QCD Vacuum Topology and Glueballs

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Hilmar Forkel
U. Heidelberg & IFT-UNESP

- A few facts about glueballs
- OPE of glueball correlators with nonperturbative Wilson coefficients
- 0^{++} Gb sum rules & direct instantons
- 0^{-+} Gb & topological charge screening
- Wrapping up...

Mini-introduction to glueballs

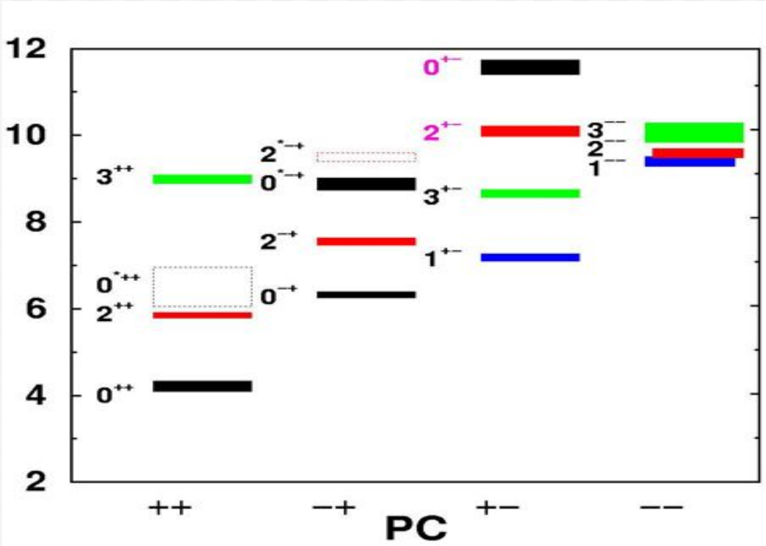
30ys of study - still theoretically and experimentally far from understood**:**

- Quarkonium admixtures subtle (unquenched lattice not yet predictive...)
- Unique expt. signatures? (e.g. rad. decays of heavy quarkonia...)
- Crucial for interpretation of light scalar meson spectrum

Benchmark: quenched lattice spectrum:

- level ordering: $0^{++} < 2^{++} < 0^{-+}$
- scalar glueball properties:
  - $m_{0++} \sim 1.5$ GeV
  - $r_{0++} \sim 0.2\text{-}0.4$ fm ($\ll r_{2++}$)

$\Rightarrow$



(Spectrum by Morningstar/Peardon)

Structure & dynamics of bound IR glue:

- Mostly coherent, nonperturbative "valence glue" ~ elusive glue in classical hadrons?
- ∃ essential gluonic (collective) dofs., contribs from known gluonic effects?
- E.g. monopoles, vortices, branes, instantons...?

Glueball correlation functions and (I)OPE

Propagate gluonic vacuum excitations:

$$\Pi_G\left(Q^2\right) = i\int d^4q\, e^{iqx}\left\langle 0\left|T\, g_G(x)\, g_G(0)\right|0\right\rangle$$

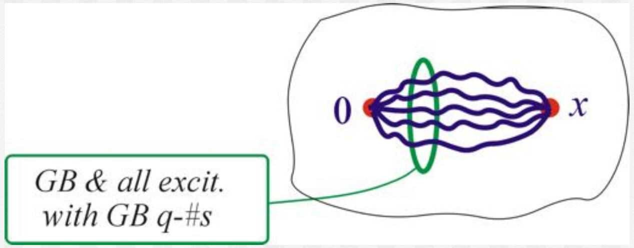
Created by interpolators:

$$g_S(x) = \alpha_s G^a_{\mu\nu} G^{a,\mu\nu}$$

$$g_P(x) = \alpha_s G^a_{\mu\nu} \widetilde{G}^{a,\mu\nu}$$

$$g_T(x) = \Theta_{\mu\nu}$$

Linked to glueball spectrum:



Dispersive representation (Σ over hadr. states) **vs. (I)OPE**:

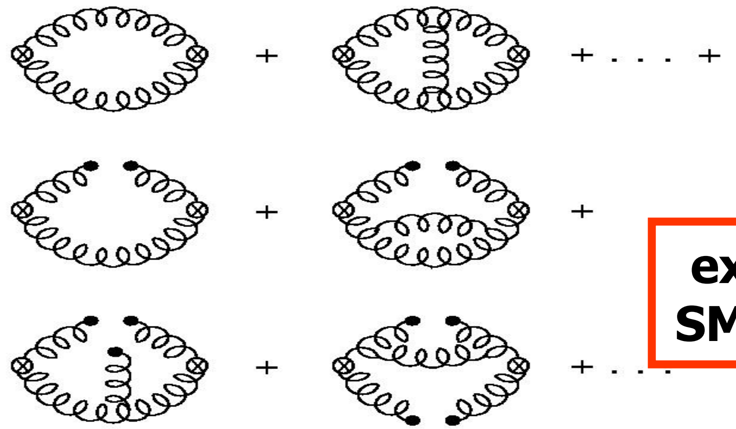
$$\Pi\left(Q^2\right) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\,\Pi^{(ph)}(-s)}{s+Q^2} \quad \mathrm{mod}(subtr.) \quad \xrightarrow{Q^2 \gg \Lambda^2_{QCD}} \sum_n C_{\hat{O}_n}\left(Q^2;\mu\right)\left\langle :\hat{O}_n:\right\rangle$$

Borel moments (for sum rules):

$$\mathcal{L}_k(\tau) \equiv \hat{B}_\tau\left[\left(-Q^2\right)^k \Pi\left(Q^2\right)\right] = \frac{1}{\pi}\int_0^\infty ds\, s^k\, \mathrm{Im}\,\Pi(-s)\, e^{-s\tau}$$

Perturbative Wilson coefficients

- Up to N^2LO: $O(\alpha^2)$, 3-loop
- Incl. interior quark loops
- Incl. all operators up to d=8
- IR cutoff μ usually neglected
- Long history of improvements...



except. SMALL !!

