Essays on Financial Frictions and Macroeconomics

A DISSERTATION
SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL
OF THE UNIVERSITY OF MINNESOTA
BY

Sewon Hur

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
FOR THE DEGREE OF
Doctor of Philosophy

Timothy J. Kehoe, Adviser, and Fabrizio Perri, Adviser

July, 2012

Acknowledgements

I am grateful to my advisors Timothy Kehoe and Fabrizio Perri for their guidance and support. This dissertation would not exist without their assistance. I would like to thank my coauthors Kim Ruhl, Illenin Kondo, and Jose Asturias. I would also like to thank Cristina Arellano, Santiago Bazdresch, Jonathan Heathcote, Erzo Luttmer, Ellen McGrattan, Jose-Victor Rios-Rull, Terry Roe, Kjetil Storesletten, and seminar participants at the University of Minnesota, the 2011 Vigo Workshop on Dynamic Macroeconomics, the Society for Economic Dynamics 2011 meetings, the Federal Reserve Bank of Minneapolis, NYU Stern School of Business, the Centre de Recerca en Economia Internacional, University of Pittsburgh, Indiana University, National University of Singapore, Carleton University, and the Korea Development Institute.

Dedication

To my loving parents, Hyunkyoung and Goosaing

Abstract

This dissertation consists of three essays. The first essay analyzes the effects of the Great Recession on different generations. While older generations have suffered the largest decline in wealth due to the collapse in asset prices, younger generations have suffered the largest decline in labor income. Potentially, the young may benefit from the purchase of cheaper assets, especially if they have access to credit. To analyze the impact of these channels, I construct an overlapping generations model with borrowing constraints in which households choose a portfolio over housing as well as risk-free and risky financial assets. Shocks to labor efficiency and uncertainty regarding the return on risky assets generate a recession with a drop in asset prices and cross-sectional changes in consumption, investment, and wealth that are consistent with the recent recession. In particular, younger generations experience large declines in nondurable consumption and housing investment, a fact that is supported by the data. Overall, the young suffer the largest welfare losses, equivalent to a 5 percent reduction in lifetime consumption.

In the second essay, Illenin Kondo and I study the foreign reserves accumulation of emerging economies. Emerging economies, unlike advanced economies, have accumulated large foreign reserve holdings. We argue that this policy is an optimal response to an increase in foreign debt rollover risk. In our model, reserves play a crucial role in reducing debt rollover crises ("sudden stops"), akin to the role of bank reserves in preventing bank runs. An unexpected increase in rollover risk leads to a global rise in sudden stops, prompting emerging economies to update their priors about the risk they face. We show that a global increase in the rollover risk faced by emerging economies explains the outburst of sudden stops in the late 1990s, the subsequent increase in foreign reserves holdings, and the salient resilience of these countries to sudden stops ever

since.

In the third essay, Jose Asturias and I examine the role of entry barriers on firm entry and exit, aggregate productivity and output. Using cross-country data, we document that gross domestic product (GDP) per capita is positively correlated with firm entry rates, and that firm entry rates are positively correlated with barriers to firm entry. We develop a model, based on Asturias, Hur, Kehoe, and Ruhl (2012) where aggregate productivity growth is driven by the endogenous entry of productive firms and the endogenous exit of unproductive firms. Differences in entry policy lead to different levels of entry and output, while all economies grow at a balanced growth path with identical growth rates. In the quantitative extension, we show that reforms to entry costs can generate transition paths that resemble that of high-growth emerging economies.

Contents

List of Tables

List of Figures

Chapter 1

The Lost Generation of the Great Recession

1.1 Introduction

The Great Recession of 2007-2009 has been one of the largest contractions in the United States since the Great Depression. However, the recession has not impacted all households equally. On the one hand, older generations have suffered the largest decline in wealth due to the collapse in asset prices. Glover et al. (2011) estimate that the average American household whose household head is between 60 and 69 years of age experienced a decline in wealth of $310,000, while the average household between 20 and 29 years of age experienced a $30,000 decline in wealth. On the other hand, younger generations have suffered the largest decline in labor income. Potentially, the young may benefit from the purchase of cheaper assets, offsetting this drop in labor income. The effects of these large changes in labor income and asset prices in the Great Recession may have lasting effects on the welfare of households beyond the duration of the recession. This paper evaluates the joint impacts of these channels on lifetime welfare.

Much of the recent literature on generational welfare over the Great Recession has focused on labor outcomes that emphasize the high unemployment suffered by the young generation (see for example Bell and Blanchflower (2010); Elsby et al. (2010)). Others such as Pynoos and Liebig (2009) have focused on the collapse in assetprices and its effect on retirement savings. Glover et al. (2011) analyze the joint effects of asset prices and labor income on lifetime welfare. Using a general equilibrium overlapping generations model, they find that old generations suffer the largest decline in welfare, equivalent to a 10 percent decline in lifetime consumption, while younger generations are welfare-neutral, largely because of their ability to take advantage of depressed asset prices. However, I document that, as of 2009, young households have *less* housing and *less* securities, in real terms, compared to 2007. Hence, it seems that many young households are not able to take full advantage of the cheaper assets. Motivated by this empirical evidence, this paper modifies and extends Glover et al. (2011) by investigating the role of borrowing constraints in household ability to finance asset purchases. Another important feature of the data is that there is great variation in household debt-to-asset ratios both across and within age cohorts. While the average debt-to-asset ratio of young households is only 34 percent, 14 percent of young households have debt-to-asset ratios exceeding 100 percent. This suggests that modeling within-age heterogeneity is essential for understanding the role of borrowing constraints.

I construct an overlapping generations model with borrowing constraints in which households choose a portfolio over risk-free and risky assets. Households are heterogeneous in portfolio, income, and wealth both across and within age cohorts. The calibrated model fits the data very well along important dimensions such as wealth profile and risky asset profile by household age, as well as population wealth distribution. Shocks to labor income and uncertainty regarding the return on risky assets generate a recession with a drop in asset prices and cross-sectional changes in consumption, investment, and wealth that are consistent with the recent recession. In particular, younger generations experience large declines in nondurable consumption and housing investment, a fact that is supported by the data. Moreover, I show that the interaction between borrowing constraints and wealth heterogeneity plays a crucial role; although the average young household is not credit-constrained, a significant fraction of young households are constrained, especially so during the recession. Overall, I find that the young suffer the largest welfare losses, equivalent to a 5 percent reduction in lifetime consumption.

This paper builds on a large literature on the distributional consequences of asset prices, models of housing and borrowing constraints, and heterogeneous agent models. Li and Yao (2007) use a life-cycle model with housing to show that housing price declines benefit young households, while Kiyotaki et al. (2010) find a similar result if the housing price decline is driven by productivity shocks but not interest rate shocks. Glover et al. (2011) focus on the welfare effects of the Great Recession to find that the young benefit from a drop in asset prices, enabling them to offset the welfare losses from a large decline in labor income. This welfare improving channel of asset price declines is also present in this paper; however, credit-constrained households are limited in their ability to take advantage of this channel. This paper follows Iacoviello and Pavan (2009), Fernandez-Villverde and Krueger (2010), and Favilukis et al. (2011) by modeling housing (durable) goods as providing both consumption services and collateral in a life-cycle model with endogenous borrowing constraints. It also builds on a large class of heterogeneous agent models that have been developed since seminal works by Aiyagari (1994) and Huggett (1996).

This chapter is structured as follows. Section 1.2 documents changes in consumption and asset positions over the recent recession, and the large heterogeneity in household leverage (debt-to-asset ratios) across and within age cohorts. Section 1.3 presents a model economy which is used to interpret the empirical findings and to formally analyze the lifetime welfare implications of this recession. The calibration strategy is discussed in section 1.4. Section 1.5 presents the quantitative results and the welfare implications of the Great Recession. Section 1.6 concludes.

1.2 Empirical Analysis

This section documents several features of the data that suggest that the young have not fared well over this recession. In particular, the young have suffered large declines in labor income, nondurable consumption, durables expenditure, and asset wealth including housing and securities. The severe reduction in nondurable consumption, durable expenditure, and asset wealth of young households suggests that credit frictions may be important. I also document the heterogeneity in household leverage both across as well as within age cohorts, which is important for disciplining the role of borrowing constraints. Statistics documenting changes in nondurable consumption, durables expenditure, and securities over the recession are based on Consumer Expenditure Survey (CEX) data (2007, 2009), changes in labor income are computed using the Current Population Survey (CPS) March supplements (2008, 2010), and changes in housing wealth is computed from the American Community Survey (ACS 2007, 2009). Statistics on leverage reported in this section are computed from the Survey of Consumer Finances (SCF 2007).

1.2.1 Labor income

Young households (ages 20-44) suffered the largest decline in labor income over the Great Recession. Table 1.1 reports the percent changes in real labor income, linearly detrended, from 2007 to 2009.[1] Labor income is defined as the sum of wages, salaries, and two-thirds of self-employment income.[2]

Table 1.1: Changes in Labor Income

age	labor income (percent change from 2007 to 2009)
all	-7.93
20-44	-8.71
45-64	-6.38

1.2.2 Household consumption

In addition to suffering the largest decline in labor income, young households experienced very large declines in nondurable consumption and durables expenditure. Table 1.2 reports changes in real nondurable consumption and real durables expenditure, linearly detrended, from 2007 to 2009.[3] Nondurable consumption includes expenditures on food, beverages, utilities, apparel, education, tobacco, etc., while durables expenditure includes home furniture and appliances, and net outlay on cars and trucks.

1.2.3 Asset wealth

Potentially, younger households may benefit from the purchase of cheap assets, offsetting the drop in labor income. I present some supporting evidence that the young have not been able to take advantage of this channel. Table 1.3 reports changes in real housing and securities held by households.[4] First, the young have less housing in 2009, in real

[1] Labor income has been adjusted for inflation, and linearly detrended by a growth rate of 2 percent per year.

[2] Two-thirds of self-employment income is treated as labor income and one third as capital income.

[3] Both nondurable consumption and durables expenditure has been deflated by respective price changes, and linearly detrended by a growth rate of 2 percent per year.

[4] Housing and securities have been deflated by the respective aggregate price declines.

Table 1.2: Changes in Consumption Expenditures

age	nondurable consumption	durables expenditure
	(percent change from 2007 to 2009)	
all	-9.76	-14.87
20-44	-9.93	-22.79
45-64	-10.36	-8.50
65-84	-7.58	-3.86

terms, compared to 2007. Second, the young also have less securities in 2009, in real terms, compared to 2007.[5] Hence, it does not seem to be the case that the young are taking advantage of these cheaper assets.

Table 1.3: Changes in Asset Wealth

age	housing	securities
	(percent change from 2007 to 2009)	
20-44	-8.17	-17.06
45-64	1.15	-1.85
65-84	6.71	1.44

1.2.4 Heterogeneity across and within age cohorts

Households also vary to large degree in their the level of indebtedness. Table 1.4 reports the debt-to-asset ratios of households across age cohorts. To be more specific, the debt-to-asset ratio reported for young households (20-44) is the total value of debt held by young households divided by the total value of assets held by young households.

[5] However, because securities include risky assets such as stocks as well as safe assets such as government bonds, one must be cautious in interpreting these results. The Consumer Expenditure Survey does not collect information on detailed items within securities.

Heterogeneity of leverage within age cohorts is even greater. In particular, Table 1.5 reports some statistics summarizing the heterogeneity of household leverage within young households. It is worth noting that more than 14 percent of young households have negative net wealth, i.e. more debt than assets, even before the decline in asset prices.

Table 1.4: Household Leverage across Age Cohorts

age	debt-to-asset ratio
20-44	0.34
45-64	0.14
65-84	0.06

Table 1.5: Household Leverage within Young Households

percentile	debt-to-asset ratio
25	0.13
50	0.46
75	0.78

In sum, the data suggests the young have suffered large declines in nondurable consumption, durables expenditure, and asset wealth, while the old have experienced the smallest decline in nondurable consumption. There is also substantial heterogeneity in household leverage across age cohorts, and more importantly, within young households. The next section presents a model that is consistent with the empirical facts documented in this section and provides a framework to evaluate the welfare consequences of this recession.

1.3 The Model

This section presents a model economy which allows us to interpret the empirical findings and to formally analyze the lifetime welfare implications of this recession. The setting is similar to ones used in recent works that use calibrated life-cycle heterogeneous agents economies (see for example Conesa et al. (2008); Heathcote et al. (2008); Del Negro et al. (2010); Heathcote et al. (2010); Glover et al. (2011)). I consider a discrete time, small open economy inhabited by overlapping generations of finitely lived households. Households face borrowing constraints and choose portfolios over housing and non-housing risky assets, as well as risk-free bonds. There are idiosyncratic shocks to housing, non-housing assets, and labor income that help generate heterogeneity in wealth and portfolio holdings across and within age cohorts. This heterogeneity is crucial: not all old households have large holdings of risky assets, and not all young households are credit-constrained. I now describe in more detail the environment and the equilibrium.

1.3.1 Households

There is a continuum of finitely lived households indexed by i. Households of age $j \in \{1, 2, .., J\}$ face conditional survival probabilities given by $\{\psi_j\}$. Newborns are endowed with $\{\omega_i\}$ which is exogenous and time invariant. Population grows at rate g, and the aggregate measure of households is normalized to one. Preferences are given by

$$\mathbb{E}\left[\sum_{j=1}^{J} \beta^{j-1} \left(\prod_{a=1}^{j-1} \psi_a\right) u_j(c_{ij}, s_{ij})\right]$$

where c_{ij} is consumption of nondurable goods, s_{ij} is services of housing (and consumer durables) at age j, and β is the time discount factor. Note that the period utility function u_j depends on age. This captures the change in consumption needs of different household sizes along the life cycle.[6] Changes in household size are exogenously given. I assume that $u_j(c_{ij}, s_{ij}) = u(\frac{c_{ij}}{e_j}, \frac{s_{ij}}{e_j})$ where e_j is the number of adult equivalents in age j households, and $u : R^2_+ \to R$ is increasing, strictly concave, and homothetic.

Portfolio choice

Households can choose a portfolio that consists of two risky assets and one risk-free asset. The first risky asset is housing, denoted by h. Housing h yields a flow of housing services s. I assume that the flow of housing services s is a linear function of the stock of housing h, and without loss of generality, $s = h$. Investing in housing is risky because, each period, housing is subject to an idiosyncratic quality shock ξ_{it} with $E(\xi) = 1$. Housing does not depreciate, but it requires $\delta_h h$ units of consumption goods to cover maintenance costs. The second risky asset is the non-housing risky asset, given by x. This asset yields d units of consumption goods as a dividend. Each period, the non-housing risky asset is subject to an idiosyncratic shock ζ_{it} with $E(\zeta) = 1$. A simplifying assumption is that while households form rational expectations over the idiosyncratic shocks, the aggregate shock, i.e. the Great Recession, is modeled as an unexpected shock.[7] Holding this asset requires a participation cost of f which is proportional to labor income. This is intended to capture the limited participation in the risky asset market observed in the data, and is a reduced form way of modeling transaction fees, monitoring costs, etc.[8] The prices of housing and non-housing risky assets are given by p_{ht} and p_{xt}, respectively.

Households also have access to a standard risk-free bond b . This asset yields an

[6] See Bick and Choi (2011) for a discussion on the importance of modeling household size and the economies of scale within households.

[7] The quantitative implication of this assumption is discussed in the concluding section. It is worth noting that household investment in equity, especially in private equity is highly concentrated, as documented by Moskowitz and Vissing-Jorgensen (2002). Non-diversification can be one source of idiosyncratic shocks.

[8] See, for example, work by Attanasio and Paiella (2006); Vissing-Jorgensen (2002) that document the significance of participation costs in accounting for limited stock market participation.

exogenously given interest rate r, and is subject to a borrowing constraint, given by $-b_{ijt} \leq \lambda p_{ht} h_{ijt}$ where λ denotes the fraction of the value of housing that can be collateralized. This borrowing constraint is motivated by the maximum loan-to-value ratios that lenders of mortgages, car loans, and home equity loans consider in their loan decisions and is consistent with household borrowing constraints widely used in the literature (see for example Rios-Rull and Sanchez-Marcos (2008); Iacoviello and Neri (2010)).[9]

Household labor income

Household labor income has two determinants: a deterministic age-specific component $\{\eta_j\}$, and an idiosyncratic component $z_{it} \in \left\{z_1, ..., z_{n_z}\right\}$ which follows a Markov process with transition matrix $\Gamma_{zz'} = \Pr(z_{t+1} = z'|z_t = z)$. The age specific component η_j captures the income profile of households over the life cycle, while the idiosyncratic component z_{it} captures the heterogeneity of labor income within age cohorts and the risky nature of labor income over time. There is mandatory retirement at age j^*, after which households receive retirement pension payments of S.[10] Thus household i of age j with shock z_{it} earns:

$$y_j(z_{it}) = \begin{cases} w(1-\tau)\eta_j z_{it} & \text{if } j < j^* \\ S & \text{if } j \geq j^* \end{cases}$$

where w is the wage rate, and τ is the labor income tax.

[9] Alternatively, one may use endogenous debt limits as in Kehoe and Levine (1993), or explicit mortgage contracts as in Chambers et al. (2009).

[10] As in Heathcote et al. (2010) and Iacoviello and Pavan (2009), I assume that pension payments are uniform across households for computational tractability.

Household problem

Let $a_{it} = b_{it}(1+r) + p_{ht}h_{it}\xi_{it} + x_{it}\zeta_{it}(p_{xt}+d)$ denote the "wealth" of household i. Then the problem of the household of age j with wealth a and labor productivity shock z can be written recursively as:

$$\begin{aligned} V_{jt}(a,z) = \quad & \max_{c,b',h',x'} \quad u_j(c,h') + \beta\psi_j E_{z',\xi',\zeta'} V_{j+1,t+1}(a',z') \\ & \text{s.t.} \quad c + h'(p_{ht}+\delta_h) + p_{xt}x' + b' \leq y_j(z)(1-\mathbf{1}_{x'>0}f) + a \\ & \qquad -b' \leq \lambda p_{ht}h' \\ & \qquad a' = b'(1+r) + p_{h,t+1}h'\xi' + x'\zeta'(p_{x,t+1}+d) \\ & \qquad c \geq 0,\ h' \geq 0,\ x' \geq 0. \end{aligned}$$

Since $\{j,a,z\}$ are sufficient to characterize household i, we can omit the dependence on i. The solution to this problem can be represented by age-dependent policy functions for nondurable consumption $c_{jt}(a,z)$, housing $h'_{jt}(a,z)$, non-housing risky assets $x'_{jt}(a,z)$, and risk-free bonds $b'_{jt}(a,z)$.

1.3.2 Production

There is a representative firm that produces nondurable goods with a constant returns to scale technology given by

$$Y_t = AL_t$$

where A is productivity and L_t is the firm's labor demand. Given the wage rate w, the firm's problem is to maximize profit, $Y_t - wL_t$.

The per capita stock of housing and stocks are assumed to be fixed at $\bar{H}$ and $\bar{X}$, respectively. I also assume that housing and non-housing risky assets can be traded only by domestic households.[11]

[11] These assumptions are for computational tractability. Moreover, neither the stock of housing nor the foreign ownership of US housing or stocks has changed dramatically over the recession.

