Holographic Wave Functions, Meromorphization and Counting Rules

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Hadronic form factors: $(1/Q^2)^{n_q - 1}$ counting rules

Expectation: some fundamental/easily visible reason

Most natural suspect: scale invariance

Implementation: hard exchange in a theory with
dimensionless coupling constant.

QCD: $(\alpha_s/Q^2)^{n_q - 1}$

Suppression: $F_\pi(Q^2) = (2\alpha_s/\pi)s_0/Q^2 \quad \left[s_0 = 4\pi^2 f_\pi^2 \approx 0.7\,\mathrm{GeV}^2\right]$

Looks like $\mathcal{O}(\alpha_s)$ correction to VMD's $F_\pi^{\mathrm{VMD}}(Q^2) \sim 1/(1 + Q^2/m_\rho^2)$

Known: $\alpha_s/\pi \sim 0.1$ is penalty for an extra loop

Growing consensus: pQCD gives small correction,
dominant contribution comes from soft terms modeled by
GPDs $\mathcal{F}(x, Q^2)$ with exponential fall-off $e^{-Q^2 g(x)}$ for fixed x

AdS/CFT claims: nonperturbative explanation of quark counting rules

Reason: conformal invariance & short-distance behavior of normalizable modes $\Phi(\zeta)$

Form factor in AdS/CFT:

$$F(Q^2) = \int_0^{1/\Lambda} \frac{d\zeta}{\zeta^3} \, \Phi_{P'}(\zeta) J(Q,\zeta) \Phi_P(\zeta)$$

Nonnormalizable mode: $J(Q,\zeta) = \zeta Q K_1(\zeta Q) \equiv \mathcal{K}_1(\zeta Q)$

For large Q: $\mathcal{K}_1(\zeta Q) \sim e^{-\zeta Q} \Rightarrow$ only small $\zeta \lesssim 1/Q$ are important

Normalizable modes: $\Phi(\zeta) = C \zeta^2 J_L(\beta_{L,k} \zeta \Lambda)$

In light-cone formalism:

$$F_{(2)}(Q^2) = \int_0^1 dx \int d^2\mathbf{b}_\perp e^{i(1-x)\mathbf{b}_\perp\cdot\mathbf{q}_\perp} \left|\Psi_2(x,\mathbf{b}_\perp)\right|^2$$

$\Rightarrow$

$$2\pi \int_0^1 dx \int_0^\infty b db J_0(\bar{x} b Q) |\Psi_2(x,b)|^2$$

$\Rightarrow$

$$2\pi \int_0^1 \frac{dx}{x\bar{x}} \int_0^\infty z dz J_0\left(\sqrt{\frac{\bar{x}}{x}}\, z\, Q\right) |\phi(x,z)|^2$$

$\bar{x} \equiv 1 - x$, $z \equiv \sqrt{x\bar{x}} b$

Observation: $\int_0^1 dx J_0\left(\sqrt{\frac{\bar{x}}{x}}\, z\, Q\right) = \mathcal{K}_1(zQ)$

Need: $|\phi(x,z)|^2 = x\bar{x}\chi^2(z) = x\bar{x}\Phi^2(\zeta)/\zeta^4$

"SJB/GdT" correspondence $\Rightarrow$ Holographic LFWF

Normalized to 1:

$$\int_0^1 dx \int d^2\mathbf{b}_\perp |\Psi_{\text{eff}}(x, \mathbf{b}_\perp)|^2 \;= 1$$

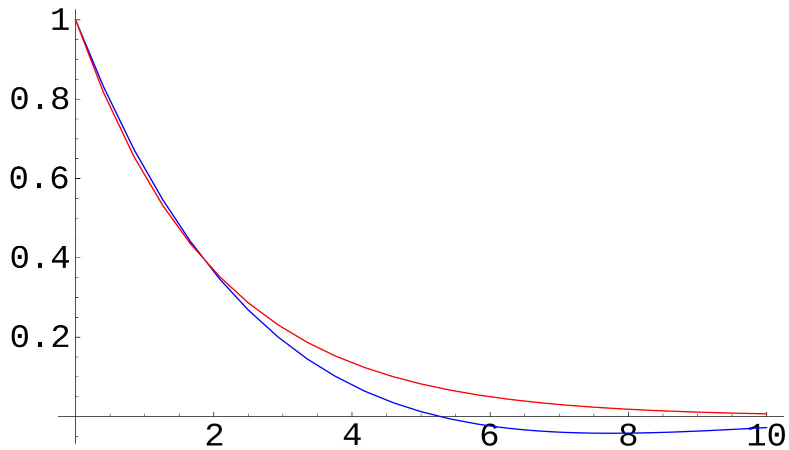
$\Rightarrow$ Effective wave functions

Lowest meson state: $M = \beta_{0,1}\Lambda$

$$\Psi_M(x, b) = \frac{M\sqrt{x\bar{x}/\pi}}{\beta_{0,1} J_1(\beta_{0,1})}\, J_0(\sqrt{x\bar{x}}Mb)\,\theta(\sqrt{x\bar{x}}b \le \beta_{0,1}/M)$$

In $\mathbf{k}_\perp$-representation:

$$\widetilde{\Psi}_M(x, \mathbf{k}_\perp) = \frac{M}{\sqrt{\pi x \bar{x}}} \frac{J_0(\beta_{0,1} k_\perp / \sqrt{x\bar{x}} M)}{M^2 - k_\perp^2 / x\bar{x}}$$



