Holographic model
of the
phases of thermal QCD

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Follow up work with K. Peeters and M. Zamaklar

Related work A. Parachev, D. Sahakyan CAQCD-06 Minneapolis

Introduction

- QCD admits at low energies: Confinement and chiral symmetry breaking.
- Apriori there is no relation between the two phenomena.
- Except of lattice simulations the arsenal of non perturbative field theory tools is quite limited.
- Gauge/gravity duality is a powerful method to deal with strongly coupled gauge theories.
- There are several stringy (gravitational) models with a dual field theory in the same universality class as QCD
- Confinement is easily realized, flavor chiral symmetry is not.

- Fundamental quarks can be incorporated via probe branes. First were introduced in duals of coulomb phase. (Karch Katz).

- Quarks in confining scenarios were introduced into the KS confining background using D7 braens (Sakai Sonnenscehin)

- Models based on Witten's near extremal D4 branes with D6 (Kruczenski et al). Also Erdmenger et al

- A model of admits full flavor chiral symmetry breaking by incorporating D8 and anti D8 branes to Witten's model. Sakai and Sugimoto

- In this work we themalized the SS model and analyzed its phase structure. We study the phase transitions of
- confinement/deconfinement
- chiral symmetry breaking/ restoring

Outline

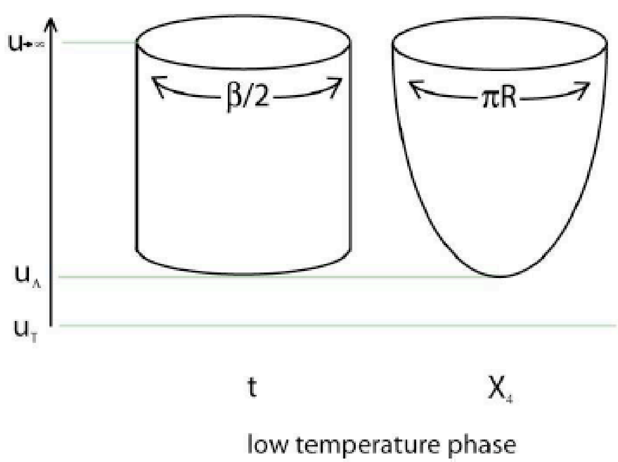
- Bulk thermodynamics of Witten's model- phases of YM theory
- Adding quarks in the fundamental representation
- The low temperature phase of the SS model – confinement,
- The high temperature phase –deconfinement.
- The phase diagram, intermediate phase of deconfinement and chiral symmetry breaking
- The spectrum of the thermal mesons of the various phases

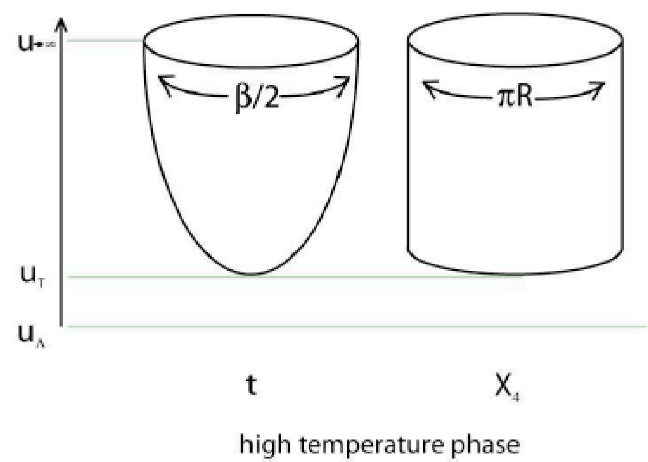
Review of the bulk thermodynamics

- We introduce temperature by compactifying the Euclidean time direction with periodicity $\beta=1/T$ and imposing anti-periodic boundary conditions on the fermions.

- In Witten's model of near extremal D4 branes there is already a compact direction x_4 so in our thermal model (t, x4) are compact.

- There are only two such smooth SUGRA backgrounds





• The low temperature background

$$ds^2 = \left(\frac{u}{R_{D4}}\right)^{3/2}\left[-dt^2 + \delta_{ij}dx^i dx^j + f(u)dx_4^2\right] + \left(\frac{R_{D4}}{u}\right)^{3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right],$$

$$F_{(4)} = \frac{2\pi N_c}{V_4}\epsilon_4, \quad e^{\phi} = g_s\left(\frac{u}{R_{D4}}\right)^{3/4}, \quad R_{D4}^3 \equiv \pi g_s N_c l_s^3, \quad f(u) \equiv 1 - \left(\frac{u_\Lambda}{u}\right)^3,$$

• The periodicity of x_4

$$\delta x_4 = \frac{4\pi}{3}\left(\frac{R_{D4}^3}{u_\Lambda}\right)^{1/2} = 2\pi R$$

• The high temperature background

$$ds^2 = \left(\frac{u}{R_{D4}}\right)^{3/2}\left[f(u)dt^2 + \delta_{ij}dx^i dx^j + dx_4^2\right] + \left(\frac{R_{D4}}{u}\right)^{3/2}\left[u^2 d\Omega_4^2 + \frac{du^2}{f(u)}\right]$$

$$F_{(4)} = \frac{2\pi N_c}{V_4}\epsilon_4, \quad e^{\phi} = g_s\left(\frac{u}{R_{D4}}\right)^{3/4}, \quad R_{D4}^3 \equiv \pi g_s N_c l_s^3, \quad f(u) \equiv 1 - \left(\frac{u_T}{u}\right)^3.$$

• The periodicity of t

$$\delta t = \frac{4\pi}{3}\left(\frac{R_{D4}^3}{u_T}\right)^{1/2} = \beta$$

• At any given T the background that dominates is the one that has a lower free energy, namely, lower classical SUGRA action (times T).