1.3.3 Equilibrium

Definition 1.1. *A recursive competitive equilibrium is*

- *policy functions of the households* $\{c_{jt}(a,z), b'_{jt}(a,z), h'_{jt}(a,z), x'_{jt}(a,z)\}_{j=1,..,J,\ t=0,..,\infty}$ *and of the firms* $\{L_t(w)\}_{t=0,..,\infty}$,
- *prices* $\{w_t, p_{ht}, p_{xt}\}_{t=0,..,\infty}$,
- *and distributions* $\{\mu_{jt}(a,z)\}_{j=1,..,J,\ t=0,..,\infty}$

such that:

1. *Given prices, the policy functions solve the problem of the households and the firms*
2. *Distribution of new born agents* $\{\mu_{1t}(\cdot)\}_t$ *is given, and is consistent with initial wealth endowments. Additional distributions are induced by policy functions and by transition functions for exogenous states*
3. *Markets clear:*

   (a) $\sum_{j\leq J}\int h'_{jt}(a,z)\mu_{jt}(da\times dz) = \bar{H}$

   (b) $\sum_{j\leq J}\int x'_{jt}(a,z)\mu_{jt}(da\times dz) = \bar{X}$

   (c) $L = \sum_{j<j^*}\eta_j\int z\mu_{jt}(da\times dz)$

1.4 Calibration

This section explains the calibration of the model. In sections 1.4.1-1.4.3, I discuss the parameters set outside of the model, followed by parameters that require solving for equilibrium allocations in section 1.4.4. I then show that the calibrated model matches the data along some important dimensions in section 1.5.

1.4.1 Demographics and Income

A period in the model is 5 years. Households enter the labor market at age 20 (model age $j = 1$), and retire at age 65 ($j^* = 9$), and die by age 100 ($J = 16$). Survival probabilities $\{\psi_j\}_{j=1,..,J}$ are taken from the 2004 US Life Tables, and the population growth rate g is set to 1.2 percent.[12] Adult equivalent sizes $\{e_j\}_{j=1,..,J}$ are calculated using household characteristics from the Consumer Expenditure Survey 2007 (CEX) and the OECD-modified scale, which assigns a value of 1 to the household head, of 0.5 to each additional adult member and of 0.3 to each child.[13] The initial wealth endowments ω_i are such that the top five 25 bins of initial wealth match those of households aged 16-24, calculated from the Survey of Consumer Finances 2007. The rest begin with zero wealth.[14] Figure 1.1 depicts the net wealth of households, aged 16-24, and the initial wealth endowments used in the model.

The age-specific component of labor income $\{\eta_j\}_{j=1,..,J}$ is taken from household earnings from the CEX, while the idiosyncratic stochastic component z is assumed to follow an order-one autoregressive process as follows:

$$\log z_t = \rho_Z \log z_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_z^2),$$

with the persistence parameter ρ_Z set to 0.9, and variance parameter σ_z set to 0.3.[15]

This process is approximated with a three-state Markov process using the procedure

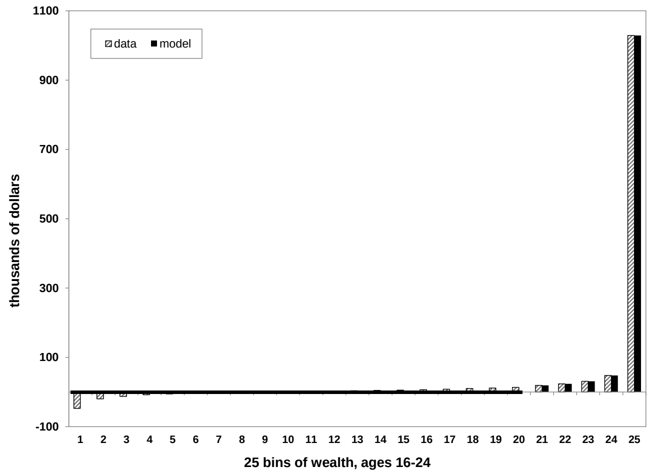
[12] This implies that in the steady state equilibrium, each cohort measure is determined by $\mu_j = \frac{\psi_j}{1+g}\mu_{j-1}$.

[13] This scale, first proposed by Hagenaars et al. (1994), and adopted by the Statistical Office of the European Union (EUROSTAT) in the late 1990s, is called the "OECD-modified equivalence scale".

[14] In the data, the bottom quantiles have significantly large negative net wealth. The model is not well-equipped to work with large negative wealth endowments because the borrowing constraint implies that those households would have to drastically reduce their debt. Moreover, in the data, the bottom 80% in the wealth distribution have a cumulative net wealth of zero. Hence, the wealth endowment is truncated at the 80th percentile so that the total wealth endowment in the model equals the wealth of households, aged 16-24, in the data.

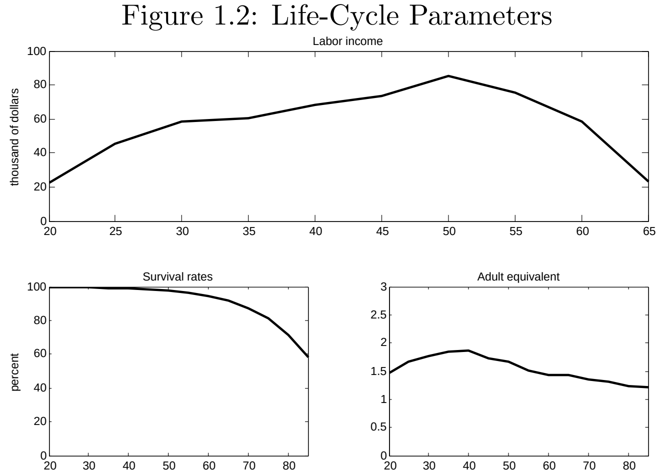
[15] These parameter choices are in the range typically used in the literature. For example, see Heaton and Lucas (2000); Storesletten et al. (2004); Scholz et al. (2006); Iacoviello and Pavan (2009).

Figure 1.1: Initial Wealth Endowments



described in Tauchen (1986), and then adjusted to reflect the five year period of the model. The income tax rate τ is set to 8.4 percent so that it fully funds the retirement pension payment S which is set to 40 percent of the average wage in the economy. Figure 1.2 summarizes the key demographics and income parameters.

Figure 1.2: Life-Cycle Parameters



1.4.2 Assets

The collateral constraint λ is set to 0.8 to be consistent with a 20 percent down payment requirement. [16] The annualized housing maintenance parameter δ_h is set to 7 percent, which is computed from the depreciation rates of housing and durables, given by 2 and 19 percent, respectively. [17] The house shock ξ is assumed to be a two-state i.i.d. process with $\xi_H = 1.16$, $\xi_L = 0.84$, with an implied variance which is consistent with the variance of housing capital gains shocks estimated by Chambers et al. (2009). The stochastic process for the non-housing asset follows a three-state i.i.d process with $\zeta \in \left\{1-\bar{\zeta}, 1, 1+\bar{\zeta}\right\}$. The variance parameter of the non-housing asset $\bar{\zeta}$, the dividend d and participation cost f are discussed in section 1.4.4.

1.4.3 Preferences

Household preferences are given by

$$u(c,s) = \frac{\left(c^{1-\gamma}s^{\gamma}\right)^{1-\sigma} - 1}{1-\sigma},$$

where γ is the preference weight on housing services, and σ is the risk aversion parameter. Following Glover et al. (2011), I set $\sigma = 3$ for the baseline calibration.The calibration of γ is discussed below.

1.4.4 Parameters Jointly Calibrated

The housing weight γ, dividend d, discount factor β, stock market participation cost f, and variance parameter of the non-housing asset $\bar{\zeta}$ are jointly calibrated to match five moments: the total value of housing risky assets, the total value non-housing risky

[16] Using the 1995 American Housing Survey, Chambers et al. (2009) find that the down payment fraction for first-time home purchases is 19.79 percent.

[17] The stocks of housing and durables are separately constructed using the perpetual inventory method. δ_h is then computed as a weighted average.

assets, the leverage ratio [18] of young households, overall stock market participation, and the 95th-quantile-to-median wealth ratio. Of particular importance is the leverage ratio since it disciplines to what extent young households are constrained. Using the Survey of Consumer Finances 2007, I find that households, aged 20-44, have an average leverage ratio of 48 percent, and that 59 percent of all households hold positive amounts of stocks, and/or mutual investment funds. The total value of housing risky assets is 4 times aggregate labor income, while that of non-housing risky assets is 3.4 times aggregate labor income, and the 95-to-median wealth ratio is 17.53.

1.5 Quantitative Results and Analysis

The calibrated model generates age profiles of wealth and risky assets that fits the data very well, as well as a wealth distribution that matches the data reasonably well. The model also generates changes is asset prices, nondurable consumption, and portfolio allocations over the Great Recession that are consistent with the changes documented in section 1.2. This section describes the main results.

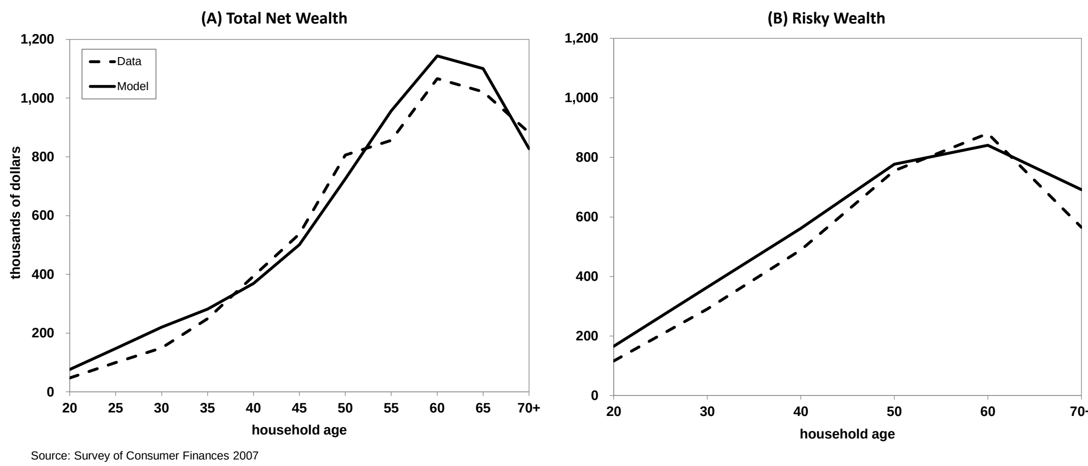
1.5.1 Steady state

Before moving on to the quantitative analysis of the Great Recession, we must verify that the model is consistent with the important dimensions of the data. Indeed, Figures 1.3 panel A and panel B show that the wealth profile and risky assets profile generated by the model closely resemble those in the data. Net wealth in the data is total assets minus total debt, and risky wealth is total assets minus safe assets such as bonds.

Figure 1.4 panel A shows the Lorenz curves of the wealth distribution in the data and that generated by the model. Note that the model does not generate the same

[18] In the model, the leverage ratio is defined to be $\frac{-b}{h}$. The data counterpart is net debt (debt minus bonds) divided by the value of residential housing and cars.

Figure 1.3: Wealth Profile



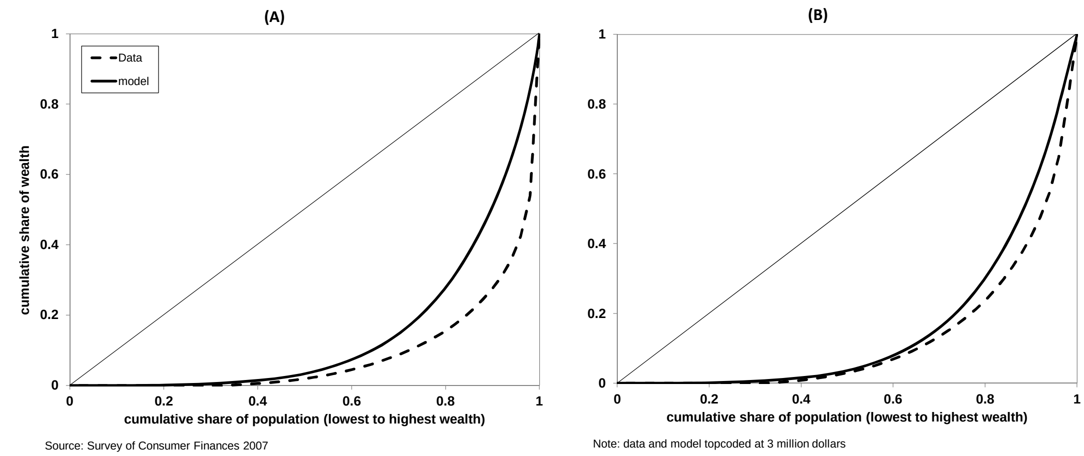
magnitude of wealth inequality as the data. This is because the model is not well-equipped to match the wealth of the top 2 percent of the wealth distribution.[19] Figure 1.4 panel B shows the Lorenz curves for which both the data and the model has been top-coded at 3 million dollars, i.e. the wealth shares have been constructed by assigning 3 million to all wealth levels that exceed 3 million dollars. The curves generated from the model and the data are better aligned in Panel B, because of the model's inability to match the top of the distribution, a common shortcoming in existing models.

1.5.2 Quantitative analysis

This section evaluates the welfare implications of the Great Recession. The shocks to the economy are an exogenous drop in labor income and a one-period increase in uncertainty regarding the risky assets. More specifically, there is an exogenous shift in the labor income distribution such that the labor income of households aged 20-44 drops 8.7 percent, while that of households aged 45-64 drops 6.4 percent, consistent

[19] One reason for this inability to generate "extremely" rich households lies in the finite state approximation of the shocks to income and risky assets.

Figure 1.4: Wealth Distribution



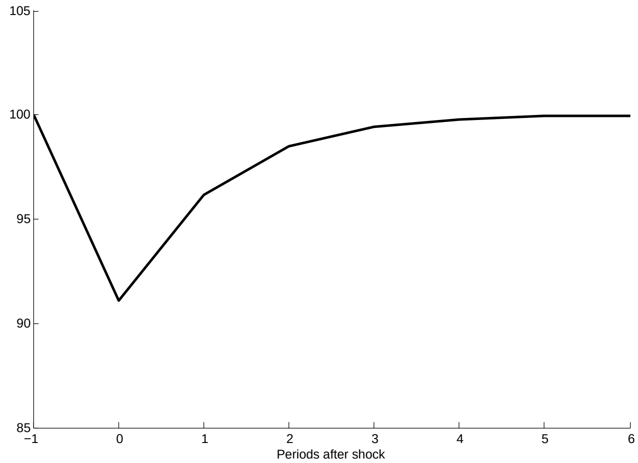
with the income changes across age groups in the data. This induces a higher fraction of low income households compared to the steady state.[20] In the periods following the recession, it is assumed that the individual labor income processes follow the auto-regressive income process described in section 1.4. This implies that it takes many periods for aggregate income to fully recover, as can be seen in Figure 1.5.

Both the income drop and uncertainty shocks, i.e. mean-preserving spreads to the stochastic processes for risky assets, drive the asset price declines in the model. The first channel is that as households have less income, their demand for both housing and non-housing risky assets fall. The second channel is that the more uncertain an asset's return becomes, the less that asset is demanded. This lower demand leads to an equilibrium fall in asset prices. Using this channel, the uncertainty shocks are calibrated such that the model recession generates price declines of 20 percent for both housing and non-housing assets.[21] The actual decline in prices of housing and stocks range

[20] The shock to labor income is modeled as a shift in the labor income distribution rather than an economy-wide drop in labor income. This is motivated by the fact that the drop in hours worked was much larger than the drop in labor productivity over the recession. One can interpret low-income state in the model as unemployment or part-time employment.

[21] Recent works have documented an increase in uncertainty regarding firm growth rates (see, for example, Arellano et al. (2011); Schaal (2010)), and this suggests a potential way to identify the

Figure 1.5: Aggregate Income



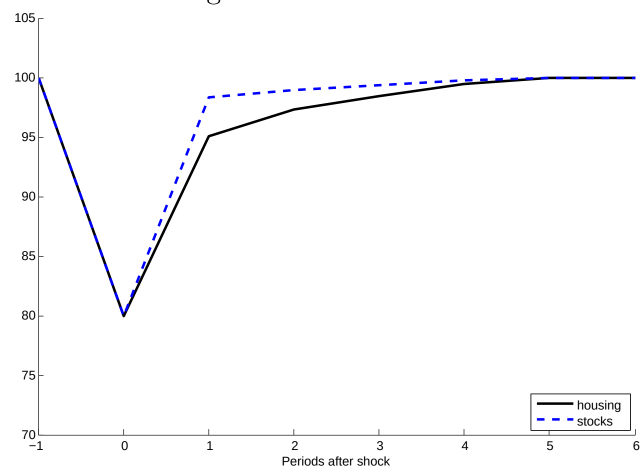
from 10 to 50 percent, depending on the data source and time length chosen.

Figure 1.6 plots the time series of asset prices. The model generates a one-period drop in asset prices, followed by a recovery over time, with non-housing risky asset prices recovering faster than housing prices. It is worth noting that non-housing asset prices are less sensitive than housing asset prices to movements in labor income. This is because, unlike housing assets, non-housing assets are held primarily by wealthy households who are less dependent on labor income. Since housing prices are more correlated with labor income, and since the shock to labor income has some persistence, the housing price falls on impact and takes some time to recover the pre-recession prices. However, as the non-housing risky asset price is less correlated with labor income, the non-housing price falls on impact, but recovers most of its value once the uncertainty has been resolved. Note that five year periods imply that if 2003-2007 is interpreted as period $t = -1$, then the model predicts asset prices will have recovered most of the losses by model period $t = 1$, which would be 2013-2017.

The welfare gains of the different generations are presented in Table 1.6. The young

magnitude of the uncertainty shocks.

Figure 1.6: Asset Prices



generation suffers the largest welfare losses, equivalent to a 5.4 percent decline in remaining lifetime consumption. Table 1.7 shows that the young also suffer a large decline in nondurable consumption in the recession, similar in magnitude to the decline in nondurable consumption in the data. As can be seen in Table 1.8, the young purchase less housing assets, as is consistent with the data, and they purchase more non-housing risky assets, but their net investment in risky assets is negligible. The young are not able to take full advantage of cheaper assets because a significant fraction of young households are credit-constrained in the model, especially so during the recession.

Table 1.6: Welfare Gains

age	consumption equivalent (remaining lifetime)
20-44	-5.4%
45-64	-3.5%
65-84	-3.8%

The middle-aged generation, ages 45-64, suffers the smallest welfare losses, equivalent to a 3.5 percent decline in remaining lifetime consumption. Although this cohort also

Table 1.7: Changes in Nondurable Consumption

age	model	data
20-44	-10.8%	-9.9%
45-64	-9.0%	-10.4%
65-84	-8.1%	-7.6%

Table 1.8: Risky Asset Wealth

age	housing	non-housing
	(percent change of steady state risky wealth)	
20-44	-2.2	2.4
45-64	1.7	-0.7
65-84	0.3	-1.7

suffers a large reduction in nondurable consumption, they enjoy a larger flow of housing services, and more importantly, the larger housing investment results in higher expected consumption in future periods due to the realized capital gains on the housing asset. Hence, it is the middle-aged cohort that is able to take advantage of the cheaper assets. The old generation also suffers large welfare losses, albeit smaller than the young. They do enjoy larger housing, but suffer a large decline in their overall risky asset investments, which decreases expected consumption in future periods.

Table 1.9: Model Comparisons

age	baseline	no income heterogeneity
25-44	-5.4%	-2.5%
45-64	-3.5%	-2.6%
65-84	-3.8%	-3.4%

Finally, it is worth noting that heterogeneity within cohorts and borrowing constraints jointly play a key role in the model. In the absence of labor income heterogeneity, borrowing constraints are not binding for any households; this reverses the welfare outcomes.[22] As can be seen Table 1.9, in the model calibrated without labor income heterogeneity, and hence no borrowing constraints, the old generation suffers the largest welfare losses while the young generation suffers the smallest welfare losses. This is because the young have the ability to offset part of their welfare losses from a large drop in labor income with the welfare gains of purchasing cheap risky assets.