Note: No singularity for $k_\perp^2 / x\bar{x} = M^2$,

zeros when $k_\perp^2 / x\bar{x} = (\text{mass})^2$ of higher state

Oscillates for large $k_\perp$, magnitude decreases as $1/k_\perp^{5/2}$

Below first zero: $\sim \exp(-k_\perp^2 / 2x\bar{x}M^2)$

Lowest state form factor:

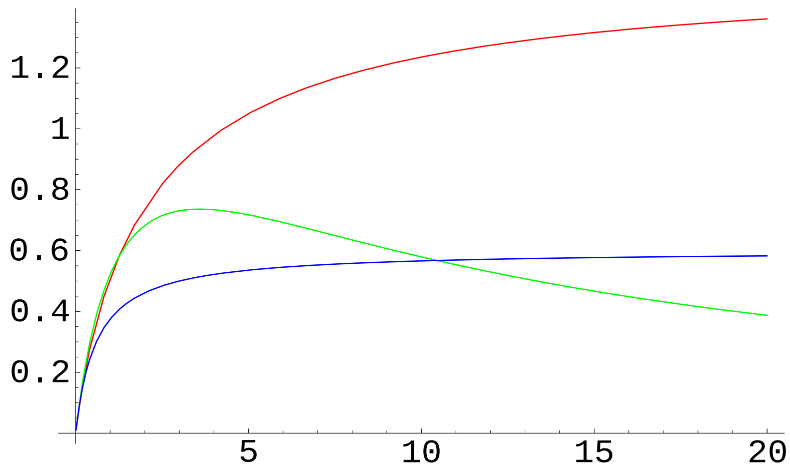
$$F_M(Q^2) = \frac{2M^2}{Q^2\gamma^2} \int_0^{Q/\Lambda} \xi \, d\xi \, \mathcal{K}_1(\xi) \, J_0^2(\xi M/Q)$$

$(\gamma \equiv \beta_{0,1} J_1(\beta_{0,1}) \approx 1.2)$

Asymptotic behavior:

$$F_M(Q^2) = \frac{4M^2}{Q^2\gamma^2} \left[1 - \frac{4M^2}{Q^2} + \frac{9}{8} \left(\frac{4M^2}{Q^2} \right)^2 + \mathcal{O}(M^6/Q^6) \right]$$

$$\sim \frac{0.64}{1 + Q^2/4M^2}$$



Note: Unlike VMD $Q^2/(1+Q^2/m_\rho^2)$ (blue),

$Q^2 F_M(Q^2)$ (red) is not constant

in accessible region $Q^2 \lesssim 10\,\mathrm{GeV}^2$

Question: What mechanism generates these huge corrections?

Observation: if ζ is interpreted as $\sqrt{x\bar{x}}b$,

then $\zeta = 0$ may correspond to $x = 1$

Use Drell-Yan formula:

$$F_M(Q^2) = \int_0^1 dx \int d^2\mathbf{k}_\perp \, \widetilde{\Psi}^*_M(x, \mathbf{k}_\perp + \bar{x}\mathbf{q}_\perp)\widetilde{\Psi}_M(x, \mathbf{k}_\perp)$$

Two possible asymptotic regimes:

- finite x and small $|\mathbf{k}_\perp|$, e.g., region $|\mathbf{k}_\perp| \ll \bar{x}|\mathbf{q}_\perp|$, where $\widetilde{\Psi}_M(x, \mathbf{k}_\perp)$ is maximal. Then

$$F_M(Q^2) \sim 2 \int_0^1 dx \, |\widetilde{\Psi}^*_M(x, \bar{x}\mathbf{q}_\perp) \, \varphi(x)|$$

$\Rightarrow$ form factor repeats large-$\mathbf{k}_\perp$ behavior of WF

$\Rightarrow$ second possibility:

- x is close to 1, so that $|\bar{x}\mathbf{q}_\perp| \sim |\mathbf{k}_\perp|$, and $|\mathbf{k}_\perp|$ is small

$\Rightarrow$ both $\widetilde{\Psi}_M(x, \mathbf{k}_\perp)$ and $\Psi^*_M(x, \mathbf{k}_\perp + \bar{x}\mathbf{q}_\perp)$ are maximal.

DY estimate: dominant contribution comes from

$$\bar{x}|\mathbf{q}_\perp| \lesssim m = \mathrm{const}$$

$\Rightarrow$ large-Q^2 behavior of form factor reflects phase space available for such configurations

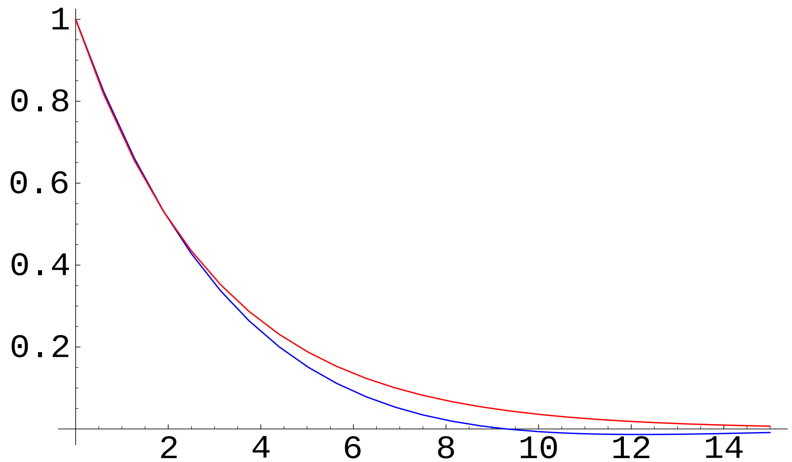
Dedicated estimate: represent form factor as x-integral of GPD

