Recursive Computation of One-Loop Amplitudes in QCD

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with

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Continuous Advances in QCD – May 13th, 2006

References

[1] Z. Bern, L. J. Dixon, D. A. Kosower,
Phys. Rev. D 71, 105013 (2005) [hep-th/0501240];
Phys. Rev. D 72, 125003 (2005) [hep-ph/0505055];
Phys. Rev. D 73, 065013 (2006) [hep-ph/0507005].

[2] CFB, Z. Bern, L. J. Dixon, D. Forde, D. A. Kosower,
hep-ph/0604195.

[3] CFB, Z. Bern, L. J. Dixon, D. Forde, D. A. Kosower,
to appear.

and Darren Forde's talk

The (In)Famous Les Houches 2005 Wishlist

process wanted at NLO ($V \in \{Z, W, \gamma\}$)	background to
1. $pp \to VV +$ jet	$t\bar{t}H$, new physics
2. $pp \to H + 2$ jets	H production by vector boson fusion (VBF)
3. $pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$
7. $pp \to V + 3$ jets	new physics
8. $pp \to VVV$	SUSY trilepton

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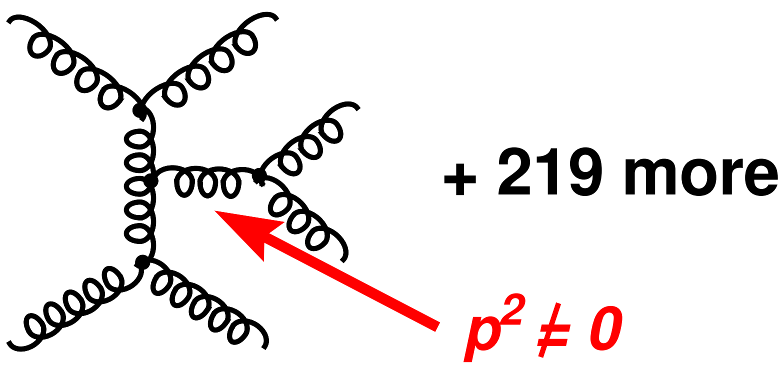
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$
7. $pp \to V + 3$ jets	new physics
8. $pp \to VVV$	SUSY trilepton

Large number of high-multiplicity processes that need to be computed!
The LHC turns on in 2007!

Feynman Graphs

- Feynman rules are too general, not optimized, do not take into account all symmetries of the theory
- Vertices and propagators involve gauge-dependent off-shell states
- In real kinematics no on-shell 3-point vertex
- Explosive growth of number of diagrams/terms



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Time to panic??

Feynman Graphs

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Time to panic?? – No!

➨ **(Semi)Numerical approaches and automatization**

MadEvent, ALPGEN, CompHEP, GRACE, HELAC/PHEGAS, . . .

Kramer, Soper, Nagy; Ellis, Giele, Glover, Zanderighi; Binoth, Ciccolini, Guillet, Heinrich, Kauer, Pilon, Schubert; Czakon; Anastasiou, Daleo; . . .

➨ **Recursion relations**

Color Ordering, Spinors and Twistors

- Strip color information, only calculate diagrams with cyclic color ordering
  $\Rightarrow$ **36 diagrams instead of 220 for $n = 6$ gluons**
- Use the "right variables" to expose more symmetries - spinor helicity formalism

$$\lambda_i = u_+(p_i) = \frac{1}{2}(1+\gamma_5)u(p_i) \qquad \tilde{\lambda}_i = u_-(p_i) = \frac{1}{2}(1-\gamma_5)u(p_i)$$

$$\langle i\, j\rangle = \langle i^-|j^+\rangle = \bar{u}_-(p_i)u_+(p_j) \quad [i\, j] = \langle i^+|j^-\rangle = \bar{u}_+(p_i)u_-(p_j)$$

Transformation to **Penrose's twistor space** ("half Fourier transform" in $\tilde{\lambda}$)
$\Rightarrow$ **amazingly simple structure of scattering amplitudes**

Witten; Nair; Roiban, Spradlin, Volovich

- "Recycle" known amplitudes via **recursion relations**

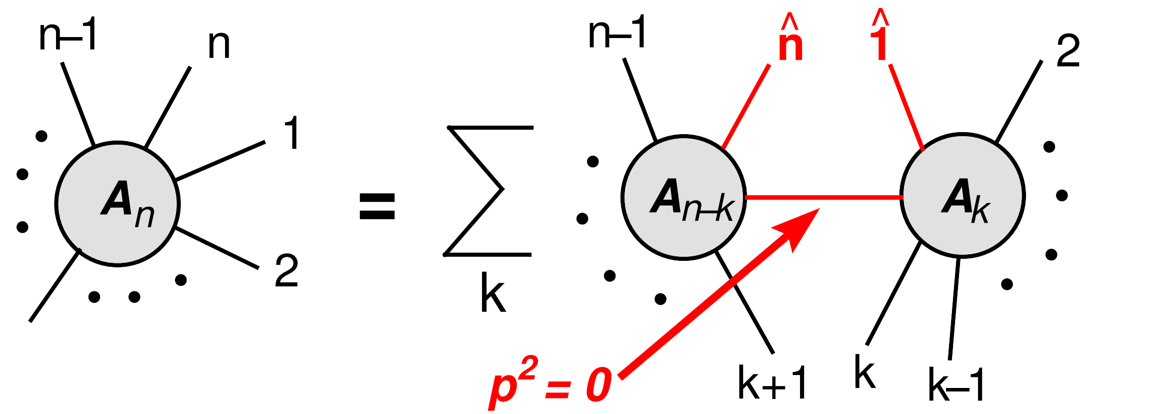
Berends, Giele; Mahlon; Cachazo, Svrcek, Witten; Britto, Cachazo, Feng, Witten

