Composite bosons and fractional quantum Hall state due to spin-orbit coupling

Tigran A. Sedrakyan

Fine Theoretical Physics Institute,
School of Physics & Astronomy,
University of Minnesota

Collaboration: ***Alex Kamenev***

Leonid Glazman

UNIVERSITY OF MINNESOTA

YALE

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LETTER

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Spin–orbit-coupled Bose–Einstein condensates

Y.-J. Lin[1], K. Jiménez-García[1,2] & I. B. Spielman[1]

[1]Joint Quantum Institute, National Institute of Standards and Technology, and University of Maryland, Gaithersburg, Maryland, 20899, USA. [2]Departamento de Física, Centro de Investigación y Estudios Avanzados del Instituto Politécnico Nacional, México DF, 07360, México.

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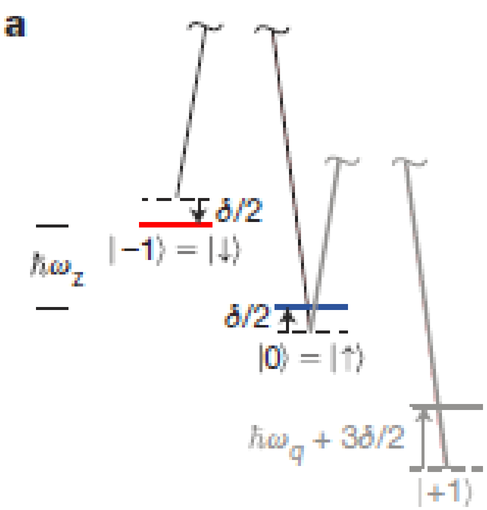

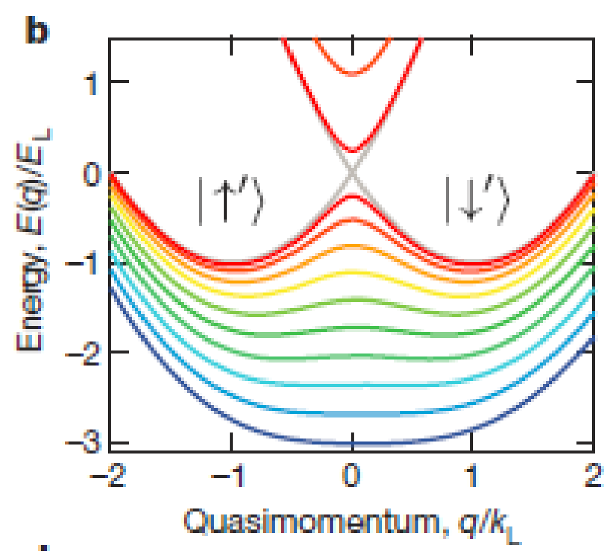

$$\hat{H} = \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m}\breve{1} - \left[\mathbf{B} + \mathbf{B}_{SO}\left(\hat{\mathbf{k}}\right)\right]\cdot\boldsymbol{\mu} = \frac{\hbar^2 \hat{\mathbf{k}}^2}{2m}\breve{1} + \frac{\Omega}{2}\breve{\sigma}_z + \frac{\delta}{2}\breve{\sigma}_y + 2\alpha\hat{k}_x\breve{\sigma}_y$$

PHYSICAL REVIEW A **84**, 025602 (2011)

Realistic Rashba and Dresselhaus spin-orbit coupling for neutral atoms

D. L. Campbell,[1] G. Juzeliūnas,[2] and I. B. Spielman[1]

[1]*Joint Quantum Institute, National Institute of Standards and Technology, and University of Maryland, Gaithersburg, Maryland 20899, USA*

[2]*Institute of Theoretical Physics and Astronomy, Vilnius University, A. Goštauto 12, Vilnius LT-01108, Lithuania*

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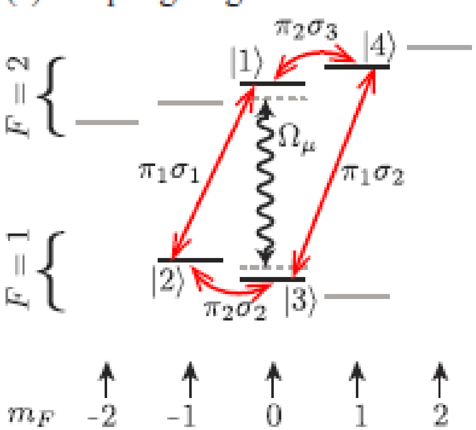
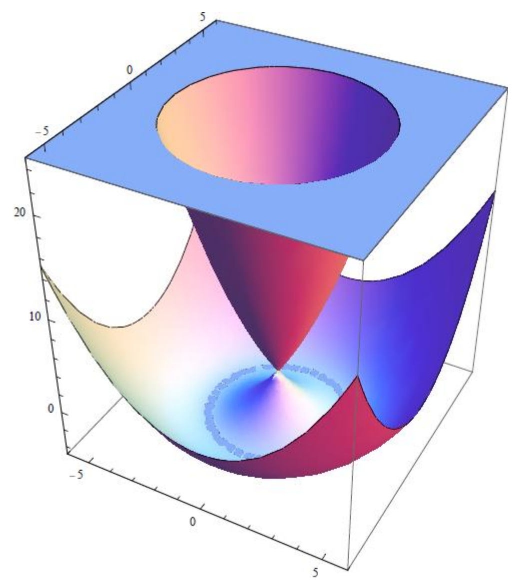
(a) Coupling diagram