$$\mathcal{F}_M(x, Q^2) = \frac{2}{\beta_{0,1}^2 J_1^2(\beta_{0,1})} \int_0^{\beta_{0,1}} y dy \, J_0\left(\sqrt{\frac{\bar{x}}{x}} \frac{Q}{M} \, y\right) J_0^2(y)$$

$$\equiv \mathcal{G}(\sigma) \quad , \quad \sigma \equiv \bar{x}Q^2/xM^2$$

$$F_M(Q^2) = \frac{M^2}{Q^2} \int_0^\infty d\sigma \, \frac{\mathcal{G}(\sigma)}{(1 + \sigma M^2/Q^2)^2}$$

$\Rightarrow$ Asymptotic $1/Q^2$ term given by 0^{th} σ-moment of $\mathcal{G}(\sigma)$



Blue line: $\mathcal{G}(\sigma)$. Red line: $e^{-\sigma/3} = e^{-\bar{x}Q^2/3xM^2}$

$\Rightarrow$ comes from $\sigma \lesssim 10 \Rightarrow \bar{x} \lesssim 10M^2/Q^2$

Note: Asymptotic estimate applicable only if $Q^2 \gg 4M^2$

Conclusion: large-Q^2 asymptotics of $F_M(Q^2)$ is governed by soft Feynman mechanism

$\Rightarrow$ Power law is determined by $x \to 1$ behavior of

$$f(x) = \mathcal{F}(x, Q^2 = 0) = \int d^2\mathbf{b}_\perp \, |\Psi(x, \mathbf{b}_\perp)|^2 = 1 \ ,$$

$f(x)$ = parton distribution function of the model

Extra $\bar{x}^N$ $\Rightarrow F_M(Q^2) \sim (\Lambda^2/Q^2)^{N+1}$

Question: Is interpretation of

holographic variable ζ as product

$\sqrt{x\bar{x}}b$ of light-cone variables justified?

MEROMORPHIZATION

Erlich et al.: some results of holographic approach are reproduced in Migdal's program

Meromorphization substitutes correlator

$$\Pi(p^2) = \frac{1}{\pi}\int_0^\infty \frac{\rho(s)}{s-p^2}\, ds$$

by

$$\Pi_{\mathcal{M}}(p^2) = \Pi(p^2) - \frac{1}{\pi \mathcal{Q}(p^2)}\int_0^\infty \frac{\rho(s)\,\mathcal{Q}(s)}{s-p^2}\, ds$$

$\Rightarrow$ cut of $\Pi(p^2)$ is eliminated

zeros of $\mathcal{Q}(p^2)$ $\Rightarrow$ poles of $\Pi_{\mathcal{M}}(p^2)$

$\Rightarrow$ hadronic bound states

Explicit Padé construction: $\mathcal{Q}(p^2) \Rightarrow J_0(\beta_{0,1}\sqrt{p^2}/M)$

for $j = \varphi\varphi$ and $j_\mu = \bar\psi\gamma_\mu\psi$, $j_{5\mu} = \bar\psi\gamma_5\gamma_\mu\psi$

Coupling of the lowest state

$$f_M^2 = \lim_{p^2 \to M^2} (M^2 - p^2)\, \Pi_{\mathcal{M}}(p^2) = \frac{1}{\pi \mathcal{Q}'(M^2)} \int_0^\infty \frac{\rho(s)\, \mathcal{Q}(s)}{s - M^2}\, ds$$

Explicit calculation with $\rho(s) = \rho_0\, \theta(s)$

$$f_M^2 = \frac{2\rho_0 M^2}{\pi \beta_{0,1} J_1(\beta_{0,1})} \int_0^\infty \frac{J_0(\beta_{0,1}\sqrt{s}/M)}{M^2 - s}\, ds = \frac{4\rho_0 M^2}{\pi [\beta_{0,1} J_1(\beta_{0,1})]^2}$$

$\rho_0 = 1/16\pi$ for $j = \varphi\varphi$, and $\rho_0 = N_c/12\pi$ for $\bar{u}\gamma_\mu(\gamma_5)d$

Dosch et al.: Meromorphize 3-point function to get form factors

Lowest order: triangle diagram has only double spectral density $\rho(s_1, s_2, Q^2) \Rightarrow$ build function

$$\mathcal{T}(p_1^2, p_2^2, Q^2) = T(p_1^2, p_2^2, Q^2)$$

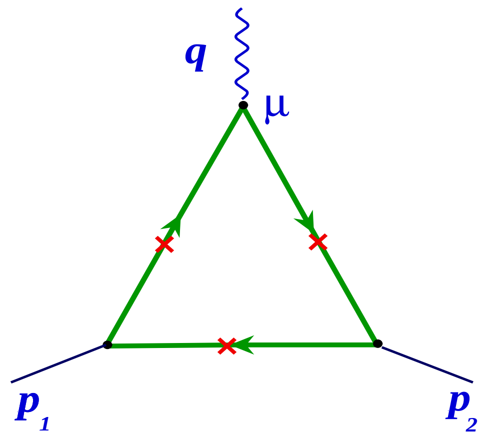
$$+\frac{1}{\pi^2 \mathcal{Q}(p_1^2)\mathcal{Q}(p_2^2)} \int_0^\infty ds_1 \int_0^\infty ds_2 \, \frac{\rho(s_1, s_2, Q^2)\, \mathcal{Q}(s_1)\, \mathcal{Q}(s_2)}{(s_1 - p_1^2)(s_2 - p_2^2)}$$