Recursion Relations at Tree Level

Complex continue spinors and momenta

(1) $$[j,l\rangle: \quad \tilde\lambda_j \to \tilde\lambda_j - z\tilde\lambda_l \qquad \lambda_l \to \lambda_l + z\lambda_j$$

(2) $$p_j^\mu \quad \to p_j^\mu(z) \equiv \hat{p}_j^\mu = p_j^\mu - \frac{z}{2}\langle j^-|\,\gamma^\mu\,|l^-\rangle$$
$$p_l^\mu \quad \to p_l^\mu(z) \equiv \hat{p}_l^\mu = p_l^\mu + \frac{z}{2}\langle j^-|\,\gamma^\mu\,|l^-\rangle$$



Britto, Cachazo, Feng

Proof at Tree-Level

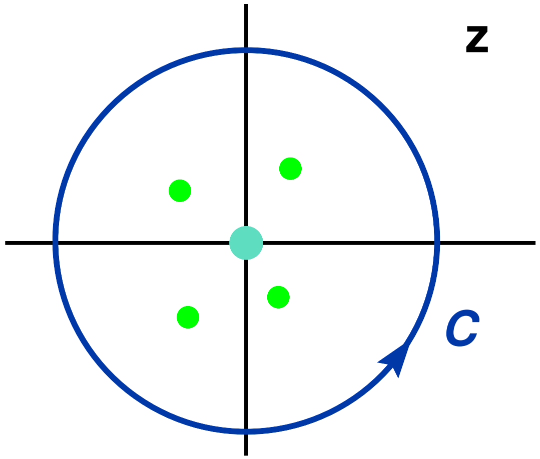
Amplitude function of complex parameter

$$A(z) = A(p_1, \ldots, p_j(z), p_{j+1}, \ldots, p_l(z), \ldots, p_n)$$

If $A(z \to \infty) \to 0$ - Cauchy's theorem

(3) $$\frac{1}{2\pi i} \oint_C \frac{dz}{z} A(z) = 0$$

(4) $$A(0) = - \sum_{\text{poles } \alpha} \operatorname*{Res}_{z=z_\alpha} \frac{A(z)}{z}$$



Poles in z correspond to physical factorizations

(5) $$\frac{1}{\hat{P}^2_{l\ldots m}} = \frac{1}{P^2_{l\ldots m} - z \langle j^- | \not{P}_{l\ldots m} | k^- \rangle}$$

Britto, Cachazo, Feng, Witten

On-Shell Recursions

Proof at tree level only relies on Cauchy's theorem and basic factorization properties.

See also: Draggiotis, Kleiss, Lazopoulos, Papadopoulos; Vaman, Yao

$\Rightarrow$ **Many applications**

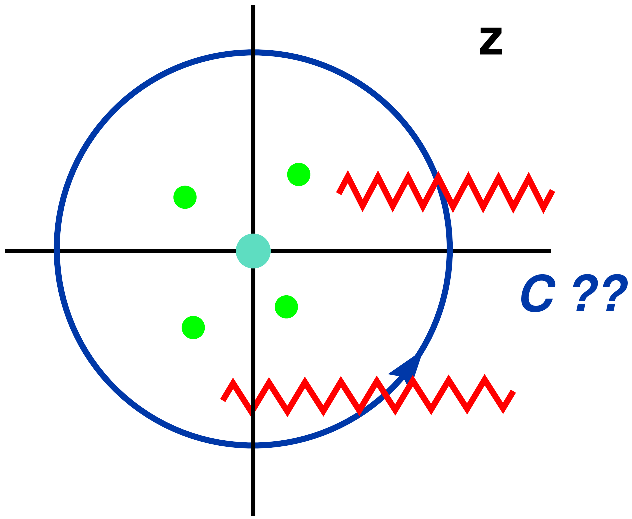
- **SUSY - processes with massless fermions** Luo, Wen
- **QCD - QCD is supersymmetric at tree level**
- **Gravity**

  Bedford, Brandhuber, Spence, Travaglini; Cachazo, Svrcek; Bjerrum-Bohr, Dunbar, Ita, Perkins, Risager
- **Massive scalars and fermions**

  Badger, Glover, Khoze, Svrcek; Forde, Kosower; Schwinn, Weinzierl; Ferrario, Rodrigo, Talavera

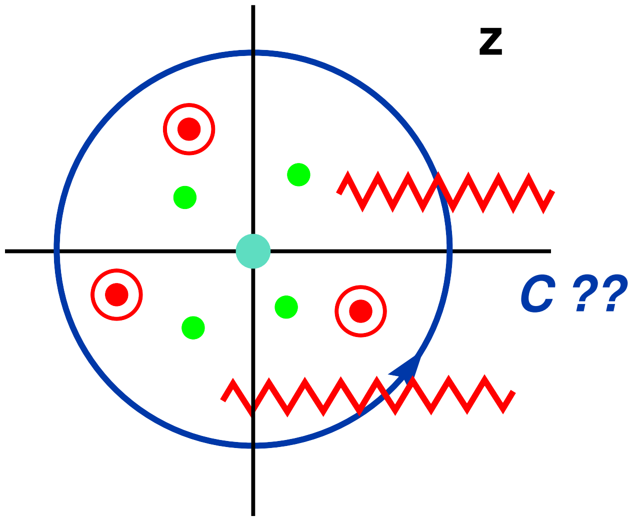
  $\Rightarrow$ **Darren Forde's talk**

QCD at One Loop - A Disaster?



