# Essays in Family and Labor Economics

A DISSERTATION
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

Satoshi Tanaka

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Doctor of Philosophy

José Víctor Ríos Rull, Advisor

August, 2012

# Acknowledgements

I am deeply grateful for helpful advice and continuous support from my advisor, José Víctor Ríos Rull. Víctor has given his time generously in helping me develop the theme of the dissertation. This dissertation would not exist without his help. I would like to thank my friend, Naoki Takayama. He helped me a lot when writing this dissertation and the discussion with him was always fruitful. I would like to extend my thanks to the rest of my thesis committee members: Alessandra Fogli, Kjetil Storesletten, and Fatih Guvenen. I also thank Elena Pastorino, Nancy Stokey, V.V. Chari, Larry E. Jones, Jonathan Heathcote, Sam Schulhofer-Wohl, Harold Cole, Yuichiro Waki, and the participants of the University of Minnesota Labor Workshop and the Bag Lunch Seminar at the Federal Reserve Bank of Minneapolis for helpful comments. In the first essay of this dissertation, Naoki Takayama helped me especially when writing a computation code for Gale and Shapley algorithm. In the second essay of this dissertation, I used some result from the previous work done by José Víctor Ríos Rull and Shannon Seitz. I couldn't write this dissertation without their help.

# Dedication

To my wife, Ayako, who has always been there through the hard times.

## Abstract

This dissertation consists of two essays. The first essay explores the child support enforcement policies and their implications. The child support enforcement (CSE) policies, aimed at protecting out-of-wedlock children from financial disadvantages, brought unexpected changes in individuals' marriage and fertility behaviors during the 1980s and the 1990s. Our estimates from state-year panel data show that in states with strict CSE there has been a significant decrease in non-marital births and a significant increase in marital births. Taking into account all these changes, what are the effects of CSE on children's welfare? To answer this question, we build a heterogeneous-agent model that features endogenous marriage and child-investment decisions. Exploiting the state-level variation in enforcement, we estimate it using the National Vital Statistics Report data. We find that men's increased willingness to marry is the driving force behind the shift from non-marital births to marital births. As evidence for the mechanism, we show that the number of marriages has risen in the states with strict CSE during the same period, consistent with the model's implication. Our model predicts that a large increase in child investment comes through a secondary effect of CSE: the shift from non-marital births to marital births increases child investment through its income effect.

In the second essay, we ask to what extent changes to the age and sex structure of the population account for the changes in the marriage behavior observed in the last century (from 1900 to 1980). The decrease in mortality, especially for women, and the changes in immigration patterns have increased the female to male ratio. With respect to marriage, there has been i) an increase in its incidence, ii) a reduction in the gender gap of the median age at first marriage, and iii) an increase in the divorce rate. We pose a model of marriage and divorce in which preferences over spouses depend on their age and on love (an idiosyncratic shock) and where frictions make it difficult to get new partners. We estimate our model using marital and population patterns of the 1950-1959 birth cohorts. Using the preference parameters estimated on the 1950's cohort and the population patterns of the 1870's cohorts, we find marriage patterns are quite similar to those observed in the earlier period. By making divorce costly for the 1870's cohort, the resemblance is more stronger. In particular, we find that these features account for i) 94.5% of the increase in the incidence of marriage ii) 140.8% of the shrink the gender age gap in the median age at first marriage.

# Contents

# List of Tables

# List of Figures

# Chapter 1

# Implications of the Child Support Enforcement Policy for Marriage, Fertility, and Long-Term Inequality

## 1.1 Introduction

Since the late 1970s, the U.S. federal government has taken a number of steps to strengthen the state-regulated, private child support systems. In the mid-1970s, it created the Office of Child Support Enforcement (OCSE), required all states to establish comparable state offices, and raised federal funding for three-quarters of the states' expenditures on child support enforcement. It also passed major federal regulations in the 1980s requiring states to strengthen paternity establishment, to create legislative guidelines for states' child support orders, and to withhold obligations from fathers' wages. As a consequence, many states increased their child support collections significantly (as shown in Figure 1.1) by establishing child support enforcement (CSE) policies through the 1980s and 1990s.[1]

[1] CSE policies consist of child support legislation and plans of state's expenditures for CSE cases. The major CSE laws are on genetic testing, paternity establishment, wage withholding under delinquency, immediate wage withholding for new cases, universal wage withholding, and state income tax interception. The years when these legislations were approved, vary across states. See the work of Huang (2002) for more details.

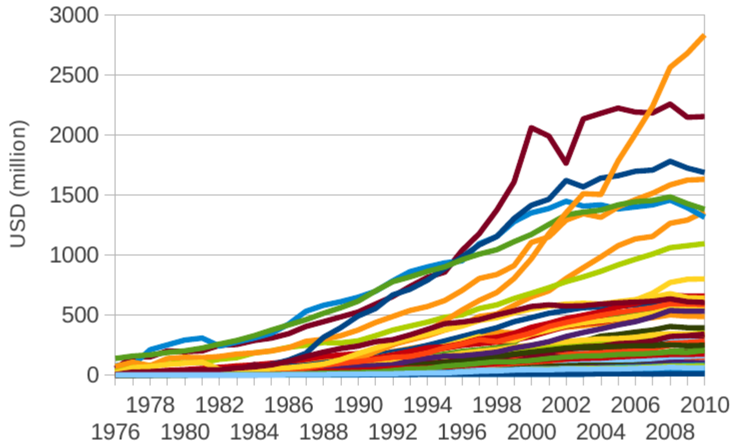


Figure 1.1: Child Support Collection Amount by States. Source: OCSE

However, these CSE policies have also brought unexpected changes in people's marriage and fertility behaviors in the United States. In addition to a reduction of non-marital births, which is well-known in the literature,[2] we find significant increases in the number of marital births and also in marriages in the states with strict CSE. (See Table 1.1 for a summary of our estimates.[3], [4]) Why did these changes happen after the CSE policies were strengthened? Furthermore, what was the effect of CSE on children's welfare if we take into account all these changes?

Table 1.1: The Effects of a 10% Increase in the Child Support Collection Rate

| **Variable** | **Level in 1980** | **Changes in Level** | **Changes in %** |
|---|---|---|---|
| Total Fertility Rate for Non-Marital Births | 0.304 | $-\Delta$ 0.0297 | $-\Delta$ 9.8% |
| Total Fertility Rate for Marital Births | 1.582 | $+\Delta$ 0.0172 | $+\Delta$ 1.1% |
| Marriage Rate | 0.042 | $+\Delta$ 0.0013 | $+\Delta$ 3.1% |

To address these issues, we build a heterogeneous-agent model which features marriage and child investment decisions. In our model, people form marital or non-marital relationships in a

[2] See the work of Case (1998), Huang (2002), Garfinkel, Huang, McLanahan, and Gaylin (2003), and Aizer and McLanahan (2006), for example.

[3] In Table 1, the *marriage rate* is defined as the number of marriages per population. To create the total fertility rate (total period fertility rate), first we calculate age-specific marital and non-marital birth rates for six age groups. Then we sum them up and multiply the sum by five.

[4] According to our estimate, a 10% increase in the child support collection rate has decreased the total fertility rate for non-marital births by 9.8%, increased the total fertility rate for marital births by 1.1%, and increased the marriage rate by 3.1% (as summarized in Table 1) relative to the trends.

stable matching equilibrium and also choose the number and the quality of children within each relationship. As in the work of Weiss and Willis (1985), here marriage allows couples to achieve the efficient level of public good investment. If a couple chooses a non-marital relationship, however, only the mother can determine the level of child investment; the father transfers child support payments to the mother but often at inefficiently low levels because of the lack of coordination. We compute our model by extending the Gale and Shapley (1962) algorithm. And we estimate using the total fertility rate for marital and non-marital births (the National Vital Statistics Report: Natality data published by the Centers for Disease Control and Prevention), exploiting the exogenous variations in CSE across the states during the period 1980 - 1997.

We find that men's increased willingness to marry is the driving force behind the shift from non-marital births to marital births. After the strengthening of CSE policies, facing the larger cost per child due to the mandatory child support payment, men in non-marital relationships may

1. Reduce the number of children and, instead, increase investments in child quality.
2. Reduce the number of children and, instead, increase the private consumption.
3. Get married to avoid the child cost change.

We find that option (1) is not attractive to unmarried fathers, because to increase a child's quality investment they have to transfer money to the mothers. But these transfers involve two types of inefficiency: First, in non-marital relationships, since the mothers are not considering the fathers' utility, they don't invest all the money in children's quality. They use some of it for their private consumption. Second, if the mother in a non-marital relationship is on a welfare program, then the state government takes away a significant portion of child support payments made by the biological father.[5] Therefore, fathers' investments don't really increase a child's quality. And unmarried fathers are thus left with options (2) or (3). Through our estimation, we show that men split between reducing the number of children and increasing marriage when facing the increased degree of CSE. This result hinges upon finding parameters that govern the elasticity between the utility from consumption and from children.

After estimating the model's underlying parameters using marital and non-marital total fertility rates in the state-year panel data, we check the identification of the model by predicting the increase of marriages found in the data. This is crucial to distinguish our story from the other alternatives. From the reduced-form regression, a 10% increase in the child support collection rate induces a 3.1% increase in the marriage rate and a 1.93% increase of the number of ever-married people at age 45. The model predicts a rise of 1.39% in the number of ever-married

[5] In most states, child support payments to mothers on Aid to Families with Dependent Children (AFDC) are now taxed 100%. Some states allow a $50 pass-through per month.

people, accounting for 72% of the increase in marriages in the data.

Finally, using the estimated parameters, we find that there are secondary, positive effects of CSE on child investment. CSE was originally supposed to protect out-of-wedlock children and, thus, to improve child investment in non-marital relationships. However, we find that a large gain in the average child investment comes through a shift from non-marital births, when we allow for the income effects on child quality investment. Our model predicts that a 10% increase in the child support collection rate will increase average child investment by 1.1%. Assuming a general human capital transmission function, we find that stronger CSE will men's decrease 90-10% income ratio of the next generation by 2.2%. We find that the effects are especially strong for the bottom group of the income distribution.

## Related Literature

Our work here is related to a sizable number of studies in the sociology and economics literature that examine the effect of CSE on non-marital births. Case (1998) and Garfinkel, Huang, McLanahan, and Gaylin (2003) analyze state level data similar to ours. Case (1998) finds significantly lower non-marital birth rates in the states where legislation allows genetic testing to establish paternity, permits paternity establishment up to age 18, and establishes presumptive guidelines for setting child support awards. Extending her framework, Garfinkel, Huang, McLanahan, and Gaylin (2003) use paternity the establishment rate[6] and child support collection amounts per cases,[7] [8] and show significantly negative effects of their CSE measures on non-marital births. Huang (2002), Aizer and McLanahan (2006), and Plotnick, Garfinkel, McLanahan, and Ku (2007) look into microeconomic data. Huang (2002) and Aizer and McLanahan (2006) use the U.S. Labor Department's National Longitudinal Survey of Youth 1979 (NLSY79) to examine whether CSE is related to the likelihood that a women's first birth is premarital. And both studies show that CSE reduces the risk of out-of-wedlock births. So do Plotnick, Garfinkel, McLanahan, and Ku (2007), but they use the University of Michigan's Panel Study of Income Dynamics (PSID) for their analysis.

Compared to studies of non-marital births, not much research has been done on the effect of CSE on marital births and marriages. Huang (2002) is one of the exceptions; he uses the multinomial logit model for NLSY79 and finds a significant increase in the likelihood of marital births in the states with strict CSE. Our work is motivated by his work, but we use state-year panel data constructed from the CDC's National Vital Statistics Report (NVSR). We show

[6] The paternity establishment rate is defined as the number of paternity establishments for non-marital births over the total number of non-marital births.

[7] More precisely, they consider only cases with single mothers on Aid to Families with Dependent Children (AFDC).

[8] Those are calculated from the Office of Child Support Enforcement (OCSE) 1980 - 1997 Annual Reports to Congress.

that there is an increase in marital births also in our state panel data. For marriage, the work of Acs and Nelson (2004) is the only research, as far as we know, which reports the effect of strengthened CSE on marital statistics. They show that two-parent families have increased in the states where CSE has been strengthened, especially among low-income people. They use the University of Michigan's 1997 and 1999 National Surveys of America's Families and apply the difference-in-difference estimation method to derive their results. Unlike their approach, we don't analyze the 'stock' of married people. Instead, we look at the marriage rate in the state-year panels. Our data cover the longer period, and our result is more robust than theirs.

In terms of theory, our study is related to the growing literature on family economics. One of the most relevant works is Weiss and Willis (1985). They show that non-marital childbearing potentially involves inefficiencies in child investment. This is because single mothers don't take into account fathers' utility when investing, and if fathers know that, they don't transfer much money to mothers. Del Boca and Flinn (1995) apply Weiss and Willis (1985)'s framework to explain why child support payments are low in the U.S. data. Our study is also based on the work of Weiss and Willis (1985). And we extend their model to the stable matching problem. Other relevant studies which analyze child support and/or CSE are those of Chiappori and Weiss (2006), Chiappori and Weiss (2007), and Greenwood, Guner, and Knowles (2003). All these use two-period models, which enable them to analyze the divorce situations. Other relevant works are those which apply the two-sided matching problem to an analysis of the marriage market. Del Boca and Flinn (2005) compute a stable matching equilibrium applying Gale and Shapley (1962)'s algorithm. We further extend their framework, allowing for marital and non-marital relationships. Finally, our work is related to those which study the intergenerational transmission of human capital. These include the work of Aiyagari, Greenwood, and Guner (2000a), Greenwood, Guner, and Knowles (2003), and Kocharkov (2010), who analyze the effects of the government's family policies on the next generation's income distribution. Also, in the growth context, De La Croix and Doepke (2003) and Moav (2005) show that economies with a less equitable income distribution have a lower rate of economic growth as the consequence of the quantity-quality trade-off of child investment.

The next section describes our economic model and defines an equilibrium. Sections 3 and 4 describe the data and the estimation methodology. Section 5 presents the results and the implications. Section 6 concludes.

## 1.2 The Model Economy

We develop a structural economic model in order to identify the channels through which changes in the degree of CSE have effects on marriage decisions and fertility choices. In our model, equal population of women and men enter the marriage market only once in their life, form either marital or non-marital unions in a stable matching equilibrium, and choose about a quantity and quality of children. Our model is static in the sense that people make a decision about marriage and fertility only once in their lives.

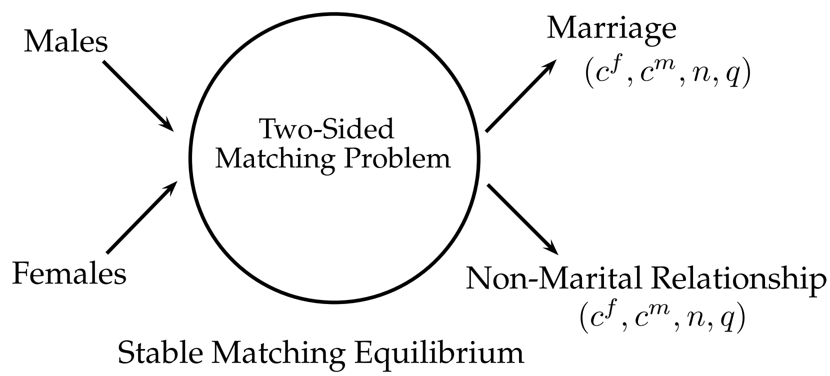


Figure 1.2: Structure of the Model Economy

### 1.2.1 Setup

The agents in the economy are unit masses of females ($f$) and males ($m$), who each live for one period. Among each of these types of agents, individuals differ in their human capital level, $h^f \in \mathcal{H}^f \subset \mathbf{R}_+$ and $h^m \in \mathcal{H}^m \subset \mathbf{R}_+$ , and their charm level, $a^f \in \mathcal{A} \subset \mathbf{R}$ and $a^m \in \mathcal{A} \subset \mathbf{R}$. Assume that there is only a finite number of types of people in the economy and that the number of types is the same for women and men. The sets of all types of people are denoted as $\{(h_i^f, a_i^f)\}_{i \in \mathcal{I}^f}$ for women and $\{(h_j^m, a_j^m)\}_{j \in \mathcal{I}^m}$ for men, where $\mathcal{I}^f$ is the set of all indices for women's type and $\mathcal{I}^m$ is the set of all indices for men's type. We assume that $n(\mathcal{I}^f) = n(\mathcal{I}^m) = N_h \times N_a$, where $n(X)$ denotes the cardinality of a set $X$, $N_h$ is the number of possible human capital levels, and $N_a$ is the number of possible charm levels. Both $N_h$ and $N_a$ are common across sex. We assume that each individual possesses only one sexual identity, and thus let $\mathcal{I}^f \cap \mathcal{I}^m = \emptyset$. Finally, assume that people are equally populated across each type $(h_i, a_i)$.

After they are born, both men and women enter the frictionless marriage market, where each person chooses one partner and forms a relationship - either marriage or a non-marital relationship. As we will discuss later, people can form a relationship with whomever they want as long as the partner agrees. But, by assumption, they cannot have more than one relationship.

Once people form a relationship, they determine allocations, $(c^f, c^m, n, q)$, where $c^f \in \mathbf{R}_+$ is women's private consumption, $c^m \in \mathbf{R}_+$ is men's private consumption, $n \in \mathbf{R}_+$ is the number of children for the couple, and $q \in \mathbf{R}_+$ is the quality of each child for the couple. We assume that $n$ and $q$ are local public goods within couples.

**Preferences** Preferences are identical across sex, and are denoted as $u(c, a') + v(nq)$. People get utility from their private consumption $c$ and from the number of children times the quality of children $nq$. Also, they get utility from their partner's charm $a'$. For men who choose non-marital relationship, probably, fathers are not always staying with their children.[9] And, thus, we take into account the possibility of a discount of their utility from children as $u(c, a')+\delta v(nq)$ where $\delta \in [0, 1]$.

**Technology** Couples in either marriage or a non-marital relationship invest in the number and the quality of children. Increasing one unit of the number, it requires fixed amounts of time $\phi$ and consumption good $\psi$. Increasing the quality requires an input of per-child educational investment $s$. Then children's quality is determined by a function, $q = f_1(s)$ for married couples, and $q = f_0(s)$ for non-married couples.

**Child Support** Men in non-marital relationships make child support payments to the mothers of their children. The payments consist of a mandatory portion and a voluntary portion. Let $wh^m\gamma\tau^{cs}(n)$ be the amount of mandatory child support payments for men whose human capital level is $h^m$ and who have $n$ children out of wedlock, where $\tau^{cs}(n)$ is the child support order, which depends on the number of children and which determines the payment rate from their income. Assume that $\gamma \in [0, 1]$ is the strength of CSE, which state governments can control. And, $w$ is the market price of human capital.