$\Rightarrow$ removes cuts & has poles at $\Pi(p^2)$ locations

Elastic form factor of the lowest state:

$$f_M^2 F_M(Q^2) = \frac{1}{\pi^2 [\mathcal{Q}'(M^2)]^2} \int_0^\infty ds_1 \int_0^\infty ds_2$$

$$\times \frac{\rho(s_1, s_2, Q^2)\, \mathcal{Q}(s_1)\, \mathcal{Q}(s_2)}{(s_1 - M^2)(s_2 - M^2)}$$



Spectral densities: use Cutkosky rules

$$\rho(s_1, s_2, Q^2) = \rho_0 \int_0^1 dx\, \frac{n(x)}{x\bar{x}} \int d^2\mathbf{k}_\perp$$

$$\times\delta\left(s_1 - \frac{\mathbf{k}_\perp^2}{x\bar{x}}\right)\delta\left(s_2 - \frac{(k_\perp + \bar{x}\mathbf{q}_\perp)^2}{x\bar{x}}\right)$$

$n(x) = 1$ for $j = \varphi\varphi\,,\ j^\mu = i\varphi\,\overleftrightarrow{\partial^\mu}\,\varphi$

Form factor:

$$f_M^2 F_M^{\rm scalar}(Q^2) = \frac{\rho_0}{\pi^2[\mathcal{Q}'(M^2)]^2} \int_0^1 \frac{dx}{x\bar{x}} \int d^2\mathbf{k}_\perp$$
$$\times \frac{\mathcal{Q}(\mathbf{k}_\perp^2/x\bar{x})}{M^2 - \mathbf{k}_\perp^2/x\bar{x}} \; \frac{\mathcal{Q}((\mathbf{k}_\perp + \bar{x}\mathbf{q}_\perp)^2/x\bar{x})}{M^2 - (\mathbf{k}_\perp + \bar{x}\mathbf{q}_\perp)^2/x\bar{x}}$$

$\Rightarrow$ has structure of Drell-Yan formula

Using $\mathcal{Q}(s) = J_0(\beta_{0,1}\sqrt{s}/M)$ and

$$f_M^2 = \frac{4\rho_0 M^2}{\pi[\beta_{0,1} J_1(\beta_{0,1})]^2}$$

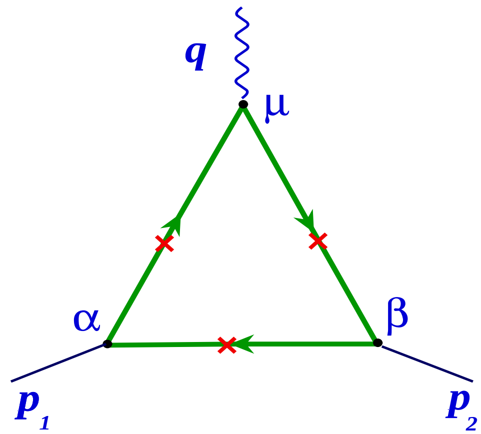
gives

$$\widetilde{\Psi}_M^{\text{scalar}}(x, \mathbf{k}_\perp) = \frac{M}{\sqrt{\pi x \bar{x}}} \frac{J_0(\beta_{0,1} k_\perp / \sqrt{x\bar{x}} M)}{M^2 - k_\perp^2 / x\bar{x}}$$

$\Rightarrow$ Same expression as in holographic model

$\Rightarrow$

Meromorphization supports interpretation of ζ
in terms of light-cone variables x, b



Spinor case: vector currents j_α, j_β for hadronized vertices

$\Rightarrow$ tensor amplitude $T^{\mu}_{\alpha\beta}(p_1, p_2)$

$\Rightarrow$ choose tensor structure to meromorphize

Simplest projection: $T^{\mu}_{\alpha\beta} n_\mu n^\alpha n^\beta$, with $n^2 = 0$, $(nq) = 0$

picks out $F_1(Q^2) + \kappa F_2(Q^2) - \kappa^2 F_3(Q^2)$, $\kappa \equiv Q^2/2m_\rho^2$

Compare to leading pQCD:

$F_{LL}(Q^2) = F_1(Q^2) - \kappa F_2(Q^2) + (\kappa^2 + 2\kappa) F_3(Q^2)$

(expects $F_1(Q^2) \sim F_2(Q^2) \sim 1/Q^4$ and $F_3(Q^2) \sim 1/Q^6$)

For this projection: $n(x) = 6x\bar{x}$

$\Rightarrow$ Extra $\bar{x}$ factor in GPD $\mathcal{F}(x, Q^2)$ should result in $1/Q^4$ asymptotics

Note: Extra $6x\bar{x}$ gives

$$6\int_0^1 dx\, x\bar{x}\, J_0\left(\sqrt{\frac{1-x}{x}}\, z\, Q\right)$$

$$= \frac{3}{2}\, z^2 Q^2\, K_2(zQ) - \frac{1}{4}\, z^3 Q^3 K_3(zQ) \equiv \mathcal{K}_2(zQ)$$

instead of nonnormalizable mode $\mathcal{K}_1(zQ) = zQK_1(zQ)$

Still, $\mathcal{K}_2(\xi) \sim e^{-\xi}$, and $z \sim 1/Q$ dominates for large Q

$\Rightarrow$ Apparently, we should get $1/Q^2$ again!