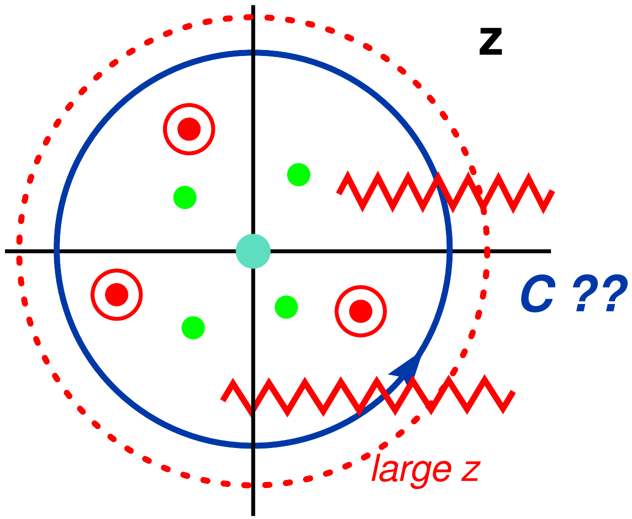
- Branch cuts with spurious singularities $\sim \frac{\ln(s_1/s_2)}{(s_1-s_2)^2}$

QCD at One Loop - A Disaster?



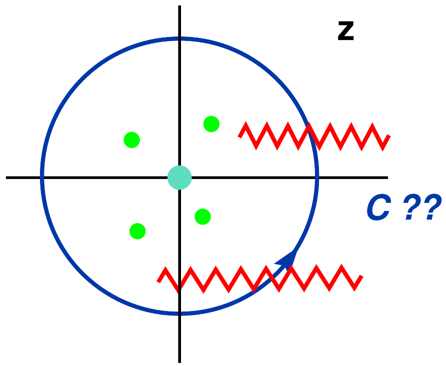
- **Branch cuts with spurious singularities** $\sim \frac{\ln(s_1/s_2)}{(s_1-s_2)^2}$
- **Double poles** $\sim \frac{\langle a\,b\rangle}{[a\,b]^2}$, **'unreal poles'** $\sim \frac{\langle a\,b\rangle}{[a\,b]}$, **and nonstandard factorizations**

QCD at One Loop - A Disaster?



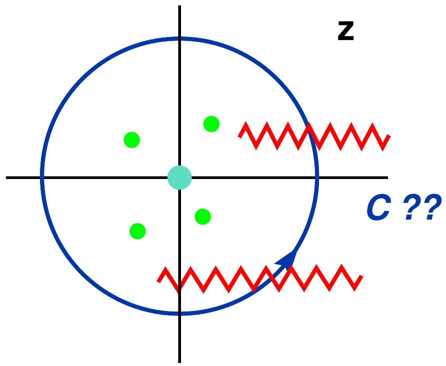
- **Branch cuts with spurious singularities** $\sim \frac{\ln(s_1/s_2)}{(s_1-s_2)^2}$
- **Double poles** $\sim \frac{\langle a\,b\rangle}{[a\,b]^2}$, **'unreal poles'** $\sim \frac{\langle a\,b\rangle}{[a\,b]}$, **and nonstandard factorizations**
- $A(z \to \infty) \neq 0$

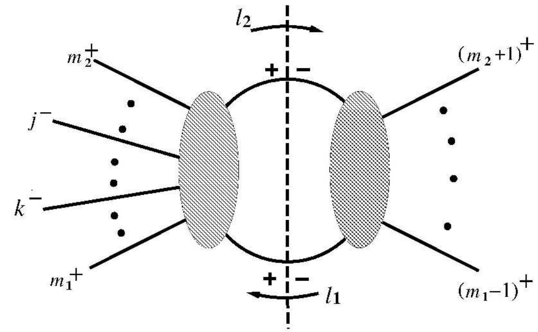
Cut Parts



$$A(z) = c_\Gamma \left[C(z) + R(z) \right] \quad (6)$$

Cut Parts





(6) $$A(z) = c_\Gamma \left[C(z) + R(z) \right]$$

$C(0)$ contains only Li, ln, π^2 – cut-constructible!

$$\int d\text{LIPS}(-l_1, l_2) A^{\text{tree}}(-l_1, m_1, .., m_2, l_2) A^{\text{tree}}(-l_2, m_2+1, .., m_1-1, l_1)$$

SUSY: $R = 0$ – fully cut-constructible via (generalized) unitarity

Bern, Dixon, Dunbar, Kosower; Bedford, Brandhuber, McNamara, Spence, Travaglini; Quigley, Rozali; Britto, Buchbinder, Cachazo, Feng, Mastrolia; Bern, Bidder, Bjerrum-Bohr, Dixon, Dunbar, Ita, Perkins

Cut Parts from Unitarity

C contains spurious singularities, $\frac{\ln(s_1/s_2)}{(s_1-s_2)^2}$. These cancel in the full amplitude. Reshuffle (= complete cut) terms between C and R,

