Scattering of Glue by Glue on the Lightcone Worldsheet[1]

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[1]The work described here was done in collaboration with Jian Qiu and Dipankar Chakrabarti

1 Outline

1. Motivation: The QFT Lightcone Worldsheet
2. Lightcone Feynman Rules and K Identities
3. Gluon-Gluon Scattering: The simplest on-shell process.
4. Bremsstrahlung Diagrams and Large N_c
5. Definition of Jets and Resolution
6. 34 and 12 Brem
7. 14 and 23 Brem and Disconnected Diagrams
8. Summary of Brem Results on the Lightcone
9. Conclusion: Enumeration of one loop gluon scattering results on the lightcone

References:

D. Chakrabarti, J. Qiu, C.B. Thorn:

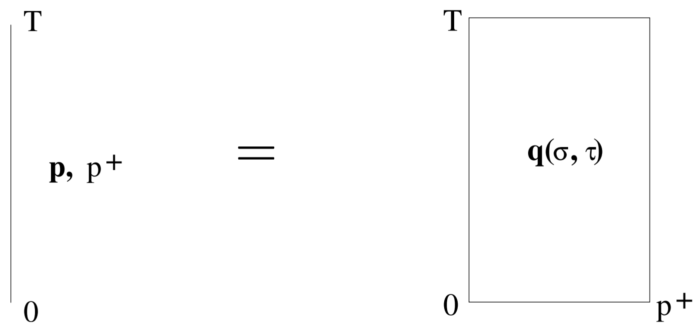
Scattering of Glue by Glue on the Light-cone Worldsheet

I. Helicity Non-conserving Amplitudes,
Phys.Rev.D72:065022,2005, hep-th/0507280;

II. Helicity Conserving Amplitudes
hep-th/0602026

2 QFT Lightcone Worldsheet

Bardakci-Thorn(NPB626:287,2002)
Master Formula for Massless Propagator:

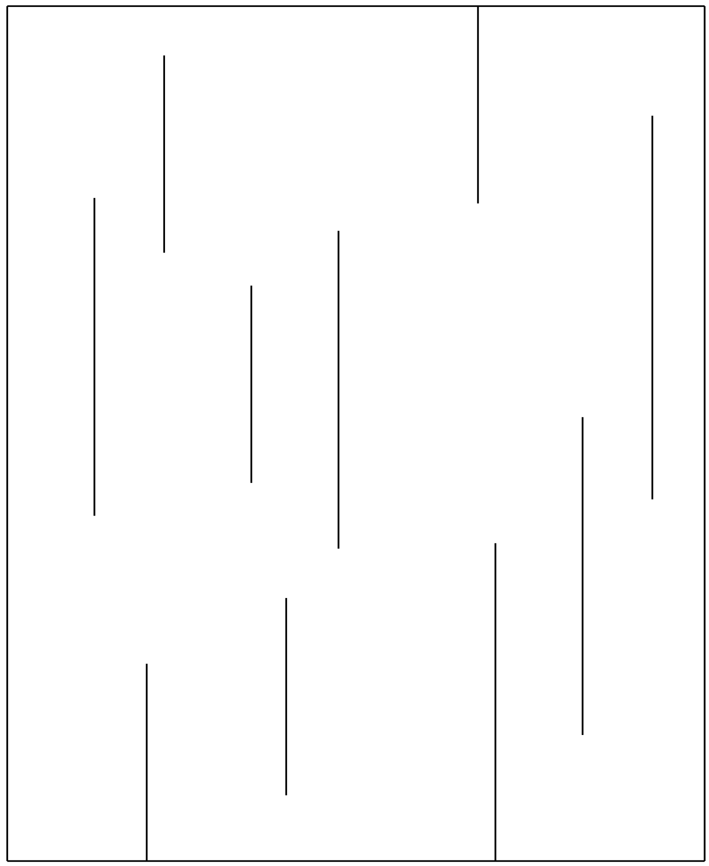


$$\exp\left\{-\frac{T}{2p^+}\boldsymbol{p}^2\right\} = \int_{\substack{\boldsymbol{q}(0,\tau)=0 \\ \boldsymbol{q}(p^+,\tau)=\boldsymbol{p}}} DcDbD\boldsymbol{q}\; e^{iS_0}$$

$$iS_0 = \int_0^T d\tau \int_0^{p^+} d\sigma \left(b'c' - \frac{1}{2}\boldsymbol{q}'^2\right)$$

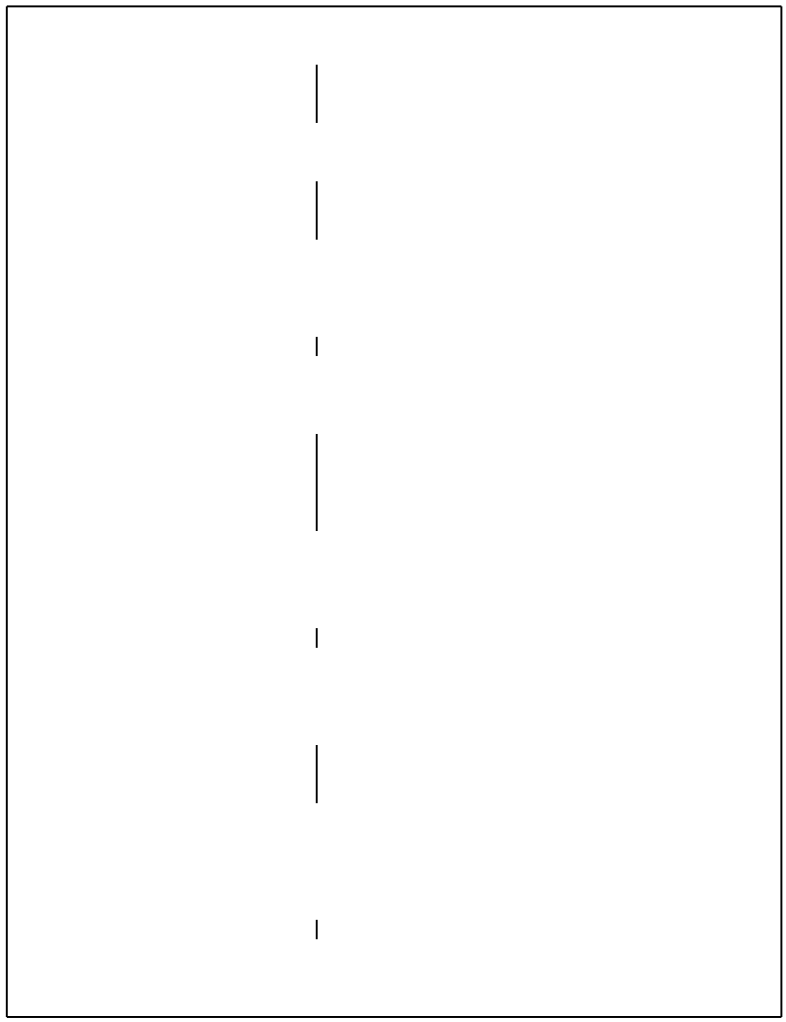
- Dirichlet b.c.'s. Cf. string in momentum space
- Represent a field quantum as a composite of String Bits
- Total $p^+ =$ (Number of bits)$\times m$.

