

The Nucleon Electron Dipole Moment in Light-Front QCD

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Context

Relating the electric dipole moments of leptons and baryons to fundamental CP-violating parameters is crucial to determining the nature of CP violation.

Estimates of hadronic electric dipole moments depend on the hadron's non-perturbative structure.

For example, in the SM (CKM mechanism of CP violation), long-distance effects (π -loop) give for the neutron

$$d_n^{\text{KM}} \simeq 10^{-32} \text{e-cm} \quad [\text{Gavela et al., PLB 1982; Khriplovich \& Zhitnitsky, PLB 1982}]$$

whereas a LL computation in three-loops yields

$$d_d^{\text{KM}} \simeq 10^{-34} \text{e-cm}. \quad [\text{Czarnecki \& Krause, PRL 1997}]$$

Here we analyze the nucleon electric dipole moment in the light-front formalism of QCD.

Evaluating d_n and d_p is also important to interpreting the ^2H EDM.

[Lebedev et al., PRD 2004]

Electric Dipole Form Factor on the Light Front

We consider the electric dipole form factor $F_3(q^2)$ in the light-front formalism of QCD, to complement earlier studies of the Dirac and Pauli form factors. [Drell, Yan, PRL 1970; West, PRL 1970; Brodsky, Drell, PRD 1980]

Recall

$$\langle P', S'_z | J^\mu(0) | P, S_z \rangle = \bar{U}(P', \lambda') \left[F_1(q^2) \gamma^\mu + F_2(q^2) \frac{i}{2M} \sigma^{\mu\alpha} q_\alpha + F_3(q^2) \frac{-1}{2M} \sigma^{\mu\alpha} \gamma_5 q_\alpha \right] U(P, \lambda)$$

We ignore the anapole form factor and define

$$\kappa = \frac{e}{2M} [F_2(0)] , \quad d = \frac{e}{M} [F_3(0)]$$

We will find a close connection between κ and d , as long recognized. [Bigi, Uralstev, NPB 1991]

Electromagnetic Form Factors on the Light Front

We work in the Drell ($q^+ = 0$) frame:

$$q = (q^+, q^-, \mathbf{q}_\perp) = (0, -q^2/P^+, \mathbf{q}_\perp)$$

$$P = (P^+, P^-, \mathbf{P}_\perp) = (P^+, M^2/P^+, \mathbf{0}_\perp)$$

$$F_1(q^2) = \left\langle P + q, \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \uparrow \right\rangle = \left\langle P + q, \downarrow \left| \frac{J^+(0)}{2P^+} \right| P, \downarrow \right\rangle$$

$$\frac{F_2(q^2)}{2M} =$$

$$\frac{1}{2} \left[-\frac{1}{q^L} \left\langle P + q, \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \downarrow \right\rangle + \frac{1}{q^R} \left\langle P + q, \downarrow \left| \frac{J^+(0)}{2P^+} \right| P, \uparrow \right\rangle \right]$$

$$\frac{F_3(q^2)}{2M} =$$

$$\frac{i}{2} \left[-\frac{1}{q^L} \left\langle P + q, \uparrow \left| \frac{J^+(0)}{2P^+} \right| P, \downarrow \right\rangle - \frac{1}{q^R} \left\langle P + q, \downarrow \left| \frac{J^+(0)}{2P^+} \right| P, \uparrow \right\rangle \right]$$

$$q^{R/L} \equiv q^1 \pm iq^2.$$

Both $F_2(q^2)$ and $F_3(q^2)$ are helicity-flip form factors.

Electromagnetic Form Factors on the Light Front

Interaction picture for $J^+(0)$ and assumed simple vacuum imply:

$$\frac{F_2(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{1}{2} \times$$
$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(\mathbf{x}_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^{\downarrow}(\mathbf{x}_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow*}(\mathbf{x}_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^{\uparrow}(\mathbf{x}_i, \mathbf{k}_{\perp i}, \lambda_i) \right],$$

$$\frac{F_3(q^2)}{2M} = \sum_a \int [dx][d^2\mathbf{k}_\perp] \sum_j e_j \frac{i}{2} \times$$
$$\left[-\frac{1}{q^L} \psi_a^{\uparrow*}(\mathbf{x}_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^{\downarrow}(\mathbf{x}_i, \mathbf{k}_{\perp i}, \lambda_i) - \frac{1}{q^R} \psi_a^{\downarrow*}(\mathbf{x}_i, \mathbf{k}'_{\perp i}, \lambda_i) \psi_a^{\uparrow}(\mathbf{x}_i, \mathbf{k}_{\perp i}, \lambda_i) \right],$$

$$\mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

for the struck constituent j and

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i\mathbf{q}_\perp$$

for each spectator ($i \neq j$). $q^+ = 0 \implies$ only $n' = n$.