Four-level ring coupling scheme in 87Rb involving hyperfine states $|F,m_F\rangle$ *Raman-coupled by a total of five* lasers marked *σ1, σ2, σ3, π1, and π2*



$$\boxed{\hat{\mathcal{H}}_{SO} = \frac{\mathbf{p}^2}{2m} + v\,(p_x\hat{\sigma}_x + p_y\hat{\sigma}_y)}$$

Spin-orbit coupling constant

Rashba spin-orbit-coupling in Bose-Einstein condensates : Theory

Interaction Hamiltonian:

Different pseudospin components are superpositions of atomic hyperfine states
Consider a simplified interaction borrowed from conventional spinor BEC:

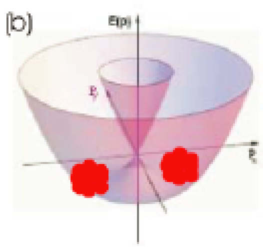
$$H_{\text{int}} = \frac{1}{2m}\int d^2r\left(g_0(n_\uparrow + n_\downarrow)^2 + g_2(n_\uparrow - n_\downarrow)^2\right)$$

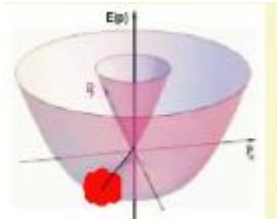
density operators

Chunji Wang, Chao Gao, Chao-Ming Jian, and Hui Zhai, PRL 105, 160403 (2010)

Spin-density wave

TRSB





g_2

$g_2 \le 0$

$$\Psi_k \sim \frac{1}{\sqrt{2}}\begin{pmatrix}\cos(kr)\\ i\sin(kr)\end{pmatrix}$$

0

$\mu \sim n$

$g_2 > 0$

$$\Psi_k \sim \frac{1}{\sqrt{2}}e^{ikr}\begin{pmatrix}1\\ -e^{i\arg(k)}\end{pmatrix}$$

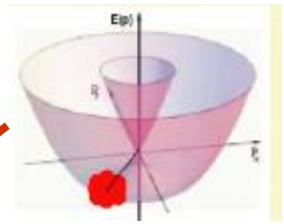
Renormalization in the dilute-gas limit

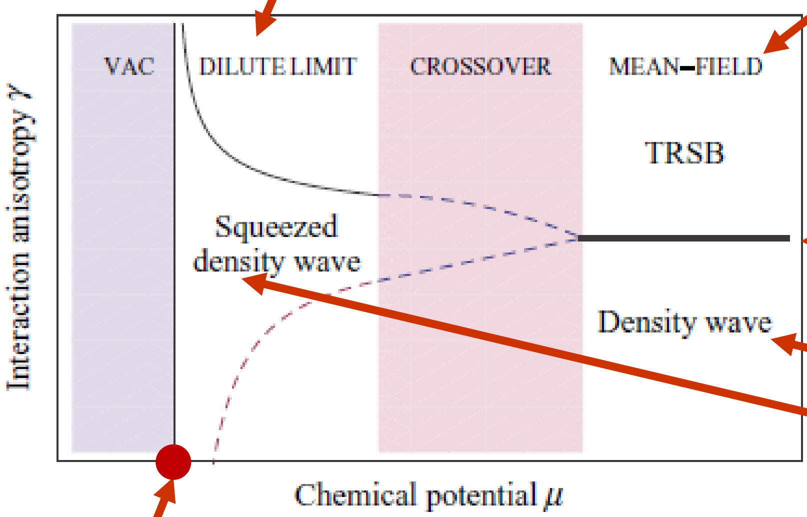
Renormalization of the interaction vertex in the vicinity of QCP at $\mu = 0$, corresponding to the phase transition from the empty vacuum to a BEC leads to:

Sarang Gopalakrishnan, Austen Lamacraft, and Paul M. Goldbart, Phys. Rev. A 84, 061604(R)