•The classical actions diverge. We regulate them by computing the difference between the two actions.

•It is obvious that the two actions are equal for β =2 R , thus at T_d =1/2π R there is a first order phase transition .

•The transition is first order since the two solutions continue to exist both below and above the transition.

•The difference in free energies is

$$\boxed{\frac{\Delta F}{V_3} \equiv \frac{F_{low} - F_{high}}{V_3} = 20 R l_s \left(\frac{4}{9}\right)^3 \pi^6 (g_s N_c) N_c^2 [(2\pi T)^6 - M_{gb}^6]}$$

•It is easy to see that for T<1/2 πR the background with a thermal factor on X_4 dominates, and above it the one with the thermal factor on t.

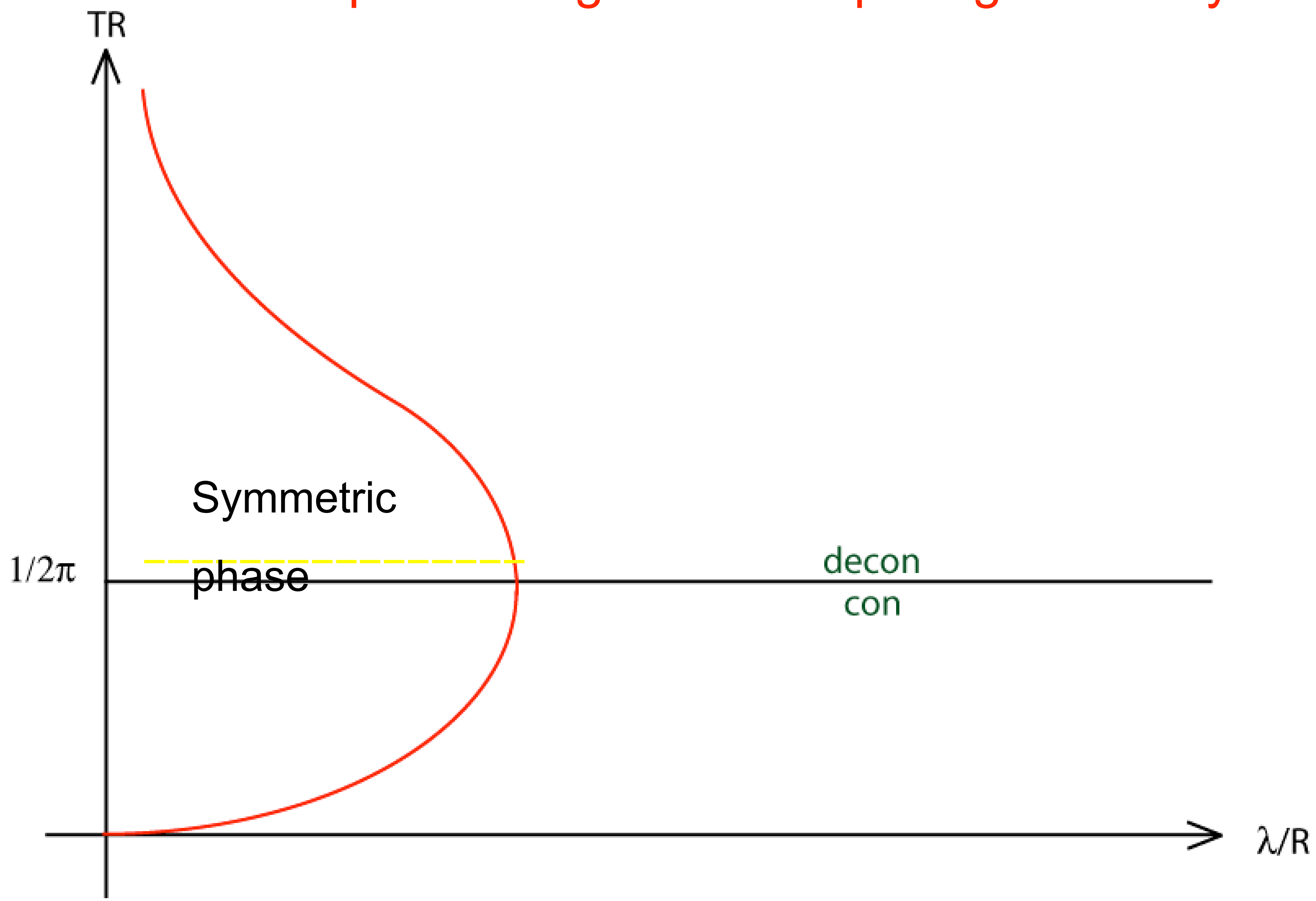
•The interpretation of the phase transition is clear.

low temperature the string tension $T_{st} \sim g_{tt}\, g_{xx}(u_m = u_\Lambda) > 0$ → confinement

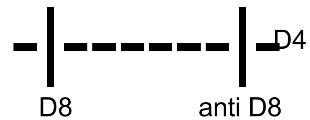
high temperature the string tension $Tst \sim gtt\, g_{xx}(u_m = u_\Lambda) = 0$ → deconfinement

•The dominant phase for small λ/R, due to the symmetry under $T \leftrightarrow 1/2\, \pi R$, is a symmetric phase

The phase diagram of the pure glue theory



Adding fundamental quarks

- The basic underlying brane configuration is 

- In the limit of $N_f < N_c$ the Sugra background is that of the near horizon limit of the near extremal D4 branes with
N_f probe D8 branes and N_f probe anti D8 branes.

