Some issues of AdS/QCD

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Plan of the talk

- Review of holographic models of QCD
- The $m_n^2 \sim n$ growth: semiclassical argument
- Holographic model with improved infrared behavior

Reference:
A. Karch, E. Katz, D.T.S, M. Stephanov, hep-ph/0602229

Holographic modeling of QCD

See Stephanov's talk

- A bottom-up approach, inspired by the AdS/CFT correspondence, but also by the phenomenological "hidden local symmetry"
- 5d Lagrangian in AdS space, with a cutoff to model confinement
- Hadrons $=$ normalizable modes: $\Box_{\text{5D}}\psi_n = m_n^2\psi_n$, eigenvalues $=$ masses square
- Decay constants $\langle 0|J_\mu|n\rangle = f_n\epsilon_\mu$,

$$f_n = \lim_{z\to 0}\frac{\psi_n'(z)}{z}$$

- Formfactors, meson coupling constants $=$ overlap integrals of wave functions

Good features of holographic models

- Automatic implementation of quark-hadron duality
  - At large Euclidean Q^2: parton-model logs, power corrections can be modeled (also: Hirn's talk)
  - At the same time, correlations functions $=$ sum over poles: from spectral representation of 5D Green's functions

$$\Pi(Q^2) = \sum_n \frac{f_n^2}{m_n^2} \frac{1}{Q^2 + m_n^2}$$

  - With enough terms in 5D Lagrangian, it seems that a lot of OPE relations can be satisifed.
- Phenomelogically good for low-masses resonances

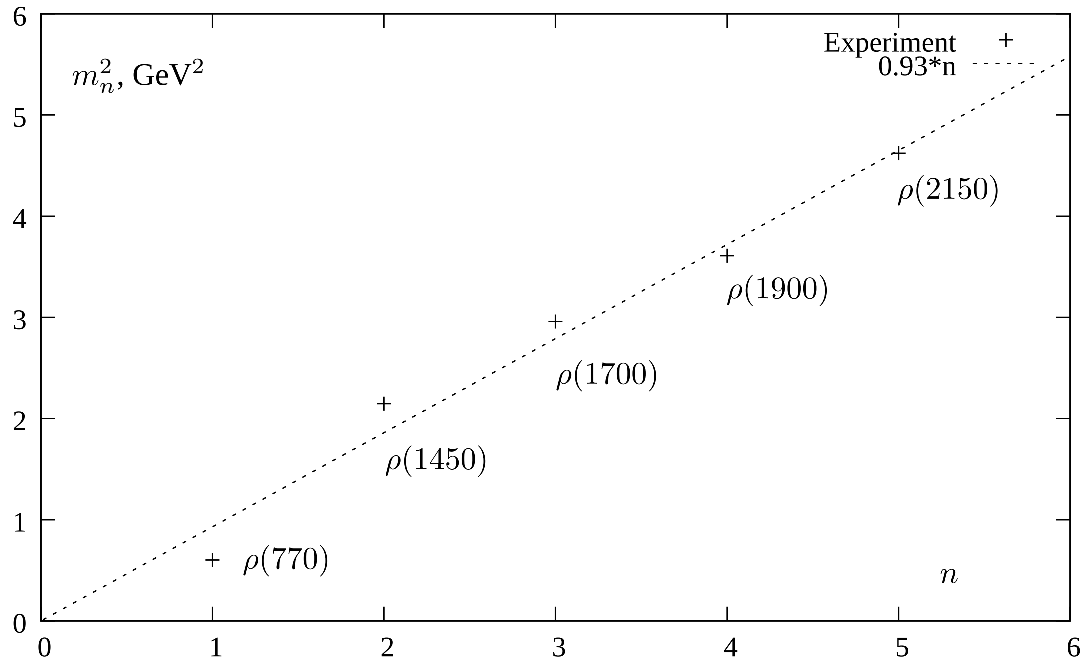
What is bad

- Only a model: but not worse than many other models
- Not unique: higher-order terms in 5D Lagrangian, background...: We want to reproduce qualitative and semiquantitative features of hadronic physics
- In one place it fails even qualitatively: resonsance mass in a tower grows two fast: $m_n^2 \sim n^2$ (emphasized by M. Shifman last year)

We shall see that the last issue is not a generic unavoidable feature of AdS/QCD modeling.

Why $m_n \sim n^{1/2}$ is preferred?

- Semiclassical quantization of a radially oscillating flux tube: maximal length l, $p \sim l$, $x \sim l$, $px \sim l^2 \sim n$
- The behavior observed in 't Hooft model
- Phenomelogically works much better than $m_n \sim n$:



- The semiclassical argument ignores transverse excitations of the (fat) flux tube, thus one should expect much more large-mass states then $m_n^2 \sim n$ would predict. We cannot see that from present data.