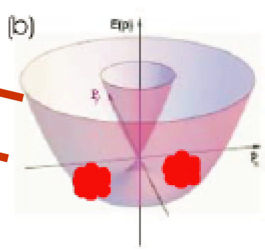
$$\mu \sim n^{4/3}$$

density





$$\gamma = \frac{g_2}{g_0} = 0$$



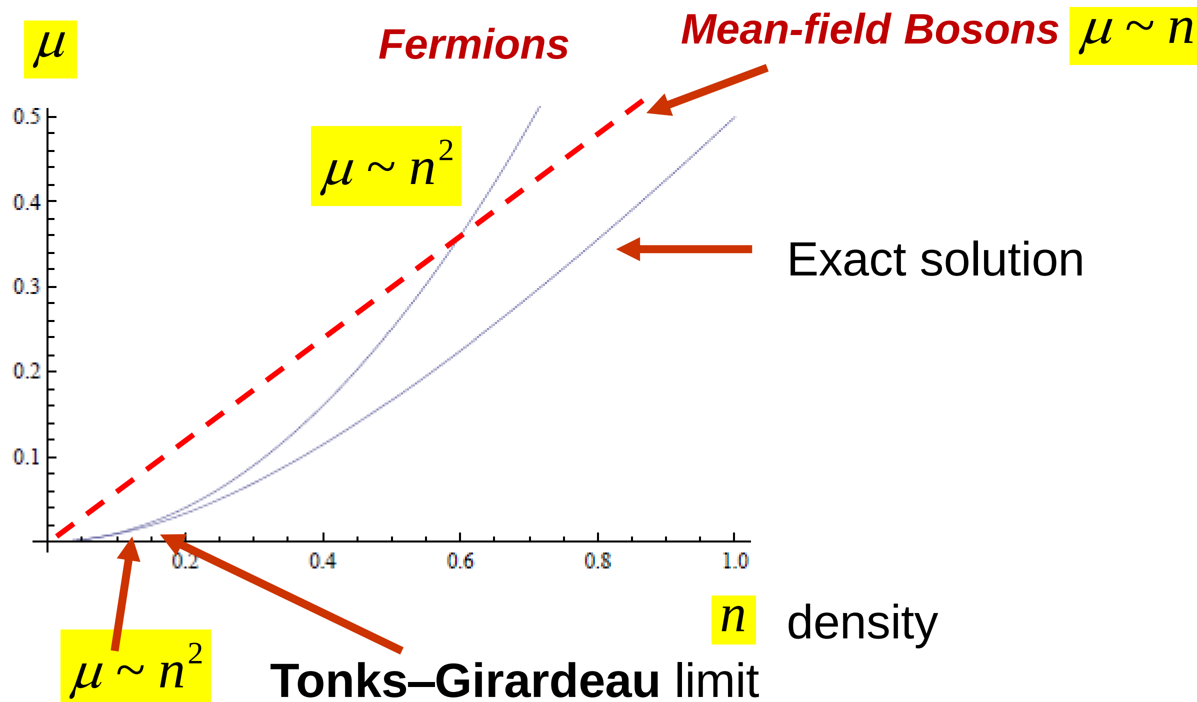
$$\mu = 0$$

Question: **Do Spin-1/2 (!) spin-orbit-coupled bosons always condense in 2D?**

Let us try to gain some intuition from ***1D***:

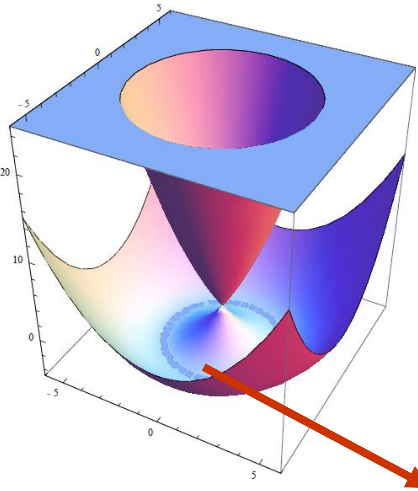
Fermions vs Bosons?

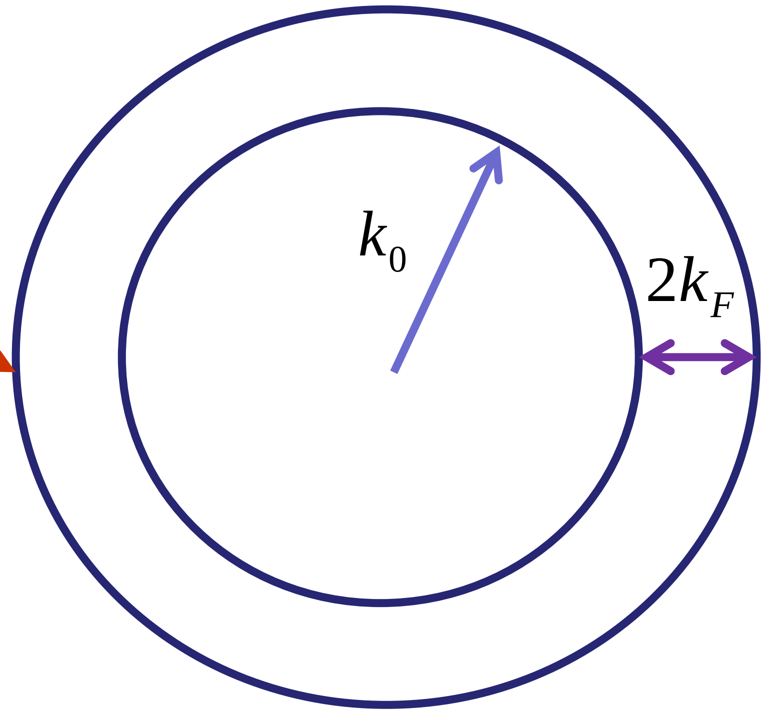
Chemical potential μ



Do we have an analogous situation for <u>spinless fermions</u> in 2D?

