (X_{it} - AF_{it}) \mathbf{1}_{[X_{it} \geq AF_{it}]}}{\sum_i X_{it}} \quad (2.2)$$

The statistic in equation (2.2) represents the average net financial inflow to firms whose investment is greater than their available funds as a fraction of total investment.

The statistic in equation (2.1) informs us about what fraction of aggregate investment must be financed externally among a subset of firms, and, therefore, depends on well-functioning financial markets.

2.2.2 Data Description

Our data sources for the U.S. include the Flow of Funds, the Statistics of Income, and Compustat. Our data sources for the U.K. include the U.K. Economic Accounts, Compustat Global, and Amadeus. We now describe each data set and how we measure available funds and investment in each source.

U.S. Data

Aggregate Data. Aggregate U.S. data are from the Flow of Funds. Our measure of aggregate available funds in the U.S. is the sum of Internal Funds (Table F.102 line 9) and Dividends (Table F.201 line 3) of all non-farm nonfinancial corporate business. Note that we add dividends here because the definition of Internal Funds has excluded them. Our measure of aggregate investment is given by Capital Expenditures (Table F.102 line 11) of non-farm nonfinancial corporate businesses.

Our second source of aggregate data is the Statistics of Income, specifically the Corporation Income Tax Returns, from 1991 to 2008. The Corporation Income Tax Returns data set contains information on assets, liabilities, and income of all incorporated firms in the U.S. classified by industry and by size of total assets. We focus on data classified by size of total assets, specifically Table 2, which details the balance sheet and income statement of active corporations. The advantage of using data from the Statistics of Income is that we can analyze aggregate financial flows to firms of various sizes. In other words, we can analyze whether the average small firm receives financial inflows even though in the aggregate all firms make financial outflows. There are three limitations to working with data from the Statistics of Income. The first is that the data cover all active corporations in the United States, including agricultural and financial firms. The second limitation is that data are not firm-level, so we can only analyze the net financial flows to an aggregated small firm. The final limitation is that the size classes for the data are presented in nominal terms which can introduce measurement bias (due to inflation and trend growth) as discussed in [Gertler and Gilchrist, 1994].

In any year, an observation in this data set represents the aggregate over all firms with assets within some nominal bounds. We measure available funds as total receipts less total deductions plus deductions for depreciation, depletion, and amortization. To measure investment, we must construct a measure of the change in fixed assets from one year to the next. Our measure of fixed assets in any year is the sum of depreciable assets, depletable assets, intangible assets, and land less accumulated depreciation, depletion, and amortization. To measure fixed asset growth among firms in any size class, we must make an educated guess about how firms move across size classes from one year to the next. Our analysis parallels that of [Gertler and Gilchrist, 1994]. First, we re-aggregate the size categories into two groups for "small" and "large." Our cutoff for small firms is the threshold in assets below which firms account for 30 percent of sales. The annual growth rate of assets for small firms, then, is a weighted average of the growth rate of the cumulative asset classes on either side of the thirtieth percentile at the start of the period. We then make adjustments to the growth rate to correct for bias introduced by the fact that some firms may have shifted asset classes. We also perform this procedure to construct a measure of the change in accumulated depreciation, depletion, and amortization, which we then use as our measure of depreciation over the year. Our measure of investment for small firms, then, is given by the growth in fixed assets plus depreciation. Investment for large firms is constructed similarly.

Firm Level Data. Our primary source of firm-level data in the U.S. is Compustat. Compustat provides financial data on firms actively traded on stock exchanges. We focus on data from 1974 to 2010. We restrict attention to firms headquartered in the U.S. (location code is USA) and we omit firms in Financial, Real Estate and Insurance industries (SIC codes 60-67) and Government (SIC codes 91-99).

We construct available funds and investment for firms in Compustat as in the literature on firm dependence on external financing (see [Rajan and Zingales, 1998]). In particular, we measure available funds for a firm i in period t, as Operating Activities - Net Cash Flow (or Funds from operations depending format of the statement of cash flows). Because our model does not distinguish between physical investment in existing assets or acquisition of new assets, we wish to include merger and acquisition activity as well as sales of property, plant and equipment (as negative investment) in our measure of investment. We define investment as the sum of capital expenditures and acquisition less sale of property plant and equipment.

Firms remain in our sample if we have sufficient data to construct our measures of available funds and investment. We also require firms to have reported positive sales and non-negative total liabilities. Our Compustat sample in the U.S. consists of about 51,000 firm-year observations, with roughly 1400 firm level observations in a typical year.

U.K. Data

Aggregate Data. Our source of aggregate data in the U.K. is the National Economic Accounts. We measure available funds as the sum of gross disposable income and dividends of non-financial corporations. Again, we add dividend payments back in because they have been previously excluded in the construction of gross disposable income. We measure investment as the sum of gross fixed capital formation, change in inventories, and acquisitions less disposals of valuables and non-financial non-produced assets.

Firm Level Data. We have two sources of panel data on financial statements of firms in the U.K. Our first data source is Compustat Global and only covers firms that are actively traded on a stock exchange. We treat data from Compustat Global exactly as we do for data from Compustat U.S.. Compustat U.K. sample consists of roughly 10,000 firm-year observations between 1992 and 2009, with 550 firm level observations in a typical year.

Our primary source of firm level data for firms not actively traded on a stock exchange is Amadeus. Amadeus contains financial information on over 18 millions private and public companies in Europe with a focus on private companies. We restrict attention to the sample of firms located in the United Kingdom from 2001 to 2009. We focus on data from the balance sheet and profit and loss account. Data limitations prevent us from measuring available funds and investment as we do in our Compustat sample, but we attempt to be as consistent as the data allow us. Specifically, we measure available funds as the sum of profit (loss) for period, which is roughly equivalent to income before extraordinary items, and depreciation. We measure investment as the change in tangible assets plus depreciation. [1] While we focus on the privately held sample of firms

[1] We have also measured investment by additionally including intangible assets, but this addition had

in Amadeus, we also report statistics for the publicly traded firms that are available to provide a comparison to the more comprehensive sample of publicly traded firms from Compustat Global.[2]

Our sample of privately held firms consists of all non-government, non-financial firms whose legal status is Private Limited Company or Public Limited Company and which are not quoted on a stock exchange. We also restrict attention to firms that report non-negative total liabilities (defined as the sum of current and non-current liabilities), non-negative total assets, and non-negative sales. Because one component of our measure of investment is the change in tangible assets, we must have at least two consecutive years of data for a firm to remain in our sample. Our sample consists of over 980,000 firm-year observations with roughly 100,000 firm-level observations in each year. Our Amadeus sample of Public Limited Companies that are quoted on a stock exchange consists of roughly 3700 firm-year observations with 400 firm-level observations in a typical year.

2.2.3 Facts about Financial Flows and External Financing

We now describe the facts on financial flows and external financing. We begin by describing the lessons from aggregate or aggregated data, and then discuss the evidence on firm-level dependence on external financing.

Aggregate Facts on Financial Flows

In figures A.4 and A.4 we plot available funds and investment for the U.S. and the U.K normalized by nonfinancial corporate business GDP (in the U.S.) and by aggregate GDP (in the U.K.). On average, available funds are roughly 1.25 times as large as investment in the U.S. and 1.6 times as large in the U.K. Moreover, in the U.S. available funds exceed investment by roughly 3% on average over the entire sample. In this sense, the

negligible effects on our measure of external financing.

[2] We have also measured available funds and investment in this manner for both of our Compustat samples. We found that this measure of investment is roughly consistent with our measure of investment from Compustat, while this measure of available funds tends to understate the amount stated as Operating Activities - Net Cash Flow. The primary discrepancy between these measures of available funds arises from the treatment of unclassified funds. To the extent that income before extraordinary items and depreciation listed on the income statement of firms in Amadeus and Compustat are measured comparably, this finding suggests that our measure of Available Funds in Amadeus may be biased downward, and thus our measure of external financing in Amadeus may be biased upwards.

aggregate firm in the U.S. and the U.K. does not rely on outside financing to fund investment.

We have argued that this observation presents a challenge for macroeconomics models with a representative firm that faces a binding collateral constraint. There are two primary responses to this challenge. First, to the extent that shareholders refuse to accept delayed dividend payments, firms may not be able to self-finance their investment using available funds net of dividends. In figures A.4 and A.4 we plot *internal funds*, or available funds net of dividends, and investment in the U.S. and in the U.K. This data shows that internal funds are on average roughly 95% of investment in the U.S. (though they are still 1.05 times as large as investment in the U.K.). Representative agent models of financial frictions, then, must rely on some mechanism to ensure that dividends in the aggregate do not respond to aggregate shocks. In our model, because dividends of privately held firms are simply consumption of the undiversified owners of the firm, standard consumption smoothing motives will cause dividends to not adjust to aggregate shocks.

The second response to the fact that available funds exceed investment in the aggregate is that the aggregate statistics do not reveal information about how much individual firms rely on external financing. The Statistics of Income data, by reporting data by asset size classes, yields information on whether there different size classes of firms rely on external financing. We find, however, that even small firms, in the aggregate, have sufficient available funds to finance their investment. From 1992 to 2008, on average, available funds for large firms are roughly 1.8 times as large as investment, and for small firms, available funds are roughly 4 times as large as investment. We conclude that it is necessary to look at firm-level evidence in order to gauge which firms rely on external financing and how much external financing these firms need for investment.

Firm Level Evidence on External Financing

In this section, we provide evidence on the importance of external financing in firm level data. In Compustat in the U.S.. we find that on average roughly 23% of investment undertaken by publicly held firms is financed externally. Similarly, in the U.K., we find that roughly 20% of investment by publicly held firms is financed externally.[3]

Analyzing U.K. data for privately held firms, we find that between 70% and 95% of aggregate (Amadeus) investment is financed externally from 2001-2008. We provide a range of estimates because our measure of external financing is sensitive to different treatments of potential outliers. On the entire sample, we find that roughly 95% of investment is externally finance. If we remove the three largest and smallest observations over the entire sample, then we find that roughly 90% of investment is externally financed. When we winsorize the sample at the 0.1% and 99.9% levels, removing observations in the smallest 0.1 percentile and largest 99.9 percentile of available funds and investment, we find that roughly 85% of investment is externally financed; winsorizing at the 1 and 99 percentiles yields that on average 70% of investment is externally financed.

It is possible that our splitting of data by privately and publicly held firms is merely serving as a proxy for firm size. To address this issue, we construct a sample of privately held and publicly traded firms of approximately the same size over the same horizon (2001-2009). Specifically, we begin by following [Gertler and Gilchrist, 1994] to develop a sample of "large" privately held firms. In any year, we classify a firm in our Amadeus sample as "large" if its assets are over a threshold above which firms account for 70% of (Amadeus) sales. We then construct a sample of "small" publicly held firms in the U.K. similarly, except we classify a firm as "small" if its assets are below the threshold in which firms account for 30% of (Compustat) sales.

Our sample of small publicly traded firms consists of roughly 6700 firm year observations with firms holding on average £184 million in total assets (with a standard deviation of £3.75 billion). Our sample of large privately held firms consists of roughly 18000 firms with assets of roughly £840 million on average with a comparable standard deviation (roughly £3.6 billion). Notice that our large, privately held firms are on average larger than our small, publicly traded firms. Of course, the small publicly traded sample comprises the majority of the data in our overall Compustat UK sample, while the large privately held sample comprises roughly 2% of our Amadeus sample.

It is not surprising then that our sample of small publicly traded firms externally

[3] This average excludes the year 1999 because of a lack of data availability for acquisitions. Alternatively, if we ignore acquisitions, then over the entire sample, roughly 8% of investment is externally financed.

finance roughly 21% of their investment. However, even after cleaning the large sample of the most extreme values of investment and available funds, we find that large privately held firms externally finance roughly 90% of their investment (small privately held firms externally finance an even larger fraction of their investment – roughly 110%). Our conclusion is that privately held firms externally finance a substantially larger fraction of their investment than do publicly traded firms. As a result, we expect privately held firms to be much more sensitive to exogenous changes in financing conditions than publicly held firms.

2.3 A Dynamic Model of Publicly Traded and Privately Held Firms

In this section, we develop a dynamic model of publicly traded and privately held firms and define a symmetric stationary equilibrium. In our model, all firms face constraints in accumulating capital, have a monopoly in producing differentiated goods and require a bundle of goods produced by other firms as an input to production. Publicly held firms are owned by diversified households; privately held firms are owned by individual entrepreneurs.

2.3.1 Model and Equilibrium Definition

Environment. Time is discrete, lasts forever, and is indexed by $t = 1, 2, \ldots$. Agents in the model include households, final good producers, and intermediate good producers. There is a single consumption good in the economy which is a composite good produced by a sector of competitive final good producers. We normalize the price of the final good to be 1 in each period. Final good producers aggregate the output of the intermediate good producers.

Each intermediate good producer has a monopoly in producing a differentiated output. There are two classes of intermediate good producers: privately-held and publicly-held firms. We normalize the total measure of intermediate good producing firms to be 1, and we assume there is a fixed measure of privately held firms, s. A firm's type is exogenously given and fixed for the lifetime of the firm. Let firms $i \in [0, s]$ denote the names of the s privately held firms and $i \in [s, 1]$ denote the names of the $1 - s$ publicly held firms in any period. Firms exogenously exit at rate $1 - \zeta$. Upon exit, firms are replaced by an otherwise identical firm endowed with the exiting firm's assets.

In our model, we distinguish between publicly and privately held firms by assuming that publicly held firms are owned by and rebate dividends to diversified households. Privately held firms are owned by individual entrepreneurs for whom the cost of delayed dividend payments is forgone consumption.

In each period, a firm of either type can produce a output according to the constant returns to scale production function

$$y_{it} = z_{it}^{\frac{1}{\rho-1}} \left(k_{it}^{\alpha} l_{it}^{1-\alpha}\right)^{\eta} I_{it}^{1-\eta}$$

where $z_{it}, k_{it}, l_{it}, I_{it}$ are firm i's productivity, capital input, labor input, and intermediate good input in period t respectively, and ρ represents the elasticity of substitution across all goods. The intermediate input is a composite of the output of every other intermediate good firm:

$$I_{it} = \left(\int_0^1 I_{itk}^{1-\frac{1}{\phi}} dk\right)^{\frac{\phi}{\phi-1}}.$$

For simplicity, we assume that the elasticity of substitution for firms' inputs and households' consumption are the same, i.e. $\phi = \rho$ (see [Basu, 1995] for an example of this type of input-output production structure). We assume that the firm-level process for productivity follows the stochastic process $\Psi(z_{it}|z^{it-1})$ where z^{it-1} represents the history of productivity shocks that firm i has received from it's initial period to period $t-1$.

We assume that labor markets are competitive and both types of firms have access to competitive financial intermediaries who receive deposits and rent capital at rate R_t to firms. We assume that lending to firms is not perfectly enforceable. After production, firms can choose to default on their loan from the financial intermediary and, with probability $1/\lambda$ they are able to retain their undepreciated capital stock (isomorphically, they are able to retain a fraction $1/\lambda$ of their undepreciated capital). If a firm defaults, the financial intermediary seizes the financial wealth of the firm, but the loss of financial wealth is the only punishment firms face from defaulting on their debt obligations. The zero-profit condition for financial intermediaries implies that the capital rental rate is given by $r_t + \delta$.

We now describe the problem and constraints of each type of agent in turn.

Final Good Producers. There are a large number of competitive final good producers. Each of these producers can combine the output of the intermediate producers to produce a composite final good according to the production function

$$Q_t = \left[\int_i q_{it}^{1-\frac{1}{\rho}} di \right]^{\frac{\rho}{\rho-1}} \tag{2.3}$$

where q_{it} is the input of firm i in period t, and ρ is the elasticity of substitution across all goods in the economy. Perfect competition among final good producers ensures that we can focus on a representative firm that solves

$$\max_{Q, q_{ji}} Q - \int_i p_{it} q_{it} di$$

subject to the Q is given by the production function in (2.3) in each period. As before, the final good producer's problem gives rise to an inverse demand curve for each intermediate good as a function of prices:

$$p_i = Q_t^{\frac{1}{\rho}} q_{it}^{\frac{-1}{\rho}}.$$

Financing and Timing of Production. Here we describe the timing of production by intermediate good firms and its interaction with firms' financing decision. Throughout, we assume that there is no equity financing. This assumption does not have any effect on production decisions of publicly held firms. Figure 2.3.1 describes the timing of financing and production decision of the intermediate good firms.

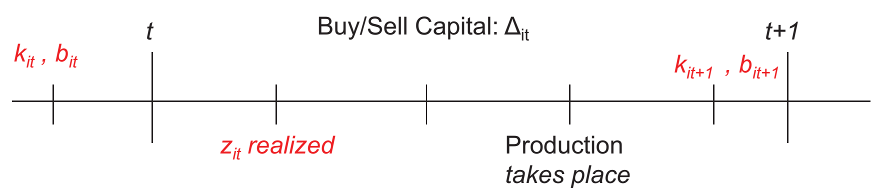

Figure 2.1: Timing of Production and Financing

As it is shown, consider a firm that enters period t with capital stock k_{it} and debt b_{it} determined at $t-1$. The owner/owners of the firm, learn about the productivity shock at t, z_{it}. Upon learning its productivity, the firm can adjust its capital stock by issuing debt. This adjustment to capital stock is subject to a collateral constraint:

$$b_{it} + \Delta_{it} \leq \frac{\lambda - 1}{\lambda} (k_{it} + \Delta_{it}) \tag{2.4}$$

The firm, then, hires labor and purchases intermediate inputs in order and production takes place. Once production has taken place, the firm decides on how much new debt to issue as well as how much to invest. Hence, the budget constraint for this firm can be written as

$$\begin{aligned} d_{it} + x_{it} \quad \leq \quad & p_{it} z_{it} \left[(k_{it} + \Delta_{it})^{\alpha} l_{it}^{1-\alpha} \right]^{\eta} I_{it}^{1-\eta} - I_{it} - w_t l_{it} \\ & + (b_{it+1} - b_{it} - \Delta_{it}) - r_t (b_{it} + \Delta_{it}) \end{aligned} \tag{2.5}$$

Note that there are two types of investment taking place in period t, Δ_{it} and x_{it}. Since Δ_{it} is being used in production in that period, it is subject to depreciation while x_{it} is not. Hence, the end of period t, capital stock for the firm is given by

$$k_{i,t+1} = x_{it} + (k_{it} + \Delta_{it})(1 - \delta)$$

The above assumtion about production and financing, helps us significantly simplify the problem. In particular, it is easy to see that we do not need to keep track of k_{it}, b_{it} jointly and the only state variable other than productivity is net worth or $a_{it} = k_{it} - b_{it}$.

Privately Held Firms. We assume that privately held firms face incomplete markets. Specifically, we assume that ownership of the privately held firms are concentrated and that they cannot issue any equity. We assume that the owners of privately held firms that are born in period j have the following preferences

$$E_j \sum_{t \geq j} (\beta \zeta)^{t-j} \log(d_{it}).$$

We assume that in making their production and financing choices, the owners of privately held firms are subject to the budget constraint described in (2.5), the collateral constraint (2.4), and the inverse demand for their production.

Publicly Held Intermediate Good Producers. As stated above, publicly held firms have the same production opportunities as a privately held firms. The only difference between publicly held firms and privately held firms arises from differences in ownership. Because publicly held firms are owned by diversified households, they maximize the expected present discounted value of their dividends under the stochastic discount factor of the households, which, in the economy with no aggregate risk is simply β^t. The objective of a publicly held firm is given by the expected present discounted value of dividends, or

$$E_0 \sum_t \beta^t d_{it}.$$

Similar to privately held firms, the owners of the publicly held firms are subject to a budget constraint, collateral constraint and inverse demand. Furthermore, we assume that they cannot issue any equity, i.e., $d_{it} \geq 0$.

Bankruptcy and Entry. As it is clear above, public and private firms have different discount factor. We interpret this as uninsurable bankruptcy by privately held firms. That is we assume that every period, privately held firms becom bankrupt with probably $1-\zeta$. Furthermore, in each period an equal measure of firms are borne and draw assets and productivities from the joint stationary distribution of assets and productivities. That is we assume that there are no losses in the value of the assets from bankruptcy. Later we perform robustness on this assumption and assume that newly born firms assets are a fraction of th amount of assets in the economy.

Households. In each period, households decide how much to work, how much to consume and how much to save in the risk-free bond. They maximize lifetime expected utility

$$E_0 \sum_t \beta^t U(C_{ht}, L_{ht})$$

subject to the sequence of budget constraints

$$C_{ht} + A_{t+1}^{h} \leq w_t L_{ht} + (1 + r_t) A_t^h + \int_s^1 c_{it} di.$$

Equilibrium Definition. The aggregate state of the economy in any period can be summarized by the distributions over debt, b, capital stock, a, and productivity, z, of privately held firms which we denote by $G_u(b, k, z)$, of publicly held firms which we denote by $G_l(b, k, z)$ and household assets A^{h}[4]. In a stationary equilibrium, we can summarize the market clearing constraints using these distributions and the decisions made by privately held and publicly held firms as functions of their individual states. We have four market clearing constraints. Bond market clearing is given by

$$\sum_{i=l,u} n_i \int_{a,z} \left[b_i(b, k, z) + \Delta_i (b, k, z) \right] G_i(db, dk, dz) = A^h \tag{2.6}$$

where n_i represents the measure of type i firms and $b_i (b, k, z) + \Delta_i (b, k, z)$ is the amount of debt issued by a firm of type i (either publicly or privately held) with individual state b, k, z, and A^h represents asset holdings of households. The market clearing constraint for final goods is given by

$$C_h + n_u \int_{a,z} d_u(b, k, z) G_u(db, dk, dz) + K' - (1 - \delta) K = Q - \sum_{i=l,u} n_i \int_{a,z} I_i(a, z) G_i(da, dz). \tag{2.7}$$

where K is the aggregate capital stock given by

$$K = \sum_{i=l,u} n_i \int_{a,z} \left[k_i(b, k, z) + \Delta_i (b, k, z) \right] G_i(db, dk, dz)$$

Intermediate goods market clearing requires for each $i = l, u$

$$q_i(b, k, z) = z^{\frac{1}{\rho - 1}} \left(k_i(b, k, z)^{\alpha} l_i(b, k, z)^{1-\alpha} \right)^{\eta} I_i(b, k, z)^{1-\eta}, \tag{2.8}$$

[4]Notice that given our assumptions about the timing of production and financing, net worth or $k - b$ is a sufficient state variable and one need not keep track of both b and k. In other words, the distribution function G_u and G_l are only a function of $k - b$.

and labor market clearing requires

$$L_h = \sum_{i=l,u} n_i \int_{a,z} l_i(b,k,z) G_i(db,dk,dz). \tag{2.9}$$

Finally, a stationary equilibrium entails that G_i is stationary for $i = l, u$, or for any $\mathcal{B}, \mathcal{K}$ and $\mathcal{Z}$

$$\begin{aligned} G_i(\mathcal{B},\mathcal{K},\mathcal{Z}) &= \int_{a,z} Q_i\left((b,k,z), \mathcal{B}\times\mathcal{K}\times\mathcal{Z}\right) G_i(db,dk,dz) \\ Q_i\left((b,k,z), \mathcal{B}\times\mathcal{K}\times\mathcal{Z}\right) &= \int_z \mathbf{1}\left\{b_i'(b,k,z)\in\mathcal{B}\right\} \mathbf{1}\left\{k_i'(b,k,z)\in\mathcal{K}\right\} \psi(z)dz \end{aligned}$$

where $\psi(z)$ represents the stationary distribution over idiosyncratic productivity and, in addition, household savings is stationary, or $A'^h = A^h$.

Definition 2.1 *A stationary recursive competitive equilibrium consists of prices, r, w, and $p_i(b,k,z)$, aggregate output Q, distributions G_l, G_u, value functions V_l, V_u, policy functions for firms $(d_i(b,k,z), b_i'(b,k,z), k_i'(b,k,z), l_i(b,k,z), I_i(b,k,z))_{i\in l,u}$, policy functions for households, $C_h(A^h), L_h(A^h), A'^h(A^h)$ such that*

1. *Given aggregate output, the wage, the interest rate, and the inverse demand curve of the final good producer , $(c_u(b,k,z), b_u'(b,k,z), k_u'(b,k,z), l_u(b,k,z))$ solves the problem of a privately held firm given by*

$$\begin{aligned} V_u(b,k,z) &= \max_{d,b',k',l,I,x,\Delta} \log(d) + \beta\zeta \int_{z'} V_u(b',k',z')d\Psi(z'|z) \quad (2.10) \\ & \quad \textit{subject to} \\ d + x &\leq p_i(b,k,z) z^{\frac{1}{\rho-1}} \left(\left(k+\Delta\right)^\alpha l^{1-\alpha}\right)^\eta I^{1-\eta} - wl - I \\ & \quad + \left(b' - b - \Delta\right) - r\left(b+\Delta\right)(1+r)a \quad (2.11) \\ b + \Delta &\leq \frac{\lambda-1}{\lambda}\left(k+\Delta\right) \\ k' &= \left(k+\Delta\right)(1-\delta) + x \quad (2.12) \\ p_i(b,k,z) &= Q^{\frac{1}{\rho}} \left(z^{\frac{1}{\rho-1}} \left(\left(k+\Delta\right)^\alpha l^{1-\alpha}\right)^\eta I^{1-\eta}\right)^{-\frac{1}{\rho}} \end{aligned}$$

2. *Given aggregate output, the wage, the interest rate, and the inverse demand curve of the final good producer , $(d_l(b,k,z), b_l'(b,k,z), k_l'(b,k,z), l_l(b,k,z), I_l(b,k,z))$ solve*

the problem of a publicly held firm given by

$$
\begin{aligned}
V_l(b,k,z) &= \max_{d,b',k',l,I,x,\Delta} d + \beta E_{z'} V_l(b',k',z') && (2.13)\\
& \quad \textit{subject to} \\
d + x &\leq p_i(b,k,z) z^{\frac{1}{\rho-1}} \left((k+\Delta)^{\alpha} l^{1-\alpha} \right)^{\eta} I^{1-\eta} - wl - I \\
& \quad + \left(b' - b - \Delta\right) - r\,(b+\Delta)\,(1+r)a && (2.14)\\
b + \Delta &\leq \frac{\lambda - 1}{\lambda} (k+\Delta) \\
k' &= (k+\Delta)(1-\delta) + x && (2.15)\\
p_i(a,z) &= Q^{\frac{1}{\rho}} \left(z^{\frac{1}{\rho-1}} \left(k^{\alpha} l^{1-\alpha}\right)^{\eta} I^{1-\eta} \right)^{-\frac{1}{\rho}}
\end{aligned}
$$

3. *Given the policy functions of publicly and privately held firms, the value functions V_u and V_l satisfy (2.10) and (2.13)*

4. *Given dividend payments of publicly held firms and the wage rate, household consumption and labor solve*

$$
\begin{aligned}
& \max U(C_h, L_h) \\
& \quad \textit{subject to} \\
C_h &= wL_h + (1-s) \int d(b,k,z) G_l(db,dk,dz)
\end{aligned}
$$

5. *The market clearing conditions (2.7), (2.8), (2.9) hold,*

6. *G_l and G_u satisfy the stationarity conditions.*

In A.3, we provide a partial characterization of the optimal decision rules of firms. By choosing to model all debt as intra-period, the optimal capital and labor decisions are purely static. As a result, the problem of a privately held firm can be simplified by two-stage budgeting. In any period, the firm chooses capital and labor; after making these decisions, the firm then decides how much to consume and save. The reaming consumption-savings decision is essentially the same as the one studied in [Huggett, 1993] or [Aiyagari, 1994]. The key difference from those models is that savings affects future profits of the firm by potentially relaxing the collateral constraint in future periods.

Before discussing theoretical and quantitative results from our model, we find it useful to demonstrate how we compute external financing in the model. Mechanically, we think of the model as generating balance sheet and income statement data and analyze the data from the model exactly as we do in the data. To see this process, let us rewrite the budget constraint in terms of $\hat{k}_t = k_t + \Delta_t$ and $\hat{b}_t = b_t + \Delta_t$, the within period capital and debt.With this notation, the budget constraint of the firm is given by

$$d_t + x_t = p_t y_t(z_t, \hat{k}_t, l_t) - wl_t - I_t + b_{t+1} - \hat{b}_t - r_t \hat{b}_t$$

or, adding Δ_{t+1} to both sides

$$d_t + X_t = p_t y_t(z_t, \hat{k}_t, l_t) - wl_t - I_t + \hat{b}_{t+1} - \hat{b}_t - r_t \hat{b}_t$$

At this point, we can define available funds and investment as we do in the data. We have

$$\begin{aligned} AF_t &= p_t y_t - w_t l_t - I_t - r_t b_t \\ X_t &= \hat{k}_{t+1} - (1-\delta)\hat{k}_t = x_t + \Delta_{t+1} \end{aligned}$$

Since the state at t is given by b_t, k_t, z_t, we may define investment X_t as a function only of b_t, k_t, z_t, and z_{t+1}. We can then define external financing in the model exactly as in the data (with a slight abuse of notation defining the distribution over b, k, z, z'):

$$\frac{\int_{b,k,z,z'} \left(X_t(b,k,z,z') - AF_t(b,k,z)\right) \mathbf{1}_{[X_t \geq AF_t]} dH(b,k,z,z')}{\int_{b,k,z,z'} X_t(b,k,z,z') dH(b,k,z,z')}.$$

2.3.2 Theoretical Results

In this section, we describe theoretical results that shed light on how the collateral constraints affect publicly held firms, the general equilibrium effects that arise in our model due to differentiated goods and the input-output structure of productions, and how the responsiveness of the economy to collateral constraint shocks depends on how much external financing occurs in the steady state equilibrium. Throughout this section, we denote net worth by $a_t = k_t - b_t$ and note that the set of constraints the firms are facing becomes

$$\begin{aligned} d_t + a_{t+1} &= p_t y_t - w_t l_t - (r_t + \delta)\,\hat{k}_t + (1 + r_t)\,a_t \\ k_t &\leq \lambda a_t \end{aligned}$$

In this section, we use the above formulation for simplicty of exposition.

Equilibrium External Financing by Publicly Held Firms

In this section, we argue that because publicly held firms discount at the same rate as household, in any equilibrium in which household consumption is stationary, publicly held firms never face binding collateral constraints. To see this. consider the problem faced by publicly held firms. If the non-negative dividend constraint or the collateral constraint ever bind along any future history with positive probability, then the value of funds inside the firm are worth more than the value of funds outside the firm. This result combined with the fact that publicly held firms are risk neutral implies that in any such period, dividends in that period must be zero. We state this result as the following lemma (the proof is in A.3).

Lemma 2.2 *In any period t, if either the non-negative dividend constraint or the collateral constraint in any history z^s with $s \geq t$ then dividends in period t are zero.*

The consequence of this lemma is that as long as either of a publicly held firm's constraints bind, the firm's asset level increases. To see this, note that stationarity of household consumption implies that $\beta(1 + r) = 1$ and thus $1 + r > 1$. The budget equation of a publicly held firm with zero dividends implies

$$a_{t+1} = \Pi(a_t, z_t) + (1 + r)a_t > a_t$$

since profits are (weakly) positive and the interest rate is positive. Of course, as long as $z_t \in [\underline{z}, \bar{z}]$ then there is a maximal optimal scale for the firm. Call this value $\bar{k}$. Once the publicly held firm's assets satisfy $a_t \geq \frac{1}{\lambda}\bar{k}$ then the firm can simply save a_t in every future period, rebate any profits and interest income to households in the form of dividends and never face binding constraints again. Since the firm's assets grow monotonically, they must cross this threshold. As a result, in any stationary equilibrium, each publicly traded firm's assets must lie above $\frac{1}{\lambda}\bar{k}$.We then have the following proposition.

Proposition 2.3 *In any stationary equilibrium, the collateral constraint never binds for any publicly held firm.*

An immediate consequence of the proposition is that there are a continuum of equilibria indexed by asset holdings of publicly held firms above $\frac{1}{\lambda}\bar{k}$ and a corresponding asset holding of households so that capital markets clear at the given rental rate of capital, $r = \frac{1}{\beta} - 1$. Since available funds for a given firm satisfy

$$AF_t = p_t y_t - w_t l_t - r_t(k_t - a_t),$$

publicly traded firms in stationary equilibria always operate at optimal scale, we have $AF_t = \Pi^*(z_t) + r_t a_t$. Since there is an equilibrium for any $\bar{a} \geq \frac{1}{\lambda}\bar{k}$, available funds are indeterminate. Consequently, the amount of external funds used by publicly held firms for investment is indeterminate. We state this result as the following corollary.

Corollary 2.4 *In any stationary equilibrium, the amount of external funds used by publicly held firms to finance investment is indeterminate.*

The Effects of Collateral Shocks on Unconstrained Firms

In any period in any stationary equilibrium of our model, some firms face binding collateral constraints and others do not. Some of the firms that do not face binding constraints are publicly held and others are privately held. One consequence of a shock that leads to a tightening of the collateral constraint is that firms that do not face currently binding collateral constraints become more productive relative to those firms for whom the collateral constraint binds. As a result, these firms, absent any general equilibrium effects, would increase their demand for production inputs and generate more output. This mechanism dampens the effects of shocks to the collateral constraints.

We argue that general equilibrium effects cause shocks to the collateral constraints to spill-over to these unconstrained firms and can dampen and even overturn their incentives to increase production in response to a tightening of the constraints. In A.1, we analyze a static, partial equilibrium version of our model where households value consumption and labor according to

$$U(C,L) = u\left(C - \frac{\psi}{1+\frac{1}{\varepsilon}} L^{1+\frac{1}{\varepsilon}}\right)$$

as in [Greenwood et al., 1988]. We develop sufficient conditions for the equilibrium output of every firm to be decreasing in the tightness of the collateral constraints. The key parameters that determine the strength of these general equilibrium spill-overs are the elasticity of household labor supply,ε, the elasticity of substitution across goods, ρ, the labor share parameter, α, and the intermediate input share parameter, η. We now state our main result from A.1.

Proposition 2.5 *Suppose there exists a positive measure set of constrained firms. If* $1+\varepsilon \geq \eta\rho(1-\alpha)$*, then output of all firms is increasing in the collateral constraint parameter,* λ*.*

The intuition for this result follows our argument in the introduction. A tightening of the collateral constraint causes constrained firms to reduce their demand for capital. Since capital and labor are complements in production, constrained firms also decrease their demand for labor. The decrease in the demand for labor causes the wage rate to fall. The decline in the wage leads to a decline in demand for the final good. Our functional form assumptions on the production function and household preferences then imply the result in the proposition. The elasticity of output of an unconstrained firm with respect to the wage rate, under the Cobb-Douglas form of the production function that we have assumed is simply $\eta\rho(1-\alpha)$ (in our static version, we hold the interest rate fixed). By assuming household preferences are of the GHH form the elasticity of aggregate demand with respect to the wage is just $1+\varepsilon$. The condition, $1+\varepsilon > \eta\rho(1-\alpha)$ simply ensures that the demand and intermediate input effects dominate the reduced marginal cost arising from the reduction in the wage.

This result is useful for understanding our results in the dynamic economy below. If we assume a labor supply elasticity of 2.6, labor share of .66, input share of .5 and an elasticity of substitution of 10, then we would expected shocks to the collateral constraint to generate an aggregate recession where all firms decrease output (at least steady state to steady state).

The Sensitivity of Output with respect to Collateral Shocks Depends on External Financing

We argue that data on external financing is a useful source of information to discipline the importance of the role financial markets play in reallocating funds from cash-rich, low productivity firms to cash-poor, high productivity firms and that the responsiveness of economic output to financial market shocks depends on the importance of this role. In this section, we use a stylized version of our dynamic model to illustrate this point theoretically.

In A.2, we analyze a simplified version of our dynamic model in which there are only privately held firms, goods are perfect substitutes, households do not save, and the productivity process is i.i.d across firms and time. This version of our model corresponds loosely to the theoretical models considered by [Kiyotaki and Moore, 2008] and, more specifically, [Kocherlakota, 2009]. Specifically, we assume that in every period, each firm has a probability π of having productivity equal to 1 and probability $1 - \pi$ of having probability equal to 0. By assuming that shocks are i.i.d. and goods are perfect substitutes, we are able to construct closed form solutions for the equilibrium wage, output, and steady-state wealth as well as compute external financing by hand.

We then consider the impact of changes in the tightness of the collateral constraint, λ, in economies with different probability of being productive, π. One difficulty in this analysis is that for economies with different productivity probabilities, π, a given change in λ represents a different size "shock" to the economy. To account for this effect, for each π-economy, we choose the collateral constraint parameter, $\lambda(\pi)$ so that the aggregate debt-to-assets ratio in the model is the same across all π-economies. Then, for each π-economy, we compare steady state wealth and output in the $\lambda(\pi)$ economy to that in the economy when $\lambda = 1$, in other words, the autarkic version of that economy. Therefore, we are considering shocks to the collateral constraint in each π economy that cause the aggregate debt-to-asset ratio to fall from some constant to 0. We show that the difference in steady state wealth between the $\lambda(\pi)$, high debt economy, and the $\lambda = 1$, no debt economy is monotonically decreasing in the probability of receiving a high productivity shock. At the same time, the amount of external financing in the $\lambda(\pi)$ high debt economy is monotonically decreasing in π. In this sense, data on external financing is useful for disciplining the macroeconomic impact of shocks to collateral constraints. We state this result from A.2 as the following proposition.[5]

Proposition 2.6 *Suppose* $0 < r < \frac{1}{\beta} - 1$. *Let* $\pi \in [\underline{\pi}, \bar{\pi}]$ *and define* $\lambda(\pi)$ *such that the debt-to-asset ratio in the* π*-economy with parameter* $\lambda(\pi)$ *is equal to* $\bar{B}$. *If for all* $\pi, \frac{1}{\beta} < \lambda(\pi) < \frac{1}{\beta(1-\pi)}$ *then external financing is decreasing in* π *and* $\log(\bar{A}(\lambda(\pi), \pi) - \log(\bar{A}(1), \pi)$ *is decreasing in* π. *(The result is the same for output).*

The intuition for this result is straightforward. Firms earn higher interest on financial wealth when they are productive. The greater is the probability of being productive, the higher is the financial wealth of firms in steady state. A fixed stock of debt relative to total assets generates a larger amount of wealth relative to autarky for an economy with a lower probability of being productive since firms cannot accumulate as much financial wealth. As a result, the difference between the autarky and the credit economy is largest when the probability of being productive is the smallest. Of course, in economies where the probability of being productive is low, when firms do become productive, they have typically experienced a long spell of being unproductive. As a result, their assets have declined and thus their available funds are low exactly when their investment exhibits a large increase. As a result, economies with low probability of becoming productive exhibit a large degree of external financing.

2.4 Calibration and Quantitative results

In this section, we calibrate the model and undertake exercises intended to illustrate the contribution of changes in financial frictions to business cycle frequency fluctuations. We calibrate the steady state of the model and then perform impulse response analysis. We compare our results to those from a standard real business cycle model.

2.4.1 Calibration

Here we describe our calibration strategy. We have two sets of parameters: parameters that are typically used in macroeconomic models, and parameters that we use our model

[5] We have proved a similar result for local changes in λ however this result requires additional sufficient conditions.

to pin down. As for the first set of parameters, we fix the discount rate to 0.96, targeting an annual real interest rate of 4%. We set the annual depreciation rate, δ, to be 0.07 and we assume that the exit rate of privately held firms, $1-\zeta$, is 10%. This exit rate implies a ten year survival rate of 34%, and is consistent with estimates from [Dunne et al., 1988]. Since ζ is a major determinant of financial flows for privately held firms[6], later, we perform robustness checks on ζ. Furthermore, we set α, one of the parameters in the firm production function to 0.3 and choose the elasticity of substitution across firms or goods to be $\rho = 4$ in line with estimates from micro data evidence (see [Burstein and Hellwig, 2008]).

We parameterize household preferences as

$$U(c,l) = \log\left(c - \frac{\psi}{1+\frac{1}{\varepsilon}} l^{1+\frac{1}{\varepsilon}}\right)$$

We choose an elasticity of labor supply, ε, to be 2.6. We assume that there are no wealth effects on the labor-leisure tradeoff to highlight the role of the complementarity between publicly held and privately held firms in our model, but note that choosing a more standard form will reduce the sensitivity of output to changes in the severity of financial frictions because declines in output by constrained firms will generate increases in output by unconstrained firms. Our choice of ε is in the range of macro estimates as documented by [Chetty et al., 2011], among others.

Next, we describe the set of parameters that are calibrated using our model. The key parameters of the steady state calibration are the tightness of the collateral constraint and the process for idiosyncratic firm level productivity. We calibrate λ, the tightness of the collateral constraint to match average aggregate debt to total assets in the U.S. economy since 1986, which is 0.49. We assume that firm level productivity follows an AR(1) process so that

$$\log z_{it} = \rho_z \log z_{it-1} + \varepsilon_{it}, \varepsilon_{it} \sim N(0, \sigma_z^2).$$

We calibrate the standard deviation of the innovations to log productivity to match the

[6] When ζ is high, firms have stronger incentives to accumulate assets in order to overcome their collateral constraint in the future. This could lower the amount of debt issued by privately held firms.This can dampen the effect of a shock to λ on aggregate output.

variance in the firm-level debt-to-asset ratio in our Amadeus sample, which is 0.28. We calibrate the persistence of the productivity process, ρ_z, so that in the model, 93% of investment by privately held firms is externally financed (as in our benchmark Amadeus sample).

The measure of privately held firms, s, is chosen so that privately held firms account for 40% of corporate gross output. We compute this share by calculating the share of firms in Compustat from gross output.[7] Figure A.4, plots this share from 1987 to 2009.

We choose the value of η (from the firm's production function) so that input's share of gross output is .43 ([Jones, 2011]). Lastly, we choose ψ, which scales household's disutility of labor supply to generate aggregate hours of 0.3. The calibrated parameters are summarized in table 2.4.1.