1.6 Conclusion

This paper develops a model of the Great Recession that is consistent with the age wealth profile, the cross-sectional wealth distribution, changes in asset prices, and changes in labor income across age groups. I use this model to evaluate the welfare consequences for the different generations. The young suffer the largest welfare losses, equivalent to a 5.4 percent decline in lifetime consumption. In the model, the young are unable to take full advantage of cheaper assets as many of them are credit-constrained, especially so during the recession. The model predicts that the young suffer large declines in nondurable consumption and housing/durables investment; these predictions are consistent with the data.

Although this paper focuses on the effects of this recession by age, there is another important dimension: leverage. The model predicts that highly leveraged households, i.e. households with very large amounts of debt relative to their assets, are more likely to suffer large welfare losses because of their limited ability to smooth consumption over the recession and to invest in cheap assets due to a binding borrowing constraint.

[22] The leverage ratio of the average young household in the data is 48 percent. Because the model is calibrated to match this moment, young households in the model without labor income heterogeneity are not constrained.

This result is related to recent empirical work by Mian and Sufi (2010) and Midrigan and Philippon (2011) who find that US regions that experienced large increases in household leverage prior to the Great Recession were also regions that experienced large declines in output, employment, and durable consumption during the recession. As documented in Section 2, young households are typically more leveraged than older households. The fact that young households are highly leveraged at the onset of the Great Recession, coupled with the fact that the young suffer the largest declines in labor income, induces the large welfare losses of the young. These facts are consistent with Hurd and Rohwedder (2010) who document that 48% of households under age 50 are under financial distress, compared to 16% for age above 64, where financial distress is defined as an indicator for any of the following: unemployed, negative equity in house, behind more than two months on mortgage, in foreclosure.

Another important dimension is the potential long-term labor market consequences for young households. Kahn (2010) uses the National Longitudinal Survey of Youth to find large, negative, and persistent wage effects of graduating into a bad economy. This dimension of adverse long-term labor market consequences is also captured in the quantitative exercise presented in this paper. In the model, there is a larger fraction of low income households in the recession period, especially for the young, compared to non-recession periods. Due to the auto-regressive properties of the labor income process, the economy eventually returns to the pre-recession labor income distribution, but as shown in Figure 5, the "scars" from the recession persist for many periods.

This paper abstracts from two dimensions that may have quantitative significance. The first is the rent-own margin. One may argue that young households who were renters at the start of the recession can potentially benefit by becoming homeowners when housing prices are cheap. This channel may be significant. However, in the data, home ownership for young households actually decreases from 51 percent in 2007 to

48 percent in 2009. The second is household expectations over aggregate shocks such as the declines in aggregate labor income and asset prices experienced in the Great Recession. If households form expectations that large aggregate shocks can happen, this would provide an additional precautionary saving motive for households. However, since the calibration strategy involves targeting the average debt-to-asset ratio of young households, the calibrated model with and without expectations over aggregate shocks would generate the same level of leverage. Still, it can be the case that the steady state fraction of households close to the borrowing constraint could be smaller. Hence whether the inclusion of rational expectations over aggregate shocks can significantly change the results remains debatable, and is left for future research.

Chapter 2

A Theory of Sudden Stops, Foreign Reserves, and Rollover Risk in Emerging Economies

2.1 Introduction

Obstfeld, Shambaugh, and Taylor (2010) noted that the sustained accumulation of massive international reserves in emerging economies constitutes a puzzle. Standard models predict that emerging economies should hold very little reserves or none at all, while in the data these economies hold as much as 50 percent of GDP in reserves. Because of this disconnect between theory and practice, the management of international reserves remains one of the main topics in policy debates on global imbalances. Gourinchas and Obstfeld (2012) reiterated the need for research on foreign reserves and financial crises after finding that the emerging economies with more foreign reserves were more resilient during the 2007-2009 global crisis.

In this paper, we ask why emerging economies have massively accumulated foreign reserves. Using a novel model of sudden stops[1] and rollover risk[2], we show that the buildup in reserves is an optimal response to endogenous sudden stops arising from an increase in foreign debt rollover risk. We argue that the outburst of sudden stops in the late 1990s reflected an unanticipated increase in rollover risk. Sudden stops have since come under control as monetary authorities and governments optimally increased their foreign reserves.

In our model of optimal reserves and rollover risk, the external debt of a small open economy is subject to the rational rollover decision of foreign lenders. Foreign reserves play a crucial role in inducing the foreign lenders to roll over debt. This role is akin to the role of reserves in banks to prevent bank-runs. In fact, consistent with Gourinchas and Obstfeld (2012), we find that foreign reserves are associated with a reduced sudden stop probability in the data.

Following an unexpected increase in the foreign debt rollover risk, our model can quantitatively account for both the increase in reserves and sudden stop occurrences in emerging economies. This is precisely because, given an underlying rollover risk, the model predicts an endogenous relation between sudden stop probabilities and foreign reserves. Therefore, an unexpected increase in rollover risk first causes an outburst in sudden stops. However, these unexpected crises allow governments to rationally update their beliefs and thereby increase their reserves accordingly. Sudden stops subside as reserve levels are appropriate. But did the rollover risk actually increase? Our model indicates that the volatility of gross flows of external liabilities reflects the rollover risk. In fact, we find that flows of gross external liabilities have become more volatile in emerging economies since the mid-1990s.

[1] A sudden stop is a sudden reversal of external capital inflows which is typically associated with a fall in output.

[2] Our rollover risk is similar to the time-preference liquidity shocks à la Diamond and Dybvig (1983). However, ours is more akin to a country-preference liquidity shock that prevents some lenders from agreeing to roll over the debt.

This paper is structured as follows. In the following subsection, we briefly relate our work to the existing literature. Section 2.2 empirically analyzes foreign reserves, external debt liabilities, and sudden stops in emerging economies from 1990 to 2007. Section 2.3 provides a simple three-period model of reserves allocation that delivers an optimal reserves-to-debt ratio with endogenous sudden stop probabilities. Section 2.4 presents a multi-country dynamic extension with Bayesian learning and regime change. In the calibrated model, we show it can quantitatively account for the outburst of sudden stops experienced by emerging economies, the subsequent accumulation of reserves, and the resilience of emerging economies to sudden stops since then. Section 2.5 concludes.

2.1.1 Relation to the Literature

This paper builds on a large body of literature on reserves and sudden stops. For a long time, reserves were seen as an integral part of a country's export promotion strategy: they promote export by slowing appreciation. Dooley, Folkerts-Landau and Garger (2004) recently reiterated this explanation to justify the large foreign reserve holdings of emerging economies, in particular China. As documented by Aizenman and Lee (2007), this export promotion view cannot explain the recent increases in reserves of most countries, including China. In fact, reserves mostly increased long after exports started growing. If reserves mainly served to promote exports, they should have grown during the export growth.

Heller (1966) and Frenkel and Jovanovic (1981) model reserves as a buffer against exogenous stochastic balance-of-payments deficits. In Frenkel and Jovanovic (1981), the government seeks to minimize the one-time adjustment costs that are incurred when reserves dry up. Higher reserves increase the distance-to-adjustment because the exogenous adjustment threshold is hit less often. Reserves however have an opportunity cost represented by the forgone interest earnings. This trade-off determines the optimal reserves held by a government. Numerous papers follow this inventory approach to the role of reserves, e.g. Flood and Marion (2001).

More recently, precautionary motives have been explored as a potential key determinant of reserve allocations (see for example Aizenman and Marion (2003); Durdu, Mendoza, Terrones (2009)). In Jeanne and Ranciere (2008) and Alfaro and Kanczuk (2007), reserves serve as a consumption smoothing mechanism since reserves can be used even after a sudden stop or default. However, these consumption smoothing models of reserves can neither account for the rise in reserve holdings nor the pattern of sudden stop occurrences. In fact, Alfaro and Kanczuk (2007) prescribe that emerging countries should hold no foreign reserves at all.

Our work is closely related to Aizenman and Lee (2007) who use a simple Diamond-Dybvig framework with exogenous interest rate, investment scale, and exogenous sudden stop probability to model reserve hoarding. In Aizenman and Lee (2007), countries face exogenous balance of payments deficits which must be financed with reserves or by liquidating domestic investments. Reserves hence serve as a cushion against the costly liquidation of productive domestic projects. Our work departs from Aizenman and Lee (2007) by crucially endogenizing the probability of sudden stops. These endogenous sudden stops probabilities also relate our work to the large literature on income fluctuations, rollover risk and sovereign default in incomplete markets following Arellano (2008).

Obstfeld, Shambaugh, and Taylor (2010) document the predictive failure of the existing "sudden stop" theories of reserves as they are not able to rationalize the level of reserves accumulated by emerging economies. In contrast to these existing theories, our model generates time series of reserves and sudden stop occurrences that are consistent with the data. The main difference in our work is that reserves serve more than just to smooth consumption; they also play an essential role in preventing sudden stops.

Furthermore, Obstfeld, Shambaugh, and Taylor (2010) show that an empirical specification of reserves with "financial stability" outperforms a specification with traditional motives[3] . However, the large reserves held since the 2000s remain largely unexplained by this empirical "financial stability" specification. Nonetheless, their empirical work indicates the need for better models of reserves and the potential connection between financial crises and reserves. In fact, Gourinchas and Obstfeld (2012) find that, in emerging economies, foreign reserves were associated with greater resilience during the global 2007-2009 crisis.

2.2 Facts on Reserves and Sudden Stops in Emerging Economies

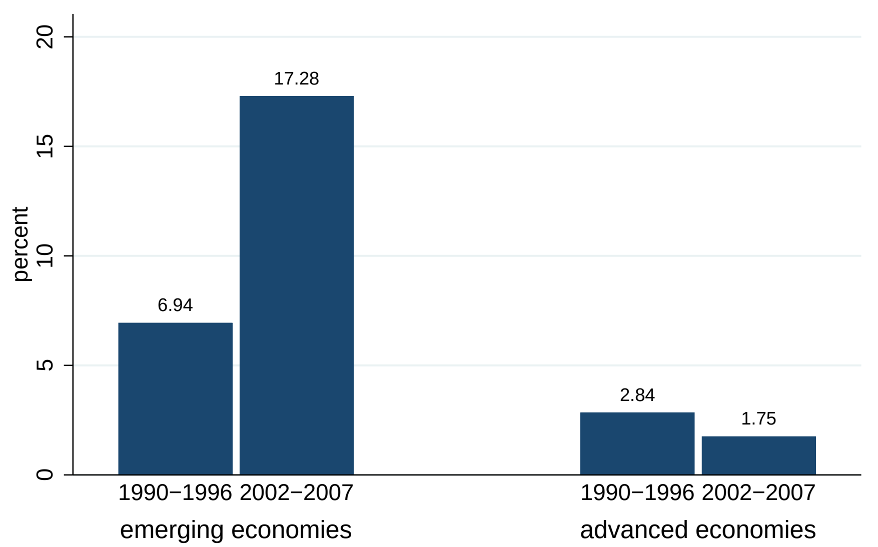
In this section, we document a set of stylized facts regarding foreign reserves, external debt liabilities, and sudden stops in 23 emerging economies during 1990-2007. We use data on international liquidity from the International Monetary Fund dataset on International Financial Statistics (IFS) in conjunction with the updated and extended version of the dataset constructed by Lane and Milesi-Ferretti (2007). The list of emerging economies we consider includes Argentina, Brazil, Chile, China, Colombia, the Czech Republic, Egypt, Hungary, India, Indonesia, Malaysia, Mexico, Morocco, Pakistan, Peru, Philippines, Poland, Romania, Russia, South Africa, South Korea, Thailand, and Turkey. This list includes countries appearing in most classifications of emerging countries with the exception of Taiwan for which the available data is limited.

[3] Their "financial stability" specification entails the inclusion of regressors such as domestic financial liabilities (M2), financial openness, access to foreign currency through debt markets, and exchange rate policy.

2.2.1 Foreign Reserves over GDP

In the IFS dataset, foreign reserves are defined as *all official public sector foreign assets, except Gold, that are readily available to and controlled by the monetary authorities.*[4] We highlight two notable facts regarding foreign reserves holdings. The first fact is that foreign reserves as a percent of GDP in emerging economies are significantly higher than those in advanced economies.[5] The second fact is that these ratios have increased in emerging economies while they have decreased in advanced economies. These facts are summarized in figure 2.1 which shows the cross-country median foreign-to-GDP ratio for emerging and advanced economies respectively.

Figure 2.1: Foreign Reserves over GDP



Note: The value for each period and each group of economies is derived as the median across economies of the period-average of each economy's ratio of reserves-to-GDP.

[4] This definition of foreign reserves includes convertible foreign exchange, SDR holdings, and IMF reserve position.

[5] We use advanced economies to refer to the United States, the United Kingdom, France, and Germany.

It is worth noting that this phenomenon of increasing reserves is not limited to just a few countries or just driven by China as one might think. In fact, foreign reserves are increasing in almost all emerging economies with Chile and Egypt being the exceptions. This robust observation is shown in the detailed table (table B.1) of average foreign reserves by country and by period.

2.2.2 Foreign Reserves and External Debt Liabilities

We document two additional facts on foreign reserves using the external debt liabilities measures.[6] The first fact is that reserves-to-liabilities ratios are also much higher in emerging economies than in advanced economies. For the period 2002-2007, these ratios for emerging economies are almost 40 times higher than that for advanced economies. The second fact is that reserves-to-liabilities ratios have been increasing in emerging economies while they have been decreasing in advanced economies.

These facts are shown succinctly in figure 2.2 which depicts the median reserves-to-liabilities ratio within each group of countries. Again, this observation holds in most emerging economies. Table B.1 details the average reserves-to-liabilities ratios by country and era.

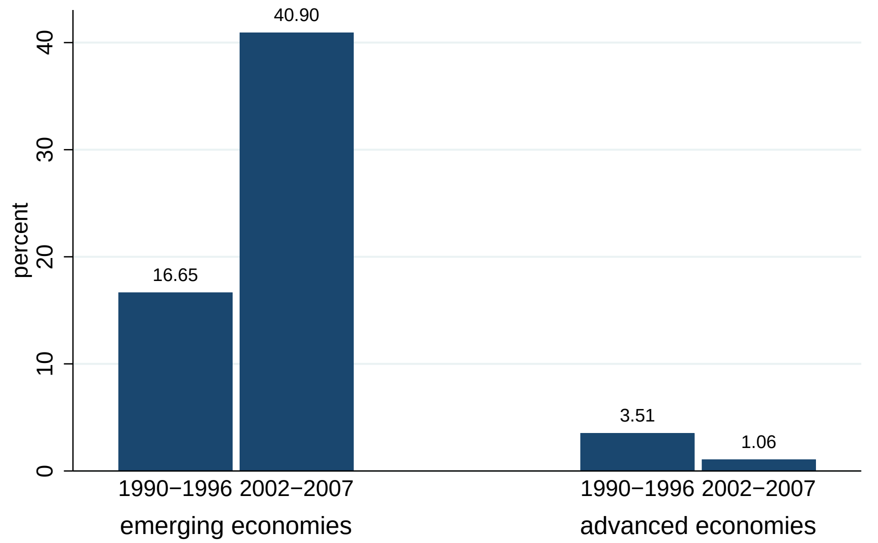
2.2.3 Sudden Stops in Emerging Economies

Following Calvo et al. (2004), we define a sudden stop episode as a spell with exceptionally large current account reversals and a recession. We find 12 sudden stop experiences during 1990-2007 across the 23 emerging economies with an outburst of 10 sudden stops between 1997 and 2001.[7]

[6] The measure of "External Debt Liabilities" include the "Debt Securities" item under "Portfolio Investment Liabilities" as well as "Other Investment Liabilities".

[7] Our sudden stop episodes are: Turkey (1994), Mexico (1995), Thailand (1997), Czech Republic, Indonesia, Philippines, South Korea (1998), Chile, Peru, Russia (1999), Argentina, Turkey (2001). Durdu, Mendoza, Terrones (2009) report other episodes which do not meet the criteria extended from Calvo et al. (2004): Argentina (1994), Malaysia (1997), Brazil, Colombia, Pakistan (1999). Whether

Figure 2.2: Foreign Reserves over External Debt Liabilities



Note: The value for each period and each group of economies is derived as the median across economies of the period-average of each economy's ratio of reserves over external debt liabilities.

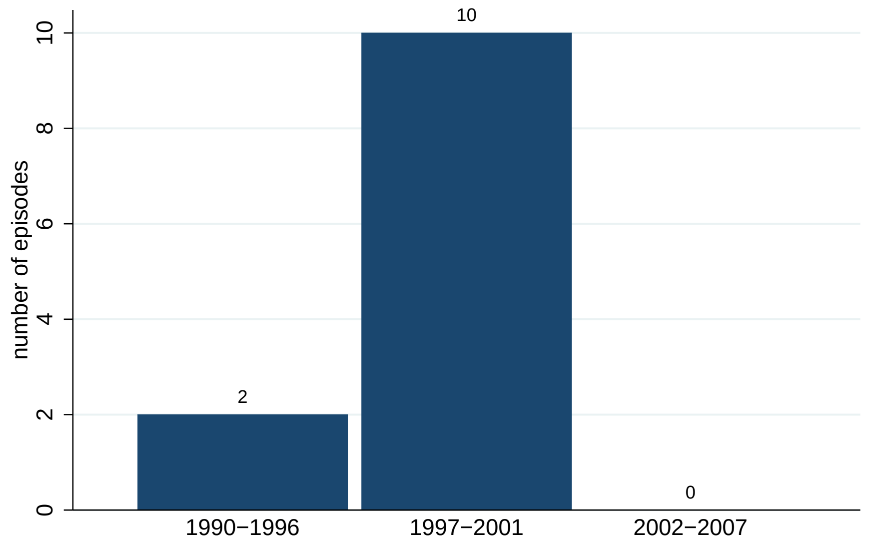
We therefore divide this time frame into three periods as shown in figure 2.3: 1990-1996 is a period of low-frequency sudden stops (with 2 occurrences), 1997-2001 is a period of high-frequency sudden stops (with 10 occurrences), and 2002-2007 is a period of low-frequency sudden stops (with no occurrence).

2.2.4 Reserves and Sudden Stop Probabilities

Following Gourinchas and Obstfeld (2012), we use a panel discrete-choice model to document the effect of foreign reserves on sudden stops. They documented that foreign reserves are associated with reduced banking crisis, currency crisis, or sovereign

we use sudden stops reported by other authors or the ones we measured, there was an outburst in sudden stops between 1997 and 2001. Our methodology for constructing sudden stop episodes is further explained in the data appendix.

Figure 2.3: Sudden Stops in Emerging Economies



default. We find that higher foreign reserves are also associated with reduced sudden stop likelihood.

As in Gourinchas and Obstfeld (2012), we use a panel logit model with country fixed effects:

$$\Pr\left(S_k^i = 1 \,|\, x_i\right) = \frac{\exp\left(\alpha_i + \beta x_i\right)}{1 + \exp\left(\alpha_i + \beta x_i\right)}$$

where S_k^i denotes whether country i is in a sudden stop episode in the next k years and x_i are foreign reserves in country i during a year that is not 0 to 3 years after a sudden stop episode (that is, "tranquil" times using the terminology of Gourinchas and Obstfeld (2012)).

The results of the panel logit estimation across emerging economies are reported in table 2.1. Foreign reserves are significantly associated with resilience to sudden stops. For instance, one standard deviation increase in the ratio of foreign reserves to external debt liabilities (around 26 percent) is associated with a fall of 11.5 percent in the probability of a sudden stop in the next three years.