Discrete Symmetries on the Light Front

Particles remain on their energy shell in analogy to the on-mass-shell condition of the equal-time formalism.

Consider transformations on \mathbf{k}_\perp ; thus $|\mathbf{k}_\perp|^2$, k^+ , k^- are unchanged.

Parity \mathcal{P}_\perp :

A vector d^μ transforms as $d^R \rightarrow -d^L$, $d^L \rightarrow -d^R$, $d^\pm \rightarrow d^\pm$.

$$\mathcal{P}_\perp \mathbf{a}_{p^L, p^R}^\lambda \mathcal{P}_\perp^\dagger = \eta_a \mathbf{a}_{-p^R, -p^L}^{-\lambda},$$

$$\mathcal{P}_\perp \mathbf{b}_{p^L, p^R}^\lambda \mathcal{P}_\perp^\dagger = \eta_b \mathbf{b}_{-p^R, -p^L}^{-\lambda},$$

$$-\frac{1}{q^L} \langle P+q, \uparrow | J^+(0) | P, \downarrow \rangle \stackrel{\mathcal{P}_\perp}{\leftrightarrow} \frac{1}{q^R} \langle P+q, \downarrow | J^+(0) | P, \uparrow \rangle$$

$$\psi_a^\downarrow(\mathbf{k}_\perp, x_i, \lambda_i) \stackrel{\mathcal{P}_\perp}{\leftrightarrow} \psi_a^\uparrow(\mathbf{k}'_\perp, x_i, -\lambda_i) \quad \text{with} \quad \mathbf{k}'_\perp = (-k_i^1, k_i^2)$$

$F_2(q^2)$ is **even** and $F_3(q^2)$ is **odd** under \mathcal{P}_\perp .

Discrete Symmetries on the Light Front

Time Reversal \mathcal{T}_\perp :

Momentum q^μ transforms as $q^R \rightarrow -q^L$, $q^L \rightarrow -q^R$, $q^\pm \rightarrow q^\pm$.
Thus $x^\mu = (x^+, x^-, x^L, x^R) \rightarrow (-x^+, -x^-, x^R, -x^L)$.

\mathcal{T}_\perp is antiunitarity but

$$\mathcal{T}_\perp a_{p^L, p^R}^\lambda \mathcal{T}_\perp^\dagger = \tilde{\eta}_a a_{-p^R, -p^L}^\lambda,$$

$$\mathcal{T}_\perp b_{p^L, p^R}^\lambda \mathcal{T}_\perp^\dagger = \tilde{\eta}_b b_{-p^R, -p^L}^\lambda,$$

$$\begin{aligned} \langle P + q, \uparrow | J^+(0) | P, \downarrow \rangle &\xrightarrow{\mathcal{T}_\perp} (\langle P + \tilde{q}, \uparrow | J^+(0) | P, \downarrow \rangle)^* \\ &= -\langle P + q, \uparrow | J^+(0) | P, \downarrow \rangle, \end{aligned}$$

$$\begin{aligned} \langle P + q, \downarrow | J^+(0) | P, \uparrow \rangle &\xrightarrow{\mathcal{T}_\perp} (\langle P + \tilde{q}, \downarrow | J^+(0) | P, \uparrow \rangle)^* \\ &= -\langle P + q, \downarrow | J^+(0) | P, \uparrow \rangle, \end{aligned}$$

with $\tilde{q} = (q^+, q^-, \tilde{\mathbf{q}}_\perp)$ and $\tilde{\mathbf{q}}_\perp = (-q^1, q^2)$.