Resolution of paradox:

$$F_M^{\text{spinor}}(Q^2) = \frac{2M^2}{Q^2\gamma^2}\int_0^\infty d\xi\,\xi\,\mathcal{K}_2(\xi)\left[1 - \frac{1}{2}\,\xi^2\frac{M^2}{Q^2} + \frac{3}{32}\,\xi^4\frac{M^4}{Q^4} \right.$$

$$\left. - \frac{5}{576}\,\xi^6\frac{M^6}{Q^6} + \mathcal{O}(M^8/Q^8)\right]$$

$$= \frac{2M^2}{Q^2\gamma^2}\left[0 + 24\,\frac{M^2}{Q^2} - 288\,\frac{M^4}{Q^4} + 2400\,\frac{M^6}{Q^6} + \mathcal{O}(M^8/Q^8)\right]$$

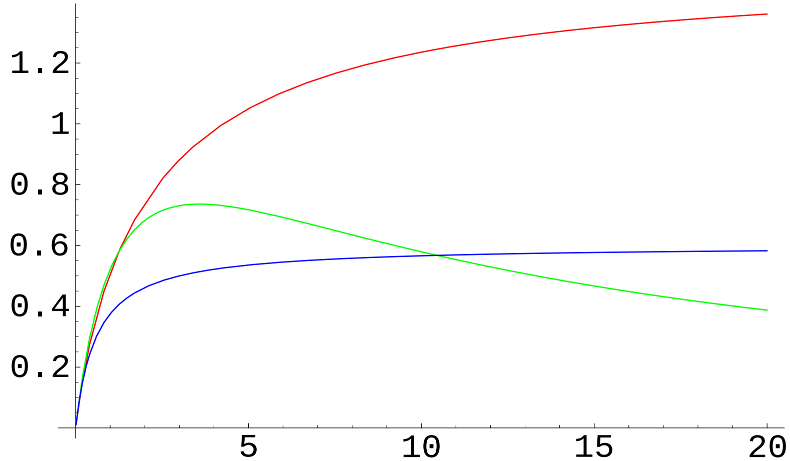
$\Rightarrow$ First term vanishes because $\int_0^\infty d\xi\,\xi\,\mathcal{K}_2(\xi) = 0$

$\Rightarrow$ leading term has $1/Q^4$ behavior

But: $1/Q^6$ correction exceeds it up to $Q^2 = 12M^2 \sim 7$ GeV^2

$\Rightarrow$ $1/Q^4$ asymptotics establishes above 20 GeV^2

Now, delayed $1/Q^4$ asymptotics is good news!



Here: VMD-like $Q^2/(1+Q^2/m_\rho^2)$,

holographic (scalar meromorphization) $Q^2 F_M(Q^2)$

spinor meromorphization $Q^2 F_M^{\text{spinor}}(Q^2)$

$\Rightarrow$ In the region of a few GeV^2 $F_M^{\text{spinor}}(Q^2)$ shows "power counting" $1/Q^2$ behavior more successfully than $F_M(Q^2)$ that displays its nominal $1/Q^2$ asymptotics outside the few GeV^2 region.

Miraculous cancellation of moments for $K_2(\xi)$ and $K_3(\xi)$ producing $F_M^{\text{spinor}}(Q^2) \sim M^4/Q^4$ can be traced to $1/Q^4$ asymptotic behavior of double spectral density $\rho(s_1, s_2, Q^2)$.

Consider double Borel transform of 3-point function

$$\Phi(\tau_1, \tau_2, Q^2) = \frac{1}{\pi^2} \int_0^\infty ds_1 \int_0^\infty ds_2 \; \rho(s_1, s_2, Q^2)\, e^{-s_1\tau_1 - s_2\tau_2}$$

For triangle diagram

$$\Phi(\tau_1, \tau_2, Q^2) = \frac{N_c}{2\pi^2(\tau_1 + \tau_2)} \int_0^1 dx\, x\bar{x}\, \exp\left[-Q^2 \frac{\bar{x}\,\tau_1\tau_2}{x(\tau_1 + \tau_2)}\right]$$

Note: $x\bar{x}$ factor (absent for scalar quarks)

$\Rightarrow$ $1/Q^4$ behavior of $\Phi(\tau_1, \tau_2, Q^2)$

$\Rightarrow$ $1/Q^4$ behavior of spectral density

$$\rho(s_1, s_2, Q^2) = N_c\, \theta(s_1)\, \theta(s_2)\, (s_1 + s_2)/2Q^4 + \ldots$$

LOCAL QUARK-HADRON DUALITY

Combines, in simplified form, some ideas of Migdal's program and QCD sum rule approach

QCD Sum Rules: only lowest state is narrow

$\Rightarrow$ model spectrum: first resonance plus perturbative "continuum" from $s = s_0$

$\Rightarrow$ Transform correlator

$$\Pi(p^2) = \frac{1}{\pi} \int_0^\infty ds \, \frac{\rho^{\mathrm{pert}}(s)}{s - p^2}$$

into

$$\Pi^{\mathrm{LD}}(p^2) = \frac{F_M^2}{M^2 - p^2} + \frac{1}{\pi} \int_{s_0}^\infty \frac{\rho^{\mathrm{pert}}(s)}{s - p^2} \, ds$$

Then: try to reach the best possible agreement in deep spacelike region $p^2 \equiv -P^2$

For axial currents: $F_M \to f_\pi$, $M \to m_\pi \approx 0$, $\rho^{\rm pert}(s) = 1/4\pi$

$$\Pi(p^2) - \Pi^{\rm LD}(p^2) = \frac{f_\pi^2}{p^2} + \frac{1}{4\pi^2} \int_0^{s_0} ds \, \frac{\rho^{\rm pert}(s)}{s - p^2}$$