- The strings between the D4 branes and the D8 and anti D8 branes

D4- D8 strings ψ_L – left chiral fermions in (N_f ,N_c) of $U(N_f) \times U(N_c)$

D4- antiD8 strings ψ_R – right chiral fermions in (N_f ,N_c) of $U(N_f) \times U(N_c)$

- Note that it is a chiral symmetry and not an $U(N_f) \times U(N_c)$ of Dirac fermions.
This is due to the fact that the there is no transverse direction to the D8 branes.
The same applies to D4 branes in 6d non critical model (Casero Paredes J.S)

•The parameters of the gauge theory are given in the sugra

$$g_5^2 = (2\pi)^2 g_s l_s, \qquad g_4^2 = \frac{g_5^2}{2\pi R} = 3\sqrt{\pi}\left(\frac{g_s u_\Lambda}{N_c l_s}\right)^{1/2}, \qquad M_{gb} = \frac{1}{R},$$

$$T_{st} = \frac{1}{2\pi l_s^2}\sqrt{g_{tt}g_{xx}}|_{u=u_\Lambda} = \frac{1}{2\pi l_s^2}\left(\frac{u_\Lambda}{R_{D4}}\right)^{3/2} = \frac{2}{27\pi}\frac{g_4^2 N_c}{R^2} = \frac{\lambda_5}{27\pi^2 R^3},$$

•The gravity is valid only provided that $\lambda_5 >> R$

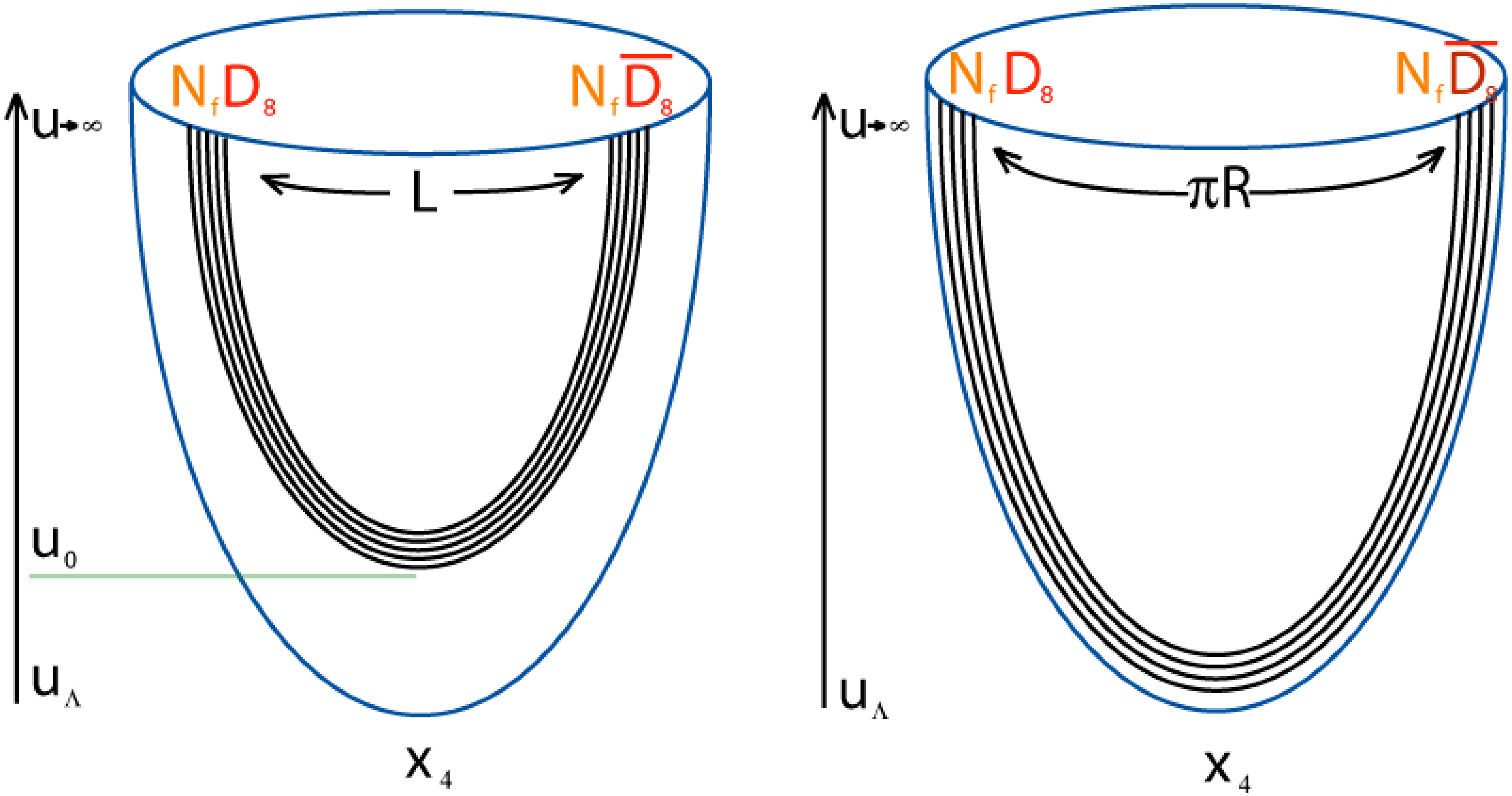
•In fact near the D8 branes the condition is $\lambda_5 >> L$

•At energies $E << 1/R$ the theory is effectively 4d.