**Aid to Families with Dependent Children (AFDC)** Mothers in non-marital relationships who meet an income test are eligible for AFDC and receive monetary payments. AFDC is a welfare program provided by each state government. In the model, we assume that there are no differences across states. Mathematically, $g(e, n)$ is the amount of receipts from AFDC, which is decreasing in the mother's income $e$ and increasing in the number of her children $n$.

[9] According to McLanahan, Garfinkel, Reichman, and Teitler (2001), more than half of new unwed parents are not cohabiting in the 1998-1999 data in the U.S..

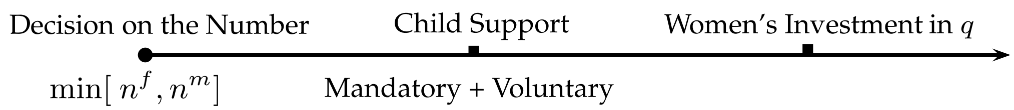


Figure 1.3: Timeline of Decisions in Non-Marital Relationships

### 1.2.2 Couples in Non-Marital Relationships

Allocations within the couples in non-marital relationships are determined through three stages, shown in Figure 1.3. In the first stage, women and men decide on the number of children they will have together. In the second stage, men pay mandatory child support, and choose the amounts of their private consumption and voluntary child support payments. In the third stage, women choose their private consumption and child investment.

Let us first focus on the last stage and go backward. Given the number of children $n$, the characteristics of the couple $\Phi \equiv (h^f, a,^f h^m, a^m)$, the strength of CSE $\gamma$, and a voluntary payment of child support from the biological father $T^{cs}$, a woman in a non-marital relationship (that is, still single, $S$) solves the following problem:

$$V_S^f(n, \Phi, \gamma, T^{cs}) \equiv \max_{c^f, s \geq 0} \; u\left(c^f, a^m\right) + v\left(nq\right)$$

subject to

$$q = f_0(s)$$

$$c^f + (\psi + s)n = wh^f(1 - \phi n) + \max\left[\underbrace{g\left(wh^f(1 - \phi n), n\right)}_{\text{AFDC}}, \; \underbrace{wh^m \gamma \tau^{cs}(n) + T^{cs}}_{\text{Mandatory+Voluntary CS}}\right],$$

where $V_S^f$ is the woman's utility value, $c^f$ is her consumption, $s$ is the amount of investment per child, $q$ is the quality of each child, $\psi$ is the consumption good cost per child, $\phi$ is the time cost per child, and $g(w^f, n)$ is the AFDC receipt. If there is a child support payment to a woman on AFDC from the biological father, it is taxed 100%. Denote the solution to the quality of the children in the above problem as $q_0^*(T^{cs}; n, \Phi, \gamma, )$.

The problem for a man in a non-marital relationship is defined in the following way. Given the number of children $n$, the characteristics of the couple $\Phi$, the degree of CSE $\gamma$, and the woman's response function $q_0^*(T^{cs}; n, \Phi, \gamma)$, a man chooses his private consumption $c^m$ and voluntary child support payments $T^{cs}$:

$$
\begin{aligned}
V_S^m(n,\Phi,\gamma;q_0^*) \equiv \max_{c^m,T^{cs}\geq 0}\ & u\left(c^m,a^f\right)+\delta v\left(nq_0^*(T^{cs};n,\Phi,\gamma)\right) \\
\text{subject to}& \\
c^m + T^{cs} =& wh^m\left(1-\gamma\tau^{cs}(n)\right),
\end{aligned}
$$

Denote the solution for the child support payments in the above problem as $T^{cs*}\left(n,\Phi,\gamma;q_0^*\right)$.

Finally, in the first stage, the number of children is determined as the minimum of the numbers which each partner wants:

$$
n_0^*(\Phi,\gamma) \equiv \min\left[\arg\max_n V_S^f(n,\Phi,\gamma,T^{cs*}), \arg\max_n V_S^m(n,\Phi,\gamma;q_0^*)\right].
$$

Once the number is determined, the utility values for each member of the couple in a non-marital relationship are well-defined. The set of utility values is denoted as $\left\{\hat{V}_S^f(\Phi,\gamma),\hat{V}_S^m(\Phi,\gamma)\right\}_{\Phi\in\mathcal{F}}$, where $\mathcal{F}$ is the set of all possible patterns of a couple's characteristics, which includes $N_h^2\times N_a^2$ patterns of coupling. Let $\hat{V}_s^f(\Phi,\gamma)$ be defined as $\hat{V}_S^f(\Phi,\gamma)\equiv V_S^f(n_0^*,\Phi,\gamma,T^{cs*})$ and $\hat{V}_s^m(\Phi,\gamma)$ be defined as $\hat{V}_S^m(\Phi,\gamma)\equiv V_S^m(n_0^*,\Phi,\gamma;q_0^*)$.

### 1.2.3 Married Couples

Unlike couples in non-marital relationships, married couples stay together with their children for a long time; thus, we assume that the allocation within marriage is determined through a Nash bargaining problem:

$$
\begin{aligned}
\max_{c^f,c^m,n,q,t^f,t^m\geq 0} & \left[V_M^f(c^f,n,q;\Phi)-\hat{V}_S^f(\Phi,\gamma)\right]^{\frac{1}{2}} \times \left[V_M^m(c^m,n,q;\Phi)-\hat{V}_S^m(\Phi,\gamma)\right]^{\frac{1}{2}} \\
\text{subject to} & \\
& q = f_1(s) \\
& \phi n = t^f + t^m \\
& c^f + c^m + (\psi+s)n = wh^f(1-t^f) + wh^m(1-t^m) \\
& V_M^f(c^f,n,q;\Phi) \geq \hat{V}_S^f(\Phi,\gamma) \\
& V_M^m(c^m,n,q;\Phi) \geq \hat{V}_S^m(\Phi,\gamma)
\end{aligned}
$$

and

$$
\begin{aligned}
V_M^f(c^f,n,q;\Phi) &\equiv u\left(c^f,a^m\right)+v\left(nq\right)+\kappa \\
V_M^m(c^m,n,q;\Phi) &\equiv u\left(c^m,a^f\right)+v\left(nq\right)+\kappa
\end{aligned}
$$

where $t^f$ and $t^m$ are the time spent for child nurture by each member of the couple, and which must sum to $\phi n$. Here $\kappa \in \mathbf{R}$ is the utility gain of marriage, which is common across couples. We denote the solution of the utility values for the above problem for married ($M$) couples as $\left\{\hat{V}_M^f(\Phi,\gamma), \hat{V}_M^m(\Phi,\gamma)\right\}_{\Phi\in\mathcal{F}}$. It is often true that there doesn't exist a solution to the above problem. In that case, we simply assume that $\hat{V}_M^f(\Phi,\gamma) = \hat{V}_M^m(\Phi,\gamma) = -\infty$, so that couples choose non-marital relationships in the stable matching equilibrium.

As in the work of Weiss and Willis (1985), here marriage allows couples to attain the efficient level of public good investment through Nash bargaining. However, in non-marital relationships, mothers choose their private consumption and child investment without taking into account fathers' utility; thus, mothers under-invest in children. Furthermore, if fathers know their payments will not fully used for their children by mothers, they will not transfer enough child support payments to the mothers. Through these steps, inefficiency in public good investment arises in non-marital relationships.

### 1.2.4 Stable Matching Equilibrium

In this economy, women and men look for a partner in the frictionless marriage market. Again, they can form a marital or a non-marital relationship with whomever they want as long as the partner agrees, but we assume that they cannot form more than one relationship at the same time. Also, in equilibrium, all the agents must form some relationship with a partner.

Formally, we consider a set of matchings $(\mu_S, \mu_M)$, where $\mu_S$ is a matching for non-marital relationships (single $S$) and $\mu_M$ is a matching for marital ($M$) relationships. Mathematically, $\mu_S$ and $\mu_M$ are mappings from $\mathcal{I}^f \cup \mathcal{I}^m$ onto itself.[10] In particular here we only consider the sets of mappings which satisfy the following properties.

**Definition 1.** *A pair $(\mu_S, \mu_M)$ is defined as an **acceptable pair of matchings** if it satisfies the following properties:*

1. *$\forall R \in \{S, M\}$, if $\mu_R(x) \neq \emptyset$, then $\mu_R(\mu_R(x)) = x$.*
2. *$\forall R \in \{S, M\}$, if $x \in \mathcal{I}^g$ and $\mu_R(x) \neq \emptyset$, then $\mu_R(x) \in \mathcal{I}^{g'}$, where $g, g' \in \{f, m\}$ and $g \neq g'$.*
3. *$\forall R, R' \in \{S, M\}$ with $R \neq R'$, if $\mu_R(x) \neq \emptyset$, then $\mu_{R'}(x) = \emptyset$ for all $x \in \mathcal{I}^f \cup \mathcal{I}^m$.*

In short, an acceptable pair of $\mu_S$ and $\mu_M$ is a set of mappings which specify the couples in each relationship and in which no one in the pairs has more than one relationship. Next, we define an equilibrium of the economy using these matchings $\mu_S$ and $\mu_M$.

[10] Remember that we assumed $\mathcal{I}^f \cap \mathcal{I}^m = \emptyset$ at the beginning of this section. Thus, women and men are indexed by different numbers. And, $\mathcal{I}^f \cup \mathcal{I}^m$ denotes the entire set of types in the population.

**Definition 2.** *Given $\gamma \in [0,1]$, a* ***stable matching equilibrium*** *is an acceptable pair of matchings, $(\mu_S, \mu_M)$, which satisfies these two conditions:*

1. *$\forall R \in \{S, M\}$, a woman $i \in \mathcal{I}^f$ with $\mu_R(i) \neq \emptyset$ receives utility, $\hat{V}_R^f\left(h_i^f, a_i^f, h_{\mu_R(i)}^m, a_{\mu_R(i)}^m, \gamma\right)$, and a man $j \in \mathcal{I}^m$ with $\mu_R(j) \neq \emptyset$ receives utility, $\hat{V}_R^m\left(h_{\mu_R(j)}^f, a_{\mu_R(j)}^f, h_j^m, a_j^m, \gamma\right)$.*

2. *(No Blocking) There doesn't exist a pair of couples $(i, \mu_R(i))$,$(\mu_{R'}(j), j)$, and relationships $R, R', R'' \in \{S, M\}$ such that*

$$\hat{V}_{R''}^f\left(h_i^f, a_i^f, h_j^m, a_j^m, \gamma\right) > \hat{V}_R^f\left(h_i^f, a_i^f, h_{\mu_R(i)}^m, a_{\mu_R(i)}^m, \gamma\right),$$
$$\hat{V}_{R''}^m\left(h_i^f, a_i^f, h_j^m, a_j^m, \gamma\right) > \hat{V}_{R'}^m\left(h_{\mu_{R'}(j)}^f, a_{\mu_{R'}(j)}^f, h_j^m, a_j^m, \gamma\right).$$

The second condition in the equilibrium definition requires that there are no pairs of a woman and a man who jointly deviate from their current relationships and obtain higher utility by starting a new relationship with the partner. In other words, the equilibrium is a *core of matching game.*

As in the last part of this section, we now pose a theorem for the existence of a stable matching equilibrium. The proof is an extension of Gale and Shapley (1962)'s.

**Theorem 3.** *A stable matching equilibrium exists in the economy.*

*Proof.* In Appendix A.2.

□

### 1.2.5 Computing an Equilibrium

**Definition 4.** *A stable matching equilibrium $(\mu_S, \mu_M)$ is* ***M-optimal*** *if every man likes it at least as well as any other stable matching equilibria. Similarly, a stable matching equilibrium $(\nu_S, \nu_M)$ is* ***W-optimal*** *if every woman likes it at least as well as any other stable matching equilibria.*

Based on the approach of Del Boca and Flinn (2005), we focus on two extreme stable matching equilibria, the one that is most beneficial to men (the $M$-optimal stable matching equilibrium) and the one most beneficial to women (the $W$-optimal stable matching equilibrium). A straight-forward extension of the Gale and Shapley algorithm enables us to compute at least these two equilibria. In addition, because assuming that each individual has strict preference over mates, each of these equilibria turns out to be unique.

**Assumption 5.** *All the agents have strict preference over partners' types.*

To satisfy the above assumption, differences in human capital level have to create strictly different utility values for potential partners.[11] When couples choose non-marital relationships, it is not obvious because some low-income men might not make any child support payments. Then if those men's charm levels are the same, women become indifferent among the different types of men. To exclude this situation, we assume that $\gamma > 0$ always holds. If $\gamma$ is strictly positive, then women get better off by having a non-marital relationship with a man with higher income because the state government transfers child support payments proportionally to men's income. Thus, Assumption 5 is always satisfied.[12]

**Theorem 6.** *Under Assumption 5, both the M-optimal stable matching equilibrium and the W-optimal stable matching equilibrium are unique.*

*Proof.* See Appendix A.3. □

In Appendix A.1, we also show that these two equilibria can be computed by extending Gale and Shapley (1962)'s algorithm. In Section 1.4, we actually compute and estimate an unique M-optimal stable matching equilibrium. Hereafter, we refer to a stable matching equilibrium as an $M$-optimal stable matching equilibrium.[13]

## 1.3 The Data

To estimate the equilibrium which we defined in the previous section, we construct state-year panel data from the birth and the marriage records of the CDC's National Vital Statistics Report (NVSR). In this section, we discuss the details of the construction of our variables: the total fertility rate for marital and non-marital births and the marriage rate. We also describe how we create the CSE measures and discuss some other control variables like state characteristics.

[11] For charm, the utility function specified in Section 1.4.6 automatically creates strict differences.

[12] Here, we implicitly assuming that there don't exist $(h_j^m, a_j^m)$ and $(h_{j'}^m, a_{j'}^m)$ such that $h_j^m > h_{j'}^m$ , $a_j^m < a_{j'}^m$, and $\hat{V}_R^f(h_i^f, a_i^f, h_j^m, a_j^m) = \hat{V}_R^f(h_i^f, a_i^f, h_{j'}^m, a_{j'}^m)$ for some $i$ and $R$. When we actually compute an equilibrium and discretizing the state spaces $\mathcal{H}^g$ and $\mathcal{A}$, it is the case only as a measure zero event. Thus, we exclude the possibility of the case from our analysis.

[13] In this paper, we only analyze the $M$-optimal stable matching equilibrium. But, there may well exist other stable matching equilibria including the $W$-optimal stable matching equilibrium. Del Boca and Flinn (2005) explore how much other equilibria could be different from the M-optimal stable matching equilibrium by counting the pairs which exist both in the M-optimal stable and W-optimal matching equilibria. In their case, the same pairs are matched in over 96% of the cases in the male-preferred and female-preferred equilibria. And, they conclude that even though other equilibria exist, they are not so different from the M-optimal stable matching equilibrium. We will leave the application of this stability exercise for our future work.

### 1.3.1 Dependent Variables

We use three dependent variables for our main analysis: the total fertility rate (more precisely, the total period fertility rate) for marital births, the total fertility rate for non-marital births, and the marriage rate. For the marital and non-marital total fertility rates, first we calculate age-specific (the mother's age) marital and non-marital birth rates for six age groups for each state and year from the NVSR 1980 - 1997, restricting the mother's age to 15 - 44. And we sum them up and multiply the sum by five. For the marriage rate, we define it as the number of marriages per 15 - 44 year-old female population.[14] And, we calculate it also from the NVSR (1980 - 1995). The summary statistics for the NVSR: Natality data are in Table 1.2.

Table 1.2: Summary Statistics for National Vital Statistics Report Natality Data, 1980 - 1997

| **Name** | | |
|---|---|---|
| | | **Num. of Observations** |
| State-Year Panels | | 908 |
| Total Number of Births in the Data | | 69,315,940 |
| Average Number of Births within a Panel | | 76,339 |
| | **Mean** | **Std. Deviation** |
| Age of Mother at Child's Birth | 26.19 | 5.68 |
| Age of Father at Child's Birth | 26.78 | 5.50 |
| Mother's Education | **Num. of Observations** | **% in the Sample** |
| HS ¡ | 13,360,589 | 19.27 |
| HS = | 23,428,527 | 33.80 |
| HS ¿ | 23,463,224 | 33.85 |
| Mother's Race | | |
| White | 55,610,558 | 79.79 |
| Black | 10,940,109 | 15.78 |
| Ohters | 2,765,173 | 4.43 |

### 1.3.2 Child Support Enforcement Measures

Our main concern is to analyze how people adjust their marriage and fertility decisions to a change in the cost of non-marital births. Therefore, we think that the aggregate collection

[14] Although the best measure of the marriage rate would be the number of marriages per non-married population, the stock of (non)-married people is only available for the years in which the decimal U.S. Census has been conducted.

rate of child support defined for each state and year is the most suitable measure for our main analysis.[15] The aggregate collection rate is the total amount of child support collected over the total eligible amount of child support. For the numerator, we pick the numbers from the Office of Child Support Enforcement (OCSE) 1980 - 1997 Annual Reports to Congress (U.S. Department of Health and Human Services, 1980 - 1997). For the denominator, we calculate a number from the Current Population Survey (CPS) (U.S. Bureau of Labor Statistics) in the following way.

1. Consider the sample of the married couples. Regress the husband's real annual income on the wife's demographic characteristics and the characteristics of the residential state, age, age-squared, education, ethnicity, whether or not living in a central city, the unemployment rate of the state, and the average income of the state.

2. Predict the income of the single mother's partner from the above regression.

3. Based on the number of children the single mother has, apply the Wisconsin guideline to determine the eligible amount of child support.[16]

4. Calculate the total eligible amount for each state and year by summing up the amount obtained in step 3, and then adjust the population of single mothers in each state to match the numerator by using the Surveillance, Epidemiology, and End Results (SEER) data published by the National Center Institute.

Our result is displayed in Table 1.3. Also in Table 1.3, two other child support enforcement measures are calculated to test the robustness of our estimation result, . The first one is the state's expenditure for the child support enforcement policies per single mother.[17] The state's expenditure data are from the OCSE's Annual Reports, and the population of single mothers is calculated from CPS and SEER. The second alternative measure is paternity establishment rate, defined as the number of paternities established for non-marital births over the total number of non-marital births in a given year. The data for the number of paternities established are from the OCSE's Annual Reports. And the data for the total number of non-marital births are from the NVSR.

---

[15] One of the difficulties with CSE measures is the possibility of their endogeneity. Case (1998) applies the instrumental variable method by using the percentage of the state's House and Senate members that are women as an instrumental variable and still finds a significant negative impact of CSE on the non-marital birth rate. Miller and Garfinkel (1999) employ the same strategy to control the endogeneity. Without exception, our CSE measures might potentially involve endogeneity. We leave the problem for future work.

