Baryons and Skyrmions in QCD with Quarks in Higher Representations

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Large N QCD

$$\mathcal{L} = -\frac{1}{2}\operatorname{tr} FF + \sum_{1}^{N_f} \bar{q}_i \left(i\not{D} - m \right) q_i$$

SU(N) QCD

'‘t Hooft large N $\quad g^2 \sim \frac{1}{N}$

$$\left\{ \begin{array}{l} \square \\ \boxminus \qquad \square\!\square \\ \text{Adj} \end{array} \right.$$

ASV Planar Equivalence!

Baryon-Skyrmion Identification

Baryon $\qquad \epsilon_{\alpha_1 \cdots \alpha_N}\, q^{\alpha_1} \cdots q^{\alpha_N}$

Large N limit $\qquad R \propto N^0 \qquad M \propto N$

Skyrmion $\qquad M \propto F_\pi^2 \sim N$

The Puzzle

$$\epsilon_{\alpha_1 \cdots \alpha_N}\, \epsilon_{\beta_1 \cdots \beta_N}\, q^{\alpha_1 \beta_1} \cdots q^{\alpha_N \beta_N}$$

Naively we would say $M \propto N$ (?)

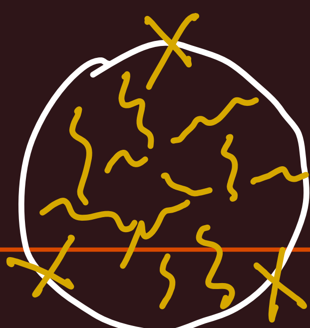
But the Skyrmion $M \propto F_\pi^2 \sim N^2$

Chiral Effective Lagrangian

$m_q = 0 \qquad SU(N_f)_L \times SU(N_f)_R \longrightarrow SU(N_f)_V$

Nambu-Goldstone bosons $\qquad U(x) = e^{i\frac{\Pi}{F_\pi}}$

$$\mathcal{L}_{eff} = F_\pi^2 \, \mathrm{tr}\, \partial U \partial U^{-1} + \dots$$



$$\begin{cases} \sim N \quad (\square) \\ \sim N^2 \quad \left(\begin{smallmatrix}\square\\ \square\end{smallmatrix} \text{ or } \square\square\right) \end{cases}$$

Skyrmion

Point-like texture

$$\pi_3\left(SU(N_f)\right) = \mathbb{Z}$$

$$\mathcal{L}_{eff} = F_\pi^2 \left\{ \mathrm{tr}\, \partial U \partial U^{-1} + h.d. \right\}$$

$$E(R) = R^3 \cdot \frac{1}{R^2} + R^3 \cdot \frac{1}{R^4}$$

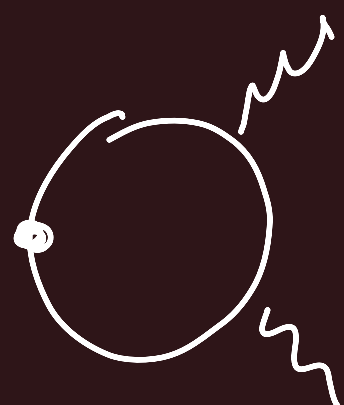
Contraction

Repulsion

Wess-Zumino-Witten

If I gauge $U(1)_B$ where is the anomaly $\pi^0 \to 2\gamma$?

$$\Gamma_{WZW} = -\frac{i\,n}{240\pi^2}\int_{M_5} \mathrm{tr}\left(\partial U U^{-1} \ldots \partial U U^{-1}\right)$$



$$n = \begin{cases} N \\ \dfrac{N(N-1)}{2} \\ \dfrac{N(N+1)}{2} \end{cases}$$

Quantum Numbers of Skyrme

Baryon # $n = \begin{cases} N \\ \frac{N(N-1)}{2} \\ \frac{N(N+1)}{2} \end{cases}$