•However it is not really QCD since $M_{gb} \sim M_{KK}$

•In the opposite limit of $\lambda_5 << R$ we approach QCD

The low temperature phase



(a) (b)

Low temperature phase

- At the UV the D8 and anti D8 are separated → $U(N_f)_L \times U(N_f)_R$
- In the IR they merge together → spontaneous breaking $U(N_f)_D$

- To verify this we analyze the DBI probe brane action

$$\boxed{S_{DBI} = T_8 \int dt d^3x dx_4 d^4\Omega e^{-\phi}\sqrt{-\det(\hat{g})} = \frac{\hat{T}_8}{g_s}\int dx_4 u^4 \sqrt{f(u) + \left(\frac{R_{D4}}{u}\right)^3 \frac{u'^2}{f(u)}},}$$

- The solution of the corresponding equation of motion is

$$\boxed{\frac{u^4 f(u)}{\sqrt{f(u) + \left(\frac{R_{D4}}{u}\right)^3 \frac{u'^2}{f(u)}}} = \text{constant} = u_0^4\sqrt{f(u_0)},}$$

•Thus there is a family of solutions parametrized by $u_0 > u_\Lambda$.

•A special case is the $u_0 = u_\Lambda$, or $L=\pi R$ (Sakai Sugimoto)

•We can parameterize the solution instead in terms of L

•For small values of L the action depends on L as follows

$$\boxed{S_{DBI} \propto \frac{\hat{T}_8 R_{D4}^{3/2}}{g_s L^7}.}$$

The high temperature deconfining phase

- Recall that the action has now the thermal factor on t direction

$$ds^2 = \left(\frac{u}{R_{D4}}\right)^{3/2}\left[f(u)dt^2 + \delta_{ij}dx^i dx^j + dx_4^2\right] + \left(\frac{R_{D4}}{u}\right)^{3/2}\left[u^2 d\Omega_4^2 + \frac{du^2}{f(u)}\right]$$

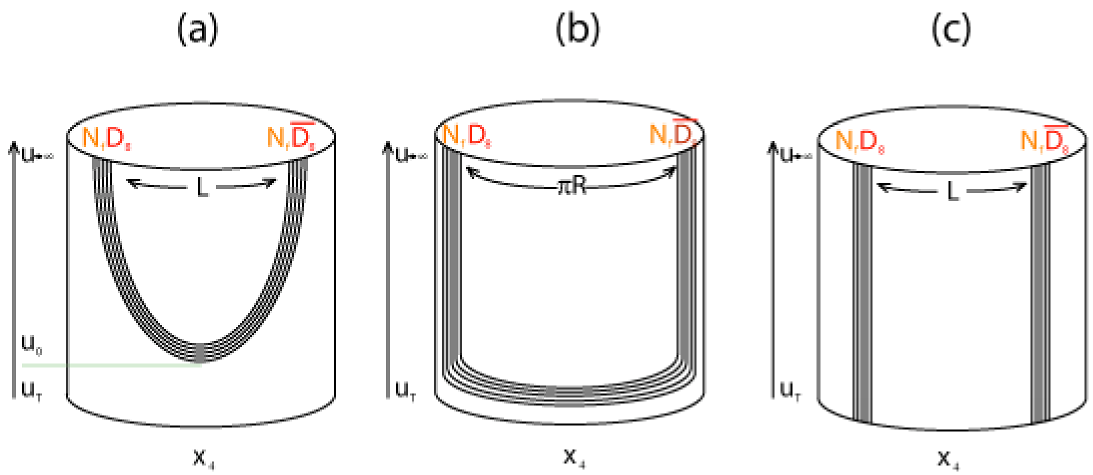
- The equation of motion admits a solution similar to the one of the low temperature domain, namely with chiral symmetry breaking

- However there is an additional stable solution of two disconnected stacks of branes.

This obviously corresponds to chiral restoration.

This is possible since at $u=u_T$ the t cycle shrinks to zero and the D8 branes can smoothly end there.

Chiral symmetry breaking/restoring



•The configuration with the lower free energy ~sugra action dominates

•The action diverges but can be regulated by computing the difference between the low temperature and high temperature phases

$$
\begin{aligned}
\Delta S &\equiv \frac{g_s(S-S^0)}{2\hat{T}_8 R_{D4}^{3/2} u_0^{7/2}} = \left\{ \int_1^\infty dy\, y^{5/2}\left[\frac{1}{\sqrt{1-\frac{f(1)}{f(y)}y^{-8}}}-1\right] - \int_{y_T}^1 dy\, y^{5/2}\right\} \\
&= \frac{1}{3}\left\{\int_0^1 dz \frac{1}{z^{13/6}}\left[\frac{\sqrt{1-y_T^3 z}}{\sqrt{1-y_T^3 z-(1-y_T^3)z^{8/3}}}-1\right]\right\} - \frac{2}{7}(1-y_T^{7/2})
\end{aligned}
$$