[16] The Wisconsin guideline is a "percentage-of-income guideline" in which fathers' child support obligation is 17% of fathers' gross income for one child and 25%, 29%, 31%, and 34% for two, three, four, and five or more children, respectively. This percentage-of-income guideline have been adopted by 15 states, whereas other 31 have adopted variants of an income-shares guideline which takes into account the incomes of both parents when determining the award amount. Garfinkel, Miller, McLanahan, and Hanson (1998) examine the two types of guideline and find that state rankings on the collection rate are not sensitive to the guideline used.

[17] The numbers in Table 1.3 are based on 2000 dollars.

Table 1.3: Summary Statistics for CSE Measures, 1980 - 1997.

| **CSE Measures** | **1980 Mean** | **1997 Mean** | **1980-1997 Mean** |
|---|---|---|---|
| Collection Rate | 4.1 % | 18.5 % | 10.0 % |
| Expenditure per Single Mother | 2.38 | 4.14 | 3.04 |
| Paternity Establishment Rate | 20.4 % | 55.0 % | 36.2 % |

(The mean values are calculated across states.)

### 1.3.3 Other Independent Variables

State average wages of full-time workers and state unemployment rates are included as independent variables in our regression analysis because they potentially affect people's marriage and fertility decisions. The former is from CPS, and the latter is from the Local Area Unemployment Statistics (LAUS) (U.S. Bureau of Labor Statistics). The gender wage gap as a fraction of the average wage of female full-time workers over the average wage of male full-time workers is also included since it might affect, especially, female's marriage and fertility decisions.[18] Three demographic statistics - the fraction of blacks, the fraction of Hispanics, and the fraction of high-school dropouts - are the other controls considered in the analysis. Finally, previous studies show that generous welfare benefits make single motherhood more affordable and increase the number of non-marital child births (Rosenzweig (1999); Neal (2004)). Thus, we include the generosity measure of state welfare policy. The sum of the maximum amount of the Aid to Families with Dependent Children (AFDC) grant and food stamp benefit is used as this measure.[19]

All the monetary values are converted to the 2000 dollar values.

## 1.4 Structural Estimation

In this section, we describe our econometric methodology to estimate the equilibrium values.

[18] See the work of Regalia, Ríos-Rull, and Short (2010a), for example.

[19] These data come from the *Overview of Entitlement Program: Green Book* edited by the U.S. House of Representatives, Committee on Ways and Means.

### 1.4.1 Two-Step Estimation Procedure

We apply simulated method of moment (SMM) to estimate the parameters of the structural model laid out in Section 2. In particular, we apply a two-step procedure similar to that used by Voena (2010). The method is closely related to (but not the same as) the indirect inference method, which is a simulation-based method for estimating the parameters of economic models, first introduced by Smith (1990, 1993) and extended by Gourieroux, Monfort, and Renault (1993) and Gallant and Tauchen (1996).[20]

Our method consists of two stages of minimization. In the first stage, we employ the standard fixed effects regression model to obtain the coefficients of the variables of interest to the changes in the CSE measure. In the second-stage regression, we estimate the structural model's parameters by SMM targeting on the coefficients which we obtained in the first stage. More precisely, using reduced-form regressions, first, we estimate the effect of CSE on non-marital and marital total fertility rates and obtain the regression coefficients, $(\hat{\beta}_{\gamma}^{S,Data}, \hat{\beta}_{\gamma}^{M,Data})$. Then, in the second stage, we calculate structural counterparts for $\left(\hat{\beta}_{\gamma}^{S}(\boldsymbol{\theta}), \hat{\beta}_{\gamma}^{M}(\boldsymbol{\theta})\right)$ and estimate a set of structural parameters $\boldsymbol{\theta}$ with other targets $Z(\boldsymbol{\theta})$ by

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} \left[\; m(\boldsymbol{\theta})^T \times \mathbf{W} \times m(\boldsymbol{\theta})\right],$$

where $m(\boldsymbol{\theta})$ is a column vector that equals to $\left(\hat{\beta}_{\gamma}^{S}(\boldsymbol{\theta}) - \hat{\beta}_{\gamma}^{S,Data}, \hat{\beta}_{\gamma}^{M}(\boldsymbol{\theta}) - \hat{\beta}_{\gamma}^{M,Data}, Z(\boldsymbol{\theta}) - Z^{Data}\right)^T$. Here $\mathbf{W}$ is an arbitrary weighting matrix. The other targets $Z(\boldsymbol{\theta})$ are chosen so that the model captures the important characteristics of the real data, like the total fertility rate or the total educational expenditure in the economy.

As the last step of our estimation, we test our model's performance by predicting the marriage rate. As we have said, the marriage rate has also sharply risen in the states with strict CSE. This increase in marriages is crucial to identify our story from other alternatives. Therefore, we use it as an over-identification device for the estimates obtained from the structural estimation.

### 1.4.2 First Stage: The Fixed Effects Regression Model

In the first-stage estimation, we use the fixed effects regression model. Moreover, we include state-specific time trends[21] in our benchmark model. Focusing on the period 1980 - 1997, we

[20] The difference between our method and indirect inference is as follows. In indirect inference, the regressions in the first stage (called an 'auxiliary model') and those in the second stage must be the same. But, in our case, the first-stage regression includes the state fixed effects, the state characteristics, and the state-specific trends, which are not included in the regressions in the second stage. In the indirect inference method, even if the independent variables are endogenous in the first-stage regression, one can obtain the consistent estimator in the structural estimation. But, in our case, if the independent variables are endogenous, then the estimated structural parameters are no longer consistent.

[21] The literature includes several discussions on the inclusion of state-specific time trends. Friedberg (1998a) talks about the importance of them to measure the impacts of the divorce law reform on the divorce rate. In our

run the following regressions on the state-year panel data:

$$Y^S_{s,t} = \beta^S_0 + \beta^{S\prime}_1 X_{s,t} + \sum_s \beta^S_{2,s} D_s + \sum_s \beta^S_{3,s} (D_s \times t) + \beta^S_\gamma \gamma_{s,t}$$
$$Y^M_{s,t} = \beta^M_0 + \beta^{M\prime}_1 X_{s,t} + \sum_s \beta^M_{2,s} D_s + \sum_s \beta^M_{3,s} (D_s \times t) + \beta^M_\gamma \gamma_{s,t},$$

In the above regressions, $Y^S_{s,t}$ and $Y^M_{s,t}$ are the total fertility rate for non-marital and marital births in state $s$ in year $t$. Here $X_{s,t}$ includes the characteristics of state $s$ in year $t$: the average wage of full-time workers, the state unemployment rate, the gender wage gap, the fraction of black people, the fraction of Hispanic people, the fraction of people without a high-school diploma, and also the measure of the generosity of welfare for single mothers. Here $D_s$ is a state dummy and $(D_s \times t)$ is a state-specific time trend. Finally, $\gamma_{s,t}$ is the three-year moving average of the CSE measure.[22]

### 1.4.3 First Stage: Results

The results of the first-stage regressions are summarized in Table 1.4. For the non-marital fertility rate, all three CSE measures have negative effects which are significant at the 1% level. Looking at the coefficient for the fraction of black people, you may wonder why it is significantly negative. But this is true when we include the state fixed effects, and other studies are also getting the same result.[23] For marital births, the effects of CSE seem to be a bit weak, but they are still positive at the 10% level for the collection rate measure and positive at the 5% level for the expenditure and paternity establishment rate measures.

### 1.4.4 Marriage Rate Regression

We also run the same regression for the marriage rate. The increase in the marriage rate is crucial to identify our story from other alternatives. So we will use this estimate later to check the model's performance. As for the regression result, we find significantly positive effects of CSE (at the 1% level) on the marriage rate for all three CSE measures as summarized in Table 1.5.

case, it also turns out to be important, especially for the estimations of the total fertility rate for marital births and the marriage rate. The results without the state-specific time trends are discussed in Section 4.4.

[22] We also consider the five-year moving average of the measures in Section 1.4.5, in order to check the robustness of the result.

[23] See the work of Garfinkel, Huang, McLanahan, and Gaylin (2003), for example.

Table 1.4: First Stage Regressions: The Total Fertility Rate for Non-Marital and Marital Births

| Dependent Variable | | Non-Marital Total Fertility Rate | | | Marital Total Fertility Rate | |
|---|---|---|---|---|---|---|
| CSE Measures (3-Year Moving Average) | | | | | | |
| 1) Collection | -0.29737$^{**}$ | | | 0.17234$^{\dagger}$ | | |
| | (0.07936) | | | (0.09502) | | |
| 2) Expenditure | | -0.01548$^{**}$ | | | 0.00738$^{*}$ | |
| | | (0.00258) | | | (0.00313) | |
| 3) Paternity | | | -0.04132$^{**}$ | | | 0.03481$^{*}$ |
| | | | (0.01451) | | | (0.01550) |
| Average Wage | -0.00014$^{**}$ | -0.00013$^{**}$ | -0.00012$^{**}$ | -0.00008 | -0.00008 | -0.00009$^{\dagger}$ |
| | (0.00004) | (0.00004) | (0.00004) | (0.00005) | (0.00005) | (0.00005) |
| Unemp. Rate | 0.00425$^{**}$ | 0.00421$^{**}$ | 0.00592$^{**}$ | -0.00577$^{**}$ | -0.00570$^{**}$ | -0.00116 |
| | (0.00128) | (0.00126) | (0.00139) | (0.00153) | (0.00153) | (0.00149) |
| Gender Gap | -0.00723 | -0.00548 | -0.00674 | 0.00507 | 0.00398 | 0.00467 |
| | (0.01463) | (0.01442) | (0.01440) | (0.01751) | (0.01747) | (0.01538) |
| Frac. Black | -0.57926$^{**}$ | -0.59053$^{**}$ | -0.50151$^{**}$ | -0.05292 | -0.05067 | -0.03460 |
| | (0.11592) | (0.11432) | (0.12300) | (0.13878) | (0.13850) | (0.13138) |
| Frac. Hisp. | -0.25437 | -0.23141 | 0.29846 | -1.18736$^{**}$ | -1.19020$^{**}$ | -0.31875 |
| | (0.16167) | (0.15908) | (0.18415) | (0.19355) | (0.19273) | (0.19670) |
| Frac. HS DP | 0.20511$^{*}$ | 0.16742$^{\dagger}$ | 0.12462 | 0.17183 | 0.19072 | 0.18766$^{\dagger}$ |
| | (0.09920) | (0.09798) | (0.10492) | (0.11877) | (0.11871) | (0.11207) |
| Max AFDC | 0.04237$^{**}$ | 0.04171$^{**}$ | 0.07972$^{**}$ | 0.02951$^{**}$ | 0.03013$^{**}$ | 0.07517$^{**}$ |
| | (0.00370) | (0.00359) | (0.00555) | (0.00443) | (0.00434) | (0.00593) |
| Intercept | -58.19391$^{**}$ | -56.04314$^{**}$ | -56.52598$^{**}$ | 18.04203$^{**}$ | 16.54161$^{**}$ | 9.01692 |
| | (4.96011) | (4.75847) | (5.45825) | (5.93830) | (5.76504) | (5.83009) |
| State Fixed Effects | Yes | Yes | Yes | Yes | Yes | Yes |
| State-Specific Trends | Yes | Yes | Yes | Yes | Yes | Yes |
| N | 908 | 908 | 806 | 908 | 908 | 806 |
| $R^2$ | 0.95963 | 0.96069 | 0.96057 | 0.97126 | 0.97135 | 0.97615 |
| F | 174.02847 | 178.92167 | 155.563 | 247.45659 | 248.18175 | 261.29123 |
| Significance levels : | $\dagger$ : 10% | $*$ : 5% | $**$ : 1% | | | |

(Standard errors are in parentheses.)

Table 1.5: Marriage Rate Regressions

| Dependent Variable | Marriage Rate | | |
|---|---|---|---|
| CSE Measures (3-Year Moving Average) | | | |
| 1) Collection | 0.01315** | | |
| | (0.00419) | | |
| 2) Expenditure | | 0.00061** | |
| | | (0.00015) | |
| 3) Paternity | | | 0.00302** |
| | | | (0.00076) |
| Average Wage | 0.00018 | -0.00025 | 0.00041† |
| | (0.00024) | (0.00023) | (0.00023) |
| Unemp. Rate | -0.00047** | -0.00046** | -0.00028** |
| | (0.00006) | (0.00006) | (0.00006) |
| Gender Gap | -0.00211 | -0.00186 | -0.00014 |
| | (0.00226) | (0.00224) | (0.00228) |
| Frac. Black | 0.00873 | 0.00853 | 0.00536 |
| | (0.00581) | (0.00577) | (0.00605) |
| Frac. Hisp. | -0.01984 | -0.01995 | -0.01791 |
| | (0.01275) | (0.01266) | (0.01317) |
| Frac. HS DP | 0.00148 | 0.00292 | 0.00300 |
| | (0.00481) | (0.00480) | (0.00500) |
| Max AFDC | -0.00077** | -0.00074** | 0.00027 |
| | (0.00018) | (0.00017) | (0.00029) |
| Intercept | 2.27407** | 2.16429** | 1.97683** |
| | (0.24523) | (0.23862) | (0.26663) |
| State Fixed Effects | Yes | Yes | Yes |
| State-Specific Trends | Yes | Yes | Yes |
| N | 681 | 681 | 591 |
| $R^2$ | 0.96773 | 0.96814 | 0.97179 |
| F | 180.25471 | 182.62504 | 184.0761 |
| Significance levels : | † : 10% | ∗ : 5% | ∗∗ : 1% |

(Standard errors are in parentheses.)

### 1.4.5 First Stage: Checking the Robustness

In this subsection, we briefly talk about the robustness of our first-stage estimation results. Details are available in Appendix A.2.

### Five-Year Moving Average of the CSE Measures

Since we have been taking only three-year moving averages of the CSE measures, it might be true that the dependent variables react too much to the change in the degree of CSE in the short term. In the long run, stronger CSE might affect the dependent variables more modestly. To check if that is true or not, we also consider the five-year moving averages of the CSE measures and run the same regressions. The results are summarized in Appendix A.4. As shown there, the five-year moving averages of the measures increase the effects of stronger CSE on the dependent variables. Therefore, we conclude that the CSE effects don't disappear even if we consider the longer time period. (They last for at least 5 years.)

### The Regressions without State-Specific Time Trends

Through our first-stage estimation, it turns out that the state-specific time trends, which we include in our bench-mark regressions, are important to capture the correct effects of stronger CSE on the dependent variables (In particular, for the marital total fertility rate and the marriage rate). To emphasize this point, we show the regression results without the state-specific time trends in Appendix A.4. In those regressions, instead, we include an aggregate time trend with standard state dummies. As you see in the tables, the results of the non-marital total fertility don't change signs or significance. However, the results of the marital total fertility do change signs. And for the marriage rate, the results become no longer significant. Friedberg (1998a) talks about the importance of state-specific time trends in her divorce rate regression. She reports that the effects of the divorce law reform on the divorce rate couldn't be observed without including state-specific time trends in the regression. That applies to our analysis as well.

## 1.4.6 Second Stage: Structural Estimation

Now we turn to the second-stage regression, where we estimate the structural model's parameters. In this subsection, we talk about the parameters, the moments to match, and the estimation procedure in our second-stage estimation.

### Parametrization

First, let us assume the following functional forms for utility and child investment functions.

$$u(c, a') + v(nq) = \ln(c) + a' + \alpha \ln(nq)$$
$$f_0(s) = (\iota + s)^{\eta_0}, f_1(s) = (\iota + s)^{\eta_1} \quad 0 < \eta_0, \eta_1 < 1.$$

These functional forms give us five parameters to be estimated, $(\alpha, \iota, \eta_0, \eta_1, \psi)$. Also, we assume that people's charm is normally distributed, with mean 0 and variance $\sigma_a^2$ for both women and men; $a \sim N(0, \sigma_a^2)$, Then $\sigma_a$ is another parameter to be estimated. Other parameters determined through estimation are $\delta$: utility discount for fathers out of wedlock, $\kappa$: utility value of marriage, and $\psi$: good cost per child. Then we end up with eight parameters to be estimated. We list them in Table 1.6. Other parameters in the model can be exactly identified from the data. Those are summarized in Table 1.7 and discussed below.

Table 1.6: Parameters to be Estimated in the Second Stage

| **Name** | **in Model** |
|---|---|
| Uitlity Discount for Fathers out of Wedlock | $\delta$ |
| Utility Value of Marriage | $\kappa$ |
| Variance of People's Charm | $\sigma_a$ |
| Parameter for Utilility Weight | $\alpha$ |
| Parameter for Child Investment Function | $\iota$ |
| Parameter for Child Investment Function (Non-Marital) | $\eta_0$ |
| Parameter for Child Investment Function (Marital) | $\eta_1$ |
| Goods Cost per Child | $\psi$ |

Table 1.7: Exactly Identified Parameters

| **Name** | **Symbol** | **Value** | **Source** |
|---|---|---|---|
| AFDC Benefit (1980) | $g(e, n)$ | 0.81×(Poverty Threshold) | Congressional Green Book |
| Child Support Order | $\tau^{cs}(n)$ | $0.17 \sim 0.31$ | Wisconsin Guideline |
| Time Cost for Children | $\phi$ | 0.075 | De La Crox et al. (2003) |
| Women's Mean Wage | $\mu_h^f$ | 2.20 | Greenwood et al. (2003) |
| Men's Mean Wage | $\mu_h^m$ | 2.58 | Greenwood et al. (2003) |
| Log Var. of Wages | $\sigma_h$ | 0.57 | Greenwood et al. (2003) |

**Aid to Families with Dependent Children (AFDC)** Eligibility and benefit levels of AFDC vary across states and time periods. In order to simplify this criteria, we assume that the maximum AFDC benefit is 81% of of the Federal poverty threshold, where the number (81%) comes from the average eligibility criteria across states in 1980.[24] We also assume that if a single mother's income is below the poverty threshold, the portion is compensated by the AFDC benefit, so that her income is equal to the one at the poverty threshold. This poverty threshold is set as $0.27\times$(Men's mean income) calculated from the 1980 data.[25] The poverty line increases by 25% every time a household gets an additional member.

**Child Support Order** We follow the Wisconsin guideline to determine the eligible amount of child support. The guideline says 17% of a payer's income if there is only one child, 25% for two children, 29% for three children, 31% for four children, and 34% for five or more.

**Time Cost for Children** We follow De La Croix and Doepke (2003) to determine the time cost for children. Based on their calculation, parent spends 7.5% of time per child in parent's entire life.