The standard string theory story

- In confining theory with gravity duals, stringy modes are much more massive than supergravity modes: $m_{\rm string} \sim (g^2 N)^{1/4} m_\rho$, $g^2 N \gg 1$
- So very high in the spectrum one should see the stringy behavior. Problem: in real world $m_n^2 \sim n$ starts right away
- The problem should go away when we know how to treat the regime $g^2 N \sim 1$: strings in highly curved space. Be patient!
- However simple 5D field-theory models provide such a simple way to interpolate between the IR and the UV of QCD. I don't know how to compute decay constants of low-spin highly excited strings (equivalence of $f_n = \psi_n'(z)/z|_{z\to 0}$.)
- So it may be useful to decouple the AdS/QCD approach from the string theory story for a moment, and try to see what can be done in in 5D field theory to get $m_n^2 \sim n$

A Schrödinger equation analogy

Consider a warped metric of the form

$$ds^2 = e^{2A(z)}(dz^2 + dx^\mu dx_\mu)$$

Consider a vector field propagating in ths metric and in background scalar field $\Phi(z)$:

$$I = \int d^5x \sqrt{g}\, e^{-\Phi} \mathcal{L}$$

The equation for the normalizable mode, after a change of variable, becomes a Schrödinger equation:

$$-\psi''(z) + V(z)\psi(z) = m^2\psi(z), \quad V(z) = \frac{1}{4}(B')^2 - \frac{1}{2}B'', \quad B(z) = \Phi(z) - A(z)$$

Cutoff AdS: $B(z) = \ln z$:

$$V(z) = \frac{3}{4z^2}\,, \qquad 0 < z < z_m$$

For high excitations: practially a cavity $(0, z_m)$

A better IR cutoff

The reason for the behavior $m_n \sim n$ is in the finiteness of the cavity. How to correct the problem?

- We don't want to change the small z behavior: guarantees the correct large Euclidean Q^2 behavior of correlators
- We can on the other hand make the behavior at large z less drastic than a hard wall
- $m_n^2 \sim n$ fixes the asymptotic behavior: $V(z) \sim z^2$ as $z \to \infty$
- This implies $B(z) \sim z^2$ at large z

Minimal model is then: $B(z) = z^2 + \ln z$, corresponding to

$$V(z) = z^2 + \frac{3}{4z^2}$$

Schrödinger equation solved exactly, solution expressed in Laguerre polynomials.

$$m_n^2 = 4(n+1), \qquad f_n^2 = \frac{8}{g_5^2}(n+1)$$

The growth of m_n and f_n is what needed to reproduce parton model log

A lesson

- A lesson is that the masses of highly excited mesons is an IR property, not an UV property
- Qualitative explanation: high mass, but large size
- The UV behavior of the theory is still determined by the AdS metric near $z = 0$
- Precise form of spectrum depends on background, but the large-n asymptotics depends only on large z behavior
- $e^{-B} = e^{-z^2}/z^2$ has $1/Q^2$ power corrections, but this can be avoided by a more sophisticated choice (e.g., $e^{z^2} \to \cosh(z^2)$).

Higher spin mesons

We introduce fields corresponding to operator of spin S: $\phi_{M_1 \ldots M_S}$ (symmetric traceless)

$$I = \frac{1}{2}\int d^5x\,\sqrt{g}\,e^{-\Phi}\left\{\nabla_N \phi_{M_1 \ldots M_S}\nabla^N \phi^{M_1 \ldots M_S} + M^2(z)\phi^2 + \cdots\right\}$$

Requires invariance under

$$\phi_{M_1 \ldots M_s} \to \phi_{M_1 \ldots M_s} + \nabla_{(M1}\xi_{M_2 \ldots M_S)}$$

Field equation becomes Schrödinger equation with potential

$$V(z) = \frac{1}{4}(B_S')^2 - \frac{1}{2}B_S'', \qquad B_S = \Phi - (2S-1)A$$

$m_{n,S}^2$ grows linearly with n if $B_S \sim z^2$ at large z

Higher spin mesons (continued)

To keep the slope the same for all spins we have to chose $A = -\ln z$, $\Phi = z^2$: purely AdS space, cutoff done by a scalar field.

$$V(z) = z^2 + 2(S-1) + \frac{S^2 - 1/4}{z^2}$$

Spectrum:

$$m^2_{n,S} = 4(n+S)$$

matches the semiclassical picture. Parallel Regge trajectories.
The operator $O^{\cdots}$ that couples to $\phi_{\ldots}$ has dimension $2+S$: twist-2 operators.

But only one field per spin: no Hagedorn behavior.

Axial vector meson

The split between the masses of axial vectors a_1's from vectors ρ's comes from a 5D Higgs mechanism.

$$\partial_z(e^{A-\Phi}\partial_z a_n) + (m_n^2 - g_5^2 e^{2A} X) e^{A-\Phi} a_n = 0$$

Asymptotics of X:

$$X(z) \stackrel{z\to 0}{\longrightarrow} \frac{1}{2} M z + \frac{1}{2} \Sigma z^3,$$

but the asymptotics at $z \to \infty$ depends on the potential in the Lagrangian for X.
If we assume $X(z) \to X_\infty = \text{const}$ when $z \to \infty$, then in the Schrödinger equation

$$V(z) = z^2 + \frac{g_5^2 X(z) + 3/4}{z^2}$$

we modify the potential only at small z (short distances): still a linear growth of $m_{n,A}^2$:

$$m_{n,A}^2 = m_{n_V}^2 + O(n^{-1/2})$$

Summary

- The asymptotic growth of masses of highly excited mesons is an IR issue, not a UV issue
- There exist a 5D field-theoretical framework that yield $m_n^2 \sim n$,
- At the same time, the deep Euclidean region is not touched by a modification of the background in the IR: still AdS space when $z \to 0$
- High spin fields: can obtain $m_{n,S}^2 \sim n + S$ naturally
- Not a picture expected from string-theory, but nothing wrong as a model interpolating the IR and the UV in QCD

Further things to understand

- Relationship of this picture to semiclassical picture of flux tubes
  - Not immediately clear: semiclassical motion in fifth dimension in one picture and in 3d space in another picture
  - The size of highly excited states from form factors: computable from formfactor
  - Minimal model has size growing as $\ln n$, any higer-dimensional term gives $n^{1/2}$.
- Structure functions: have twist-2 operators, need to know coupling of high-spin fields to low spins ones