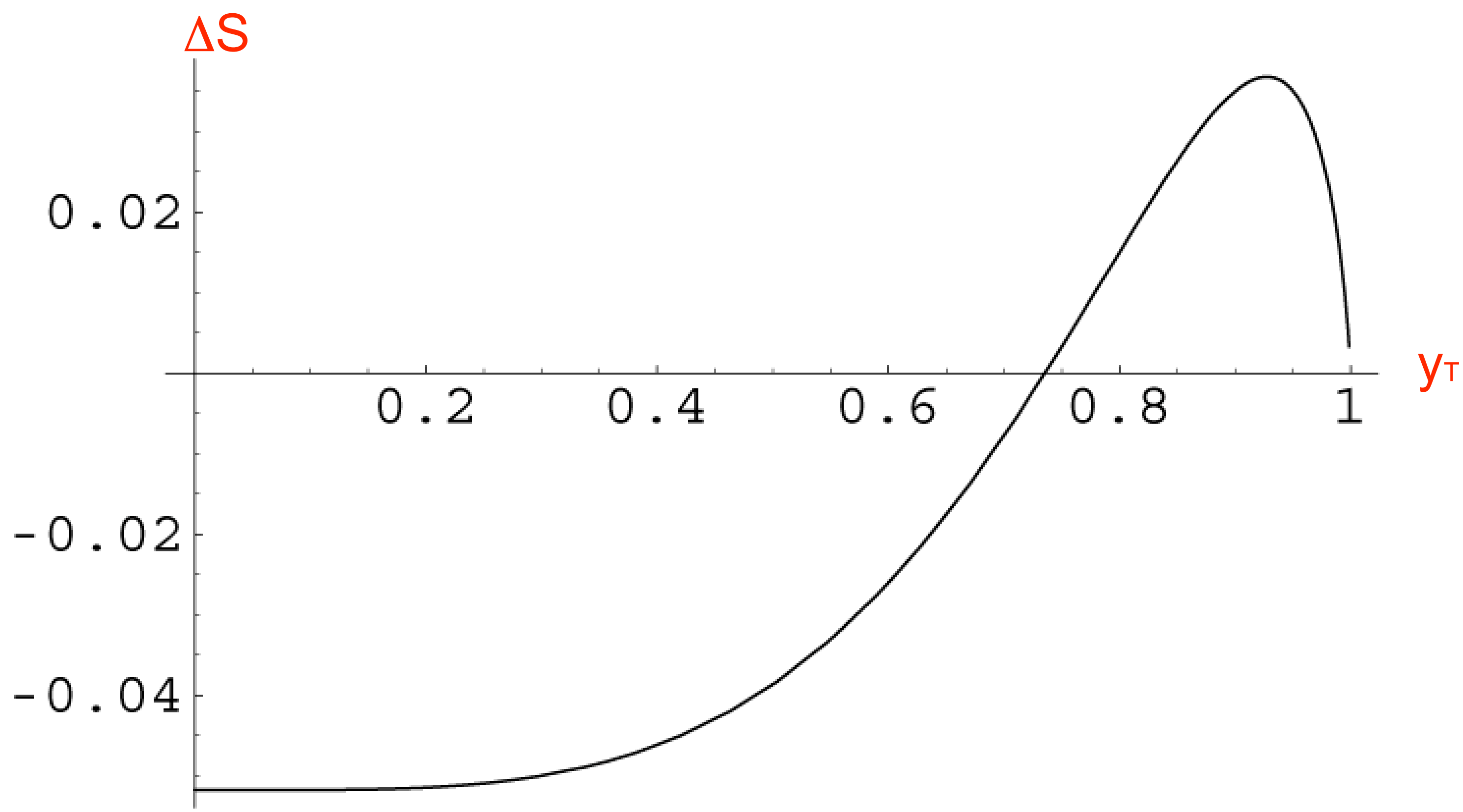
where $y=u/u_0$

•We solve it numerically and find

•For $y_T > y_{Tc} \sim 0.735$ $\Delta S > 0$ | |

•For $y_T < y_{Tc} \sim 0.735$ $\Delta S > 0$ U

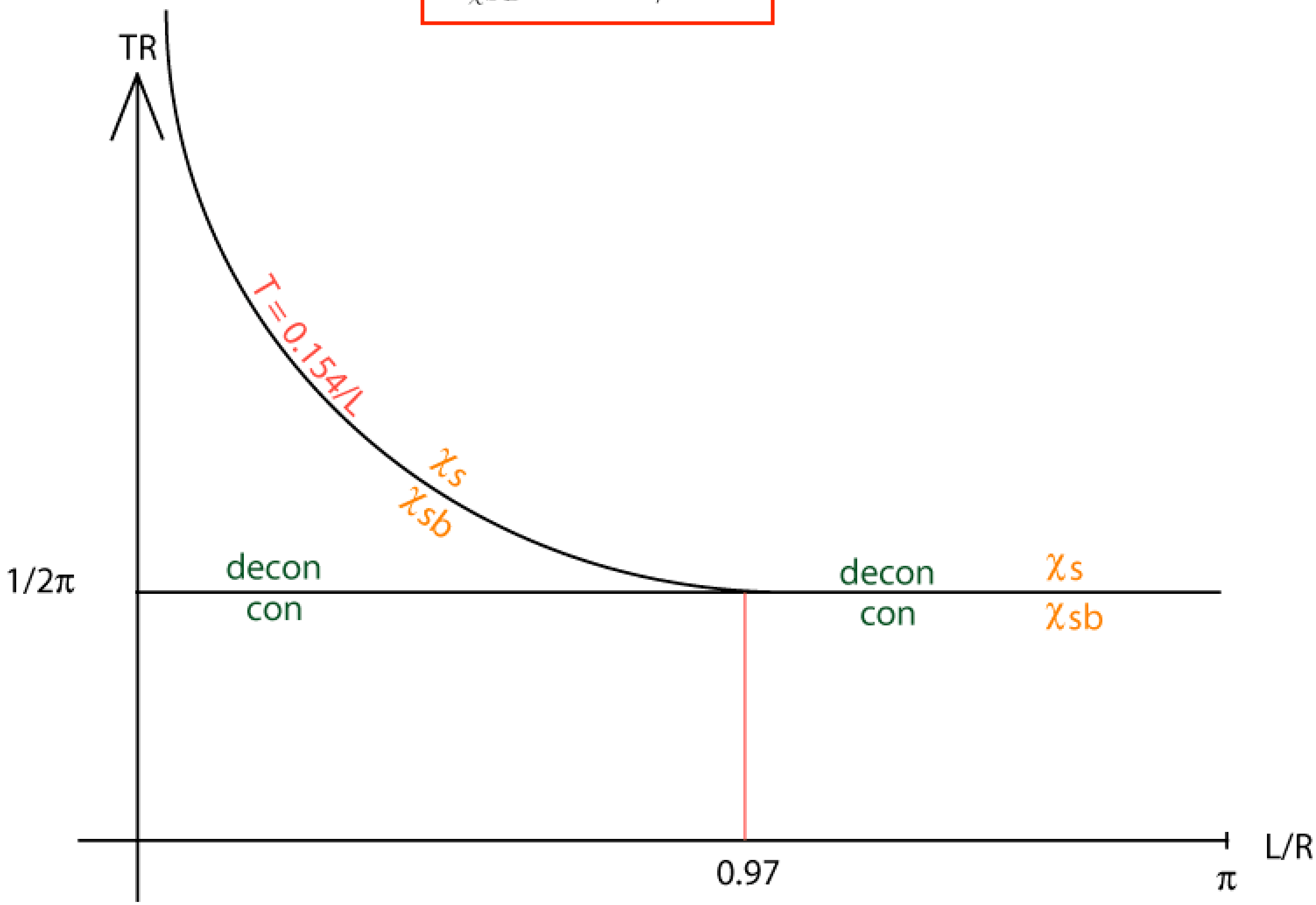
The action difference ΔS as a function of yT (~LT)