Parameter	Explanation	Value	Target
β	Discount rate	0.96	Annual interest rate = 0.04
ϵ	labor supply elasticity	2.6	Frisch elasticity
ρ	Elasticity of substitution	4	[Burstein and Hellwig, 2008]
α	Share of capital	0.3	
δ	Depreciation rate	0.07	
ζ	Survival rate	0.9	[Dunne et al., 1988]
	calibrated using the model		
ψ	Coefficient on leisure	0.10	Aggregate hours = 0.3
s	Measure of private firms	0.42	Share of gross output by pvt. corporations=0.40
η	Share of Intermediated Inputs	0.43	Input share of gross output = 0.43 (Jones, 2011)
λ	Tightness of collateral constraint	3.50	Average debt to asset ratio = 0.49
ρ_z	Persistence of productivity shocks	0.70	Net financial inflow = 0.93
σ_z	Variance of productivity shocks	0.65	Variance of debt to asset

Table 2.1: Calibrated Parameter Values

Before discussing the results of our quantitative exercise, it is useful to discuss how well the model does in capturing some of the key moments in data. In particular, we are

[7]We have calculated gross output by calculating the gross output at industry levels. This data is available from BEA only from 1987. Moreover, this figure includes gross output by non-corporations as well. That makes the share of privately held firms a bit biased upward.

interested in statistics relevant for privately held firms. In our calibration strategy, we have calibrated the productivity process for firms to match external financing by private firm as well the variance of debt to assets. One way to check the validity of our estimates is to compare employment fluctuations in our model to data, as documented by [Davis et al., 2007]. In our calibrated model, the cross-sectional dispersion in employment growth is 0.3 for privately held firms while in data it is 0.4. We take this as a fairly close estimate and the amount of idiosyncratic risk faced by our firms are not very far from that presented by [Davis et al., 2007].

2.4.2 Shocks to Collateral Constraints

We now analyze the response of the economy to a purely unanticipated shock to λ. We impose an initial shock to λ which then returns back to its steady state value. On impact, agents in the economy immediately learn the entire path of λ. Our goal is to understand how large a response in output is generated by a "typical" financial shock when agents place zero probability on this event occurring.

We fix the decay of the impulse so that the (annual) half-life of the shock to λ is 1 year. We choose the size of the initial impulse so that on impact, the shock generates a one standard deviation decline in aggregate debt-to-total assets in our model economy. In the U.S., since 1986, the standard deviation of the aggregate debt-to-total assets ratio for non-farm non-financial corporate businesses is 0.015 or roughly 3% (after using the HP-filter to filter out longer frequency movements). Figure (A.4) displays the residuals of the aggregate debt-to-assets ratio at an annual frequency.

Figure A.4 displays the path of deviations from steady state values of the collateral constraint parameter, λ, the aggregate debt-to-total asset ratio, and measured productivity.

We find that the impulse to the economy generates a roughly .45% decline in gross value added on impact (we define gross value added as Gross Output less the aggregate use of intermediate inputs). As we will discuss below, this is roughly 70% as large as the response of output in our model to a "typical" productivity shock; ie: an aggregate productivity shock that causes measured productivity to fall by one standard deviation of the measured solow residuals in the U.S. economy. In this sense, we view the effect of the financial shock as sizeable.

Given that the aggregate capital stock is fixed on impact, it is perhaps surprising that output and labor fall on impact of the shock. This fall occurs because the tighter collateral constraint leads constrained firms to reduce demand for capital and unconstrained firms to increase their demand for capital (the rental rate of capital falls). The set of unconstrained firms is typically made up of all publicly held firms and those privately held firms with low productivity (relative to their assets). Thus, this reallocation of capital to unconstrained firms implies more capital is installed by unproductive firms, leading to a decline in aggregate productivity. This can be seen in the fourth panel of figure A.4, which shows that measured productivity on impact falls by 0.65%.

Second, we find that output does not recover by half until 2.5 years after the impulse even though the shock has recovered by half after 1 year. In this sense, financial shocks have persistent effects on output.

Third, we find that a financial shock causes consumption, employment, and investment to move in the same direction of output. The paths for these objects, along with the paths for wages and interest rates are depicted in Figure A.4. The decline in employment is driven by the reduction in the wage rate which, in turn, falls due to the decline in measured productivity. The interest rate declines because the financial shock causes aggregate demand for capital to fall. The decline in the wage and rental rate of capital, which is also the rate of return households earn on their savings make final good consumers poorer and lead them to reduce their consumption.

Fourth, Figure A.4 displays the responses of sales by publicly and privately held firms as well as the gross output share of the publicly held firms. We find that although sales of privately held firms fall by roughly 1.5% on impact and remain below steady state for over 10 years, sales by publicly held firms actually rise by roughly .2% on impact of the shock. By the second year after the impulse, sales of publicly held firms return to steady state, and by the third year of the shock are below trend. The output share of publicly held firms, however, remains above trend for the duration of the impulse.

The different response of sales by publicly and privately held firms initially is driven by the increased use of capital by publicly held firms, none of which face binding collateral constraints, and the fact that in response to the roughly 0.1% decline in the wage rate, aggregate labor does not decline dramatically. Within two periods, however, as the supply of capital falls, in response to the tightening of the collateral constraints, sales of both types of firms are below the initial steady state. In this sense, our model is capable of generating co-movement across the publicly held and privately held sectors, at least in the medium term.

To comparing the response of these sectors in our model to data, in Figure A.4 we plot gross output of all non-financial firms in Compustat in the U.S. and gross output of all non-financial firms in the U.S. (data from the BEA) as percentage deviations from a linear trend. This figure shows that output of publicly held firms in compustat is highly correlated with that of all non-financial firms, but not perfectly so.

In terms of the share of output accounted for by publicly held firms in Compustat, which we plot (again as deviations from trend) in figure A.4, we observe that this share varies by roughly 6% but not only around business cycle dates. For example, although the share of output by Compustat firms is above trend from 2000 to 2001 and rises from 2007 to 2008, there are also significant movements (both increases and decreases) in the middle of business cycle recoveries. Given these irregulat movements, we choose not use this evidence to discipline our model.

The fact that output of publicly held firms falls below trend results only from the complementarity in production across firms that we have discussed theoretically above. We have performed sensitivity analysis with respect to the elasticity of substitution across goods, ρ, the labor supply elasticitiy ϵ and the input-output structure governed by the parameter η.

However, in each of these cases, when we have re-calibrated our model to be consistent with our target moments, we find that our benchmark result, that the response of aggregate output to our calibrated financial shock is roughly 0.45% does not vary. What does change is the composition of the response of privately versus publicly held firms. For example, with a lower labor supply elasticitiy, a higher elasticity of substitution across goods, or no input-output structure, we find that the gross output of publicly held firms does not fall below trend at any point along the impulse response path. In this sense, we view trade linkages as an important mechanism for generating co-movement of firms in response to a financial shock.

Figure A.4 displays the effect of the financial shock on the use of external funds measured in the model as in our statistic in equation (2.2). Use of external funds declines on impact and recovers back to its steady level. This result is driven primarily by the fact that most firms that use external funds are constrained firms, and investment of constrained firms falls faster than aggregate investment since most unconstrained firms actually increase investment in response to lower wages and capital rental rates.

2.4.3 Shocks to Aggregate Productivity

We now compare the effects of financial shocks to the effects of aggregate productivity shocks in our model. We perform a similar exercise as when we analyze financial shocks, only in this section we consider the transition dynamics the result from a purely unanticipated decline in aggregate productivity which slowly returns to steady state. Again, we fix the half-life of the impulse to 1 year. In order to compare the magnitudes of the effects, we choose the size of the shock so that measured productivity in the model falls on impact by one standard deviation of the measured solow residual in the United States. This corresponds to roughly a 1.% decline in measured productivity (at an annual rate).

We also compare the effects of productivity shocks in a version of our model without collateral constraints. Specifically, we analyze our model economy in the case where there are no privately held firms, but the process for idiosyncratic risk for publicly held firms is the same as in our calibrated model above.

Figure A.4 displays the impulse path for measured productivity following the shock to aggregate productivity and gross value added (and, as in all of these pictures, the dashed line represents the corresponding path from the response of our model economy to the calibrated financial shock). First, observe that a financial shock generates a decline in measured productivity roughly 70% as large as this shock to measured productivity and that measured productivity recovers faster in response to an aggregate productivity shock than to a financial shock.

Regarding gross value added, we find that a typical financial shock are roughly 70% as large as the effects of a typical productivity shock. Note that in this model, the effects of productivity shocks are dampened. In other words, a 1% decline in measured productivity causes gross value added to fall by only roughly 0.7%. This is due primarily to the monopoly distortions, which become less severe in response to a decline in aggregate productivity. Also note that the model with and without collateral constraints generate roughly the same effect on output. This is a well known result going back at least to

Kocherlakota (2000).

Finally, figure A.4 depicts the response of aggregate debt-to-total assets ratio as well as the use of external funds in the model (with collateral constraints). Notice that debt-to-assets do not move at all and the use of external funds as defined in equation (2.2) rises in response to the shock. Demand for debt by firms falls as the level of capital they optimally want to install is lower in response to the aggregate productivity shock, but their demand for capital has also fallen by roughly an equivalent amount, causing the debt-to-asset ratio to remain roughly stable. The response of the use of external funds, on the other hand, is in the opposite direction of the response following a financial shock. The reason for this is that when aggregate productivity declines, the collateral constraints for constrained firms are relaxed as the unconstrained level of investment they want to undertake is lower. As a result, investment by constrained firms falls by less than the aggregate, leading the use of external funds to rise.

In this sense, although productivity shocks and financial shocks have similar size effects on output, they affect financial flows in very different ways. It is not surprising that time series variation in aggregated financial flows statistics may be useful in distinguishing between productivity and financial shocks. Given longer time series data on financial flows for privately held firms, these differences could be useful in undertaking just such an analysis.

2.4.4 Sensititivy with Respect to Exit Risk

We conclude our quantitative section with a brief discussion of the sensitivity of our results to the magnitude of the exit risk in our model, which is governed by the parameter ζ. First, consider an increase in the survivial rate, holding the remaining parameters of the model fixed. IN this example, we increase ζ from 0.9 to 0.95.

The steady state of this economy differs significantly along the key financial flows moments that we target from that of our benchmark economy. In particular, in this economy, the aggregate debt-to-assets ratio is 0.30% (instead of 0.49) and the use of external funds is 1.63 (instead of .93). The main difference between these economies is that the average wealth of owners of privately held firms is higher. In fact, the book value of the net worth of owners of privately held firms is 60% of aggregate capital; in our benchmark, this value is 20% of aggregate capital.

Somewhat surprisingly, the use of external funds in this model is significantly higher than in our benchmark economy. The difference is that in this version of the model, most use of external funds comes in the form of changes in negative debt holdings; in other words, firms in this economy use external funds when they draw down against their net worth. One way to see this difference is to note that in this economy, rougly 20% of privately held firms face binding collateral constraints in any period (as opposed to 40% in our benchmark), but 26% of firms use external funds for investment (as opposed to 17%). In addition, over 35% of owners of privately held firms have a negative debt position. These results combine to ensure that the aggregate debt-to-assets ratio is too low relative to our benchmark economy.

The consequence of this difference is that if we impose the same impulse response as in our benchmark model, aggregate output falls by only 0.2%. We view this kind of sensitivity of our result that financial shocks have sizeable effects as arising for the wrong reasons. That is, the effects are small only because these firms do not use as much debt as they do in the data.

2.5 Conclusion

We have analyzed the effect of disturbances to financial markets in a quantitative model where financial frictions operate through collateral constraints. We used data on firm-level financial flows to discipline the importance of financial markets in the model and found that the decline in macroeconomic output generated by a calibrated tightening in the collateral constraint is roughly consistent with that generated by a calibrated decline in aggregate productivity in a standard real business cycle model.

These results suggest that shocks to collateral constraints may contribute to a significant fraction of business cycle volatility. A quantitative version of our model with aggregate shocks to collateral constraints and productivity is in progress.

An important question for future analysis is the extent to which our quantitative results are sensitive to our decision to model collateral constraints as working capital constraints as opposed to constraints on new investment. We also intend to study this question by analyzing a similar version of our model where capital is not fully mobile and the financial frictions manifest as constraints on new investment.

Chapter 3

Efficient Financial Crises

3.1 Introduction

Banks and other financial firms typically rely heavily on short-term debt to finance their assets. Non-financial firms typically do not. The short-term debt heavy capital structure of banks and other financial firms naturally exposes them to runs or other panic-like phenomena as in [Diamond and Dybvig, 1983] for example. This chapter analyzes the reasons why banks, in particular, choose to expose themselves to bank runs and the role short-term debt, as opposed to other types of capital structures, plays in achieving this exposure. In this chapter, we develop and analyze a model in which financial firms rely more heavily on short-term debt than do non-financial firms. In the model, such differences in capital structure arise from differences in the kind and quality of information lenders have about the balance sheets of financial and non-financial firms. Lenders, in the model, cannot commit to a full set of contingent contracts. We show how short-term debt with many small lenders introduces ex-post coordination problems that effectively commit the lenders to implement the optimal ex-ante contract under commitment. The reliance of financial firms on short-term debt leads to occasional bank runs, or financial crises. Such equilibrium outcomes are optimal in the sense that any alternative arrangement leads to lower welfare. Consequently, government interventions to mitigate financial crises are not desirable.

The simple observation that financial firms rely heavily on short-term debt without being required to do so suggests that such capital structures play a desirable social role.

This implies that in order to analyze the welfare implications of policy interventions aimed at mitigating or eliminating financial crises, one needs a model in which it is optimal for financial firms to issue short-term debt. One common view in the literature on banking suggests that short-term debt can be useful because the threat of bank runs provide discipline to bank managers (see [Calomiris and Kahn, 1991] or [Diamond and Rajan, 2001] for examples of this role). However, data on publicly traded corporations in the United States since 1970 suggests that heavy reliance on short-term debt is, by and large, only a property of the capital structure of financial firms. In other words, non-financial firms do not rely heavily on short-term debt. Therefore, such a model should have the property that the gains to a short-term debt heavy capital structure are specific to the business of financial firms.

Following the literature in corporate finance, we develop a model in which the capital structure of firms is optimally designed to solve incentive problems (see [Biais et al., 2007] for an example). Specifically, we develop a model in which lenders to a firm must design compensation contracts for managers to provide incentives to exert effort in a dynamic environment. The manager is protected by limited liability. In the model, this effort affects the distribution of both future outcomes and future signals lenders receive about the manager's previous effort level. High effort implies that good outcomes and good signals are likely, and low effort implies that poor outcomes and poor signals are likely (see [Hölmstrom, 1979] for an example of this kind of incentive problem). One way of providing incentives to exert high effort is to commit to dismissing the manager if both outcomes and signals are poor. Such dismissal, typically, is costly not just for the manager but for the lenders as well.

In the model, differences in the capital structure of financial and non-financial firms arise due to differences in the information lenders have about the balance sheets of financial and non-financial firms. One difference between financial and non-financial firms is that financial firms can change the types of assets they hold on their balance sheets more quickly than non-financial firms. It is also more difficult for outside lenders to monitor the types of assets a financial institution holds than for non-financial firms. For example, it is more difficult to know the quality of, say a portfolio of mortgage backed securities acquired by a financial institution than it is to tell whether a non-financial firm has built a plant in a particular location.

We interpret this difference in the ability of lenders to infer the investment actions of the manager in the model as implying that the signals lenders receive are more informative for non-financial firms than for financial firms. We show that the optimal provision of incentives depends critically on the quality of the signals lenders receive. When signal quality about managerial effort is relatively low, it is efficient to dismiss the manager when outcomes and signals are poor in spite of the costs of such dismissal to the lenders. When signal quality is high, however, it is no longer efficient to provide incentives to the manager with dismissal after poor outcomes and signals. We then demonstrate that this result implies that it is efficient for lenders in a financial firm to lend via short-term debt contracts, while for non-financial firms lenders will prefer long-term debt contracts.

To see these results, consider more specifically the tradeoffs involved with dismissing the manager after poor outcomes. In any history in which the lenders continue the project, the manager receives strictly positive expected value (net of effort costs), or a "rent," due to the combination of moral hazard and limited liability. As signals become more precise the value of the manager's rent decreases – intuitively, the moral hazard problem becomes less severe as lenders can more easily observe the manager's effort choice. As the manager's rent declines, the expected profit the lenders can receive by continuing the project increases. Since the loss to lenders from dismissal after poor outcome and signal histories is the forgone profit from continuing the investment opportunity, the loss becomes larger as signal quality improves.

The benefit to lenders from such a dismissal strategy is cost savings in terms of providing the manager with incentives to exert effort. When the lenders dismiss the manager, the manager does not receive the rents involved with continuing the investment. By dismissing the manager after poor outcomes, lenders align the manager's ex-ante incentives with their own, and, therefore, can save on how much they must compensate the manager after good outcomes and signals in order to provide appropriate incentives for effort. Because the manager's rent is decreasing in signal quality, this incentive savings is also decreasing in signal quality. As a result, the total benefit to using such a dismissal strategy becomes smaller as signal quality improves.

Thus, when signals are very precise, the value of dismissal after poor outcomes is low and lenders prefer to always continue the investment, using the compensation scheme only to provide the manager with incentives to exert effort. When signals are very imprecise, however, the value of such a dismissal strategy is highest, and, under sufficient conditions on the returns of the project, is the optimal way for lenders to provide incentives to the manager.

We go on to show how these differences in the optimal provision of incentives can be used to explain why financial institutions rely more heavily on short-term debt than do non-financial businesses. To allow for a meaningful distinction between short and long-term debt, we analyze our environment under an assumption that lenders cannot commit to the entirety of their long-term contract. A consequence of assuming that lenders lack full commitment, however, is that implementing a contract which calls for dismissal after poor outcomes may not be feasible. The reason is that after such outcomes, both the lenders and the manager stand to gain by renegotiating their contract and allowing the manager to continue. If the manager and the lenders expect such renegotiation, then the manager rationally chooses a low level of effort (ex ante). Thus, if managers and lenders cannot commit to carrying out their contracts, outcomes on average are worse than with commitment. In this sense, lack of commitment creates a *time inconsistency problem* (see [Kydland and Prescott, 1977] for an example of this problem).

Our main theoretical contribution in this chapter is to show that the use of short-term debt introduces a coordination problem among lenders that can help solve the time inconsistency problem. In particular, if an agreement to renegotiate the contract requires all or a substantial fraction of lenders to agree to a renegotiation, we show that the time inconsistency problem can be resolved. The basic idea is that such an agreement to renegotiate creates incentives on the part of each lender to threaten to disagree unless that lender is paid a large fraction of the firm's assets. Such incentives make it difficult for lenders to renegotiate the terms of the contract and help ensure that the original contract is implemented even when it is undesirable from the perspective of the collective interests of the lenders. Since the manager anticipates the likelihood of such disagreement, the manager expects the original contract to be implemented and rationally chooses to exert high effort to reduce the likelihood of poor outcomes and signals.

Coordination problems of the kind studied here are well known in the literature on the problem of providing public goods which serve common purposes such as military defense or pollution control (see [Rob, 1989] or [Mailath and Postlewaite, 1990] for examples). This literature has emphasized that requiring all or most citizens to agree to an appropriate level of defense or pollution control is difficult and has emphasized that government action might be desirable in such circumstances. The theoretical result, that coordination problems can be used to resolve time inconsistency problems demonstrates how such coordination problems can actually serve a desirable social role.

Short-term debt plays a desirable social role by introducing a coordination problem among lenders that allow them to, in effect, commit to dismiss the manager after poor project outcomes. This is only (ex ante) efficient when future signal quality about manager effort is low enough. As a result, it implies that in some cases (when the project yields a poor outcome) along the equilibrium path, outcomes trigger actions that look like they could not be part of an ex ante efficient contract – e.g., bank runs. Each lender refuses to roll-over their debt even though it is in the collective interest of the lenders to do so.

The same short-term debt contracts, however, are not optimal when the optimal provision of incentives calls for lenders to never dismiss the manager. When signal quality is high, short-term debt contracts introduce the same coordination problem and prevent lenders from continuing profitable investment opportunities. As a result, when signal quality is high, lenders prefer to use long-term debt, which, in effect, commits them to continue the project after all outcome histories.

To the extent that the information lenders have about the balance sheets of financial businesses is less precise than those of non-financial businesses, this chapter provides one reason that financial institutions are more exposed to coordination problems than non-financial institutions. As a result, not surprisingly, financial institutions are more fragile and more susceptible to crises. In the model, such fragility and susceptibility serves an important purpose by providing managers of financial institutions the incentives needed to achieve good outcomes. Equilibrium outcomes in the model are efficient in the sense that no planner confronted with the same informational structures as other agents could achieve a better outcome. In this sense, government interventions can only be harmful. In future work, we plan to introduce various spillover effects which are likely to imply that equilibrium outcomes are inefficient. Such an extended model could be useful for analyzing the best way to mitigate the probability of financial crises and to address them appropriately when they do occur.

3.1.1 Related Literature

This chapter is related to an extensive literature on bank runs and the role of demand deposits or short-term debt (see [Cole and Kehoe, 2000] and [Diamond and Dybvig, 1983]). The theoretical results on the use of short-term debt as a commitment device are closest in nature to those found in [Diamond and Rajan, 2001], [Diamond and Rajan, 2000], [Calomiris and Kahn, 1991], and [Bolton and Scharfstein, 1990], and we view our results as an important generalization of the ones in these papers. Specifically, in [Diamond and Rajan, 2001], bank runs do not occur along the equilibrium path; in [Diamond and Rajan, 2000], inefficient dismissal of the manager is not a feature of the optimal contract; and in [Calomiris and Kahn, 1991], when the optimal contract is, in their terminology, "short-term debt," dismissal of the manager is desirable from the perspective of the collective interest of the lenders. [Bolton and Scharfstein, 1990] consider an arbitrary ex-post coordination game between two lenders. When terminating the firm is costly from the collective interests of the lenders, the lenders have strong incentives to coordinate and roll over their debt. In this chapter, we allow the lenders to coordinate to develop arbitrary incentive-feasible contracts that induce the lenders as a whole to roll their debt over and demonstrate that even when the option to do so exists, short-term debt prevents them from doing so. Generalizing these results to the case where bank runs occur in equilibrium and are a feature of the optimal contract is a necessary first step in building a framework to analyze the effects of regulatory policy on the capital structure of banks.

The idea that bank runs may be a feature of optimal lending arrangements is related to results in [Allen and Gale, 1998] and [Allen and Gale, 2004]. In these papers, when intermediaries are restricted to offer demand deposits, bank default or crises allow intermediaries to share risk and effectively offer fully state-contingent contracts. In this chapter, we analyze general optimal contracts and show under what conditions particular frictions of moral hazard on the part of the bank manager and incomplete information regarding lenders' liquidity shocks give rise to crises as a feature of optimal lending arrangements.

The idea that a coordination problem can resolve a time inconsistency problem is related to the results in [Laffont and Tirole, 1988] and [Netzer and Scheuer, 2010]. In their environments, a risk-neutral principal wants to provide both incentives for effort and insurance to a risk-averse principal. Under commitment, the principal provides incentives by delivering less than full insurance to the agent. In both of these papers, when the principal (or markets) lacks commitment, the optimal contract introduces an adverse selection problem ex-post, which limits the ability of the principal to provide full insurance after effort has been provided. This adverse selection problem allows the principal to commit to deliver less than full insurance and is the efficient way to provide ex-ante incentives. In our problem, because the agent or manager is risk-neutral, a different type of ex-post informational problem is necessary for the principal (in our case, lenders) to commit to deliver the appropriate incentives.

Additionally, this chapter provides new results regarding the optimality of short-term contracts in long term agency relationships. [Fudenberg et al., 1990] develop conditions under which spot contracts implement optimal commitment outcomes in a long-run relationship. One key condition for their result is that the utility frontier describing payoffs of the principal and payoffs of the agent must be decreasing. In other words, after each history, continuation utilities for the principal and the agent lie on the set of efficient continuation allocations. The main result in this chapter demonstrates that short-term contracts may implement long-run commitment outcomes even when long-run commitment outcomes feature histories where continuation outcomes are ex-post inefficient. In this sense, our results differ from those found in [Brunnermeier and Oehmke, 2010], where a lack of commitment causes short-term contracts to deliver worse outcomes than long-run commitment outcomes.

Lastly, this chapter is related to an extensive literature on the optimal maturity structure of firm debt (see [Diamond, 1991], [Flannery, 1986], [Myers, 1977] for examples). Each of these papers is primarily concerned with variation of maturity across non-financial firms, whereas we focus on differences in maturity structure between financial and non-financial firms.

The remainder of the chapter is organized as follows. Section 3.2 contains a benchmark moral hazard problem between a single principal and an agent where managerial effort affects the distribution over outcomes and signals. This sections demonstrates how optimal incentive provision depends on signal quality. Section 3.3 contains the same model with a large number of lenders, each of which receive liquidity shocks. This section contains the main result about efficiency of short-term debt. In section 3.4, we demonstrate empirically that financial firms in the U.S. do in fact have a much more fragile capital structure than do non-financial firms in the U.S.. Section 3.5 concludes.

3.2 A Moral Hazard Model with Signals of Managerial Actions

In this section we study a benchmark moral hazard problem. In this model, a single principal must design compensation contracts in order to provide the agent, or manager of the investment, with incentives to exert effort. This effort affects the distribution over future outcomes and signals the principal will receive about the manager's previous effort level. We show that when signal quality is sufficiently high, the principal does not dismiss the manager after any history (in the optimal contract) but when signal quality is sufficiently low, (in the optimal contract) the principal dismisses the manager after a poor project outcome.

3.2.1 The Model with 1 Lender, Direct Signals of Managerial Effort, and Full Commitment

There are three periods, indexed by $t = 0, 1, 2$. There is a single principal and a single agent we call a manager. Both the principal and the manager are risk neutral, but they gave dufferent discount rates. The manager's preferences over consumption streams are given by $c_0 + c_1 + \beta c_2$. The principal's preferences are given by $c_0 + c_1 + \delta c_2$. We restrict consumption of the manager to be positive in all periods and assume $\delta > \beta$.

The manager has access to a project which, in period 0, requires I units of resources and effort of the manager $e_0 \in \{\pi_l, \pi_h\}$ which causes disutility to the manager $q(e_0)$. In period 1, the project yields both a gross return $I + y_1, y_1 \in \{y_l, y_h\}$ and a signal of the manager's effort level s_1. If the project is continued from period 1 to 2, it again requires I units of resources and effort of the manager, e_1. In period 2, the project yields gross output y_2 and another signal, s_2. Effort affects the distribution over returns, y_t and signals s_t $(t \geq 1)$ according to

$$y_{t+1} = \begin{cases} y_h \text{ w/prob } e_t \\ y_l \text{ w/prob } 1 - e_t \end{cases}$$

and

$$s_{t+1} = \begin{cases} s_h \text{ w/prob } \sigma_{e_t} \\ s_l \text{ w/prob } 1 - \sigma_{e_t} \end{cases}$$

where $\sigma_{\pi_h} \geq \frac{1}{2} \geq \sigma_{\pi_l}$. For simplicity, assume $\sigma_h = 1 - \sigma_l$. The timeline of events in the model can be depicted as follows:

e_0	y_1, v	I_1, e_1	y_2
$t = 0$	$t = 1$	$t = 1$	$t = 2$
initial effort	project returns & effort signal	Investment cost & effort if continued	output, signal if continued

Figure 3.1: Timeline for the model described in Section 3.2.1.

We restrict attention to direct revelation mechanisms in which the principal recommends an action to the agent, and the contract is designed so that the recommended action is incentive compatible for the manager. A contract (with initial investment) in this environment consists of the following collection of functions:

$$\{e_0, c_1(y_1, s_1), x(y_1, s_1), e_1(y_1, s_1), c_2(y_1, s_1, y_2, s_2)\}$$

which specify effort in period t, e_t, the continuation rule, x, and consumption of the manager, c_t as functions of the relevant history.

The value of an investment contract to the principal is given by

$$E_{e_0}\left[y_1 + I - c_1(y_1, s_1) + x(y_1, s_1)\left(-I + \delta E_{e_1(y_1,s_1)}\left[y_2 - c_2(y_1, s_1, y_2, s_2)\right]\right)\right].$$

The value to the agent is

$$-q(e_0) + E_{e_0}\left[c_1(y_1, s_1) + x(y_1, s_1)\left(-\psi(e_1(y_1, s_1)) + \beta E_{e_1(y_1,s_1)} c_2(y_1, s_1, y_2, s_2)\right)\right]$$

Let $\psi = q(\pi_h)$ and normalize $q(\pi_l) = 0$. Assuming effort is valuable, the optimal investment contract solves the following problem.

$$
\begin{aligned}
&\max E_{\pi_h}\left[y_1 + I - c_1(y_1,s_1) + x(y_1,s_1)\left(-I + \delta E_{\pi_h}\left[y_2 - c_2(y_1,s_1,y_2,s_2)\right]\right)\right] \\
&\text{subject to} \\
&\beta E_{\pi_h} c_2(y_1,s_1,y_2,s_2) - \psi \geq \beta E_{\pi_l} c_2(y_1,s_1,y_2,s_2) \\
&U_1(y_1,s_1) = x(y_1,s_1)\left[\beta E_{\pi_h} c_2(y_1,s_1,y_2) - \psi\right] \\
&E_{\pi_h}\left[c_1(y_1,s_1) + U_1(y_1,s_1)\right] - \psi \geq E_{\pi_l}\left[c_1(y_1,s_1) + U_1(y_1,s_1)\right] \\
&c_t \geq 0
\end{aligned}
$$

The first constraint contains the incentive constraint for the manager in period 1 for every period 1 history. The second and third constraints contain the incentive constraint for the manager in period 0. The final constraint $c_t \geq 0$ is the limited liability constraint that arises by restricting the manager's consumption to be positive. This constraint plays a key role, as is standard in the moral hazard literature, in the sense that it ensures that the full information optimum is not attainable. We now characterize key features of the optimal contract in this environment.

3.2.2 Characterization of the Optimal Contract

Given that the principal is more patient than the agent, standard front-loading arguments (the principal retains as much utility as possible in the final period) allow me to reduce to problem to one of only determining the continuation rule in different histories. In front-loading the contract, the principal only delivers consumption to the manager after a high output and a high signal realization in period 2. Then, we use the period 1 incentive constraint of the manager to define consumption after such a history in period 2. In other words, we have the following set of lemmas.

Lemma 3.1 *The optimal contract satisfies* $c_2(y_1, s_1, y_2, s_2) = 0$ *if* $(y_2, s_2) \neq (y_h, s_h)$ *and* $c_2(h_1, y_h, s_h) = \frac{\psi}{\beta(\pi_h\sigma_h - \pi_l\sigma_l)}$.

Then, the problem reduces to choosing only period 1 consumption and continuation rule, $(c_1(h_1), x(h_1))$ to solve

$$\max E_{\pi_h}\left[y_1 + I - c_1(y_1,s_1) + x(y_1,s_1)\left(-I + \delta\left(E_{\pi_h} y_2 - \frac{\psi\pi_h\sigma_h}{\beta\left(\pi_h\sigma_h - \pi_l\sigma_l\right)}\right)\right)\right]$$

subject to

$$E_{\pi_h}\left[c_1(y_1,s_1) + x(y_1,s_1)\frac{\psi\pi_l\sigma_l}{\pi_h\sigma_h - \pi_l\sigma_l}\right] - \psi$$
$$\geq E_{\pi_l}\left[c_1(y_1,s_1) + x(y_1,s_1)\frac{\psi\pi_l\sigma_l}{\pi_h\sigma_h - \pi_l\sigma_l}\right]$$
$$c_1 \geq 0$$

It is useful to define Ψ as the expected rent the manager receives in any continuation contract (arising from limited liability):

$$\Psi = \frac{\pi_l\sigma_l}{\pi_h\sigma_h - \pi_l\sigma_l}\psi$$

and a term ζ, the expected gain to the principal of continuing the project under high effort of the manager,

$$\zeta = -I + \delta\left(E_{\pi_h} y_2 - \frac{\pi_h\sigma_h\psi}{\beta\left(\pi_h\sigma_h - \pi_l\sigma_l\right)}\right)$$

Then, the next result characterizes c_1:

Lemma 3.2 *The optimal contract satisfies* $c_1(y_h, s_l) = c_1(y_l, s_1) = 0$ *and*

$$\begin{aligned}
(\pi_h\sigma_h - \pi_l\sigma_l)c_1(y_h,s_h) \quad = \quad & \psi - x(y_h,s_h)(\pi_h\sigma_h - \pi_l\sigma_l)\Psi \\
& -x(y_h,s_l)(\pi_h(1-\sigma_h) - \pi_l(1-\sigma_l))\Psi \\
& -x(y_l,s_h)((1-\pi_h)\sigma_h - (1-\pi_l)\sigma_l))\Psi \\
& -x(y_l,s_l)((1-\pi_h)(1-\sigma_h) - (1-\pi_l)(1-\sigma_l))\Psi
\end{aligned}$$

Using the above lemma to simplify the problem yields an optimization problem only in $x(h_1)$:

$$\begin{aligned}
&\max_{x(h_1)} E_{\pi_h} y_1 + I - \frac{\pi_h \sigma_h}{\pi_h \sigma_h - \pi_l \sigma_l}\psi \\
&+x(y_h, s_h)\left[\pi_h \sigma_h (\zeta + \Psi)\right] \\
&+x(y_h, s_l)\left[\pi_h (1-\sigma_h)\zeta + \frac{\pi_h \sigma_h}{\pi_h \sigma_h - \pi_l \sigma_l}(\pi_h(1-\sigma_h) - \pi_l(1-\sigma_l))\Psi\right] \\
&+x(y_l, s_h)\left[(1-\pi_h)\sigma_h \zeta + \frac{\pi_h \sigma_h}{\pi_h \sigma_h - \pi_l \sigma_l}((1-\pi_h)\sigma_h - (1-\pi_l)\sigma_l)\Psi\right] \\
&+x(y_l, s_l)\left[(1-\pi_h)(1-\sigma_h)\zeta + \frac{\pi_h \sigma_h}{\pi_h \sigma_h - \pi_l \sigma_l}((1-\pi_h)(1-\sigma_h) - (1-\pi_l)(1-\sigma_l))\Psi\right]
\end{aligned}$$

Now, we consider extreme cases, when the signal is uninformative, $\sigma_h = \sigma_l = \frac{1}{2}$ and when the signal is perfectly informative.

Lemma 3.3 *If the signal of effort is perfectly informative, i.e.* $\sigma_h = 1$ *and* $\sigma_l = 0$*, then* $x(y_1, s_1) = 1$.

When $\sigma_h = 1$, $\Psi = 0$, the terms multiplying x must be positive. In this extreme case, effort is observable (through the signal) and therefore there is no moral hazard problem. In this sense, this result is not surprising. What is important is that in a neighborhood of perfect signal quality the same result holds, as we demonstrate below.

In the other extreme, signal quality is completely uninformative. In this case, there is no reason to condition the continuation rule on the signal, so the problem simplifies to

$$\max_{x(y_1)} E_{\pi_h} y_1 + I - \frac{\pi_h \psi}{\pi_h - \pi_l} + x(y_h)\pi_h(\zeta + \Psi) + x(y_l)\left[(1-\pi_h)\zeta - \pi_h \Psi\right]$$

This equation illustrates the tradeoffs involved with continuation after different histories. Typically, the term ζ, the expected gain to continuing the project net of the manager's rent will be positive. Thus, continuation after either a high or low outcome will yield this continuation gain, ζ. However, the incentive costs of continuing the investment after a high or low outcome are asymmetric. Continuing after a high outcome and dimissing the manager after a low outcome relaxes the period 0 incentive constraint of the manager because it introduces more spread in the manager continuation payoffs.

This spread provides incentives for the manager to exert high effort in period 0. The cost of introducing this kind of spread, however, is forgone continuation value for the principle, ζ, which occurs with probability $(1-\pi_h)$ if the manager exerts high effort in period 0. These kind of tradeoffs, in the sense that the ex-ante optimal contract may require ex-post inefficient outcomes are standard in the moral hazard literature.

We place sufficient conditions on the technologies so that dismissal after a low outcome is optimal. In other words, we assume that

$$(1-\pi_h)\zeta < \pi_h \Psi$$

so that when signals are uninformative, the optimal contract calls for dismissal after low project outcomes. We then have the following lemma.

Lemma 3.4 *Suppose*

$$(1-\pi_h)\zeta < \pi_h \Psi.$$

If $\sigma_h = \sigma_l = 1/2$, *then* $x(y_h, s_1) = 1, x(y_l, s_1) = 0$.

We now demonstrate that results similar to the previous two lemmas hold in neighborhood of these signal quality cases.

Proposition 3.5 *Suppose*

$$\delta E_{\pi_h} y_2 - I \leq \frac{\pi_h \psi}{\pi_h - \pi_l}\left[\frac{\delta}{\beta} + \frac{\pi_l}{1-\pi_h}\right]$$

and $\delta(1-\pi_h) \geq \beta$. *There exists* $\bar{\sigma}_1 < \bar{\sigma}_2 \in [1/2, 1]$ *s.t. when* $\sigma_h \in \left[\frac{1}{2}, \bar{\sigma}_1\right]$, *the optimal contract satisfies*

$$x(y_h, s_1) = 1, x(y_l, s_1) = 0.$$

Furthermore, when $\sigma_h \in [\bar{\sigma}_2, 1]$, *the optimal contract satisfies*

$$x(y_1, s_h) = 1, x(y_1, s_l) = 1.$$

That a lower portion exists is straightforward from continuity.[1] For the upper portion, the key idea is that as σ_h gets large, ζ increases and Ψ decreases. In words, the limited liability rent the manager receives (in both periods) declines as signals become more informative because the principal need only deliver consumption to the manager when observing high output and a high signal. As the rent declines in period 2, the loss from discontinuing the project from period 1 to 2 becomes larger. Moreover, because the rent the manager receives in future periods is lower, continuing the project in period 1 has lower ex-ante costs (from the principal's perspective) with respect to providing incentives for the manager to exert effort in period 0. These effects re-inforce each other, and a sufficient condition ensures that for σ_h near 1, the gains outweigh the costs.

One minor difficulty that complicates the proof arises because as σ_h increases, the likelihood of the history (y_l, s_l) (when the agent exerts effort in period 0) converges to zero, so the expected opportunity cost of such a dismissal strategy also converges to zero. In the extreme, the costs of liquidation after a low signal of effort are exactly zero because this history never occurs (under high effort) and the incentive costs are zero because the manager receives zero rents. The additional condition, which relates δ to β and π_h simply ensures that for σ_h near 1, the expected loss from not continuing the project is large relative to the expected savings in incentives and guarantees the existence of an upper dominance region.

3.2.3 The Value of Commitment

In this type of moral hazard model, there are two primary tools which the principal can use to provide incentives for the manager to exert effort. First, the principal can create spread in transfers following different histories, rewarding the manager with high transfers after good outcomes and punishing the manager with low transfers after poor outcomes. However, because the manager receives "rents" in subsequent periods when the project is continued, there is a limit to the amount of spread that principal can deliver using only transfers. When these spreads are not sufficient, the principal must rely on dismissing the manager (and terminating the investment) to provide incentives. We have developed sufficient conditions under which when signal quality is sufficiently

[1]More stringent sufficient conditions can be developed to show fully characterize the optimal continuation rules in general, but they do not alter the key content of the characterization.

low, such dismissal is necessary for providing incentives, while when signal quality is high, such dismissal is not necessary.

One problem with contracts that call for dismissal is that they require commitment on the part of the principal. As is well known in these types of moral hazard problems, lack of commitment can be a severe problem and lead to underinvestment ex-ante. To be precise, consider the case of uninformative signals and suppose that

$$\delta E_{\pi_h} y_2 - I \geq \frac{\delta \pi_h \psi}{\beta \pi_h - \pi_l}.$$

Then, if the principal cannot commit to the continuation rule and is free to re-write the contract in period 1 following any history of y_1, under the above condition, the principal will always continue the project (since it is profitable to do so even though the principal must pay the manager additional compensation so that the manager will exert effort in period 1). As a result, the value of the optimal contract (including the initial investment cost) when the principal cannot commit to dismiss the manager is given by

$$E_{\pi_h} y_1 - \frac{\pi_h}{\pi_h - \pi_l}\psi + \pi_h(\zeta + \Psi) + (1 - \pi_h)\zeta - \pi_h \Psi.$$

When the principal can commit to dismiss the manager, this value is given by

$$E_{\pi_h} y_1 - \frac{\pi_h}{\pi_h - \pi_l}\psi + \pi_h(\zeta + \Psi).$$

Hence, the value of commitment can be defined as

$$\pi_h \Psi - (1 - \pi_h)\zeta$$

which we have assumed to be strictly positive. The fact that the principal earns strictly higher value under commitment can be made more stark by assuming that without commitment and the ability to provide optimal incentives to the manager, the principal would not undertake any investment ex-ante. We now show that when the principal acts as a stand in for a large group of lenders, each of which must contribute to finance the investment, the lenders as a block can recapture the value of commitment even when they lack the same commitment as the principal in this example.

3.3 Benchmark Model of Investment with Many lenders and Limited Commitment

In this section, we focus on the case with uninformative signals and show how short-term contracts among many small lenders can help resolve the time inconsistency problem. We set up the problem with N lenders designing optimal compensation contracts for a manager who operates the firm and is subject to moral hazard and limited liability. We develop conditions analogous to the above case under which the optimal commitment contract calls for lenders to discontinue their investment in the project after poor project outcomes. We show that when there are sufficiently many lenders and re-negotiating the contract requires all or almost all of the lenders to agree, then the lenders can effectively commit not to re-negotiate the contract.

3.3.1 Environment of the Benchmark Model

Consider again a three period environment with $N+1$ agents. Let the periods be indexed by $t = 0, 1, 2$. We call the $N+1$st agent a manager and the remaining agents lenders. All agents are risk neutral, but we assume that they discount consumption in the final period at different rates. The manager's preferences over consumption streams are given by $c_0+c_1+\beta c_2$. Each lender's utility over consumption is given by $c_0+c_1+v_i c_2$ where we restrict consumption to be positive in all periods. We assume that v_i is an i.i.d. preference shock realized in period 1 with $v_i \sim G_i(v_i)$ having support $[\underline{v}, \bar{v}]$. Let G denote the joint distribution over v. The preference shocks can be thought of as liquidity shocks to the lenders, causing them to have a stronger preference for period 1 consumption when they realize lower values of v_i. Additionally, the preference shocks are privately known by each individual lender and $\beta < \underline{v}$.