Statistics $(-1)^n$

"Rigid" quatities

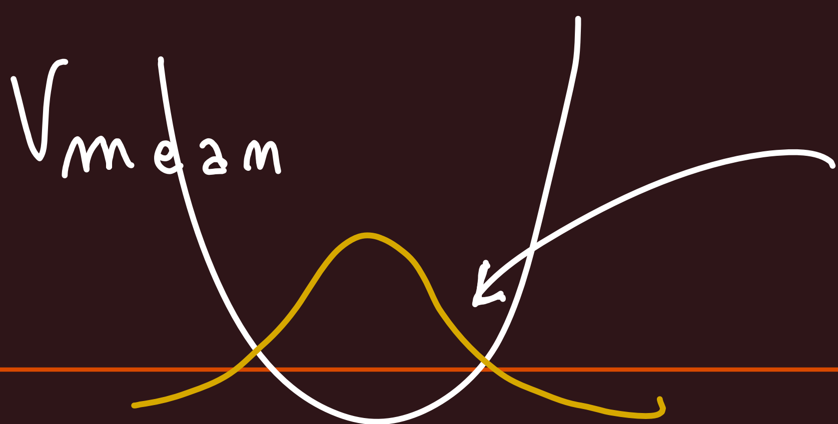
Mass $\propto N^2$

Baryons at Large N

$$\epsilon_{\alpha_1 \cdots \alpha_N}\, q^{\alpha_1} \cdots q^{\alpha_N}$$

$$\boxed{\begin{array}{ccc} \psi_{\text{gauge}} & \psi_{\text{spin}} & \psi_{\text{space}} \\ - & + & + \end{array}}$$

Bose-Einstein condensate in a mean potential



Ground state

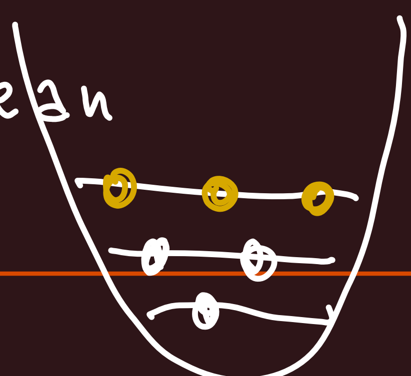
“Minimal” Baryon in Higher Rep.

$$\epsilon_{\alpha_1 \cdots \alpha_N}\, \epsilon_{\beta_1 \cdots \beta_N}\, Q^{\alpha_1 \beta_1} \cdots Q^{\alpha_N \beta_N}$$

4_{gauge}	4_{spin}	4_{space}
$+$	$+$	$-$

Fermi degenerate gas

V_{mean}



$$M \propto N^{4/3} \; !$$

Statistical pressure

Solution of the puzzle

Suppose we find a gauge wave function completely antisymmetric which contains $\frac{N(N\pm1)}{2}$ quarks

$\boxed{\text{This would solve the puzzle!}}$

Minimal baryons would form bound states

$$\left[N^{4/3} \frac{N\pm1}{2} \right] \gg \left[\frac{N(N\pm1)}{2} \right]$$

Symmetric Representation

We split the quark into two fundamental quarks

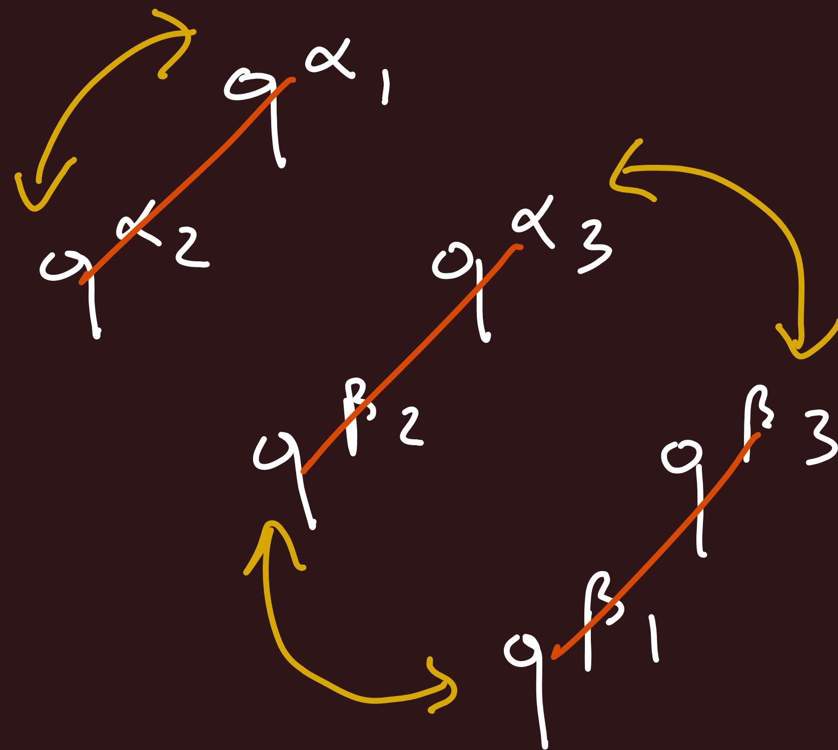
$$Q^{\{\alpha\beta\}} \longrightarrow q^{\alpha}, q^{\beta}$$