Phase diagram-

•We express the critical point in terms of the physical quantities T,L

$$\boxed{T_{\chi SB} \simeq 0.154/L.}$$



On the spectrum of Thermal Mesons

- In general mesons are strings that start and end on a D8 brane

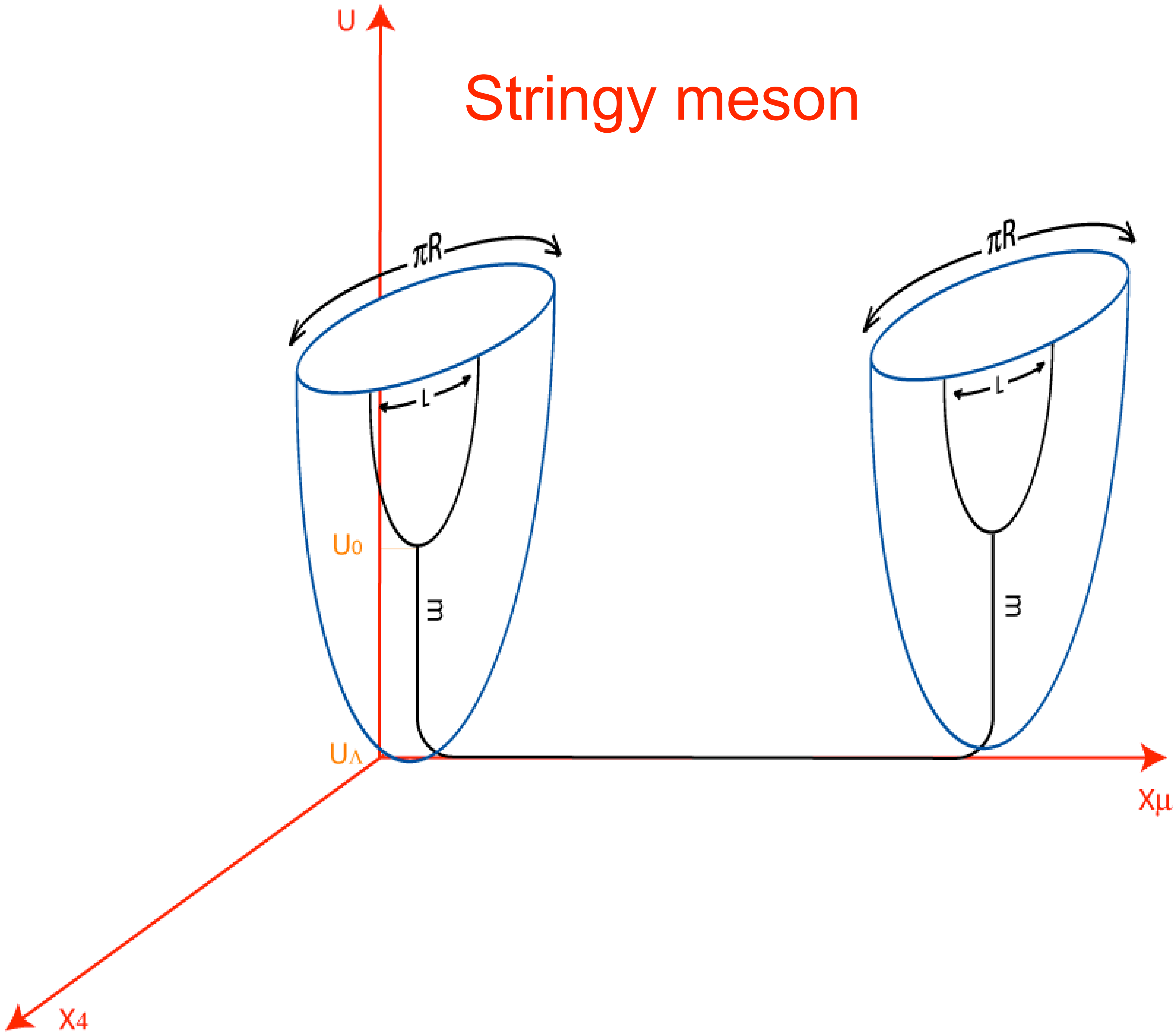
- For low spin these mesons correspond to the fluctuations of the fields that reside on the probe branes.