The following is the worldsheet diagram describing a 7 loop 5 gluon diagram:



- LCWS treats:
  $\boldsymbol{q}_i$ as worldsheet fields
  x^+, p^+ as moduli of worldsheet
- In string theory one usually first integrates over worldsheet fields, then over moduli.
- Insight on Field/String duality may come from treating loop integrals similarly.

Example: A novel way to resum diagrams (K. Bardakci): First do loop sum at fixed σ in a background mean field:



Then determine mean field self-consistently.
Note that the diagrams summed here would be tiny bits of self energy bubble diagrams.

Comments:

- Worldsheet system that sums QFT planar diagrams is a two-dimensional dynamical system of scalar fields $\boldsymbol{q}(\sigma, \tau)$, Grassmann ghosts $\boldsymbol{b}(\sigma, \tau)$, $\boldsymbol{c}(\sigma, \tau)$, and Ising spins $s(\sigma, \tau)$.

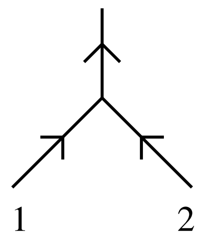
- These degrees of freedom have a fairly complicated but *local* worldsheet action.

- The scalar and ghost fields enter the worldsheet action quadratically, but with coefficients that depend on the Ising spins.

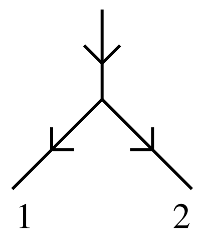
- The role of "string tension" in this worldsheet system is played by a quantity that depends on the Ising spin configuration. Its "value" fluctuates locally and can't be regarded as a fixed parameter.

- The fluctuating string tension is the crucial difference between the string description of a field theory and the Nambu-Goto string. We can hopefully trace the well-known short-comings of the Nambu-Goto string for describing hadrons to this difference.

3 Feynman Rules for Light-cone gauge Yang-Mills



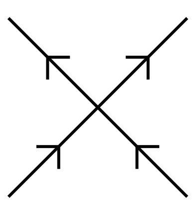
$$= \quad \frac{2gp_3^+}{p_1^+ p_2^+}\left(p_1^+ p_2^\wedge - p_2^+ p_1^\wedge\right) = \frac{2gp_3^+}{p_1^+ p_2^+} K_{12}^\wedge$$



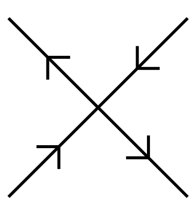
$$= \quad \frac{2gp_3^+}{p_1^+ p_2^+}\left(p_1^+ p_2^\vee - p_2^+ p_1^\vee\right) = \frac{2gp_3^+}{p_1^+ p_2^+} K_{12}^\vee$$

$p^\wedge = (p^x + ip^y)/\sqrt{2}, \qquad p^\vee = (p^x - ip^y)/\sqrt{2}$

$g = g_s\sqrt{N_c/2}$: these rules are for $N_c = \infty$!



$$= \quad -2g^2 \frac{p_1^+ p_3^+ + p_2^+ p_4^+}{(p_1^+ + p_4^+)^2}$$



$$= \quad +2g^2\left(\frac{p_1^+ p_2^+ + p_3^+ p_4^+}{(p_1^+ + p_4^+)^2} + \frac{p_1^+ p_4^+ + p_2^+ p_3^+}{(p_1^+ + p_2^+)^2}\right)$$

4 K Identities

$$K_{ij}^{\mu} \equiv p_i^+ p_j^\mu - p_j^+ p_i^\mu$$

(Cf. spinor matrix elements in Parke-Taylor amplitudes)

$$\sum_j K_{ij}^{\mu} = 0$$

$$p_i^+ K_{jk}^{\mu} + p_k^+ K_{ij}^{\mu} + p_j^+ K_{ki}^{\mu} = 0$$

$$K_{li}^{\wedge} K_{jk}^{\wedge} + K_{lk}^{\wedge} K_{ij}^{\wedge} + K_{lj}^{\wedge} K_{ki}^{\wedge} = 0$$

$$\sum_j \frac{K_{ij}^{\wedge} K_{jk}^{\vee}}{p_j^+} = -p_i^+ p_k^+ \sum_j \frac{p_j^2}{2p_j^+}$$

Derivation of last K identity

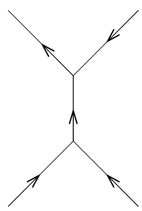
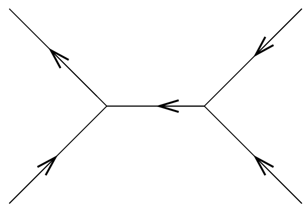
$$
\begin{aligned}
\sum_j \frac{K^{\wedge}_{ij} K^{\vee}_{jk}}{p^+_j} &= \sum_j \frac{K^{\wedge}_{ij}(p^+_j p^{\vee}_k - p^+_k p^{\vee}_j)}{p^+_j} \\
&= -p^+_k \sum_j \frac{K^{\wedge}_{ij} p^{\vee}_j}{p^+_j} \\
&= -p^+_k \sum_j \frac{(p^+_i p^{\wedge}_j - p^+_j p^{\wedge}_i) p^{\vee}_j}{p^+_j} \\
&= -p^+_i p^+_k \sum_j \frac{p^{\wedge}_j p^{\vee}_j}{p^+_j} \\
&= -p^+_i p^+_k \sum_j \frac{p^2_j + 2p^+_j p^-_j}{2p^+_j} \\
&= -p^+_i p^+_k \sum_j \frac{p^2_j}{2p^+_j}
\end{aligned}
$$

Here we used the lightcone parametrization of the square of a Lorentz 4-vector:

$$p^2 = \boldsymbol{p}^2 - 2p^+ p^- = 2p^{\wedge} p^{\vee} - 2p^+ p^-$$

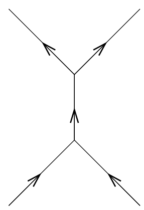
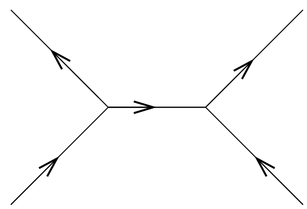
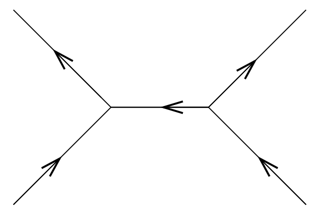
5 Scattering of Glue by Glue

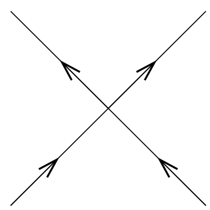
$$\Gamma^{\wedge\wedge\wedge\wedge} = 1 \text{ Loop} + \cdots$$