***No* contact interactions**





$$k_0 = mv$$

Spin-orbit coupling constant

Estimate chemical potential: $(2\pi k_0)(2k_F) \sim 4\pi^2 n$ $\Rightarrow$ $\boxed{E_F = \frac{k_F^2}{2m} \sim n^2}$

As in 1D!

Near the band bottom, the density of states diverges as $\rho(E) \sim 1/\sqrt{E}$

Fermions with spin-1/2 do interact in 2D

local densities at position ***r***

$$H_{\text{int}} = \frac{1}{2m}\int d^2r\left(g_0(n_\uparrow + n_\downarrow)^2 + g_2(n_\uparrow - n_\downarrow)^2\right) \Rightarrow \frac{(g_0 - g_2)}{m}\int d^2r\, n_\uparrow(r)n_\downarrow(r)$$

Fermions with the same spin do not interact via contact interaction

---

The kinetic energy per particle in the homogeneous state is

$$\boxed{E_{kin} = \frac{k_F^2}{2m} \sim \frac{n^2}{mk_0^2}}$$

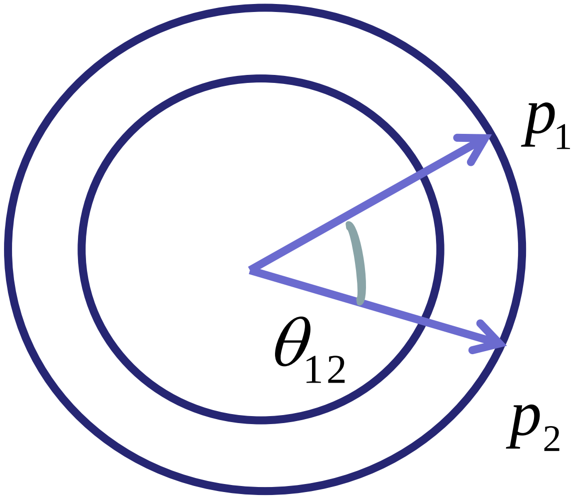
The potential energy per particle

$$\boxed{E_{pot} \sim \frac{g}{mN}\int d^2r\left\langle n_\uparrow(r)n_\downarrow(r)\right\rangle \sim \frac{gn}{m}}$$

Dimensionless interaction parameter: $g = (g_0 - g_2)$

Naively one could expect that the interaction leads to the scaling ***$\mu \sim n$,***
but the competition between potential energy ~n and kinetic energy ~n^2 at low densities suggests that the uniform state is unstable to forming some order .

Two-particle interaction:

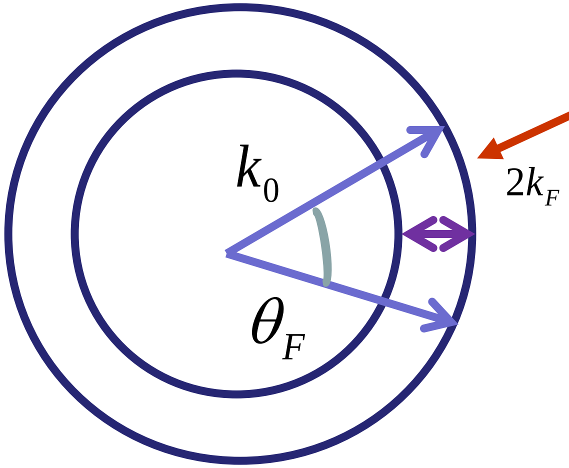


interaction energy is small if the angle is small

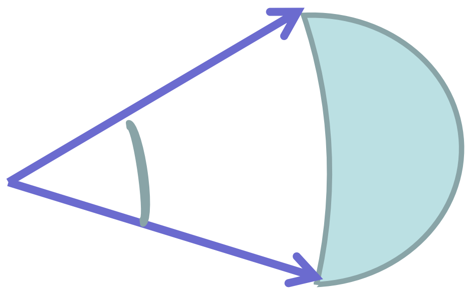
$$\boxed{E_{\text{int}} = \langle H_{\text{int}} \rangle_{p_1 p_2} \sim \frac{g}{mV}\left(1-\cos\theta_{12}\right)}$$

Ferromagnetic-Nematic state:

We start with a variational wave function, which minimizes the interaction energy, taking each electron to be confined to a strip of dimensions: $\sim\left(\theta_F\, k_0\right)\times 2k_F$

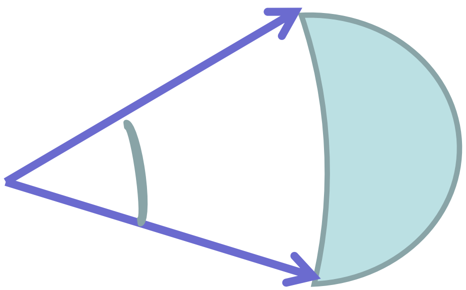


Thus we have a single variational parameter θ_F



The energy per particle in the variational state is given by the ground-state energy of a single particle in a strip with angle θ_F

Kinetic energy: $\theta k_0 (2k_F) \sim 4\pi^2 n$ $\Rightarrow$ $\boxed{E_{kin} = \frac{k_F^2}{2m} \sim \frac{n^2}{m\theta^2 k_0^2}}$



Interaction energy: $\boxed{E_{pot} \sim \frac{gn}{m}(1-\cos\theta) \sim \frac{gn\theta^2}{m}}$

$$E(\theta) = E_{kin}(\theta) + E_{pot}(\theta)$$

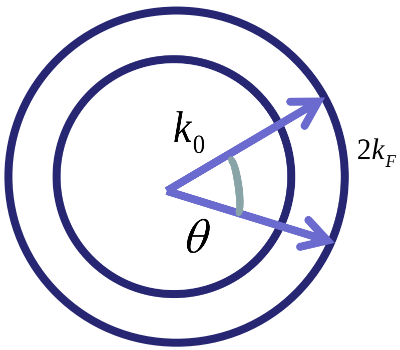
Minimization yields:

$$\theta_F \sim \left(\frac{n}{gk_0^2}\right)^{1/4} < \pi \quad \Rightarrow \quad E_{\min} \sim \frac{n^{3/2}\sqrt{g}}{mk_0}$$

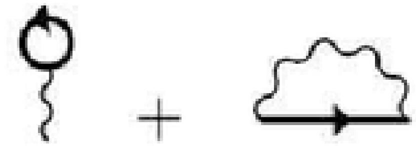
In the Ferromagnetic-Nematic state.

Berg, Rudner, and Kivelson, PRB 85, 035116 (2012)