Table 2.1: Panel Logit Estimation across Emerging Economies

		Sudden Stop $\partial p/\partial x$	
x	$s.d.\,(x)$	1 year ahead	1-3 years ahead
Reserves over External Debt Liabilities	26.06	-0.0090*** (.0025)	-0.0044** (.0019)
Sudden Stops Mean		0.05	0.26

Note: **, and *** denote significance at 5%, and 1%. $\partial p/\partial x$ is the marginal effect in percentage at "tranquil" sample mean. $s.d.\,(x)$ is the unconditional standard deviation of x over "tranquil" times. Robust standard errors in parentheses are computed using the delta-method. The estimation sample is an unbalanced panel that spans 15 emerging countries between 1973 and 2007.

2.2.5 Volatility of Gross External Liabilities

Finally, we document that the volatility of quarterly gross debt flows in emerging economies has increased over time. Using the series on external liabilities relative to output, we find that gross external liabilities were less volatile prior to the outburst of sudden stops.

Table 2.2: Quarterly Gross Flows of Total External Liabilities

	Period		
	1990-1996	1997-2001	2002-2007
Standard deviation	3.23	4.97	4.83
Mean	7.90	9.75	9.88
Coefficient of variation	0.41	0.51	0.49

Table 2.2 summarizes the increase in the volatility of global liquidity flows. Since the late nineties, there has been an increase in both the standard deviation and the coefficient of variation of gross total external liabilities flows.

2.3 A Three-Period Model of Optimal Reserves Allocation

In this section, we provide a theory of optimal reserves allocation in an environment where governments face rollover risk. The model highlights the endogenous role of reserves in determining sudden stop probabilities. In the next section, we provide a dynamic extension to explain the increase in reserves and the pattern of sudden stops in emerging countries documented in the previous section.

2.3.1 Environment

We consider a small open economy model with three periods: $t = 1$ (initial), 2 (interim), 3 (final). There is a unit measure of risk neutral foreign lenders who can choose to lend to the domestic country.[8] The domestic country has a representative agent who has linear preferences $u(C) = C$ over the final period consumption C. The government chooses allocations and debt arrangements to maximize the expected utility of the domestic agent.

2.3.2 Technologies

The domestic country has access to two technologies à la Diamond and Dybvig (1983). The first transforms the investment K made in the initial period into AK units in the final period if production is uninterrupted. However, if production is interrupted in the interim through the liquidation of $L \in [0, K]$ units of investment, the technology yields λL in the interim and $A(K - L)$ in the final period. We assume $\lambda < 1$, reflecting the idea that it is costly to divest from the long-term investment in the interim. We also impose that there is no partial interim liquidation, that is: $L \in \{0, K\}$. This assumption of full liquidation is made for analytical tractability and is relaxed in the next section. The second technology transforms a unit of investment into a unit of output in the subsequent period. This technology is referred to as the "reserves" technology.

These technologies are summarized by the following table:

Technologies	$t = 1$	$t = 2$	$t = 3$
Production and liquidation	$-K$ investment	λL liquidation	$A(K - L)$ final output
Reserves	$-R_1$ initial reserves	R_1	
		$-R_2$ interim reserves	R_2

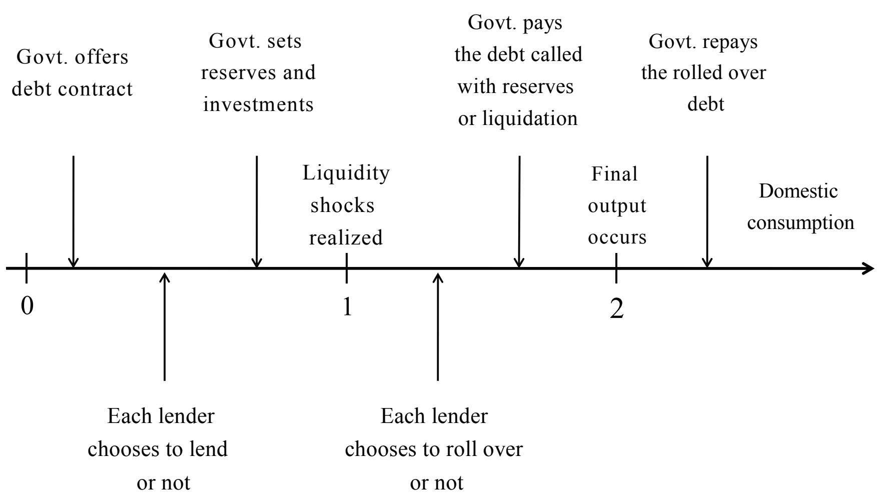
[8] For technical reasons, we assume that the foreigners' capital endowment is finite and large enough.

2.3.3 International Financial Markets: Timeline and Constraints

Timing of actions and shocks

In the initial period, the domestic government borrows D from foreign lenders to finance its initial period investments. An overview of the sequence of actions taken by the government and the lenders is presented in figure 2.4. In the interim, a fraction φ of the foreign lenders receive liquidity shocks, denoted by $\varphi_i = 1$, meaning that they must call the loan and be repaid back. The fraction φ is stochastic and has a cumulative distribution function that follows the bounded Pareto distribution given by $F_\sigma(\varphi) = 1 - (1-\varphi)^{\frac{1}{\sigma}}$. The remaining fraction $(1-\varphi)$ of lenders with $\varphi_i = 0$ can choose to call or roll over their loans.

Figure 2.4: Timeline



We denote $\psi_i = 0$ if lender i chooses to roll over the loan and $\psi_i = 1$ otherwise. We assume that each individual lender i takes the actions $\{\psi_j(\varphi, \varphi_j)\}_{j \neq i}$ of the other lenders $j \in [0,1]$ as given. This implies that once the aggregate liquidity shock is realized, the fraction of lenders calling the loan $\psi(\varphi) \triangleq \int \psi_j(\varphi) dj$ can be inferred by each individual lender. We call it a *sudden stop* when all lenders refuse to roll over in the interim, that is, when $\psi(\varphi) = 1$.

Payments Schedule

We allow the debt contract to be contingent on whether or not the economy is facing a sudden stop. During normal times, foreign lenders receive $P_1 = D$ if they call the loan in the interim, and $P_2 = (1 + r_N) D$ in the final period if they roll over the loan.[9] During a sudden stop, however, all the lenders call the debt and receive $P_1 = (1 + r_S) D$ in the interim. The debt payment schedule is then summarized given by:

	Interim payment P_1	Final payment P_2
Normal times($\psi(\varphi) < 1$)	D	$(1 + r_N) D$
Sudden stop ($\psi(\varphi) = 1$)	$(1 + r_S) D$	-

Because the interest rate is different when the economy is in sudden stop, the government can choose to partially default during sudden stop episodes by setting $r_S < r_N$. However, there is a limit to the haircut the lenders can suffer because the lenders can collectively bargain and extract a fraction θ of the interim resources available $(R + \lambda K)$.

Feasible Debt Contracts

We now define the feasibility constraints that the debt contract offered by the government must satisfy in this environment. First, we define a debt contract as a list of:

- four scalars: $\{R_1,\ K,\ r_N,\ r_S\}$ representing the initial reserves, the normal interest rate, and the sudden stop interest rate, and

[9] The assumption that lenders receive zero net return on debt called in the interim is not essential; in fact, it can be any arbitrary return that does not exceed the world interest rate.

- four state-contingent functions: $\{C(\varphi),\ R_2(\varphi),\ L(\varphi),\ \psi_i(\varphi,\varphi_i)\}$, which denote the consumption, the interim reserves, the interim liquidation, and the individual rollover policies, respectively.

Resource Feasibility A debt contract is *resource feasible* if it satisfies the following constraints:

$$\begin{aligned}
R_1 + K &\leq D && (2.1)\\
R_2(\varphi) + \psi(\varphi) P_1(\psi(\varphi)) &\leq R_1 + \lambda L(\varphi) \quad \forall\varphi && (2.2)\\
C(\varphi) + (1-\psi(\varphi)) P_2(\psi(\varphi)) &\leq R_2(\varphi) + A(K - L(\varphi)) \quad \forall\varphi && (2.3)\\
L(\varphi) &\in \{0, K\} \quad \forall\varphi && (2.4)\\
0 &\leq R_1,\ R_2(\varphi),\ C(\varphi) \quad \forall\varphi && (2.5)
\end{aligned}$$

In other words, initial reserves and invested capital cannot exceed the loan amount; interim reserves and interim payments cannot exceed initial reserves and interim output; and consumption and final payments cannot exceed interim reserves and final output.

Individual Rationality A debt contract is *individually rational* if, in the interim, an individual rolls over the loan if and only if this yields a higher payoff than calling the loan:

$$\psi_i^*(\varphi,\varphi_i) \in \quad \arg\max V(\cdot) \qquad (2.6)$$

$$\text{where} \quad V(\psi_i \,|\, \varphi, \varphi_i) = \begin{cases} P_1(\psi(\varphi)) & \text{if } \psi_i = 1 \\ \mathbb{1}_{\varphi_{i=0}} \cdot P_2(\psi(\varphi)) & \text{if } \psi_i = 0 \end{cases}$$

Participation Constraint A debt contract satisfies the *participation constraint* if ex ante the debt contract is as profitable as investing at the world interest rate r_W:

$$\mathbf{E}\left[V\left(\psi_i \,|\, \varphi, \varphi_i\right)\right] \quad \geq \quad (1 + r_W)D \tag{2.7}$$

Renegotiation Proofness Finally, a debt contract is *renegotiation-proof* if it satisfies:

$$\left(1 + r_S\right) D \quad \geq \quad \min\left\{D, \theta\left(R_1 + \lambda K\right)\right\} \tag{2.8}$$

This condition arises as foreigners bargain over domestic interim output during sudden stops. In this section, we impose $\theta = 1$. This assumption can be relaxed in the next section.

2.3.4 Optimal Debt Contract

An *optimal debt contract* is a tuple $B^* = \{R_1^*,\, K^*,\, r_N^*,\, r_S^*,\, C^*\left(\varphi\right),\, R_2^*\left(\varphi\right),\, L^*\left(\varphi\right),\, \psi_i^*\left(\varphi, \varphi_i\right)\}$ which maximizes the expected utility of the domestic representative agent subject to resource feasibility, individual rationality, the participation constraint, and renegotiation-proofness. In other words, the government solves the following problem:

$$\begin{aligned} \max\nolimits_B \qquad & \mathbf{E}_\varphi\left[C\left(\varphi\right)\right] \\ \text{subject to} \quad & (2.1) - (2.8). \end{aligned}$$

2.3.5 Optimal Contract Characterization

We now characterize the solution to the optimal debt contract problem.

Proposition 2.1. Optimal Debt Contract

If B^* solves the government's problem then:

(i) Interim payments are paid exclusively with reserves until they are depleted, i.e.,

$$\exists \varphi_R \in [0,1] \ s.t. \ \begin{cases} R_2(\varphi) > 0 & \iff \varphi \in [0, \varphi_R) \\ L(\varphi) = 0 & \iff \varphi \in [0, \varphi_R) \end{cases}$$

Furthermore,

$$\varphi_R^* = \frac{R_1}{D} = 1 - \left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right]^{\sigma}$$

(ii) For sufficiently large aggregate shocks, all lenders call their loans in the interim, i.e.,

$$\exists \varphi_S \in [0,1] \ s.t. \ \begin{cases} \psi(\varphi) = \varphi & \forall \varphi \in [0, \varphi_S) \\ \psi(\varphi) = 1 & \forall \varphi \in [\varphi_S, 1] \end{cases}$$

Furthermore, $\varphi_S = \varphi_R$.

Discussion Proposition 2.1(i) and 2.1(ii) establish that there are cutoff rules for reserves, liquidation, and sudden stops. In Proposition 2.1(i), φ_R is the liquidity shock at which reserves are depleted and the government must start liquidating the invested capital to meet the promised payments. In Proposition 2.1(ii), φ_S is the liquidity shock above which all lenders exit; we identify this phenomena as a *sudden stop*.

Because $\lambda < 1$, the government always uses existing reserves to meet payments before eventually liquidating the invested capital. Proposition 2.1(i) also establishes that the optimal reserves-to-liabilities ratio is φ_R.

Corollary 2.2. Endogenous Sudden Stop Probability

The optimal contract B^* induces an ex ante endogenous probability that a sudden stop occurs.

Furthermore, $\Pr(\psi = 1) = 1 - F(\varphi_R^*)$.

Proof. This follows immediately from Proposition 2.1. □

Discussion Together, Proposition 2.1 and Corollary 2.2 highlight the endogenous relation between the reserves-to-liabilities and the probability of sudden stops. In this environment, reserves are set to balance the liquidation costs incurred when reserves are not high enough, the sudden stop risks associated with excessive liquidation, and the cost of holding idle reserves.

2.3.6 Comparative Statics

In this subsection, we discuss how reserves and sudden stop probabilities are affected by changes in the underlying rollover risk, that is, changes in σ.

Proposition 2.3. Reserves and Sudden Stop Probability

(i) The optimal reserves-to-debt liabilities is increasing in rollover risk:

$$\frac{\partial \varphi_R^*}{\partial \sigma} > 0$$

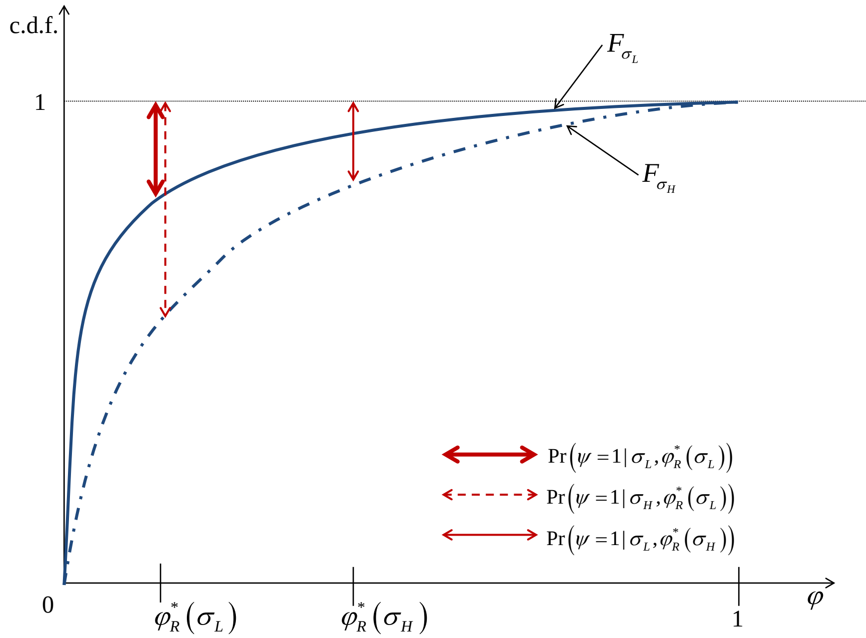
(ii) The sudden stop probability is increasing in rollover risk:

$$\frac{\partial \Pr\left(\psi\left(\varphi\right) = 1 \mid \sigma\right)}{\partial \sigma} > 0$$

Proposition 2.3 establishes that both the reserves-to-liabilities ratio and sudden stop probability are increasing in rollover risk. A larger σ implies larger interim shocks, prompting the domestic government to invest in higher reserves. However, the increase in reserves does not completely offset the higher probability of larger shocks, thus leading to an increase in the sudden stop probability.

A crucial question that we address in this paper is: what happens during an unexpected increase in rollover risk fromσ_L to $\sigma_H > \sigma_L$? It is easy to see that an unexpected increase in σ leads to a large increase in sudden stop probability not only due to the fact that larger shocks are more likely, but also because the government underinvests in

Figure 2.5: Illustration of Sudden Stop Surge



reserves. This relationship between beliefs, reserve-to-debt liabilities, and actual sudden stop probabilities is illustrated in Figure 2.5. Corollary 2.4 states this result formally.

Corollary 2.4. Sudden Stop Surge

Sudden stops increase following an unanticipated increase in σ from σ_L to σ_H, i.e.

$$\Pr(\psi = 1 \mid \sigma_H, \varphi_R^*(\sigma_L)) > \Pr(\psi = 1 \mid \sigma_L, \varphi_R^*(\sigma_L))$$

The previous corollary indicates that a surge in sudden stops may be an indirect evidence of an increase in the underlying rollover risk. A more direct way to identify the increase in rollover risk is by examining the volatility of gross interim flows. In our model, the gross interim flows in external liabilities is given by $-\varphi D$ during normal times. The next corollary shows that in our model, gross external flows do become more volatile following an increase in rollover risk, consistent with our findings in the empirical section.

Corollary 2.5. Increased Interim Volatility

Gross interim flows are more volatile following an increase in σ from σ_L to σ_H, i.e.,

$$Var(\varphi \mid \sigma_H) > Var(\varphi \mid \sigma_L)\,.$$

2.4 A Multi-Country Dynamic Extension with Learning and Regime Change

The previous section highlighted the main forces determining the optimal foreign reserves: costly sudden stop probabilities and productive capital use. We now propose a dynamic model with N small (emerging) economies to explain the increase in foreign reserves holdings and the transitory outburst in sudden stops.

As shown in the empirical section, gross flows in external liabilities increased after the mid-1990s in emerging economies. In our model, this is consistent with an increase in rollover risk as shown in Corollary 2.5. We showed in Corollary 2.4 that an unexpected increase in the rollover risk will lead to an outburst of sudden stops. This unusual rise in sudden stops prompts governments to update their beliefs and thereby rein in sudden stops.

In fact, using this extension, we can quantitatively account for the increase in reserves and the temporary outburst of sudden stops following an unexpected switch in rollover risk.

2.4.1 Environment

We consider N identical small economies indexed by $j = 1, \ldots, N$. Time is infinite, discrete and indexed by $t = 0, 1, \ldots, \infty$. Each country is populated by an infinitely-lived representative agent and a welfare-maximizing domestic government. The domestic agents in country j order consumption sequences according to $\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t C_t^j\right]$ where

β is the discount factor. There is a continuum of infinitely lived risk-neutral foreign lenders $i \in [0,1]$.

2.4.2 Timing

Each time period t is divided into three stages, $s = 0, 1, 2$ and encapsulates the three stages of the previous model:

- $s = 0$ is the initial contracting stage
- $s = 1$ is the interim stage when liquidity shocks are realized and rollovers decided
- $s = 2$ is the final production and consumption stage.

2.4.3 Shocks and Information Structure

The aggregate interim liquidity shock in country j at time t is denoted by $\varphi_t^j \in [0,1]$. The N aggregate shocks $\left\{\varphi_t^j \ : \ j = 1 \ldots N\right\}_{t=0}^{\infty}$ are independent and identically distributed across countries and time. These aggregate liquidity shocks follow a common stochastic process with cumulative distribution function F_{σ_t}. As in the basic model, a fraction φ_t^j of foreign lenders lending to country j receive liquidity shocks and must call the debt in the interim. We assume $\sigma_t \in \{\sigma_L, \sigma_H\}$ with $\sigma_L < \sigma_H$. This regime parameter σ_t is unobserved and unknown to the agents. However, all agents share a common belief ρ_t at time t:

$$\rho_t \triangleq \Pr\left(\sigma_t = \sigma_L\right)$$

At the end of each period t, agents observe the sudden stop occurrences in the N countries. Using these sudden stop occurrences and the endogenous sudden stop probabilities, agents update their beliefs according to Bayes' rule, detailed in section 2.4.5.