Trad. SVZ-OPE & QCD sum rules built on these coefficients!

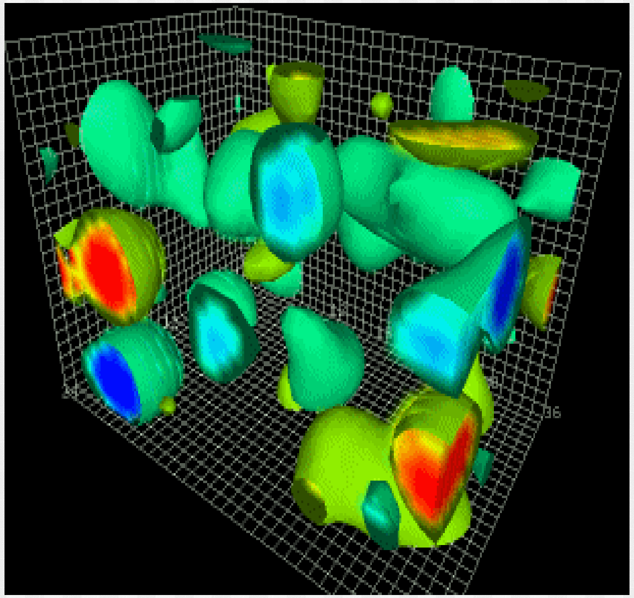
- **Notorious inconsistencies** in spin-0 glueball sum rules:
  - $\mathcal{L}_{-1}$ ΣR predicts $m_{0++} \sim 0.2 - 0.6$ GeV,
  - $\mathcal{L}_{0,1,\ldots}$ ΣRs predict $m_{0++} \geq 1.2$ GeV,
  - $\mathcal{L}_{-1}$ ΣRs badly violate LETs in both spin-0 channels!
- Nonpert. contributions underrepresented: $\mathbf{C}_{\langle GGG\ldots\rangle}$ **except. small!**

Ideal hunting ground for **addtl. (semi-hard) npert. physics in OPE-coeffs!**

Qualitative indications for relevance of instantons

A) Strength:

- $\mathbf{0^{PC}}$ GBs couple except. strongly to Is.
- "Classical enhancement": $G^{(I)4} \sim 1/\alpha_s^2$
- 0^{++} GB except. small: $r_G \sim \rho \ll \Lambda_{QCD}^{-1}$
- 0^{-+} GB couples to topological charge of Is



(Picture by Adelaide group)

B) Channel dependence:

(Anti-) Self-duality:

$$G_{\mu\nu}^{(I,\bar{I})} = \pm\, i\tilde{G}_{\mu\nu}^{(I,\bar{I})}$$

$$(:= \pm \tfrac{i}{2}\varepsilon_{\mu\nu\alpha\beta} G^{(I,\bar{I})\alpha\beta})$$

$$\Rightarrow \quad \boxed{\begin{array}{l} \left\langle TG\tilde{G}(x)G\tilde{G}(0)\right\rangle_{I,\bar{I}} = -\left\langle TGG(x)GG(0)\right\rangle_{I,\bar{I}} \\ \left\langle T\Theta_{\mu\nu}(x)\Theta_{\rho\sigma}(0)\right\rangle_{I,\bar{I}} = 0 \quad \text{since } \Theta_{\mu\nu}^{(I,\bar{I})} = 0 \end{array}} \quad \Rightarrow$$

- attraction (repulsion) in 0^{++} (0^{-+})
- no interaction in 2^{++}
- $\Rightarrow$ reproduces qual. level order!

Semiclassical treatment of direct instantons

Hard **=** **"direct"** $\rho \le \mu^{-1}$ **instanton contributions** to

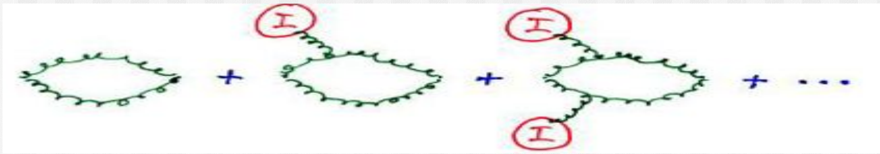
$$\Pi_G(x) = \left\langle 0 \middle| T\, g_G(x) g_G(0) \middle| 0 \right\rangle = \sum_n C_{\hat{O}_n}^{(pt \& I,\bar{I})}(x; k \ge \mu) \times \left\langle \hat{O}_n \right\rangle (k < \mu)$$

at $|x| \ll \Lambda_{QCD} \sim R$ (nearest instanton dominates) **semiclassically** (analytically):

$$\Pi_G(x) = \int D\left(q\bar{q}\tilde{G}_\mu\right) g_G(x) g_G(0) \exp\left(-\frac{S_E[q\bar{q}G]}{\hbar}\right)$$

expanded around $I,\bar{I}$ saddle points: $G_\mu = G_\mu^{(I,\bar{I})} + \tilde{G}_\mu$

i.e.:



$\Rightarrow$

$$\Pi_S^{(I,\bar{I})}(x) = \sum_{I,\bar{I}} \int dU \int d\rho\, n(\rho) \int d^4x_0\, \pi(x; x_0, \rho, \mu) = \frac{2^8 3}{7} \int d\rho \frac{n(\rho)}{\rho^4}\, {}_2F_1\left(4, 6, \frac{9}{2}; -\frac{x^2}{4\rho^2}\right) + \dots$$

- Quark admixtures through $n(\rho)$, quark loops, condensates
- Analytical structure (cuts) determines large-Q behavior
- Requires $n(\rho)$ as input from lattice, ILM

Instanton size distribution

$$n_I(\rho) = n_{\bar{I}}(\rho) = \frac{d^5 N_I}{d^4 x_0\, d\rho}$$

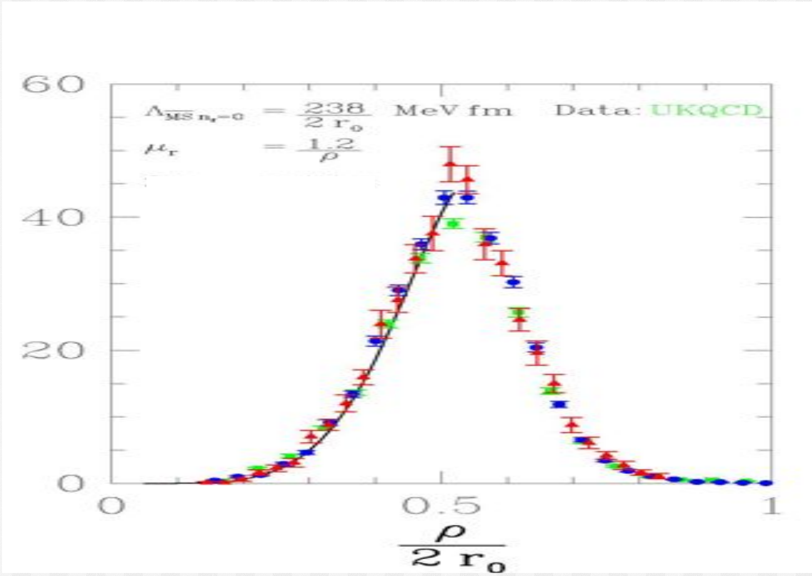
Quenched SU(3) data from UKQCD, analyzed by
A. Ringwald, F. Schrempp, PLB 459 (1999) 249

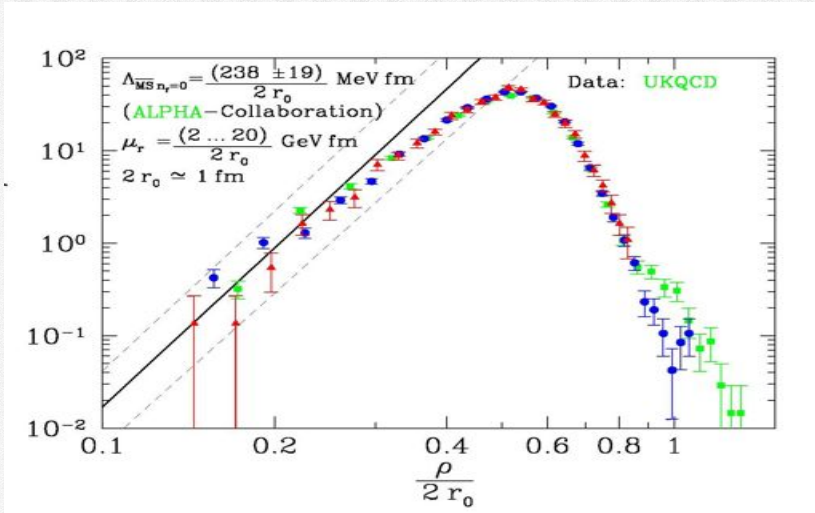
$$\bar{n} := \int_0^\infty n_I(\rho) d\rho \simeq 0.5\,\mathrm{fm}^{-4} \qquad \text{(lattice, ILM, phen.)}$$

$$\bar{\rho} := \frac{1}{\bar{n}} \int_0^\infty n_I(\rho) \rho\, d\rho \simeq 0.33\,\mathrm{fm} \qquad \text{(lattice, ILM, phen.)}$$

$$n_I(\rho) \to C_s\, \rho^{b_0 - 5} \qquad \text{for } \rho \to 0 \qquad \text{(backgrnd. PT, latt.)}$$

$$n_I(\rho) \to C_l \exp\left(C_e \frac{\rho^2}{\bar{\rho}^2} \right) \qquad \text{for } \rho \to \infty \qquad \text{(latt., ILM, } N_f\text{-indep.)}$$

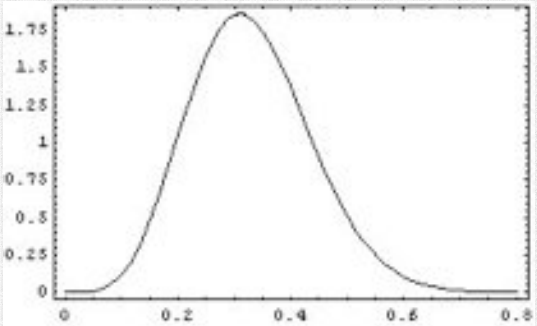




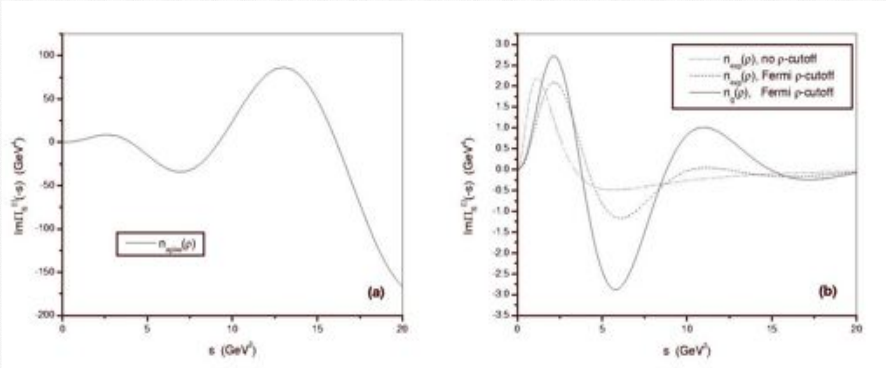
Realistic size distribution & renormalization

