A General Model to Determine the Difference Limen of Frequency (DLF)

A Thesis

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Abstract

Many studies over several decades have studied the relationship between the difference limen for frequency (DLF) for pure tones and various parameters of the pure tones, including frequency, duration, and level. The present study analyzes several data sets in an attempt to provide a general equation to predict DLFs as a function of these three primary parameters. The results show that the data are well fitted by assuming that a power function governs the relationship between log(DLF) and all three parameters (as measured in Hz, ms, and dB for frequency, duration, and level, respectively). An additional improvement in the fits was obtained by including a frequency-duration interaction term that accounted for the sometimes large increase in DLFs when the number of presented cycles of a sinusoid becomes small.

Keywords: difference limen of frequency (DLF), frequency, duration, level

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1. Introduction
A. Background

The minimum detectable change, or difference limen, of frequency of pure tone (DLF) is a fundamental limit of frequency resolution and is, hence, an important criterion to examine general theories of hearing. There have been two common ways of measuring frequency discrimination. One involves the presentation of two successive steady tones with slightly different frequencies. The subject is asked to judge whether the first or the second had the higher frequency. The order of the tones is varied randomly from trial to trial, and the DLF is usually taken as that frequency separation between the pulses for which the subject achieves a certain percentage of correct responses, such as 71%. A second measure uses tones which are frequency modulated (FM) at a low rate (For example: 20-Hz modulation frequency with a 500-Hz carrier). Usually, two successive tones are presented, one modulated and the other unmodulated. The amount (or depth) of modulation necessary for detection is determined. This measure is called the frequency modulation detection limen (FMDL).

Frequency is defined as the number of times per second a sinusoidal pattern repeats. For a pure tone (a sinusoid), there is only one frequency; any other sound is referred to as a complex sound, and can be represented by a sum of sinusoids with different frequencies, amplitudes, and phases. Periodic complex sounds are known as harmonic tone complexes and are comprised of sinusoids with frequencies that are all integer multiples of the fundamental frequency (F0), which corresponds to the repetition rate of the overall waveform. There is evidence that humans are able to hear out the harmonics of a periodic sound wave individually to a limited extent (Moore, 2003). For this reason, understanding how pure tones are processed is a fundamental element in our understanding the mechanisms of human pitch perception. There are a variety of theories and computational models of pitch, only some of which have been validated by testing their ability to predict the resolution of pure tone coding. Prevailing theories are championed by temporal theory and place (or spectral) theory (e.g., Moore, 2003). The temporal theory states that the pitch information is extracted from stimulus-driven, phase-locked temporal patterns with which neurons respond to sound in cochlea (e.g., Rose et al., 1967). The place theory states that the place of maximum vibration along the basilar membrane provides the information from which the pitch of a pure tone is extracted (e.g., Zwicker, 1962). Both of the theories can be used to construct models which can in some cases result in different predictions in the influence of various parameters on DLFs. Therefore, a clear understanding of how, and how well, pure tones are coded in the human auditory system, can help us to test the feasibility and relative strengths and weakness of different models of pitch.

B. Experiments on the difference limen for frequency

There are a variety of studies that have tested pure-tone frequency discrimination. Moore (1973) obtained DLFs on three normal-hearing subjects over a broad range of stimulus durations and frequencies. He analyzed the product of the DLF and tone duration ($\Delta f*d$), and showed that it was about one order of magnitude smaller than the value predicted by the “place” model of Zwicker (1970), and concluded that “a temporal mechanism is the most efficient for frequencies below 5 kHz. Above this frequency a "place" mechanism becomes relatively more efficient.”

Wier et al. (1977) measured DLFs corresponding to 70.7% correct with a two-interval adaptive procedure from three normal-hearing listeners over the range of stimulus levels, from 5-80 dB SL, in octave frequency steps from 250 to 8000 Hz. Based on their data, the concluded that $log_{10}(DLF)$ is related linearly to $\sqrt{F}$:

$$\mathrm{logDLF} = \mathrm{a}\sqrt{\mathrm{F}} + \mathrm{b} \qquad (1)$$

where F is the frequency of the tone (in Hz), and a and b are free parameters. Later, Freyman and Nelson (1983) measured DLFs corresponding to 70.7% correct in a four-interval adaptive task as a function of frequency and level in three normal-hearing subjects. In addition, these authors reanalyzed the data of Wier et al. (1977), and the data from an earlier study by Harris (1952). They confirmed Wier et al.’s (1977) conclusion that $log_{10}(DLF)$ increases linearly with $\sqrt{F}$, and they determined that $log_{10}(DLF)$ is inversely related to the stimulus sensation level (in dB). Moreover, they determined that the slope of the line relating $log_{10}(DLF)$ to $\sqrt{F}$ did not vary significantly across sensation levels (SL):

$$log10\ DLF = a'\sqrt{F} + k' + m'(SL^{-1}) \qquad (2)$$

In a subsequent study, Freyman and Nelson (1986) measured DLFs by varying the stimulus duration in the conditions with a fixed level and frequency, and compared the data to the predictions of Zwicker's excitation-pattern model (Zwicker, 1970). In contrast to Moore (1973), Freyman and Nelson (1986) found good generally good correspondence between the model predictions and the data. Freyman and Nelson ascribed the different conclusions to details in the modeling assumptions, in particular the estimates of the spectral slopes of the stimuli.