Each lender has an identical endowment stream $\left(k_0^i, k_1^i, k_2^i\right)$ with $k_0^i = k_0(N), k_1^i = 0, k_2^i = 0$. For simplicity, we will further assume that $k_0^i = I/N, k_1^i = k_2^i = 0$. This assumption ensures that each lender must participate in the investment project in order for the project to be undertaken. This assumption can be relaxed by appropriately modifying some of the assumptions that follow.

The manager has access to a project which requires I units of resources in period 0 and effort of the manager $e_0 \in \{\pi_l, \pi_h\}$ which causes disutility $q\left(e_0\right)$ to the manager in

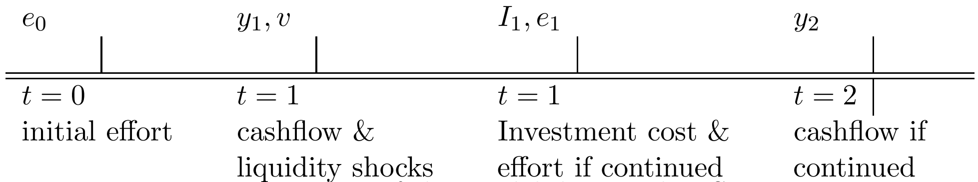

Figure 3.2: Timeline of the Model discussed in Section 3.3.1

period 0. The project yields a gross return $I + y_1 \in \{y_l, y_h\}$ in period 1 where $y_1 = y_h$ with probability e_0 and $y_1 = y_l$ with prob. $1 - e_0$. If the project is continued from period 1 to 2, it requires resource inputs I again and additional effort of the manager e_1. The project then yields output $y_2 \in \{y_l^1, y_h^1\}$ with $y_j^h > y_j^l$ and is $y_2^1 = y_h^1$ with probability e_1. Notice, then, the problem is slightly different from the earlier example as there is some persistence in project outcomes. The time-line of the physical attributes of the environment are as follows

A contract in this environment consists of transfers (to or from the lenders and to the manager), recommended effort levels for the manager, and a continuation rule in period 1 as a function of the relevant history. We will focus on investment contracts which call for investment in period 0 and we will ensure that investment is superior to autarky from the ex-ante perspective of each of the lenders. To be specific, an *investment contract* consists of the following collection of functions:

$$\Big\{ \big(t_0^i, t_1^i(y_1), t_{1c}^i(y_1, v), t_{1n}^i(y_1, , v), t_2^i(y_1, v)\big)_{i \in \{1, \ldots, N\}},$$
$$x(y_1, v), e_0, e_1(y_1, v), c_1(y_1), c_2(y_1, y_2^1, v) \Big\}.$$

Let $\mathbf{C}$ denote the set of all contracts. A contract consists of transfers to lenders at each feasible history. Notice, a contract may specify transfers in period 1 both before lenders' report their types and after. In a model with full commitment, this distinction is meaningless, however in the model with limited commitment, this distinction will play an important role which we describe carefully below. In effect, the distinction will allow me to distinguish between short and long term debt contract. In addition, we have indexed period 2 transfers independently of the period 2 cash flow outcome because lenders are risk neutral.

The additional components of the contract consist of a continuation rule in period 1 (as a function of the cash flow and lenders' types, $x(y_1, v)$), recommended effort levels for the manager, $(e_0, e_1(y_1, v))$, and managerial consumption in period 1 (in expectation with respect to the continuation rule and the lenders' reports) $c_1(y_1)$, and as a function of the history in period 2 $(c_2(y_1, y_2, v))$. Ex-ante welfare of the lenders (net of initial investment costs) under any contract is given by

$$E_{e_0} \sum_{i=1}^{N} \left[t_1^i(y_1) + \int_v \left[x(y_1, v) \left(t_{1c}^i(y_1, v) + v_i t_2^i(y_1, v) \right) + (1 - x(y_1, v)) t_{1n}^i(y_1, v) \right] dG(v) \right]. \tag{3.1}$$

Given the structure of the environment, a contract will have to satisfy several constraints to be considered feasible. These constraints include resource feasibility in all periods, capacity constraints of the lenders in period 1, incentive compatibility of the lenders in period 1 with respect to their privately known liquidity shock, voluntary participation of the lenders in period 0, and incentive compatibility and participation of the manager in each period (where we appeal to the revelation principle to restrict attention to incentive compatible contracts[2]).

A contract is *resource feasible* if it satisfies

$$I + \sum_{i=1}^{N} t_0^i \leq 0 \tag{3.2}$$

$$\sum_{i=1}^{N} \left[t_1^i(y_1) + x(y_1, v) t_{1c}^i(y_1, v) + (1 - x(y_1, v)) t_{1n}^i(y_1, v) \right] + c_1(y_1)$$
$$\leq y_1 + I - I x(y_1, v)$$
$$\sum_{i=1}^{N} t_2^i(y_1, v) \leq E_{e_1(y_1, v)}(y_2^1 - c_2(y_1, y_2^1, v)).$$

A contract satisfies the lenders' *capacity constraints* (or positive consumption requirement) if for $j = c, n$

$$t_{1j}^i(y_1, v) + t_1^i(y_1) \geq 0. \tag{3.3}$$

To define incentive compatibility, it is useful to define the continuation utility of an

[2] Although restricting attention to incentive compatible contracts is not without loss of generality in general in environments with limited commitment, the fact that the manager's effort level in period 0 does not independently effect the likelihood of of period 2 outcomes ensures that the standard version of the revelation principle applies in our environment. See [Sleet and Yeltekin, 2006] for an extended discussion of such an application of the revelation principle.

lender conditional on realizing a preference shock v_i and reporting preference shock $\hat{v}_i$, which we denote by $w_i(y_1, \hat{v}_i, v_i)$ and is given by

$$\begin{aligned} w_i(y_1, \hat{v}_i, v_i) &= \int_{v_{-i}} x(y_1, \hat{v}_i, v_{-i}) \left(t^i_{1c}(y_1, \hat{v}_i, v_{-i}) + v_i t^i_2(y_1, \hat{v}_i, v_{-i}) \right) dG_{-i}(v_{-i}) \\ &\quad + \int_{v_{-i}} (1 - x(y_1, \hat{v}_i, v_{-i}) t^i_{1n}(y_1, \hat{v}_i, v_{-i}) dG_{-i}(v_{-i}) \end{aligned}$$

A contract is *incentive compatible* with respect to the lenders if

$$w_i(y_1, v_i, v_i) \geq \max_{\hat{v}_i \in [\underline{v}, \bar{v}]} w_i(y_1, \hat{v}_i, v_i) \tag{3.4}$$

and satisfies lender *voluntary participation* if

$$E_{e_0} \left[t^i_1(y_1) + \int_{v_i} w_i(y_1, v_i, v_i) dG_i(v_i) \right] \geq \frac{I}{N}. \tag{3.5}$$

A contract is *incentive compatible* with respect to the manager if

$$\begin{aligned} &\beta e_1(y_1, v) c_2(y_1, y_h, v) + \beta(1 - e_1(y_1, v)) c_2(y_1, y_l, v) - q(e_1(y_1, v)) \qquad (3.6) \\ &\quad \geq \max_{e'} \beta e' c_2(y_1, y_h, v) + \beta(1 - e') c_2(y_1, y_l, v) - q(e') \\ &U_1(y_1, v) = x(y_1, v) \left[\beta E_{e_1(y_1, v)} c_2(y_1, y_2, v) - q(e_1(y_1, v)) \right] \\ &E_{e_0} \left[c_1(y_1) + \int_v U_1(y_1, v) dG(v) \right] - q(e_0) \\ &\quad \geq \max_{e'} E_{e'} \left[c_1(y_1) + \int_v U_1(y_1, v) dG(v) \right] - q(e') \end{aligned}$$

We now describe the notion of limited commitment which we impose on the environment. We assume that in period 1, after period 1 transfers $t^i_1(y_1)$ and $c_1(y_1)$ have been allocated, the lenders may freely choose to alter the remaining components of the contract. The only restrictions on the new contract are that it cannot deliver negative (total) consumption to any agent in period 1 and no agent can be coerced into participating. The first constraint limits any single lender (or a small block of lenders) from fully financing the investment project in period 1. The second constraint serves to define the outside option of any individual lender, which is simply $t^i_1(y_1)$.

Explicitly, the lenders may choose a new *continuation contract* defined as

$$\left\{\left(\hat{t}^i_{1c}(v), \hat{t}^i_{1n}(v), \hat{t}^i_2(v)\right)_{i\in\{1,\ldots,N\}}, \hat{x}(v), \hat{e}_1(v), \hat{c}_2(y^1_2, v)\right\}.$$

This notion of limited commitment gives rise to an *enforceability* constraint which any contract must additionally satisfy to be considered feasible. Specifically, a contract $C \in \mathbf{C}$ is *enforceable* if there exists no sub-contract

$$\hat{C} = \left\{\left(\hat{t}^i_{1c}(v), \hat{t}^i_{1n}(v), \hat{t}^i_2(v)\right)_{i\in\{1,\ldots,N\}}, \hat{x}(v), \hat{e}_1(v), \hat{c}_2(y^1_2, v)\right\}$$

satisfying (3.6) such that

$$\int_v \hat{x}(v)\left[\beta E_{\hat{e}_1(v)}\hat{c}_2(y^1_2, v) - q(\hat{e}_1(v))\right] dG(v) \geq U_1(y_1, v) \tag{3.7}$$

$$\sum_{i=1}^N \int_v \left[\hat{x}(v)\left(\hat{t}^i_{1c}(v) + v_i\hat{t}^i_2(v)\right) + (1-\hat{x}(v))\hat{t}^i_{1n}(v)\right] dG(v) \tag{3.8}$$

$$> \sum_{i=1}^N \int_v \left[x(y_1, v)\left(t^i_{1c}(y_1, v) + v_i t^i_2(y_1, v)\right) + (1 - x(y_1, v))t^i_{1n}(y_1, v)\right] dG(v)$$

$$\hat{w}^i(v_i, v_i) \geq \max_{\hat{v}_i\in[\underline{v},\bar{v}]} \hat{w}_i(\hat{v}_i, v_i) \tag{3.9}$$

$$\hat{w}_i(v_i, v_i) \geq 0 \tag{3.10}$$

$$\sum_{i=1}^N \hat{t}^i_2(v) \leq E_{\hat{e}_1(v)}\left[y^1_2 - \hat{c}_2(y^1_2, v)\right] \tag{3.11}$$

$$\sum_{i=1}^N \int_v \left[\hat{x}(v)\hat{t}^i_{1c}(v) + (1-\hat{x}(v))\hat{t}^i_{1n}(v)\right] dG(v)$$

$$\leq y_1 + I - \sum_{i=1}^N t^i_1(y_1) - c_1(y_1) + I\int_v \hat{x}(v)dG(v) \tag{3.12}$$

$$t^i_1(y_1) + \hat{x}(v)\hat{t}^i_{1c}(v) + (1-\hat{x}(v))\hat{t}^i_{1n}(v) \geq 0 \tag{3.13}$$

In other words, a contract is enforceable if there is no other contract that increases expected utility of the manager and the total expected utility of the collection of lenders in from period 1 onwards. Observe that elements of the original contract, C, appear only in the objective (equation (3.8)), the resource constraint (equation (3.12)), the capacity constraint (equation (3.13)), and the manager's participation constraint (equation

(3.7)). This notion of enforceability makes clear the distinction between period 1 transfers made before agents realize their preference shock (early transfers) and period 1 transfers made after agents realize their type (late transfers). Because we assume that early transfers are made before new continuation contracts can be designed, these transfers affect the set of feasible continuation contracts. By allocating positive early transfers in period 1, the limited liability constraints of the lenders and the resource constraints in any continuation contract become more stringent. If early transfers are all equal to zero, then the limited liability constraints are weak – any contract (including contracts that call for continuation) are feasible as long as they deliver at least 0 transfers to each agent.[3]

3.3.2 Optimal Contracts

The optimal contract with *full* commitment maximizes ex-ante welfare in (3.1) and satisfies the resource constraints in (3.2), the incentive constraints and voluntary participation constraints in (3.4), (3.6), and (3.5), and satisfies the positive consumption constraint for lenders given by (3.3). The optimal contract with *limited* commitment maximizes ex-ante welfare in (3.1) and satisfies the resource constraints in (3.2), the incentive constraints and voluntary participation constraints in (3.4), (3.6), and (3.5), satisfies the positive consumption constraint for lenders given by (3.3), and is enforceable (there exist no subcontracts satisfying (3.7)-(3.13)).

The optimal contract with full commitment provides a benchmark that is useful in characterizing optimal contracts with limited commitment. Indeed, the main result in this section provides assumptions under which the optimal commitment contract is enforceable in the environment with limited commitment. We state this result here including all additional assumptions and then discuss the outline of its proof and its significance.

Proposition 3.6 *Define* $C(y_1) = E_{\pi_h} y_2^1 - \frac{\pi_h \psi}{\beta \Delta}$. *Suppose that* $\{G^n\}$ *defines a sequence of economies where* $G^n = (G_1^n, \ldots, G_n^n)$ *such that*

[3] Note that in this formulation, re-negotiated contracts do not necessarily have to be pareto improving. Requiring pareto improvements, that is, ensuring each lender receives at least weakly greater utility under the new contract than under the status quo, is a stronger assumption and the following results would still hold.

(i) *$\beta < \underline{v}$,*

(ii) $0 < \pi_h \frac{\pi_l \psi}{\pi_h - \pi_l} + (1 - \pi_h)\left[I - \bar{v}C(y_l)\right]$,

(iii) *$I < \tilde{v}C(y_l)$,*

(iv) *$\underline{v}C(y_l) < I < \underline{v}C(y_h)$,*

(v) *$\frac{1-G_i(v_i)}{v_i^2 g_i(v_i)}$ is decreasing in v_i*

(vi) *there exists $\kappa > 0$ such that $g_i^n(v_i) > \kappa$*

(vii) *there exists v^* such that $\bar{v}_i^n < v^*$.*

Then, as $N \to \infty$, the optimal contract under commitment is enforceable.

The basic idea is that assumptions $(i) - (ii)$ ensure that the continuation rule in the optimal commitment contract satisfies $x(y_h) = 1, x(y_l) = 0$. Assumption (iii) ensures that that choosing $x(y_l) = 0$ is ex-post inefficient (on average) and, therefore, would typically suffer from the same time inconsistency problem discussed above in section 3.2.3. In other words, the first three assumptions ensure that whether the optimal contract under commitment is enforceable with limited commitment is an interesting question.

Assumptions $(iv) - (vii)$ are useful in characterizing the nature of the coordination problem that arises in period 1 following low or high outcomes. Indeed, assumption (iv) asserts that if all of the lenders have the lowest rate of time preference, then it would be ex-post inefficient for them to continue the project. Of course, as the number of agents becomes large, the probability of this outcome becomes arbitrarily small (by assumption (iii)). Nonetheless the fact that it is possible for all of the lenders to have a preference shock of $\underline{v}$ implies that each individual lender must be provided with incentives to reveal their type truthfully, and these incentives do not become arbitrarily small as N to ∞. As we will show, only by choosing t_1^i to be sufficiently large do these incentives become relevant.

The incentive problem facing the lenders following high period 1 outcomes is different however, due primarily to the right-most inequality in assumption (iv). After high period 1 outcomes, all of the lenders know (with probability 1) that it is ex-post efficient to continue the project. As a result, even with a probability of continuation equal to 1, there exist transfers satisfying voluntary participation and incentive compatibility of the lenders. For example, a constant transfer scheme will satisfy these constraints. As a result, the lenders will always efficiently continue the project after high period 1 project outcomes; in effect, the public good problem that enforces liquidation of the project following poor period 1 outcomes is weaker (because future project returns following high period 1 outcomes are higher) and can be overcome by the lenders.

The outline of the proof of Proposition 3.6 can be summarized by the results:

1. Under full commitment, the optimal continuation rule satisfies $x(y_h) = 1$, $x(y_l) = 0$ (we also characterize optimal consumption of the manager),

2. There exists period 1 transfers, t_1^i such that $x(y_h) = 1, x(y_l) = 0$ is enforceable as $N \to \infty$

3. Given the transfers t_1^i and continuation rules $x(y_h) = 1, x(y_l) = 0$ (and optimal managerial consumption), the remaining optimization problem with limited commitment coincides with the optimization problem with full commitment, implying that they have the same solution.

We now discuss the proofs of each of these results in turn (in all of the following lemmas and propositions, assumptions $(i) - (vii)$ are maintained).

Optimal Continuation and Managerial Consumption under Commitment

The first step in demonstrating our main result is to characterize key features of the optimal contract under commitment. To do so, we first adapt techniques from the dynamic moral hazard literature to our economy and characterize optimal consumption allocations for the manager as a function of the continuation rule.

Characterizing Consumption of the Manager. Since the manager is more impatient than the most impatient lender, optimal contracts are front-loaded. That is, the manager's effort constraint in the last period always binds (except possibly on a set of measure 0 realizations of lenders types). As a result, the manager's continuation utility, conditional on the continuation probability is a fixed number independent of the history at time 1.[4] Then, by assuming that the manager's participation constraint is slack (which is satisfied, for example, if the manager's outside option yields 0 utility), the manager's incentive constraint must bind and implies $c_1(y_l) = 0$. We state these results in the following lemmas. (All proofs are in the appendix).

Lemma 3.7 *If $\beta < \underline{v}$, then it is without loss of generality to restrict attention to contracts that satisfy*

$$c_2(y_1, y_h, v) = \frac{\psi}{\beta(\pi_h - \pi_l)}$$

and $c_2(y_1, y_2, v) = 0$ otherwise.

This result is standard in environments with moral hazard and limited liability (see [Hölmstrom, 1979] for an example). Here, the result is simply extended to allow for dependence of the consumption function on the realization of v. Let $\Delta = \pi_h - \pi_l$ and define Ψ to be the expected "rent" the manager in any state in period 1 in which the project is continued. The value of this rent is given by

$$\Psi = \beta E c_2 - \psi = \frac{\pi_l \psi}{\Delta}.$$

We use Ψ to characterize consumption allocated to the manager in period 1 conditional on a realization of high output.

Lemma 3.8 *If $\beta < \underline{v}$, then it is without loss of generality to restrict attention to contracts that satisfy $c_1(y_l) = 0$ and*

$$c_1(y_h) = \frac{\psi}{\Delta} + \Psi \int_v [x(y_l, v) - x(y_h, v)]\, dG(v). \tag{3.14}$$

Lemma 3.8 illustrates the tradeoff of continuing the investment project following different period 1 project outcomes. Contracts that call for continuation after a low output realization provides the manager with utility because the manager will receive the rent in the second period. As a result, to induce high effort in period zero, the manager

[4]What is important is not that the manager's utility conditional on continuation is a fixed number, but that it is strictly positive. In our model, the continuation utility is strictly positive in period 1 because the moral hazard problem is present in period 2. Alternative assumptions are interpretations could be made to ensure that the model has the same feature.

must be compensated with more period 1 consumption after a high output. Contracts that call for continuation after a high output realization in period 1 can deliver lower period 1 consumption to the manager because such contracts deliver future utility to the manager.

The previous two lemmas have fully specified the manager's consumption as a function of only the continuation rule x. It remains to show how the choice of the continuation rule affects lenders' payoffs.

The Optimal Continuation Rule. It remains to develop additional sufficient conditions so that the optimal continuation rule has the property that when $y_1 = y_h$, it is optimal to continue the project and when $y_1 = y_l$, it is optimal to shut down the project. Conditions analogous to those in Section 3.2.2 ensure this is the case. To prove that this continuation rule is feasible, given the resource feasibility constraints and the lender incentive compatibility and participation constraints, we demonstrate that such a continuation rule can be implemented with transfers that depend on the outcome y_1, but not on the reported discount rate of any individual lender v_i.

Assumption (*iii*) ensures that even after poor period 1 project outcomes, in expectation (over the lenders' preference shocks) continuing the project is a profitable venture (even net of the cost of providing the manager with incentives to exert effort). Assumption (*ii*) is analogous to that assumed in Lemma (3.4) and ensures that the gains the incentive associated with dismissing the manager after low period 1 outcomes outweigh the resources lost by forgoing the investment for another period.

We then have the following result concerning the optimal continuation rule.

Lemma 3.9 *Under the maintained assumptions, if U_0^* sufficiently small, any optimal contract under full commitment satisfies $e_0 = e_1 = \pi_h, x(y_h, v) = 1, x(y_l, v) = 0$.*

This proof mirrors that of Lemma (3.4). The only additional concern is whether such a continuation rule is feasible with respect to the lenders incentive constraints. This is easily demonstrated, however, by verifying that there exist transfer schemes such that combined with the continuation rule the contracts are incentive compatible. When the contract calls for dismissal ($x(y_l, v) = 0$) it is clear that fixed transfers will implement the optimal contract. When the contract calls for continuation ($x(y_h, v) = 1$), again, fixed transfers are feasible, though not necessarily optimal. The optimal transfer scheme satisfies

$$
\begin{aligned}
&\max \sum_i \int_v \left[t_1^i(y_h) + t_{1c}^i(y_h, v) + v_i t_2^i(y_h, v)\right] dG(v)\\
&\text{subject to}\\
&t_1^i(y_h) + t_{1c}^i(y_h, v) \geq 0\\
&\sum_i \left[t_1^i(y_h) + t_{1c}^i(y_h, v)\right] + c_1(y_h) \leq y_h\\
&\sum_i t_2^i(y_h, v) \leq E_{\pi_h} y_2(y_h) - c_2\\
&w_i(v_i, \hat{v}_i) = \int_v \left[t_{1c}^i(y_h, \hat{v}_i, v_{-i}) + v_i t_2^i(y_h, \hat{v}_i, v_{-i})\right] dG_{-i}(v_{-i})\\
&w_i(v_i, v_i) \geq w_i(v_i, \hat{v}_i)\\
&w_i(v_i, v_i) \geq 0.
\end{aligned}
$$

Enforcing the Optimal Commitment Continuation Rule

In period 1, after any history, without commitment lenders are free to choose a new contract satisfying the constraints in the definition of enforceable contracts. In words, the constraints facing lenders are limited liability, which depends on t_1^i, resource constraints (continuation requires I to be re-invested, whereas liquidation can pay out the initial principal, I), the resource constraint in period 2, and incentive compatibility and voluntary participation for lenders.

It is easiest to prove enforceability by considering extreme period 1 transfers, $t_1^i = I/N$. In this case, following a low period 1 outcome, by limited liability, it must be that

$$t_{1c}^i = -I/N, t_{1n}^i = 0.$$

Consider, then, the constraints facing the lenders after a low realization of output in the first period. These constraints can be summarized as

$$w_i(\hat{v}_i, v_i) = \int_{v_{-i}} \left[x(\hat{v}_i, v_{-i}) \left(\frac{-I}{N} + v_i t_2^i(\hat{v}_i, v_{-i}) \right) \right] G_{-i}(dv_{-i}) \tag{3.15}$$

$$w_i(v_i, v_i) \geq w_i(\hat{v}_i, v_i) \tag{3.16}$$

$$w_i(v_i, v_i) \geq 0 \tag{3.17}$$

$$\sum_i t_2^i(v) \leq E_{\pi_h} y_2^1 - \frac{\pi_h \psi}{\beta \Delta} \tag{3.18}$$

Notice, the manager's incentive constraint is nested in the resource constraint in period 2 (in equation (3.18)).

Now, we appeal to results from [Myerson, 1981] and [Myerson and Satterthwaite, 1983] which allow me to characterize the global incentive constraints as local constraints and allow me to eliminate period 2 transfers from the problem (albeit under a weaker, ex-ante version of the period 2 resource constraint). These results are stated in the following lemmas.

Lemma 3.10 *A contract satisfies lender incentive compatibility if and only if the function $\rho_i(v_i)$ defined by*

$$\rho_i(v_i) = \int_{v_{-i}} x(v_{-i}, v_i) dG_{-i}(v_{-i}) test$$

is increasing in v_i for all i, v_i, y_1 and

$$u^i(v_i) \equiv w^i(v_i, v_i) = v_i \left[\frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N} \int_{\underline{v}}^{v_i} \frac{1}{z^2} \rho_i(z) dz \right].$$

Moreover, the contract satisfies voluntary participation if and only if $u^i(\underline{v}) \geq 0$.

The proof is in the appendix. Combining this lemma with the ex-ante version of the period 2 resource constraint, we have the following result.

Lemma 3.11 *Suppose that x is such that ρ^i is increasing in v_i. There exist payments t_2^i such that (t_2^i, x) satisfy lender incentive compatibility, voluntary participation, and* *the period 2 resource constraint if and only if*

$$\int_v x(v)\left[E_{\pi_h} y_2^1 - \frac{\pi_h \psi}{\beta \Delta} - \frac{I}{N}\sum_i \left[\frac{1-G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i}\right]\right] G(dv) \geq 0.$$

Using this lemma, following a result in [Mailath and Postlewaite, 1990], it is straightforward to demonstrate that as as $N \to \infty$, the maximal probability of continuation for which there exist transfers so that the continuation rule and transfers satisfy the constraints of the renegotiation problem converges to zero (following low period 1 outcomes). We state this result in the following proposition, which makes use of the regularity conditions of assumptions (v)-(vii) in Proposition 3.6.

Lemma 3.12 *Following a low period 1 outcome, the maximum probability that the project is continued, or $\hat{x}(n)$ satisfying*

$$\begin{aligned} \hat{x}(n) \quad &= \quad sup\left\{E\rho(v) : \exists (t_2^i), (\rho, t_2^i) \text{ satisfies lender IC, participation} \right. \\ &\left. \quad\quad , \text{and the RC in the } n\text{-agent economy } ((3.15)\text{-}3.18)\right\} \end{aligned}$$

converges to 0 as n goes to infinity. Furthermore, the probability that it is ex-post efficient to continue the project goes to 1.

To understand the result, consider the tradeoffs a single lender faces when determining which discount rate to report, $\hat{v}_i$. The benefit of under-reporting is that the agent's transfers become larger (if the agent were that type, for a fixed probability of continuation a higher discount rate would require higher transfers to satisfy the participation constraint of the lender). The cost to that lender is that by under-reporting, the probability of continuation declines (since ρ_i is increasing).

The key idea behind the limiting result, as in [Mailath and Postlewaite, 1990], is that as $N \to \infty$, the cost of under-reporting shrinks to zero since the likelihood that a single lender is pivotal, and thus by under-reporting v_i would cause the investment not to be undertaken, becomes arbitrarily small. On the other hand, the costs of providing incentives do not shrink to zero. As a result, in the limit the incentive costs are so large as to make the value of continuation, net of incentive costs, equivalent to that which would occur in a full information economy where all lenders had the highest discount rate; at this rate, lenders prefer not to undertake the investment.

The constraints also make plain why by using all long-term debt (i.e. $t_1^i = 0$), even when lenders have the option to walk away from the contract (and force liquidation of the project) cannot implement the optimal continuation rule. With this contract, an individual lender's payoff (from walking away) is simply 0. Thus, any contract that delivers 0 transfers in period 1 and constant transfers in period 2 will be incentive compatible, satisfy the limited liability constraints and the participation constraints. In particular, if the lenders maximize the ex-ante value of all the lenders, it will be optimal for the lenders to continue the project after low outcomes.

After high period 1 outcomes, it is straightforward to demonstrate the existence of transfers which support $x(y_h) = 1$ for arbitrary transfer rules $t_1^c(y_h)$. In this sense, the public goods problem that enforces dismissal of the manager following low period 1 outcomes is much weaker following high period 1 outcomes.

Optimal Transfers with Limited Commitment

We have demonstrated that the optimal continuation rule (and managerial consumption) under commitment is enforceable even with limited commitment as $N \to \infty$. It remains to prove that the remaining features of the optimal contract with the limited commitment coincide with those under full commitment. Conditional on the continuation rule and managerial consumption, the optimal transfer scheme with limited commitment must coincide with that under full commitment (up to the indeterminacy between t_1 and t_{1c} and t_{1n}). Moreover, optimality of transfers under full commitment ensures that these transfers are enforceable with limited commitment (were they not enforceable, then there would exist superior transfers under full commitment). As a result, we have proved that the solutions to the commitment and limited commitment problem coincide (as $N \to \infty$).

3.3.3 Discussion

We conclude this section with a summary of the above results. Note that we have proved that short-term contracts, freely negotiated in each period after project outcomes realized on behalf of the collective interests of the lenders implement long-term commitment outcomes. One feature of these outcomes is that after a realization of low period 1 project outcomes, the investment project is terminated. We call this feature of optimal contracts a bank run. There are two reasons for this. First, for almost every realization of lenders' types, every lender could be made better off by continuing the investment project if the lenders' types were public information. Second, lenders choose not to continue investment projects because each lender has an incentive to "run" or threaten not to roll their funds over unless given a larger share of the future project returns than the remaining lenders. Even under arbitrary general mechanisms, lenders cannot avoid the "run" outcome and enforce termination of the investment project. Of course, viewed from the perspective of period zero, these ex-post inefficient outcomes are actually ex-ante efficient, and allow for more efficient investment ex-ante.

One feature of this environment is that, under the maintained assumptions, a contract with the property that $t_1^i(y_l) = 0$ will not enforce the optimal continuation rule. In other words, if short-term contracts are not used and the period 1 resources of the firm are not remitted to the lenders before the realization of their private types, v_i, then the lenders can re-negotiate the contract continue with probability 1. The key difference between these contracts is that the implementability constraint in Lemma 3.11 is weaker as the resources I are still available to the agents. In this sense, long-term lending contracts which only deliver the net return to the lenders in each period are worse than short-term claims which return the gross return to the lenders in each period. We interpret this result, in the context of [Fudenberg et al., 1990], as a sample economy in which short-term contracts can implement long-term commitment outcomes even though the utility possibility frontier features ex-post inefficiencies.

Finally, in an economy with sufficiently informative signals, the optimal contract under commitment has the feature that $x(y_h) = x(y_l) = 1$. In such an economy, if the contract returned the gross payout to the lenders in each period, the same public goods problem would arise after low period 1 project outcomes, causing lenders to "run" and not continue their investment projects. Such contracts would not implement optimal long-term outcomes. Thus, we interpret differences in signal quality as leading to predictions about optimal debt maturity. With low signal quality, the optimal contract is short-term and makes full principal payments in each period; whereas, with sufficiently informative signals, the optimal contract is longer-term, and only returns net payments to the lenders in period 1. Such a payout structure under informative signals effectively commits the lenders to continue investment projects regardless of period 1 project outcomes which is both ex-ante and ex-post efficient.

3.4 Facts about Debt Structure of Financial and Non-Financial Firms

In this section, we provide documentation that demonstrates how the capital structure of financial firms differs from those of non-financial firms. In particular, we show first that financial firms use much more short-term debt than do non-financial firms. Second, we show that the debt structure of financial firms is much shorter in maturity than is the maturity structure of their assets, while the opposite holds for non-financial firms.

The data are from Compustat and cover both active and inactive publicly traded firms in the United States since 1950. We define financial firms broadly as firms that are banks or undertake bank-like investment activities. Specifically, we restrict the sample of financial firms to be those in the 4 digit SIC industries 6000-6299, which cover Depository Institutions (6000), Non-depository Credit Institutions (6100), and Security And Commodity Brokers, Dealers, Exchanges, And Services (6200). Of the remaining firms, we exclude only those in public administration (9000), and label the rest non-financial firms.

We now present two sets of statistics that describe first how much short-term debt a sample of firms uses and second how mis-matched a firm's debt and assets are. First, we define short-term debt as the sum of notes and accounts payable on the firm's balance sheet. This definition of short-term debt explicitly excludes long-term debt due in one year as well as taxes payable and other accrued expenses. Our definition of short-term assets is the sum of cash and short-term investments and receivables. We restrict attention to firms that report positive total assets, less short-term debt than total assets, positive short-term debt and positive short-term assets.

Table (3.1) reports the ratios of short-term debt to total assets and short-term debt to short-term assets for financial and non-financial firms. The first finding is that the median use of short-term debt by financial financial firms is roughly 78% of their total assets whereas for non-financial firms it is only 9% of total assets. Figures B.2 and B.2

	Short-Term Debt to Total Assets	Short-Term Debt to Short-Term Assets
Financial Firms	78%	108%
Non-Financial Firms	9%	42%

Table 3.1: Comparison of Financial and Non-Financial Firms.

display histograms of these ratios for each set of firms for the entire sample.[5] Figure (B.2) displays empirical cumulative distributions of this ratio for each set of firms. Notice, roughly 80% of non-financial firms have less than 20% of their assets backed by short-term debt claims, whereas roughly 70% of financial firms have more than 80% of their assets backed by short-term debt claims.

Figures B.2 and B.2 display histograms of the ratio of short-term debt to short-term assets of financial and non-financial firms. We have truncated the sample at a ratio of 3, as 95% of firms in both samples have a ratio below this value. Again, there is a large mass of financial firms with more short-term debt than short-term assets but relatively few non-financial firms with this property.

3.5 Conclusion

In this chapter, we have asked the following question: Is there any value to having banks and other financial institutions rely heavily on short-term debt to finance their assets when this use of short-term debt exposes them to bank run or panic like phenomena? The answer in this chapter is that short-term debt can serve as an efficient means to provide firms with appropriate incentives only when it is sufficiently difficult to monitor the investment choices of firms. To the extent that the types of investments banks make are more opaque than those of non-financial firms, we should expect banks to rely more heavily on short-term debt.

[5] While we including data for all years, the statistics and figures look similar if we focus on any particular year.

Chapter 4

Adverse Selection, Reputation, and Sudden Collapses in Secondary Loan Market

4.1 Introduction

Following the sharp decline in the volume of new issuances in the U.S. secondary loan market in the fall of 2007, policymakers argued that the market was not functioning normally and proposed and carried out a variety of policy interventions intended to restore the normal functioning of this market. Here we present evidence on sudden collapses and motivated by that evidence, construct a model in which new issuances in the secondary loan market abruptly collapse. This collapse, in our model, is associated with an increase in inefficiency. We also argue that reductions in the value of the collateral used to secure the underlying loans are particularly likely to trigger sudden collapses associated with increased inefficiency. Since sudden collapses are associated with increased inefficiency, our model is consistent with policymakers' views that the market was functioning poorly. We use this model to analyze proposed and actual policy interventions and argue that these interventions typically do not remedy the inefficiency associated with the market collapse.

In our model, the main economic function of the secondary loan market is to allocate originated loans to institutions that have a *comparative advantage* in holding and managing the loans. This economic function is disrupted by informational frictions. In our model, loan originators differ in their ability to originate high-quality loans. The originators are better informed about their ability to generate high-quality loans than are potential purchasers. This informational friction creates an *adverse selection* problem. The focus of our analysis is to examine the extent to which *reputational* considerations ameliorate or intensify the adverse selection problem in these markets. In order to analyze these reputational considerations, we develop a dynamic adverse selection model of the secondary loan market.

Our main finding is that our model has fragile outcomes in which sudden collapses in the volume of new issuances in secondary loan markets are associated with increased inefficiency. We say that outcomes are fragile if the model has multiple equilibria or if a large number of originators change their decisions in response to small changes in aggregate fundamentals.

In terms of fragility as multiplicity, we show that our baseline dynamic adverse selection model with reputation has multiple equilibria for a range of reputation levels. In one of these equilibria, labeled the *positive reputational equilibrium*, high-quality loan originators have incentives to sell at a current loss in order to improve their reputations and command higher prices for future loans. In the other equilibrium, labeled the *negative reputational equilibrium*, loan originators who sell their loans are perceived by future buyers to have low-quality loans. These perceptions induce high-quality loan originators to hold on to their loans. Since low-quality originators always sell their loans, the volume of new issuances is larger in the positive reputational equilibrium than in the negative reputational equilibrium. Clearly, with multiple equilibria sunspot like shocks can generate sudden collapses. We show that the positive equilibrium Pareto dominates the negative equilibrium for a range of reputation levels. In this sesnse, sudden collapses are associated with increased inefficiency.

Although the multiplicity of equilibria has the attractive feature that it implies that the model can be consistent with observations of sudden collapses, such multiplicity makes it difficult to conduct policy analysis. We propose a refinement adapted from the global games literature (see [Carlsson and Van Damme, 1993] and [Morris and Shin, 2003]). Our refinement is also motivated by the idea that sudden collapses in the volume of new issuances in loan markets are associated with falls in the value of the collateral that supports the underlying loans. These considerations lead us to add aggregate shocks to collateral values and to assume that the collateral value is observed with an arbitrarily small error.

We show that shocks to collateral values make the outcomes of our model consistent with our second notion of fragility, namely, a large fraction of loan originators choose to change their decisions on whether to sell or hold their loans in response to small changes in collateral values. In this sense, reductions in collateral values can induce sudden collapses in the volume of new issuances for the market as a whole.

Both adverse selection and the dynamics induced by reputation acquisition play central roles in generating sudden collapses from small changes in collateral values. A simple way of seeing the role of adverse selection is to note that the version of our model with symmetrically informed originators and buyers does not produce sudden collapses in new issuances. With asymmetrically informed agents, originators with high reputations receive higher prices for their loans and are therefore more willing to sell their loans. We show that a fall in collateral values makes high-quality originators who were close to being indifferent about selling versus holding to hold. Small changes in collateral values can induce a large number of originators to switch to holding from selling only if they are all close to the point of indifference. In a static model, we have no reason to expect that the distribution of originators by reputation levels will be concentrated close to the indifference point.

In a dynamic model with learning by market participants, we argue that originators' reputations are likely to be clustered. The reason is that in models like ours, the reputation levels of high-quality originators have an upward trend over time, resulting in the reputation levels of many high-quality originators tending to become similar in the long run. We show that in an infinitely repeated version of our model, the long run or invariant distribution of reputation levels displays significant clustering. This clustering in turn implies that small changes in fundamentals can lead a large number of originators to change their decisions when the fundamentals are close to the point of indifference. A related result is that small changes in collateral values, when these values are far away from the point of indifference, do not lead to large changes in the volume of new issuances.

We have argued that our model is consistent with abrupt collapses in secondary loan markets and with the widespread view among policymakers that such abrupt collapses were associated with sharp increases in the inefficiency of the operation of such markets. In the wake of the 2007 collapse of secondary loan markets, policymakers proposed a variety of programs intended to remedy inefficiencies in the market for securitized assets. Some of these programs, such as the proposed Public-Private Partnership and TALF, were implemented at least in part. The TALF program allows participants to purchase securitized assets by borrowing from the Federal Reserve and using the assets as collateral. We use our model to evaluate the effects of various policies. In terms of purchase policies, we show that if the purchase price is set at or below the level that prevails in the positive reputational equilibrium, the equilibrium outcomes do not change and in this sense the policy is ineffective. If the purchase price is set at a sufficiently high level, the policy implies that the government makes negative profits.

We also analyze policies that change the time path of interest rates. We show that temporary decreases in interest rates worsen the adverse selection problem. Interestingly, anticipated decreases in interest rates in the future can have beneficial current effects by reducing the range of reputations over which the economy has multiple equilibria.

4.1.1 Related Literature

Our work here is related to an extensive literature on adverse selection in asset markets, such as [Myers and Majluf, 1984], [Glosten and Milgrom, 1985], [Kyle, 1985], and [Garleanu and Pedersen, 2004] as well as to the related securitization literature, specifically, the work of [DeMarzo and Duffie, 1999] and [DeMarzo, 2005]. See also [Eisfeldt, 2004], [Kurlat, 2009], [Guerrieri et al., 2010] and [Guerrieri and Shimer, 2011] for analyses of adverse selection in dynamic environments.We add to this literature by analyzing how reputational incentives affect adverse selection problems.

Our assumption that buyers have less information concerning the loan quality of a bank is in line with a descriptive literature that argues that secondary loan markets feature adverse selection (see, for example, the work of [Dewatripont and Tirole, 1994], [Ashcraft and Schuermann, 2008], and [Arora et al., 2009]). Also, a growing literature provides data on the presence of adverse selection in asset markets. For example, [Ivashina, 2009] finds evidence of adverse selection in the market for syndicated loans. [Downing et al., 2009] find that loans that banks held on their balance sheets yielded more on average relative to similar loans which they securitized and sold. [Drucker and Mayer, 2008] argue that underwriters of prime mortgage-backed securities are better informed than buyers and present evidence that these underwriters exploit their superior information when trading in the secondary market. Specifically, the tranches that such underwriters avoid bidding on exhibit much worse than average ex post performance than the tranches that they do bid on.

A recent paper by [Elul, 2011] presents evidence that is consistent with our model. [Elul, 2011] shows that returns on securitized loans and loans held by originators were similar before 2006 and that returns on securitized loans were lower than returns on comparable loans after 2006. This evidence is consistent with our model in the following sense. Our model implies that when collateral values underlying loans are relatively high, most high-quality banks with high costs of managing the loans choose to sell their loans; but when collateral values are relatively low, such banks choose to hold their loans. Before 2006, land values were rising, so it seems reasonable to suppose that collateral values were relatively high. After 2006, land values stopped rising and in some cases fell, so it seems reasonable to suppose that collateral values were lower than they had been.

Finally, [Mian and Sufi, 2009] present evidence that securitized loans were more likely to default than nonsecuritized loans. This evidence is consistent with our model in the sense that for all realizations of the aggregate shock, the default rate of securitized loans is at least as high as that of held loans, and for some realizations the default rate of securitized loans is higher than that of held loans.

Our work is also related to the literature on reputation. [Kreps and Wilson, 1982] and [Milgrom and Roberts, 1982] argue that equilibrium outcomes are better in models with reputational incentives than in models without them. In the banking literature, [Diamond, 1989] develops this argument. More recently, [Mailath and Samuelson, 2001] analyze the role of reputational incentives in infinite horizon economies and provide conditions under which they can improve outcomes. In contrast, [Ely and Välimäki, 2003] and [Ely et al., 2008] describe models in which reputational incentives can worsen outcomes. Our work here combines the results in this literature by showing that reputational models can have multiple equilibria. In some of these equilibria, reputational incentives can generate better outcomes; in others, they can generate worse. Furthermore, using techniques from the global games literature, we develop a refinement that produces a unique, fragile equilibrium. Perhaps the work most closely related to ours is that of [Ordoñez, 2008]. An important difference between our work and his is that our model has equilibria that are worse than the static equilibrium, so that reputational incentives can lead to outcomes that are ex post less efficient than those in a model without these incentives.