Minimal implementation :

$$n_I(\rho) = \frac{2^{18}}{3^6 \pi^3} \frac{\bar{n}\,\rho^4}{\bar{\rho}^5} \exp\left(-\frac{2^6}{3^2 \pi} \frac{\rho^2}{\bar{\rho}^2} \right) \qquad (N_c = N_f = 3)$$

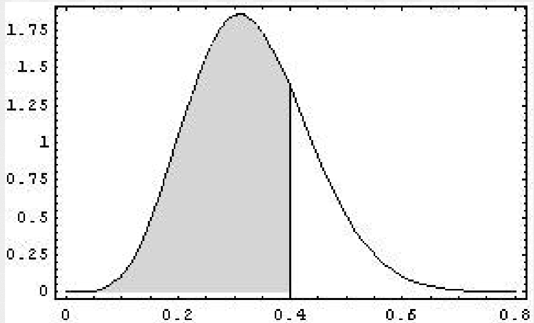


Impact on IOPE, Gb sum rules:



- Wilson coeffs. & ΣRs insensitive to details of n(ρ)
- Spike artefacts gone, positivity restored
- Π(Q) decays slower at large Q → smaller m_G
- Fiducial domains larger → more accurate ΣRs

→ **Renorm. of inst.-ind. Wilson coefficients :**



Gauge-inv. exclusion of instantons with $\rho > \mu^{-1} \Rightarrow$

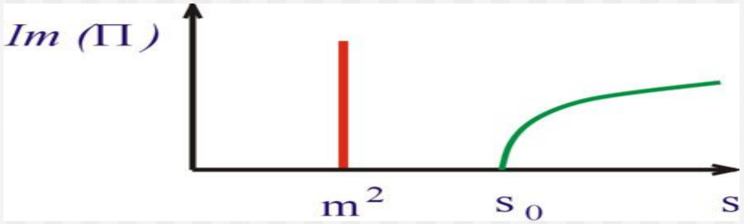
- Improved consistency of all ΣRs
- LET in scalar channel fully restored
- (Approx. "pragmatic" OPE with spike only)

Continuum-subtracted Borel moments & sum rules

Hadronic dispersive **representation**

$$\Pi^{(ph)}(Q^2) = \frac{1}{\pi}\int_0^\infty ds \frac{\mathrm{Im}\,\Pi^{(ph)}(-s)}{s+Q^2}$$

Based on **GB pole** & **parton-hadron duality**:



$$\mathrm{Im}\,\Pi^{(ph)}(-s) = \mathrm{Im}\,\Pi^{(res)}(-s) + \mathrm{Im}\,\Pi^{(cont)}(-s)$$

$$\mathrm{Im}\,\Pi^{(res)}(-s) = \pi\sum_i f_{Gi}^2 m_{Gi}^4 \delta\left(s - m_{Gi}^{\ 2}\right); \quad \mathrm{Im}\,\Pi^{(cont)}(-s) = \left[\mathrm{Im}\,\Pi^{(OPE)} + \mathrm{Im}\,\Pi^{(I+\bar{I})}\right](-s)\,\theta(s - s_0)$$

For 0^{++}:

$$\mathrm{Im}\,\Pi^{(I+\bar{I})}(-s) = -(2\pi)^4 \int_0^\infty d\rho\, n(\rho)\, \rho^4 s^2 J_2\left(\sqrt{s}\rho\right) Y_2\left(\sqrt{s}\rho\right)$$