C. Models describing the difference limen for frequency as a function of frequency, level, and duration

Studies show that the pure tone difference limen of frequency (DLF) can vary between different studies. Hence, it is worthy of find a general equation that can optimally represent the DLF under different combinations of frequency, duration and level.

There have been some previous attempts to develop general equations to predict the change in DLF as a function of frequency, duration and level. Siebert (1970) developed an ideal-observer model, which combines all of the information that is assumed to be present at the level of the auditory nerve to discriminate the frequency of the pure tones. By using information theory, he developed a formula for DLFs as a function of level, duration and frequency. A comparison of the ideal observer predictions with experimental data revealed that the overall performance of the model was generally much better than that of real listeners. In spite of this, the author also points out that DLF can generally be described by the first part of the formula as Eq.(3)

$$DLF = F/T^{-1/2} \quad (3)$$

Where F is the frequency and T is the stimulus duration. Taking the logarithm of both sides, we will see $log_{10}(DLF)$ is linearly related to $log_{10}(F)$ and $log_{10}(T)$ with slopes of 1 and -1/2, respectively.

The variation with frequency derived by Siebert's theoretical analysis is somewhat different from that derived from the empirical data by Freyman and Nelson (1983), as described above (Eq. 2). Indeed, Freyman and Nelson provided a test which showed that $\sqrt{F}$ had a higher correlation coefficient with $log_{10}(DLF)$ than did $log(F)$. Their data and analysis also indicate that in most cases the relationship between $log_{10}(DLF)$ and $\sqrt{F}$ remained the same at different overall sound levels.

Despite the wealth of earlier studies, some questions remain, and further improvements could be made to test the generality and robustness of the predictions concerning the dependence of the DLF on the pure-tone parameters. First of all, Freyman and Nelson's (1983) general equation did not incorporate the effects of stimulus duration on DLF. Secondly, the choice of square-root or log functions was made based on mathematical convenience, rather than goodness of fit. For instance, in the plots of Freyman and Nelson (1983; Fig. 4), the high-frequency region tends to be steeper than the predicted curve and the low-frequency region tends to be shallower than the curve, suggesting that a power function with an exponent higher than 0.5 may provide a better fit the data. Thirdly, the previous studies have not fully considered the possibility of interactions between the various parameters. For instance, in Wier et al.

(1977), although a linear regression of $log_{10}(DLF)$ versus $\sqrt{F}$ was chosen, it was also pointed out that "In selecting coordinates, we found that the goodness of fit could be improved slightly at higher sensation levels by using exponents for F that were greater than 0.5," suggesting a possible interaction between frequency and level. Finally, considerably more data is available now than was the case in 1983, when the last published meta-analysis was undertaken.

D. Specific aims

In this project, data were gathered from several past papers that have measured DLFs at various frequencies, durations and levels, using various measurement techniques. The primary sources for the data are listed in Table I:

Table I: Data for different studies with the tested range of frequency, duration and loudness.

Study	Frequency(Hz)	Effective duration (ms)	Loudness: SL or SPL(dB)
Freyman and Nelson (1983)	300-8000	300	SL:10-70
Moore (1973)	125-8500	8.25-202	Equal loudness[1]: 60dB 1kHz
Moore and Glasberg (1989)	500-6500	220	SPL:70

Wier et al. (1977)	200-8000	500	SL:5-80
Dai and Micheyl (2011)	200-8000	190	SL:35-85
Dai (1995)	250-8000	90	SPL: 60-78
Moore and Sek (1995)	250-8000	220	SPL:70
Freyman and Nelson (1986)	500-2000	7-302	SPL:60-80
Micheyl et al. (1998)	250-2000	5-320	SPL:70
Nelson and Stanton (1981)	1200		SL:10-80

In order to maintain uniformity, we selected only data from conditions of fixed level (no roving), quiet (no background noise), and single component of frequency (pure tone), collected in listeners with normal hearing. The aim of the project was to provide a general summary of the available data on pure-tone DLFs and to derive a descriptive equation that can provide a good estimate of DLF at any arbitrary value of frequency, duration and level.

2. Model description and results

In order to make a direct comparison of the data from different studies, certain transformations were necessary. First, because in some experiments level was defined as sound pressure level (SPL), while in others it was defined as sensation level (SL), we transformed into SL values by subtracting the absolute threshold – the standard minimum audible pressure (MAP) from the SPL values. Second, duration was defined in different ways in the different studies, with some studies citing the overall duration, and other studies using just the steady-state duration (not including onset and offset ramps) (Moore and Sek, 1995). Therefore, is this study we calculated the effective durations of each group of data using period over which the amplitude of the waveform is greater than or equal to half the peak amplitude (termed the half-amplitude duration). Third, different levels of d' at threshold were compensated for by assuming that ΔF is proportional to d' and adjusting the threshold levels to the same value of d', corresponding to 70.7% correct in a two-interval two-alternative forced-choice (2AFC) procedure (d'=0.78).

In fitting curve parameters to the data, the Matlab function, nlmefit[2] , was used. Data from experiments listed in Table I were used. During simulation of each model, data from different studies are set as different groups. For each study, the DLFs for different subjects under the same condition (frequency, duration, loudness) are averaged as one point of data. For the paper Moore (1973), subject T.C. was tested using a different paradigm from that of subject B.M and subject R.S, so data from this paper were split into two groups (subject T.C and the averaged value of subject B.M and subject R.S).