- Embedding scalars ←→ pseudo scalar mesons
- U(N_f) gauge fiedls ← → pseudo vector mesons

- Higher spin mesons are described by semi-classical stringy Configurations Kruczenski, Pando Zayas, J.S Vaman

- Now for the mesons it is more appropriate to switch to Minkowski

Stringy meson



Mesons in the confining phase

- The structure of the mesonic spectrum is like in zero temperature.
- The four dimensional action of the vector fluctuations reads

$$S = -\int d^4x \mathrm{tr}\left[\frac{1}{2}\partial_\mu \pi^{(0)}\partial^\mu \pi^{(0)} + \sum_{n\geq 1}\left(\frac{1}{4}F^{(n)}_{\mu\nu}F^{\mu\nu\,(n)} + \frac{1}{2}m_n^2 B^{(n)}_\mu B^{\mu\,(n)}\right)\right]$$

- The spectrum includes massless "Goldstone pions" associated with the χ symmetry breaking
- The mass eigenvalues are determined from

$$-u^{1/2}\gamma^{-1/2}\partial_u\left(u^{5/2}\gamma^{-1/2}\partial_u\psi_{(n)}\right) = R^3\, m_n^2\, \psi_{(n)}$$

- There are no deconfined quarks
- The spectrum of massive mesons is discrete

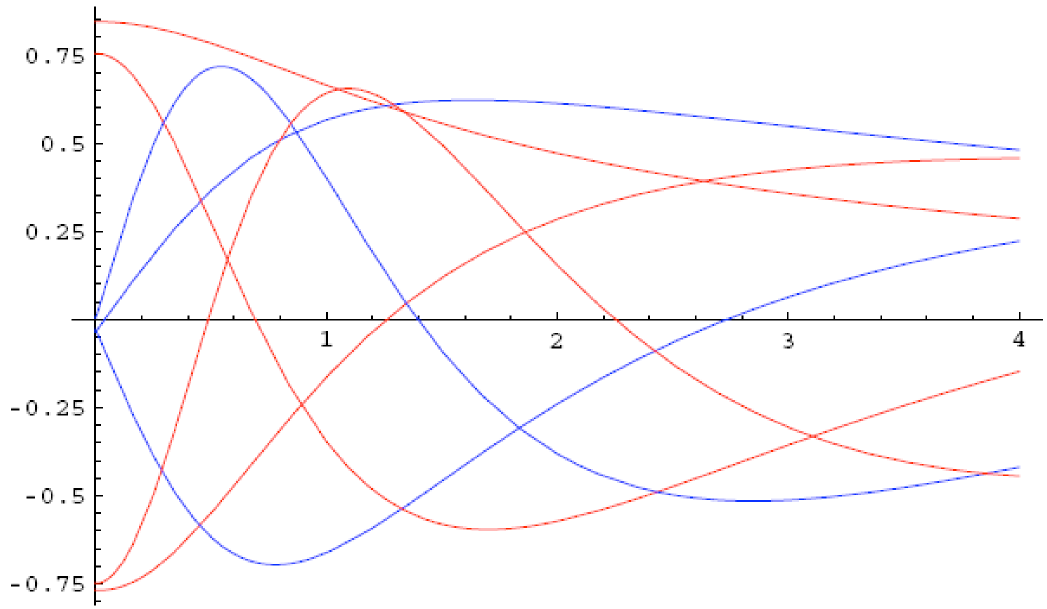


Figure 2: The first few meson modes for the case $u_0 = u_\Lambda$. Red curves are C-odd, blue curves are C-even.