$\text{Re}(F_2)$ and $\text{Im}(F_3)$ are **even** and $\text{Re}(F_3)$ and $\text{Im}(F_2)$ are **odd** under \mathcal{T}_\perp .

Electromagnetic Form Factors on the Light Front

β_a violates \mathcal{P}_\perp and \mathcal{T}_\perp .

$$\begin{aligned}\psi_a^\uparrow(x_i, \mathbf{k}_\perp i, \lambda_i) &= \phi_a^\uparrow(x_i, \mathbf{k}_\perp i, \lambda_i) e^{+i\beta_a/2}, \\ \psi_a^\downarrow(x_i, \mathbf{k}_\perp i, \lambda_i) &= \phi_a^\downarrow(x_i, \mathbf{k}_\perp i, \lambda_i) e^{-i\beta_a/2}\end{aligned}$$

$$\frac{F_2(q^2)}{2M} = \sum_a \cos(\beta_a) \Xi_a$$

$$\frac{F_3(q^2)}{2M} = \sum_a \sin(\beta_a) \Xi_a,$$

$$\Xi_a = \int \frac{[d^2\vec{k}_\perp dx]}{16\pi^3} \sum_j e_j \frac{1}{-q^1 + iq^2} \left[\phi_a^{\uparrow*}(x_i, \vec{k}'_{\perp j}, \lambda_i) \phi_a^\downarrow(x_i, \vec{k}_{\perp i}, \lambda_i) \right].$$

For Fock component a :

$$[F_3(q^2)]_a = (\tan \beta_a) [F_2(q^2)]_a$$

$$d_a = (\tan \beta_a) 2\kappa_a \quad \text{or} \quad d_a = 2\kappa_a \beta_a \quad \text{as} \quad q^2 \rightarrow 0$$

Implications for Models of CP Violation

CP violation via a QCD θ -term.

In a $q(qq)_0$ model of the nucleon

$$d^n \approx e\beta^n \kappa^n (2 \cdot 10^{-14} \text{ cm}), \quad d^p \approx e\beta^p \kappa^p (2 \cdot 10^{-14} \text{ cm}),$$

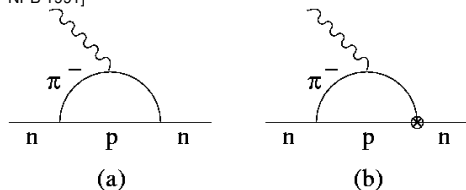
Since δL_{CP} is isoscalar, $\beta^n = \beta^p$ and

$$(d^n + d^p)/(d^p - d^n) = (\kappa^n + \kappa^p)/(\kappa^p - \kappa^n) \approx -0.12/3.70 \approx -0.03.$$

Smaller than leading-order QCD sum rule estimate. [Pospelov and Ritz, PRL 1999]

In a chiral Lagrangian framework [Baluni, PRD 1979; Crewther et al., PLB 1979; Pich & de Rafael,

NPB 1991]



d^n (and d^p) determined from b) as π -loop is logarithmically enhanced. Here $d^n = -d^p$; no isoscalar component.

Can we estimate d^n ?

Assume Fock state sum saturated by $uddu\bar{u}$ Fock component:

$$|\beta_a| \approx 2 \frac{|\bar{g}_{\pi NN}|}{|g_{\pi NN}|} \log(M_N/M_\pi) \approx 4 \left(\frac{0.027}{13.4} \right) |\bar{\theta}|$$

$\bar{g}_{\pi NN}$ is the CP-violating $g_{\pi NN}$ coupling constant

$$d^n \sim \bar{\theta} e(3 \cdot 10^{-16} \text{cm})$$

Comparable to existing estimates.

It has long been recognized that the hadronic matrix elements in the anomalous magnetic and electric dipole moments must be related, up to CP-violating effects. [Bigi, Uralstev, 1991]

Here, however, using the light-front formalism we find a general equality, based on first principles.

- Relation holds for spin-1/2 systems, in general, not only neutron and is independent of the mechanism of CP violation.
- Both the EDM and anomalous magnetic moment should be calculated in a given model, to test for consistency.
- We argue that d^n and d^p (in the SM) should echo the isospin structure of the anomalous magnetic moments.

Summary and Outlook

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