**Human Capital Distribution** We follow Greenwood, Guner, and Knowles (2003) to determine the human capital distributions for women and men. They assume log-normal distributions for human capital and match their mean and standard deviation to the wage data. More precisely, they assume that women's human $h^f$ follows a log-normal distribution with its parameters $\mu_h^f = 2.20$ and $\sigma_h = 0.57$. For men, they assume the parameters $\mu_h^m = 2.58$ and $\sigma_h = 0.57$. When computing the model, we discretize those distributions so that each type has the same number of people as we assumed in Section 1.2.1.

[24] This data come from the *Overview of Entitlement Program: Green Book* edited by the U.S. House of Representatives, Committee on Ways and Means.

[25] See the website; http://www.census.gov/hhes/www/poverty/data/threshld/.

Table 1.8: Targeted Values in the Second-Stage Estimation

| Name | Value | Data Source |
|---|---|---|
| (1) Regression Coefficient for Non-Marital Births: $\beta^S$ | -0.297 | First Stage Regression |
| (2) Regression Coefficient for Marital Births: $\beta^M$ | 0.172 | First Stage Regression |
| (3) 1980: Fraction of Non-Marital Births in Total Births | 0.162 | NVSR (1980) |
| (4) 1980: Total Fertility Rate | 1.887 | NVSR (1980) |
| (5) 1980: Total Educational Expenditure in Cons. Exp. | 0.104 | U.S. Dept. of Education |
| (6) 1980: Fertility Ratios Between | | |
| Non-Married and Married | 0.727 | NSFG (1979) |
| (7) 1980: Educational Investment Ratio Between | | |
| Non-Married and Married | 2.764 | U.S. Bureau of Labor Stat. |
| (8) 1980: Income Correlation among Married | 0.440 | CPS (1980) |
| (9) Fraction of Child's Consumption | | |
| in Married Household with One Child (OECD) | 0.147 | |

**Moments**

Listed in Table 1.8 the eight moments in the second-stage estimation. The seven parameters are thus over-identified. The regression coefficients for (1) the non-marital and (2) marital total fertility rates, $(\beta^S, \beta^M)$, and (3) the fraction of non-marital births are the most crucial targets, which identify $(\delta, \kappa)$. (4) The total fertility rate, (5) the total educational expenditure,[26] (6) the fertility ratio between the non-married and the married, and (7) the educational investment ratio between the non-married and the married jointly determine the four parameters, $(\alpha, \iota, \eta_0, \eta_1)$.[27]

(8) The fraction of child's consumption in married household with one child identifies $\psi$: good cost per child. (9) The income correlation among married couples determine $\sigma_a$. In the model, all the statistics (3) - (9) are calculated in the equilibrium in which CSE $\gamma \approx 0$. That is, assuming that CSE is very small in 1980, we calculate the model's counterparts for the pre-CSE targets.

[26] *Digest of Education Statistics*, edited by the Office of Educational Research and Improvement, U.S. Department of Education.

[27] The fertility ratio between the non-married and the married is calculated from the CDC's National Survey of Family Growth (NSFG) 1976. The educational investment ratio between the non-married and the married is calculated from *Household Expenditure on Children 2007-2008* in *the Monthly Labor Review*, U.S. Bureau of Labor Statistics.

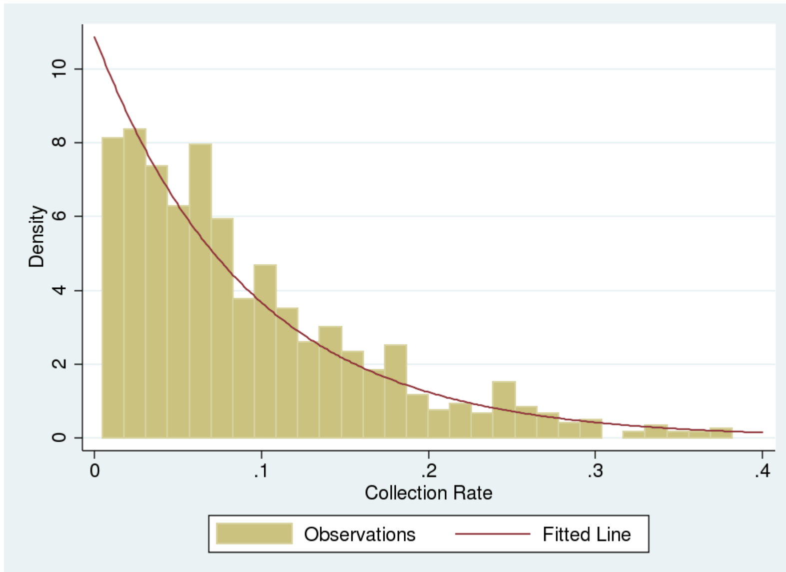


Figure 1.4: Distribution of Child Support Collection Rate, 1980-1997

**Second-Stage Estimation Procedure**

In the second-stage estimation, we simulate the model, run regressions, and obtain the model's counterpart for the regression coefficients $(\hat{\beta}^S_\gamma, \hat{\beta}^M_\gamma)$. In order to run this simulation, we first need to approximate the distribution of the child support collection rate in order to draw random policy values $\{\gamma_t\}$. Figure 1.4 shows such an approximation. The distribution of the collection rate is replicated by an exponential distribution with the same mean as in the data. Then we run the second-stage estimation by taking the following steps.

1. In every loop in the estimation, simulate the policy from the exponential distribution 908 times, $\{\gamma_t\}_{t=1}^{908}$. This is the actual number of the state-year observations in the first-stage regression.

2. Then, given a set of parameters $\theta$, compute the equilibrium for each given $\gamma_t$. In particular, when computing the equilibrium, we follow the steps below.

   (a) Discretize the spaces for human capital and charm.

   (b) After calculating the utility values for all possible pairs, $\left\{\hat{V}^f_S(\Phi,\gamma_t), \hat{V}^m_S(\Phi,\gamma_t)\right\}_{\Phi\in\mathcal{F}}$ and $\left\{\hat{V}^f_M(\Phi,\gamma_t), \hat{V}^m_M(\Phi,\gamma_t)\right\}_{\Phi\in\mathcal{F}}$, apply the extended version of Gale and Shapley's algorithm as described in Appendix A.1. Compute the distribution of the pairs in non-marital relationships and marital relationships. Then calculate the total fertility rate in each relationship.

3. Run the following regressions for the obtained marital and non-marital total fertility rates for each of $\{\gamma_t\}_{t=1}^{908}$.

$$Y_t^S = \beta_0 + \beta^S(\boldsymbol{\theta})\gamma_t$$
$$Y_t^M = \beta_0 + \beta^M(\boldsymbol{\theta})\gamma_t.$$

4. Construct other targets, $Z(\boldsymbol{\theta})$, and evaluate the model's performance by calculating

$$m(\boldsymbol{\theta})^T \times \mathbf{W} \times m(\boldsymbol{\theta})$$

where $m(\boldsymbol{\theta}) \equiv \Big(\hat{\beta}_\gamma^S(\boldsymbol{\theta}) - \hat{\beta}_\gamma^{S,Data}, \hat{\beta}_\gamma^M(\boldsymbol{\theta}) - \hat{\beta}_\gamma^{M,Data}, Z(\boldsymbol{\theta}) - Z^{Data}\Big)$.

5. Repeat steps 1 - 4 until the set of parameters attains the minimum of the above objective.

In the computation of the model, we discretize the state space for human capital and charm using 20 and 5 grids, respectively. Thus, there are 100 types of women and men in the economy.

## 1.5 Results

In this section, we talk about our estimation results. We also derive the CSE's implications for the next generation's income distribution.

### 1.5.1 Estimation Result

The model's performance and the estimation result for the parameters are summarized in Table 1.9 and Table 1.10. In Table 1.9, as you see in the data and model columns, our model is performing well; it closely matches most of the targets in the data including the coefficients, $\beta_\gamma^S$ and $\beta_\gamma^M$. One exception is the fraction that educational expenditures are of the total consumer expenditures. In the data, education is about 10% of total consumer expenditures, but the model suggests about 20%. One possible explanation for this difference is the lack of public education in the model. If we include this amount, the model might perform better.

In the model, men's increased willingness to marry is the driving force behind the decrease in non-marital births ($\beta_\gamma^S < 0$) and the increase in marital births ($\beta_\gamma^M > 0$). After the strengthening of CSE policies, facing the larger cost per child due to the mandatory child support payment, men in non-marital relationships may

1. Reduce the number of children and, instead, increase investments in child quality.

2. Reduce the number of children and, instead, increase the private consumption.

3. Get married to avoid the child cost change.

Option (1) is not attractive to unmarried fathers, because to increase a child's quality investment they have to transfer money to the mothers. But these transfers involve two types of inefficiency in our model: First, in non-marital relationships, since the mothers are not considering the fathers' utility, they don't invest all the money in children's quality. They use some of it for their private consumption. Second, if the mother in a non-marital relationship is on a welfare program, then the state government takes away a significant portion of child support payments made by the biological father. Therefore, fathers' investments don't really increase a child's quality. And unmarried fathers are thus left with options (2) or (3). As a result, men split between reducing the number of children and increasing marriage when facing the increased degree of CSE. This result hinges upon the parameter, $\delta$.

Table 1.9: The Match Between the Model and the Data

| **Name** | **Data** | **Model** |
|---|---|---|
| (1) Regression Coefficients for M Births: $\beta_\gamma^S$ | -0.297 | -0.272 |
| (2) Regression Coefficients for NM Births: $\beta_\gamma^M$ | 0.172 | 0.169 |
| (3) 1980: Fraction of Non-Marital Births in Total Births | 0.162 | 0.170 |
| (4) 1980: Total Fertility Rate | 1.887 | 1.899 |
| (5) 1980: Total Educational Expenditure in Cons. Exp. | 0.104 | 0.182 |
| (6) 1980: Fertility Ratios Between Non-Married and Married | 0.727 | 0.699 |
| (7) 1980: Educational Investment Ratio Between Non-Married and Married | 2.764 | 2.566 |
| (8) 1980: Income Correlations among Married | 0.440 | 0.434 |
| (9) Fraction of Child's Consumption (OECD) | 0.147 | 0.140 |

Table 1.10: Estimated Parameters

| Name | Parameter | Estimates |
|---|---|---|
| Uitlity Discount for Fathers out of Wedlock | $\delta$ | 0.491 |
| Utility Cost of Marriage | $\kappa$ | -0.185 |
| Parameter for Utilility Weight | $\alpha$ | 3.900 |
| Parameter for Child Investment | $\iota$ | 0.814 |
| Parameter for Child Investment (Non-Marital) | $\eta_0$ | 0.251 |
| Parameter for Child Investment (Marital) | $\eta_1$ | 0.610 |
| Variance of People's Charm | $\sigma_a$ | 0.128 |
| Goods Cost per Child | $\psi$ | 1.125 |

### 1.5.2 Over-Identification: Model's Performance for Marriage

We check the model's performance through its prediction for the increase of marriages after CSE strengthens. We calculate the changes in the number of the ever-married at age 45 after 10% increase in the collection rate by using the estimated coefficient for the marriage rate from the previous regression. According to our estimate in Table 1.5, a 10% gain in the collection rate will increase the marriage rate by 0.0013 points. (Using the 1980's average marriage rate, 0.042, this turns out to be a 3.1% increase of the marriage rate) If we calculate the number of the ever-married using the marriage rate in 1980 and the one after the 10% increase of the collection rate, the change from the former to the latter is about 1.93%. And, the change in the model is 1.39%. Thus, the model accounts for 72% of the increase in the ever-married in the data, which implies a good performance of our model to account for the changes. (See Table 1.11)

| | Num of Ever-Married (Data) | Num of Ever-Married (Model) |
|---|---|---|
| 1980 | 0.940 | 0.860 |
| CSE Δ10% | 0.958 | 0.871 |
| Change | Δ1.93% | Δ1.39% |

Table 1.11: Model's Performance for the Increase of Marriage

### 1.5.3 The Effects of CSE on Child Investment and Individual's Welfare

Next, we use the model to quantify the effects of CSE on child investment and individual's welfare. In 1.12, we calculate the changes in the amount of child investment, the quality of child, and the number of child after 10% increase in the collection rate. As found in the table, CSE increases the amount of child investment significantly among the bottom-income group. This change is driven by the change in people's marital status. As more couples start getting married, they pool their income together, and access to the better child investment technology, which results in the increase in child investment.

Table 1.12: The Changes in Child Investment After a 10% Increase in the Collection Rate

| **Mother's Income Group** | **Child Investment** $\Delta s$ | **Child Quality** $\Delta q$ | **Number of Child** $\Delta n$ |
|---|---|---|---|
| Top 0-20 % | -Δ0.1% | -Δ0.1% | +Δ0.0% |
| 20-40 % | -Δ0.1% | -Δ0.1% | +Δ0.0% |
| 40-60 % | +Δ0.0% | +Δ0.0% | +Δ0.0% |
| 60-80 % | +Δ1.1% | +Δ0.9% | -Δ0.5% |
| 80-100 % | +Δ5.0% | +Δ4.8% | -Δ2.0% |
| **Average** | +Δ1.2% | +Δ1.1% | -Δ0.5% |

Table 1.13 shows the associated changes in the number of the married, and women and men's welfare after 10% increase in the collection rate. As shown in the table, more people start getting married, especially, in the bottom-income group of people. This is caused by the men's increased willingness to marry; the mechanism we talk in the previous sections. And, that change of the increase in marriage is the driving force behind the increase in child investment.

Table 1.13 shows the changes in individual's welfare by income level. You will notice that there are significant transfers of utility from men to women even in the high-income groups of people. This is because women obtain more consumption within household after men's outside option values (the values of being single) decrease. This effect is not quite strong for women in the bottom-income group since not all the women in that group get married after 10% increase in the collection rate. Most of those unmarried women are on the welfare program (AFDC), and thus, they cannot enjoy the child support transfers from men because the government takes them away.

Table 1.13: The Changes in Marriage and Individual's Welfare After a 10% Increase in the Collection Rate

| | **Marriage** | | **Welfare** | |
|---|---|---|---|---|
| **Income Group** | **Women** | **Men** | **Women** | **Men** |
| Top 0-20 % | -$\Delta$0.0% | -$\Delta$0.0% | +$\Delta$2.6% | -$\Delta$2.2% |
| 20-40 % | -$\Delta$0.0% | -$\Delta$0.0% | +$\Delta$2.6% | -$\Delta$2.2% |
| 40-60 % | +$\Delta$0.0% | +$\Delta$0.0% | +$\Delta$2.7% | -$\Delta$2.3% |
| 60-80 % | +$\Delta$2.5% | +$\Delta$0.0% | +$\Delta$3.0% | -$\Delta$2.5% |
| 80-100 % | +$\Delta$5.0% | +$\Delta$7.5% | +$\Delta$1.5% | -$\Delta$5.1% |
| **Average** | +$\Delta$1.5% | +$\Delta$1.5% | +$\Delta$2.5% | -$\Delta$2.9% |

### 1.5.4 Inter Generational Human Capital Transmission

Finally, we look into the changes in the next generation's income distribution. Assume human capital in the next generation is log-normally distributed around child quality. The conditional mean is $\mu^f_{h|q}$ and $\mu^m_{h|q}$, and conditional variance, $\sigma_{h|q} = \sigma_g$:

$$\mu^f_{h|q} = \log\left(\epsilon_1 \times q^{\epsilon_2}\right)$$
$$\mu^m_{h|q} = \log\left(\epsilon_1 \times q^{\epsilon_2} + \mu_g\right).$$

To predict the next generation's income distribution, we first calibrate the parameters $(\epsilon_1, \epsilon_2, \sigma_g, \mu_g)$ so that (1) human capital distribution in the next generation is the same as in the previous generation in 1980, and (2) the correlation between son and father's income is $\rho_g = 0.73$ (Knowles (1999)). Then we use those human capital transmission functions to generate the next generation's income distribution. The calibration result of $(\epsilon_1, \epsilon_2, \sigma_g, \mu_g)$ is summarized in Appendix A.5.

Table 1.14 and Table 1.15 summarize the result of a 10% increase in the child support collection rate. Our model predicts that assuming a general human capital transmission function, the model predicts that the increased collection rate will increase people's income, especially, in the bottom group, and decrease the 90-10% income ratio of the next generation by 3.1%.

Table 1.14: The Changes in Income After a 10% Increase in the Collection Rate

| Income Group | Men's Income in the Next Generation $\Delta wh^m$ |
|---|---|
| Top 0-20 % | $+\Delta 0.0\%$ |
| 20-40 % | $+\Delta 0.1\%$ |
| 40-60 % | $+\Delta 1.3\%$ |
| 60-80 % | $+\Delta 1.7\%$ |
| 80-100 % | $+\Delta 3.0\%$ |
| **Average** | $+\Delta 1.22\%$ |

Table 1.15: The Changes in Income Distribution After a 10% Increase in the Collection Rate

| **Name** | **Before** | **After** | **Changes** |
|---|---|---|---|
| Men's 90-10 Income Ratio | 1.963 | 1.904 | -$\Delta$3.1% |
| Men's Gini Coefficient | 0.348 | 0.345 | -$\Delta$0.8% |

## 1.6 Conclusion

In this paper, we have analyzed the effects of the strengthened U.S. Child Support Enforcement policies on people's marriage and fertility decisions and long-term inequality. Despite their original purposes, the CSE policies have brought unexpected changes in people's marriage and fertility behaviors. Based on our new empirical findings, we propose a mechanism which accounts for the changes of non-marital births, marital births, and the marriage rate. We develop a novel stable matching model which features the choices of marital or non-marital relationships, and structurally estimate the model using the CDC's National Vital Statistics Report Natality data. Our results show that strengthened CSE increases child investment through secondary effect; the shift from non-marital births to marital births. And our model predicts that there will be a significant reduction in the poverty in the next generation through this change.

# Chapter 2

# Sex Ratios and Long-Term Marriage Trends

## 2.1 Introduction

The last century (from 1920 to 1990) has seen four important changes in the characteristics of marriage. The following features of the historical trends have been well documented in the literature: (i) The *incidence* of marriage *increased*, the fraction of never married women by age 50 went from .114 to .055. (ii) A delay in the median age of women, from 21 to 24. (iii) A reduction of the pervasiveness of the married state, resulting in a lower number of people living in a household formed by a married couple. In 1920, 59% of women was married compared to 52% in 1990. (iv) An increase in the divorce rate of women from 8% to 21%. Simultaneously, there have been important changes in the sex structure of the population both in the relative number of men and women and in the age characteristics of each sex. (a) The ratio of males to females aged 15 years and older fell from 1.05 to 0.926. (b) The fraction of women under 40 in the population went from 0.66 to 0.60 while for men went from 0.64 to 0.62. Another important secular change in the twentieth century was: (c) The increased easiness with which divorce was attainable.