Our analysis of policy is closely related to recent work by [Philippon and Skreta, 2011] who analyze a variety of policies in a model with adverse selection. The main difference with our work is that we focus on the incentives induced by reputation, whereas they analyze a static model.

4.2 Evidence on Sudden Collapses

Here we present evidence on sudden collapses in the market for new issuances of asset-backed securities. Figure C.4 displays the volume of new issuances of asset-backed securities for various categories from the first quarter of 2000 to the first quarter of 2009. The figure shows that the total volume of new issuances of asset-backed securities rose from roughly \$50 billion in the first quarter of 2000 to roughly \$300 billion in the fourth quarter of 2006. The volume of new issuances fell abruptly to roughly \$100 billion in the third quarter of 2007 and then fell again to near zero in roughly the fourth quarter of 2008. The figure also shows similar large fluctuations in the volume of new issuances for each category.

[Ivashina and Scharfstein, 2008] document a similar pattern for new issues of syndicated loans. Figure 1, Panel A of their paper shows that syndicated lending rose from roughly \$300 billion in the first quarter of 2000 to roughly \$700 billion in the second quarter of 2007. This lending declined sharply thereafter and fell to roughly \$100 billion by the third quarter of 2008.

The reduction in the volume of new issuances in the secondary market roughly coincided with a reduction in collateral values. One way of seeing this coincidence is to consider the Case-Shiller home price index[1]. This index stopped growing in late 2006 and declined through 2007. The coincidence of the reduction in the volume of new issuances and the reduction in collateral values is consistent with our model.

[White, 2009] has argued that in the 1920s, the United States experienced a boom-bust cycle in securitization of real estate assets that was similar to its recent experience. Figure C.4 displays the change in the outstanding stock in real estate bonds in the 1920s based on data in [Carter and Sutch, 2006]. Such bonds were issued against single large commercial mortgages or pools of commercial or real estate mortgages and were publicly traded. To make this data comparable to more recent data, we scale the data from the 1920s by nominal GDP in 2009. Specifically, we multiply the change in the nominal stock of outstanding debt in each year by the ratio of the nominal GDP in 2009 to that in the relevant year. This figure shows that the changes in the stock rose dramatically from essentially 0 in 1919 to an average of \$145 billion in the period from 1925 to 1928. The market then collapsed sharply, and changes in the stock fell to roughly \$50 billion in 1929. Such large changes in the stock are likely to have been associated with similar large changes in the volume of new issuances.

4.3 Reputation in a Secondary Loan Market Model

We develop a finite horizon model of the secondary loan market and use the model to demonstrate how adverse selection and reputation interact to yield abrupt collapses with increased inefficiency. We show that for every history, the last period of the model has a unique equilibrium which we use to construct equilibria in previous periods. We show that equilibria of the multi period model typically exhibit dynamic coordination problems in the sense that for a wide range of parameters, the game has multiple equilibria. Although reputation is always valued, loan originators choose different actions across the different equilibria based on the different inferences future buyers draw from the current actions of originators.

[1] Available at http://www.standardandpoors.com/indices

4.3.1 Static Model: A Unique Equilibrium

We start with a static model which should be interpreted as describing the last period of a finite horizon model. We show that the static model has a unique equilibrium in which the equilibrium outcomes depend on the informed originator's reputation.

Agents. The model has three types of agents: a loan originator referred to as a bank, a continuum of buyers, and a continuum of lenders. All agents are risk neutral.

The bank is endowed with a risky loan indexed by π. The loan can also be thought of more generally as an investment opportunity such as a project, a mortgage, or an asset-backed security. Each loan requires q units of inputs, which represents the loan's size. A loan of type π yields a return of $v = \bar{v}$ if the borrower does not default and a return of $v = \underline{v}$ if the borrower does default. We refer to $\underline{v}$ as the *collateral value* of the loan. The probability that the borrower does not default is denoted by π. For the analysis in this section, we normalize $\underline{v}$ to 0. Later, when we allow for aggregate shocks and introduce our refinement, we will allow $\underline{v}$ to be a random variable, possibly different from zero. We assume that $\pi \in \{\underline{\pi}, \bar{\pi}\}$ with $\underline{\pi} < \bar{\pi}$. We refer to a bank that has a loan of type $\bar{\pi}$ as a *high -quality bank* and one with a loan of type $\underline{\pi}$ as a *low-quality bank*. We assume that $\underline{\pi}\bar{v} \geq q$ so that each loan has positive net present value if sold.

The bank either can sell the loan in a secondary market or can hold the loan. Selling the loan at a price p yields a payoff to the bank of $p - q$. The purchaser of the loan is entitled to the resulting return. If the bank chooses to hold the loan, it must borrow q from lenders to finance the loan and repay $q(1 + r)$ at the end of the period, where r is the within-period interest rate paid to lenders. We allow r to be positive or negative in order to examine the effects of various policy experiments described below. If the bank holds the loan, it is entitled to the return from its projects; however, the bank then incurs a cost of holding the loan, c, in addition to the cost of repaying its debt, $q(1+r)$.

Besides the quality of its loan, the bank is indexed by a cost type, which represents the costs, relative to the marketplace, that the bank incurs when it holds the loan to maturity. We intend the cost of the loan to represent funding liquidity costs, servicing costs, renegotiation costs in the event of a loan default, and costs associated with holding a loan that may be correlated in a particular way with the rest of the bank's portfolio, among other potential factors. We assume that $c \in \{\underline{c}, \bar{c}\}$ with $\underline{c} < -qr < 0 < \bar{c}$. We refer to a bank of type $\bar{c}$ as a *high-cost bank* and a bank of type $\underline{c}$ as a *low-cost bank*.

We normalize the cost of holding and managing the loan for the market to be zero.

Hence, the model has four types of banks: $(\pi, c) \in \{\underline{\pi}, \bar{\pi}\} \times \{\underline{c}, \bar{c}\}$. We refer to the different types of banks, $(\bar{\pi}, \bar{c}), (\bar{\pi}, \underline{c}), (\underline{\pi}, \bar{c}), (\underline{\pi}, \underline{c})$, as HH, HL, LH, LL banks, respectively.

Timing of the Static Game. We formalize the interactions in this economy as an extensive form game with the following timing. First, nature draws the quality and cost types of the bank. Then, buyers simultaneously offer a price, p, to purchase the loan. Finally, the bank sells the loan to one of the buyers or holds the loan to maturity.

We assume that, as perceived by buyers and lenders, the bank has quality type $\bar{\pi}$ with probability μ_2 and quality type $\underline{\pi}$ with probability $1 - \mu_2$. (The subscript 2 on the probability is meant to indicate that these are the beliefs of lenders associated with the second period of our two-period model described below.) Following the work of [Kreps and Wilson, 1982] and [Milgrom and Roberts, 1982], we refer to μ_2 as the bank's reputation. Also, buyers believe that the bank has cost type $\underline{c}$ with probability α and cost type $\bar{c}$ with probability $1-\alpha$. The cost and quality types are independently drawn.

Strategy and Equilibrium. A strategy for the bank consists of a decision of whether to sell or hold its loan as a function of prices offered by buyers, and which buyer to sell to if the bank chooses to sell. Clearly, the bank will choose the buyer offering the highest price if the bank decides to sell, so we suppress this aspect of the bank's strategy. Let $a = 1$ denote the decision of the bank to sell the loan, and let $a = 0$ denote the decision to hold the loan. A strategy for the bank is a function $a(\cdot)$ that maps the highest offered price, p, into a decision of whether to sell or hold the loan. The payoffs to a type (π, c) bank are given by

$$w_2(a|p, \pi, c) = a(p - q) + (1 - a)\left[\pi\bar{v} - q(1 + r) - c\right].$$

A strategy for a buyer consists of the choice of a price to offer a bank for its loan. The payoffs to a buyer with an accepted price p and a strategy $a_2(\cdot|\pi, c)$ for each type of bank is

$$u_2(p|a_2) = E_{\pi,c}[v|a_2(p|\pi, c) = 1] - p.$$

Since buyers move simultaneously, they engage in a form of Bertrand competition, so that the price is equal to the expected return on the loan.

A (pure strategy) Perfect Bayesian Equilibrium is a price p_2 and a strategy for each bank type, $a_2(\cdot|\pi, c)$, such that for all p, each bank type chooses the optimal loan decision and buyers offer the highest price that yields a payoff of 0; i.e., $p_2 = \max\{p|u_2(p|r, a_2) = 0\}$.

With full information, when the bank's type is known by buyers, under the assumption that $\underline{c} < -qr$, it is easy to show that the high cost bank sells its loan and a low cost bank holds its loan. In particular, the decision of whether to sell or hold the loan does not depend on the quality type of the bank. The reason is that the return on the loan, ignoring the holding cost, is the same for both the bank and the buyers. Notice that the equilibrium allocation under full information is ex post efficient. Low-cost banks have a comparative advantage (over buyers) in holding loans to maturity, while buyers have a comparative advantage over high-cost banks. The full information equilibrium allocates loans to agents with a comparative advantage in holding and managing the loan.

Next, we show that the private information model has a unique equilibrium. For expositional simplicity, we focus on the decisions of the high-quality, high-cost bank (HH) and restrict the strategy sets of the low-cost bank as well as the low-quality, high-cost bank (LH). Specifically, we assume that HL and LL banks hold their loans and the LH bank sells its loan. In Appendix C.2, we show that if $\underline{c}$ is sufficiently negative, the assumed strategies for these three types of banks are indeed optimal.

To show uniqueness of equilibrium, we show that the HH bank sells its loan for reputation levels higher than a critical threshold, μ_2^*, and holds its loan otherwise. To see this result, note that facing price p, the HH bank sells its loan if and only if

$$p - q \geq \bar{\pi}\bar{v} - q(1 + r) - \bar{c}. \tag{4.1}$$

Bertrand competition among buyers implies that buyers must make zero profits so that any candidate equilibrium price at which the HH bank sells must satisfy the following equality:

$$\hat{p}(\mu_2) := [\mu_2\bar{\pi} + (1 - \mu_2)\underline{\pi}]\,\bar{v}. \tag{4.2}$$

To determine the threshold, μ_2^*, above which the equilibrium involves the HH selling its loan, subsitute from (4.2) into (4.1) and find the thresholds for μ_2 at which (4.1) holds with equality. We obtain

$$\mu_2^* = 1 - \frac{qr + \bar{c}}{(\bar{\pi} - \underline{\pi})\bar{v}}. \tag{4.3}$$

To see that when $\mu_2 \geq \mu_2^*$ the equilibrium must have the HH bank selling, note that if $\mu_2 \geq \mu_2^*$ and the offered price is below $\hat{p}(\mu_2)$, one of the buyers can deviate and offer a price just below $\hat{p}(\mu_2)$ and induce the HH bank to sell. This deviation yields strictly positive profits. For reputation levels below μ_2^*, the HH bank holds even if offered $\hat{p}(\mu_2)$. Thus the equilibrium must have the HH bank holding at these reputation levels and only the low-quality bank selling, so that the equilibrium price must satisfy

$$p = \underline{\pi}\bar{v}. \tag{4.4}$$

We use this characterization of the static equilibrium to calculate the payoffs associated with a given level of reputation μ_2 at the beginning of the period before a bank's cost type is realized. These payoff calculations play a crucial role in our dynamic game. They are given by

$$V_2(\mu_2) = \begin{cases} \bar{\pi}\bar{v} - q(1+r) - Ec, & \mu_2 < \mu_2^* \\ (1-\alpha)\left\{[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\bar{v} - q\right\} + \alpha[\bar{\pi}\bar{v} - q(1+r) - \underline{c}], & \mu_2 \geq \mu_2^*. \end{cases} \tag{4.5}$$

Similarly, we can define the value of the equilibrium for a low-quality bank:

$$W_2(\mu_2) = \begin{cases} (1-\alpha)\left[\underline{\pi}\bar{v} - q\right] + \alpha[\underline{\pi}\bar{v} - q(1+r) - \underline{c}], & \mu_2 < \mu_2^* \\ (1-\alpha)\left\{[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\bar{v} - q\right\} + \alpha[\underline{\pi}\bar{v} - q(1+r) - \underline{c}], & \mu_2 \geq \mu_2^*. \end{cases}$$

It is clear that V_2 is weakly increasing and convex in μ_2. We have proved the following proposition.

Proposition 4.1 *If $\underline{\pi}\bar{v} > q$ and $qr + \bar{c} > 0$, then for any $\mu \in [0,1]$, the static model has a unique equilibrium. Let μ_2^* be defined by (4.3). For $\mu_2 < \mu_2^*$, the HH bank holds its loan and for $\mu_2 \geq \mu_2^*$, the HH bank sells its loan.*

Note that we have modeled buyers as behaving strategically. This modeling choice plays an important role in ensuring that the static game has a unique equilibrium. Suppose that rather than modeling buyers as behaving strategically, we had instead simply required that market prices satisfy a zero profit condition. One rationale for this requirement is that buyers take prices as given and choose how many loans to buy as in a competitive equilibrium. It is easy to show that with this requirement the economy has multiple equilibria in the static game if $\mu_2 \geq \mu_2^*$. One of these equilibria corresponds to the unique equilibrium of our game. In the other equilibrium, the buyers offer a price of $\underline{\pi}\bar{v}$. At this offered price, the HH bank holds its loan and only the low-quality, high-cost bank sells its loan. We find multiplicity of this kind unattractive in our model because obvious bilateral gains to trade are not being exploited. Each of the buyers has a strong incentive to offer a price slightly below $[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\,\bar{v}$. At this offered price, the HH bank strictly prefers to sell, and the buyer making such an offer makes strictly positive profits. In our formulation, with strategic behavior by the buyers, this low price outcome cannot be an equilibrium.

Although we prefer our strategic formulation, we emphasize that our results that reputational incentives induce multiplicity do not rely on the static game having a unique equilibrium. We chose a formulation in which the static game has a unique equilibrium in order to argue that reputational incentives by themselves can induce multiplicity.

4.3.2 Two-Period Benchmark Model

Consider now a two-period repetition of our static game in which the bank's quality type is the same in both periods. We assume that the bank's second-period payoffs are discounted at rate β. In period 1, a continuum of buyers who are present in the market for only one period choose to offer prices for loans sold in that period. In period 2, a new set of buyers each offer prices for loans sold in that period. This new set of buyers observes whether the bank sold or held its loan in the previous period, and, if the bank sold its loan, buyers observe the realized value of the loan. If the loan is held, we assume that period 2 buyers do not observe the realized value of the loan.

The assumption that period 2 buyers receive no information about the realized value of the loan is convenient but not essential in generating multiplicity of equilibria. Our multiplicity results go through if period 2 buyers receive a sufficiently noisy signal of the realized value of the loan. The critical assumption in generating multiplicity is that the market receives more precise information about the value of the loan if it is sold than if it is held. We think this assumption is natural in that market participants typically receive information only about aggregate returns to bank portfolios and do not receive information on the returns to specific assets. Banks typically hold a variety of assets in their portfolios, some of which can be securitized and others which cannot. In such a setting, the information investors receive about returns on specific assets is typically not as precise if a bank holds an asset as it would be if the bank sold the asset.

The timing of the game is an extension of that described in the static game. As in that game, at the beginning of period 1, nature draws the bank's quality and cost type. We assume that the bank's quality type is fixed for both periods. At the beginning of period 2, nature draws a new cost type for the bank. In any period, the bank's quality and cost types are unknown to buyers. The timing within each period is the same as in the static game. We also assume that the returns to successful loans, $v = \bar{v}$, and to unsuccessful loans, $v = 0$, are the same in both periods.

In order to define an equilibrium in this repeated game, we must develop language that will allow us to describe how second-period buyers update their beliefs about the bank's type based on observations from period 1. To do so, we let the public history at the beginning of period 2 be denoted by θ_1, where $\theta_1 \in \{h, s0, s\bar{v}\}$ where $\theta_1 = h$ denotes that the bank held its loan in period 1, $\theta_1 = s0$ denotes that the bank sold its loan and the loan paid off $v = 0$, and $\theta_1 = s\bar{v}$ denotes that the bank sold its loan and the loan paid off $v = \bar{v}$.

As in the static game, we focus on the strategic incentives of the HH bank and restrict the strategy sets of the low-cost bank as well as the low-quality, high-cost bank. Specifically, we assume that the low-cost bank must hold its loan and the LH bank must sell its loan. A strategy for the high-quality, high-cost bank is now given by a pair of functions, $a_1(p_1)$ representing the decision in period 1 and $a_2(p_2, \theta_1)$ representing the loan decision in period 2 if the bank realizes a high cost in period 2, as a function of offered prices.

Consider next how the buyers in the last period update their beliefs about the bank's type. This updating depends through Bayes' rule on the prior belief of the buyers, the loan decision of the bank and the loan return realization if the bank sold, as well as on the first-period strategies chosen by the HH bank and period 1 buyers. From Bayes' rule, these posterior probabilities are given by

$$\mu_2(\mu_1, \theta_1 = h, a_1(\cdot), p_1) = \frac{\mu_1 \left(\alpha + (1-\alpha)(1 - a_1(p_1))\right)}{\mu_1 \left(\alpha + (1-\alpha)(1 - a_1(p_1))\right) + (1-\mu_1)\alpha} \tag{4.6}$$

$$\mu_2(\mu_1, \theta_1 = s\bar{v}, a_1(\cdot), p_1) = \frac{\mu_1 a_1(p_1)(1-\alpha)\bar{\pi}}{\mu_1 a_1(p_1)(1-\alpha)\bar{\pi} + (1-\mu_1)(1-\alpha)\underline{\pi}} \tag{4.7}$$

$$\mu_2(\mu_1, \theta_1 = s0, a_1(\cdot), p_1) = \frac{\mu_1 a_1(p_1)(1-\alpha)(1-\bar{\pi})}{\mu_1 a_1(p_1)(1-\alpha)(1-\bar{\pi}) + (1-\mu_1)(1-\alpha)(1-\underline{\pi})}. \tag{4.8}$$

For notational convenience, we suppress the dependence on strategies and priors and let μ_h denote the posterior associated with the bank holding its loan, and $\mu_{s\bar{v}}$ and μ_{s0} denote the posteriors associated with selling and yielding a high or low return.

Given the updating rules, the period 1 payoffs for the HH bank are given by

$$\begin{aligned} w_1(a|p) =& a\left[p - q + \beta\left(\bar{\pi} V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0})\right)\right] \\ &+ (1-a)\left[(\bar{\pi}\bar{v} - q(1+r) - \bar{c}) + \beta V_2(\mu_h)\right] \end{aligned}$$

where $\mu_h, \mu_{s\bar{v}}$, and μ_{s0} are given by equations (4.6), (4.7), and (4.8). Buyers' payoffs associated with an accepted price, p, in period t are given by

$$u_t(p|r, a_t, \mu_t) = \frac{\mu_t(1-\alpha)a_t(p)\bar{\pi} + (1-\mu_t)(1-\alpha)\underline{\pi}}{\mu_t(1-\alpha)a_t(p) + (1-\mu_t)(1-\alpha)}\bar{v} - p.$$

A Perfect Bayesian Equilibrium is a first-period price, p_1, a first-period loan decision for the high-quality, high-cost bank $a_1(\cdot)$ that maps accepted prices into loan decisions, updating rules $\mu_h, \mu_{s\bar{v}}, \mu_{s0}$ that map observations on loan decisions into posterior beliefs, a second-period price, p_2, that maps second-period beliefs into prices, and a second-period loan decision $a_2(\cdot)$ that maps accepted prices and histories into loan decisions such that *(i)* for all p, the HH bank chooses the optimal action in period 1 so that $w_1(a_1(p)|p) \geq \max_{a'} w_1(a|p)$, *(ii)* for all p, the HH bank chooses the optimal action in period 2 so that $w_2(a_1(p)|p) \geq \max_{a'} w_2(a|p)$, *(iii)* the first-period price, p_1, satisfies $p_1 \in \max\{p|u_1(p|a_1) = 0\}$, *(iv)* the second-period price, p_2, satisfies $p_2 \in \max\{p|u_2(p|a_2) = 0\}$, *(v)* the updating rules, $\mu_h, \mu_{s\bar{v}}, \mu_{s0}$, satisfy Bayes' rule, namely, (4.6), (4.7), and (4.8).

To show that our model has mutliplicity of equilibria, we begin by showing that the game has two (pure strategy) equilibria when prior beliefs in period 1, μ_1, are equal to the static threshold, μ_2^*. Continuity of payoffs then implies that the game has two equilibria in an interval around the static threshold.

In one equilibrium, labeled the positive reputational equilibrium, the HH bank chooses to sell its loan in period 1. To see that such a choice is part of an equilibrium, note that in this case, the period 1 price is given by

$$\hat{p}(\mu_2) := [\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}]\,\bar{v}. \tag{4.9}$$

Given this price, selling is optimal if the following incentive constraint is satisfied:

$$(\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi})\,\bar{v} - q + \beta\left(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0})\right) \geq \bar{\pi}\bar{v} - q(1+r) - \bar{c} + \beta V_2(\mu_h) \tag{4.10}$$

where the posterior beliefs are obtained from (4.6) through (4.8) substituting $a_1(p_1) = 1$ so that

$$\mu_h = \mu_1,\ \mu_{s\bar{v}} = \frac{\mu_1\bar{\pi}}{\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi}},\ \text{and}\ \mu_{s0} = \frac{\mu_1(1-\bar{\pi})}{\mu_1(1-\bar{\pi}) + (1-\mu_1)(1-\underline{\pi})}. \tag{4.11}$$

Notice from (4.11) that if a bank holds the loan, the posterior beliefs are unchanged. The reason is that the beliefs of period 2 buyers is that low cost banks of both qualities hold their loans and period 2 buyers receive no information about the return to the loan if it is held.

We show that at μ_2^*, the incentive constraint (4.10) holds as a strict inequality. Note that using (4.9), (4.10) evaluated at μ_2^* can be written as

$$\beta\left(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0})\right) \geq \beta V_2(\mu_h)$$

Further, from the updating rules for posterior beliefs, (4.11), $\mu_{s\bar{v}} > \mu_h = \mu_2^* > \mu_{s0}$. Hence, using the second period payoffs from (4.5), it follows that $V_2(\mu_{s\bar{v}}) > V_2(\mu_{s0}) = V_2(\mu_h)$ so that (4.10) holds as a strict inequality at μ_2^*. Thus, the HH bank has a strict incentive to sell. The reason for this strict incentive is that the worst outcome associated with selling is that the loan is unsuccesful and this payoff is the same as that associated with holding the loan. If the loan is succesful, the HH bank's payoff is strictly higher than the payoff to holding the loan. Not surprisingly, this result suggests that for reputation levels in an interval around μ_2^*, given beliefs that the HH bank sells in period 1, the bank finds it optimal to do so, and hence the model has a positive equilibrium for an interval around μ_2^*.

In the second type of equilibrium, labeled the negative reputational equilibrium, the HH bank chooses to hold its loan. In this case the equilibrium price is given by $\underline{\pi}\bar{v}$ using (4.4). A bank holds its loan if and only if

$$(\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi})\,\bar{v} - q + \beta\,(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0})) \leq \bar{\pi}\bar{v} - q(1+r) - \bar{c} + \beta V_2(\mu_h), \tag{4.12}$$

where

$$\mu_h = \frac{\mu_1}{\mu_1 + (1-\mu_1)\alpha}, \text{ and } \mu_{s\bar{v}} = \mu_{s0} = 0. \tag{4.13}$$

Note that in the negative equilibrium, only low quality banks sell, and uninformed agents assign a posterior reputation of zero if the bank sells and rationally disregard the information from the realized value of the loans. Note also that if a bank chooses to hold its loan, buyers perceive that it is more likely to be a high quality bank and the posterior belief rises.

The argument that at μ_2^*, the incentive constraint (4.12) holds as a strict inequality parallels that of the positive equilibrium. Using the updating rules in (4.13), it follows that $\mu_{s\bar{v}} = \mu_{s0} < \mu_2^* < \mu_h$. Hence, using the second period payoffs given in (4.5), it follows that $V_2(\mu_{s\bar{v}}) = V_2(\mu_{s0}) = V_2(\mu_2^*) < V_2(\mu_h)$ so that the incentive constraint (4.12) holds as a strict inequality at μ_2^*. This result suggests that for reputation levels in an interval around μ_2^*, given beliefs that the HH bank holds in period 1, the bank finds it optimal to do so, and hence the model has a negative equilibrium.

Continuity of payoffs implies that (4.10) and (4.12) hold as strict inequalities in some interval of prior beliefs around μ_2^* so that our model has multiple equilibria in this interval. In Appendix C.2, we show that our model has unique equilibria outside this interval under the assumption that $\beta(1-\alpha) \leq 1$.

Proposition 4.2 *(Multiplicity of Equilibria) Suppose* $0 < \mu_2^* < 1$*. Then, there exist* $\underline{\mu}$ *and* $\bar{\mu}$ *with* $\underline{\mu} < \mu_2^* < \bar{\mu}$ *such that if* $\mu_1 \in [\underline{\mu}, \bar{\mu}]$*, the model has two equilibria: in one the HH bank sells its loan, and in the other the HH bank holds its loan in the first period.*

In the proposition, we have shown that introducing reputation as a device for mitigating lemons problems results in equilibrium multiplicity, that is, reputation can be both a blessing and a curse. The game has a positive reputational equilibrium in which, encouraged by reputational incentives, banks with a high-quality asset sell their asset. In this equilibrium, reputation helps sustain market activity in a market that would be illiquid without reputational incentives. The game also has a negative reputational equilibrium in which reputational incentives discourage selling and banks with a high-quality asset hold on to their asset. In this equilibrium, reputation helps depress market activity in a market that would be liquid without reputational incentives.

In terms of the relationship to the literature on reputation, our model nests features of the model in [Mailath and Samuelson, 2001] and [Ordoñez, 2008] as well as that of [Ely and Välimäki, 2003]. In [Mailath and Samuelson, 2001] and [Ordoñez, 2008], strategic types are good and want to separate from nonstrategic types, although in [Mailath and Samuelson, 2001] reputation generally fails to deliver this type of equilibria. Nevertheless, in their environments, there is no long-run reputational loss from good behavior. [Ely and Välimäki, 2003] share the property that strategic types are good and want to separate; however, the structure of learning is such that good behavior never implies long-run positive reputational gains, and therefore reputational incentives exacerbate bad behavior in equilibrium.

4.3.3 Sudden Collapses and Increased Inefficiency

In this section, we study the efficiency properties of the positive and negative reputational equilibria. We provide sufficient conditions under which the positive reputational equilibrium Pareto dominates the negative reputational equilibrium in the sense of interim utility (see [Holmström and Myerson, 1983]), and sufficient conditions under which the positive equilibrium dominates the negative equilibrium in the sense of ex ante utility. In this sense, sudden collapses of trade volume in our model due to switches between equilibria are associated with increased inefficiency.

In order to develop these sufficient conditions, suppose that $\mu_1 \in [\underline{\mu}, \mu_2^*]$. Consider the welfare of the HH bank. Let μ_h^n denote the posterior beliefs in the negative equilibrium respectively, conditional on future buyers observing a hold decision by a bank in the first period. Suppose μ_h^n is less than the static cutoff, μ_2^*. (In Appendix C.2, we show that $\mu_2^* < \left(\frac{\underline{\pi}}{\overline{\pi}\alpha - \underline{\pi}} + \beta\overline{\pi}\right) / \left(1 + \beta\overline{\pi}(1-\alpha)\right)$ and μ_1 close to $\underline{\mu}$ is a sufficient condition for μ_h^n to be less than or equal to μ_2^*.) Since μ_h^n is less than the static cutoff, μ_2^*, using the form of second period payoffs (4.5), it follows that the present value of payoffs in the negative equilibrium is given by the right side of the incentive constraint in the positive equilibrium, (4.10). The left side of (4.10) is the equilibrium payoff in the positive equilibrium. Clearly, the payoff for the HH bank is higher in the positive equilibrium than it is in the negative equilibrium.

Consider next the low quality, high cost, or LH bank. This bank sells in both equilibria in the first period, but receives a higher price in the positive equilibrium than in the negative equilibrium. In terms of continuation values, note that the reputation level in the negative equilibrium falls to zero and is positive in the positive equilibrium. It follows that this bank is strictly better off in the positive equilibrium than in the negative equilibrium. Since $\mu_h^n \leq \mu_2^*$, the continuation values for low-cost types is the same in the two equilibria, and since they are holding in the first period, their utility levels are the same. Since buyers make zero profits in both equilibria, we have established the following proposition.

Proposition 4.3 *Suppose that* $0 < \mu_2^* < \frac{\beta\overline{\pi} - \frac{\underline{\pi}}{\overline{\pi}\alpha - \underline{\pi}}}{1 + \beta\overline{\pi}(1-\alpha)}$ *and that* $\mu_2^* < 1$. *Then for all* μ_1 *in some neighborhood of* $\underline{\mu}$, *the utility level for each type of bank and the buyers in the positive equilibrium is at least as large as the utility level for the corresponding type of bank and the buyers in the negative equilibrium.*

If $\mu_h^n > \mu_2^*$, one can show that the utility level of the low-cost types is lower in the positive reputational equilibrium than in the negative reputational equilibria. Hence, the two equilibria are not comparable in interim utility terms. However, under appropriate sufficient conditions, the positive equilibrium yields a higher ex ante utility than the negative equilibrium. Consider the allocations in the two equilibria in the first period. The only difference in allocations is that in the positive equilibrium the high-quality, high-cost type sells, whereas in the negative equilibrium this type holds. Thus, the difference in ex ante utility (or social surplus) in the first period between the two equilibria is given by $(1-\alpha)\mu(qr + \bar{c})$. Clearly, first-period utility is higher in the positive equilibrium than in the negative equilibrium. However, in the second period social surplus is higher in the negative equilibrium than in the positive equilibrium because the high-cost types always sell in the negative equilibrium, whereas in the positive equilibrium they hold the asset some fraction of the time – when the signal quality is bad in the first period or after a hold decision in the first period. Therefore, the change in social surplus in the second period is given by $-\mu(1-\alpha)((1-\alpha)(1-\bar{\pi})+\alpha)(qr+\bar{c})$. Thus, the overall change in the social surplus is given by

$$\mu(1-\alpha)(1-\beta(1-\bar{\pi}(1-\alpha)))(qr+\bar{c}).$$

Clearly, this overall change is positive if and only if $\beta(1-\bar{\pi}(1-\alpha)) < 1$. We have established the following proposition.

Proposition 4.4 *Suppose that $\beta(1-\bar{\pi}(1-\alpha)) < 1$. Then the ex-ante utility of the bank is higher in the positive reputational equilibrium than in the negative reputational equilibrium and the ex-ante utility of the buyers is the same in the two equilibria.*

4.4 Aggregate Shocks and Uniqueness

In this section, we show that with two perturbations our model has a unique equilibrium which is fragile. The perturbations add aggregate shocks to collateral values and assume that past aggregate shocks are imperfectly observed. With these perturbations, we show that the model has a unique equilibrium in which small fluctuations in collateral values in a critical region lead to sudden collapses in the volume of trade.

Adding aggregate shocks with imperfect observability ensures that our model has a unique equilibrium and is, in this sense, a type of refinement. This device is in the spirit of the refinement literature on static coordination games (see, for example, [Carlsson and Van Damme, 1993], [Morris and Shin, 2003]). One reason for using such a refinement is to compare outcomes under various policies. Uniqueness is desirable because such comparison is difficult in models with multiple equilibria. Furthermore, we want to develop a well-defined notion of fragility. In many macroeconomic environments with multiple equilibria, small shocks to the environment can cause sudden changes in behavior. Without a selection device, multiplicity leads to a lack of discipline on how equilibrium behavior changes in response to shocks. Techniques adapted from the literature on coordination games, however, enable us to impose such discipline. We show that small aggregate shocks to collateral values near a critical range induce sudden collapses in trade while similar small shocks far from the critical range do not induce significant changes in the volume of trade.

We assume that aggregate shocks affect collateral values. Specifically, we assume that the collateral value, $\underline{v}$, is affected by an aggregate shock common for all banks. One example of the situation in which collateral values are subject to aggregate shocks is a mortgage on a residential or a commercial property. The value of real estate is often subject to aggregate shocks.

Consider the following model with aggregate shocks and imperfect observability. In each period $t = 1, 2$, an aggregate shock $\underline{v}_t \sim F_t(\underline{v}_t)$ is drawn. These shocks are drawn independently across periods. Banks and buyers at the beginning of each period observe a noisy signal of $\underline{v}_t$ given by $v_t = \underline{v}_t + \sigma\varepsilon_t$, where $\varepsilon_t \sim G(\varepsilon_t)$ with $E[\varepsilon_t] = 0$ is i.i.d. across periods. When $\sigma > 0$ the aggregate shock is imperfectly observed. We assume that F_t and G have full support over $\mathbb{R}$.

We assume that the distributions F_1 and G satisfy a monotone likelihood property. To develop this property note that, when $\sigma > 0$, the updating rules for the signal of the aggregate shock are given by

$$
\begin{aligned}
\Pr(v_1 \quad &\le \quad \hat{v}_1 | \underline{v}_1) = \Pr(\underline{v}_1 + \sigma\varepsilon_1 \le \hat{v}_1) = G\left(\frac{\hat{v}_1 - \underline{v}_1}{\sigma}\right) \\
\Pr(\underline{v}_1 \quad &\le \quad \hat{\underline{v}}_1 | v_1) = \frac{\int_{-\infty}^{\hat{\underline{v}}_1} f_1(v) g\left(\frac{v_1 - v}{\sigma}\right) dv}{\int_{-\infty}^{\infty} f_1(v) g\left(\frac{v_1 - v}{\sigma}\right) dv} = H(\hat{\underline{v}}_1 | v_1)
\end{aligned}
$$

Assumption 4.5 *(Monotone Likelihood Ratio) The posterior belief function* $H(\underline{v}_1|v_1)$ *is a decreasing function of* v_1*.*

This assumption implies that when the signal, v_1, about the shock is high, the value of the shock, $\underline{v}_1$, is likely to be high. Straightforward algebra can be used to show that this assumption is satisfied if a monotone likelihood ratio property on g holds, namely, that for any $v_1 > v_1'$, $g(v_1 - \underline{v}_1)/g(v_1' - \underline{v}_1)$ is increasing in $\underline{v}_1$.

The timing of the game is as follows: *(i)* At the beginning of each period t, agents observe the aggregate shock in the previous period $\underline{v}_{t-1}$. Buyers do not observe previous period signals v_{t-1} or the market price p_{t-1}. (We believe that our uniqueness result goes through if future buyers receive a noisy signal about previous prices.), *(ii)*The new aggregate state $\underline{v}_t$ is drawn, the bank and current period buyers do not observe the current state, $\underline{v}_t$, but they do observe the noisy signal, v_t, *(iii)* Buyers offer prices, *(iv)* The bank decides whether to sell or hold.

With aggregate shocks and perfect observability, $\sigma = 0$, it is immediate that a version of Proposition 4.2 applies and the two period model has multiple equilibria.

To establish uniqueness in our two period model with imperfect observability we begin from the last period. We will show that in the last period, the unique equilibrium is characterized by a cutoff threshold $\mu_2^*(v_2)$ such that banks with reputation levels above $\mu_2^*(v_2)$ sell their loans and banks below this threshold hold their loans and a fall in v_2 raises $\mu_2^*(v_2)$. In this sense, a fall in collateral values worsens the adverse selection problem. To see this result, note that an HH bank sells its loan if and only if

$$\hat{p}(\mu_2; v_2) - q \geq \bar{\pi}\bar{v} + (1-\bar{\pi})E\left[\underline{v}_2|v_2\right] - q(1+r) - \bar{c}, \tag{4.14}$$

where

$$\hat{p}(\mu_2; v_2) := \left[\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi}\right]\bar{v} + \left[\mu_2(1-\bar{\pi}) + (1-\mu_2)(1-\underline{\pi})\right]E\left[\underline{v}_2|v_2\right]. \tag{4.15}$$

Substituting for $\hat{p}(\mu_2; v_2)$ from (4.15) into (4.14) and noting that $E\left[\underline{v}_2|v_2\right] = v_2$, we obtain that the threshold reputation at which the HH bank is just indifferent between holding and selling is given by

$$\mu_2^*(v_2) = 1 - \frac{qr + \bar{c}}{(\bar{\pi} - \underline{\pi})(\bar{v} - v_2)} \tag{4.16}$$

whenever the right hand side of (4.16) is between zero and one and at the appropriate extreme points otherwise. Clearly $\mu_2^*(v_2)$ is decreasing in v_2. We summarize this discussion in the following proposition.

Proposition 4.6 *In the second period, given a reputation level μ_2 and a default value signal v_2, there is a unique equilibrium outcome in which the HH bank's decision is to* *sell if* $\mu_2 \geq \mu_2^*(v_2)$ *and to hold otherwise, where*

$$\mu_2^*(v_2) = \max\left\{\min\left\{1 - \frac{qr + \bar{c}}{(\bar{\pi} - \underline{\pi})(\bar{v} - v_2)}, 1\right\}, 0\right\}.$$

Given this characterization of the second period equilibrium, we can calculate the payoff to the HH bank before the aggregate shock (as well as the second period signal) or the cost type is realized for every value of reputation at the beginning of the second period. These payoffs are given by

$$V_2(\mu_2) = \int\int \hat{V}_2(\mu_2, v_2) dG\left(\frac{v_2 - \underline{v}_2}{\sigma}\right) dF_2(\underline{v}_2) \tag{4.17}$$

where

$$\hat{V}_2(\mu_2, v_2) = \alpha\left[\bar{\pi}\bar{v} - q(1+r) - \underline{c}\right] + (1-\alpha)\max\{\hat{p}(\mu_2; v_2) - q, \bar{\pi}\bar{v} + (1-\bar{\pi})v_2 - q(1+r) - \bar{c}\}.$$

Next, we use the characterization of the payoffs given in (4.17) to prove that the two period model has a unique equilibrium. Proving that the perturbed game has a unique equilibrium is easiest when F_1 is an improper uniform distribution, $U[-\infty, \infty]$. In Section 4.4.2, we prove uniqueness as $\sigma \to 0$, when F_1 is a proper distribution.

We have the following proposition:

Proposition 4.7 *For each* $\sigma > 0$ *and* $V_2(\mu_2)$ *given by (4.17), the game with uniform improper priors has a unique equilibrium in which in period 1, HH bank's action is characterized by a cutoff* $v_1^*(\sigma) \in \mathbb{R}$ *above which the HH bank sells and below which the HH bank holds.*

We prove this proposition using a method similar to [Carlsson and Van Damme, 1993]. We begin by restricting attention to switching strategies in which the bank sells for all default values above a threshold and holds for all default values below that threshold. We show that the game has a unique equilibrium in switching strategies. We then prove that the equilibrium switching strategy is the only strategy that survives iterated elimination of strictly dominant strategies so that we have a unique equilibrium.

The intuition for the iterated elimination argument is as follows. Note that we can define equilibrium as a strategy for the bank in period 1, and a belief – about the bank's action in period 1 – by period 2 buyers used for Bayesian updating. In equilibrium beliefs have to coincide with strategies. Obviously reputational incentives depend on future buyers' beliefs. When v_1 is very large, independent of future buyers' beliefs, an HH bank sells the asset. Similarly, when v_1 is very low, an HH bank holds on to the asset, independent of future beliefs. This argument establishes two bounds $\hat{v}^1 > \tilde{v}^1$, such that any equilibrium strategy must prescribe a sale for v_1 higher than $\hat{v}^1$ and holding for v_1 lower than $\tilde{v}^1$. This result means that the set of beliefs by future buyers have to satisfy the same property. Limiting the set of beliefs puts tighter upper and lower bounds on reputational incentives, which in turn implies new bounds $\hat{v}^2 > \tilde{v}^2$. We show that iterating in this manner implies that the bounds $\hat{v}^n$ and $\tilde{v}^n$ converge to a common limit.

Here we sketch the key steps of the proof and leave the details to Appendix C.1.