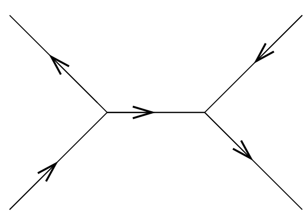
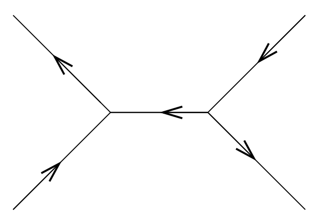
$\Gamma^{\wedge\wedge\wedge\vee} =$  $+$  $+ \; 1 \text{ Loop} \; + \cdots$

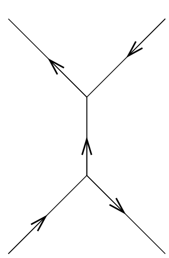
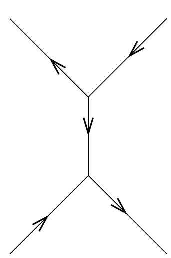
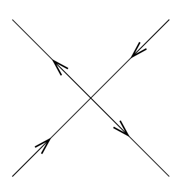
$$= 0 + \; 1 \text{ Loop} \; + \cdots$$

$$\begin{aligned}
\frac{K^{\wedge}_{12}K^{\wedge}_{43}}{p^2_{12}} + \frac{K^{\wedge}_{14}K^{\wedge}_{23}}{p^2_{14}} & \\
&= \quad -2p_1^+ \left(p_2^+ \frac{K^{\wedge}_{43}}{K^{\vee}_{12}} + p_4^+ \frac{K^{\wedge}_{23}}{K^{\vee}_{14}} \right) \\
&= \quad -\frac{2p_1^+ p_2^+ p_4^+}{K^{\vee}_{12}K^{\vee}_{14}} \left(\frac{K^{\vee}_{14}K^{\wedge}_{43}}{p_4^+} + \frac{K^{\vee}_{12}K^{\wedge}_{23}}{p_2^+} \right) \\
&= \quad -\frac{2p_1^+ p_2^+ p_4^+}{K^{\vee}_{12}K^{\vee}_{14}} \sum_j \frac{K^{\vee}_{1j}K^{\wedge}_{j3}}{p_j^+} = 0
\end{aligned}$$

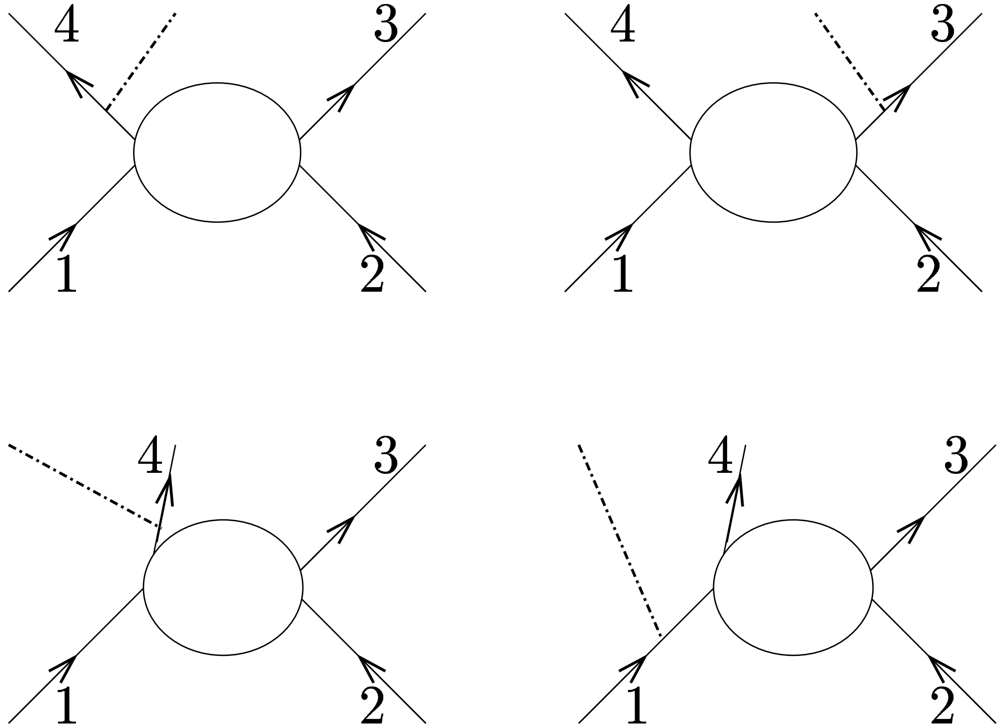
$\Gamma^{\wedge\wedge\vee\vee} =$  $+$  $+$ 

$+$  $+\ 1$ Loop $+\cdots$

$\Gamma^{\wedge\wedge\vee\vee} =$  $+$  $+$

$+$  $+$  $+$  $+\ 1$ Loop $+\cdots$

6 Bremsstrahlung and large N_c



At $N_c = \infty$ Diagrams on second line do not interfere with those on first line.

Therefore can analyze process independently in 12,23,34,41 sets of diagrams.

Lee-Nauenberg: Extra gluons in initial as well as final states (but at $N_c = \infty$ only for 23 and 41!)

7 Self Energy

$$\Pi^{\wedge\vee} = \frac{g^2}{4\pi^2}\frac{p^2}{|p^+|}\sum_{k^+}\left(x(1-x)+\frac{x}{1-x}+\frac{1-x}{x}\right)$$
$$\ln(x(1-x)p^2\delta e^\gamma)$$

where $x = k^+/p^+$, δ is UV cutoff $(e^{-\delta \boldsymbol{q}^2})$

$\Pi/p^2 \to \infty$ on shell (collinear divergence)
Including bremstrahlung will fix this.
Give gluons on internal lines a small mass μ.

$$\Pi_\mu^{\wedge\vee} = \frac{g^2}{4\pi^2}\frac{1}{|p^+|}\sum_{k^+}(\mu^2 + x(1-x)p^2)$$
$$\left(1+\frac{1}{(1-x)^2}+\frac{1}{x^2}\right)\ln((\mu^2+x(1-x)p^2)\delta e^\gamma)$$

$$\to \text{Constant} +$$

$$\frac{p^2}{|p^+|}\frac{g^2}{4\pi^2}\sum_{k^+}\left(x(1-x)+\frac{x}{1-x}+\frac{1-x}{x}\right)\ln(\mu^2\delta e^{\gamma+1})$$

as $p^2 \to 0$. This is just $(Z-1)p^2$.

8 Jets and Resolution

$$-(k+p_i)^2 = \frac{(p_i^+ \boldsymbol{k} - k^+ \boldsymbol{p}_i)^2}{k^+ p_i^+} = 2|\vec{k}||\vec{p}_i|(1-\cos\theta)$$

vanishes for $k \to 0$ or k collinear with p_i.