In this paper, we ask the extent to which changes in the structure of the population have contributed to shape the changes in the characteristics of marriages. We start by posing a model of marital choice, where men and women differ in their numbers, and in their aging process. Men and women meet (not all since there are different numbers of them) and may or not choose to marry or to continue to be married, an outcome that depends on the investments made by agents who are guided by how much they like the specific partner they are matched with. In the second step of our analysis, we map the model to the born in 1950 by using a method of moments estimator of the relevant model parameters (for preferences, meeting technology, and mechanisms to create and maintain marriages and demographic turnover). The model works very well. In the third step, we substitute the demographic turnover characteristics of the model for the 1950 cohort with those that generate the population structure of 1870. We keep preferences and the mating process as estimated for 1950. The new predictions of the model are in the general direction of those in the data. In the fourth step, we add to the change in demographic structure a minimal change (one parameter) in the process to maintain marriages. We estimate this parameter alone using the other parameters at the values of the previous exercise to match the divorce rate of the 1870 cohort. This is a way to estimate the extent of the social changes that eased the divorce process. Finally, we ask what are the implications of the model for the other three characteristics of marriage for the 1870 cohort. We find that the model essentially matches all the other features of the data: (i) 94.5% of the increase in the incidence of marriage. (ii) 140.8% of the shrink the gender age gap in the median age at first marriage.

**Features of the model** We use an overlapping generations model of marriage with stochastic aging, where agents are randomly matched with partners and invest costly effort to form and maintain marriages. Demographics play two roles in determining marital status: men and women age at different rates within the model, in essence capturing biological constraints that determine the gains to marriage and the sex imbalance in the population determines the rate at which men and women meet, where the sex in short supply meets partners at a relatively fast rate. Together, biological constraints and differential mortality rates across gender determine the timing and attractiveness of marriage.

**Properties of estimates** Our parameter estimates suggest: (i) Men become attractive marriage partners later in life than women. (ii) Women lose their attractiveness in the marriage market earlier than men. (iii) Men are attractive for a longer portion of their lives than women.

**Papers with related question** Many alternative theories have been advanced in an attempt to explain the delay in marriage and rise in divorce. The most prominent explanations for the delay in marriage point to a fall in the gains to marriage. In Greenwood and Güner (2004), technological innovations in home production reduced the gains to specialization within marriage. Goldin and Katz (2002) suggest the introduction of the birth control pill reduced the gains to marriage as pre-marital sex became less costly. Both explanations are consistent with a decline in marriage and a rise in divorce. What existing theories cannot explain is a lesser known fact that changes considerably our understanding of the trends in marriage:

**Papers with related models** Our work builds on several strands of the literature on marriage. As in Siow (1998), biological constraints such as gender differences in fertility horizons play an important role in determining the timing of marriage. A large literature (see Becker (1981); Wilson and Neckerman (1986); Brien (1997); Angrist (2002); Seitz (2004)) studies the relationship between sex ratios, marriage and divorce. The model framework we adopt here is similar in spirit to recent equilibrium marriage models used to study marriage and divorce (Aiyagari, Greenwood, and Guner (2000b)), single motherhood (Regalia, Ríos-Rull, and Short (2010b)), and marital sorting (Fernandez, Güner, and Knowles (2004); Choo and Siow (2003)). Our work is also complementary to a recent literature on that examines the economic implications of the demographic transition (Nardi, Imrohoroglu, and Sargent (1999); Attanasio and Violante (2005)).

**Questions to which our model and findings can contribute to** The remainder of the paper is structured as follows. We describe the model that we use to study marriage and divorce in Section 2.2. Section 2.3 outlines the calibration procedure and provides evidence on the performance of our baseline model for 1990. In Section 2.4 we conduct model experiments to determine the extent to which the demographic transition, divorce liberalization, and changes in the gains to marriage can account for the trends in marriage and divorce. Section 2.5 concludes and discusses directions for future research.

## 2.2 The Model

The model has four main features: First, the model is an overlapping generations model with stochastic aging, where men and women have different life expectancies (a full description of demographics is contained in Section 2.2.1 and the aggregate state is described in Section 2.2.3). Second, agents match randomly, where the matching rate depends on the relative supplies of unmarried men and women (outlined in Section 2.2.4). Third, the quality of men and women as marriage partners changes as individuals age over time. Combined with the differential in life expectancies of men and women, this feature allows us to capture gender differences in the incentives to delay marriage and to divorce (preferences and individual's decisions are presented in Sections 2.2.5 and 2.2.6, respectively). Fourth, agents invest in costly effort to form and maintain relationships. The investment decisions agents make are described in Section 2.2.7. A steady state is defined in Section 2.2.8.

### 2.2.1 Demographics

At each point in time there are many agents differing in sex (male and female), $g \in \{m, f\}$, and maturity (adolescent, young, and old), $i \in \{a, y, o\}$. While sex is a permanent fixture of agents, an individual's maturity is stochastic with transition probabilities $\Gamma^g_{i,i'}$. All agents begin their lives as adolescents. Agents of any maturity can make contacts in the marriage market, but only the young and the old can form matches. From the point of view of the model, death or leaving the matching environment are equivalent, and this happens to agents of different maturities with probability $\pi^g_i$. The fact that men and women die at different rates generates differences in the age and sex distribution of the population. We normalize the measure of females to 1, and we denote the total number of males by $x^m$. To keep the population stationary, each period there is an inflow of newborn females ($n^f$) that equal the outflow of women through death. The measure of newborn males is equal to that for females. The measure of newborns is

$$n^f = n^m = \frac{\left[1 - \Gamma^f_{a,a}(1 - \pi^f_a)\right]\left[1 - \Gamma^f_{y,y}(1 - \pi^f_y)\right]}{1 - \Gamma^f_{y,y}(1 - \pi^f_y) + \Gamma^f_{y,o}(1 - \pi^f_y)\Gamma^f_{a,y}(1 - \pi^f_a)}. \tag{2.1}$$

Male adolescent immigrants $i_m$ also enter the market in every period. Immigration is introduced to allow us to account for exogenous changes in the aggregate stocks of men and women that we observe in the data but cannot attribute directly to mortality. Mortality and immigration determine both the age structure of the population and the sex imbalance in the model by determining the rate at which individuals exit the matching environment.

At the beginning of each period an agent can be in one of three marital states: single ($z = 0$), dating ($z = 1$) or married ($z = 2$). All couples must date for one period before becoming married.

### 2.2.2 Match Quality

In the first half of the period, each member of a couple draws match quality $a$. The match quality has two components, a Markov component and an i.i.d. component as $q = \mu + \epsilon$. A Markov component $\mu \in \{\mu_G, \mu_B\}$ has age-dependent transition probability $\Lambda^i$, and $\lambda$ is the initial probability of $\mu = \mu_G$. $\epsilon$ is drawn from a normal distribution as $\epsilon \sim N(0, \sigma^2)$. We denote it's cumulative distribution function as $\Phi(\hat{\epsilon}) = Prob(\epsilon < \hat{\epsilon})$. Whether the pair becomes a marriage depends on the realization of each member's match quality $(q^f, q^m)$.

### 2.2.3 Aggregates

We denote by $x^{g,i}(z, i^*, \mu, \mu^*)$ the measure of agents of gender $g$ and maturity $i$ that are paired in a type $z$ relationship with a partner of maturity $i^*$. The state variables $\mu$ and $\mu^*$ are the current regimes of match quality for partners respectively. Since every paired male must be matched with a paired female, a feasibility constraint is

$$x^{f,i}(z,i^*,\mu,\mu^*) = x^{m,i^*}(z,i,\mu^*,\mu) \qquad \forall\, z,i,i^*, \tag{2.2}$$

where

$$x^g(z,i^*,\mu,\mu^*) = x^{g,a}(z,i^*,\mu,\mu^*) + x^{g,y}(z,i^*,\mu,\mu^*) + x^{g,o}(z,i^*,\mu,\mu^*) \tag{2.3}$$

and

$$x^g(0) = x^{g,a}(0) + x^{g,y}(0) + x^{g,o}(0). \tag{2.4}$$

### 2.2.4 Matching Technology

At the end of each period there is a measure of available males and a measure of available females, composed of those agents who were single, those who were dating but who did not marry in the previous period, and those who were married and subsequently separated. All these agents meet via a constant-returns-to-scale matching function described by:

$$\psi^f = \min\left\{1, \frac{x^m(0) + x^m(1,.)}{x^f(0) + x^f(1,.)}\right\} \tag{2.5}$$

for women, and

$$\psi^m = \min\left\{1, \frac{x^f(0) + x^f(1,.)}{x^m(0) + x^m(1,.)}\right\} \tag{2.6}$$

for men. The matching technology depends directly on the sex ratio of available agents, the ratio of potential spouses to potential competitors. In particular, the gender in short supply meets a potential spouse with certainty, while the opposite sex meets partners at a rate equal to the size of the sex imbalance in the population of available men and women. The measures of the single population $\{x^f(0), x^m(0)\}$ and the paired population $\{x^f(z,i^*,\mu,\mu^*), x^m(z,i^*,\mu,\mu^*)\}$ of women and men, respectively, refer to the situation after the meetings have occurred and we refer to this as the beginning of the period.

### 2.2.5 Preferences

Preferences differ by gender and current marital status. The utility function for a single or dating individual of gender $g$ is described by $u^g(0)$. If married, preferences also depend on two other factors: *(i)* the maturity of the spouse and *(ii)* match quality as $u^g(i^*) = \alpha^g_{i^*} + q$.

### 2.2.6 Value Functions

In this section, we describe the problems faced by agents of different ages and in different marital states. In each instance, we denote by $V^g(i,z,i^*,\mu,\mu^*)$ the value of a gender $g$ person of maturity

$i$, with marital status $z$, that has a pairing with an agent of maturity $i^*$, where $i^*$ is equal to zero for unpaired agents. The states $\mu$ and $\mu^*$ are the current regimes of match quality. The future is discounted at rate $\beta$. Note that we are implicitly assuming stationarity in the sense that agents assume meeting rates and the behavior of other agents to be time invariant.

**Single agents**

The value for a single agents of gender $g$ and age $i$ is

$$\begin{aligned} V^{g,i}(0,0,0,0) \;=\; & u^g(0) \;+\beta\;(1-\pi^g)\;\sum_{i'}\Gamma^g_{i,i'}\Bigg\{\;(1-\psi^g)\;V^{g,i'}(0,0,0,0) \\ & + \psi^g \sum_{i^{*\prime},\mu,\mu^*} \frac{x^{g^*,i^{*\prime}}(1,.)}{x^{g^*}(0)+x^{g^*}(1,.)}\;\lambda(\mu)\lambda(\mu^*)\;V^{g,i'}(1,i^{*\prime},\mu,\mu^*)\Bigg\} \end{aligned} \tag{2.7}$$

for $i, i', i^{*\prime} \in \{a, y, o\}$, where $g^*$ denotes the gender of the opposite sex. The first term is the period utility of being single. The second term, the expected value of entering the next period unmarried, is composed of two parts. The first is the value of being unpaired and the second is the value of dating, conditional on meeting a spouse of age $i^*$.

**Paired agents**

The value functions for paired agents depend on the actions of both members of the couple. The couple is indexed by $\{z, i, i^*, \mu, \mu^*\}$: marital status (dating or married), the maturities of the female and her spouse, and the current regimes of match quality for the female and her spouse. The value of being a paired individual is given by:

$$\begin{aligned} V^{g,i}(z,i^*,\mu,\mu^*;\hat{\epsilon}_{g^*,i^*}) = & \max_{\hat{\epsilon}_{g,i}} \left\{V^{g,i}(0,0,0,0)-\omega 1_{[z=2]}\right\}\Phi(\hat{\epsilon}_{g,i})\;\Phi(\hat{\epsilon}_{g^*,i^*}) \\ & + \int_{\hat{\epsilon}_{g,i}}^{\infty}\int_{\hat{\epsilon}_{g^*,i^*}}^{\infty}\Bigg\{\alpha^g_{i^*}\;+\;\mu\;+\;\epsilon_g\;+ \\ & + \;\beta(1-\pi^g)\Big[(1-\pi^{g^*})\sum_{i',i^{*\prime},\mu',\mu^{*\prime}}\Gamma^g_{i,i'}\Gamma^{g^*}_{i^*,i^*}\Lambda^{i'}_{\mu,\mu'}\Lambda^{i^{*\prime}}_{\mu^*,\mu^{*\prime}}V^{g,i'}(2,i^{*\prime},\mu',\mu^{*\prime}) \\ & + \;\beta\pi^{g^*}\sum_{i'}\Gamma^g_{i,i'}V^{g,i'}(0,0,0,0)\Big]\Bigg\}\;d\Phi(\epsilon_g)\;d\Phi(\epsilon_{g^*}) \end{aligned} \tag{2.8}$$

for $i, i^* \in \{a, y, o\}$, where $\omega$ is the divorce cost for individuals.

In the above problem, a member of the couple with gender $g$ and age $i$ chooses the cutoff value $\hat{\epsilon}_{g,i}(z, i^*, \mu, \mu^*)$ for the realization of the i.i.d. component of match quality given his/her parner's cutoff value $\hat{\epsilon}_{g^*,i^*}(z, i, \mu^*, \mu)$: If the realized value of $\epsilon_g$ is greater than $\hat{\epsilon}_{g,i}(z, i^*, \mu, \mu^*)$, the indivual accepts being married in the next period. The probablity that the couple will break up in the next period is, then, $\Phi(\hat{\epsilon}_{g,i}) \times \Phi(\hat{\epsilon}_{g^*,i^*})$. The first term on the right hand side in the equation (2.8) is the utility the individual would get when breaking-up with the current partner. With probability $1 - \Phi(\hat{\epsilon}_{g,i}) \times \Phi(\hat{\epsilon}_{g^*,i^*})$, the pair continues and the agents are married in the next period.

### 2.2.7 Cutoff Strategies

The decisions on the cutoff values are taken in pairwise meetings by agents that play Nash. A female member of each couple solves:

$$\begin{aligned}
\hat{\epsilon}_{f,i}(z, i^*, \mu, \mu^*, \hat{\epsilon}) = \operatorname{argmax}_{\hat{\epsilon}_{f,i}} & \left\{ V^{f,i}(0,0,0,0) - \omega 1_{[z=2]} \right\} \Phi(\hat{\epsilon}_{f,i})\ \Phi(\hat{\epsilon}) \\
& + \int_{\hat{\epsilon}_{f,i}}^{\infty} \int_{\hat{\epsilon}}^{\infty} \Bigg\{ \alpha^f_{i^*} \ +\ \mu\ +\ \epsilon_f\ + \\
& + \ \beta(1-\pi^f) \Big[ (1-\pi^m) \sum_{i', i^{*\prime}, \mu', \mu^{*\prime}} \Gamma^f_{i,i'} \Gamma^m_{i^*,i^*} \Lambda^{i'}_{\mu,\mu'} \Lambda^{i^{*\prime}}_{\mu^*,\mu^{*\prime}} V^{f,i'}(2, i^{*\prime}, \mu', \mu^{*\prime}) \\
& + \ \beta \pi^m \sum_{i'} \Gamma^f_{i,i'} V^{f,i'}(0,0,0,0) \Big] \Bigg\}\ d\Phi(\epsilon_f)\ d\Phi(\epsilon_m)
\end{aligned} \tag{2.9}$$

The male solves a similar problem that yields the solution $\hat{\epsilon}_{m,i^*}(z, i, \mu^*, \mu, \hat{\epsilon})$. A Nash equilibrium is a pair of values that are a fixed point denoted, as above with a slight abuse of notation, by $\hat{\epsilon}_{f,i}(z, i^*, \mu, \mu^*)$ and $\hat{\epsilon}_{m,i^*}(z, i, \mu^*, \mu)$ that satisfy

$$\begin{aligned}
\hat{\epsilon}_{f,i}(z, i^*, \mu, \mu^*) &= \hat{\epsilon}_{f,i}(z, i^*, \mu, \mu^*, \hat{\epsilon}_{m,i^*}(z, i, \mu^*, \mu, \hat{\epsilon}_{f,i})) \\
\hat{\epsilon}_{m,i^*}(z, i, \mu^*, \mu) &= \hat{\epsilon}_{m,i^*}(z, i, \mu^*, \mu, \hat{\epsilon}_{f,i}(z, i^*, \mu, \mu^*, \hat{\epsilon}_{m,i^*})).
\end{aligned} \tag{2.10}$$

### 2.2.8 Steady states

A steady state requires that agents maximize and that the allocation is stationary. Formally,

**Definition 7.** *A steady state is a distribution of the population across sex, maturity, marital status, and spousal maturity* $\{\tilde{x}^g(z, i^*, \mu, \mu^*), \tilde{x}^g(0)\}$, *a set of value functions* $\{\tilde{V}^{g,i}(z, i^*, \mu, \mu^*)\}$, *and a set of cutoff strategies* $\{\tilde{\epsilon}_{g^*,i^*}(z, i^*, \mu, \mu^*)\}$ *for* $g \in \{f, m\}$, $i, j \in \{a, y, o\}$, $z \in \{1, 2\}$, *and* $\mu, \mu^* \in \{\mu_G, \mu_B\}$ *such that:*

1. *The value functions satisfy (2.7) and (2.8).*

2. *Agents play Nash (2.9) and (2.10).*

3. *Individual and aggregate behavior are consistent; agent's choices yield the stationary distribution which for single females is:*

$$\widehat{x}^{f,i'}(0) = \Big[1 \,-\, \psi^f\Big]\Big\{ \sum_{i,\mu,\mu^*} \Gamma^f_{i,i'} \, \Lambda^{i'}_{\mu,\mu'} \Lambda^{i^{*\prime}}_{\mu^*,\mu^{*\prime}} \, (1-\pi^f) \, \Big\{\widehat{x}^{f,i}(0) + \sum_{z,i^*} \pi^m \, \widehat{x}^{f,i}(z,i^*,\mu,\mu^*) + \sum_{z,i^*} (1-\pi^m) \, \Phi(\hat{\epsilon}_{g,i}) \, \Phi(\hat{\epsilon}_{g^*,i^*}) \;\; x^{f,i}(z,i^*,\mu,\mu^*)\Big\}\Big\}, \quad (2.11)$$

*for dating mature and old females is*

$$\widehat{x}^{f,i'}(1,i^{*\prime},\mu',\mu^{*\prime}) = \psi^f \Big\{ \sum_{i} \Gamma^f_{i,i'} \, \Lambda^{i'}_{\mu,\mu'} \Lambda^{i^{*\prime}}_{\mu^*,\mu^{*\prime}} \, (1-\pi^f) \, \Big\{\widehat{x}^{f,i}(0) + \sum_{z,i^*} \pi^m \, \widehat{x}^{f,i}(z,i^*,\mu,\mu^*) + \sum_{z,i^*} (1-\pi^m) \, \Phi(\hat{\epsilon}_{g,i}) \, \Phi(\hat{\epsilon}_{g^*,i^*}) \;\; \widehat{x}^{f,i}(z,i^*,\mu,\mu^*)\Big\}\Big\}, \quad (2.12)$$

*and for mature and old married females is*

$$\widehat{x}^{f,i'}(2,i^{*\prime},\mu',\mu^{*\prime}) = (1-\pi^f) \, (1-\pi^m) \, \sum_{i,j,z} \Gamma^f_{i,i'} \, \Gamma^m_{i^*,i^{*\prime}} \, \Lambda^{i'}_{\mu,\mu'} \Lambda^{i^{*\prime}}_{\mu^*,\mu^{*\prime}} \times [1 - \Phi(\hat{\epsilon}_{g,i}) \, \Phi(\hat{\epsilon}_{g^*,i^*})] \;\; \widehat{x}^{f,i}(z,i^*,\mu,\mu^*). \quad (2.13)$$

*Similar conditions are required for males.*

4. *The meeting probabilities $\psi^g$ are consistent with the number of people that end a period single and with the meeting technology.*

### 2.2.9 Taking Stock

In the following section, we assess the extent to which our parsimonious model can account for the trends in marriage. Before proceeding to our quantitative analysis, it is worth discussing the role of the two main demographic mechanisms in shaping marital status in the model.