Eliminate leading $1/p^2$ term $\Rightarrow$ $f_\pi^2 = s_0/4\pi^2$

$\Rightarrow$ Local duality relation

$$\int_0^{s_0} \rho_\pi(s) \, ds = \int_0^{s_0} \rho^{\rm pert}(s) \, ds$$

Three-point function $\Rightarrow$ local duality relation for pion form factor

$$f_\pi^2 F_\pi^{\rm LD}(Q^2) = \frac{1}{\pi^2}\int_0^{s_0} ds_1 \int_0^{s_0} ds_2\ \rho^{\rm pert}(s_1, s_2, Q^2)$$

$\Rightarrow$ Drell-Yan-type formula

$$F_\pi^{\rm LD}(Q^2) = \frac{6}{\pi s_0}\int_0^1 dx \int d^2\mathbf{k}_\perp$$

$$\times\theta\left(\mathbf{k}_\perp^2 \le x\bar{x}s_0\right)\theta\left((\mathbf{k}_\perp + \bar{x}\mathbf{q}_\perp)^2 \le x\bar{x}s_0\right)$$

with effective local duality wave function

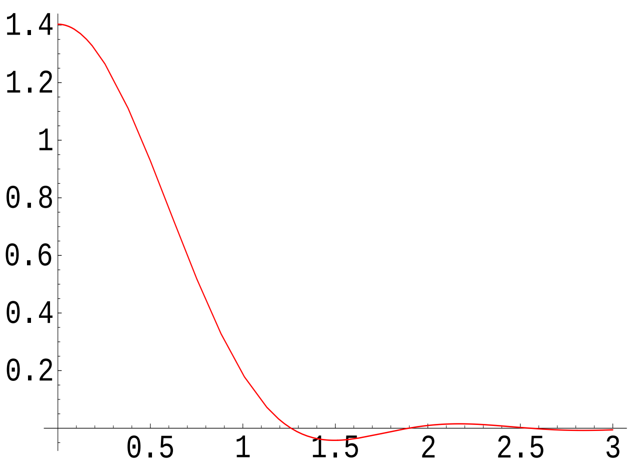
$$\widetilde{\Psi}_\pi^{\rm LD}(x, \mathbf{k}_\perp) = \sqrt{6/\pi s_0}\ \theta(k_\perp^2 \le x\bar{x}s_0)$$

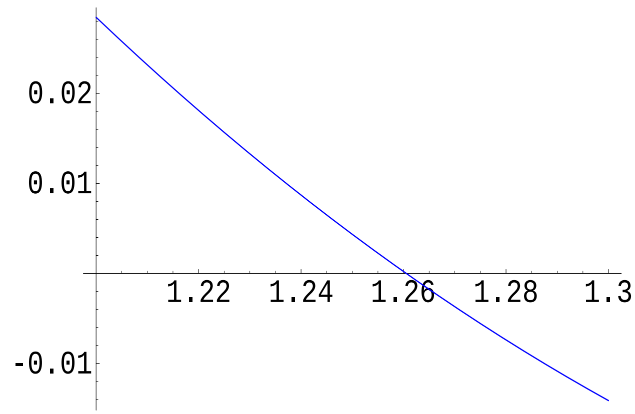
In the impact parameter representation

$$\Psi_\pi^{\rm LD}(x, \mathbf{b}_\perp) = \sqrt{6x\bar{x}/\pi}\, J_1(b\sqrt{x\bar{x}s_0})/b$$

$b \to 0$ limit: "LD" distribution amplitude $\varphi_\pi^{\rm LD}(x) = 6f_\pi x\bar{x}$ coincides with asymptotic pion DA.

"Holographize" $\Psi_\pi^{\rm LD}(x,\mathbf{b}_\perp)$ by cut-off $\theta(b\sqrt{x\bar{x}s_0}\le\beta_{1,1})$





First zero of $\widetilde{\Psi}_\pi^{\rm LD}(x,\mathbf{k}_\perp)$ is located at $k_\perp^2/x\bar{x}\approx(1.26\,\mathrm{GeV})^2$, "unexpectedly close" to A_1 position.

Explicit expression for LD form factor:

$$F_\pi^{\mathrm{LD}}(Q^2) = 1 - \frac{1 + 6s_0/Q^2}{\left(1 + 4s_0/Q^2\right)^{3/2}}$$

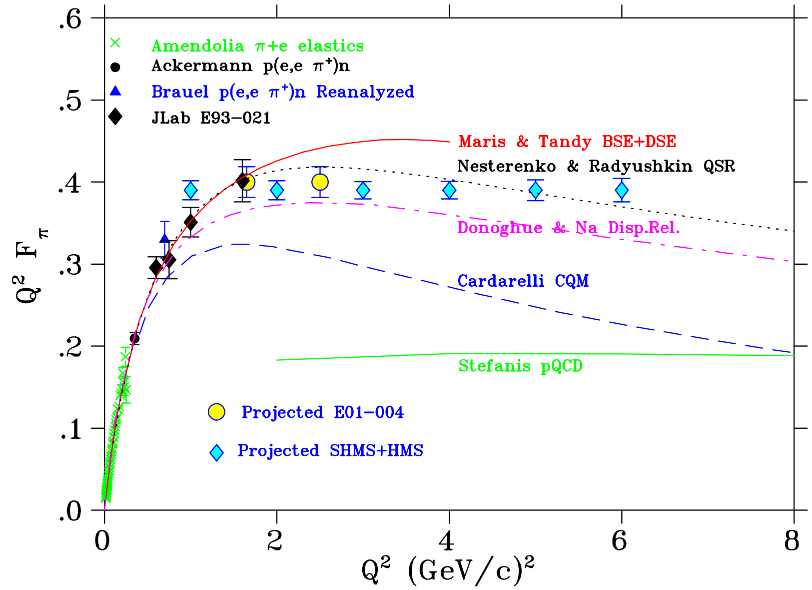
$$= \frac{6s_0^2}{Q^4} - \frac{40s_0^3}{Q^6} + \frac{210s_0^4}{Q^8} - \frac{1008s_0^4}{Q^8} + \mathcal{O}(s_0^5/Q^{10})$$