Fermions in Ferromagnetic-Nematic state



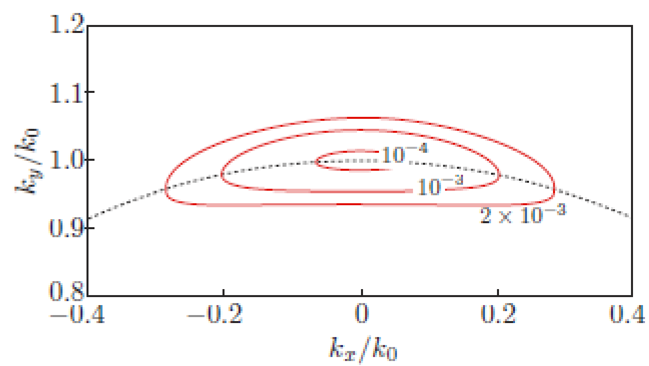
Hartree-Fock $\Sigma_{MB} =$



$$n < g k_0^2$$

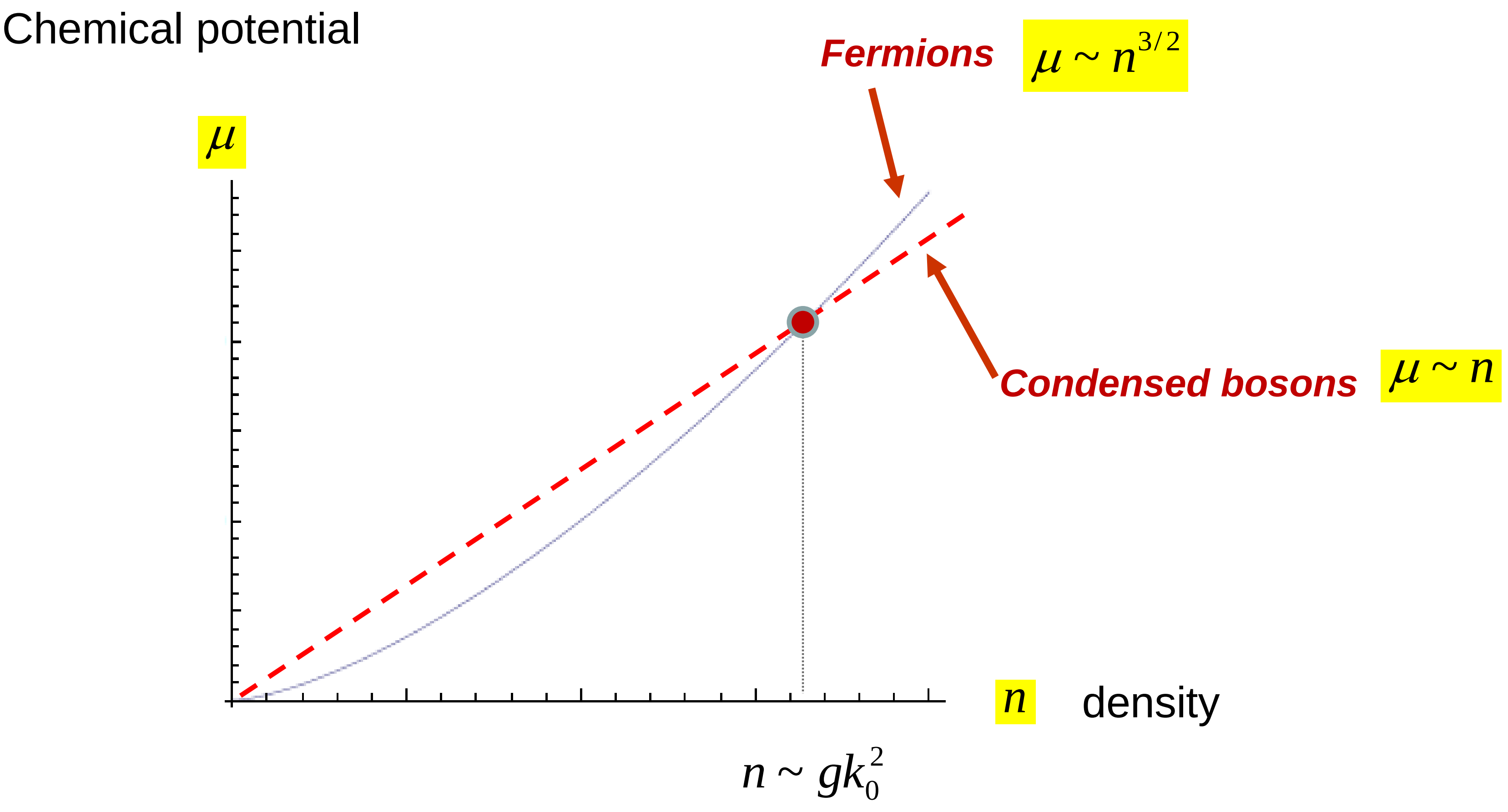
$$E(\vec{k}) = \frac{k_x^2}{2m} + \frac{k_y^2}{2m^*} \qquad m^* \sim m\left(\frac{k_0}{\sqrt{ng}}\right)^2$$

Constant-energy contours of the dispersion:



Erez Berg, Mark S. Rudner, and Steven A. Kivelson, PRB 85, 035116 (2012)

Conclusion (intermediate):



thus at low densities spinfull fermions are still better than condensed bosons

What is the analog of Tonks–Girardeau limit for 2D spin-orbit-coupled bosons?

Composite bosons:

Halperin, Lee, Read, PRB 47, 7312 (1993)
Jain PRL 63, 199 (1989)

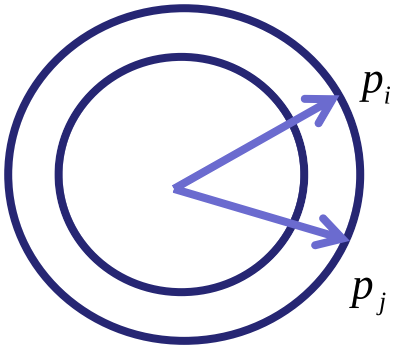
$$\Phi(x_1 s_1; x_2 s_s \dots x_N s_N) = \exp\left\{ i \sum_{i<j}^{N} \arg(x_j - x_i) \right\} \Psi(x_1 s_1; x_2 s_2 \dots x_N s_N)$$

$$s = \uparrow, \downarrow$$

Bosonic wave function

Fermionic wave function

where $$\Psi(\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \chi_1(\mathbf{x}_1) & \chi_2(\mathbf{x}_1) & \cdots & \chi_N(\mathbf{x}_1) \\ \chi_1(\mathbf{x}_2) & \chi_2(\mathbf{x}_2) & \cdots & \chi_N(\mathbf{x}_2) \\ \vdots & \vdots & \ddots & \vdots \\ \chi_1(\mathbf{x}_N) & \chi_2(\mathbf{x}_N) & \cdots & \chi_N(\mathbf{x}_N) \end{vmatrix} \equiv |\chi_1 \quad \chi_2 \quad \cdots \quad \chi_N|,$$



$$e^{i\arg(x_j - x_i)} = -e^{i\arg(x_i - x_j)} = \frac{\vec{x}_j - \vec{x}_i}{|\vec{x}_j - \vec{x}_i|}$$

Single particle eigenfunction:

$$\chi_m(x_n) = \psi_{p_m}(s_m) e^{i\vec{p}_m \vec{x}_n} = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm e^{i\arg(p_m)} \\ 1 \end{pmatrix} e^{i\vec{p}_m \vec{x}_n}$$