4.4.1 Outline of Proof with Improper Priors

1. Unique Equilibrium in Switching Strategies: We begin by restricting attention to switiching strategies of the form:

$$d_k(v_1) = \begin{cases} 1 & v_1 \geq k \\ 0 & v_1 < k, \end{cases}$$

where k represents the switching point. We characterize the best response of the HH bank when future buyers use d_k to form their posteriors over the bank's type. To do so, we use Bayes rule. Consider an arbitrary belief $\hat{a}_1(\cdot)$ by period 2 buyers about the HH bank's period 1 action. Based on the observed history and signal $\underline{v}_1$, Bayes rule implies the following updating formulas:

$$
\begin{aligned}
\mu_{sg}(\underline{v}_1;\hat{a}_1) &= \frac{\mu_1\bar{\pi}\int \hat{a}_1(v_1)dG\left(\frac{v_1-\underline{v}_1}{\sigma}\right)}{\mu_1\bar{\pi}\int \hat{a}_1(v_1)dG\left(\frac{v_1-\underline{v}_1}{\sigma}\right)+(1-\mu_1)\underline{\pi}} \\
\mu_{sd}(\underline{v}_1;\hat{a}_1) &= \frac{\mu_1(1-\bar{\pi})\int \hat{a}_1(v_1)dG\left(\frac{v_1-\underline{v}_1}{\sigma}\right)}{\mu_1(1-\bar{\pi})\int \hat{a}_1(v_1)dG\left(\frac{v_1-\underline{v}_1}{\sigma}\right)+(1-\mu_1)(1-\underline{\pi})} \\
\mu_{h}(\underline{v}_1;\hat{a}_1) &= \frac{\mu_1\left[(1-\alpha)\int\left[1-\hat{a}_1(v_1)\right]dG\left(\frac{v_1-\underline{v}_1}{\sigma}\right)+\alpha\right]}{\mu_1\left[(1-\alpha)\int\left[1-\hat{a}_1(v_1)\right]dG\left(\frac{v_1-\underline{v}_1}{\sigma}\right)+\alpha\right]+(1-\mu_1)\alpha}.
\end{aligned}
$$

For switching strategies, these formulas simplify to

$$
\begin{aligned}
\mu_{sg}(\underline{v}_1;d_k) &= \frac{\mu_1\bar{\pi}\left[1-G\left(\frac{k-\underline{v}_1}{\sigma}\right)\right]}{\mu_1\bar{\pi}\left[1-G\left(\frac{k-\underline{v}_1}{\sigma}\right)\right]+(1-\mu_1)\underline{\pi}} \\
\mu_{sd}(\underline{v}_1;d_k) &= \frac{\mu_1(1-\bar{\pi})\left[1-G\left(\frac{k-\underline{v}_1}{\sigma}\right)\right]}{\mu_1(1-\bar{\pi})\left[1-G\left(\frac{k-\underline{v}_1}{\sigma}\right)\right]+(1-\mu_1)(1-\underline{\pi})} \\
\mu_{h}(\underline{v}_1;d_k) &= \frac{\mu_1\left[(1-\alpha)G\left(\frac{k-\underline{v}_1}{\sigma}\right)+\alpha\right]}{\mu_1\left[(1-\alpha)G\left(\frac{k-\underline{v}_1}{\sigma}\right)+\alpha\right]+(1-\mu_1)\alpha}.
\end{aligned}
\tag{4.18}
$$

Next, given any belief $\hat{a}_1$ and noting that with improper priors $H(\underline{v}_1|v_1) = G\left(\frac{v_1-\underline{v}_1}{\sigma}\right)$, we define the gain from reputation as

$$
\begin{aligned}
&\Delta(v_1;\hat{a}_1) = \\
&\quad\beta\int\left[\bar{\pi}V_2(\mu_{sg}(\underline{v}_1;\hat{a}_1))+(1-\bar{\pi})V_2(\mu_{sd}(\underline{v}_1;\hat{a}_1))-V_2(\mu_h(\underline{v}_1;\hat{a}_1))\right]dG\left(\frac{v_1-\underline{v}_1}{\sigma}\right).
\end{aligned}
$$

In Appendix C.1 we prove the following Lemma, which characterizes the gain from reputation for general strategies and switching strategies.

Lemma 4.8 *The gain from reputation $\Delta(v_1;\hat{a}_1)$ is uniformly bounded and strictly increasing in $\hat{a}_1$ according to a point-wise ordering on beliefs. In particular, if $\hat{a}_1$ is a switching strategy, d_k, then $\Delta(v_1;d_k)$ is strictly decreasing in k. Moreover, when $\hat{a}_1$ is* *a switching strategy,* $\Delta(v_1; \hat{a}_1)$ *is strictly increasing in* v_1*.*

Facing a switching strategy belief of future buyers, d_k, clearly, the HH bank sells if and only if

$$\hat{p}(\mu_1; v_1) - q + \Delta(v_1; d_k) \geq \bar{\pi}\bar{v} + (1-\bar{\pi})v_1 - q(1+r) - \bar{c}. \tag{4.19}$$

Note that the value of selling, given by the left side of (4.19), is increasing in v_1 and its partial derivative with respect to v_1 is at least the derivative of $\hat{p}(\mu_1; v_1)$, given by $\mu_1(1-\bar{\pi})+(1-\mu_1)\underline{\pi}$. The value of holding, given by the right side of (4.19), is increasing in v_1 and its derivative is $1-\bar{\pi}$. Since the derivative for the value of selling is greater than the value of holding, there exists a unique solution, $b(k)$, that solves the equation

$$\hat{p}(\mu_1; b(k)) - q + \Delta(b(k); d_k) = \bar{\pi}\bar{v} + (1-\bar{\pi})b(k) - q(1+r) - \bar{c}.$$

Hence, the best response of the HH bank to a switching strategy belief of future buyers, d_k, is a switching strategy, $d_{b(k)}$, in which the bank sells for all returns above $b(k)$ and holds for all return values below $b(k)$. An equilibrium in switching strategies must be a fixed point of the above equation, so an equilibrium switching point, k^*, satisfies

$$\hat{p}(\mu_1; k^*) - q + \Delta(k^*; d_{k^*}) = \bar{\pi}\bar{v} + (1-\bar{\pi})k^* - q(1+r) - \bar{c}.$$

In Appendix C.1, we prove the following lemma.

Lemma 4.9 *The best response function* $b(k)$ *has a unique fixed point* k^* *which is globally stable.*

Hence, the game with switching strategies has a unique equilibrium.

2. Restriction to Switching Strategies Is without Loss of Generality: We follow [Morris and Shin, 2003] in showing that the restriction to switching strategies is without loss of generality. We do so by showing that regardless of future buyers' belief functions, the bank has a dominant strategy for extreme values of default values. Consider two numbers $\hat{v} < \tilde{v}$. We define an *extreme monotone strategy* to be a strategy that calls for selling when $v_1 \geq \tilde{v}$ and holding for $v_1 \leq \hat{v}$. We define $A_{\hat{v},\tilde{v}}$ to be the set of such strategies. Notice that $A_{-\infty,\infty}$ is the set of all strategies. Define the best response set operator on a subset of beliefs, A, as

$$\begin{aligned} BR(A) &= \{a_1 | \exists \hat{a}_1 \ni a_1(v_1) = 1 \Leftrightarrow \\ & \qquad \hat{p}(\mu_1; v_1) - q + \Delta(v_1; \hat{a}_1) \geq \bar{\pi}\bar{v} + (1-\bar{\pi})v_1 - q(1+r) - \bar{c}\} . \end{aligned}$$

We show that there exist bounds $\hat{v}^0 < \tilde{v}^0$ such that the HH bank holds for $v_1 \leq \hat{v}^0$ and it sells the asset for $v_1 \geq \tilde{v}^0$, independent of future buyers' belief function $\hat{a}_1$. That is,

$$\begin{aligned} \forall \hat{a}_1, v_1 &\geq \tilde{v}^0; \hat{p}(\mu_1; v_1) - q + \Delta(v_1; \hat{a}_1) \geq \bar{\pi}\bar{v} + (1-\bar{\pi})v_1 - q(1+r) - \bar{c} \qquad (4.20) \\ \forall \hat{a}_1, v_1 &\leq \hat{v}^0; \hat{p}(\mu_1; v_1) - q + -q + \Delta(v_1; \hat{a}_1) \leq \bar{\pi}\bar{v} + (1-\bar{\pi})v_1 - q(1+r) - \bar{c}. \end{aligned}$$

Using the result from Lemma (4.8) that $\Delta(v_1; \hat{a}_1)$ is uniformly bounded in (4.20), it follows that these bounds exist. We have established that any equilibrium strategy must be an extreme monotone strategy with cutoffs $\hat{v}^0 < \tilde{v}^0$. That is,

$$BR(A_{-\infty,\infty}) \subseteq A_{\hat{v}^0,\tilde{v}^0}.$$

Thus, we can restrict attention to extreme monotone strategies without loss of generality.

Next, we show that the best response set operator is decreasing in the sense that it induces a best response set, which is a strict subset of any arbitrary set of extreme monotone beliefs. Repeatedly applying this operator induces a decreasing sequence of sets, which converges to a unique equilibrium.

To show that the best response set operator is decreasing, we show that for any $\hat{v} < \tilde{v}$, $BR(A_{\hat{v},\tilde{v}}) \subseteq A_{b(\hat{v}),b(\tilde{v})} \subset A_{\hat{v},\tilde{v}}$. Since $\Delta(v_1; \hat{a}_1)$ is increasing in $\hat{a}_1$, for all $\hat{a}_1 \in A_{\hat{v},\tilde{v}}$ we have

$$\hat{p}(\mu_1; v_1) - q + \Delta(v_1; d_{\tilde{v}}) \leq \hat{p}(\mu_1; v_1) - q + \Delta(v_1; \hat{a}_1) \leq \hat{p}(\mu_1; v_1) - q + \Delta(v_1; d_{\hat{v}})$$

because $\hat{a}_1$ first order stochastically dominates $d_{\tilde{v}}$ and is dominated by $d_{\hat{v}}$. This result implies that

$$\bar{\pi}\bar{v} + (1-\bar{\pi})v_1 - q(1+r) - \bar{c} \geq \hat{p}(\mu_1; v_1) - q + \Delta(v_1; \hat{a}_1)$$

if

$$\bar{\pi}\bar{v} + (1-\bar{\pi})v_1 - q(1+r) - \bar{c} \geq \hat{p}(\mu_1; v_1) - q + \Delta(v_1; d_{\hat{v}}).$$

This result implies that if a_1 is the best response to $\hat{a}_1$, then

$$\forall v_1 < b(\hat{v}), \quad a_1(v_1) = 0.$$

Similarly, we can show that the best response to $\hat{a}_1$ must satisfy $a_1(v_1) = 1$ for all $v_1 \geq b(\tilde{v})$. We have proved that $BR(A_{\hat{v},\tilde{v}}) \subseteq A_{b(\hat{v}),b(\tilde{v})}$. Since $b(k)$ is globally stable, $A_{b(\hat{v}),b(\tilde{v})} \subset A_{\hat{v},\tilde{v}}$ so that $BR(A_{\hat{v},\tilde{v}}) \subseteq A_{b(\hat{v}),b(\tilde{v})} \subset A_{\hat{v},\tilde{v}}$. Finally, because $b(k)$ has a unique fixed point, $A^n_{b(\hat{v}),b(\tilde{v})}$ converges to $A_{k^*,k^*} = \{d_{k^*}\}$ so that $BR^n(A_{-\infty,\infty})$ also converges to $\{d_{k^*}\}$.

4.4.2 Uniqueness Result with Proper Priors

In this section, we provide a characterization of equilibria in the limiting perturbed game with general proper priors. In particular, we prove that in the perturbed game as $\sigma \to 0$, the set of period 1 equilibrium strategies converges to a unique strategy. We use the method of Laplacian beliefs introduced by [Frankel et al., 2003] and reviewed by [Morris and Shin, 2003] to prove our uniqueness result. In fact, we show that the game described above is equivalent to a game discussed by [Morris and Shin, 2003]. We then use their result to prove the following theorem. The proof is in Appendix C.1.

Theorem 4.10 *Given the value function $V_2(\mu_2)$ given by (4.17), as $\sigma \to 0$ the set of first period equilibrium strategies in the game with proper priors converges to a unique strategy by the HH bank in which the bank sells if $v_1 \geq v_1^*$ and holds if $v_1 < v_1^*$ where v_1^* satisfies*

$$\begin{aligned}\hat{p}(\mu_1; v_1^*) - q + \beta \int_0^1 \left[\bar{\pi} V_2\left(\hat{\mu}_{sg}\left(l\right)\right) + (1-\bar{\pi}) V_2\left(\hat{\mu}_{sd}\left(l\right)\right) - V_2\left(\hat{\mu}_h\left(l\right)\right)\right] dl \\ = \bar{\pi}\bar{v} + (1-\bar{\pi})v_1^* - q(1+r) - \bar{c}\end{aligned}$$

and

$$\begin{aligned}
\hat{\mu}_{sg}(l) &= \frac{\mu_1 \bar{\pi} l}{\mu_1 \bar{\pi} l + (1-\mu_1)\underline{\pi}} \\
\hat{\mu}_{sd}(l) &= \frac{\mu_1 (1-\bar{\pi}) l}{\mu_1 (1-\bar{\pi}) l + (1-\mu_1)(1-\underline{\pi})} \\
\hat{\mu}_{h}(l) &= \frac{\mu_1 \left[(1-\alpha)(1-l)+\alpha\right]}{\mu_1 \left[(1-\alpha)(1-l)+\alpha\right] + (1-\mu_1)\alpha}.
\end{aligned}$$

4.5 The Multi-Period Model

In this section, we extend the model to many periods. The qualitative properties of the model are very similar to the model with two periods. In particular, we show that the game with noisy signals has a unique equilibrium in the limit as the observation error converges to zero.

The extension of the model to multi periods is as follows: time is discrete and $t = 1, \cdots, T$, $T < \infty$. The bank's quality type is drawn at the beginning of period 1. The bank's cost type is drawn independently over time and is independent of the quality type. The collateral value $\underline{v}_t$ is drawn from a distribution function $F(\underline{v}_t)$ and is independent across periods. A new set of buyers arrives each period and lives only for that period. The information structure of the game is as in the two-period model in Section 4.4. In each period before trading occurs, all agents in the economy observe $v_t = \underline{v}_t + \sigma_t \varepsilon_t$ where ε_t is i.i.d. and distributed according to $G(\varepsilon)$. They do not, however, observe $\underline{v}_t$. Given this information, the agents trade in the market. After the trade, the collateral value $\underline{v}_t$ becomes public information. Previous prices are not observed by current buyers. Based on observables, agents update their beliefs at the end of period t.

In Appendix C.1, we recursively construct the payoff of the HH bank and its equilibrium strategy and prove the following proposition.

Proposition 4.11 *Suppose that for some period $t+1$ and for any μ_{t+1}, the multiperiod model has a unique equilibrium with payoff for the HH bank given by $V_{t+1}(\mu_{t+1})$. If $V_{t+1}(\mu_{t+1})$ is increasing in μ_{t+1}, there is a unique equilibrium strategy in period t as $\sigma_t \to 0$ for all μ_t. The equilibrium strategy for the HH bank in period t is given by a* *cutoff strategy in which the HH bank sells if $v_t \geq v_t^*(\mu_t)$ and holds if $v_t < v_t^*(\mu_t)$ where $v_t^*(\mu_t)$ satisfies the following equation*

$$\hat{p}(\mu_t; v_t^*) - q + \beta \int_0^1 \left[\bar{\pi} V_{t+1}(\hat{\mu}_{sg}(l)) + (1-\bar{\pi}) V_{t+1}(\hat{\mu}_{sb}(l)) - V_{t+1}(\hat{\mu}_h(l))\right] dl$$
$$= \bar{\pi}\bar{v} + (1-\bar{\pi}) v_t^* - q(1+r) - \bar{c}.$$

Furthermore, the model has a unique equilibrium in the last period.

This proposition shows that the finite horizon version of the model has a unique equilibrium under the assumption that the value function is increasing in the reputation of the bank. This assumption can be replaced by assumptions on parameter values. One such assumption is that α, the probability that the bank's cost type is low, is sufficiently small. In the numerical examples described below, we found that the value function is increasing in the reputation of the bank for all of the parameter values we studied.

4.6 Fragility

We think of equilibrium outcomes as *fragile* in two ways. One notion of fragility is simply that the economy has multiple equilibria so that sunspot-like fluctuations can induce changes in outcomes. A second notion of fragility is that small changes in fundamentals induce large changes in aggregate outcomes.

Equilibrium outcomes in our unperturbed game are clearly fragile under the first notion because that game has multiple equilibria. They are also fragile under the second notion if agents in the model coordinate on different equilibria depending on the realization of the fundamentals and if a large mass of agents have reputation levels in the multiplicity region.

Since our perturbed game has a unique equilibrium, it is not fragile under the first notion. We argue that it is fragile under our second notion. In our multi-period model, the history of past outcomes induces dispersion in the reputation levels of different banks. In order for our equilibrium to display fragility under the second notion, we must have that either banks with a wide variety of reputation levels change their actions in the same way in response to aggregate shocks or that the reputation levels of banks cluster close to each other. We conducted a wide variety of numerical exercises and found that the clustering effect is very strong in our model. This clustering effect clearly depends on the details of the history of exogenous shocks. To abstract from these details, we consider the invariant distribution associated with our model and show that this invariant distribution displays clustering. The invariant distribution is that associated with the infinite horizon limit of our multi-period model. We allow for a small probability of replacement in order to ensure that the invariant distribution is not concentrated at a single point.

Figure C.4 displays the cutoff values for each reputation type for the ergodic set associated with the invariant distribution.[2] This ergodic set contains reputation levels between roughly 0.25 and 0.85. For collateral values above the cutoffs shown in Figure C.4, banks sell their loans and below the cutoffs banks hold their loans. This figure illustrates that as the collateral value falls, the adverse selection problem worsens in the sense that banks with a wider range of reputations hold their loans. For example, at a collateral value of 5, banks with reputation levels below roughly 0.4 hold their loans and the banks with higher reputation levels sell their loans. At a collateral value of 4, banks with reputation levels below roughly 0.65 hold their loans and banks with higher reputation levels sell their loans. Thus, a fall in collateral values from 5 to 4 induces banks with reputation levels roughly between 0.4 and 0.65 to switch from selling to holding their loans.

Figure C.4 displays the invariant distribution of reputation levels for high-quality banks. This figure shows that the invariant distribution displays significant clustering. Roughly 70 percent of high-quality banks have reputation levels between 0.8 and 0.85. Small fluctuations in the default value of loans around the cutoff values for such banks can induce a large mass of banks to alter their behavior.

Figure C.4 plots the volume of trade, measured as the fraction of all banks that sell their loans. A decrease in the default value from 1.3 to 1.1 induces a 50 percent decrease in the volume of trade. In this sense, Figure C.4 suggests that equilibrium outcomes in our model are fragile under the second notion.

Next we analyze the forces that induce clustering in our model. Bayes' rule implies

[2] The parameters used in this simulation are the following: $\bar{\pi} = 0.8, \underline{\pi} = 0.3, \bar{v} = 7, \bar{c} = 0.5, \underline{c} = -3, \alpha = 0.15, q = .1, r = 0.5, \beta(1-\lambda) = .99, \lambda = .4, \mu_0 = .6$, where λ represents the exogenous probability of replacement and μ_0 is the reputation of a newly replaced bank. The distribution of $\underline{v}$ is $N(0, 2)$.

that $\frac{1}{\mu_t}$ is a martingale. Since $\frac{1}{\mu_t}$ is a convex function, Jensen's inequality implies that the reputation of a bank, μ_t, is a submartingale so that μ_t tends to rise. Conditional on a high-quality, high-cost bank holding, the analysis of our equilibrium implies that the reputation of such a bank also rises. These forces imply that the reputation of a high-quality bank displays an upward trend. This upward trend is dampened by replacement. Since all high-quality banks tend to have an upward trend in their reputations, these reputations tend to cluster toward each other.

This reasoning suggests that fragility under the second notion does not depend on the particular equilibrium that we have selected. In both the positive and negative reputational equilibria, the reputations of high-quality banks rise over time and tend to cluster together eventually. This clustering tends to make them react in the same way to fluctuations in the default value of the underlying loans. We conjecture that any continuous selection procedure will produce periods of high volumes of new issuances followed by sudden collapses.

We have analyzed the effect of other aggregate shocks in our model. In particular, we allowed the comparative advantage cost, $\bar{c}$, to be subject to aggregate shocks. In that version of the model, we found that banks with a wide variety of reputations tend to have cutoffs that are very close to each other. That model displays fragility under our second notion because small fluctuations in holding costs around a critical value induce large changes in actions by banks with a wide variety of reputations. (Details are available upon request.)

4.7 Policy Exercises

In this section, we use our model to evaluate the effects of various policies intended to remedy problems of credit markets – policies that have been proposed since the 2007 collapse of secondary loan markets in the United States. We focus on the effects of policies in which the government would purchase asset-backed securities at prices above existing market value, such as the Public-Private Partnership plan, as well as on policies that decreased the costs of holding loans to maturity, including changes in the Federal Funds target rate, the Term Asset-Backed Securities Loan Facility (TALF), and increased FDIC insurance.

These policies were motivated by perceived inefficiencies in secondary loan markets. For example, the Treasury Department asserts, in its Fact Sheet dated March 23, 2009, releasing details of a proposed Public-Private Investment Program for Legacy Assets,

> Secondary markets have become highly illiquid, and are trading at prices below where they would be in normally functioning markets. ([of Treasury, 2009])

Similarly, the Federal Reserve Bank of New York asserts, in a White Paper dated March 3, 2009, making the case for the Term Asset-Backed Securities Loan Facility (TALF),

> Nontraditional investors such as hedge funds, which may otherwise be willing to invest in these securities, have been unable to obtain funding from banks and dealers because of a general reluctance to lend. (TALF White Paper 2009)

Note that in our model sudden collapses are associated with increased inefficiency so that our model is consistent with policy makers concerns that the market had become more inefficient. In this sense, our model is an appropriate starting point for analyzing policies intended to remedy inefficiencies.

We first consider policies in which the government attempts to purchase so-called toxic assets at above-market values. Consider the following government policy in the limiting version of the perturbed game as $\sigma \to 0$. The government offers to buy the asset at some price p in the first period.

Suppose first that $p \leq \hat{p}(\mu_1; v_1)$. We claim that the unique equilibrium without government is also the unique equilibrium with this government policy. To see this claim, note that the equilibrium in the second period is the same with and without the government policy so that the reputational gains are the same with and without the government policy. Consider the first period and a realization of first-period return $v_1 < v_1^*$. In the game without the government, the HH bank found it optimal not to sell at a price $\hat{p}(\mu_1; v_1)$. Since the reputational gains are the same with and without the government policy, in the game with the government, it is also optimal for the HH not to sell at this price. A similar argument implies that the equilibrium strategy of the

HH bank is unchanged for $v_1 > v_1^*$. Thus, this government policy has no effect on the equilibrium strategy of the HH bank. Of course, under this policy, the government ends up buying the asset from low-quality banks. The only effect of this policy is to make transfers to low-quality banks.

Suppose next that the price set by the government, p, is sufficiently larger than $\hat{p}(\mu_1; v_1)$. Then, the HH bank will find it optimal to sell and will enjoy the reputational gain associated with a policy of selling. In this sense, if the government offers a sufficiently high price, it can ensure that reputational incentives work to overcome adverse selection problems. Note, however, that this policy necessarily implies that the government must earn negative profits.

Consider now a policy that reduces interest rates in period 1 and leaves period 2 interest rates unchanged. We begin the analysis with the unperturbed game. Such a policy increases the static payoff in period 1 from holding loans which worsens the static incentives for the HH bank to sell its loan. Specifically, this policy raises both the threshold $\underline{\mu}$ below which banks find it optimal to hold in the positive reputational equilibrium and the threshold $\bar{\mu}$ below which banks find it optimal to hold their loans in the negative reputational equilibrium. Thus, this policy serves only to aggravate the lemons problem in secondary loans markets.

Consider next a policy under which the government commits to reducing period 2 interest rates but leaves period 1 interest rates unchanged. Obviously, this policy increases incentives for banks to hold their loans in period 2 and thereby increases the threshold below which banks hold their loans, μ_2^*. In this sense, it makes period 2 allocations less efficient. We will show that this policy reduces the region of multiplicity in period 1 and in this sense can improve period 1 allocations. To show the reduction in the region of multiplicity, consider the reputational gain in the positive reputational equilibrium evaluated at $\underline{\mu}$:

$$\beta \left(\bar{\pi} V_2(\mu_{s\bar{v}}) + (1 - \bar{\pi}) V_2(\mu_{s0}) - V_2(\mu_h) \right).$$

Using (4.5), it is straightforward to see that an arbitrarily small reduction in interest rates of dr in period 2 reduces $V_2(\mu_{s\bar{v}})$ by $\alpha q dr$ since $\mu_{s\bar{v}} > \mu_2^*$. Moreover, since μ_{s0} and μ_h are strictly less than μ_2^*, $V_2(\mu_{s0})$ and $V_2(\mu_h)$ fall by qdr. As a result, the reputational gain falls by $\beta\bar{\pi}(1-\alpha)qdr$. This decline in reputational gain induces an increase in the threshold $\underline{\mu}$. Similarly, we can show that the policy induces a fall in the threshold $\bar{\mu}$. Thus, the region of multiplicity shrinks and in this sense can improve period 1 allocations. Interestingly, such a policy is time inconsistent because the government has a strong incentive in period 2 not to make period 2 allocations less efficient.

An alternative policy that has not been proposed is to consider forced asset sales in which the government randomly forces banks to sell their loans. Such a policy in our model would mitigate the lemons problem in secondary loan markets by generating a pool of loans in secondary markets consistent with the ex ante mix of loan types. Although this is a standard intervention directed at increasing the price and volume of trade in markets that suffer from adverse selection, in our model such an intervention comes at the cost of misallocating loans to those without comparative advantage. Specifically, some banks with low costs of holding loans will be forced to sell to the marketplace.

It is straightforward to show that a policy under which the government commits to purchase assets in period 2 at prices that are contingent on the realization of the signals can eliminate the multiplicity of equilibria and support the positive reputational equilibrium. Although such a policy would be desirable, the feasibility of such a policy can be analyzed only by developing a model in which private agents cannot commit but the government can.

4.8 Conclusion

This chapter is an attempt to make three contributions: a theoretical contribution to the literature on reputation, a substantive contribution to the literature on the behavior of financial markets during crises, and a contribution to analyses of proposed and actual policies during the recent crisis. In terms of the theoretical contribution, we have combined insights from the literature that emphasizes the positive aspects of reputational incentives (see [Mailath and Samuelson, 2001]) with the literature that emphasizes the negative aspects of reputational incentives (see [Ely and Välimäki, 2003]) to show that multiplicity of equilibria naturally arise in reputation models like ours. We have also shown how techniques from the coordination games literature can be adapted to develop a refinement method that produces a unique equilibrium. In terms of the literature on the behavior of financial markets during crises, we have argued that sudden collapses in secondary loan market activity are particularly likely when the collateral value of the underlying loan declines. In terms of policy, we have argued that a wide variety of proposed policy responses would not have averted either the sudden collapse or the associated inefficiency. An important avenue for future work is to analyze policies that might in fact remedy the inefficiencies.

Another important avenue for future work is to introduce loan origination as a choice for banks in the model so that the model can be used to analyze the effects of sudden collapses on investment and other macroeconomic aggregates.

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Appendix A

Appendix to Chapter 2

A.1 Static Version of our Model

In this Appendix, we develop a static version of our model in order to establish the main economic mechanism of the model. We use the static model to illustrate how shocks to the collateral constraint affects both firms for whom the constraint binds and also those for whom the constraint does not bind. We derive a sufficient condition on the model parameters under which a tightening of the collateral constraint leads all firms to reduce output in equilibrium.

Consider a static economy populated by a continuum of intermediate good firms, a representative final good producer, and a representative household.

Intermediate Good Producers. Each intermediate good firm $i \in [0, 1]$ has an asset level a_i and productivity z_i. Moreover, (a_i, z_i) is distributed according to $F(a, z)$. An intermediate good firm with productivity z_i and assets a_i, rents labor, l, and capital, k, rents out its assets a_i and purchases an amount of the final good, I, to be used as an input to production according to the production function

$$q_i = z_i^{\frac{1}{\rho-1}} \left(k^\alpha l^{1-\alpha}\right)^\eta I^{1-\eta}$$

Each firm may rent capital up to a multiple of the value of the firm's assets. Specifically, we impose a *collateral constraint* so that the amount of capital rented by a firm with asset level a_i is bounded by λa_i where $\lambda \geq 1$. One can rationalize this type of constraint by a model of moral hazard or limited enforcement. In line with the rest of the literature, we impose this constraint and do not provide a formal micro foundation for it.

Final Good Producer. The final good producer uses a bundle of inputs purchased from intermediate good firms and takes their prices as given. Given a bundle $\{q_i\}_{i\in[0,1]}$, the final good producer uses the following Dixit-Stiglitz production function:

$$Q = \left[\int_0^1 q_i^{\frac{\rho-1}{\rho}} dF(i)\right]^{\frac{\rho}{\rho-1}}$$

with $\rho > 1$.

Households. We assume that there is a representative households who buys the final good and provides labor to intermediate good producers. Following [Greenwood et al., 1988], we assume that household preferences are given by

$$U\left(c - \psi \frac{l^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}}\right)$$

Markets. We assume that the labor market is competitive at wage level w. As for capital market, we assume that the economy is small and open. That is, there exist suppliers of capital that have deep pockets and inelastically supply capital at a given interest rate r. This is an assumption that simplifies the analysis; under this assumption, when we perform comparative statics, we do not need to consider general equilibrium effects that arise from changes in the interest rate for capital. Similar analysis can be done in a closed economy.

Moreover, we assume that there is monopolistic competition across intermediate good firms and prices are given by p_i. The final good producer takes these prices as given and intermediate good producers take the demand function for intermediate output as given. We normalize the price of final good to 1.

Given the above market structure, the final good producer's maximization problem is given by

$$\max_{q_i} \left[\int_0^1 q_i^{\frac{\rho-1}{\rho}} dF(i)\right]^{\frac{\rho}{\rho-1}} - \int_0^1 p_i q_i dF(i).$$

The resulting demand for intermediate good i is given by

$$q_i^{-\frac{1}{\rho}} Q^{\frac{1}{\rho}} = p_i.$$

Given this demand function, each intermediate good firm maximizes its profit subject to its collateral constraint:

$$\pi_i = \max_{k,l,I,p_i} p_i z_i^{\frac{1}{\rho-1}} \left(k^\alpha l^{1-\alpha}\right)^\eta I^{1-\eta} - wl - rk - I + ra_i \tag{A.1}$$

subject to

$$\begin{aligned} p_i &= Q^{\frac{1}{\rho}} \left(z_i^{\frac{1}{\rho-1}} \left(k^\alpha l^{1-\alpha}\right)^\eta I^{1-\eta} \right)^{-\frac{1}{\rho}} \\ k &\leq \lambda a_i \end{aligned}$$

We say that a firm is *financially constrained* if the collateral constraint is binding for a firm in equilibrium and is *financially unconstrained* otherwise.

To complete the definition of competitive equilibrium, we need to specify the household's optimization problem as well as market clearing conditions. The representative household's optimization problem is given by

$$\max_{c,L} U \left(c - \psi \frac{L^{1+\frac{1}{\varepsilon}}}{1+\frac{1}{\varepsilon}} \right)$$

subject to

$$c \leq wL + \int_0^1 \pi_i dF(i).$$

Labor market and product market clearing are given by

$$\begin{aligned} \int_0^1 l_i dF(i) &= L \\ c + \int_i I_i dF(i) &= Q. \end{aligned}$$

Hence a competitive equilibrium of this economy is given by

$$\left\{ \{k_i, l_i, I_i, p_i\}_{i\in[0,1]}, c, L, Q, w \right\}$$

that satisfies the above conditions.

Because of monopolistic competition, the revenue function of the firm exhibits decreasing returns to scale. As a result, for every z, there is an unconstrained optimal scale, which is increasing in z. Not surprisingly, then, every z, there is a threshold in assets, say $a^*(z)$ such that firms with assets and productivity (a, z) with $a \geq a^*(z)$ are financially unconstrained and if $a < a^*(z)$ the firm is financially constrained. We state this result along with optimal capital, labor, and intermediate input decisions for firms in the following lemma (the proof is omitted).

Lemma A.1 *For every z, there exists $a^*(z)$ such that for $a \geq a^*(z)$, $k(a,z) \leq \lambda a$ and for $a < a^*(z), k = \lambda a$. Furthermore, $a^*(z)$ satisfies*

$$a^*(z) = \frac{1}{\lambda}\left[\nu(1-\eta)\right]^{\frac{(1-\eta)\nu}{1-\nu}} \left(\frac{\alpha\eta\nu}{\hat{r}}\right)^{1+\frac{\alpha\eta\nu}{1-\nu}} \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{(1-\alpha)\eta\nu}{1-\nu}} Qz.$$

If $a \geq a^*(z)$ then

$$k(a,z) = \left[\nu(1-\eta)\right]^{\frac{(1-\eta)\nu}{1-\nu}} \left(\frac{\alpha\eta\nu}{\hat{r}}\right)^{1+\frac{\alpha\eta\nu}{1-\nu}} \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{(1-\alpha)\eta\nu}{1-\nu}} Qz$$

if $a < a^*(z)$ then $k(a,z) = \lambda a$. Finally,

$$\begin{aligned}
l &= \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{1-(1-\eta)\nu}{1-(1-\alpha\eta)\nu}} \left(\nu(1-\eta)\right)^{\frac{(1-\eta)\nu}{1-(1-\alpha\eta)\nu}} \left(Qz\right)^{\frac{(1-\nu)}{1-(1-\alpha\eta)\nu}} k^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} \\
I &= \left[\nu(1-\eta)(Qz)^{1-\nu}\left(k^{\alpha}l^{1-\alpha}\right)^{\eta\nu}\right]^{\frac{1}{1-(1-\eta)\nu}}
\end{aligned}$$

where $\nu = 1 - \frac{1}{\rho}$.

Given the decisions of firms along with optimal labor supply of households, we can characterize equilibrium output, Q and the wage rate, w using the production function of the final good producer and the labor market clearing conditions. Given these equilibrium values, we can show that the equilibrium wage rate is decreasing in the collateral constraint parameter λ. This final result follows because a relaxing of the constraint must increase aggregate capital demand and therefore labor demand causing the wage to rise. We have the following lemma.

Lemma A.2 *Any competitive equilibrium must satisfy the following:*

1. $Q = \frac{1}{(1-\alpha)\eta\left(1-\frac{1}{\rho}\right)}\psi^{-\varepsilon}w^{1+\varepsilon}$,

2. *The equilibrium wage, w, is decreasing in λ.*

Proof

Given capital, labor and intermediate input demand, we can construct aggregate excess output and labor as functions of prices r, w, and output Q. Specifically, let $A^* = \{(a,z) : a \geq a^*(z)\}$

$$\begin{aligned} Q^\nu &= \int q^\nu G(da,dz) \qquad \text{(A.2)} \\ &= \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\nu\eta(1-\alpha)}{1-\nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-\nu}} \left(\frac{\alpha\eta\nu}{\hat{r}}\right)^{\frac{\alpha\eta\nu}{1-\nu}} Q^\nu \int_{(a,z)\in A^*} zG(da,dz) \\ &+ \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\nu\eta(1-\alpha)}{1-(1-\alpha\eta)\nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-(1-\alpha\eta)\nu}} Q^{\frac{\nu(1-\nu)(1-\alpha\eta)}{1-(1-\alpha\eta)\nu}} \lambda^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} \\ &\times \int_{(a,z)\notin A^*} z^{\frac{(1-\nu)}{1-(1-\alpha\eta)\nu}} a^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} G(da,dz) \end{aligned}$$

Labor Demand is just a function of output. We have

$$l = \left(\frac{(1-\alpha)\eta\nu}{w}\right) Q^{1-\nu} q^\nu.$$

Thus

$$\int lG(da,dz) = \left(\frac{(1-\alpha)\eta\nu}{w}\right) Q^{1-\nu} \int q^\nu G(da,dz)$$

Household labor supply given our assumed form for household preferences (GHH) is just $\psi^{-\varepsilon}w^{\varepsilon}$.Thus labor market clearing is simply

$$\psi^{-\varepsilon}w^{\varepsilon} = \left(\frac{(1-\alpha)\eta\nu}{w}\right) Q$$

and, therefore, aggregate output in equilibrium satisfies

$$Q = \psi^{-\varepsilon}((1-\alpha)\eta\nu)^{-1}w^{1+\varepsilon}.$$

Re-write Q from (A.2) as

$$\begin{aligned} 1 &= \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\nu\eta(1-\alpha)}{1-\nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-\nu}} \left(\frac{\alpha\eta\nu}{\hat{r}}\right)^{\frac{\alpha\eta\nu}{1-\nu}} \int_{(a,z)\in A^*} zG(da,dz) \quad \text{(A.3)} \\ &+ \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\nu\eta(1-\alpha)}{1-(1-\alpha\eta)\nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-(1-\alpha\eta)\nu}} Q^{\frac{-\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} \lambda^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} \\ &\times \int_{(a,z)\notin A^*} z^{\frac{(1-\nu)}{1-(1-\alpha\eta)\nu}} a^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} G(da,dz) \end{aligned}$$

Analyzing the derivative with respect to λ,we have

$$\begin{aligned} 0 &= \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\nu\eta(1-\alpha)}{1-\nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-\nu}} \left(\frac{\alpha\eta\nu}{\hat{r}}\right)^{\frac{\alpha\eta\nu}{1-\nu}} \int_{(a,z)\in A^*} zG(da,dz) \\ &\times \left[-\frac{\nu\eta(1-\alpha)}{1-\nu}\frac{1}{w}\frac{dw}{d\lambda}\right] \\ &+ \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\nu\eta(1-\alpha)}{1-(1-\alpha\eta)\nu}} (\nu(1-\eta))^{\frac{\nu(1-\eta)}{1-(1-\alpha\eta)\nu}} Q^{\frac{-\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} \lambda^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} \\ &\times \int_{(a,z)\notin A^*} z^{\frac{(1-\nu)}{1-(1-\alpha\eta)\nu}} a^{\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}} G(da,dz) \\ &\times \left[-\frac{\nu\eta(1-\alpha)}{1-(1-\alpha\eta)\nu}\frac{1}{w}\frac{dw}{d\lambda} - \frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}\frac{1}{Q}\frac{dQ}{d\lambda} + \frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}\frac{1}{\lambda}\right] \end{aligned}$$

Suppose $\frac{dw}{d\lambda} \leq 0$. Then $\frac{dQ}{d\lambda} \leq 0$. Then we must have

$$-\frac{\nu\eta(1-\alpha)}{1-(1-\alpha\eta)\nu}\frac{1}{w}\frac{dw}{d\lambda} - \frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}\frac{1}{Q}\frac{dQ}{d\lambda} + \frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}\frac{1}{\lambda} \leq 0$$

and since

$$\frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}\frac{1}{\lambda} > 0$$

we have

$$\frac{\nu\eta(1-\alpha)}{1-(1-\alpha\eta)\nu}\frac{1}{w}\frac{dw}{d\lambda} + \frac{\alpha\eta\nu}{1-(1-\alpha\eta)\nu}\frac{1}{Q}\frac{dQ}{d\lambda} > 0$$

but the coefficients are all positive so this is a contradiction. As a result, the wage must be increasing in λ.

Q.E.D

We are now ready to state our necessary and sufficient condition for a tightening of the collateral constraint to cause both financially constrained and financially unconstrained firms to decrease output. Using the optimal production decisions of firms, we can show that a financially unconstrained firm's output satisfies

$$\begin{aligned} q_i &= \left(\frac{\nu(1-\alpha)\eta}{w}\right)^{\frac{\eta(1-\alpha)}{1-\nu}} (\nu(1-\eta))^{\frac{1-\eta}{1-\nu}} \left(\frac{\alpha\eta\nu}{\hat{r}}\right)^{\frac{\alpha\eta}{1-\nu}} Qz^{\frac{1}{\nu}} \\ &= \kappa w^{-\frac{\eta(1-\alpha)}{1-\nu}} Q \end{aligned}$$

A one percent increase in w causes Q to increase by $1+\varepsilon$ percent and $w^{-\frac{\eta(1-\alpha)}{1-\nu}}$ to decrease by $\frac{\eta(1-\alpha)}{1-\nu}$. Hence, output of financially unconstrained firms is increasing in w or λ if and only if $1+\varepsilon \geq \eta\rho(1-\alpha)$. We then have the following proposition.

Proposition A.3 *Suppose there exists a positive measure set of constrained firms. If $1+\varepsilon \geq \eta\rho(1-\alpha)$, then output of all firms is increasing in the collateral constraint parameter, λ.*

A.2 External Financing in a Simplified Version of our Dynamic Model

In this section, we analyze a version of our model with perfect competition, perfect substitutes, and an i.i.d. process for firm level productivity. Specifically, we assume that in every period, each firm has a probability π of having productivity equal to 1 and probability $1-\pi$ of having probability equal to 0. We solve analytically for the equilibrium and the amount of external financing used by firms in the model. We then compare the effect of changes in the collateral constraint parameter, λ, across economies with different probabilities of high productivity, π.

In particular, for each π-economy, we choose the collateral constraint parameter, $\lambda(\pi)$ so that the aggregate debt-to-assets ratio in the model is the same across all π-economies. We show that even though the debt-to-asset ratio is held constant, the amount of external financing is decreasing the probability of receiving a high productivity shock. Then, for each π-economy, we compare steady state wealth in the $\lambda(\pi)$ economy to that in the economy when $\lambda = 1$, in other words, the autarkic version of that economy. We show that the difference in steady state wealth between the $\lambda(\pi)$, high debt economy, and the $\lambda = 1$, no debt economy monotonically decreasing in the probability of receiving a high productivity shock.

Model and Solution

In this simplified version of our model, firms are identical, produce a homogeneous final output good, and the process for firm level productivity is given by

$$z_t = \begin{cases} 1 \text{ w.prob } \pi \\ 0 \text{ w.prob } 1-\pi \end{cases}$$

where the shocks are independent and identically drawn across firms and time. We assume a small open economy with a fixed interest rate that satisfies $0 \leq r \leq \frac{1}{\beta} - 1$.

The problem of a firm in any period can be written recursively as

$$V(a, z) = \max \ln(c) + \beta E V(a', z')$$

subject to

$$c + a' \leq (1+r)a + \max_{k \leq \lambda a, l} z k^{\alpha} l^{1-\alpha} - wl - (r+\delta)k$$

Clearly a firm with $z = 0$ chooses $k = 0, l = 0$. It is straightforward to show that profits for a firm with $z = 1$ are given by

$$\left[\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} - (r+\delta)\right]\lambda a.$$

Using this result, we may write the recursive problem of the firm as

$$V(a,z) = \max \ln(c) + \beta E V(a', z')$$

s.t.

$$c + a' \leq \left\{z\lambda\left[\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} - (r+\delta)\right] + (1+r)\right\} a.$$

Given our assumed form of preferences along with i.i.d. shock process for productivity, we immediately have that the savings functions are linear in asset holdings and given by

$$\begin{aligned} a'(a,1) &= \beta\left\{\lambda\left[\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} - (r+\delta)\right] + (1+r)\right\} a \qquad \text{(A.4)} \\ a'(a,0) &= \beta(1+r)a. \end{aligned}$$

The law of motion for assets in a steady state equilibrium yields the equilibrium wage rate which must satisfy

$$1 = \beta(1+r) + \beta\pi\lambda\left[\alpha\left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} - (r+\delta)\right].$$

Labor market clearing (with aggregate labor normalized to 1), then, defines steady state wealth:

$$\bar{A}(\lambda,\pi) = \left[\frac{\alpha\beta(\pi\lambda)^{\alpha}}{1-\beta(1+r)+\beta(r+\delta)\pi\lambda}\right]^{\frac{1}{1-\alpha}}.$$

We now turn to analyzing the amount of external financing firms rely on as well as the amount of aggregate debt and assets.