Collinear point is

$$\boldsymbol{k} = k^+ \frac{\boldsymbol{p}_i}{p_i^+} \equiv k^+ \boldsymbol{v}_i$$

Gluon(k) and Gluon(p_i) in a jet of resolution Δ if

$$-(k+p_i)^2 < \Delta^2 \qquad \text{or} \qquad (\boldsymbol{k} - k^+ \boldsymbol{v}_i)^2 < \frac{k^+ \Delta^2}{p_i^+}$$

On light cone, this gives a clean constraint on transverse momentum.

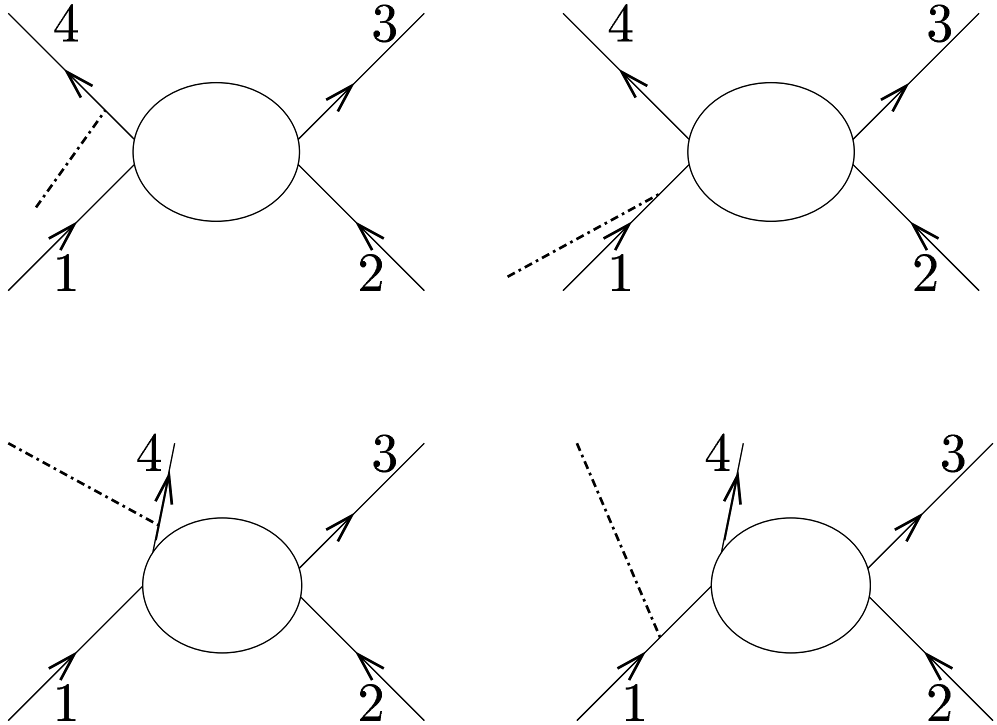
9 34 Bremsstrahlung

$$
\begin{aligned}
|\mathcal{M}_{34}^{\mathrm{Brem}}|^2 &= \frac{g^2}{8\pi^2}\sum_{i=3,4}|A^i_{\mathrm{Core}}|^2\sum_{|k^+|}\frac{1}{|k^+|} \\
&\left(\frac{P_i^+}{P_i^+-k^+}+\frac{(P_i^+-k^+)^3}{P_i^{+3}}\right)\ln\frac{k^+(P_i^+-k^+)\Delta^2}{P_i^{+2}\mu^2 e} \\
&+\frac{g^2|A_{\mathrm{Core}}|^2}{4\pi^2}\sum_{|k^+|<\Delta^2/|P^+_{min}|v_{34}^2}\frac{1}{|k^+|}\ln\frac{k^{+2}\boldsymbol{v}_{34}^4|p_3^+p_4^+|}{\Delta^4}
\end{aligned}
$$

$$
\begin{aligned}
&\approx \frac{g^2}{8\pi^2}\sum_{i=3,4}|A^i_{\mathrm{Core}}|^2\sum_{|k^+|}\frac{1}{|k^+|} \\
&\left(\frac{P_i^+}{P_i^+-k^+}+\frac{(P_i^+-k^+)^3}{P_i^{+3}}\right)\ln\frac{k^+(P_i^+-k^+)\Delta^2}{P_i^{+2}\mu^2 e} \\
&+\frac{g^2|A_{\mathrm{Core}}|^2}{4\pi^2}\left[\sum_{|k^+|<A}\frac{1}{|k^+|}\ln\frac{k^{+2}\boldsymbol{v}_{34}^4|P_3^+P_4^+|}{\Delta^4}\right. \\
&\left. -\ln\frac{\Delta^2}{A|P_4^+|v_{34}^2}\;\ln\frac{\Delta^2}{A|P_3^+|v_{34}^2}\right]
\end{aligned}
$$

where $v_{34}^2 \equiv (\boldsymbol{v}_3-\boldsymbol{v}_4)^2 = -(p_3+p_4)^2/p_3^+p_4^+$