A. Determining the power index

Wier et al. (1977) have shown that the DLFs at any particular SL can be described in the frequency domain with a square-root transformation of frequency (Eq. 1). Later, Freyman and Nelson (1983) extended the model by providing the relationship between DLF and sensation level (SL) (Eq. 2). In this research, the power index of frequency, duration and level are all set as free parameters. Confidence intervals of the power indexes are shown as the final results. Considering that it is worthy of proving that the equation is better in prediction than Siebert (1970)'s prediction (Eq. 3), Eq. 4 is introduced to combine a power index transformation and a logarithm transformation together:

$$\begin{cases} f(\mathrm{F,c}) = \begin{cases} \log(\mathrm{F}) & \mathrm{c} = 0 \\ \frac{\mathrm{F^c} - 1}{\mathrm{c}} & \mathrm{c} \neq 0 \end{cases} \\ \log\mathrm{DLF} = \mathrm{a} * \mathrm{f(F,c)} + \mathrm{b} \end{cases} \tag{4}$$

Here $\mathrm{a, b, c}$ are coefficients.

It is clear that this equation is the same as Wier et al.'s (1977) prediction (Eq. 1) with the power index set as the free parameter when $\mathrm{c} \neq 0$. On the other hand, $c = 0$ indicates the Siebert's (1970) prediction. Assuming that difference among studies only influence the DLF by multiplying a constant, a mixed effects model (Eq. 5.1) is introduced to describe the relationship between DLF and F. The same functions were used to describe the variation of the DLF with level and duration.

$$\begin{cases} \log\mathrm{DLF} = \mathrm{a} * \mathrm{f(F,c)} + \mathrm{b} + \mathrm{b'} & \text{(I)} \\ \log\mathrm{DLF} = \mathrm{a} * \mathrm{f(T,c)} + \mathrm{b} + \mathrm{b'} & \text{(II)} \\ \log\mathrm{DLF} = \mathrm{a} * \mathrm{f(SL,c)} + \mathrm{b} + \mathrm{b'} & \text{(III)} \end{cases} \tag{5}$$

Here a, b, c are the fixed effects and b' is the random effect.

First, to find the best-fitting power function to describe the variation of DLF with frequency, data from several studies were extracted, and only the data of long-duration (100-200-ms) tones were used to avoid any potential interactions between frequency and duration at short durations (see below). For the purpose of weighting each group equally, when a study testing DLF versus both frequency and sensation level, only the data of a median sensation level was used. The results are shown in Table II. The mean squared error (Rmse), the BIC[3], the standard error of coefficients, and the degrees of error freedom (Dfe) are also listed.

Next, a similar procedure was used to derive the best-fitting function for the parameters of duration (T) and sensation level (SL) (Eq. 5.2 and 5.3). The data used were from the middle frequency region (1000-2000 Hz) and the results are shown in Table II. The curve fits are summarized in Figure 1.

The model shows that data overall are fit better by a power function (0.71) that is different from both the functions proposed by Freyman and Nelson (1983) and Siebert (1970), as evidenced by the fact that neither power-index values 0.5 or 0 fall within the confidence interval of the fit. A similar conclusion can be drawn for level, where the confidence interval for the power index (-0.75) does not quite span the value of -1, suggesting that the overall data do not support the inverse relationship postulated by Freyman and Nelson. In contrast, the confidence interval for the power-index for duration includes the value of -0.5, in line with the model of Wier et al.

Table II: Values of fixed effects and random effects as shown in Eq.(5). The Rmse is the mean squared error of the residual, Dfe is the error degree of freedom.

Table IIa. DLF versus Frequency

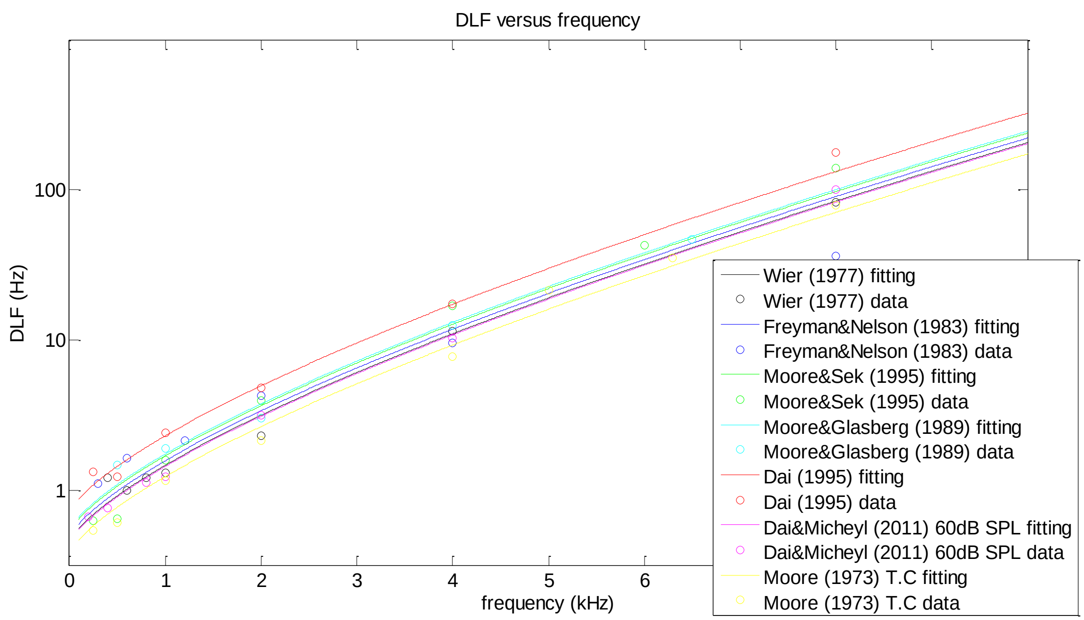
Fixed effect	a mean(SE)	B mean(SE)	c mean(SE)	Confidence interval of c		Rmse	BIC	Dfe
				Lower bound	Higher bound			
	0.37(0.02)	0.20(0.04)	0.71(0.05)	0.61	0.81	0.1047	-64.3	49