SO-bosons = Fermions in Ferromagnetic-Nematic state + Chern-Simons Magnetic Field

$$g = (g_0 - g_2) > 0 \qquad n < g k_0^2$$

Kinetic energy **:**

$$\boxed{E_{kin} = \frac{k_F^2}{2m} \sim \frac{n^{3/2}\sqrt{g}}{m k_0}}$$

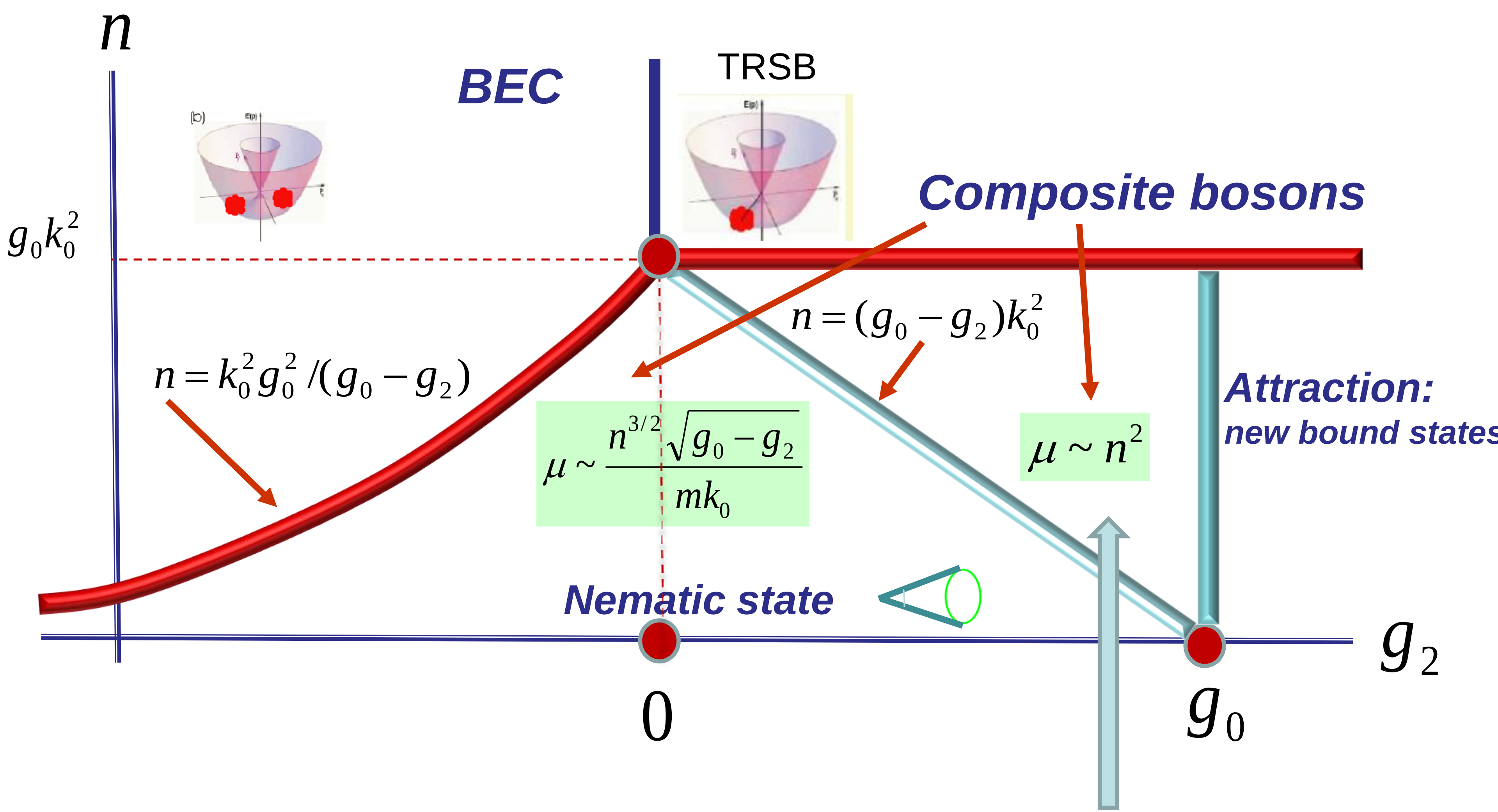
Interaction energy between up-down (Hartree-Fock):

$$\boxed{E_{\uparrow\downarrow} = \frac{k_y^2}{2m^*} \sim \frac{n^{3/2}\sqrt{g}}{m k_0}}$$

Thus the ground state energy per particle of our spin-orbit coupled Bose-system at low densities is:

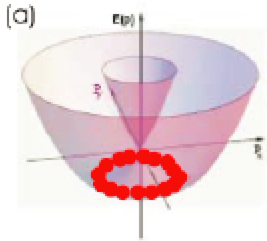
$$\mu \sim \frac{n^{3/2}\sqrt{g}}{m k_0}$$