2.4.4 Technologies

Within each period t, the technologies available at a stage s are identical to those in the previous section.[10] We now allow for partial liquidation in the interim: $L_t^j\left(\varphi_t^j\right) \in \left[0,\ K_t^j\right]$. Also, at the end of each period, the government can save $R_{0,t+1}^j\left(\varphi_t^j\right)$ reserves for the next period. These saved reserves are available at the initial stage ($s = 0$) during the next period. These inter-temporal reserves are allocated from the reserves remaining in the final stage ($s = 2$) of the current period t:

$$R_{0,t+1}^j\left(\varphi_t^j\right) \in \left[0\ ,\ R_{2,t}^j\left(\varphi_t^j\right)\right]$$

2.4.5 Optimal Recursive Debt Contracts

An important difference with the basic model is the endogeneity of reserve endowments. In the basic model, the reserve endowment was zero; in the dynamic model, governments will face a consumption/savings decision and will choose the reserve endowments of the following period. Another difference is the relaxation of the full liquidation constraint; this implies that sudden stops need not necessarily occur as soon as reserves are depleted.

Given an incoming level of reserves R_0 and a belief ρ, a debt contract is defined to be:

- five scalars $\{R_1,\ K,\ r_N,\ r_S,\ \varphi_S\}$, representing the initial reserves, the normal interest rate, the sudden stop interest rate, and the sudden stop cutoff; and
- five state-contingent functions $\{C(\varphi)\ ,\ R_2(\varphi)\ ,\ L(\varphi)\ ,\ R_0'(\varphi)\ ,\ \psi_i(\varphi,\varphi_i)\}$, which denote the consumption, the interim reserves, the interim liquidation, the inter-temporal reserves savings, and the individual rollover policies, respectively.

As before, let

[10] The superscript j and the subscript t are therefore added to the variables from the previous model to denote the country and the period. We keep the subscripts indicating the stage s when necessary.

$$\psi(\varphi) = \int \psi_j(\varphi)dj$$

$$P_1(\varphi) = \begin{cases} D & \text{if } \psi(\varphi) < 1 \\ (1+r_S)\,D & \text{otherwise} \end{cases}$$

$$P_2(\varphi) = \begin{cases} (1+r_N)\,D & \text{if } \psi(\varphi) < 1 \\ 0 & \text{otherwise} \end{cases}$$

Resource Feasibility A debt contract is *resource feasible* if it satisfies the following constraints:

$$R_1 + K \leq D + R_0 \tag{2.9}$$

$$R_2(\varphi) + \psi(\varphi)\,P_1(\psi(\varphi)) \leq R_1 + \lambda L(\varphi) \quad \forall\varphi \tag{2.10}$$

$$C(\varphi) + (1-\psi(\varphi))\,P_2(\psi(\varphi)) \leq R_2(\varphi) - R_0'(\varphi) + A\,(K - L(\varphi)) \quad \forall\varphi \tag{2.11}$$

$$L(\varphi) \in [0\,,\ K] \quad \forall\varphi \tag{2.12}$$

$$R_0'(\varphi) \in [0\,,\ R_2(\varphi)] \quad \forall\varphi \tag{2.13}$$

$$0 \leq R_1,\ R_2(\varphi)\,,\ C(\varphi) \quad \forall\varphi \tag{2.14}$$

Individual Rationality A debt contract is *individually rational* if it satisfies the following constraints:

$$\psi_i^*(\varphi, \varphi_i) \in \arg\max V(\cdot) \tag{2.15}$$

$$\text{where} \qquad V(\psi_i \,|\, \varphi, \varphi_i) = \begin{cases} P_1(\psi(\varphi)) & \text{if } \psi_i = 1 \\ \mathbb{1}_{\varphi i=0} \cdot P_2(\psi(\varphi)) & \text{if } \psi_i = 0 \end{cases}$$

$$\psi_i^*(\varphi, \varphi_i) = \begin{cases} 0 & \text{if } \varphi < \varphi_S \\ 1 & \text{otherwise} \end{cases} \tag{2.16}$$

The first condition, which requires an individual to roll over the loan if and only if this yields a higher payoff than calling the loan, is identical to the basic model. The second condition requires an individual to roll over the loan if and only if the aggregate liquidity shock, φ, is larger than the sudden stop cutoff, φ_S.

Participation Constraint A debt contract satisfies the *participation constraint* if ex ante the debt contract is as profitable as investing at the world interest rate r_W:

$$\mathbf{E}_{\varphi}\left[V\left(\psi_i \,|\, \varphi, \varphi_i\right)\right] \;\geq\; (1+r_W)D \tag{2.17}$$

Renegotiation Proofness Finally, a debt contract is *renegotiation-proof* if it satisfies the following constraints:

$$(1+r_S)\,D \geq \min\left\{D, \theta\left(R_1 + \lambda K\right)\right\} \tag{2.18}$$

Recursive Problem and Beliefs Given a belief ρ about the liquidity shock regime, we define the optimal recursive contracts as the solution to the following recursive problem:

$$\begin{aligned} W\left(R_0;\ \rho\right) = \quad & \max\nolimits_B \quad \mathbf{E}_{\varphi|\,\rho}\left[u\left(C\left(\varphi\right)\right) + \beta W\left(R_0'\left(\varphi\right);\ \rho\right)\right] \\ & \text{subject to} \qquad\qquad (2.9) - (2.18) \end{aligned}$$

The common belief is dynamically updated using the sudden stop occurrences and sudden stop probabilities in the N countries. Let us denote $\chi_t \in \{0,1\}^N$ as the vector of sudden stops. Bayes' Rule implies that:

$$\rho_{t+1} = \frac{\rho_t \Pr\left(\chi_t \mid \sigma_L\right)}{\rho_t \Pr\left(\chi_t \mid \sigma_L\right) + \left(1-\rho_t\right)\Pr\left(\chi_t \mid \sigma_H\right)}$$

with

$$\begin{cases} \Pr\left(\chi_t \mid \sigma_L\right) & \triangleq \displaystyle\prod_{j=1}^{N}\left\{\left[1 - F_{\sigma_L}\left(\underline{\varphi_{s,t}^j}\right)\right]\cdot \chi_t^j + F_{\sigma_L}\left(\underline{\varphi_{s,t}^j}\right)\cdot\left(1-\chi_t^j\right)\right\} \\ \Pr\left(\chi_t \mid \sigma_H\right) & \triangleq \displaystyle\prod_{j=1}^{N}\left\{\left[1 - F_{\sigma_H}\left(\overline{\varphi_{s,t}^j}\right)\right]\cdot \chi_t^j + F_{\sigma_H}\left(\overline{\varphi_{s,t}^j}\right)\cdot\left(1-\chi_t^j\right)\right\} \end{cases}$$

where $\underline{\varphi^j_{S,t}}$ is the sudden stop cutoff for$(R_0;\ \rho) = \left(R^j_{0,t},;\ \rho_t = 1\right)$ and $\overline{\varphi^j_{S,t}}$ is the sudden stop cutoff for $(R_0;\ \rho) = \left(R^j_{0,t},;\ \rho_t = 0\right)$. Hence, given a sequence of sudden stop occurrences $\{\chi_t\}_t$, an initial belief ρ_0, and initial reserve endowments $\left\{R^j_-\right\}_{j=1}^N$, the realized sequence of optimal contracts is well-defined using the functional equation solutions and Bayes' rule.

2.4.6 Quantitative Analysis

In this section, we discuss the quantitative results of a carefully parametrized model. Our simulations show that our extended model can account for the stylized facts we documented.

In particular, we simulate the following thought experiment. We assume that the period of 1990-1996 was an era of relatively low volatility in international capital movements, i.e. a σ_L regime. By 1997, globalization and widespread financial liberalization allowed less restrictive capital movements but governments and investors underestimated the increase in capital mobility, i.e. there is an unexpected change to a σ_H regime. Based on our theory, this will cause an underinvestment in reserve holdings which increases the probability of sudden stops. Governments and investors, seeing the rise in sudden stops, update their common belief about the prevailing regime. By 2002, agents have fully learned the new regime; as a result, reserves-to-debt is higher and sudden stops decrease.

Parametrization and Functional Forms A period in the model is assumed to be a quarter. We choose $N = 23$ as we have 23 emerging economies in our dataset. The liquidity shock process is an important element of the model. We assume the aggregate liquidity shock distributions $(F_{\sigma_L}, F_{\sigma_H})$ belong to the class of Generalized Bounded

Pareto distributions on $[0,1]$:

$$F_\sigma(\varphi) \triangleq 1 - (1-\varphi)^{\frac{1}{\sigma}}$$

An increase in σ shifts the cumulative distribution function F_σ to the right. The switch from σ_L to σ_H therefore reflects the increase in interim capital mobility. The parameters β, r_W, σ_L, and σ_H are then set to match some facts regarding international liquidity. In particular, we set β to match average interest rates of 2% in emerging economies over 1990-2007, r_W to match the risk-free rate of 1%, while σ_L and σ_H are set to match median reserves-to-debt ratios in the emerging economies for the periods of 1990-1996 and 2002-2007 respectively. We follow Ennis and Keister (2003) to set the divestment cost $1-\lambda$ to be 30%. We assign an arbitrary value of A to 1.2. The parameters are summarized in table 2.3.

Table 2.3: Parameter values

Name	Symbol	Value	Target
Discount factor	β	0.98	average interest rates in emerging economies over 1990-2007
World interest rate	r_W	0.01	risk-free rate
Low rollover risk parameter	σ_L	0.06	average reserves-to-external debt liabilities median among emerging economies 1990-1996
High rollover risk parameter	σ_H	0.16	average reserves-to-external debt liabilities median among emerging economies 2002-2007
Divestment parameter	λ	0.7	Ennis and Keister (2003)
Productivity	A	1.2	-
Bargaining parameter	θ	1	-
Number of economies	N	23	number of emerging countries studied

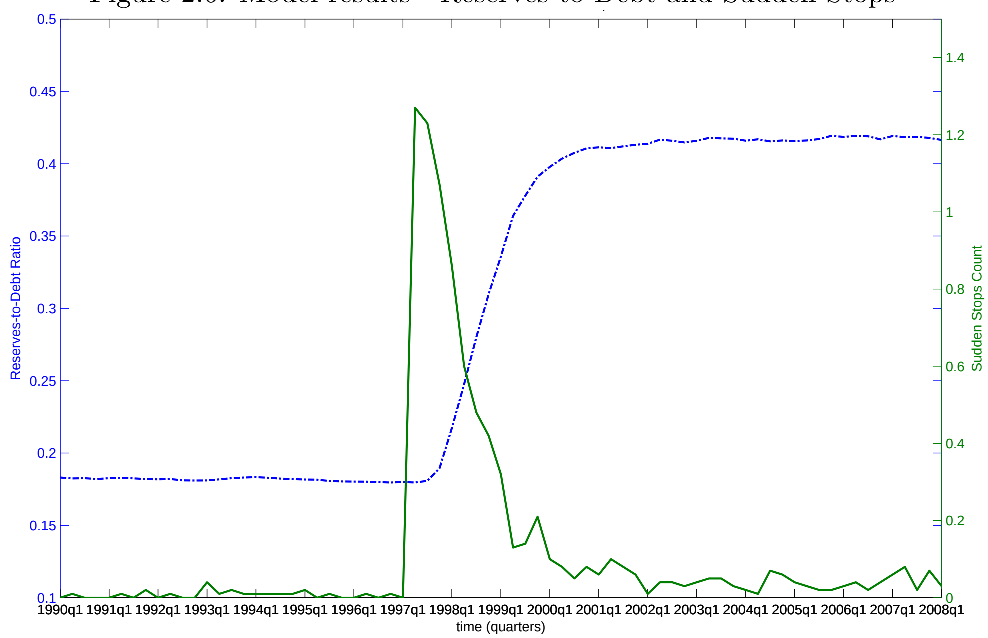
Quantitative Results We consider $N = 23$ identical economies starting with different initial foreign reserves.[11] As the N economies experience different aggregate liquidity paths, their reserves holdings and sudden stops paths also evolve differently. The results shown are the average across a large number of simulated paths for the N countries.

As can be seen in figure 2.6, our model is able to replicate the pattern of low

[11] The initial levels of foreign reserves were generated by simulating, for a few periods, the model with initial zero foreign reserves.

frequency sudden stops during 1990-1996, high frequency sudden stops in the transition (1997-2001), and low frequency after the transition (2002-2007). During the transition, governments under-invest in reserves, thereby increasing the probability of sudden stops. Once the governments have learned of the regime change to higher liquidity shocks, they choose to hold a higher level of reserves, thus returning sudden stop probabilities to lower levels.

Figure 2.6: Model results - Reserves-to-Debt and Sudden Stops



In our theory, misaligned beliefs beget "abnormal" sudden stop occurrences. As sudden stop occurrences increase, reserves dry up more often but governments keep building up their foreign holdings as a result of updated beliefs. In this model, model reserves do not serve as a *post*-sudden stop insurance. Instead, in contrast to most consumption smoothing theories of reserves allocation, reserves play an active role of preventing sudden stop occurrences and they do not help increase consumption after sudden stops.

Table 2.4 summarizes our key results. One drawback of the results is that the speed at which agents learn the true process is quite fast: this leads to governments increasing reserves faster and sudden stops ceasing sooner than 2002 as seen in the data. Also, since $\sigma_L < \sigma_H$, the post-crisis era is characterized by slightly more sudden stops than the pre-crisis era. Of course, both periods feature much less sudden stops than the crisis/adjustment era.

Table 2.4: Summary of Numerical Results

	1990-1996	1997-2001	2002-2007
Data			
Reserves-to External Debt Liabilities	0.167	0.282	0.409
Sudden Stops	2	10	0
Model			
Reserves-to External Debt Liabilities	0.177	0.326	0.416
Sudden Stops	0.32	7.19	0.95

2.5 Conclusion

In this paper, we have studied empirically and theoretically the joint dynamics foreign reserves and sudden stops in emerging economies. Using international liquidity data for 23 emerging economies, we document that foreign reserves have dramatically increased in emerging countries from 1990 to 2007. We also present the time series of sudden stops in these emerging economies: there were virtually no sudden stops except during 1997-2001.

We then develop a small open economy model where reserves endogenously affect the probability of sudden stops. In our model, "patient" foreign lenders choose to roll over their loans as long as their returns are not undermined by the divestment made to repay lenders calling in the interim. Sudden stops occur when all foreign lenders choose to call the loans. On one hand, reserves protect domestic projects from liquidation and make foreign lenders calmer as the country is solvent in more states of the world. On the other hand, foreign reserves reduce the capital used in the productive sector. Consequently, the model yields an endogenous probability of sudden stop and an optimal reserves-to-debt ratio. Furthermore, we show that foreign reserve holdings increase with the rollover risk.

We then extend our model to explain the accumulation of foreign reserves and the transitory surge in sudden stops in emerging economies. In our model, any underestimation of the underlying rollover risk will temporarily induce an outburst of sudden stops. We present a dynamic multi-country model with Bayesian learning and a regime switch in the rollover risk. With the gradual learning of the true regime, we obtain a transition path during which sudden stops surge. The optimal reserves-to-debt ratios are higher at the end of the transition, and sudden stops subside, as seen in the data.

This paper therefore provides a useful theory of reserves, rollover risk, and sudden stops. Our model however is highly stylized. It does not include many potentially relevant features. For instance, Cole and Kehoe (2000) indicate that the composition of a country's debt portfolio matters for sudden stops and Obstfeld, Shambaugh, and Taylor (2010) highlighted other relevant dimensions of financial stability. We leave these directions for future research.

Chapter 3

Entry Barriers and Creative Destruction

3.1 Introduction

Economist Joseph Schumpeter viewed capitalism as the "process of industrial mutation ... that incessantly revolutionizes the economic structure from within, incessantly destroying the old one, incessantly creating a new one."[1] In other words, innovative entry by entrepreneurs is the disruptive force that sustains economic growth. Indeed, using cross-country data, we find that GDP per capita is positively correlated with firm entry rates. We also find that firm entry rates are positively correlated with barriers to firm entry.

We develop a model where aggregate productivity growth is driven by the continual entry of young, productive firms and the exit of old, unproductive firms. The model, based on Asturias et al. (2012a,b), features a tractable balanced growth path where aggregate productivity, consumption, and output all grow at the same rate. Lower entry

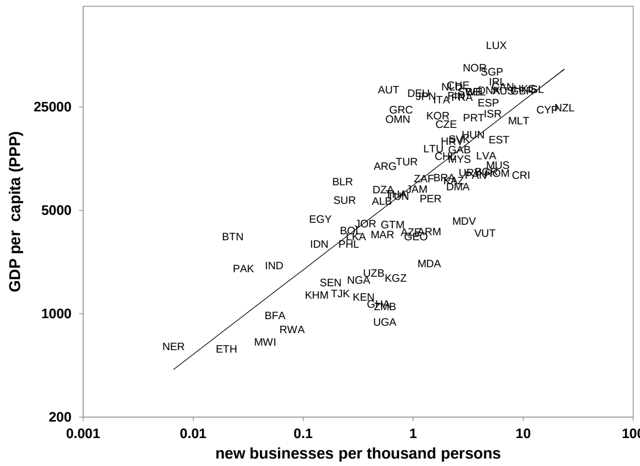
[1] Schumpeter (1942)

costs lead to a larger entry of new firms, and due to general equilibrium effects, lead to a larger exit of old, existing firms. Hence, differences in entry policy lead to different levels of entry and output, while all economies grow at a balanced growth path with identical growth rates. Moreover, in the quantitative extension, we show that reforms to entry costs can generate transition paths that resemble that of high-growth emerging economies.

Our work is closely related to Barseghyan and DiCecio (2009) who construct a model in which cross country differences in entry costs lead to differences in output and productivity. A key departure of our model is that we have an analytically tractable balanced growth path that features endogenous firm entry and exit. Aggregate productivity growth in our model is determined by entry and exit decisions of individual firms. Building up aggregate differences from firm level decisions, as we do, has been researched by several authors, such as Parente and Prescott (1999) and Herrendorf and Teixeira (2009). In these models a coalition secures monopoly rights that allow the coalition to block the adoption of better technologies. In our theory better technology is embodied in new firms and barriers to entry drive suboptimal levels of new firm entry. Our model of heterogeneous firms is also based on Hopenhayn, H. A. (1992). We have abstracted from firm level uncertainty, which allows us to develop a framework that generates an extremely tractable balanced growth path, rather than the stationary distributions considered in other models of heterogeneous firms.

This chapter is structured as follows. Section 3.2 documents the relationship between business entry, GDP, and barriers to entry. Section 3.3 presents the model, characterizes the balanced growth path, and discusses how the economy is affected by changes to entry barriers. Section 3.4 presents the quantitative results, and section 3.5 concludes.

Figure 3.1: Business entry and GDP per capita



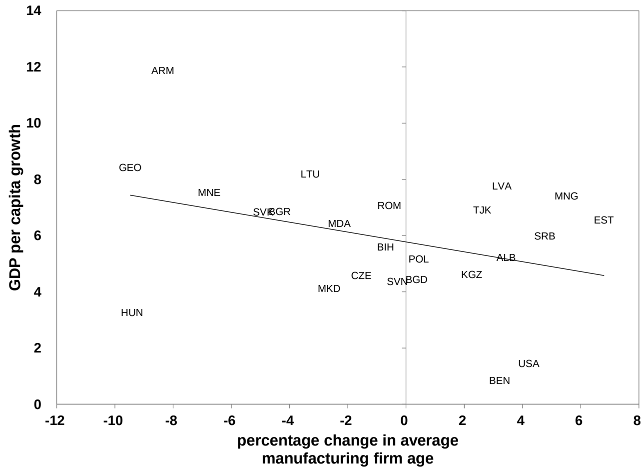
3.2 Empirical Analysis

In this section we document three patterns in the data: (i) firm entry rate is positively correlated with GDP per capita across countries, (ii) declining average manufacturing firm age is associated with higher growth rates, and (iii) the firm entry rate is decreasing in the level of entry costs.