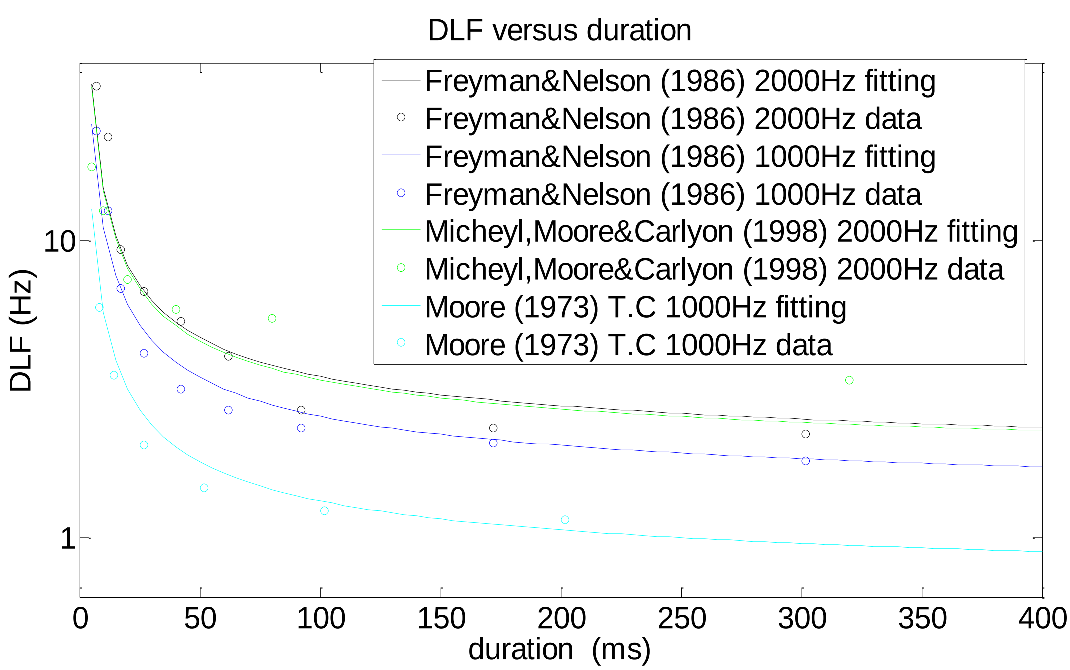
Table IIb. DLF versus Duration

Fixed effect	a mean(SE)	b mean(SE)	c Mean (SE)	Confidence interval of c		Rmse	BIC	Dfe
				Lower bound	Higher bound			
	1.15(0.52)	2.73(0.55)	-0.43(0.14)	-0.69	-0.16	-25.9	0.117	26

Table IIc. DLF versus Level

Fixed effect	a	b	c	Confidence interval of c		Rmse	BIC	Dfe
				Lower bound	Higher bound			
	2.96(0.76)	-1.86(0.58)	-0.75(0.11)	-0.97	-0.53	-50.6	0.043	14


DLF versus frequency
DLF (Hz)
100
10
1
0
1
2
3
4
5
6
frequency (kHz)
Wier (1977) fitting
Wier (1977) data
Freyman&Nelson (1983) fitting
Freyman&Nelson (1983) data
Moore&Sek (1995) fitting
Moore&Sek (1995) data
Moore&Glasberg (1989) fitting
Moore&Glasberg (1989) data
Dai (1995) fitting
Dai (1995) data
Dai&Micheyl (2011) 60dB SPL fitting
Dai&Micheyl (2011) 60dB SPL data
Moore (1973) T.C fitting
Moore (1973) T.C data


DLF versus duration
DLF (Hz)
10
1
0
50
100
150
200
250
300
350
400
duration (ms)
Freyman&Nelson (1986) 2000Hz fitting
Freyman&Nelson (1986) 2000Hz data
Freyman&Nelson (1986) 1000Hz fitting
Freyman&Nelson (1986) 1000Hz data
Micheyl,Moore&Carlyon (1998) 2000Hz fitting
Micheyl,Moore&Carlyon (1998) 2000Hz data
Moore (1973) T.C 1000Hz fitting
Moore (1973) T.C 1000Hz data