(7) $$A(z) = c_\Gamma \left[\widehat{C}(z) + \widehat{R}(z)\right]$$

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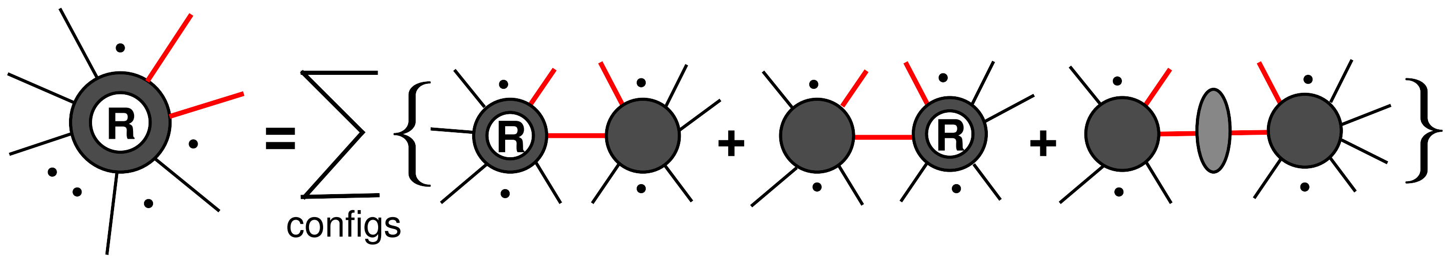
(7) $$A(z) = c_\Gamma\left[\widehat{C}(z) + \widehat{R}(z)\right]$$

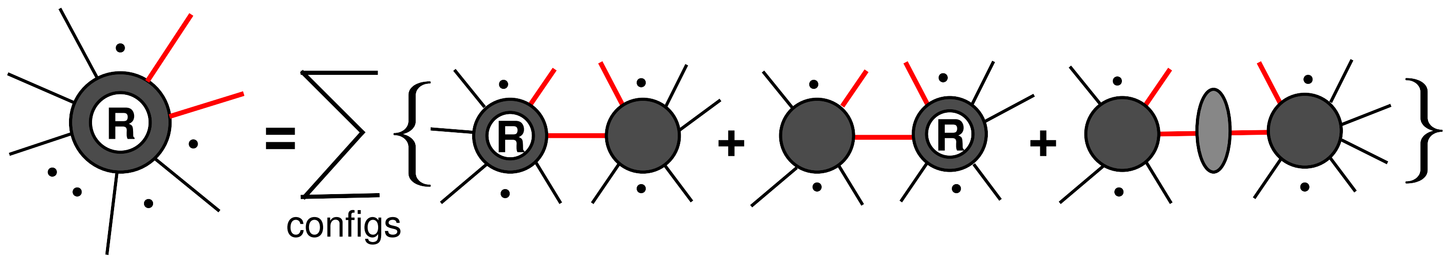
Merge unitarity with recursion

(8) $$A(0) = c_\Gamma\left[-\int \frac{dz}{z}\,\text{Disc}\,\widehat{C}(z) - \sum_{\text{poles }\beta} \mathop{\text{Res}}_{z=z_\beta} \frac{\widehat{C}(z)}{z} - \text{Inf}\,\widehat{C} - \sum_{\text{poles }\alpha} \mathop{\text{Res}}_{z=z_\alpha} \frac{\widehat{R}(z)}{z}\right]$$

$$= c_\Gamma\left[\widehat{C}(0) - \text{Inf}\,\widehat{C} - \sum_{\text{poles }\alpha} \mathop{\text{Res}}_{z=z_\alpha} \frac{\widehat{R}(z)}{z}\right]$$


R
=
configs
R
+
R
+

On-Shell Recursion for Rational Parts



$\widehat{C}$ has a rational part from **cut completion**, $\widehat{CR}$.

Recursion for rational parts over **all poles**, including those already included in $\widehat{C}$.

$\Rightarrow$ avoid double counting by subtracting off **overlap terms O** (not unique)

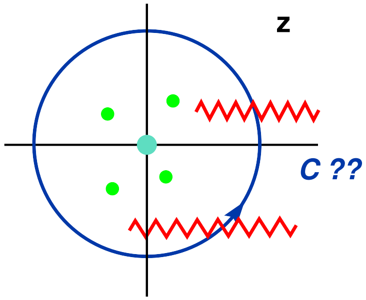
$$R = \widehat{R} + \widehat{CR} \tag{9}$$

$$-\sum_{\text{poles }\alpha} \operatorname*{Res}_{z=z_\alpha} \frac{\widehat{R}(z)}{z} = -\sum_{\text{poles }\alpha} \operatorname*{Res}_{z=z_\alpha} \frac{R(z)}{z} + \sum_{\text{poles }\alpha} \operatorname*{Res}_{z=z_\alpha} \frac{\widehat{CR}(z)}{z}$$

$$\equiv -\sum_{\text{poles }\alpha} \operatorname*{Res}_{z=z_\alpha} \frac{R(z)}{z} + O \tag{10}$$

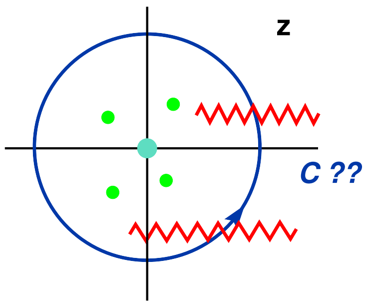
Bern, Dixon, Kosower

Non-Standard Factorizations



$$A(0) = c_\Gamma [\widehat{C}(0) - \mathrm{Inf}\,\widehat{C} - \sum_{\text{poles }\alpha} \mathop{\mathrm{Res}}_{z=z_\alpha} \frac{R(z)}{z} + O] \tag{11}$$