N=2 first

$$\frac{N(N+1)}{2} = 3$$

$$Q^{\{\alpha_1 \beta_1\}}$$

$$Q^{\{\alpha_2 \beta_2\}} \quad Q^{\{\alpha_3 \beta_3\}}$$



$$\epsilon_{\alpha_1\alpha_2}\,\epsilon_{\beta_2\alpha_3}\,\epsilon_{\beta_1\beta_3}\; Q^{\{\alpha_1\beta_1\}}\, Q^{\{\alpha_2\beta_2\}}\, Q^{\{\alpha_3\beta_3\}}$$

Proposition

There is one and only one gauge function that is gauge invariant and completely antisymmetric

It is the antisymmetric subspace of the Tensor product of $\frac{N(N+1)}{2}$ quarks

Proof

FORBIDDEN

$$\epsilon_{\alpha_i \beta_i \cdots} \qquad Q^{\{\alpha_i \beta_i\}}$$

$$\begin{cases} \epsilon_{\alpha_i \alpha_j \cdots} \qquad Q^{(\alpha_i \beta_i)} \\ \epsilon_{\beta_i \beta_j \cdots} \qquad Q^{[\alpha_j \beta_j]} \end{cases}$$

q^{α_N}

q^{α_1}

q^{β_1}

$\cdots$

q^{β_N}

$\frac{N(N+1)}{2}$ Quarks!

Antisymmetric representation

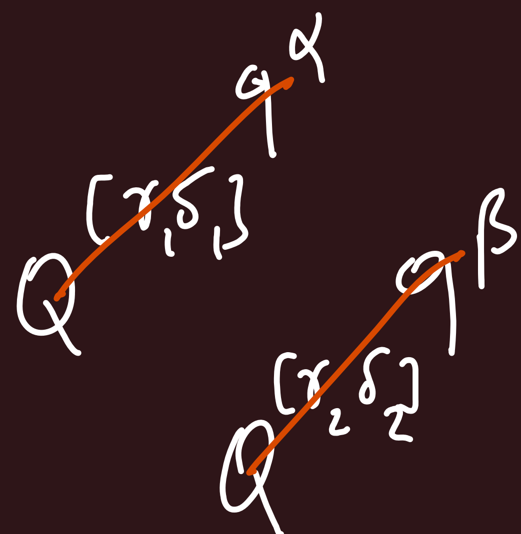
N=2 $\qquad \epsilon_{\alpha\beta} \, Q^{[\alpha\beta]}$

N=3 $\qquad$ Is ordinary QCD!

$$\tilde{Q}_{\gamma} = \frac{1}{2} \epsilon_{\gamma\alpha\beta} \, Q^{[\alpha\beta]} \qquad \overline{\square} = \begin{array}{|c|}\hline \\ \hline \\ \hline \end{array}$$

Baryon $\Rightarrow \epsilon^{\gamma\rho\tau} \, \tilde{Q}_{\gamma} \tilde{Q}_{\rho} \tilde{Q}_{\tau}$

N=3



Diagram

$$\frac{1}{2}\left[\epsilon_{\gamma_1 \delta_1 \alpha}\,\epsilon_{\gamma_2 \delta_2 \beta} - \epsilon_{\gamma_2 \delta_2 \alpha}\,\epsilon_{\gamma_1 \delta_1 \beta}\right] Q^{[\alpha\beta]}\, Q^{[\gamma_1 \delta_1]}\, Q^{[\gamma_2 \delta_2]}$$