10 41 Bremsstrahlung

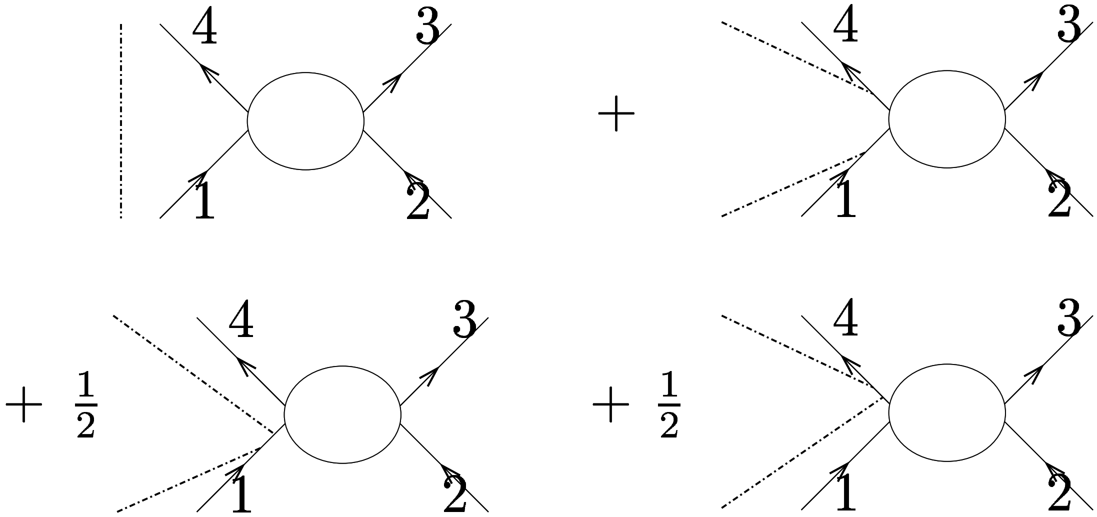


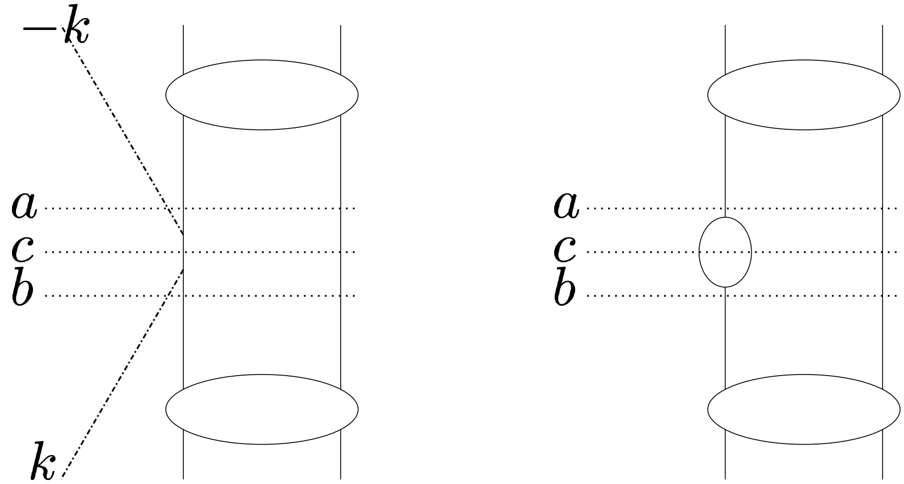
For these processes:

- Too many collinear divergences:
  Out brem collinear with p_1 separated from p_3, p_4
  In brem collinear with p_4 separated from p_1, p_2.

- Twice too much soft brem

11 Disconnected Diagrams





Interference terms are same order as single brem gluon $|amp|^2$. They cancel unwanted collinear divergences and soft brem, leaving terms that combine to 34 result with p_3 replaced by $-p_1$.

12 Total Bremsstrahlung + S.E.

Simply add the wave function renormalization to the sum of 12, 23, 34, 41 bremsstrahlung contributions.

$$\sum_{i=1}^{4} \sum_{|k^+|<|P_i^+|} \frac{g^2|A_{\rm Core}|^2}{8\pi^2}\left(\frac{|P_i^+|}{|k^+(P_i^+-k^+)|}+\frac{|P_i^+-k^+|^3}{|k^+P_i^{+3}|}\right.$$

$$\left.+\frac{|k^+|^3}{|(P_i^+-k^+)P_i^{+3}|}\right)\ln\frac{|k^+(P_i^+-k^+)|\Delta^2\delta e^\gamma}{|P^+|^2}$$

$$+\sum_{i=1}^{4}\frac{g^2|A_{\rm Core}|^2}{4\pi^2}\left[\sum_{|k^+|<A}\frac{1}{|k^+|}\ln\frac{k^{+2}\boldsymbol{v}^4_{i,i+1}|P_i^+P_{i+1}^+|}{\Delta^4}\right.$$

$$\left.-\ln\frac{\Delta^2}{A|P_i^+|v^2_{i,i+1}}\ \ln\frac{\Delta^2}{A|P_{i+1}^+|v^2_{i,i+1}}\right]$$

Here $v_{ij}^2=(\boldsymbol{v}_i-\boldsymbol{v}_j)^2=-\dfrac{(p_i+p_j)^2}{P_i^+P_j^+}=\dfrac{|(p_i+p_j)^2|}{|P_i^+P_j^+|}$

We have shown that the k^+ divergences in this expression cancel against corresponding divergences in the one loop corrections to the elastic process.
The final answer is Lorentz invariant.

13 Cancellation of IR divergence and Lorentz invariance

$$\text{Probability} \quad = \quad |\text{Tree}|^2 F$$

$$\begin{aligned} F_{\wedge\wedge\vee\vee} \quad &= \quad 1 + \frac{g^2}{4\pi^2}\Bigg[-2\log^2\frac{\Delta^2}{s} - 2\log^2\frac{\Delta^2}{|t|} \\ +4\cdot\frac{\pi^2}{3} &+ \frac{67}{9} - \frac{11}{3}\left[\log\left(\Delta^2\delta e^{\gamma}\right) + \log\frac{\Delta^2}{|t|}\right] + \log^2\frac{s}{|t|}\Bigg] \end{aligned}$$

$$\begin{aligned} F_{\wedge\vee\wedge\vee} \quad &= \quad 1 + \frac{g^2}{4\pi^2}\Bigg[-2\log^2\frac{\Delta^2}{s} - 2\log^2\frac{\Delta^2}{|t|} \\ +4\cdot\frac{\pi^2}{3} &+ \frac{67}{9} - \frac{11}{3}[\log\left(\Delta^2\delta e^{\gamma}\right) + \frac{1}{2}\log\frac{\Delta^4}{s|t|}] \\ &+\frac{(s^2+st+t^2)^2}{(t+s)^4}\log^2\frac{s}{|t|} \\ &+\frac{(5st^2-5s^2t+11t^3-11s^3)}{6(t+s)^3}\cdot\log\frac{s}{|t|} - \frac{ts}{(t+s)^2}\Bigg] \end{aligned}$$

We see that all IR divergences have cancelled, that the UV divergences are exactly those dictated by asymptotic freedom, and Lorentz invariance is manifest.

14 Counterterms on the Lightcone Worldsheet

Counterterms for correct mhv amps:

$$
\begin{aligned}
\Pi^{\wedge\wedge}_{\mathrm{C.T.}} &= -\frac{g^2}{12\pi^2}[k_0^{\wedge 2} + k_0^{\wedge} k_1^{\wedge} + k_1^{\wedge 2}] \\
\Pi^{\wedge\vee}_{\mathrm{C.T.}} &= -\frac{g^2}{24m\delta}p^+ + \frac{g^2}{4\pi^2\delta} + \frac{g^2}{24\pi^2}p^2 \\
\Gamma^{\wedge\wedge\vee}_{\mathrm{C.T.}} &= \frac{g^3}{12\pi^2}[k_0^{\wedge} + k_1^{\wedge} + k_2^{\wedge}]
\end{aligned}
$$

Each a polynomial in momenta of correct degree.