Non-Standard Factorizations



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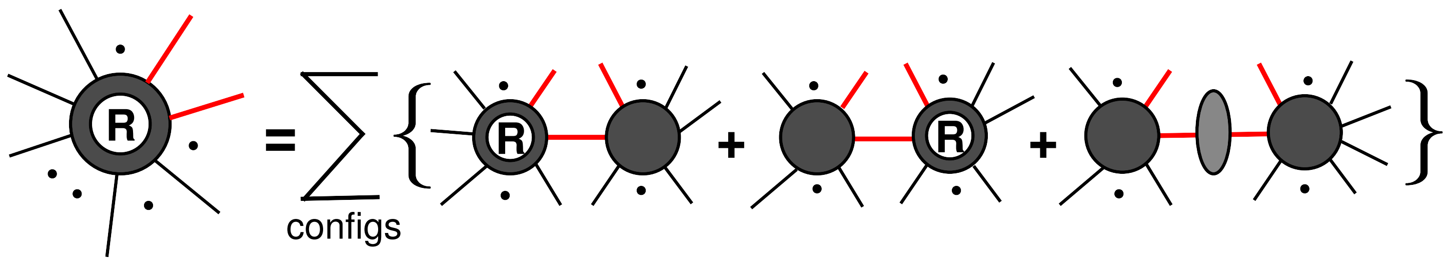
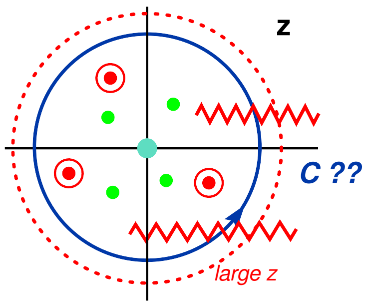


Non-Standard Factorizations



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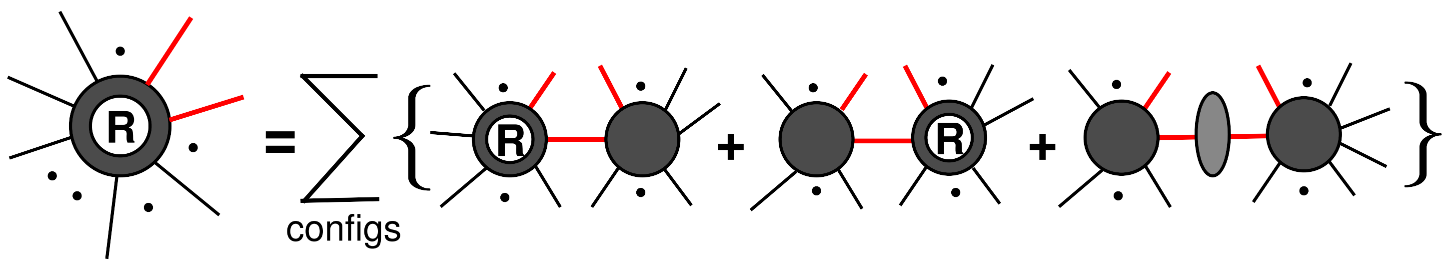
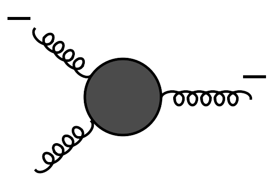
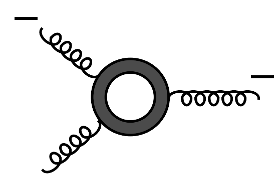
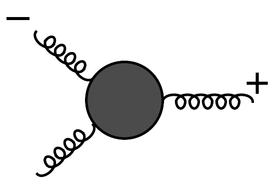




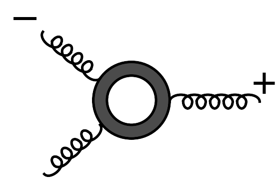
$= 0$



$\sim \dfrac{\langle a\, b\rangle}{[a\, b]^2}$



$\sim \dfrac{1}{[a\, b]}$



$\sim$ **rational** $\times \dfrac{1}{[a\, b]}$

Factorization properties unclear at one loop.

Large-z Contributions

Can pick shifts to avoid either non-standard factorizations or $z \to \infty$ contributions, but in general not both!

- $[j, l\rangle$ avoids non-standard factorizations

$$A(0) = \operatorname*{Inf}_{[j,l\rangle} A + c_\Gamma \left[\widehat{C}(0) - \operatorname*{Inf}_{[j,l\rangle} \widehat{C} + R_D^{[j,l\rangle} + O^{[j,l\rangle} \right]$$

(12)

Can pick shifts to avoid either non-standard factorizations or $z \to \infty$ contributions, **but in general not both!**

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- $[a, b\rangle$ has no large-parameter contributions

$$A(0) = c_\Gamma \Big[\widehat{C}(0) - \mathop{\mathrm{Inf}}_{[a,b\rangle} \widehat{C} + R_D^{[a,b\rangle} + \text{non-standard channels}^{[a,b\rangle} + O^{[a,b\rangle} \Big] \tag{13}$$

Solution $\Rightarrow$ use two shifts!