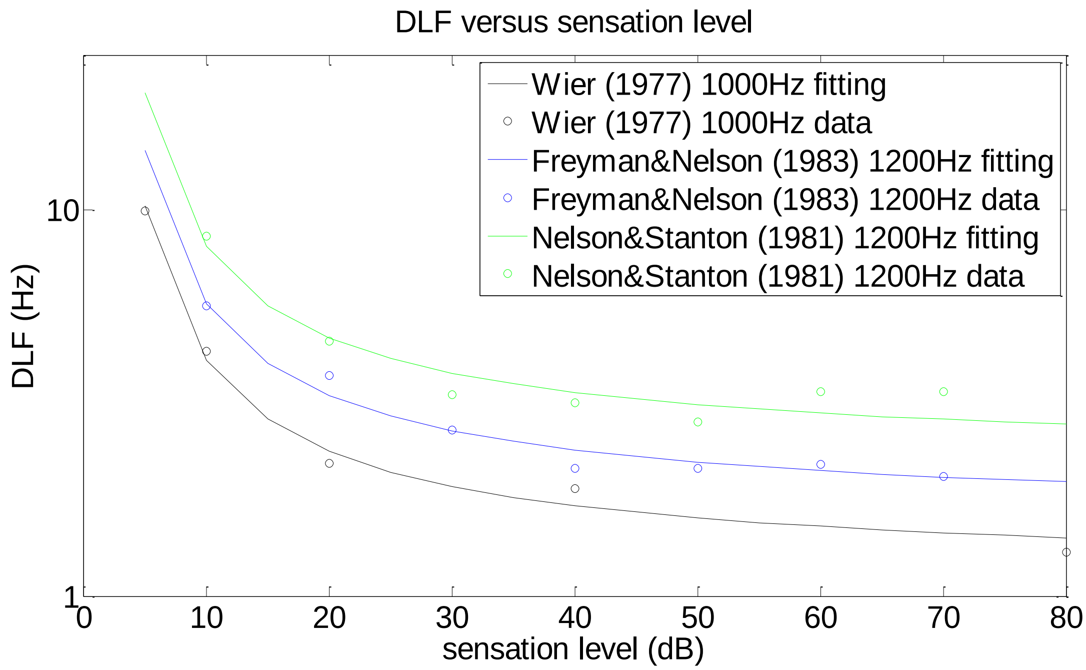


Figure 1: Data and model fits for several studies with the mixed effect model as Eq.(5). Different colors indicate fitting curves for different studies, the dots indicates the data from these studies. The three figures are the fittings for frequency, duration and level.

B. Modeling the interaction between frequency and duration

For the purpose of discovering a general model that contains different independent variables, an interaction term involving frequency and duration (F*T) was introduced, which can be thought of as representing the number of stimulus cycles. The formula is shown in Eq.(6). Results are shown in Table III and fits of the curves are shown in Figure 2.

$$\log DLF = a * F^{e} + b * T^{f} + c * (F * T)^{g} + (d + d') \qquad (6)$$

Where a, b, c, d, e, f, g are the fixed effects and d' is the random effect.

Table III: Values of fixed effects and random effects given by Eq. (6). The Rmse is the residual mean standard error and the Dfe is the error degree of freedom.

Fixed effects	a	b	c	d	e	f	g
Mean	0.46	1.58	2.13	-0.86	0.80	-0.25	-1
Standard error	0.07	0.25	0.66	0.38	0.06	0.15	0.33
	Dfe	BIC	Rmse				
	61	-117	0.108				