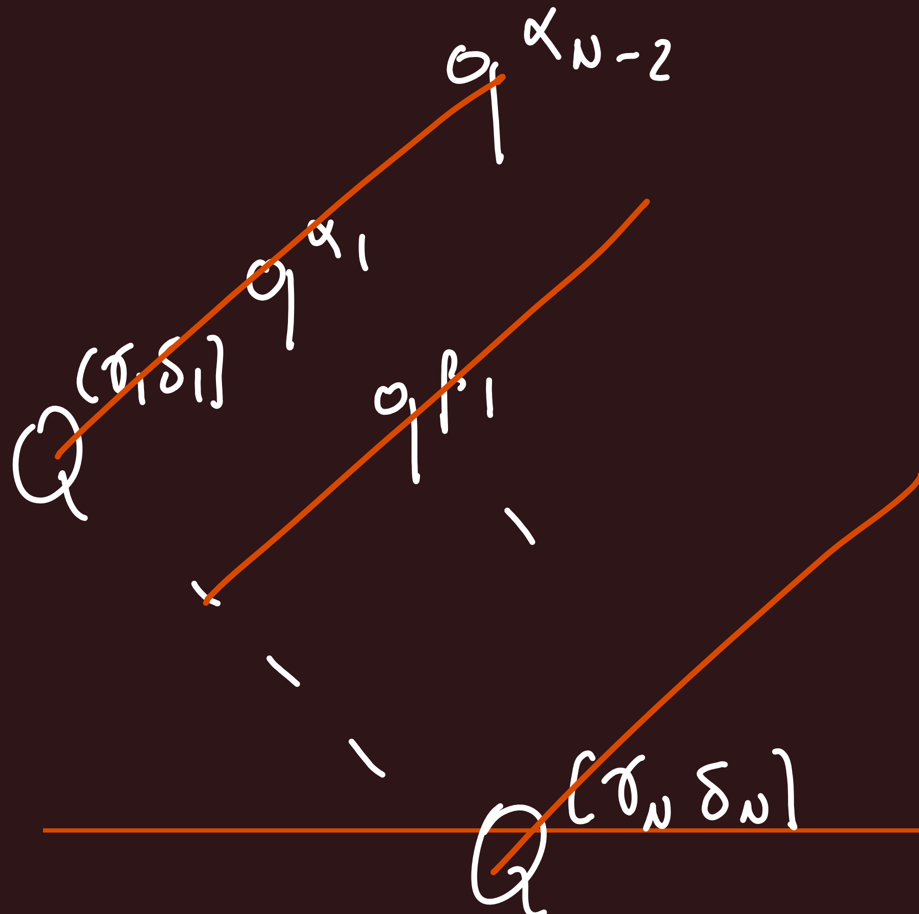
Proposition

There is one and only one gauge function that is gauge invariant and completely antisymmetric

$\boxed{\text{It is the antisymmetric subspace of the Tensor product of } \frac{N(N-1)}{2} \text{ quarks}}$

Proof

FORBIDDEN $\epsilon_{\gamma_1\delta_1\gamma_2\delta_2\dots}\, Q^{[\gamma_1\delta_1]}\, Q^{[\gamma_2\delta_2]} \dots$



$\frac{N(N-1)}{2}$ Quarks!

More on the antisymmetric representation

$\boxed{\text{There is a dinstinction between N even and N odd}}$

For N even the minimal baryon is another one

$$\epsilon_{\alpha_1 \ldots \alpha_{N/2}\, \beta_1 \ldots \beta_{N/2}}\, Q^{[\alpha_1 \beta_1]} \ldots Q^{[\alpha_N \beta_N]}$$

For N odd there are no symmetric gauge wave functions at all!

$$\epsilon_{\alpha_1 \cdots \alpha_N} \epsilon_{\beta_1 \cdots \beta_N} Q^{[\alpha_1 \beta_1]} \cdots Q^{[\alpha_N \beta_N]}$$

$$= - \epsilon_{\alpha_1 \cdots \alpha_N} \epsilon_{\beta_1 \cdots \beta_N} Q^{[\beta_1 \alpha_1]} \cdots Q^{[\beta_N \alpha_N]}$$

$$= - \epsilon_{\beta_1 \cdots \beta_N} \epsilon_{\alpha_1 \cdots \alpha_N} Q^{[\alpha_1 \beta_1]} \cdots Q^{[\alpha_N \beta_N]}$$

Adjoint representation

N_f Dirac adjoint $\qquad \lambda^{\alpha}{}_{\beta}$

$$\pi_3\left(\frac{SU(N_f)}{O(N_f)}\right) = \mathbb{Z}_2$$

$$\begin{matrix} \lambda^{\alpha_{11}}{}_{\beta_{11}} & \cdots & \lambda^{\alpha_{1N}}{}_{\beta_{1N}} \\ \vdots & & \vdots \\ \lambda^{\alpha_{N1}}{}_{\beta_{N1}} & \cdots & \lambda^{\alpha_{NN}}{}_{\beta_{NN}} \end{matrix} \qquad M \sim N^2$$

Conclusions

Skyrmion $\in$ Spectrum

At $N \to \infty$ is the most stable baryonic bound state!