Extract large-parameter contributions of primary shift from auxiliary relation (13)

$$(14)\quad \operatorname*{Inf}_{[j,l\rangle} A = c_\Gamma \operatorname*{Inf}_{[j,l\rangle} \left[\widehat{C}(0) - \operatorname*{Inf}_{[a,b\rangle} \widehat{C} + R_D^{[a,b\rangle} + O^{[a,b\rangle} \right]$$

$$(15)\quad \text{if} \quad \operatorname*{Inf}_{[j,l\rangle} \left[\text{non-standard channels}^{[a,b\rangle}\right] = 0$$

The Bootstrap Formalism

Solution $\Rightarrow$ use two shifts!

Extract large-parameter contributions of primary shift from auxiliary relation (13)

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(15) **if** $\operatorname*{Inf}_{[j,l\rangle} [\text{non-standard channels}^{[a,b\rangle}] = 0$

The complete bootstrap

$$\boxed{A(0) = \operatorname*{Inf}_{[j,l\rangle} A^{[a,b\rangle} + c_\Gamma[\widehat{C}(0) - \operatorname*{Inf}_{[j,l\rangle} \widehat{C} + R_D^{[j,l\rangle} + O^{[j,l\rangle}]}$$

Passes all nontrivial checks!

CFB, Bern, Dixon, Forde, Kosower

Example: $A^{(1)}(1^-,2^-,3^-,4^+,5^+,6^+)$

(16) $$X(1,2,3,4,5,6)\Big|_{\text{flip 1}} \equiv X(3,2,1,6,5,4)$$

$$\widehat{C}_6(1^-,2^-,3^-,4^+,5^+,6^+) = \frac{1}{3c_\Gamma} A_{6;1}^{\mathcal{N}=1}(1^-,2^-,3^-,4^+,5^+,6^+)$$

(17) $$+\ \frac{2}{9} A_6^{\text{tree}}(1^-,2^-,3^-,4^+,5^+,6^+) + \hat{C}_6^a + \hat{C}_6^a\Big|_{\text{flip 1}}$$

$$\hat{C}_6^a =$$

$$\frac{i}{3}\Bigg[\frac{\langle 1\,2\rangle\langle 2\,3\rangle[2\,4]\langle 1^-|\,(3+4)\,|2^-\rangle\Big[\langle 3^-|\,42\,|1^+\rangle P_{234}^2 - \langle 3^-|\,2(3+4)\,|1^+\rangle P_{34}^2\Big]}{\langle 3\,4\rangle\,\langle 5\,6\rangle\,\langle 6\,1\rangle\,[2\,3]\,\langle 5^-|\,(3+4)\,|2^-\rangle} \frac{\mathrm{L}_2\!\left(\frac{-P_{234}^2}{-P_{34}^2}\right)}{(P_{34}^2)^3}$$

$$+\ \frac{\langle 3\,5\rangle[4\,5][5\,6]\langle 5^-|\,(1+2)\,|6^-\rangle\Big[\langle 3^-|\,(5-4)\,|6^-\rangle P_{345}^2 + \langle 3^-|\,(4+5)\,|6^-\rangle P_{34}^2\Big]}{\langle 4\,5\rangle\,[1\,2]\,[1\,6]\,\langle 5^-|\,(3+4)\,|2^-\rangle} \frac{\mathrm{L}_2\!\left(\frac{-P_{345}^2}{-P_{34}^2}\right)}{(P_{34}^2)^3}$$

$$\mathrm{L}_2(r) = \frac{\ln(r)-(r-1/r)/2}{(1-r)^3}$$

Bern, Bjerrum-Bohr, Dunbar, Ita

Example: $A^{(1)}(1^-, 2^-, 3^-, 4^+, 5^+, 6^+)$

$$X(1,2,3,4,5,6)\Big|_{\text{flip }1} \equiv X(3,2,1,6,5,4) \tag{16}$$

$$\begin{aligned}\widehat{C}_6(1^-,2^-,3^-,4^+,5^+,6^+) &= \frac{1}{3c_\Gamma} A_{6;1}^{\mathcal{N}=1}(1^-,2^-,3^-,4^+,5^+,6^+) \\ &+ \frac{2}{9} A_6^{\text{tree}}(1^-,2^-,3^-,4^+,5^+,6^+) + \hat{C}_6^a + \hat{C}_6^a\Big|_{\text{flip }1}\end{aligned} \tag{17}$$

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$$+\frac{\langle 3\,5\rangle[4\,5][5\,6]\langle 5^-|\,(1+2)\,|6^-\rangle\Big[\langle 3^-|\,(5-4)\,|6^-\rangle P_{345}^2 + \langle 3^-|\,(4+5)\,|6^-\rangle P_{34}^2\Big]}{\langle 4\,5\rangle\,[1\,2]\,[1\,6]\,\langle 5^-|\,(3+4)\,|2^-\rangle}\frac{\mathsf{L}_2\left(\frac{-P_{345}^2}{-P_{34}^2}\right)}{(P_{34}^2)^3}$$

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Bern, Bjerrum-Bohr, Dunbar, Ita

Shift $[1,2\rangle$

Example: $A^{(1)}(1^-,2^-,3^-,4^+,5^+,6^+)$ contd.