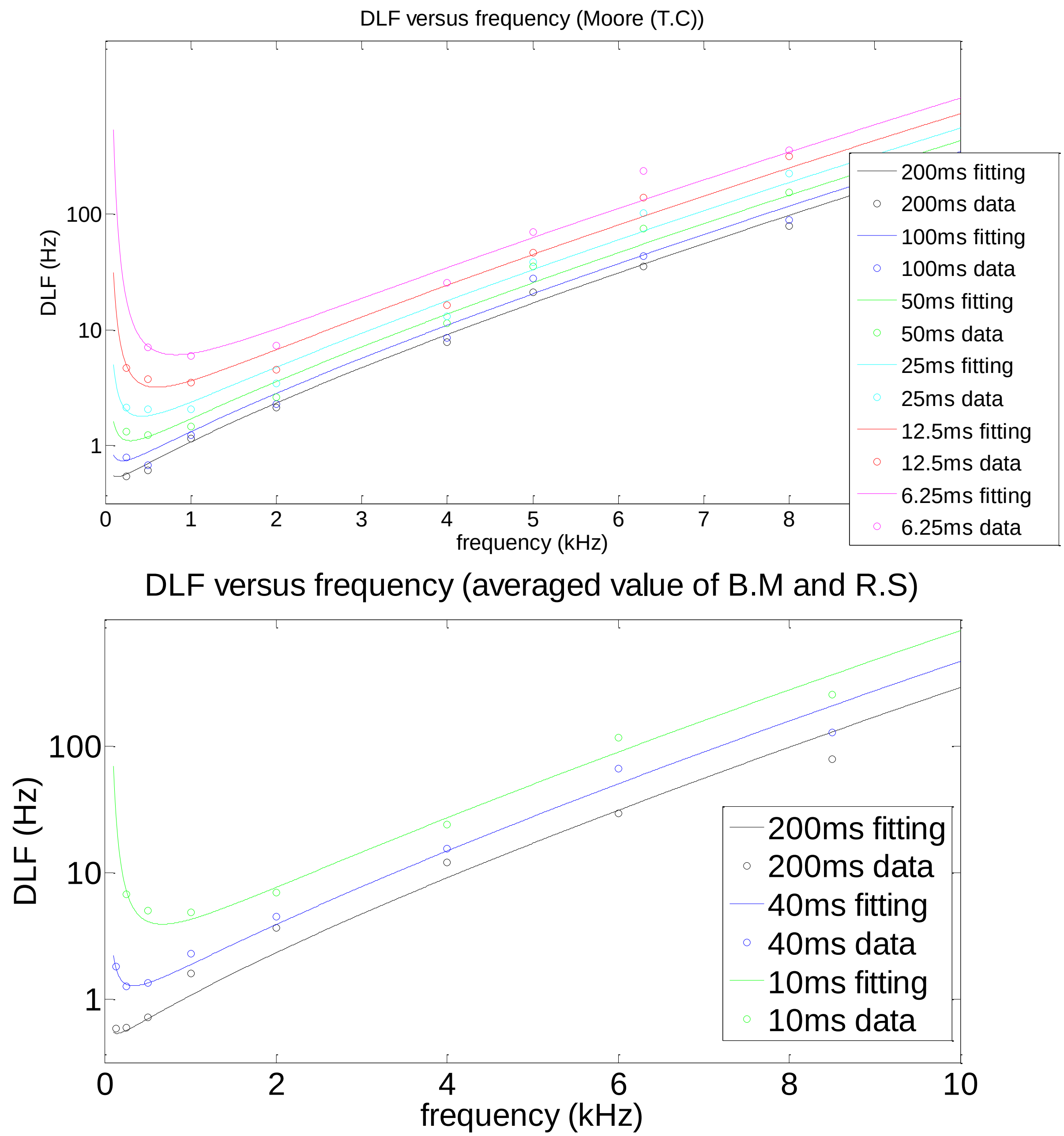


Figure 2: the change of DLF versus frequency for three different subjects in Moore (1973)'s data. Lines are the fitted curves by Eq.(6).

The results show that the curves fit well with the experimental data. And the interaction term also well contributes to the inverse change of DLF versus F in the low-frequency region. There is no significant change of power index for frequency and duration when combining the effect of frequency and duration together.

C. Model combining frequency, duration and level

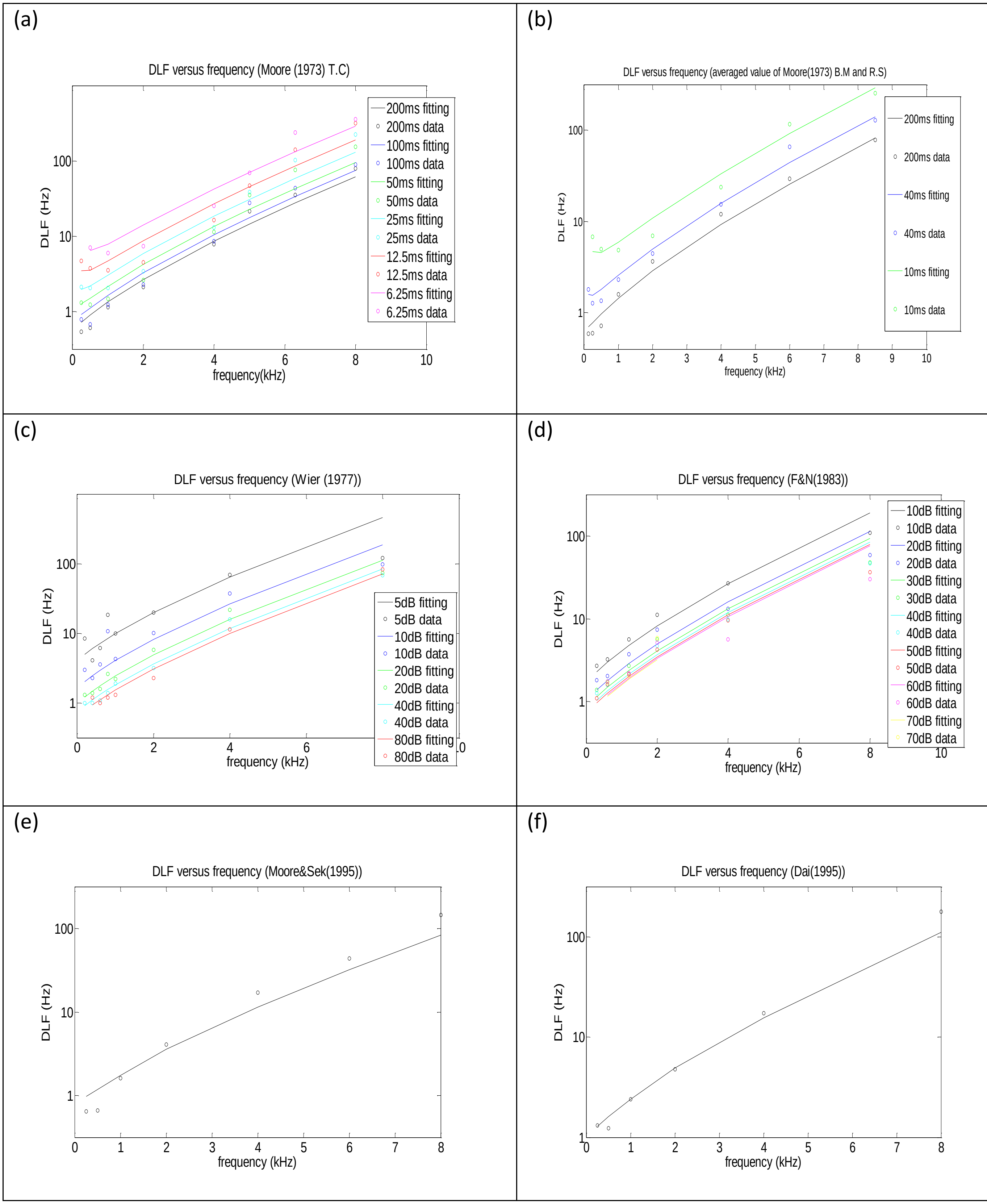
Since the goal of this investigation was to find a general equation to describe the change of DLFs versus frequency, duration and level between different studies, we extended the model as Eq. (7) by including frequency, duration and level together. Results of this fitting are shown in Table IV and the fits of the curves are shown in figure III.

$$logDLF = a * F^f + b * T^g + c * (F * T)^h + d * SL^i + (e + e') \quad (7)$$

Where $a, b, c, d, e, f, g, \mathrm{h}, \mathrm{i}$ are fixed effects, and e' is a random effect.