Phase diagram



Fractional Quantum Hall state with homogeneous distribution of momenta on the circle

Chemical potential for SO bosons with $g_0 k_0^2 > n > (g_0 - g_2) k_0^2$



Non-interacting fermions + magnetic field

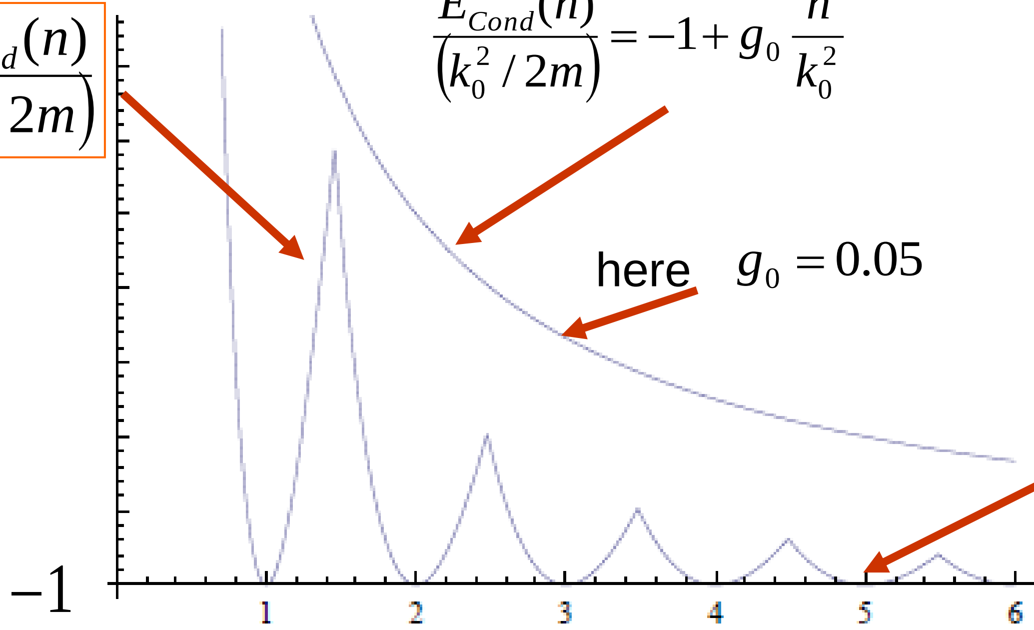
$$g = (g_0 - g_2) \to 0 \quad \Rightarrow \quad H_{\text{int}} = \frac{g_0}{m} \int d^2 r \left(n_\uparrow^2 + n_\downarrow^2 \right)$$

Landau quantization: $\dfrac{E_l}{\left(k_0^2 / 2m\right)} = l \dfrac{n}{k_0^2} - 2 \sqrt{l \dfrac{n}{k_0^2}}$

$\dfrac{E_{Land}(n)}{\left(k_0^2 / 2m\right)}$

$\dfrac{E_{Cond}(n)}{\left(k_0^2 / 2m\right)} = -1 + g_0 \dfrac{n}{k_0^2}$

here $g_0 = 0.05$



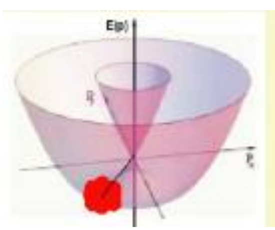
$\dfrac{k_0^2}{n}$

Minima correspond to $n = k_0^2 / l$

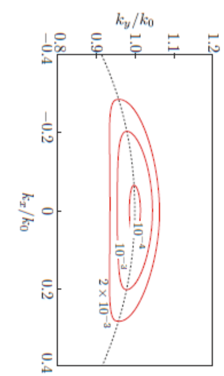
$l = 1,2,3\ldots$

Possible experimental manifestations

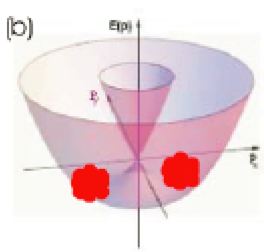
We propose to utilize density-density correlations of an expanding gas to probe complex many-body states of trapped spin-orbit coupled gases

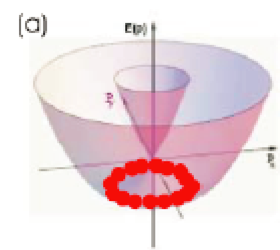


BEC



Nematic state





Homogeneous FQH state

Conclusion:

Question: **Do Spin-1/2 spin-orbit-coupled bosons always condense?**

Answer: **No**

Many-Body Physics of
Spin-Orbit-Coupled Cold Atoms

Ferromagnetic-Nematic state

Fractional Quantum Hall state (homogeneous distribution of momenta on the circle)

Effective attraction: new bound states