#### Mortality Rates

Mortality plays three roles in the analysis. First, a fall in mortality rates for the opposite sex implies an increase in the expected future value of marriage, as the probability of remaining married in the future increases. The second role of mortality is through its effect on the marriage opportunities for men versus women through the sex ratio. If mortality rates fall to a greater extent for women than for men, as we observe in the data, then men are predicted to experience an improvement in marriage market conditions. Thus, we expect the fall in mortality to benefit men more than women along two dimensions: the value of marriage increases because one's current spouse is more likely to survive and the value of being single increases as one's marriage market improves. Third, the age composition of the population is determined by mortality rates, where a fall in mortality rates is consistent with an increase in the average age in the population.

#### Immigration Rates

We model immigration as an inflow of young men into the marriage market in every period. This assumption, although restrictive, is consistent with the fact that men are more likely to immigrate than women and the young are more likely to immigrate than the old. An increase in the immigration rate serves two roles in the model, similar to the effect of a fall in the male mortality rate. First, marriage market conditions for women improve as immigration for men increases, as more potential husbands become available. Second, an increase in the immigration rate results in a decrease in the average age of men in the model.

## 2.3 Mapping the Model to the Data

In this section, we describe how to specify the model so that its equilibrium yields the demographic structure and marriage behavior for the cohort born in 1950, i.e. the calibration. We start by choosing the functional forms and listing the parameters that we need to calibrate in Section 2.3.1. In Section 2.3.2, we describe the set of targets we use to solve for the parameter values. We present and interpret the parameter values in Section 2.3.3. Evidence on the ability of the baseline model to match the targets and other features of the data is presented in Section 2.3.4

### 2.3.1 Parameters

In addition to the discount factor, which we set equal to 0.95, the model has 24 parameters. We divide the parameters into three groups: *(i)* demographic parameters (3), *(ii)* preference parameters (11), and *(iii)* parameters determining the match quality (10). Each set of parameters is discussed in turn below.

#### Demographics

Agents remain alive at rate $\pi^g$ ($g \in \{f, m\}$). There is also a immigration parameter for men ($i^m$).

#### Preferences

The current period utility function takes the form: $u^g(j) = \alpha_j^g$ for paired men and women and $u^g(0) = 0$ for single agents. Preferences for a paired agent depend only on gender and on the age of the agent's spouse. We assume all men and women start out young and age stochastically over time. As a result, there are eight preference parameters to be determined $\{\alpha_a^f, \alpha_y^f, \alpha_o^f, \alpha_a^m, \alpha_y^m, \alpha_o^m, \Gamma_{a,y}^f, \Gamma_{y,o}^f, \Gamma_{a,y}^m, \Gamma_{y,o}^m\}$.

#### Match Quality

Agents decide wheather they get married or not observing the realization of match quality. The parameters, which govern the initial distribution and the transition of match quality, are $\{\mu_G, \mu_B, \sigma, \lambda, \Lambda_{GG}^a, \Lambda_{GG}^y, \Lambda_{GG}^o, \Lambda_{BB}^a, \Lambda_{BB}^y, \Lambda_{BB}^o\}$.

### 2.3.2 Targets

We choose the parameters to match three sets of targets: *(i)* 3 demographic targets summarizing the age and sex structure of the population, *(ii)* 28 statistics summarizing detailed marriage behavior and divorce behavior by age.

#### Demographics

We want the model to match the age and sex structure of the population for the 1950 cohort. To this end, we set the demographic parameters so that they match the life expectancies for men and women and sex ratios for the population that is active in the matching environment. We target the life expectancy of men and women at age 15 (two targets) and the number of men per 100 women aged 15 and above (one target).[1] The values of the demographic targets are presented in Table 2.1.

Table 2.1: Demographic Targets

| | 1950's |
|---|---|
| Men per 100 women (aged 15 and above) | 92.9 |
| Life expectancy of women (at age 15) | 61.0 |
| Life expectancy of men (at age 15) | 54.4 |

#### Marital Status

There are three sets of marital status statistics that we want the baseline model economy to match, resulting in a total of fourteen marital status targets. First, we want the model to capture the marriage rates for agents at different ages. For each sex, we therefore target the marriage

[1] Sex ratios are computed from Table 094 of the International Data Base of the U.S. Bureau of the Census.

rates, per 1,000 in the relevant unmarried population for six age groups and both genders for the 1950 birth cohort (twelve targets). Second, to match the incidence of marriage, we target the fraction of men and women that never marry by the age of 50 for each sex (two targets). The last sixteen targets we consider are the divorce rates per 1,000 married couples for six age groups (twelve targets), the first age at marriage (two targets), and the percent of the people aged 16 to 49 that are married, (two targets). The values for the marriage and divorce targets are presented in Table 2.2.

Table 2.2: Marriage Rates and Divorce Statistics by age

| Age | Women | Men |
|---|---|---|
| **Marriage rates by age, per 1,000 unmarried** | | |
| 16-19 in 1965 | 22.8 | 29.6 |
| 20-24 in 1970 | 15.8 | 18.8 |
| 25-29 in 1975 | 15.1 | 14.5 |
| 30-34 in 1980 | 15.9 | 13.3 |
| 35-39 in 1985 | 16.6 | 13.0 |
| 40-44 in 1990 | 17.0 | 13.0 |
| **Divorce rates by age, per 1,000 married** | | |
| 16-19 in 1965 | 19.9 | 29.8 |
| 20-24 in 1970 | 19.3 | 17.3 |
| 25-29 in 1975 | 18.1 | 16.4 |
| 30-34 in 1980 | 17.4 | 15.1 |
| 35-39 in 1985 | 15.5 | 13.0 |
| 40-44 in 1990 | 15.4 | 11.2 |
| **Marriage incidence** | | |
| % of women never-married by age 50 in 1990 | | 5.5 |
| % of men never-married by age 50 in 1990 | | 6.4 |
| **Age at marriage** | | |
| Median first age at marriage for women | | 22.0 |
| Median first age at marriage for men | | 24.7 |
| **Marriage preverence** | | |
| % of Women aged 16 to 49 that are married | | 56.7 |
| % of Women aged 16 to 49 that are married | | 52.8 |

There are three features of the marriage data in particular that we want the model to replicate. First, marriage rates rise then fall with age, peaking between the ages of 25 to 29 for both men and women. Second, despite the fact that marriage rates for men and women peak during the same age range, marriage rates for men are low relative to those of women before the age of 35, and high relative to those of women after the age of 35. Both trends are consistent with the well documented fact that men tend to marry younger women. Both trends are reflected in the stocks through the higher fraction of women that are married prior to age 35 and the higher fraction of men that are married thereafter. Finally, as illustrated in Table 2.2, the data on marriage incidence and divorce indicate that men have both higher rates of entry into and exit from marriage.

### 2.3.3 Parameter Values

In this section, we briefly summarize the estimation results. We start with the parameters that describe the biological aging process in the model and the parameters that determine how the gains to marriage change as individuals age, presented in Table 2.3. The parameter estimates indicate that *(i)* there exists a peak age of attractiveness for men and women in the marriage market, *(ii)* women become attractive marriage partners at an earlier age than men, and that *(iii)* men remain attractive marriage partners for a longer period of time than women. All three features of the estimates are consistent with the biological differences across gender, where men mature more slowly than women and women become infertile earlier than men. In fact, the ages at which men and women transit from middle-aged to old in the model are strikingly consistent with the ages at which reproductive ability starts to fall: between the ages of 27 to 29 for women and after the age of 35 for men (Dunson, Colombo, and Baird (2002)).

Table 2.3: Estimated Values of the Preference and Aging Parameters in the Baseline Model

| Parameter | Value |
|---|---|
| Female's preferences over adolescent spouse ($\alpha_a^f$) | -14.08 |
| Female's preferences over young spouse ($\alpha_y^f$) | -2.29 |
| Female's preferences over old spouse ($\alpha_o^f$) | -3.50 |
| | |
| Male's preferences over adolescent spouse ($\alpha_a^m$) | -14.96 |
| Male's preferences over young spouse ($\alpha_y^m$) | 11.15 |
| Male's preferences over old spouse ($\alpha_o^m$) | -1.71 |
| | |
| Average age at which women become young | 21.5 |
| Average age at which women become old | 25.3 |
| | |
| Average age at which men become young | 21.4 |
| Average age at which men become old | 27.7 |

Table 2.4 presents the calibrated values of the parameters determining the ease with which agents enter and exit marriage. The estimates indicate that everyone starts with bad regime when dating. Also, the transition probability of switching to a good match ($\mu_G$) from a bad match ($\mu_B$) is higher for the adolescent and for the young. Furthermore, the estimates show that the Probability of swithcing to a bad match ($\mu_B$) from a good match ($\mu_G$) is higher for the young. These properties of the match quality captures the patterns of marriage and divorce in the data; both the marriage rate and the divorce rate are higher for the young.

Table 2.4: Estimated Values of the Match-Quality Parameters in the Baseline Model

| Parameter | Value |
|---|---|
| Mean of match quality in good regime,$\mu_G$ | 8.28 |
| Mean of match quality in bad regime,$\mu_B$ | -20.5 |
| Variance of match quality, $\sigma$ | 7.01 |
| | |
| Initial dist. of good match, $\lambda$ | 0.000 |
| Initial dist. of bad match, $1-\lambda$ | 1.000 |
| Transition probability of regimes, $\Lambda^a_{G,G}$, for adolescent | 0.996 |
| Transition probability of regimes, $\Lambda^a_{B,B}$, for adolescent | 0.066 |
| Transition probability of regimes, $\Lambda^y_{G,G}$, for young | 0.980 |
| Transition probability of regimes, $\Lambda^y_{B,B}$, for young | 0.011 |
| Transition probability of regimes, $\Lambda^o_{G,G}$, for old | 1.000 |
| Transition probability of regimes, $\Lambda^o_{B,B}$, for old | 0.647 |
| | |
| Cost of divorce | 2.36 |

### 2.3.4 Performance of the Baseline Model Economy

In this section we assess the extent to which the baseline model matches the statistics in the data. It is worth emphasizing that, in addition to the discount factor and demographic parameters, the benchmark model has 21 parameters and 29 targets; thus the model is over-identified. Overall, our parsimonious model is able to replicate the data very well. Table 2.5 shows the model is able to simultaneously match the divorce rates and the incident rates of marriage, including the fact that men have both higher exit and entry rates than women.

Table 2.5: Model Performance: Marriage and Divorce Statistics

| | Women | | Men | |
|---|---|---|---|---|
| | Data | Model | Data | Model |
| **Marriage Rates by Age, per 1,000 Unmarried** | | | | |
| 16-19 in 1965 | 127.7 | 135.9 | 57.8 | 74.2 |
| 20-24 in 1970 | 220.6 | 206.1 | 184.4 | 167.7 |
| 25-29 in 1975 | 129.4 | 151.1 | 145.6 | 156.4 |
| 30-34 in 1980 | 105.6 | 95.5 | 124.1 | 120.4 |
| 35-40 in 1985 | 68.9 | 61.5 | 80.9 | 90.0 |
| 40-44 in 1990 | 60.1 | 43.8 | 75.3 | 69.7 |
| **Divorce Rates by Age, per 1,000 Married** | | | | |
| 16-19 in 1965 | 19.9 | 22.8 | 29.8 | 29.6 |
| 20-24 in 1970 | 19.3 | 15.8 | 17.3 | 18.8 |
| 25-29 in 1975 | 18.1 | 15.1 | 16.4 | 14.5 |
| 30-34 in 1980 | 17.4 | 15.9 | 15.1 | 13.3 |
| 35-39 in 1985 | 15.4 | 16.6 | 11.7 | 13.0 |
| 40-44 in 1990 | 15.4 | 17.0 | 11.2 | 13.0 |
| **Marriage Incidence** | | | | |
| % Never-Married by Age 50 in 1990 | 5.5 | 5.4 | 6.5 | 6.5 |
| **Age at Marriage** | | | | |
| | 22.0 | 22.0 | 24.7 | 24.7 |
| **Percent aged 16 to 49 that are Married** | | | | |
| | 56.7 | 50.9 | 52.8 | 50.7 |

Turning to Table 2.5, the model is able to replicate two important features of the data on marriage rates: the hump-shaped marriage rate profiles by age and the peak ages of marriage for men and women. The model is also able to generate the high marriage rates of women relative to men before the age of 35 and the opposite trend at later ages.

## 2.4 Can Demographics Account for the Trends in Marriage Since the 1870's Birth Cohort?

### 2.4.1 Changes in Demographics

In this section, we consider the extent to which changes in demographics alone account for the changes in marriage and divorce since the birth of the 1870 cohort (exactly 80 years earlier). The demographic trends on which we focus are the rise in life expectancy and the fall in the ratio of men to women in the population, as presented in Table 2.6.

Table 2.6: The Demographic Transition: 1870's to 1950's Birth Cohort

| | 1870 | 1950 |
|---|---|---|
| Men per 100 women (aged 15 and above) | 104.3 | 92.9 |
| Life expectancy of women (at age 15) | 45.6 | 61.0 |
| Life expectancy of men (at age 15) | 44.5 | 54.4 |

Targetting on those values, we recalibrate the demographic parameters. Then, we ask the extent to which changes in the structure of the population have contributed to the changes in the characteristics of marriages. When recalibrating the demographic parameters for the 1870's, we set $\pi^f(a)$ and $\pi^f(o)$ so that the rate of the change of the mortality to 1950's is same as men's. We adjust $\pi^f(y)$ for 1870's to match women's life expectancy in those years as shown in Table B.2.[2] Immigration is then set to match the sex ratio.

Table 2.7: The Change in Mortality rate: 1870's to 1950's Birth Cohort

| Mortality Rate | $\pi^f(a)$ | $\pi^f(y)$ | $\pi^f(o)$ | $\pi^m$ |
|---|---|---|---|---|
| 1950's | 0.0166 | 0.0166 | 0.0166 | 0.0187 |
| 1930's | 0.0173 | 0.0228 | 0.0173 | 0.0194 |
| 1870's | 0.0205 | 0.0338 | 0.0205 | 0.0230 |

### 2.4.2 Model's Performance to Account for the Long-Run Trend

We study the effect of the demographic changes presented above on a set of four statistics: the fraction of individuals aged 16 to 49 that are married, the divorce rate for women aged 16 to

[2] Albanesi and Olivetti (2010) talk on significant improvements in maternal health that started in the mid 1930s.

49, the fraction never-married by age 50, and the age at first marriage. We focus on these statistics presented in Table 2.6, as they summarize the main characteristics of the trends we wish to examine. We begin by considering the effects of the aging of the population and the fall in the sex ratio between the 1870 cohort and the 1950 cohort. The result of the experiment is presented in Table 2.8.

How important is the structure of the population in explaining the trends in marriage? Together, the aging of the population and the fall in the ratio of men to women can simultaneously explain (i) the decrease in age at marriage for men (139.1%), and no change on age at marriage for women, (ii)the decrease of the gap in age at marriage, and (iii) the increased incidence of marriage for women (18.4%) and for men (93.8%). The intuition of the mecahnism is simple: The population shifted from a high sex ratio/low life expectancy regime in 1870's to a low sex ratio/high life expectancy regime in 1950's. This represents a move towards an environment where; the average gains to marriage rise for women and fall for men, and, men drive the marriage decisions. As a result, the model predicts the earlier age at marriage for men because it became easier to find a wife, and the rise in marriage prevalence and incidence for women because there are larger average gains of marriage as their life expectancy has increased. In summary, the combination of increased longevity and a scarcity of men in 1990 served to increase the incentives of men to delay marriage but participate in marriage to a greater extent in 1990 than in 1920.

Table 2.8: Demographic Experiments: 1870's to 1950's

| | Data | | Model | |
|---|---|---|---|---|
| | 1870's | 1950's | 1870's | 1950's |
| **Age at Marriage** | | | | |
| Women | 21.9 | 22.0 | 21.9 | 22.0 |
| (%Δ) | | (+0.5) | | (+0.5) |
| Men | 25.9 | 24.7 | 26.1 | 24.7 |
| (%Δ) | | (-4.6) | | (-6.4) |
| **% Aged 16 to 49 that are Married** | | | | |
| Women | 55.2 | 56.7 | 40.8 | 50.9 |
| (%Δ) | | (+2.7) | | (+24.8) |
| **% of Never-Married by Age 50** | | | | |
| Women | 10.2 | 5.5 | 5.9 | 5.4 |
| (%Δ) | | (-46.1) | | (-8.5) |
| Men | 14.4 | 6.5 | 13.4 | 6.5 |
| (%Δ) | | (-54.9) | | (-51.5) |
| **Divorce Rate, per 1,000** | | | | |
| | 0.7 | 5.2 | 4.4 | 4.4 |
| (%Δ) | | (+742.9) | | (+0.0) |

### 2.4.3 The Shift Toward Unilateral Divorce

Although demographics can simultaneously account for both the delay and rise in marriage, it fails to explain the large rise in divorce observed in the data. The reason for this is simple: although changing demographics altered the gains to marriage at different points in an individual's lifetime, the ease with which agents could divorce in the model remained unchanged. In reality, however, there were large changes in the ease with which men and women could obtain a divorce through the liberalization of divorce laws. For this reason, it is of interest to consider the introduction of unilateral divorce changes in the face of an aging population. To study unilateral divorce in our framework, we change the parameter governing the cost of divorce ($\omega$) to match the divorce rate for the 1870 corhort. This exercise is consistent with the introduction of unilateral divorce.