As in Asturias et al. (2012a), we use the World Bank's World Development Indicators business entry statistic, which measures the number of new firms officially registered per year, to construct the business entry rate as the number of newly registered firms per thousand working age people. We plot GDP per capita, measured at purchasing power parity, against firm entry rates in figure 3.1. The data are for the year 2004. Firm entry rates are positively correlated with income per capita, with a correlation coefficient of 0.4.

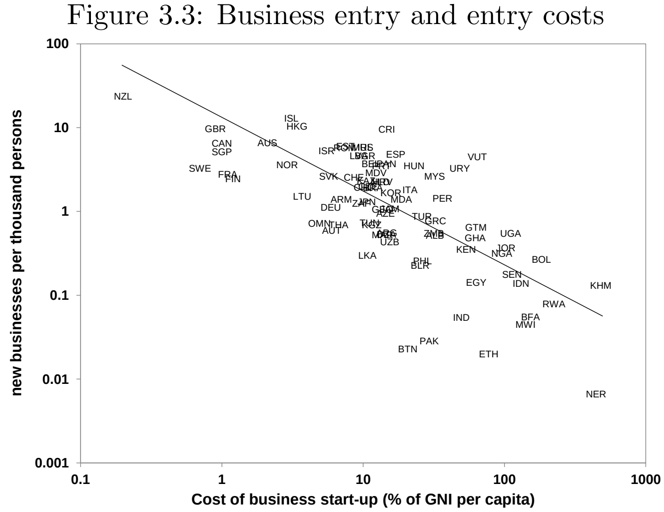
The second fact that we document is that economies that are growing faster also have declining average firm ages for manufacturing firms. We use data from the World Bank Enterprise Surveys, a program that has collected firm level survey data in developing

Figure 3.2: Change in manufacturing firm age and GDP per capita



countries since 2002. Among many questions, the survey asks firms the age of their establishment. Typically 1200-1800 interviews are conducted in larger economies, 360 interviews are conducted in medium-sized economies, and 150 interviews are conducted in smaller economies. After removing resource rich countries, we find that declining average manufacturing firm age is associated with higher growth rates, as can be seen in figure 3.2. This relationship is statistically significant at the 10 percent level.

Figure 3.3: Business entry and entry costs



Finally, as in Asturias et al. (2012a), we document that the rate of firm entry is positively correlated with policy induced barriers to entry. In particular, we use the World Bank's Doing Business surveys, to construct the measure of entry barriers which estimates the costs of starting up a representative industrial or commercial business. As can be seen in figure 3.3, the relationship is clearly positive and significant.

3.3 The Model

The model is closely related to the one developed in Asturias et al. (2012a,b). A key departure from the original model is the abstraction from financial frictions. This allows us to look at not only the effects of entry barriers on firm entry, but also endogenous exit. The model is a closed economy general equilibrium model comprised of a representative household, a representative final good producer, and a continuum of monopolistically competitive intermediate goods producers.

3.3.1 Households

The representative household is endowed with labor, L, which is inelastically supplied to firms. The household chooses consumption of composite good, C_t, and bond holdings, B_{t+1}, to solve

$$\max_{c_t, B_{t+1}} \sum_{t=0}^{\infty} \beta^t \log C_t$$

$$P_t C_t + q_{t+1} B_{t+1} = w_t L + \Pi_t + B_t \tag{3.1}$$

$$C_t \geq 0, \quad B_t \geq -g^t B, \quad B_0 \text{given},$$

where β is the time discount factor and Π_t is the aggregate dividends paid by firms in the economy. We normalize the wage so that $w_t = 1$ in each period. The constraint $B_t \geq -g^t B$ rules out Ponzi schemes but we choose the constant B large enough so that the constraint does not otherwise bind in equilibrium; $g \geq 1$ is the growth factor for the economy.

3.3.2 Producers

The representative final good firm, which operates in perfect competition, purchases intermediate goods from intermediate firm z, $y(z)$, to solve

$$\min_{y_t(z)} \int_0^{\eta_t} p_t(z) y_t(z) di$$

$$\text{s.t.} \left(\int_0^{\eta_t} y_t(z)^\rho di \right)^{\frac{1}{\rho}} = Y_t$$

The measure of goods available, η_t, is an important variable in this model. We show that entry costs and financial frictions decrease the measure of intermediate goods available. The elasticity of substitution between intermediate good varieties is $1/(1-\rho)$; the varieties are close, but imperfect, substitutes, $\rho < 1$. Solving the final good producer's problem, we obtain the demand function for good z,

$$y_t(z) = \frac{Y_t}{p_t(z)^{\frac{1}{1-\rho}} P_t^{\frac{-\rho}{1-\rho}}} \tag{3.2}$$

with

$$P_t = \left(\int_0^{\eta_t} p_t(z)^{\frac{-\rho}{1-\rho}} \, dz \right)^{\frac{1-\rho}{-\rho}} \tag{3.3}$$

There is a continuum of intermediate good firms which live for a maximum of N periods, unless they choose to exit the market earlier. The firm producing good z that is of age j produces according to

$$y_t(z) = x(z) \ell_t(z) \tag{3.4}$$

where $x(z)$ is the marginal productivity of firm z. Production is subject to a fixed cost of operating, κ_j, which is denominated in units of labor, and may be conditional on the age of the firm, j. If a firm chooses not to pay κ_j the firm exits the economy forever.

Conditional on choosing to produce, the firm chooses $p_t(j)$ to maximize profit, taking as given the final good price, P_t, and aggregate output Y_t,

$$\pi_{jt}(x(z), Y_t, P_t) = \max_{p_{jt}(z)} \; p_{jt}(z) \frac{Y_t}{p_{jt}(z)^{\frac{1}{1-\rho}} P_t^{\frac{-\rho}{1-\rho}}} - \frac{Y_t}{x(z) p_{jt}(z)^{\frac{1}{1-\rho}} P_t^{\frac{-\rho}{1-\rho}}} - \kappa_j \tag{3.5}$$

The solution to this problem yields the standard markup over marginal cost pricing, $p_{jt}(z) = (\rho x(z))^{-1}$. We assume that firms can costlessly differentiate their products, so no two firms will choose to produce the same good z. Every firm with productivity x, however, chooses the same price and sells the same identical quantity. In what follows, we no longer characterize a good by its label, z, but by the productivity, x, of the firm that produces it.

Every period, a mass μ of potential entrants draw their productivities from a Pareto distribution that improves each period before deciding to produce-an entry- or to exit immediately. A potential entrant in period t draws its productivity from the distribution

$$F_t(x) = 1 - \left(\frac{x}{g^t} \right)^{-\gamma},$$

which is characterized by a mean that is improving at rate $g - 1$. Additionally, we require that $\gamma(1 - \rho) - \rho > 0$. We assume that new entrants pay a higher fixed cost than existing firms. In what follows, we consider a simple structure for the fixed costs of operating: a new entrant pays κ and, regardless of a firm's age, all continuing firms pay $f < \kappa$.

In equilibrium, the behavior of firm entry and exit can be summarized by simple cutoff rules. Let $\hat{x}_{jt}$ be the productivity of the firm which has the lowest productivity of all firms of age j that are operating in period t. Profits, $\pi(x)$, increase monotonically with productivity, so every firm of age j with $x \geq \hat{x}_{jt}$ produces in period t. Since entering firms must finance any fixed costs from the operating profits during the period in which they enter, the productivity cutoff level of entering firms, denoted as $\hat{x}_{1t}$ is given by the period no-profit condition:

$$p(\hat{x}_{1t})y_t(\hat{x}_{1t}) - \frac{y_t(\hat{x}_{1t})}{\hat{x}_{1t}} - \kappa = 0 \tag{3.6}$$

For firms of age $j > 2$, the cutoff productivity $\hat{x}_{jt}$, in the case of endogenous exit, is determined by the no profit condition,

$$p(\hat{x}_{jt})y_t(\hat{x}_{jt}) - \frac{y_t(\hat{x}_{jt})}{\hat{x}_{jt}} - f = 0 \tag{3.7}$$

or in the case of exogenous exit, is given by the entry cutoff level,

$$\hat{x}_{jt} = \hat{x}_{1,t-j+1}. \tag{3.8}$$

For the rest of this section, we assume

$$\frac{\kappa}{f} < g^{N\frac{\rho}{1-\rho}},$$

which ensures that firm exit is endogenous. Given this notation, the mass of firms that are producing at time t is

$$\eta_t = \mu \sum_{i=1}^{N} (1 - F_{t-i+1}(\hat{x}_{i,t-i+1})). \tag{3.9}$$

3.3.3 Equilibrium

In this section, we focus on balanced growth paths. To define a balanced growth path, we first define an equilibrium. The definition is also useful for potential future research that studies transition paths of the model. To specify the equilibrium, we need to provide initial conditions on the number of firms entering period 0 that have ages $j > 1$, given by the cutoff levels $\hat{x}_{j0}$, $j = 2, ..., N$, and the bond holdings by households, B_0.

Definition 3.1. *Given these initial conditions, an equilibrium is sequences of entry-exit threshold values, $\{\hat{x}_{jt}\}_{t=0}^{\infty}$, $j = 1, \ldots, N$, prices and allocations for intermediate* *firms,* $\{p_t(x), y_t(x), \ell_t(x)\}_{t=0}^{\infty}$*, for all* $x \geq \hat{x}_{jt}$*,* $j = 1, \ldots, N$*, bond prices and bond holdings,* $\{q_{t+1}, B_{t+1}\}_{t=0}^{\infty}$*, aggregate dividends and final output,* $\{D_t, Y_t\}_{t=0}^{\infty}$*, and household consumption,* $\{C_t\}_{t=0}^{\infty}$*, such that:*

1. *Given* $\{P_t, D_t, q_{t+1}\}_{t=0}^{\infty}$*,* $\{C_t\}_{t=0}^{\infty}$ *and* $\{B_{t+1}\}_{t=0}^{\infty}$ *solve the household's problem in* (3.1)

2. *Given* $\{P_t, Y_t, q_{t+1}\}_{t=0}^{\infty}$ *,*$\{p_t(x), \ell_t(x)\}_{t=0}^{\infty}$ *solves the problem of the firm with productivity* x*and age* j *in* (3.5)*, for all* $x \geq \hat{x}_{jt}$

3. *The labor market clears for all* $t \geq 0$*,*

$$L = \mu \sum_{i=1}^{N} \int_{\hat{x}_{jt}}^{\infty} \left(\ell_t(x) + \kappa_i\right) dF_{t-i+1}(x)dx \tag{3.10}$$

4. *Entry and exit cutoffs satisfy conditions* (3.6)*,* (3.7)*, and* (3.8) *for all* $j = 1, \ldots, N$*, and all* $t \geq 0$

5. *The bond market clears for all* $t \geq 0$*,*

$$B_{t+1} = 0 \tag{3.11}$$

6. *Dividend payments satisfy for all* $t \geq 0$*,*

$$\Pi_t = \sum_{i=1}^{N} \int_{\hat{x}_{it}}^{\infty} \pi_{it}(x)\, dF_{t-i+1}(x)dx \tag{3.12}$$

3.3.4 Balanced Growth Path

In this section, we prove that the model has a balanced growth path, and we characterize the behavior of the key variables along the balanced growth path.

Definition 3.2. *A balanced growth path is a path of entry-exit threshold values,* $\{\hat{x}_{jt}\}_{t=0}^{\infty}$*,* $j = 1, \ldots, N$*, real aggregate profits,* $\left\{\tilde{\Pi}_t\right\}_{t=0}^{\infty}$ *where* $\tilde{\Pi}_t = \frac{\Pi_t}{P_t}$*, household consumption,*

$\{C_t\}_{t=0}^{\infty}$, *and final good output*, $\{Y_t\}_{t=0}^{\infty}$, *such that* $\hat{x}_{jt}$, $j = 1, \ldots, N$, *and*, $\tilde{\Pi}_t$, C_t, Y_t, *grow at the same rate*

$$\frac{\hat{x}_{jt+1}}{\hat{x}_{jt}} = \frac{\tilde{\Pi}_{t+1}}{\tilde{\Pi}_t} = \frac{C_{t+1}}{C_t} = \frac{Y_{t+1}}{Y_t} = g \quad, j = 1, \ldots, N.$$

We will denote $\hat{x}_{1t}$ as a potential entrant's minimum productivity necessary to operate and $\hat{x}_{jt}$ as the minimum productivity necessary for an incumbent with age $j \geq 2$ to remain in the market. Furthermore, let $n_t(x)$ be the last age at which a firm of productivity x born at time t remains in the market.

Lemma 3.3. *Let* $\frac{\kappa}{f} < g^{N\frac{\rho}{1-\rho}}$. *In any balanced growth path, the cutoffs* $\hat{x}_{jt}$ *are characterized by*

$$\hat{x}_{1t} = \kappa[(1-\rho)(L + \Pi_t)]^{\frac{1-\rho}{-\rho}} P_t^{-1} \rho^{-1} \tag{3.13}$$

$$\hat{x}_{jt} = f[(1-\rho)(L + \Pi_t)]^{\frac{1-\rho}{-\rho}} P_t^{-1} \rho^{-1} \tag{3.14}$$

The proof of lemma 3.3 can be found in the appendix. We can now state proposition 3.4 which establishes a balanced growth equilibrium.

Proposition 3.4. *The economy has a balanced growth.*

The proof of proposition 3.4 is similar to that in Asturias et al. (2012a, 2012b), and can be found in the appendix.

We can now characterize $n_t(\hat{x}_{1t})$. An existing firm of age $2 \leq j \leq N$ will remain in the market as long as it remains profitable. This implies that

$$\hat{x}_{j,t+j-1} = \max\left\{\kappa^{\frac{1-\rho}{\rho}}[(1-\rho)(L+\Pi_t)]^{\frac{1-\rho}{-\rho}} P_t^{-1}\rho^{-1} \ , \ f^{\frac{1-\rho}{\rho}}[(1-\rho)(L+\Pi_{t+j-1})]^{\frac{1-\rho}{-\rho}} P_{t+j-1}^{-1}\rho^{-1}\right\} \tag{3.15}$$

The first term is the cutoff productivity for firm age j and the second term is the minimum productivity to be profitable that period. Using the balanced growth path conditions that $\Pi_{t+j-1} = \Pi_t$ and $P_{t+j-1} = g^{1-j} P_t$, we get that if $n_t(\hat{x}_{1t}) < N$ then the following condition must be satisfied

$$g^{(n_t(\hat{x}_{1t})-1)\frac{\rho}{1-\rho}} \leq \frac{\kappa}{f} < g^{n_t(\hat{x}_{1t})\frac{\rho}{1-\rho}} \tag{3.16}$$

and if $n_t(\hat{x}_{1t}) = N$ then

$$\frac{\kappa}{f} > g^{(N-1)\frac{\rho}{1-\rho}} \tag{3.17}$$

Note that $n_t(\hat{x}_{1t})$ does not depend on t in the balanced growth path. Thus, we will denote $\hat{n}(\kappa) = n_t(\hat{x}_{1t}; \kappa)$.

Lemma 3.5. *If $g > 1$, then in any balanced growth path, $\hat{n}(\kappa)$ is an increasing step function of κ.*

Lemma 3.6. *Continuity of the aggregate variables*

(i) $P_t(\kappa)$ is continuous in κ

(ii) $Y_t(\kappa)$ is continuous in κ

(iii) $\Pi(\kappa)$ is continuous in κ

(iv) $M(\kappa)$ is continuous in κ.

The proofs of lemma 3.5 and lemma 3.6 are similar to that in Asturias et al. (2012b), and can be found in the appendix. Lemma 3.6 implies that the aggregate variables are continuous even though $\hat{n}(\kappa)$ is not continuous. This comes from the fact that when there is a jump in $\hat{n}(\kappa)$, only an infinitesimally small number of firms change the age at which they exit. Lemma 3.6 is extremely useful for comparative statics. Since $Y_t(\kappa)$ and $M(\kappa)$ are continuous, and differentiable everywhere except at the jump points of $\hat{n}(\kappa)$, it suffices to consider only the points at which $Y_t(\kappa, \theta)$ and $M(\kappa, \theta)$ are differentiable in the comparative statics analysis.

3.3.5 Comparative Statics

In this section, we prove that firm entry and output are decreasing in entry costs.

Proposition 3.7. *Firm entry and output are decreasing in entry costs, i.e.*

(i) $\log Y_t(\kappa)$ *is decreasing in* κ*, and*

(ii) $\log M(\kappa)$ *is decreasing in* κ*.*

Proof. From lemma 3.6, we know that $Y_t(\kappa)$ and $M(\kappa)$ is continuous in κ. It suffices to show that $\frac{d\log Y_t(\kappa)}{d\kappa} < 0$ and $\frac{d\log M(\kappa)}{d\kappa} < 0$ for $\kappa \neq \kappa(m)$, $\forall m = 2, ..., N$. We can show these conditions are true. Refer to the technical appendix of Asturias et al. (2012a, 2012b) for details. □

3.4 Quantitative Results and Analysis

We present here the quantitative results of the baseline model. For this section, we relax the assumption, $\frac{\kappa}{f} < g^{N\frac{\rho}{1-\rho}}$, which we made for analytical tractability in previous sections. We choose standard values for the curvature parameter γ and elasticity of substitution σ. g is set to match the productivity growth of the United States, and firms are assumed to live up to 30 years. The parameters used are summarized below in table 3.1.

To compute this equilibrium, we first use the zero profit conditions in (3.6) and (3.7) as an initial guess for the cutoff conditions for firms of different cohorts and compute the aggregate variables. We then simulate forward one period and calculate the cutoff conditions for each cohort. Then, as specified in (3.15) we set the cutoff condition for each cohort to be the maximum of the zero profit condition productivity and the previous period's zero profit condition for that cohort. If the latter is greater than the

Table 3.1: Calibration and parameters

Parameter		Value
γ	Curvature parameter	4
σ	Elasticity of substitution	2.5
g	Annual growth rate of CDF	2 percent
$\frac{\kappa}{f}$	Entry cost relative to operational cost	4
t	Number of years in period %	5
n	Max periods of firm operation	6

former, this means that no firms from that cohort exit. We continue to iterate until we converge to the steady state.