•M^2 as a function of the radial excitation number

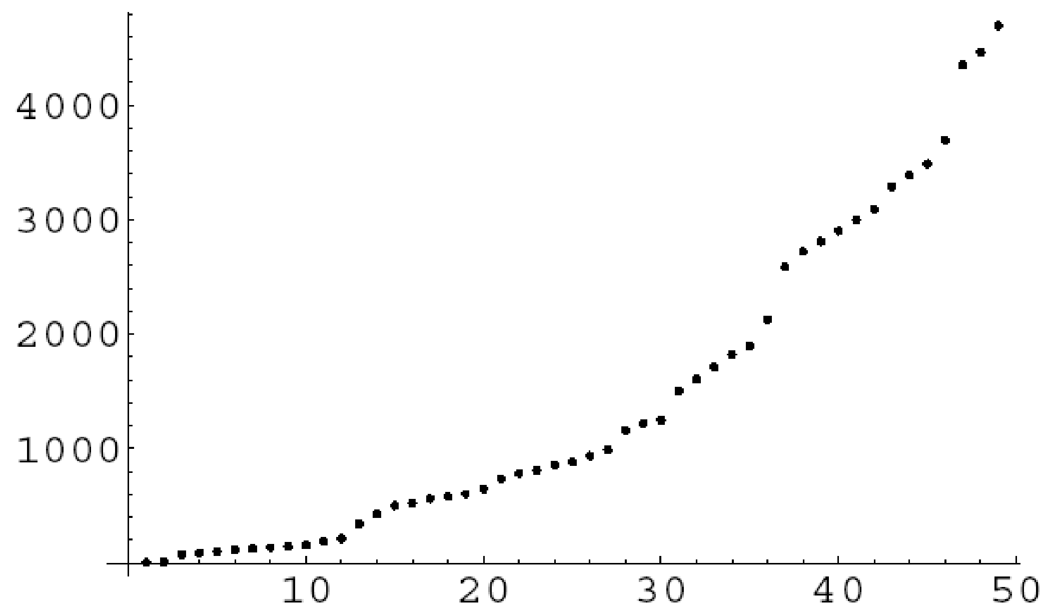
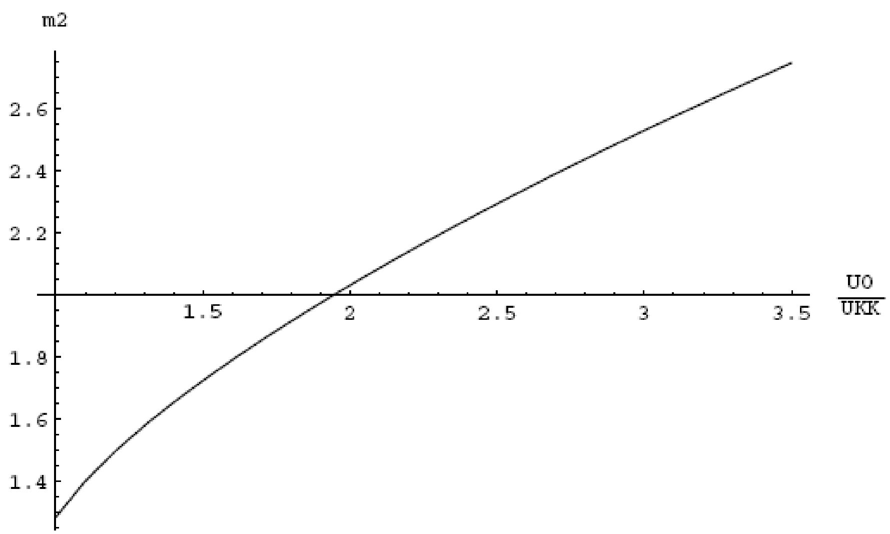
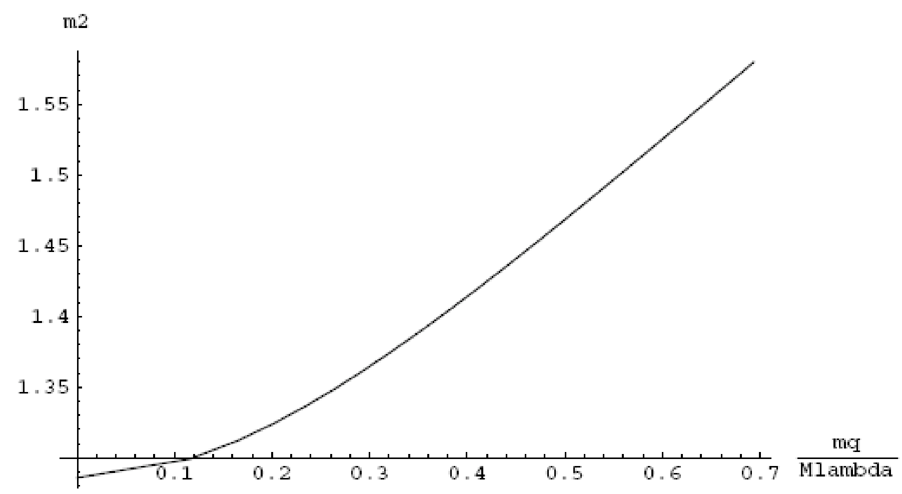


Figure 3: Behaviour of the mass-square of the first C-even meson as a function of the excitation level n, for $u_0 = u_\Lambda$. A numerical fit shows that the exponent is very close to 2.

Mass square of first state as a function of u_0



Meson mass as a function of the constituent quark mass



Summary

- We constructed a holographic model of the thermal phases of QCD

- It is based on thermalizing the Sakai Sugimoto mode

- Both the conf/deconf and χ breaking/restoring

are first oder phase transition

- For small L/R there is an intermediate phase of

deconfinement and chiral symmetry breaking

- One can compute the thermal spectrum of mesons

Adding quarks in the fundamental representation- The Sakai Sugimoto model model

$$
\begin{aligned}
ds^2 &= \left(\frac{u}{R_{D4}}\right)^{3/2}\left[-dt^2+\delta_{ij}dx^i dx^j+f(u)dx_4^2\right]+\left(\frac{R_{D4}}{u}\right)^{3/2}\left[\frac{du^2}{f(u)}+u^2 d\Omega_4^2\right],\\
F_{(4)} &= \frac{2\pi N_c}{V_4}\epsilon_4, \quad e^{\phi}=g_s\left(\frac{u}{R_{D4}}\right)^{3/4}, \quad R_{D4}^3\equiv \pi g_s N_c l_s^3, \quad f(u)\equiv 1-\left(\frac{u_\Lambda}{u}\right)^3,
\end{aligned}
$$

$$
\delta x_4 = \frac{4\pi}{3}\left(\frac{R_{D4}^3}{u_\Lambda}\right)^{1/2} = 2\pi R
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\begin{aligned}
g_5^2 &= (2\pi)^2 g_s l_s, \qquad g_4^2=\frac{g_5^2}{2\pi R}=3\sqrt{\pi}\left(\frac{g_s u_\Lambda}{N_c l_s}\right)^{1/2}, \qquad M_{gb}=\frac{1}{R},\\
T_{st} &= \frac{1}{2\pi l_s^2}\sqrt{g_{tt}g_{xx}}|_{u=u_\Lambda}=\frac{1}{2\pi l_s^2}\left(\frac{u_\Lambda}{R_{D4}}\right)^{3/2}=\frac{2}{27\pi}\frac{g_4^2 N_c}{R^2}=\frac{\lambda_5}{27\pi^2 R^3},
\end{aligned}
$$