External Financing

First, as in our quantitative model, we define available funds and investment by re-writing the budget

$$c_t + a_{t+1} = z_t k_t^{\alpha} l_t^{1-\alpha} - wl_t - (r+\delta)k_t + (1+r)a_t$$

of a firm with an explicit definition of debt $b_t = k_t - a_t$. We then have

$$c_t + k_{t+1} - (1-\delta)k_t = z_t^{\alpha} k_t^{\alpha} l_t^{1-\alpha} - wl_t - rb_t + b_{t+1} - b_t.$$

We then define available funds, debt, and investment as

$$\begin{aligned} AF_t &= k_t^{\alpha} l_t^{1-\alpha} - wl_t - rb_t \\ b_t &= k_t - a_t \\ X_t &= k_{t+1} - (1-\delta)k_t \end{aligned}$$

Available funds for specific firms depends on their asset holdings and their productivity in any period. All of the derivations are included below. Available funds for firms with assets a_t and productivity z_t satisfy

$$AF_t(a_t, z_t) = \begin{cases} a_t \left[\frac{1-\beta(1+r)}{\beta\pi} + \lambda\delta + r\right] & \text{if } z_t = \bar{z} \\ a_t r & \text{if } z_t = 0 \end{cases}$$

Investment, of course, depends on productivity and assets in period $t+1$ since these factors determine the amount of capital a firm uses in period $t+1$. Since assets in period $t+1$ are functions of assets and productivity in period t we may define investment as functions only of a_t, z_t and z_{t+1}. We have

$$X_t(a_t, z_t, z_{t+1}) = \begin{cases} a_t\lambda\left[\delta + \frac{1-\pi}{\pi}(1-\beta(1+r))\right] & \text{if } z_t = \bar{z}, z_{t+1} = \bar{z} \\ -a_t(1-\delta)\lambda & \text{if } z_t = \bar{z}, z_{t+1} = 0 \\ a_t\lambda\beta(1+r) & \text{if } z_t = 0, z_{t+1} = \bar{z} \\ 0 & \text{if } z_t = 0, z_{t+1} = 0 \end{cases}.$$

To aid us in defining external financing, it is useful to define the amount of excess available funds a firm has for investment. Define $AF - X$ for each firm:

$$AF - X = \begin{cases} a_t\left[r + (1-\beta(1+r))\left[\frac{1-\beta\lambda(1-\pi)}{\beta\pi}\right]\right] & \text{if } z_t = \bar{z}, z_{t+1} = \bar{z} \\ a_t\left[\frac{1-\beta(1+r)}{\beta\pi} + \lambda + r\right] & \text{if } z_t = \bar{z}, z_{t+1} = 0 \\ a_t\left[r(1-\lambda\beta) - \lambda\beta\right] & \text{if } z_t = 0, z_{t+1} = \bar{z} \\ a_t r & \text{if } z_t = 0, z_{t+1} = 0 \end{cases}$$

We use this expression to get a sense of which firms are likely to rely on external financing. Clearly unproductive firms in period $t+1$ will not typically rely on outside funds since both firms have 0 or negative investment. Typically, the firm that switches from unproductive in period t to productive in period $t+1$ ($z_t = 0, z_{t+1} = \bar{z}$) will rely on outside funds since that firm has low available funds in period t but a high amount of investment (when λ is sufficiently large). Finally the firm that is productive in two consecutive periods ($z_t = \bar{z}, z_{t+1} = \bar{z}$) will typically not rely on external funds since that firm's available funds are large in period t, however this is sensitive to the choice of λ since, as λ becomes large, even though the firm has high available funds, the amount the firm invests grows as well.

Before turning to the effects of changes in λ, we point out that in the aggregate, independent of the collateral constraint and the probability of being productive, in the aggregate firms can self finance all of their investment. To see this, notice that in the aggregate, investment is simply

$$\delta\pi\lambda\bar{A}$$

as productive firms are maintaining the capital stock, and available funds are given by

$$\bar{A}\left[r + \frac{1-\beta(1+r)}{\beta} + \pi\lambda\delta\right]$$

Thus, in the aggregate, firms can self-finance all of their investment as the aggregate excess is given by

$$\bar{A}\left[r + \frac{1-\beta(1+r)}{\beta}\right]$$

Finally, we have the aggregate debt-to-asset ratio:

$$\frac{\pi(\lambda-1)}{\pi\lambda+1-\pi}. \tag{A.5}$$

To see this final result, note that debt is just $k-a$ for firms with $k \geq a$. The only firms with $k \geq a$ are those with $z = 1$. Hence, aggregate debt is simply $\pi\bar{A}(\lambda-1)$. Total assets, however, is not simply wealth, or $\bar{A}$. Total assets are capital installed by firms with $z = 1$ and assets of firms with $z = 0$ since these firms, in effect, have claims to financial assets. Hence, total assets are given by $\pi\lambda\bar{A} + (1-\pi)\bar{A}$. Note that the aggregate debt-to-asset ratio for any π varies from 0 to 1 as λ varies from 1 to ∞.

Relating External Financing to the Importance of Financial Markets

Consider the following exercise. For any π, choose λ so that the debt-to-asset ratio is constant (same amount of aggregate debt relative to assets in every π economy). Then, consider the difference in steady state wealth when $\lambda = \lambda(\pi)$ and when $\lambda = 1$ (or when debt-to-assets falls from the constant level to 0). I do this because the metric is easier to analyze (with respect to π) than is the derivative of steady state wealth. We have the following proposition.

Proposition A.4 *Suppose* $0 < r < \frac{1}{\beta} - 1$. *Let* $\pi \in [\underline{\pi}, \bar{\pi}]$ *and define* $\lambda(\pi)$ *such that the debt-to-asset ratio in the* π*-economy with parameter* $\lambda(\pi)$ *is equal to* $\bar{B}$. *If for all* $\pi, \frac{1}{\beta} < \lambda(\pi) < \frac{1}{\beta(1-\pi)}$ *then external financing is decreasing in* π *and* $\log(\bar{A}(\lambda(\pi), \pi) - \log(\bar{A}(1), \pi)$ *is decreasing in* π. *(The result is the same for output).*

Proof

The assumptions of the proposition ensure that the only firm relying on external funds for investment is the firm switching from unproductive to productive. Formally, these assumptions place bounds on π for a given $\bar{B}$. To see this, using the definition of debt-to-assets in equation (A.5), we have that

$$\lambda(\pi) = \frac{\bar{B}}{\pi(1-\bar{B})} + 1. \tag{A.6}$$

Since $\lambda(\pi)$ is decreasing in π, we can replace the assumption on $\lambda(\pi)$ by ensuring that

$$\frac{1}{\beta} \leq \frac{\bar{B}}{\bar{\pi}(1-\bar{B})} + 1$$

and

$$\frac{\bar{B}}{\underline{\pi}(1-\bar{B})} + 1 \leq \frac{1}{\beta(1-\underline{\pi})}$$

It can be shown that these conditions are consistent with $\bar{\pi} > \underline{\pi}$.

Recall the definitions of external financing for the $(z_t = 0, z_{t+1} = \bar{z})$ and the $(z_t = \bar{z}, z_{t+1} = \bar{z})$ firms:

$$\begin{array}{ll} a_t\left[r(1-\lambda\beta) - \lambda\beta\right] & \text{if } z_t = 0, z_{t+1} = \bar{z} \\ a_t\left[r + (1-\beta(1+r))\left[\frac{1-\beta\lambda(1-\pi)}{\beta\pi}\right]\right] & \text{if } z_t = \bar{z}, z_{t+1} = \bar{z} \end{array}$$

Since $0 \geq 1 - \lambda\beta$, it must be that firm switching from unproductive to productive $(z_t = 0, z_{t+1} = \bar{z})$ uses external funds for investment and since $1 - \beta\lambda(\pi)(1-\pi) \geq 0$ the firm that is productive for two consecutive periods does not use external funds for investment. Therefore, our statistic on the amount of external funds used for investment satisfies

$$\begin{aligned} & \frac{\pi(1-\pi)\left[\beta(1+r)\lambda - r\right]}{\pi\delta\lambda} \\ = \; & \frac{(1-\pi)}{\delta}\left[\beta(1+r) - \frac{r}{\lambda}\right] \end{aligned}$$

Using the definition of λ in (A.6), we have

$$\frac{1}{\lambda} = \frac{\pi(1-\bar{B})}{\bar{B}+\pi(1-\bar{B})}$$

so that our external financing statistic satisfies

$$\begin{aligned}
&\frac{(1-\pi)}{\delta}\left[\beta(1+r) - \frac{r\pi(1-\bar{B})}{\bar{B}+\pi(1-\bar{B})}\right] \\
= &\frac{(1-\pi)}{\delta}\left[\beta(1+r) - \frac{r}{\frac{\bar{B}}{\pi(1-\bar{B})}+1}\right]
\end{aligned}$$

As a result, we immediately see that our statistic is decreasing in π. Consider now the definition of steady state wealth. We have

$$\bar{A}(\lambda,\pi) = \left[\frac{\alpha\beta\,(\pi\lambda)^{\alpha}}{1-\beta(1+r)+\beta(r+\delta)\pi\lambda}\right]^{\frac{1}{1-\alpha}}$$

and

$$\bar{A}(1,\pi) = \left[\frac{\alpha\beta\pi^{\alpha}}{1-\beta(1+r)+\beta(r+\delta)\pi}\right]^{\frac{1}{1-\alpha}}$$

Then (recalling that $\pi\lambda = \frac{\bar{B}}{1-\bar{B}} + \pi$)

$$\begin{aligned}
&(1-\alpha)\left[\log(\bar{A}(\lambda(\pi),\pi)) - \log(\bar{A}(1,\pi))\right] \\
= \;&\alpha\log\left(1+\frac{\bar{B}}{(1-\bar{B})}\frac{1}{\pi}\right) - \log\left(1+\frac{\bar{B}}{1-\bar{B}}\frac{1}{\frac{1-\beta(1+r)}{\beta(r+\delta)}+\pi}\right)
\end{aligned}$$

Analyzing this equation, let $c = \frac{\bar{B}}{1-\bar{B}}, d = \frac{1-\beta(1+r)}{\beta(r+\delta)}$. Then I claim that

$$f(\pi) = \log\left(1+c\pi^{-1}\right) - \log(1+c(d+\pi)^{-1})$$

is decreasing in π. To see this, we have

$$\begin{aligned}
f'(\pi) &= \frac{-c\pi^{-2}}{1+c\pi^{-1}} - \frac{-c(d+\pi)^{-2}}{1+c(d+\pi)^{-1}} \\
&= c\left[(d+\pi)^{-2}(1+c(d+\pi)^{-1})^{-1} - \pi^{-2}(1+c\pi^{-1})^{-1}\right]
\end{aligned}$$

And we must have

$$(d+\pi)^{-2}(1+c(d+\pi)^{-1})^{-1} \leq \pi^{-2}(1+c\pi^{-1})^{-1}$$

since

$$\begin{aligned} \pi^2 + c\pi &\leq (d+\pi)^2 + c\,(d+\pi) \\ 0 &\leq d^2 + 2d\pi + cd \end{aligned}$$

And we have that $\beta(1+r) \leq 1$ so that $d \geq 0$ and $c \geq 0$.
Q.E.D

As for output, we have that

$$\begin{aligned} Y(\pi,\lambda) &= \left(\frac{1-\alpha}{w}\right)^{\frac{1-\alpha}{\alpha}} \pi\lambda\bar{A}(\pi,\lambda) \\ &= \left(\pi\lambda\bar{A}(\pi,\lambda)\right)^{\alpha} \\ &= \left[\frac{\alpha\beta\pi\lambda}{1-\beta(1+r)+\beta(r+\delta)\pi\lambda}\right]^{\frac{\alpha}{1-\alpha}} \\ &= \left[\frac{\alpha\beta}{(1-\beta(1+r))\,(\pi\lambda)^{-1}+\beta(r+\delta)}\right]^{\frac{\alpha}{1-\alpha}} \end{aligned}$$

Then

$$\begin{aligned} &\frac{1-\alpha}{\alpha}\left[\log Y(\pi,\lambda(\pi)) - \log(Y(\pi,1))\right] \\ = \ &\log\left(\frac{1}{\pi}(1-\beta(1+r)+\beta(r+\delta)\right) - \log\left(\frac{1}{\pi\lambda}(1-\beta(1+r))+\beta(r+\delta)\right) \\ = \ &\log\left(\frac{1}{\pi}c_1+c_2\right) - \log\left(\frac{1}{\bar{c}+\pi}c_1+c_2\right) \end{aligned}$$

Differentiating with respect to π we have

$$\begin{aligned} & \frac{-c_1\pi^{-2}}{\pi^{-1}c_1 + c_2} - \frac{-c_1(\bar{c}+\pi)^{-2}}{(\bar{c}+\pi)^{-1}c_1 + c_2} \\ = & \; c_1 \left[\frac{1}{(\bar{c}+\pi)\, c_1 + (\bar{c}+\pi)^2 c_2} - \frac{1}{\pi c_1 + \pi^2 c_2} \right] \end{aligned}$$

which must be negative.

Derivations for External Financing

- Available Funds:

  - If $z_t = 1$ then

$$\begin{aligned} AF_t &= \alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} k_t - r(k_t - a_t) \\ &= a_t \left[\lambda\alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} - r(\lambda - 1) \right] \\ &= a_t \left[\frac{1-\beta(1+r)}{\beta\pi} + \lambda(r+\delta) - r\lambda + r \right] \\ &= a_t \left[\frac{1-\beta(1+r)}{\beta\pi} + \lambda\delta + r \right] \end{aligned}$$

  If $z_t = 0$ then

$$AF_t = -rb_t = -r(-a_t) = ra_t$$

- Investment

– If $z_t = z_{t+1} = \bar{z}$,

$$\begin{aligned} X_t &= \lambda a_{t+1} - (1-\delta)\lambda a_t \\ &= \lambda\beta \left\{ \lambda \left[\alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} - (r+\delta) \right] + (1+r) \right\} a_t - (1-\delta)\lambda a_t \\ &= \lambda a_t \left[\beta\lambda\alpha \left(\frac{1-\alpha}{w} \right)^{\frac{1-\alpha}{\alpha}} - \beta\lambda(r+\delta) + \beta(1+r) - (1-\delta) \right] \\ &= \lambda a_t \left[\frac{1-\beta(1+r)}{\pi} + \beta\lambda(r+\delta) - \beta\lambda(r+\delta) + \beta(1+r) - (1-\delta) \right] \\ &= \lambda a_t \left[\delta + \frac{1-\pi}{\pi}(1-\beta(1+r)) \right] \end{aligned}$$

– If $z_t = \bar{z}, z_{t+1} = 0$,

$$X_t = -(1-\delta)\lambda a_t$$

– If $z_t = 0, z_{t+1} = \bar{z}$,

$$X_t = \lambda a_{t+1} = \lambda\beta(1+r)a_t$$

– If $z_t = z_{t+1} = 0, X_t = 0$.

- AF - X

– If $z_t = z_{t+1} = 0$,

$$AF_t - X_t = ra_t$$

– If $z_t = 0, z_{t+1} = \bar{z}$,

$$\begin{aligned} AF_t - X_t &= ra_t - \lambda\beta(1+r)a_t \\ &= a_t \left[r - \lambda\beta(1+r) \right] \end{aligned}$$

– If $z_t = \bar{z}, z_{t+1} = 0$,

$$\begin{aligned} AF_t - X_t &= a_t \left[\frac{1-\beta(1+r)}{\beta\pi} + \lambda\delta + r + (1-\delta)\lambda \right] \\ &= a_t \left[\frac{1-\beta(1+r)}{\beta\pi} + \lambda + r \right] \end{aligned}$$

– If $z_t = z_{t+1} = \bar{z}$,

$$\begin{aligned} AF_t - X_t &= a_t \left[\frac{1-\beta(1+r)}{\beta\pi} + \lambda\delta + r - \lambda \left[\delta + \frac{1-\pi}{\pi}(1-\beta(1+r)) \right] \right] \\ &= a_t \left[r + (1-\beta(1+r)) \left[\frac{1-\beta\lambda(1-\pi)}{\beta\pi} \right] \right] \end{aligned}$$

A.3 Other Proofs

Proof of Lemma (2.2), Proposition (2.3) and Corollary (2.4).

Publicly held firms solve

$$\begin{aligned}
&\max E_0 \sum_t \beta^t d_t \\
&d_t + a_{t+1} \leq f(z_t, k_t) + (1+r)a_t \\
&k_t \leq \lambda_t a_t \\
&d_t \geq 0
\end{aligned}$$

or

$$\max E_0 \sum_t \beta^t \left[(f_t(z_t, k_t) + (1+r)a_t - a_{t+1})(1 + \eta(z^t)) + \mu(z^t)\left(\lambda a_t - k_t\right)\right]$$

FOCs

$$\begin{aligned}
f'(z_t, k_t) &= \mu(z^t) \\
(1 + \eta(z^t)) &= \beta E_t \left[(1+r)\left(1 + \eta(z_{t+1}, z^t)\right) + \mu(z_{t+1}, z^t)\lambda \right]
\end{aligned}$$

where the last can be re-written using $\beta(1+r) = 1$:

$$\eta(z^t) = \beta E_t \left[(1+r)\eta(z_{t+1}, z^t) + \mu(z_{t+1}, z^t)\lambda \right]$$

Thus, if $\eta(z_{t+1}, z^t)$ or $\mu(z_{t+1}, z^t)$ are positive for any z_{t+1} with strictly positive probability following z^t then $\eta(z^t) = 0$ and

$$a_{t+1}(z^t) = f(z_t, k_t) + (1+r)a_t$$

Since $f \geq 0$ and $1 + r > 1$ we have that assets grow as long as constraints ever bind in the future.

Now, optimal unconstrained capital scale satisfies

$$f'(z_t, k_t) = 0.$$

Let the optimal scale be defined as

$$k^*(z) = \left(f'\right)^{-1}(0, z).$$

We can easily show that $k^*(z)$ is increasing in z. Suppose $z \in [\underline{z}, \bar{z}]$. Then, if $a_t \geq \lambda k^*(\bar{z})$, the constraint does not bind currently and choosing any $a_{t+1} \geq a_t$ will ensure that the constraint never binds again in the future. Suppose at time s the firm's assets satisfy $a_s \geq \lambda k^*(z)$. In this case, the firm solves

$$\max E_s \sum_{t \geq s} \beta^{t-s} \left[(f_t(z_t, k_t) + (1+r)a_t - a_{t+1}) \right]$$

and since $\beta(1+r) = 1$ this simplifies to

$$E_s \sum_{t \geq s} \beta^{t-s} \left[(f_t(z_t, k_t) \right] + a_t(1+r)$$

Note then, in steady state that no publicly held firms will be constrained (assets are increasing until the constraint never binds and then are indeterminate above $\bar{a}$). Then, investment for any firm is simply

$$k^*(z_{t+1}) - (1-\delta)k^*(z_t)$$

Thus, investment is bounded above by

$$k^*(\bar{z}) - (1-\delta)k^*(\underline{z}).$$

However, available funds, in steady state, are not pinned down since they are equal to

$$f(z_t, k^*(z_t)) - r(k^*(z_t) - a_t)$$

Since the firm is indifferent between any $a_t \geq \bar{a}$, there are a continuum of equilibria with different steady state asset holdings of publicly held firms each corresponding to different amounts of external financing.

Optimal Production and Savings Decisions of Privately Held Firms

In this appendix, we provide a characterization of the privately held firm's optimal capital, labor, and savings decisions. The problem of a surviving privately held firm can be written recursively as

$$
\begin{aligned}
&V_u(a,z) = \max_{c,a',k,l} u(c) + \beta\zeta E[V_u(a',z')|z] \\
&\text{s.t.} \\
&c + a' \leq p(a,z) z^{\frac{1}{\rho-1}} \left(k^{\alpha} l^{1-\alpha}\right)^{\eta} I^{1-\eta} - wl - (r+\delta)k - I + (1+r)a \\
&p(a,z) = Q^{\frac{1}{\rho}} \left(z^{\frac{1}{\rho-1}} \left(k^{\alpha} l^{1-\alpha}\right)^{\eta} I^{1-\eta}\right)^{-\frac{1}{\rho}} \\
&k \leq \lambda a \\
&a_0 \text{ given}
\end{aligned}
$$

The decision of how much capital to install and how much labor to hire is a static one. We therefore can use the results from our static model, namely Lemma (A.1) to define profits of a privately held firm, which we denote $\Pi_u(a,z;w,r,Q)$. Then the problem of an unlisted firm can be simplified to a consumption and savings problem written as

$$
\begin{aligned}
V^u(a,z) &= \max_{c,a'} u(c) + \beta\zeta E\left[V^u(a',z')|z\right] \\
& \quad \text{s.t.} \\
c + a' &\leq \Pi_u\left(a,z;w,r,Q\right) + (1+r)a
\end{aligned}
$$

As in [Aiyagari, 1994], the only intertemporal effect of the borrowing constraint comes from distorting the savings decisions. The nature of the borrowing constraint ensures that this happens in a smooth way. Thus, the optimal savings decision can be solved as in [Aiyagari, 1994], except for the extra term that captures the effect of savings on the next period's profit function. We then have the following proposition, in which we suppress the dependence of Π_u on equilibrium parameters w, r, and Q and write it only as $\Pi_u(a,z)$.

Lemma A.5 *For an unlisted firm, the optimal asset position policy is given by a function $a'(a,z)$ that satisfies*

$$\frac{1}{\beta\zeta}u'(\Pi(a,z)+(1+r)a-a'(a,z))=$$
$$E\left\{\left[\Pi_a^U(a'(a,z),z')+(1+r)\right]u'(\Pi(a'(a,z),z')+(1+r)a'(a,z)-a'(a'(a,z),z'))|z\right\}$$

A.4 Figures

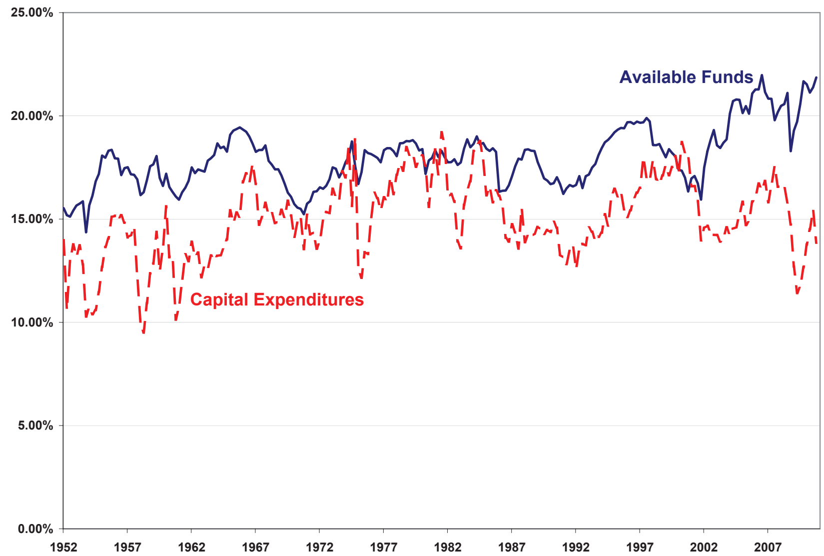


Figure A.1: Available Funds and Capital Expenditures normalized by Nonfinancial Corporate Business GDP, U.S. Nonfarm Nonfinancial Corporate Businesses. Source: U.S. Flow of Funds and BEA.

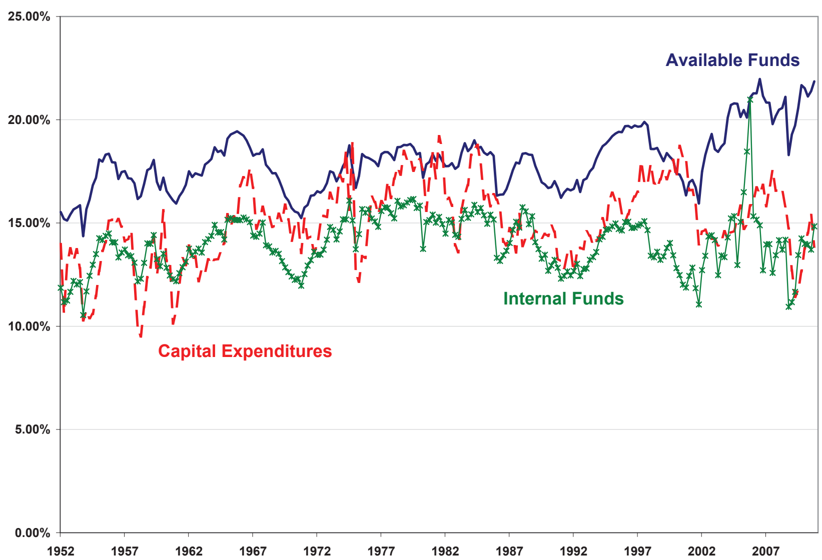


Figure A.2: Available Funds, Internal Funds and Capital Expenditures normalized by Nonfinancial Corporate Business GDP, U.S. Nonfarm Nonfinancial Corporate Businesses. Source: U.S. Flow of Funds and BEA.

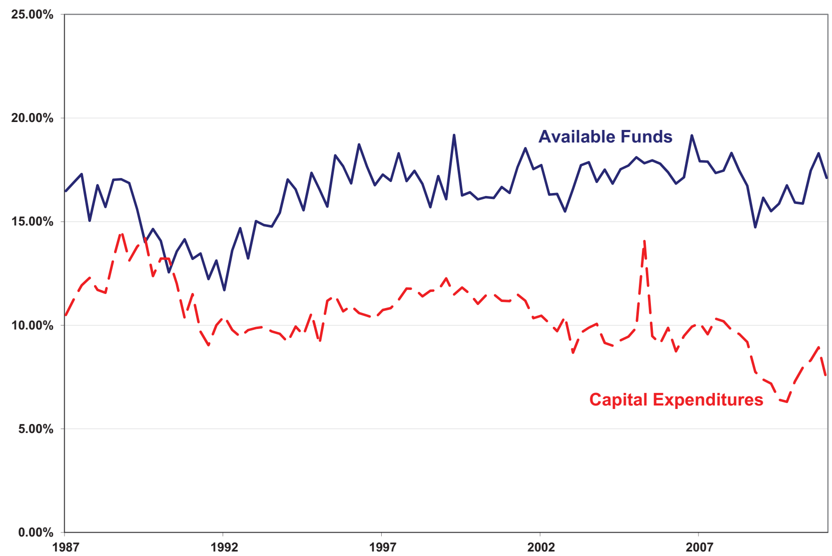


Figure A.3: Available Funds and Capital Expenditures normalized by GDP, U.K. Nonfarm Nonfinancial Corporate Businesses. Source: U.K. National Accounts.

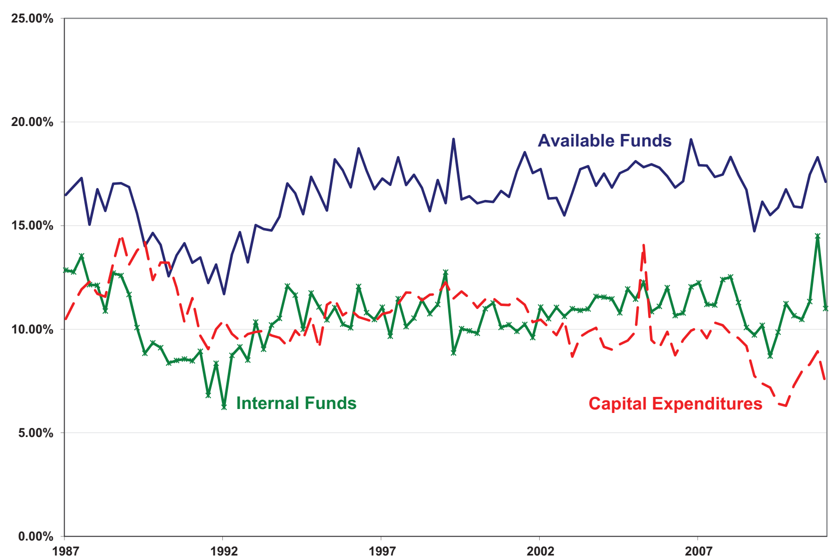


Figure A.4: Available Funds, Internal Funds and Capital Expenditures normalized by GDP, U.K. Nonfarm Nonfinancial Corporate Businesses. Source: U.K. National Accounts.

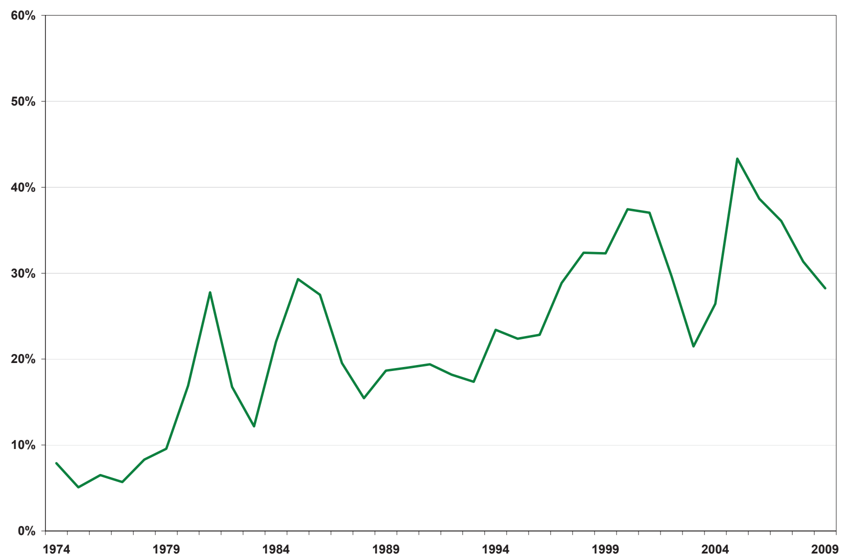


Figure A.5: Fraction of Investment by Compustat U.S. Firms Financed by External Funds. Source: Compustat and Authors' calculations.

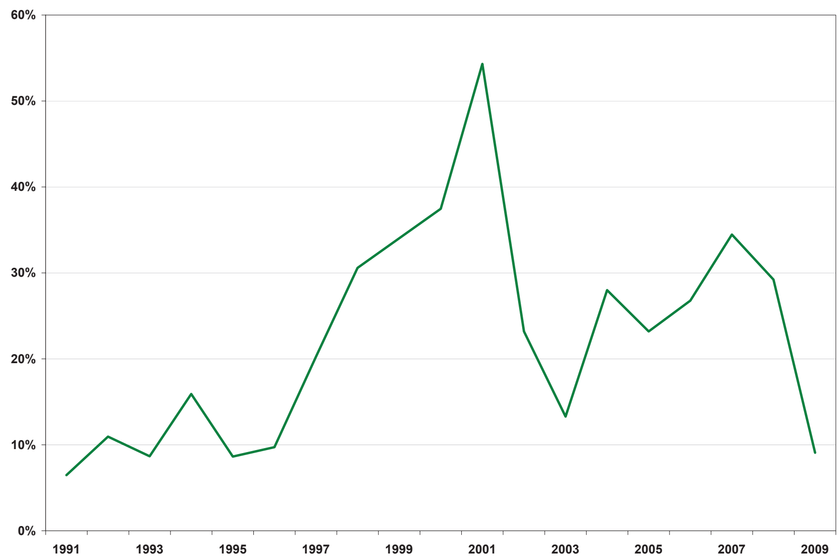


Figure A.6: Fraction of Investment by Compustat U.K. Firms Financed by External Funds. Source: Compustat Global and Authors' calculations.

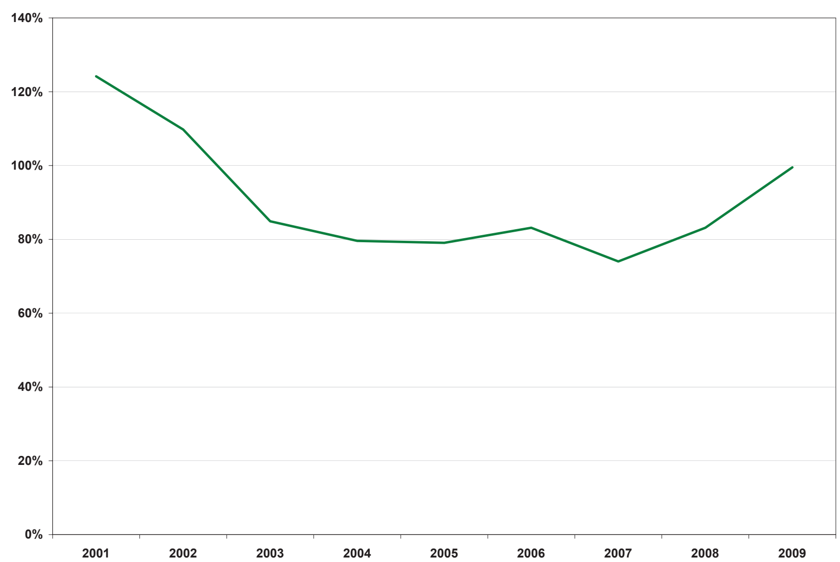


Figure A.7: Fraction of Investment by Privately Held Firms in the U.K. Financed by External Funds. Source: Amadeus.

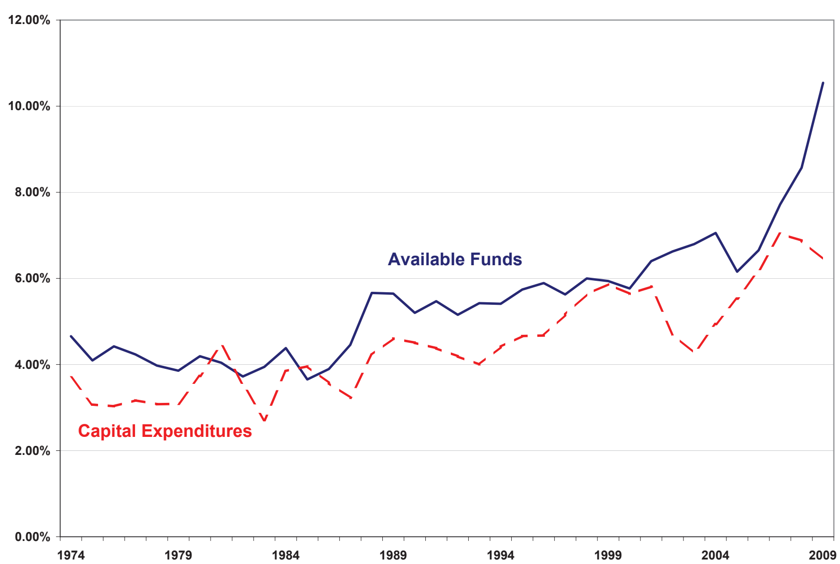


Figure A.8: Available Funds and Investment of Compustat U.S. Firms normalized by U.S. Non-farm Non-financial Corporate Business GDP. Source: Compustat and BEA.

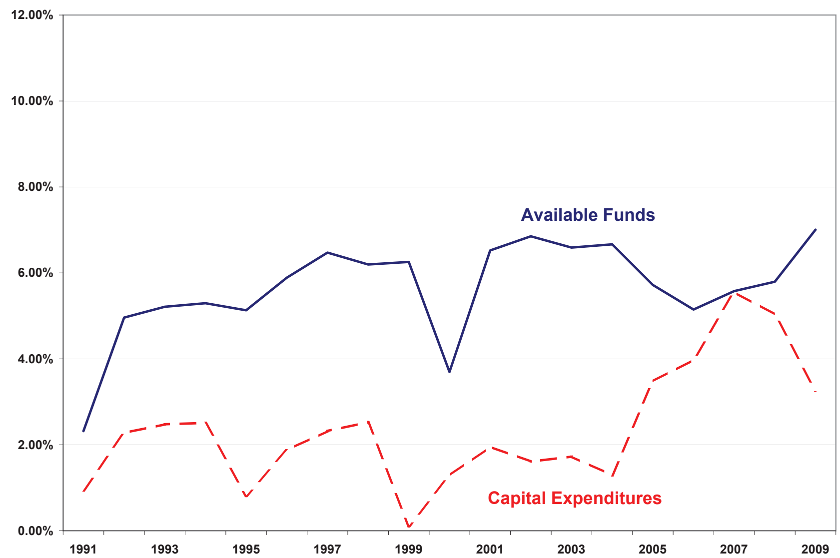


Figure A.9: Available Funds and Investment of Compustat U.K. Firms normalized by GDP. Source: Compustat Global and U.K. National Accounts.

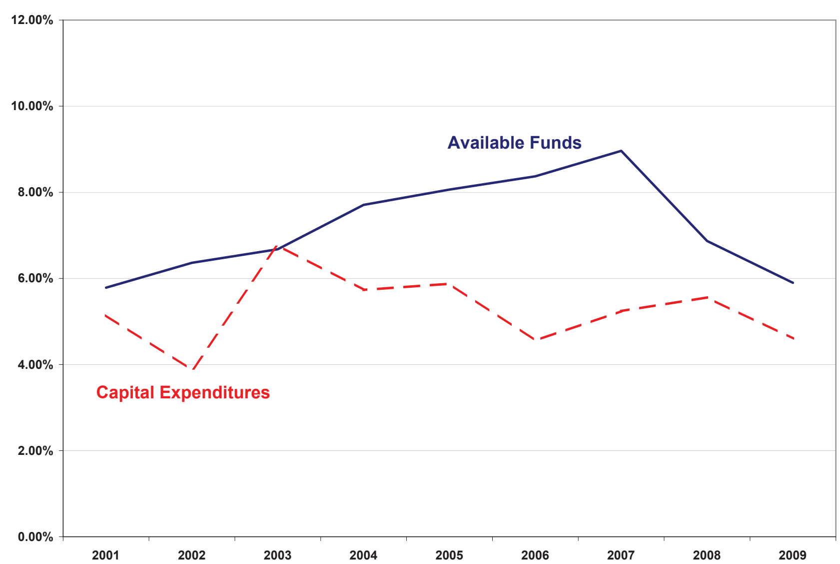


Figure A.10: Available Funds and Investment of Privately Held Firms in the U.K. normalized by Aggregate Sales of such firms. Source: Amadeus.

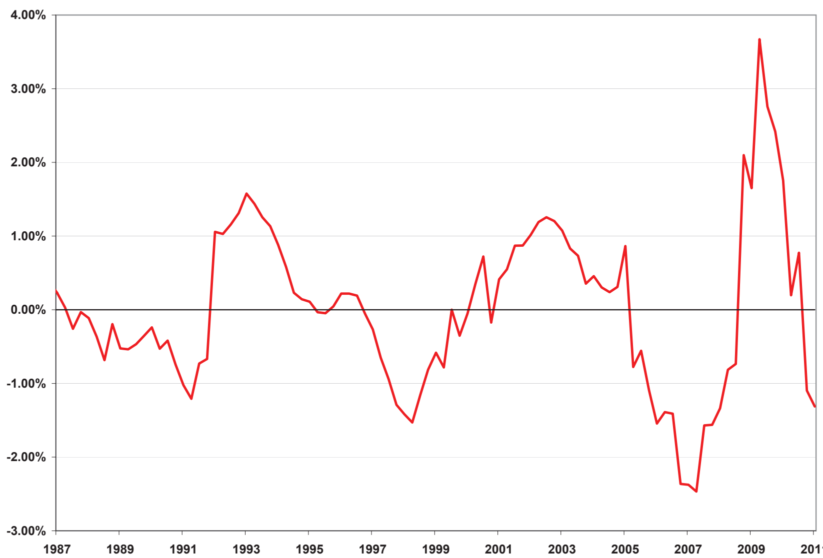


Figure A.11: Share of Gross Output Accounted for by Privately Held Firms. Source: Compustat, BEA, and Authors' calculations.

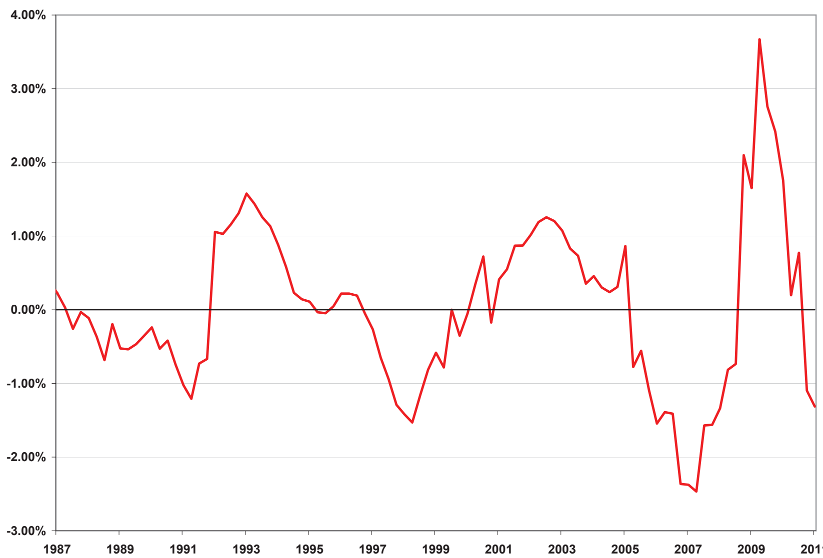


Figure A.12: Aggregate Debt-to-Assets of U.S. Nonfarm Nonfinancial Corporate Businesses, deviations from HP-filtered trend. Source: U.S. Flow of Funds.