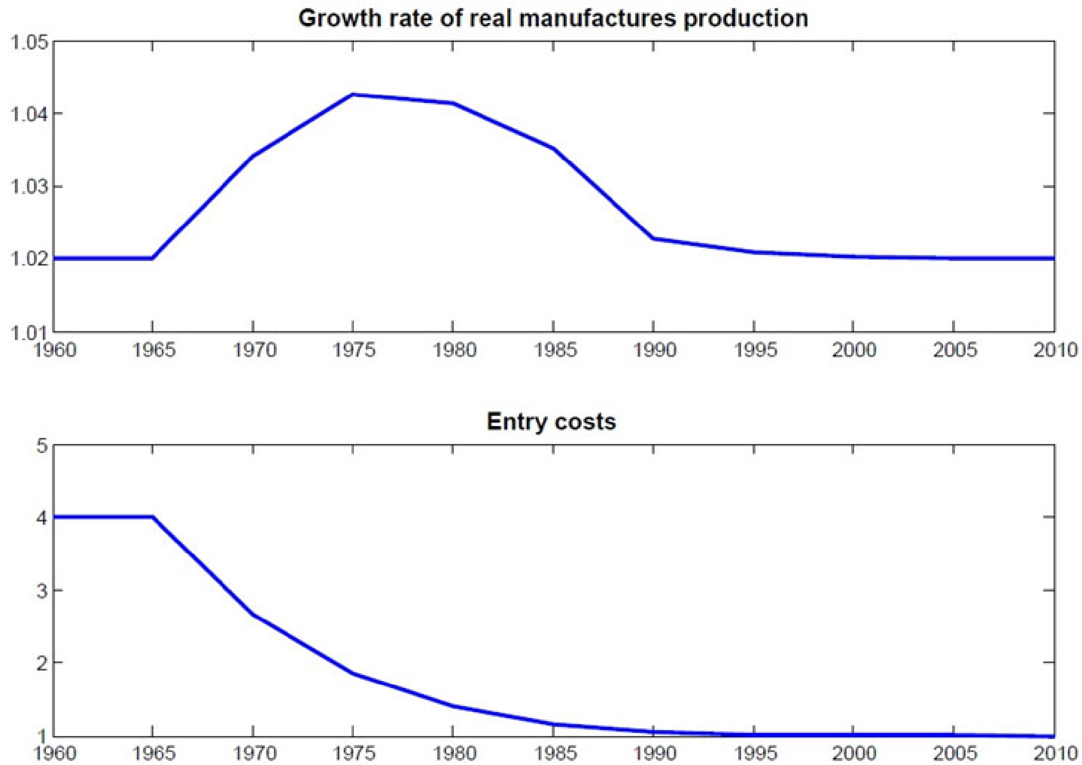
As can be seen in figure 3.4, the growth rate of real manufactures consumption is 2 percent in steady state. As the rate of entry cost relative to operational cost decreases gradually from 4 to 1, the economy exhibits rapid growth, averaging 1.5 to 2 percentage points faster than the steady state 2 percent rate of growth.

3.5 Conclusion

In this paper, we have developed a model where aggregate productivity growth is driven by the continual entry of young, productive firms and the exit of old, unproductive firms. The model features a balanced growth path where aggregate productivity, consumption, and output all grow at the same rate. We show that lower entry costs lead to a larger entry of new and productive firms, a larger exit of old and productive firms, which leads to greater productivity, output, and consumption.

In the calibrated version of the model, we find that reforms to entry costs generate periods of high growth before the economy converges to a new balanced growth path, at a higher level of income. Lower entry barriers induce more entry and exit along the transition, which is consistent with the facts we documented in the empirical section.

Figure 3.4: Transition



This paper focused on the effect of entry costs on firm entry, exit, and growth. Asturias et al. (2012a) explores the interaction between entry costs and financial frictions. We have abstracted from other potentially important barriers to growth such as flexible labor markets, rule of law, and good institutions. We leave this for future research.

Chapter 4

References

Aiyagari, S. Rao. 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving." *The Quarterly Journal of Economics*, 109(3): 659-84.

Aizenman, Joshua, Menzie D. Chinn, and Hiro Ito. 2009. "Surfing the Waves of Globalization: Asia and Financial Globalization in the Context of the Trilemma." *ADB Economics Working Paper Series*, No. 180.

Aizenman, Joshua, and Jaewoo Lee. 2007. "International Reserves: Precautionary versus Mercantilist views, Theory and Evidence." *Open Economies Review*, May 2007.

Aizenman, Joshua and Nancy Marion. 2003. "The High Demand for International Reserves in the Far East: What is Going on?" *Journal of the Japanese and International Economies,* 17(3): 370–400.

Alfaro, Laura, and Fabio Kanczuk. 2007. "Optimal Reserve Management and Sovereign Debt." NBER Working Paper No. 13216.3.

Arellano, Cristina. 2008. "Default Risk and Income Fluctuations in Emerging Markets." *American Economic Review*, 98(3): 690–712.

Asturias, Jose, Sewon Hur, Timothy J. Kehoe, and Kim Ruhl. 2012a. "The Interaction between Entry Barriers and Financial Frictions in Growth." University of Minnesota.

Asturias, Jose, Sewon Hur, Timothy J. Kehoe, and Kim Ruhl. 2012b. "Firm Entry, Exit, and Barriers to Growth." University of Minnesota.

Attanasio, Orazio P., and Monica Paiella. 2006. "Intertemporal Consumption Choices, Transaction Costs and Limited Participation to Financial Markets: Reconciling Data and Theory." NBER Working Paper No. 12412.

Arellano, Cristina, Yan Bai, and Patrick Kehoe. 2011. "Financial Markets and Fluctuations in Uncertainty." Federal Reserve Bank of Minneapolis Research Department Staff Report.

Barseghyan, Levon, and Riccardo DiCecio. 2009. "Entry Costs, Industry Structure, and Cross-country Income and TFP Differences." Federal Reserve Bank of St. Louis Working Paper 2009-005D.

Bell, David N.F., and David G. Blanchflower. 2010. "Young people and recession. A lost generation?" Working paper.

Bick, Alexander, and Sekyu Choi. 2011. "Life-Cycle Consumption: Can Single Agent Models Get it Right?" Universitat Autonoma de Barcelona.

Calafell, Javier; Rodolfo Padilla del Bosque. 2002. "The Ratio of International Reserves to Short-Term External Debt as as Indicator of External Vulnerability: Some Lessons From the Experience of Mexico and Other Emerging Economies." Working Paper.

Calvo, G. Guillermo A. 1998. "Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops." *Journal of Applied Economics*, 1(1): 35–54.

Calvo, Guillermo A., Alejandro Izquierdo, and Luis-Fernando Mejia. 2004. "On the Empirics of Sudden Stops: The Relevance of Balance-Sheet Effects." Working Papers 10520. NBER, Cambridge, MA.

Campbell, John, and Joao Cocco. 2007. "How Do House Prices Affect Consumption? Evidence from Micro Data." *Journal of Monetary Economics* 54(3): 591-621.

Chambers, Matthew S., Carlos Garriga, and Don Schlagenhauf. 2009. "Accounting For Changes in the Homeownership Rate." *International Economic Review*, 50(3): 677–726.

Chang, Roberto, and Andres Velasco. 2000. "Banks, Debt Maturity and Financial Crises". *Journal of International Economics*, 51(1): 169-194.

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan. 2005. "Sudden Stops and Output Drops." *American Economic Review*, 95(2): 381–87.

Cole, Hal L. and Timothy J. Kehoe. 2000. "Self-Fulfilling Debt Crises". *Review of Economic Studies*, 67(1): 91–116.

Conesa, Juan Carlos, Sagiri Kitao, and Dirk Krueger. 2008. "Taxing Capital? Not a Bad Idea After All!" *American Economic Review*, 99(1): 25-48.

Del Negro, Marco, Fabrizio Perri, and Fabiano Schivardi. 2010. "Tax buyouts." *Journal of Monetary Economics,* 57(5): 576-595.

Diamond, Douglas W., and Philip H. Dybvig. 1983. "Bank Runs, Deposit Insurance, and Liquidity." *Journal of Political Economy* 91(3): 401–419.

Dooley, Michael, David Folkerts-Landau, and Peter Garber. 2004. "An Essay on the Revived Bretton Woods system." NBER Working Paper No. 515225.

Durdu, Ceyhun Bora, Enrique G. Mendoza, and Marco E. Terrones. 2009. "Precautionary Demand for Foreign Assets in Sudden Stop Economies: An Assessment of the New Mercantilism." *Journal of Development Economics*, 89(2): 194-209.

Elsby, Michael W., Bart Hobijn, and Aysegul Sahin. 2010. "The Labor Market in the Great Recession." NBER Working Paper No. 15979.

Ennis, Huberto M., and Todd Keister. 2003. "Economic Growth, Liquidity, and Bank Runs." *Journal of Economic Theory*, 109(2): 220–245.

Favilukis, Jack, Sydney Ludvigson, and Stijn Van Nieuwerburgh. 2011. "The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk-Sharing in General Equilibrium." NBER Working Paper No. 15988.

Fernandez-Villverde, Jesus, and Dirk Krueger. 2011. "Consumption and Saving over the Life Cycle: How Important are Consumer Durables?" *Macroeconomic Dynamics*, 15: 725-770.

Flood, Robert, and Nancy Marion. 2001. "Holding International Reserves in an Era of High Capital Mobility." Brookings Trade Forum: 2001.

Frenkel, Jacob, and Boyan Jovanovic. 1981. "Optimal International Reserves: A Stochastic Framework." *The Economic Journal*, 91(362): 507-514.

Glover, Andrew, Jonathan Heathcote, Dirk Krueger, and Jose-Victor Rios-Rull. 2011. "Inter-generational Redistribution in the Great Recession." Unpublished.

Gourinchas, Pierre-Olivier and Maurice Obstfeld. 2012. "Stories of the Twentieth Century for the Twenty-First." *American Economic Journal: Macroeconomics*, 4(1): 226–65.

Hagenaars, Aldi, Klaas de Vos, and M. Asghar Zaidi. 1994. "Poverty Statistics in the Late 1980s: Research Based on Micro-data." Office for Official Publications of the European Communities. Luxembourg.

Heathcote, Jonathan, Kjetil Storesletten, and Gianluca Violante. 2008. "The Macroeconomic Implications of Rising Wage Inequality in the United States." NBER Working paper 14052 33.

Heathcote, Jonathan, Fabrizio Perri, and Gianluca Violante. 2010. "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States, 1967-2006." *Review of Economic Dynamics*, 13(1): 15-51

Heaton, John, and Deborah Lucas. 2000. "Portfolio Choice and Asset Prices: The Importance of Entrepreneurial Risk." *Journal of Finance.* 55(3): 1163-1198.

Heller, Heinz Robert. 1966. "Optimal International Reserves." *The Economic Journal*, 76(302): 296-311.

Herrendorf, Berthold, and Arilton Teixeira. 2009. "Barriers to Entry and Development." Arizona State University.

Hopenhayn, Hugo. 1992. "Entry, Exit, and Firm Dynamics in Long Run Equilibrium." *Econometrica*, 60(5): 1127-50.

Huggett, Mark. 1996. "Wealth distribution in life-cycle economies." *Journal of Monetary Economics*, 38(3): 469-494.

Hurd, Michael D., and Susann Rohwedder. 2010. "Effects of the Financial Crisis and Great Recession on American Households." NBER Working paper.

Iacoviello, Matteo. 2005. "House Prices, Borrowing Constraints and Monetary Policy in the Business Cycle." *American Economic Review*, 95(3): 739-764.

Iacoviello, Matteo, and Stefano Neri. 2010. "Housing Market Spillovers: Evidence from an Estimated DSGE Model." *American Economic Journal: Macroeconomics*, 2(2): 125-64.

Iacoviello, Matteo, and Marina Pavan. 2009. "Housing and debt over the life cycle and over the business cycle." Working Papers 09-12, Federal Reserve Bank of Boston.

Jeanne, Olivier and Romain Ranciere. 2008. "The Optimal Level of International Reserves for Emerging Market Countries: A New Formula and Some Applications." *CEPR Discussion Paper*, No. DP6723.

Kahn, Lisa B. 2010. "The long-term labor market consequences of graduating from college in a bad economy." *Labour Economics* 17(2): 303-316.

Kehoe, Timothy J. and David. K. Levine. 1993. "Debt Constrained Asset Markets." *Review of Economic Studies* 60: 865-888.

Kiyotaki, Nobuhiro, Kalin Nikolov, and Alexander Michaelides. 2011. "Winners and Losers in Housing Markets." *Journal of Money, Credit and Banking*, 43(2-3): 255-296.

Lane, Philip R., and Gian Maria Milesi-Ferretti (2007). "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970–2004." *Journal of International Economics*, 73(4): 223–25.

Li, Wenli, and Rui Yao. 2007. "The Life-cycle Effects of House Price changes." *Journal of Money, Credit, and Banking.* 39: 1375-1409.

Mian, Atif, and Amir Sufi. 2010. "Household Leverage and the Recession of 2007–09." *IMF Economic Review*, 58(1): 74-117.

Midrigan, Virgiliu and Thomas Philippon. 2011. "Household Leverage and the Recession." NBER Working Paper 16965.

Moskowitz, Tobias J., and Annette Vissing-Jorgensen. 2002. "The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?" *American Economic Review*, 92(4): 745-778.

Nakajima, Makoto 2005. "Rising Earnings Instability, Portfolio Choice, and Housing Prices." University of Illinois, Urbana-Champaign.

Obstfeld, Maurice, Jay C. Shambaugh, and Alan M. Taylor. 2010. "The Trilemma in History: Tradeoffs among Exchange Rates, Monetary Policies, and Capital Mobility." *American Economic Journal: Macroeconomics*, 2(2): 57–94.

Parente, Stephen L., and Edward C. Prescott. 1999. "Monopoly Rights: A Barrier to Riches." *American Economic Review*, 89(5): 1216-33.

Pynoos, Jon, and Phoebe Liebig. 2009. "Changing Work, Retirement, and Housing Patterns," *Generations*, 33(3).

Rios-Rull, Jose-Victor, and Virginia Sanchez-Marcos. 2008. "An Aggregate Economy with Different Size Houses." *Journal of the European Economic Association*, 6(2-3): 705-714.

Rodrik, Daniel (2006). "The Social Cost of Foreign Exchange Reserves," *International Economic Journal* 20(3): 253–66.

Schaal, Edouard. 2010. "Uncertainty, Productivity and Unemployment during the Great Recession." Job market paper.

Scholz, John Karl, Ananth Seshadri, Surachai Khitatrakun. 2006. "Are Americans Saving "Optimally" for Retirement?" *Journal of Political Economy*, 114(4).

Schumpeter, Joseph. 1942. *Capitalism, Socialism and Democracy.* London: Routledge.

Storesletten, Kjetil, Christopher Telmer, and Amir Yaron. 2004. "Consumption and risk sharing over the life cycle." *Journal of Monetary Economics*, 51(3): 609-633.

Tauchen, George. 1986. "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions." *Economics Letters*, 20(2): 177-181.

Vissing-Jorgensen, Annette. 2002. "Towards an Explanation of Household Portfolio Choice Heterogeneity: Nonfinancial Income and Participation Cost Structures," NBER Working Paper.

Appendix A

Appendix to Chapter 1

A.1 Computational Appendix

The computations strategy involves jointly solving for equilibrium and calibration procedures. The household problem is characterized by three states: (i) age, (ii) wealth, and (iii) labor productivity shock and three decisions: (i) risk-free bonds, (ii) housing risky assets, and (iii) non-housing risky assets. I discretize the state space for wealth and the decision variables by choosing a finite grid, and use interpolation methods when the level of next-period wealth implied by decisions and shocks to the risky assets is not on the grid. The measure of households over age, wealth, and labor productivity shock, denoted by $\mu_{jt}(a, z)$ can then be represented by a finite-dimensional array.

Algorithm for solving steady state equilibrium and calibration

1. Guess a vector of parameters $\left\{\gamma, \beta, d, f, \bar{\zeta}\right\}$ and a vector of equilibrium objects$\{p_h, p_x\}$
2. Starting from age J backward, compute value functions and policy functions
3. Given policy functions, the shock processes, and the initial distribution of newborns, calculate the implied distributions
4. Continue steps 1-3 until markets clearing conditions for housing and non-housing

risky assets have been satisfied, and the difference between model moments and corresponding data targets are less than a specified threshold.

Algorithm for solving transition path

Let t_0 be the period of the recession as defined in Section 5.

1. Guess that the transition takes T periods, i.e. the equilibrium is in steady state from $t_0 + T$ onwards.

2. Guess a vector of parameters $\left\{\bar{\zeta}_{t0+1}, \bar{\xi}_{t0+1}\right\}$ and a sequence of prices $\{p_{ht}, p_{xt}\}_{t=t_0+1,..,t_0+T-1}$

3. Starting from period $t_0 + T - 1$ backward, compute value functions and policy functions for all ages $j = 1, .., J$

4. Given policy functions, the shock processes, and the initial steady state distribution, calculate the implied distributions for $t = t_0, .., t_0 + T - 1$

5. Continue steps 2-4 until markets clearing conditions for housing and non-housing risky assets for $t = t_0, .., t_0 + T - 1$ have been satisfied.

6. If the distribution at period $t_0 + T - 1$ differs from the steady state distribution, let $T = T + 1$ and repeat steps 1-5.

Appendix B

Appendix to Chapter 2

B.1 Table of average ratios by country

Table B.1: Detailed Foreign Reserves

	Average Foreign Reserves to GDP		Average Foreign Reserves to External Debt Liabilities	
	1990-1996	2002-2007	1990-1996	2002-2007
Emerging				
Argentina	4.9	13.5	14.3	21.4
Brazil	4.9	8.6	19.5	35.3
Chile	20.4	16.5	50.9	39.0
China	8.4	33.2	54.3	271.4
Colombia	9.9	10.8	35.6	35.1
Czech Republic	17.4	25.3	57.2	77.5
Egypt	20.9	19.8	35.2	65.5
Hungary	15.5	16.6	26.6	26.2
India	3.5	18.4	11.8	101.0
Indonesia	6.5	12.4	11.7	25.2
Korea	5.4	24.6	28.0	97.4
Malaysia	28.5	47.1	77.0	125.6
Mexico	4.6	8.2	12.2	40.9
Morocco	10.4	28.6	15.9	99.1
Pakistan	1.8	10.4	4.3	28.2
Peru	11.2	18.4	16.7	49.1
Philippines	8.0	17.3	13.3	28.4
Poland	6.9	14.2	15.9	35.8
Romania	4.2	18.5	22.7	53.9
Russia	3.0	23.2	7.3	68.5
South Africa	1.0	7.1	4.7	34.9
Thailand	19.3	30.6	41.8	112.1
Turkey	3.9	10.9	13.0	25.5
Advanced				
France	2.1	1.7	3.9	1.3
Germany	3.8	1.8	8.7	1.5
United Kingdom	3.6	1.9	2.4	0.7
United States	1.0	0.5	3.2	0.8

B.2 Proofs

Proof of Proposition 2.1. The proof involves guessing and verifying an equilibrium that is characterized by the cutoff conditions specified in Proposition 2.1(i) and 2.1(ii). We then derive a condition under which the equilibrium is unique.

The cutoff conditions imply that the state-contingent policy and payment functions can be written as:

$$
\begin{aligned}
L(\varphi) &= \begin{cases} 0 & \text{if } \varphi < \varphi_R \\ K & \text{otherwise} \end{cases} \\
R_2(\varphi) &= \begin{cases} R_1 - \varphi D & \text{if } \varphi < \varphi_R \\ 0 & \text{otherwise} \end{cases} \\
\psi_i(\varphi, \varphi_i) &= \begin{cases} 0 & \text{if } \varphi < \varphi_R \text{ and } \varphi_i = 0 \\ 1 & \text{otherwise} \end{cases} \\
P_1(\varphi) &= \begin{cases} D & \text{if } \varphi < \varphi_R \\ (1 + r_S) D & \text{otherwise} \end{cases} \\
P_2(\varphi) &= \begin{cases} (1 + r_N) D & \text{if } \varphi < \varphi_R \\ 0 & \text{otherwise} \end{cases}
\end{aligned}
$$

Individual rationality implies that $1 + r_N \geq 1$. Since $\lambda < 1$, we know that $1 + r_N > \frac{R_1 + \lambda K}{D}$. Combining (2.2) and (2.8), we get that $1 + r_S = \frac{R_1 + \lambda K}{D}$.

The participation constraint, holding with equality, can be written as $(1 + r_W) = G(\varphi_R) + (1 + r_N)(F(\varphi_R) - G(\varphi_R)) + (1 - F(\varphi_R))(1 + r_S)$ where $G(x) = \int_0^x \varphi dF(\varphi)$.