Table IV: The value of fixed effects and random effects of coefficients in Eq. (7). Rmse is the residual mean squared error, Dfe is the error degrees of freedom.

Fixed effect	Mean	SE	Random effect	Intercept
a	0.45	0.06	Moore(1973)T.C	-0.11
b	2.06	0.27	Moore(1973)B.M and R.S	-0.08
c	0.67	0.19	Wier(1977)	0.08
d	3.29	0.81	Freyman and Nelson(1983)	0.04
e	-0.64	0.17	Moore and Sek(1995)	0.03
f	0.75	0.05	Dai(1995)	0.03
g	-0.37	0.12	Dai and Micheyl(2011)	-0.08
h	-0.79	0.37	Moore and Glasberg(1989)	0.03
i	-0.80	0.18	Micheyl et.al (1998)	0
BIC	-184.5		Freyman and Nelson (1986)	0.06
Rmse	0.1480			
Dfe	216			


(a)
DLF versus frequency (Moore (1973) T.C)
DLF (Hz)
frequency(kHz)
200ms fitting
200ms data
100ms fitting
100ms data
50ms fitting
50ms data
25ms fitting
25ms data
12.5ms fitting
12.5ms data
6.25ms fitting
6.25ms data
(b)
DLF versus frequency (averaged value of Moore(1973) B.M and R.S)
DLF (Hz)
frequency (kHz)
200ms fitting
200ms data
40ms fitting
40ms data
10ms fitting
10ms data
(c)
DLF versus frequency (Wier (1977))
DLF (Hz)
frequency (kHz)
5dB fitting
5dB data
10dB fitting
10dB data
20dB fitting
20dB data
40dB fitting
40dB data
80dB fitting
80dB data
(d)
DLF versus frequency (F&N(1983))
DLF (Hz)
frequency (kHz)
10dB fitting
10dB data
20dB fitting
20dB data
30dB fitting
30dB data
40dB fitting
40dB data
50dB fitting
50dB data
60dB fitting
60dB data
70dB fitting
70dB data
(e)
DLF versus frequency (Moore&Sek(1995))
DLF (Hz)
frequency (kHz)
(f)
DLF versus frequency (Dai(1995))
DLF (Hz)
frequency (kHz)