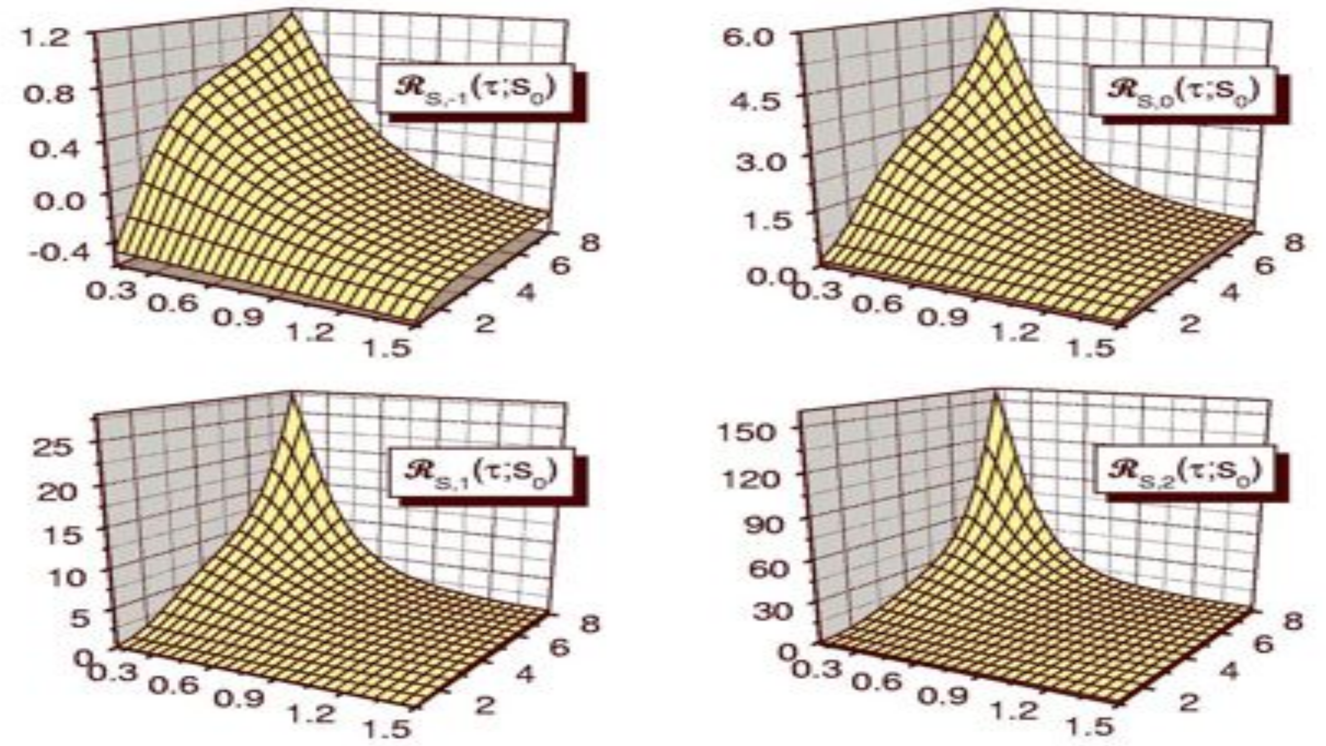
$$\mathrm{Im}\,\Pi^{(OPE)}(-s) = -\pi[a_1 s^2 + 2a_2 s^2 \ln\left(s/\mu^2\right) + b_1 \left\langle \alpha G^2 \right\rangle - c_1 \left\langle g G^3 \right\rangle / s + \ldots]$$

GB props. to be calc.!

Matched over fiducial τ domain **to IOPE** moments $\Rightarrow$ **sum rules**:

$$\boxed{\begin{aligned} \mathcal{R}_{G,k}(\tau; s_0) &\equiv \hat{B}_\tau \left\{ (-Q^2)^k \left[\Pi_G^{(IOPE)}(Q^2) - \Pi_G^{(non-res)}(Q^2; s_0) \right] \right\} \\ &= \frac{1}{\pi}\int_0^{s_0} ds\, s^k\, \mathrm{Im}\,\Pi_G^{(IOPE)}(-s)\, e^{-s\tau} - \delta_{k,-1}\Pi^{(IOPE)}(0) = f_G^{\ 2} m_G^{\ 2(k+2)} e^{-\tau m_G^{\ 2}} \end{aligned}}$$

Impact of direct instantons on 0^{++} IOPE



- positivity restored
- Fiducial domain enlarged
- No hint of 2nd resonance

- $\mathcal{R}_{-1}$ special: subtraction $\Pi(0)$
- Same scale of exp. tail !
- No hint of m < 1 GeV res. !

0^{++} Borel sum rule analysis

Direct instantons indispensable at $x \lesssim 0.5$ fm:

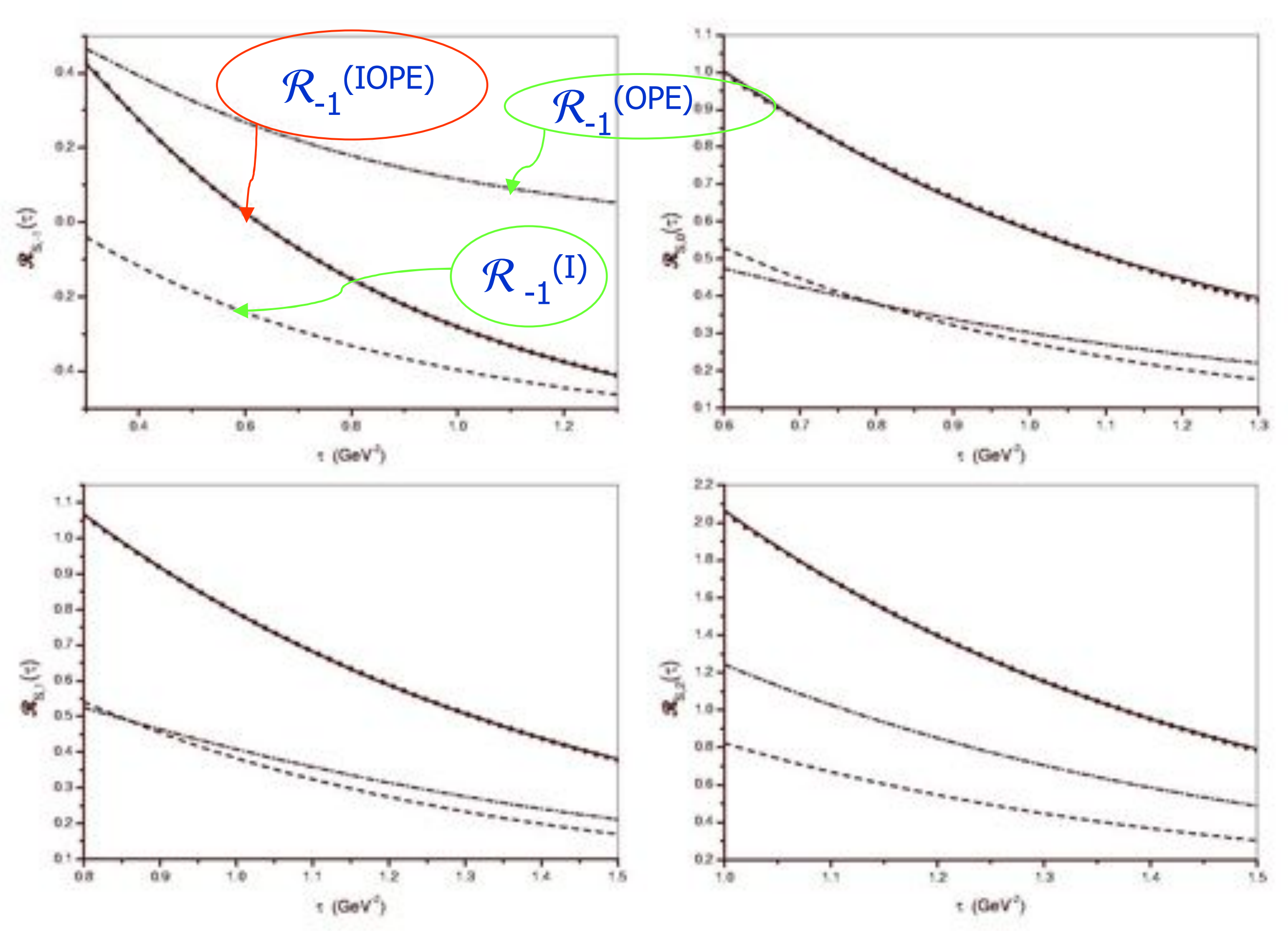
Optimal match of $\mathcal{R}_0$ to GB pole with $s_0 = 5.0\ \mathrm{GeV}^2$:

- 0^{++} mass reduced:

  $m_G = 1.25 \pm 0.2$ GeV
- 5x larger coupling:

  $f_G = 1.05 \pm 0.1$ GeV

Resolve old puzzle of strongly bound 0^{++} GB



- mutual and LET **inconsistencies resolved!**
- semi-hard npert. instantons bind & gen. mass

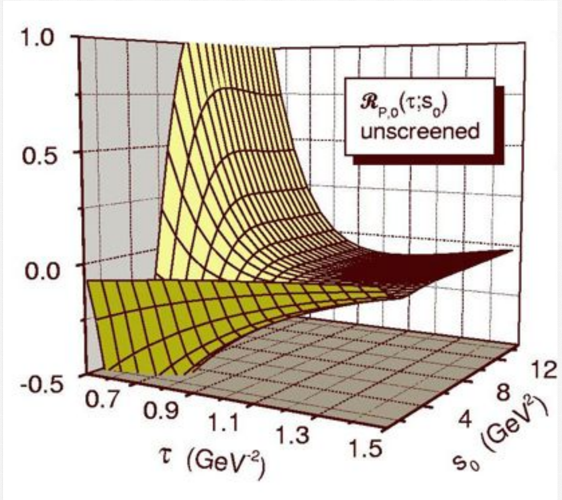
$0^{++} \rightarrow \; 0^{-+} \;\; \Rightarrow$ success into failure?

Now the analogous study for

$$\Pi_P(x) = \langle 0 | T g_P(x) g_P(0) | 0 \rangle = (8\pi)^2 \langle 0 | T Q(x) Q(0) | 0 \rangle$$

Dir-I contributions just change sign ⇒

- strong repulsion ⇒
- **0^{-+}** GB bound state (without I) disappears
- positivity (unitarity) bound violated
- ps. LET badly violated



What goes wrong here? I.e.

1) What is qual. difference to scalar channel?
2) Why seemed ΣRs with purely pert. Wilson coefficients to "work" ?

Answer 1): $\Pi_P \propto$ top. charge correlator ⇒ **max. sensitive to sign of Q!**

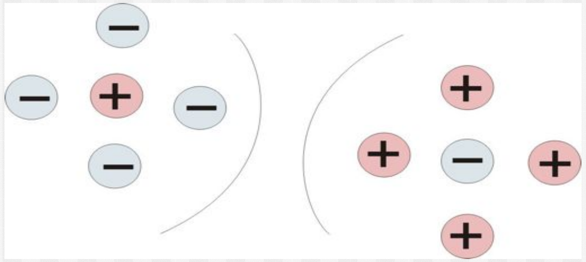
since $\nu = \int d^4x Q(x) \in \mathbb{Z}$ "topological charge" distinguishes $I \; (\nu = +1)$ from $\bar{I} \; (\nu = -1)$!