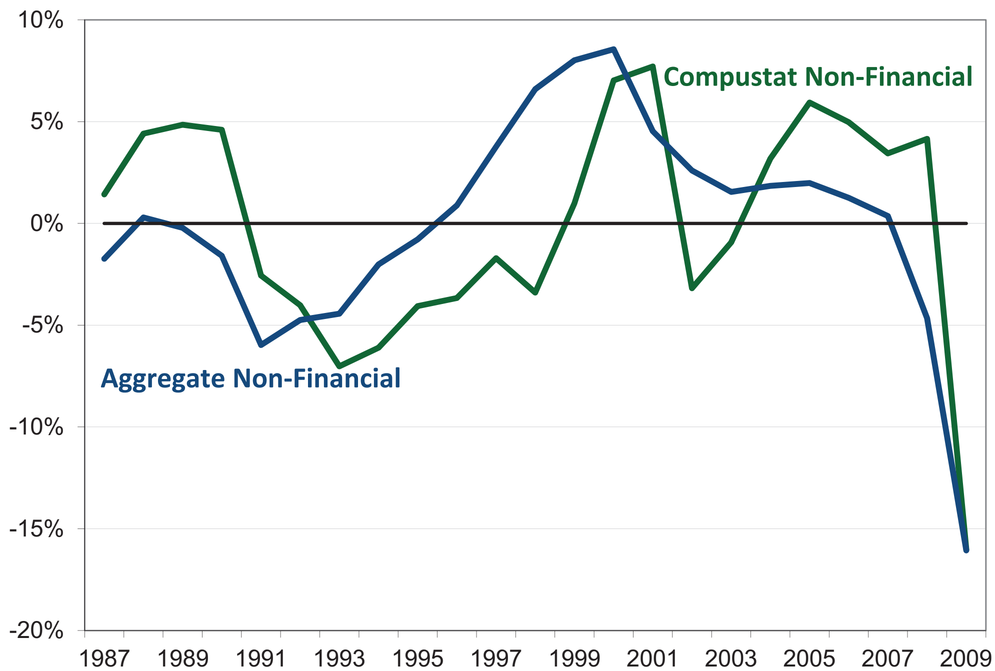


Figure A.13: Gross Output of all Non-Financial Firms and all Non-Financial Firms in Compustat. Source: U.S. BEA and Compustat

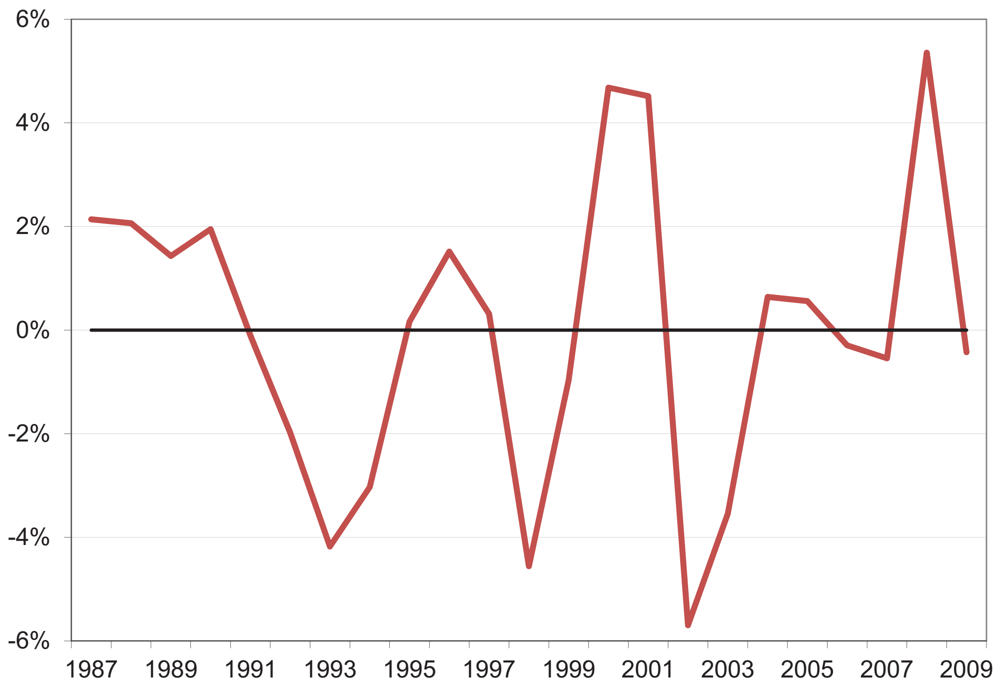


Figure A.14: Share of all Non-Financial Gross Output Accounted for by all Non-Financial Firms in Compustat, devaitions from a linear trend Source: U.S. BEA and Compustat.

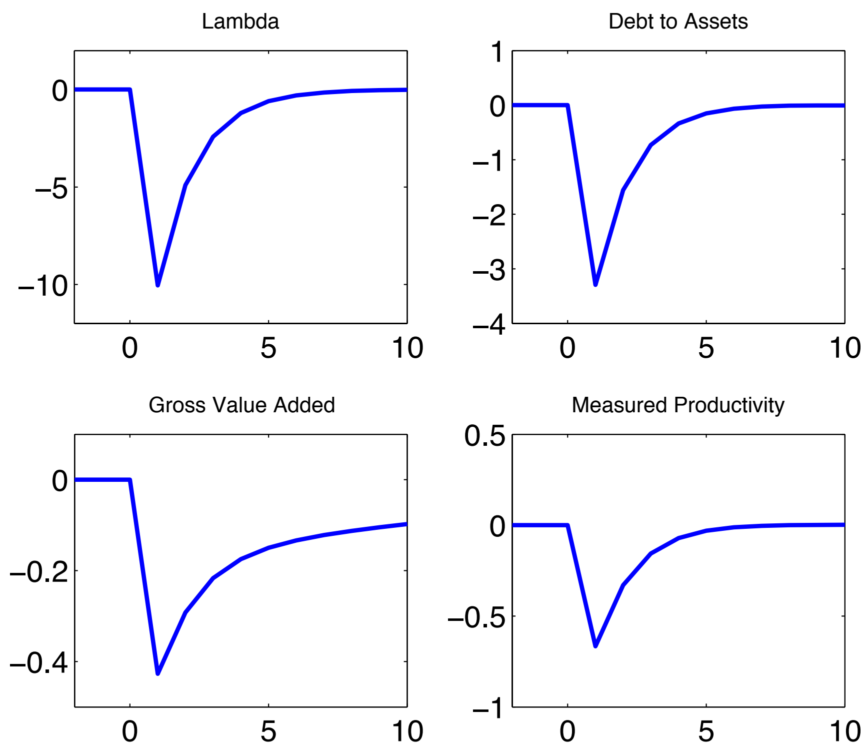


Figure A.15: Impulse Responses to Negative Shock to Collateral Constraint. (The x-axis of each plot measures time in years; the y-axis of each plot measures percentage point deviations from the steady state.)

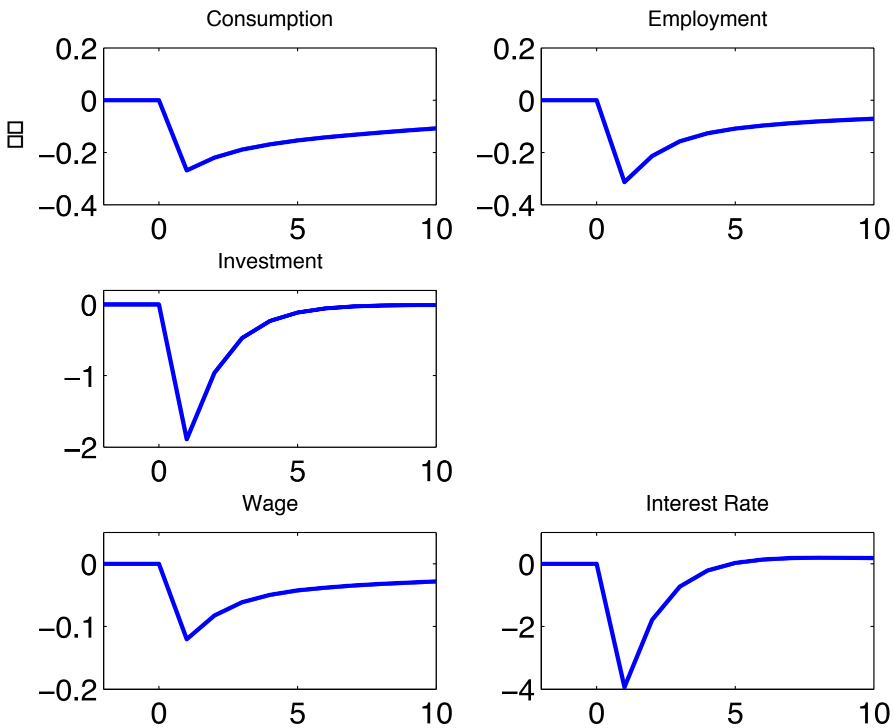


Figure A.16: Impulse Responses to Negative Shock to Collateral Constraint. (The x-axis of each plot measures time in years; the y-axis of each plot measures percentage point deviations from the steady state.)

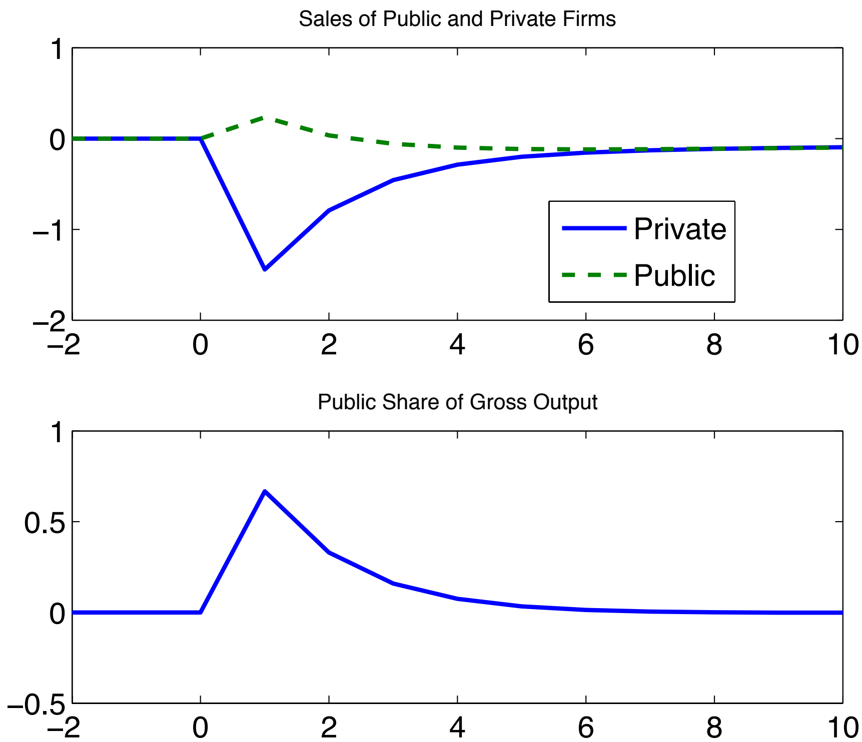


Figure A.17: Impulse Responses to Negative Shock to Collateral Constraint. (The x-axis of each plot measures time in years; the y-axis of each plot measures percentage point deviations from the steady state.)

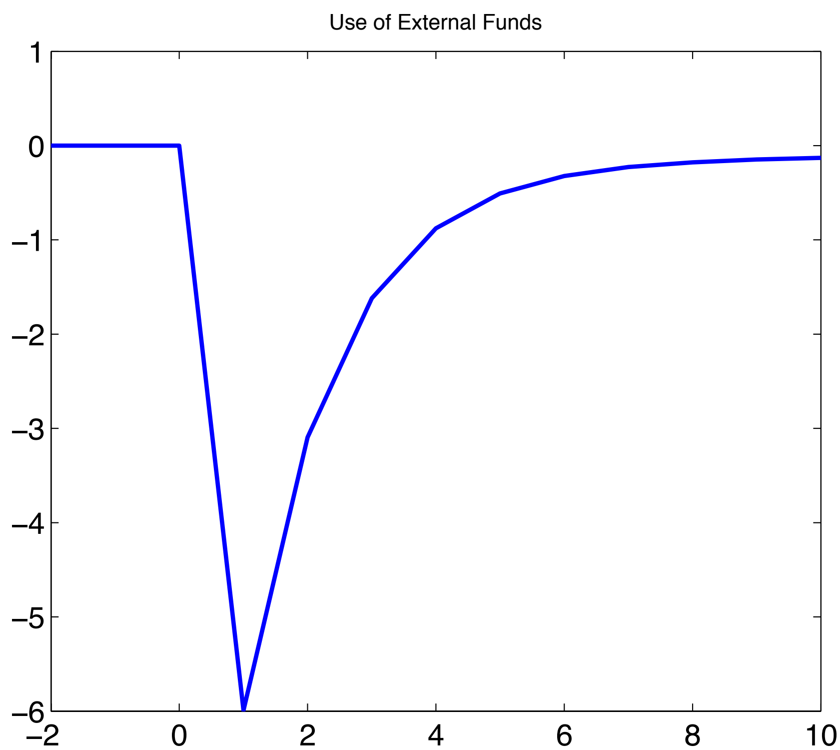


Figure A.18: Impulse Responses to Negative Shock to Collateral Constraint. (The x-axis of each plot measures time in years; the y-axis of each plot measures percentage point deviations from the steady state.)

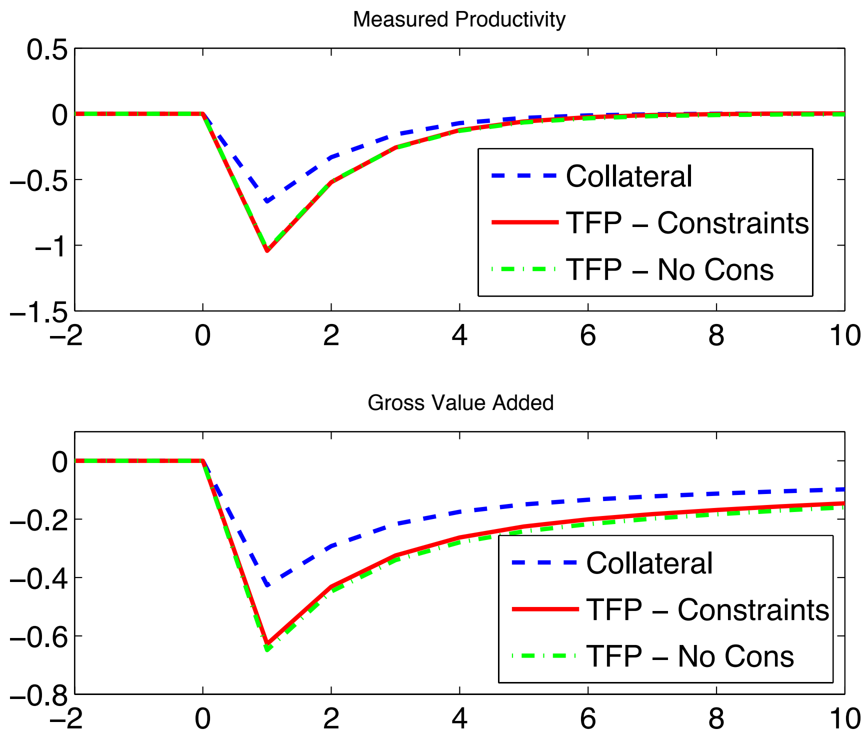


Figure A.19: Impulse Responses to Negative Aggregate Productivity Shock. (The x-axis of each plot measures time in years; the y-axis of each plot measures percentage point deviations from the steady state.)

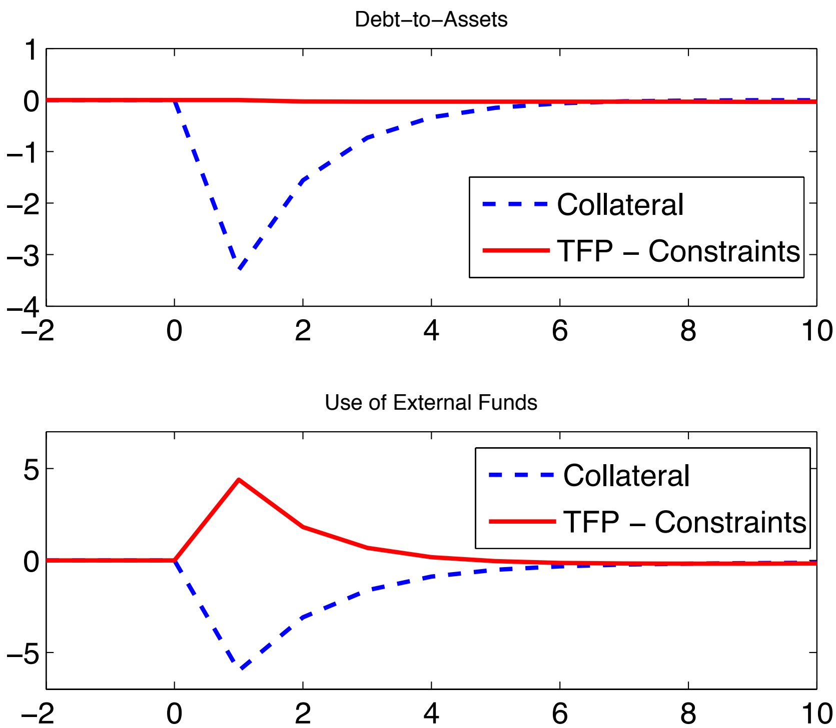


Figure A.20: Impulse Responses to Negative Aggregate Productivity Shock. (The x-axis of each plot measures time in years; the y-axis of each plot measures percentage point deviations from the steady state.)

Appendix B

Appendix to Chapter 3

B.1 Proofs

Proof of Lemma 3.10

Define

$$\begin{aligned} \zeta_i(v_i) &= \int_{v_{-i}} x(v_i, v_{-i}) t_2^i(v_i, v_{-i}) G_{-i}(dv_{-i}) \\ \rho_i(v_i) &= \int_{v_{-i}} x(v_i, v_{-i}) G_{-i}(dv_{-i}) \end{aligned}$$

Then

$$u_i(v_i) = -\frac{I}{N}\rho_i(v_i) + v_i \zeta_i(v_i)$$

or

$$\begin{aligned} \frac{1}{v_i} u_i(v_i) &= -\frac{I}{N v_i}\rho_i(v_i) + \zeta_i(v_i) \\ &\geq \frac{-I}{N v_i}\rho_i(\hat{v}_i) + \zeta_i(\hat{v}_i) \\ &= \frac{-I}{N v_i}\rho_i(\hat{v}_i) + \zeta_i(\hat{v}_i) + \frac{I}{N \hat{v}_i}\rho_i(\hat{v}_i) - \frac{I}{N \hat{v}_i}\rho_i(\hat{v}_i) \\ &= \frac{1}{\hat{v}_i} u_i(\hat{v}_i) + \rho_i(\hat{v}_i)\frac{I}{N}\left[\frac{1}{\hat{v}_i} - \frac{1}{v_i}\right] \end{aligned}$$

So

$$\begin{aligned}
\frac{1}{v_i}u_i(v_i) &\geq \frac{1}{\hat{v}_i}u_i(\hat{v}_i) + \rho_i(\hat{v}_i)\frac{I}{N}\left[\frac{1}{\hat{v}_i} - \frac{1}{v_i}\right] \\
\frac{1}{\hat{v}_i}u_i(\hat{v}_i) &\geq \frac{1}{v_i}u_i(v_i) + \rho_i(v_i)\frac{I}{N}\left[\frac{1}{v_i} - \frac{1}{\hat{v}_i}\right]
\end{aligned}$$

So

$$\begin{aligned}
\frac{1}{\hat{v}_i}u_i(\hat{v}_i) + \rho_i(v_i)\frac{I}{N}\left[\frac{1}{\hat{v}_i} - \frac{1}{v_i}\right] &\geq \frac{1}{v_i}u_i(v_i) \geq \frac{1}{\hat{v}_i}u_i(\hat{v}_i) + \rho_i(\hat{v}_i)\frac{I}{N}\left[\frac{1}{\hat{v}_i} - \frac{1}{v_i}\right] \\
\rho_i(v_i)\frac{I}{N}\left[\frac{1}{\hat{v}_i} - \frac{1}{v_i}\right] &\geq \frac{1}{v_i}u_i(v_i) - \frac{1}{\hat{v}_i}u_i(\hat{v}_i) \geq \rho_i(\hat{v}_i)\frac{I}{N}\left[\frac{1}{\hat{v}_i} - \frac{1}{v_i}\right] \\
\rho_i(v_i)\frac{I}{N}\frac{v_i - \hat{v}_i}{\hat{v}_i v_i} &\geq \frac{1}{v_i}u_i(v_i) - \frac{1}{\hat{v}_i}u_i(\hat{v}_i) \geq \rho_i(\hat{v}_i)\frac{I}{N}\frac{v_i - \hat{v}_i}{\hat{v}_i v_i} \\
\frac{\rho_i(v_i)}{\hat{v}_i v_i}\frac{I}{N} &\geq \frac{\frac{1}{v_i}u_i(v_i) - \frac{1}{\hat{v}_i}u_i(\hat{v}_i)}{v_i - \hat{v}_i} \geq \frac{\rho_i(\hat{v}_i)}{\hat{v}_i v_i}\frac{I}{N}
\end{aligned}$$

This implies that $\rho_i(v_i)$ is increasing in v_i. Taking limits, we have

$$\frac{1}{v_i^2}\rho_i(v_i)\frac{I}{N} = \frac{d}{dv_i}\left[\frac{1}{v_i}u_i(v_i)\right] = \frac{1}{v_i}u_i'(v_i) - \frac{1}{v_i^2}u_i(v_i)$$

So that

$$\frac{1}{v_i}\rho_i(v_i)\frac{I}{N} = u_i'(v_i) - \frac{1}{v_i}u_i(v_i)$$

Then, solving the differential equation we have

$$u_i(v_i) = v_i\left[\frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N}\int_{\underline{v}}^{v_i}\frac{1}{z^2}\rho_i(z)dz\right]$$

This concludes the "If" portion of the proof. The "Only if" portion follows standard arguments.

Proof of Lemma 3.11 First, suppose the constraints of the original problem are satisfied. Now, the resource constraint is

$$\sum_i \int_v x(v)t_2^i(v)G(dv) \leq C\int_v x(v)G(dv)$$

Focusing on the LHS, and using the definitions from the previous proof, we have

$$\begin{aligned}
& \int_v x(v)t_2^i(v)G(dv) \\
= & \int_{v_i} \zeta_i(v_i)G_i(dv_i) \\
= & \int_{v_i} \left[\frac{u_i(v_i)}{v_i} + \frac{I}{Nv_i}\rho_i(v_i)\right] G_i(dv_i) \\
= & \int_{v_i} \left[\frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N}\int_{\underline{v}}^{v_i} \frac{1}{z^2}\rho_i(z)dz + \frac{I}{Nv_i}\rho_i(v_i)\right] G_i(dv_i) \\
= & \frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N}\int_{v_i}\int_{\underline{v}}^{v_i} \frac{\rho_i(z)}{z^2} dz G_i(dv_i) + \frac{I}{N}\int_{v_i} \frac{\rho_i(v_i)}{v_i} G_i(dv_i) \\
= & \frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N}\left[\int_{v_i} \frac{\rho_i(v_i)}{v_i^2} dv_i - \int_{v_i} G_i(v_i)\frac{\rho_i(v_i)}{v_i^2} dv_i + \int_{v_i} \frac{\rho_i(v_i)}{v_i} G_i(dv_i)\right] \\
= & \frac{u_i(\underline{v})}{\underline{v}} + \frac{I}{N}\left[\int_{v_i} \rho_i(v_i)\left[\frac{1-G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i}\right] G_i(dv_i)\right]
\end{aligned}$$

Summing over i and combining with the resource constraint yields the desired result. The "Only if" can be demonstrated using a transfer scheme similar to that considered in [Mailath and Postlewaite, 1990].

Proof of Proposition 3.12 The general idea is to consider the following auxiliary problem:

$$\max \int_v x(v)G(dv)$$

subject to

$$\int_v x(v)\left[C - \frac{I}{N}\sum_i \left[\frac{1-G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i}\right]\right] G(dv) \geq 0$$

Then, the optimal continuation rule has the property that

$$x(v) = 1 \Leftrightarrow C \geq \frac{I}{N}\sum_i \left[\frac{1-G_i(v_i)}{v_i^2 g_i(v_i)} + \frac{1}{v_i}\right]$$

In any case, forming the lagrangian, we have that $x(v) = 1$ if and only if the condition above modified by incorporating the inverse of the lagrange multiplier on the implementability constraint. An argument from [Mailath and Postlewaite, 1990] ensures that the lagrange muliplier converges to ∞ as $N \to \infty$ so that this term vanishes in the limit.

Then, the term multiplying I, by a law of large numbers, converges to

$$E\left[\frac{1-G_i(v_i)}{v_i^2 g_i(v_i)}+\frac{1}{v_i}\right]=\frac{1}{\underline{v}}$$

Thus, as $N \to \infty$, the RHS converges to $\frac{I}{\underline{v}}$ and $\underline{v}\left(E_{\pi_h} y_2^1 - \frac{\pi_h \psi}{\beta\Delta}\right) < I$. Therefore, $x(v) \to 0$ for all v.

B.2 Figures

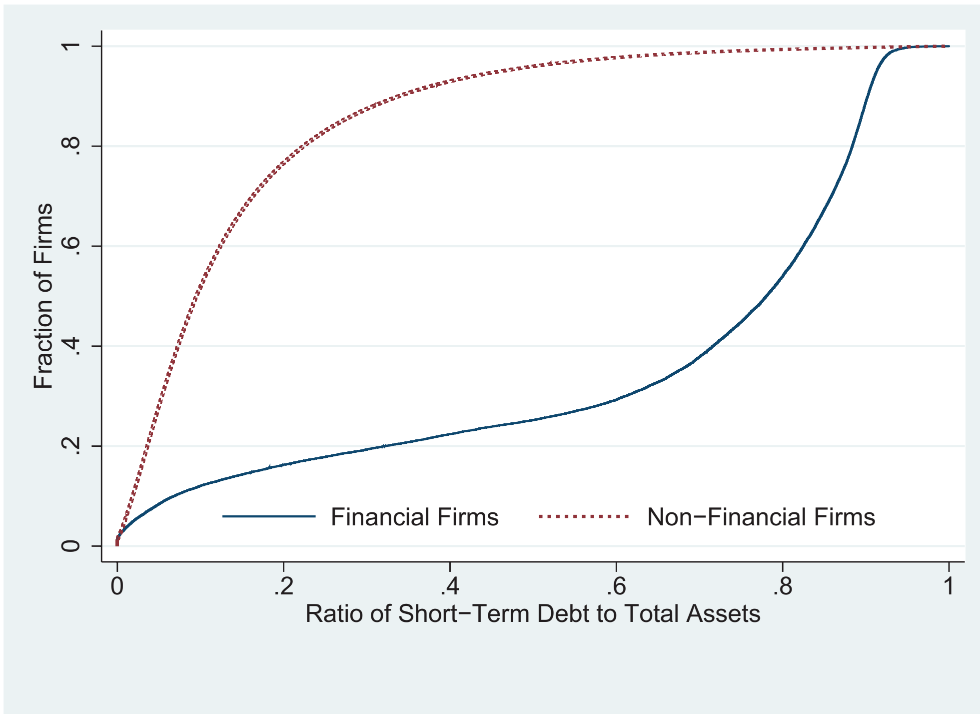


Figure B.1: Empirical Cumulative Distributions of Short-Term Debt to Total Assets for Financial and Non-Financial Firms. Source: Compustat and Author's calculations.

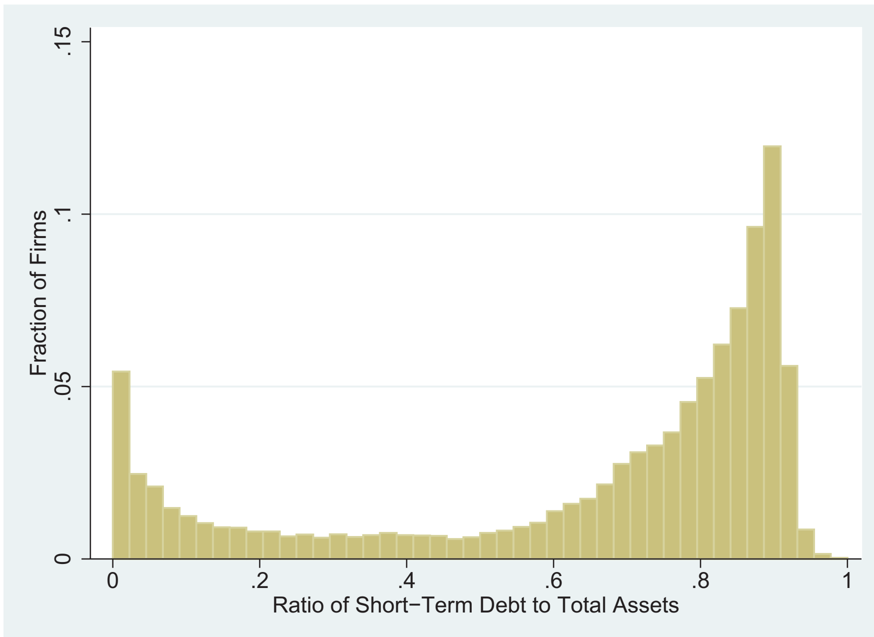


Figure B.2: Short-Term Debt to Total Assets Ratios, U.S. Financial Firms. Source: Compustat and Author's calculations.

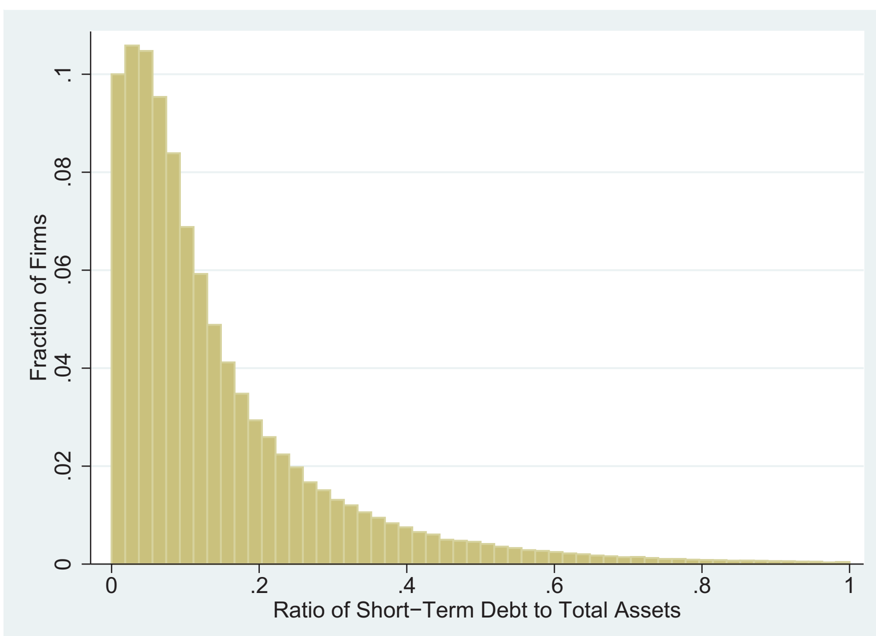


Figure B.3: Short-Term Debt to Total Assets Ratios, U.S. Non-Financial Firms. Source: Compustat and Author's calculations.

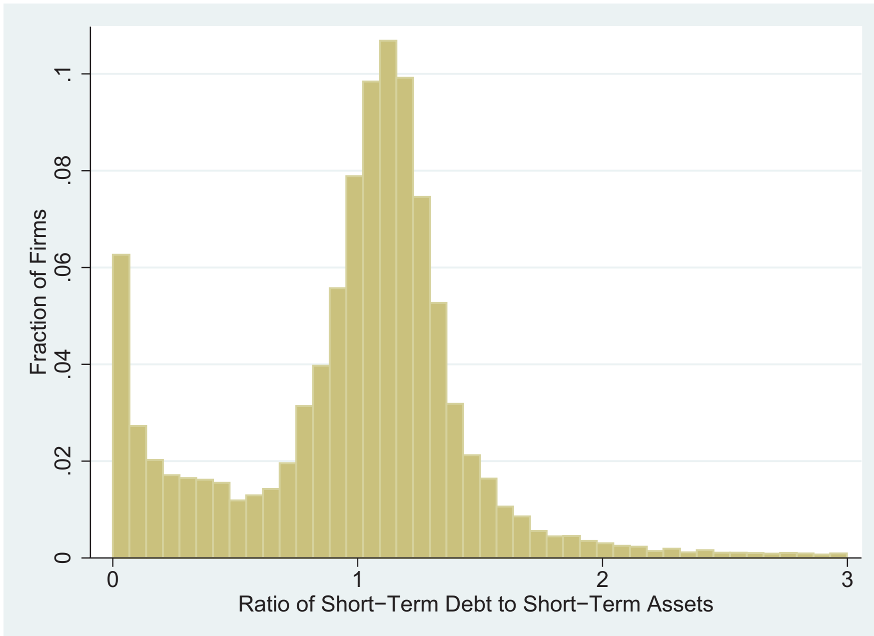


Figure B.4: Short-Term Debt to Short-Term Assets Ratios, U.S. Financial Firms. Source: Compustat and Author's calculations.

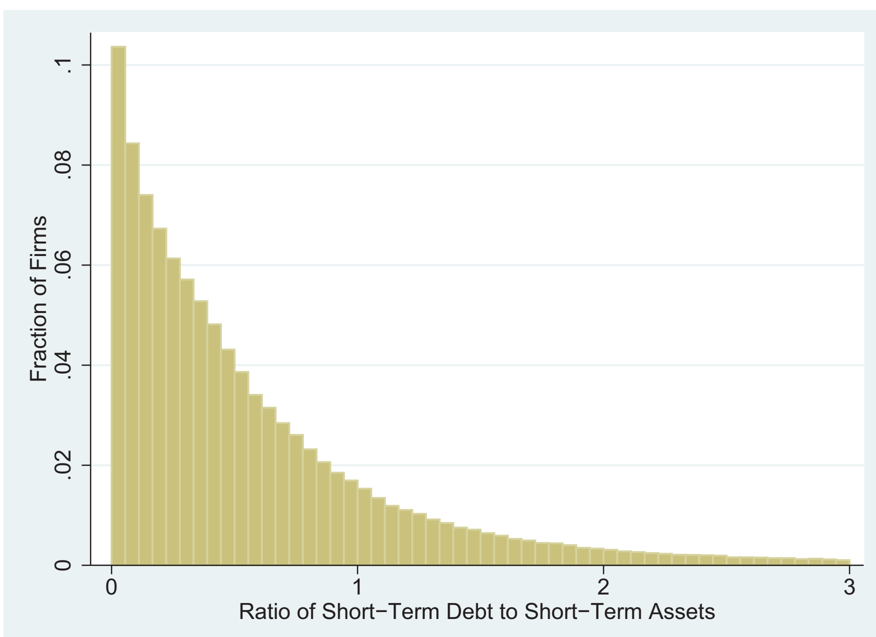


Figure B.5: Short-Term Debt to Short-Term Assets Ratios, U.S. Non-FInancial Firms. Source: Compustat and Author's calculations.

Appendix C

Appendix to Chapter 4

C.1 Proofs

C.1.1 Proof of Lemma 4.8

First, consider the set $A = \{v_1; \hat{a}_1(v_1) > \hat{a}'_1(v_1)\}$. Then

$$\int \hat{a}_1(v_1) dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right) - \int \hat{a}'_1(v_1) dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right) = \int_A dG\left(\frac{v_1 - \underline{v}_1}{\sigma}\right) \geq 0$$

with equality only if A is measure zero. Given the Bayesian updating formulas, this inequality implies that for any $\underline{v}_1$,

$$\mu_{sg}(\underline{v}_1; \hat{a}_1) \geq \mu_{sg}(\underline{v}_1; \hat{a}'_1), \mu_{sd}(\underline{v}_1; \hat{a}_1) \geq \mu_{sd}(\underline{v}_1; \hat{a}'_1), \mu_h(\underline{v}_1; \hat{a}_1) \leq \mu_h(\underline{v}_1; \hat{a}'_1)$$

with strict inequalities only if A is zero measure. Therefore, for each v_1, the integrand in (4.19) is higher for $\hat{a}_1$ and therefore $\Delta(v_1; \hat{a}_1) \geq \Delta(v_1; \hat{a}'_1)$ with equality only if A is measure zero.

Second, if $\hat{a}_1$ is a switching strategy with switching point k, from (4.18) it is straightforward to see that $\mu_{sg}(\underline{v}_1; \hat{a}_1), \mu_{sd}(\underline{v}_1; \hat{a}_1)$ are strictly increasing and $\mu_h(\underline{v}_1; \hat{a}_1)$ is strictly decreasing in $\underline{v}_1$. Thus the integrand in (4.19) is increasing in $\underline{v}_1$. Since we have assumed that $H(\hat{v}_1|v_1)$ is decreasing in v_1, from first-order stochastic dominance, it follows that $\Delta(v_1; \hat{a}_1)$ is strictly increasing.

Finally, to show boundedness, we first show that for all μ_2, $V_2(\mu_2)$ is well defined and continuous. Since μ_2 lies in a compact set, it follows that $V_2(\mu_2)$ is bounded. To show continuity, note that when $v_2 \geq (\mu^*)^{-1}(\mu_2)$, $V_2(\mu_2, v_2) = \hat{p}(\mu_2; v_2) - q$ and if $v_2 < (\mu^*)^{-1}(\mu_2)$, $V_2(\mu_2, v_2) = \bar{\pi}\bar{v} + (1-\bar{\pi})v_2 - q(1+r) - \bar{c}$. Therefore,

$$\begin{aligned} V_2(\mu_2) &= \int_{-\infty}^{\infty}\left[\int_{\mu^{*-1}(\mu_2)}^{\infty}\{[\hat{p}(\mu_2; v_2) - q\}\,dG\left(\frac{v_2 - \underline{v}_2}{\sigma}\right)\right. \qquad \text{(C.1)} \\ &\quad \left. + \int_{-\infty}^{\mu^{*-1}(\mu_2)}\{\bar{\pi}\bar{v} + (1-\bar{\pi})v_2 - q(1+r) - \bar{c}\}\,dG\left(\frac{v_2 - \underline{v}_2}{\sigma}\right)\right] dF(\underline{v}_2) \\ &= \{\hat{p}(\mu_2; \underline{v}_2) - q\} \\ &\quad + \int_{-\infty}^{\infty}\int_{-\infty}^{\mu^{*-1}(\mu_2)}\{(1-\mu_2)(\bar{\pi} - \underline{\pi})(\bar{v} - v_2) - qr - \bar{c}\}\,dG\left(\frac{v_2 - \underline{v}_2}{\sigma}\right) dF(\underline{v}_2). \end{aligned}$$

Using our assumption that the random variable v_2 has a finite mean with respect to G in (C.1), it follows that $V_2(\mu_2)$ is bounded. Continuity follows by inspection of (C.1) noting that so that G and F are continuous functions. Thus, there exist bounds $\underline{\Delta} \leq \bar{\Delta}$ such that for any $v_1, \hat{a}_1$

$$\underline{\Delta} \leq \bar{\pi}V_2(\mu_{s\bar{v}}(\underline{v}_1; \hat{a}_1)) + (1-\bar{\pi})V_2(\mu_{s0}(\underline{v}_1; \hat{a}_1)) - V_2(\mu_h(\underline{v}_1; \hat{a}_1)) \leq \bar{\Delta}.$$

Q.E.D.

C.1.2 Proof of Lemma 4.9.

We start by showing that $b(k)$ is continuous and strictly increasing. Note that $b(k)$ satisfies the following:

$$\hat{p}(\mu_1; b(k)) - q + \Delta(b(k); d_k) = \bar{\pi}\bar{v} + (1-\bar{\pi})b(k) - q(1+r) - \bar{c} \qquad \text{(C.2)}$$

Since $\Delta(b; d_k)$ is continuous in b and k, it is obvious that $b(k)$ is continuous. An increase in k causes the function $\Delta(b; d_k)$ to decrease by Lemma 4.8. Since $\hat{p}(\mu_1; b) - (1-\bar{\pi})b$ is increasing in b, from (C.2), $b(k)$ must be an increasing function of k.

Next, we show that the fixed point of $b(k)$ is unique. To see this, note that any fixed point of $b(k)$, v_1^* must satisfy

$$\hat{p}(\mu_1; v_1^*) - q + \Delta(v_1^*; d_{v_1^*}) = \bar{\pi}\bar{v} + (1-\pi)v_1^* - q(1+r) - \bar{c}.$$

Now, notice that under $d_{v_1^*}$, from the Bayesian updating rules, the updating rules are functions of only $1 - G\left(\frac{v_1^* - \underline{v}_1}{\sigma}\right)$. Therefore, we can rewrite $\Delta(v_1^*; d_{v_1^*})$ as the following:

$$\begin{aligned} \Delta(v_1^*; d_{v_1^*}) \quad = \quad & \beta \int_{-\infty}^{\infty} \left\{ \bar{\pi} V_2 \left(\hat{\mu}_{sg} \left(1 - G\left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \right) \right) \right. \\ & + (1-\bar{\pi}) V_2 \left(\hat{\mu}_{sd} \left(1 - G\left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \right) \right) \\ & \left. - V_2 \left(\hat{\mu}_{h} \left(1 - G\left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \right) \right) \right\} dG\left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \end{aligned}$$

Let $l = 1 - G\left(\frac{v_1^* - \underline{v}_1}{\sigma}\right)$. Then the above integral becomes

$$\Delta(v_1^*; d_{v_1^*}) = \beta \int_0^1 \left[\bar{\pi} V_2 \left(\hat{\mu}_{sg}(l)\right) + (1-\bar{\pi}) V_2 \left(\hat{\mu}_{sd}(l)\right) - V_2\left(\hat{\mu}_h(l)\right)\right] dl$$

and v_1^* must satisfy

$$\begin{aligned} -q + \beta \int_0^1 & \left[\bar{\pi} V_2 \left(\hat{\mu}_{sg}(l)\right) + (1-\bar{\pi}) V_2 \left(\hat{\mu}_{sd}(l)\right) - V_2\left(\hat{\mu}_h(l)\right)\right] dl \\ & = \bar{\pi}\bar{v} + (1-\underline{\pi})v_1^* - \hat{p}(\mu_1; v_1^*) - q(1+r) - \bar{c}. \end{aligned}$$

The left side of the above equation does not depend on v_1^* and the right side is strictly decreasing in v_1^*. Since the right side ranges from plus infinity to minus infinity, there exist a unique v_1^* that satisfies the above equation. Now, notice that under $d_{v_1^*}$, from the Bayesian updating rules, the updating rules are functions of only $1 - G\left(\frac{v_1^* - \underline{v}_1}{\sigma}\right)$. Therefore, we can rewrite $\Delta(v_1^*; d_{v_1^*})$ as the following:

$$\begin{aligned} \Delta(v_1^*; d_{v_1^*}) \quad = \quad & \beta \int_{-\infty}^{\infty} \left\{ \bar{\pi} V_2 \left(\hat{\mu}_{sg} \left(1 - G\left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \right) \right) \right. \\ & + (1-\bar{\pi}) V_2 \left(\hat{\mu}_{sd} \left(1 - G\left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \right) \right) \\ & \left. - V_2 \left(\hat{\mu}_{h} \left(1 - G\left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \right) \right) \right\} dG\left(\frac{v_1^* - \underline{v}_1}{\sigma} \right) \end{aligned}$$

and v_1^* must satisfy

$$-q+\beta\int_0^1\left[\bar{\pi}V_2\left(\hat{\mu}_{sg}\left(l\right)\right)+(1-\bar{\pi})V_2\left(\hat{\mu}_{sd}\left(l\right)\right)-V_2\left(\hat{\mu}_h\left(l\right)\right)\right]dl=$$
$$\bar{\pi}\bar{v}+(1-\underline{\pi})v_1^*-\hat{p}(\mu_1;v_1^*)-q(1+r)-\bar{c}.$$

The left side of the above equation does not depend on v_1^* and the right side is strictly decreasing in v_1^*. Since the right side ranges from plus infinity to minus infinity, there exists a unique v_1^* that satisfies the above equation.