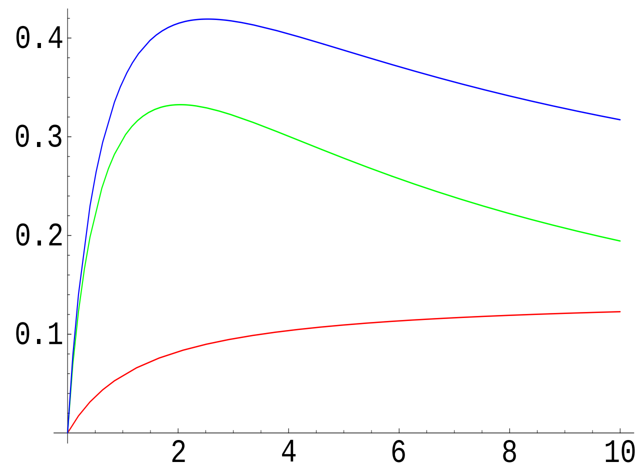
As expected, has $1/Q^4$ asymptotics

But: $1/Q^4$ estimate should not be used at accessible Q^2

At accessible Q^2, successfully imitates $1/Q^2$ behavior

and is close to data



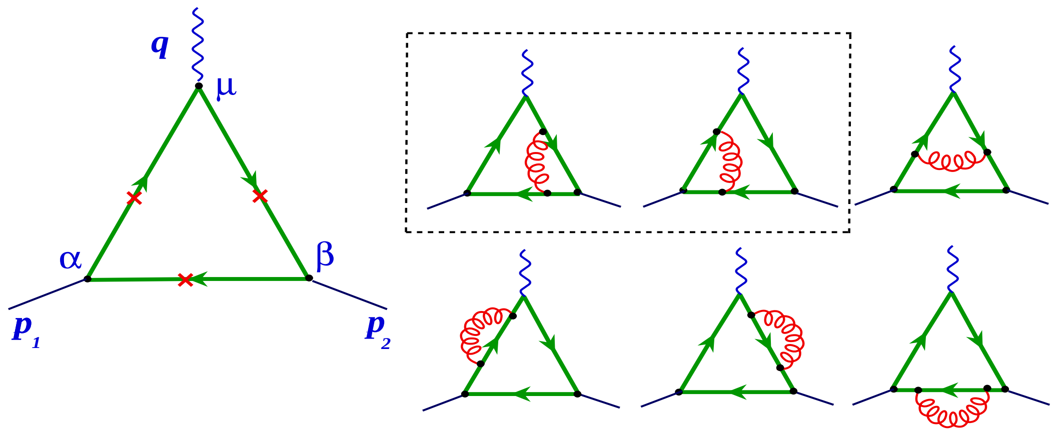


Right panel: Pion form factor in local quark-hadron duality model

$Q^2 F_\pi^{\mathrm{LD}}$: lowest-order, (green)

$Q^2 F_\pi^{\mathrm{LD}(\alpha_s)}$: $\mathcal{O}(\alpha_s)$ correction (red)

total contribution (blue)



Higher orders and transition to pQCD: include higher order α_s corrections to spectral densities

There appear gluon-exchange diagrams with large-Q^2 behavior determined by hard pQCD mechanism

As a result:

$$\rho_{\alpha_s}(s_1, s_2, Q^2) = 2\pi\alpha_s \frac{C_F}{N_c} \int_0^1 dx \int_0^1 dy \; \frac{\rho(x,s_1)\rho(y,s_2)}{xyQ^2}$$

$$+\mathcal{O}(1/Q^4)$$

$\rho(x, s_1)$: x-unintegrated 2-point spectral density,

$$\rho(x, s_1) = \frac{N_c}{2\pi^2} \int \delta\left(s_1 - \frac{k_{1\perp}^2}{x\bar{x}}\right) d^2\mathbf{k}_{1\perp} = \frac{N_c}{2\pi}\,\theta(s_1)\, x\bar{x}$$

Its integral over duality interval $0 \leq s \leq s_0$ gives
$\varphi_\pi^{\mathrm{LD}}(x) = 6 f_\pi x\bar{x}$ for the pion DA

$\Rightarrow$ Local duality at $\mathcal{O}(\alpha_s)$ order gives pQCD result

$$F_\pi^{\mathrm{pQCD}}(Q^2) = 8\pi\alpha_s f_\pi^2/Q^2$$

calculated for asymptotic shape of pion DA

Writing

$$F_\pi^{\mathrm{pQCD}}(Q^2) = 2(s_0/Q^2)(\alpha_s/\pi)$$

reveals its nature as α_s correction to soft contribution

Note: $F_\pi^{\mathrm{LD}(\alpha_s)}(0) = \alpha_s/\pi$ from Ward identity

$\Rightarrow$ Interpolation:

$$F_\pi^{\mathrm{LD}(\alpha_s)}(Q^2) = (\alpha_s/\pi)/(1 + Q^2/2s_0)$$

(very accurate)