Topological charge screening

- LE int. of localized Q(x) lumps in vacuum
- via η & η' exchange given by $U_A(1)$ anomaly:
- $\Rightarrow$ opposite signs attract, equal signs repel

$$\Delta\mathcal{L}_{an} = -i\gamma_{\eta_0} Q(x)\,\eta_0(x)$$

$\Rightarrow$ **small Debye screening clouds** form around Q(x):



$\Rightarrow$

I) Isolated Q's: $\quad m_{\eta_0} = 0 \quad \Rightarrow \quad U_{12} \propto \dfrac{Q(x)Q(y)}{(x-y)^2}$

II) Q plasma: $\quad m_{\eta_0} = \sqrt{\dfrac{2N_f \bar{n}}{f_{\eta_0}^2}} \quad \Rightarrow \quad U_{12} \propto \dfrac{Q(x)Q(y)}{(x-y)^2} e^{-m_{\eta_0}|x-y|}$

- $\Rightarrow$ η' acquires a **screening mass ~ 1 GeV** (solves $U_A(1)$ problem)
- $\lambda_D = m_\eta^{-1} < \mu^{-1} \Rightarrow$ screening effects in Wilson coeffs.!
- With expl. chiral SB & η_0 - η_8 mixing Π dictated by $U_A(1)$ Ward Identity $\Rightarrow$

$$\Pi_P^{(scr)}(x) = -\left(2\bar{n}\gamma_{\eta_0}\right)^2 \left[\cos^2\phi \frac{m_{\eta'} K_1(m_{\eta'} x)}{4\pi^2 x} + \sin^2\phi \frac{m_\eta K_1(m_\eta x)}{4\pi^2 x}\right]$$

Pseudoscalar glueball IOPE & ΣRs incl. screening

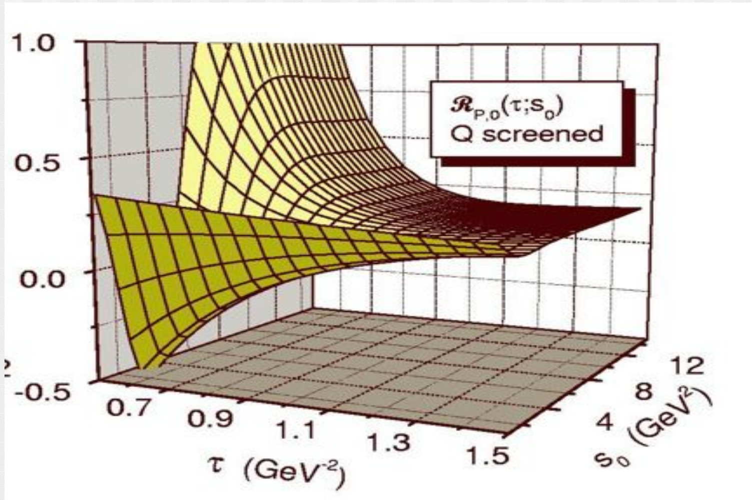
Screening contributions to 0^{-+} OPE coefficients have to be **included**...:

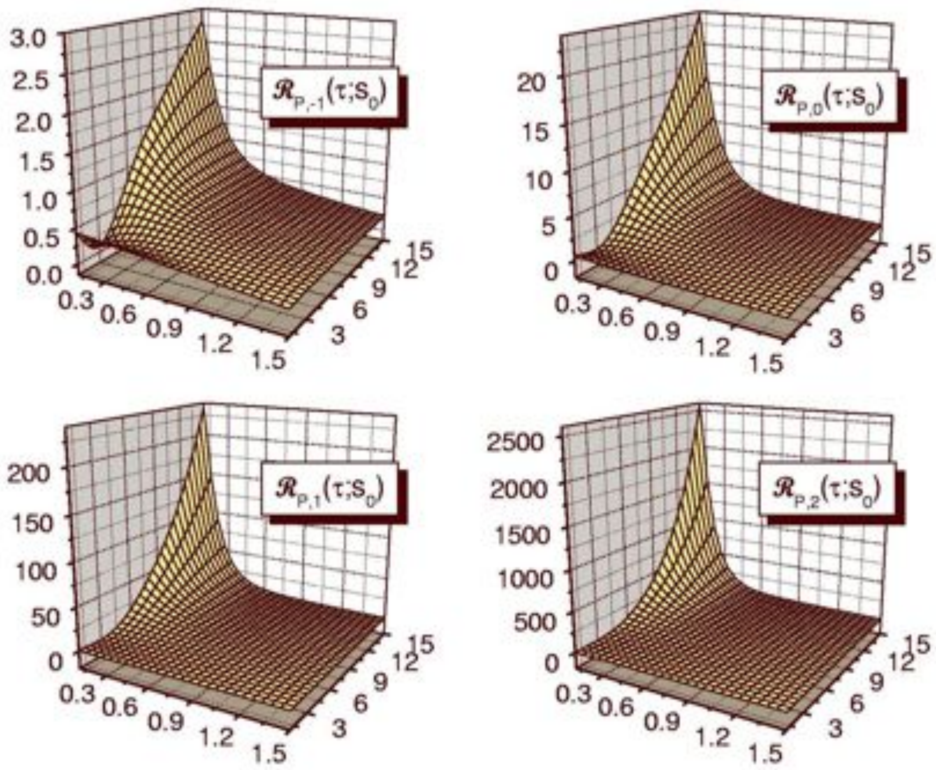
Q-screening results in:

- Unitarity (positivity) restored

  (for spike only in fiducial domain)