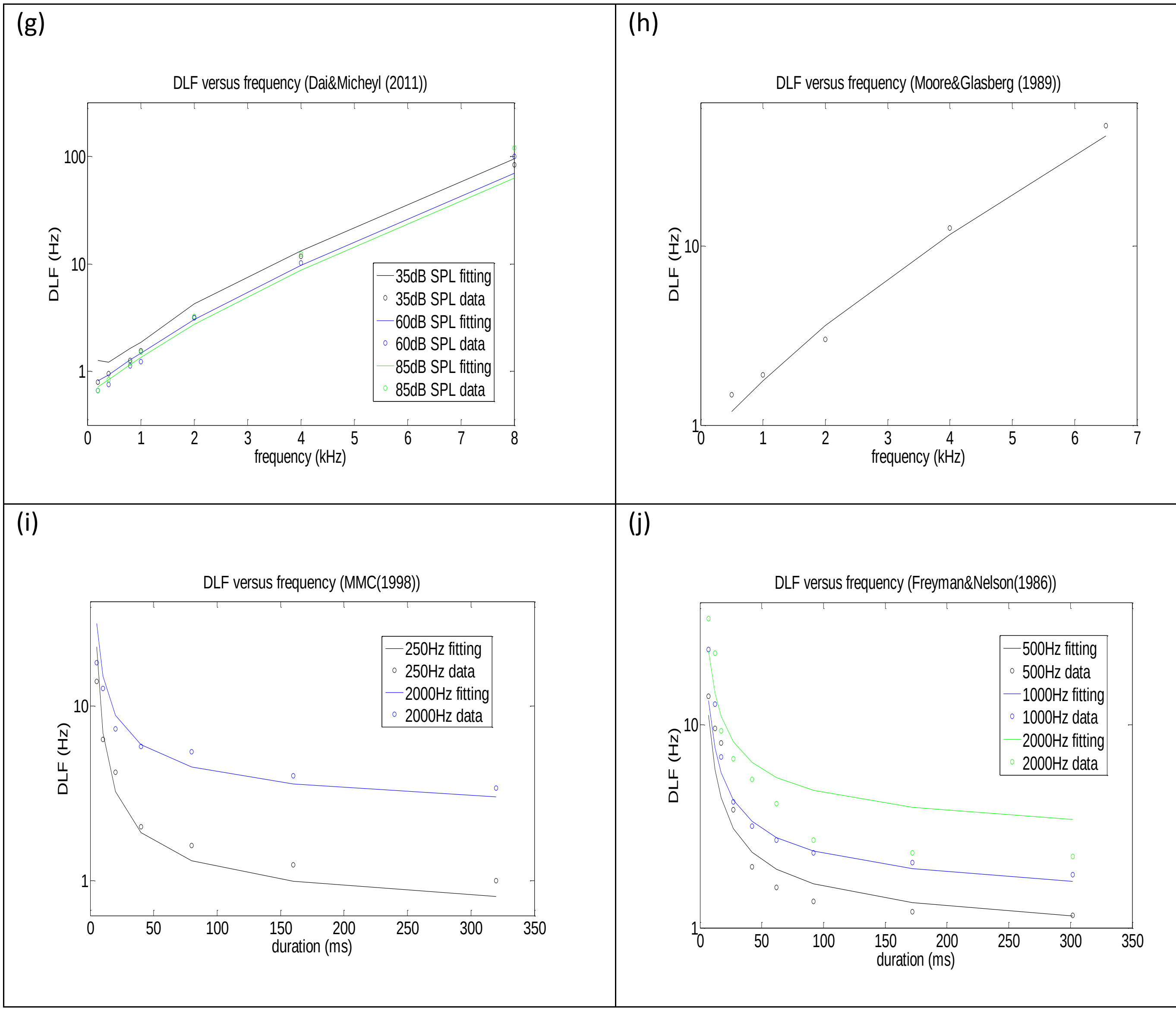


Figure 3: Data and fittings of different studies by using the model of Eq. (7). The ten panels illustrate the ten studies used in simulating the model: (a) Subject T.C of Moore (1973), (b) Averaged value of Subject B.M and Subject R.S of Moore (1973), (c) Wier et al. (1977), (d) Freyman and Nelson (1983), (e) Moore and Sek (1995),(f) Dai (1995), (g) Dai and Micheyl (2011), (h) Moore and Glasberg (1989), (i) Micheyl et al. (1998), and (j) Freyman and Nelson (1986).

D. Accuracy of the prediction equation

Tables II,III,IV shows that as the complexity of the model increases, the power indexes of frequency, duration and level still fall into the confidence interval that predicted by fitting each variables individually. This indicates that the power index for each independent variable is relatively robust and independent from the condition of other independent variables.

Figure 3 shows some error exists between the predicted value and the data, which can be attributed to the variability of the slope of frequency and duration between different studies. Considering the various kinds of testing conditions and paradigms, and only one random effect as intercept for the difference between different studies, from a subjective of viewpoint, these curves provide a good description of the actual data.

3.Discussion

The goal of this research was to determine a general equation which can be used to predict DLFs as a function of frequency, duration and level. Previous work was extended by modeling the relationship between $logDLF$ and these different to stimulus parameters as power functions, the exponents of which were treated as free parameters. In this way, our conclusions are not predicted on a priori assumption concerning the relationship. Meanwhile, a term of power function of the number of cycles ($\mathrm{F} * \mathrm{T}$) was introduced to account for potential interactions between F and T, particularly at low values of F.

Comparing to the result of Freyman and Nelson's (1983) equation, this research shows that 0.7 is an optimal value for the power index of frequency, and the value of 0.5 which used in the former research does not fall within the confidence intervals of power indexes this research has predicted. Meanwhile, this research also provides a model to combine frequency, duration and level together, which provides a good fit to the data.

Some problems remain beyond the scope of this research. First of all, a single term for SL in the formula cannot account for the fact that DLF seems to increase as the level increases for long-duration, high-level tones. One possibility of solving this problem is to introduce an interaction term between frequency and level. One potential candidate for this term is the ERB bandwidth. It is known that the ERB bandwidth becomes broader as the level increases. Thus, a prediction of the place model of pitch is that DLFs will be increasingly poor as the filter bandwidth increases. Hence, a term of SL by a power function and a term of ERB bandwidth may cooperate together to obtain a better fit for DLF versus F. Meanwhile, Figure 3 illustrates some deviation of fittings for the data of Freyman and Nelson (1986), this may because that an interaction term $F * T$ is still not enough in accounting for the change of DLF versus F. To solve this problem, one possible way would be to divide the frequency range into several parts, and fit each part with an individual equation. However, the increase in the number of free parameters, makes this option unattractive.

To summarize, this paper provides models to describe pure tone DLFs by combining frequency, duration and level together, and this model has some advantage over the models in Freyman and Nelson (1983). However, there are still some limitations based on the results of the model. Further work may be done on two directions. One is to develop a model according to some theoretical basis and physiological procedure. Another one is to extend the model to describe the change of DLFs on a noisy environment, the change of DLFs for different excitation patterns and the change of DLFs for cochlear implant listeners.

1 An equal-loudness contour is a measure of sound pressure (dB SPL), over different frequencies, for which a listener perceive equal loudness when presented with pure steady tones.

2 In statistics, an effect indicates something that influences the value of a response variable at a particular setting of the predictor variables. Effects are expressed as model parameters. For a linear model, effects become coefficients, while for a nonlinear model, effects often have specific physical interpretations. Mixed-effects models account for both fixed and random effects. As with all regression models, their purpose is to describe a response variable as a function of the predictor variables. On the other hand, Mixed-effects models also recognize correlations within sample subgroups. Hence, they provide a compromise between ignoring data groups and fitting each group with a separate model.

nlmefit fits the model by maximizing an approximation to the marginal likelihood with random effects integrated out, two assumption holds during the simulation: 1-Random effects are multivariate normally distributed and independent between group, 2-observation errors are independent, identically normally distributed, and independent of the random effects

3 Bayesian information criterion (BIC) is a criterion for model selection among a finite set of models. BIC is a value which balance the likelihood function and the degree of freedom. Given any two estimated models, the model with the lower value of BIC is the one to be preferred.

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