SUMMARY

Studied large-Q^2 behavior of meson form factor $F_M(Q^2)$ constructed using holographic model

Observed that, despite its $1/Q^2$ asymptotic behavior, combination $Q^2 F_M(Q^2)$ is not flat in accessible region $Q^2 \lesssim 10\ \text{GeV}^2$

Found that asymptotic $1/Q^2$ result for $F_M(Q^2)$ is governed by Feynman mechanism

Using (scalar) meromorphization approach reproduced wave functions of holographic model

For spin-1/2 quarks, demonstrated that extra $(1-x)$ factor, results in $F_M^{\text{spinor}}(Q^2) \sim 1/Q^4$ asymptotic behavior

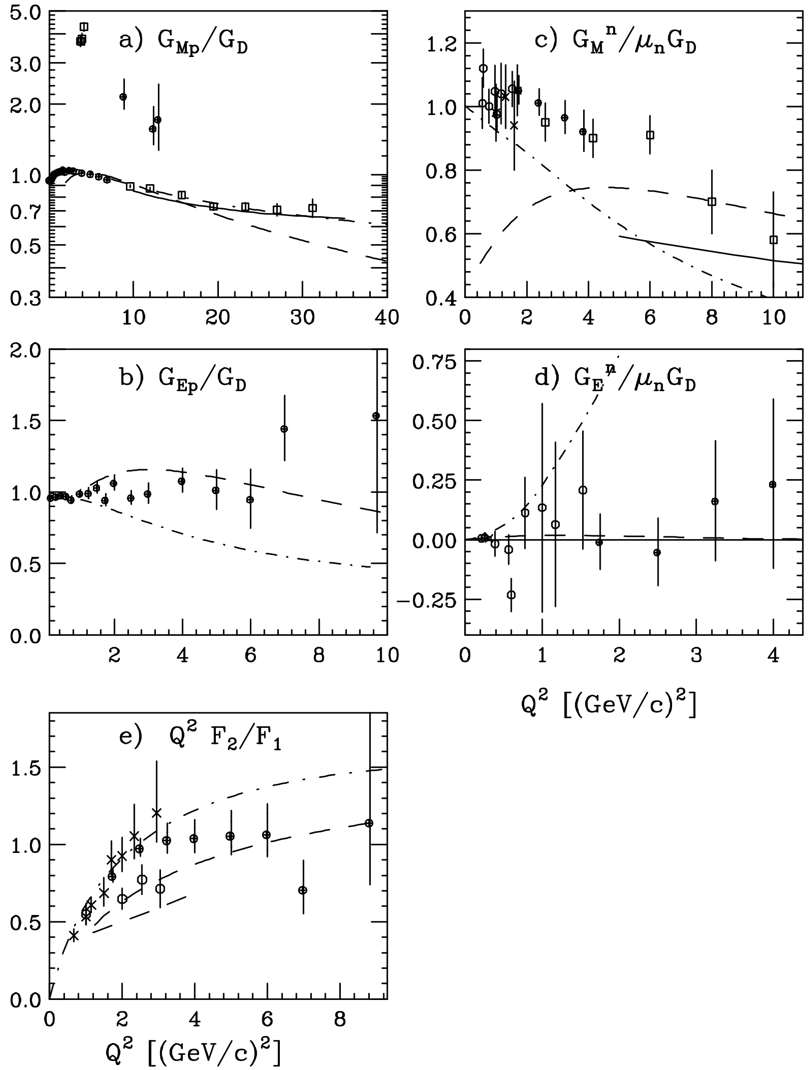
Due to late onset of the asymptotic pattern $Q^2 F_M^{\text{spinor}}(Q^2)$ is flat in few GeV^2 region

Presented results for pion form factor in local quark-hadron duality model: the lowest-order term again has nominally $1/Q^4$ asymptotics, but it imitates $1/Q^2$ behavior in the few GeV^2 region

Showed that $\mathcal{O}(\alpha_s)$ term of $\rho(s_1, s_2, Q^2)$ brings in hard pQCD contribution having the dimensional counting $1/Q^2$ behavior.

Argued that at accessible Q^2, the $\mathcal{O}(\alpha_s)$ term is a small fraction of total result, because of small $\alpha_s/\pi \sim 0.1$ factor associated with each higher order correction

Did not discuss nucleons form factors. But want to mention that the lowest-order local duality result for $G_M^p(Q^2)$ closely follows the dipole shape of the data up to $Q^2 \sim 15\ \text{GeV}^2$.



Local duality predictions is shown by dashed lines

CONCLUSIONS

Power of $(1-x)$ in perturbative versions of relevant parton densities $f(x)$ increases with number of quarks like $f_n(x) \sim (1-x)^{n-1}$

$\Rightarrow$ Probability that total momentum of $n-1$ spectators is x_{sp} goes like x_{sp}^{n-1}, which looks OK

Feynman mechanism then gives $(1/Q^2)^n$ asymptotic behavior for form factors

Because of late onset of asymptotic regime, form factors imitate $(1/Q^2)^{n-1}$ behavior in a rather wide preasymptotic region

In this scenario, quark counting rules (if any) is approximate and transitional phenomenon dominated by nonperturbative, long-distance aspects of hadron dynamics