- Resonances appear: η' & 0^{-+} Gb





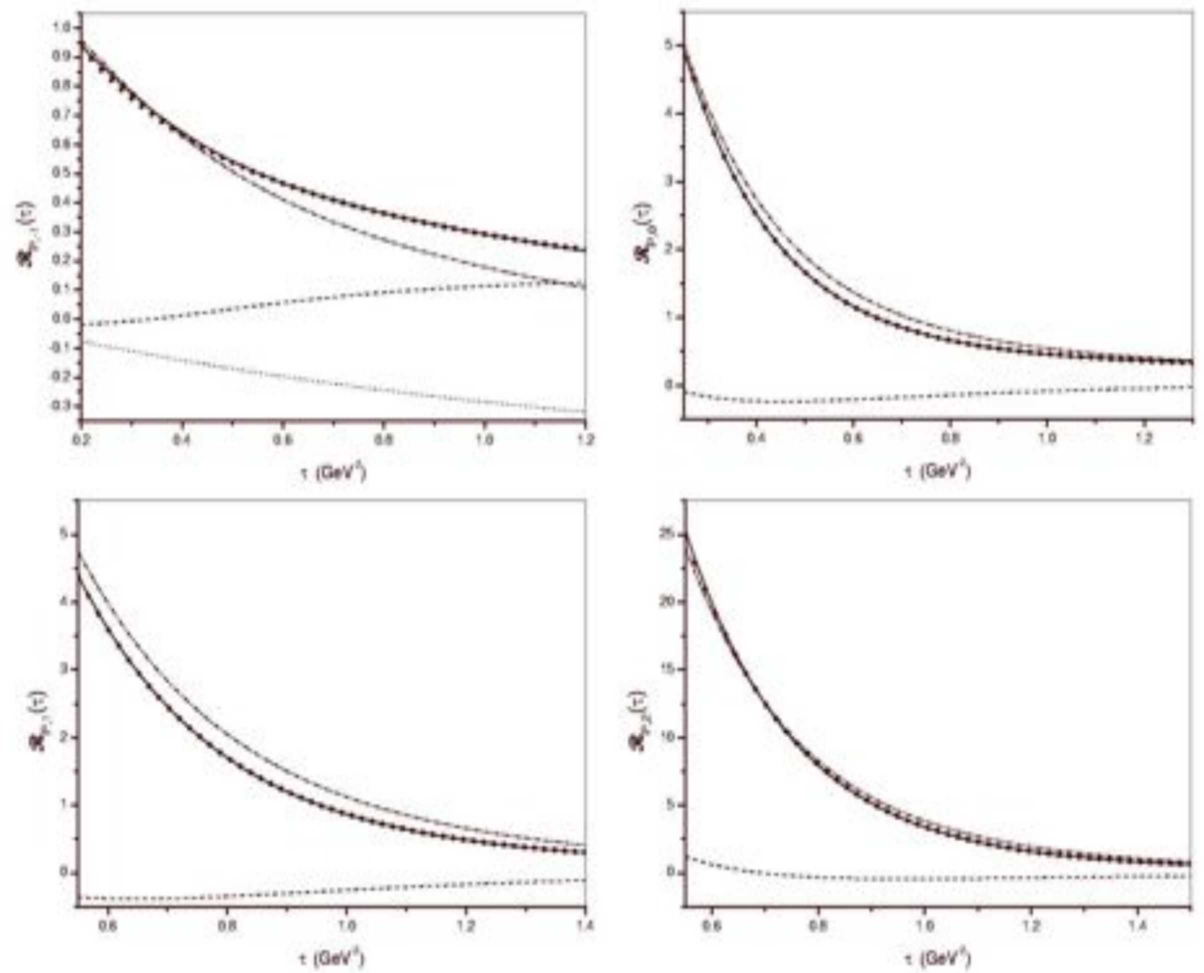
0^{-+} Borel sum rule analysis

- 2 resonances (0^{-+} Gb & η') strongly preferred
- Q screening reduces inst-induced repulsion, $r_P \sim \Lambda^{-1}$
- Consistency with anom. Ward Id. & LET restored !

  (qual. different but still ok at large N, pure YM)

- $m_P = 2.2 \pm 0.2$ GeV
- $f_P = 0.6 \pm 0.25$ GeV
- $s_0 \sim 9 - 10$ GeV^2 ,

  (in "duality inverval")
- η' props. at phys. values



Low-energy theorems & sum rule consistency

LETs in 0⁺⁺ and 0⁺⁻ glueball channels:

$$\boxed{\Pi_S(Q=0) = \frac{32\pi}{b_0}\langle \alpha_S G^2 \rangle \simeq 0.6\,\mathrm{GeV}^4}$$

$$\boxed{\Pi_P(Q=0) = (8\pi)^2 \frac{m_u m_d}{m_u + m_d}\langle \bar{q}q \rangle \simeq -0.022\,\mathrm{GeV}^4}$$

Direct-instanton contribution:

$$\boxed{\Pi_{S/P}(0) = \pm 2^7 \pi^2 \bar{n}_{dir} \simeq 0.63\,\mathrm{GeV}^4}$$

Top. screening contribution:

$$\boxed{\Pi_P^{(scr)}(0) = \varsigma \left(\frac{F_\eta^2}{m_\eta^2} + \frac{F_{\eta'}^2}{m_{\eta'}^2} \right) \simeq 0.59\,\mathrm{GeV}^4}$$

(Quenched 0⁺⁻ : LET:

$$\Pi_P^{(qn)}(0) = -(8\pi)^2 \chi_t^{(qn)} \simeq -0.66\,\mathrm{GeV}^4$$

Top. charge screening disappears:)

$$\Pi_P^{(scr,qn)}(0) = 0$$

The main points:

First fully consistent analysis of 8 QCD 0^{P+} Gb sum rules:

- **dominant npert. physics is hard: in OPE coefficients (of top. origin: instantons, top. screening)**

- **long-standing inconsistencies (among ΣRs, LETs) resolved**

New mass, coupl. & width predictions for 0^{++} and 0^{-+} with

- generally smaller m, larger f ("unquenching"?)
- **low-lying** ($m_G \ll 1$ GeV) **0^{++} GB violates LET ⇒ obsolete**
- instanton scales set scales of 0^{++} - Gb - partly bound by instantons?