The results of this exercise, presented in Table 2.9 indicate that the introduction of unilateral divorce, in combination with demographic changes can account for (i) the decrease in age at marriage for men (206.5%) and almost no change on age at marriage for women, (ii) the increased incidence of marriage for women (29.1%) and for men (94.0%), and (iii) the increase in prevalence of marriage (255.5%). Note that with the easing of restrictions on divorce, the model also well explains the rise in the prevalence of marriage, that is over-predicted in the benchmark case. Overall, with divorce liberalization, the demographic transition from 1870's to 1950's can account for most of the transition in marital status.

Table 2.9: Unilateral Divorce: 1870's to 1950's

| | Data | | Model | |
|---|---|---|---|---|
| | 1870's | 1950's | 1870's | 1950's |
| **Age at Marriage** | | | | |
| Women | 21.9 | 22.0 | 21.67 | 22.0 |
| (%Δ) | | (+0.5) | | (+1.5) |
| Men | 25.9 | 24.7 | 27.3 | 24.7 |
| (%Δ) | | (-4.6) | | (-9.5) |
| **% Aged 16 to 49 that are Married** | | | | |
| Women | 55.2 | 56.7 | **47.6** | 50.9 |
| (%Δ) | | (+2.7) | | (+6.9) |
| **% of Never-Married by Age 50** | | | | |
| Women | 10.2 | 5.5 | 6.3 | 5.4 |
| (%Δ) | | (-46.1) | | (-16.3) |
| Men | 14.4 | 6.5 | 13.7 | 6.5 |
| (%Δ) | | (-56.9) | | (-53.5) |
| **Divorce Rate, per 1,000** | | | | |
| | 0.7 | 5.2 | **0.7** | 4.4 |
| (%Δ) | | (+742.9) | | (+642.9) |

### 2.4.4 Model's Performance for the Short-Run Trend

In appendix, we also assess model's performance to account for the short-term trend in marital statistics. We recalibrate the demographic parameters so that they capture the age and sex structure of the population in the 1930's birth cohort. As a conclusion of the analysis in appendix, we find that demographics alone are not able to account for the delay in age at marriage and the decreased prevalence of marriage from the 1930's cohort to the 1950's cohort. We also find that the changes of divorce isn't an answer.

## 2.5 Conclusion

In this paper, we make three contributions to the economic literature on marriage. We document an important but overlooked feature of the data: the incidence of marriage has increased over time despite the well-documented delay in marriage. We examine the role of the demographic transition in explaining the trends in marital status. In combination with the liberalization of divorce laws, we find that demographics can quantitatively account for much of the increased incidence and delay of marriage and the rise in divorce. Changes in the gains to marriage over time, the standard explanation for the delay in marriage, cannot account for the increased incidence of marriage.

An appealing feature of this analysis is that the mechanism we consider here, namely changes in mortality rates, are directly observed and quantified in the data. Thus, it is straightforward to assess whether the demographic transition can account for the trends in marriage for other time periods and in other countries. It is also possible to extend our analysis to the other prominent feature of the demographic transition: the large decline in fertility. The role of demographics in accounting for the trends in fertility is the focus of our future work.

# References

Acs, G., and S. Nelson (2004): "Changes in Living Arrangements During the Late 1990s: Do Welfare Policies Matter?," *Journal of Policy Analysis and Management*, 23(2), 273–290.

Adda, J., and R. Cooper (2003): *Dynamic Economics: Quantitative Methods and Applications*. The MIT Press.

Aiyagari, S., J. Greenwood, and N. Guner (2000a): "On the State of the Union," *Journal of Political Economy*, 108(2), 213–244.

——— (2000b): "On the State of the Union," *Journal of Political Economy*, 108(2), 213–244.

Aizer, A., and S. McLanahan (2006): "The Impact of Child Support Enforcement on Fertility, Parental Investments, and Child Well-Being," *Journal of Human Resources*, 41(1), 28–45.

Akerlof, G., J. Yellen, and M. Katz (1996): "An Analysis of Out-of-Wedlock Childbearing in the United States," *The Quarterly Journal of Economics*, pp. 277–317.

Albanesi, S., and C. Olivetti (2010): "Maternal Health and the Baby Boom," *NBER Working Paper*.

Altonji, J., and L. Segal (1996): "Small-Sample Bias in GMM Estimation of Covariance Structures," *Journal of Business & Economic Statistics*, pp. 353–366.

Angrist, J. (2002): "How Do Sex Ratios Affect Marriage and Labor Markets? Evidence From America's Second Generation," *Quarterly Journal of Economics*, 117, 997–1038.

Attanasio, O., and G. L. Violante (2005): "The Demographic Transition in Closed and Open Economy: A Tale of Two Regions," Mimeo, New York University.

Becker, G. (1973): "A Theory of Marriage: Part I," *Journal of Political Economy*, 81(4), 813–46.

Becker, G. (1981): *A Treatise on the Family*. Harvard University Press.

Becker, G., and N. Tomes (1976): "Child Endowments and the Quantity and Quality of Children," *The Journal of Political Economy*, 84(4), 143–162.

Becker, G. S., and R. J. Barro (1988): "A Reformulation of the Economic Theory of Fertility," *Quarterly Journal of Economics*, 103, 1–25.

Blundell, R., P. Chiappori, and C. Meghir (2005): "Collective Labor Supply with Children," *Journal of Political Economy*, 113(6), 1277–1306.

Bohn, H. (1999): "Will Social Security and Medicare Remain Viable as the U.S. Population is Aging?," *Carnegie-Rochester Conference Series on Public Policy*, 50, 1–53.

Brien, M. (1997): "Racial Differences in Marriage and the Role of Marriage Markets," *Journal of Human Resources*, pp. 741–778.

Brown, M., and C. Flinn (2011): "Family Law Effects on Divorce, Fertility and Child Investment," *mimeo*.

Burdett, K., and M. Coles (1997): "Marriage and Class," *Quarterly Journal of Economics*, 112(1), 141–168.

——— (1999): "Long-Term Partnership Formation: Marriage and Employment," *Economic Journal*, pp. 307–334.

Burdett, K., and J. Ermisch (2003): "Single Mothers," *mimeo*.

Burdett, K., and R. Wright (1998): "Two-Sided Search with Nontransferable Utility," *Review of Economic Dynamics*, 1(1), 220–245.

Carlson, M., I. Garfinkel, S. McLanahan, R. Mincy, and W. Primus (2005): "The Effects of Welfare and Child Support Policies on Union Formation," *Population Research and Policy Review*, 23(5), 513–542.

Case, A. (1998): "The Effects of Stronger Child Support Enforcement on Nonmarital Fertility," p. 191, in *Fathers Under Fire: The Revolution in Child Support Enforcement*.

Chiappori, P., and Y. Weiss (2006): "Divorce, Remarriage, and Welfare: A General Equilibrium Approach," *Journal of the European Economic Association*, pp. 415–426.

——— (2007): "Divorce, Remarriage, and Child Support," *Journal of Labor Economics*, 25(1), 37–74.

Choi, S. (2011): "Fertility Risk in the Life-Cycle," *mimeo*.

Choo, E., and A. Siow (2003): "Who Marries Whom and Why," Mimeo, University of Toronto.

De La Croix, D., and M. Doepke (2003): "Inequality and Growth: Why Differential Fertility Matters," *The American Economic Review*, 93(4), 1091–1113.

Del Boca, D., and C. Flinn (1995): "Rationalizing Child-Support Decisions," *American Economic Review*, 85(5), 1241–62.

——— (2005): "Household Time Allocation and Modes of Behavior: A Theory of Sorts," *mimeo*.

Dunson, D., B. Colombo, and D. Baird (2002): "Changes with age in the level and duration of fertility in the menstrual cycle," *Human Reproduction*, 17.

Edlund, L. (2005): "The Role of Paternity Presumption and Custodial Rights for Understanding Marriage Patterns," *mimeo*.

Fernandez, R., N. Güner, and J. Knowles (2004): "Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality," Mimeo, New York University.

Fernandez, R., N. Guner, and J. Knowles (2005): "Love and Money: A Theoretical and Empirical Analysis of Household Sorting and Inequality," *Quarterly Journal of Economics*, 120(1), 273–344.

Fernpndez-Villaverde, J., J. Greenwood, and N. Guner (2009): "From Shame to Game in One Hundred Years: An Economic Model of the Rise in Premarital Sex and its De-Stigmatization," *mimeo*.

Friedberg, L. (1998a): "Did Unilateral Divorce Raise Divorce Rates? Evidence from Panel Data," *The American Economic Review*, 88(3), 608–627.

Friedberg, L. (1998b): "Did Unilateral Divorce Raise Divorce Rates? Evidence from Panel Data," *American Economic Review*, 88, 608–627.

Gale, D., and L. Shapley (1962): "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 69(1), 9–15.

Gallant, A., and G. Tauchen (1996): "Which Moments to Match?," *Econometric Theory*, 12(04), 657–681.

Garfinkel, I., C. Huang, S. McLanahan, and D. Gaylin (2003): "The Roles of Child Support Enforcement and Welfare in Non-Marital Childbearing," *Journal of Population Economics*, pp. 55–70.

Garfinkel, I., S. Mclanahan, and D. Meyer (1998): "A Brief History of Child Support Policies in the United States," p. 14, in *Fathers Under Fire: The Revolution in Child Support Enforcement*.

Garfinkel, I., C. Miller, S. McLanahan, and T. Hanson (1998): "Deadbeat Dads or Inept States?," *Evaluation Review*, 22(6), 717.

Giolito, E. (2003): "A Search Model of Marriage with Differential Fecundity," Mimeo, University of Maryland.

Goldin, C., and L. F. Katz (2002): "The Power of the Pill: Oral Contraceptives and Women's Career and Marriage Decisions," *Journal of Political Economy*, 110, 730–770.

Gourieroux, C., A. Monfort, and E. Renault (1993): "Indirect Inference," *Journal of Applied Econometrics*, 8(S), S85–118.

Greenwood, J., and N. Güner (2004): "Marriage and Divorce Since World War II: Analyzing the Role of Technological Progress on the Formation of Households," Mimeo, University of Rochester.

Greenwood, J., N. Guner, and J. Knowles (2003): "More on Marriage, Fertility, and the Distribution of Income," *International Economic Review*, 44(3), 827–862.

Greenwood, J., N. Güner, and J. Knowles (2003): "More on Marriage, Fertility and the Distribution of Income," *International Economic Review*, 44.

Gruber, J., and D. Wise (2001): "An International Perspective on Policies for an Aging Society," *NBER Working Paper 8103*.

Güner, N., and J. Knowles (2003): "Why is the Rate of Single Parenthood Lower in Canada than in the U.S.? A Dynamic Equilibrium Analysis of Welfare Policies," Mimeo, University of Pennsylvania.

Guvenen, F., and A. Smith Jr. (2010): "Inferring Labor Income Risk from Economic Choices: An Indirect Inference Approach," *NBER Working Paper*.

Guyer, J., C. Miller, and I. Garfinkel (1996): "Ranking States Using Child Support Data: A Cautionary Note," *Social Service Review*, 70(4), 635–652.

Huang, C. (2002): "The Impact of Child Support Enforcement on Nonmarital and Marital Births: Does It Differ by Racial and Age Groups?," *The Social Service Review*, 76(2), 275–301.

Hutchinson, E. (1956): *Immigrants and Their Children, 1950-1950*. New York: John Wiley and Sons and The Social Science Research Council.

Jones, L., A. Schoonbroodt, and M. Tertilt (2008): "Fertility Theories: Can They Explain the Negative Fertility-Income Relationship?," *NBER Working Paper*.

Jones, L., and M. Tertilt (2006): "An Economic History of Fertility in the US: 1826-1960," *NBER Working Paper*.

Kennes, J., and J. Knowles (2010): "Marriage, Fertility and Step Families: An Equilibrium Analysis," *mimeo*.

Knowles, J. (1999): "Can Parental Decisions Explain US Income Inequality?," PhD dissertation, University of Pennsylvania.

Kocharkov, G. (2010): "Abortions, Inequality and Family Formation," *mimeo*.

Lam, D. (1988): "Marriage Markets and Assortative Mating with Household Public Goods: Theoretical Results and Empirical Implications," *Journal of Human Resources*, 23(4), 462–487.

McLanahan, S., I. Garfinkel, N. Reichman, and J. Teitler (2001): "Unwed Parents or Fragile Families? Implications for Welfare and Child Support Policy," p. 202, In *Out of wedlock: Causes and Consequences of Nonmarital Fertility*.

Miller, C., and I. Garfinkel (1999): "The Determinants of Paternity Establishment and Child Support Award Rates among Unmarried Women," *Population Research and Policy Review*, 18(3), 237–260.

Moav, O. (2005): "Cheap Children and the Persistence of Poverty," *The Economic Journal*, 115(500), 88–110.

Moffitt, R. (1992): "Incentive Effects of the U.S. Welfare System," *Journal of Economic Literature*, 30, 1–61.

Nardi, M. D., S. Imrohoroglu, and T. Sargent (1999): "Projected US Demographics and Social Security," *Review of Economic Dynamics*, 2.

Neal, D. (2004): "The Relationship between Marriage Market Prospects and Never-Married Motherhood," *Journal of Human Resources*, pp. 938–957.

Pissarides, C. (1985): "Short-Run Equilibrium Dynamics of Unemployment, Vacancies, and Real Wages," *American Economic Review*, 75, 676–690.

Plotnick, R., I. Garfinkel, S. McLanahan, and I. Ku (2007): "The Impact of Child Support Enforcement Policy on Nonmarital Childbearing," *Journal of Policy Analysis and Management*, 26(1), 79–98.

Rasul, I. (2006): "The Economics of Child Custody," *Economica*, 73(289), 1–25.

Regalia, F., J. Ríos-Rull, and J. Short (2010a): "What Accounts for the Increase in the Number of Single Households?," *mimeo.*

——— (2010b): "What Accounts for the Increase in the Number of Single Households?," *mimeo.*

Rosenzweig, M. (1999): "Welfare, Marital Prospects, and Nonmarital Childbearing," *Journal of Political Economy*, 107(S6), 3–32.

Roth, A., and M. Sotomayor (1990): *Two-Sided Matching. A Study in Game-Theoretic Modeling and Analysis*. Econometric Society Monographs, 18.

Seitz, S. (2004): "Accounting for Racial Differences in Marriage and Employment," Mimeo, Queens' University.

Siow, A. (1998): "Differential Fecundity, Markets and Gender Roles," *Journal of Political Economy*, 106, 334–354.

Smith, A. (1990): "Three Essays on the Solution and Estimation of Dynamic Macroeconomic Models," PhD dissertation, Duke University.

——— (1993): "Estimating Nonlinear Time-Series Models Using Simulated Vector Autoregressions," *Journal of Applied Econometrics*, 8(S), S63–84.

Thomson, E., T. Hanson, and S. McLanahan (1994): "Family Structure and Child Well-Being: Economic Resources vs. Parental Behaviors," *Social Forces*, pp. 221–242.

Voena, A. (2010): "Yours, Mine, and Ours: Do Divorce Laws Affect the Intertemporal Behavior of Married Couples?," *mimeo.*

Weiss, Y., and R. Willis (1985): "Children as Collective Goods and Divorce Settlements," *Journal of Labor Economics*, 3(3), 268–92.

Willis, R. (1999): "A Theory of Out-of-Wedlock Childbearing," *Journal of Political Economy*, 107(S6), 33–64.

Wilson, W. (1990): *The Truly Disadvantaged: The Inner City, the Underclass, and Public Policy*. University of Chicago Press.

Wilson, W. J., and K. M. Neckerman (1986): "Poverty and Family Structure," in *Fighting Poverty: What Works and What Doesn't*, Cambridge, Mass: Harvard University Pres.

Wolfers, J. (2003): "Did Unilateral Divorce Laws Raise Divorce Rates? A Reconciliation and New Results," *NBER Working Paper*.

# Appendix A

# Appendix to Chapter 1

## A.1 Gale and Shapley Algorithm

After women and men enter the marriage market, their marital status is determined in a stable pair of matchings $(\mu_S, \mu_M)$. Here, we describe how to compute such a pair of matchings by applying the Gale and Shapley (1962) algorithm. As we have said, we are going to focus on a $M$-optimal stable matching equilibrium. But, with a small change, the method can be also applied for a $W$-optimal equilibrium. To begin with, we have the following lemma, which is easily derived from the participation constraints of marriage problem.

**Lemma 8.** $\forall i \in \mathcal{I}^f, \forall j \in \mathcal{I}^m$,

$$
\begin{aligned}
& \hat{V}^f_M(h^f_i, a^f_i, h^m_j, a^m_j, \gamma) \geq \hat{V}^f_S(h^f_i, a^f_i, h^m_j, a^m_j, \gamma), \\
\Longleftrightarrow & \\
& \hat{V}^m_M(h^f_i, a^f_i, h^m_j, a^m_j, \gamma) \geq \hat{V}^m_S(h^f_i, a^f_i, h^m_j, a^m_j, \gamma).
\end{aligned}
$$

Now we consider the situation in which each male proposes to a female in a given round, say the $n$-th round. Let $\lambda(j) = 1$ if a type-$j$ male is tentatively matched with a partner from the previous round, and $\lambda(j) = 0$ if he is not matched at the beginning of the $n$-th round.

1. In the $n$-th round, if $\lambda(j) = 0$, then a type-$j$ male proposes to a type-$i$ female with a relationship $R \in \{S, M\}$, who gives him the highest utility value $\hat{V}^m_R(h^f_i, a^f_i, h^m_j, a^m_j, \gamma)$ among the females who have never received his proposal ($\rho(i, j) = 0$).

2. Each woman accepts the proposal which gives her the highest utility value $\hat{V}^f_R(h^f_i, a^f_i, h^m_j, a^m_j, \gamma)$ among the proposals which she received in the $n$-th round plus the one she carried over

from the previous round. The selected male changes his status to $\lambda(j) = 1$. All other rejected males (these might include her partner from the previous round) change their status to $\lambda(j) = 1$. All the males $j' \in \mathcal{I}^m$ who newly proposed to her change their status to $\rho(i, j) = 1$.