Quartic counterterms (helicity conserving amps)

$$
\Gamma^{\wedge\wedge\vee\vee}_{\mathrm{C.T.}} = -\frac{g^4}{12\pi^2}, \qquad \Gamma^{\wedge\vee\wedge\vee}_{\mathrm{C.T.}} = -\frac{g^4}{12\pi^2}
$$

- Quartic CT's spin independent.
- Quartics local in target space but non-local on lc worldsheet

$\Pi^{\wedge\wedge}_{\text{C.T.}}$ contribution to ws path integral:

$$\begin{aligned}\frac{T\Pi^{\wedge\wedge}_{\text{C.T.}}}{2p^+} &= \frac{g^2}{16\pi^2}\int d\tau \frac{q^{\wedge 2}(0)+q^{\wedge 2}(p^+)}{p^+} \\ &\quad + \frac{g^2}{48\pi^2}\int d\tau d\sigma \left(\frac{\partial q^\wedge}{\partial \sigma}\right)^2\end{aligned}$$

- 1st is a boundary contribution to $-S_{ws}$
- 2nd is a new bulk contribution
- Together cancel angular momentum violation caused by bare $\Pi^{\wedge\wedge} \neq 0$

New contact contributions from $\Pi^{\wedge\wedge}_{\text{C.T.}}$:

$$\begin{aligned}\Gamma_\Pi^{\wedge\wedge\vee\vee} &= -\frac{g^4}{12\pi^2}\left[\frac{p_1^+p_3^+ + p_2^+p_4^+}{p_{14}^{+2}} + 1\right] \\ \Gamma_\Pi^{\wedge\vee\wedge\vee} &= \frac{g^4}{12\pi^2}\left[\frac{p_1^+p_2^+ + p_3^+p_4^+}{p_{14}^{+2}} + \frac{p_1^+p_4^+ + p_2^+p_3^+}{p_{12}^{+2}} + 2\right]\end{aligned}$$

Nonlocal terms handled by altering $\Pi^{\wedge\vee}_{\mathrm{C.T.}}$:

$$\Pi^{\wedge\vee}_{\mathrm{C.T.}} \quad\to\quad \Pi^{\wedge\vee}_{\mathrm{C.T.ws}} \;=\; -\frac{g^2}{24m\delta}p^+ + \frac{g^2}{4\pi^2\delta} + \frac{g^2}{12\pi^2}p^2$$

- First term absorbed in ws boundary cosmological constant.
- Second term absorbed in ws mass counterterm
- Third term a finite cont to Z
- Effect of Z combined with $g \to g/Z$:

  $$\text{Multiply } \Gamma_3 \text{ by } \sqrt{Z}; \qquad \Gamma_4 \text{ untouched}$$
- WS formalism automatically generates quartic from cubic: Can prevent this by altering the ghost worldsheet action near the interaction point.

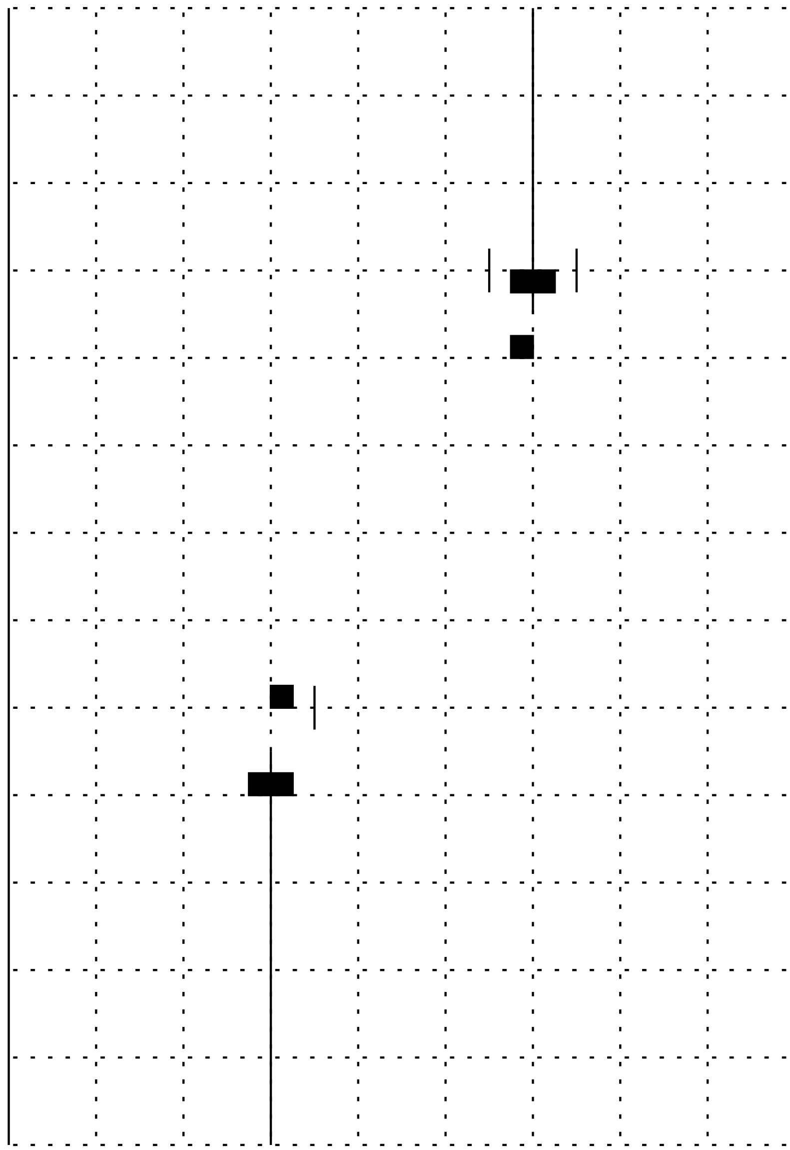


Figure 1: Discretized worldsheet for a four gluon tree. The solid squares indicate where $\partial q/\partial\sigma$ insertions can be located. The short vertical lines indicate the links to be deleted in the worldsheet ghost action. All indicated ghost link deletions are present regardless of the insertion location.