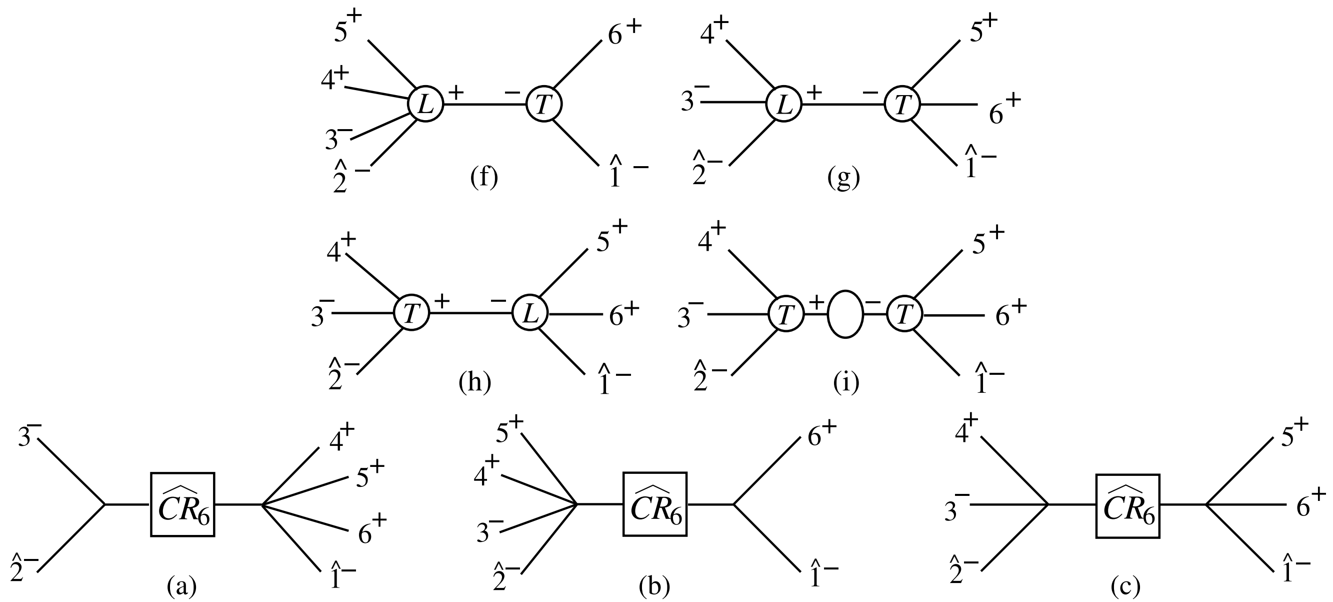
$$\Rightarrow \mathrm{Inf}_{[1,2\rangle}\, \widehat{C}_6 = \lim_{z\to\infty} \widehat{C}_6$$

Recursive and overlap contributions in channels

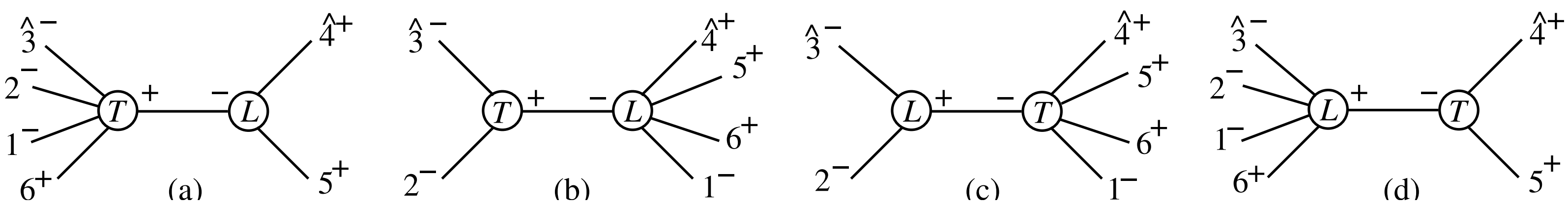
$$P_{61}^2 \quad\to\quad P_{61}^2 - z\,\langle 1^-|\,\not{P}_{61}\,|2^-\rangle \tag{18}$$

$$P_{23}^2 \quad\to\quad P_{23}^2 + z\,\langle 1^-|\,\not{P}_{23}\,|2^-\rangle \tag{19}$$

$$P_{234}^2 \quad\to\quad P_{234}^2 + z\,\langle 1^-|\,\not{P}_{234}\,|2^-\rangle \tag{20}$$



Auxiliary recursion relation for $\mathrm{Inf}_{[1,2\rangle}\, A$



$$\underset{[1,2\rangle}{\mathrm{Inf}}\; A_{6;1}(1^-,2^-,3^-,4^+,5^+,6^+) =$$

$$\underset{[1,2\rangle}{\mathrm{Inf}}\; A_{5;1}(1^-,2^-,\hat{3}^-,\hat{K}_{45}^+,6^+)\,\frac{i}{P_{45}^2}\,A_3^{\mathrm{tree}}(-\hat{K}_{45}^-,\hat{4}^+,5^+) \qquad (21)$$

Example: $A^{(1)}(1^-,2^-,3^-,4^+,5^+,6^+)$ contd.