Substituting the resource constraints and the condition $\varphi_R = \dfrac{R_1}{D}$, the optimal debt contract problem can be written as:

$$
\max_{\varphi_R} D \int_0^{\varphi_R} \left[A(1 - \varphi_R) + \varphi_R - \varphi + (1 - \varphi) \frac{G(\varphi_R) + (1 - F(\varphi_R))(\lambda + (1 - \lambda)\varphi_R) - (1 + r_W)}{F(\varphi_R) - G(\varphi_R)} \right] dF(\varphi)
$$

The first order condition is given by:

$$
(1 - \varphi_R) f(\varphi_R) + 1 - F(\varphi_R) = \frac{A - 1}{A - \lambda}
$$

Using the bounded Pareto distribution, we get:

$$
\varphi_R^* = 1 - \left[\frac{A - 1}{A - \lambda} \left(\frac{\sigma}{\sigma + 1} \right) \right]^\sigma
$$

To verify the equilibrium, it suffices to show that $C^*(\varphi) \geq 0 \ \forall \varphi \in [0, \varphi_R)$. Since $C^*(\varphi)$ is strictly increasing in φ, it suffices to show $C^*(0) \geq 0$.

$$
\begin{aligned}
C^*(0) &= A\,(1-\varphi_R)+\varphi_R+\left(\frac{G(\varphi_R)+(1-F(\varphi_R))\,(\lambda+(1-\lambda)\varphi_R)-(1+r_W)}{F(\varphi_R)-G(\varphi_R)}\right)\\
&= \frac{((A\,(1-\varphi_R)+\varphi_R)\,(F(\varphi_R)-G(\varphi_R))+G(\varphi_R)+(1-F(\varphi_R))\,(\lambda+(1-\lambda)\varphi_R)-(1+r_W))}{F(\varphi_R)-G(\varphi_R)}\\
&= \frac{((A-1)\,(1-\varphi_R)\,(F(\varphi_R)-G(\varphi_R))+F(\varphi_R)+(1-F(\varphi_R))\,(\lambda+(1-\lambda)\varphi_R)-(1+r_W))}{F(\varphi_R)-G(\varphi_R)}\\
&= \frac{1}{F(\varphi_R)-G(\varphi_R)}\,((A-1)\,(1-\varphi_R)\,(F(\varphi_R)-G(\varphi_R))-(1-\lambda)\,(1-\varphi_R)\,(1-F(\varphi_R))-r_W)\\
&= (A-1)\,(1-\varphi_R)-\frac{1}{F(\varphi_R)-G(\varphi_R)}\,((1-\lambda)\,(1-\varphi_R)\,(1-F(\varphi_R))+r_W)\\
&= (A-1)\,(1-\varphi_R)-\frac{\sigma+1}{1-(1-\varphi_R)^{\frac{1}{\sigma}+1}}\left((1-\lambda)\,(1-\varphi_R)\,(1-\varphi_R)^{\frac{1}{\sigma}}+r_W\right)\\
&= (A-1)\,(1-\varphi_R)-\frac{\sigma+1}{1-(1-\varphi_R)^{\frac{1}{\sigma}+1}}\left((1-\lambda)(1-\varphi_R)^{\frac{1}{\sigma}+1}+r_W\right)\\
&= (A-1)\left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right]^{\sigma}-\frac{\sigma+1}{1-\left(\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right)^{\sigma+1}}\left((1-\lambda)\left(\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right)^{\sigma+1}+r_W\right)
\end{aligned}
$$

Note that

$$
\lim_{A\to\infty}\left[(A-1)\left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right]^{\sigma}-\frac{\sigma+1}{1-\left(\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right)^{\sigma+1}}\left((1-\lambda)\left(\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right)^{\sigma+1}+r_W\right)\right]=+\infty
$$

Hence $\exists A^*(\lambda,\sigma,r_W)$ such that $\forall A\geq A^*$, $C^*(0)\geq 0$. □

Proof of Proposition 2.3. (i) From Proposition 1, we know that

$$
\varphi_R^* = 1-\left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right]^{\sigma}.
$$

Then,

$$
\begin{aligned}
\frac{\partial\varphi_R^*}{\partial\sigma} &> 0\\
-\left\{\log\left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right]+\frac{1}{\sigma+1}\right\}\left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right]^{\sigma} &> 0\\
\log\left[\frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)\right]+\frac{1}{\sigma+1} &< 0\\
\log\left[\frac{A-1}{A-\lambda}\right] &< -\left[\log\left(\frac{\sigma}{\sigma+1}\right)+\frac{1}{\sigma+1}\right]
\end{aligned}
$$

It suffices to show $0<-\left[\log\left(\frac{\sigma}{\sigma+1}\right)+\frac{1}{\sigma+1}\right]$, which is true since $h(\sigma)\equiv\log\left(\frac{\sigma}{\sigma+1}\right)+\frac{1}{\sigma+1}$ is increasing in σ, $\lim_{\sigma\to+\infty}h(\sigma)=0^+$, and $\lim_{\sigma\to 0^+}h(\sigma)=-\infty$, which implies that $h(\sigma)<0$ for all $\sigma>0$.

(ii) From Corollary 2.2, we know that

$$\Pr(\psi = 1) = 1 - F(\varphi_R^*)$$

Substituting for φ_R^*, we get

$$\Pr(\psi = 1) = \frac{A-1}{A-\lambda}\left(\frac{\sigma}{\sigma+1}\right)$$

The result is obvious. □

Proof of Corollary 2.4. That $\Pr(\psi = 1 \mid \sigma_H, \varphi_R^*(\sigma_H)) > \Pr(\psi = 1 \mid \sigma_L, \varphi_R^*(\sigma_L))$ follows from Proposition 2.3(ii). That $\Pr(\psi = 1 \mid \sigma_H, \varphi_R^*(\sigma_L)) > \Pr(\psi = 1 \mid \sigma_H, \varphi_R^*(\sigma_H))$ follows from Proposition 2.3(i). □

Appendix C

Appendix to Chapter 3

C.1 Proofs

Proof of Lemma 3.3. From equation (3.6),

$$p(\hat{x}_{1t})y_t(\hat{x}_{1t}) - \frac{y_t(\hat{x}_{1t})}{\hat{x}_{1t}} - \kappa = 0.$$

Substituting the solution from (3.5),

$$(1-\rho)(L+\Pi_t)P_t^{\frac{\rho}{1-\rho}}(\hat{x}_{1t}\rho)^{\frac{\rho}{1-\rho}} - \kappa = 0.$$

Applying the balanced growth path conditions ($P_t = g^{-1}P_{t-1},\ \ \Pi_t = \Pi_{t-1}$), we get

$$(1-\rho)(L+\Pi_t)P_t^{\frac{\rho}{1-\rho}}(\hat{x}_{1t}\rho)^{\frac{\rho}{1-\rho}} = \kappa.$$

Rearranging terms yields the desired result.

From equation (3.7),

$$p(\hat{x}_{jt})y_t(\hat{x}_{jt}) - \frac{y_t(\hat{x}_{jt})}{\hat{x}_{jt}} - f = 0.$$

Substituting the solution from (3.5),

$$(1-\rho)(L+\Pi_t)P_t^{\frac{\rho}{1-\rho}}(\hat{x}_{jt}\rho)^{\frac{\rho}{1-\rho}} - f = 0.$$

Applying the balanced growth path conditions ($P_t = g^{-1}P_{t-1},\ \ \Pi_t = \Pi_{t-1}$),

$$(1-\rho)(L+\Pi_t)P_t^{\frac{\rho}{1-\rho}}(\hat{x}_{jt}\rho)^{\frac{\rho}{1-\rho}} = f.$$

Rearranging terms yields the desired result.

It remains to show that $\hat{x}_{jt} \geq \hat{x}_{1,t-j+1}$ for $j = 2, .., N$. We know that,

$$\hat{x}_{jt} = f^{\frac{1-\rho}{\rho}} [(1-\rho)(L+\Pi_t)]^{\frac{1-\rho}{-\rho}} P_t^{-1} \rho^{-1}$$

$$\hat{x}_{1,t-j+1} = \kappa^{\frac{1-\rho}{\rho}} [(1-\rho)(L+\Pi_{t-j+1})]^{\frac{1-\rho}{-\rho}} P_{t-j+1}^{-1} \rho^{-1}$$

Applying the balanced growth path conditions ($P_t = g^{-1} P_{t-1}, \quad \Pi_t = \Pi_{t-1}, \quad \beta = q_t$), we get

$$\hat{x}_{1,t-j+1} = \kappa^{\frac{1-\rho}{\rho}} [(1-\rho)(L+\Pi_t)]^{\frac{1-\rho}{-\rho}} \left(P_t g^{j-1}\right)^{-1} \rho^{-1}.$$

Hence,

$$\begin{aligned}
\frac{\hat{x}_{1,t-j+1}}{\hat{x}_{jt}} &= \frac{\kappa^{\frac{1-\rho}{\rho}} [(1-\rho)(L+\Pi_t)]^{\frac{1-\rho}{-\rho}} \left(P_{t-j+1}\right)^{-1} \rho^{-1}}{f^{\frac{1-\rho}{\rho}} [(1-\rho)(L+\Pi_t)]^{\frac{1-\rho}{-\rho}} P_t^{-1} \rho^{-1}} \\
&= \frac{\kappa^{\frac{1-\rho}{\rho}} [(1-\rho)(L+\Pi_t)]^{\frac{1-\rho}{-\rho}} \left(P_t g^{j-1}\right)^{-1} \rho^{-1}}{f^{\frac{1-\rho}{\rho}} [(1-\rho)(L+\Pi_t)]^{\frac{1-\rho}{-\rho}} P_t^{-1} \rho^{-1}} \\
&= \frac{\kappa^{\frac{1-\rho}{\rho}} \left(g^{j-1}\right)^{-1}}{f^{\frac{1-\rho}{\rho}}} \\
&= \left(\frac{\kappa}{f g^{(j-1)\frac{\rho}{1-\rho}}}\right)^{\frac{1-\rho}{\rho}} \\
&< 1
\end{aligned}$$

where the last inequality follows from our assumption that $\frac{\kappa}{f} < g^{N\frac{\rho}{1-\rho}}$. □

Proof of Propotion 3.4. The proof of proposition 3.4 involves guessing and verifying the existence of an equilibrium with a balanced growth path.

The expression for aggregate profits is given by

$$\begin{aligned}
\Pi_t &= \sum_{i=1}^{N} \mu \int_{\hat{x}_{it}}^{\infty} \pi_{it}(x)\, dF_{t-i+1}(x) \\
&= \sum_{i=1}^{N} \mu \left\{ (1-\rho)(L+\Pi_t) P_t^{\frac{\rho}{1-\rho}} \rho^{\frac{\rho}{1-\rho}} \frac{\gamma g^{\gamma(t-i+1)}(1-\rho)}{\gamma(1-\rho)-\rho} \hat{x}_{i,t}^{\frac{\rho-\gamma(1-\rho)}{1-\rho}} - g^{\gamma(t-i+1)} \hat{x}_{i,t}^{-\gamma} \kappa_i \right\}
\end{aligned}$$

The expression for the intermediate good price is given by

$$\begin{aligned}
P_t^{\frac{-\rho}{1-\rho}} &= \sum_{i=1}^{N} \mu \int_{\hat{x}_{it}}^{\infty} p(x)^{\frac{-\rho}{1-\rho}}\, dF_{t-i+1}(x) \\
&= \frac{\rho^{\frac{\rho}{1-\rho}} \gamma(1-\rho)}{\gamma(1-\rho)-\rho} \mu \sum_{i=1}^{N} g^{\gamma(t-i+1)} \hat{x}_{it}^{\frac{\rho-\gamma(1-\rho)}{1-\rho}}
\end{aligned}$$

Using the expression for cutoffs from lemma 1 which are given by,

$$\hat{x}_{1t} = \kappa^{\frac{1-\rho}{\rho}}[(1-\rho)(L+\Pi_t)]^{\frac{1-\rho}{-\rho}} P_t^{-1}\rho^{-1}$$
$$\hat{x}_{jt} = f^{\frac{1-\rho}{\rho}}[(1-\rho)(L+\Pi_t)]^{\frac{1-\rho}{-\rho}} P_t^{-1}\rho^{-1},$$

the balanced growth path conditions ($P_t = g^{-1}P_{t-1}, \quad \Pi_t = \Pi_{t-1}$) and solving for P_t and Π_t yields

$$P_t(\kappa)^{-\gamma} = g^{\gamma t}[(1-\rho)(L+\Pi(\kappa))]^{\frac{\rho-\gamma(1-\rho)}{-\rho}} \rho^{\gamma} \frac{\gamma(1-\rho)}{\gamma(1-\rho)-\rho}\mu\omega(\kappa)$$
$$\Pi(\kappa) = \frac{L\xi(\kappa)}{1-\xi(\kappa)}$$

where

$$\xi(\kappa) = 1-\rho-\frac{\gamma(1-\rho)-\rho}{\gamma\omega(\kappa)}\left(\kappa^{-\frac{1-\rho}{\rho}\gamma}\sum_{i=1}^{\hat{n}(\kappa)}\kappa_i + f^{\frac{\rho-\gamma(1-\rho)}{\rho}}\sum_{i=\hat{n}(\kappa)+1}^{N} g^{\gamma(1-i)}\right)$$
$$\omega(\kappa) = \kappa^{1-\frac{1-\rho}{\rho}\gamma}\sum_{i=1}^{\hat{n}(\kappa)} g^{(1-i)\frac{\rho}{1-\rho}} + f^{1-\frac{1-\rho}{\rho}\gamma}\sum_{i=\hat{n}(\kappa)+1}^{N} g^{\gamma(1-i)}.$$

Real output is given by

$$Y_t(\kappa) = \frac{L+\Pi(\kappa)}{P_t(\kappa)}$$

Thus, our guess has been verified. □

Proof of Lemma 3.5. We will show that there are $N-1$ jump points. For m such that $2 \leq m \leq N$, define $\kappa(m)$ to be $\kappa(m) = fg^{(m-1)\frac{\rho}{1-\rho}}$. We show that $\kappa(m)$ are the jump points of the step function. We show this in two steps. First, it is clear that for small $\epsilon > 0$,

$$\frac{\kappa(m)-\epsilon}{f} < g^{(m-1)\frac{\rho}{1-\rho}},$$

which violates the condition that the marginal entrant lives to the age of m. Second, it is clear that for small $\epsilon > 0$,

$$g^{(m-2)\frac{\rho}{1-\rho}} \leq \frac{\kappa(m)-\epsilon}{f} < g^{(m-1)\frac{\rho}{1-\rho}},$$

which satisfies the condition that the marginal entrant lives to the age of $m-1$. Finally, it is clear that for small $\epsilon > 0$,

$$g^{(m-1)\frac{\rho}{1-\rho}} \leq \frac{\kappa(m)+\epsilon}{f} < g^{m\frac{\rho}{1-\rho}},$$

which satisfies the condition that the marginal entrant lives to the age of m. Hence, $\hat{n}(\kappa)$ is an increasing step function of κ. □

Proof of Lemma 3.6. We know that

$$\begin{aligned}
P_t(\kappa)^{-\gamma} &= g^{\gamma t}[(1-\rho)(L+\Pi(\kappa))]^{\frac{\rho-\gamma(1-\rho)}{-\rho}}\rho^{\gamma}\frac{\gamma(1-\rho)}{\gamma(1-\rho)-\rho}\mu\omega(\kappa)\\
M(\kappa) &= \kappa^{-\frac{\gamma(1-\rho)}{\rho}}\left[L+\Pi(\kappa)\right]\frac{\gamma(1-\rho)-\rho}{\gamma}\mu^{-1}\omega(\kappa)^{-1}\\
Y_t(\kappa) &= \left[L+\Pi(\kappa)\right]P_t(\kappa,\theta)^{-1}\\
\Pi(\kappa) &= \frac{L\xi(\kappa)}{1-\xi(\kappa)}
\end{aligned}$$

where

$$\begin{aligned}
\xi(\kappa) &= 1-\rho-\frac{\gamma(1-\rho)-\rho}{\gamma\omega(\kappa)}\left(\kappa^{-\frac{1-\rho}{\rho}\gamma}\sum_{i=1}^{\hat{n}(\kappa)}\kappa_i + f^{\frac{\rho-\gamma(1-\rho)}{\rho}}\sum_{i=\hat{n}(\kappa)+1}^{N} g^{\gamma(1-i)}\right)\\
\omega(\kappa) &= \kappa^{1-\frac{1-\rho}{\rho}\gamma}\sum_{i=1}^{\hat{n}(\kappa)} g^{(1-i)\frac{\rho}{1-\rho}} + f^{1-\frac{1-\rho}{\rho}\gamma}\sum_{i=\hat{n}(\kappa)+1}^{N} g^{\gamma(1-i)}
\end{aligned}$$

It suffices to show that $\xi(\kappa)$ and $\omega(\kappa)$ are continuous in κ at the jump points.

(i) $\omega(\kappa)$: We know that $\omega(\kappa) = \lim_{\epsilon\to 0_+}\omega(\kappa(m)-\epsilon)$, $\forall n = 2, ..., N$ since

$$\begin{aligned}
\omega(\kappa(m)) &= \kappa(m)^{1-\frac{1-\rho}{\rho}\gamma}\sum_{i=1}^{m} g^{(1-i)\frac{\rho}{1-\rho}} + f^{1-\frac{1-\rho}{\rho}\gamma}\sum_{i=m+1}^{N} g^{\gamma(1-i)}\\
&= \kappa(m)^{1-\frac{1-\rho}{\rho}\gamma}\sum_{i=1}^{m-1} g^{(1-i)\frac{\rho}{1-\rho}} + f^{1-\frac{1-\rho}{\rho}\gamma}g^{\gamma(1-m)} + f^{1-\frac{1-\rho}{\rho}\gamma}\sum_{i=m+1}^{N} g^{\gamma(1-i)}\\
&= \kappa(m)^{1-\frac{1-\rho}{\rho}\gamma}\sum_{i=1}^{m-1} g^{(1-i)\frac{\rho}{1-\rho}} + f^{1-\frac{1-\rho}{\rho}\gamma}\sum_{i=m}^{N} g^{\gamma(1-i)}\\
&= \lim_{\epsilon\to 0_+}\omega(\kappa(m)-\epsilon)
\end{aligned}$$

(ii) $\xi(\kappa, \theta)$: We know that $\xi(\kappa) = \lim_{\epsilon \to 0_+} \xi(\kappa(m) - \epsilon)$, $\forall m = 2, ..., N$ since

$$\xi(\kappa(m)) = 1 - \rho - \frac{\gamma(1-\rho) - \rho}{\gamma\omega(\kappa)} \left(\kappa(m)^{-\frac{1-\rho}{\rho}\gamma} \sum_{i=1}^{m} \kappa_i + f^{\frac{\rho - \gamma(1-\rho)}{\rho}} \sum_{i=m+1}^{N} g^{\gamma(1-i)} \right)$$

$$= 1 - \rho - \frac{\gamma(1-\rho) - \rho}{\gamma\omega(\kappa(m))} \left(\kappa(m)^{-\frac{1-\rho}{\rho}\gamma} \sum_{i=1}^{m-1} \kappa_i + g^{\gamma(1-m)} f^{\frac{\rho - \gamma(1-\rho)}{\rho}} + f^{\frac{\rho - \gamma(1-\rho)}{\rho}} \sum_{i=m+1}^{N} g^{\gamma(1-i)} \right)$$

$$= 1 - \rho - \frac{\gamma(1-\rho) - \rho}{\gamma\omega(\kappa(m)} \left(\kappa(m)^{-\frac{1-\rho}{\rho}\gamma} \sum_{i=1}^{m-1} \kappa_i + f^{\frac{\rho - \gamma(1-\rho)}{\rho}} \sum_{i=m}^{N} g^{\gamma(1-i)} \right)$$

$$= \lim_{\epsilon \to 0_+} \xi(\kappa(m) - \epsilon)$$

□