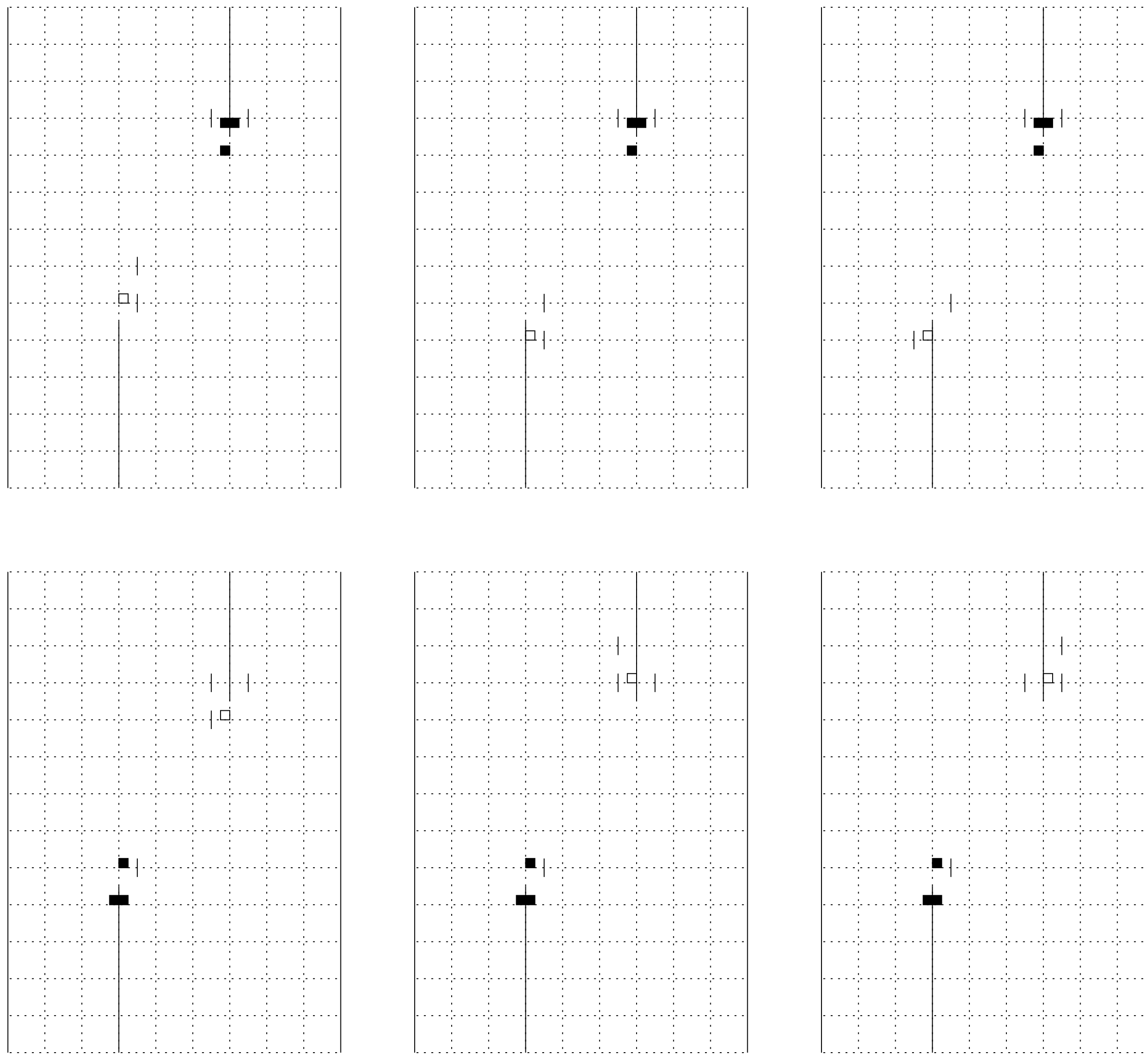


Figure 2: Ghost deletion scheme for cubic C.T.

Ghost deletion scheme allows scaling the cubic vertex without affecting the quartic vertex. Can convert of the nonlocal parts of quartic counterterms to complete trees.

Also aids translating cubic counterterm ws:

$$\begin{aligned}
\Gamma^{\wedge\wedge\vee}_{\text{C.T.}} &= \frac{g^3}{12\pi^2}[k_0^\wedge + k_1^\wedge + k_2^\wedge] \\
&= \frac{g^3}{12\pi^2}[k_0^\wedge - k_1^\wedge + k_2^\wedge - k_1^\wedge + 3k_1^\wedge] \\
&= \frac{g^3}{12\pi^2}[p_2^\wedge - p_1^\wedge + 3k_1^\wedge] \\
&= \frac{g^3}{12\pi^2}\left\langle p_2^+ \frac{\partial q^\wedge}{\partial\sigma}(B) - p_1^+ \frac{\partial q^\wedge}{\partial\sigma}(A) + 3q^\wedge(A)\right\rangle
\end{aligned}$$

Here A and B label worldsheet points just to the left and right of the internal boundary separating the two gluon propagators that fuse to or fission from the third gluon propagator (see Fig. 3).

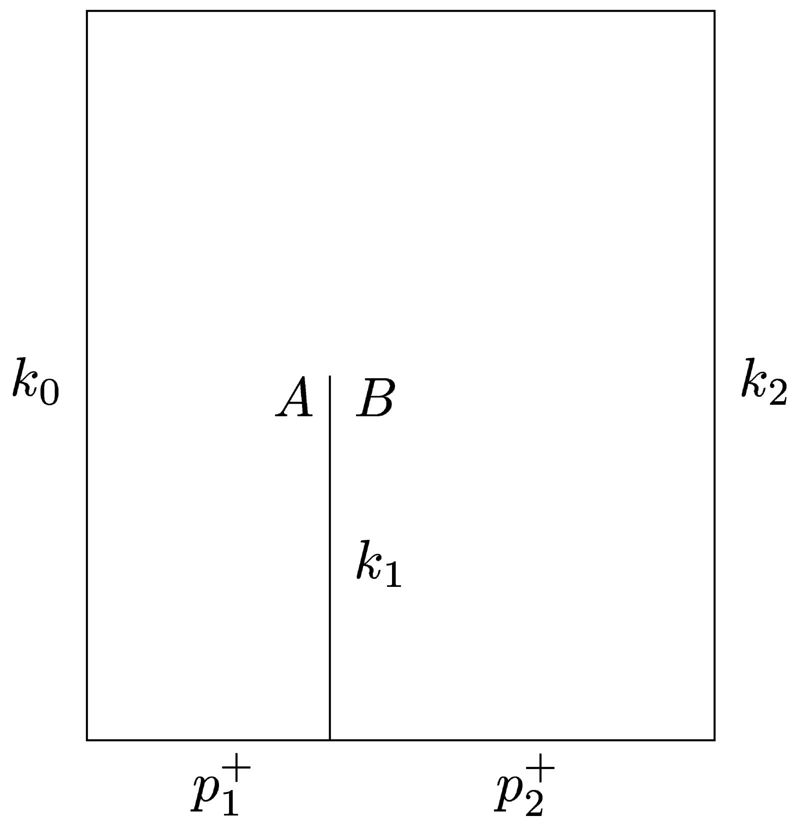


Figure 3: Worldsheet for cubic fusion vertex.

It remains to find a local worldsheet description of the purely constant parts of the quartic counterterms

$$\Gamma^{\wedge\wedge\vee\vee}_{\rm C.T.ws} = 0, \qquad \Gamma^{\wedge\vee\wedge\vee}_{\rm C.T.ws} = -\frac{g^4}{4\pi^2}$$

- Must generate these from cubics, e.g.

$$\begin{aligned} C^{\wedge\wedge\vee} &= g^3\xi(p_2^\wedge - p_1^\wedge) \\ &\rightarrow g^3\xi\left\langle p_2^+\frac{\partial q^\wedge}{\partial\sigma}(2) - p_1^+\frac{\partial q^\wedge}{\partial\sigma}(1)\right\rangle \end{aligned}$$

  leads to

$$C^{\wedge\wedge\vee\vee} = -g^4\xi, \qquad C^{\wedge\vee\wedge\vee} = +2g^4\xi$$

- Ratio of two polarization structures same as from $[A_\mu, A_\nu]^2$ term, in the field theoretic Lagrangian.

- Can suppress the new cubic couplings:

$$0 = C^{\wedge\wedge\vee} - C^{\wedge\wedge\vee}$$

  and apply ghost link deletions on the second term. Then the exchange graphs will cancel leaving only the contact quartic vertex!