$$\widehat{R}_6 = \widehat{R}_6^a + \widehat{R}_6^a\Big|_{\text{flip }1} \tag{22}$$

$$\begin{aligned}
\widehat{R}_6^a = \frac{i}{6}\frac{1}{[2\,3]\,\langle 5\,6\rangle\,\langle 5^-|\,(3+4)\,|2^-\rangle}\Bigg\{ & -\frac{[4\,6]^3\,[2\,5]\,\langle 5\,6\rangle}{[1\,2]\,[3\,4]\,[6\,1]} - \frac{\langle 1\,3\rangle^3\,\langle 2\,5\rangle\,[2\,3]}{\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 6\,1\rangle} \\
& + \frac{\langle 1^-|\,(2+3)\,|4^-\rangle^2}{[3\,4]\,\langle 6\,1\rangle}\left(\frac{\langle 1^-|\,2\,|4^-\rangle - \langle 1^-|\,5\,|4^-\rangle}{P_{234}^2} + \frac{\langle 1\,3\rangle}{\langle 3\,4\rangle} - \frac{[4\,6]}{[6\,1]}\right) \\
& - \frac{\langle 1\,3\rangle^2(3\,\langle 1^-|\,2\,|4^-\rangle + \langle 1^-|\,3\,|4^-\rangle)}{\langle 3\,4\rangle\,\langle 6\,1\rangle} \\
& + \frac{[4\,6]^2(3\,\langle 1^-|\,5\,|4^-\rangle + \langle 1^-|\,6\,|4^-\rangle)}{[3\,4]\,[6\,1]}\Bigg\}
\end{aligned}$$

$$\widehat{R}_6 = \widehat{R}_6^a + \widehat{R}_6^a\Big|_{\text{flip }1} \tag{22}$$

$$\begin{aligned}\widehat{R}_6^a = \frac{i}{6}\frac{1}{[2\,3]\langle 5\,6\rangle\langle 5^-|(3+4)|2^-\rangle}\Bigg\{ &-\frac{[4\,6]^3[2\,5]\langle 5\,6\rangle}{[1\,2][3\,4][6\,1]} - \frac{\langle 1\,3\rangle^3\langle 2\,5\rangle[2\,3]}{\langle 3\,4\rangle\langle 4\,5\rangle\langle 6\,1\rangle}\\ &+\frac{\langle 1^-|(2+3)|4^-\rangle^2}{[3\,4]\langle 6\,1\rangle}\left(\frac{\langle 1^-|2|4^-\rangle-\langle 1^-|5|4^-\rangle}{P_{234}^2}+\frac{\langle 1\,3\rangle}{\langle 3\,4\rangle}-\frac{[4\,6]}{[6\,1]}\right)\\ &-\frac{\langle 1\,3\rangle^2(3\langle 1^-|2|4^-\rangle+\langle 1^-|3|4^-\rangle)}{\langle 3\,4\rangle\langle 6\,1\rangle}\\ &+\frac{[4\,6]^2(3\langle 1^-|5|4^-\rangle+\langle 1^-|6|4^-\rangle)}{[3\,4][6\,1]}\Bigg\}\end{aligned}$$

$\Rightarrow$ **all-n solution** $\Rightarrow$ **Darren Forde's talk**

CFB, Bern, Dixon, Forde, Kosower

Summary and Outlook

"One of the most remarkable discoveries in elementary particle physics has been that of the existence of the complex plane." **in J. Schwinger, "Particles, Sources, and Fields", Vol. I.**

Summary and Outlook

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process wanted at NLO ($V \in \{Z, W, \gamma\}$)	background to
1. $pp \to VV +$ jet	$t\bar{t}H$, new physics
2. $pp \to H + 2$ jets	H production by vector boson fusion (VBF)
3. $pp \to t\bar{t}b\bar{b}$	$t\bar{t}H$
4. $pp \to t\bar{t} + 2$ jets	$t\bar{t}H$
5. $pp \to VVb\bar{b}$	VBF $\to H \to VV$, $t\bar{t}H$, new physics
6. $pp \to VV + 2$ jets	VBF $\to H \to VV$
7. $pp \to V + 3$ jets	new physics
8. $pp \to VVV$	SUSY trilepton

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Run II Monte Carlo Workshop 2001

Single boson	Diboson	Triboson	Heavy flavor
$W+\leq 5j$	$WW+\leq 5j$	$WWW+\leq 3j$	$t\bar{t}+\leq 3j$
$W+b\bar{b}+\leq 3j$	$WW+b\bar{b}+\leq 3j$	$WWW+b\bar{b}+\leq 3j$	$t\bar{t}+\gamma+\leq 2j$
$W+c\bar{c}+\leq 3j$	$WW+c\bar{c}+\leq 3j$	$WWW+\gamma\gamma+\leq 3j$	$t\bar{t}+W+\leq 2j$
$Z+\leq 5j$	$ZZ+\leq 5j$	$Z\gamma\gamma+\leq 3j$	$t\bar{t}+Z+\leq 2j$
$Z+b\bar{b}+\leq 3j$	$ZZ+b\bar{b}+\leq 3j$	$WZZ+\leq 3j$	$t\bar{t}+H+\leq 2j$
$Z+c\bar{c}+\leq 3j$	$ZZ+c\bar{c}+\leq 3j$	$ZZZ+\leq 3j$	$t\bar{b}+\leq 2j$
$\gamma+\leq 5j$	$\gamma\gamma+\leq 5j$		$tb\bar{b}+\leq 3j$
$\gamma+b\bar{b}+\leq 3j$	$\gamma\gamma+b\bar{b}+\leq 3j$		
$\gamma+c\bar{c}+\leq 3j$	$\gamma\gamma+c\bar{c}+\leq 3j$		
	$WZ+\leq 5j$		
	$WZ+b\bar{b}+\leq 3j$		
	$WZ+c\bar{c}+\leq 3j$		
	$W\gamma+\leq 3j$		
	$Z\gamma+\leq 3j$		

Summary and Outlook

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$\Rightarrow$ **Darren Forde's talk**