3. Go back to step 1 until $\forall j \in \mathcal{I}^m, \lambda(j) = 1$.

Here unlike in the original Gale and Shapley algorithm, men choose a type of relationship (marriage or a non-marital relationship) every time they make an offer. Also, Lemma 7 assures that a woman doesn't have incentives to reject the highest offer she receives.[1]

## A.2 Proof of Theorem 3

*Proof.* We will show that a pair of matchings $(\mu_S, \mu_M)$ obtained through the above algorithm always satisfies the two conditions in Definition 2. Since condition 1 holds obviously, we will only check whether the condition 2 holds or not. Suppose, to the contrary, that within the matchings $(\mu_S, \mu_M)$, there exist a pair of couples $(i, \mu_R(i))$,$(\mu_{R'}(j), j)$, and relationships $R, R', R'' \in \{S, M\}$ such that

$$\hat{V}^f_{R''}\left(h^f_i, a^f_i, h^m_j, a^m_j, \gamma\right) > \hat{V}^f_R\left(h^f_i, a^f_i, h^m_{\mu_R(i)}, a^m_{\mu_R(i)}, \gamma\right),$$
$$\hat{V}^m_{R''}\left(h^f_i, a^f_i, h^m_j, a^m_j, \gamma\right) > \hat{V}^m_{R'}\left(h^f_{\mu_{R'}(j)}, a^f_{\mu_{R'}(j)}, h^m_j, a^m_j, \gamma\right).$$

Then one of the following two must be true: (1) type-$j$ male didn't propose to a type-$i$ female when she gave the highest utility value among available mates, or (2) a type-$i$ female didn't accept a type-$j$ male's offer when he gave her the highest utility value among the offers she received. Both of these contradict the algorithm described above.

□

## A.3 Proof of Theorem 6

*Proof.* We will show that a pair of matchings $(\mu_S, \mu_M)$ obtained through the above algorithm is the unique $M$-optimal pair. In particular, we will prove that in the above algorithm, no man is ever rejected by an achievable woman. Consequently, the stable pair of matchings $(\mu_S, \mu_M)$ that is produced in the above algorithm matches each man to his most preferred achievable woman,

[1] If Lemma 7 doesn't hold, then, women might have strategic motives to reject the offer which gives her the highest utility value among those she receives in the current round.

and is, therefore, the unique $M$-optimal stable pair of matchings. This proof is based on the work of Roth and Sotomayor (1990).

The proof is by induction. Assume that up to a given step in the procedure no man has yet been rejected by a woman who is achievable for him. At this step, suppose woman $i$ rejects man $j$. If she rejects $j$ in favor of man $j'$, whom she keeps engaged, then she prefers $j'$ to $j$. Then we must show that $i$ is not achievable for $j$.

We know $j'$ prefers $i$ to any women except for those who have previously rejected him and hence (by inductive assumption) are unachievable for him. Consider a hypothetical pair of matchings $(\mu'_S, \mu'_M)$ that matches $j$ to $i$ and everyone else to an achievable partner. Then $j'$ prefers $i$ to his partner at $(\mu'_S, \mu'_M)$. So, the pair $(\mu'_S, \mu'_M)$ is unstable, since it is blocked by $j'$ and $i$, who each prefer the other to their partner at $(\mu'_S, \mu'_M)$. Therefore, there is no stable matching that matches $i$ and $j$, and so they are unachievable for each other, which completes the proof.

□

## A.4 Checking the Robustness of the Estimation

Here, we present tables to check the robustness of the estimation.

Table A.1: First Stage Regressions: Total Fertility Rate for Non-Marital and Marital Births with the 5-Years Moving Average of the CSE Measures

| Dependent Variable | Non-Marital Total Fertility Rate | | | Marital Total Fertility Rate | | |
|---|---|---|---|---|---|---|
| CSE Measures (5-Year Moving Average) | | | | | | |
| 1) Collection | -0.46814** | | | 0.39790** | | |
| | (0.09619) | | | (0.10756) | | |
| 2) Expenditure | | -0.01813** | | | 0.01962** | |
| | | (0.00345) | | | (0.00383) | |
| 3) Paternity | | | -0.08773** | | | 0.03272 |
| | | | (0.02273) | | | (0.02242) |
| Average Wage | -0.01403** | -0.01301** | -0.01501** | -0.00663 | -0.00713 | -0.00724 |
| | (0.00419) | (0.00415) | (0.00466) | (0.00468) | (0.00462) | (0.00460) |
| Unemp. Rate | 0.00633** | 0.00578** | 0.00863** | -0.00391** | -0.00354* | -0.00165 |
| | (0.00132) | (0.00130) | (0.00184) | (0.00147) | (0.00145) | (0.00181) |
| Gender Gap | -0.01025 | -0.00844 | -0.01077 | 0.00521 | 0.00387 | 0.00657 |
| | (0.01423) | (0.01418) | (0.01483) | (0.01591) | (0.01577) | (0.01463) |
| Frac. Black | -0.54892** | -0.56211** | -0.44155** | 0.01986 | 0.03852 | -0.01728 |
| | (0.11791) | (0.11771) | (0.13617) | (0.13185) | (0.13088) | (0.13433) |
| Frac. Hisp. | 0.27465 | 0.23108 | 0.33257† | -0.55493** | -0.53392** | -0.23748 |
| | (0.17962) | (0.17815) | (0.19925) | (0.20085) | (0.19809) | (0.19655) |
| Frac. HS DP | 0.13233 | 0.12098 | 0.09186** | 0.14240 | 0.15326 | 0.18223 |
| | (0.10080) | (0.10054) | (0.11732) | (0.11271) | (0.11179) | (0.11574) |
| Max AFDC | 0.06693** | 0.06504** | 0.09186** | 0.06043** | 0.06288** | 0.07658** |
| | (0.00496) | (0.00499) | (0.00623) | (0.00555) | (0.00554) | (0.00615) |
| Intercept | -62.26066** | -58.74095** | -54.16074** | 17.10579** | 14.79859** | 11.26589† |
| | (5.22574) | (5.07200) | (6.52845) | (5.84324) | (5.63974) | (6.44007) |
| State Fixed Effects | Yes | Yes | Yes | Yes | Yes | Yes |
| State Specific Trends | Yes | Yes | Yes | Yes | Yes | Yes |
| N | 857 | 857 | 705 | 857 | 857 | 705 |
| $R^2$ | 0.96154 | 0.96174 | 0.95876 | 0.97529 | 0.97135 | 0.97720 |
| F | 171.33623 | 172.25725 | 126.92143 | 270.51605 | 248.18175 | 233.96588 |
| Significance levels : † : 10% ∗ : 5% ∗∗ : 1% | | | | | | |

(Standard errors are in parenthesis.)

Table A.2: Marriage Rate Regression without State-Specific Time Trends.

| Dependent Variable | Marriage Rate | | |
|---|---|---|---|
| CSE Measures | | | |
| (5-Year Moving Average) | | | |
| 1) Collection | 0.02396** | | |
| | (0.00525) | | |
| 2) Expenditure | | 0.00112** | |
| | | (0.00020) | |
| 3) Paternity | | | 0.00190† |
| | | | (0.00101) |
| Average Wage | -0.00017 | -0.00027 | -0.00034 |
| | (0.00024) | (0.00024) | (0.00021) |
| Unemp. Rate | -0.00046** | -0.00043** | -0.00049** |
| | (0.00006) | (0.00006) | (0.00007) |
| Gender Gap | -0.00168 | -0.00117 | 0.00142 |
| | (0.00229) | (0.00226) | (0.00200) |
| Frac. Black | 0.00816 | 0.00813 | -0.00372 |
| | (0.00609) | (0.00603) | (0.00552) |
| Frac. Hisp. | -0.02013 | -0.01856 | -0.00783 |
| | (0.01341) | (0.01327) | (0.01181) |
| Frac. HS DP | 0.00443 | 0.00425 | 0.00519 |
| | (0.00502) | (0.00496) | (0.00458) |
| Max AFDC | -0.00047† | -0.00036 | 0.00018 |
| | (0.00025) | (0.00025) | (0.00027) |
| Intercept | 2.34269** | 2.20281** | 1.96177** |
| | (0.26385) | (0.25569) | (0.26319) |
| State Fixed Effects | Yes | Yes | Yes |
| State Specific Trends | Yes | Yes | Yes |
| N | 634 | 634 | 506 |
| $R^2$ | 0.96874 | 0.96933 | 0.98172 |
| F | 179.96616 | 183.53104 | 237.96394 |
| Significance levels : | † : 10% | ∗ : 5% | ∗∗ : 1% |

(Standard errors are in parenthesis.)

Table A.3: First Stage Regressions: Total Fertility Rate for Non-Marital and Marital Births without State-Specific Time Trends.

| Dependent Variable | Non-Marital Total Fertility Rate | | | Marital Total Fertility Rate | | |
|---|---|---|---|---|---|---|
| CSE Measures (3-Year Moving Average) | | | | | | |
| 1) Collection | -0.27859** | | | -0.21389* | | |
| | (0.05941) | | | (0.09147) | | |
| 2) Expenditure | | -0.01735** | | | -0.01440** | |
| | | (0.00218) | | | (0.00341) | |
| 3) Paternity | | | -0.03714** | | | -0.07411** |
| | | | (0.01239) | | | (0.01682) |
| Average Wage | -0.00714† | -0.00756* | -0.00795* | 0.02292** | 0.02241** | 0.02541** |
| | (0.00367) | (0.00356) | (0.00391) | (0.00565) | (0.00558) | (0.00531) |
| Unemp. Rate | 0.00132 | 0.00216† | 0.00013 | -0.00399* | -0.00330† | -0.00207 |
| | (0.00128) | (0.00125) | (0.00135) | (0.00197) | (0.00196) | (0.00183) |
| Gender Gap | 0.01130 | 0.01111 | -0.00189 | -0.03819 | -0.03859 | -0.05314* |
| | (0.01651) | (0.01612) | (0.01635) | (0.02542) | (0.02522) | (0.02221) |
| Frac. Black | -0.53101** | -0.56611** | -0.46910** | 0.17598 | 0.14454 | 0.25145 |
| | (0.11912) | (0.11648) | (0.12910) | (0.18341) | (0.18224) | (0.17534) |
| Frac. Hisp. | 0.45978** | 0.46487** | 0.90602** | 0.96065** | 0.95865** | 0.84388** |
| | (0.13544) | (0.13121) | (0.15132) | (0.20853) | (0.20530) | (0.20552) |
| Frac. HS DP | 0.25737** | 0.25041** | 0.07038 | -0.49354** | -0.49574** | -0.52108** |
| | (0.09159) | (0.08890) | (0.09845) | (0.14101) | (0.13909) | (0.13371) |
| Max AFDC | 0.03307** | 0.03569** | 0.05378** | 0.02653** | 0.02868** | 0.04181** |
| | (0.00357) | (0.00350) | (0.00501) | (0.00549) | (0.00547) | (0.00680) |
| Intercept | -62.40616** | -62.74655** | -55.96561** | 39.13232** | 38.45340** | 38.05713** |
| | (2.55343) | (2.27437) | (2.37166) | (3.93135) | (3.55851) | (3.22118) |
| State Fixed Effects | Yes | Yes | Yes | Yes | Yes | Yes |
| State Specific Trends | No | No | No | No | No | No |
| Aggregate Trend | Yes | Yes | Yes | Yes | Yes | Yes |
| N | 908 | 908 | 806 | 908 | 908 | 806 |
| $R^2$ | 0.93801 | 0.94084 | 0.93834 | 0.92703 | 0.92807 | 0.93968 |
| F | 174.02847 | 228.5594 | 192.40494 | 182.59391 | 185.45679 | 196.96413 |

Significance levels : † : 10% $*$ : 5% $**$ : 1%

(Standard errors are in parenthesis.)

Table A.4: Marriage Rate Regression without State-Specific Time Trends.

| Dependent Variable | Marriage Rate | | |
|---|---|---|---|
| CSE Measures | | | |
| (3-Year Moving Average) | | | |
| 1) Collection | -0.00957* | | |
| | (0.00411) | | |
| 2) Expenditure | | 0.00014 | |
| | | (0.00017) | |
| 3) Paternity | | | -0.00087 |
| | | | (0.00077) |
| Average Wage | -0.00042† | -0.00027 | -0.00038† |
| | (0.00024) | (0.00024) | (0.00023) |
| Unemp. Rate | -0.00035** | -0.00036** | -0.00028** |
| | (0.00008) | (0.00008) | (0.00007) |
| Gender Gap | 0.00036 | 0.00049 | 0.00043 |
| | (0.00324) | (0.00327) | (0.00307) |
| Frac. Black | -0.01253† | -0.01222 | -0.01592* |
| | (0.00581) | (0.00762) | (0.00754) |
| Frac. Hisp. | -0.01693 | -0.01203 | -0.01354 |
| | (0.01112) | (0.01112) | (0.01116) |
| Frac. HS DP | -0.01542* | -0.01594** | -0.00984† |
| | (0.00599) | (0.00601) | (0.00581) |
| Max AFDC | 0.00002 | -0.00004 | 0.00072* |
| | (0.00022) | (0.00023) | (0.00030) |
| Intercept | 1.09778** | 1.34034** | 1.05778** |
| | (0.17297) | (0.15891) | (0.15130) |
| State Fixed Effects | Yes | Yes | Yes |
| State Specific Trends | No | No | No |
| Aggregate Time Trend | Yes | Yes | Yes |
| N | 681 | 681 | 591 |
| $R^2$ | 0.91392 | 0.91327 | 0.93437 |
| F | 120.64919 | 119.65849 | 150.47517 |

Significance levels : † : 10% $*$ : 5% $**$ : 1%

(Standard errors are in parenthesis.)

## A.5 Calibration Result for Intergenerational Analysis

This appendix lists the result of the calibration for intergenerational analysis. The followings are the estimated value of the parameters, and the values for the targets.

Table A.5: Estimated Parameters for Intergenerational Exercise

| **Name** | **Parameter** | **Estimates** |
|---|---|---|
| Parameter for Conditional Mean | $\epsilon_1$ | 3.385 |
| Parameter for Conditional Mean | $\epsilon_2$ | 0.856 |
| Parameter for Gender Gap | $\mu_g$ | 2.896 |
| Parameter for Conditional Variance | $\sigma_g$ | 0.125 |

Table A.6: The Match Between the Model and the Data for Intergenerational Exercise

| **Name** | | **Data** | **Model** |
|---|---|---|---|
| Log Mean of Human Capital for Women | $\mu_h^f$ | 2.200 | 2.217 |
| Log Mean of Human Capital for Men | $\mu_h^m$ | 2.580 | 2.555 |
| Log Variance of Human Capital for Women | $\sigma_h^f$ | 0.755 | 0.801 |
| Log Variance of Human Capital for Men | $\sigma_h^m$ | 0.755 | 0.721 |
| Intergenerational Correlation of Income | $\rho_g$ | 0.730 | 0.711 |

# Appendix B

# Appendix to Chapter 2

## B.1 Can Demographics Account for the Trends in Marriage Since the 1930's Birth Cohort?

In this appendix, we ask to what extent changes in demographics alone account for the changes in marriage and divorce since the birth of the 1930 cohort (exactly 20 years).

### B.1.1 Demographics

The demographic targets we used in this analysis is listed in Table B.1.

Table B.1: The Demographic Transition: 1930's to 1950's Birth Cohort

| | 1930 | 1950 |
|---|---|---|
| Men per 100 women (aged 15 and above) | 98.4 | 92.9 |
| Life expectancy of women (at age 15) | 56.7 | 61.0 |
| Life expectancy of men (at age 15) | 52.5 | 54.4 |

Table B.2: The Change in Mortality Rate 1930's to 1950's Birth Cohort

| Mortality Rate | $\pi^f(a)$ | $\pi^f(y)$ | $\pi^f(o)$ | $\pi^m$ |
|---|---|---|---|---|
| 1950's | 0.0166 | 0.0166 | 0.0166 | 0.0187 |
| 1930's | 0.0173 | 0.0228 | 0.0173 | 0.0194 |

### B.1.2 The Result for the Short-Run Trend

The demographic transition from 1930's to 1950's can explain only a few of the transition in marital status for women and none of the transition in marital status for men (See in Table B.3). The model with changes in the age and sex structure between the 1930's and 1950's birth cohorts is consistent with: (i) The delay in marriage for women (15.4%). (ii) The fall in the incidence of marriage for women (128.8%). However, the model can't explain the delay in marriage, the fall in incidence of marriage for men and the decreased prevalence of marriage, and any of the rise in divorce.

Table B.3: Demographic Experiments: 1930's to 1950's

| | Data | | Model | |
|---|---|---|---|---|
| | 1930's | 1950's | 1930's | 1950's |
| **Age at Marriage** | | | | |
| Women | 20.3 | 22.0 | 21.8 | 22.0 |
| (%Δ) | | (+8.4) | | (+1.3) |
| Men | 22.8 | 24.7 | 24.8 | 24.7 |
| (%Δ) | | (+8.3) | | (-0.5) |
| **% Aged 16 to 49 that are Married** | | | | |
| Women | 71.0 | 56.7 | 49.2 | 50.9 |
| (%Δ) | | (-20.1) | | (+3.5) |
| **% of Never-Married by Age 50** | | | | |
| Women | 4.5 | 5.5 | 4.2 | 5.4 |
| (%Δ) | | (+22.2) | | (+28.6) |
| Men | 6.2 | 6.5 | 7.2 | 6.5 |
| (%Δ) | | (+4.8) | | (-9.7) |
| **Divorce Rate, per 1,000** | | | | |
| | 2.2 | 5.2 | 4.6 | 4.4 |
| (%Δ) | | (+136.4) | | (-4.4) |

### B.1.3 The Shift Toward Unilateral Divorce

As in 2.4.3, we run change the parameter governing the cost of divorce ($\omega$) to match the divorce rate for the 1870 corhort. The result is shown in Table B.4.

Even if we adjust the costs of divorce, the model cannot account for the data from 1930's to 1950's. Especially, the model cannot match the following at the same time: (i) An increase of age at marriage both for men and for women, and (ii) a decrease of prevalence of marriage. Furthermore, the model can account for (iii) the change in the incidence of marriage.

Table B.4: Unilateral Divorce: 1930's to 1950's

| | Data | | Model | |
|---|---|---|---|---|
| | 1930's | 1950's | 1930's | 1950's |
| **Age at Marriage** | | | | |
| Women | 20.3 | 22.0 | 22.0 | 22.0 |
| (%Δ) | | (+8.4) | | (+0.0) |
| Men | 22.8 | 24.7 | 25.1 | 24.7 |
| (%Δ) | | (+8.3) | | (-1.6) |
| **% Aged 16 to 49 that are Married** | | | | |
| Women | 71.0 | 56.7 | 57.9 | 50.9 |
| (%Δ) | | (-20.1) | | (-12.1) |
| **% of Never-Married by Age 50** | | | | |
| Women | 4.5 | 5.5 | 3.8 | 5.4 |
| (%Δ) | | (+22.2) | | (+42.1) |
| Men | 6.2 | 6.5 | 4.4 | 6.5 |
| (%Δ) | | (+4.8) | | (+47.7) |
| **Divorce Rate, per 1,000** | | | | |
| | 2.2 | 5.2 | 2.2 | 4.4 |
| (%Δ) | | (+136.4) | | (-104.5) |