Quartic C.T.'s from Extra Dimensions

- Increase flexibility of the worldsheet formalism by increasing the dimensionality of the worldsheet fields $\boldsymbol{q}(\sigma, \tau)$.

- AdS/CFT correspondence: string theory ten dimensional whereas the equivalent susy gauge theory is four dimensional.

- Same for lightcone ws description of $\mathcal{N} = 4$ supersymmetric gauge theories: the index of q^i took the values $i = 1, 2, \ldots, 8$, and add three new sets of b, c ghosts. All new boundary conditions were strict Dirichlet $q^i = c = b = 0$ on all boundaries, internal or external.

- Since the extra q's and ghosts share identical boundary conditions, their contributions to the path integral exactly cancel.

- To locally produce the necessary quartic counterterms in pure gauge theories, two extra dimensions and one extra set of b, c ghosts suffice: ws describes 6 dimensional QCD string.

- Call new dimensions r^k, $k = 1, 2$ or in complex basis, $r^\wedge, r^\vee$. But here $\wedge, \vee$ do *not* represent helicity, but rather an analogous charge in the extra dimensions. Next we allow spurions with values ± 1 of this charge to couple to two gluons as indicated in the top line Fig. 4.

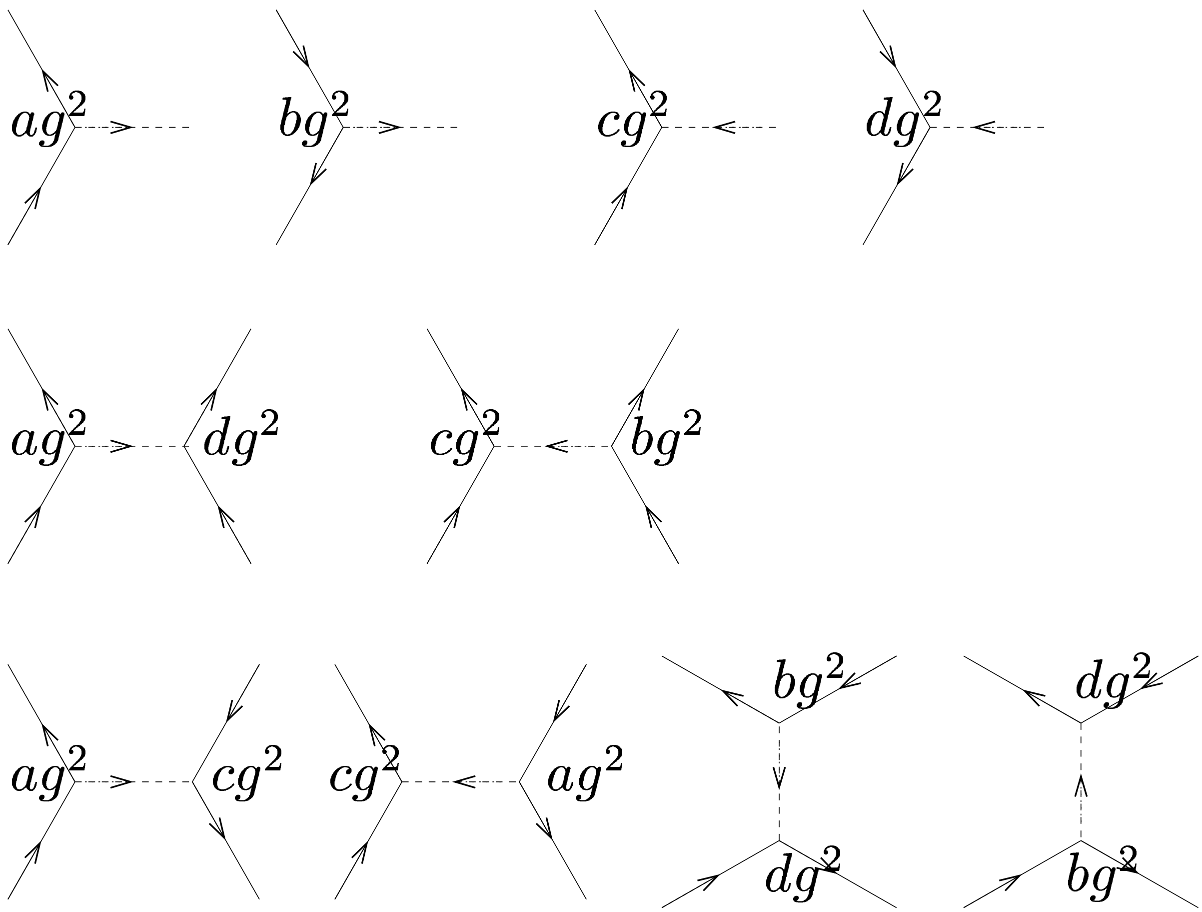


Figure 4: New cubic counterterms, with a spurion (dashed line) coupling to two gluons.

- Vertex insertion: $p_i^+\partial r/\partial\sigma$ on the spurion propagator near the interaction point. Because $r^k = 0$ on all worldsheet boundaries, ws average vanishes, unless another such factor is on the same time slice.

- Only contact contributions remain. We find:

$$\begin{aligned} \Gamma_{spur}^{\wedge\wedge\vee\vee} &= -(ad+bc)g^4 \\ \Gamma_{spur}^{\wedge\vee\wedge\vee} &= -2(ac+bd)g^4 \end{aligned}$$

- Five parameters for two counterterms:

$$\begin{aligned} \Gamma_{\mathrm{C.T.}}^{\wedge\wedge\vee\vee} &= -(ad+bc+\xi)g^4 \\ \Gamma_{\mathrm{C.T.}}^{\wedge\vee\wedge\vee} &= -2(ac+bd-\xi)g^4 \end{aligned}$$

- For instance employing extra dimensions only to cancel the part of the anomaly due to UV artifacts requires spin independence for this part: $ad+bc = 2(ac+bd)$. Then $\xi = -(ad+bc) = -1/12\pi^2$.

15 Other Results

- Our first paper showed that this method gives the known results for the one loop helicity violating amplitudes, $\Gamma^{\wedge\wedge\wedge\wedge}$ and $\Gamma^{\wedge\wedge\wedge\vee}$.

- In our second paper we show that the ratio of 1 loop helicity conserving amplitudes calculated with these methods $\Gamma^{\wedge\wedge\vee\vee}/\Gamma^{\wedge\vee\wedge\vee}$ agrees with the known answers.

- After inclusion of bremsstrahlung all IR and collinear divergences cancel in the helicity conserving amplitudes and the net UV divergences agree with the known one loop β function. Furthermore all results are Lorentz covariant.

- The regulation of $p^+ = 0$ singularities by discretization *with no zero modes* leads in all cases to the correct answers.

- As is to be expected our brute force UV cutoff δ does introduce Lorentz violating artifacts, but they are all polynomials of the external momentum whose order matches that dictated by power counting. The counterterms that cancel them have all been determined. They modify the "bare" worldsheet action which can be seen to be worldsheet local if target space is holographically extended to 6 dimensional spacetime.