Finally, we conclude by showing that when $k > v_1^*$, $b(k) < k$ and when $k < v_1^*$, $b(k) > k$. Suppose $k < v_1^*$ and $b(k) \leq k$. Since $\lim_{k\to-\infty} b(k) = \hat{v}^0 > -\infty$. Then by continuity of $b(\cdot)$, there must exist $k \in (-\infty, k]$ such that $b(\hat{k}) = \hat{k}$, contradicting part 2. Similarly, we can show that for all $k > v_1^*$, $b(k) < k$. *Q.E.D.*

C.1.3 Proof of Theorem 4.10.

We show that our environment can be mapped into that described in [Morris and Shin, 2003] and show that their requirements for existence of a unique equilibrium in the limit are satisfied.

Given a value function $V_2(\mu_2)$, consider an equilibrium strategy profile in the first period $(a_1(\cdot), \hat{a}_1(\cdot), p_1(\cdot))$. In a game with full information about shocks to returns, when agents in period 2 believe that the HH bank sells with probability l in the first period 1, the HH bank's differential gain from selling is given by

$$\hat{\pi}(v_1,l)=\hat{p}(\mu_1;v_1)+qr+\bar{c}-\bar{\pi}\bar{v}-(1-\bar{\pi})v_1+\beta\left[\bar{\pi}V_2(\hat{\mu}_{sg}(l))+(1-\bar{\pi})\,V_2(\hat{\mu}_{sd}(l))-V_2(\hat{\mu}_h(l))\right].$$

Then, in the game with private information, $l = \int \hat{a}_1(v_1)dH(v_1|\underline{v}_1)$ is a random variable. We then show that $\hat{\pi}$ satisfies the conditions A1–A3, A4*, A5, and A6 in [Morris and Shin, 2003]. We then can apply Theorem 2.2 in [Morris and Shin, 2003], and that completes the proof of our Proposition. It is easy to see that $\hat{\mu}_{sg}(l)$ and $\hat{\mu}_{sd}(l)$ are increasing in l and $\hat{\mu}_h(l)$ is decreasing in l. Since $V_2(\mu_2)$ is nondecreasing in μ_2, $\hat{\pi}(v_1, l)$ is nondecreasing in l – condition A1. Obviously $\hat{\pi}(v_1, l)$ is increasing in v_1– condition A2.

Since $\hat{\pi}(v_1, l)$ is separable in v_1 and l, and $\hat{\pi}(v_1, l)$ is linearly increasing in v_1, there must exist a unique v_1^* such that $\int \hat{\pi}(v_1^*, l)dl = 0$ – condition A3. Since $V_2(\mu_2)$ is a continuous function over a compact set $[0, 1]$, $\beta\,[\bar{\pi}V_2(\hat{\mu}_{sg}(l)) + (1-\bar{\pi})\,V_2(\hat{\mu}_{sd}(l)) - V_2(\hat{\mu}_h(l))]$ is bounded above and below by $\underline{\Delta}$ and $\bar{\Delta}$, respectively. Now let $\tilde{v}_1$ and $\hat{v}_1$ be defined by

$$\begin{aligned} 0 &= -\hat{p}(\mu_1; \tilde{v}_1) - qr + \bar{\pi}\bar{v} + (1-\bar{\pi})\tilde{v}_1 - \bar{c} - \bar{\Delta} - \varepsilon, \\ 0 &= -\hat{p}(\mu_1; \hat{v}_1) - qr + \bar{\pi}\bar{v} + (1-\bar{\pi})\hat{v}_1 - \bar{c} - \underline{\Delta} + \varepsilon. \end{aligned}$$

Then, if $v_1 \leq \tilde{v}_1$, $\hat{\pi}(c_1, l) \leq -\varepsilon$ for all $l \in [0, 1]$. Moreover, if $v_1 \geq \hat{v}_1$, $\hat{\pi}(v_1, l) \geq -\varepsilon$ for all $l \in [0, 1]$ – condition A4*. Continuity of V_2 implies that $\hat{\pi}(v_1, l)$ is a continuous function of v_1 and l. Therefore, $\int_0^1 g(l)\hat{\pi}(v_1, l)dl$ is a continuous function of $g(\cdot)$ and v_1 – condition A5. Moreover, by definition of $F(\cdot)$ and $G(\cdot)$, noisy signal v_1 has a finite expectation, $E[v_1] \in R$ – condition A6. Therefore, we can rewrite Proposition 2.2 in [Morris and Shin, 2003] for our environment as follows:

Proposition Let v_1^* satisfy $\int \hat{\pi}(v_1^*, l)dl = 0$. For any $\delta > 0$, there exists a $\bar{\sigma} > 0$ such that for all $\sigma \leq \bar{\sigma}$, if strategy a_1 survives iterated elimination of dominated strategies, then $a_1(v_1) = 1$ for all $v_1 \geq v_1^* + \delta$ and $a_1(v_1) = 0$ for all $v_1 \leq v_1^* - \delta$.

Q.E.D.

C.1.4 Proof of Proposition 4.11.

We proceed by induction. As described in Proposition 4.1, the game has a unique equilibrium in period T. The equilibrium strategy in the last period is a cutoff strategy with cutoff $v_T^*(\mu_T)$ given by

$$v_T^*(\mu_T) = \bar{v} - \frac{qr + \bar{c}}{(1-\mu_T)(\bar{\pi} - \underline{\pi})}.$$

Using the equilibrium strategy, we define the last period's ex-ante value function, $V_T(\mu_T)$ according to

$$\begin{aligned} V_T(\mu_T) &= (1-\alpha)\int_{-\infty}^{v_T^*(\mu_T)} \{\bar{\pi}\bar{v} + (1-\underline{\pi})\underline{v}_t - q(1+r) - \bar{c}\}\, dF(\underline{v}_t) \\ &\quad +(1-\alpha)\int_{v_T^*(\mu_T)}^{\infty} \{\hat{p}(\mu_T;\underline{v}_t) - q\}\, dF(\underline{v}_t). \end{aligned}$$

From Theorem 4.10, as σ_{T-1} converges to zero, the set of equilibrium strategies in period $T-1$ converges to a cutoff strategy with cutoff $v_{T-1}^*(\mu_{T-1})$ given by

$$v_{T-1}^*(\mu_{T-1}) = \bar{v} - \frac{qr + \bar{c} + \beta\int_0^1 [\bar{\pi}V_T(\hat{\mu}_{sg}(l;\mu_{T-1})) + (1-\bar{\pi})V_T(\hat{\mu}_{sg}(l;\mu_{T-1})) - V_T(\hat{\mu}_h(l;\mu_{T-1}))]\, dl}{(1-\mu_{T-1})(\bar{\pi}-\underline{\pi})}$$

Notice that for σ_{T-1} small and given the above cutoff strategy, the value function at period $T-1$, $V_{T-1}(\mu_{T-1};\sigma_{T-1})$ is given by

$$\begin{aligned} V_{T-1}(\mu_{T-1};\sigma_{T-1}) = (1-\alpha)\int_{\underline{v}_t}\int_{-\infty}^{\frac{v_{T-1}^*(\mu_{T-1})-\underline{v}_t}{\sigma_{T-1}}} &\{\bar{\pi}\bar{v} + (1-\underline{\pi})\underline{v}_t - q(1+r) - \bar{c} \\ &+\beta V_T\left(\hat{\mu}_h\left(1 - G\left(\frac{v_{T-1}^*(\mu_{T-1}) - \underline{v}_t}{\sigma_{T-1}}\right)\right)\right)\right\} dG(\varepsilon_{T-1})dF(\underline{v}_t) \\ &+(1-\alpha)\int_{\underline{v}_t}\int_{-\infty}^{\frac{v_{T-1}^*(\mu_{T-1})-\underline{v}_t}{\sigma_{T-1}}} \{\hat{p}(\mu_{T-1};\underline{v}_t) - q \\ &+\beta\bar{\pi}V_T\left(\hat{\mu}_{sg}\left(1 - G\left(\frac{v_{T-1}^*(\mu_{T-1}) - \underline{v}_t}{\sigma_{T-1}}\right)\right)\right) \\ &+\beta(1-\bar{\pi})V_T\left(\hat{\mu}_{sb}\left(1 - G\left(\frac{v_{T-1}^*(\mu_{T-1}) - \underline{v}_t}{\sigma_{T-1}}\right)\right)\right)\right\} dG(\varepsilon_{T-1})dF(\underline{v}_t) \\ &+\alpha\int_{\underline{v}_t}\int_{-\infty}^{\infty} \{\bar{\pi}\bar{v} + (1-\underline{\pi})\underline{v}_t - q(1+r) - \bar{c} \\ &+\beta V_T\left(\hat{\mu}_h\left(1 - G\left(\frac{v_{T-1}^*(\mu_{T-1}) - \underline{v}_t}{\sigma_{T-1}}\right)\right)\right)\right\} dG(\varepsilon_{T-1})dF(\underline{v}_t), \end{aligned}$$

and hence, the above formula becomes the following as $\sigma_{T-1} \to 0$:

$$V_{T-1}(\mu_{T-1}) = \tag{C.3}$$
$$(1-\alpha)\int_{-\infty}^{v^*_{T-1}(\mu_{T-1})} \{\bar{\pi}\bar{v} + (1-\bar{\pi})\underline{v}_t - q(1+r) - \bar{c} + \beta V_T(\hat{\mu}_h(0))\}\, dF(\underline{v}_t)$$
$$+(1-\alpha)\int_{v^*_{T-1}(\mu_{T-1})}^{\infty} \{\hat{p}(\mu_{T-1};\underline{v}_t) - q +$$
$$\beta\bar{\pi}V_T(\hat{\mu}_{sg}(1)) + \beta(1-\bar{\pi})V_T(\hat{\mu}_{sd}(1))\}\, dF(\underline{v}_t)$$
$$+\alpha\int_{-\infty}^{v^*_{T-1}(\mu_{T-1})} \{\bar{\pi}\bar{v} + (1-\bar{\pi})\underline{v}_t - q(1+r) - \underline{c} + \beta V_T(\hat{\mu}_h(0))\}\, dF(\underline{v}_t)$$
$$+\alpha\int_{v^*_{T-1}(\mu_{T-1})}^{\infty} \{\bar{\pi}\bar{v} + (1-\bar{\pi})\underline{v}_t - q(1+r) - \underline{c} + \beta V_T(\hat{\mu}_h(1))\}\, dF(\underline{v}_t)$$

Similarly, suppose for some period $t+1$ and any μ_{t+1}, the multi period model has a unique equilibrium with payoff for the HH bank given by $V_{t+1}(\mu_{t+1})$. If $V_{t+1}(\mu_{t+1})$ is increasing in μ_{t+1}, then the proof of Theorem 4.10 can be applied. As a result, as $\sigma_t \to 0$, the set of equilibrium strategies in period t converges to a cutoff strategy with cutoff $v^*_t(\mu_t)$ satisfying the properties defined in Proposition 4.11. In addition, this cutoff strategy can be used to construct the value function in period t, $V_t(\mu_t)$ in fashion similar to (C.3).
Q.E.D.

Proof of Proposition 4.3. We shall prove that when

$$\mu^*_2 < \frac{\beta\bar{\pi} - \frac{\underline{\pi}}{\bar{\pi}\alpha - \underline{\pi}}}{1 + \beta\bar{\pi}(1-\alpha)}$$

then if $\mu_1 = \underline{\mu}$, we must have $\mu^n_h < \mu^*_2$. Note that from (4.13),

$$\begin{aligned}
\mu^n_h &= \frac{\underline{\mu}}{\underline{\mu} + (1-\underline{\mu})\alpha} < \mu^*_2 \\
&\Leftrightarrow \underline{\mu} < \mu^*_2\left[\underline{\mu} + (1-\underline{\mu})\alpha\right] \\
&\Leftrightarrow \underline{\mu}\,(1 - \mu^*_2(1-\alpha)) < \mu^*_2\alpha \\
&\Leftrightarrow \underline{\mu} < \frac{\mu^*_2\alpha}{1 - \mu^*_2 + \mu^*_2\alpha}
\end{aligned} \tag{C.4}$$

Hence, we must show that the above inequality holds. Notice that from (4.10), $\underline{\mu}$ is defined by

$$\hat{p}(\underline{\mu}) + \beta \left[\bar{\pi} V(\mu_{s\bar{v}}) + (1-\bar{\pi}) V(\mu_{s0})\right] = \bar{\pi}\bar{v} - \bar{c} - qr + \beta V(\mu_h).$$

Since $\hat{p}(\mu_2^*) = \bar{\pi}\bar{v} - \bar{c} - qr$ and $V(\mu_h) = V(\mu_{s0}) = V(\mu_2^*)$, the above equality can be written as

$$\hat{p}(\underline{\mu}) + \beta\bar{\pi}\left[V(\mu_{s\bar{v}}) - V(\mu_2^*)\right] = \hat{p}(\mu_2^*).$$

Moreover, since low cost types always hold their assets, we must have

$$V(\mu_{s\bar{v}}) - V(\mu_2^*) = (1-\alpha)\left[\hat{p}(\mu_{s\bar{v}}) - \hat{p}(\mu_2^*)\right].$$

Therefore, (4.10) becomes

$$\hat{p}(\underline{\mu}) + \beta\bar{\pi}(1-\alpha)\left[\hat{p}(\mu_{s\bar{v}}) - \hat{p}(\mu_2^*)\right] = \hat{p}(\mu_2^*),$$

Using the fact that, $\hat{p}(\cdot)$ is a linear function and definition of $\mu_{s\bar{v}}$ from (4.11),

$$\underline{\mu} + \beta\bar{\pi}(1-\alpha)\left[\frac{\underline{\mu}}{\underline{\mu} + (1-\underline{\mu})\frac{\underline{\pi}}{\bar{\pi}}} - \mu_2^*\right] = \mu_2^*$$

Given that the right hand side of the above equation is increasing in $\underline{\mu}$, (C.4) is equivalent to the following inequality

$$\frac{\mu_2^*\alpha}{1-\mu_2^*+\mu_2^*\alpha} + \beta\bar{\pi}(1-\alpha)\left[\frac{\frac{\mu_2^*\alpha}{1-\mu_2^*+\mu_2^*\alpha}}{\frac{\mu_2^*\alpha}{1-\mu_2^*+\mu_2^*\alpha} + (1-\frac{\mu_2^*\alpha}{1-\mu_2^*+\mu_2^*\alpha})\frac{\underline{\pi}}{\bar{\pi}}} - \mu_2^*\right] > \mu_2^*$$

The above inequality can be further simplified in the following steps:

$$\begin{aligned}
& \frac{\mu_2^*\alpha}{1-\mu_2^*+\mu_2^*\alpha} + \beta\bar{\pi}(1-\alpha)\left[\frac{\mu_2^*\alpha}{\mu_2^*\alpha + (1-\mu_2^*)\frac{\underline{\pi}}{\bar{\pi}}} - \mu_2^*\right] > \mu_2^* \\
\Leftrightarrow \quad & \beta\bar{\pi}(1-\alpha)\mu_2^*\frac{\alpha(1-\mu_2^*) - (1-\mu_2^*)\frac{\underline{\pi}}{\bar{\pi}}}{\mu_2^*\alpha + (1-\mu_2^*)\frac{\underline{\pi}}{\bar{\pi}}} > \mu_2^* - \frac{\mu_2^*\alpha}{1-\mu_2^*+\mu_2^*\alpha} \\
\Leftrightarrow \quad & \beta\bar{\pi}(1-\alpha)\mu_2^*\frac{\alpha(1-\mu_2^*) - (1-\mu_2^*)\frac{\underline{\pi}}{\bar{\pi}}}{\mu_2^*\alpha + (1-\mu_2^*)\frac{\underline{\pi}}{\bar{\pi}}} > \mu_2^*\frac{1-\mu_2^*-\alpha(1-\mu_2^*)}{1-\mu_2^*+\mu_2^*\alpha}.
\end{aligned}$$

Since $0 < \mu_2^* < 1$, we can divide both sides of the above inequality by $\mu_2^*(1-\mu_2^*)$ and we have

$$
\begin{aligned}
& \beta\bar{\pi}(1-\alpha)\frac{\alpha - \frac{\underline{\pi}}{\bar{\pi}}}{\mu_2^*\alpha + (1-\mu_2^*)\frac{\underline{\pi}}{\bar{\pi}}} > \frac{1-\alpha}{1-\mu_2^*+\mu_2^*\alpha} \\
\Leftrightarrow \quad & \beta\bar{\pi}\frac{\alpha - \frac{\underline{\pi}}{\bar{\pi}}}{\mu_2^*\alpha + (1-\mu_2^*)\frac{\underline{\pi}}{\bar{\pi}}} > \frac{1}{1-\mu_2^*+\mu_2^*\alpha} \\
\Leftrightarrow \quad & \beta\bar{\pi}(\alpha - \frac{\underline{\pi}}{\bar{\pi}})\left(1-\mu_2^*(1-\alpha)\right) > \mu_2^*\left(\alpha - \frac{\underline{\pi}}{\bar{\pi}}\right) + \frac{\underline{\pi}}{\bar{\pi}} \\
\Leftrightarrow \quad & \beta\bar{\pi}(\alpha - \frac{\underline{\pi}}{\bar{\pi}}) - \frac{\underline{\pi}}{\bar{\pi}} > \mu_2^*\left(\alpha - \frac{\underline{\pi}}{\bar{\pi}}\right)[1+\beta\bar{\pi}(1-\alpha)]
\end{aligned}
$$

The above inequality is equivalent to

$$
\frac{\beta\bar{\pi} - \frac{\underline{\pi}}{\bar{\pi}\alpha - \underline{\pi}}}{1+\beta\bar{\pi}(1-\alpha)} > \mu_2^*
$$

and this completes the proof. *Q.E.D.*

C.2 Characterization of Equilibria in Two Period Game

Proposition C.1 *Suppose $\beta(1-\alpha) \leq 1$ and $0 < \mu_2^* < 1$. Then, there exist $\underline{\mu}$ and $\bar{\mu}$ with $\underline{\mu} < \mu_2^* < \bar{\mu}$ such that*

1. *if $\mu_1 \in [\underline{\mu}, \bar{\mu})$, the model has two equilibria: in one the HH bank sells its loan, and in the other the HH bank holds its loan,*
2. *if $\mu_1 < \underline{\mu}$, the model has a unique equilibrium in which the HH bank holds its loan in period 1,*
3. *if $\mu_1 \geq \bar{\mu}$, the model has a unique equilibrium in which the HH bank sells its loan in period 1.*

Proof. We show that our economy has a positive reputational equilibrium. As an implication of Bayes Rule, if the HH bank sells its loan in the first period, the reciprocal of the posterior beliefs is a martingale. Formally, we have

$$\frac{\bar{\pi}}{\mu_{s\bar{v}}} + \frac{1-\bar{\pi}}{\mu_{s0}} = \frac{1}{\mu_1} = \frac{1}{\mu_h}$$

Since $1/\mu$ is a convex function, it follows that

$$\bar{\pi}\mu_{s\bar{v}} + (1-\bar{\pi})\mu_{s0} \geq \mu_1 = \mu_h. \tag{C.5}$$

Let the reputational gain be defined as

$$\Delta^g(\mu_1) = \beta\left(\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0}) - V_2(\mu_h)\right)$$

Recall from (4.5) that V_2 is a convex and increasing function, so that

$$\bar{\pi}V_2(\mu_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu_{s0}) \geq V_2(\bar{\pi}\mu_{s\bar{v}} + (1-\bar{\pi})\mu_{s0}).$$

This convexity together with (C.5) implies that $\Delta^g(\mu_1) \geq 0$.

Next we show that there is some critical value of μ_1 denoted $\mu_g < \mu_2^*$ such that for all μ_1 in the interval $\mu_g < \mu_1 \leq \mu_1^*$, $\Delta^g(\mu_1)$ is strictly positive and increasing in μ_1 and

$\Delta^g(\mu_1) = 0$ for $\mu_1 \leq \mu_g$. To obtain these results, define μ_g implicitly by

$$\mu_2^* = \frac{\mu_g \bar{\pi}}{\mu_g \bar{\pi} + (1 - \mu_g)\underline{\pi}}.$$

That is μ_g denotes that initial reputation level such that if the HH bank sells and receives a good signal, its reputation level would rise to μ_2^*.Since $\bar{\pi} > \underline{\pi}$, $\mu_g < \mu_2^*$. To see that for all $\mu_g < \mu_1 \leq \mu_1^*$, $\Delta^g(\mu_1)$ is strictly positive and increasing in μ_1, rewrite the reputational gain as

$$\Delta^g(\mu_1) = \beta\left(\bar{\pi}(V_2(\mu_{s\bar{v}}) - V_2(\mu_h)) + (1 - \bar{\pi})(V_2(\mu_{s0}) - V_2(\mu_h))\right).$$

Since $\mu_h = \mu_1$ and $\mu_{s0} < \mu_1$, from Proposition 1 it follows that for all $\mu_g < \mu_1 \leq \mu_1^*, V_2(\mu_{s0}) = V_2(\mu_h)$ Since $\mu_{s\bar{v}} > \mu_h = \mu_1$, it follows that $\Delta^g(\mu_1)$is positive and since $\mu_{s\bar{v}}$ is strictly increasing in μ_1 it follows that $\Delta^g(\mu_1)$ is strictly increasing. To see that $\Delta^g(\mu_1) = 0$ for $\mu_1 \leq \mu_g$, note that $\mu_{s\bar{v}} \leq \mu_2^*$ so that $V_2(\mu_{s\bar{v}}) = V_2(\mu_h)$.

Next, rewrite (4.10) as

$$(\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi})\,\bar{v} - q + \Delta^g(\mu_1) \geq \bar{\pi}\bar{v} - q(1 + r) - \bar{c} \tag{C.6}$$

Consider $\mu_1 \leq \mu_2^*$. Since $\Delta^g(\mu_1)$ is a nondecreasing function of μ_1 in this range and $(\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi})\,\bar{v}$ is a strictly increasing function of μ_1, it follows that the left side of (C.6) is strictly increasing in this range. Since $\Delta^g(\mu_1^*)$ is strictly positive, using (4.3) the left side of (C.6) is strictly greater than the right side of this inequality at μ_1^*. Since $\Delta^g(\mu_g) = 0$ and $\mu_g < \mu_2^*$, the left side is strictly less than the right side at μ_g. Thus, there is a unique value of μ at which (C.6) holds as an equality. For $\mu_1 > \mu_2^*$, $(\mu_1\bar{\pi} + (1 - \mu_1)\underline{\pi})\,\bar{v} - q > \bar{\pi}\bar{v} - q(1 + r) - \bar{c}$ and $\Delta^g(\mu_1) \geq 0$ so that (C.6) is satisfied. We have established that our model has an equilibrium in which all HH banks with reputation levels above $\mu_1 \geq \underline{\mu}$ sell.

To obtain the negative reputational equilibrium, define μ_b implicitly by

$$\mu_2^* = \frac{\mu_b}{\mu_b + (1 - \mu_b)\alpha}.$$

That is μ_b denotes that initial reputation level such that if the HH bank holds, its reputation level would rise to μ_2^*. Clearly $\mu_b < \mu_2^*$.

Since $\mu_h = \mu_1/(\mu_1 + (1-\mu_1)\alpha)$ is greater than μ_1, it follows that $\Delta^b(\mu_1)$ is negative for $\mu_1 > \mu_b$. If $\mu_1 \in [\mu_b, \mu_2^*]$, selling has a static cost, i.e. $\hat{p}(\mu_2) - q \leq \bar{\pi}\bar{v} - q(1+r) - \bar{c}$ as well as a loss from reputation, i.e. $\Delta^b(\mu_1) < 0$ so that the HH bank prefers to hold the asset. If $\mu_1 \in (\mu_2^*, 1]$, there are benefits from selling the asset, i.e. $\hat{p}(\mu_2) - q \geq \bar{\pi}\bar{v} - q(1+r) - \bar{c}$, while there is a loss from reputation $\Delta^b(\mu_1) < 0$. Our assumption that $\beta(1-\alpha) \leq 1$ ensures that when $\mu_1 = 1$, the static benefit outweighs the loss from reputation, i.e. (4.12) is reversed at $\mu_1 = 1$. Moreover, Since $\mu_h = \mu_1/(\mu_1 + (1-\mu_1)\alpha)$,it is easy to show that $(\mu_2\bar{\pi} + (1-\mu_2)\underline{\pi})\,\bar{v} - q + \Delta^b(\mu_1)$ is a strictly convex function of μ_1 for $\mu_1 \in [\mu_2^*, 1]$. Since the value of this function is strictly less than $\bar{\pi}\bar{v} - q(1+r) - \bar{c}$ at $\mu_1 = \mu_2^*$ and weakly higher when $\mu_1 = 1$, there exists a unique $\bar{\mu} \in (\mu_2^*, 1]$, at which (4.12) holds with equality. For $\mu_1 \leq \bar{\mu}$, (4.12) holds and for $\mu_1 > \bar{\mu}$ (4.12) is violated.

Q.E.D.

C.3 Strategic Types

Proposition C.2 *Suppose $\beta(1-\alpha) \leq 1$ and*

$$(\bar{\pi} - \underline{\pi})\,\bar{v} + qr + \max_{\mu_1 \in [0,1]} \Delta^g(\mu_1) < -\underline{c}. \tag{C.7}$$

Then the unique equilibrium of the static game described in Proposition 1 and the multiple equilibria of the dynamic game described in Proposition 2 are also equilibria of the associated games when all bank types behave strategically.

Proof. Consider the static game. It is sufficient to show that given the constructed equilibrium and specified strategies for all agents, there is no profitable deviation by any agent. Note that in the proof of Proposition 2 we show that $\Delta^g(\mu_1) \geq 0$ for all $\mu_1 \in [0,1]$. Hence, (C.7) implies that

$$\mu_1\,(\bar{\pi} - \underline{\pi})\,\bar{v} + qr < -\underline{c}$$

or

$$[\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi}]\bar{v} - q < \underline{\pi}\bar{v} - q(1+r) - \underline{c} \tag{C.8}$$

Inequality (C.8) implies that facing break even prices the low cost type bank would like to hold. Moreover a deviation by a buyer must attract these types of bank and (C.8) implies that buyers must offer a price higher than the actuarially fair price. Hence, there is no deviation by any buyer or a low cost bank type. Moreover, an LH bank wants to sell even at the lowest possible price, $\underline{\pi}\bar{v}$, since $\bar{c} > 0$. Thus there are no profitable deviation from the specified strategies in the static game.

Consider the positive equilibrium of the dynamic game. Given future beliefs, the value of selling to a low quality bank adjusted by the future reputational gain from holding is given by

$$[\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi}]\bar{v} - q + \beta\,[\underline{\pi}V_2(\mu^g_{s\bar{v}}) + (1-\underline{\pi})V_2(\mu^g_{s0}) - V_2(\mu)]$$

where $\mu_{s\bar{v}} = \bar{\pi}\mu_1/(\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi})$ and $\mu^g_{s0} = (1-\bar{\pi})\mu_1/((1-\bar{\pi})\mu_1 + (1-\underline{\pi})(1-\mu_1)$.

The value of selling to a high quality bank is given by

$$[\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi}]\bar{v} - q + \Delta^g(\mu_1)$$

From (C.7) and $\beta\left[\underline{\pi}V_2(\mu^g_{s\bar{v}}) + (1-\underline{\pi})V_2(\mu^g_{s0}) - V_2(\mu)\right] = \Delta^g(\mu_1)$, we have

$$\begin{aligned}
[\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi}]\bar{v} - q + \beta\left[\underline{\pi}V_2(\mu^g_{s\bar{v}}) + (1-\underline{\pi})V_2(\mu^g_{s0}) - V_2(\mu)\right] &\leq \underline{\pi}\bar{v} - q(1+r) - \underline{c} \\
[\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi}]\bar{v} - q + \Delta^g(\mu_1) &\leq \bar{\pi}\bar{v} - q(1+r) - \underline{c}
\end{aligned}$$

Hence, there is no profitable deviation by the low cost types. As for the LH type bank, note that in the positive equilibrium

$$[\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi}]\bar{v} - q + \beta\left[\bar{\pi}V_2(\mu^g_{s\bar{v}}) + (1-\bar{\pi})V_2(\mu^g_{s0}) - V_2(\mu)\right] \geq \bar{\pi}\bar{v} - q(1+r) - \bar{c} \quad \text{(C.9)}$$

We use the above inequality to show that the LH type bank does not have a profitable deviation. There are two possible cases: Case 1. $\bar{c} + qr \geq (\bar{\pi} - \underline{\pi})\bar{v}$. In this case, $\mu_2^* = 0$ and $V_2(\mu)$ is a constant function. Therefore, $\Delta^g(\mu_1) = 0$ for all μ_1 and $\beta\left[\underline{\pi}V_2(\mu^g_{s\bar{v}}) + (1-\underline{\pi})V_2(\mu^g_{s0}) - V_2(\mu)\right] = 0$. In this case, we are back to the static game and as we have shown before, the LH bank finds it optimal to sell always. Case 2. $\bar{c} + qr < (\bar{\pi} - \underline{\pi})\bar{v} < \bar{v}$. In this case, we have

$$\begin{aligned}
\beta\left[V_2(\mu_{s\bar{v}}) - V_2(\mu_{s0})\right] &\leq \beta(1-\alpha)\left\{[\mu_{s\bar{v}}\bar{\pi} + (1-\mu_{s\bar{v}})\underline{\pi}]\bar{v} - q - \bar{\pi}\bar{v} + q(1+r) + \bar{c}\right\} \\
&= \beta(1-\alpha)\left\{-(1-\mu_{s\bar{v}})(\bar{\pi} - \underline{\pi})\bar{v} + qr + \bar{c}\right\}
\end{aligned}$$

The last expression is increasing in μ_1 and therefore maximized at $\mu_1 = 1$. Hence, we must have

$$\beta\left[V_2(\mu_{s\bar{v}}) - V_2(\mu_{s0})\right] \leq \beta(1-\alpha)(qr + \bar{c}) < \bar{v}$$

Therefore,

$$-\beta(\bar{\pi} - \underline{\pi})\left[V_2(\mu_{s\bar{v}}) - V_2(\mu_{s0})\right] > -\bar{v}(\bar{\pi} - \underline{\pi})$$

Adding this inequality to (C.9) , we get

$$[\mu_1\bar{\pi} + (1-\mu_1)\underline{\pi}]\bar{v} - q + \beta\left[\underline{\pi}V_2(\mu^g_{s\bar{v}}) + (1-\underline{\pi})V_2(\mu^g_{s0}) - V_2(\mu)\right] \geq \bar{\pi}\bar{v} - q(1+r) - \bar{c}$$

which implies that the LH type bank does not have a profitable deviation in the constructed equilibrium.

As for the negative equilibrium, it is clear that a bank with low cost does not want to sell its loan, since selling only punishes the bank. Therefore, it is sufficient to show that the LH bank wants to sell its loan. That is, we need to show that for all $\mu_1 \in [0, \bar{\mu}]$, we have

$$\underline{\pi}\bar{v} - q + \beta[V_2(0) - V_2(\mu_h^b)] \geq \underline{\pi}\bar{v} - q(1+r) - \bar{c} \tag{C.10}$$

where $\mu_h^b = \mu_1/(\mu_1 + (1-\mu_1)\alpha)$. To do so, we first show that this inequality is satisfied at $\mu_1 = \bar{\mu}$. Now, since $\Delta^b(\mu_1) = \beta[V_2(0) - V_2(\mu_h^b)]$ is decreasing, this implies that (C.10) holds for all $\mu_1 \in [0, \bar{\mu}]$. By definition, $\bar{\mu}$ satisfies

$$\underline{\pi}\bar{v} - q + \beta[V_2(0) - V_2(\mu_h^b)] = \bar{\pi}\bar{v} - q(1+r) - \bar{c}$$

Obviously, this equality leads to the above inequality. Therefore, we have shown that LH bank still finds it optimal to sell in the negative equilibrium.

Q.E.D.

C.4 Figures

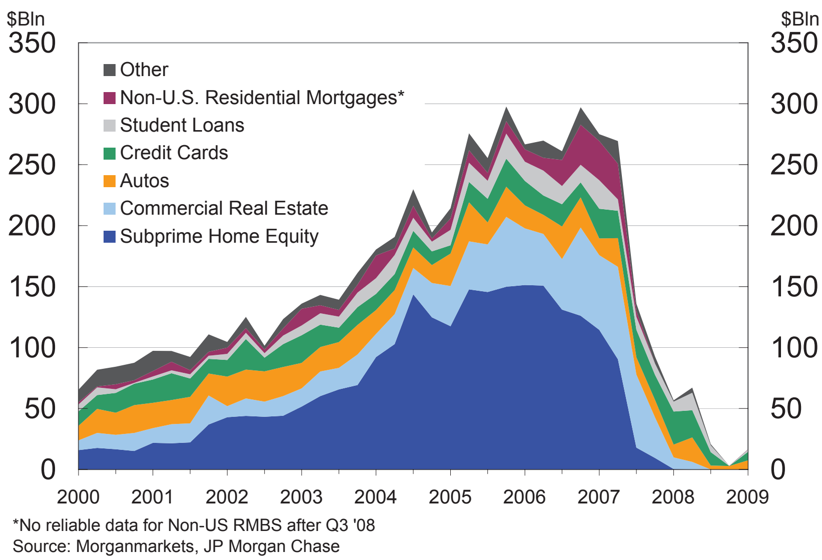
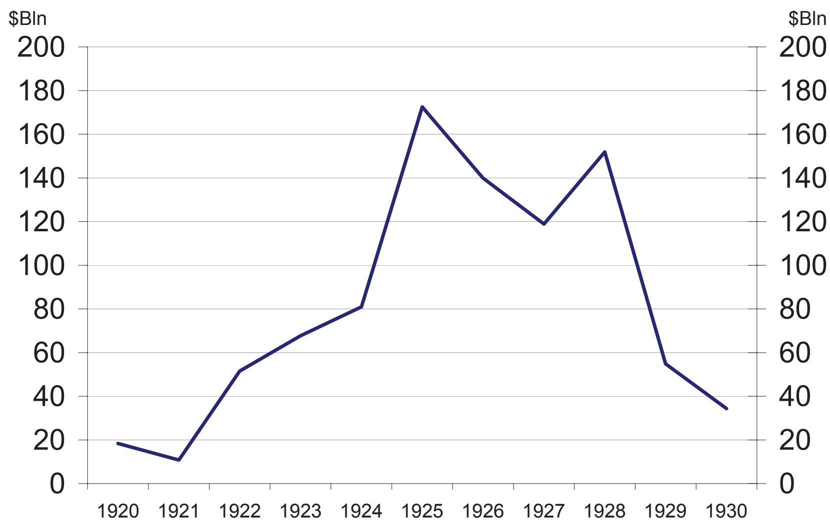


Figure C.1: New Issuance of Asset-Backed Securities (Source: JP Morgan Chase)



Note: Data is annual change in real estate bonds divided by Nominal GDP at relevant year multiplied by Nominal GDP 2009.
Source: Carter, et. al., Historical Statistics, (2006)Series Dc904

Figure C.2: Change in Stock of Real Estate Bonds, 1920-1930

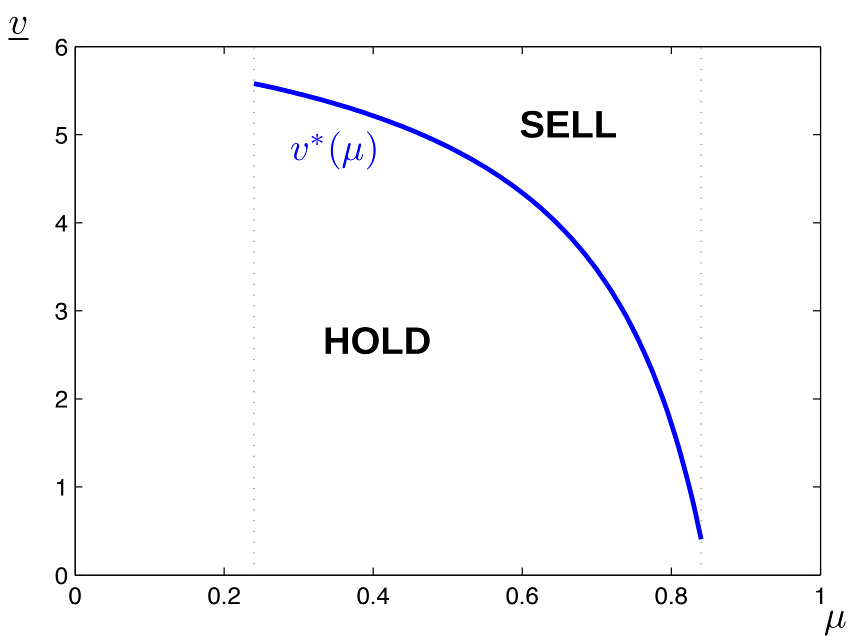


Figure C.3: Cutoff Thresholds for High-Quality Banks.

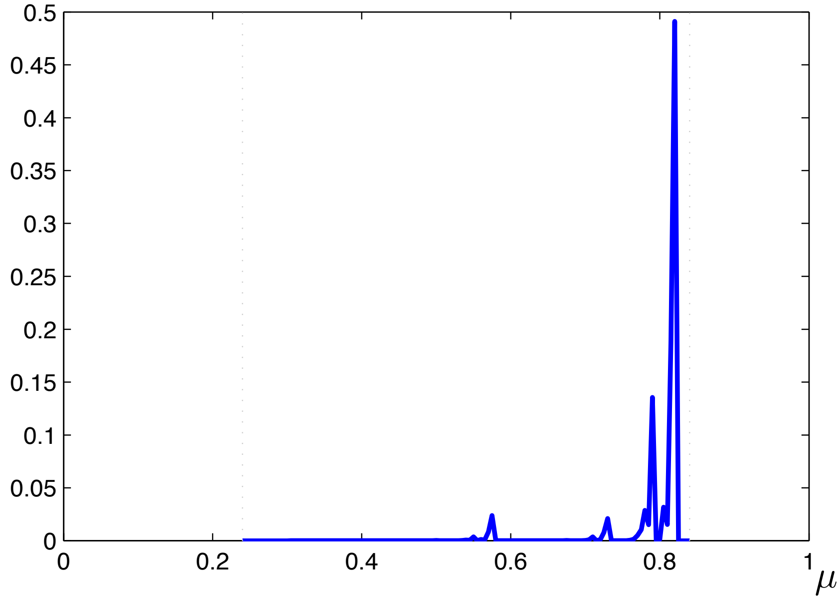


Figure C.4: Invariant Distribution of Reputations of High-Quality Banks

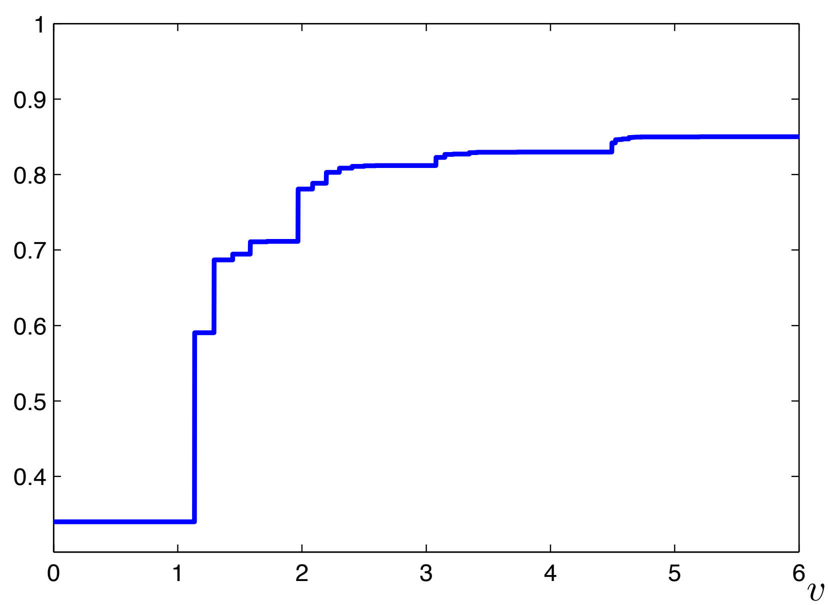


Figure C.5: Volume of Trade as a Function of shock to Default Value.